Credit, Complexity and Systemic Risk
Case 2 (2024): CDS Stripping

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1 Question 1 – Simplified CDS stripping

Maturity	Average hazard rate	Forward Hazard Rate	Default Probability
1	0.02500	0.02500	0.02469
3	0.02750	0.02875	0.07919
5	0.03000	0.03375	0.13929
7	0.03000	0.03000	0.18942
10	0.03125	0.03417	0.26838

Table 1: CDS Risk metrics for the simplified case

Using the data values provided and the provided formula, we calculate the table above, as a rule of thumb or approximation. The main idea is to be used as benchmark values for the detailed model to follow.

$$\lambda_{\text{average}}(T) = \frac{R(T)}{\text{LGD}} \tag{1}$$

For the Forward Hazard Rate the formula below is used:

$$\text{Forward Hazard Rate} = \frac{\lambda_{average}(T_i) \times T_i - \lambda_{average}(T_{i-1}) \times T_{i-1}}{T_i - T_{i-1}} \tag{2}$$

It worth noting that the average hazard rate at maturity 1Y or λ_1 is used for the initial interval [0,1].

Lastly, for the Cumulative Default Probability (5), the interval survival probability (3) is calculated using the forward hazard rates (2).Then updating the cumulative survival probability (4) iteratively for each time interval. The cumulative default probability (5) up to each maturity is then derived from the cumulative survival probability (4).

$$Q_{\text{survival}}(T_{i-1}, T_i) = \exp\left(-\lambda_{\text{forward}}(T_{i-1}, T_i) \cdot (T_i - T_{i-1})\right) \tag{3}$$

$$Q_{\text{cumulative survival}}(T_i) = \prod_{j=1}^{i} Q_{\text{survival}}(T_{j-1}, T_j) \tag{4}$$

$$P_{\text{default}}(T_i) = 1 - Q_{\text{cumulative survival}}(T_i) \tag{5}$$

2 Question 2 – Involved CDS stripping

Maturity	Average Hazard Rate	Forward Hazard Rate	Default Probability
1	0.02492	0.02492	0.00996
3	0.02793	0.03689	0.08037
5	0.03046	0.03426	0.14127
7	0.03030	0.02989	0.19110
10	0.03174	0.03512	0.27199

Table 2: CDS Risk metrics for the involved case

For better readability, the provided formula for the CDS is divided into smaller - more manageable pieces. This was also the approach used while coding it.

The general formula for pricing a CDS contract, considering the present value of expected premium payments (Premium Leg) and the present value of expected loss in the event of default (Protection Leg), is:

$$\text{PV}_{\text{Premium Leg}} - \text{PV}_{\text{Protection Leg}} = 0$$

$$\text{Sum}_1 = \sum_{i=0}^{N} \exp(-rT_i) \cdot (T_i - T_{i-1}) \cdot Q(\tau > T_i)$$

$$\text{Sum}_2 = \sum_{i=0}^{N} \exp(-rT_{\text{mid}_i}) \cdot (Q(\tau > T_{i-1}) - Q(\tau > T_i)) \cdot \frac{(T_i - T_{i-1})}{2}$$

$$\text{Sum}_3 = \sum_{i=0}^{N} \exp(-rT_{\text{mid}_i}) \cdot (Q(\tau > T_{i-1}) - Q(\tau > T_i))$$

Then:

$$\text{PV}_{\text{Premium Leg}} = R(T) \cdot (\text{Sum}_1 + \text{Sum}_2)$$

$$\text{PV}_{\text{Protection Leg}} = \text{LGD} \cdot \text{Sum}_3$$

Using the *fsolve* function from *scipy* library of Python, the equation is set equal to zero and solve for λ(u). Lastly, the average Hazard Rate is calculated using the *quad* function from *scipy* library to integrate over our λ's for each time interval. Its worth noting thta this time the forward hazard rates are not calculated using a formula but directly come from solving the above equation. The Cumulative Default Probability is calculated in a similar way as before.

3 Question 3 – Involved CDS stripping

i) In order to give a definite answer to the above investigation we would like to firstly represent the aforementioned values graphically to better visualise their behaviour allowing for an easier comparison.

Firstly, we have the Average Hazard Rate (1) as defined by the equation of Q2. It is obvious from the graph the Average Hazard rates between Q1 and Q2 follow a very similar pattern and are in general identically behaved. They start at a single common point and then they start having differing values while maintaining the same increasing pattern. They have similar behaviour and values.

For the Forward Hazard Rates (2) we do see differences between the two Questions. Q2 in particular seems to have a more explosive behaviour for maturities between 1 and 5 years , rapidly increasing in value and having an all time peak at 3Y maturity while the equivalent of Q1 has a linear increase between 1 and 5 with 3Y being relatively low compared to its all-time maximum. From maturities of 5Y to 10Y we see once again a similar pattern in both behaviour and values between the 2 questions reaching an (almost) identical value state right on the 7Y maturity.

Finally, the Cumulative Default Probabilities we see the biggest and most definite overlap between the 2 Questions . As it can be seen the way we treat our Hazard (Q1 vs Q2) has little effect on the ultimate computation of the default probability. Regardless of approach we have the (almost) same probability values for both questions. It is interesting however to notice that while Q1 stars with an increased probability at maturity of 1Y it slowly converges to that of Q2 on the 3Y maturity only to eventually be slightly undervalued thereafter from the Q2 approach all the way till 10Y maturity.

As a conclusion, we would like to point out the 1-year maturity. Despite the hazard rates being almost identical, the difference in default probability is substantial. This could be interpreted that in the involved case we do have intermediate payments thus reducing the 'duration of the exposure'. Meaning that since payments do occur the probability of defaulting on the next payment are expected to be less. In larger maturities we do not see that pattern as the maturity risk this effect. This becomes more apparent for the following maturities (3, 5 year) as the results are quite close to each other. So this security the intermediate payments offer, seems to disappear as maturity increases, inline with the economic intuition.

ii) To determine the circumstances that would affect and cause a difference in the two approaches it is only reasonable to go back to the points where we spot differences between the 2 approaches in the previous question i) . Another way is to deduct the differences in variables between the formulas utilised in Q1 and Q2 respectively.

The answer lies in the value of λ and this is dependent as we can deduct from the formula (2, 2, 2) this in turn depends to the condition of our summation and the value of r. In essence the difference lies and can be adjusted by changing the frequency and level of compounding in our CDS formula . In this way the solutions of λ will be different and thus the CDS as a whole.

In summary, changes in input parameters, such as an adjustment in the Loss Given Default (LGD) or the risk-free interest rate (r), could lead to noticeable differences in the calculated probabilities. For example, a higher LGD would increase default probabilities, while a higher risk-free rate would decrease the present value of expected future cash flows, potentially altering the balance between premium payments and protection legs in the CDS valuation. For the longer maturities (7 and 10 years), even small differences in hazard rates can lead to relatively larger differences in default probabilities due to the compounding effect of risk over time.

Concluding, the close alignment of the rates and probabilities for the 1-year, 3-year, and 5-year maturities suggests that the market's risk perception for these intervals is relatively stable. The slight divergence in rates for the 7-year and 10-year maturities might indicate a market expectation of changing risk conditions in the longer term, which could be due to economic forecasts, credit conditions, or other macroeconomic factors. Lastly, the methodology and precision in calculations (simple vs involved) can affect the default probabilities, especially when dealing with exponential functions over multiple periods.

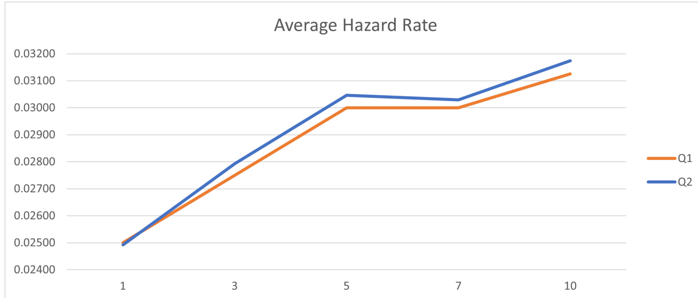

Figure 1: Average Hazard Rate between simple and involved case.

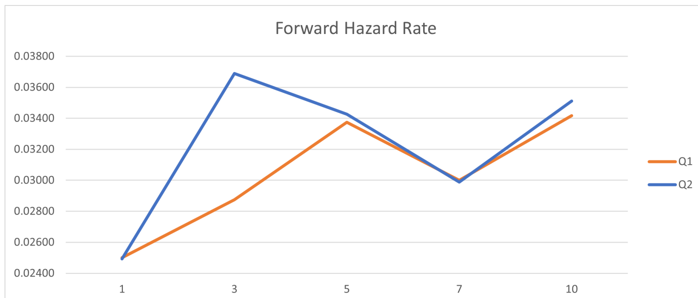

Figure 2: Forward Hazard Rates between simple and involved cases.

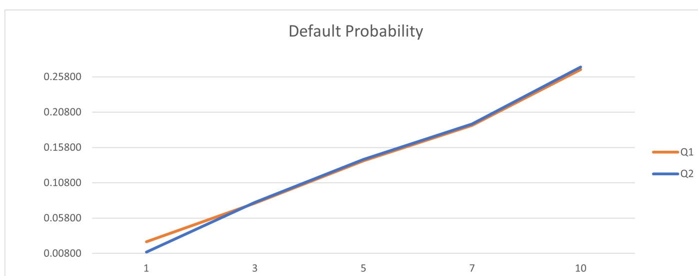

Figure 3: Cumulative Default Probability between simple and involved cases.