The Cost of Algorithmic Trading: A First Look at Comparative Performance

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The goal of advanced trade execution services is performance, and algorithmic trading is currently in the spotlight. A trip to an Internet search engine can yield up to 24,500 hits on the term.[1] Tabb [2004] reports that 61 percent of U.S. buy-side firms employ model-based execution vehicles of some sort and, based on survey evidence, forecasts 144 percent growth in the use of algorithmic trading by the end of 2006. A survey of European investment managers suggests that 58 percent process up to 50 percent of their trading volumes using algorithmic trading programs.[2] Yet, despite all the talk and activity, little is known about the performance of algorithmic trading engines. This paper provides preliminary evidence on this point, comparing performance with a control sample of trades executed by non-algorthmic means and across a subset of model-based trading service providers.

The term *algorithmic trading* is used to describe different functions and support multiple positions with respect to market structure. *Smart routing, program trading,* and *rules-based trading* are some of the other terms used to describe algorithmic trading. Like Grossman [2005], we generally define *algorithmic trading* as the automated, computer-based execution of equity orders via direct market-access channels, usually with the goal of meeting a particular benchmark.

Literature on the topic is thin at this stage. General discussion and survey evidence with respect to systems, vendors, and usage appear largely in consultative pieces, such as Tabb [2004], Giraud [2004], and Grossman [2005]. The issues surrounding algorithmic trading have not yet directly penetrated the academic literature, largely due to the unavailability of public data for analysis.

In our analysis, we concentrate on performance along the dimension of transaction costs; that literature is covered substantively in Domowitz, Glen, and Madhavan [2001]. The work most relevant to this paper is the consulting report of QSG [2004]. The QSG study, based on trade data from February through June of 2004 for a single institutional client of QSG, concentrates on the higher costs incurred by non-random (relative to randomized), algorithmic trading strategies. The sample includes 7,676 orders with a total dollar value of approximately $1.5 billion. As one would expect, the results suggest that front-running behavior is abetted by lack of randomization and results in higher transaction costs.

In contrast, our analysis is broader in topic and coverage. Our sample includes over 2.5 million orders consisting of almost 10 billion shares traded from January through December of 2004 from over 40 institutions. One million of those orders—accounting for roughly 2.6 billion shares with a value of $82 billion—are executed via algorithmic means. The remainder of the sample is divided into matched and random samples of orders not traded algorithmically. The data permit us not only to compare algorithmic executions with a broader universe of trades, but also to examine performance across providers of model-based trading services.

Our main results can be summarized as follows. In the aggregate, algorithmic trading is less expensive than the alternative means represented in our samples, based on a measure of implementation shortfall. This conclusion holds even when we take into account trade difficulty, differences in market, side of trade, and volatility regime. The superiority of algorithm performance applies only for order sizes up to 10 percent of ADV. The evidence suggests that algorithms may not yet be sophisticated enough for very large orders. However, it is difficult to conclude based on the analysis conducted here that algorithm performance simply degrades relative to other methods for larger order sizes without explicitly modeling the choices between alternative means of execution.

Algorithmic trading performance relative to a volume participation measure is quite good, averaging only about 2 basis points off the benchmark. As size increases, degradation of performance along this dimension is not as great as for implementation shortfall, although certainty of outcome declines sharply with size. Such differences in uncertainty are highlighted in the comparison of performance across vendors of the service. While this comparison shows no great differences in average performance, it does show 100 percent swings in benchmark standard deviation across service providers. This result is confirmed by examining the distributions of outcomes across cost deciles and providers. A clear link between performance and variability in performance relative to the benchmark appears to be lacking, however. Further, equality of average performance across providers breaks down, once order sizes exceed one percent of ADV.

Data and Methodology

We employ data taken from ITG's Transaction Cost Analysis Peer Group Database from January 2004 through December 2004. To facilitate comparisons across the sample and eliminate issues associated with multi-day benchmarks, the data are limited to orders completed within a single day. The limitation is without loss of generality, given the nature of our data. Fully 97 percent of orders traded algorithmically in the Peer Group data are completed within a single day. This information is complemented by publicly available market data over the period and model-based expected cost estimates, discussed further below. Exhibit 1 contains some simple summary statistics.[3]

Algo trades represent all orders specifically identified as executed through computerized model-based trading strategies and implemented through algorithmic server technology.[4] As the results presented later suggest, the vast majority of these executions follow a volume weighted average price (VWAP) strategy.[5] These data represent 1,017,000 orders, with an average order size of 2,548 shares. We also use a slightly reduced sub-sample in comparisons of relative performance across vendors of execution services, since the majority of, but not all of, algorithmic trades are identified by broker in the data. Six vendors are represented, but are not identified for reasons of confidentiality. Similarly, we aggregate all institutional data (here and with respect to the samples discussed below) and do not provide any information that might identify participants.

Non-random, non-algo trades constitute a sample matched to the characteristics of the algorithmic trades. These data comprise trades in the same stocks within a 20-day window before or after the algorithmic trade was completed in the same name. To be included in this control group, the order size of such a trade must also be within 20 percent of the algorithm order. The results of the study are robust with respect to reasonable variations in the matching procedure. The set includes roughly 1.2 million observations, representing the trading of 3.8 billion shares with a dollar value of $125 billion.

Finally, *Random non-algo trades* are randomly selected orders that do not appear in either of the preceding samples. We include 304,000 such orders for comparison with the other samples, but their characteristics are indeed different. In particular, the average order size for this group is 11,314--more than four times that found in the algorithmic trading sample.

In our implementation shortfall analysis of trading costs, we sometimes employ a trade "handicap" based on the ITG ACE™ (Agency Cost Estimator) model. ACE is a mathematical/econometric model that provides a pre-trade forecast of the cost of executing an order. ACE recognizes that there is no such thing as "the" cost estimate of a trade. Cost is a function of the strategy behind the trade, as well as of market conditions and the idiosyncrasies of a particular stock. Although the ACE model accounts for a variety of strategies, the estimates used here are predicated on a VWAP strategy using a 20 percent market participation rate.

Aggregate ACE numbers appear in Exhibit 1 for all three samples. Average expected cost for algorithmic trades is 14 basis points (bps), suggesting slightly more difficult orders than for the control sample, at 11 bps. The random sample, for different stocks and order sizes than appear in the first two sets of data, exhibits expected costs of approximately 30 bps, however. Such differences in trade difficulty play a role in the work to follow.

Performance comparisons require benchmarks. In order to standardize the analysis, we limit ourselves to two benchmarks that we can apply across the entire sample of available orders. The first is the midpoint of the bid and ask prices at the time the order was received by the trading desk. Since this measure is of the implementation shortfall variety, it can be compared directly to, and adjusted by, the ACE handicap for trade difficulty. This benchmark is shown as *MBA* (midpoint of bid and ask) in the exhibits. The second is Volume Weighted Average Price (VWAP), the most commonly used volume participation benchmark in the industry. Full-day VWAP is used, consistent with basing the selected sample on completion of the order within a single day.

Implementation Shortfall Costs of Algorithmic Trades

Aggregate statistics for performance relative to the MBA are provided in Exhibit 1. Algorithmic orders generate average costs of 10 bps relative to this benchmark, compared with 18 bps for the control sample and 28 bps for the random sample of non-algorithmic trades. The algorithmic trades are expected to cost 14 bps, however, based on the ACE handicap for trade difficulty, resulting in over-performance of 4 bps on average. This result is slightly higher than for the random sample, but the difference in difficulty-adjusted performance of algorithmic trades relative to the control sample is a full 11 bps.

Breakdowns by market, side, and volatility appear in Exhibit 2. As shown by the ACE numbers—11 and 22 bps for listed and OTC shares, respectively—OTC names are generally more difficult to trade than their listed counterparts. Implementation shortfall costs for algorithms are quite similar across markets, however. Algorithmic trading appears less costly than trades for the orders in the other two samples, with and without adjustments for trade difficulty. There are few differences across buy and sell orders, regardless of sub-sample.[6]

Panel 3 of Exhibit 2 contains results across different volatility regimes as an additional proxy for difficult market conditions.[7] As expected, the MBA costs and the ACE handicaps increase with volatility for all three samples. Algorithmic trades exhibit under-performance of 6 to 13 bps, while the costs of the control group range from 14 to 26 bps as volatility increases. The random sample, consisting of larger order sizes on average, has correspondingly higher costs. Once trade difficulty is taken into account, algorithmic trading continues to perform better than methods employed for the control group and for the random sample of larger orders. The algorithmic trades now beat the benchmark by 1 to 11 bps, with out-performance increasing with volatility.

Exhibit 3 contains results with respect to orders' demand for liquidity, as measured by percentage of average daily volume (ADV).[8] Roughly 80 percent of orders consigned to algorithmic trading fall in the 0 to 1 percent ADV category, while 14 percent are between 1 and 5 percent of ADV. Another 5 percent of orders are in the range of 5 to 10 percent of ADV. These numbers illustrate the growing pains associated with the new technology. Tabb [2004] notes, for example, that most institutions responding to his survey have model-based trading capabilities on the desk, while sources generally agree that only about 5 to 7 percent of volume is traded algorithmically.[9] People are still learning how to use the capabilities and are looking for assurance that performance will not degrade substantially with order size.

With only about 8,200 orders in the algorithmic trading sample larger than 10 percent of ADV, it is difficult to draw firm conclusions about larger order sizes.[10] Nevertheless, some insight can be gained from Exhibit 3. For orders up to 10 percent of ADV, algorithmic engines outperform the control group and random sample, with or without adjustment for trade difficulty. Once we take the trade difficulty handicap into account, however, trades in the control and random samples generally outperform the algorithms for orders in excess of 10 percent of ADV. It appears that larger orders are intelligently selected for manual handling, possibly based on additional characteristics that we don't consider here. The evidence suggests that algorithms are not yet sophisticated enough for very large orders. However, without explicit modeling of the choices involved, it is difficult to conclude based on this evidence that algorithm performance simply degrades relative to other methods for larger order sizes.

Performance Relative to VWAP

We now briefly turn to some results with respect to the VWAP volume participation benchmark, summarized in Exhibit 4. For comparison purposes, this exhibit also includes statistics relating to the MBA benchmark. We concentrate in this section solely on the absolute performance of the algorithmic orders, as opposed to comparing those orders with samples containing trades completed by alternative means—the vast majority of which do not involve VWAP participation. We return to comparisons when we examine relative performance across providers of algorithmic trading services in the next section.

The average under-performance of algorithmic orders relative to the VWAP benchmark is 2 bps, with under-performance increasing to 4 bps as order sizes range from less than 1 percent to 25 percent of ADV. These differences in performance are mirrored by the MBA measure on a non-ACE-adjusted basis, although the percentage decrease in performance across size categories is much larger than for the VWAP benchmark. In other words, degradation of algorithm performance based on order size is less for a volume participation benchmark than for an implementation shortfall measure.

The last conclusion does not hold, however, with respect to certainty of outcome. For both VWAP and MBA measures, the standard deviation jumps by almost 100 percent from orders less than 1 percent of ADV to those in the 1 to 5 percent category. The increase—though much smaller—is similar in percentage terms across benchmarks as orders increase to 10 percent. At this point, the standard deviation basically stabilizes across larger order sizes. Uncertainty of outcome is an important issue in algorithmic trading, and we return to it below when we compare performance across service providers.

Comparisons Across Providers of Algorithmic Trading Services

Grossman [2005] surveys the algorithmic offerings of seven brokers and two technology firms, accounting for approximately half of the service providers at the time of this writing. Our data identify six brokers who performed algorithmic trades in 2004.[11] Summary statistics for VWAP and MBA performance across brokers appear in Exhibit 5.

The first striking result is that there is virtually no difference in absolute performance versus the VWAP benchmark across providers. Five of the six brokers exhibit under-performance by a mere 2 bps, and one provider is practically on the benchmark. Although precise strategies are rarely defined in our aggregate data for all brokers, it is clear that the vast majority of algorithmic trades in our sample use a VWAP engine.

The issue in VWAP trading, however, is certainty of outcome. In this respect, brokers vary considerably. The standard deviation of VWAP costs ranges from 12 to 24 bps, a considerable difference in both absolute and percentage terms. One can certainly argue that less certainty of outcome can be a reasonable price to pay for superior performance. However, given the similarity in VWAP average costs across brokers, the data do not support this conclusion.

ACE-adjusted costs for the MBA benchmark also tend to be similar across providers, despite more pronounced differences in unadjusted implementation shortfall costs. This result underscores the need to correct implementation shortfall numbers for trade difficulty. The adjusted shortfalls range from 0 to 7 bps, but are clustered in the 1 to 3 bps range. There is a reasonably large spread, however, in the standard deviations associated with adjusted cost, with outcomes ranging from 25 to 47 bps. Further, there is little correlation between the measures of absolute performance and the degree of uncertainty with respect to a low-cost outcome. For example, brokers 3 and 6 exhibit the highest average costs, but with standard deviations of 25 and 47 bps, respectively, while a 2-bps performance on the part of broker 4 yields a standard deviation of 29 bps. As in the case of VWAP, a clear link between performance and uncertainty is lacking.

We provide another perspective on performance variability in Exhibit 6, which breaks down MBA performance by decile in Panel 1 and presents results similarly for VWAP in Panel 2. For VWAP, the difference between the first and tenth deciles, by broker, ranges from 39 to 82 bps—a gap of approximately 100 percent. It can certainly be argued, however, that the highest and lowest performance deciles represent outliers. When we compare the difference between the second and ninth deciles, the gap narrows considerably—to a range of 13 to 27 bps across algorithmic engine providers. A comparison between best- and worst-performing brokers by this measure continues to suggest a 100 percent difference, however. Thus, once outliers are accounted for, relative certainty performance differences are maintained.

Exhibit 6 shows qualitatively similar results for the MBA benchmark. Here, the difference between the highest and lowest deciles is between 93 and 168 bps—roughly 81 percent. Discarding the extreme deciles as outliers, this alternative measure of uncertainty now exhibits values of 26 to 54 bps across providers—a difference of roughly 100 percent, similar to that observed for the VWAP benchmark.

We break down the comparative results by order percentage of ADV, including the standard deviations of MBA and VWAP performance, in Exhibit 7. Equality of performance across providers tends to break down for orders greater than 1 percent of ADV. In the 1 to 5 percent category, for example, VWAP performance now ranges from 2 bps out-performance to 8 bps under-performance, with corresponding increases in the standard deviation of outcome. A similar spread is evident for implementation shortfall. Once order sizes are greater than 5 percent of ADV, these differences increase sharply. In the 5 to 10 percent category, the cost versus VWAP is 0 in one case, increasing to as much as 25 bps. Implementation shortfall costs follow a similar pattern, but with sharper increases in the standard deviation of outcome than observed for VWAP performance. This theme is becoming familiar as we take different perspectives on the data.

Concluding Remarks

The advantages of algorithmic trading have helped popularize the idea to the extent that virtually every major broker, and some technology providers, offer the service in some form. Survey results suggest that productivity is the major driver, followed by control over the trading process, the ability to focus on the most difficult trades, and cost control.[12] This paper has provided a first look at the quantitative evidence for relatively large sample sizes in an effort to shed light on some of these points.

The evidence is admittedly preliminary in nature, and we rely on simple tabular data, as opposed to sophisticated econometric models. Nevertheless, the data permit comparison with control samples and across providers of algorithmic trading services, and the sample sizes are large enough that all differences are "statistically significant." Our interest is, therefore, not in statistical significance, but in the economic significance of the tabulated findings. Much of that economic significance lies in the numbers themselves. Rather than repeat numerical results, however, we now attempt to summarize the qualitative lessons learned from the preceding analysis.

1. *Beyond the productivity enhancements inherent in using model-based engines, algorithmic trading is a cost-effective technique.* This lesson holds for both implementation shortfall and volume participation benchmarks and is robust once trade difficulty measures are taken into account. On an implementation shortfall basis, however, algorithm performance is superior only for orders up to ten percent of ADV. Absolute performance relative to the VWAP benchmark also tends to break down even before the 10 percent boundary is reached—which brings us to the next point.
2. *Are algorithmic engines ready for prime time with respect to large order sizes?* The lesson here is to ask this question intelligently. Implementation shortfall performance is more sensitive to increases in order size than volume participation, for example. Declines in certainty of outcome tend to be sharper than degradation of average performance. Overall, the empirical evidence suggests that algorithms are not yet sophisticated enough for large order sizes. It is not possible to reach this conclusion without further evidence, however, since the factors driving selection of an algorithm or an alternative execution technique affect the conclusion. Econometricians refer to this problem as *sample selection bias*.
3. *The algorithmic trader must live by the law of large numbers.* We all have heard anecdotes to the effect that a trader wishing to use any model-based technique

must trade frequently in order to realize the desired result. The evidence presented here demonstrates that the distribution of outcomes for algorithmic trades is quite diffuse and appears to have decidedly fat tails. Nevertheless, the distribution is fairly symmetric, implying very good as well as very bad outcome possibilities. Average results for the costs of algorithmic trades are encouraging, but one needs to do a lot of them to reach the average. Similarly, judging a particular algorithm by a small set of sample trades can be quite misleading.

4. *Shop carefully for algorithmic trading services.* You don't really need us to tell you this. With that said, there is a fair amount of marketing hype these days, and there are many ways to sell algorithm characteristics. The intent of this paper is to help direct questions to the right areas in terms of performance. Average performance differences across providers for very small orders are few, but gaps between providers grow as order size grows. The biggest differentiator is in the standard deviation of outcome, and the empirical link between performance and risk of under-performance is weak—which brings us to the next point.

5. *Is there value in diversifying algorithmic trading across different providers?* The answer is a qualified "maybe." At a simple level, there are two potential types of diversification. The first is across benchmarks, which may have value in the context of larger execution strategies. We have nothing to say on this point, however, given the nature of the data currently available to us. The second balances expected performance against risk, much as one would do in stock portfolio selection. There are clear differences in this area among algorithmic providers, suggesting that diversification could be valuable. On the other hand, the link between performance and risk is weak at best and inconsistent at worst. The implication is that a well-defined "efficient frontier" of outcomes may not exist empirically, ruling out portfolio balance across algorithmic execution possibilities.

6. *Algorithmic trading is simply another piece of the overall execution strategy puzzle, and there is value in considering alternatives as execution portfolio choices.* The empirical evidence pertaining to differences in costs across our various samples validates this piece of anecdotal knowledge. Unlike our point with respect to diversification across algorithm providers, the emphasis here is on the word "portfolio" and on the potential to optimize performance across various trading strategies. There are clear differences in risk and performance, and further research in this area would be particularly valuable.

Endnotes

[1] Yahoo® search as of February 15, 2005.
[2] Giraud [2004], based on 68 institutions, managing in excess of 4,000bn Euros.
[3] Standard errors of estimates are not included in any of the tabulated results. The sample sizes are so large as to make any differences discussed in the paper statistically significant at any typical level of statistical confidence.
[4] The considerable leeway in Grossman's [2005] definition forces us to be a bit more precise in our empirical analysis. For example, automated trading strategies can be programmed directly into front-end trading products, permitting extensive customization. It is not possible for us to systematically isolate trades through such sources. Therefore, in our empirical work, we adhere to the Grossman definition, with the additional proviso that the algorithms are executed through centralized server technology. Such technology typically is maintained at the broker, as opposed to being a customized part of the institutional trade blotter, although it is possible for the institution to reach the algorithmic trading engine without manual intervention of a broker.
[5] It should be understood, however, that a limitation of the data set is that exact strategies followed are rarely identified separately.
[6] There have been studies in the past, cited in Domowitz, Glen, and Madhavan (2001), which have noted higher costs for sells relative to buys. This result is indeed reflected in the ACE numbers for the random sample, which is most reflective of samples used in other work, for which the ACE adjustment for sells is a multiple of that for buy orders.
[7] Groupings in Panel 3 use a forward-looking daily volatility calculation based on the last 60 days of the stock's closing price. *Low volatility* is defined as volatility up to 2 percent; *medium volatility* is defined as movement from 2 percent to 10 percent; and, *high volatility* represents measures of greater than 10 percent in the volatility calculation.
[8] The 21 day ADV of any particular stock, prior to order submission, is used as the denominator in the calculation of order percentage of ADV.
[9] See also Financial Insights [2005], which highlights the vast majority of usage as being by hedge funds.
[10] We note, however, that 8,200 observations remains a relatively large number for statistical purposes. By comparison, even this smaller sample is larger than the full set of data used in the previously cited QSG [2004] study.
[11] The sample used in this section is slightly smaller than that used previously, since not all orders are identified by broker; hence, some of the aggregate numbers will differ. We also note that, it cannot be assumed by the reader that the brokers examined anonymously here correspond to those surveyed by Grossman [2005], which covers only about half of service providers.
[12] See, for example, Tabb [2004].

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Exhibit 1
Aggregate Summary Statistics Across Samples

Order Type	**# Orders (000's)**	**# Shares (MM)**	**Avg Order Size**	**Total Value (MM)**	**Cost v. MBA**	**Cost v. MBA ACE Adj**	**ACE Cost Adj.**
Algo Trades	1,017	2,591	2,548	81,877	(10)	4	14
Non-Algo Non-Random Trades	1,208	3,813	3,155	125,915	(18)	(7)	11
Non-Algo Random Trades	304	3,444	11,314	102,521	(28)	2	30
All costs in basis points							

Exhibit 2
Comparative trading costs by Market, Side and Price Volatility

	Algo Trades			Non-Algo Trades Non-Random			Non-Algo Trades Random		
Panel 1									
Market	**Cost v. MBA**	**Cost v. MBA ACE Adj**	**ACE Cost Adj.**	**Cost v. MBA**	**Cost v. MBA ACE Adj**	**ACE Cost Adj.**	**Cost v. MBA**	**Cost v. MBA ACE Adj**	**ACE Cost Adj.**
OTC	(11)	11	22	(22)	(9)	13	(34)	1	35
Listed	(9)	2	11	(16)	(6)	10	(26)	2	28
Panel 2									
Side									
Buy	(10)	4	14	(18)	(7)	11	(28)	1	29
Sell	(9)	4	13	(16)	(5)	11	(27)	6	33
Panel 3									
Price Volatility									
Low	(6)	1	7	(14)	(7)	7	(22)	(4)	18
Medium	(10)	2	12	(16)	(6)	10	(25)	2	27
High	(13)	11	24	(26)	(8)	18	(37)	7	44

Exhibit 3
Trading Cost by Order Percentage of Average Daily Volume

	Algo Trades			**Non-Algo Trades Non-Random**			**Non-Algo Trades Random**		
Pct of ADV	**Cost v. MBA**	**ACE Cost Adj**	**Cost v. MBA ACE Adj**	**Cost v. MBA**	**ACE Cost Adj.**	**Cost v. MBA ACE Adj**	**Cost v. MBA**	**ACE Cost Adj.**	**Cost v. MBA ACE Adj**
Less than 1%	(4)	5	1	(15)	5	(10)	(20)	7	(13)
1 - 5%	(14)	19	5	(23)	18	(6)	(28)	22	(6)
5 - 10%	(21)	33	12	(27)	34	7	(31)	37	6
10 - 25%	(28)	45	17	(20)	53	33	(31)	55	24
25 - 50%	(19)	55	36	(30)	73	43	(51)	74	24
More than 50%	(25)	65	40	(55)	91	36	(39)	121	82
All costs in basis points									

Exhibit 4
Costs for Algorithmic Trades by Order Percentage of ADV

All Algorithmic Trades							
Pct of ADV	**% Orders**	**Cost v. VWAP**	**StdDev Cost v. VWAP**	**Cost v. MBA**	**StdDev Cost v. MBA**	**Cost v. MBA ACE Adj**	**StdDev Cost v. MBA ACE Adj**
Less than 1%	80.21%	(2)	32	(5)	51	0	51
1 - 5%	14.01%	(3)	59	(14)	106	6	106
5 - 10%	4.95%	(4)	68	(19)	122	14	122
10 - 25%	0.78%	(4)	70	(21)	125	24	126
25 - 50%	0.04%	(2)	60	(4)	127	41	123
More than 50%	0.00%	(2)	26	(17)	108	43	123
Grand Total	**100.00%**	**(2)**	**40**	**(9)**	**68**	**4**	**67**
All costs in basis points							

Exhibit 5
Trading Costs and Standard Deviation by Algorithmic Server Providers

All Algorithmic Trades						
Provider	**Cost v. VWAP**	**StdDev Cost v. VWAP**	**Cost v. MBA**	**StdDev Cost v. MBA**	**Cost v. MBA ACE Adj**	**StdDev Cost v. MBA ACE Adj**
Broker 1	(2)	22	(8)	46	0	46
Broker 2	(0)	23	(10)	43	(3)	43
Broker 3	(2)	12	(15)	25	(6)	25
Broker 4	(2)	14	(7)	30	(2)	29
Broker 5	(2)	18	(14)	37	(2)	37
Broker 6	(2)	24	(16)	47	(7)	47
Total	**(2)**	**20**	**(9)**	**42**	**(1)**	**42**
All Costs in basis points						

Exhibit 6

Trading Costs for Different Algorithmic Server Providers by Decile

Panel 1

Benchmark = Order Mid Point Bid/Ask

Cost Decile	**Broker1**	**Broker2**	**Broker3**	**Broker4**	**Broker5**	**Broker6**
1	(95)	(77)	(49)	(51)	(58)	(91)
2	(33)	(29)	(16)	(17)	(18)	(31)
3	(18)	(17)	(9)	(9)	(10)	(17)
4	(10)	(10)	(5)	(5)	(5)	(10)
5	(5)	(6)	(2)	(2)	(2)	(4)
6	(1)	(2)	(1)	0	(1)	(0)
7	4	1	1	3	2	4
8	10	6	4	6	5	10
9	21	15	10	13	12	20
10	73	62	44	44	45	65

Panel 2

Benchmark = VWAP

Cost Decile	**Broker1**	**Broker2**	**Broker3**	**Broker4**	**Broker5**	**Broker6**
1	(44)	(40)	(21)	(27)	(27)	(42)
2	(16)	(15)	(8)	(9)	(9)	(14)
3	(9)	(9)	(5)	(5)	(5)	(8)
4	(5)	(5)	(3)	(3)	(3)	(5)
5	(2)	(3)	(1)	(1)	(1)	(2)
6	(0)	(1)	(0)	0	(0)	(0)
7	2	1	1	1	1	2
8	5	5	2	3	3	5
9	11	11	5	7	6	11
10	38	38	18	21	24	35

All costs in basis points

Exhibit 7

Cost and Standard Deviation by Provider and Order Percentage of ADV

Pct of ADV	**Provider**	**Cost v. VWAP**	**StdDev Cost v. VWAP**	**Cost v. MBA**	**StdDev Cost v. MBA**	**Cost v. MBA ACE Adj**	**StdDev Cost v. MBA ACE Adj**
Less than 1%	Broker 1	(2)	23	(8)	45	(4)	45
	Broker 2	(3)	18	(3)	31	2	31
	Broker 3	(1)	11	(5)	24	(2)	24
	Broker 4	(2)	14	(7)	29	(3)	28
	Broker 5	(1)	28	(2)	49	3	49
	Broker 6	(1)	25	(19)	48	(13)	48
Less than 1% Total		**(2)**	**20**	**(8)**	**40**	**(3)**	**40**
1 - 5%	Broker 1	(2)	29	(12)	79	5	79
	Broker 2	(7)	19	(14)	55	6	54
	Broker 3	(8)	19	(42)	52	(20)	54
	Broker 4	0	28	(25)	65	(0)	66
	Broker 5	2	35	(24)	63	(6)	68
	Broker 6	(4)	33	(39)	65	(22)	63
1 - 5% Total		**(2)**	**29**	**(16)**	**75**	**1**	**76**
5 - 10%	Broker 1	(9)	28	(25)	95	10	97
	Broker 2	(25)	37	(35)	114	17	109
	Broker 3	(23)	28	(97)	84	(57)	78
	Broker 4	(0)	30	(9)	73	34	74
	Broker 5	6	28	(27)	76	2	87
	Broker 6	(9)	15	(115)	87	(59)	51
5 - 10% Total		**(8)**	**28**	**(30)**	**91**	**6**	**92**
More than 10%	Broker 1	(0)	31	(29)	94	17	110
	Broker 3	(7)	11	(96)	74	(39)	70
	Broker 4	3	34	(22)	68	32	89
	Broker 5	3	33	(37)	82	17	86
	Broker 6	32	41	(3)	15	37	50
More than 10% Total		**(1)**	**32**	**(45)**	**86**	**5**	**100**
Total		**(2)**	**21**	**(11)**	**42**	**(2)**	**42**