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Website: www.aero.iitb.ac.in/satlab

README

Guidance, Navigation and Controls Subsystem

General Practice: *"Main" function should be written first*

co_dynamic_function ()

Code author: Ajay Tak
Created on: 12/06/2022
Last modified: 18/06/2022
Reviewed by: -
Description:
The function just takes in different constants and variables, the definition of whose will be provided below, and evaluates a function dependent on these constants and variables. This function is nothing but the derivative of a component of the CubeSat's angular velocity with respect to the Earth's inertial frame expressed in the CubeSat's body frame.

Formula & References:
H: angular momentum

$$H_{\text{total}} = H_{\text{CubeSat}} + H_{\text{Reaction Wheel}}$$

$$H_{\text{CubeSat}} = (I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z)\hat{\imath} + (-I_{xy}\omega_x + I_{yy}\omega_y - I_{yz}\omega_z)\hat{\jmath} + (-I_{zx}\omega_x - I_{zy}\omega_y + I_{zz}\omega_z)\hat{k}$$
$$H_{\text{Reaction Wheel}} = (I'\Omega_1 + 2(I' + mb^2)\omega_x)\hat{\imath} + (I'\Omega_2 + 2(I' + mb^2)\omega_y)\hat{\jmath} + (I'\Omega_3 + 2(I' + mb^2)\omega_z)\hat{k}$$

where

- I_{xx}, I_{yy}, I_{zz} : diagonal elements of inertia matrix
- I_{xy}, I_{yz}, I_{xz} : off-diagonal elements of inertia matrix
- $\omega_x, \omega_y, \omega_z$: component of angular velocity expressed in body frame
- I' : moment of inertia of the reaction wheel about the spin axis
- $\Omega_1, \Omega_2, \Omega_3$: the angular velocities of each of the reaction wheels
- m : mass of reaction wheel
- b : distance between centres of reaction wheels and satellite

$$
\begin{aligned}
H_{\text{total}} =& (I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z + I'\Omega_1 + 2(I' + mb^2)\omega_x)\hat{i} \\
& (-I_{xy}\omega_x + I_{yy}\omega_y - I_{yz}\omega_z + I'\Omega_2 + 2(I' + mb^2)\omega_y)\hat{j} \\
& (-I_{xy}\omega_x - I_{yy}\omega_y + I_{yz}\omega_z + I'\Omega_3 + 2(I' + mb^2)\omega_y)\hat{k}
\end{aligned}
$$

then

$$
\frac{dH_{\text{Total}}}{dt} = \left.\frac{dH_{\text{Total}}}{dt}\right|_{rel} + \vec{\omega} \times H_{\text{total}}
$$

this gives

$$
\begin{aligned}
I_{xx}\dot{\omega_x} + I'\dot{\Omega_1} + 2(I' + mb^2)\dot{\omega_x} + \omega_y\omega_z(I_{zz} - Iyy) + I'(\Omega_3\omega_y - \Omega_2\omega_z) = 0 \\
I_{yy}\dot{\omega_y} + I'\dot{\Omega_2} + 2(I' + mb^2)\dot{\omega_y} + \omega_z\omega_x(I_{xx} - Izz) + I'(\Omega_1\omega_z - \Omega_3\omega_x) = 0 \\
I_{zz}\dot{\omega_z} + I'\dot{\Omega_3} + 2(I' + mb^2)\dot{\omega_z} + \omega_x\omega_y(I_{yy} - Ixx) + I'(\Omega_2\omega_x - \Omega_1\omega_y) = 0
\end{aligned}
$$

Assuming $I_{xy}, I_{yz}, I_{zx} = 0$ (Safe Assumption)

Actuator dynamics :

$$
\begin{aligned}
I'\dot{\Omega}_1 = k_{R_1} i_{R_1} - b_{R_1}\Omega_{R_1} - I'\dot{\omega}_x \\
I'\dot{\Omega}_2 = k_{R_2} i_{R_2} - b_{R_2}\Omega_{R_2} - I'\dot{\omega}_y \\
I'\dot{\Omega}_3 = k_{R_3} i_{R_3} - b_{R_3}\Omega_{R_3} - I'\dot{\omega}_z
\end{aligned}
$$

- k_{R_i} = motor torque constant of i^{th} reaction wheel
- i_{R_i} = armature current of i^{th} reaction wheel
- b_{R_i} = viscous friction coefficient of i^{th} reaction wheel

Substituting actuator dynamics into satellite dynamics we finally have:

$$
\begin{aligned}
I_{xx}\dot{\omega_x} + k_{R_1} i_{R_1} - b_{R_1}\Omega_1 - I'\dot{\omega_x} + 2(I' + mb^2)\dot{\omega_x} + \omega_y\omega_z(I_{zz} - Iyy) + I'(\Omega_3\omega_y - \Omega_2\omega_z) = 0 \\
I_{yy}\dot{\omega_y} + k_{R_2} i_{R_2} - b_{R_2}\Omega_2 - I'\dot{\omega_y} + 2(I' + mb^2)\dot{\omega_y} + \omega_z\omega_x(I_{xx} - Izz) + I'(\Omega_1\omega_z - \Omega_3\omega_x) = 0 \\
I_{zz}\dot{\omega_z} + k_{R_3} i_{R_3} - b_{R_3}\Omega_3 - I'\dot{\omega_z} + 2(I' + mb^2)\dot{\omega_z} + \omega_x\omega_y(I_{yy} - Ixx) + I'(\Omega_2\omega_x - \Omega_1\omega_y) = 0
\end{aligned}
$$

Input parameters:

1. **K** : (Float) - It's just a term which involves arithmetic over other variables. *kg* m^2
2. **kr_i** : (Float) - It is the torque constant of the i'th motor driving the i'th reaction wheel. *N m* A^{-1}
3. **Ir_i** : (Float) - It is the control variable/armature current of the motor driving the reaction wheel. *A*
4. **br_i** : (Float) - It is the friction co-efficient of the i'th motor driving the i'th reaction wheel. *kg* m^2 s^{-1}
5. **W_i0, W_i1, W_i2** : (Float) - These are the angular velocity of reaction wheels about spin axis. *rad* s^{-1}
6. **w_i1, w_i2** : (Float) - These are 2 components of cubesat's angular velocity w.r.t to earth's inertial frame expressed in body frame. *rad* s^{-1}

7. **I_i1, I_i2** : (Float) - These are the two principle elements of the moment of inertia matrix. *kg* m^2

8. **I_dash** : (Float) - Moment of inertia of the reaction wheel about the spin axis. *kg* m^2

Output:
This function returns the value of rate of change of a component of angular velocity of the body frame w.r.t inertial frame.

co_propagating_w_i ()

Code author: Ajay Tak
Created on: 12/06/2022
Last modified: 18/06/2022
Reviewed by: -
Description:
This function is just used for propagating w vector at the next time step. It uses the RK4 method of numerical integration for the same.
Formula & References:

$$
\begin{aligned}
\dot{\bar{\mathbf{x}}} &= \bar{\mathbf{f}}(\bar{\mathbf{x}}) \\
\bar{\mathbf{x}}_n &= (x_{1,n}, x_{2,n}, ..., x_{m,n}) \\
\bar{\mathbf{x}}_{n+1} &= (x_{1,n+1}, x_{2,n+1}, ..., x_{m,n+1}) \\
\bar{\mathbf{a}}_n &= \bar{\mathbf{f}}(\bar{\mathbf{x}}) \\
\bar{\mathbf{b}}_n &= \bar{\mathbf{f}}(\bar{\mathbf{x}} + (h/2)\bar{\mathbf{a}}_n) \\
\bar{\mathbf{c}}_n &= \bar{\mathbf{f}}(\bar{\mathbf{x}} + (h/2)\bar{\mathbf{b}}_n) \\
\bar{\mathbf{d}}_n &= \bar{\mathbf{f}}(\bar{\mathbf{x}} + (h)\bar{\mathbf{c}}_n) \\
\bar{\mathbf{x}}_{n+1} &= \bar{\mathbf{x}}_n + (h/6)(\bar{\mathbf{a}} + 2\bar{\mathbf{b}} + 2\bar{\mathbf{c}} + \bar{\mathbf{d}})
\end{aligned}
$$

reference: https://www.myphysicslab.com/explain/runge-kutta-en.html, last accessed on 19/06/2022
Input parameters:

1. **w_i0, w_i1, w_i2** : (Float) - These are components of cubesat's angular velocity w.r.t to earth's inertial frame expressed in body frame. *rad* s^{-1}

2. **I_i0, I_i1, I_i2** : (Float) - These are the three principle elements of the moment of inertia matrix. *kg* m^2

3. **kr_i** : (Float) - It is the torque constant of the i'th motor driving the i'th reaction wheel. *N m* A^{-1}

4. **Ir_i0, Ir_i1, Ir_i2** : (Float) - These are our control variables/ armature currents of the motor driving the reaction wheels. *A*

5. **br_i** : (Float) - It is the friction co-efficient of the i'th motor driving the i'th reaction wheel. *kg* m^2 s^{-1}

6. **W_i0, W_i1, W_i2** : (Float) - These are reaction wheel angular velocities. *rad* s^{-1}

7. **I_dash** : (Float) - Moment of inertia of the reaction wheel about the spin axis. *kg* m^2

8. **m** : (Float) - Mass of the reaction wheel. *kg*

9. **b** : (Float) - It is distance between centre of reaction wheel and centre of cubesat. *m*

10. **h** : (Float) - It is the time step. *s*

Output:
This function returns the 3 components of the angular velocity of the body frame w.r.t inertial frame evaluated at the next time step.

co_kinematic_function ()

Code author: Ajay Tak
Created on: 14/06/2022
Last modified: 18/06/2022
Reviewed by: -
Description:
This function gives out the rate of change of quaternion (which takes a vector from body frame to inertial frame) at a time step, given that we have the quaternion and the components of angular velocity of body frame w.r.t inertial frame at the same time step.
Formula & References:

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -P & -Q & -R \\ P & 0 & R & -Q \\ Q & -R & 0 & P \\ R & Q & -P & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Input parameters:

1. **q_0, q_1, q_2, q_3** : (Float) - These are the components of quaternion from body frame to inertial frame. *none*

2. **P, Q, R** : (Float) - These are the components of the angular velocity of body frame with respect to inertial frame. *rad* s^{-1}

Output:
Output of the function is an array containing the rate of change of components of quaternion from body frame to earth frame.

co_propagating_q_IB ()

Code author: Ajay Tak
Created on: 14/06/2022
Last modified: 18/06/2022
Reviwed by: -
Description:
This function is just used for propagating quaternion at the next time step. It uses the RK4 method of numerical integration for the same.
Formula & References:

$$\dot{\bar{\mathbf{x}}} = \bar{\mathbf{f}}(\bar{\mathbf{x}})$$
$$\bar{\mathbf{x}}_n = (x_{1,n}, x_{2,n}, ..., x_{m,n})$$
$$\bar{\mathbf{x}}_{n+1} = (x_{1,n+1}, x_{2,n+1}, ..., x_{m,n+1})$$
$$\bar{\mathbf{a}}_n = \bar{\mathbf{f}}(\bar{\mathbf{x}})$$
$$\bar{\mathbf{b}}_n = \bar{\mathbf{f}}(\bar{\mathbf{x}} + (h/2)\bar{\mathbf{a}}_n)$$
$$\bar{\mathbf{c}}_n = \bar{\mathbf{f}}(\bar{\mathbf{x}} + (h/2)\bar{\mathbf{b}}_n)$$
$$\bar{\mathbf{d}}_n = \bar{\mathbf{f}}(\bar{\mathbf{x}} + (h)\bar{\mathbf{c}}_n)$$
$$\bar{\mathbf{x}}_{n+1} = \bar{\mathbf{x}}_n + (h/6)(\bar{\mathbf{a}} + 2\bar{\mathbf{b}} + 2\bar{\mathbf{c}} + \bar{\mathbf{d}})$$

reference: https://www.myphysicslab.com/explain/runge-kutta-en.html, last accessed on 19/06/2022
Input parameters:

1. **q_0, q_1, q_2, q_3** : (Float) - These are the components of quaternion from body frame to inertial frame. *none*
2. **P, Q, R** : (Float) - These are the components of the angular velocity of body frame with respect to inertial frame. *rad* s^{-1}
3. **h** : (Float) - It is the time step. *s*

Output:
This function returns the quaternion from body to inertial frame, evaluated at the next time step.

co_actuator_model ()

Code author: Ajay Tak
Created on: 18/06/2022
Last modified: 18/06/2022
Reviewed by: -
Description:
This function is derived based on the relation ship between rate of change of angular velocity of the reaction wheel about spin axis with moment of inertia of reaction wheel about the spin axis, armature current driving the reaction wheel, angular velocity of reaction wheel, and the component of the satellite's angular velocity which is along the direction of spin axis.
Formula & References:

$$I'\dot{\Omega}_1 = k_{R_1} i_{R_1} - b_{R_1}\Omega_{R_1} - I'\dot{\omega}_x$$
$$I'\dot{\Omega}_2 = k_{R_2} i_{R_2} - b_{R_2}\Omega_{R_2} - I'\dot{\omega}_y$$
$$I'\dot{\Omega}_3 = k_{R_3} i_{R_3} - b_{R_3}\Omega_{R_3} - I'\dot{\omega}_z$$

Input parameters:

1. **I_dash** : (Float) - Moment of inertia of the reaction wheel about the spin axis. *kg* m^2
2. **kr_i** : (Float) - It is the torque constant of the i'th motor driving the i'th reaction wheel. *N m* A^{-1}

3. **br_i** : (Float) - It is the friction co-efficient of the i'th motor driving the i'th reaction wheel. *kg* m^2 s^{-1}

4. **Ir_i** : (Float) - This is our control variable/ armature current of the i'th motor driving the i'th reaction wheel. *A*

5. **W_i** : (Float) - This is the i'th reaction wheel angular velocity. *rad* s^{-1}

6. **w_i_dot** : (Float) - It is the rate of change of the i'th component of the satellite's angular velocity w.r.t inertial frame. *rad* s^{-1}

Output:
This function returns the rate of change of the i'th reaction wheel's angular velocity about its spin axis.

co_propagating_w_rw_i ()

Code author: Ajay Tak
Created on: 18/06/2022
Last modified: 26/06/2022
Reviewed by: -
Description:
This function is just used for propagating reaction wheel angular velocity at the next time step. It uses the RK4 method of numerical integration for the same.
Formula & References:

$$
\begin{aligned}
\dot{\bar{\mathbf{x}}} &= \bar{\mathbf{f}}(\bar{\mathbf{x}}) \\
\bar{\mathbf{x}}_n &= (x_{1,n}, x_{2,n}, ..., x_{m,n}) \\
\bar{\mathbf{x}}_{n+1} &= (x_{1,n+1}, x_{2,n+1}, ..., x_{m,n+1}) \\
\bar{\mathbf{a}}_n &= \bar{\mathbf{f}}(\bar{\mathbf{x}}) \\
\bar{\mathbf{b}}_n &= \bar{\mathbf{f}}(\bar{\mathbf{x}} + (h/2)\bar{\mathbf{a}}_n) \\
\bar{\mathbf{c}}_n &= \bar{\mathbf{f}}(\bar{\mathbf{x}} + (h/2)\bar{\mathbf{b}}_n) \\
\bar{\mathbf{d}}_n &= \bar{\mathbf{f}}(\bar{\mathbf{x}} + (h)\bar{\mathbf{c}}_n) \\
\bar{\mathbf{x}}_{n+1} &= \bar{\mathbf{x}}_n + (h/6)(\bar{\mathbf{a}} + 2\bar{\mathbf{b}} + 2\bar{\mathbf{c}} + \bar{\mathbf{d}})
\end{aligned}
$$

reference: https://www.myphysicslab.com/explain/runge-kutta-en.html, last accessed on 19/06/2022
Input parameters:

1. **I_dash** : (Float) - Moment of inertia of the reaction wheel about the spin axis. *kg* m^2

2. **kr_i** : (Float) - It is the torque constant of the i'th motor driving the i'th reaction wheel. *N m* A^{-1}

3. **br_i** : (Float) - It is the friction co-efficient of the i'th motor driving the i'th reaction wheel. *kg* m^2 s^{-1}

4. **Ir_i** : (Float) - This is our control variable/ armature current of the i'th motor driving the i'th reaction wheel. *A*

5. **W_i** : (Float) - This is the i'th reaction wheel angular velocity. *rad* s^{-1}

6. **w_i_present** : (Float) - It is the i'th component of the satellite's angular velocity w.r.t inertial frame at the present time step. *rad* s^{-1}

7. **w_i_previous** : (Float) - It is the i'th component of the satellite's angular velocity w.r.t inertial frame at the previous time step. *rad* s^{-1}

8. **h** : (Float) - It is the time step. *s*

Output:
This function returns the i'th reaction wheel's angular velocity about its spin axis at the next time step.

co_state ()

Code author: Ajay Tak
Created on: 26/06/2022
Last modified: 27/06/2022
Reviewed by: -
Description:
The state, which is to be controlled, is a function of error quaternion. error quaternion is just the quaternion representing the rotation from the actual attitude to the desired attitude.
Formula & References:

$$\begin{bmatrix} q_{0e} \\ q_{1e} \\ q_{2e} \\ q_{3e} \end{bmatrix} = \begin{bmatrix} q_{0c} & q_{1c} & q_{2c} & q_{3c} \\ -q_{1c} & q_{0c} & q_{3c} & -q_{2c} \\ -q_{2c} & -q_{3c} & q_{0c} & q_{1c} \\ -q_{3c} & q_{2c} & -q_{1c} & q_{0c} \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$\sigma_R = 2q_{0e} \begin{bmatrix} q_{1e} \\ q_{2e} \\ q_{3e} \end{bmatrix}$$

Input parameters:

1. **q_actual** : (Numpy Array) - The quaternion that represents a rotation from current/actual body frame to the inertial frame. *None*

2. **q_command** : (Numpy Array) - The quaternion that represents a rotation from the desired orientation of the body frame to the inertial frame. *None*

Output:
The output is the state which is to be controlled.

co_armature_current ()

Code author: Ajay Tak
Created on: 26/06/2022
Last modified: 27/06/2022
Reviewed by: -

Description:

This function is the implementation of the PID controller. It requires user to provide values of omega_n, zeta, and T, which will be required to find the gains of the controller.

Formula & References:

$$u_R = K_{PR}\sigma_R + K_{DR}\omega_e + K_{IR}\int \sigma_R dt$$
$$K_{PR} = ({\omega_n}^2 + \frac{2\zeta\omega_n}{T})R^{-1}I$$
$$K_{DR} = (2\zeta\omega_n + \frac{1}{T})R^{-1}I$$
$$K_{IR} = (\frac{{\omega_n}^2}{T})R^{-1}I$$

Input parameters:

1. **q_actual** : (Numpy Array) - The quaternion that represents a rotation from current/actual body frame to the inertial frame. *None*
2. **q_command** : (Numpy Array) - The quaternion that represents a rotation from the desired orientation of the body frame to the inertial frame. *None*
3. **w_actual** : (Numpy Array) - The current/actual angular velocity of the body frame of the cubesat w.r.t the inertial frame. *rad* s^{-1}
4. **w_command** : (Numpy Array) - The required angular velocity of the body frame of cubesat w.r.t to inertial frame. *rad* s^{-1}
5. **h** : (Float) - It is the time step *s*
6. **omega_n** : (Float) - It is the linear control bandwidth of the integral controller. *rad* s^{-1}
7. **zeta** : (Float) - It is the damping ratio of the integral controller. *None*
8. **T** : (Float) - It is the time constant of the integral controller. *s*
9. **I** : (Numpy Matrix) - It is the moment of inertial matrix of the cubesat. *kg* m^2
10. **kr_i** : (Float) - It the motor torque constant of the i'th motor running the i'th reaction wheelt. *N m* A^{-1}

Output:

The output is a numpy array whose elements are the armature currents of the corresponding reaction wheel motors.