$$x(t) = \sum_{k=1}^{K} \alpha_k \delta(t - t_k) \quad t \in [0, \tau)$$

$$\mathrm{sinc}(t) = \frac{\sin(t)}{t} \quad \leftrightarrow \quad \pi \mathrm{rect}(\omega/2)$$

$$\begin{aligned}
y_\ell &= \langle x(t), \mathrm{sinc}(\pi B(t'_\ell - t)) \rangle_t \\
&= \int x(t) \mathrm{sinc}(\pi B(t'_\ell - t)) \mathrm{d}t \\
&= \int \sum_{m \in \mathbb{Z}} \hat{x}_m \mathrm{e}^{j2\pi m t/\tau} \mathrm{sinc}(\pi B(t'_\ell - t)) \mathrm{d}t \\
&= \sum_{m \in \mathbb{Z}} \hat{x}_m \int \mathrm{sinc}(\pi B(t'_\ell - t)) \mathrm{e}^{j2\pi m t/\tau} \mathrm{d}t \\
\text{change of variable} \quad &= \sum_{m \in \mathbb{Z}} \hat{x}_m \mathrm{e}^{j2\pi m t'_\ell/\tau} \underbrace{\int \mathrm{sinc}(\pi B t) \mathrm{e}^{-j2\pi m t/\tau} \mathrm{d}t}_{\text{FT of } \mathrm{sinc}(\pi B t) \text{ at } \omega = 2\pi m/\tau} \\
&= \sum_{m \in \mathbb{Z}} \hat{x}_m \mathrm{e}^{j2\pi m t'_\ell/\tau} \cdot \frac{1}{B} \mathrm{rect}\left(\frac{2\pi m/\tau}{2\pi B} \right) \\
&= \frac{1}{B} \sum_{|m| \leqslant \lfloor B\tau/2 \rfloor} \hat{x}_m \mathrm{e}^{j2\pi m t'_\ell/\tau}.
\end{aligned}$$