Práctica 6

2do cuatrimestre 2021 Álgebra I

Integrante	LU	Correo electrónico
Yago Pajariño	546/21	ypajarino@dc.uba.ar

Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires
Ciudad Universitaria - (Pabellón I/Planta Baja)
Intendente Güiraldes 2610 - C1428EGA
Ciudad Autónoma de Buenos Aires - Rep. Argentina
Tel/Fax: (++54 +11) 4576-3300
http://www.exactas.uba.ar

Índice

6. Práctica 6

6.1. Ejercicio 1

6.1.A. Pregunta i

Paso a polares:

- $5i = 5(\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}))$
- $(1+i)^4 = (\sqrt{2})^4.(\cos(\pi) + i\sin(\pi))$

Luego,

$$\begin{aligned} z &= 4.5(\cos(\pi + \frac{\pi}{2}) + i\sin(\pi + \frac{\pi}{2})) \\ &= 20(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2})) \end{aligned}$$

Así,

- $Re(z) = 20.\cos(\frac{3\pi}{2}) = 0$
- $Im(z) = 20.\sin(\frac{3\pi}{2}) = -20$
- $|z| = 20$
- $Re(z^{-1}) = 0$
- $iz = 20(\cos(2\pi) + i\sin(2\pi)) \implies Im(iz) = 0$

6.1.B. Pregunta ii

$$\begin{aligned} z &= (\sqrt{2} + \sqrt{3}i)^2.(\overline{1-3i}) \\ &= (2 + 2\cdot\sqrt{2}\cdot\sqrt{3}i - 3).(1+3i) \\ &= (-1 + 2\cdot\sqrt{6}i).(1+3i) \\ &= -1 - 3i + 2\cdot\sqrt{6}i - 6\cdot\sqrt{6} \\ &= -1 - 6\cdot\sqrt{6} + (2\cdot\sqrt{6} - 3)i \end{aligned}$$

- $Re(z) = -1 - 6\cdot\sqrt{6}$
- $Im(z) = 2\cdot\sqrt{6} - 3$
- $|z| = \sqrt{(-1-6\cdot\sqrt{6})^2 + (2\cdot\sqrt{6}-3)^2} = \sqrt{250} = 5\cdot\sqrt{10}$

6.1.C. Pregunta iii

Paso a polares,

$i^{17} = \cos(17.\frac{\pi}{2}) + i\sin(17\frac{\pi}{2})$

$\frac{1}{2} = \frac{1}{2}.(\cos(0) + i\sin(0))$

$i = \cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2})$

$(1-i)^3 = (\sqrt{2})^3(\cos(3.\frac{7}{4}\pi) + i\sin(3.\frac{7}{4}\pi))$

Luego,

$$\frac{1}{2}.i.(1+i)^3 = \frac{1}{2}.(\sqrt{2})^3.\left(\cos(\frac{\pi}{2}+\frac{21}{4}\pi)+i\sin(\frac{\pi}{2}+\frac{21}{4}\pi)\right)$$
$$= \frac{\sqrt{2}^3}{2}.\left(\cos\left(\frac{23}{4}\pi\right)+i\sin\left(\frac{23}{4}\pi\right)\right)$$

Entonces,

$$z = \frac{\sqrt{2}^3}{2}.\left(\cos\left((\frac{17}{2}+\frac{23}{4})\pi\right)+i\sin\left((\frac{17}{2}+\frac{23}{4})\pi\right)\right)$$
$$= \frac{\sqrt{2}^3}{2}.\left(\cos\left(\frac{57}{4}\pi\right)+i\sin\left(\frac{57}{4}\pi\right)\right)$$

- $Re(z) = \frac{\sqrt{2}^3}{2}.\cos\left(\frac{57}{4}\pi\right)$
- $Im(z) = \frac{\sqrt{2}^3}{2}.\sin\left(\frac{57}{4}\pi\right)$
- $|z| = \frac{\sqrt{2}^3}{2}$
- $Re(z^{-1}) = \cos\left(\frac{57}{4}\pi\right)$
- $Im(iz) = \frac{\sqrt{2}^3}{2}.\sin\left(\frac{59}{4}\pi\right)$

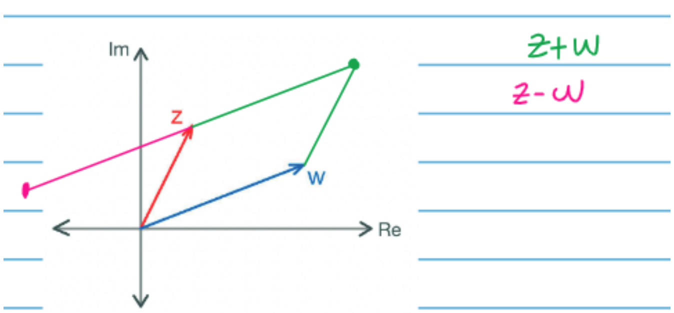
6.1.D. Pregunta iv

TODO

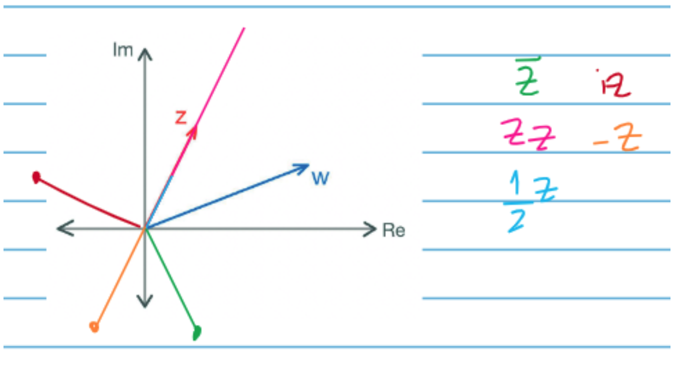
6.1.E. Pregunta v

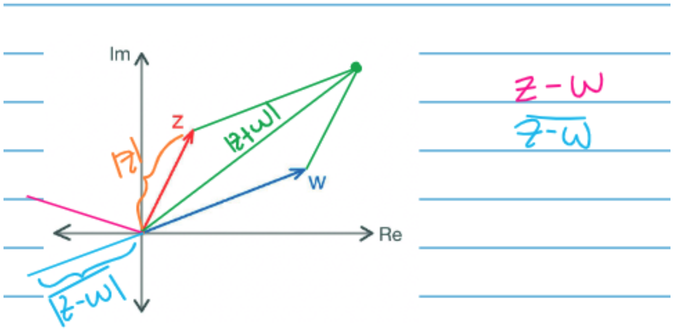
TODO

6.2. Ejercicio 2



1.

2. 

3. 

6.3. Ejercicio 3

6.3.A. Pregunta i

$z^2 = -36$

Se que $z = a + bi$ con $(a, b) \in \mathbb{R}^2$

Luego busco los z tales que $z^2 = -36$

$$
\begin{aligned}
z^2 = -36 \iff z^2 &= (a+bi)^2 \\
&= a^2 - b^2 + 2abi
\end{aligned}
$$

También se que el módulo debe ser igual $|z^2| = |-36|$,

$|z^2| = |z|^2 = (\sqrt{a^2+b^2})^2 = a^2 + b^2$
$|-36| = 36$

Usando la igualdad de números complejos,

$$
\begin{cases}
a^2 - b^2 = -36 \\
2ab = 0 \\
a^2 + b^2 = 36
\end{cases}
$$

Sumando (1) y (3) $2a^2 = 0 \iff a = 0$

Restando (1) a (3) $2b^2 = 2.36 \iff b = \pm 36$

Luego $z = a + bi$ con los valores de a y b hallados resulta en

Rta.: $z_1 = 6i; z_2 = -6i$

6.3.B. Pregunta ii

$z^2 = i$

Se que $z = a + bi$ con $(a, b) \in \mathbb{R}^2$

También se que el módulo debe ser igual $|z^2| = |i|$,

$|z^2| = |z|^2 = (\sqrt{a^2 + b^2})^2 = a^2 + b^2$
$|i| = 1$

Usando la igualdad de números complejos,

$\begin{cases} a^2 - b^2 = 0 \\ 2ab = 1 \\ a^2 + b^2 = 1 \end{cases}$

Sumando (1) y (3) $2a^2 = 1 \iff a = \pm\frac{1}{\sqrt{2}}$

Restando (1) a (3) $2b^2 = 1 \iff b = \pm\frac{1}{\sqrt{2}}$

Usando (2) se que $2ab > 0 \iff (a > 0 \wedge b > 0) \vee (a < 0 \wedge b < 0)$

Rta.: $z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i; z_2 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

6.3.C. Pregunta iii

$z^2 = 7 + 24i$

Se que $z = a + bi$ con $(a, b) \in \mathbb{R}^2$

También se que el módulo debe ser igual $|z^2| = |7 + 24i|$,

$|z^2| = |z|^2 = (\sqrt{a^2 + b^2})^2 = a^2 + b^2$

$|7 + 24i| = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = 25$

Usando la igualdad de números complejos,

$\begin{cases} a^2 - b^2 = 7 \\ 2ab = 24 \\ a^2 + b^2 = 25 \end{cases}$

Sumando (1) y (3) $2a^2 = 32 \iff a = \pm 4$

Restando (1) a (3) $2b^2 = 18 \iff b = \pm 3$

Usando (2) se que $2ab > 0 \iff (a > 0 \wedge b > 0) \vee (a < 0 \wedge b < 0)$

Rta.: $z_1 = 4 + 3i; z_2 = -4 - 3i$

6.3.D. Pregunta iv

$z^2 + 15 - 8i = 0 \iff z^2 = -15 + 8i$

Se que $z = a + bi$ con $(a, b) \in \mathbb{R}^2$

También se que el módulo debe ser igual $|z^2| = |-15 + 8i|$,

$|z^2| = |z|^2 = (\sqrt{a^2 + b^2})^2 = a^2 + b^2$

$|-15 + 8i| = \sqrt{(-15)^2 + 8^2} = \sqrt{225 + 64} = 17$

Usando la igualdad de números complejos,

$\begin{cases} a^2 - b^2 = -15 \\ 2ab = 8 \\ a^2 + b^2 = 17 \end{cases}$

Sumando (1) y (3) $2a^2 = 2 \iff a = \pm 1$

Restando (1) a (3) $2b^2 = 16 \iff b = \pm 4$

Usando (2) se que $2ab > 0 \iff (a > 0 \wedge b > 0) \vee (a < 0 \wedge b < 0)$

Rta.: $z_1 = 1 + 4i; z_2 = -1 - 4i$

6.4. Ejercicio 4

6.4.A. Pregunta i

$z = (2+2i)(\sqrt{3}-i)$

Busco la forma polar de cada factor.

$$2+2i = \sqrt{8}.e^{\frac{\pi}{4}i}$$
$$\sqrt{3}-i = 2.e^{\frac{11}{6}\pi i}$$

Por DeMoivre,

$$\begin{aligned} z &= (2+2i)(\sqrt{3}-i) \\ &= \sqrt{8}.2.e^{\frac{\pi}{4}i+\frac{11}{6}\pi i} \\ &= 2\cdot\sqrt{8}e^{\frac{1}{12}\pi i} \end{aligned}$$

Luego,

- $|z| = 4\cdot\sqrt{2}$
- $\theta = \frac{1}{12}\pi$

6.4.B. Pregunta ii

$z = (-1+\sqrt{3}i)^5$

$$\begin{aligned} -1+\sqrt{3}i &= 2\cdot e^{(\pi-\frac{1}{3}\pi)i} \\ &= 2\cdot e^{\frac{2}{3}\pi i} \end{aligned}$$

Luego,

$$\begin{aligned} z &= (-1+\sqrt{3}i)^5 \\ &= (2\cdot e^{\frac{2}{3}\pi i})^5 \\ &= 2^5\cdot e^{\frac{10}{3}\pi i} \end{aligned}$$

Por lo tanto,

- $|z| = 2^5$
- $\theta = \frac{4}{3}\pi$

6.4.C. Pregunta iii

$z = (-1+\sqrt{3}i)^{-5}$

$$\begin{aligned} z &= (-1+\sqrt{3}i)^{-5} \\ &= (2\cdot e^{\frac{2}{3}\pi i})^{-5} \\ &= 2^{-5}\cdot e^{\frac{-10}{3}\pi i} \end{aligned}$$

Por lo tanto,

- $|z| = \frac{1}{2^5}$
- $\theta = \frac{2}{3}\pi$

6.4.D. Pregunta iv

$z = \frac{1+\sqrt{3}i}{1-i}$
Busco las expresiones polares.

$$\begin{aligned} 1 + \sqrt{3}i &= 2.e^{\frac{\pi}{3}i} \\ 1 - i &= \sqrt{2}.e^{\frac{7}{4}\pi i} \end{aligned}$$

Luego,

$$\begin{aligned} z &= \frac{1+\sqrt{3}i}{1-i} \\ &= \frac{2}{\sqrt{2}} \cdot e^{(\frac{1}{3}-\frac{7}{4})\pi i} \\ &= \frac{2}{\sqrt{2}} \cdot e^{\frac{-17}{12}\pi i} \end{aligned}$$

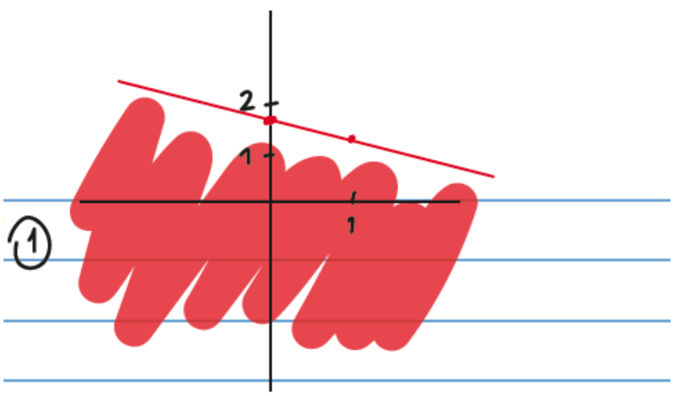
Por lo tanto,

- $|z| = \frac{2}{\sqrt{2}}$
- $\theta = \frac{7}{12}\pi$

6.5. Ejercicio 5

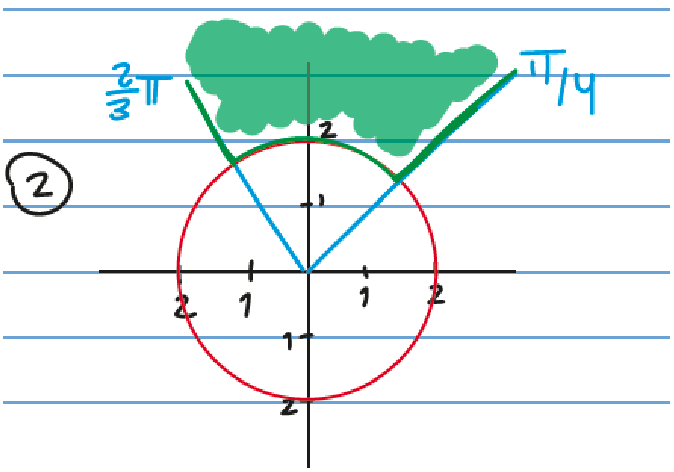
6.5.A. Pregunta i

$Re(z) = x \wedge Im(z) = y \implies x + 5y \leq 8 \iff y \leq -\frac{1}{5}x + \frac{8}{5}$

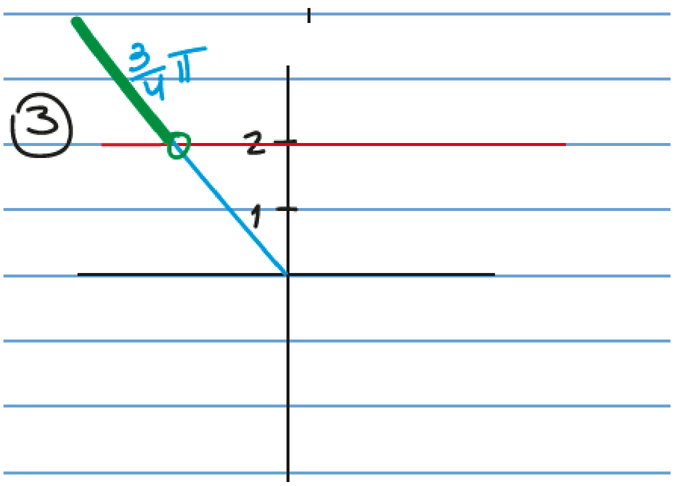


6.5.B. Pregunta ii

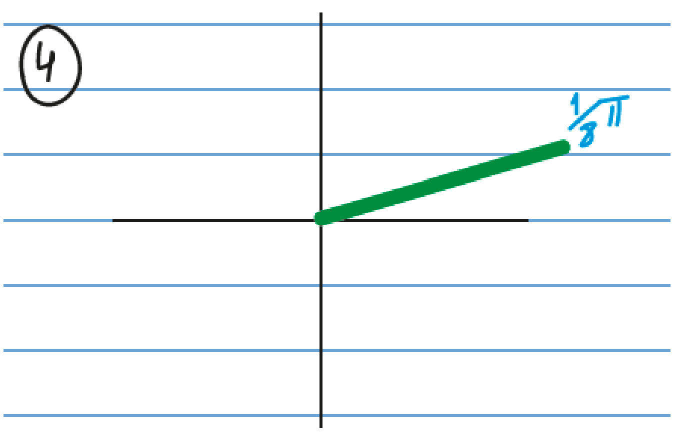
- $|z| = 2$ define una circunferencia de radio 2.
- $\frac{\pi}{4} \leq arg(z) \leq \frac{2\pi}{3}$ define un arco de angulo barrido.



6.5.C. Pregunta iii



6.5.D. Pregunta iv



6.6. Ejercicio 6

6.6.A. Pregunta i

$z = (\frac{1+\sqrt{3}i}{1-i})^{17}$

Sea $w = \frac{1+\sqrt{3}i}{1-i}$

Busco las expresiones polares.

$$\begin{aligned} 1 + \sqrt{3}i &= 2.e^{\frac{\pi}{3}i} \\ 1 - i &= \sqrt{2}.e^{\frac{7}{4}\pi i} \end{aligned}$$

Luego,

$$\begin{aligned} w &= \frac{1+\sqrt{3}i}{1-i} \\ &= \frac{2}{\sqrt{2}} \cdot e^{(\frac{1}{3}-\frac{7}{4})\pi i} \\ &= \frac{2}{\sqrt{2}} \cdot e^{\frac{7}{12}\pi i} \end{aligned}$$

Por lo tanto,

$$\begin{aligned} z &= w^{17} \\ &= \left(\frac{2}{\sqrt{2}} \cdot e^{\frac{7}{12}\pi i}\right)^{17} \\ &= \left(\frac{2}{\sqrt{2}}\right)^{17} \cdot e^{\frac{17.7}{12}\pi i} \\ &= \left(\frac{2}{\sqrt{2}}\right)^{17} \cdot e^{\frac{23}{12}\pi i} \end{aligned}$$

Por lo tanto, $z = \left(\frac{2}{\sqrt{2}}\right)^{17} \cdot \cos(\frac{23}{12}\pi i) + \left(\frac{2}{\sqrt{2}}\right)^{17} \cdot \sin(\frac{23}{12}\pi i)$

6.6.B. Pregunta ii

$z = (-1 + \sqrt{3}i)^n$

$-1 + \sqrt{3}i = 2 \cdot e^{\frac{2}{3}\pi i}$

$(-1 + \sqrt{3}i)^n = 2^n \cdot e^{\frac{2n}{3}\pi i}$

Luego $0 \leq \frac{2n}{3}\pi i < 2\pi \iff 0 \leq n < 3$

Por lo tanto,

- $n = 0 \implies z = 1$
- $n = 1 \implies z = 2 \cdot e^{\frac{2}{3}\pi i} = -1 + \sqrt{3}i$
- $n = 1 \implies z = 4 \cdot e^{\frac{4}{3}\pi i} = -2 + 2 \cdot \sqrt{3}i$

6.7. Ejercicio 7

6.7.A. Pregunta i

Busco los $n \in \mathbb{N}$ tales que $(\sqrt{3} - i)^n = 2^{n-1} \cdot (-1 + \sqrt{3}i)$

Luego,

$$\begin{aligned}(\sqrt{3}-i)^n = 2^{n-1}\cdot(-1+\sqrt{3}i) &\iff 2(\sqrt{3}-i)^n = 2^n\cdot(-1+\sqrt{3}i)\\ &\iff \left(\frac{\sqrt{3}-i}{2}\right)^n = \frac{-1+\sqrt{3}i}{2}\end{aligned}$$

Busco expresiones polares,

- $-1+\sqrt{3}i = 2\cdot e^{\frac{2}{3}\pi i}$
- $2 = 2\cdot e^0$
- $\sqrt{3}-i = 2\cdot e^{\frac{11}{6}\pi i}$

Así,

$$\begin{aligned}\frac{-1+\sqrt{3}i}{2} &= 1\cdot e^{\frac{2}{3}\pi i}\\ &= e^{\frac{2}{3}\pi i}\end{aligned}$$

Y,

$$\begin{aligned}\frac{\sqrt{3}-i}{2} &= 1\cdot e^{\frac{11}{6}\pi i}\\ &= e^{\frac{11}{6}\pi i}\\ \implies \left(\frac{\sqrt{3}-i}{2}\right)^n &= e^{\frac{11}{6}n\pi i}\end{aligned}$$

Luego sea $z = e^{\frac{2}{3}\pi i}$ y $w = e^{\frac{11}{6}n\pi i}$ por definición de números complejos, $z = w \iff \begin{cases}|z| = |w|\\ \frac{2}{3}\pi = \frac{11}{6}n\pi + 2k\pi\end{cases}$

Luego,

$$\begin{aligned}\frac{2}{3}\pi &= \frac{11}{6}n\pi + 2k\pi\\ \frac{2}{3} &= \frac{11}{6}n + 2k\\ 4 &= 11n + 12k\\ 11n &= -12k + 4\\ \iff 11n &\equiv 4(12)\\ -n &\equiv 4(12)\\ n &\equiv 8(12)\end{aligned}$$

Rta.: $w = z \iff n \equiv 8(12)$

6.7.B. Pregunta ii

Busco los $n \in \mathbb{N}$ tales que $(-\sqrt{3}+i)^n \cdot \left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)$ es un real negativo

Busco expresiones polares.

- $-\sqrt{3}+i = 2\cdot e^{\frac{5}{6}\pi i}$
- $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right) = \left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\cdot e^0$

Luego, $z = 2^n \cdot \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) \cdot e^{\frac{5}{6}\pi n i}$

Por definición de la forma polar, $z \in \mathbb{R}_{<0} \iff arg(z) = \pi$

Así,

$$\begin{aligned} \frac{5}{6}\pi n &= \pi + 2k\pi \\ \frac{5}{6}n &= 1 + 2k \\ 5n &= 6 + 12k \\ \iff 5.5n &\equiv 5.6(12) \\ \iff n &\equiv 6(12) \end{aligned}$$

Rta.: z es real negativo $\forall n \in \mathbb{N} : n \equiv 6(12)$

6.7.C. Pregunta iii

Busco los $n \in \mathbb{N}$ tales que $arg((-1+i)^{2n}) = \frac{\pi}{2}$ y $arg((1-\sqrt{3}i)^{n-1}) = \frac{2}{3}\pi$

Busco expresiones polares.

- $(1-i) = \sqrt{2} \cdot e^{\frac{3}{4}\pi i}$
- $(1-i)^{2n} = \left(\sqrt{2}\right)^{2n} \cdot e^{2n\frac{3}{4}\pi i} = 2^n \cdot e^{\frac{3}{2}\pi n i}$
- $(1-\sqrt{3}i) = 2 \cdot e^{\frac{5}{3}\pi i}$
- $(1-\sqrt{3}i)^{n-1} = 2^{n-1} \cdot e^{(n-1)\frac{5}{3}\pi i}$

Luego resolviendo la primer igualdad,

$$\begin{aligned} arg((-1+i)^{2n}) = \frac{\pi}{2} \iff \frac{3}{2}\pi n &= \frac{\pi}{2} + 2k\pi \\ \frac{3}{2}n &= \frac{1}{2} + 2k \\ 3n &= 1 + 4k \\ 3n &\equiv 1(4) \\ n &\equiv 3(4) \end{aligned}$$

Y la segunda,

$$\begin{aligned} arg((1-\sqrt{3}i)^{n-1}) = \frac{2}{3}\pi \iff (n-1)\frac{5}{3}\pi &= \frac{2}{3}\pi + 2k\pi \\ (n-1)\frac{5}{3} &= \frac{2}{3} + 2k \\ (n-1)5 &= 2 + 6k \\ 5n - 5 &= 2 + 6k \\ 5n &= 7 + 6k \\ 5n &\equiv 7(6) \\ -n &\equiv 7(6) \\ n &\equiv 5(6) \end{aligned}$$

Juntando ambas soluciones, $\begin{cases} n \equiv 3(4) \implies n \equiv 1(2) \\ n \equiv 5(6) \end{cases}$

La segunda implica la primera, luego los n que cumplen lo pedido son $\{n \in \mathbb{N} : n \equiv 5(6)\}$

6.8. Ejercicio 8

6.8.A. Pregunta i

Busco los $w : w^6 = 8$

Se que, $\begin{cases} 8 = 8 \cdot e^0 \\ w^6 = |w|^6 \cdot e^{6\theta i} \end{cases}$

Luego por igualdad de números complejos, $w^6 = 8 \iff \begin{cases} |w|^6 = 8 \\ \theta = \frac{0+2k\pi}{6} \end{cases}$

Con $0 \leq \theta < 2\pi \iff 0 \leq \frac{k\pi}{3} < 2\pi \iff 0 \leq k < 6 \implies k \in \{0, 1, 2, 3, 4, 5\}$

Así, las raíces sextas de 8 son:

- $n = 0 \implies w_0 = \sqrt[6]{8} \cdot e^0$
- $n = 1 \implies w_1 = \sqrt[6]{8} \cdot e^{\frac{2}{6} \cdot \pi i}$
- $n = 2 \implies w_2 = \sqrt[6]{8} \cdot e^{\frac{4}{6} \cdot \pi i}$
- $n = 3 \implies w_3 = \sqrt[6]{8} \cdot e^{\frac{6}{6} \cdot \pi i}$
- $n = 4 \implies w_4 = \sqrt[6]{8} \cdot e^{\frac{8}{6} \cdot \pi i}$
- $n = 4 \implies w_5 = \sqrt[6]{8} \cdot e^{\frac{10}{6} \cdot \pi i}$

6.8.B. Pregunta ii

Busco los $w : w^3 = -4$

Se que, $\begin{cases} -4 = 4 \cdot e^{\pi i} \\ w^3 = |w|^3 \cdot e^{3\theta i} \end{cases}$

Luego por igualdad de números complejos, $w^3 = -4 \iff \begin{cases} |w|^3 = 4 \implies |w| = \sqrt[3]{4} \\ \theta = \frac{\pi+2k\pi}{3} \end{cases}$

Con $0 \leq \theta < 2\pi \iff 0 \leq \frac{\pi+2k\pi}{3} < 2\pi \iff 0 \leq k < 3 \implies k \in \{0, 1, 2\}$

Así, las raíces cubicas de -4 son:

- $n = 0 \implies w_0 = \sqrt[3]{4} \cdot e^{\frac{1}{3}\pi i}$
- $n = 1 \implies w_1 = \sqrt[3]{4} \cdot e^{\pi i}$
- $n = 2 \implies w_2 = \sqrt[3]{4} \cdot e^{\frac{5}{3}\pi i}$

6.8.C. Pregunta iii

Busco los $w : w^7 = -1 + i$

Se que, $\begin{cases} -1 + i = \sqrt{2} \cdot e^{\frac{3}{4}\pi i} \\ w^7 = |w|^7 \cdot e^{7\theta i} \end{cases}$

Luego por igualdad de números complejos, $w^7 = -1 + i \iff \begin{cases} |w|^7 = \sqrt{2} \implies |w| = \sqrt[14]{2} \\ \theta = \frac{3/4\pi+2k\pi}{7} \end{cases}$

Con $0 \leq \theta < 2\pi \iff 0 \leq \frac{3/4\pi+2k\pi}{7} < 2\pi \iff 0 \leq k < 7 \implies k \in \{0, 1, 2, 3, 4, 5, 6\}$

Así, las raíces séptimas de $-1 + i$ son:

- $n = 0 \implies w_0 = \sqrt[14]{2} \cdot e^{\frac{3}{28}\pi i}$
- $n = 1 \implies w_1 = \sqrt[14]{2} \cdot e^{\frac{11}{28}\pi i}$

- $n = 2 \implies w_2 = \sqrt[14]{2} \cdot e^{\frac{19}{28}\pi i}$
- $n = 3 \implies w_3 = \sqrt[14]{2} \cdot e^{\frac{27}{28}\pi i}$
- $n = 4 \implies w_4 = \sqrt[14]{2} \cdot e^{\frac{35}{28}\pi i}$
- $n = 5 \implies w_5 = \sqrt[14]{2} \cdot e^{\frac{43}{28}\pi i}$
- $n = 6 \implies w_6 = \sqrt[14]{2} \cdot e^{\frac{51}{28}\pi i}$

6.9. Ejercicio 9

Busco todos los $z \in \mathbb{C}$ tales que $3z^5 + 2|z|^5 + 32 = 0$

Pero, $3z^5 + 2|z|^5 + 32 = 0 \iff 3z^5 = -2|z|^5 - 32$

Luego por igualdad de números complejos, $arg(3z^5) = arg(-2|z|^5 - 32)$

$$\begin{aligned} arg(3z^5) &= arg(-2|z|^5 - 32) \\ arg(3z^5) &= arg(-2(|z|^5 - 16)) \\ arg(3) + arg(z^5) &= arg(-2) + arg(|z|^5 - 16) + 2k\pi \\ 0 + 5\theta &= \pi + 0 + 2k\pi \\ \theta &= \frac{\pi}{5} + \frac{2k\pi}{5} \end{aligned}$$

Con $0 \leq \frac{\pi}{5} + \frac{2k\pi}{5} < 2\pi \iff 0 \leq 1 + 2k < 10 \iff \frac{-1}{2} \leq k \leq \frac{9}{2} \implies k \in \{0, 1, 2, 3, 4\}$

Ahora busco $|z|$

$$\begin{aligned} |3||z|^5 &= |-2|||z|^5 - 16| \\ 3|z|^5 &= 2|z|^5 + 32 \\ |z|^5 &= 32 \\ |z| &= \sqrt[5]{32} \\ |z| &= 2 \end{aligned}$$

Luego $z = 2 \cdot e^{\theta i} = 2 \cdot e^{\frac{\pi}{5} + \frac{2k\pi}{5}}$ con $k \in \{0, 1, 2, 3, 4\}$

Los z que cumplen lo pedido son:

- $k = 0 \implies z_0 = 2 \cdot e^{\frac{1}{5}\pi i}$
- $k = 1 \implies z_1 = 2 \cdot e^{\frac{3}{5}\pi i}$
- $k = 2 \implies z_2 = 2 \cdot e^{\pi i}$
- $k = 3 \implies z_3 = 2 \cdot e^{\frac{7}{5}\pi i}$
- $k = 4 \implies z_4 = 2 \cdot e^{\frac{9}{5}\pi i}$