$P_0(x), P_1(x), \ldots$ – wielomiany ortogonalne

$\langle P_i, P_j \rangle = \emptyset \quad i \neq j$

$\Pi_n \quad P_0, P_1, \ldots P_n$ – baza przestrzeni.

$\forall\, w \in \Pi_n \ \exists\, \alpha_i \quad w(x) = \sum_{j=0}^{n} \alpha_j P_j(x)$

$\langle P_{n+1}(x), w(x) \rangle = \sum_{j=0}^{n} \alpha_j \underbrace{\langle P_{n+1}(x), P_j(x) \rangle}_{=\emptyset} = \emptyset$

Teza: $\underline{P_{n+1} \perp \Pi_n}$

$w(x) = a_k x^k + \ldots$

$w^2(x) = (a_k x^k + \ldots)^2$

$\deg(w) = k \Rightarrow \underline{\deg(w^2 + 2023) = 2k}$

$8 \to 3$

$7 \to 3$

$\lfloor \frac{k-1}{2} \rfloor$

$\langle f, g \rangle = \sum_{k=0}^{N} f(x_k) g(x_k)$

$N+1$ mnożeń $\quad N$ dodawań – koszt iloczynu skalarnego.

$\begin{cases} P_0(x) = 1 \quad P_1(x) = x - c_1 \\ P_k(x) = (x - c_k) P_{k-1}(x) - d_k P_{k-2}(x) \end{cases} \quad k = 0 \ldots n$

Jeżeli znamy $\{c_k\}_{k=0}^{l}$, $\{d_k\}_{k=0}^{l}$ $\quad$ 2 mnożenia 2 dodawania. (b)

$k = 0, 1, \ldots, n$

$c_k = \frac{\langle x P_{k-1}, P_{k-1} \rangle}{\langle P_{k-1}, P_{k-1} \rangle} \qquad d_k = \frac{\langle P_{k-1}, P_{k-1} \rangle}{\langle P_{k-2}, P_{k-2} \rangle}$

liczymy 1 raz

było obliczone poprzednio.

$\langle P_{k-1}, P_{k-1} \rangle$ $\quad N+1$ mnożeń $\quad N$ dodawań (a)

$\langle x P_{k-1}, P_{k-1} \rangle = \sum_{j=0}^{N} x_j P_{k-1}(x_j) P_{k-1}(x_j)$

$2N+2$ mnożenia $\quad N$ dodawań (a)

$k = 0, 1, \ldots, n$

$O(nN) \quad O(N^2)$

Mnożeń

$3(N+1)(n+1) + 2(n+1)$

Dodawania

$(n+1)(a+b)$

$\langle P_0, P_0 \rangle = \sum_{j=0}^{N} 1 = N+1$

$\langle x P_0, P_0 \rangle = \sum x_i \qquad N$ dodawań

$c_1 = \frac{\langle x P_1, P_1 \rangle}{\langle P_0, P_0 \rangle} = \frac{\sum x_i}{N+1}$

obliczenie $P_0(x)$ bez kosztu

$P_1(x) = (x - c_1) P_0 \qquad 1$ mnoż. $\quad 1$ dodawanie.

$$P_0(x) = 1 \qquad P_1(x) = x - c_1 = x\cdot P_0 - c_1 P_0$$

$$\underline{\underline{0}} = \langle P_0, P_1\rangle = \langle P_0, x\cdot P_0 - c_1 P_0\rangle = \langle P_0, xP_0\rangle - c_1 \underbrace{\langle P_0, P_0\rangle}_{>\emptyset}$$

$$c_1 = \frac{\langle xP_0, P_0\rangle}{\langle P_0, P_0\rangle} \quad \checkmark$$

Zał. że $P_0, P_1, \dots, P_k$ – ortogonalne $\quad P_k(x) = x^k + \dots$

(2) $$P_{k+1}(x) = x\cdot P_k(x) + \sum_{j=0}^{k} \beta_j P_j(x)$$

Co chcemy pokazać $\beta_0 = \beta_1 = \dots = \beta_{k-2} = 0$,

czyli $P_{k+1}(x) = xP_k(x) + \beta_k P_k(x) + \beta_{k-1} P_{k-1}(x)$

czyli $P_{k+1}(x) = (x+\beta_k) P_k(x) + \beta_{k-1} P_{k-1}(x)$

(1) $\langle xf, g\rangle = \langle f, xg\rangle$

$$\sum x_i f(x_i) g(x_i) = \sum f(x_i) x_i g(x_i)$$

$\checkmark$ $$0 = \langle P_{k+1}, P_i\rangle \overset{(2)}{=} \langle xP_k, P_i\rangle + \sum_{j=0}^{k} \beta_j \langle P_j, P_i\rangle =$$

$i = 0, 1, \dots, k-2$

$$= \underbrace{\langle \underset{\text{st } k}{P_k}, \underset{\le k-1}{xP_i}\rangle}_{=\emptyset} + \sum_{j=0}^{k} \beta_j \langle P_j, P_i\rangle = \beta_i \underbrace{\langle P_i, P_i\rangle}_{>\emptyset}$$

Z zał.

$\boxed{\text{Teza: } P_{n+1} \perp \Pi_n}$

$$0 = \beta_i \underbrace{\langle P_i, P_i\rangle}_{\neq 0}$$

$$\beta_i = 0 \quad (i = 0, \dots, k-2)$$

$$0 = \langle P_{k+1}, P_k\rangle = \langle xP_k, P_k\rangle + \beta_k \underbrace{\langle P_k, P_k\rangle}_{>\emptyset} + \beta_{k-1} \underbrace{\langle P_{k-1}, P_k\rangle}_{=\emptyset}$$

ponieważ $P_{k+1}(x) = xP_k(x) + \beta_k P_k(x) + \beta_{k-1} P_{k-1}(x)$

$$\beta_k = -\frac{\langle xP_k, P_k\rangle}{\langle P_k, P_k\rangle} = -c_{k+1}$$

$$0 = \langle P_{k+1}, P_{k-1}\rangle = \langle xP_k, P_{k-1}\rangle + \beta_k \underbrace{\langle P_k, P_{k-1}\rangle}_{=\emptyset} + \beta_{k-1} \underbrace{\langle P_{k-1}, P_{k-1}\rangle}_{>\emptyset} =$$

$$\overset{(1)}{=} \langle P_k, xP_{k-1}\rangle + \beta_{k-1} \langle P_{k-1}, P_{k-1}\rangle = (*)$$

$$xP_{k-1}(x) = P_k(x) + \underset{\in \Pi_{k-1}}{s(x)}$$

$$(*) = \langle P_k, P_k\rangle + \underbrace{\langle \underset{\text{st. } k}{P_k}, \underset{\text{st.} \le k-1}{s(x)}\rangle}_{=\emptyset} + \beta_{k-1} \langle P_{k-1}, P_{k-1}\rangle = \emptyset$$

$$\beta_{k-1} \overset{\cdot}{=} -\frac{\langle P_k, P_k\rangle}{\langle P_{k-1}, P_{k-1}\rangle} = -d_{k+1}$$

$$P_{k+1}(x) = (x+\beta_k) P_k(x) + \beta_{k-1} P_{k-1}(x)$$

$$\beta_k = -\frac{\langle x P_k, P_k\rangle}{\langle P_k, P_k\rangle} \qquad \beta_{k-1} = -\frac{\langle P_k, P_k\rangle}{\langle P_{k-1}, P_{k-1}\rangle}$$

$$= -c_{k+1} \qquad\qquad -d_{k+1}$$

$$P_{k+1}(x) = (x - c_{k+1}) P_k(x) - d_{k+1} P_{k-1}(x)$$

1 2 80

x_0	$y_0 ..$		
x_1	y_1	y_{01}	
			y_{012}
x_{81}	y_{81}	- - -	$y_{0..81}$

```
for i=1:80
  for j=81:-1:i+1
    y(j) = (y(j)-y(j-1))/(x(j) - x(j-i))
```

$$y_0 + y_1 (x-x_0) + y_2 (x-x_0)(x-x_1) + \ldots + y_{81} \cdot \prod_{j=0}^{80} (x-x_j)$$

$y(0)$ $y(1)$ $y(2)$

```
w = y81
for j=80:-1:0
w = w*(x-x(j))+y(j)
end
```

$w_5^*(x)$

$\mathrm{lin}\{P_0, P_1, \ldots P_5\}$

lub

$\mathrm{lin}\{1, x, \ldots x^5\}$

$$\begin{bmatrix} \langle 1,1\rangle & \langle 1,x\rangle & \ldots & \langle 1,x^5\rangle \\ \langle x,1\rangle & \langle x,x\rangle & \ldots & \langle x,x^5\rangle \\ \vdots & \vdots & & \vdots \\ \langle x^5,1\rangle & \langle x^5,x\rangle & \ldots & \langle x^5,x^5\rangle \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \vdots \\ \alpha_5 \end{bmatrix} = \begin{bmatrix} \langle 1,f\rangle \\ \langle x,f\rangle \\ \vdots \\ \langle x^5,f\rangle \end{bmatrix}$$

$Ax=b$

$x = inv(A)*b$

$$\langle x^2, x^3\rangle = \sum x_i^2 x_i^3 = \sum x_i^5$$

$$\langle x^1, x^4\rangle = \qquad = \sum x_i^5$$

$81 \quad \sum x_i \quad \sum x_i^3$

$\sum x_i \quad \sum x_i^2$

$\sum x_i^2$