

CS476 Bioinformatics Quiz-Test 2

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CS-476

I provide a mammalian phylogenetic tree involving five organisms: **camel, dog, rat, whale, porpoise**. In your last two exercise sets, we dove into two algorithms to construct phylogenetic trees from a given distance matrix: **UPGMA** and **Neighbor Joining**. Yet, you have not yet ran these algorithms on realistic distance data. I know this week is hectic for you all, so I thought I'd take the opportunity in this "quiztest 2" to give you some small feeling of *closure* with respect to this class, especially phylogenetic trees.

The following is the distance matrix I have computed by applying the Jukes-Cantor evolutionary distance model upon a multiple alignment of the five mammalian species:

Mammal-DM	camel	Dog	porpoise	rat	whale
camel	0	0.30958	0.180443	0.567132	0.183449
dog	0.30958	0	0.230048	0.580985	0.249529
porpoise	0.180443	0.230048	0	0.51628	0.046206
rat	0.567132	0.580985	0.51628	0	0.521058
whale	0.183449	0.249529	0.046206	0.521058	0

Please submit your answers to all questions that follow into a single document (MS Word, LaTeX, OpenOffice, Pages, etc.) and export/compile that document into a PDF entitled QT2<lastname>.pdf (where you replace "<lastname>" with your own last name). You may insert screenshots, diagrams, or photos of hand-drawn work directly into that document. And, it is only that document that you need to submit for this Quiz-Test 2.

Now the questions follow on the next pages.

Part I: Run UPGMA algorithm on the Mammal-DM matrix to obtain a phylogenetic tree.

- a) If you compute the UPGMA based phylogenetic tree by hand, please show your computation steps. Otherwise, if you use either your own UPGMA project or the perl code I uploaded, then please show a screenshot of the execution, with explanation of what you used to compute it.

```

Merged (porpoise, whale) with distances: [0.181946 0.2397885 0.518669 ]
      camel      dog      rat      (porpoise, whale)
camel      0.000000 0.309580 0.567132      0.181946
dog        0.309580 0.000000 0.580985      0.239789
rat        0.567132 0.580985 0.000000      0.518669
(porpoise, whale) 0.181946 0.239789 0.518669      0.000000
Merged (camel, (porpoise, whale)) with distances: [0.27468425 0.5429005 ]
      dog      rat      (camel, (porpoise, whale))
dog      0.000000 0.580985      0.274684
rat      0.580985 0.000000      0.542901
(camel, (porpoise, whale)) 0.274684 0.542901      0.000000
Merged (dog, (camel, (porpoise, whale))) with distances: [0.56194275]
      rat      (dog, (camel, (porpoise, whale)))
rat      0.000000      0.561943
(dog, (camel, (porpoise, whale))) 0.561943      0.000000
Merged (rat, (dog, (camel, (porpoise, whale)))) with distances: []
      (rat, (dog, (camel, (porpoise, whale))))
(rat, (dog, (camel, (porpoise, whale)))) 0.0
Final UPGMA based phylogenetic tree in Newick format: (rat, (dog, (camel, (porpoise, whale))))

```

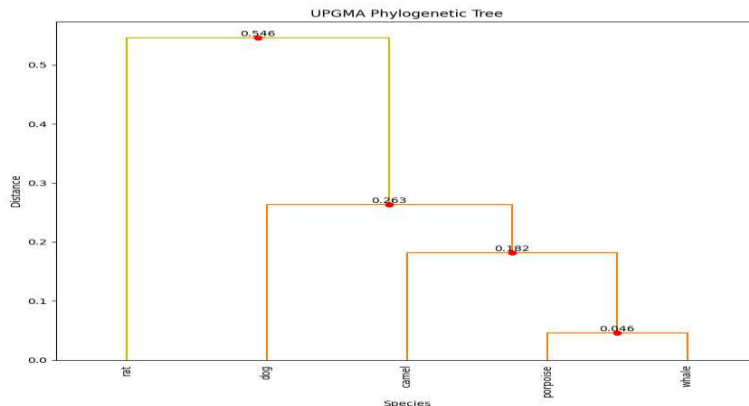
```

1 import numpy as np
2 import pandas as pd
3
4 # Define the initial distance matrix
5 distance_matrix = np.array([
6     [0, 0.30958, 0.180443, 0.567132, 0.183449],
7     [0.30958, 0, 0.238648, 0.580985, 0.249529],
8     [0.180443, 0.238648, 0, 0.51628, 0.046206],
9     [0.567132, 0.580985, 0.51628, 0, 0.521058],
10    [0.183449, 0.249529, 0.046206, 0.521058, 0]
11 ])
12
13 labels = ["camel", "dog", "porpoise", "rat", "whale"]
14
15 def compute_upgma(distance_matrix, labels):
16     cluster_history = {}
17     while len(distance_matrix) > 1:
18         # Find the two closest clusters
19         min_dist = np.inf
20         min_i, min_j = -1, -1
21         for i in range(len(distance_matrix)):
22             for j in range(i + 1, len(distance_matrix)):
23                 if distance_matrix[i, j] < min_dist:
24                     min_dist = distance_matrix[i, j]
25                     min_i, min_j = i, j
26
27         # Merge the two clusters
28         new_label = f"({labels[min_i]}, {labels[min_j]})"
29
30         # Calculate the distances to the new cluster
31         new_distances = []
32         for k in range(len(distance_matrix)):
33             if k != min_i and k != min_j:
34                 new_distance = (distance_matrix[min_i, k] + distance_matrix[min_j, k]) / 2
35                 new_distances.append(new_distance)
36
37         # Update the distance matrix
38         distance_matrix = np.delete(distance_matrix, [min_i, min_j], axis=0)
39         distance_matrix = np.delete(distance_matrix, [min_i, min_j], axis=1)
40
41         new_distances = np.array(new_distances)
42         new_distance_matrix = np.zeros((len(distance_matrix) + 1, len(distance_matrix) + 1))
43         new_distance_matrix[-1, :-1] = distance_matrix
44         new_distance_matrix[:-1, -1] = new_distances
45         new_distance_matrix[-1, -1] = 0
46
47         distance_matrix = new_distance_matrix
48         distance_matrix[-1, -1] = 0
49
50         # Update labels
51         label = [label for i, label in enumerate(labels) if i != min_i and i != min_j]
52         labels.append(new_label)
53
54         print(f"Merged {new_label} with distances: {new_distances}")
55         print(pd.DataFrame(distance_matrix, index=labels, columns=labels))
56
57     return labels[0]
58
59 # Run the UPGMA algorithm
60 result_tree = compute_upgma(distance_matrix, labels)
61 print(f"Final UPGMA based phylogenetic tree in Newick format: {result_tree}")

```

Python: Simple UPGMA algo to update distance matrix, run the algo and even output the newick formatted tree. Used given data from table and excel.

- Show the UPGMA based phylogenetic tree in Newick format ignoring branch distances. ((camel, (porpoise, whale)), dog, rat)
- Draw the UPGMA based phylogenetic tree (either using diagramming/drawing tool or a photo of your hand-drawn tree) showing branch distances.



Part II: Run the Neighbor Joining algorithm on the Mammal-DM matrix to obtain a phylogenetic tree.

- a) If you compute the Neighbor Join based phylogenetic tree by hand, please show all of your computation steps. Otherwise, if you use either your own Neighbor Joining project or the perl code I uploaded, then please show a screenshot of the execution, with explanation of what you used to compute it.

```
Merged (porpoise, whale) with distances: [0.01855883333333316, 0.02764716666666668]
      camel      dog      rat      (porpoise:0.018559, whale:0.027647)
camel      0.000000      0.000000      0.000000      0.158843
dog         0.000000      0.000000      0.000000      0.216686
rat         0.000000      0.000000      0.000000      0.495566
(porpoise:0.018559, whale:0.027647) 0.158843 0.216686 0.495566 0.000000
Merged (camel, (porpoise:0.018559, whale:0.027647)) with distances: [-0.098641375, 0.257484375]
      dog      rat      (camel:-0.098641, (porpoise:0.018559, whale:0.027647):0.257484)
dog         0.000000      0.000000      0.028921
rat         0.000000      0.000000      0.168362
(camel:-0.098641, (porpoise:0.018559, whale:0.027647):0.257484) 0.028921 0.168362 0.000000
Merged (dog, rat) with distances: [-0.06972012500000001, 0.06972012500000001]
      (camel:-0.098641, (porpoise:0.018559, whale:0.027647):0.257484) (dog:-0.069720, rat:0.069720)
(camel:-0.098641, (porpoise:0.018559, whale:0.027647):0.257484) 0.000000 0.098641
(dog:-0.069720, rat:0.069720) 0.098641 0.000000
Final Neighbor Joining based phylogenetic tree in Newick format: ((camel:-0.098641, (porpoise:0.018559, whale:0.027647):0.257484), (dog:-0.069720, rat:0.069720))
```

```
import numpy as np
import pandas as pd
# Function to define the distance matrix
def distance_matrix(labels):
    n = len(labels)
    distance_matrix = np.zeros((n, n))
    # Fill the distance matrix with values
    distance_matrix[0, 1] = 0.158843
    distance_matrix[1, 0] = 0.158843
    distance_matrix[0, 2] = 0.216686
    distance_matrix[2, 0] = 0.216686
    distance_matrix[0, 3] = 0.495566
    distance_matrix[3, 0] = 0.495566
    distance_matrix[1, 2] = 0.000000
    distance_matrix[2, 1] = 0.000000
    distance_matrix[1, 3] = 0.000000
    distance_matrix[3, 1] = 0.000000
    distance_matrix[2, 3] = 0.000000
    distance_matrix[3, 2] = 0.000000
    return distance_matrix

labels = ["camel", "dog", "porpoise", "rat", "whale"]

def print_matrix(matrix, labels):
    df = pd.DataFrame(matrix, index=labels, columns=labels)
    print(df)

def neighbor_joining(dist_matrix, labels):
    while len(labels) > 2:
        n = dist_matrix.shape[0]
        total_dist = np.sum(dist_matrix, axis=1)
        q_matrix = np.zeros((n, n))

        for i in range(n):
            for j in range(i + 1, n):
                q_matrix[i, j] = (total_dist[i] + total_dist[j] - dist_matrix[i, j]) / (n - 2)
                q_matrix[j, i] = q_matrix[i, j]

        min_index = np.unravel_index(np.argmin(q_matrix, axis=None), q_matrix.shape)
        i, j = min_index

        delta = (total_dist[i] - total_dist[j]) / (n - 2)
        limb_length_i = 0.5 * (dist_matrix[i, j] - delta)
        limb_length_j = 0.5 * (dist_matrix[i, j] + delta)

        new_label = f"({labels[i]}:{limb_length_i:.6f}, {labels[j]}:{limb_length_j:.6f})"
        print(f"Merged ({labels[i]}, {labels[j]}) with distances: [{limb_length_i}, {limb_length_j}]")

        new_dist_matrix = np.zeros((n - 1, n - 1))
        new_labels = []

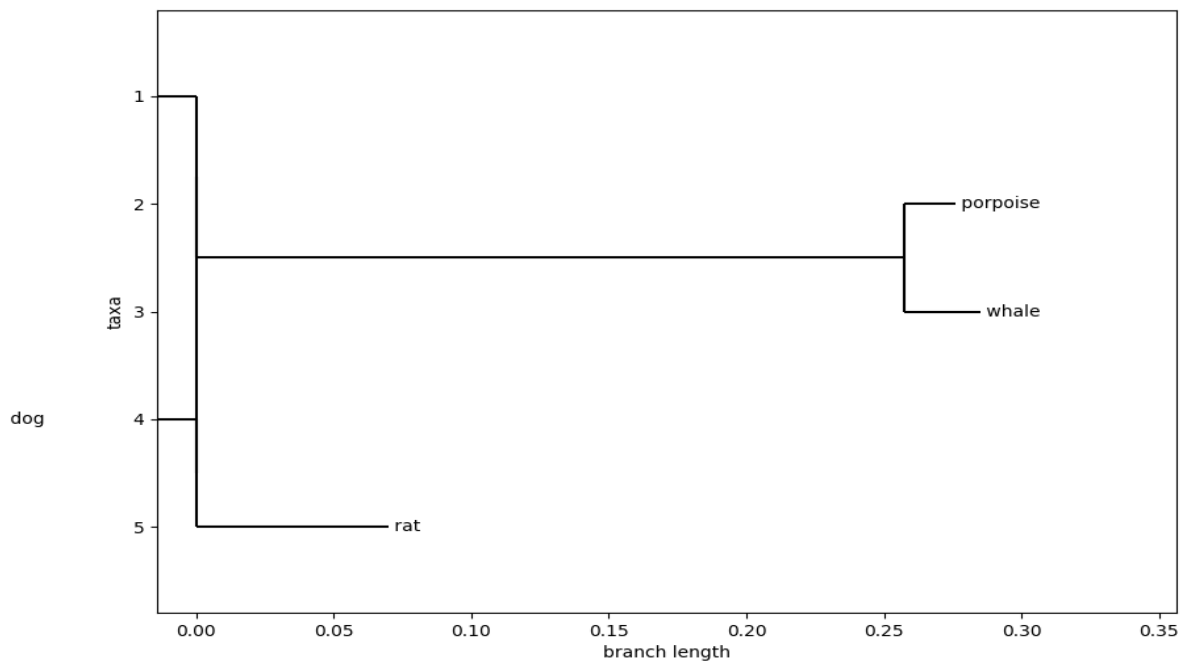
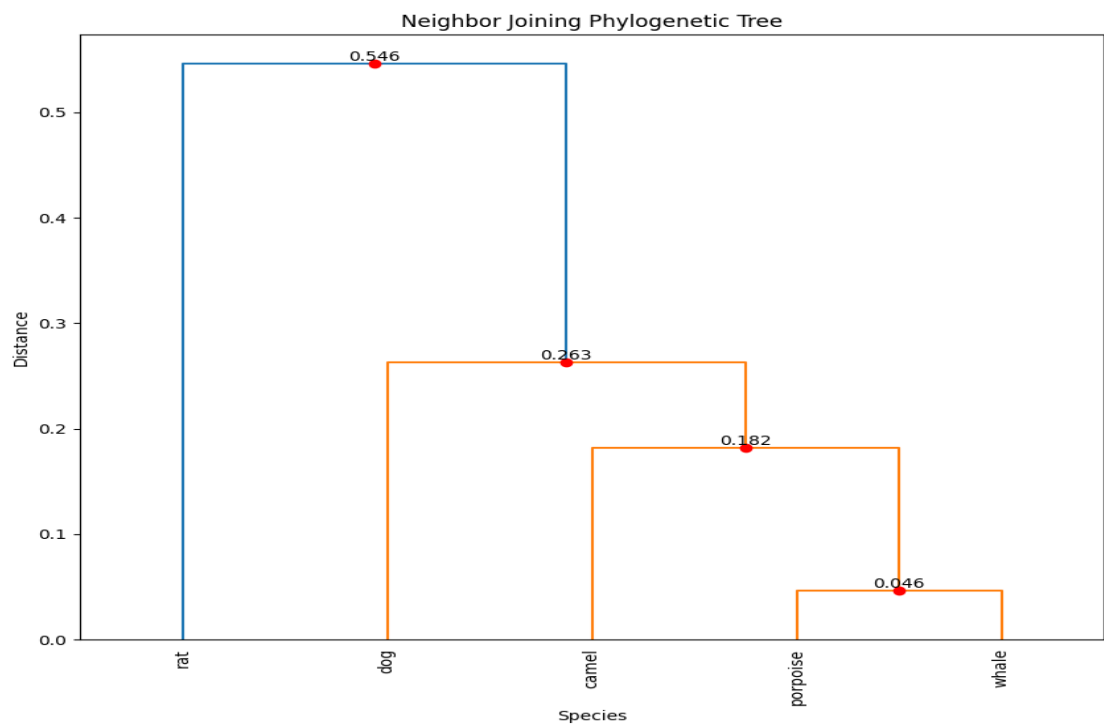
        idx = 0
        for k in range(n):
            if k != i and k != j:
                new_dist_matrix[idx, -1] = new_dist_matrix[-1, idx] = 0.5 * (dist_matrix[i, k] + dist_matrix[j, k] - dist_matrix[i, j])
                idx += 1

        idx = 0
        for k in range(n):
            if k != i and k != j:
                new_labels.append(labels[k])
                idx += 1

        labels = new_labels
        dist_matrix = new_dist_matrix
```

Python: Simple Neighbor-Joining Algo that outputs the intermediate steps used to compute and even outputs Newick formatted tree.

- b) Show the Neighbor Join based phylogenetic tree in Newick format ignoring branch distances. ((rat, (dog, (camel, (porpoise, whale))))))
- c) Draw the Neighbor Join based phylogenetic tree (either using diagramming/drawing tool or a photo of your hand-drawn tree) showing branch distances. We have covered branch distance computation at length for Neighbor Joining in both lecture videos and exercises, and they are also output as part of the perl implementation.



Part III: Comparison of Phylogenetic Trees computed.

- a) Consider the UPGMA and Neighbor Join based phylogenetic trees you computed in Parts I.c and II.c. What are the similarities? What are the differences?

Similarities: Both trees grouped 'whale' and 'porpoise' together, showing close relation.

Clustering of taxa, is closely linked compared to others, is similar.

Difference: UPGMA, Groups '(camel, (porpoise, whale))' first, then groups 'dog, & 'rat' with the whole cluster, whereas Neighbor Joining, Groups '((rat, (dog, (camel, (porpoise, whale))))', showing different evolutionary path and distance calculations.

- b) Do you have any additional comments or opinions on the two different trees?

Nope.

Part IV: A little time complexity.

- a) What is the time complexity of the UPGMA algorithm?

Each iteration, pair of clusters take $O(n^2)$ time since scanning the entirety of the distance matrix. There are $n-1$ iterations, each iteration involves $O(n^2)$ operations for finding the closest pair and then matrix updates. Therefore, total time complexity is $O(n \cdot n^2) = O(n^3)$.

- b) What is the time complexity of the Neighbor Joining algorithm (as you are to implement for Project 2)?

Similar to UPGMA, $n \times n$ distance matrix that initialized with $O(n^2)$ time. Finding min value in the Q-matrix, calculating Q-matrix and updating the distance matrix all take $O(n^2)$ time.

There are $n-1$ iterations, with each iteration involving $O(n^2)$ operations for Q-matrix calculations, therefore we end up with $O(n \cdot n^2) = O(n^3)$.