Expansion of MARTA to Local Suburbs Through Optimized Combinatorial Algorithms

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MATH 3012-QHS: Applied Combinatorics

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Abstract

MARTA currently only provides transportation to downtown Atlanta, leaving out highly populated areas of Metro Atlanta like the suburbs. The aim of this project is to first expand MARTA by generating several new stations based on population density maps and then create optimized routes between all vertices, given the requirement of being an Eulerian circuit. 122 additional MARTA stations were added to generate a fuller web. An overarching Eulerian circuit for the graph as a whole was generated to maximize efficiency, using a hybrid of a K-Nearest Neighbors model with Dijikstra's algorithm to find the optimal circuit with shortest paths between stations. By targeting the high-population-density locations, a successful Eulerian circuit was developed, creating a feasible route that maximizes convenience to boost Metro Atlanta's accessibility for all.

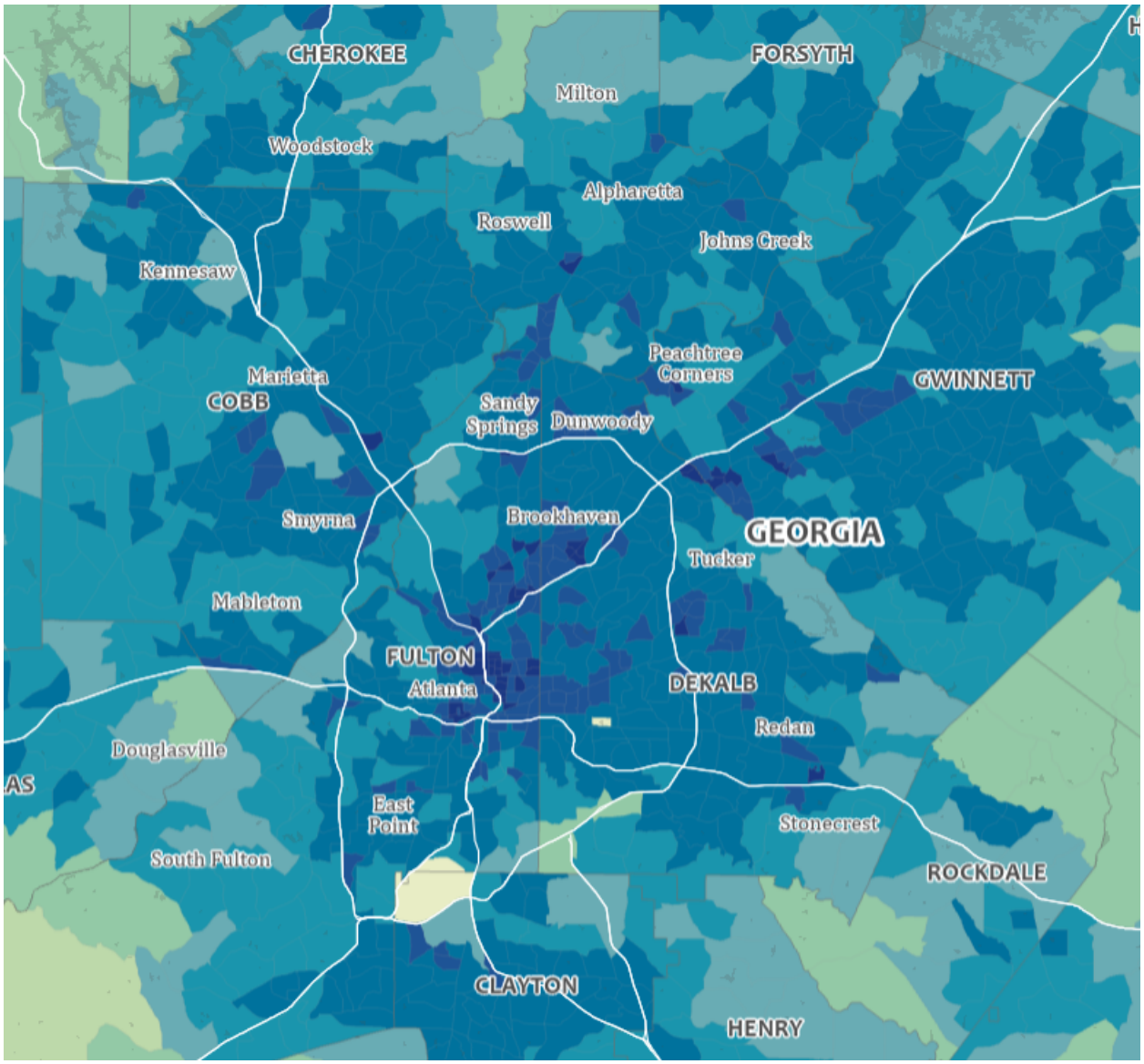
Background

MARTA is the premier subway system for Metro Atlanta; however, it does not currently extend into the suburban areas surrounding Atlanta. Expanding public transportation into suburban areas can solve issues like congestion, carbon emissions, and social disparity. A recent INRIX study ranked Atlanta as the tenth most congested urban area in the United States, resulting in a $1,257 cost per driver and a $3.1 billion cost for the city (Inrix, 2023). Traffic congestion can be easily relieved by optimizing and expanding public transportation since that will help free up the roads and offer alternative modes of transportation. By accomplishing this, residents and the city can save money, which can be invested into other programs rather than dealing with worsening congestion issues. For suburban counties, such as Gwinnett County, emissions from transportation significantly exceed emissions from other sectors, alone tripling the second-largest sector (Drawdown, 2023). By expanding MARTA, the need for individual

transportation can be greatly reduced with the onset of more widespread and accessible public transportation. Not only will this expansion help to alleviate tensions among residents, the city, and the globe, but it will also help to solve societal issues. By granting everybody equal access to transportation across Metro Atlanta, more residents can commute to their jobs and other places of importance, even without personal transportation. This can, in turn, better address the socioeconomic gap between classes while combating regionalized discrimination.

To alleviate these current problems in MARTA, the proposed solution is to expand the current train system in Metro Atlanta. To combinatorially optimize MARTA routes for the expanded system, an algorithm made up of a combination of the machine learning model K-Nearest Neighbors and Dijkstra's algorithm is created. The K-Nearest Neighbors model is a supervised learning classifier that essentially allows for groupings of the data points to be made based on their proximity to one another, discovering similarities across multiple classifications. This model goes hand-in-hand with Dijkstra's algorithm, which, according to Keller & Trotter (2017) searches for the shortest path between two nodes. By applying a K-Nearest neighbors model with Dijkstra's algorithm, the shortest pathways between a starting and ending location can be discovered and through multiple phases of training, develop a high level of confidence in the generated graphs. The basis of the pathways are Eulerian circuits, as Eulerian paths travel along each edge only once, reducing redundancy (Keller & Trotter, 2017). By applying this multifaceted model, an optimal and cost-minimizing solution for MARTA's expansion can be discovered.

Methods



*A population density map for the Metro-Atlanta area, deeper shades of blue are more densely populated (*2020 Census Demographic Data Map Viewer, n.d.).

This map was utilized to find the most ideal locations for new stations. More densely populated areas demand more stations to satisfy the transportation needs of the local population. After finding the most populated regions, a novel K-Nearest Neighbors algorithm using a distance function was established (Hu et al., 2016). In the algorithm, each vertex could have a

maximum of between one and twenty edges to prevent congestion of railways. The edges generated via the K-Nearest Neighbors algorithm were weighted by distance, allowing for only the shortest edges to be considered. Using Euclidean distance, the initial set of railways to connect stations was generated. After establishing this initial network, observational analysis was employed to remove redundant edges, thus reducing complexity. This process computes with a time complexity of O(nk), where n and k represent the number of stations and number of edges out of each station.

$$d = |\mathbf{x} - \mathbf{y}| = \sqrt{\sum_{i=1}^{n} |x_i - y_i|^2}$$

Euclidean distance formula.

Following this process, an Eulerian function was applied to produce the graph and its Eulerian edges, allowing us to discover the Eulerian circuit. Alongside this was an implementation of Dijkstra's algorithm for efficient pathfinding between any two stations on the map. Using the distance-weighted edges and a Greedy-based algorithm, the program takes in two user inputs (starting location and destination) and returns the shortest path from point A to point B.

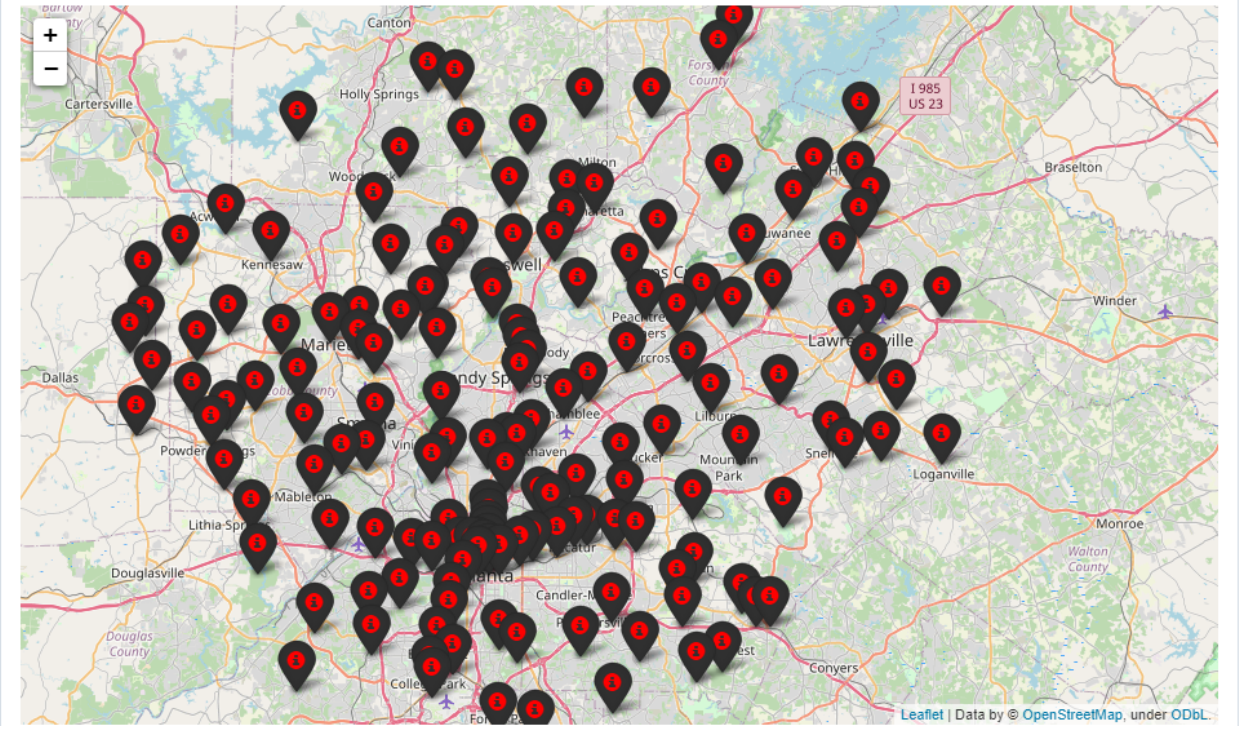
```python
class Graph():

    def __init__(self, vertices):
        self.V = vertices
        self.graph = [[0 for column in range(vertices)]
                      for row in range(vertices)]

    def printSolution(self, dist):
        print("Vertex \t Distance from Source")
        for node in range(self.V):
            print(node, "\t\t", dist[node])

    # A utility function to find the vertex with minimum distance value
    def minDistance(self, dist, sptSet):

        # Initialize minimum distance for next node
        min = 1e7

        for v in range(self.V):
            if dist[v] < min and sptSet[v] == False:
                min = dist[v]
                min_index = v

        return min_index

    # Function that implements Dijkstra's single source shortest path algorithm
    def dijkstra(self, src):

        dist = [1e7] * self.V
        dist[src] = 0
        sptSet = [False] * self.V

        for cout in range(self.V):

            # Pick the minimum distance vertex from the set of vertices not yet processed.
            # u is always equal to src in first iteration
            u = self.minDistance(dist, sptSet)

            # Put the minimum distance vertex in the shortest path
            sptSet[u] = True

            # Update dist value of the adjacent vertices
            # of the picked vertex only if the current
            # distance is greater than new distance and
            # the vertex in not in the shortest path tree
            for v in range(self.V):
                if (self.graph[u][v] > 0 and
                   sptSet[v] == False and
                   dist[v] > dist[u] + self.graph[u][v]):
                    dist[v] = dist[u] + self.graph[u][v]

        self.printSolution(dist)
```

Example function that conducts Dijkstra's algorithm (not implemented in project as we used NetworkX, this is purely for mathematical clarity).

Lastly, a feed-forward graphical neural network was created to create node embeddings. These node embeddings are vector representations of the nodes in the graph, each of which contain identifiable markers that can be used by passengers to optimize transfers, routing, and updates through updated encoded information about station connectivities, delays, crowding, and accessibility (Dehghan-Kooshkghazi, 2022).

Results

```
import folium

mapmarta=folium.Map(location=[33.934150, -84.246706])

for ind in range(len(martabase["Station"])):
    name = martabase.iloc[ind,1]
    lat = martabase.iloc[ind,2]
    longi = martabase.iloc[ind,3]
    folium.Marker([lat,longi],tooltip=name, icon=folium.Icon(color='black',icon_color='red')).add_to(mapmarta)
mapmarta
```



Folium program that prints a map of the new MARTA stations with a total of 160 stations (all vertices on the map represent stations—both old and new).

This map presents our dataset of stations: the most populated areas in the Metro Atlanta area for efficient transportation. With these locations, most residents in Metro Atlanta will have access to MARTA subways and be able to get from one location to another without having to make excessive stops. Certain locations were chosen for their popularity and significance, like local parks or malls.

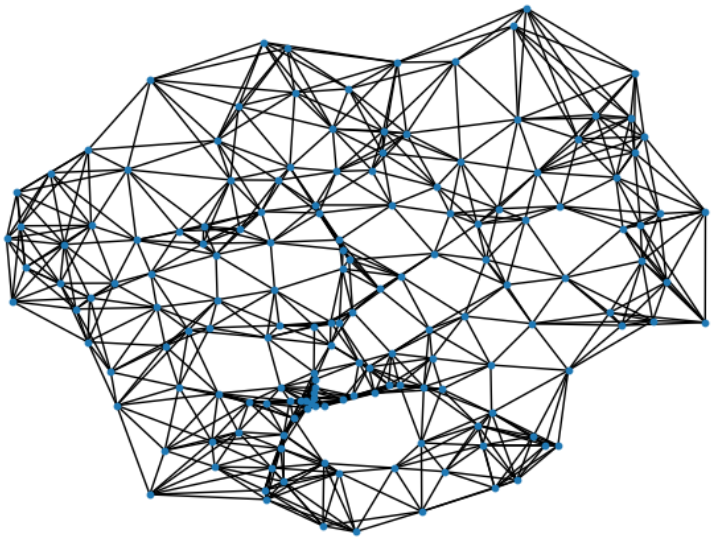
Any new stations are regional approximations, not precise locations for new stations. Approximations allow for focus exclusively on new and optimal routes without considering factors such as residential areas, privately owned land, and other logistical considerations. As such, economic factors like the cost of these new stations were also not considered.

```python
def distance_formula(x1, y1, x2, y2):
   dist = math.sqrt( (math.pow( (x1 - x2) ,2)) + (math.pow( (y1 - y2) ,2)) )
   return dist

def knn_edges(n, k, vertices):
    edges = []
    for key, v in vertices.items():
        # Values for x1,x2
        name = key
        y1 = v[0]
        x1 = v[1]

        # Calculate distances to other vertices
        distances = []
        for key2, u in vertices.items():
            if u[0]!=y1 and u[1]!=x1:
                try:
                    distances.append((u, distance_formula(x1, y1, u[1], u[0])))
                except Exception as e:
                    print(f"Error calculating distance: {e}")

        # Sort distances
        distances.sort(key=lambda x: x[1])

        # Select k nearest neighbors and add edges
        for j in range(k):
            u, _ = distances[j]
            val = {i for i in martagraph if martagraph[i]==u}
            va = val.pop()
            edges.append((name, va, distances[j][1]))

    return edges
```

```python
G = nx.Graph()

# Step 3: Add nodes and edges
for name, coordinates in martagraph.items():
    G.add_node(name, pos=coordinates)  # Add node with 'pos' attribute

edges = knn_edges(160, 8, martagraph)
#print(edges)
G.add_weighted_edges_from(edges)

# Plot the graph
pos = nx.get_node_attributes(G, 'pos')
nx.draw(G, pos, with_labels=False, node_size=10)
plt.show()
```

Graphing vertices and adding edges through KNN on graph

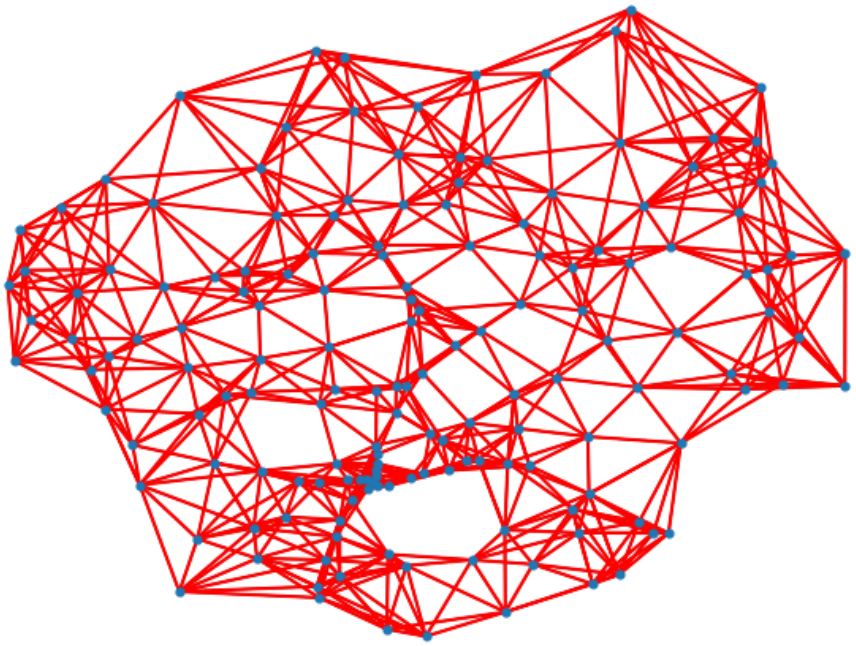
Then, each station is graphed using Networkx, a Python library that supports graph theory visualizations such as Dijstrka's Algorithm, with its coordinates serving as vertices. After that, the Euclidean distance formula is applied to each vertex against all other vertices to find the closest neighbors using K-Nearest-Neighbors approximations. Edges are created between each of these neighboring vertices, producing the graph that will be used for Euler circuits, pathfinding algorithms, and travel optimizations, as shown below.



Unlabeled graph with distance-weighted edges added

```
Ge = nx.euler.eulerize(G)

eulerian_circuit = list(nx.eulerian_circuit(Ge))

# Print the Eulerian circuit
print("Eulerian Circuit:", eulerian_circuit)

# Visualize the graph and the Eulerian circuit
pos = nx.get_node_attributes(Ge, 'pos')
nx.draw(Ge, pos, with_labels=False, node_size=12)
nx.draw_networkx_edges(Ge, pos, edgelist=eulerian_circuit, edge_color='r', width=1.5)
plt.show()
```

Eulerizing and computing Eulerian circuit for graph



Final Euler graph with Euler circuit embedded

```
# example path provided by app from Avalon to Airport station using weighted Dijkstra's algo
lp = True
while lp:
    try:
        start = input("Enter your starting location: ")
        end = input("Enter your destination: ")
        print(nx.dijkstra_path(G, start, end))
        lp = False
    except Exception as e:
            print(f"Error: {e}")
```

User input-based Dijkstra's/Greedy pathfinding algorithm

This text-based application allows a passenger to enter any two locations and receive the most optimized path weighted on both distance and travel time to get from one station to another using the graph, presenting the following route as the solution to get from Avalon to Airport

Station:

```
Enter your starting location: Avalon
Enter your destination: Airport
['Avalon', 'North Point', 'Horseshoe Bend ', 'North Springs', 'Medical Center', 'Buckhead', 'Lindbergh Center', 'Arts Ce
nter', 'Vine City', 'West End', 'Lakewood/Fort McPherson', 'Airport']
```

```python
import torch
import torch.nn as nn
import torch.optim as optim
import torch_geometric
import tensorflow as tf

node_features = torch.randn(160, 16)
data = torch_geometric.utils.from_networkx(G)
data.x = node_features
print(data)

# Define and train a GNN model
class GNN(nn.Module):
    def __init__(self, input_size, hidden_size, output_size):
        super(GNN, self).__init__()
        self.conv1 = torch_geometric.nn.GraphConv(input_size, hidden_size)
        self.conv2 = torch_geometric.nn.GraphConv(hidden_size, output_size)

    def forward(self, x, edge_index):
        x = self.conv1(x, edge_index)
        x = torch.relu(x)
        x = self.conv2(x, edge_index)
        return x

model = GNN(input_size=16, hidden_size=64, output_size=2)
optimizer = optim.Adam(model.parameters(), lr=0.01)
criterion = nn.CrossEntropyLoss()

# Train the model on graph data
for epoch in range(20):
    optimizer.zero_grad()
    output = model(data.x, data.edge_index)
    target = torch.ones_like(output)
    loss = criterion(output, target)
    loss.backward()
    optimizer.step()

    print(f'Epoch {epoch + 1}/{20}, Loss: {loss.item()}')

# uses the learned embeddings to guide Hamiltonian path search
node_embeddings = model.conv1(data.x, data.edge_index)
print(node_embeddings)
```

Graphical Neural Network (GNN) for Node Embeddings and applications

The novel GNN serves several key purposes for the user, making the new MARTA experience mathematically optimized and efficient for traveling at a low loss of approximately 1.7. The GNN consists of two graph convolutional layers, a Rectified Linear Unit (ReLU) activation function, an Adam optimizer, and a CrossEntropyLoss function, making it optimized for this set of parameters and node features.

Conclusions

After generating the optimized Eulerian circuit, each station is visited once, and a sufficient number of rails/connections are present such that complexity is reduced while increasing connectivity across Metro-Atlanta. The establishment of a novel network of MARTA stations and connection pathways allows for a more affordable, accessible transportation system to be present for all Georgia residents. While the political and social implementations of expanding MARTA have been widely considered, this study aims to provide a new perspective on the most efficient and socially optimal expansion of MARTA. This project, however, would require a substantial infrastructural investment. As such, revenue-generating methods and municipality budgets must be deeply considered and researched to implement the conclusions of this study. Furthermore, individual stations must be considered more carefully to ensure that local residents are neither displaced nor disturbed. Beyond the initial MARTA expansion, this system would be extended beyond just the highly populated regions, eventually creating a state-wide railway system to solve problems of congestion, carbon emissions, and social disparity across Georgia. The proposed system of railways in this study offers a feasible solution to guide the state of Georgia towards a more sustainable future.

References

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Dehghan-Kooshkghazi, A., Kamiński, B., Kraiński, Ł., Prałat, P., & Théberge, F. (2022, August 4). *Evaluating node embeddings of complex networks*. Academic.oup.com. https://academic.oup.com/comnet/article-abstract/10/4/cnac030/6655565

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Keller, M. T., & Trotter, W. T. (2017). Applied combinatorics. Mitchel T. Keller, William T. Trotter

Appendix (Inputs and Respective Outputs)

Source code

12/1/23, 6:03 PM marta

In [ ]:
```
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
```

In [ ]:
```
martabase = pd.read_csv("martabase_final.csv")
```

In [ ]:
```
martabase.head()
```

Out[ ]:

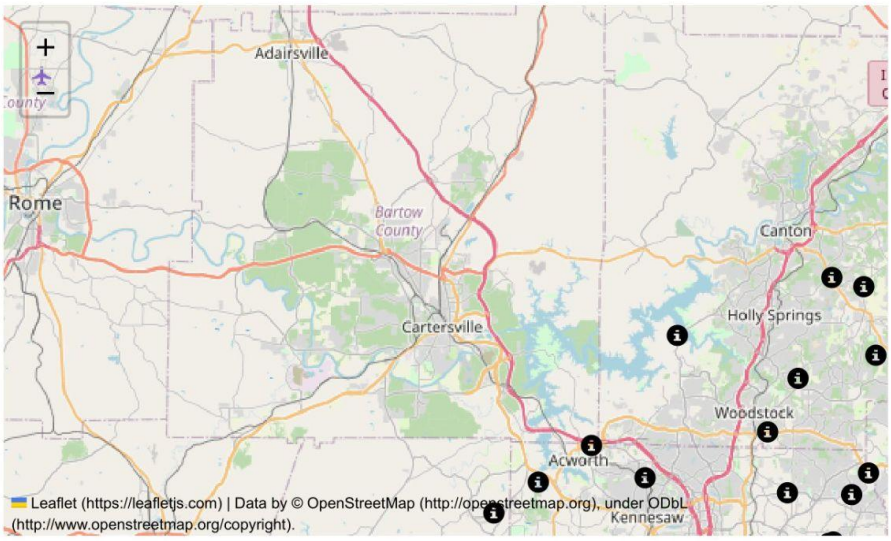
	ID	Station	Latitude	Longitude	Color
0	1	Airport	33.640758	-84.446341	NaN
1	2	Arts Center	33.789705	-84.387789	NaN
2	3	Ashby	33.756346	-84.417556	NaN
3	4	Avondale	33.775277	-84.281903	NaN
4	5	Bankhead	33.771890	-84.428840	NaN

In [ ]:
```
martabase.shape
```

Out[ ]:
```
(160, 5)
```

In [ ]:
```
import folium

mapmarta=folium.Map(location=[33.934150, -84.246706])

for ind in range(len(martabase["Station"])):
    name = martabase.iloc[ind,1]
    lat = martabase.iloc[ind,2]
    longi = martabase.iloc[ind,3]
    folium.Marker([lat,longi],tooltip=name, icon=folium.Icon(color='black',icon_col
mapmarta
```

Out[ ]:



In [ ]:

```
from collections import defaultdict

class Graph:
    def __init__(self, vertices):
        self.V = vertices
        self.graph = defaultdict(list)

    def addEdge(self, v1, v2):
        self.graph[v1].append(v2)
        self.graph[v2].append(v1)

    def DepthFirst(self, v, visited_list):
        visited_list[v] = True
        for i in self.graph[v]:
            if visited_list[i] == False:
                self.DepthFirst(i, visited_list)

    def isConnected(self):
        visited_list = False*(self.V)
        for i in range(self.V):
            if(len(self.graph[i]) != 0):
                break
        if i == self.V-1:
            return True

        self.DepthFirst(i, visited_list)

        # Check if all non-zero degree vertices are visited
        for i in range(self.V):
            if visited_list[i] == False and len(self.graph[i]) > 0:
                return False

        return True
```

```
    def isEulerian(self):
        # Check if all non-zero degree vertices are connected
        if self.isConnected() == False:
            return 0
        else:
            # Count vertices with odd degree
            odd = 0
            for i in range(self.V):
                if len(self.graph[i]) % 2 != 0:
                    odd += 1
            #number of odds:
                # 0 -> euler cycle
                # 2 -> euler path
                # >2 -> not eulerian
            if odd == 0:
                return 2 #euler cycle
            elif odd == 2:
                return 1 #euler path
            elif odd > 2:
                return 0 #no euler

    def test(self):
        res = self.isEulerian()
        if res == 0:
            print("Graph is not Eulerian")
        elif res == 1:
            print("Graph has a Euler path")
        else:
            print("Graph has a Euler cycle")
```

In [ ]:
```
g = Graph(160)
g.addEdge(1,2)
```

In [ ]:
```
names = martabase.iloc[:,1]
latitudes = martabase.iloc[:,2]
longitudes = martabase.iloc[:,3]
longitudes.head()
```

Out[ ]:
```
0   -84.446341
1   -84.387789
2   -84.417556
3   -84.281903
4   -84.428840
Name: Longitude, dtype: float64
```

In [ ]:
```
npnames = names.to_numpy()
nplats = latitudes.to_numpy()
nplongs = longitudes.to_numpy()
```

In [ ]:
```
from sklearn.preprocessing import MinMaxScaler

nplats = nplats.reshape(-1, 1)
nplongs = nplongs.reshape(-1, 1)
```

```
lats_fin = MinMaxScaler().fit_transform(nplats)
longs_fin = MinMaxScaler().fit_transform(nplongs)
```

In [ ]:
```
lats_fin[0]
```

Out[ ]:
```
array([0.06162806])
```

In [ ]:
```
longs_fin[0]
```

Out[ ]:
```
array([0.37190522])
```

In [ ]:
```
longs = longs_fin.flatten()
lats = lats_fin.flatten()
```

In [ ]:
```
vertices_finnn = pd.DataFrame({'Station': npnames, 'Latitude': lats, 'Longitude': l

martagraph = vertices_finnn.set_index('Station')[['Longitude', 'Latitude']].apply(t
```

In [ ]:
```
vertices_f = martabase.iloc[:,2:4]
vertices_f.head()
```

Out[ ]:

	Latitude	Longitude
0	33.640758	-84.446341
1	33.789705	-84.387789
2	33.756346	-84.417556
3	33.775277	-84.281903
4	33.771890	-84.428840

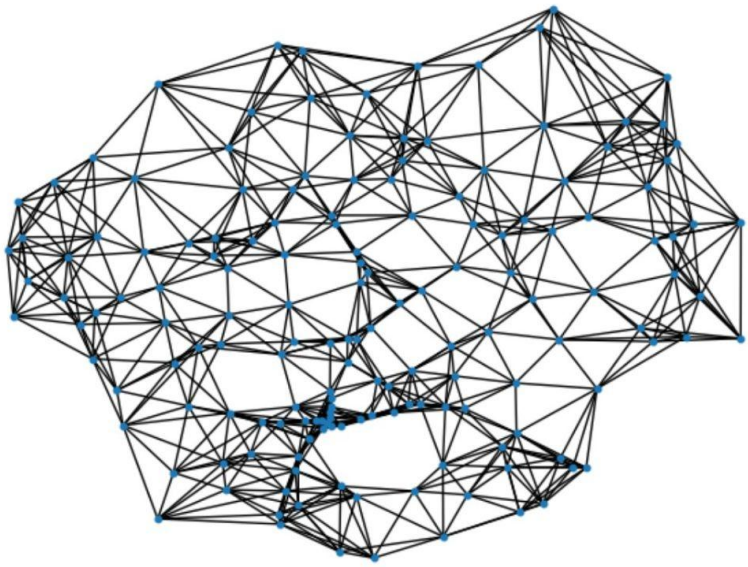
In [ ]:
```
import numpy as np
import networkx as nx
import matplotlib.pyplot as plt
import math
```

In [ ]:
```
vertices_final = vertices_f.to_numpy().tolist()
vertices_final[0]
```

Out[ ]:
```
[33.640758, -84.446341]
```

In [ ]:
```
def distance_formula(x1, y1, x2, y2):
   dist = math.sqrt( (math.pow( (x1 - x2) ,2)) + (math.pow( (y1 - y2) ,2)) )
   return dist

def knn_edges(n, k, vertices):
    edges = []
    for key, v in vertices.items():
        # Values for x1,x2
        name = key
        y1 = v[0]
        x1 = v[1]
```

```
        # Calculate distances to other vertices
        distances = []
        for key2, u in vertices.items():
            if u[0]!=y1 and u[1]!=x1:
                try:
                    distances.append((u, distance_formula(x1, y1, u[1], u[0])))
                except Exception as e:
                    print(f"Error calculating distance: {e}")

        # Sort distances
        distances.sort(key=lambda x: x[1])

        # Select k nearest neighbors and add edges
        for j in range(k):
            u, _ = distances[j]
            val = {i for i in martagraph if martagraph[i]==u}
            va = val.pop()
            edges.append((name, va, distances[j][1]))

    return edges

G = nx.Graph()

# Step 3: Add nodes and edges
for name, coordinates in martagraph.items():
    G.add_node(name, pos=coordinates)  # Add node with 'pos' attribute

edges = knn_edges(160, 8, martagraph)
#print(edges)
G.add_weighted_edges_from(edges)

# Plot the graph
pos = nx.get_node_attributes(G, 'pos')
nx.draw(G, pos, with_labels=False, node_size=10)
plt.show()
```



In [ ]:
```python
for k in range(20):
    G2 = nx.Graph()

    # Step 3: Add nodes and edges
    for name, coordinates in martagraph.items():
        G2.add_node(name, pos=coordinates)  # Add node with 'pos' attribute

    edges = knn_edges(160, k, martagraph)
    G2.add_weighted_edges_from(edges)
    try:
        eulerian_circuit = list(nx.eulerian_circuit(G))
        # Print the Eulerian circuit
        print("Eulerian Circuit:", eulerian_circuit)

        # Visualize the graph and the Eulerian circuit
        pos = nx.spring_layout(G2)
        nx.draw(G2, pos, with_labels=True, font_weight='bold')
        nx.draw_networkx_edges(G2, pos, edgelist=eulerian_circuit, edge_color='r',
        plt.show()
    except Exception as e:
        print(f"Error not Eulerian: {e}")
```

```
Error not Eulerian: G is not Eulerian.
Error not Eulerian: G is not Eulerian.
Error not Eulerian: G is not Eulerian.
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Error not Eulerian: G is not Eulerian.
Error not Eulerian: G is not Eulerian.
```

In [ ]:
```
'''def hamiltonian(graph, start, path=[]):
    path = path + [start]
    if len(path) == len(graph.nodes):
        return path
    for node in graph.neighbors(start):
        if node not in path:
            new_path = hamiltonian(graph, node, path)
            if new_path:
                return new_path
    return None

starting_node = list(G.nodes)[0]

hamiltonian_path = hamiltonian(graph=G, start=starting_node)

print("Hamiltonian Path:", hamiltonian_path)
'''
# Visualize the graph and the Hamiltonian path
'''
pos = nx.spring_layout(G)
nx.draw(G, pos, with_labels=True, font_weight='bold')
nx.draw_networkx_nodes(G, pos, nodelist=hamiltonian_path, node_color='r')
nx.draw_networkx_edges(G, pos, edgelist=[(hamiltonian_path[i], hamiltonian_path[i +
plt.show()
'''
```

Out[ ]:
```
"\npos = nx.spring_layout(G)\nnx.draw(G, pos, with_labels=True, font_weight='bol
d')\nnx.draw_networkx_nodes(G, pos, nodelist=hamiltonian_path, node_color='r')\nn
x.draw_networkx_edges(G, pos, edgelist=[(hamiltonian_path[i], hamiltonian_path[i +
1]) for i in range(len(hamiltonian_path) - 1)], edge_color='r', width=2)\nplt.show
()\n"
```

In [ ]:
```
Ge = nx.euler.eulerize(G)

eulerian_circuit = list(nx.eulerian_circuit(Ge))
```

```
# Print the Eulerian circuit
print("Eulerian Circuit:", eulerian_circuit)

# Visualize the graph and the Eulerian circuit
pos = nx.get_node_attributes(Ge, 'pos')
nx.draw(Ge, pos, with_labels=False, node_size=12)
nx.draw_networkx_edges(Ge, pos, edgelist=eulerian_circuit, edge_color='r', width=1.
plt.show()
```

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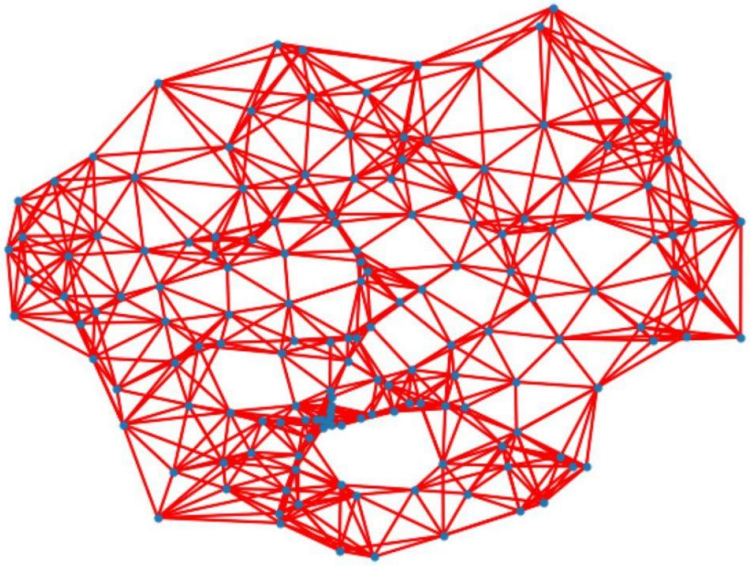

In [ ]:
```python
# example path provided by app from Avalon to Airport station using weighted Dijkst
lp = True
while lp:
    try:
        start = input("Enter your starting location: ")
        end = input("Enter your destination: ")
        print(nx.dijkstra_path(G, start, end))
        lp = False
    except Exception as e:
            print(f"Error: {e}")
```

```
Error: Node io not found in graph
['Avalon', 'North Point', 'Horseshoe Bend ', 'North Springs', 'Medical Center', 'B
uckhead', 'Lindbergh Center', 'Arts Center', 'Vine City', 'West End', 'Lakewood/Fo
rt McPherson', 'Airport']
```

In [ ]:
```python
for u, v, data in G.edges(data=True):
    if 'weight' not in data:
        G[u][v]['weight'] = 0.05
    #print(f"Edge ({u}, {v}): {data}")
```

In [ ]:
```python
import torch
import torch.nn as nn
```

```
import torch.optim as optim
import torch_geometric
import tensorflow as tf

# Convert your NetworkX graph to a PyTorch Geometric Data object
node_features = torch.randn(160, 16)
data = torch_geometric.utils.from_networkx(G)
data.x = node_features
print(data)

# Define and train a GNN model
class GNN(nn.Module):
    def __init__(self, input_size, hidden_size, output_size):
        super(GNN, self).__init__()
        self.conv1 = torch_geometric.nn.GraphConv(input_size, hidden_size)
        self.conv2 = torch_geometric.nn.GraphConv(hidden_size, output_size)

    def forward(self, x, edge_index):
        x = self.conv1(x, edge_index)
        x = torch.relu(x)
        x = self.conv2(x, edge_index)
        return x

model = GNN(input_size=16, hidden_size=64, output_size=2)
optimizer = optim.Adam(model.parameters(), lr=0.01)
criterion = nn.CrossEntropyLoss()

# Train the model on your graph data
for epoch in range(20):
    optimizer.zero_grad()
    output = model(data.x, data.edge_index)
    target = torch.ones_like(output)
    loss = criterion(output, target)
    loss.backward()
    optimizer.step()

    print(f'Epoch {epoch + 1}/{20}, Loss: {loss.item()}')

# uses the learned embeddings to guide Hamiltonian path search
node_embeddings = model.conv1(data.x, data.edge_index)
print(node_embeddings)
```

```
Data(edge_index=[2, 1540], pos=[159, 2], weight=[1540], x=[160, 16])
Epoch 1/20, Loss: 3.6754250526428223
Epoch 2/20, Loss: 7.075736045837402
Epoch 3/20, Loss: 4.351600170135498
Epoch 4/20, Loss: 5.848458290100098
Epoch 5/20, Loss: 3.195577621459961
Epoch 6/20, Loss: 3.5237343311309814
Epoch 7/20, Loss: 4.00560998916626
Epoch 8/20, Loss: 2.653991937637329
Epoch 9/20, Loss: 2.611069917678833
Epoch 10/20, Loss: 3.016585350036621
Epoch 11/20, Loss: 2.355003833770752
Epoch 12/20, Loss: 2.118072032928467
Epoch 13/20, Loss: 2.5813546180725098
Epoch 14/20, Loss: 2.4784207344055176
Epoch 15/20, Loss: 1.8085826635360718
Epoch 16/20, Loss: 1.8909556865692139
Epoch 17/20, Loss: 2.1892223358154297
Epoch 18/20, Loss: 1.8698699474334717
Epoch 19/20, Loss: 1.697495460510254
Epoch 20/20, Loss: 1.9151763916015625
tensor([[ 1.2764, -0.0851,  4.0211,  ...,  0.6951, -0.8406, -1.1633],
        [ 0.5514,  0.5048,  1.0761,  ..., -0.4511, -0.9251,  1.0614],
        [ 2.7418, -4.4167,  2.8022,  ..., -1.3299, -2.9913, -3.1469],
        ...,
        [ 0.1665,  1.2033, -1.5127,  ...,  0.8876, -0.6083,  0.1930],
        [ 3.8695,  0.4998,  1.3280,  ..., -0.4611,  1.4111, -2.7711],
        [-0.5052,  0.2717,  1.0724,  ..., -0.5386,  0.3535, -1.3361]],
       grad_fn=<AddBackward0>)
```