British Algorithmic Olympiad

Round 1 Report 2022

5th January 2023

1 Binary Rotors

Part A

This question threw some candidates due to the large constraints involved. The problem statement itself required some deciphering due to a range of formulae and new ideas being introduced. Full marks were gained by candidates towards the top 50% of students.

Part B

A surprising number of candidates failed to answer this question correctly, possibly due to skipping question 1 entirely. It is clear that many candidates realised that squaring a term has no effect unless the term can be -1. One approach was to write a program to run each option, but for only 1 mark, a collection of simple truth tables was sufficient.

Part C

This question was to test candidates' problem solving techniques. There are around 3×10^{13} different combinations of shift values and hence this problem cannot be solved by brute force within a reasonable time frame.

The approach was to look at smaller test cases. Let u_n denote the number of check strings consisting of an even number of 1s with length n. For $2 \leq n \leq 10$, we can use part **A**:

$$\bigcup_{n=2}^{10} u_n = \{1,\ 3,\ 6,\ 10,\ 15,\ 21,\ 28,\ 36,\ 45\}$$

It is possible to deduce that, for $n > 1$:

$$u_{n+1} = u_n + n.$$

We can now calculate the value of the sequence at $n = 2^{23}$, computable by writing a simple program. Alternatively, we could simplify the recurrence relation:

$$\begin{aligned} u_n &= u_{n-1} + (n-1) \\ &= u_{n-2} + (n-2) + (n-1) \\ &= \cdots \\ &= u_2 + 2 + 3 + \cdots + (n-1) \\ &= 1 + \sum_{r=2}^{n-1} r \\ &= \tfrac{1}{2} n(n-1) \end{aligned}$$

We could now simply substitute 2^{23} for n.

Part D

This question looked more challenging than it turned out to be. Once candidates had recognised that the series is circular with a period of 6, the most challenging part of this question was splitting up the infinite series into 6 geometric series, with one being equal to zero. Candidates were required to recall the formula for the sum of an infinite geometric series.

2 Sonar

Part A

This question was well answered by many candidates. After reading the problem statement, the question could be heavily abstracted to become a simpler problem. A maximum of $O(n^2)$ time complexity was required to gain full marks.

Part B

Most candidates that had at least partially solved part **A** were able to gain full marks.

Part C

This was statistically one of the hardest problems on the exam paper. Candidates that had achieved 4 out of 5 marks did not see the single right angle triangle located in the bottom left of the grid. This triangle was evident in the example given. A combinatorial or recursive approach was rewarded.

3 Pairs and Triples

Part A
This question was well answered.

Part B
Statistically quite difficult, this question required exploration and a clear logical process. We knew that $a^2 + b^2 = c^2$. This can be rearranged to $a^2 = c^2 - b^2$. We have

$$a^2 = (c-b)(c+b) \tag{1}$$

We can rearrange (1) to achieve

$$\frac{c+b}{a} = \frac{a}{c-b} = \frac{m}{n}$$

and hence

$$\frac{c-b}{a} = \frac{n}{m}$$

We can now find

$$\frac{2b}{a} = \frac{c+b}{a} - \frac{c-b}{a} = \frac{m}{n} - \frac{n}{m}$$

to reach

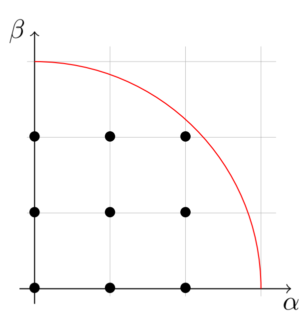
$$\frac{b}{a} = \frac{1}{2}\left(\frac{m}{n} - \frac{n}{m}\right) = \frac{m^2 - n^2}{2mn}$$

$$\frac{c}{a} = \frac{1}{2}\left(\frac{m}{n} + \frac{n}{m}\right) = \frac{m^2 + n^2}{2mn}$$

We know from part **A** that $m^2 - n^2$ will always be odd or a multiple of 4. Hence, $m^2 - n^2$ and $2mn$ will either have a greatest common divisor of 1 (if $m^2 - n^2$ is odd) or 2 (using the multiple of 4 case from part **A**). If $m^2 - n^2$ has that factor of 2, then it must have a factor of 4 (from part **A**). This would mean that b is even, a contradiction as the question states to assume b is odd.

Hence, $\gcd(m^2 - n^2, 2mn) = 1$ so $a = 2mn$, $b = m^2 - n^2$ and $c = m^2 + n^2$.

Part C
The large constraints of this question should have indicated that an $O(n)$ solution is not viable to gain full marks. We can note that the given expression $\alpha^2 + \beta^2 < n$ is very similar to the equation of a circle centred at the origin, $x^2 + y^2 = r^2$. For this problem, we're interested in the integer points inside this circle of radius $\sqrt{n}$. For $n = 9$:



There are a total of 9 different pairs (α, β) such that $\alpha^2 + \beta^2 < n$. It is possible to write an $O(\sqrt{n})$ program which will gain full marks.

Part D

This was statistically the hardest problem on the exam paper. This question was solved by a small number of candidates. As $n \to \infty$, $c(n)$ will closely approximate the area inside a quarter of the circle. We know the circle has an area of $n\pi$, therefore the area of the top right quadrant is $\frac{1}{4}n\pi$.

Hence

$$\lim_{n \to \infty} \left(\frac{c(n)}{n} \right) = \frac{\frac{1}{4}n\pi}{n} = \frac{\pi}{4}$$

Part E

This question was solved by candidates that had solved parts **C** and **D**. Caution was required when considering the constraints on α and β.

4 Wolves

Part A
This was the toughest programming question on the exam paper, but many candidates still achieved some marks. A longest increasing sub-sequence algorithm proved to be useful when reaching the higher test cases.

Part B
Candidates were not required to gain full marks in part **A** for this problem. Many gained full marks.

Part C
This proved to be a difficult problem. Many students did not reach this question in time. The problem style allowed any valid interpretation of a given code. There were some more straightforward ways to interpret it, but more complex approaches were still rewarded. There was no defined correct answer in the mark scheme.