Security and Privacy, Blatt 5

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10. Juli 2018

Problem 1: Schnorr's protocol - special honest verifier zero-knowledge

It is easy to see, that (P, V) as given for Schnorr's protocol has the form of a Σ-protocol with commitment a, challenge e and response z. The "special honest verifier ZK" property requires: $\exists$ ppt simulator M such that $\forall x \in L_R$ and $e \in \{0,1\}^t : M(x,e) = Trans^P_{V^e}(x)$ where $Trans^P_{V^e}(x)$ is the Transcript of an interaction between (honest) P and V using challenge e on input x and equality refers to an identical distribution.

M works as follows:

- Receive inputs $(\mathcal{G}, q, g, h)$ and e.
- Select $z \in \mathbb{Z}_q$ randomly.
- Calculate $a = \frac{g^z}{h^e} \mod q$.
- Output transcript (a, e, z).

On the calculation of a:
We know from the protocol, that P calculates z as follows:

$$z = r + e \cdot w \mod q$$

Since $\mathcal{G}$ is cyclic with generator g:

$$g^z = g^{r+e\cdot w \mod q} = g^r \cdot g^{e\cdot w \mod q}$$

We also know:

$$g^r = a; g^w = h$$
$$\implies g^z = a \cdot h^e \mod q$$
$$a = \frac{g^z}{h^e} \mod q$$

As for the probability distribution of a transcript of (P, V^e), we know that $r \in \mathbb{Z}_q$ is selected randomly by the honest prover and thus $a = g^r$ is indirectly selected randomly as well with the same distribution as for r. Note again, that $\mathcal{G}$ is cyclic. z on the other hand directly depends on the other values and can be calculated by a deterministic function, as it is actually performed by P.

We have the same probability distribution for M, as for M $z \in \mathbb{Z}_q$ is selected at random. a is then deterministically calculated from e and z. Just the other way around.
More precisely we have the same probability space $(\Omega, 2^\Omega, P)$ in both cases, where

- $\Omega = \mathbb{Z}_q$
- $P : 2^\Omega \to [0, 1]$ is a uniform distribution.

It follows from the above, that Schnorr's protocol fulfills the "special honest verifier ZK" property.

Problem 2: Homomorphic properties of algorithms

$p = (\mathcal{G}, q, g, h)$ fixed, $r_0, r_1, v_0, v_1 \in \mathbb{Z}_q$, we show:

$$com^{r_0+r_1}(p, v_0 + v_1) \stackrel{!}{=} com^{r_0}(p, v_0) \cdot com^{r_1}(p, v_1)$$

By just applying the definition of com in the Pedersen commitment scheme, we know:

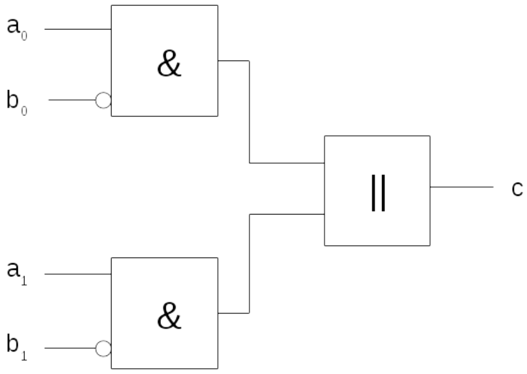
$$com^{r_0+r_1}(p, v_0 + v_1) = g^{v_0+v_1} \cdot h^{r_0+r_1}$$
$$com^{r_0}(p, v_0) = g^{v_0} \cdot h^{r_0}$$
$$com^{r_1}(p, v_1) = g^{v_1} \cdot h^{r_1}$$

Thus:

$$\begin{aligned} com^{r_0}(p, v_0) \cdot com^{r_1}(p, v_1) &= g^{v_0} \cdot h^{r_0} \cdot g^{v_1} \cdot h^{r_1} \\ &= g^{v_0} \cdot g^{v_1} \cdot h^{r_0} \cdot h^{r_1} \\ &= g^{v_0+v_1} \cdot h^{r_0+r_1} \\ &= com^{r_0+r_1}(p, v_0 + v_1) \end{aligned}$$

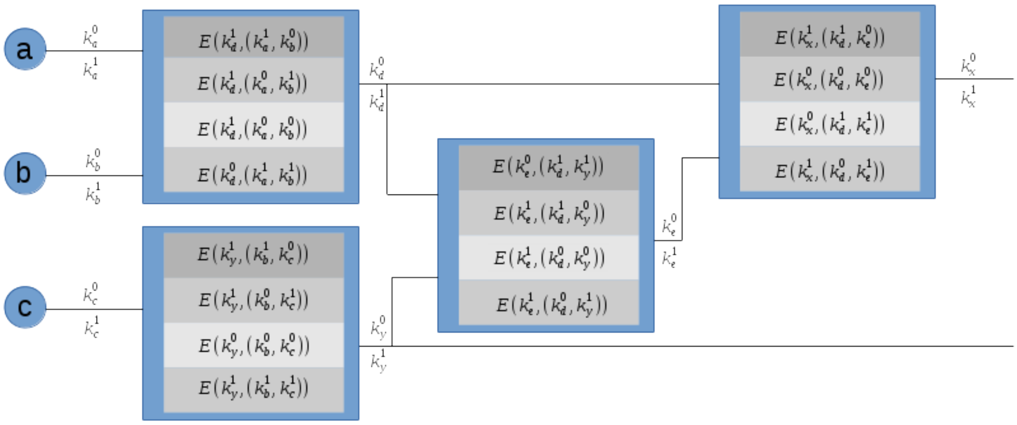
Problem 3: Building circuits for functions

Find the drawing of our circuit for f below.



Problem 4: Garbled circuits

Find a garbled circuit for the given circuit below.



Problem 5: 51%-Attack on Bitcoin

Preliminary thoughts before the actual lecture on the topic:

(a)

With 49% of the hash-power a block is found within 20 minutes on average, i.e. 3 blocks per hour. Thus 51% of the hash-power can find $\frac{3}{49\%} \cdot 51\% \approx 3.12245$ blocks per hour. The attacker needs to find 6 blocks more than the honest part of the network in the same time. This is: $t \cdot 0.12245 = 6$, which leads to $t \approx 49$ hours needed to catch up with the honest branch.

(b)

In the calculated 49 hours the attacker generates $3.12245 \cdot 49 \approx 153$ new blocks. Obviously the difference between the earnings and costs for finding a block is $100,000€ - 12.5 \cdot 6,000€ = 25,000€$. Thus the attacker would need to make an additional $25,000€ \cdot 153 = 3,825,000€$ via double-spending to compensate his costs. Under the given circumstances $3,825,000€$ were worth approximately 637.5₿. To make a profit, the attacker would need to double-spend at least a bit more than the aforementioned amount.

(c)

No idea so far.