Credit, Complexity and Systemic Risk
Case 3 (2024): CVA Equity Derivatives

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1 Question 1 - Equity Model Simulation

For part 1 of the assignment, two separate procedures where followed. Both illustrate and verify the correctness of the Equity Simulation model implementations from different paths. For the sake of completeness and robustness, it was decided to present both methods.

1.1 Forwards

In this approach, the initial value of the underlying is set equal to the given Base Level. Using the underlying equity processes given, 100,000 simulations are conducted, until maturity t=5 years. Then the average value out of these simulations is taken and discounted to t=0. Using this method, we can verify that the Present Value of the underlying equity is almost identical to the theoretical value set to be the Base Level.

In more details, we start by using the parameters given by the assignment. A seed equal to the #Student_id# is set for replicability and the stochastic equity processes are given by:

Parameter	Value
Risk-free rate, r	0.03
Dividend yield, q	0.02
Volatility for SX5E, σ_{SX5E}	0.15
Volatility for AEX, σ_{AEX}	0.15
Maturity, T (years)	5
Initial price for SX5E, $S_{\text{SX5E},0}$	4235
Initial price for AEX, $S_{\text{AEX},0}$	770
Number of simulations, num_simulations	100000
Correlation, ρ	0.8

Table 1: Parameters for Monte Carlo Simulations

$$S_{\text{SX5E},T} = S_{\text{SX5E},0} \exp\left(\left(r - q - \frac{1}{2}\sigma_{\text{SX5E}}^2\right)T + \sigma_{\text{SX5E}} Z_{\text{correlated},0}\sqrt{T}\right)$$

$$S_{\text{AEX},T} = S_{\text{AEX},0} \exp\left(\left(r - q - \frac{1}{2}\sigma_{\text{AEX}}^2\right)T + \sigma_{\text{AEX}} Z_{\text{correlated},1}\sqrt{T}\right)$$

where $S_{\text{SX5E},0}$ and $S_{\text{AEX},0}$ represent the initial values of the SX5E and AEX indices at time $t = 0$, T is the time step to maturity, σ_{SX5E} and σ_{AEX} are the volatilities of the SX5E and AEX indices respectively, r is the risk-free rate, q is the dividend yield, and $Z_{\text{correlated},0}$ and $Z_{\text{correlated},1}$ are the transformed standard normal random variables that incorporate the specified correlation between these two financial assets. These variables are used in Monte Carlo simulations to model the future values of the indices considering their initial values, expected returns, volatilities, and the correlation between their movements.

In the context of the Monte Carlo simulation for estimating future asset prices with correlation, $Z_{\text{correlated},0}$ and $Z_{\text{correlated},1}$ refer to the correlated random variables that are derived from originally uncorrelated standard normal random variables through a process involving the Cholesky decomposition of the correlation matrix between the two assets being simulated.

Initially, two independent standard normal random variables are generated, denoted as Z_0 and Z_1. These variables are drawn from a normal distribution with a mean of 0 and a standard deviation of 1. In financial modeling, these random variables are used to simulate the randomness inherent in asset price movements.

In reality, as in the assignment, the returns (and hence the price movements) of financial assets are often correlated. For instance, if two stocks are in the same industry, they might react similarly to industry news. This correlation needs to be incorporated into the simulation to make it realistic. This is where $Z_{\text{correlated},1}$ and $Z_{\text{correlated},0}$ come into play.

The Cholesky decomposition is a mathematical tool that transforms a positive definite matrix (in this case, the correlation matrix) into a lower triangular matrix (denoted as L) and its transpose. This transformation is particularly useful for generating correlated random variables from uncorrelated ones. The correlation matrix is constructed such that it reflects the correlation (ρ) between the two assets.

The next step is to use the lower triangular matrix L from the Cholesky decomposition to transform the uncorrelated standard normal random variables (Z_0 and Z_1) into correlated ones. The original correlation matrix would have looked something like this:

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

where ρ is the correlation coefficient between the two assets. Given that the Cholesky decomposition result L has values of 1, 0, 0.8, and 0.6, it suggests a correlation coefficient (ρ) of 0.8 between the assets.

Applying L to Generate Correlated Random Variables: If you have a vector of independent standard normal random variables, say $Z = (Z_0, Z_1)$, then multiplying this vector by L will give you a new vector of correlated random variables:

$$\begin{pmatrix} Z_{\text{correlated},0} \\ Z_{\text{correlated},1} \end{pmatrix} = L \cdot Z = \begin{pmatrix} 1 & 0 \\ 0.8 & 0.6 \end{pmatrix} \cdot \begin{pmatrix} Z_0 \\ Z_1 \end{pmatrix}$$

This results in:

$$Z_{\text{correlated},0} = Z_0$$
$$Z_{\text{correlated},1} = 0.8Z_0 + 0.6Z_1$$

The first variable, $Z_{\text{correlated},0}$, remains unchanged, indicating it serves as the base for generating the correlation. The second variable, $Z_{\text{correlated},1}$, is a linear combination of Z_0 and Z_1, with coefficients derived from L. This linear combination introduces the specified correlation to the simulations, allowing the second asset's simulated price movements to be influenced by both its own randomness and the movements of the first asset, in accordance with the correlation coefficient $\rho = 0.8$.

In this formula, $Z_{\text{correlated},0}$ and $Z_{\text{correlated},1}$ are the outcomes of this multiplication, representing the new, correlated random variables. These variables now embody the specified correlation (ρ) between the two assets, making them suitable for simulating correlated asset price movements.

Finally, $Z_{\text{correlated},0}$ and $Z_{\text{correlated},1}$ are used in the formula for geometric Brownian motion to simulate the future prices of the assets. Since these variables are correlated, they ensure that the simulated price paths of the assets reflect the real-world observation that the movements of some assets are related to each other.

After we have calclulated the future value random paths via the MC, the resulting values provide us with a mean value. Then, this mean value is discounted by a factor:

$$\text{Discount factor} = \exp\left[(-r + q) \cdot T\right]$$

Note the addition of the dividend yield in this case, as dividends are paying throughout our underlying equity portfolio life-cycle and therefore do not need to discount these earnings to t=0.

Parameter	**Initial Value**	**Simulated Value**	**Difference**	**Confidence Interval**
SX5E	4235	4236.32467	1.32467	[4226.82051, 4245.82884]
AEX	770	770.28465	0.28465	[768.55099, 772.01830]

Table 2: Comparison of Initial, Simulated Values, and Confidence Intervals

Given the standard deviations of the simulated outcomes for SX5E and AEX assets, denoted as σ_{SX5E} and σ_{AEX} respectively, the standard error of the mean (SE) for both simulations is calculated using the formula:

$$SE = \frac{\sigma}{\sqrt{n}}$$

where:

- σ is the standard deviation of the simulated outcomes,
- n is the number of simulations ($num\_simulations$).

For a 95% confidence interval, we use the Z-score associated with the desired confidence level, which is 1.96 for 95%. The confidence interval (CI) for the present value (PV) of the average expected prices at maturity for both SX5E and AEX assets is then calculated as:

$$CI = PV \pm Z \cdot SE$$

where:

- PV is the present value of the average expected price at maturity,
- Z is the Z-score for the desired confidence level (1.96 for 95%),
- SE is the standard error of the mean.

Thus, the 95% confidence intervals for the present values of SX5E and AEX are given by:

$$CI_{SX5E} = (PV_{SX5E} - Z \cdot SE_{SX5E}, PV_{SX5E} + Z \cdot SE_{SX5E})$$

$$CI_{AEX} = (PV_{AEX} - Z \cdot SE_{AEX}, PV_{AEX} + Z \cdot SE_{AEX})$$

This calculation assumes a normal distribution of the mean of the simulated outcomes, allowing us to estimate the range within which the true mean is likely to lie with 95% confidence.

In this first part we also used another method to compare the theoretical and the simulation values of the contracts. Basically the other way was using the Forward formula to theoretically calculate the value of the contract as

$$S(t) = (S(0)e^{(r-q)\tau} - K)e^{-r\tau}$$

Where τ stands for the remaining time to maturity of the contract from 0 discounted back to the current time. Then the empirical way through the simulation constituted of us generating 100000 simulations of the values of S(5) substituting them to the above formula and getting the average. This just one of the random executions.

The results are in the following table

Parameter	Theoretical Value	Empirical Value	**Confidence Interval**
SX5E	186.88	184.269	[176.073, 192.464]
AEX	33.979	34.0038	[32.545, 35.521]

Table 3: Comparison of Values based on Theoretical and Empirical approach and Confidence Intervals

1.2 Put Options

The Black-Scholes model provides a theoretical estimate for the price of European-style options, utilizing the underlying asset's current price, the option's strike price, the risk-free interest rate, the time to expiration, the dividend yield, and the volatility of the underlying asset. For put options, the formula is given by:

$$P = Ke^{-rT}N(-d_2) - S_0e^{-qT}N(-d_1)$$

where:

- P is the price of the put option,
- S_0 is the current price of the underlying asset,
- K is the strike price of the option,
- T is the time to expiration (in years),
- r is the risk-free interest rate (annual),
- q is the annual dividend yield,
- σ is the volatility of the underlying asset's returns (annual),
- $N(\cdot)$ is the cumulative distribution function of the standard normal distribution,
- d_1 and d_2 are intermediate calculations used in the model, defined as:

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The terms $N(-d_1)$ and $N(-d_2)$ represent the probabilities, adjusted for risk, that the option will end in the money, factored by the present value of the payoff at expiration. Specifically:

- $N(-d_2)$ is the risk-adjusted probability that the option will be exercised (i.e., that the underlying asset's price will be below the strike price at expiration).
- $N(-d_1)$ adjusts this probability for the present value of the price differential between the underlying asset and the strike price.

The Black-Scholes formula for put options thus reflects the present value of the expected payoff of the option at expiration, under the assumption of log-normal distribution of asset prices and continuous compounding.

The empirical process for estimating the values of put options on the SX5E and AEX indices using MC simulations involves several steps integrated into a cohesive flow. The process starts by setting a replicable seed for the random number generation, ensuring consistency across simulation runs. Arrays are initialized to store the payoff of each simulation for both the SX5E and AEX put options.

Correlated random variables are generated for each simulation iteration by applying the Cholesky decomposition to two independent standard normal random variables. This step is crucial for simulating price paths that reflect the specified correlation between the SX5E and AEX assets. The simulated stock prices at maturity for both assets are calculated using the geometric Brownian motion formula:

$$S_T = S_0 e^{(r-q-\frac{1}{2}\sigma^2)T + \sigma Z\sqrt{T}},$$

where S_T represents the stock price at maturity, S_0 is the initial stock price, r is the risk-free rate, q is the dividend yield, σ is the asset's volatility, T is the time to maturity, and Z is the correlated random variable.

The payoff for each put option simulation is determined by the formula:

$$\text{Payoff} = \max(K - S_T, 0),$$

where K is the strike price, and S_T is the simulated stock price at maturity. The average of these simulated payoffs is then calculated to estimate the expected payoff of the put options.

Finally, the expected payoff is discounted to its present value using the exponential discount factor e^{-rT}, where r is the risk-free rate, and T is the time to maturity. This results in the empirical estimates of the put option values for the SX5E and AEX indices, integrating the effects of correlations between asset prices.

Parameter	**Theoretical Value**	**Simulated Value**	**Difference**	**Confidence Interval**
SX5E	130.6817	130.30975	0.37195	[128.59614, 132.76726]
AEX	23.78584	23.69268	0.09315	[23.40605, 24.16562]

Table 4: Comparison of Expected Payoffs and Confidence Intervals

Finally for the last part, given the simulated final prices of SX5E and AEX assets, we calculate their log-returns as follows:

$$\text{log\_returns}_{SX5E} = \log\left(\frac{S_{SX5E,T}}{S_{SX5E,0}}\right), \quad \text{log\_returns}_{AEX} = \log\left(\frac{S_{AEX,T}}{S_{AEX,0}}\right)$$

where $S_{SX5E,T}$ and $S_{AEX,T}$ are the simulated prices at maturity, and $S_{SX5E,0}$ and $S_{AEX,0}$ are the initial prices.

The correlation coefficient between these log-returns is computed, resulting in a value of 0.799907533689217, which is very close to the original correlation coefficient of 0.8, with a negligible difference of -0.000092466.

To assess the significance of this correlation, we apply the Fisher transformation:

$$z = 0.5 \log\left(\frac{1 + \text{correlation\_log\_returns}}{1 - \text{correlation\_log\_returns}}\right)$$

and calculate the standard error for the Fisher-transformed value as:

$$SE_z = \frac{1}{\sqrt{n-3}}$$

where n is the number of simulations.

Using the Z-score for a 95% confidence interval, we calculate the lower and upper bounds of the confidence interval for the Fisher-transformed value and then apply the inverse Fisher transformation to these bounds to obtain the 95% confidence interval for the original correlation coefficient:

$$r_{CI\_lower} = \frac{\exp(2 \cdot z_{CI\_lower}) - 1}{\exp(2 \cdot z_{CI\_lower}) + 1}, \quad r_{CI\_upper} = \frac{\exp(2 \cdot z_{CI\_upper}) - 1}{\exp(2 \cdot z_{CI\_upper}) + 1}$$

The Fisher transformation is used primarily to transform the distribution of the Pearson correlation coefficient, which inherently does not follow a normal distribution, into one that is approximately normally distributed. Without transformation, the variance of correlation coefficients can depend on the underlying population correlation, leading to non-constant variance across different samples. The distribution of the correlation coefficient, especially with small sample sizes, can be skewed and limited to the range between -1 and 1. The Fisher transformation helps stabilize this variance, making it more uniform across different correlation values.

Parameter	**Theoretical Value**	**Simulated Value**	**Difference**	**Confidence Interval**
ρ	0.8	0.79991	-0.00009	[0.79742, 0.81125]

Table 5: Comparison of Theoretical and Simulated Correlation Coefficients

2 Question 2 - Exposures & CVA

We calculate the values of a diversified portfolio comprising forwards and puts on SX5E and AEX indices over time, leading to the assessment of Expected Positive Exposure (EPE). The portfolio consists of forward contracts and put options characterized by their types, quantities (N), and strike prices (K).

The calculation proceeds by determining the value of each contract within the portfolio at successive time intervals until maturity. For forward contracts, the value at any time t is given by $N \cdot (S_t - K)$, with S_t denoting the index level at time t. Conversely, for put options, the value is represented as $N \cdot \max(K - S_t, 0)$, capturing the intrinsic value of the option at time t.

Addition of these individual contract values across the portfolio for each time step yields the total portfolio value at that point. Subsequently, we calculate the Expected Positive Exposure (EPE) as the mean of positive values of the portfolio's worth over all simulations at each time step. This metric signifies the expected exposure when the portfolio holds a positive value, serving as a critical indicator for credit risk assessment.

Below is the visualization of the EPE across the contract duration, with the EPE profile depicted over monthly intervals from inception to maturity. This profile provides insight into the potential exposure the portfolio could face due to fluctuations in the underlying assets' prices. As expected, the exposure varies over time, reflecting the dynamics of the simulated equity processes and the nature of the financial instruments involved.

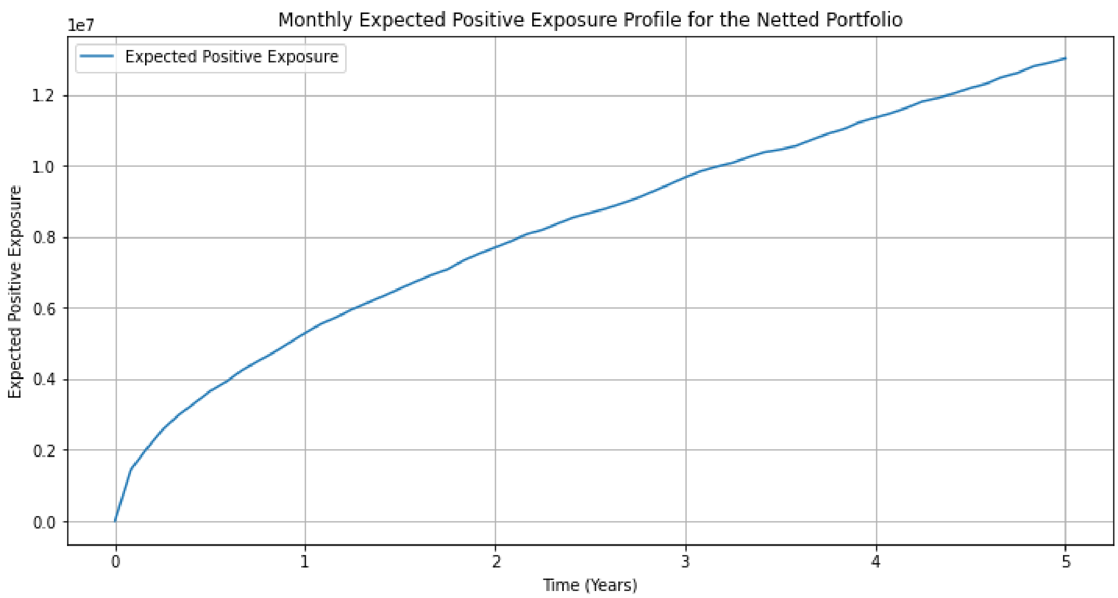

Figure 1: Monthly Expected Positive Exposure Profile for the Netted Portfolio

To calculate the CVA charges, three separate cases are distinguished:

A) For each underlying contract separetely,

B) The complete portfolio assuming non-netting agreement,

C) The complete portfolio assuming netting agreement.

For all cases, we first need to calculate the cumulative intensity and then the $Q(t_{i-1}, t_i)$ probabilities. Then, we can compute the Average Discount Loss and thus the CVA for each case.

The calculation process involves two main steps: computing the cumulative intensity $\Lambda(T_i)$ for each year based on given hazard rates (λ) and then determining the Q values for each interval T_{i-1} to T_i. The given time points are $T = [0, 1, 2, 3, 4, 5]$, and the corresponding hazard rates for each period are $\lambda = [0.02, 0.02, 0.0215, 0.0215, 0.022, 0.022]$.

Step 1: Cumulative Intensity Calculation

The cumulative intensity $\Lambda(T_i)$ represents the total hazard accumulated up to time T_i and is defined for discrete intervals as the sum of the products of hazard rates and the length of each interval:

$$\Lambda(T_i) = \sum_{j=1}^{i} \lambda_{j-1} \cdot (T_j - T_{j-1}),$$

with $\Lambda(T_0) = 0$. This cumulative intensity is calculated iteratively for each time point T_i, starting with an initial value of 0.

Step 2: Calculation of Q Values

The Q values, representing the probability of surviving from T_{i-1} to T_i, are derived using the cumulative intensities. Specifically, the Q value for an interval is calculated as the difference in survival probabilities at the start and end of the interval:

$$Q(T_{i-1}, T_i) = \exp(-\Lambda(T_{i-1})) - \exp(-\Lambda(T_i)).$$

This formula utilizes the exponential function of the negative cumulative intensity, reflecting the principle that survival probability decreases as the cumulative hazard increases.

By applying these steps, we compute the cumulative intensity for each year and the corresponding Q values, which together offer insights into the risk and timing of events as governed by the specified hazard rates. We end pu with the following values which will be used to calculate the CVA charges for each sub-category:

Year (T_i)	λ Values	Cumulative Intensity ($\Lambda(T_i)$)	Interval	Q Values
0–1	0.02	0.02	[0, 1]	0.01980133
1–2	0.02	0.0415	[1, 2]	0.19409234
2–3	0.0215	0.063	[2, 3]	0.02043649
3–4	0.0215	0.085	[3, 4]	0.0200018
4–5	0.022	0.107	[4, 5]	0.02002662

Table 6: Cumulative Intensity and Q Values

Following these two initiation steps, we assess the potential exposures for both forwards and puts. For forwards, positive exposure is evaluated as the difference between the market and strike prices, considering only positive outcomes. For puts, exposure hinges on the intrinsic value, applicable when the strike price surpasses the market value. These exposures are adjusted for Loss Given Default (LGD) and discounted to present values using a risk-free rate (r), encapsulated in the formula:

$$\text{Discounted Exposure} = \text{Exposure} \cdot \text{LGD} \cdot e^{-r \cdot t}.$$

The transition to annualized figures is essential for matching the exposure analysis with the timeline of default risk, requiring aggregation of monthly exposures into annual metrics.

CVA calculation for each contract leverages these annual discounted losses and the survival probability differences (Q values), the latter derived from the cumulative intensities. The integration of market exposure and default probability into the CVA equation offers a comprehensive credit risk perspective:

$$\text{CVA} = \sum \left(\text{Annual Discounted Loss} \times Q\right).$$

The aggregation of CVA across all periods reveals the total credit cost inherent in each contract, encapsulating the counterparty credit risk over the contract's duration. This methodology highlights the importance of credit risk in financial valuations, enabling the adjustment of derivatives pricing to account for the risk of counterparty default. Note that each instrument is multiplied by its corresponding number of contracts. Also for Case C (netting portfolio) we assume that the position is long for the Forwards and short for the Put Options. Meaning in the final calculations we add the CVA for the forwards but deduct the CVA for the Puts.

Description	Instrument	CVA / Total CVA
	Case A	
Forward SX5E		1,902,345.48
Forward AEX		1,898,090.40
Put SX5E		305,559.19
Put AEX		303,385.78
Total Independent CVA		**4,409,380.84**
	Case B	
Total CVA without Netting		**4,409,380.84**
	Case C	
Total CVA Netted Portfolio		**3,191,490.90**

Table 7: Credit Valuation Adjustment (CVA) Totals for Various Cases

3 Question 3: Impact of Model Parameters

For the next section, we are asked to loosen some restrictions for the initial parameters, while focusing only on the last case (C) of the netted portfolio.

A) Increase volatility of the equity contracts from 0.15 to 0.3,

B) Reduce the correlation between the log-returns of the equity contracts from 0.8 to 0.4.

Using the same Q values and Cumulative Intensity calculations, we compute the following:

Time Point (T)	CVA (\$)
1	75,944.20
2	110,990.31
3	133,866.00
4	155,212.34
5	171,474.84
Total	647,487.69

Table 8: Case A: CVA for the Netted Portfolio at Each T with Increased Volatility

Time Point (T)	CVA (\$)
1	83,047.01
2	125,146.07
3	151,931.37
4	176,274.25
5	194,829.23
Total	678,227.93

Table 9: Case B: CVA for the Netted Portfolio at Each T with Reduced Correlation

From the tables above we can draw some interesting conclusions.

Increased Volatility Results: The CVA values are higher across all future time steps compared to the baseline scenario. This increase is due to the higher volatility ($\sigma = 0.3$ or 30%) used in the simulations. Higher volatility leads to a greater range of potential outcomes for the underlying asset prices, increasing the potential for both higher gains and losses. In the context of CVA, which focuses on positive net exposures that could be lost in the event of counterparty default, higher volatility increases the potential positive net exposure, thus increasing the CVA.

Decreased Correlation Results: Lower CVA values are observed across all time steps when the correlation between the underlying assets is decreased from 0.8 to 0.4. Decreasing the correlation means the price movements of the underlying assets become less synchronized. In a diversified portfolio, this can lead to a natural hedging effect where losses in one asset may be offset by gains in another, reducing the net exposure of the portfolio to positive outcomes that are at risk in the event of a counterparty default. Consequently, the CVA, which quantifies the risk of these positive net exposures, is lower.

The results reinforce the principle that higher volatility increases the credit risk quantified by CVA, as it amplifies the range of potential positive exposures at risk in case of default. The decrease in correlation demonstrates how less synchronized movements between assets can reduce credit risk in a netted portfolio, showcasing the importance of diversification and the hedging potential within a portfolio.

The outcomes illustrate the critical roles both volatility and correlation play in determining the credit risk of a portfolio under a netting agreement. While higher volatility increases CVA by widening the potential distribution of exposures, lower correlation can mitigate credit risk by reducing the portfolio's net positive exposure, leading to lower CVA charges. These insights emphasize the need for careful risk management and the benefits of diversification within financial portfolios.

4 Question 4 - Collateral Impact on CVA

For this part, we have to adjust our process to take into account the new time steps of [1, 2, 3, 4, 5, 6, 12, 24, 36, 48, 60] months, converts these to years for the simulation, and incorporates the monthly collateral posting mechanism. After simulating the forward prices for SX5E and AEX, calculating the net exposures, and applying the collateral posting logic, the remaining exposures are used to calculate the Credit Valuation Adjustment (CVA) for the netted portfolio.

The CVA values for the netted portfolio at each time step (after the first month, since the first month's CVA is implicitly zero due to initial conditions) are extremely low, close to zero in most cases. This result reflects the significant impact of the collateral posting mechanism on reducing credit exposure. By posting collateral equal to the positive net exposure each month, the model effectively minimizes the residual exposure that would contribute to the CVA, leading to these minimal CVA values.

This outcome demonstrates how effective collateral management can be in mitigating counterparty credit risk in derivative portfolios. The CVA, representing the cost of counterparty credit risk, is greatly reduced when collateral is accurately and frequently posted to offset positive exposures.

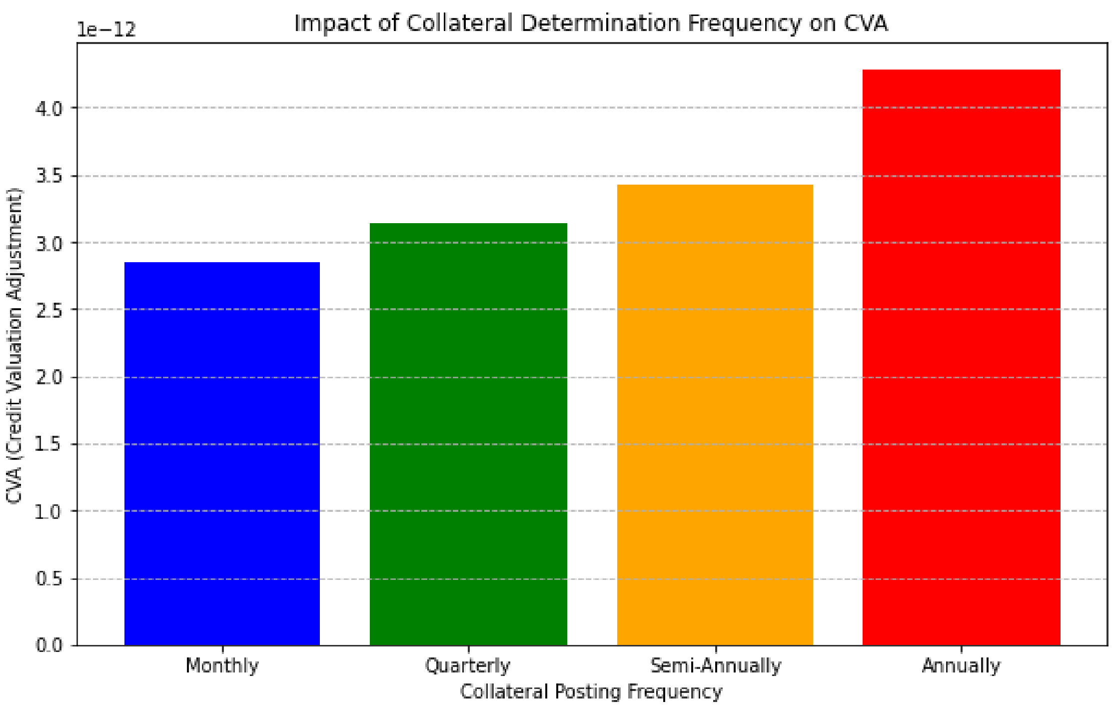

Figure 2: CVA impact based on different posting frequencies.

The plot above illustrates the impact of collateral determination frequency on the CVA change. As the frequency of collateral posting decreases (from monthly to quarterly, semi-annually, and annually), the CVA increases. This trend highlights the significance of frequent collateral postings in reducing counterparty credit risk. More frequent collateral adjustments more effectively mitigate potential exposure, thereby reducing the CVA. Conversely, less frequent postings lead to higher residual exposure and, consequently, a higher CVA, reflecting the increased risk of counterparty default.

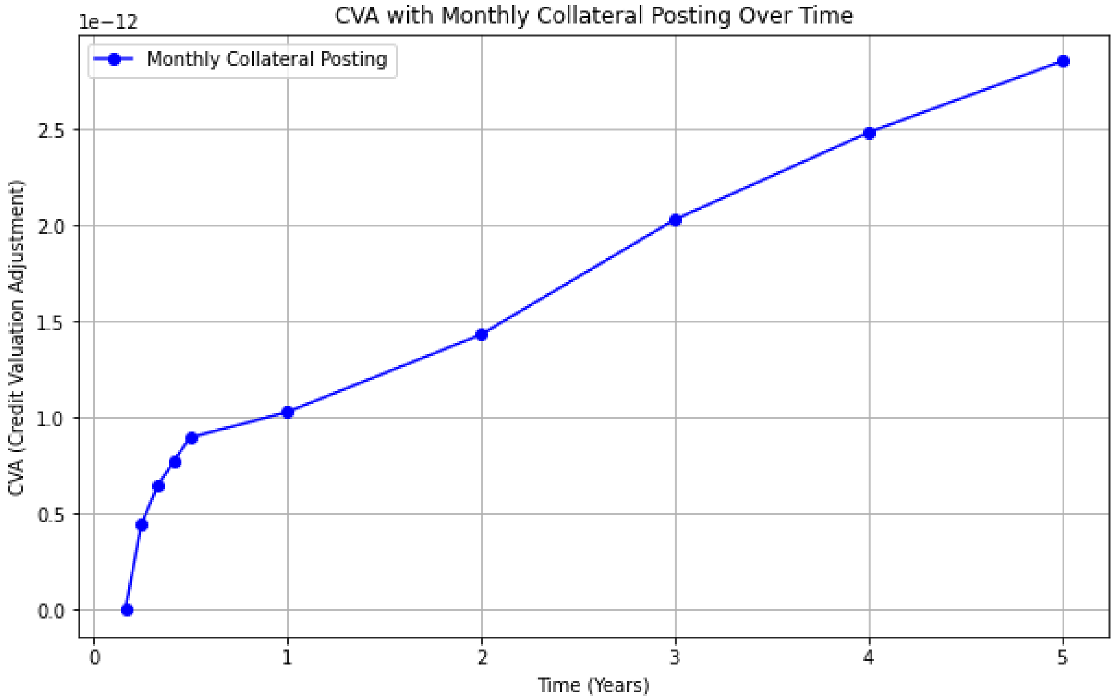

Figure 3: CVA impact based on different posting frequencies.

Similarly in the second plot when collateral is posted monthly, any increase in the value of the derivative position that would increase potential exposure to counterparty risk is offset by posting additional collateral. This means if the market moves in such a way that the value of the derivative increases (indicating a potential gain and, thus, an increased exposure to loss if the counterparty defaults), additional collateral is posted to match this increase. Conversely, if the market value decreases, excess collateral can be withdrawn. This dynamic adjustment effectively keeps the net exposure—and thus the potential loss in the event of default—low. When this process is done frequently (in this case monthly) we observe that the CVA is (close) zero. The CVA exposure increases as the collateral posting also increase in time steps or periods.

An alternative method is to post an initial margin. We will be examining the cases that 1mln, 10mln and 100mln EUR are posted as initial margin.

This analysis shows a significant impact of the initial margin on the CVA. As the initial margin increases, the CVA decreases substantially, indicating a lower credit risk associated with the counterparty. Specifically, posting a higher initial margin (e.g., 100 million EUR) almost completely mitigates the counterparty credit risk, reducing the CVA to a negligible amount. This underscores the effectiveness of using an initial margin as a risk management tool to safeguard against potential future exposures in derivative transactions.

Scenario	CVA with Initial Margins ($)
1	387,694.44
2	152,502.74
3	99.80

Table 10: CVA Values with Initial Margins

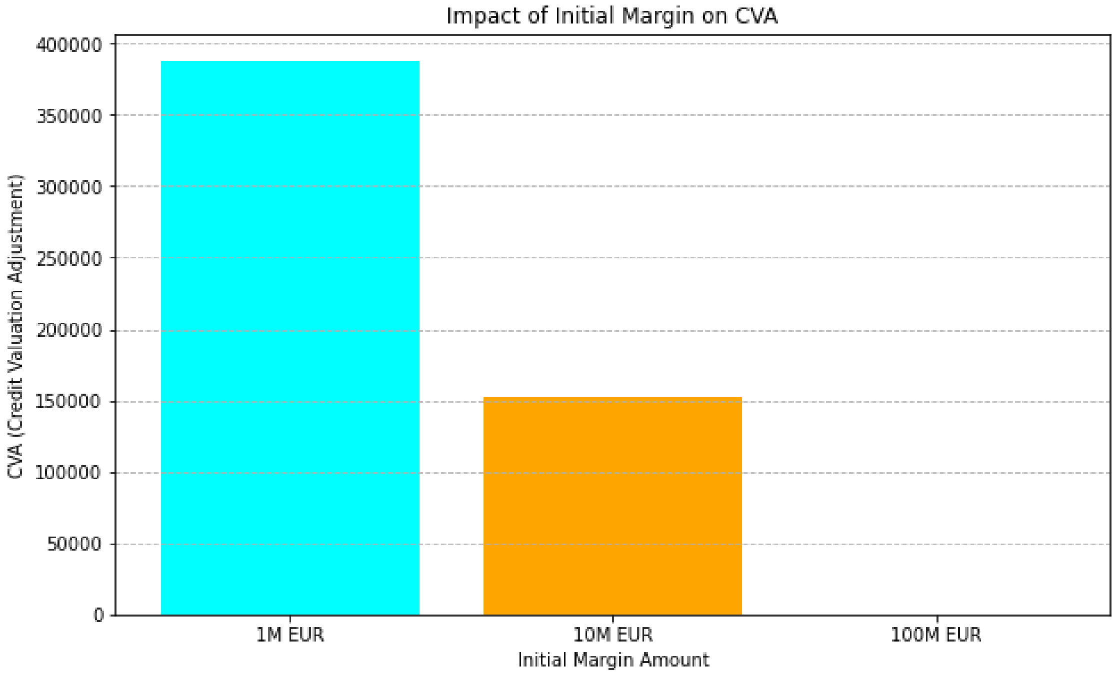

Figure 4: CVA impact based on different posting frequencies.

5 Question 5 - Credit Risk Exposure Hedging

In this part we investigate the influence of incremental adjustments in hazard rates by 10 basis points (bps) across specified intervals on the Credit Valuation Adjustment (CVA) for a netted portfolio. The exploration is structured through a sequential process, each stage integrating specific adjustments and culminating in the evaluation of their impact on CVA.

The investigation begins by making adjustments to the hazard rates, set to increase by 0.001 (or 10 bps) for distinct time intervals. These adjustments aim to simulate a scenario analysis across three different periods:

Initially, the survival probabilities, denoted as Q_{values}, are adjusted according to the specific interval targeted in each scenario, employing the formula:

$$\text{Adjusted } Q_{\text{values}}[i] = Q_{\text{values}}[i] + \text{adjustment},$$

where the adjustment is the specified increase, and i represents the interval undergoing adjustment.

Subsequently, with the adjusted Q_{values}, we proceed to recalculate the net exposures and, by extension, the CVA for the netted portfolio. This step reflects the altered risk profile post-adjustment, formulating the CVA as:

$$\text{Adjusted CVA}_{\text{netted portfolio}} = \sum_{i=1}^{n} (\text{Discounted Net Exposure}_i \times \text{LGD} \times \text{Adjusted } Q_{\text{values}}[i-1]),$$

where LGD represents the Loss Given Default.

The final phase involves contrasting the recalculated CVA against the original to ascertain the variance instigated by the hazard rate adjustments. This differential insight elucidates the CVA's sensitivity to fluctuations in forward hazard rates across varying intervals, underscoring the pivotal role of precise hazard rate estimations in counterparty credit risk management.

The findings, 'adjusted_cva_changes', clarifies the consequential impact of hazard rate adjustments on the portfolio's credit risk valuation, thereby emphasizing the criticality of accurate forward hazard rate projection in credit risk mitigation strategies.

Moving to the second part of the question, we need to set out initial parameters, including the fixed rate of the CDS ($R = 0.01$), the Loss Given Default (LGD $= 0.4$), and the risk-free interest rate ($r = 0.03$). The maturity of the CDS is established at 5 years ($T = 5$), with an assumption of annual payments over 5 periods ($N = 5$).

The annual time points for premium payments, denoted T_i, are evenly distributed over the term of the CDS, starting from year 1 to year 5. To facilitate calculations, midpoints between these time points, T_{mid_i}, are also determined, providing the temporal framework for our pricing model.

Scenario	Increase Forward Hazard Rate Interval	CVA Charge ($)
1	[0,1]	2,102.06
2	[1,3]	6,428.02
3	[3,5]	8,408.95

Table 11: CVA Charge and Price Change Due to Increasing Forward Hazard Rate

The core of our pricing methodology hinges on the survival probability $Q(\tau > T)$, which is derived from piecewise constant hazard rates, encapsulated by the formula:

$$Q(\tau > T) = \exp(-\lambda \cdot T),$$

where λ represents the average hazard rate over the specified interval.

The CDS pricing model integrates this survival probability into both the premium and protection legs of the swap. The valuation process is articulated through the equations:

$$\text{PV}_{\text{Premium Leg}} = R \cdot \left(\sum \exp(-r \cdot T_i) \cdot (T_i - T_{i-1}) \cdot Q(\tau > T_i) + \sum \exp(-r \cdot T_{\text{mid}_i}) \cdot (Q(\tau > T_{i-1}) - Q(\tau > T_i)) \cdot \frac{T_i - T_{i-1}}{2} \right)$$

$$\text{PV}_{\text{Protection Leg}} = \text{LGD} \cdot \sum \exp(-r \cdot T_{\text{mid}_i}) \cdot (Q(\tau > T_{i-1}) - Q(\tau > T_i))$$

The net CDS price is the difference between these two present values.

Upon evaluating the CDS pricing under an initial hazard rate of 0.02, adjustments are made by increasing the hazard rate by 10 bps in specific intervals, and the resultant CDS prices are recalculated. The change in CDS price attributable to this hazard rate increase is meticulously computed, illustrating the sensitivity of CDS valuations to underlying credit risk dynamics.

Subsequent calculations further dissect the impact across different maturity segments (1-year, 3-year, and 5-year CDS) under distinct interval adjustments, offering a nuanced view of how hazard rate fluctuations within varying temporal scopes affect CDS pricing.

The findings reveal discernible shifts in CDS valuations contingent on the timing and magnitude of hazard rate adjustments, underscoring the critical interplay between credit risk assessments and derivative pricing mechanisms.

CDS Maturity	**Interval**	**Initial Price**	**Adjusted Price**	**Price Change**
Overall CDS Price Change				
Overall	-	0.0082	0.0064	-0.0018
CDS Price Changes by Maturity and Interval				
1Y CDS	[0,1]	0.0082	-	-0.0004
1Y CDS	[1,3]	0.0082	-	0.0000
1Y CDS	[3,5]	0.0082	-	0.0000
3Y CDS	[0,1]	0.0082	-	-0.0011
3Y CDS	[1,3]	0.0082	-	-0.0022
3Y CDS	[3,5]	0.0082	-	0.0000
5Y CDS	[0,1]	0.0082	-	-0.0018
5Y CDS	[1,3]	0.0082	-	-0.0036
5Y CDS	[3,5]	0.0082	-	-0.0036

Table 12: Impact of a 10 bps Increase in Hazard Rate on CDS Prices for Various Maturities

Hedging CVA Charge Movements Using CDS Contracts

Credit Valuation Adjustment (CVA) represents the market value of counterparty credit risk inherent in over-the-counter (OTC) derivatives. As the credit quality of the counterparty "C" fluctuates, so does the CVA charge, reflecting changes in the perceived risk of default. Credit Default Swaps (CDS) are financial derivatives that allow market participants to transfer the credit risk of a reference entity to another party. When used strategically, CDS contracts can effectively hedge against movements in CVA charges, particularly those arising from changes in the underlying forward hazard rates.

The essence of this hedging strategy lies in the inverse relationship between a counterparty's creditworthiness and the CVA charge: as the counterparty's credit risk increases (implied by a rise in forward hazard rates), the CVA charge also increases. Conversely, the value of a CDS contract purchased on counterparty "C" would increase under the same conditions, since the CDS serves as insurance against "C's" default.

To implement a hedge using CDS contracts, an entity would purchase CDS protection on counterparty "C" corresponding to the notional amount and maturity of the underlying derivative exposure. The hedge aims to neutralize the impact of forward hazard rate fluctuations on the CVA charge in the following manner:

1. As the forward hazard rate increases, signaling a higher risk of default from counterparty "C", the CVA charge associated with the derivative exposure to "C" increases.

2. Simultaneously, the mark-to-market value of the CDS protection purchased on "C" increases, as the market now deems "C's" default more likely. This increase in the CDS value offsets the rise in the CVA charge.

3. In the event of "C's" default, the CDS contract pays out, covering losses that would otherwise be incurred due to the counterparty's failure to meet its obligations under the derivative contract.

By aligning the notional amount, maturity, and reference entity of the CDS protection with the underlying exposure, the increase in CVA charge due to heightened credit risk can be offset by gains in the CDS contract's value, effectively hedging the credit risk of the OTC derivative position.

It is crucial, however, to continuously monitor and adjust this hedge, as discrepancies in recovery rates, maturities, and basis risk such as interest rate risk between the CDS contract and the underlying exposure can lead to an imperfect hedge. Additionally, the dynamic nature of market conditions and credit risk requires regular reassessment of the hedging strategy's effectiveness.

The objective is to neutralize the impact of changes in forward hazard rates on the CVA charge associated with a netted portfolio of derivative instruments. We specifically focus on utilizing CDS contracts of 1-year (1Y), 3-year (3Y), and 5-year (5Y) maturities as hedging instruments. The approach involves backward solving for the notional values of these contracts that effectively offset the delta of the CVA due to hazard rate fluctuations.

Given are the changes in CVA charges resulting from a 10 basis points (bps) increase in the hazard rates across different intervals. The task is to compute the notional adjustments required for CDS contracts at different maturities to counterbalance these CVA charge movements.

We commence with predetermined changes in CVA charges for specified hazard rate intervals, alongside the changes in CDS prices for contracts of varying maturities due to these hazard rate adjustments. The initial and adjusted CDS prices provide a foundation for understanding the sensitivity of CDS valuations to hazard rate changes.

We begin with the 5Y CDS contract, recognizing its broad impact on the portfolio due to its longer maturity. The notional adjustment is solved by inversely relating the total CVA impact to the 5Y CDS's specific impact.Proceeding to the 3Y CDS contract, the remaining impact after accounting for the 5Y hedge is further neutralized. The notional required for the 3Y contract is calculated to offset the residual CVA movement.Finally, the 1Y CDS contract is adjusted to hedge against any remaining sensitivity in the shortest interval. The cumulative effect of prior hedges is considered to solve for the 1Y notional adjustment.

The calculated notional adjustments for each maturity level are then interpreted, providing insight into the effective hedging strategy across the maturity spectrum of CDS contracts. This strategy elucidates a structured approach to managing the counterparty credit risk embedded in the valuation of derivative instruments through dynamic adjustments in CDS notional values.

CDS Maturity	Notional Adjustment (million USD)
5Y	1.88
3Y	0.00
1Y	0.00

Table 13: Notional Adjustments for CDS Contracts to Hedge CVA Movements

Delta hedging, a dynamic strategy, necessitates periodic adjustments to maintain its effectiveness, particularly as market conditions evolve over time. Financial markets are inherently volatile, with asset prices, interest rates, and credit spreads fluctuating continuously. Since delta—the sensitivity of the hedged position's value to changes in the underlying asset price—is not static, these market movements can render a previously perfect hedge ineffective. Additionally, as time progresses, the characteristics of financial instruments, including their time to maturity, change. This affects options and derivatives' value and, consequently, the delta. For instance, as options approach expiry, their delta can change significantly, necessitating adjustments to the hedge. As we have already seen in previous sections, volatility is a critical factor in determining the price of derivatives and their delta. An increase in market volatility can increase the delta of options, requiring the hedge to be adjusted to account for the higher sensitivity of the derivative's price to movements in the underlying asset. Lastly, for CVA hedging, changes in the counterparty's credit quality can alter the forward hazard rates, impacting the CVA charge and the delta of the hedge. Adjustments may be needed to account for revised expectations of default risk.

A cautious agent should be aware that such adjustments should be considered at regular intervals, based on a pre-determined hedging strategy or risk management policy. This could be daily, weekly, or at any other suitable frequency. Of course, one should not forget to do so following significant market events that lead to substantial price movements, changes in volatility, or alterations in the creditworthiness of counterparties. Finally, if the delta of the hedged position deviates beyond a certain pre-defined threshold from its ideal value, indicating that the hedge's effectiveness has diminished sufficiently to warrant rebalancing.