Práctica 6

2do cuatrimestre 2021 Álgebra I

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6. Práctica 6

6.1. Ejercicio 1

6.1.A. Pregunta i

Paso a polares:

- $5i = 5(\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}))$
- $(1+i)^4 = (\sqrt{2})^4.(\cos(\pi) + i\sin(\pi))$

Luego,

$$\begin{aligned} z &= 4.5(\cos(\pi + \frac{\pi}{2}) + i\sin(\pi + \frac{\pi}{2})) \\ &= 20(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2})) \end{aligned}$$

Así,

- $Re(z) = 20.\cos(\frac{3\pi}{2}) = 0$
- $Im(z) = 20.\sin(\frac{3\pi}{2}) = -20$
- $|z| = 20$
- $Re(z^{-1}) = 0$
- $iz = 20(\cos(2\pi) + i\sin(2\pi)) \implies Im(iz) = 0$

6.1.B. Pregunta ii

$$\begin{aligned} z &= (\sqrt{2} + \sqrt{3}i)^2.(\overline{1-3i}) \\ &= (2 + 2\cdot\sqrt{2}\cdot\sqrt{3}i - 3).(1+3i) \\ &= (-1 + 2\cdot\sqrt{6}i).(1+3i) \\ &= -1 - 3i + 2\cdot\sqrt{6}i - 6\cdot\sqrt{6} \\ &= -1 - 6\cdot\sqrt{6} + (2\cdot\sqrt{6} - 3)i \end{aligned}$$

- $Re(z) = -1 - 6\cdot\sqrt{6}$
- $Im(z) = 2\cdot\sqrt{6} - 3$
- $|z| = \sqrt{(-1-6\cdot\sqrt{6})^2 + (2\cdot\sqrt{6}-3)^2} = \sqrt{250} = 5\cdot\sqrt{10}$

6.1.C. Pregunta iii

Paso a polares,

$i^{17} = \cos(17.\frac{\pi}{2}) + i\sin(17\frac{\pi}{2})$

$\frac{1}{2} = \frac{1}{2}.(\cos(0) + i\sin(0))$

$i = \cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2})$

$(1-i)^3 = (\sqrt{2})^3(\cos(3.\frac{7}{4}\pi) + i\sin(3.\frac{7}{4}\pi))$

Luego,

$$\begin{aligned}
\frac{1}{2}.i.(1+i)^3 &= \frac{1}{2}.(\sqrt{2})^3.\left(\cos(\frac{\pi}{2}+\frac{21}{4}\pi)+i\sin(\frac{\pi}{2}+\frac{21}{4}\pi)\right)\\
&= \frac{\sqrt{2}^3}{2}.\left(\cos\left(\frac{23}{4}\pi\right)+i\sin\left(\frac{23}{4}\pi\right)\right)
\end{aligned}$$

Entonces,

$$\begin{aligned}
z &= \frac{\sqrt{2}^3}{2}.\left(\cos\left((\frac{17}{2}+\frac{23}{4})\pi\right)+i\sin\left((\frac{17}{2}+\frac{23}{4})\pi\right)\right)\\
&= \frac{\sqrt{2}^3}{2}.\left(\cos\left(\frac{57}{4}\pi\right)+i\sin\left(\frac{57}{4}\pi\right)\right)
\end{aligned}$$

- $Re(z) = \frac{\sqrt{2}^3}{2}.\cos\left(\frac{57}{4}\pi\right)$
- $Im(z) = \frac{\sqrt{2}^3}{2}.\sin\left(\frac{57}{4}\pi\right)$
- $|z| = \frac{\sqrt{2}^3}{2}$
- $Re(z^{-1}) = \cos\left(\frac{57}{4}\pi\right)$
- $Im(iz) = \frac{\sqrt{2}^3}{2}.\sin\left(\frac{59}{4}\pi\right)$

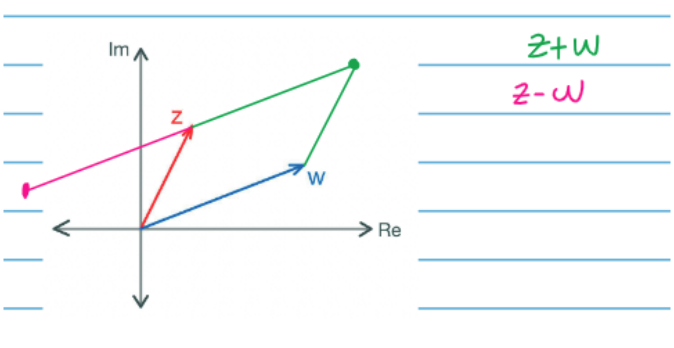
6.1.D. Pregunta iv

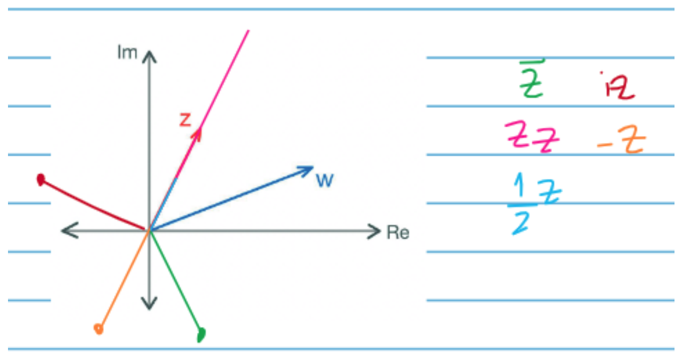
TODO

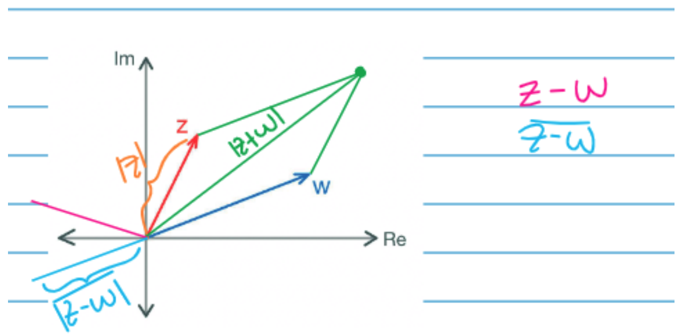
6.1.E. Pregunta v

TODO

6.2. Ejercicio 2

1. 

2. 

3. 

6.3. Ejercicio 3

6.3.A. Pregunta i

$z^2 = -36$

Se que $z = a + bi$ con $(a, b) \in \mathbb{R}^2$

Luego busco los z tales que $z^2 = -36$

$$\begin{aligned} z^2 = -36 \iff z^2 &= (a + bi)^2 \\ &= a^2 - b^2 + 2abi \end{aligned}$$

También se que el módulo debe ser igual $|z^2| = |-36|$,
$|z^2| = |z|^2 = (\sqrt{a^2 + b^2})^2 = a^2 + b^2$
$|-36| = 36$

Usando la igualdad de números complejos,

$$\begin{cases} a^2 - b^2 = -36 \\ 2ab = 0 \\ a^2 + b^2 = 36 \end{cases}$$

Sumando (1) y (3) $2a^2 = 0 \iff a = 0$

Restando (1) a (3) $2b^2 = 2.36 \iff b = \pm 36$

Luego $z = a + bi$ con los valores de a y b hallados resulta en

Rta.: $z_1 = 6i; z_2 = -6i$

6.3.B. Pregunta ii

$z^2 = i$

Se que $z = a + bi$ con $(a, b) \in \mathbb{R}^2$

También se que el módulo debe ser igual $|z^2| = |i|$,

$|z^2| = |z|^2 = (\sqrt{a^2+b^2})^2 = a^2 + b^2$
$|i| = 1$

Usando la igualdad de números complejos,

$$\begin{cases} a^2 - b^2 = 0 \\ 2ab = 1 \\ a^2 + b^2 = 1 \end{cases}$$

Sumando (1) y (3) $2a^2 = 1 \iff a = \pm\frac{1}{\sqrt{2}}$

Restando (1) a (3) $2b^2 = 1 \iff b = \pm\frac{1}{\sqrt{2}}$

Usando (2) se que $2ab > 0 \iff (a > 0 \wedge b > 0) \vee (a < 0 \wedge b < 0)$

Rta.: $z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i; z_2 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

6.3.C. Pregunta iii

$z^2 = 7 + 24i$

Se que $z = a + bi$ con $(a,b) \in \mathbb{R}^2$

También se que el módulo debe ser igual $|z^2| = |7 + 24i|$,

$|z^2| = |z|^2 = (\sqrt{a^2+b^2})^2 = a^2 + b^2$

$|7 + 24i| = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = 25$

Usando la igualdad de números complejos,

$$\begin{cases} a^2 - b^2 = 7 \\ 2ab = 24 \\ a^2 + b^2 = 25 \end{cases}$$

Sumando (1) y (3) $2a^2 = 32 \iff a = \pm 4$

Restando (1) a (3) $2b^2 = 18 \iff b = \pm 3$

Usando (2) se que $2ab > 0 \iff (a > 0 \wedge b > 0) \vee (a < 0 \wedge b < 0)$

Rta.: $z_1 = 4 + 3i; z_2 = -4 - 3i$

6.3.D. Pregunta iv

$z^2 + 15 - 8i = 0 \iff z^2 = -15 + 8i$

Se que $z = a + bi$ con $(a,b) \in \mathbb{R}^2$

También se que el módulo debe ser igual $|z^2| = |-15 + 8i|$,

$|z^2| = |z|^2 = (\sqrt{a^2+b^2})^2 = a^2 + b^2$

$|-15 + 8i| = \sqrt{(-15)^2 + 8^2} = \sqrt{225 + 64} = 17$

Usando la igualdad de números complejos,

$$\begin{cases} a^2 - b^2 = -15 \\ 2ab = 8 \\ a^2 + b^2 = 17 \end{cases}$$

Sumando (1) y (3) $2a^2 = 2 \iff a = \pm 1$

Restando (1) a (3) $2b^2 = 16 \iff b = \pm 4$

Usando (2) se que $2ab > 0 \iff (a > 0 \wedge b > 0) \vee (a < 0 \wedge b < 0)$

Rta.: $z_1 = 1 + 4i; z_2 = -1 - 4i$

6.4. Ejercicio 4

6.4.A. Pregunta i

$z = (2+2i)(\sqrt{3}-i)$

Busco la forma polar de cada factor.

$$2+2i = \sqrt{8}.e^{\frac{\pi}{4}i}$$
$$\sqrt{3}-i = 2.e^{\frac{11}{6}\pi i}$$

Por DeMoivre,

$$\begin{aligned} z &= (2+2i)(\sqrt{3}-i) \\ &= \sqrt{8}.2.e^{\frac{\pi}{4}i+\frac{11}{6}\pi i} \\ &= 2\cdot\sqrt{8}e^{\frac{1}{12}\pi i} \end{aligned}$$

Luego,

- $|z| = 4\cdot\sqrt{2}$
- $\theta = \frac{1}{12}\pi$

6.4.B. Pregunta ii

$z = (-1+\sqrt{3}i)^5$

$$\begin{aligned} -1+\sqrt{3}i &= 2\cdot e^{(\pi-\frac{1}{3}\pi)i} \\ &= 2\cdot e^{\frac{2}{3}\pi i} \end{aligned}$$

Luego,

$$\begin{aligned} z &= (-1+\sqrt{3}i)^5 \\ &= (2\cdot e^{\frac{2}{3}\pi i})^5 \\ &= 2^5\cdot e^{\frac{10}{3}\pi i} \end{aligned}$$

Por lo tanto,

- $|z| = 2^5$
- $\theta = \frac{4}{3}\pi$

6.4.C. Pregunta iii

$z = (-1+\sqrt{3}i)^{-5}$

$$\begin{aligned} z &= (-1+\sqrt{3}i)^{-5} \\ &= (2\cdot e^{\frac{2}{3}\pi i})^{-5} \\ &= 2^{-5}\cdot e^{\frac{-10}{3}\pi i} \end{aligned}$$

Por lo tanto,

- $|z| = \frac{1}{2^5}$
- $\theta = \frac{2}{3}\pi$

6.4.D. Pregunta iv

$z = \frac{1+\sqrt{3}i}{1-i}$

Busco las expresiones polares.

$$1 + \sqrt{3}i = 2.e^{\frac{\pi}{3}i}$$
$$1 - i = \sqrt{2}.e^{\frac{7}{4}\pi i}$$

Luego,

$$\begin{aligned} z &= \frac{1+\sqrt{3}i}{1-i} \\ &= \frac{2}{\sqrt{2}} \cdot e^{(\frac{1}{3}-\frac{7}{4})\pi i} \\ &= \frac{2}{\sqrt{2}} \cdot e^{\frac{-17}{12}\pi i} \end{aligned}$$

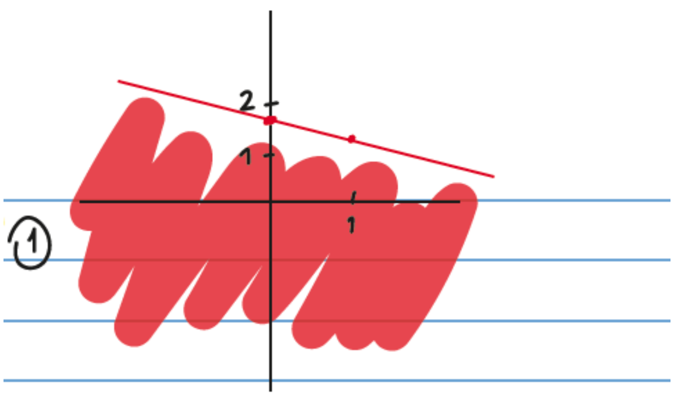
Por lo tanto,

- $|z| = \frac{2}{\sqrt{2}}$
- $\theta = \frac{7}{12}\pi$

6.5. Ejercicio 5

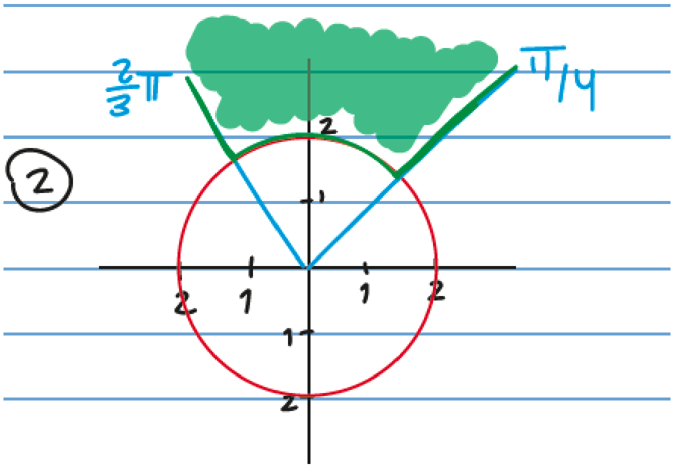
6.5.A. Pregunta i

$Re(z) = x \wedge Im(z) = y \implies x + 5y \leq 8 \iff y \leq -\frac{1}{5}x + \frac{8}{5}$

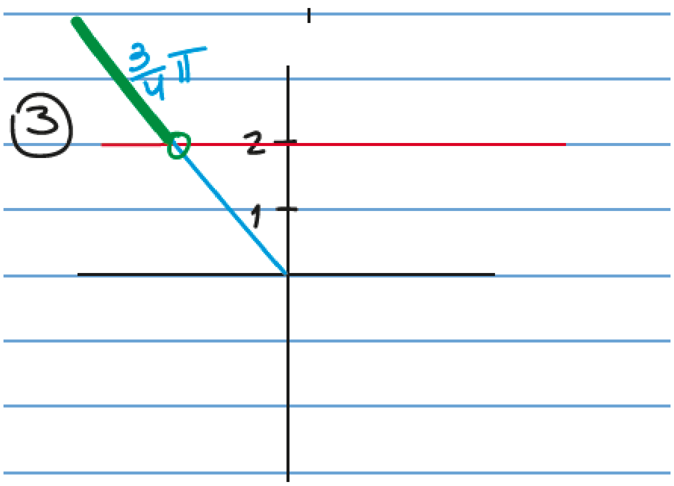


6.5.B. Pregunta ii

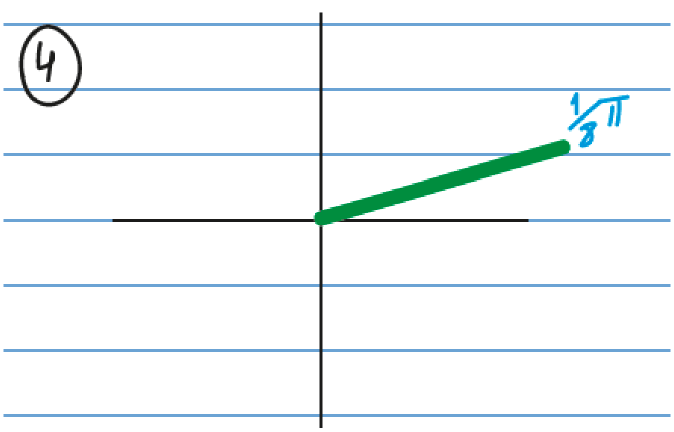
- $|z| = 2$ define una circunferencia de radio 2.
- $\frac{\pi}{4} \leq arg(z) \leq \frac{2\pi}{3}$ define un arco de angulo barrido.



6.5.C. Pregunta iii



6.5.D. Pregunta iv



6.6. Ejercicio 6

6.6.A. Pregunta i

$z = (\frac{1+\sqrt{3}i}{1-i})^{17}$

Sea $w = \frac{1+\sqrt{3}i}{1-i}$

Busco las expresiones polares.

$$\begin{aligned} 1+\sqrt{3}i &= 2.e^{\frac{\pi}{3}i} \\ 1-i &= \sqrt{2}.e^{\frac{7}{4}\pi i} \end{aligned}$$

Luego,

$$\begin{aligned} w &= \frac{1+\sqrt{3}i}{1-i} \\ &= \frac{2}{\sqrt{2}} \cdot e^{(\frac{1}{3}-\frac{7}{4})\pi i} \\ &= \frac{2}{\sqrt{2}} \cdot e^{\frac{7}{12}\pi i} \end{aligned}$$

Por lo tanto,

$$\begin{aligned} z &= w^{17} \\ &= \left(\frac{2}{\sqrt{2}} \cdot e^{\frac{7}{12}\pi i}\right)^{17} \\ &= \left(\frac{2}{\sqrt{2}}\right)^{17} \cdot e^{\frac{17.7}{12}\pi i} \\ &= \left(\frac{2}{\sqrt{2}}\right)^{17} \cdot e^{\frac{23}{12}\pi i} \end{aligned}$$

Por lo tanto, $z = \left(\frac{2}{\sqrt{2}}\right)^{17} \cdot \cos(\frac{23}{12}\pi i) + \left(\frac{2}{\sqrt{2}}\right)^{17} \cdot \sin(\frac{23}{12}\pi i)$

6.6.B. Pregunta ii

$z = (-1+\sqrt{3}i)^n$

$-1+\sqrt{3}i = 2 \cdot e^{\frac{2}{3}\pi i}$

$(-1+\sqrt{3}i)^n = 2^n \cdot e^{\frac{2n}{3}\pi i}$

Luego $0 \leq \frac{2n}{3}\pi i < 2\pi \iff 0 \leq n < 3$

Por lo tanto,

- $n = 0 \implies z = 1$
- $n = 1 \implies z = 2 \cdot e^{\frac{2}{3}\pi i} = -1+\sqrt{3}i$
- $n = 1 \implies z = 4 \cdot e^{\frac{4}{3}\pi i} = -2 + 2 \cdot\sqrt{3}i$