Equilibrium Contracts In a Bilateral Monopoly with Unequal Bargaining Powers

Article *in* International Economic Journal · February 2002

DOI: 10.1080/10168730200080003 · Source: RePEc

Stephen Devadoss
Texas Tech University

EQUILIBRIUM CONTRACTS IN A BILATERAL MONOPOLY WITH UNEQUAL BARGAINING POWERS

Siddhartha Dasgupta
Aquaculture/Fisheries Center
University of Arkansas at Pine Bluff
Pine Bluff, AK 71611
Phone: 870-543-8537
Fax: 208-543-8532
email: dasgupta_s@vx4500.uapb.edu

and

Stephen Devadoss
Department of Agricultural Economics
University of Idaho
Moscow, ID 83844-2334
Phone: 208-885-6806
Fax: 208-885-5759
E-mail: sdevadoss@uidaho.edu

Authors acknowledge the helpful comments of an anonymous reviewer. This paper is based upon work partially supported by the Cooperative State Research, Education, and Extension Service of U.S. Department of Agriculture, under Agreement No. 00-35400-9202.

The work was completed when the first author was post-doctoral research associate at the University of Idaho.

EQUILIBRIUM CONTRACTS IN A BILATERAL MONOPOLY WITH UNEQUAL BARGAINING POWERS

Abstract

Real-world bilateral monopolies often indicate that one party exercises slightly superior bargaining power than the other party. We analyze long-term, cooperative contracts in bilateral monopolies with unequal bargaining powers. We assume that the two parties bargain for a determinate price and quantity of the intermediate product by optimizing a joint objective which takes into account the profits and bargaining power of each party. We use a Bowley price leadership model to develop the multi-period contracts and derive conditions that induce a Nash equilibrium at the jointly determined points of operation.

JEL Classification: C71 and C78

EQUILIBRIUM CONTRACTS IN A BILATERAL MONOPOLY WITH UNEQUAL BARGAINING POWERS

1. INTRODUCTION

In a bilateral monopoly, a downstream monopsonist (buyer) purchases a commodity produced or service supplied by an upstream monopolist (seller). This commodity or service, which forms an essential input of the buyer, is called an intermediate product. Since both parties exercise power in the market for the intermediate product, the price and quantity at which the intermediate product is transacted requires joint negotiation by the buyer and seller. Bilateral monopolies often disagree on the price for the intermediate product. As Kreps (1990: 551) suggests the indeterminacy of the price of the intermediate product can be illustrated using the Edgeworth box, "although no predictions are offered as to the chosen point of operation along the contract curve." Kreps (1990: 551) further summarizes that studies which dealt with the economics of bilateral monopolies have concluded "sharp predictions as to what will happen cannot be offered," and "the price at which this exchange (of the intermediate product) takes place is indeterminate." Blair et al. (1989) using the contract curve approach also conclude the price in a bilateral monopoly is indeterminate.

Since both buyer and seller exercise power over the market for the intermediate product, the process of joint determination of price and quantity invariably involves bargaining between the two parties and thus depends on bargaining strengths of each party. Truett and Truett using the same contract curve approach as in Blair et al. (1989) show that the buyer and seller eventually obtain a joint agreement on a price where the buyer's marginal value product and the seller's marginal cost of the intermediate product are equal. The implicit assumption in the Truett and Truett (1993) analysis is that both buyer and seller have equal bargaining power. The reason for the incorrect conclusions of indeterminate price by Blair et al. (1989) is that their analysis is incomplete as they did not fully carry out the bargaining process as executed by Truett and Truett (1993). Devadoss and Cooper

(2000) present a dynamic optimization of the bilateral monopoly, which allows for bargaining by both parties, and show both equilibrium price and quantity are determinate.

Although equal bargaining power in a bilateral monopoly is often assumed, it is not implausible for one party to have slightly greater bargaining power than the other party. For instance, Pindyck and Rubenfield (1992: 361) state that in a bilateral monopoly, "... monopoly power might be large, for example, and the monopsony power small, so that residual monopoly power would still be significant." Devadoss (2001) incorporates the unequal bargaining powers between the buyer and seller and shows the effect of each party's bargaining power in determining the price.

Unequal bargaining power arises from superior market power exerted by one of the parties. The cause of differential market power can be attributed to supply demand sources of intermediate product substitutes. In real-world bilateral monopolies, the buyer may have supply sources for such substitutes. Similarly, the seller may find other buyers who can use the intermediate product as an imperfect substitute. For example, in the mid 1980s, when players of the National Football League (NFL) went on strike, NFL team owners used replacement players. Hiring of replacement players gave the NFL team owners a slightly superior bargaining power. We assume that such substitutes are imperfect, which is consistent with the consequences of the NFL strike where the team owners had to negotiate with the player's union because of the unpopularity of the replacement players. Therefore, despite the availability of intermediate product substitutes, the underlying bilateral monopoly structure is unchanged beyond inducing unequal bargaining powers. Other examples of bilateral monopolies include the players' union and team owners' association in pro-sports (the National Basketball Association and the Major League Baseball). In fact, union-management bilateral monopolies exist whenever the firm hiring labor behaves as a monopsonist and the labor union acts as a single entity, i.e., a monopolist, in selling their service (Bandyopadhyay, 1995).

The aim of this paper is to derive a determinate equilibrium price and quantity of the intermediate product transacted in a bilateral monopoly with each party having different bargaining strength. We develop a multiple-period contract between the buyer and seller with differing degree of bargaining powers, specify strategies for each party, analyze their incentives of operating within the contract, and derive contract terms that induce a Nash equilibrium at their 'negotiated' point of operation. The 'negotiated' point of operation is determined from a joint profit maximization objective which indicates cooperation between the two parties. Such cooperation is imperative because, if the two parties do not cooperate, the bilateral monopoly disintegrates and results in zero disagreement payoffs (Rubinstein, 1982; and Kreps, 1990). We design the contract by using a Bowley price leadership model where one party controls the price and the other party selects the quantity of the intermediate product to be transacted at that price (Fouraker and Siegel, 1963).

Studies on bilateral monopoly (Bowley, 1924; Fellner, 1947; Machlup and Taber, 1960; Truett and Truett, 1993; and Devadoss; 2001) are often based on the assumption that the buyer and seller maximize the sum of their profits. We diverge from this assumption by specifying an objective that incorporates not only the profits of each party but also their bargaining powers. For clarity of the exposition, the sum of the buyer's and seller's profits is called 'total profits' while 'joint profits,' as explained later, is derived from the collusive profit maximization objective.

The remainder of this paper contains a description of the objectives of the buyer and seller in a bilateral monopoly, followed by specification and analysis of multiple-period contracts that induce a Nash equilibrium at jointly negotiated points of operation. We analyze the incentives of each party to operate within the bounds of the contract, the incentives of each party to deviate from their negotiated point of operation, and derive of contract terms which induces the Nash equilibrium. We conclude with an intuitive discussion and comments on the equilibrium terms of the contracts.

2. THEORETICAL MODEL

In a bilateral monopoly, the seller produces q amount of the intermediate product at a cost of $c(q)$ while the buyer purchases it at price p, and uses it to produce $f(q)$ amount of output which is sold at the price of w. Assuming all other markets except the market for q are perfectly competitive, the buyer and seller's profits are, respectively, denoted as:

$$\Pi_B(p,q)=wf(q)-pq \quad \text{and } \Pi_S(p,q)=pq-c(q) \quad . \tag{1}$$

If the bilateral monopoly is organized into a vertically integrated firm, the profit is given by:

$$\Pi_V(q)=\Pi_B+\Pi_S=wf(q)-c(q). \tag{2}$$

Hence, $\Pi_V(q)$refers to the 'total' profit, as noted in the introduction. Let β ($0<\beta<1$) and ($1-\beta$) represent the seller's and buyer's bargaining power, respectively. Next, we posit that the buyer and seller decide the optimal price and quantity profile of the intermediate product by maximizing the product of each party's profits, weighted by their respective bargaining powers.[1]

$$\mathrm{Max}_{p,q}\Pi_S^{\beta}\cdot\Pi_B^{(1-\beta)}=[pq-c(q)]^{\beta}\cdot[wf(q)-pq]^{(1-\beta)} \ . \tag{3}$$

---

1. Binmore et al. (1986) have shown that the limiting equilibria of alternative-offer bargaining processes that have an infinite horizon are equivalent to solving an appropriate Nash product. We use a bargaining power-weighted Nash product as a joint objective for negotiations between the two parties. A version of this joint objective has been commonly used in the trade union literature (Espinosa and Rhee, 1989; and Bandyopadhyay, 1995). Our goal diverges from the framework of Binmore et al. (1986) in that we derive negotiated price and quantity profiles of the intermediate product and equilibrium strategies and conditions for a bilateral monopoly with unequal bargaining powers, given the non-cooperative incentives of either party.

Thus, 'joint' profits are $\Pi_S^{\beta} \cdot \Pi_B^{(1-\beta)}$. Thus, (3) is a Nash product incorporating payoffs of both parties and their bargaining powers, and indicating zero payoffs to each party in the event of a breakdown in negotiations. The first-order condition with respect to price of the intermediate product is:

$$\frac{1}{(1-\beta)}\left(\frac{\Pi_B}{\Pi_S}\right)^{1-\beta} = \frac{1}{\beta}\left(\frac{\Pi_S}{\Pi_B}\right)^{\beta}. \tag{4}$$

The first-order condition with respect to the quantity of intermediate product is:

$$c'(\_\_\_\_) = wf\_(q). \tag{5}$$

The solution to (5), q^*, is a determinate quantity and is independent of the bargaining powers of the two parties. However, if we substitute q^* into (4) and solve for the price of the intermediate product, the solution is dependent on β:

$$p^* = \beta p_1 + (1-\beta) p_2 \text{ where } p_1 = \frac{wf(q^*)}{q^*} \text{ and } p_2 = \frac{c(q^*)}{q^*}. \tag{4'}$$

Here, p_1 and p_2 are, respectively, prices at which the buyer's and seller's profits are zero. Buyer's and seller's profits corresponding to p^* and q^* are such that they divide total profits proportional to their bargaining power. Hence,

$$\begin{aligned} \Pi_S(p^*, q^*) &= p^* q^* - c(q^*) = [(1-\beta)c(q^*) + \beta wf(q^*)] - c(q^*) \\ &= \beta[wf(q^*) - c(q^*)] = \beta \Pi_V(q^*) \end{aligned} \tag{6}$$

and similarly,

$$\Pi_B(p^*, q^*) = (1-\beta)\Pi_V(q^*). \tag{6'}$$

Thus, equal bargaining power translates to equal optimal profits for each party, while unequal bargaining powers implies that the party with superior bargaining power receives a greater share of the total profit [$\Pi_V(q^*)$], proportional to its bargaining power. It is worth noting that total profits are maximized if a vertically integrated firm produces q^* amount of intermediate product.

$$\text{Max}_q \Pi_V = wf(q) - c(q). \tag{7}$$

That is, (7) yields optimal q which is equal to q^* since the necessary condition for (7) is identical to (5), indicating that both bilateral monopoly and a vertically integrated firm optimally produces the same amount of the intermediate product. It is also interesting to note that the result in (5) defines the Pareto-efficient contract curve along which both parties transact the intermediate product efficiently. This can be easily shown by deriving the locus of price and quantity profiles (p, q) of the intermediate product where the iso-profit curves of both parties are tangent to one another, i.e.,

$$\left.\frac{dp}{dq}\right|_{[\Pi_B = constant]} = \left.\frac{dp}{dq}\right|_{[\Pi_S = constant]}. \tag{8}$$

This simplifies to (5), indicating that the contract curve in a bilateral monopoly is a vertical line through q^* (figure 1).[2] Hence, a bilateral monopoly operates Pareto-efficiently when it trades q^*

---

2. Here, we show that the contract curve in a bilateral monopoly is given by (5). The slope of the buyer's iso-profit curve can be obtained by totally differentiating the buyer's profit function in (1): $wf\_(q)dq - pdq - qdp = 0$, which simplifies to $\left.\frac{dp}{dq}\right|_{\Pi_B = constant} = \frac{wf\_(q) - p}{q}$. Similarly, the slope of the seller's iso-profit function can be obtained by totally differentiating the seller's profit function: $qdp + pdq - c$ ĩ____$)dq = 0$, which simplifies to $\left.\frac{dp}{dq}\right|_{\Pi_S = constant} = \frac{c\_(q) - p}{q}$. Since the contract curve is defined as the locus of points for which: $\left.\frac{dp}{dq}\right|_{\Pi_B = constant} = \left.\frac{dp}{dq}\right|_{\Pi_S = constant}$, the equation of the contract curve is $\frac{wf\_(q) - p}{q} = \frac{c\_(q) - p}{q}$, which simplifies to: $wf\_(q) = c\_(q)$, as in (5).

amount of the intermediate product.

Non-cooperative behavior is characterized by each party maximizing its own-profits after assuming that the other party does the same. In our model non-cooperation is important in providing direction for rational deviations from contracted actions (given by (3)), should there be any incentives to do so. The best-response correspondence (or best-response function) of each party is its profit-maximizing marginal condition which gives its respective demand and supply schedules for the intermediate product. For instance, the buyer's demand curve is the locus of price and quantity profiles of the intermediate product at which his profit is maximized:

$$\mathrm{Max}_q \Pi_B = wf(q) - pq. \tag{9}$$

The first-order condition with respect to q: $p=wf\_(q)$, yields the buyer's demand curve. Thus, if the seller determines p (q), the corresponding q (p) along the demand curve is the buyer's best response. This result, which can be also shown for the seller's supply curve, leads to the conclusion that there exists a Nash equilibrium at the price and quantity profile for which the two reaction functions (the demand and supply functions) intersect (point *A* in figure 1). It is from these arguments that one can justify the stability of Truett and Truett's equilibrium which is characterized by "a price that will maximize the profits of both parties at an unique quantity (q^*)" (Truett and Truett, 1993: 267). Their implicit assumption of equal bargaining power is evident from (4') and (5), which indicates a point of operation at *A* if and only if $\beta = ½$. Hence, equal bargaining power implies a Nash equilibrium at *A* in every time period which, therefore, requires no inducement through a multiple-period contract.

Let us now consider the case where the bargaining power of the two parties are not equal (β*ne* ½). From expressions in (4_), (5), (6) and (6_), it is clear that for β*ne*½, the price is not determined at *A* but at a point above or below *A*. If the seller (buyer) has superior bargaining power,

i. e., $\beta > ½$ ($\beta < ½$), the price is determined at a point such as B_1 in figures 1 and 2 (B_2 in figures 4 and 5). This conclusion is further strengthened by Truett and Truett's comment that negotiating a different price along the contract curve (other than that at *A*) indicates " . . . that one firm has a degree of power over the other . . ." (Truett and Truett, 1993: 261)."

Nash (1953) and Roth (1979) conclude that a solution to a bargaining game needs to be Pareto-optimal such that no other feasible outcome is preferred by both parties. Since the disagreement payoffs of either party are zero in a bilateral monopoly, the jointly determined points of operation, contingent on the relative bargaining strengths of the two parties (*A*, B_1 or B_2), represent Pareto-optimal solutions to the bilateral bargaining game.

Since the points B_1 and B_2 are not on either the buyer's demand or seller's supply function, actions taken by each party to operate at these points do not represent their best-response to the actions of the other party. Hence, both parties have incentives of taking actions to deviate from B_1 or B_2 in order to increase own-profits by operating along their best-response functions. Consistent with the Bowley price leadership model, when bargaining powers are unequal, we first consider the case where the party with superior bargaining power determines the price of the intermediate product, and next, the case where the party with inferior bargaining power determines the price of the intermediate product. We show that actions taken by one party to maximize own-profits, by controlling either price or quantity of the intermediate product, increases its profits only at the expense of the other. Hence, such strategies induce retaliatory behavior in the other party, which can lead to a breakdown in the bargaining process and dissolution of the bilateral monopoly. Since we assume zero disagreement payoffs, any point of operation along the contract curve is significantly more profitable (except for points D_1 and D_2 in figures 2 and 5, respectively, where the buyer and seller respectively earn zero profits). In this light, we develop a multiple-period contract, using a dynamic game theory model, to show that a Nash equilibrium can exist at B_1 or B_2. Specifically, we develop terms of a contract between both parties such that, conditional on their bargaining powers, when the seller (buyer) operates at B_1 (B_2), it is in the interest of the buyer (seller) to operate at the same point.

3. EQUILIBRIUM CONTRACT WHEN PRICE IS CONTROLLED BY THE PARTY WITH SUPERIOR BARGAINING POWER

Let us assume that the seller has superior bargaining power (β > ½). From (4_), (5), (6) and (6_), the bilateral monopoly operates at B_1 where the seller receives a greater share of the total profits than the buyer (figure 1). Since B_1 does not reside on either the buyer's demand function or the seller's supply function, both parties have incentives of moving the point of operation in order to earn a higher profit. The buyer, being unable to change the price, can restrict purchase of the intermediate product and shift the point of operation to C_1 along his demand curve and derive a higher profit at the expense of the seller.[3] Similarly, the seller can raise the price and shift the point of

---

3. The buyer shifts the point of operation from B_1 to C_1 by reducing the amount of purchase to $q|_{C_1}$ $(<q^*)$, at price p^*. Since, C_1 is on the buyer's demand function and $\frac{\partial \Pi_B}{\partial q} = wf\_(q) - p < 0$ between B_1 and C_1, the buyer's profit is greater at C_1 than at B_1. Furthermore, since $\frac{\partial \Pi_S}{\partial q} = p - c\_(q) >$ 0 between B_1 and C_1 (inclusive), the seller's profit at B_1 is greater than his profit at C_1. Hence in shifting the point of operation from B_1 to C_1, the buyer is better off and the seller is worse off. From the iso-profit curves in figure 1, a movement in the northeast (southwest) increases the seller's (buyer's) profit: $\Pi_S|_{B_1} > \Pi_S|_{C_1}$ and $\Pi_B|_{B_1} < \Pi_B|_{C_1}$.

operation to D_1, obtaining a profit higher than that at $B_1$and giving the buyer a zero profit (figure 2). In fact, given the buyer's profit must always be non-negative (that is, at least his disagreement payoff), the seller's profit is maximized at D_1, as proved in Appendix I.

Let the buyer and seller enter into a multi-period contract and employ the following strategies.

Seller's strategy:

(i) The seller announces his decision to operate at the point $B_1$by fixing price of the intermediate product at p^*.

(ii) In the next period the seller checks if the buyer purchased q^*amount of the intermediate product in the previous period. If the buyer purchased q^*, the seller acts according to his best response to the buyer's strategy. If his best response is repeating (i), he then proceeds to (ii). If the seller's best response is not repeating (i), he deviates from his contracted action (p^*) and increases the price to $p|_{D_1}$(figure 2), which is his only rational alternative. If the buyer did not purchase q^*amount of the intermediate product in (i), the seller proceeds to (iii).

(iii) If the buyer did not purchase q^*amount of intermediate product, the seller triggers a punishment phase for M_1consecutive periods by raising price to $p|_{E_1}$ such that the point of operation shifts to E_1, where$\Pi_B \,\big|_{E_1}$=0 (figures 1 and 2). In the (1+M_1)th period the seller acts according to his best response to the buyer's strategy.

Buyer's strategy:

(i) The buyer's actions are always his best response to the seller's actions.

(ii) If the buyer observes that the seller deviated from operating at B_1 (except during a seller-triggered punishment phase), he triggers a N_1 period punishment phase by purchasing only $q|_{F_1}$ amount of the intermediate product such that the point of operation shifts to F_1 (figure 2). Similar to the definition of C_1, F_1 is a point at which the buyer's (seller's) profit is greater (less) than his profit at D_1.

We now consider the rationality and credibility of the threats issued by the seller and buyer. If a rational threat materializes, one that inflicts the punishment must not be worse off. If a credible threat materializes, the one that is punished must be worse off. Considering only the incentives of each party to deviate from B_1, the consequent threats of punishment, and not taking into account the dynamic nature of the contract, one can specify the following rationality and credibility conditions:

Buyer's threat: $\Pi_S\Big|_{F_1} \leq \Pi_S\Big|_{B_1} \leq \Pi_S\Big|_{D_1}$ and $\Pi_B\Big|_{D_1} \leq \Pi_B\Big|_{F_1}$

(10)

Seller's threat: $\Pi_S\Big|_{C_1} \leq \Pi_S\Big|_{E_1} \leq \Pi_S\Big|_{B_1}$ and $\Pi_B\Big|_{E_1} < \Pi_B\Big|_{B_1} \leq \Pi_B\Big|_{C_1}$.

(10')

The above inequalities can be interpreted as follows: since the seller's profits are maximized at D_1, he has incentive to deviate from B_1 to D_1 by raising the price from p^* to $p|_{D_1}$. The buyer threatens the seller by restricting his purchase to $q|_{F_1}$ and moving the point of operation to F_1. Since F_1 is along the buyer's demand curve, by triggering punishment the buyer maximizes profits at the price $p|_{D_1} = p|_{F_1}$, indicating the buyer's profits at D_1 are less than his profits at F_1 (figure 2). Since the seller's profits are maximized at D_1, his profits at F_1 are less than the profits at D_1. Hence, (10) follows from the definition of credible and rational threats. Similarly, if the buyer deviates from B_1 to C_1, the seller punishes him by shifting the point of operation to E_1, such that the buyer receives zero profits at a point along the demand curve from which the buyer, being unable to control the price, will have no incentive to deviate. Since the buyer's profits are maximized at C_1 and E_1, given price of the intermediate product is fixed at p^* and $p|_{E_1}$ respectively, (10_) follows from the definition of credible and rational threats. Further, if one does not take the dynamic aspect of the contract into account, the seller has incentive of operating at B_1 provided $\Pi_S\,|_{E_1} \leq \Pi_S|_{B_1}$. We now investigate the credibility (and rationality) conditions of threats issued by either party by taking the dynamic aspect of the repeated contract into consideration.

3.1. Credibility of the Buyer's Threat of Punishment

As illustrated in figure 2, if the seller unilaterally deviates from B_1 to D_1 by raising price of the intermediate product from p^* to $p|_{D_1}$, the buyer threatens to punish the seller by purchasing only $q|_{F_1}$ amount of intermediate product for N_1 consecutive periods. Without loss of generality, let us assume that the seller deviates to D_1 during the first period of operation, triggering punishment from the buyer.[4] If a period of deviation, followed by periods of punishment are collectively called an epoch, we assume that the seller deviates to D_1 for T (>0) consecutive epochs, after which he never deviates from B_1. Further, we assume that the seller's threat of punishment is credible, that is, the buyer never unilaterally deviates from B_1. If δ and γ are discount factors, $B(\delta)$ and $S(\gamma)$ are the normalized, discounted, current and future stream of profits of the buyer and seller respectively, we can define:

---

4. Although the seller's strategy indicates that he tries to operate at B_1 in the first period, our assumption that he deviates from B_1 in the first period is only to simplify the exposition. Analysis that the seller begins deviation from the second or third period will produce identical results.

$$\sum_{t=1}^{infinity} \delta^{t-1}B(\delta)=\{\Pi_B\big|_{D_1}+\sum_{t=2}^{N_1+1}\delta^{t-1}\Pi_B\big|_{F_1}\}+\delta^{N_1+1}\{\Pi_B\big|_{D_1}+\sum_{t=2}^{N_1+1}\delta^{t-1}\Pi_B\big|_{F_1}\}+\delta^{2(N_1+1)}\{\Pi_B\big|_{D_1}+\sum_{t=2}^{N_1+1}\delta^{t-1}\Pi_B\big|_{F_1}\}+DOTSAXIS$$
$$+\delta^{(T-1)(N_1+1)}\{\Pi_B\big|_{D_1}+\sum_{t=2}^{N_1+1}\delta^{t-1}\Pi_B\big|_{F_1}\}+\sum_{t=T(N_1+1)+1}^{infinity}\delta^{t-1}\Pi_B\big|_{B_1},$$

(11)

which simplifies to

$$B(\delta)=\{\delta(1-\delta^{N_1})\Pi_B\big|_{F_1}\}\left\{\frac{1-\delta^{T(N_1+1)}}{1-\delta^{(N_1+1)}}\right\}+\delta^{T(N_1+1)}\Pi_B\big|_{B_1} \qquad \text{since } \Pi_B\big|_{D_1}=0.$$

(11')

Similarly,

$$\sum_{t=1}^{infinity} \gamma^{t-1}S(\gamma)=\{\Pi_S\big|_{D_1}+\sum_{t=2}^{N_1+1}\gamma^{t-1}\Pi_S\big|_{F_1}\}+\gamma^{N_1+1}\{\Pi_S\big|_{D_1}+\sum_{t=2}^{N_1+1}\gamma^{t-1}\Pi_S\big|_{F_1}\}+\gamma^{2(N_1+1)}\{\Pi_S\big|_{D_1}+\sum_{t=2}^{N_1+1}\gamma^{t-1}\Pi_S\big|_{F_1}\}+DOTSAXIS$$
$$+\gamma^{(T-1)(N_1+1)}\{\Pi_S\big|_{D_1}+\sum_{t=2}^{N_1+1}\gamma^{t-1}\Pi_S\big|_{F_1}\}+\sum_{t=T(N_1+1)+1}^{infinity}\gamma^{t-1}\Pi_S\big|_{B_1},$$

(12)

which simplifies to

$$S(\gamma)=\{(1-\gamma)\Pi_S\big|_{D_1}+\gamma(1-\gamma^{N_1})\Pi_S\big|_{F_1}\}\left\{\frac{1-\gamma^{T(N_1+1)}}{1-\gamma^{(N_1+1)}}\right\}+\gamma^{T(N_1+1)}\Pi_S\big|_{B_1}. \qquad (12')$$

If the buyer never punishes the seller, the rational seller always operates at D_1causing $B(\delta)=\Pi_B\big|_{D_1}=0$. Since the buyer's threat is rational, i. e., by punishing the seller he is not made worse off, $B(\delta)$from (11_) is required to be non-negative. This is satisfied since both $\Pi_B \big|_{F_1}$ and $\Pi_B \big|_{B_1}$ are positive. Further, credibility of the buyer's threat requires the seller to be worse off as a result of the punishment, that is, $S(\gamma) < \Pi_S\big|_{B_1}$,i.e.,

$$\begin{aligned} S(\gamma) &= \{(1-\gamma)\Pi_S\big|_{D_1} + \gamma(1-\gamma^{N_1})\Pi_S\big|_{F_1}\}\left\{\frac{1-\gamma^{T(N_1+1)}}{1-\gamma^{(N_1+1)}}\right\} + \gamma^{T(N_1+1)}\Pi_S\big|_{B_1} < \Pi_S\big|_{B_1} \\ &\Rightarrow \{(1-\gamma)\Pi_S\big|_{D_1} + \gamma(1-\gamma^{N_1})\Pi_S\big|_{F_1}\}\left\{\frac{1-\gamma^{T(N_1+1)}}{1-\gamma^{(N_1+1)}}\right\} < (1-\gamma^{T(N_1+1)})\Pi_S\big|_{B_1} \end{aligned} \tag{13}$$

$$\frac{(1-\gamma)\Pi_S\big|_{D_1} + \gamma(1-\gamma^{N_1})\Pi_S\big|_{F_1}}{1-\gamma^{(N_1+1)}} < \Pi_S\big|_{B_1}. \tag{13'}$$

The left hand side of (13_) is a weighted average of $\Pi_S \big|_{D_1}$ and $\Pi_S \big|_{F_1}$.[5] Since from the definition

---

5. If the seller deviates for an infinite number of epochs, his payoff is given by:

$$\sum_{t=1}^{infinity} \gamma^{t-1}S(\gamma) = \{\Pi_S\big|_{D_1} + \sum_{t=2}^{N_1+1}\gamma^{t-1}\Pi_S\big|_{F_1}\} + \gamma^{N_1+1}\{\Pi_S\big|_{D_1} + \sum_{t=2}^{N_1+1}\gamma^{t-1}\Pi_S\big|_{F_1}\} + \gamma^{2(N_1+1)}\{\Pi_S\big|_{D_1} + \sum_{t=2}^{N_1+1}\gamma^{t-1}\Pi_S\big|_{F_1}\} + DOTSAXIS$$

which simplifies to

$$S(\gamma) = (1-\gamma)\{\Pi_S\big|_{D_1} + \sum_{t=2}^{N_1+1}\gamma^{t-1}\Pi_S\big|_{F_1}\} + \{1 + \gamma^{N_1+1} + \gamma^{2(N_1+1)} + DOTSAXIS\}$$

which simplifies to

of D_1, $\Pi_S \big|_{D_1}$ is greater than $\Pi_S \big|_{B_1}$, a necessary condition for $S(\gamma)$ to be less than $\Pi_S \big|_{B_1}$ is $\Pi_S \big|_{F_1} <$ $\Pi_S \big|_{B_1}$ which simplifies (13_) to:

$$\gamma^{N_1+1} \leq \frac{(1-\gamma)\Pi_S\big|_{D_1} + \gamma\Pi_S\big|_{F_1} - \Pi_S\big|_{B_1}}{\Pi_S\big|_{F_1} - \Pi_S\big|_{B_1}} \text{ or} \tag{14}$$

$$N_1 \geq \frac{1}{\ln\gamma} \cdot \ln\left[\frac{(1-\gamma)\Pi_S\big|_{D_1} + \gamma\Pi_S\big|_{F_1} - \Pi_S\big|_{B_1}}{\Pi_S\big|_{F_1} - \Pi_S\big|_{B_1}}\right] - 1. \tag{14'}$$

The necessary condition for credibility of the buyer's threat of punishment, ($\Pi_S \big|_{F_1} < \Pi_S \big|_{B_1}$), imposes restrictions on the bilateral monopoly and require further qualification. If $\beta > ½$, it is clear from (6) that the seller's profit at B_1 is greater than that at *A*. Hence, it is sufficient to show that the seller's profit increases as we move from F_1 to *A* along the demand curve, in order for $\Pi_S \big|_{F_1}$ to be less than $\Pi_S \big|_{B_1}$. To examine the change in the seller's profit as the point of operation shifts from F_1 to *A* along the demand curve, we evaluate the directional derivative of $\Pi_S(p, q)$ along the demand curve in the direction of *A* from F_1 (figure 3). If $p = wf\_\_(q_0)q + [p_0 - wq_0 f\_\_(q_0)]$ is the

---

$$S(\gamma) = \frac{(1-\gamma)\,\Pi_S\big|_{D_1} + \gamma(1-\gamma^{N_1})\,\Pi_S\big|_{F_1}}{1-\gamma^{N_1+1}}.$$

Hence, if (13_) is satisfied, the seller never deviates for an infinite number of epochs.

equation of the tangent to the demand curve at the point (p_0, q_0), which makes an angle θ_1 to the positive abscissa in figure 3:

$$\mathrm{Tan}(\theta_1) = wf'(q_0) < 0 \text{ and } \mathrm{Sin}(\theta_1) = \frac{-wf'(q_0)}{\sqrt{1+w^2 f'^2(q_0)}} > 0, \text{ because } \pi/2 < \theta_1 < \pi. \tag{15}$$

The directional derivative of Π_S along the unit vector $u_1 = [\cos\theta_1]i + [\sin\theta_1]j$ in figure 3 at any point (p, q) is given by:

$$\frac{\partial \Pi_S}{\partial u_1} = \left(\frac{\partial \Pi_S}{\partial q}\frac{1}{\mathrm{Tan}\theta_1} + \frac{\partial \Pi_S}{\partial p}\right)\mathrm{Sin}\theta_1 = \left(\frac{p-c'(q)}{wf'(q)} + q\right)\left(\frac{-wf'(q)}{\sqrt{1+(wf'(q))^2}}\right) \tag{16}$$

Since $f'(q) < 0$, $\frac{p-c'(q)}{wf'(q)} < 0$ along the demand curve between F_1 and A. For small enough $|wf'(q)|$, (16) is negative for all price and quantity profiles along the demand curve from A to F_1 (exclusive of A). Thus, if $|wf'(q)|$ is small enough, say for a sufficiently elastic demand curve, the seller's profit increases along the demand curve from F_1 to A, indicating the condition $\Pi_S|_{F_1} \leq \Pi_S|_{B_1}$ is satisfied. Therefore, given that the buyer's threat is credible in every time period (i. e., $\Pi_S|_{F_1} \leq \Pi_S|_{B_1}$), and the length of the buyer's punishment phase is long enough (as per $(14')$), if the buyer operates at B_1, the seller's best response to the buyer's strategy is to also operate at B_1.

3.2. Credibility of the Seller's Threat of Punishment

As illustrated in figure 1, if the buyer deviates from B_1 to C_1 by restricting the quantity of intermediate product purchased at from q^* to $q|_{C_1}$ price p^*, the seller threatens to punish the buyer for

M_1 consecutive periods by raising prices to $p|_{E_1}$ such that $\Pi_B|_{E_1}=0$. We assume that the buyer deviates to C_1 during the first period of operation, triggering punishment from the seller. We also assume that the buyer deviates for T (>0) consecutive epochs, after which he never deviates from B_1. Further, the buyer's threat of punishment is credible, i. e., the seller never unilaterally deviates from B_1. Hence, $B(\delta)$ $S(\gamma)$ are defined as follows.

$$\sum_{t=1}^{infinity} \delta^{t-1}B(\delta)=\{\Pi_B|_{C_1}+\sum_{t=2}^{M_1+1}\delta^{t-1}\Pi_B|_{E_1}\}+\delta^{M_1+1}\{\Pi_B|_{C_1}+\sum_{t=2}^{M_1+1}\delta^{t-1}\Pi_B|_{E_1}\}+\delta^{2(M_1+1)}\{\Pi_B|_{C_1}+\sum_{t=2}^{M_1+1}\delta^{t-1}\Pi_B|_{E_1}\}+\dots$$
$$+\delta^{(T-1)(M_1+1)}\{\Pi_B|_{C_1}+\sum_{t=2}^{M_1+1}\delta^{t-1}\Pi_B|_{E_1}\}+\sum_{t=T(M_1+1)+1}^{infinity}\delta^{t-1}\Pi_B|_{B_1},$$

(17)

which simplifies to

$$B(\delta)=\left\{\frac{1-\delta^{T(M_1+1)}}{1-\delta^{(M_1+1)}}\right\}(1-\delta)\Pi_B|_{C_1}+\delta^{T(M_1+1)}\Pi_B|_{B_1} \text{ since } \Pi_B|_{E_1}=0.$$

(17')

Similarly,

$$\sum_{t=1}^{infinity} \gamma^{t-1}S(\gamma)=\{\Pi_{S\,0}+\sum_{t=2}^{M_1+1}\gamma^{t-1}\Pi_S|_{E_1}\}+\gamma^{M_1+1}\{\Pi_{S\,0}+\sum_{t=2}^{M_1+1}\gamma^{t-1}\Pi_S|_{E_1}\}+\gamma^{2(M_1+1)}\{\Pi_{S\,0}+\sum_{t=2}^{M_1+1}\gamma^{t-1}\Pi_S|_{E_1}\}+\dots$$
$$+\gamma^{(T-1)(M_1+1)}\{\Pi_{S\,0}+\sum_{t=2}^{M_1+1}\gamma^{t-1}\Pi_S|_{E_1}\}+Sigmafromt=T(M_1+1)+1?infinity\gamma^{t-1}\Pi_S|_{B_1}, \quad (18)$$

which simplifies to

$$S(\gamma)=\{(1-\gamma)\Pi_{S\,0}+\gamma(1-\gamma^{M_1})\Pi_S\big|_{E_1}\}\left\{\frac{1-\gamma^{T(M_1+1)}}{1-\gamma^{(M_1+1)}}\right\}+\gamma^{T(M_1+1)}\Pi_S\big|_{B_1}. \tag{18'}$$

In (18) and (18') , assuming that the buyer does not pre-announce his deviations from B_1, $\Pi_{S\,0}=p^*q|_{C_1}-c(q^*)$ represents the seller's profit when he produces q^* to be sold at the price p^*, of which the buyer only purchases $q|_{C_1}$. Since $q|_{C_1}<q^*$, $\Pi_{S\,0}$ is less than $\Pi_S\big|_{C_1}$.

In order to show rationality and credibility of the seller's threat, we derive conditions which makes the seller not worse off and the buyer worse off if the seller's threat of punishment is triggered. If the seller never punishes the buyer, the latter consistently operates at C_1resulting in S(γ) to be $\Pi_{S\,0}$. Similarly, if the buyer never deviates from B_1, $B(\delta)=\Pi_B\big|_{B_1}$. Hence, the rationality and credibility conditions for the seller's threat are:

$$S(\gamma)\geq\Pi_{S\,0}\text{and } B(\delta)<\Pi_B\big|_{B_1}. \tag{19}$$

In order to show $S(\gamma)\geq\Pi_{S\,0}$, from (18_):

$$\{(1-\gamma)\Pi_{S\,0}+\gamma(1-\gamma^{M_1})\Pi_S\big|_{E_1}\}\left\{\frac{1-\gamma^{T(M_1+1)}}{1-\gamma^{(M_1+1)}}\right\}+\gamma^{T(M_1+1)}\Pi_S\big|_{B_1}\geq\Pi_{S\,0} \tag{20}$$

which simplifies to

$$\gamma(1-\gamma^{M_1})\left\{\frac{1-\gamma^{T(M_1+1)}}{1-\gamma^{T(M_1+1)}}\right\}\Pi_S\big|_{E_1}+\gamma^{T(M_1+1)}\Pi_S\big|_{B_1}\geq\left[1-(1-\gamma)\frac{1-\gamma^{T(M_1+1)}}{1-\gamma^{(M_1+1)}}\right]\Pi_{S\,0}. \tag{20'}$$

From the definition of C_1, it is clear that $\Pi_S\big|_{B_1}\geq\Pi_S\big|_{C_1}\geq\Pi_{S\,0}$. It is noteworthy that as γ approaches one, the difference in the coefficients of $\Pi_S\big|_{B_1}$and $\Pi_{S\,0}$ approaches zero:

$$0 > \left[\gamma^{T(M_1+1)} - 1 + (1-\gamma)\frac{1-\gamma^{T(M_1+1)}}{1-\gamma^{(M_1+1)}}\right] \rightarrow 0 \text{ as } \gamma \rightarrow 1. \tag{21}$$

Therefore, (20') is satisfied for large enough γ. Clearly, for (20') to hold, the condition $\Pi_S \big|_{E_1} \geq \Pi_S\big|_{C_1} \geq \Pi_{S0}$ from (10'), is not necessary. In other words, if one takes the dynamic nature of the contract into account, the seller's threat is credible even if the threat is not rational to him in each time period, i. e.,$\Pi_S \big|_{E_1} < \Pi_{S0}$.

In order to show $B(\delta) < \Pi_B\big|_{B_1}$, we note from (17'):

$$\left\{\frac{1-\delta^{T(M_1+1)}}{1-\delta^{(M_1+1)}}\right\}(1-\delta)\Pi_B\big|_{C_1}+\delta^{T(M_1+1)}\Pi_B\big|_{B_1}<\Pi_B\big|_{B_1}$$
$$\Rightarrow\left\{\frac{1-\delta^{T(M_1+1)}}{1-\delta^{(M_1+1)}}\right\}(1-\delta)\Pi_B\big|_{C_1}<\{1-\delta^{T(M_1+1)}\}\Pi_B\big|_{B_1}$$
$$\Rightarrow\left\{\frac{1-\delta}{1-\delta^{(M_1+1)}}\right\}\Pi_B\big|_{C_1}<\Pi_B\big|_{B_1}$$
$$\Rightarrow\delta^{1+M_1}<1-(1-\delta)\frac{\Pi_B\big|_{C_1}}{\Pi_B\big|_{B_1}} \tag{22}$$

The right-hand side of the above simplified inequality is positive if δ is close enough to one. Hence, we conclude that the rationality and credibility conditions for the seller's threat are:

$$M_1>\frac{1}{\ln\delta}\cdot\ln\left[1-(1-\delta)\frac{\Pi_B\big|_{C_1}}{\Pi_B\big|_{B_1}}\right]-1 \tag{22'}$$

and both δ and γ are close enough to one. Thus, if the length of the seller's punishment phase is long enough (as per (22')) and both parties impute sufficiently high current value to their future income (i.e., both δ and γ are close enough to one), if the seller operates at B_1, the buyer's best response is to also operate at B_1. Following an analysis similar to that in endnote 5, we can show that if (22) is satisfied, the buyer will not deviate for an infinite number of epochs.

Proposition 1. Given that the seller has superior bargaining power ($\beta > ½$), $f\_\geq 0$, and $f\_\_<0$, the jointly negotiated point of operation B_1 is a Nash equilibrium provided the following conditions are satisfied:

(i) $N_1 \geq \frac{1}{\ln\gamma}\cdot\ln\left[\frac{(1-\gamma)\Pi_S|_{D_1}+\gamma\Pi_S|_{F_1}-\Pi_S|_{B_1}}{\Pi_S|_{F_1}-\Pi_S|_{B_1}}\right]-1$, i.e., the buyer's punishment phase is long enough,

(ii) $M_1 \geq \frac{1}{\ln\delta}\cdot\ln\left[1-(1-\delta)\frac{\Pi_B|_{C_1}}{\Pi_B|_{B_1}}\right]-1$, i. e., the seller's punishment phase is long enough,

(iii) $\Pi_S|_{F_1}<\Pi_S|_{B_1}$, which is satisfied when $|wf\_\_(q)|$ is small enough such that $\partial\Pi_S/\partial u_1<0$ for all (p, q) between A and F_1, where $u_1=[\cos\theta\_\_\_\_i+[\sin\theta\_\_\_\_j$, $\theta_1=$Arctan[$wf\_\_(q)$] (figure 3), and

(iv) the seller's and buyer's discount factors, γ and δ, are close enough to 1, i. e., both parties do not highly devalue their future income.

Corollary 1. Among the static credibility conditions outlined in (10) and (10'), only $\Pi_S|_{F_1}\leq\Pi_S|_{B_1}$ needs to be satisfied in the repeated contract. The remaining conditions either follow from definitions of points C_1, D_1, E_1 and F_1 or are not required for a Nash equilibrium at B_1.

Proof. Some of the inequalities contained in conditions (10) and (10_) follow from the definitions of points C_1, D_1, E_1 and F_1. Namely, the following inequalities are always satisfied:

from the definition of C_1: $\Pi_S|_{C_1}\leq\Pi_S|_{B_1}$ and $\Pi_B|_{C_1}\geq\Pi_B|_{B_1}$, from the definition of D_1: $\Pi_S\big|_{B_1} \leq \Pi_S\big|_{D_1} = \Pi_V(q^*)$ and $\Pi_B\big|_{B_1} \geq \Pi_B\big|_{D_1} = 0$,

from the definition of E_1: $\Pi_S\big|_{E_1} = \Pi_V(q|_{E_1}) \leq \Pi_V(q^*) = \Pi_S\big|_{D_1}$ and $\Pi_B\big|_{E_1} = 0$, and

from the definition of F_1: $\Pi_S\big|_{F_1} \leq \Pi_S\big|_{D_1}$ and $\Pi_B\big|_{F_1} \geq \Pi_B\big|_{D_1} = \Pi_B\big|_{E_1} = 0$.

The following inequalities given in (10) and (10_) are not implied from the above definitions:

$$\Pi_S\big|_{C_1} \leq \Pi_S\big|_{E_1},\ \Pi_S\big|_{E_1} \leq \Pi_S\big|_{B_1},\ \text{and } \Pi_S\big|_{F_1} \leq \Pi_S\big|_{B_1}. \tag{23}$$

Among these inequalities, $\Pi_S\big|_{F_1} \leq \Pi_S\big|_{B_1}$ is a necessary condition for a Nash equilibrium to exist at B_1, and it indicates that, if the seller has superior bargaining power, the buyer's threat must be credible to the seller in every period of time. The remaining inequalities in (23) are not necessary for a Nash equilibrium at B_1, and hence, we can draw the following conclusions:

> $\Pi_S\big|_{C_1} \leq \Pi_S\big|_{E_1}$ indicates that the dynamic contract and the strategies of both parties are such that when the buyer deviates from B_1, the seller is better off triggering punishment and shifting the point of operation to E_1, even if in a single period the seller is better off not triggering punishment and allowing the buyer to deviate;
>
> $\Pi_S\big|_{E_1} \leq \Pi_S\big|_{B_1}$ indicates that even if the seller is better off by triggering punishment than operating at B_1(i. e., if $\Pi_S\big|_{E_1} > \Pi_S\big|_{B_1}$), the dynamic contract and the buyer's strategy prevents the seller from making any attempt to operate at E_1other than that warranted by his strategy.
>
> **Q. E. D.**

Hence, rationality of seller's threat of deviating to E_1, as defined by the inequalities: $\Pi_S\big|_{C_1} \leq \Pi_S\big|_{E_1}$ and $\Pi_S\big|_{E_1} \leq \Pi_S\big|_{B_1}$, is not a necessary condition for a Nash equilibrium at B_1.

When the buyer has superior bargaining power ($\beta < ½$), from (4_)and (5), the negotiated point of operation (B_2) is below the point A (figures 4 and 5). There is a reversal of the buyer's and seller's role when compared to the above case (β > ½) and is reflected in the reversal of strategies and incentives of both parties. For example, the buyer's strategy consists of first announcing his intention to operate at B_2 by fixing price at p^*. If the seller is detected to deviate from B_2, the buyer triggers a M_2-period punishment phase by lowering price to $p|_{E_2}$ where the seller's profits are zero (figure 4). The seller's strategy consists of playing his best response to the buyer's strategy. If the buyer is detected to deviate from B_2 (other than in triggering punishment), the seller triggers a N_2 -period punishment phase by supplying only $q|_{F_2}$ amount of the intermediate product (figure 5). Therefore, due to the reversal of the buyer's and seller's role, analysis of credibility of threats of punishment and contract terms are similar to the above analysis of β > ½, and is not presented for the sake of brevity. Here, we present only the results.

Proposition 2. Given the buyer has superior bargaining power (β < ½), $c\tilde{\imath}$____≥0and c___≥0, there exists a Nash equilibrium at B_2, provided the following conditions are satisfied:

(i) $N_2 \geq \frac{1}{\ln\delta}\cdot\ln\left[\frac{(1-\delta)\Pi_B|_{D_2}+\delta\Pi_B|_{F_2}-\Pi_B|_{B_2}}{\Pi_B|_{F_2}-\Pi_B|_{B_2}}\right]-1$, i. e., the seller's punishment phase is long enough,

(ii) $M_2 \geq \frac{1}{\ln\gamma}\cdot\ln\left[1-(1-\gamma)\frac{\Pi_S|_{C_2}}{\Pi_S|_{B_2}}\right]-1$, i. e., the buyer's punishment phase is long enough,

(iii) $\Pi_B|_{F_2}<\Pi_B|_{B_2}$, which is satisfied when c___(q) is sufficiently close to zero for points along the supply curve from F_2 to A, such that $\partial\Pi_B/\partial u_2>0$ for all (p, q) between F_2 and A, where u_2= [$cos\tilde{\imath}$____i + [$sin\tilde{\imath}$____j is the tangent vector to a point on the supply curve in the direction of A from F_2, and θ_2=arctan[c__(q)] (figure 6). This condition is equivalent to credibility of the seller's threat to the buyer in *every* time period,

(iv) the seller and buyer's discount factors, γ and δ, are close enough to 1, i. e., both parties do not highly devalue their future income.

4. EQUILIBRIUM CONTRACT WHEN PRICE IS CONTROLLED BY THE PARTY WITH INFERIOR BARGAINING POWER

Consistent with a Bowley price leadership model, we now consider the case when the party with inferior bargaining power controls the price of the intermediate product. We first assume that the buyer has inferior bargaining power ($\beta > ½$). Our goal is to develop a multi-period contract that induces a Nash equilibrium at B_1.

Since the seller can only shift the point of operation by changing the quantity of the intermediate product, assuming the buyer never deviates from B_1, the seller never produces more than q^* amount of the intermediate product. Given that the price remains fixed, if the seller produces less than q^*, his profits are reduced because of the following condition:

$$\frac{\partial \Pi_S}{\partial q} = \frac{\partial [pq - c(q)]}{\partial q} = p - c\_(q) > 0, \text{ for all } q \leq q^*. \tag{24}$$

Thus, if the buyer never deviates from B_1, it is in the interest of the seller not to deviate from B_1.

The buyer has the option of deviating from B_1 by lowering the price of the intermediate product. A rational buyer lowers the price to $p|_{D_2}$ such that his profits are maximized at D_2 (Appendix I) where the seller receives a zero profit (figure 7). The seller has the option to threaten punishment by reducing the amount of the intermediate product from q^*. If the seller implements the threat by producing q ($<q^*$) amount of the intermediate product, a rational buyer adjusts the price

such that the point of operation is along the seller's zero iso-profit curve where the buyer's profits are $\Pi_V(q)$ and the seller's profits are zero. Clearly, for all $q<q^*$, if $\Pi_V(q)$is greater than the buyer's profits at B_1 (i.e., $(1-\beta)\Pi_V(q^*)$), the seller's threat is not credible to the buyer and a Nash equilibrium at $B_1$is impossible. Thus, a necessary condition for inducing a Nash equilibrium at $B_1$is that the seller's threat must be credible to the buyer in *each* time period. This is achieved if there exists a $q|_{G_1}<q^*$ such that $\Pi_V(q|_{G_1})<(1-\beta)\Pi_V(q^*)$(figure 7). Assuming such a $q|_{G_1}$exists, we specify strategies of the two parties in a multi-period contract and evaluate conditions for inducing a Nash equilibrium at B_1.

The buyer's strategy is to act according to his best response to the seller's strategy. The seller's strategy is to operate at $B_1$by producing q^*amount of the intermediate product. If the seller detects that the buyer has lowered the price of the intermediate product from p^*, a M_3-period punishment is triggered where the seller limits production of the intermediate product to $q|_{G_1}$ and the point of operation shifts to G_1(figure 7). At G_1, the buyer and seller earn $\Pi_V(q|_{G_1})$and zero profits respectively. At the end of the punishment phase, the seller reverts to producing q^*amount of the intermediate product.

Assuming the buyer deviates to D_2 for T (> 0) consecutive epochs after which he never deviates from B_1, $B(\delta)$and $S(\gamma)$are such that:

$$\sum_{t=1}^{infinity} \delta^{t-1}B(\delta)=\{\Pi_B|_{D_2}+\sum_{t=2}^{M_3+1}\delta^{t-1}\Pi_B|_{G_1}\}+\delta^{M_3+1}\{\Pi_B|_{D_2}+\sum_{t=2}^{M_3+1}\delta^{t-1}\Pi_B|_{G_1}\}$$
$$+\delta^{2(M_3+1)}\{\Pi_B|_{D_2}+\sum_{t=2}^{M_3+1}\delta^{t-1}\Pi_B|_{G_1}\}+\ldots \tag{25}$$
$$+\delta^{(T\text{-}1)(M_3+1)}\{\Pi_B|_{D_2}+\sum_{t=2}^{M_3+1}\delta^{t-1}\Pi_B|_{G_1}\}+\sum_{t=T(M_3+1)+1}^{infinity}\delta^{t-1}\Pi_B|_{B_1},$$

which simplifies to

$$B(\delta)=\left\{\frac{1\text{-}\delta^{T(M_3+1)}}{1\text{-}\delta^{(M_3+1)}}\right\}\{(1\text{-}\delta)\Pi_B|_{D_2}+\delta(1\text{-}\delta^{M_3})\Pi_B|_{G_1}\}+\delta^{T(M_3+1)}\Pi_B|_{B_1}.$$

(25')

$$\sum_{t=1}^{infinity}\gamma^{t-1}S(\gamma)=\{\Pi_S|_{D_2}+\sum_{t=2}^{M_3+1}\gamma^{t-1}\Pi_S|_{G_1}\}+\gamma^{M_3+1}\{\Pi_S|_{D_2}+\sum_{t=2}^{M_3+1}\gamma^{t-1}\Pi_S|_{G_1}\}+\gamma^{2(M_3+1)}\{\Pi_S|_{D_2}+\sum_{t=2}^{M_3+1}\gamma^{t-1}\Pi_S|_{G_1}\}+\ldots$$
$$+\gamma^{(T\text{-}1)(M_3+1)}\{\Pi_S|_{D_2}+\sum_{t=2}^{M_3+1}\gamma^{t-1}\Pi_S|_{G_1}\}+\sum_{t=T(M_3+1)+1}^{infinity}\gamma^{t-1}\Pi_S|_{B_1},$$

(26)

which simplifies to

$$S(\gamma)=\gamma^{T(M_3+1)}\Pi_S|_{B_1}\text{because } \Pi_S|_{D_2}=\Pi_S|_{G_1}=0.$$

(26')

If the seller did not punish the buyer, the buyer would be operating at D_2 where the seller's profits are zero. Therefore, the seller's threat is credible if the following conditions are satisfied:

$$S(\gamma) > 0 \text{ and } B(\delta) < \Pi_B\big|_{B_1}. \tag{27}$$

From (26_), if $\gamma > 0$, $S(\gamma) > 0$. The condition $B(\delta) < \Pi_B\big|_{B_1}$ simplifies to:

$$\frac{(1\text{-}\delta)\Pi_B\big|_{D_2} + \delta(1\text{-}\delta^{M_3})\Pi_B\big|_{G_1}}{1\text{-}\delta^{M_3+1}} < \Pi_B\big|_{B_1}. \tag{28}$$

Since $\Pi_B\big|_{D_2} = \Pi_V(q^*) > (1\text{-}\beta)\Pi_V(q^*) = \Pi_B\big|_{B_1}$, it is necessary for $\Pi_B\big|_{G_1} < \Pi_B\big|_{B_1}$ in order to satisfy (28). However, additional necessary conditions derived from (28) are: δ is close enough to one and

$$M_3 > \frac{1}{\ln\delta}\cdot\ln\left[\frac{\Pi_B\big|_{B_1}\text{-}(1\text{-}\delta)\Pi_B\big|_{D_2}\text{-}\delta\Pi_B\big|_{G_1}}{\Pi_B\big|_{B_1}\text{-}\Pi_B\big|_{G_1}}\right]\text{-}1. \tag{29}$$

Following an analysis similar to that in endnote 5, we can show that if (28) is satisfied, the buyer will not deviate for an infinite number of epochs.

Proposition 3. Given that the party with the weaker (stronger) bargaining power controls the price (quantity) of the intermediate product, if β > ½, a Nash equilibrium at B_1 is possible if there exists a $q\big|_{G_1} < q^*$ such that the seller's threat of limiting production of the intermediate product to $q\big|_{G_1}$ (and thus shifting the point of operation to G_1 as in figure 7) is credible to the buyer in every time period. Further, if such a $q\big|_{G_1}$ exists, the following conditions are necessary and sufficient for a Nash equilibrium at B_1:

$$\Pi_B\Big|_{G_1} < \Pi_B\Big|_{B_1},\ \delta \text{ close enough to 1, and } M_3 > \frac{1}{\ln\delta}\cdot\ln\left[\frac{\Pi_B\big|_{B_1} - (1-\delta)\Pi_B\big|_{D_2} - \delta\Pi_B\big|_{G_1}}{\Pi_B\big|_{B_1} - \Pi_B\big|_{G_1}}\right] - 1,$$

where M_3 is the length of the seller's punishment phase.

The case where β < ½ is similar to the above case, and thus, for sake of brevity, we present only the results. If β < ½ and the seller controls the price while the buyer controls quantity, by following arguments similar to those for β > ½, we can show that the buyer never unilaterally deviates from B_2 (figure 8). The seller has the option to deviate from B_2 by raising the price to $p\big|_{D_1}$, where the seller's profits are maximized, $\Pi_S\Big|_{D_1} = \Pi_V(q^*)$, and the buyer's profits are zero. Further, we can show that a Nash equilibrium at $B_2$is not possible unless there exists a $q\big|_{G_2} < q^*$such that $\Pi_S\Big|_{G_2} < \Pi_S\big|_{B_2}$. Hence, if the seller deviates from operating at $B_2$to D_1, the buyer has the option of punishing the seller by purchasing only $q\big|_{G_2}$amount of the intermediate product (figure 8). Given such a $q\big|_{G_2}$exists, the following proposition gives the conditions for a Nash equilibrium at B_2.

Proposition 4. If β < ½ and the seller controls the price of the intermediate product, a Nash equilibrium at B_2 is possible if there exists a $q\big|_{G_2} < q^*$such that the seller's profit at G_2 is less than his profit at B_2 in every time period (figure 8). Given such a $q\big|_{G_2}$ exists and the buyer threatens to punish the seller for N_3 consecutive periods by shifting the point of operation to G_2 if the seller is found to deviate from B_2, the following conditions are necessary and sufficient for a Nash equilibrium to exist at B_2:

$$\Pi_S\Big|_{G_2} < \Pi_S\Big|_{B_2},\ \gamma \text{ close enough to 1 and } N_3 > \frac{1}{\ln\gamma}\cdot\ln\left[\frac{\Pi_S\big|_{B_2} - (1-\gamma)\Pi_S\big|_{D_1} - \gamma\Pi_S\big|_{G_2}}{\Pi_S\big|_{B_2} - \Pi_S\big|_{G_2}}\right] - 1.$$

5. CONCLUSIONS

In his study, Coursey (1982: 258) states that in bilateral monopolies ". . . a large number of negotiated contracts occur around the joint maximizing quantity (or the solution to the joint objective)". Here, we develop contracts that induce a Nash equilibrium at jointly negotiated points of operation in bilateral monopolies regardless of (i) which party has greater bargaining power and (ii) which party controls the price or quantity of the intermediate product. We also resolve the issue of price indeterminacy in negotiating a point of operation in a bilateral monopoly.

If bargaining powers of both parties are equal, Truett and Truett's result of an equilibrium point of operation, determined by the simultaneous satisfaction of the best-response function of both parties, is upheld. However, in real world negotiations, since both parties do not always possess equal bargaining powers, a determinate point of operation based on optimizing a joint objective can be an equilibrium provided both parties enter into a multiple-period contract, pursue the requisite strategies and the following conditions are satisfied.

If the party with the stronger bargaining power controls the price of the intermediate product or service, both parties have incentive to deviate from their jointly negotiated point of operation. Here, equilibrium conditions require that both parties should impute a sufficiently high current value to their future income, the length of the punishment phases should be long enough, and the party with stronger bargaining power must consider the other party's threat of punishment credible in every time period. This condition prevents the party with stronger bargaining power from setting the price for the intermediate product to its own profit maximizing level.

If the party with stronger bargaining power controls quantity, only the party with weaker bargaining power has incentive to deviate from the jointly negotiated point of operation. Here, the equilibrium conditions require that the party with the weaker bargaining power must impute sufficiently high current value to its future income, the length of the punishment phase threatened by the party with stronger bargaining power must be long enough, and the party with weaker bargaining power must consider the threat of punishment issued by the other party credible in every time period. The last condition prevents the party with weaker bargaining power from setting the price to its own-profit maximizing level.

Union-management organization in some industries indicates a bilateral monopoly where the intermediate product is often the services of union members. In such cases, real-world examples of bargaining between union and management give a strong indication that one party has a slightly superior bargaining power. Research in union-management bargaining often indicates that firms typically retain the right to determine the number of workers to hire. In the context of our study this means that management often controls quantity while union controls the price. The multi-period contracts outlined in this study give equilibrium employment and wage terms that are mutually agreeable, irrespective of bargaining power of either party.

APPENDIX I

Proposition. The seller's profit is maximized at D_1 under the constraint that the buyer's profit must be non-negative.

Proof: Given that the buyer's zero iso-profit curve is defined by $\Pi_B = wf(q) - pq = 0$ or $p = wf(q)/q$, the seller's profit along the buyer's zero iso-profit curve is given by $\Pi_S = pq - c(q) = \{wf(q)/q\}q - c(q) = wf(q) - c(q) = \Pi_V(q)$. From (7) and (5), profits of the vertically integrated firm are maximized when q^* amount of the intermediate product is transacted. Thus, the seller's profit, along the buyer's zero iso-profit curve, is maximized at the point where the buyer's zero iso-profit curve intersects the contract curve $q = q^*$. Let us consider any point G, characterized by the quantity and price profile $(q|_G, p|_G)$, such that the buyer's profit is non-negative at G.

Since $\left.\frac{\partial \Pi_S}{\partial p}\right|_G = q|_G \geq 0$, there exists a point $G\_$ on the buyer's zero iso-profit curve such that $q|_G = q|_{G\_}$, $p|_G \leq p|_{G\_}$, and $\Pi_S|_G \leq \Pi_S|_{G\_}$. Since $G\_$ is a point on the buyer's zero iso-profit curve, $\Pi_S|_{G\_} \leq \Pi_V(q^*) = \Pi_S|_{D_1}$. Therefore, for any point G for which the buyer's profit is non-negative, $\Pi_S|_{G\_} \leq \Pi_S|_{D_1}$, that is the seller's profit is maximized at D_1. **Q. E. D.**

The above arguments can be analogously extended to show that the buyer's profit is maximized at D_2 (figure 5), provided the seller's profit is always non-negative.

REFERENCES

Bandyopadhyay, S. C., “Wage and Employment Negotiations Between a Union and a Firm in a Dynamic Context,” *Southern Economic Journal*, October 1995, 348-358.

Blair, R. D., Kaserman, D. L., and Romano, R. E., “A Pedagogical Treatment of Bilateral Monopoly,” *Southern Economic Journal*, April 1989, 831-840.

Binmore, K., Rubinstein, A. and Wolinsky, A., “The Nash Bargaining Solution in Economic Modeling,” *RAND Journal of Economics* 17, Summer 1986, 176-88.

Bowley, A. L., “Bilateral Monopoly,” *Economic Journal* 38, December 1928, 651-59.

Coursey, D., “Bilateral Bargaining, Pareto Optimality, and the Empirical Frequency of Impasse,” *Journal of Economic Behavior and Organization* 3, June-September 1982, 243-59.

Devadoss, S., and Cooper, K., “Simultaneous Price and Quantity Determination in a Joint Profit Maximizing Bilateral Monopoly under Dynamic Optimization,” *International Economic Journal* 14(1), Spring 2000, 71-84.

Devadoss, S., “Dynamic Analysis of Price Determination under Joint Profit Maximization in Bilateral Monopoly,” Paper Presented at the 37th Annual Missouri Valley Economic Association, February 22-24, 2001, Kansas City.

Espinosa, M. P. and Rhee, C., “Efficient Wage Bargaining as a Repeated Game,” *Quarterly Journal of Economics* 104, August 1989, 565-88.

Fellner, W., “Prices and Wages Under Bilateral Monopoly,” *Quarterly Journal of Economics* 61, August 1947, 503-32.

Fouraker, L. E. and Siegel, S., *Bargaining Behavior*, McGraw Hill, New York, 1963.

Kreps, D, *A Course in Microeconomic Theory*, Princeton University Press, Princeton NJ, 1990.

Machlup, F., and Taber, M., “Bilateral Monopoly, Successive Monopoly, and Vertical Integration,” *Economica*, 27, May 1960, 101-19.

Nash, J., “Two-Person Cooperative Games,” *Econometrica*, 21, January 1953, 128-40.

Pindyck, R. S., and Rubinfield, D. L., *Microeconomics*, Macmillan Publishing Company, New York, 1992.

Roth, A. E., *Axiomatic Models of Bargaining*, Springer-Verlag, Berlin and New York, 1979.

Rubinstein, A., “Perfect Equilibrium in a Bargaining Model,” *Econometrica*, January 1982, 97-109.

Truett, D. B. and Truett., L. J., “Joint Profit Maximization, Negotiation, and the Determinacy of Price in Bilateral Monopoly,” *Journal of Economic Education*, Summer 1993, 260-70.

January 31, 2001

Professor Young Chin Kim
Department of Economics
Northern Illinois University
Dekalb, Illinois 60115

Dear Professor Kim:

Thank you for your letter of December 22, 2000 encouraging us to revise our paper MS#1239, "Equilibrium Contracts in a Bilateral Monopoly with Unequal Bargaining Powers," along the lines suggested by you and reviewers. Please find two original copies of the revised manuscript, my responses to the reviewer #1, and a disk containing the manuscripts in two formats (ASCII/DOS test and WordPerfect).

We carefully examined and incorporated all of reviewer #1's suggestions. We have enclosed a detailed response to reviewer #1 by describing how all the comments are incorporated in the text. In revising the manuscript, I tightened up the writing and reduced the length of the manuscript by two pages. I could not eliminate the figures because, as noted by the reviewer #1, explanations of figures are intertwined with the interpretations of the mathematical results. Consequently, I am concerned readers may not follow the paper if I eliminate the figures.

With regard to reviewer #2's comments, I have included in the introduction of the paper (the last paragraph on page 1 and first paragraph on page 2) some discussions of Devadoss and Cooper (IEJ, Spring-2000) and other papers, which help to explain how current study differs from these earlier studies and also presents the history of the bilateral monopoly literature. In doing so, I heeded your suggestions (rather than reviewer #2's recommendation) of revising and resubmitting to *IEJ*.

In addition, I very closely followed your suggestions on the journal's style in preparing the figures camera ready, numbering the sections, presenting the references both in the text and at the end of the paper, numbering equations, inclusion of JEL code numbers, and providing information on the cover page. Finally, extensive proofreading of the paper was done for both substance and style.

I look forward to hearing from you regarding the publication decisions of this paper.

Sincerely,

Stephen Devadoss
Professor

Response to Reviewer #1's Suggestions on the Paper,
Equilibrium Contracts in a Bilateral Monopoly with Unequal Bargaining Powers

This reviewer seems to have a good background on the issues surrounding the bilateral monopoly, and thus, has a good understanding of the paper. I found this reviewer's suggestion very helpful in revising the paper. My explanation of how I incorporated this reviewer's comments are given below.

1. Bandyopadhyay's reference is included in the reference section on page 27.

2. In abstract (line 4-5), I corrected the transpose and now it reads as "takes into account".

3. (Page 3, para 2, and line 3) I changed the article from a to an and now reads as "an objective".

4. (Page 4, para 1, and line 3) I modified the sentence by replacing "aside of" with "except".

5. (Page 25, last para, line 1) I corrected for the singular verb "indicates".

The revised manuscript benefitted from this reviewer's comments, and I thank the reviewer for reading the paper closely.