

HASH TABLES

CS340

Division Method Hash Function

- $H(k) = k \bmod m$ (k is number of keys, m is number of slots)
- **Example:** $m = 20$ and $k = 91$. $h(k) = 11$.
- **Advantage:** Fast, since requires just one division operation.
- **Disadvantage:** Have to avoid certain values of m :
 - Powers of 2 are bad. If $m = 2^p$ for integer p , then $h(k)$ is just the least significant p bits of k .
- **Good choice for m :** A prime not too close to an exact power of 2.

Multiplication method hash function

- 1. Choose constant A in the range $0 < A < 1$.
- 2. Multiply key k by A .
- 3. Extract the fractional part of kA .
- 4. Multiply the fractional part by m .
- 5. Take the floor of the result.
- **Disadvantage:** Slower than division method.
- **Advantage:** Value of m is not critical.

Open Addressing

- Store all keys in the hash table itself.
- Each slot contains either a key or NIL.
- To search for key k :
 - Compute $h(k)$ and examine slot $h(k)$. Examining a slot is known as a probe.
 - If slot $h(k)$ contains key k , the search is successful. If this slot contains NIL, the search is unsuccessful.
 - There's a third possibility: slot $h(k)$ contains a key that is not k . We compute the index of some other slot, based on k and on which probe (count from 0: 0th, 1st, 2nd, etc.) we're on.
 - Keep probing until we either find key k (successful search) or we find a slot holding NIL (unsuccessful search).

Sequence of slots probed

- We need the sequence of slots probed to be a permutation of the slot numbers (so that we examine all slots if we have to, and so that we don't examine any slot more than once).
- To insert, act as though we're searching, and insert at the first NIL slot we find.

Open Addressing

HASH-SEARCH(T, k)

$i = 0$

repeat

$j = h(k, i)$

if $T[j] == k$

return j

$i = i + 1$

until $T[j] == \text{NIL}$ or $i = m$

return NIL

HASH-INSERT(T, k)

$i = 0$

repeat

$j = h(k, i)$

if $T[j] == \text{NIL}$

$T[j] = k$

return j

else $i = i + 1$

until $i == m$

error “hash table overflow”

Probing Techniques: Linear

- Given auxiliary hash function h' , the probe sequence starts at slot $h'(k)$ and continues sequentially through the table, wrapping after slot $m - 1$ to slot 0.
- Given key k and probe number i ($0 \leq i < m$),
 $h(k,i) = (h'(k) + i) \bmod m$.
- **DISADVANTAGE:** Linear probing suffers from primary clustering: long runs of occupied sequences build up.

Interview Questions

- Consider a hash table of size 7 with hash function:
 $h(k) = k \bmod 7$
- Insert in order 19, 26, 13, 48, 17
- What is result when collisions are handled by linear probing

Probing Techniques: Quadratic

- As in linear probing, the probe sequence starts at $h'(k)$.
- Unlike linear probing, it jumps around in the table according to a quadratic function of the probe number:
 - $h(k,i) = (h'(k) + c_1i + c_2i^2) \bmod m$, where $c_1, c_2 \neq 0$ are constants.
- For example, if $h(k,i) = (h'(k) + i + i^2) \bmod m$, the sequence will be: $h(k)$, $h(k)+2$, $h(k)+6$...

Probing Techniques: Double hashing

- Use two auxiliary hash functions, h_1 and h_2 . h_1 gives the initial probe, and h_2 gives the remaining probes:
$$h(k, i) = (h_1(k) + ih_2(k)) \bmod m.$$

Interview Questions

- Consider a hash table of size 7 with hash function:
 $h(k) = k \bmod 7$
- Insert in order 19, 26, 13, 48, 17
- When collisions are handled by double hashing where the second hash function is $h'(k) = 5 - (k \bmod 5)$.

- Insert in order 19, 26, 13, 48, 17
- $H(k,i) = (k \bmod 7 + i * [5 - k \bmod 5]) \bmod 7 =$