INTRODUCTION TO GRAPHS

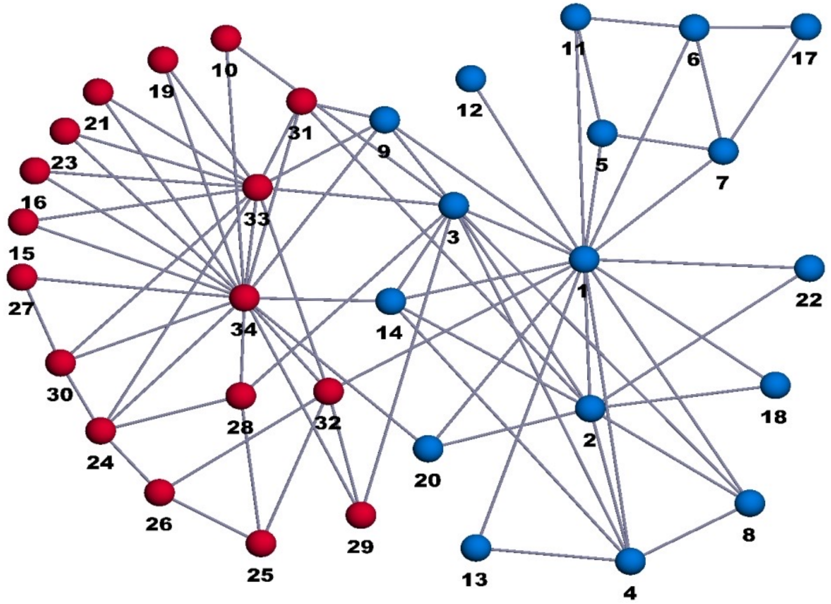
CS340

Graphs

- G = (V, E)
- A graph consists of a set of vertices, and a set of edges representing some kind of connection between specific vertices.
- Edges and vertices are abstractions that can represent virtually anything

Graphs can represent…

- Computer networks
- Design of computer chips
- Communications networks
- Water flow
- Streets and traffic
- Airline routes
- The structure of the internet
- Linguistic models
- Chemical reactions
- Protein interactions
- Molecular interactions
- Coloring problems
- Social interactions
- Rumor spreading
- Epidemiology
- Covert crime networks
- Acquaintanceships
- Friendships
- Matchmaking
- Collaborations
- Economic transactions



Terminology

- Adjacent (neighbors)
  - Vertices u and v are **adjacent** if an edge e is **incident** with the vertices u and v
- Degree
  - The **degree** of a vertex in an undirected graph is the number of edges incident with it.
- The Handshaking Theorem
  - Let G = (V ,E) be an undirected graph with m edges. The total number of edges in the graph equals half the sum of the degrees of each vertex. Mathematically,
    - $2m = \sum_{v \in V} degree(v)$

Terminology

- Complete
  - A graph that has all possible edges.
- Directed/Undirected
  - A directed edge is directed from one vertex to another. It flows in one direction.
- Weighted
  - Edges of a graph have associated weights.

Terminology

- Sparse graph
  - The size of the set of edges $|E|$ is much less than the square of the size of the set of vertices $|V|^2$. Counter-concept is a **dense** graph.
- Cycle
  - Traversal of a graph that starts and ends at the same vertex but otherwise has no repeated vertices or edges.
- Acyclic
  - Containing no cycles
- Tree
  - An acyclic graph

Representation of graphs

- Adjacency list
- Adjacency matrix

Graph is undirected, so adjacency matrix is symmetric along the diagonal.

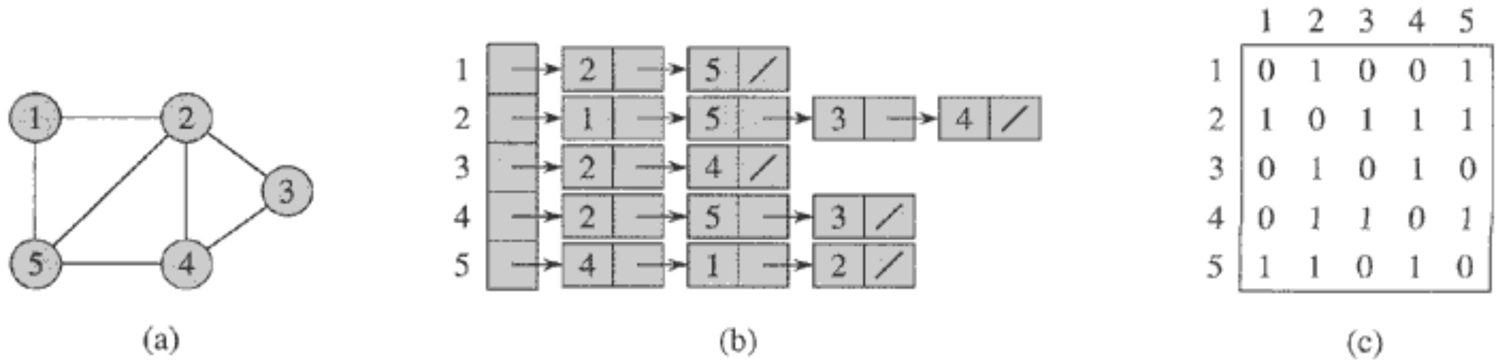
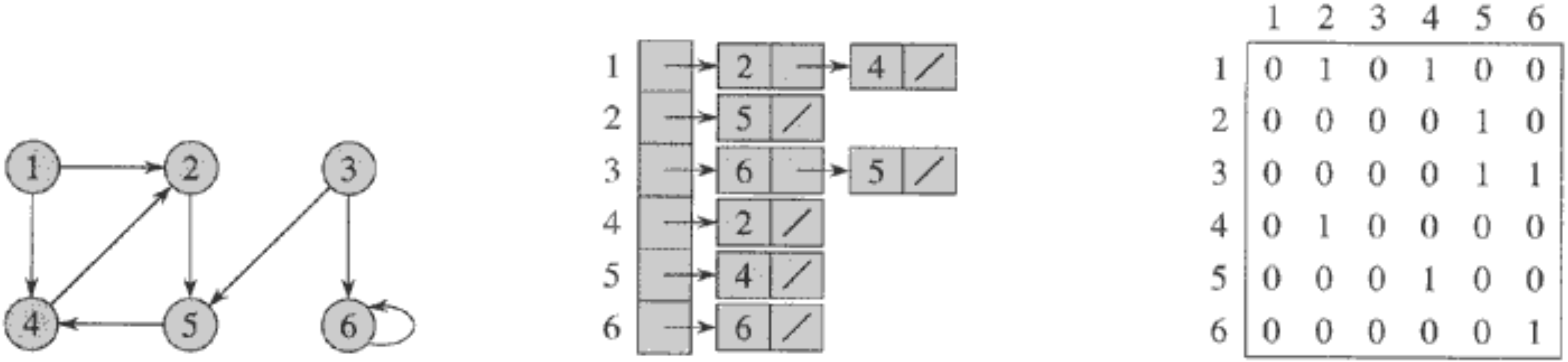


Figure 22.1 Two representations of an undirected graph. **(a)** An undirected graph G having five vertices and seven edges. **(b)** An adjacency-list representation of G. **(c)** The adjacency-matrix representation of G.

Representation of a directed graph



(a) (b) (c)

Figure 22.2 Two representations of a directed graph. **(a)** A directed graph G having six vertices and eight edges. **(b)** An adjacency-list representation of G. **(c)** The adjacency-matrix representation of G.

Representation of graphs

- Adjacency list
  - Array *Adj* of |V| lists, one per vertex.
  - Preferred for sparse graphs (why?)
  - Weights and other info can be stored with each vertex
  - Time: to list all vertices adjacent to u: $\Theta(\text{degree}(u))$
  - Time: to determine whether $(u, v) \in E$: $O(\text{degree}(u))$
- Adjacency matrix
  - 1 or weight in matrix represents an edge
  - Preferred for dense graphs
  - Preferred if we need to determine quickly if an edge exists (why?)
  - Time: to list all vertices adjacent to u: $\Theta(V)$
  - Time: to determine whether $(u, v) \in E$: $\Theta(1)$