2020

AP® CollegeBoard

From the 2019 Administration

AP® CollegeBoard

AP® Calculus BC

Practice Exam

Contents

Note: This publication shows the page numbers that appeared in the ***2018–19 AP Exam Instructions*** book and in the actual exam. This publication was not repaginated to begin with page 1.

AP Calculus AB/BC Exams

Regularly Scheduled Exam Date: Tuesday morning, May 14, 2019

Late-Testing Exam Date: Friday morning, May 24, 2019

Section I	**Total Time:** 1 hour and 45 minutes **Number of Questions:** 45 *(The number of questions may vary slightly depending on the form of the exam.)* **Percent of Total Score:** 50% **Writing Instrument:** Pencil required	**Part A:** **Number of Questions:** 30	**Time:** 1 hour *No calculator allowed*
		Part B: **Number of Questions:** 15	**Time:** 45 minutes *Graphing calculator required*
Section II	**Total Time:** 1 hour and 30 minutes **Number of Questions:** 6 **Percent of Total Score:** 50% **Writing Instrument:** Either pencil or pen with black or dark blue ink **Note:** For Section II, if students finish Part A before the end of the timed 30 minutes for Part A, they cannot begin working on Part B. Students must wait until the beginning of the timed 1 hour for Part B. However, during the timed portion for Part B, students may work on the questions in Part A without the use of a calculator.	**Part A:** **Number of Questions:** 2	**Time:** 30 minutes **Percent of Section II Score:** 33.33% *Graphing calculator required*
		Part B: **Number of Questions:** 4	**Time:** 1 hour **Percent of Section II Score:** 66.67% *No calculator allowed*

Before Distributing Exams: Check that the title on all exam covers is ***Calculus AB*** or ***Calculus BC***. Be sure to distribute the correct exam—AB or BC—to the students. If there are any exam booklets with a different title, contact the AP coordinator immediately.

What Proctors Need to Bring to This Exam

- ☐ Exam packets
- ☐ Answer sheets
- ☐ AP Student Packs
- ☐ *2018-19 AP Coordinator's Manual*
- ☐ This book—*2018-19 AP Exam Instructions*
- ☐ AP Exam Seating Chart template
- ☐ School Code and Homeschool/Self-Study Codes
- ☐ Extra graphing calculators
- ☐ Pencil sharpener
- ☐ Container for students' electronic devices (if needed)
- ☐ Extra No. 2 pencils with erasers
- ☐ Extra pens with black or dark blue ink
- ☐ Extra paper
- ☐ Stapler
- ☐ Watch
- ☐ Signs for the door to the testing room
  - "Exam in Progress"
  - "Phones of any kind are prohibited during the test administration, including breaks"

SEATING POLICY FOR AP CALCULUS AB AND CALCULUS BC EXAMS

Testing Window	Exams Administered at Schools in the United States, Canada, Puerto Rico, and the U.S. Virgin Islands	Exams Administered at Schools Outside the United States, Canada, Puerto Rico, and the U.S. Virgin Islands
Regularly Scheduled Exams	Students must be seated no less than 4 feet apart.	Students must be seated no less than 5 feet apart.
Late-Testing Exams	Students must be seated no less than 5 feet apart.	

Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list on page 53 of the *2018-19 AP Coordinator's Manual*. If a student does not have a graphing calculator from the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, he or she must hand copy, date, and sign the release statement on page 52 of the *AP Coordinator's Manual*.

During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks. Calculators may not be shared. **Calculator memories do not need to be cleared before or after the exam.** Students with Hewlett-Packard 48–50 Series and Casio FX-9860 graphing calculators may use cards designed for use with these calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. **Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use the calculator to remove exam questions and/or answers from the room may result in the cancellation of AP Exam scores.**

The AP Calculus AB Exam and the AP Calculus BC Exam should be administered simultaneously. They may be administered in separate rooms, or in the same room if it is more convenient.

SECTION I: Multiple Choice

› **Do not begin the exam instructions below until you have completed the appropriate General Instructions for your group.**

These exams include survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

Make sure you begin the exams at the designated time. Remember, you must complete a seating chart for this exam. See pages 295–296 for a seating chart template and instructions. See the *2018-19 AP Coordinator's Manual* for exam seating requirements (pages 56–59).

If you are giving the regularly scheduled exam, say:

It is Tuesday morning, May 14, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

If you are giving the alternate exam for late testing, say:

It is Friday morning, May 24, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

If you are giving the ***AP Calculus AB Exam***, say:

Look at your exam packet and confirm that the exam title is "AP Calculus AB." Raise your hand if your exam packet contains any title other than "AP Calculus AB," and I will help you.

If you are giving the ***AP Calculus BC Exam***, say:

Look at your exam packet and confirm that the exam title is "AP Calculus BC." Raise your hand if your exam packet contains any title other than "AP Calculus BC," and I will help you.

If you are giving both the ***AP Calculus AB Exam*** and ***AP Calculus BC Exam*** in the same room, say:

Look at your exam packet and confirm that the exam title is "AP Calculus AB" or "AP Calculus BC," depending upon which exam you are taking today. Raise your hand if your exam packet contains any other title and I will help you.

Once you confirm that all students have the correct exam, say:

In a moment, you will open the exam packet. By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the *2018-19 Bulletin for AP Students and Parents*.

You may now remove the shrinkwrap from the outside only of your exam packet. Do not open the Section I booklet; do not remove the shrinkwrap from the Section II materials. Put the white seals and the shrinkwrapped Section II booklet aside. . . .

Carefully remove the AP Exam label found near the top left of your exam booklet cover. Place it on page 1 of your answer sheet on the light blue box near the top right corner that reads "AP Exam Label."

If students accidentally place the exam label in the space for the number label or vice versa, advise them to leave the labels in place. They should not try to remove the label; their exam can still be processed correctly.

Listen carefully to all my instructions. I will give you time to complete each step. Please look up after completing each step. Raise your hand if you have any questions.

Give students enough time to complete each step. Don't move on until all students are ready.

Read the statements on the front cover of the Section I booklet. . . .

Sign your name and write today's date. . . .

Now print your full legal name where indicated. . . .

Turn to the back cover of your exam booklet and read it completely. . . .

Give students a few minutes to read the entire cover.

Are there any questions? . . .

You will now take the multiple-choice portion of the exam. You should have in front of you the multiple-choice booklet and your answer sheet. You may never discuss the multiple-choice exam content at any time in any form with anyone, including your teacher and other students. If you disclose the multiple-choice exam content through any means, your AP Exam score will be canceled.

Open your answer sheet to page 2. You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses beginning on page 2 of your answer sheet, one response per question. Completely fill in the circles. If you need to erase, do so carefully and completely. No credit will be given for

anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work.

Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put your calculators under your chair. Are there any questions? . . .

You have 1 hour for Part A. Part A questions are numbered 1 through 30. Mark your responses for these questions on page 2 of your answer sheet. Open your Section I booklet and begin.

Note Start Time ______________ . Note Stop Time ______________ .

Check that students are marking their answers in pencil on page 2 of their answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work.

After 50 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working on Part A and turn to page 24 in your Section I booklet. . . .

On that page, you should see an area marked "PLACE SEAL HERE." Making sure all of your other exam materials, including your answer sheet, are out of the way, take one of your seals and press it on that area and then fold the seal over the open edge to the front cover. Be sure you don't seal the Part B section of the booklet or let the seal touch anything except the marked areas. . . .

After all students have sealed Part A, say:

Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. Part B questions are numbered 76 through 90. Fold your answer sheet so only page 3 is showing and mark your responses for these questions on that page. You have 45 minutes for Part B. You may begin.

Note Start Time ______________ . Note Stop Time ______________ .

Check that students have sealed their booklets properly and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. Proctors should make sure that students are using their calculators appropriately. Proctors should also make sure Hewlett-Packard calculators' infrared ports are not facing each other.

After 35 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working and turn to page 38. You have 3 minutes to answer Questions 91–94. These are survey questions and will not affect your score. Note that each survey question has five answer options. You may not go back to work on any of the exam questions. . . .

Give students approximately 3 minutes to answer the survey questions.

Then say:

> **Close your booklet and put your answer sheet on your desk, faceup. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. Sit quietly while I collect your answer sheets.**

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label.

After all answer sheets have been collected, say:

> **Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, faceup. I will now collect your Section I booklet. . . .**

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II.

When all Section I materials have been collected and accounted for and you are ready for the break, say:

> **Please listen carefully to these instructions before we take a 10-minute break. All items you placed under your chair at the beginning of this exam, including your Student Pack, must stay there, and you are not permitted to open or access them in any way. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, notes, textbooks, or any other resources during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. You may not leave the designated break area. Remember, you may never discuss the multiple-choice exam content with anyone, and if you disclose the content through any means, your AP Exam score will be canceled. Are there any questions? . . .**
>
> **You may begin your break. Testing will resume at ________________ .**

SECTION II: Free Response

After the break, say:

> **May I have everyone's attention? Place your Student Pack on your desk. . . .**
>
> **You may now remove the shrinkwrap from the Section II packet, but do not open the Section II exam booklet until you are told to do so. . . .**
>
> **Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .**
>
> **Now take an AP number label from your Student Pack and place it on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .**
>
> **Read the last statement. . . .**
>
> **Using your pen, print the first, middle, and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .**

Now turn to the back cover. Using your pen, complete Items 1 through 3 under "Important Identification Information."...

Read Item 4. . . .

Are there any questions? . . .

If this is your last AP Exam, you may keep your Student Pack. Place it under your chair for now. Otherwise if you are taking any other AP Exams this year, leave your Student Pack on your desk and I will collect it now. . . .

Read the remaining information on the back cover of the exam booklet, paying careful attention to the bulleted statements in the instructions. Do not open the exam booklet or break the seals in the exam booklet until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs from students who are taking any other AP Exams this year.

Then say:

Are there any questions? . . .

Section II also has two parts that are timed separately. You are responsible for pacing yourself and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not break the seals for Part B at this time. Are there any questions? . . .

You have 30 minutes to answer the questions in Part A. If you need more paper to complete your responses, raise your hand. At the top of each extra sheet of paper you use, write only:

- **your AP number,**
- **the exam title, and**
- **the question number you are working on.**

Do not write your name. Open your exam booklet and begin.

Note Start Time __________ . Note Stop Time __________ .

Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for the Part A questions are marked with large 1's or 2's at the top of each page to assist you in monitoring their work.

After 20 minutes, say:

There are 10 minutes remaining in Part A.

After 10 minutes, say:

Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair. . . .

Turn to page 13. You have 1 hour for Part B. During this time you may go back to Part A, but you may <u>not</u> use your calculator. Remember to show your work and write your answer to each part of each problem in the appropriate space in the exam booklet. Are there any questions? . . .

Using your finger, break open the seals on Part B. Do not peel the seals away from the booklet. You may go on to the next page and begin Part B.

Note Start Time __________ . Note Stop Time __________ .

After 50 minutes, say:

There are 10 minutes remaining in Part B.

After 10 minutes, say:

Stop working and close your exam booklet. Place it on your desk, faceup. . . .

If any students used extra paper for a question in the free-response section, have those students staple the extra sheet(s) to the first page corresponding to that question in their free-response exam booklets. Complete an Incident Report after the exam and return these free-response booklets with the extra sheets attached in the Incident Report return envelope (see page 68 of the *2018-19 AP Coordinator's Manual* for complete details).

Then say:

Remain in your seat, without talking, while the exam materials are collected. . . .

Collect a Section II exam booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box and printed their initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

If you are giving the regularly scheduled exam, say:
You may not discuss or share the free-response exam content with anyone unless it is released on the College Board website in about two days. Your AP Exam score results will be available online in July.

If you are giving the alternate exam for late testing, say:
None of the content in this exam may ever be discussed or shared in any way at any time. Your AP Exam score results will be available online in July.

If any students completed the AP number card at the beginning of this exam, say:

Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.

Then say:

You are now dismissed.

After-Exam Tasks

Be sure to give the completed seating chart to the AP coordinator. Schools must retain seating charts for at least six months (unless the state or district requires that they be retained for a longer period of time). Schools should not return any seating charts in their exam shipments unless they are required as part of an Incident Report.

NOTE: If you administered exams to students with accommodations, review the *2018-19 AP Coordinator's Manual* and the *2018-19 AP SSD Guidelines* for information about completing the Nonstandard Administration Report (NAR) form, and returning these exams.

The exam proctor should complete the following tasks if asked to do so by the AP coordinator. Otherwise, the AP coordinator must complete these tasks:

- Complete an Incident Report for any students who used extra paper for the free-response section. (Incident Report forms are provided in the coordinator packets sent with the exam shipments.) **These forms must be completed with a No. 2 pencil.** It is best to complete a single Incident Report for multiple students per exam subject, per administration (regular or late testing), as long as all required information is provided. Include all exam booklets with extra sheets of paper in an Incident Report return envelope (see page 68 of the *2018-19 AP Coordinator's Manual* for complete details).
- Return all exam materials to secure storage until they are shipped back to the AP Program. (See page 27 of the *2018-19 AP Coordinator's Manual* for more information about secure storage.) Before storing materials, check the "School Use Only" section on page 1 of the answer sheet and:
  - Fill in the appropriate section number circle in order to access a separate AP Instructional Planning Report (for regularly scheduled exams only) or subject score roster at the class section or teacher level. See "Post-Exam Activities" in the *2018-19 AP Coordinator's Manual*.
  - Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

Name:___________________________________

Answer Sheet for AP Calculus BC
Practice Exam, Section I

No.	Answer
1	
2	
3	
4	
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7	
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9	
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12	
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26	
27	
28	
29	
30	

No.	Answer
76	
77	
78	
79	
80	
81	
82	
83	
84	
85	
86	
87	
88	
89	
90	

AP® Calculus BC Exam

SECTION I: Multiple Choice

2019

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
30
Time
1 hour
Electronic Device
None allowed

Part B

Number of Questions
15
Time
45 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 30 on the answer sheet. For Part B, fill in only the circles for numbers 76 through 90 on the answer sheet. Because Part A and Part B offer only four answer options for each question, do not mark the (E) answer circle for any question. The survey questions are numbers 91 through 94.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4PBP4-S
68

CALCULUS BC

SECTION I, Part A

Time—1 hour

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. $\int_{1}^{2} \left(4x^3 - x\right) dx =$

(A) $\frac{27}{2}$ (B) 27 (C) 36 (D) 57

2. Let f be the function defined by $f(x) = x^3 - 3x^2 - 9x + 11$. At which of the following values of x does f attain a local minimum?

(A) 3 (B) 1 (C) -1 (D) -3

3. $\frac{d}{dx}\left(2(\sin\sqrt{x})^2\right) =$

(A) $4\cos\left(\frac{1}{2\sqrt{x}}\right)$ (B) $4\sin\sqrt{x}\cos\sqrt{x}$ (C) $\frac{2\sin\sqrt{x}}{\sqrt{x}}$ (D) $\frac{2\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$

---

4. The position of a particle is given by the parametric equations $x(t) = \ln\left(t^2 + 1\right)$ and $y(t) = e^{3-t}$. What is the velocity vector at time $t = 1$?

(A) $\left\langle 1, e^2 \right\rangle$ (B) $\left\langle 1, -e^2 \right\rangle$ (C) $\left\langle \frac{1}{2}, e^2 \right\rangle$ (D) $\left\langle \frac{1}{2}, -e^2 \right\rangle$

GO ON TO THE NEXT PAGE.

5. $\sum_{n=1}^{\infty} \frac{e^n}{\pi^n}$ is

(A) $\frac{\pi}{\pi - e}$ (B) $\frac{e}{\pi - e}$ (C) $\frac{e}{\pi \ln\left(\frac{\pi}{e}\right)}$ (D) divergent

---

x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$
2	4	-3	3	-2	5	1

6. The table above gives values of the twice-differentiable functions f and g and their derivatives at $x = 2$. If h is the function defined by $h(x) = \frac{f'(x)}{g(x)}$, what is the value of $h'(2)$?

(A) $\frac{9}{4}$ (B) $\frac{3}{5}$ (C) $-\frac{3}{2}$ (D) $-\frac{21}{4}$

GO ON TO THE NEXT PAGE.

7. Which of the following is the Maclaurin series for $x\cos\left(x^2\right)$?

(A) $x - \dfrac{x^5}{2!} + \dfrac{x^9}{4!} - \dfrac{x^{13}}{6!} + \cdots$

(B) $x - \dfrac{x^3}{2!} + \dfrac{x^5}{4!} - \dfrac{x^7}{6!} + \cdots$

(C) $x^3 - \dfrac{x^7}{3!} + \dfrac{x^{11}}{5!} - \dfrac{x^{15}}{7!} + \cdots$

(D) $x^3 - \dfrac{x^5}{3!} + \dfrac{x^7}{5!} - \dfrac{x^9}{7!} + \cdots$

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8. $\displaystyle\lim_{x\to\infty} \frac{10 - 6x^2}{5 + 3e^x}$ is

(A) -2 (B) 0 (C) 2 (D) nonexistent

GO ON TO THE NEXT PAGE.

9. The function f is not differentiable at $x = 5$. Which of the following statements must be true?

(A) f is not continuous at $x = 5$.

(B) $\lim_{x \to 5} f(x)$ does not exist.

(C) $\lim_{x \to 5} \dfrac{f(x) - f(5)}{x - 5}$ does not exist.

(D) $\int_0^5 f(x)\,dx$ does not exist.

10. The second derivative of a function f is given by $f''(x) = x(x - 3)^5(x - 10)^2$. At which of the following values of x does the graph of f have a point of inflection?

(A) 3 only

(B) 0 and 3 only

(C) 3 and 10 only

(D) 0, 3, and 10

11. $\displaystyle\int_0^{\pi} \frac{e^x - 1}{e^x - x}\,dx =$

(A) $e^{\pi} - \pi - 1$ (B) $\ln(e^{\pi} - \pi) - 1$ (C) $\pi - \ln \pi$ (D) $\ln(e^{\pi} - \pi)$

x	0	4	8	12	16
$f(x)$	8	0	2	10	1

12. The table above gives selected values for the differentiable function f. In which of the following intervals must there be a number c such that $f'(c) = 2$?

(A) $(0, 4)$ (B) $(4, 8)$ (C) $(8, 12)$ (D) $(12, 16)$

13. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + 2y$ with initial condition $f(0) = 2$. What is the approximation for $f(-0.4)$ obtained by using Euler's method with two steps of equal length starting at $x = 0$?

(A) 0.76 (B) 1.20 (C) 1.29 (D) 3.96

---

14. What is the slope of the line tangent to the curve $\sqrt{x} + \sqrt{y} = 2$ at the point $\left(\frac{9}{4}, \frac{1}{4}\right)$?

(A) -3 (B) $-\frac{1}{3}$ (C) 1 (D) $\frac{4}{3}$

15. $\displaystyle\int_1^{\infty} \frac{6}{(x+3)^{3/2}}\,dx$ is

(A) $\dfrac{3}{4}$ (B) 3 (C) 6 (D) divergent

---

16. If $\dfrac{dy}{dx} = 2 - y$, and if $y = 1$ when $x = 1$, then $y =$

(A) $2 - e^{x-1}$ (B) $2 - e^{1-x}$ (C) $2 - e^{-x}$ (D) $2 + e^{-x}$

GO ON TO THE NEXT PAGE.

17. Which of the following series converges?

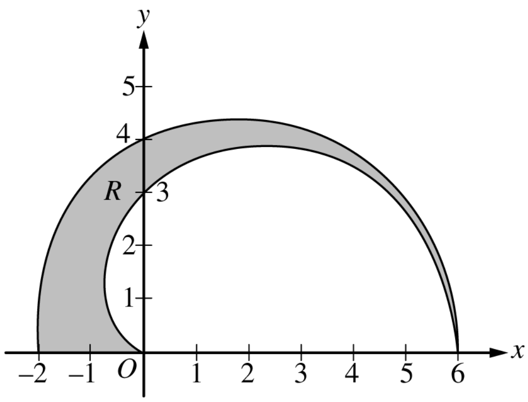
(A) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{1-n}{n}\right)$

(B) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n+1}{2n}\right)$

(C) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n^2}{3\sqrt{n}}\right)$

(D) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{2\sqrt{n}}{n}\right)$

GO ON TO THE NEXT PAGE.



18. Let R be the region in the first and second quadrants between the graphs of the polar curves $f(\theta) = 3 + 3\cos\theta$ and $g(\theta) = 4 + 2\cos\theta$, as shaded in the figure above. Which of the following integral expressions gives the area of R ?

(A) $\int_{-2}^{6} (g(\theta) - f(\theta))\, d\theta$

(B) $\int_{0}^{\pi} (g(\theta) - f(\theta))\, d\theta$

(C) $\frac{1}{2}\int_{0}^{\pi} (g(\theta) - f(\theta))^2\, d\theta$

(D) $\frac{1}{2}\int_{0}^{\pi} \left((g(\theta))^2 - (f(\theta))^2\right) d\theta$

19. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1}$ is true?

(A) The series can be shown to diverge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) The series can be shown to diverge by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series can be shown to converge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(D) The series can be shown to converge by the alternating series test.

20. If $\int_1^4 f(x)\,dx = 8$ and $\int_1^4 g(x)\,dx = -2$, which of the following cannot be determined from the information given?

(A) $\int_4^1 g(x)\,dx$

(B) $\int_1^4 3f(x)\,dx$

(C) $\int_1^4 3f(x)g(x)\,dx$

(D) $\int_1^4 (3f(x) + g(x))\,dx$

21. Which of the following is the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(x+4)^n}{n \cdot 5^{n+1}}$?

(A) $[-9, 1)$ (B) $[-5, 5)$ (C) $[1, 9)$ (D) $(-\infty, \infty)$

---

22. What is the slope of the line tangent to the polar curve $r = 3\theta$ at the point where $\theta = \frac{\pi}{2}$?

(A) $-\frac{\pi}{2}$ (B) $-\frac{2}{\pi}$ (C) 0 (D) 3

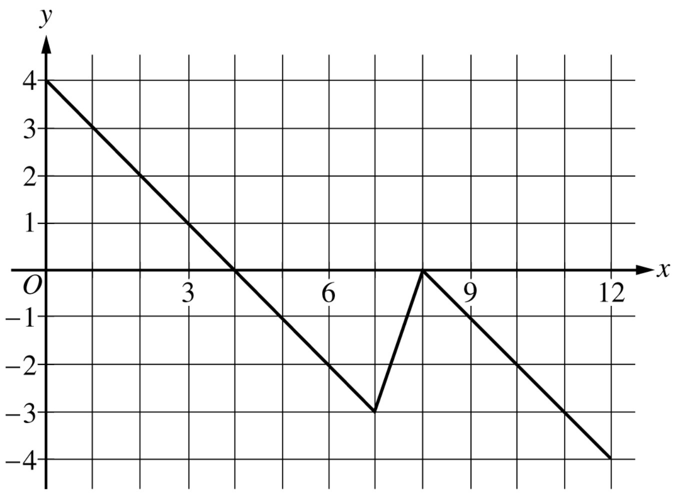
23. The definite integral $\int_0^4 \sqrt{x}\, dx$ is approximated by a left Riemann sum, a right Riemann sum, and a trapezoidal sum, each with 4 subintervals of equal width. If L is the value of the left Riemann sum, R is the value of the right Riemann sum, and T is the value of the trapezoidal sum, which of the following inequalities is true?

(A) $L < \int_0^4 \sqrt{x}\, dx < T < R$

(B) $L < T < \int_0^4 \sqrt{x}\, dx < R$

(C) $R < \int_0^4 \sqrt{x}\, dx < T < L$

(D) $R < T < \int_0^4 \sqrt{x}\, dx < L$



Graph of f

24. The graph of the piecewise linear function f is shown above. What is the value of $\int_0^{12} f'(x)\,dx$?

(A) -8 (B) -6 (C) 0 (D) 22

25. The function f has a continuous derivative. If $f(0) = 1$, $f(2) = 5$, and $\int_0^2 f(x)\,dx = 7$, what is $\int_0^2 x \cdot f'(x)\,dx$?

(A) 3 (B) 6 (C) 10 (D) 17

26. Which of the following series are conditionally convergent?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

II. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

III. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$

(A) I only (B) II only (C) II and III only (D) I, II, and III

27. Let R be the region in the first quadrant bounded by the graph of $y = \sqrt{x-1}$, the x-axis, and the vertical line $x = 10$. Which of the following integrals gives the volume of the solid generated by revolving R about the y-axis?

(A) $\pi \int_1^{10} (x-1)\, dx$

(B) $\pi \int_1^{10} (100 - (x-1))\, dx$

(C) $\pi \int_0^3 \left(10 - \left(y^2+1\right)\right)^2 dy$

(D) $\pi \int_0^3 \left(100 - \left(y^2+1\right)^2\right) dy$

28. If $\dfrac{dx}{dt} = 5$ and $\dfrac{dy}{dt} = \sin\left(t^2\right)$, then $\dfrac{d^2y}{dx^2}$ is

(A) $2t\cos\left(t^2\right)$ (B) $\dfrac{2t\cos\left(t^2\right)}{5}$ (C) $\dfrac{2t\cos\left(t^2\right)}{25}$ (D) undefined

GO ON TO THE NEXT PAGE.

29. Which of the following expressions is equal to $\lim_{n \to \infty} \frac{1}{n}\left(\frac{1}{2+\frac{1}{n}} + \frac{1}{2+\frac{2}{n}} + \frac{1}{2+\frac{3}{n}} + \cdots + \frac{1}{2+\frac{n}{n}}\right)$?

(A) $\int_1^2 \frac{1}{x}\,dx$

(B) $\int_0^1 \frac{1}{2+x}\,dx$

(C) $\int_0^2 \frac{1}{2+x}\,dx$

(D) $\int_2^3 \frac{1}{2+x}\,dx$

30. A function f has a Maclaurin series given by $2 + 3x + x^2 + \frac{1}{3}x^3 + \cdots$, and the Maclaurin series converges to $f(x)$ for all real numbers x. If g is the function defined by $g(x) = e^{f(x)}$, what is the coefficient of x^2 in the Maclaurin series for g ?

(A) $\frac{1}{2}e^2$ (B) e^2 (C) $\frac{5}{2}e^2$ (D) $\frac{11}{2}e^2$

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION I, Part B

Time—45 minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU FILL IN THE CIRCLES ON THE ANSWER SHEET THAT CORRESPOND TO QUESTIONS NUMBERED 76–90.

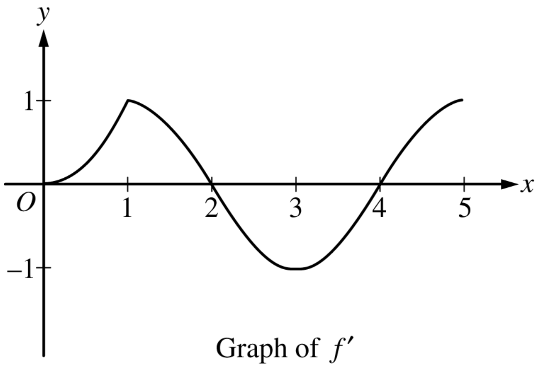
YOU MAY NOT RETURN TO QUESTIONS NUMBERED 1–30.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f'

76. The function f is continuous on the closed interval [0, 5]. The graph of f', the derivative of f, is shown above. On which of the following intervals is f increasing?

(A) [0, 1] and [2, 4]

(B) [0, 1] and [3, 5]

(C) [0, 1] and [4, 5] only

(D) [0, 2] and [4, 5]

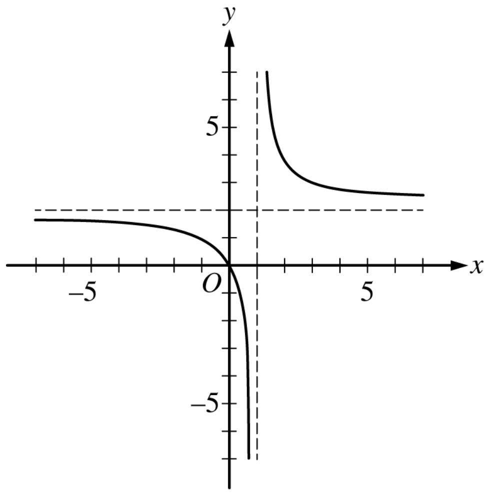
---

77. If $\dfrac{dy}{dt} = 6e^{-0.08(t-5)^2}$, by how much does y change as t changes from $t = 1$ to $t = 6$?

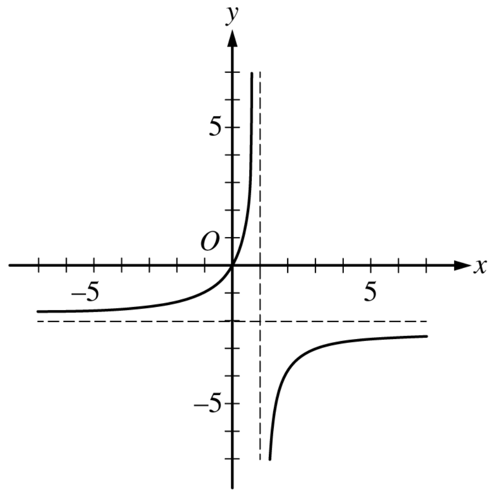
(A) 3.870 (B) 8.341 (C) 18.017 (D) 22.583

78. The function f has the property that $\lim_{x \to 1^-} f(x) = +\infty$, $\lim_{x \to 1^+} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = 2$, and $\lim_{x \to +\infty} f(x) = 2$. Of the following, which could be the graph of f ?

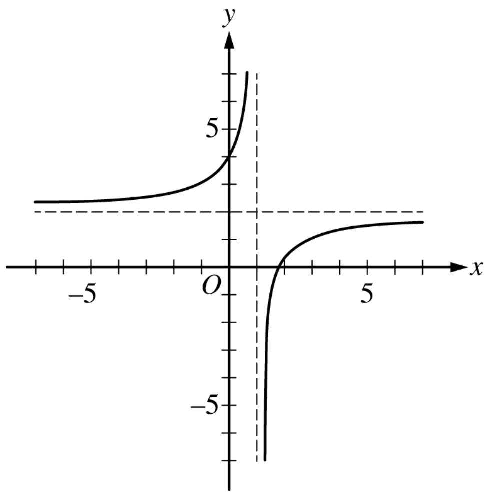
(A)



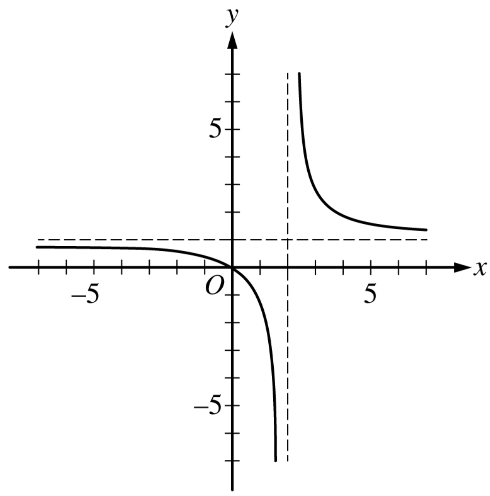
(B)



(C)



(D)



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79. Tara's heart rate during a workout is modeled by the differentiable function h, where $h(t)$ is measured in beats per minute and t is measured in minutes from the start of the workout. Which of the following expressions gives Tara's average heart rate from $t = 30$ to $t = 60$?

(A) $\int_{30}^{60} h(t)\, dt$

(B) $\frac{1}{30}\int_{30}^{60} h(t)\, dt$

(C) $\frac{1}{30}\int_{30}^{60} h'(t)\, dt$

(D) $\frac{h'(30) + h'(60)}{2}$

GO ON TO THE NEXT PAGE.

80. The function f has derivatives of all orders for all real numbers with $f(2) = -1$, $f'(2) = 4$, $f''(2) = 6$, and $f'''(2) = 12$. Using the third-degree Taylor polynomial for f about $x = 2$, what is the approximation of $f(2.1)$?

(A) -0.570 (B) -0.568 (C) -0.566 (D) -0.528

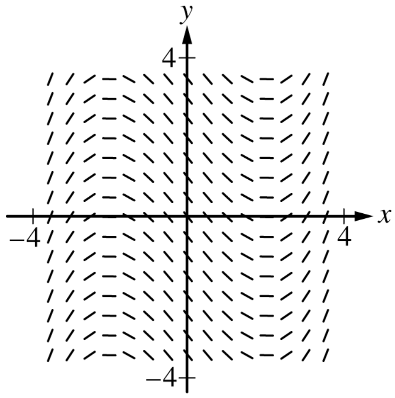
---

81. Let f be a function with derivative given by $f'(x) = \sqrt{x^3 + 1}$. What is the length of the graph of $y = f(x)$ from $x = 0$ to $x = 1.5$?

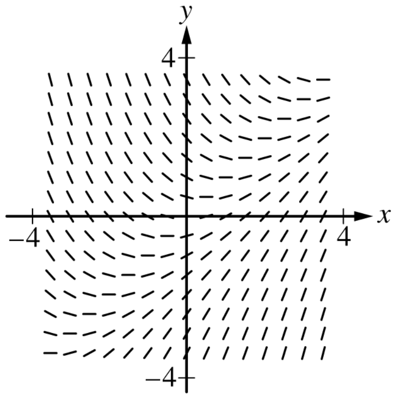
(A) 4.266 (B) 2.497 (C) 2.278 (D) 1.976

82. Let h be a continuous function of x. Which of the following could be a slope field for a differential equation of the form $\dfrac{dy}{dx} = h(x)$?

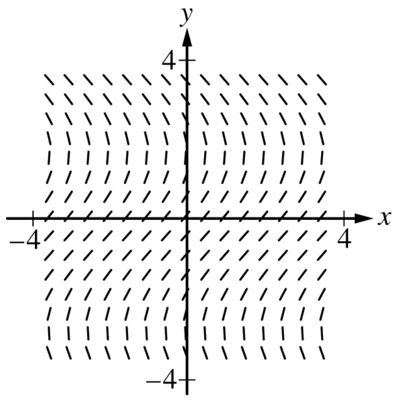
(A)



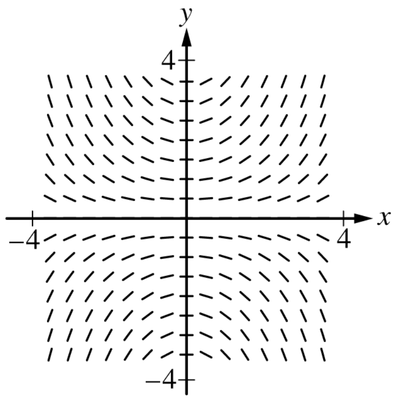
(B)



(C)



(D)



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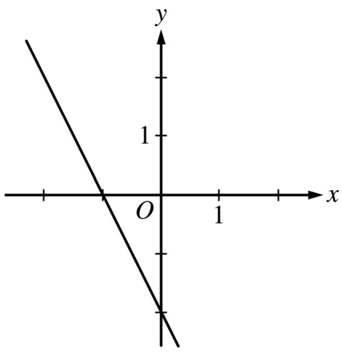
$$f(x) = \begin{cases} k^3 + x & \text{for } x < 3 \\ \dfrac{16}{k^2 - x} & \text{for } x \geq 3 \end{cases}$$

83. Let f be the function defined above, where k is a positive constant. For what value of k, if any, is f continuous?

(A) 2.081 (B) 2.646 (C) 8.550 (D) There is no such value of k.



84. Let f be a differentiable function. The figure above shows the graph of the line tangent to the graph of f at $x = 0$. Of the following, which must be true?

(A) $f'(0) = -f(0)$

(B) $f'(0) < f(0)$

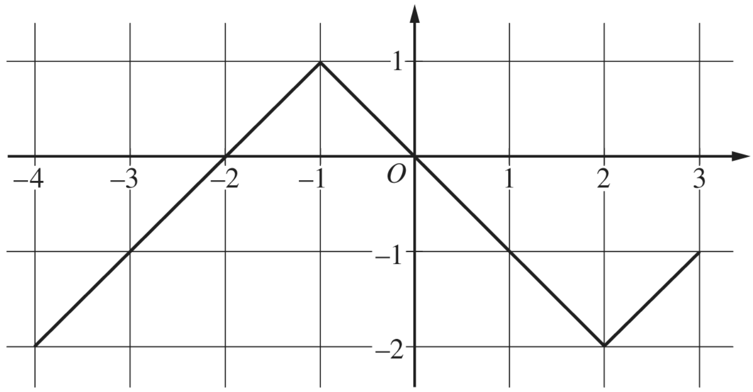
(C) $f'(0) = f(0)$

(D) $f'(0) > f(0)$

GO ON TO THE NEXT PAGE.

85. A referee moves along a straight path on the side of an athletic field. The velocity of the referee is given by $v(t) = 4(t - 6)\cos(2t + 5)$, where t is measured in minutes and $v(t)$ is measured in meters per minute. What is the total distance traveled by the referee, in meters, from time $t = 2$ to time $t = 6$?

(A) 3.933 (B) 14.578 (C) 21.667 (D) 29.156



Graph of f

86. The graph of the function f shown above consists of three line segments. Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. At what value of x does h attain its absolute maximum on the interval $[-4, 3]$?

(A) -4 (B) -2 (C) 0 (D) 3

87. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = e^{-t}$ and $y(t) = \sin(4t)$ for time $t \geq 0$. What is the speed of the particle at time $t = 1.2$?

(A) 1.162 (B) 1.041 (C) 0.462 (D) 0.221

---

88. Let f be the function defined by $f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{2}x$. For how many values of x in the open interval $(0, 1.565)$ is the instantaneous rate of change of f equal to the average rate of change of f on the closed interval $[0, 1.565]$?

(A) Zero (B) One (C) Three (D) Four

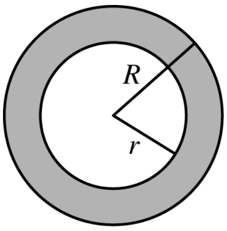
89. The population P of rabbits on a small island grows at a rate that is jointly proportional to the size of the rabbit population and the difference between the rabbit population and the carrying capacity of the population. If the carrying capacity of the population is 2400 rabbits, which of the following differential equations best models the growth rate of the rabbit population with respect to time t, where k is a constant?

(A) $\frac{dP}{dt} = 2400 - kP$

(B) $\frac{dP}{dt} = k(2400 - P)$

(C) $\frac{dP}{dt} = k\frac{1}{P}(2400 - P)$

(D) $\frac{dP}{dt} = kP(2400 - P)$



90. A region is bounded by two concentric circles, as shown by the shaded region in the figure above. The radius of the outer circle, R, is increasing at a constant rate of 2 inches per second. The radius of the inner circle, r, is decreasing at a constant rate of 1 inch per second. What is the rate of change, in square inches per second, of the area of the region at the instant when R is 4 inches and r is 3 inches?

(A) 3π (B) 6π (C) 10π (D) 22π

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AP® Calculus BC Exam

SECTION II: Free Response

2019

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name
   First letter of your first name
2. Date of birth
   Month Day Year
3. Six-digit school code
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights.

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4PBP4-S

68

CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (hours)	2	5	9	11	12
$L(t)$ (cars per hour)	15	40	24	68	18

1. The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t - 2)(t - 5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function L. Selected values of $L(t)$ are given in the table above. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time t is measured in hours after 5 A.M. ($t = 0$). Both functions are defined for $0 \le t \le 12$.

(a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.

(b) How many cars enter the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole number.

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Continue question 1 on page 5.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate $\int_2^{12} L(t)\,dt$. Using correct units, explain the meaning of $\int_2^{12} L(t)\,dt$ in the context of this problem.

(d) For $0 \le t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \le t \le 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.

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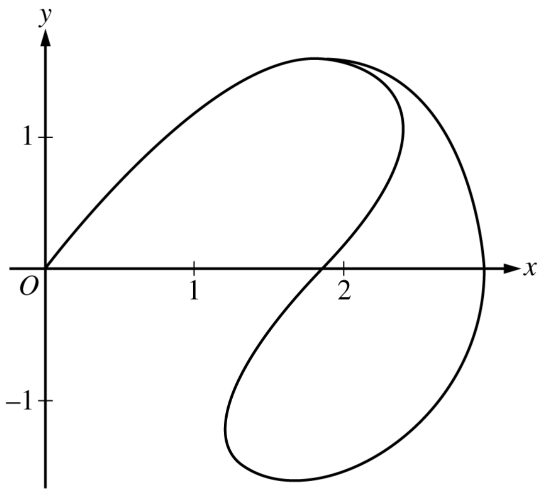

2. A laser is a device that produces a beam of light. A design, shown above, is etched onto a flat piece of metal using a moving laser. The position of the laser at time t seconds is represented by $(x(t), y(t))$ in the xy-plane. Both x and y are measured in centimeters, and t is measured in seconds. The laser starts at position $(0, 0)$ at time $t = 0$, and the design takes 3.1 seconds to complete. For $0 \le t \le 3.1$, $\frac{dx}{dt} = 3\cos\left(t^2\right)$ and $\frac{dy}{dt} = 4\cos(2.5t)$.

(a) Find the speed of the laser at time $t = 3$ seconds.

(b) Find the total distance traveled by the laser from time $t = 1$ to time $t = 3$ seconds.

Continue question 2 on page 7.

(c) The laser is farthest to the right at time $t = 1.253$ seconds. Find the x-coordinate of the laser's rightmost position.

(d) What is the difference between the y-coordinates of the laser's highest position and lowest position for $0 \le t \le 3.1$? Justify your answer.

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END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\dfrac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

3. Let f be the function defined above.

(a) Find the average rate of change of f on the interval $-3 \le x \le 4$.

(b) Write an equation for the line tangent to the graph of f at $x = 3$.

Continue question 3 on page 15.

NO CALCULATOR ALLOWED

(c) Find the average value of f on the interval $-3 \le x \le 4$.

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \le x \le 4$? Justify your answer.

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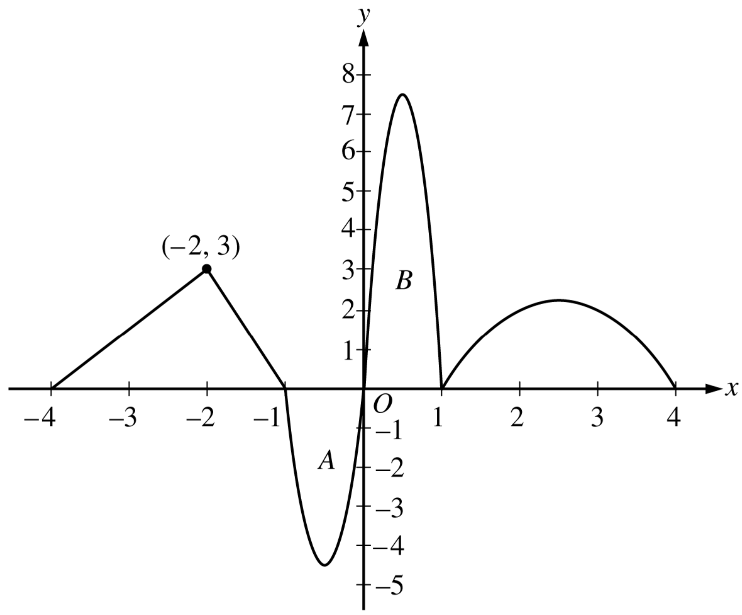
NO CALCULATOR ALLOWED



Graph of f

4. The continuous function f is defined for $-4 \le x \le 4$. The graph of f, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x = -\frac{1}{2}$, $x = \frac{1}{2}$, and $x = \frac{5}{2}$. It is known that $f(x) = -x^2 + 5x - 4$ for $1 \le x \le 4$. The areas of regions A and B bounded by the graph of f and the x-axis are 3 and 5, respectively. Let g be the function defined by $g(x) = \int_{-4}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(4)$.

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Continue question 4 on page 17.

(b) Find the absolute minimum value of g on the closed interval $[-4, 4]$. Justify your answer.

(c) Find all intervals on which the graph of g is concave down. Give a reason for your answer.

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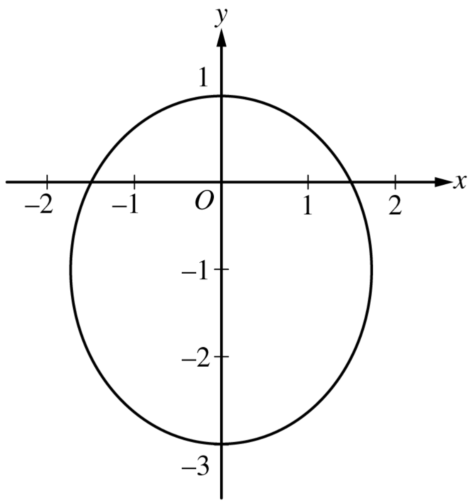
NO CALCULATOR ALLOWED



5. The graph of the curve C, given by $4x^2 + 3y^2 + 6y = 9$, is shown in the figure above.

(a) Show that $\frac{dy}{dx} = \frac{-4x}{3(y+1)}$.

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(b) Using the information from part (a), find $\frac{d^2y}{dx^2}$ in terms of x and y.

Continue question 5 on page 19.

NO CALCULATOR ALLOWED

(c) In polar coordinates, the curve C is given by $r = \dfrac{3}{2 + \sin\theta}$ for $0 \le \theta \le 2\pi$. Find $\dfrac{dr}{d\theta}$.

As θ increases, on what intervals is the distance between the origin and the point (r, θ) increasing?

Give a reason for your answer.

(d) Let S be the region inside curve C, as defined in part (c), but outside the curve $r = 2$. Write, but do not evaluate, an integral expression for the area of S.

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Do not write beyond this border.

NO CALCULATOR ALLOWED

6. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{5^n \cdot n^p}$, where p is a constant and $p > 0$.

(a) For $p = 3$ and $x = 8$, does the series converge absolutely, converge conditionally, or diverge? Explain your reasoning.

(b) For $p = 1$ and $x = 8$, does the series converge absolutely, converge conditionally, or diverge? Explain your reasoning.

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Continue question 6 on page 21.

NO CALCULATOR ALLOWED

(c) When $x = -2$, for what values of p does the series converge? Explain your reasoning.

(d) When $p = 1$ and $x = 3.1$, the series converges to a value S. Use the first two terms of the series to approximate S. Use the alternating series error bound to show that this approximation differs from S by less than $\frac{1}{300{,}000}$.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Answer Key for AP Calculus BC

Practice Exam, Section I

Question 1: A
Question 2: A
Question 3: D
Question 4: B
Question 5: B
Question 6: A
Question 7: A
Question 8: B
Question 9: C
Question 10: B
Question 11: D
Question 12: C
Question 13: A
Question 14: B
Question 15: C
Question 16: B
Question 17: D
Question 18: D
Question 19: D
Question 20: C
Question 21: A
Question 22: B
Question 23: B
Question 24: A
Question 25: A
Question 26: C
Question 27: D
Question 28: C
Question 29: B
Question 30: D

Question 76: D
Question 77: D
Question 78: C
Question 79: B
Question 80: B
Question 81: B
Question 82: A
Question 83: A
Question 84: C
Question 85: C
Question 86: A
Question 87: C
Question 88: C
Question 89: D
Question 90: D

Multiple-Choice Section for Calculus BC
2019 Course Framework Alignment and Rationales

Question 1

Skill	Learning Objective	Topic
1.E	FUN-6.B	The Fundamental Theorem of Calculus and Definite Integrals

(A)	**Correct**. This question involves using the basic power rule for antidifferentiation and correctly substituting the endpoints and evaluating, as follows. $\int_1^2 \left(4x^3 - x\right) dx = x^4 - \frac{1}{2}x^2\Big\|_1^2 = (16 - 2) - \left(1 - \frac{1}{2}\right) = 14 - \frac{1}{2} = \frac{27}{2}$
(B)	Incorrect. This response would result if the antidifferentiation was not done and the endpoints were substituted directly into the integrand, as follows. $4x^3 - x\Big\|_1^2 = (32 - 2) - (4 - 1) = 30 - 3 = 27$
(C)	Incorrect. This response would result if the integrand was differentiated rather than antidifferentiated, as follows. $12x^2 - 1\Big\|_1^2 = (48 - 1) - (12 - 1) = 47 - 11 = 36$
(D)	Incorrect. This response would result if the powers of x were not divided by the new exponent when the power rule for antiderivatives was applied, as follows. $4x^4 - x^2\Big\|_1^2 = (64 - 4) - (4 - 1) = 60 - 3 = 57$

Question 2

Skill		Learning Objective	Topic
3.D		FUN-4.A	Using the First Derivative Test to Find Relative (Local) Extrema
(A)	**Correct**. A local minimum for f will occur where the derivative f' changes sign from negative to positive. $f'(x) = 3x^2 - 6x - 9 = 3\left(x^2 - 2x - 3\right) = 3(x+1)(x-3)$ The zeros of f' are at $x = -1$ and $x = 3.$ A sign chart shows that f' changes from negative to positive at $x = 3.$ Another way to see this is to observe that the graph of f' is a parabola opening up so that the graph crosses the x-axis from negative to positive at the larger zero, $x = 3.$		
(B)	Incorrect. This is a zero of the second derivative $f''(x) = 6x - 6 = 6(x-1)$ where f'' changes sign from negative to positive. Therefore, $x = 1$ is a point of inflection of the graph of f, not the location of a local minimum. This response might also have been chosen if the derivative had been incorrectly factored as $3(x-1)(x+3).$ Then $x = 1$ is a zero of this expression where the sign changes from negative to positive.		
(C)	Incorrect. This is a zero of the derivative f' where f' changes sign from positive to negative. Therefore, $x = -1$ is the location of a local maximum of f, not a local minimum.		
(D)	Incorrect. This response might have been chosen if the derivative had been incorrectly factored as $3(x-1)(x+3)$ and the zero where the derivative changes sign from positive to negative was selected.		

Question 3

Skill	Learning Objective	Topic
1.E	FUN-3.C	The Chain Rule

(A)	Incorrect. This response might come from incorrectly applying the chain rule twice as $\frac{d}{dx}(f(g(x))) = f'(g'(x))$, as follows. $\frac{d}{dx}\left(2(\sin\sqrt{x})^2\right) = 2\cdot\left(2\cdot\frac{d}{dx}(\sin\sqrt{x})\right) = 2\cdot 2\cdot\cos\left(\frac{d}{dx}(\sqrt{x})\right) = 4\cos\left(\frac{1}{2\sqrt{x}}\right)$
(B)	Incorrect. This response might come from correctly applying the chain rule the first time but not the second, as follows. $\frac{d}{dx}\left(2(\sin\sqrt{x})^2\right) = 2\cdot 2(\sin\sqrt{x})\cdot\frac{d}{dx}(\sin\sqrt{x}) = 2\cdot 2(\sin\sqrt{x})\cdot\cos\sqrt{x}$
(C)	Incorrect. This response might come from using the chain rule only once, with the innermost "inside" function, $\sqrt{x}$, as follows. $\frac{d}{dx}\left(2(\sin\sqrt{x})^2\right) = 2\cdot 2(\sin\sqrt{x})\cdot\frac{d}{dx}(\sqrt{x}) = 2\cdot 2(\sin\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$
(D)	**Correct**. The chain rule must be used twice for this composition of three functions. $\frac{d}{dx}\left(2(\sin\sqrt{x})^2\right) = 2\cdot 2(\sin\sqrt{x})\cdot\left(\frac{d}{dx}(\sin\sqrt{x})\right)$ $= 2\cdot 2(\sin\sqrt{x})\cdot\left(\cos\sqrt{x}\cdot\frac{d}{dx}(\sqrt{x})\right)$ $= 2\cdot 2(\sin\sqrt{x})\cdot\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$ $= \frac{2\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$

Question 4

Skill	Learning Objective	Topic
1.E	FUN-8.B	(BC ONLY) Solving Motion Problems Using Parametric and Vector-Valued Functions

(A)	Incorrect. This response would result if the chain rule was not used during the differentiation of the $y(t)$ component of the position vector resulting in the loss of the negative sign, as follows. $v(t) = \left\langle \frac{d}{dt}\ln\left(t^2+1\right), \frac{d}{dt}e^{3-t} \right\rangle = \left\langle \frac{2t}{t^2+1}, e^{3-t} \right\rangle \Rightarrow v(1) = \left\langle 1, e^2 \right\rangle$
(B)	**Correct**. The components of the velocity vector are the derivatives of the components of the position vector. $v(t) = \left\langle \frac{d}{dt}\ln\left(t^2+1\right), \frac{d}{dt}e^{3-t} \right\rangle = \left\langle \frac{2t}{t^2+1}, -e^{3-t} \right\rangle \Rightarrow v(1) = \left\langle 1, -e^2 \right\rangle$
(C)	Incorrect. This response would result if the chain rule was not used during the differentiation of both components of the position vector, as follows. $v(t) = \left\langle \frac{d}{dt}\ln\left(t^2+1\right), \frac{d}{dt}e^{3-t} \right\rangle = \left\langle \frac{1}{t^2+1}, e^{3-t} \right\rangle \Rightarrow v(1) = \left\langle \frac{1}{2}, e^2 \right\rangle$
(D)	Incorrect. This response would result if the chain rule was not used during the differentiation of the $x(t)$ component of the position vector, as follows. $v(t) = \left\langle \frac{d}{dt}\ln\left(t^2+1\right), \frac{d}{dt}e^{3-t} \right\rangle = \left\langle \frac{1}{t^2+1}, -e^{3-t} \right\rangle \Rightarrow v(1) = \left\langle \frac{1}{2}, -e^2 \right\rangle$

Question 5

Skill		Learning Objective	Topic
3.D		LIM-7.A	(BC ONLY) Working with Geometric Series
(A)	Incorrect. This response might be chosen if the series is correctly identified as a convergent geometric series with common ratio $r = \frac{e}{\pi}$, but the first term was taken to be 1 rather than $\frac{e}{\pi}$, resulting in $\frac{1}{1-r} = \frac{1}{1-\frac{e}{\pi}} = \frac{\pi}{\pi - e}$.		
(B)	**Correct**. The series is a geometric series with first term $a = \frac{e}{\pi}$ and common ratio $r = \frac{e}{\pi}$. Since this ratio is less than 1, the series converges to $\frac{a}{1-r} = \frac{\frac{e}{\pi}}{1-\frac{e}{\pi}} = \frac{e}{\pi - e}$.		
(C)	Incorrect. The sum of the series might have been taken to be the value of the definite integral used in performing the integral test for convergence. Let $f(x) = \left(\frac{e}{\pi}\right)^x$. $$\int_1^{\infty} f(x)\,dx = \int_1^{\infty} \left(\frac{e}{\pi}\right)^x dx = \lim_{b\to\infty} \int_1^b \left(\frac{e}{\pi}\right)^x dx = \lim_{b\to\infty} \left.\frac{\left(\frac{e}{\pi}\right)^x}{\ln\left(\frac{e}{\pi}\right)}\right\|_1^b$$ $$= \lim_{b\to\infty}\left(\frac{\left(\frac{e}{\pi}\right)^b}{\ln\left(\frac{e}{\pi}\right)} - \frac{\frac{e}{\pi}}{\ln\left(\frac{e}{\pi}\right)}\right) = 0 - \frac{\frac{e}{\pi}}{\ln\left(\frac{e}{\pi}\right)} = \frac{\frac{e}{\pi}}{-\ln\left(\frac{e}{\pi}\right)} = \frac{e}{\pi \ln\left(\frac{\pi}{e}\right)}$$		
(D)	Incorrect. This response might be chosen if the series is correctly identified as a geometric series with common ratio $r = \frac{e}{\pi}$, but the ratio is thought to be greater than 1.		

Question 6

Skill	Learning Objective	Topic
1.E	FUN-3.B	The Quotient Rule

(A)	**Correct**. The derivative of h is found by using the quotient rule. $h'(x) = \dfrac{f''(x)g(x) - f'(x)g'(x)}{(g(x))^2}$ The values for the functions and derivatives at $x = 2$ are obtained from the table. $h'(2) = \dfrac{f''(2)g(2) - f'(2)g'(2)}{(g(2))^2} = \dfrac{(3)(-2) - (-3)(5)}{(-2)^2} = \dfrac{-6 + 15}{4} = \dfrac{9}{4}$
(B)	Incorrect. This response would result if the derivative of a quotient was taken to be the quotient of the derivatives, as follows. $h'(x) = \dfrac{f''(x)}{g'(x)} \Rightarrow h'(2) = \dfrac{f''(2)}{g'(2)} = \dfrac{3}{5}$
(C)	Incorrect. This response comes from only differentiating the numerator in the quotient, as follows. $h'(x) = \dfrac{f''(x)}{g(x)} \Rightarrow h'(2) = \dfrac{f''(2)}{g(2)} = \dfrac{3}{-2} = -\dfrac{3}{2}$
(D)	Incorrect. This response would result if the terms in the numerator were added rather than subtracted in the quotient rule, as follows. $h'(x) = \dfrac{f''(x)g(x) + f'(x)g'(x)}{(g(x))^2} \Rightarrow h'(2) = \dfrac{(3)(-2) + (-3)(5)}{(-2)^2} = \dfrac{-6 - 15}{4} = -\dfrac{21}{4}$

Question 7

Skill	Learning Objective	Topic
2.C	LIM-8.G	(BC ONLY) Representing Functions as Power Series

(A)	**Correct**. Beginning with the Maclaurin series for $\cos x$, x^2 is substituted to obtain the Maclaurin series for $\cos\left(x^2\right)$. This series is then multiplied term-by-term by x. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ $\cos\left(x^2\right) = 1 - \frac{\left(x^2\right)^2}{2!} + \frac{\left(x^2\right)^4}{4!} - \frac{\left(x^2\right)^6}{6!} + \cdots = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots$ $x\cos\left(x^2\right) = x\left(1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots\right) = x - \frac{x^5}{2!} + \frac{x^9}{4!} - \frac{x^{13}}{6!} + \cdots$
(B)	Incorrect. This response would result if the Maclaurin series for $\cos x$ was used, but there was no substitution of x^2 before multiplication by x. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ $x\cos x = x\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right) = x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$
(C)	Incorrect. This response would result if the Maclaurin series for $\sin x$ was used instead of the series for $\cos x$. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\sin\left(x^2\right) = \left(x^2\right) - \frac{\left(x^2\right)^3}{3!} + \frac{\left(x^2\right)^5}{5!} - \frac{\left(x^2\right)^7}{7!} + \cdots = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \cdots$ $x\sin\left(x^2\right) = x\left(x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \cdots\right) = x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \frac{x^{15}}{7!} + \cdots$
(D)	Incorrect. This response would result if the Maclaurin series for $\sin x$ was used, and the series was multiplied by x^2. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $x^2\sin x = x^2\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right) = x^3 - \frac{x^5}{3!} + \frac{x^7}{5!} - \frac{x^9}{7!} + \cdots$

Question 8

Skill		Learning Objective	Topic
2.B		LIM-2.D	Connecting Limits at Infinity and Horizontal Asymptotes
(A)	Incorrect. This response might come from treating the problem like the limit of a rational function as x goes to infinity when the numerator and denominator are polynomials of the same degree. If only the coefficients of the x^2 term and the e^x term are considered, it might be thought that the limit would be $\frac{-6}{3} = -2.$		
(B)	**Correct**. The numerator of $\frac{10 - 6x^2}{5 + 3e^x}$ is a translated power function and the denominator is a translated exponential function. Since the exponential function e^x grows faster than the power function x^2, the relative magnitude of the denominator compared to the numerator will result in this expression converging to 0 as x goes to infinity.		
(C)	Incorrect. This response might come from treating the problem like the limit of a rational function as x goes to $0.$ If only the constant terms are considered, it might be thought that the limit would be $\frac{10}{5} = 2.$		
(D)	Incorrect. It might be thought that the limit is nonexistent since the numerator goes to $-\infty$ and the denominator goes to $+\infty$ as x goes to infinity, but this does not take into account the relative magnitude of the exponential function in the denominator compared to the quadratic term in the numerator as x gets larger.		

Question 9

Skill	**Learning Objective**	**Topic**
3.B	CHA-2.B	Defining Average and Instantaneous Rates of Change at a Point

(A)	Incorrect. This statement could be false. A function can be continuous at a point without it being differentiable at that point. For example, $f(x) = \lvert x-5 \rvert$ is continuous at $x = 5$ but not differentiable there.
(B)	Incorrect. This statement could be false. For example, if $f(x) = \lvert x-5 \rvert$, then f is not differentiable at $x = 5$ but $\lim_{x \to 5} f(x) = 0$.
(C)	**Correct**. The expression $\lim_{x \to 5} \frac{f(x) - f(5)}{x - 5}$ is one form of the definition of the derivative of f at $x = 5$. Since f is not differentiable at $x = 5$, however, this limit does not exist.
(D)	Incorrect. This statement could be false. The definite integral can be found if f is continuous even if f is not differentiable at one of the endpoints. For example, if $f(x) = \lvert x-5 \rvert$, then f is not differentiable at $x = 5$ but $\int_0^5 f(x)\,dx = \frac{1}{2}(5)(5) = \frac{25}{2}$.

Question 10

Skill	**Learning Objective**	**Topic**
2.B	FUN-4.A	Determining Concavity of Functions over Their Domains

(A)	Incorrect. This response would result if the change of sign of the second derivative f'' at $x = 3$ is detected, but the change in sign at $x = 0$ is overlooked.
(B)	**Correct**. A point of inflection occurs where the second derivative f'' changes sign. The zeros of $f''(x)$ occur at $x = 0,$ $x = 3,$ and $x = 10.$ Constructing a sign chart with the three factors $x,$ $(x-3)^5$, and $(x-10)^2$ shows that $f''(x)$ is positive for $x < 0,$ negative for $0 < x < 3,$ positive for $3 < x < 10,$ and positive for $x > 10.$ Therefore, the graph of f has a point of inflection only at $x = 0$ and $x = 3,$ where $f''(x)$ changes from positive to negative and then from negative to positive, respectively.
(C)	Incorrect. The second derivative f'' is positive both to the left and to the right of $x = 10,$ so there is no sign change in $f''(x)$ at $x = 10.$ Therefore, the graph of f does not have a point of inflection at $x = 10.$ There is a sign change in $f''(x)$ from positive to negative at $x = 0,$ so the graph of f does have a point of inflection at $x = 0$ in addition to the one at $x = 3.$
(D)	Incorrect. These are the three zeros of the second derivative $f'',$ but $f''(x)$ only changes sign at $x = 0$ and $x = 3.$

Question 11

Skill	Learning Objective	Topic
1.E	FUN-6.D	Integrating Using Substitution

(A)	Incorrect. This response would result if the substitution $u = e^x - x$ was used in an attempt to evaluate the definite integral. $u = e^x - x \Rightarrow \dfrac{du}{dx} = e^x - 1 \Rightarrow dx = \dfrac{du}{e^x - 1}$ When $x = 0,$ $u = e^0 - 0 = 1.$ When $x = \pi,$ $u = e^\pi - \pi.$ Only the substitutions for dx and for the limits of integration were made, however, and not for $e^x - x,$ as follows. $\displaystyle\int_0^\pi \frac{e^x - 1}{e^x - x}\,dx = \int_1^{e^\pi - \pi} du = u\Big\|_1^{e^\pi - \pi} = \left(e^\pi - \pi\right) - 1$
(B)	Incorrect. This response would result if the substitution $u = e^x - x$ was correctly used in an attempt to evaluate the definite integral, but the evaluation used $\ln 1 = 1$ rather than $\ln 1 = 0.$ $u = e^x - x \Rightarrow \dfrac{du}{dx} = e^x - 1 \Rightarrow dx = \dfrac{du}{e^x - 1}$ When $x = 0,$ $u = e^0 - 0 = 1.$ When $x = \pi,$ $u = e^\pi - \pi.$ Substituting for $e^x - x,$ for $dx,$ and for the limits of integration gives $\displaystyle\int_0^\pi \frac{e^x - 1}{e^x - x}\,dx = \int_1^{e^\pi - \pi} \frac{1}{u}\,du = \ln u\Big\|_1^{e^\pi - \pi} = \ln\left(e^\pi - \pi\right) - \ln 1 = \ln\left(e^\pi - \pi\right) - 1.$
(C)	Incorrect. This response would result if the integrand was incorrectly simplified as $1 - \dfrac{1}{x}$ and then either the antiderivative was only evaluated at the upper limit or $\ln 0$ was taken to equal $0,$ as follows. $\displaystyle\int_0^\pi \frac{e^x - 1}{e^x - x}\,dx = \int_0^\pi \left(1 - \frac{1}{x}\right) dx = (x - \ln x)\Big\|_0^\pi = \pi - \ln \pi$
(D)	**Correct**. This integral can be evaluated by using substitution of variables with $u = e^x - x.$ $u = e^x - x \Rightarrow \dfrac{du}{dx} = e^x - 1 \Rightarrow dx = \dfrac{du}{e^x - 1}$ When $x = 0,$ $u = e^0 - 0 = 1.$ When $x = \pi,$ $u = e^\pi - \pi.$ Substituting for $e^x - x,$ for $dx,$ and for the limits of integration gives $\displaystyle\int_0^\pi \frac{e^x - 1}{e^x - x}\,dx = \int_1^{e^\pi - \pi} \frac{1}{u}\,du = \ln u\Big\|_0^{e^\pi - \pi} = \ln\left(e^\pi - \pi\right) - \ln 1 = \ln\left(e^\pi - \pi\right) - 0$ $= \ln\left(e^\pi - \pi\right).$

Question 12

Skill		Learning Objective	Topic
3.D		FUN-1.B	Using the Mean Value Theorem
(A)	Incorrect. This interval might be chosen because of an error in computing the average rate of change over the interval as $\frac{f(0)-f(4)}{4}=\frac{8-0}{4}=2$ rather than $\frac{f(0)-f(4)}{0-4}=\frac{8-0}{-4}=-2$.		
(B)	Incorrect. This interval might be chosen because of an error in computing the average rate of change over the interval as $\frac{8-4}{f(8)-f(4)}=\frac{4}{2-0}=2$ rather than $\frac{f(8)-f(4)}{8-4}=\frac{2-0}{4}=\frac{1}{2}$.		
(C)	**Correct**. The function f is continuous on the closed interval $[8, 12]$ and differentiable on the open interval $(8, 12)$. By the Mean Value Theorem, there is a number c in the interval $(8, 12)$ such that $f'(c)=\frac{f(12)-f(8)}{12-8}=\frac{10-2}{4}=2$.		
(D)	Incorrect. This response would result if the Intermediate Value Theorem was used instead of the Mean Value Theorem to select the open interval $(12, 16)$ where $f(c)=2$ for some number c in the interval.		

Question 13

Skill	Learning Objective	Topic
1.E	FUN-7.C	(BC ONLY) Approximating Solutions Using Euler's Method

(A)	**Correct**. Euler's method for the differential equation $\frac{dy}{dx} = f(x, y) = x + 2y$ can be written as $y_{k+1} = y_k + f(x_k, y_k)\Delta x$, where Δx is the step size. Then y_k is an approximation for $f(x_k)$. Here the step size is $\Delta x = \frac{-0.4 - 0}{2} = -0.2$, since there are two steps of equal length. $x_0 = 0;\ \ y_0 = 2$ $x_1 = -0.2;\ \ y_1 = y_0 + f(x_0, y_0)\Delta x = 2 + (0 + 2(2))(-0.2) = 2 + (4)(-0.2) = 1.2$ $x_2 = -0.4;$ $y_2 = y_1 + f(x_1, y_1)\Delta x = 1.2 + (-0.2 + 2(1.2))(-0.2) = 1.2 + (2.2)(-0.2) = 0.76$
(B)	Incorrect. This response would result if only the first step was done using Euler's method for the differential equation $\frac{dy}{dx} = f(x, y) = x + 2y$. $x_0 = 0;\ \ y_0 = 2$ $x_1 = -0.2;\ \ y_1 = y_0 + f(x_0, y_0)\Delta x = 2 + (0 + 2(2))(-0.2) = 2 + (4)(-0.2) = 1.2$
(C)	Incorrect. This response would result if the step size was taken to be $\Delta x = -0.1$ rather than $\Delta x = -0.2$. As a result, this response was the approximation for $f(-0.2)$ rather than for $f(-0.4)$. $x_0 = 0;\ \ y_0 = 2$ $x_1 = -0.1;\ \ y_1 = y_0 + f(x_0, y_0)\Delta x = 2 + (0 + 2(2))(-0.1) = 2 + (4)(-0.1) = 1.6$ $x_2 = -0.2;$ $y_2 = y_1 + f(x_1, y_1)\Delta x = 1.6 + (-0.1 + 2(1.6))(-0.1) = 1.6 + (3.1)(-0.1) = 1.29$
(D)	Incorrect. This response would result if the step size was taken to be $\Delta x = 0.2$ rather than $\Delta x = -0.2$. As a result, this response was the approximation for $f(0.4)$ rather than for $f(-0.4)$. $x_0 = 0;\ \ y_0 = 2$ $x_1 = 0.2;\ \ y_1 = y_0 + f(x_0, y_0)\Delta x = 2 + (0 + 2(2))(0.2) = 2 + (4)(0.2) = 2.8$ $x_2 = 0.4;$ $y_2 = y_1 + f(x_1, y_1)\Delta x = 2.8 + (0.2 + 2(2.8))(0.2) = 2.8 + (5.8)(0.2) = 3.96$

Question 14

Skill	**Learning Objective**	**Topic**
1.E	FUN-3.D	Implicit Differentiation

(A)	Incorrect. This response would result if the equation obtained by implicit differentiation was solved incorrectly for $\frac{dy}{dx}$, as follows. $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{x}}{\sqrt{y}}$ $\left.\frac{dy}{dx}\right\|_{\left(\frac{9}{4},\frac{1}{4}\right)} = -\frac{\sqrt{\frac{9}{4}}}{\sqrt{\frac{1}{4}}} = -\frac{\frac{3}{2}}{\frac{1}{2}} = -3$ It would also be obtained if the expression for $\frac{dy}{dx}$ had been correctly found but the x- and y-values were reversed when substituting into the first derivative, as follows. $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$ $\left.\frac{dy}{dx}\right\|_{\left(\frac{9}{4},\frac{1}{4}\right)} = -\frac{\sqrt{\frac{9}{4}}}{\sqrt{\frac{1}{4}}} = -\frac{\frac{3}{2}}{\frac{1}{2}} = -3$
(B)	**Correct**. The slope of the tangent line is the value of $\frac{dy}{dx}$ at the point $\left(\frac{9}{4}, \frac{1}{4}\right)$. During the implicit differentiation to find $\frac{dy}{dx}$, both the power rule and the chain rule are needed. $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$ The point $\left(\frac{9}{4}, \frac{1}{4}\right)$ is on the curve, since $x = \frac{9}{4}$ and $y = \frac{1}{2}$ satisfy the equation $\sqrt{x} + \sqrt{y} = 2$. At this point, $\frac{1}{2\left(\frac{3}{2}\right)} + \frac{1}{2\left(\frac{1}{2}\right)}\frac{dy}{dx} = 0 \Rightarrow \frac{1}{3} + \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{1}{3}$.
(C)	Incorrect. This response might come from observing the symmetry of the expressions for x and y in the equation of the curve and concluding that the derivative expressions will be symmetric, leading to a slope of 1, without considering that the x- and y-coordinates of the given point are not the same.
(D)	Incorrect. This response would result if the method of implicit differentiation was incorrectly applied by taking the derivative of the expression $\sqrt{x} + \sqrt{y}$ without consideration of the chain rule and setting the result equal to $\frac{dy}{dx}$, as follows. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \Rightarrow \left.\frac{dy}{dx}\right\|_{\left(\frac{9}{4},\frac{1}{4}\right)} = \frac{1}{2\sqrt{\frac{9}{4}}} + \frac{1}{2\sqrt{\frac{1}{4}}} = \frac{1}{3} + 1 = \frac{4}{3}$

Question 15

Skill	**Learning Objective**	**Topic**
1.E	LIM-6.A	(BC ONLY) Evaluating Improper Integrals

(A)	Incorrect. This response would result if the integrand was just evaluated at the lower limit, $x = 1$, rather than antidifferentiated, as follows. $$\left.\frac{6}{(x+3)^{\frac{3}{2}}}\right\|_{x=1} = \frac{6}{4^{\frac{3}{2}}} = \frac{6}{8} = \frac{3}{4}$$
(B)	Incorrect. This response would result if in applying the power rule for antiderivatives, the power of $(x + 3)$ was not divided by the new exponent, as follows. $$\int_1^{\infty} \frac{6}{(x+3)^{\frac{3}{2}}}\,dx = \lim_{b\to\infty}\int_1^{b} \frac{6}{(x+3)^{\frac{3}{2}}}\,dx = \lim_{b\to\infty}\left(\left.-\frac{6}{(x+3)^{\frac{1}{2}}}\right\|_1^b\right)$$ $$= \lim_{b\to\infty}\left(-\frac{6}{(b+3)^{\frac{1}{2}}} + \frac{6}{4^{\frac{1}{2}}}\right) = 0 + \frac{6}{2} = 3$$
(C)	**Correct**. This is an improper integral, since the region over which the integrand is being integrated is unbounded. The antidifferentiation is an application of the power rule. $$\int_1^{\infty} \frac{6}{(x+3)^{\frac{3}{2}}}\,dx = \lim_{b\to\infty}\int_1^{b} \frac{6}{(x+3)^{\frac{3}{2}}}\,dx = \lim_{b\to\infty}\left(\left.-\frac{12}{(x+3)^{\frac{1}{2}}}\right\|_1^b\right)$$ $$= \lim_{b\to\infty}\left(-\frac{12}{(b+3)^{\frac{1}{2}}} + \frac{12}{4^{\frac{1}{2}}}\right) = 0 + \frac{12}{2} = 6$$
(D)	Incorrect. This response might come from assuming that the definite integral of a function integrated over an unbounded region will always diverge.

Question 16

Skill		Learning Objective	Topic
1.E		FUN-7.D	Finding General Solutions Using Separation of Variables
(A)	Incorrect. This response would result if a chain rule error was made during the antidifferentiation of the dy term. $\frac{dy}{dx} = 2 - y \Rightarrow \frac{dy}{2 - y} = dx$ $\int \frac{1}{2 - y}\, dy = \int dx \Rightarrow \ln\lvert 2 - y\rvert = x + C$ $\ln\lvert 1\rvert = 1 + C \Rightarrow C = -1$ $\ln\lvert 2 - y\rvert = x - 1 \Rightarrow \lvert 2 - y\rvert = e^{x-1}$ Since $2 - y > 0$ at the initial value $y = 1,$ the solution would be $2 - y = e^{x-1},$ or $y = 2 - e^{x-1}.$		
(B)	**Correct**. The differential equation can be solved using separation of variables and the initial condition to determine the appropriate value for the arbitrary constant. $\frac{dy}{dx} = 2 - y \Rightarrow \frac{dy}{2 - y} = dx$ $\int \frac{1}{2 - y}\, dy = \int dx \Rightarrow -\ln\lvert 2 - y\rvert = x + C$ $-\ln\lvert 1\rvert = 1 + C \Rightarrow C = -1$ $-\ln\lvert 2 - y\rvert = x - 1 \Rightarrow \ln\lvert 2 - y\rvert = -x + 1 \Rightarrow \lvert 2 - y\rvert = e^{1-x}$ Since $2 - y > 0$ at the initial value $y = 1,$ the solution to the differential equation is $2 - y = e^{1-x},$ or $y = 2 - e^{1-x}.$		
(C)	Incorrect. This response would result if an arbitrary constant was not included during the antidifferentiation. $\frac{dy}{dx} = 2 - y \Rightarrow \frac{dy}{2 - y} = dx$ $\int \frac{1}{2 - y}\, dy = \int dx \Rightarrow -\ln\lvert 2 - y\rvert = x \Rightarrow \lvert 2 - y\rvert = e^{-x}$ Since $2 - y > 0$ at the initial value $y = 1,$ the solution would be $2 - y = e^{-x},$ or $y = 2 - e^{-x}.$		
(D)	Incorrect. This response would result if an arbitrary constant was not included during the antidifferentiation and the incorrect sign was taken for the absolute value when solving for y. $\frac{dy}{dx} = 2 - y \Rightarrow \frac{dy}{2 - y} = dx$ $\int \frac{1}{2 - y}\, dy = \int dx \Rightarrow -\ln\lvert 2 - y\rvert = x \Rightarrow \lvert 2 - y\rvert = e^{-x}$ $2 - y = -e^{-x} \Rightarrow y = 2 + e^{-x}$		

Question 17

Skill	Learning Objective	Topic
3.D	LIM-7.A	(BC ONLY) Alternating Series Test for Convergence

(A)	Incorrect. This series diverges by the nth term test since $\lim_{n\to\infty}\left\lvert(-1)^n\frac{1-n}{n}\right\rvert = \lim_{n\to\infty}\left\lvert\frac{1-n}{n}\right\rvert = 1 \neq 0.$
(B)	Incorrect. This series diverges by the nth term test since $\lim_{n\to\infty}\left\lvert(-1)^n\frac{n+1}{2n}\right\rvert = \lim_{n\to\infty}\frac{n+1}{2n} = \frac{1}{2} \neq 0.$
(C)	Incorrect. This series diverges by the nth term test since $\lim_{n\to\infty}\left\lvert(-1)^n\frac{n^2}{3\sqrt{n}}\right\rvert = \lim_{n\to\infty}\frac{\sqrt{n}}{3}$ does not exist.
(D)	**Correct**. The series $\sum_{n=1}^{\infty}(-1)^n\left(\frac{2\sqrt{n}}{n}\right) = \sum_{n=1}^{\infty}(-1)^n\left(\frac{2}{\sqrt{n}}\right)$ satisfies the three conditions: (1) the series is alternating, (2) $\lim_{n\to\infty}\frac{2}{\sqrt{n}} = 0,$ and (3) the terms $a_n = \frac{2}{\sqrt{n}}$ are decreasing since $\frac{2}{\sqrt{n+1}} < \frac{2}{\sqrt{n}}$ for all $n \geq 1.$ Therefore, the series converges by the alternating series test.

Question 18

Skill	Learning Objective	Topic
1.C	CHA-5.D	(BC ONLY) Finding the Area of the Region Bounded by Two Polar Curves

(A)	Incorrect. This response comes from using a rectangular form for area instead of a polar form for area, as if the curves were treated as functions of y in terms of x. The region appears to lie above the interval $[-2, 6]$ on the x-axis. $\int_{-2}^{6} (y_2(x) - y_1(x))\, dx = \int_{-2}^{6} (g(x) - f(x))\, dx$
(B)	Incorrect. This response comes from using a rectangular form of the area instead of a polar form for area, as if the curves were treated as functions of y in terms of x, but expressing the limits in terms of the polar angle θ. $\int_{0}^{\pi} (y_2(x) - y_1(x))\, dx = \int_{0}^{\pi} (g(x) - f(x))\, dx$
(C)	Incorrect. The square of the difference between the two polar curves was integrated rather than the difference of the squares. $\frac{1}{2}\int_{0}^{\pi} (r_2(\theta) - r_1(\theta))^2\, d\theta = \frac{1}{2}\int_{0}^{\pi} (g(\theta) - f(\theta))^2\, d\theta$
(D)	**Correct**. The area bounded by the two polar curves can be found with a definite integral. Let r_1 be the smaller radius, and let r_2 be the larger radius. The graphs of the two polar curves bound the region R over the domain $0 \le \theta \le \pi$. Then the area of R is given by $\frac{1}{2}\int_{0}^{\pi} (r_2(\theta))^2\, d\theta - \frac{1}{2}\int_{0}^{\pi} (r_1(\theta))^2\, d\theta = \frac{1}{2}\int_{0}^{\pi} \left((r_2(\theta))^2 - (r_1(\theta))^2\right) d\theta$ $= \frac{1}{2}\int_{0}^{\pi} \left((g(\theta))^2 - (f(\theta))^2\right) d\theta.$

Question 19

Skill		Learning Objective	Topic
3.B		LIM-7.A	(BC ONLY) Alternating Series Test for Convergence
(A)	Incorrect. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is the divergent harmonic series. However, $\frac{n}{n^2+1} < \frac{1}{n}$, so this inequality goes the wrong way to use the comparison test to show that the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges. In addition, the divergence of the positive series $\sum_{n=1}^{\infty} \lvert a_n \rvert$ does not imply that the series $\sum_{n=1}^{\infty} a_n$ diverges.		
(B)	Incorrect. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is the divergent harmonic series. Since $\lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1,$ the limit comparison test shows that the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ also diverges. However, it cannot be concluded from that the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ diverges. The divergence of the positive series $\sum_{n=1}^{\infty} \lvert a_n \rvert$ does not imply that the series $\sum_{n=1}^{\infty} a_n$ diverges.		
(C)	Incorrect. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series with $p = 2 > 1.$ However, $\frac{1}{n^2} < \frac{n}{n^2+1}$ for all $n > 1,$ so this inequality goes the wrong way to use the comparison test to show that the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ converges and that therefore $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ is absolutely convergent.		

(D)	**Correct**. The series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1}$ satisfies the three conditions: (1) the series is alternating, (2) $\lim_{n\to\infty} \frac{n}{n^2 + 1} = 0,$ and (3) the terms $a_n = \frac{n}{n^2 + 1}$ are decreasing. Therefore, the series converges by the alternating series test. To verify that the terms $a_n = \frac{n}{n^2 + 1}$ are decreasing, consider the following. $a_{n+1} < a_n \Leftrightarrow \frac{n + 1}{(n + 1)^2 + 1} < \frac{n}{n^2 + 1}$ $\Leftrightarrow n^3 + n^2 + n + 1 < n^3 + 2n^2 + 2n$ $\Leftrightarrow 1 < n^2 + n$ The last inequality is true for all $n \geq 1$. Alternatively, if $f(x) = \frac{x}{x^2 + 1},$ then $a_n = f(n)$ and the function f is decreasing because $f'(x) = \frac{\left(x^2 + 1\right) - 2x^2}{\left(x^2 + 1\right)^2} = \frac{1 - x^2}{\left(x^2 + 1\right)^2} < 0$ for $x > 1$.

Question 20

Skill	Learning Objective	Topic
1.C	FUN-6.A	Applying Properties of Definite Integrals

(A)	Incorrect. The value of this integral can be determined using the properties of the definite integral, as follows. $\int_4^1 g(x)\,dx = -\int_1^4 g(x)\,dx = -(-2) = 2$
(B)	Incorrect. The value of this integral can be determined using the properties of the definite integral, as follows. $\int_1^4 3f(x)\,dx = 3\cdot\int_1^4 f(x)\,dx = 3\cdot 8 = 24$
(C)	**Correct**. It is not true in general that $\int_1^4 3f(x)g(x)\,dx = \int_1^4 3f(x)\,dx\cdot\int_1^4 g(x)\,dx,$ so the individual values of $\int_1^4 f(x)\,dx$ and $\int_1^4 g(x)\,dx$ cannot be used to determine the value of $\int_1^4 3f(x)g(x)\,dx.$ For example, if $f(x) = \frac{8}{3}$ and $g(x) = -\frac{2}{3},$ then $\int_1^4 f(x)\,dx = 8,$ $\int_1^4 g(x)\,dx = -2,$ and $\int_1^4 3f(x)g(x)\,dx = \int_1^4 -\frac{16}{3}\,dx = -16.$ However, if $f(x) = \frac{16}{9}(x-1)$ and $g(x) = -\frac{4}{9}(x-1),$ then $\int_1^4 f(x)\,dx = 8$ and $\int_1^4 g(x)\,dx = -2$ as before, but now $\int_1^4 3f(x)g(x)\,dx = \int_1^4 -\frac{64}{27}(x-1)^2\,dx = -\frac{64}{3}.$
(D)	Incorrect. The value of this integral can be determined using the properties of the definite integral, as follows. $\int_1^4 (3f(x)+g(x))\,dx = \int_1^4 3f(x)\,dx + \int_1^4 g(x)\,dx$ $= 3\cdot\int_1^4 f(x)\,dx + \int_1^4 g(x)dx = 3\cdot 8 + (-2) = 22$

Question 21

Skill	Learning Objective	Topic
3.D	LIM-8.D	(BC ONLY) Radius and Interval of Convergence of Power Series

(A)	**Correct**. The ratio test can be used to determine the interval of convergence. $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert = \lim_{n\to\infty}\left\lvert\frac{(x+4)^{n+1}}{(n+1)\cdot 5^{n+2}}\cdot\frac{n\cdot 5^{n+1}}{(x+4)^n}\right\rvert = \lim_{n\to\infty}\left\lvert\frac{n}{n+1}\cdot\frac{x+4}{5}\right\rvert = \frac{1}{5}\lvert x+4\rvert < 1$ $\Rightarrow \lvert x+4\rvert < 5 \Rightarrow -9 < x < 1$ Now check the endpoints of the interval. At $x = -9,$ the series is $\sum_{n=1}^{\infty}\frac{(-5)^n}{n\cdot 5^{n+1}} = \frac{1}{5}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n},$ which converges since it is a multiple of the alternating harmonic series. At $x = 1,$ the series is $\sum_{n=1}^{\infty}\frac{5^n}{n\cdot 5^{n+1}} = \frac{1}{5}\sum_{n=1}^{\infty}\frac{1}{n},$ which diverges since it is a multiple of the harmonic series. Therefore, the interval of convergence is $-9 \le x < 1.$
(B)	Incorrect. The ratio test might have been used to correctly determine that the radius of convergence R is $5,$ but the interval was taken to be centered at $0.$ $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert = \lim_{n\to\infty}\left\lvert\frac{1}{(n+1)\cdot 5^{n+2}}\cdot\frac{n\cdot 5^{n+1}}{1}\right\rvert = \lim_{n\to\infty}\left\lvert\frac{n}{n+1}\cdot\frac{1}{5}\right\rvert = \frac{1}{5} \Rightarrow R = 5$ At $x = -5,$ the series is $\sum_{n=1}^{\infty}\frac{(-1)^n}{n\cdot 5^{n+1}},$ which converges by the alternating series test. At $x = 5,$ the series is $\sum_{n=1}^{\infty}\frac{9^n}{n\cdot 5^{n+1}},$ which diverges by the nth term test.
(C)	Incorrect. This response would result if the ratio test was not used to determine the radius of convergence. Looking at the form of the general term, the center was thought to be at $x = 5$ and the radius of convergence was taken to be $4.$ It was assumed that the series must converge at one endpoint and diverge at the other.
(D)	Incorrect. Errors in applying the ratio test might have led to the conclusion that the radius of convergence was infinite.

Question 22

Skill	Learning Objective	Topic
1.E	FUN-3.G	(BC ONLY) Defining Polar Coordinates and Differentiating in Polar Form

(A)	Incorrect. This response would result if the slope was taken to be $\frac{dy}{dx} = \frac{\frac{dx}{d\theta}}{\frac{dy}{d\theta}}$. $x = r\cos\theta = 3\theta\cos\theta \Rightarrow \left.\frac{dx}{d\theta}\right\|_{\theta=\frac{\pi}{2}} = 3\cos\theta - 3\theta\sin\theta\big\|_{\theta=\frac{\pi}{2}} = -\frac{3\pi}{2}$ $y = r\sin\theta = 3\theta\sin\theta \Rightarrow \left.\frac{dy}{d\theta}\right\|_{\theta=\frac{\pi}{2}} = 3\sin\theta + 3\theta\cos\theta\big\|_{\theta=\frac{\pi}{2}} = 3$ $\left.\frac{dy}{dx}\right\|_{\theta=\frac{\pi}{2}} = \left.\frac{\frac{dx}{d\theta}}{\frac{dy}{d\theta}}\right\|_{\theta=\frac{\pi}{2}} = \frac{-\frac{3\pi}{2}}{3} = -\frac{\pi}{2}$
(B)	**Correct**. The slope of the polar curve is $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$. $x = r\cos\theta = 3\theta\cos\theta \Rightarrow \left.\frac{dx}{d\theta}\right\|_{\theta=\frac{\pi}{2}} = 3\cos\theta - 3\theta\sin\theta\big\|_{\theta=\frac{\pi}{2}} = -\frac{3\pi}{2}$ $y = r\sin\theta = 3\theta\sin\theta \Rightarrow \left.\frac{dy}{d\theta}\right\|_{\theta=\frac{\pi}{2}} = 3\sin\theta + 3\theta\cos\theta\big\|_{\theta=\frac{\pi}{2}} = 3$ $\left.\frac{dy}{dx}\right\|_{\theta=\frac{\pi}{2}} = \left.\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right\|_{\theta=\frac{\pi}{2}} = \frac{3}{-\frac{3\pi}{2}} = -\frac{2}{\pi}$
(C)	Incorrect. This response would result if the product rule was applied incorrectly by taking the product of the derivatives. $x = r\cos\theta = 3\theta\cos\theta \Rightarrow \frac{dx}{d\theta} = -3\sin\theta \Rightarrow \left.\frac{dx}{d\theta}\right\|_{\theta=\frac{\pi}{2}} = -3\sin\theta\big\|_{\theta=\frac{\pi}{2}} = -3$ $y = r\sin\theta = 3\theta\sin\theta \Rightarrow \frac{dy}{d\theta} = 3\cos\theta \Rightarrow \left.\frac{dy}{d\theta}\right\|_{\theta=\frac{\pi}{2}} = 3\cos\theta\big\|_{\theta=\frac{\pi}{2}} = 0$ $\left.\frac{dy}{dx}\right\|_{\theta=\frac{\pi}{2}} = \left.\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right\|_{\theta=\frac{\pi}{2}} = \frac{0}{-3} = 0$
(D)	Incorrect. This response would result if the slope was taken to be $\frac{dr}{d\theta}$ rather than $\frac{dy}{dx}$. $\left.\frac{dr}{d\theta}\right\|_{\theta=\frac{\pi}{2}} = 3\big\|_{\theta=\frac{\pi}{2}} = 3$

Question 23

Skill	**Learning Objective**	**Topic**
1.F	LIM-5.A	Approximating Areas with Riemann Sums

(A)	Incorrect. It was correctly determined that the left Riemann sum is an underestimate and the right Riemann sum is an overestimate of the definite integral because the function $f(x) = \sqrt{x}$ is increasing on the interval $[0, 4]$. Because the graph of f is concave down, however, the trapezoidal sum is an underestimate, not an overestimate.
(B)	**Correct**. Because the function $f(x) = \sqrt{x}$ is increasing on the interval $[0, 4]$, the left Riemann sum is an underestimate and the right Riemann sum is an overestimate of the definite integral, so $L < \int_0^4 \sqrt{x}\, dx < R$. Since the graph of f is concave down, the trapezoidal sum is also an underestimate, but it is a closer approximation to the definite integral than the left Riemann sum. Therefore, $L < T < \int_0^4 \sqrt{x}\, dx < R$.
(C)	Incorrect. This would be the correct response for the graph of a function that is decreasing and concave up. The graph of $f(x) = \sqrt{x}$, however, is increasing and concave down, so all the inequalities are going in the wrong direction.
(D)	Incorrect. It was correctly determined that the trapezoidal sum is an underestimate of the definite integral since the graph of $f(x) = \sqrt{x}$ is concave down on the interval $[0, 4]$. Because the function f is increasing, however, the left Riemann sum is an underestimate and the right Riemann sum is an overestimate of the definite integral. This response has reversed those two inequalities.

Question 24

Skill	Learning Objective	Topic
3.D	FUN-6.B	The Fundamental Theorem of Calculus and Definite Integrals

(A)	**Correct**. By the Fundamental Theorem of Calculus, $\int_0^{12} f'(x)\,dx = f(12) - f(0) = (-4) - 4 = -8,$ where the values of f at $x = 0$ and $x = 12$ are obtained from the graph.
(B)	Incorrect. This response would result if the function f was integrated over the interval $[0, 12]$ rather than f', as follows. $\int_0^{12} f(x)\,dx = \frac{1}{2}(4)(4) - \frac{1}{2}(4)(3) - \frac{1}{2}(4)(4) = -6$
(C)	Incorrect. This response would result if the Fundamental Theorem of Calculus was incorrectly applied, as follows. $\int_0^{12} f'(x)\,dx = f'(12) - f'(0) = (-1) - (-1) = 0$
(D)	Incorrect. This response is the total area bounded by the graph of f and the x-axis over the interval $[0, 12]$. $\int_0^{12} \lvert f(x)\rvert\,dx = \frac{1}{2}(4)(4) + \frac{1}{2}(4)(3) + \frac{1}{2}(4)(4) = 8 + 6 + 8 = 22$

Question 25

Skill	Learning Objective	Topic
1.E	FUN-6.E	(BC ONLY) Using Integration by Parts

(A)	**Correct**. This definite integral can be found using integration by parts, where $\int u dv = uv - \int v\, du.$ $u = x \Rightarrow du = dx \quad dv = f'(x)\, dx \Rightarrow v = f(x)$ $\int_0^2 x \cdot f'(x)\, dx = xf(x)\Big\|_0^2 - \int_0^2 f(x)\, dx = 2f(2) - 7 = 10 - 7 = 3$
(B)	Incorrect. This response would result if the technique of integration by parts was applied incorrectly as follows. $u = x \Rightarrow du = dx \quad dv = f'(x)\, dx \Rightarrow v = f(x)$ $\int_0^2 x \cdot f'(x)\, dx = xf(x)\Big\|_0^2 - f(x)\Big\|_0^2 = (2f(2) - 0) - (f(2) - f(0)) = 10 - 4 = 6$
(C)	Incorrect. This response would result if each factor in the integrand was antidifferentiated separately, as follows. $\int_0^2 x \cdot f'(x)\, dx = \left(\frac{1}{2}x^2\right) f(x)\Big\|_0^2 = 2f(2) - 0 = 10$
(D)	Incorrect. This response would result if an error was made in the integration by parts when addition was used instead of subtraction, resulting in $\int u dv = uv + \int v\, du.$ $u = x \Rightarrow du = dx \quad dv = f'(x) dx \Rightarrow v = f(x)$ $\int_0^2 x \cdot f'(x)\, dx = xf(x)\Big\|_0^2 + \int_0^2 f(x)\, dx = 2f(2) + 7 = 10 + 7 = 17$

Question 26

Skill		**Learning Objective**	**Topic**
3.D		LIM-7.A	(BC ONLY) Determining Absolute or Conditional Convergence
(A)	Incorrect. Series I is absolutely convergent, not conditionally convergent, since $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges by the ratio test.		
(B)	Incorrect. Series II was correctly identified as being conditionally convergent. Series III is also conditionally convergent, since $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$ converges by the alternating series test but $\sum_{n=1}^{\infty} \frac{1}{n+2}$ diverges by the limit comparison test with the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.		
(C)	**Correct**. A series $\sum a_n$ is conditionally convergent if the series converges but the series of absolute terms $\sum \lvert a_n \rvert$ diverges. Each of the three series in this problem converges by the alternating series test. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ is not conditionally convergent, since $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges by the ratio test (so this series is absolutely convergent). The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is conditionally convergent because the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges, since it is a p-series with $p = \frac{1}{2} < 1$. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$ is conditionally convergent because the series $\sum_{n=1}^{\infty} \frac{1}{n+2}$ diverges by the limit comparison test with the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.		
(D)	Incorrect. This response might be chosen because all three series converge by the alternating series test. To determine whether they are conditionally convergent, however, it is also necessary to consider the series of absolute terms $\sum \lvert a_n \rvert$. Series I is absolutely convergent, not conditionally convergent, since $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges by the ratio test.		

Question 27

Skill		Learning Objective	Topic
1.C		CHA-5.C	Volume with Washer Method - Revolving Around x- or y-axis
(A)	Incorrect. This response is the volume of the solid generated by revolving R about the x-axis rather than the y-axis. A typical slice rotated about the x-axis would form a disk of radius $r = y = \sqrt{x-1}$ with cross-sectional area $A(x) = \pi r^2 = \pi(x-1)$. The volume of the solid would be $\int_0^{10} \pi(x-1)\,dx$.		
(B)	Incorrect. This response is the volume obtained by rotating about the x-axis the region bounded by the graph of $y = \sqrt{x-1}$, the horizontal line $y = 10$, and the vertical lines $x = 1$ and $x = 10$. A typical slice rotated about the x-axis would form a washer with cross-sectional area $A(x) = \pi r_2^2 - \pi r_1^2$, where the inner radius is $r_1 = \sqrt{x-1}$ and the outer radius is $r_2 = 10$.		
(C)	Incorrect. For this response, the cross-sectional area of the washer obtained by rotating a typical slice about the y-axis was taken to be $\pi(r_2 - r_1)^2$ rather than $\pi r_2^2 - \pi r_1^2$, where the inner radius is $r_1 = y^2 + 1$ and the outer radius is $r_2 = 10$.		
(D)	**Correct**. Rotating a typical slice about the y-axis will form a washer with inner radius r_1 and outer radius r_2, where each radius must be expressed in terms of y. Since $y = \sqrt{x-1}$, the inner radius is $r_1 = x = y^2 + 1$. Since the region is bounded on the right by the vertical line $x = 10$, the outer radius is $r_2 = 10$. The cross-sectional area of a typical slice rotated about the y-axis is therefore $A(y) = \pi r_2^2 - \pi r_1^2 = \pi\left(10^2 - \left(y^2+1\right)^2\right)$. The volume of the solid is found by using the definite integral of the cross-sectional area for y between 0 and $\sqrt{10-1} = 3$, as follows. $\int_0^3 \pi\left(10^2 - \left(y^2+1\right)^2\right) dy = \pi \int_0^3 \left(100 - \left(y^2+1\right)^2\right) dy$		

Question 28

Skill		Learning Objective	Topic
1.E		CHA-3.G	(BC ONLY) Second Derivatives of Parametric Equations
(A)	Incorrect. This response is the value of $\frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}\left(\sin\left(t^2\right)\right)$, not the value of $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$. The chain rule must be used to find $\frac{dy}{dx}$ and then used again to find $\frac{d^2y}{dx^2}$.		
(B)	Incorrect. The chain rule was correctly used to find $\frac{dy}{dx} = \frac{\sin\left(t^2\right)}{5}$. This response is just $\frac{d}{dt}\left(\frac{dy}{dx}\right)$, however. Another application of the chain rule would be needed to find $\frac{d^2y}{dx^2}$ by dividing $\frac{d}{dt}\left(\frac{dy}{dx}\right)$ by $\frac{dx}{dt}$.		
(C)	**Correct**. By the chain rule, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin\left(t^2\right)}{5}$. The chain rule is needed again to find the second derivative, as follows. $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{2t\cos\left(t^2\right)}{5}}{5} = \frac{2t\cos\left(t^2\right)}{25}$		
(D)	Incorrect. This response comes from thinking that $\frac{d^2y}{dx^2} = \frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$. Since $\frac{d^2x}{dt^2} = 0$, the quotient would be undefined.		

Question 29

Skill		Learning Objective	Topic
2.C		LIM-5.C	Riemann Sums, Summation Notation, and Definite Integral Notation
(A)	Incorrect. The sum inside the limit can be interpreted as a right Riemann sum in the form $\sum_{k=1}^{n}\left(\frac{1}{2+\frac{k}{n}}\right)\frac{1}{n} = \sum_{k=1}^{n} f(2 + k\Delta x)\Delta x,$ where $f(x) = \frac{1}{x}$ and $\Delta x = \frac{1}{n}.$ The value of Δx corresponds to an interval of length 1. Since $k\Delta x = 2 + \frac{1}{n}$ for the initial value $k = 1,$ the interval starts at 2. Therefore, the limit of the Riemann sum would be equal to $\int_2^3 f(x)\,dx = \int_2^3 \frac{1}{x}\,dx,$ not $\int_1^2 \frac{1}{x}\,dx.$		
(B)	**Correct**. The sum inside the limit can be interpreted as a right Riemann sum in the form $\sum_{k=1}^{n}\left(\frac{1}{2+\frac{k}{n}}\right)\frac{1}{n} = \sum_{k=1}^{n} f(k\Delta x)\Delta x,$ where $f(x) = \frac{1}{2+x}$ and $\Delta x = \frac{1}{n}.$ The value of Δx corresponds to an interval of length 1. Since $k\Delta x = \frac{1}{n}$ for the initial value $k = 1,$ the interval starts at 0. Therefore, the limit of the Riemann sum is equal to the definite integral $\int_0^1 f(x)\,dx = \int_0^1 \frac{1}{2+x}\,dx.$		
(C)	Incorrect. The sum inside the limit can be interpreted as a right Riemann sum in the form $\sum_{k=1}^{n}\left(\frac{1}{2+\frac{k}{n}}\right)\frac{1}{n} = \sum_{k=1}^{n} f(k\Delta x)\Delta x,$ where $f(x) = \frac{1}{2+x}$ and $\Delta x = \frac{1}{n}.$ The value of Δx corresponds to an interval of length 1. Since $k\Delta x = \frac{1}{n}$ for the initial value $k = 1,$ the interval starts at 0 and therefore the limits on the definite integral should go from 0 to 1, not 0 to 2.		
(D)	Incorrect. The sum inside the limit was correctly interpreted as a right Riemann sum on an interval of length 1 starting at $x = 2.$ With that choice of interval, however, the Riemann sum would be written as $\sum_{k=1}^{n}\left(\frac{1}{2+\frac{k}{n}}\right)\frac{1}{n} = \sum_{k=1}^{n} f(2 + k\Delta x)\Delta x,$ where $f(x) = \frac{1}{x}.$ The right Riemann sum using $f(x) = \frac{1}{2+x}$ would correspond to an interval starting at $x = 0,$ not $x = 2.$		

Question 30

Skill	Learning Objective	Topic
2.B	LIM-8.A	(BC ONLY) Finding Taylor Polynomial Approximations of Functions

(A)	Incorrect. This response would result if the chain rule was never used in finding the first and second derivatives of g leading to $g'(x) = e^{f(x)}$ and $g''(x) = e^{f(x)}$. As a result, the coefficient of x^2 in the Maclaurin series for g was computed as $\frac{1}{2!}g''(0) = \frac{1}{2}e^{f(0)} = \frac{1}{2}e^2$.
(B)	Incorrect. This response would result if the coefficient of x^2 in the Maclaurin series for g was taken to be $g''(0)$ without dividing by $2!$. In addition, the chain rule was never used in finding the first and second derivatives of g, leading to $g''(0) = e^{f(0)} = e^2$.
(C)	Incorrect. This response would result if the chain rule was correctly used in finding $g'(x)$ but was not used again when the product rule was used during the computation of the second derivative, as follows. $g'(x) = f'(x)e^{f(x)}$ $g''(x) = f''(x)e^{f(x)} + f'(x)e^{f(x)}$ $g''(0) = f''(0)e^{f(0)} + f'(0)e^{f(0)} = 2e^2 + 3e^2 = 5e^2$ The coefficient of x^2 in the Maclaurin series for g was found to be $\frac{1}{2!}g''(0) = \frac{5}{2}e^2$.
(D)	**Correct**. The coefficient of x^2 in the Maclaurin series for g is $\frac{1}{2!}g''(0)$. Since $g(x) = e^{f(x)}$, the chain rule gives $g'(x) = f'(x)e^{f(x)}$. Using the product rule and the chain rule again gives $g''(x) = f''(x)e^{f(x)} + f'(x)\left(f'(x)e^{f(x)}\right) = \left(f''(x) + (f'(x))^2\right)e^{f(x)}$. The values of $f(0)$, $f'(0)$, and $f''(0)$ can be determined from the Maclaurin series for f. $f(0) = \text{constant term} = 2$ $f'(0) = \text{coefficient of } x = 3$ $f''(0) = 2 \cdot \left(\text{coefficient of } x^2\right) = 2 \cdot 1 = 2$ Therefore, $g''(0) = \left(f''(0) + (f'(0))^2\right)e^{f(0)} = (2+9)e^2 = 11e^2$ and so the coefficient of x^2 in the Maclaurin series for g is $\frac{1}{2!}\left(11e^2\right) = \frac{11}{2}e^2$.

Question 76

Skill	Learning Objective	Topic
2.E	FUN-4.A	Determining Intervals on Which a Function Is Increasing or Decreasing

(A)	Incorrect. The graph of f is concave up where f' is increasing. This response might come from switching the roles of the function and its derivative and thinking that f is increasing where the graph of f' is concave up. The graph of f' is concave up on the intervals $(0, 1)$ and $(2, 4)$.
(B)	Incorrect. This response might come from treating the given graph as the graph of f rather than the graph of f'. These are the two intervals where f' is increasing.
(C)	Incorrect. These are the intervals where both f and f' are increasing.
(D)	**Correct**. The function f is increasing on closed intervals where f' is positive on the corresponding open intervals. The graph indicates that $f'(x) > 0$ on the intervals $(0, 2)$ and $(4, 5)$, so f is increasing on the intervals $[0, 2]$ and $[4, 5]$.

Question 77

Skill	**Learning Objective**	**Topic**
1.E	CHA-4.D	Using Accumulation Functions and Definite Integrals in Applied Contexts

(A)	Incorrect. This response is how much $\frac{dy}{dt}$ changes from $t = 1$ to $t = 6$; that is, $y'(6) - y'(1) = 3.870$. The amount by which y changes from $t = 1$ to $t = 6$ is $y(6) - y(1)$, which can be computed by using the Fundamental Theorem of Calculus.
(B)	Incorrect. This response is the approximation to the change in y along the line tangent to the graph of y at $t = 1$. It can also be interpreted as the approximation to the integral $\int_1^6 y'(t)\,dt = y(6) - y(1)$ by using a left Riemann sum with one interval of length $\Delta t = 5$. $\Delta y \approx y'(1)\Delta t = y'(1) \cdot 5 = 8.341$
(C)	Incorrect. This response is the approximation to the integral $\int_1^6 y'(t)\,dt = y(6) - y(1)$ by using the trapezoidal sum approximation over one interval of length $\Delta t = 5$. $\left(\frac{y'(1) + y'(6)}{2}\right) \cdot \Delta t = \left(\frac{y'(1) + y'(6)}{2}\right) \cdot 5 = 18.017$
(D)	**Correct**. The change in y from $t = 1$ to $t = 6$ is $y(6) - y(1)$. By the Fundamental Theorem of Calculus, $y(6) - y(1) = \int_1^6 y'(t)\,dt = \int_1^6 6e^{-0.08(t-5)^2}\,dt = 22.583$, where the numerical integration is done with the calculator.

Question 78

Skill	**Learning Objective**	**Topic**
2.D	LIM-2.D	Connecting Infinite Limits and Vertical Asymptotes

(A)	Incorrect. This graph displays the appropriate behavior as it approaches the horizontal asymptote at $y = 2,$ but in this graph $\lim_{x \to 1^-} f(x) = -\infty$ and $\lim_{x \to 1^+} f(x) = +\infty,$ which is the opposite behavior for what the graph of f should be doing as it approaches the vertical asymptote at $x = 1.$
(B)	Incorrect. This graph displays the appropriate behavior as it approaches the vertical asymptote at $x = 1,$ but it has $y = -2$ as a horizontal asymptote rather than $y = 2.$
(C)	**Correct**. Since $\lim_{x \to 1^-} f(x) = +\infty$ and $\lim_{x \to 1^+} f(x) = -\infty,$ the graph of f approaches the vertical asymptote at $x = 1$ in the upward direction as x approaches 1 from the left and approaches the vertical asymptote in the downward direction as x approaches 1 from the right. Since $\lim_{x \to -\infty} f(x) = 2$ and $\lim_{x \to +\infty} f(x) = 2,$ the graph of f approaches the horizontal asymptote at $y = 2$ in both horizontal directions. This graph exhibits these properties and therefore could be the graph of f.
(D)	Incorrect. This graph has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 1$ rather than a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 2.$

Question 79

Skill	**Learning Objective**	**Topic**
3.F	CHA-4.B	Finding the Average Value of a Function on an Interval

(A)	Incorrect. The definite integral was not divided by the length of the interval $[30, 60]$ over which the averaging is done.
(B)	**Correct**. The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx.$ Tara's average heart rate from $t = 30$ to $t = 60$ is the average value of the function h over the interval $[30, 60]$ and would therefore be given by the expression $\frac{1}{60-30}\int_{30}^{60} h(t)\,dt.$
(C)	Incorrect. This response is the average rate of change of Tara's heart rate from $t = 30$ to $t = 60,$ not the average of her heart rate over that interval. By the Fundamental Theorem of Calculus, this expression is equal to $\frac{h(60)-h(30)}{60-30}.$
(D)	Incorrect. This response is the average of the rate of change of Tara's heart rate at the two times $t = 30$ and $t = 60,$ not the average of her heart rate over the interval from $t = 30$ to $t = 60.$

Question 80

Skill		Learning Objective	Topic
1.E		LIM-8.B	(BC ONLY) Finding Taylor Polynomial Approximations of Functions
(A)	Incorrect. This response comes from only using three terms of the Taylor polynomial rather than the Taylor polynomial of degree 3. $\left.\left(-1+4(x-2)+\frac{6}{2}(x-2)^2\right)\right\rvert_{x=2.1}=-0.570$		
(B)	**Correct**. The third-degree Taylor polynomial for f about $x=2$ is the following. $P_3(x)=f(2)+f'(2)(x-2)+\frac{1}{2!}f''(2)(x-2)^2+\frac{1}{3!}f'''(2)(x-2)^3$ $=-1+4(x-2)+\frac{6}{2}(x-2)^2+\frac{12}{6}(x-2)^3$ $=-1+4(x-2)+3(x-2)^2+2(x-2)^3$ $f(2.1)\approx P_3(2.1)=-1+4(0.1)+3(0.1)^2+2(0.1)^3=-0.568$		
(C)	Incorrect. This response would result if the coefficient of the degree n term was taken to be $\frac{f^{(n)}(2)}{n}$ rather than $\frac{f^{(n)}(2)}{n!}$. $\left.\left(-1+4(x-2)+\frac{6}{2}(x-2)^2+\frac{12}{3}(x-2)^3\right)\right\rvert_{x=2.1}=-0.566$		
(D)	Incorrect. The coefficient of the degree n term was taken to be $f^{(n)}(2)$ rather than $\frac{f^{(n)}(2)}{n!}$. $\left.\left(-1+4(x-2)+6(x-2)^2+12(x-2)^3\right)\right\rvert_{x=2.1}=-0.528$		

Question 81

Skill		Learning Objective	Topic
1.E		CHA-6.A	(BC ONLY) The Arc Length of a Smooth, Planar Curve and Distance Traveled
(A)	Incorrect. This response comes from not taking the square root in the integrand of the definite integral for the length of a curve, as follows. $\int_0^{1.5}\left(1+(f'(x))^2\right)dx = \int_0^{1.5}\left(1+\left(\sqrt{x^3+1}\right)^2\right)dx = \int_0^{1.5}\left(x^3+2\right)dx = 4.266$		
(B)	**Correct**. The length of the graph of $y = f(x)$ from $x = 0$ to $x = 1.5$ is given by the definite integral $\int_0^{1.5}\sqrt{1+(f'(x))^2}\,dx = \int_0^{1.5}\sqrt{1+\left(\sqrt{x^3+1}\right)^2}\,dx = \int_0^{1.5}\sqrt{x^3+2}\,dx = 2.497,$ where the numerical integration is done with the calculator.		
(C)	Incorrect. This response comes from not squaring the derivative in the integrand of the definite integral for the length of a curve, as follows. $\int_0^{1.5}\sqrt{1+f'(x)}\,dx = \int_0^{1.5}\sqrt{1+\left(\sqrt{x^3+1}\right)}\,dx = 2.278$		
(D)	Incorrect. This response is the change in y from $x = 0$ to $x = 1.5$. $f(1.5) - f(0) = \int_0^{1.5} f'(x)\,dx = \int_0^{1.5}\sqrt{x^3+1}\,dx = 1.976$		

Question 82

Skill	Learning Objective	Topic
2.D	FUN-7.C	Sketching Slope Fields

(A)	**Correct**. In the slope field for a differential equation of the form $\frac{dy}{dx} = h(x)$, the slope at a point (x, y) depends only on the value of x. The line segments in the slope field at each point on a vertical line perpendicular to the x-axis should therefore all have the same slope. The line segments in this slope field show that behavior and therefore this could be a slope field for a differential equation of the form $\frac{dy}{dx} = h(x)$.
(B)	Incorrect. In the slope field for a differential equation of the form $\frac{dy}{dx} = h(x)$, the slope at a point (x, y) depends only on the value of x. The line segments in the slope field at each point on a vertical line perpendicular to the x-axis should therefore all have the same slope. The line segments in this slope field do not exhibit that behavior. The slopes of the line segments depend on both the x- and the y-values.
(C)	Incorrect. In the slope field for a differential equation of the form $\frac{dy}{dx} = h(x)$, the slope at a point (x, y) depends only on the value of x. The line segments in the slope field at each point on a vertical line perpendicular to the x-axis should therefore all have the same slope. The line segments in this slope field do not exhibit that behavior. The slope of the line segment at a point depends only on the value of y. This could be a slope field for a differential equation of the form $\frac{dy}{dx} = g(y)$.
(D)	Incorrect. In the slope field for a differential equation of the form $\frac{dy}{dx} = h(x)$, the slope at a point (x, y) depends only on the value of x. The line segments in the slope field at each point on a vertical line perpendicular to the x-axis should therefore all have the same slope. The line segments in this slope field do not exhibit that behavior. The slopes of the line segments depend on both the x- and the y-values.

Question 83

Skill	**Learning Objective**	**Topic**
3.D	LIM-2.C	Removing Discontinuities

<table>
<tr><td>(A)</td><td>Correct. The limit at $x = 3$ exists if the left-hand and right-hand limits are equal.
$\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) \Rightarrow k^3 + 3 = \dfrac{16}{k^2 - 3}$
The solution to this equation for $k > 0$ is $k = 2.081$. With this value of k, $\lim\limits_{x \to 3} f(x)$ exists and is equal to $f(3)$. Therefore, f is continuous at $x = 3$.</td></tr>
<tr><td>(B)</td><td>Incorrect. This response comes from trying to make the left-hand and right-hand limits of the derivative equal at $x = 3$, as follows.
$f'(x) = \begin{cases} 1 & \text{for } x < 3 \\ \dfrac{16}{\left(k^2 - x\right)^2} & \text{for } x > 3 \end{cases}$
$\lim\limits_{x \to 3^-} f'(x) = \lim\limits_{x \to 3^+} f'(x) \Rightarrow 1 = \dfrac{16}{\left(k^2 - 3\right)^2}$
The solution to this equation for $k > 0$ is $k = 2.646$.</td></tr>
<tr><td>(C)</td><td>Incorrect. In trying to set the left-hand and right-hand limits of f equal at $x = 3$, the 3 might have been substituted for the parameter k rather than the variable x, as follows.
$27 + x = \dfrac{16}{9 - x}$
The positive solution to this equation is $x = 8.550$.</td></tr>
<tr><td>(D)</td><td>Incorrect. This response might come from errors that lead to an equation that has no positive solution. For example, it might come from trying to make the left-hand and right-hand limits of the derivative equal at $x = 3$ but also making a chain rule error in the derivative of the piece for $x > 3$, as follows.
$f'(x) = \begin{cases} 1 & \text{for } x < 3 \\ \dfrac{-16}{\left(k^2 - x\right)^2} & \text{for } x > 3 \end{cases}$
$\lim\limits_{x \to 3^-} f'(x) = \lim\limits_{x \to 3^+} f'(x) \Rightarrow 1 = \dfrac{-16}{\left(k^2 - 3\right)^2}$
This equation has no solution for k.</td></tr>
</table>

Question 84

Skill		Learning Objective	Topic
2.D		CHA-2.C	Defining the Derivative of a Function and Using Derivative Notation
(A)	Incorrect. The slope of the line tangent to the graph of f at $x = 0$ is $f'(0)$. Therefore, $f'(0)$ is negative. Since the tangent line goes through the point $(0, f(0))$, $f(0)$ is also negative. Therefore, it cannot be true that $f'(0) = -f(0)$ since both are negative.		
(B)	Incorrect. The slope of the line tangent to the graph of f at $x = 0$ is $f'(0)$, and the tangent line goes through the point $(0, f(0))$. Therefore, $f'(0) = -2$ and $f(0) = -2$, so it is not true that $f'(0) < f(0)$.		
(C)	**Correct**. The slope of the line tangent to the graph of f at $x = 0$ is $f'(0)$, and the tangent line goes through the point $(0, f(0))$. Therefore, $f'(0) = -2$ and $f(0) = -2$, so $f'(0) = f(0)$.		
(D)	Incorrect. The slope of the line tangent to the graph of f at $x = 0$ is $f'(0)$, and the tangent line goes through the point $(0, f(0))$. Therefore, $f'(0) = -2$ and $f(0) = -2$, so it is not true that $f'(0) > f(0)$.		

Question 85

Skill		Learning Objective	Topic
1.E		CHA-4.C	Connecting Position, Velocity, and Acceleration Functions Using Integrals
(A)	Incorrect. This response is the referee's displacement over the time interval $2 \le t \le 6$ rather than the total distance the referee traveled. $\int_2^6 v(t)\,dt = \int_2^6 4(t-6)\cos(2t+5)\,dt = 3.933$		
(B)	Incorrect. This response is the absolute value of the referee's change in velocity from time $t = 2$ to time $t = 6$. $\lvert v(6) - v(2)\rvert = 14.578$		
(C)	**Correct**. The referee's total distance traveled on the time interval $2 \le t \le 6$ is $\int_2^6 \lvert v(t)\rvert\,dt = \int_2^6 \lvert 4(t-6)\cos(2t+5)\rvert\,dt = 21.667$, where the numerical integration is done with the calculator.		
(D)	Incorrect. This response comes from averaging the referee's initial and final velocities on the time interval $2 \le t \le 6$, then multiplying by the length of the time interval. $\frac{v(2)+v(6)}{2} \cdot \Delta t = \frac{v(2)+v(6)}{2} \cdot 4 = 29.156$		

Question 86

Skill	Learning Objective	Topic
2.B	FUN-5.A	Interpreting the Behavior of Accumulation Functions Involving Area

(A)	**Correct**. Because h is continuous, the Extreme Value Theorem guarantees the existence of an absolute maximum on the closed interval $[-4, 3]$ and that the maximum will occur at a critical value or at one of the endpoints. Since h is an antiderivative of f, $h'(x) = f(x) = 0$ at $x = -2$ and $x = 0$. Therefore, the candidates are $x = -4$, $x = -2$, $x = 0$, and $x = 3$. Evaluate h at each candidate and select the largest. $h(-4) = \int_0^{-4} f(t)\,dt = -\int_{-4}^{0} f(t)\,dt = -\left(-\frac{1}{2}(2)(2) + \frac{1}{2}(2)(1)\right) = -(-2 + 1) = 1$ $h(-2) = \int_0^{-2} f(t)\,dt = -\int_{-2}^{0} f(t)\,dt = -\left(\frac{1}{2}(2)(1)\right) = -1$ $h(0) = \int_0^{0} f(t)\,dt = 0$ $h(3) = \int_0^{3} f(t)\,dt = -\left(\frac{1}{2}(2)(2) + \frac{1}{2}(2 + 1)\right) = -(2 + 1.5) = -3.5$ The maximum occurs at $x = -4$.
(B)	Incorrect. Both critical values might have been found, and the relative minimum was picked while not accounting for the endpoints.
(C)	Incorrect. Both critical values might have been found, and the relative maximum was picked while not accounting for the endpoints.
(D)	Incorrect. The four candidates might have been identified, but the computation of $h(3)$ found the area of the region from $x = 0$ to $x = 3$ and did not account for the region being below the horizontal axis, as follows. $h(3) = \int_0^{3} f(t)\,dt = \left(\frac{1}{2}(2)(2) + \frac{1}{2}(2 + 1)\right) = (2 + 1.5) = 3.5$

Question 87

Skill	**Learning Objective**	**Topic**
1.E	FUN-8.B	(BC ONLY) Solving Motion Problems Using Parametric and Vector-Valued Functions

(A)	Incorrect. This response would result if the problem was treated like rectilinear motion. The velocity of the particle was taken to be the value of the derivative $\frac{dy}{dx}$ at time $t = 1.2.$ $\left.\frac{dy}{dx}\right\|_{t=1.2} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right\|_{t=1.2} = \left.\frac{4\cos(4t)}{-e^{-t}}\right\|_{t=1.2} = \frac{4\cos(4.8)}{-e^{-1.2}} = -1.162$ Then the speed was taken to be the absolute value of velocity.
(B)	Incorrect. This response is the magnitude of the position vector at time $t = 1.2,$ not the magnitude of the velocity vector at that time. $\left.\sqrt{(x(t))^2 + (y(t))^2}\right\|_{t=1.2} = \sqrt{\left(e^{-1.2}\right)^2 + (\sin(4 \cdot 1.2))^2} = 1.041$
(C)	**Correct**. The speed of the particle at time t is the magnitude of the velocity vector $\langle x'(t), y'(t)\rangle = \left\langle -e^{-t}, 4\cos(4t)\right\rangle$ at time t. $\left.\sqrt{(x'(t))^2 + (y'(t))^2}\right\|_{t=1.2} = \sqrt{\left(-e^{-1.2}\right)^2 + (4\cos(4 \cdot 1.2))^2} = 0.462$
(D)	Incorrect. This response would result if the components of the vector $\langle x'(t), y'(t)\rangle = \left\langle -e^{-t}, 4\cos(4t)\right\rangle$ were not squared when finding the magnitude of the vector at $t = 1.2.$ $\left.\sqrt{x'(t) + y'(t)}\right\|_{t=1.2} = \sqrt{\left(-e^{-1.2}\right) + (4\cos(4 \cdot 1.2))} = 0.221$

Question 88

Skill	Learning Objective	Topic
1.E	CHA-3.A	Interpreting the Meaning of the Derivative in Context

(A)	Incorrect. This response might be chosen if the calculation of the average rate of change resulted in a value that was greater than 0 or less than -0.5. It would also be chosen if the average rate of change was correctly found to be -0.39206, but the instantaneous rate of change was taken to be the second derivative of f, not the first derivative. In either case, the resulting equation would have no solution in the interval $[0, 1.565]$.
(B)	Incorrect. This response would be chosen if the average rate of change was correctly found to be -0.39206, but the graph of f, not f', was drawn to determine the number of intersection points with the horizontal line $y = -0.39206$. It would also be chosen if the instantaneous rate of change was correctly identified as the derivative of f, but the average rate of change over the interval $[0, 1.565]$ was thought to be the average at the endpoints, $\frac{f(0) + f(1.565)}{2} = -0.30678$, or the average value of the function over the interval, $\frac{1}{1.565}\int_0^{1.565} f(x)\,dx = -0.32195$. In all these cases, the resulting equation would have only one solution in the interval $[0, 1.565]$.
(C)	**Correct**. The average rate of change of f on the closed interval $[0, 1.565]$ is $\frac{f(1.565) - f(0)}{1.565 - 0} = -0.39206$. The instantaneous rate of change of f is the derivative, $f'(x) = x^3 - 2x^2 + x - \frac{1}{2}$. The graph of f', produced using the calculator, intersects the horizontal line $y = -0.39206$ three times in the open interval $(0, 1.565)$.
(D)	Incorrect. This response might be chosen because the function f is a polynomial of degree 4.

Question 89

Skill	Learning Objective	Topic
3.F	FUN-7.H	(BC ONLY) Logistic Models with Differential Equations

(A)	Incorrect. This differential equation is for a rate of change that is equal to the difference between the carrying capacity and a term that is proportional to the size of the population. There is no joint proportionality with the size of the population and the difference between the size of the population and the carrying capacity.
(B)	Incorrect. This differential equation would be for a model where the population grows at a rate that is only proportional to the difference between the carrying capacity and the size of the population. There is no joint proportionality with the size of the population.
(C)	Incorrect. This differential equation would be for a model where the population grows at a rate that is jointly proportional to the reciprocal of the size of the population, not the population itself, and the difference between the carrying capacity and the size of the population.
(D)	**Correct**. The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is $\frac{dy}{dt} = ky(a - y)$, where y represents the quantity that is changing and a is the carrying capacity. Here the population P is changing according to the logistic growth model with carrying capacity 2400. Therefore, the differential equation for the model would be $\frac{dP}{dt} = kP(2400 - P)$.

Question 90

Skill	Learning Objective	Topic
1.E	CHA-3.E	Solving Related Rates Problems

(A)	Incorrect. This response would result if the area of the region was taken to be $A = \pi(R-r)^2$ instead of $\pi R^2 - \pi r^2$. In addition, an error was made in the power rule when doing the differentiation, as follows. $\frac{dA}{dt} = \pi(R-r)\left(\frac{dR}{dt} - \frac{dr}{dt}\right) = \pi(4-3)(2-(-1)) = 3\pi$
(B)	Incorrect. This response would result if the area of the region was taken to be $A = \pi(R-r)^2$ instead of $\pi R^2 - \pi r^2$. It is given that $\frac{dR}{dt} = 2$ and $\frac{dr}{dt} = -1$ (since the inner radius r is decreasing). At the instant when $R = 4$ and $r = 3$, this gave $\frac{dA}{dt} = 2\pi(R-r)\left(\frac{dR}{dt} - \frac{dr}{dt}\right) = 2\pi(4-3)(2-(-1)) = 6\pi$.
(C)	Incorrect. This response would result if the area of the region was correctly taken as $A = \pi R^2 - \pi r^2$. Using the chain rule gave the rate of change of the area with respect to time t as $\frac{dA}{dt} = 2\pi R\frac{dR}{dt} - 2\pi r\frac{dr}{dt}$. It is given that $\frac{dR}{dt} = 2$, but $\frac{dr}{dt}$ was taken to be 1 rather than -1 by not taking into account that the inner radius r is decreasing. At the instant when $R = 4$ and $r = 3$, this gave $\frac{dA}{dt} = 2\pi(4)\frac{dR}{dt} - 2\pi(3)\frac{dr}{dt} = 2\pi(4(2) - 3(1)) = 10\pi$.
(D)	**Correct**. The area of the region is $A = \pi R^2 - \pi r^2$. Using the chain rule gives the rate of change of the area with respect to time t as $\frac{dA}{dt} = 2\pi R\frac{dR}{dt} - 2\pi r\frac{dr}{dt}$. It is given that $\frac{dR}{dt} = 2$ and $\frac{dr}{dt} = -1$ (since the inner radius r is decreasing). At the instant when $R = 4$ and $r = 3$, $\frac{dA}{dt} = 2\pi(4)\frac{dR}{dt} - 2\pi(3)\frac{dr}{dt} = 2\pi(4(2) - 3(-1)) = 22\pi$.

AP® CALCULUS BC
2019 SCORING GUIDELINES

Question 1

(a) $E'(7) = 6.164924$ The rate of change of $E(t)$ at time $t = 7$ is 6.165 (or 6.164) cars per hour per hour.	1 : answer with units
(b) $\int_0^{12} E(t)\,dt = 520.070489$ To the nearest whole number, 520 cars enter the parking lot from time $t = 0$ to time $t = 12$.	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_2^{12} L(t)\,dt \approx (5-2)\cdot\frac{L(2)+L(5)}{2} + (9-5)\cdot\frac{L(5)+L(9)}{2} + (11-9)\cdot\frac{L(9)+L(11)}{2} + (12-11)\cdot\frac{L(11)+L(12)}{2}$ $= 3\cdot\frac{15+40}{2} + 4\cdot\frac{40+24}{2} + 2\cdot\frac{24+68}{2} + 1\cdot\frac{68+18}{2}$ $= 345.5$ $\int_2^{12} L(t)\,dt$ is the number of cars that leave the parking lot in the 10 hours between 7 A.M. $(t = 2)$ and 5 P.M. $(t = 12)$.	$3 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$
(d) $5\int_0^6 E(t)\,dt + 8\int_6^{12} E(t)\,dt = 3530.1396$ To the nearest dollar, 3530 dollars are collected from time $t = 0$ to time $t = 12$.	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constants} \\ 1 : \text{answer} \end{cases}$

Question 2

(a) $\sqrt{(x'(3))^2 + (y'(3))^2} = 3.064951$

The speed of the laser at time $t = 3$ seconds is 3.065 (or 3.064) centimeters per second.

1 : answer

(b) $\int_1^3 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 7.090427$

The total distance traveled by the laser from time $t = 1$ to time $t = 3$ seconds is 7.090 centimeters.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $x(1.253) = 0 + \int_0^{1.253} \frac{dx}{dt}\, dt = 2.932354$

The x-coordinate of the laser's rightmost position is 2.932.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\frac{dy}{dt} = 0 \Rightarrow t = t_1 = 0.628319$ or $t = t_2 = 1.884956$

$y(0) = 0$

$y(t_1) = 0 + \int_0^{t_1} \frac{dy}{dt}\, dt = 1.6$

$y(t_2) = 0 + \int_0^{t_2} \frac{dy}{dt}\, dt = -1.6$

$y(3.1) = 0 + \int_0^{3.1} \frac{dy}{dt}\, dt = 1.591358$

The difference between the y-coordinates of the laser's highest position and lowest position is $y(t_1) - y(t_2) = 3.2$.

$4 : \begin{cases} 1 : \text{sets } \frac{dy}{dt} = 0 \\ 1 : \text{critical points} \\ 1 : \text{integrand} \\ 1 : \text{answer with justification} \end{cases}$

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Question 3

(a) Average rate of change $= \dfrac{f(4) - f(-3)}{4 - (-3)} = \dfrac{-1 - 0}{7} = -\dfrac{1}{7}$	1 : answer
(b) $f(3) = -3 + 3\cos\left(\dfrac{3\pi}{2}\right) = -3$ For $0 < x < 4,\ f'(x) = -1 + \left(-3\sin\left(\dfrac{\pi x}{2}\right)\right) \cdot \dfrac{\pi}{2}$ $f'(3) = -1 + \left(-3\sin\left(\dfrac{3\pi}{2}\right)\right) \cdot \dfrac{\pi}{2} = -1 + \dfrac{3\pi}{2}$ An equation for the tangent line is $y = -3 + \left(-1 + \dfrac{3\pi}{2}\right)(x - 3)$.	$2 : \begin{cases} 1 : f'(3) \\ 1 : \text{equation} \end{cases}$
(c) The average value of f on the interval $-3 \le x \le 4$ is $\dfrac{1}{4 - (-3)} \displaystyle\int_{-3}^{4} f(x)\, dx$. $\displaystyle\int_{-3}^{4} f(x)\, dx = \int_{-3}^{0} f(x)\, dx + \int_{0}^{4} f(x)\, dx$ $\displaystyle\int_{-3}^{0} f(x)\, dx = \int_{-3}^{0} \sqrt{9 - x^2}\, dx = \frac{9\pi}{4}$ $\displaystyle\int_{0}^{4} f(x)\, dx = \int_{0}^{4} \left(-x + 3\cos\left(\frac{\pi x}{2}\right)\right) dx = \left[-\frac{1}{2}x^2 + \frac{6}{\pi}\sin\left(\frac{\pi x}{2}\right)\right]_{0}^{4} = -8$ $\displaystyle\frac{1}{4 - (-3)} \int_{-3}^{4} f(x)\, dx = \frac{1}{7}\left(\frac{9\pi}{4} - 8\right)$	$4 : \begin{cases} 1 : \text{integrals of } f \text{ over} \\ \quad -3 \le x \le 0 \text{ and } 0 \le x \le 4 \\ 1 : \text{value of } \int_{-3}^{0} \sqrt{9 - x^2}\, dx \\ 1 : \text{antiderivative of} \\ \quad -x + 3\cos\left(\frac{\pi x}{2}\right) \\ 1 : \text{answer} \end{cases}$
(d) $\displaystyle\lim_{x \to 0^-} f(x) = f(0) = 3$ and $\displaystyle\lim_{x \to 0^+} f(x) = 3$, so f is continuous at $x = 0$. Because f is continuous on $[-3, 4]$, the Extreme Value Theorem guarantees that f attains an absolute maximum on $[-3, 4]$.	$2 : \begin{cases} 1 : \text{continuity at } x = 0 \\ 1 : \text{answer with justification} \end{cases}$

Question 4

(a) $g(0) = \int_{-4}^{0} f(t)\, dt = \frac{9}{2} - 3 = \frac{3}{2}$

$$\begin{aligned} g(4) &= \int_{-4}^{4} f(t)\, dt \\ &= \int_{-4}^{0} f(t)\, dt + \int_{0}^{1} f(t)\, dt + \int_{1}^{4} f(t)\, dt \\ &= \frac{3}{2} + 5 + \int_{1}^{4} \left(-t^2 + 5t - 4\right) dt \\ &= \frac{3}{2} + 5 + \left[-\frac{1}{3}t^3 + \frac{5}{2}t^2 - 4t\right]_1^4 \\ &= \frac{3}{2} + 5 + \left[\left(-\frac{1}{3}\cdot 4^3 + \frac{5}{2}\cdot 4^2 - 4\cdot 4\right) - \left(-\frac{1}{3}\cdot 1^3 + \frac{5}{2}\cdot 1^2 - 4\cdot 1\right)\right] \\ &= \frac{3}{2} + 5 + \left(\frac{8}{3} - \left(-\frac{11}{6}\right)\right) = 11 \end{aligned}$$

$4: \begin{cases} 1: g(0) \\ 1: \text{integral of } f \text{ over } 1 \le t \le 4 \\ 1: \text{antiderivative} \\ 1: g(4) \end{cases}$

(b) $g'(x) = f(x)$ is negative for $-1 < x < 0,$ and nonnegative elsewhere. Thus, the absolute minimum value of g on $[-4, 4]$ can only occur at $x = -4$ or $x = 0.$

$g(-4) = 0$

$g(0) = \frac{3}{2}$

The absolute minimum value of g on $[-4, 4]$ is $g(-4) = 0.$

$3: \begin{cases} 1: g'(x) = f(x) \\ 1: \text{identifies } x = -4 \text{ and } x = 0 \text{ as candidates} \\ 1: \text{answer with justification} \end{cases}$

(c) The graph of g is concave down on the intervals $-2 < x < -\frac{1}{2},$ $\frac{1}{2} < x < 1,$ and $\frac{5}{2} < x < 4$ because $g'(x) = f(x)$ is decreasing on these intervals.

$2: \begin{cases} 1: \text{intervals} \\ 1: \text{reason} \end{cases}$

Question 5

(a) $\frac{d}{dx}\left(4x^2 + 3y^2 + 6y\right) = \frac{d}{dx}(9)$

$\Rightarrow 8x + 6y\frac{dy}{dx} + 6\frac{dy}{dx} = 0$

$\Rightarrow (6y + 6)\frac{dy}{dx} = -8x$

$\Rightarrow \frac{dy}{dx} = \frac{-8x}{6y+6} = \frac{-4x}{3(y+1)}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $\frac{d^2y}{dx^2} = \frac{-4\cdot 3(y+1) - (-4x)\cdot 3\frac{dy}{dx}}{(3(y+1))^2} = \frac{-12(y+1) - \frac{16x^2}{y+1}}{(3(y+1))^2}$

$2 : \begin{cases} 1 : \text{form of quotient rule} \\ 1 : \frac{d^2y}{dx^2} \end{cases}$

(c) $r = \frac{3}{2+\sin\theta} > 0$ for $0 \le \theta \le 2\pi$.

$\frac{dr}{d\theta} = \frac{-3}{(2+\sin\theta)^2}\cdot\cos\theta$

$(2+\sin\theta)^2 > 0$ for all θ.

For $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$, $\cos\theta < 0$, so $\frac{dr}{d\theta} > 0$.

The distance between the origin and the point (r, θ) is increasing for $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$.

$2 : \begin{cases} 1 : \frac{dr}{d\theta} \\ 1 : \text{answer with reason} \end{cases}$

(d) $2 = \frac{3}{2+\sin\theta} \Rightarrow 2 + \sin\theta = \frac{3}{2} \Rightarrow \sin\theta = -\frac{1}{2} \Rightarrow \theta = \frac{7\pi}{6}$ or $\theta = \frac{11\pi}{6}$

$$\text{Area} = \frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \left(\left(\frac{3}{2+\sin\theta}\right)^2 - 2^2\right) d\theta$$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{form of integrand} \\ 1 : \text{integrand} \end{cases}$

Question 6

(a) The series with $p = 3$ and $x = 8$ is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5^n}{5^n \cdot n^3} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$.

$\displaystyle\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n^3} \right| = \sum_{n=1}^{\infty} \frac{1}{n^3}$ is a p-series with $p = 3 > 1$, which converges.

Therefore, the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5^n}{5^n \cdot n^3}$ converges absolutely.

2 : { 1 : considers $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$
1 : converges absolutely with explanation

(b) The series with $p = 1$ and $x = 8$ is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5^n}{5^n \cdot n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges by the alternating series test.

$\displaystyle\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n}$ is the harmonic series, which diverges.

Therefore, the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5^n}{5^n \cdot n}$ converges conditionally.

2 : { 1 : considers $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$
1 : converges conditionally with explanation

(c) The series with $x = -2$ is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (-5)^n}{5^n \cdot n^p} = \sum_{n=1}^{\infty} \frac{-1}{n^p} = -\sum_{n=1}^{\infty} \frac{1}{n^p}$.

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ is a p-series, which converges if and only if $p > 1$.

Therefore, the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (-5)^n}{5^n \cdot n^p}$ converges for $p > 1$.

2 : { 1 : considers $\displaystyle\sum_{n=1}^{\infty} \frac{-1}{n^p}$
1 : answer with explanation

(d) The series with $p = 1$ and $x = 3.1$ is

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{5^n \cdot n} (3.1 - 3)^n = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (0.1)^n}{5^n \cdot n}.$$

Using two terms, $S \approx \dfrac{0.1}{5} - \dfrac{(0.1)^2}{5^2 \cdot 2} = \dfrac{99}{5000}$.

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (0.1)^n}{5^n \cdot n}$ is an alternating series with terms that decrease in magnitude to 0.

By the alternating series error bound, the approximation $S \approx \dfrac{99}{5000}$ has absolute error bounded by the magnitude of the third term, $\dfrac{1}{5^3 \cdot 10^3 \ 3}$.

Therefore, $\left| S - \dfrac{99}{5000} \right| < \left| \dfrac{1}{5^3 \cdot 10^3 \cdot 3} \right| = \dfrac{1}{375{,}000} < \dfrac{1}{300{,}000}$.

3 : { 1 : approximation
1 : uses third term of series
1 : error bound

2019 AP Calculus BC Scoring Worksheet

Section I: Multiple Choice

_______________ × 1.2000 = _______________

Number Correct (out of 45) — Weighted Section I Score (Do not round)

Section II: Free Response

Question 1 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 2 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 3 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 4 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 5 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 6 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Sum = _______________
Weighted Section II Score (Do not round)

Composite Score

_______________ + _______________ = _______________

Weighted Section I Score — Weighted Section II Score — Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus BC

Composite Score Range	AP Score
70-108	5
59-69	4
45-58	3
28-44	2
0-27	1

2019 AP Calculus BC — AB Subscore Scoring Worksheet

Section I: Multiple Choice

Questions (1-3, 6, 8-12, 14, 16, 20, 23-24, 27, 29, 76-79, 82-86, 88, 90)

__________ × 1.0000 = __________

Number Correct (out of 27) — Weighted Section I Score (Do not round)

Section II: Free Response

Question 1 ________ (out of 9) × 1.0000 = ________ (Do not round)

Question 3 ________ (out of 9) × 1.0000 = ________ (Do not round)

Question 4 ________ (out of 9) × 1.0000 = ________ (Do not round)

Sum = ________

Weighted Section II Score (Do not round)

Composite Score

________ + ________ = ________

Weighted Section I Score — Weighted Section II Score — Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus AB Subscore

Composite Score Range	AP Score
35-54	5
29-34	4
23-28	3
15-22	2
0-14	1

2019 AP Calculus BC
Question Descriptors and Performance Data

Multiple-Choice Questions

Question	Skill	Learning Objective	Topic	Key	% Correct
1	1.E	FUN-6.B	The Fundamental Theorem of Calculus and Definite Integrals	A	95
2	3.D	FUN-4.A	Using the First Derivative Test to Find Relative (Local) Extrema	A	82
3	1.E	FUN-3.C	The Chain Rule	D	85
4	1.E	FUN-8.B	(BC ONLY) Solving Motion Problems Using Parametric and Vector-Valued Functions	B	80
5	3.D	LIM-7.A	(BC ONLY) Working with Geometric Series	B	66
6	1.E	FUN-3.B	The Quotient Rule	A	88
7	2.C	LIM-8.G	(BC ONLY) Representing Functions as Power Series	A	69
8	2.B	LIM-2.D	Connecting Limits at Infinity and Horizontal Asymptotes	B	81
9	3.B	CHA-2.B	Defining Average and Instantaneous Rates of Change at a Point	C	68
10	2.B	FUN-4.A	Determining Concavity of Functions over Their Domains	B	67
11	1.E	FUN-6.D	Integrating Using Substitution	D	70
12	3.D	FUN-1.B	Using the Mean Value Theorem	C	80
13	1.E	FUN-7.C	(BC ONLY) Approximating Solutions Using Euler's Method	A	74
14	1.E	FUN-3.D	Implicit Differentiation	B	74
15	1.E	LIM-6.A	(BC ONLY) Evaluating Improper Integrals	C	61
16	1.E	FUN-7.D	Finding General Solutions Using Separation of Variables	B	46
17	3.D	LIM-7.A	(BC ONLY) Alternating Series Test for Convergence	D	57
18	1.C	CHA-5.D	(BC ONLY) Finding the Area of the Region Bounded by Two Polar Curves	D	65
19	3.B	LIM-7.A	(BC ONLY) Alternating Series Test for Convergence	D	69
20	1.C	FUN-6.A	Applying Properties of Definite Integrals	C	78
21	3.D	LIM-8.D	(BC ONLY) Radius and Interval of Convergence of Power Series	A	71
22	1.E	FUN-3.G	(BC ONLY) Defining Polar Coordinates and Differentiating in Polar Form	B	59
23	1.F	LIM-5.A	Approximating Areas with Riemann Sums	B	67
24	3.D	FUN-6.B	The Fundamental Theorem of Calculus and Definite Integrals	A	65
25	1.E	FUN-6.E	(BC ONLY) Using Integration by Parts	A	33
26	3.D	LIM-7.A	(BC ONLY) Determining Absolute or Conditional Convergence	C	53
27	1.C	CHA-5.C	Volume with Washer Method - Revolving Around *x*- or *y*-axis	D	58
28	1.E	CHA-3.G	(BC ONLY) Second Derivatives of Parametric Equations	C	40
29	2.C	LIM-5.C	Riemann Sums, Summation Notation, and Definite Integral Notation	B	37
30	2.B	LIM-8.A	(BC ONLY) Finding Taylor Polynomial Approximations of Functions	D	22

2019 AP Calculus BC
Question Descriptors and Performance Data

Question	Skill	Learning Objective	Topic	Key	% Correct
76	2.E	FUN-4.A	Determining Intervals on Which a Function Is Increasing or Decreasing	D	92
77	1.E	CHA-4.D	Using Accumulation Functions and Definite Integrals in Applied Contexts	D	82
78	2.D	LIM-2.D	Connecting Infinite Limits and Vertical Asymptotes	C	95
79	3.F	CHA-4.B	Finding the Average Value of a Function on an Interval	B	70
80	1.E	LIM-8.B	(BC ONLY) Finding Taylor Polynomial Approximations of Functions	B	80
81	1.E	CHA-6.A	(BC ONLY) The Arc Length of a Smooth, Planar Curve and Distance Traveled	B	68
82	2.D	FUN-7.C	Sketching Slope Fields	A	71
83	3.D	LIM-2.C	Removing Discontinuities	A	72
84	2.D	CHA-2.C	Defining the Derivative of a Function and Using Derivative Notation	C	63
85	1.E	CHA-4.C	Connecting Position, Velocity, and Acceleration Functions Using Integrals	C	62
86	2.B	FUN-5.A	Interpreting the Behavior of Accumulation Functions Involving Area	A	49
87	1.E	FUN-8.B	(BC ONLY) Solving Motion Problems Using Parametric and Vector-Valued Functions	C	72
88	1.E	CHA-3.A	Interpreting the Meaning of the Derivative in Context	C	42
89	3.F	FUN-7.H	(BC ONLY) Logistic Models with Differential Equations	D	66
90	1.E	CHA-3.E	Solving Related Rates Problems	D	60

Free-Response Questions

Question	Skill	Learning Objective	Topic	Mean Score
1	1.E|3.D|3.F|4.D|4.A|4.B|4.C|4.E	CHA-2.D|CHA-4.E|LIM-5.A	2.3|8.3|6.2|8.3	7.33
2	1.D|1.E|3.B|3.D|4.D|4.E	FUN-8.B	9.6	5.05
3	1.D|1.E|3.C|3.D|4.A	CHA-2.A|CHA-2.C|CHA-4.B|FUN-1.C	2.1|2.2|8.1|5.2	3.03
4	1.D|1.E|2.B|2.E|4.A	FUN-6.A|FUN-4.A	6.6|5.5|5.6	4.89
5	1.D|1.E |3.E|3.G|4.C	FUN-3.D|FUN-4.E|FUN-3.G|CHA-5.D	3.2|5.12|9.7|9.9	5.04
6	1.E|3.B|3.D|3.E|4.A	LIM-7.A|LIM-8.C	10.9|10.5|10.12	3.64