Number	$F(s)$	$f(t), t \ge 0$
1	1	$\delta(t)$
2	$1/s$	$1(t)$
3	$1/s^2$	t
4	$2!/s^3$	t^2
5	$3!/s^4$	t^3
6	$m!/s^{m+1}$	t^m
7	$\frac{1}{s+a}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$
17	$\frac{a}{s^2+a^2}$	$\sin at$
18	$\frac{s}{s^2+a^2}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$

Number	Laplace Transform	Time Function	Comment
—	$F(s)$	$f(t)$	Transform pair
1	$\alpha F_1(s)+\beta F_2(s)$	$\alpha f_1(t)+\beta f_2(t)$	Superposition
2	$F(s)e^{-s\lambda}$	$f(t-\lambda)$	Time delay ($\lambda \ge 0$)
3	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$	$f(at)$	Time scaling
4	$F(s+a)$	$e^{-at}f(t)$	Shift in frequency
5	$s^mF(s)-s^{m-1}f(0)-s^{m-2}\dot{f}(0)-\cdots-f^{(m-1)}(0)$	$f^{(m)}(t)$	Differentiation
6	$\frac{1}{s}F(s)$	$\int_0^t f(\zeta)\,d\zeta$	Integration
7	$F_1(s)F_2(s)$	$f_1(t)*f_2(t)$	Convolution
8	$\lim_{s\to\infty} sF(s)$	$f(0^+)$	Initial Value Theorem
9	$\lim_{s\to 0} sF(s)$	$\lim_{t\to\infty} f(t)$	Final Value Theorem
10	$\frac{1}{2\pi j}\int_{\sigma_c-j\infty}^{\sigma_c+j\infty} F_1(\zeta)F_2(s-\zeta)d\zeta$	$f_1(t)f_2(t)$	Time product
11	$\frac{1}{2\pi}\int_{-j\infty}^{+j\infty} Y(-j\omega)U(j\omega)\,d\omega$	$\int_0^\infty y(t)u(t)\,dt$	Parseval's Theorem
12	$-\frac{d}{ds}F(s)$	$tf(t)$	Multiplication by time

$f_1(t)+f_2(t) = 1(t) + t$

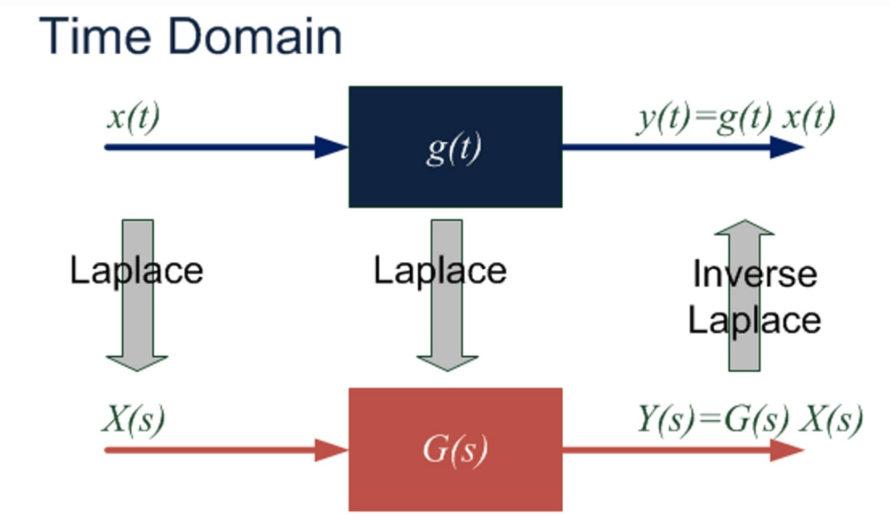
$\Rightarrow F_1(s)+F_2(s) = \frac{1}{s} + \frac{1}{s^2}$

$F_1(t)\cdot F_2(t) \neq F_1(s)\cdot F_2(s)$

$\ddot{y}(t) \xrightarrow{\mathcal{L}} s^2\cdot Y(s) - s\,y(0) - \dot{y}(0)$

$\dot{y}(t) \xrightarrow{\mathcal{L}} S\cdot Y(s) - y(0)$

$\dddot{y}(t) \xrightarrow{\mathcal{L}} s^3 Y(s) - s^2 y(0) - s\dot{y}(0) - \ddot{y}(0)$

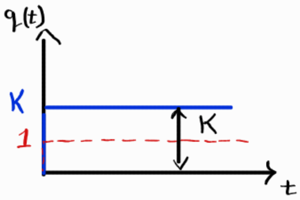


$$\left.\begin{array}{l}\text{Step: } 1(t) \longrightarrow \frac{1}{s} \\ \text{Impuls: } \delta(t) \longrightarrow 1 \\ \text{Ramp: } t \longrightarrow \frac{1}{s^2}\end{array}\right\} \text{types of input signals.}$$

Proportional: Relation between output & input is in general proportional.

a) **Ideal** : $q(t) = K\,u(t)$ $\qquad G(s) = \frac{Y(s)}{U(s)} = K$

input

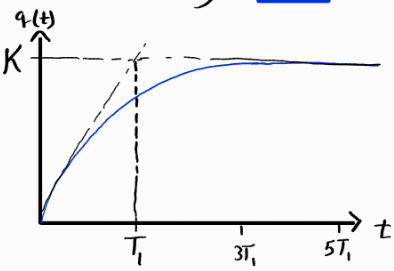


(P-System)

Examples: spring, ohms resistance, Lever, gear, ----

b) With delay: $\boxed{PT_1} \longrightarrow \boxed{T_1\,\dot{y}(t)} + y(t) = K\,u(t)$

more than the ideal



$G(s) = \frac{K}{T_1 S + 1}$ (stable)

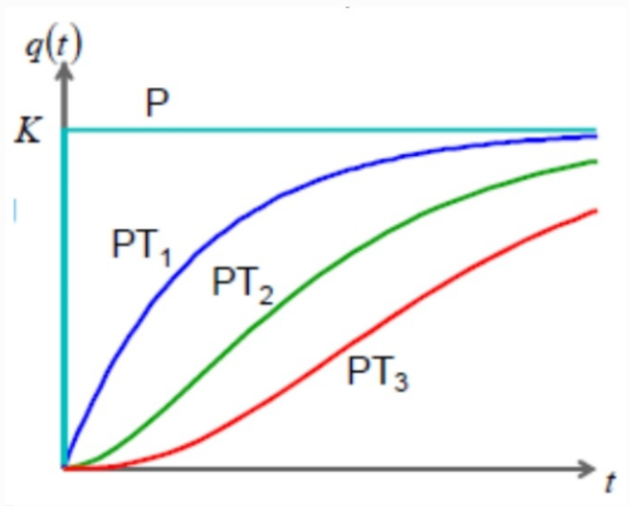
Step response: $q(t) = K\left(1 - e^{\frac{-t}{T_1}}\right)$

c) with delay: $\boxed{PT_2} \longrightarrow \boxed{T_2^2\,\ddot{y}} + T_1\,\dot{y} + y = K\,u$ or $\ddot{y} + 2DW_0\dot{y} + W_0^2\,y = \hat{K}\,u$

more than PT_1

where $W_0 = \frac{1}{T_2}$, $D = \frac{T_1}{2T_2}$

↳ damping ratio



$$G(s) = \frac{K/T_2^2}{S^2 + \frac{T_1}{T_2^2}S + \frac{1}{T_2^2}}$$

$2DW_0$ $\qquad W_0^2$

$$f(s) = S^2 + 2DW_0\,S + W_0^2$$

$$P_{1,2} = -DW_0 \pm \sqrt{D^2W_0^2 - W_0^2} = -DW_0 \pm \sqrt{W_0^2(D^2-1)}$$

$$P_{1,2} = -DW_0 \pm W_0\sqrt{(D-1)(D+1)} = W_0\left(-D \pm \sqrt{(D-1)(D+1)}\right)$$

a) overdamped → $D>1$ → under sqrt is positive & smaller than D → Re: ⊖ Im: 0

b) Critically damped → $D=1$ → sqrt$=0$ → $P_1 = P_2$ → Re: ⊖ Im: 0

c) Damped → $0<D<1$ → Re: ⊖ Im: +/-

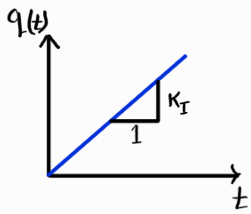
$$D = \sqrt{\frac{\ln(\text{overshoot})^2}{\pi^2 + \ln(\text{overshoot})^2}}$$

d) Undamper → $D=0$ → Re: 0 Im: +/- (Boundary stable)

Integral: Although input is constant, the output is increasing over time.

a) **Ideal**: (I-System) $\longrightarrow y = K_I\int_0^t u(t)$ or $\dot{y} = K_I\,u(t)$

$G(s) = \frac{K_I}{S}$



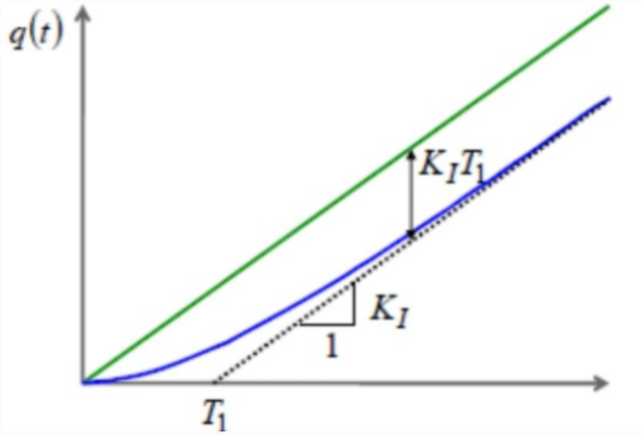
$q(t) = K_I\,t$

(Boundary stable)

b) with delay: $\boxed{IT_1}$ $\longrightarrow$ $\boxed{T_1 \dot{y}} + y = K_I \int_0^t u(t)$ or $\boxed{T_1 \ddot{y}} + \dot{y} = K_I u$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K_I}{s} \cdot \frac{1}{(T_1 s + 1)}$$

Ideal integral, PT_1-System



* IT_1 system is an Ideal integral in series with PT_1 System.

$$q(t) = K_I\left[(t - T_1) + T_1 e^{-\frac{t}{T_1}}\right]$$

(Boundary stable)

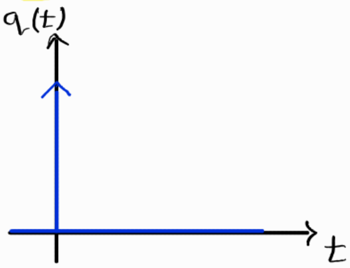
Derivative: output is a derivative of the input.

Example: System with derivative behavior $\longrightarrow$ Shock absorber

a) Ideal: (D-system) $\longrightarrow$ $y = K_D \dot{u}$

$$G(s) = K_D \cdot s$$

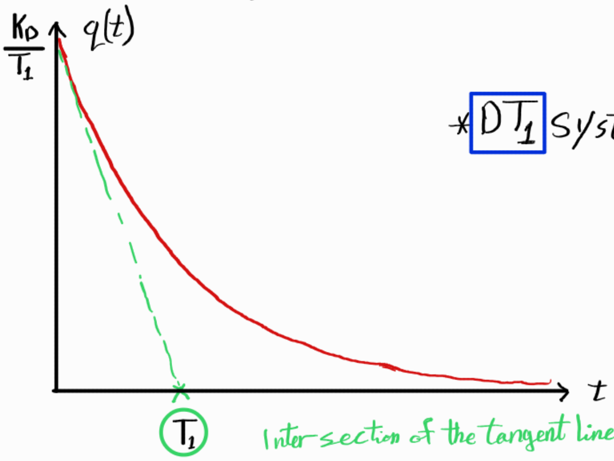
$$q(t) = K_D \cdot \delta(t)$$



b) with delay: $\boxed{DT_1}$ $\longrightarrow$ $T_1 \dot{y} + y = K_D \dot{u}$

$$G(s) = \frac{K_D s \cdot 1}{T_1 s + 1}$$

Ideal D, PT_1

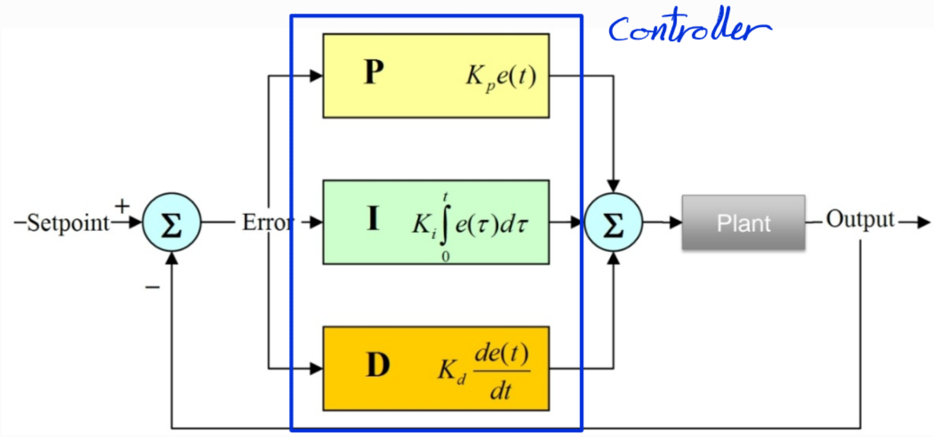


Intersection of the tangent line with the time axis gives us the time constant T_1.

* $\boxed{DT_1}$ system is an Ideal derivative system in series with PT_1 system.

Stable system

Combined System types:



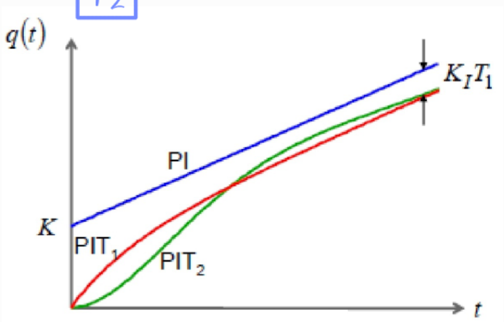
Important combined system types:

* PI-system (with delay)
* PD-system (with delay)
* PID-system (with delay)

* PIT_1 – Sytem (T_2)

$$\boxed{T_2^2\,\ddot{y}(t)} + \boxed{T_1\,\dot{y}(t)} + y(t) = \underbrace{K_P\,u(t)}_{P} + \underbrace{K_I \cdot \int_0^t u(\tau)\,d\tau}_{I} = K_P\left[u(t) + \frac{1}{T_I}\int_0^t u(\tau)\,d\tau\right]$$

where $T_I = \frac{K_P}{K_I}$



$$\boxed{T_2^2\,\dddot{y}(t)} + \boxed{T_1\,\ddot{y}(t)} + \dot{y}(t) = K_P\,\dot{u}(t) + K_I\,u(t) = K_P\left[\dot{u}(t) + \frac{1}{T_I}u(t)\right]$$

$$G(s) = \frac{K_P s + K_I}{s} = K_P + \frac{K_I}{s}$$

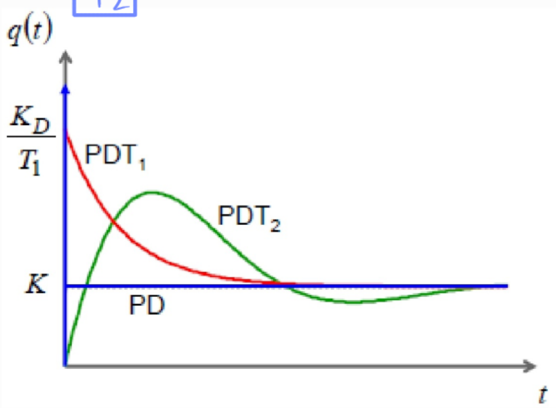
(Boundary stable)

or $G(s) = \frac{K_P\left(s + \frac{1}{T_I}\right)}{s}$ where $T_I = \frac{K_P}{K_I}$

* PDT_1 – System (T_2)

$$\boxed{T_2^2\,\ddot{y}(t)} + \boxed{T_1\,\dot{y}(t)} + y(t) = \underbrace{K_P\,u(t)}_{P} + \underbrace{K_D\,\dot{u}(t)}_{D} = K_P\left[u(t) + T_D\,\dot{u}(t)\right]$$

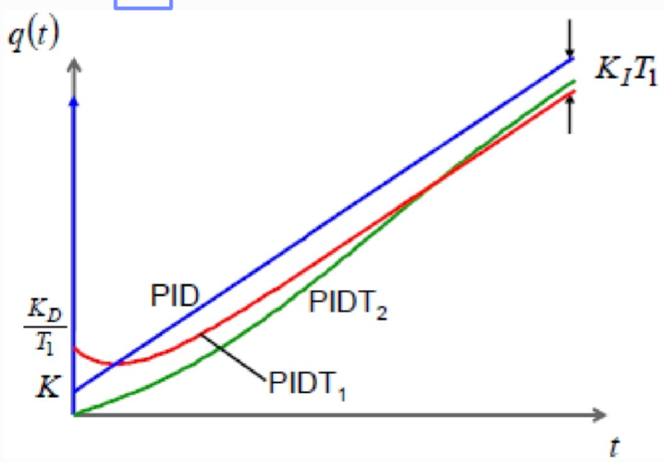
with $T_D = \frac{K_D}{K_P}$



$$G(s) = K_P + K_D \cdot s = \frac{Y(s)}{U(s)}$$

or $G(s) = K_P[1 + T_D \cdot s]$ with $T_D = \frac{K_D}{K_P}$

PID$\boxed{T_1}$ / $\boxed{T_2}$ - System $\quad \boxed{T_2^2 \dddot{y}(t)} + \boxed{T_1 \ddot{y}(t)} + \dot{y}(t) = \underbrace{K_I\, u(t)}_{I} + \underbrace{K_P\, \dot{u}(t)}_{P} + \underbrace{K_D\, \ddot{u}(t)}_{D}$

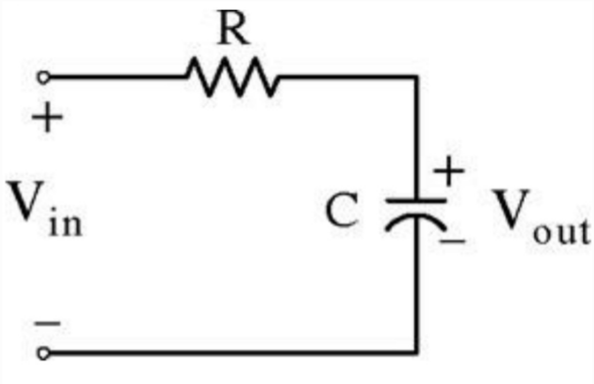


$$G(S) = \frac{K_D S^2 + K_P S + K_I}{S}$$

(Boundary stable)

Example:

$$\overbrace{V_{out}(t)} + \overbrace{RC\, \dot{V}_{out}(t)}^{\text{with } T_1 \text{ delay}} = \overbrace{V_{in}(t)}^{\text{ideal P-system}}$$

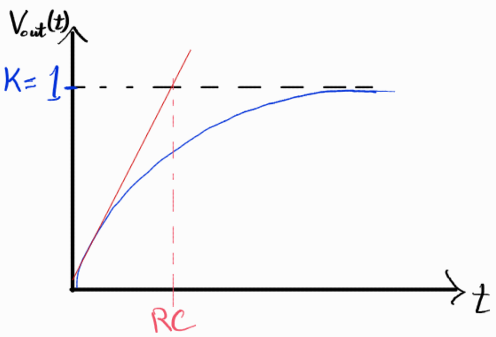


a) TF? $\quad TF = G(S) = \dfrac{V_{out}(S)}{V_{in}(S)} = \dfrac{\overset{K}{1}}{\underset{T_1}{RC}\, S + 1}$

b) Standard system type? PT_1

c) What are the prameters? $K = 1$, $T_1 = RC$

d) Draw step response:



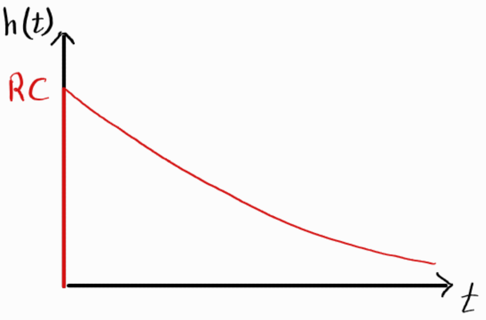
e) Impulse response?

(The derevative of the step response, where the step response is the integral of impulse.

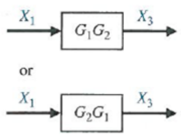
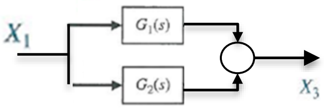
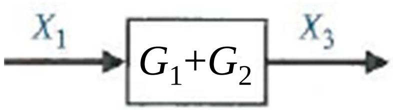
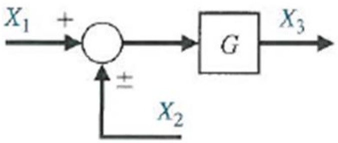
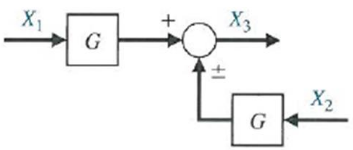
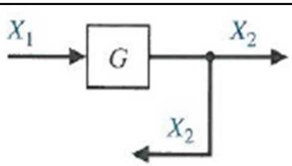
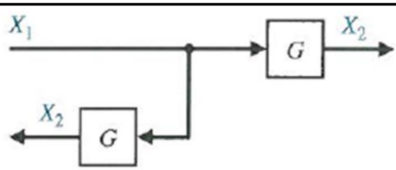
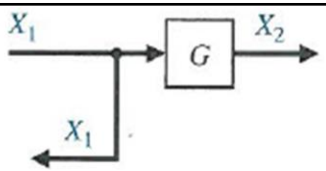
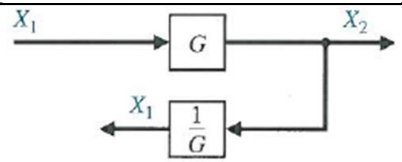
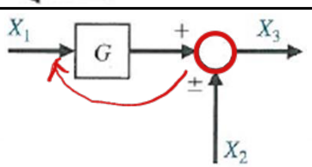
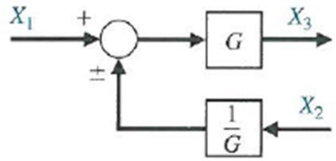
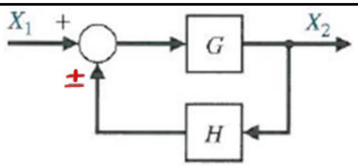
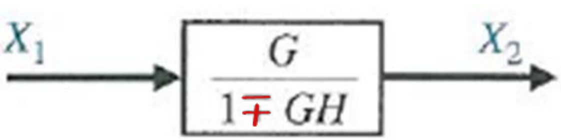
$q(t) = K(1 - e^{-\frac{t}{T_1}}) = 1 - e^{-\frac{t}{RC}}$

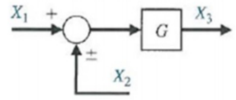
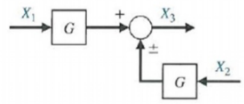
$\dot{q}(t) = h(t) = \frac{1}{RC} \cdot e^{-\frac{t}{RC}}$

f) Draw implulse response

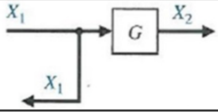
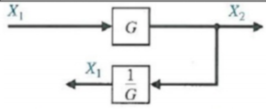


5.1 Block Diagram Transformations

Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade / series		
2. Combining blocks in parallel		
3. Moving a summation point behind a block		
4. Moving a pickoff point ahead of a block		
5. Moving a pickoff point behind a block		
6. Moving a summation point ahead of a block		
7. Eliminating a feedback loop		

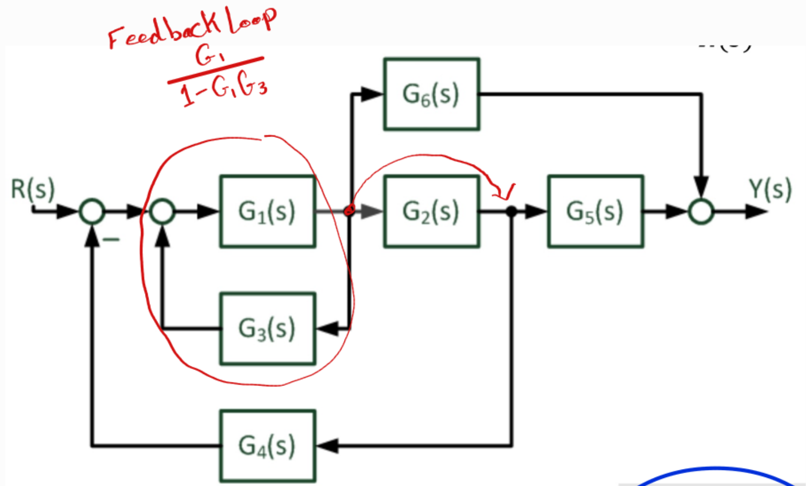
3. Moving a summation point behind a block		

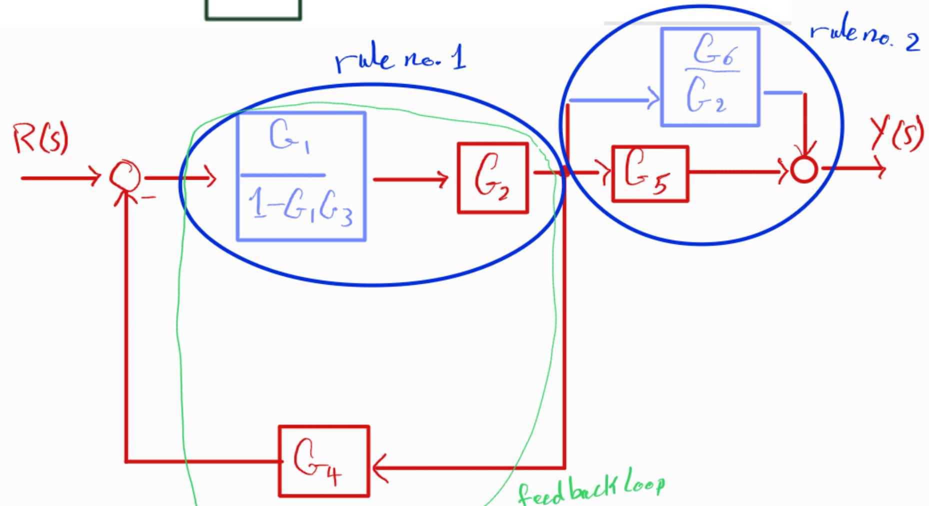
$X_3 = G(s)(X_1 \pm X_2)$ $\quad\longrightarrow\quad$ $X_3 = X_1 G(s) \pm X_2 G(s)$

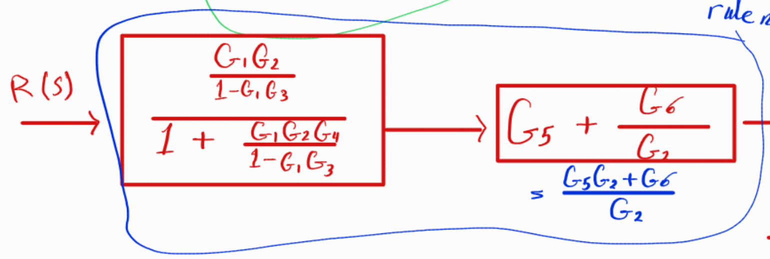
5. Moving a pickoff point behind a block		

$X_2 = X_1 G(s)$ $\qquad$ $X_2 = X_1 G(s)$

$X_1 = X_1$ $\qquad$ $X_1 = X_2 \cdot \frac{1}{G(s)} = X_1 \cancel{G(s)} \cdot \frac{1}{\cancel{G(s)}} = X_1$







$$\frac{\frac{G_1(G_5G_2+G_6)}{\cancel{1-G_1G_3}}}{\frac{1-G_1G_3+G_1G_2G_4}{\cancel{1-G_1G_3}}}$$

$$\therefore TF = \frac{Y(s)}{R(s)} = \frac{G_1(G_5G_2+G_6)}{1-G_1G_3+G_1G_2G_4}$$

5.2 Closed-Loop Behavior

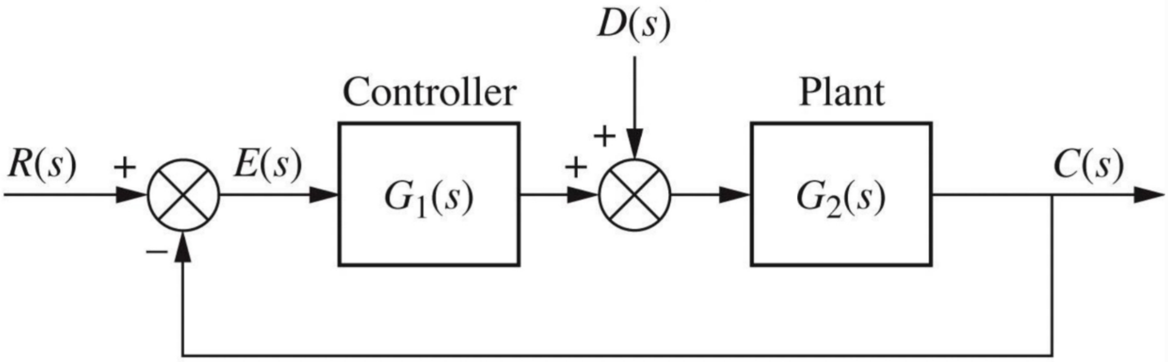


Figure 7.11

To investigate the reference TF, we set the disturbance function = 0

Reference transfer function → Disturbance $D(s) = 0$

ideally

$$G_R(s) = \frac{C(s)}{R(s)} = \frac{G_1(s)\, G_2(s)}{1 + G_1(s)\, G_2(s)} = \frac{G_C(s)\, G_P(s)}{1 + G_C(s)\, G_P(s)} \approx 1$$

5.2 Closed-Loop Behavior

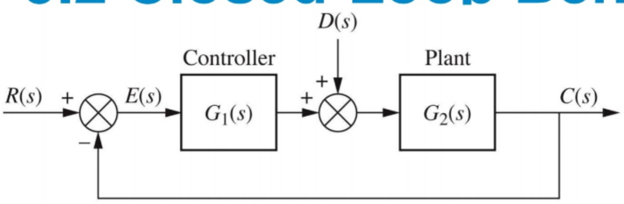
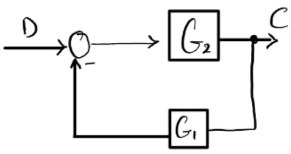


Figure 7.11



To investigate the disturbance TF, we set the reference function = 0

Disturbance transfer function → Reference $R(s) = 0$

ideally

$$\boxed{G_D(s) = \frac{C(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)\, G_2(s)} = \frac{G_P(s)}{1 + G_C(s)\, G_P(s)}} \approx 0$$

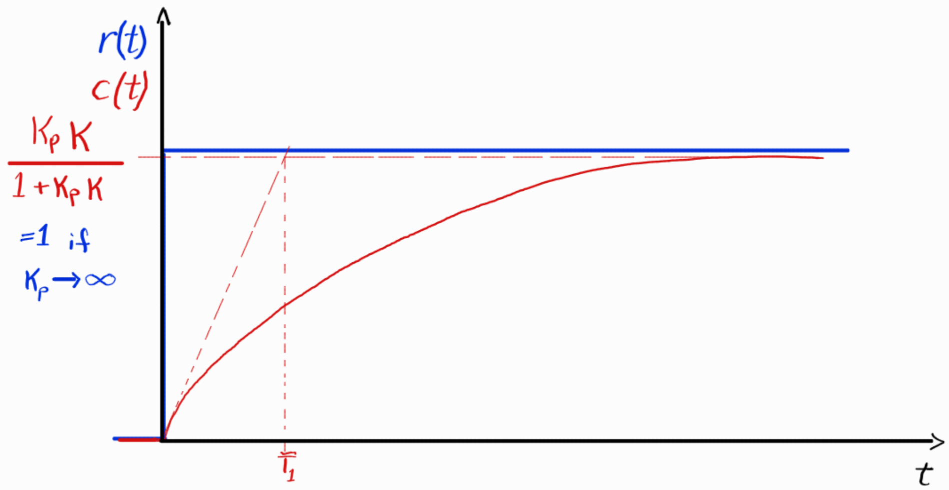
Let´s control a plant with PT1 characteristic by using a P-controller:

$$G_P(s) = \frac{K}{T_1 s + 1} \; ; \; G_C(s) = K_P$$

Reference TF:

$$G_R(S) = \frac{\frac{K_p K}{T_1 S + 1}}{1 + \frac{K_p K}{T_1 S + 1}} = \frac{K_p K}{T_1 S + 1 + K K_p} = \frac{\overbrace{\frac{K_p K}{K_p K + 1}}^{\tilde{K}}}{\underbrace{\frac{T_1}{1 + K_p K}}_{\tilde{T}_1} \cdot S + 1}$$

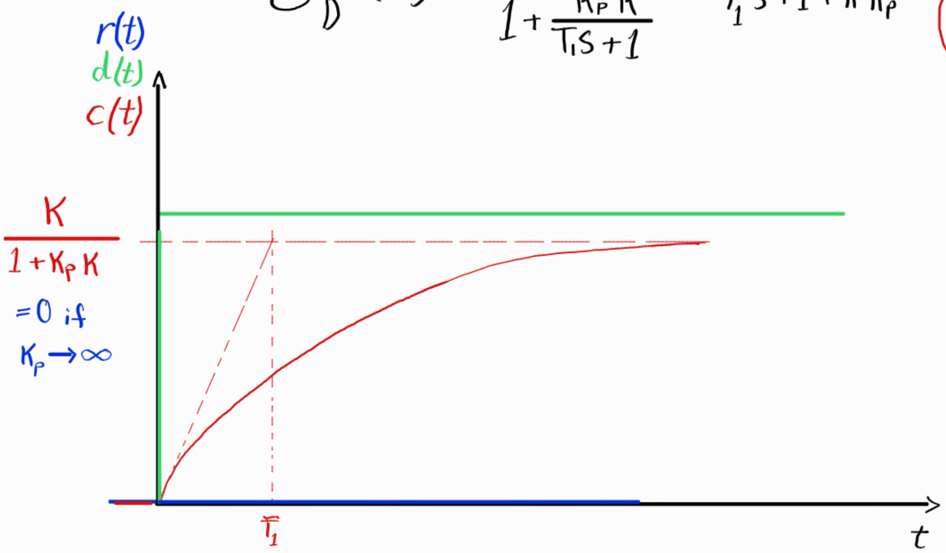
PT_1 - system



Disturbance TF:

$$G_D(S) = \frac{\frac{K}{T_1 S + 1}}{1 + \frac{K_p K}{T_1 S + 1}} = \frac{K}{T_1 S + 1 + K K_p} = \frac{\overbrace{\frac{K}{K_p K + 1}}^{\tilde{K}}}{\underbrace{\frac{T_1}{1 + K_p K}}_{\tilde{T}_1} \cdot S + 1}$$

PT_1 - system



Let´s control our plant with PT1 characteristic by using an I-controller:

$$G_P(s) = \frac{K}{T_1 s + 1} \; ; \; G_C(s) = \frac{K_I}{s}$$

Reference TF:

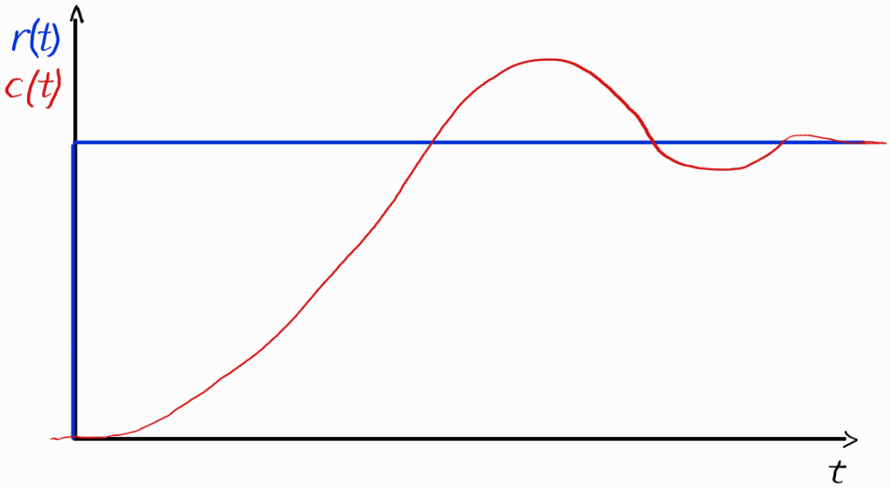
$$G_R(S) = \frac{\frac{K_I \, K}{S(T_1 S + 1)}}{1 + \frac{K_I \, K}{S(T_1 S + 1)}} = \frac{K_I \, K}{T_1 S^2 + S + K_I K} = \frac{K_I K / T_1}{S^2 + \underbrace{\frac{1}{T_1}}_{2Dw_0} S + \underbrace{\frac{K_I K}{T_1}}_{w_0^2}}$$

PT$_2$ - System

$$w_0 = \sqrt{\frac{K_I K}{T_1}}$$

$$2D\sqrt{\frac{K_I K}{T_1}} = \frac{1}{T_1} \Rightarrow D = \frac{1}{2\sqrt{T_1 K_I K}}$$

$K_I \uparrow \Rightarrow D \downarrow$

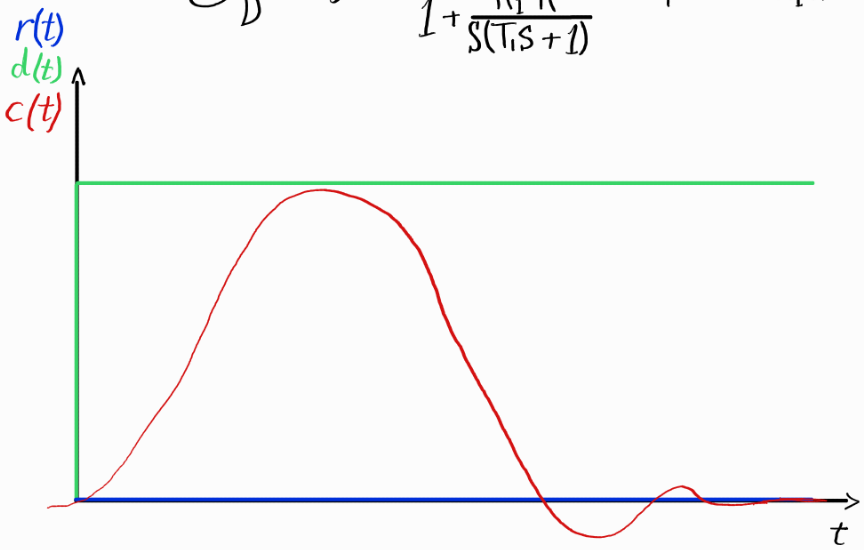


if we set $s=0$
the gain $G_R(s) = 1$
(which is good)

Disturbance TF:

$$G_D(S) = \frac{\frac{K}{T_1 S + 1}}{1 + \frac{K_I \, K}{S(T_1 S + 1)}} = \frac{K \cdot S}{T_1 S^2 + S + K_I K}$$

DT$_2$ - System



good that disturbance TF is going to zero.