Residue Environment Analysis for Collatz-like Dynamics

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Abstract

1 Introduction

1.1 Definitions

Odd core decomposition Reference table Local arithmetic constraints

Information flow - shifting is harmless - role of LSB and MSB

Radical

Natural order - Definition - Explain nested prime cycles - Piano roll visualization - Define radical, show it maps to factor sets - Gaps are symmetric - At least one per cycle when masking a bidirectional walk

Prime cycle gaps

Odd core environment - Reference table

Odd core graph - Diagram

- The odd-to-odd Collatz map can be defined as a shift left in ternary, and a shift right in binary, with a $+1$ operation in between. - Both shifts are information preserving, because they only shuffle zeroes. Pushing n around in this base or the other is reversible, so no information is lost. - The binary avalanche caused by $+1$ is the only meaningful change of state in the Collatz map. The shifts are temporary and informationally harmless. - The number of bits flipped in the avalanche is k. - The per-step information evolution of Collatz is proportional to k. The nature of it is isomorphic to the rules of carry propagation.

- The binary carry avalanche maximizes diffusion of information from odd prime rings into the dyadic hierarchy (powers of two). - Each bit flipped in the avalanche shuffles the corresponding mod $2^k class, erasing low-order memory. - Information diffused by odd primes is not she duntil the$

Symbolic progression - The LSB of n under the odd-to-odd map is always 1 (because it's odd). The must also be an MSB $_s omewhere_(because we' re still playing Collatz, so n > 1). - These are delimiters without informational value. The interior "payload" bits completely determine the state o$
$The MSB protects the leading zeroes in the payload. The LSB protects the trailing zeroes, and serves as a "stop" c$
$The interior bits as a string form a symbolic progression, isomorphic to the arithmetic by construction (you can a$

- Integers have a finite MSB, so a carry avalanche is guaranteed to terminate, but 2-adic values are infinite bit strings, and sensitive to the low order bits instead. A pure 2-adic representation does not sieve radicals, so it can't make progress, and exhibits ergodic behavior. When the avalanche reaches the MSB, it is replaced by the carry, destroying information.

- (As an aside, this is why Minsky's two-tag machine traces Collatz: it's a cleverly obfuscated carry-propagator).

References