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Exercise 3

Task 1 - Proofs

a) Show that: $\|\mathcal{T}^\pi v - \mathcal{T}^\pi v'\|_\infty \leq \gamma \|v - v'\|_\infty$

per definition:
$\|\mathcal{T}^\pi v - \mathcal{T}^\pi v'\|_\infty = \max_s |(\mathcal{T}^\pi v)(s) - (\mathcal{T}^\pi v')(s)| \rightarrow$ insert equation given on exercise sheet

$$
\begin{aligned}
&= \max_s \left| \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)[r + \gamma v(s')] - \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)[r + \gamma v'(s')] \right| \\
&= \max_s \left| \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)[r + \gamma v(s') - r - \gamma v'(s')] \right| \\
&= \gamma \max_s \left| \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)[v(s') - v'(s')] \right| \\
&\leq \gamma \max_s \left| \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a) \max_s |[v(s') - v'(s')]| \right| \\
&= \gamma \max_s |v(s) - v(s')| \\
&= \gamma \|v - v'\|_\infty
\end{aligned}
$$

b)

Task 2 - Value Iteration