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Using genetic algorithm for search of optimal schemes of entangling transformations in linear quantum optics

Final qualifying work

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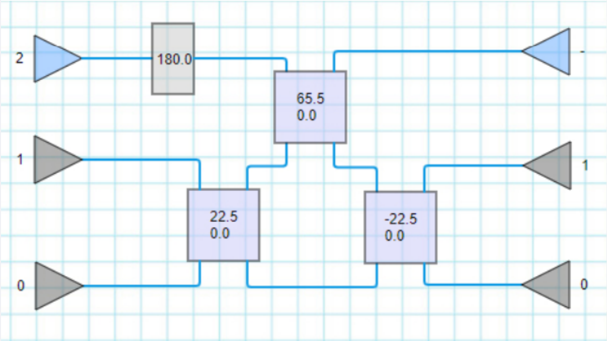
Introduction

- Protocol KLM
- CX with probability 1/9
- Genetic algorithms for search

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$



My task

1. Find a criterion for the entanglement of the state of the system and the gate
2. Find a criterion for fidelity of a matrix
3. Find a continuous criterion for how unitary is matrix
4. Write a cost function for genetic algorithm
5. Get schemes
6. Characterize results

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fidelity criterion

- Probability of getting vectors should be equal
- Conditional probabilities of each vector should be 1

$$A_{base} = |00\rangle, |01\rangle, |10\rangle, |11\rangle$$

$$B_{base} = b_2b_4, b_2b_3, b_1b_4, b_1b_3, b_1^2, b_1b_2, b_2^2, b_3^2, b_3b_4, b_4^2$$

$$\frac{P_a}{P_b} = 1 \Rightarrow P_a = P_b$$

entanglement criterion

for approximating entanglement of a transformation we find entanglement on 12 basis vectors $Z: |0\rangle, |1\rangle$; $X: |+\rangle = \frac{|0\rangle+|1\rangle}{2}, |-\rangle = \frac{|0\rangle-|1\rangle}{2}$; $Y: |y_1\rangle = \frac{|0\rangle+i|1\rangle}{2}, |y_2\rangle = \frac{|0\rangle-i|1\rangle}{2}$

$$\rho = \rho_{AB} = |\phi\rangle \langle\phi|_{AB}$$

$$\rho_A = Tr_A(\rho_{AB})$$

$$E(\rho) = S(\rho_A)$$

$$S(\rho) = Tr(\rho^2)$$

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Unitary criterion

For quantum computation it is necessary that a transform be Unitary

$$M_{in} * M_{in}^{\dagger} = Id$$

- Let us see $M_{in} * M_{in}^{\dagger} - Id$
- Get sum of squares of its elements

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Formation of fitness function

- parameters

```
fun fitness_func(tranform) {
  return (entanglement < entanglement_t) ? entanglement :
         (unitary_cond < unitary_cond_t) ? unitary_cond :
         (cond_prob < cond_prob_t) ? cond_prob :
         (fidelity < fidelity_t) ? fidelity :
         probability;
}
```

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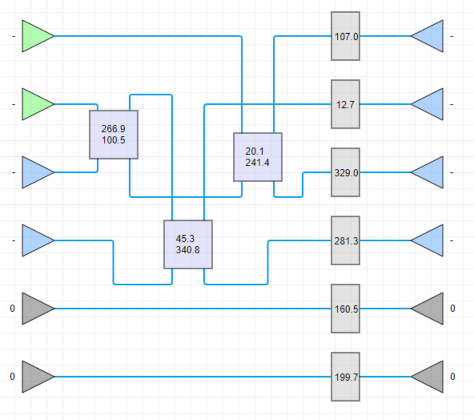
Runs without ancillary photons

Two photons encode two qubits, however without ancillary photons the conditional probability of transform won't get to 1.



$$
\begin{aligned}
|00\rangle &\to ((0.5 - 0.5j))|00\rangle \\
|01\rangle &\to ((0.5 + 0.5j))|00\rangle \\
|10\rangle &\to ((0.5 - 0.5j))|11\rangle \\
|11\rangle &\to ((-0.5 - 0.5j))|11\rangle
\end{aligned}
$$

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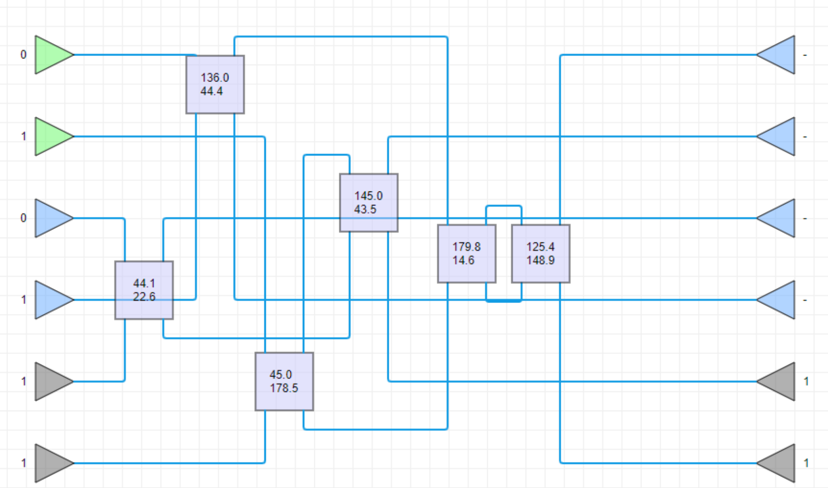
Search of maximum entangled state

It is necessary to use at least 4 photons



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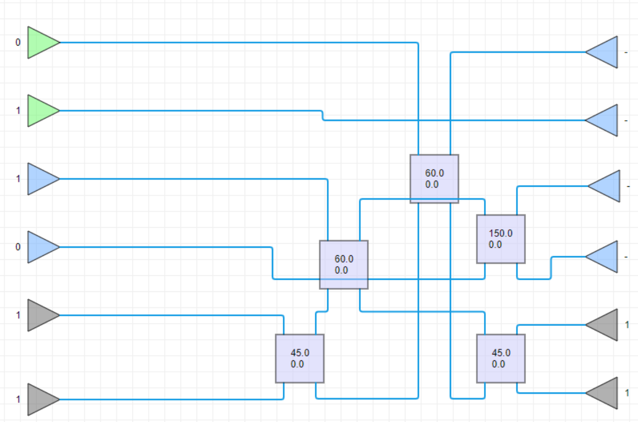
First result of unitary matrix

With addition of unitary criterion and search of entanglement by all basis vectors.



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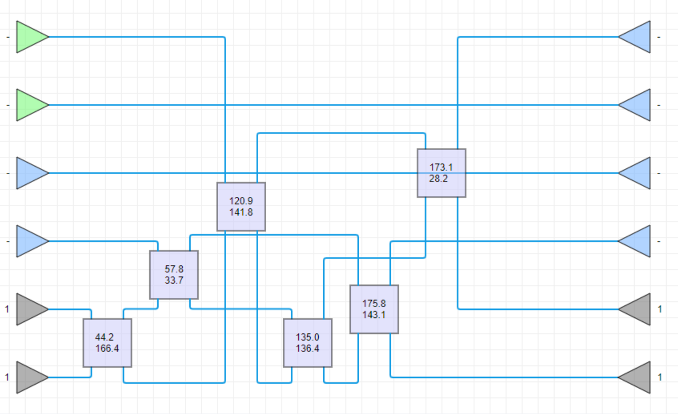
Other transforms

All other runs already had fidelity criterion and made sure that transform satisfies such criterion



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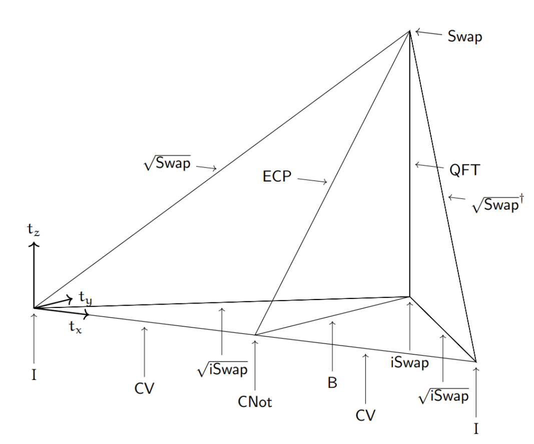
Characterization of two qubit gate

Can we get any already known gate from our by application of local operation?



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Conclusion

- A convenient criterion for entanglement of transform was found
- entanglement search algorithm was implemented.
- Scheme for generation of maximum entanglement state and new implementation of CX gate were found.
- Schemes were analyzed using Weyls Chamber
- A useful hypothesis was made

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