

Security and Privacy, Blatt 3

Franziska Hutter (3295896)

Felix Truger (3331705)

Felix Bühler (2973410)

13. Juni 2018

Problem 1: Sum of negligible functions

Definition: v is negligible $\implies \exists N \in \mathbb{N}$ such that $\forall n > N$ and for all positive polynomials p : $v(n) < \frac{1}{p(n)}$

v and v' negligible: $\exists N_1, N_2 \in \mathbb{N}$, such that:

$$\begin{aligned} \forall n > N_1 : v(n) &< \frac{1}{p(n)} \\ \forall n > N_2 : v'(n) &< \frac{1}{p(n)} \end{aligned}$$

(by Definition)

Let $w(n) = v(n) + v'(n)$: For w to be negligible, we need an $N_3 \in \mathbb{N}$, such that $\forall n > N_3 : w(n) < \frac{1}{p(n)}$. We conclude from the above, that $\forall n > (N_1 + N_2) : v(n) + v'(n) < \frac{1}{p(n)} + \frac{1}{p(n)}$, Thus $N_3 = N_1 + N_2 \implies \forall n > N_3 : w(n) < \frac{2}{p(n)}$.

Since we're looking at the inverses of *all* positive polynomials, we can easily generate $\frac{1}{p(n)}$ from $\frac{2}{p(n)}$ by multiplying $p(n)$ by 2, which makes just another polynomial $2p(n)$, at which we are looking anyways. This means, that also $\forall n > N_3 : w(n) < \frac{1}{p(n)}$ holds for all positive polynomials p .

Problem 2: Deterministic verifier in IPS

V deterministic $\implies V$ does not use any randomness. Thus the output of V for a given input (i.e. under same conditions) is always the same. That means, that one can think of a prover P' that just replays the messages from P to V . By definition there are only polynomially many messages. Thus a deterministic P' can be constructed to convince V .

Witnesses:

$x \in L$: The witness w consists just of the messages, which P' has to send to V such that V accepts on (x, w) . This witness must exist for $x \in L$, because otherwise the probability for V to accept would be 0 (in contradiction to the definition of IPS).

$x \notin L$: On the other hand there can not be a witness that leads to V accepting if $x \notin L$. Otherwise V would always accept when w is used, making the probability for V to accept 1, which again contradicts the definition of IPS.

This shows us for $L \in \mathcal{IP}$ if V is a deterministic verifier for L , that $L \in \mathcal{NP}$ holds.

Problem 3: Anonymous credentials and IPS

2. Give an IPS for L and prove its properties

IPS (P, V) . Protocol on input c :

| Prover P | Verifier V |
|--|---|
| Compute $name, age, residence, k, r$ and $x = \text{"Name: } name, \text{ Age: } age, \text{ Residence: } residence\text{"}$ such that $c = E(x, k, r) \wedge age \geq 18$ | Compute $x' = \text{"Name: } name, \text{ Age: } age, \text{ Residence: } residence\text{"}$ and $c' = E(x', k, r)$ if $(c' == c \ \&\& \ age \geq 18)$ then output 1 else output 0. |

Proof of properties:

- **Correctness:** It is obvious that for any $c \in L$, V will accept with probability 1, because V does not use any randomness and directly receives all necessary information from P to conclude $c \in L$.
Thus $Pr[\langle P, V \rangle(c) = 1 \mid c \in L] = 1 \geq \frac{2}{3}$.
- **Soundness:** $\forall c' \notin L$ and \forall ITMs P^* : Since V is directly checking the properties for c' itself, there is obviously no chance that any prover would fool V on any $c' \notin L$ to accept upon such a c' .
Thus $Pr[\langle P, V \rangle(c') = 1 \mid c' \notin L] = 0 \leq \frac{1}{3}$

1. Show: $L \in \mathcal{NP}$

In the above protocol we see, that V is deterministic (i.e. V does not use any randomness and always has the same output under same conditions). Furthermore $\langle P, V \rangle$ is an IPS for L . As we know from problem 2, these are exactly the conditions for $L \in \mathcal{NP}$.

Problem 4: Equivalent definition of computational ZK

Problem 5: Reducing the error probability 1

