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Engineering Notes

Modified Command to Line-of-Sight Intercept Guidance for Aircraft Defense

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I. Introduction

PROTECTING an aircraft from missile attack is a challenging issue. Most missile guidance algorithms [1–11] use information basically about the missile and the target *but not that* of the protected system. Considering only the kinematics with the aid of the collision triangle, Boyell [12] calculated a minimum range to intercept the attacking missile over a specified distance to protect a moving aircraft/torpedo and obtained a closed-form expression of the range for the missile to be successful against attacking missiles. Shneydor [13] developed conditions for the applicability of Boyell's technique and simplified the expression for the operational range for the defense missile. However, those analyses were based on an ideal guidance assuming a perfect collision course. Rusnak [14] applied the differential game theory (DGT) to such a three-player game, a protected aircraft, an attacking missile, and a defense missile, by changing the three-player game into a two-team game by way of grouping the same cooperative players: a protected aircraft and a defensive missile. Rusnak [14] showed that the resulting expression of the guidance law was represented by two line-of-sight (LOS) rate terms multiplied by variable gains derived from the differential game theory. One of the two LOS rates is the LOS rate of the aircraft with respect to the attacking missile, and the other is the LOS rate of the attacking missile with respect to the defense missile. The Rusnak method demands the recursive backward computations from the end. Perelman et al. [15] further modified Rusnak's method to represent the two LOS rate terms in a closed form instead of using the recursive solution. Both simulation studies showed that the required acceleration could be reduced as compared with that of the conventional two-player differential game theory. Further recent work on such DGT can be found in [16]. Shima [17] proposed an optimal guidance methodology for a defense missile assuming prior knowledge of an attacking missile's guidance law. Shaferman and Shima [18] applied a multiple model adaptive estimator algorithm with the multiple model adaptive control for such a three-player game to estimate the attacking missile's guidance law.

The proposed work here takes a basic approach to the three-player guidance law, similar to the proportional navigation (PN), in the way that it uses geometrical information of the moving object, that is, the protected aircraft's LOS and the LOS rate. This type of guidance law can be classified as a three-point guidance [19–21]; examples are the beam rider (BR) guidance [2,19–21] and the command to LOS (CLOS) guidance [2,19–21]. The "three points" in the three-point guidance usually means a missile, a target, and a reference point from which the target LOS is drawn or the target is observed. In this study, the protected aircraft is selected as one of the three points instead of the reference point. Ratnoo and Shima [22] applied the CLOS guidance for aircraft protection. Their analysis shows that the defense missile requires less lateral acceleration (latax) than that of the attacking missile. They also proposed a guidance law for the protected aircraft that would help the defense missile against the target. However, the problem is that as the range of the target becomes larger, the resolution of the target LOS angle at a tracking point will be lower. This fact degrades the system performance or demands a high-resolution radar to track a moving target from a distance.

This Note proposes a different approach as compared with the BR or the CLOS guidance though it falls under the three-point guidance classification. The proposed approach manipulates two LOS rates associated with three vehicles and has benefits over the conventional three-point guidance law in two major aspects: 1) a simple form with one gain for two LOS rates, and 2) high sensitivity to an attacking missile's maneuvers in the proximity of the attacking missile, but low sensitivity to the attacking missile's maneuvers in the proximity of the protected aircraft. This novel guidance law, called the airborne-CLOS guidance (A-CLOSG) law, is derived using optimal control theory.

II. Problem Statement

To avoid confusion, a protected aircraft and its cooperative (friendly) missile are called a blue aircraft (BA) and a blue missile (BM), respectively, while an attacking missile is called a red missile (RM) or a target in the rest of this Note. Engagement geometry of the three vehicles (the BA, the BM, and the RM) is depicted in Fig. 1. As can be seen in Fig. 1, the three vehicles' geometry forms a triangle. The three interior angles in the triangle are denoted by ϕ_{BA}, ϕ_{BM}, and ϕ_{RM} corresponding to the locations of the vehicles (the BA, the BM, and the RM).

To develop the basic guidance law, assumptions are made as follows:

1) The three vehicles are moving in a plane.
2) The autopilot and the seeker dynamics are perfect.
3) Velocities of the three vehicles' velocities are constant.
4) The BA and the RM are on a collision course.
5) Required information such as LOS and LOS rate is available.

The problem now is to find a guidance law with which the BM can intercept the RM for protecting the BA using the three-point guidance concept.

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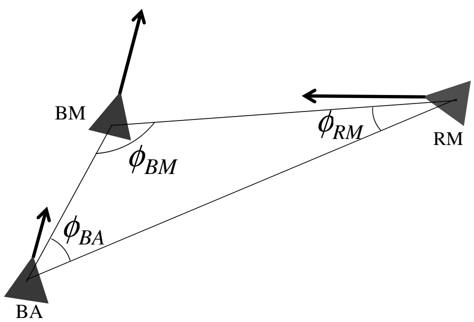


Fig. 1 Engagement geometry.

III. Development of a Guidance Law

Our goal is to protect the BA from the attacking RM by using the BM. The underlying notion for this note is that in a three-point guidance, the BM can protect the BA from the RM attack if the BM is always located inside the line segment between the BA and the RM. To achieve this, zeroing the angle, ϕ_{RM} among the three interior angles in Fig. 1 is considered to facilitate the BM to have high sensitivity to the RM's maneuvers in the proximity of the RM.

To develop the guidance law, required notations are defined in Fig. 2 using a 2-D plane. The origin of the coordinate system in Fig. 2 is fixed at the center of the BA. The x axis in Fig. 2 coincides with the LOS from the BA to the RM. For convenience, $\boldsymbol{V}^A$, $\boldsymbol{V}^D$, and $\boldsymbol{V}^{\mathrm{RM}}$ are drawn in Fig. 2, and they indicate the respective BA, BM, and RM velocity vectors in an inertial frame (not appearing in Fig. 2), whereas $\boldsymbol{V}_A^D$ and $\boldsymbol{V}_A^{\mathrm{RM}}$, respectively, indicate the BM and the RM velocity vectors with respect to the BA fixed frame. From the assumptions A3 and A4, the BA and the RM are on a collision course moving with a constant speed, and the y-axis velocity component of the RM is zero as expected from the vector $\boldsymbol{V}_A^{\mathrm{RM}}$ in Fig. 2. Therefore, the RM is moving along the x axis toward the origin, and the x axis does not rotate under the assumptions. The scalar distance η in Fig. 2 defines the error distance from the x axis (the LOS from the BA to the RM) to the BM. Our objective is to annul this error. To this end, the following optimal control problem is considered to yield a zero-miss distance with low-energy consumption:

$$I = \int_0^{t_f} u_C^2(t)\,\mathrm{d}t \tag{1}$$

subject to

$$\eta(t_f) = 0 \tag{2}$$

where, I is the performance index to be minimized, t_f is the final time of the engagement, and $u_C(t)$ denotes the BM's acceleration input that is the second time derivative of the error distance $\eta(t)$; $u_C(t) = \ddot{\eta}(t)$. Solution to this problem can easily be derived (see Bryson and Ho [5]) as

$$u_C(t) = -3\left(\frac{\dot{\eta}}{t_f - t} + \frac{\eta}{(t_f - t)^2}\right) \tag{3}$$

Equation (3) is transformed with the effective navigation constant of $N'(=3)$ as

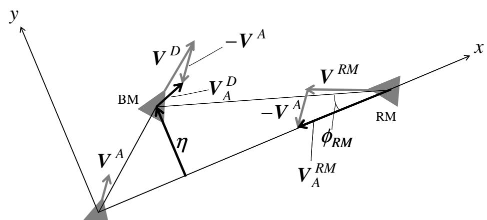


Fig. 2 Three vehicles' geometry in the BA fixed 2-D plane.

$$u_C(t) = -N'\left(\frac{\eta + \dot{\eta}(t_f - t)}{(t_f - t)^2}\right) \quad = -N'\frac{\mathrm{ZEM}_{\mathrm{LOS}}}{(t_f - t)^2} \tag{4}$$

where $\mathrm{ZEM}_{\mathrm{LOS}}$ denotes the zero-effort miss with respect to the x axis in Fig. 2. If the assumptions A3 and A4 (the BA and the RM are on a collision course moving with a constant speed) hold, the solution given by Eq. (4) is optimal. Assuming the value η is small, and letting V_C be the closing velocity between the BM and the RM, then

$$\phi_{\mathrm{RM}} \approx \frac{\eta}{V_C(t_f - t)} \tag{5}$$

Equation (5) leads to

$$u_C(t) = -N' V_C \dot{\phi}_{\mathrm{RM}} \tag{6}$$

where N' can be set as three in the perfect control and measurement environment. The following relation holds between ϕ_{RM} and the two LOS angles λ_{RA} and λ_{RD} defined in Fig. 3:

$$\phi_{\mathrm{RM}} = \lambda_{\mathrm{RA}} - \lambda_{\mathrm{RD}} \tag{7}$$

Taking the time derivative of Eq. (7) yields

$$\dot{\phi}_{\mathrm{RM}} = \dot{\lambda}_{\mathrm{RA}} - \dot{\lambda}_{\mathrm{RD}} \tag{8}$$

Substituting Eq. (8) for Eq. (6) completes the form of the A-CLOSG law as

$$u_C(t) = -N' V_C(\dot{\lambda}_{\mathrm{RA}} - \dot{\lambda}_{\mathrm{RD}}) \tag{9}$$

The LOS rate $\dot{\lambda}_{\mathrm{RA}}$ in Eq. (9) vanishes when the BA and the RM are on a collision course. Equation (9) can relax the assumption A4. In the case of the BA and the RM being on a collision course, that is, $\dot{\lambda}_{\mathrm{RA}} = 0$, then Eq. (9) represents the conventional PN guidance law exactly. Note that the LOS rate $\dot{\lambda}_{\mathrm{RA}}$ is expected to be small when the RM is far from the BA as compared with the distance between the BM and the RM. This means that the A-CLOSG law works similarly to the PN during the final part of the engagement.

The A-CLOSG law for the dynamical system, which has time delay, can be deducted easily by following the conventional technique for the PN navigation constant optimization process (see [2]).

IV. Kinematical Simulation

To investigate the performance of the A-CLOSG expressed as Eq. (9), a few simulations are made. In the simulations, the BM uses the A-CLOSG law of Eq. (9). On the other hand, the RM uses the PN guidance law to head toward the BA. In these simulations, the three vehicles' velocities (250 m/s for the BA; 800 m/s for the BM and the RM) are assumed to be constant, and the BM lateral accelerations are limited up to 50 g's.

Two sets of representative simulation results are presented here. One is called Case 1, where the BM has some initial heading error against the RM, while the RM heads toward the BA initially. This is most likely the case for an aircraft-launched defense missile. The other is called Case 2, where the BM has no heading error, while the RM, however, has a large initial heading error. This is most likely the case for a surface-to-air missile. Figures 4–6 show the Case 1

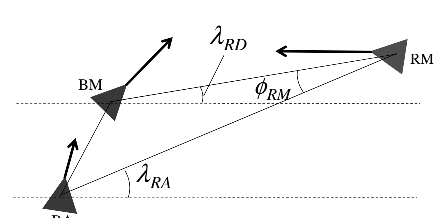
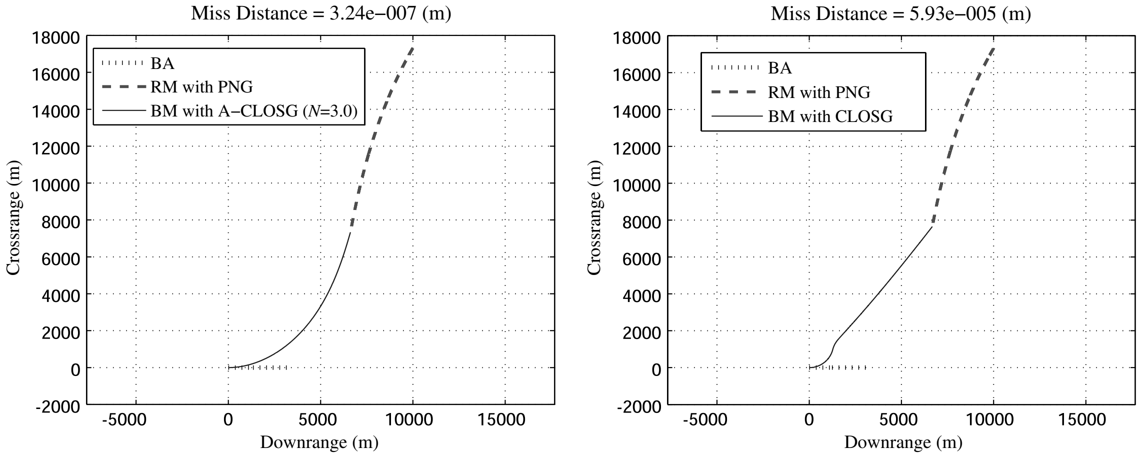
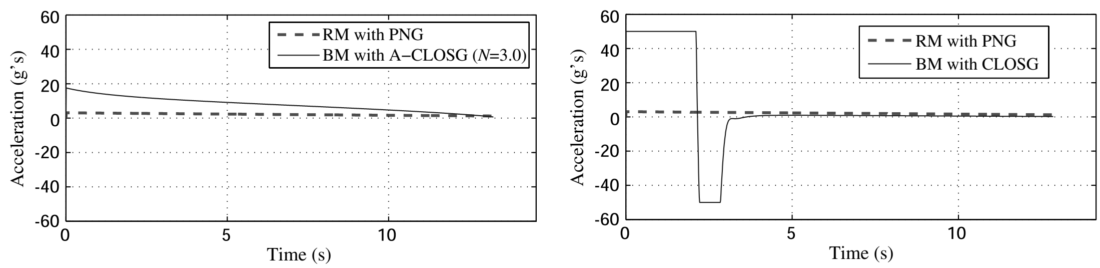


Fig. 3 Two LOS angles and one interior angle.



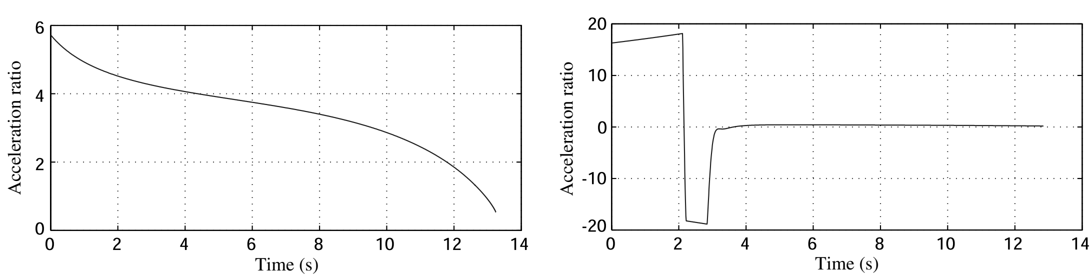
a) BM with A-CLOSG **b) BM with CLOSG**

Fig. 4 Flight trajectories of BA, BM, and RM (Case 1).



a) BM with A-CLOSG **b) BM with CLOSG**

Fig. 5 Lateral acceleration time histories (Case 1).



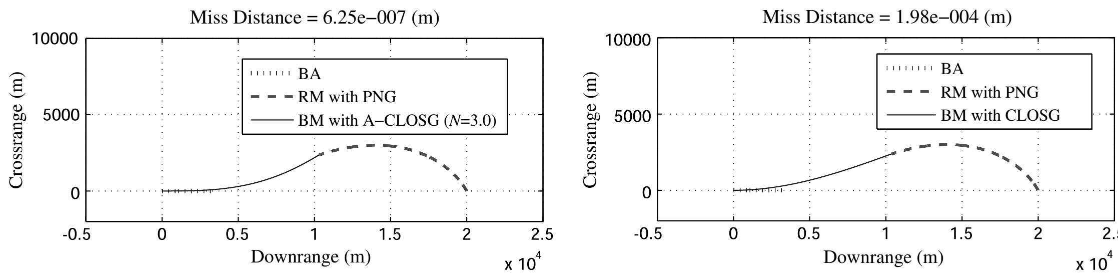
a) BM with A-CLOSG **b) BM with CLOSG**

Fig. 6 Lateral acceleration ratio vs time (Case 1).

simulation results. The labels a) and b) in these figures denote results using the A-CLOSG and the CLOS guidance (CLOSG), respectively. The gains for the CLOSG were set at the same values as in Ratnoo and Shima [22]. Figure 4 shows trajectories of the three vehicles in a 2-D plane, along with miss-distance values on top. The BA and the BM start from the origin, while the RM is launched 20 km away from the BA with an angle of 60 deg deviated from the BA heading in Fig. 4. Looking at the resulting miss distance on top of Figs. 4a and 4b, the BM with both guidance laws can intercept the RM successfully. The BM guided with the A-CLOSG gradually changes its flight path to the intercept point in a way that the flight path gradually tends to be straight, whereas the BM with the CLOSG seems to swiftly head toward the collision course, that is, after the flight-path change, the BM flight path is almost straight. From this point the CLOSG works effectively when the initial heading error is small enough. However, when having a large initial heading error like the case of this simulation, quicker change of the BM flight direction (making a large flight-path curvature as in Fig. 4b) demands high lateral acceleration. The high lateral acceleration drastically reduces the kinetic energy, which is one of the most essential factors to maintain maneuverability and extend reaching ability.

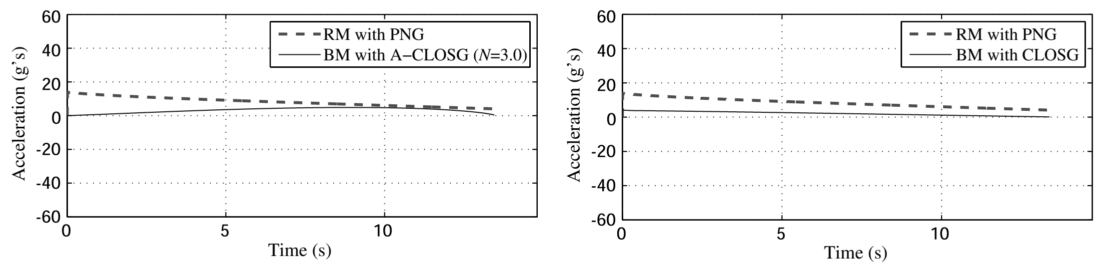
Figure 5 shows the lateral acceleration time histories. Figure 6 shows the latax ratio ($|\mathrm{BM\,latax}|/|\mathrm{RM\,latax}|$) with time. The BM guided with the CLOSG uses maximum acceleration at the beginning after launch to cancel the heading error. After that the initial phase, less acceleration is used, as shown in Figs. 5b and 6b. On the other hand, the BM with the A-CLOSG seems to use less lateral acceleration in the way that the required lateral acceleration decreases gradually, as shown in Figs. 5a and 6a. It should be noted that the maneuver advantage with A-CLOSG is far better than with CLOSG.

In Case 2 where the RM is launched 20 km ahead of the BA with a heading error of 60 deg, and the BM has no initial heading error, assumption A4 is violated, which means that the BA and the RM are not on a collision course. Therefore, the A-CLOSG law is not optimal for Case 2. Figure 7 shows three vehicles' trajectories. The BM with both guidance laws can intercept the RM successfully (see the miss distance on top in Figs. 7a and 7b). Similar to the previous simulation results, the BM guided with the A-CLOSG gradually changes its



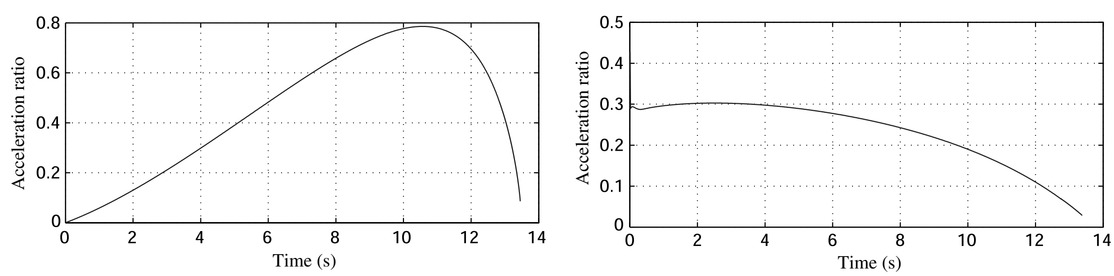
a) BM with A-CLOSG **b) BM with CLOSG**

Fig. 7 Flight trajectories of BA, BM, and RM (Case 2).



a) BM with A-CLOSG **b) BM with CLOSG**

Fig. 8 Lateral acceleration time histories (Case 2).



a) BM with A-CLOSG **b) BM with CLOSG**

Fig. 9 Lateral acceleration ratio vs time (Case 2).

flight path to the intercept point, whereas the BM trajectory made with the CLOSG seems to be almost straight toward the collision course after the initial flight-path change. Figure 8 shows the BM lateral acceleration time histories along with that of the RM. In both guidance laws, the BM with no initial heading error uses less latax compared with that of the RM. Figure 9 shows the latax ratio (|BM latax|/|RM latax|) time histories. In this case, the maneuver advantage of CLOG is almost the same as that of the A-CLOSG. It is interesting to note that the CLOSG simulation results satisfy the inequality in Ratnoo and Shima [22] (which shows that the maximum BM latax is less than a third of the maximum of the RM latax).

Comparing the number of tuning parameters, *only one gain needs to be tuned* for the A-CLOSG, whereas the CLOSG requires three. Furthermore, only two LOS rates are used in the A-CLOSG, whereas the CLOSG requires two angles and their first and second time derivatives (or velocity and acceleration of the RM). From the implementation aspect for aircraft defense, this means that the BA may need relatively more sophisticated sensing equipment to implement CLOSG.

V. Conclusions

The problem of defending a moving aircraft against an attacking missile was considered in this study. For aircraft defense, a novel guidance law, which uses two line-of-sight rates, was developed. The simulation results show that the new guidance law has desirable kinematic and performance characteristics. From the results obtained so far, it appears that the proposed guidance law has excellent potential for use in a defense missile.

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