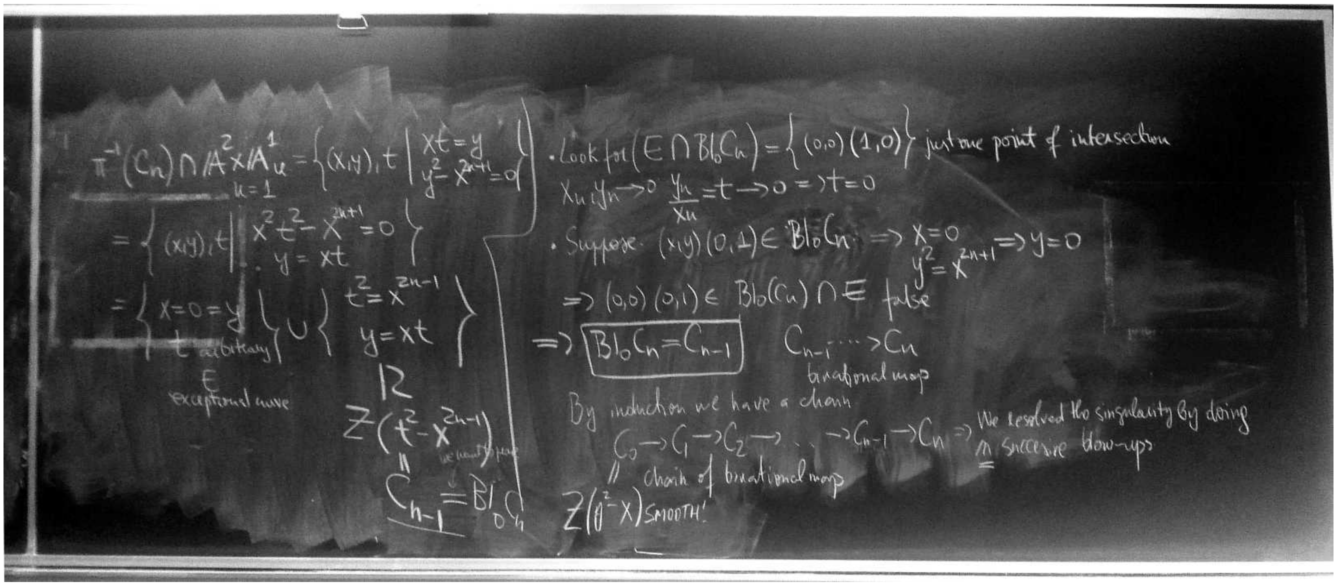
Exercise: Resolve the singularities of the following curve by subsequent blowups

$$y^2 - x^{2n+1} = 0$$

Proof: Let c_n be the curve $y^2 - x^{2n+1} = 0$. Notice that $(0,0)$ is the only singularity.

$$\mathrm{Bl}_0\mathbb{A}^2 = \{(x,y), u : t | xt = yu\}.$$

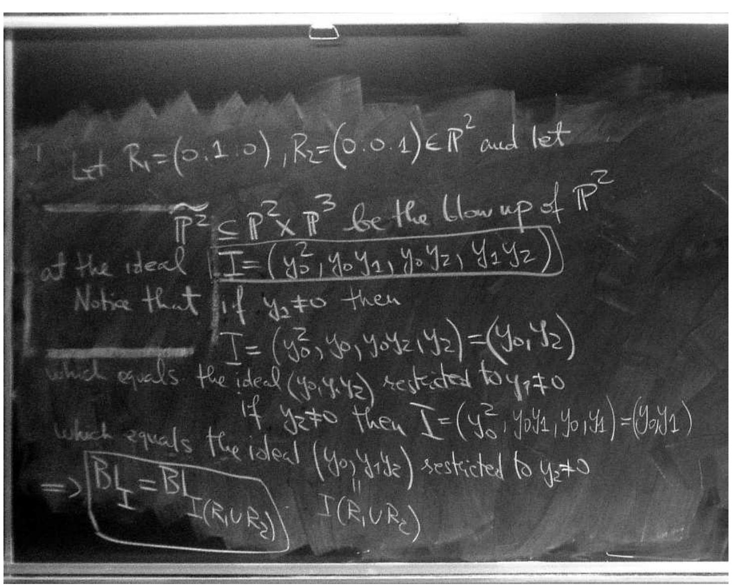
$\pi : \mathrm{Bl}_0\mathbb{A}^2 \to \mathbb{A}^2$ is the blow up map.

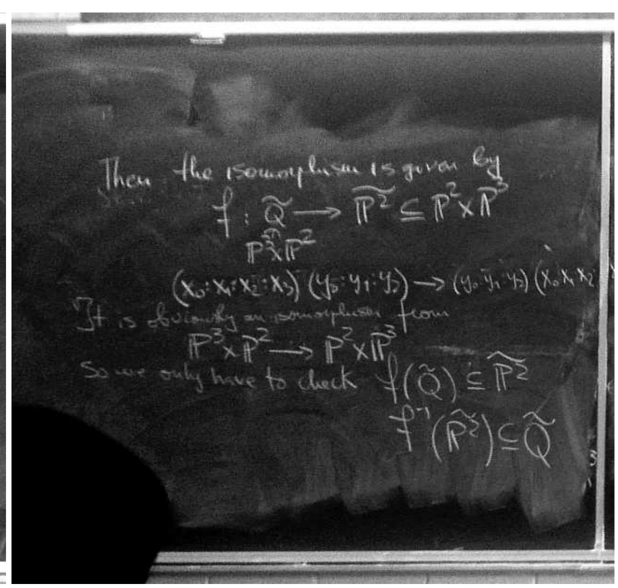


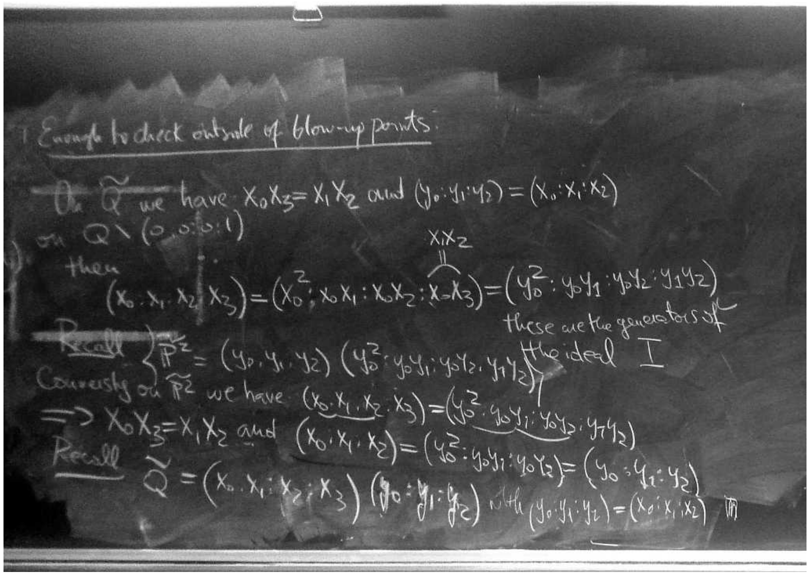
Example: Show that the blowup of $\mathbb{P}^1$ at one point is isomorphic to the blowup of $\mathbb{P}^2$ in two points.

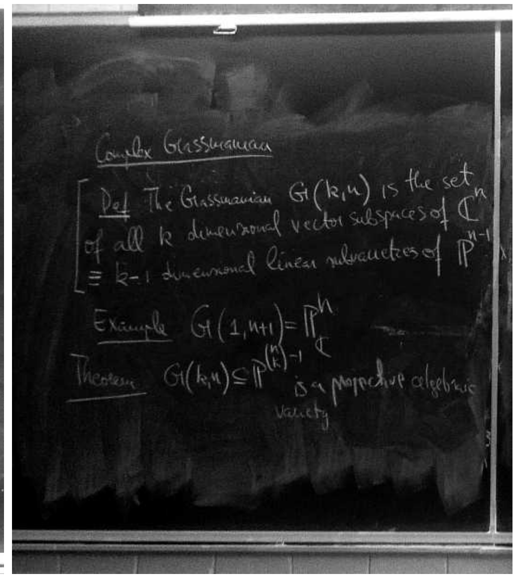
Proof: We have $\mathbb{P}^1 \times \mathbb{P}^1 \cong Q = (x_0x_3 - x_1x_2)$ in $\mathbb{P}^3$. We blowup the ideal (x_0, x_1, x_2) vanishing at $[0:0:0:1]$ to obtain

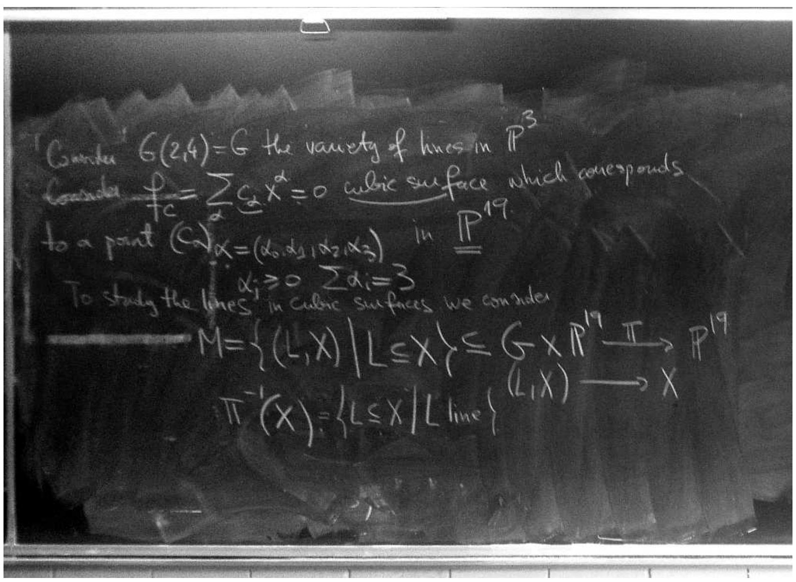
$$\mathrm{Bl}_{(x_0,x_1,x_2)}Q = \tilde{Q} \subseteq \mathbb{P}^3 \times \mathbb{P}^2.$$











Lemma:

a) M is a smooth 19-dimensional variety

b) The projection map $\pi : M \to \mathbb{P}^{19}$ is a local isomorphism i.e. covering map

$$\forall x \in \mathbb{P}^{19}\ \exists U_x \quad \pi^{-1}(U_x) = \coprod S_i$$

union of disjoint sets s.t. $S_i \to U_x$ homomorphism.

In particular all fibers have the same cardinal $\left|\pi^{-1}(x)\right|$ constant finite.

The number of lines on a cubic smooth surface is independent of the particular cubic chosen

(I have no idea what this is on about, looks like its in Gathmann though)