# Quantitative Financial Risk Management - Market Risk (Revised)

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# 1 Introduction

The purpose of this report is to develop a robust Value at Risk (VaR) system for a diversified portfolio and to evaluate various methods for calculating VaR and Expected Shortfall (ES). Our portfolio consists of a mix of stocks, indices, bonds, and commodities, chosen to ensure exposure to different currencies, interest rates, and economic conditions. Specifically, the portfolio includes Apple Inc. (AAPL), Alphabet Inc. (GOOGL), SPDR SP 500 ETF (SPY), iShares 20+ Year Treasury Bond ETF (TLT), and SPDR Gold Shares (GLD). This selection ensures a diversified exposure across different market sectors and asset classes.

Apple Inc. (AAPL) and Alphabet Inc. (GOOGL) provide exposure to technology stocks, representing the equity market. The SPDR SP 500 ETF (SPY) offers broad market exposure, capturing the performance of the largest 500 companies in the U.S. equity market. The iShares 20+ Year Treasury Bond ETF (TLT) represents long-term U.S. government bonds, providing exposure to interest rate fluctuations, particularly U.S. interest rates. The SPDR Gold Shares (GLD) provides exposure to gold, a traditional safe-haven asset, offering a hedge against market volatility and inflation. Additionally, since gold is traded in U.S. dollars, it indirectly provides exposure to exchange rate fluctuations.

To ensure exposure to Euribor rates and exchange rate fluctuations directly, we assume a portion of the SPDR SP 500 ETF (SPY) and technology stocks (AAPL and GOOGL) are held by European investors, thereby making their performance sensitive to changes in EUR/USD exchange rates and Euribor rates. This diversified selection ensures that the portfolio is well-positioned to manage risks associated with different currencies, interest rates, including Euribor, and market conditions.

# 2 Data Collection and Preparation

To analyze the portfolio, we downloaded daily adjusted closing prices for each asset from Yahoo Finance, covering the period from January 1, 2014, to January 1, 2024. The data was then cleaned and synchronized to remove any missing values. Daily returns were calculated as the percentage change in adjusted closing prices. This resulted in 2516 observations.

# 3 Variance-Covariance Method

The Variance-Covariance method assumes that returns are normally distributed and calculates VaR using the mean and standard deviation of portfolio returns. The VaR at confidence level $\alpha$ over a time horizon $t$ is given by:

$$\text{VaR} = \Phi^{-1}(\alpha) \cdot \sigma \cdot \sqrt{t}$$

where $\Phi^{-1}$ is the inverse cumulative distribution function of the standard normal distribution, and $\sigma$ is the standard deviation of the portfolio returns. We calculated VaR and ES for $\alpha = 0.975$ and $\alpha = 0.99$, and backtested the model by comparing actual violations to expected violations.

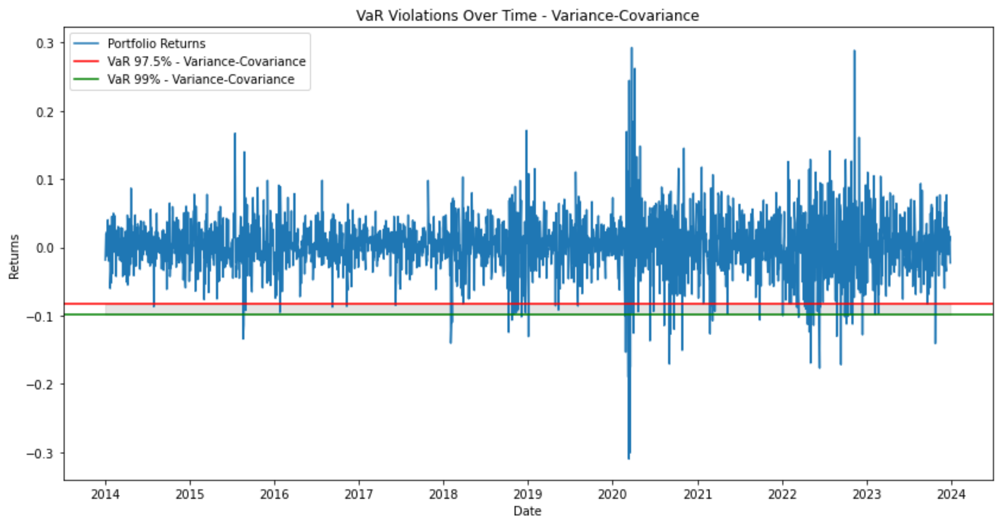



**Figure 1:** Structure of the diversified portfolio and its exposure to various risk factors.

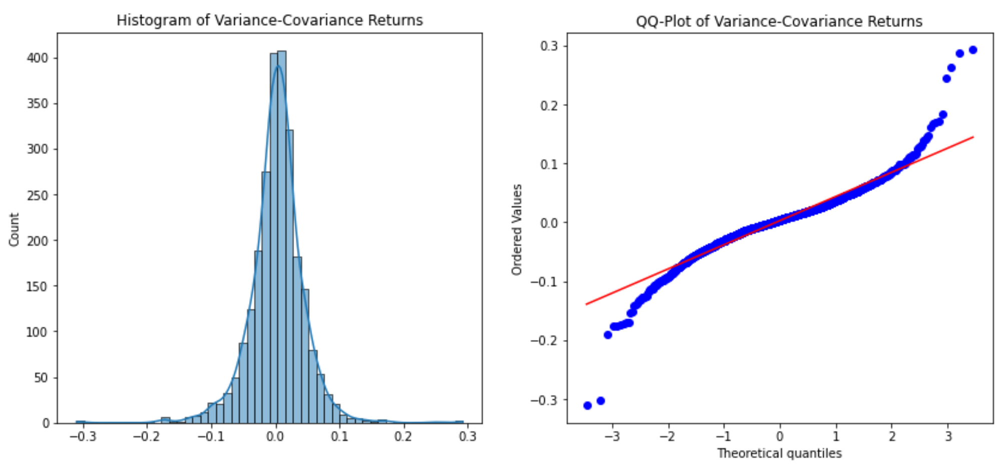


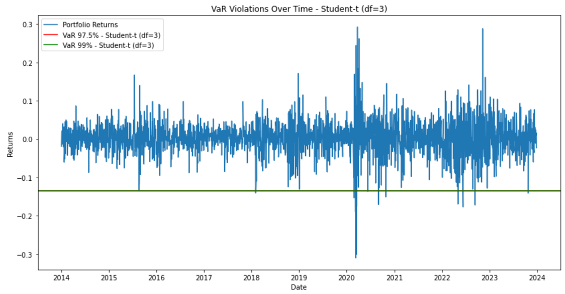

**Figure 2:** Histogram of returns and QQ-plot of Variance-Covariance method

# 4 Variance-Covariance with Student-t Distribution

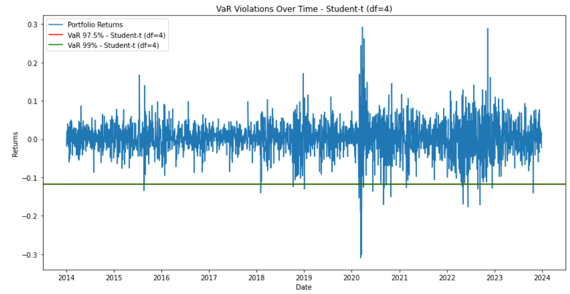
To account for fat tails in the return distribution, we also calculated VaR using the Student-t distribution with varying degrees of freedom (df). The VaR for the Student-t distribution is given by:

$$\text{VaR} = t^{-1}(\alpha, \nu) \cdot \sigma$$

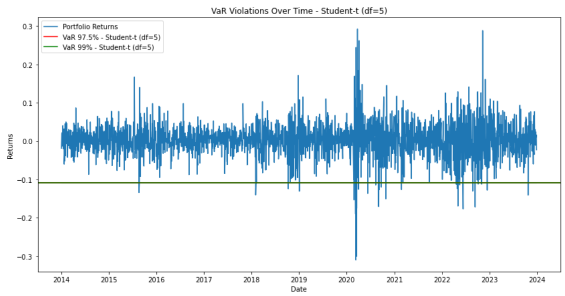
where $t^{-1}$ is the inverse cumulative distribution function of the Student-t distribution with $\nu$ degrees of freedom. We evaluated this method for $\nu = 3, 4, 5, 6$ and backtested the results.



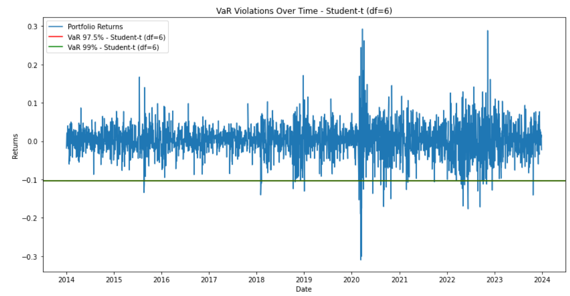

**(a)** Student-t (df=3)




**(b)** Student-t (df=4)

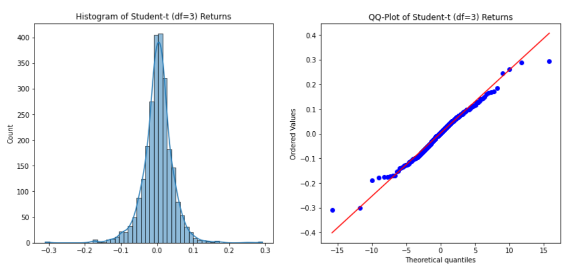



**(c)** Student-t (df=5)

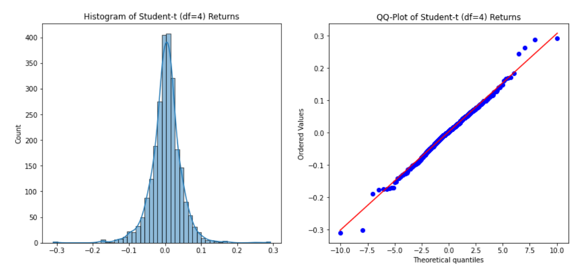



**(d)** Student-t (df=6)

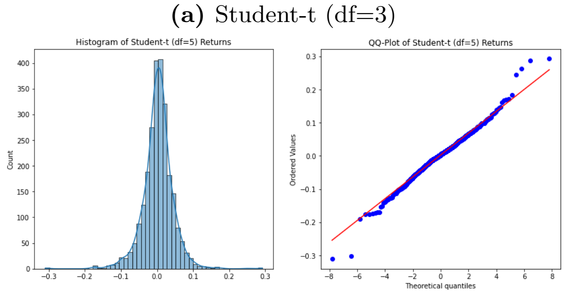
**Figure 3:** Histogram of returns and QQ-plot for Student-t distribution with different degrees of freedom.



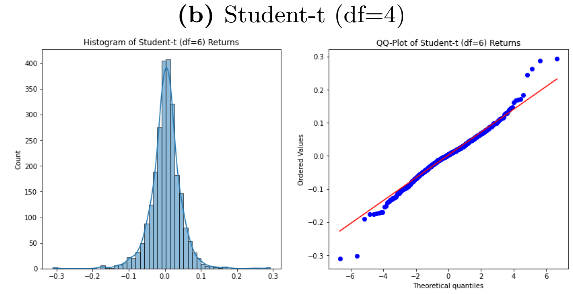

**(a)** Student-t (df=3)




**(b)** Student-t (df=4)




**(c)** Student-t (df=5)

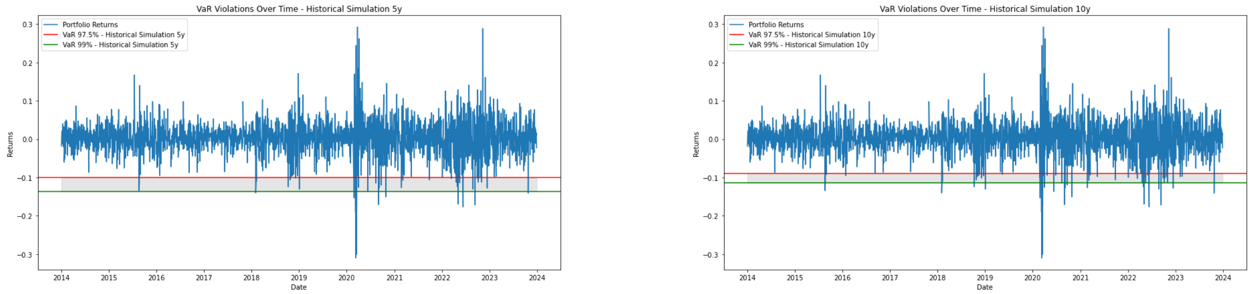



**(d)** Student-t (df=6)

**Figure 4:** VaR violations for Student-t distribution with different degrees of freedom.

# 5 Historical Simulation

The Historical Simulation method is non-parametric and uses historical return data to calculate VaR. For a confidence level $\alpha$, we sort the historical returns and select the $(1-\alpha)$-quantile. We performed the simulation over the past 5 and 10 years and compared the results to the expected violations.




**(a)** Historical Simulation 5y **(b)** Historical Simulation 10y

**Figure 5:** VaR violations for Historical Simulation over different time horizons.

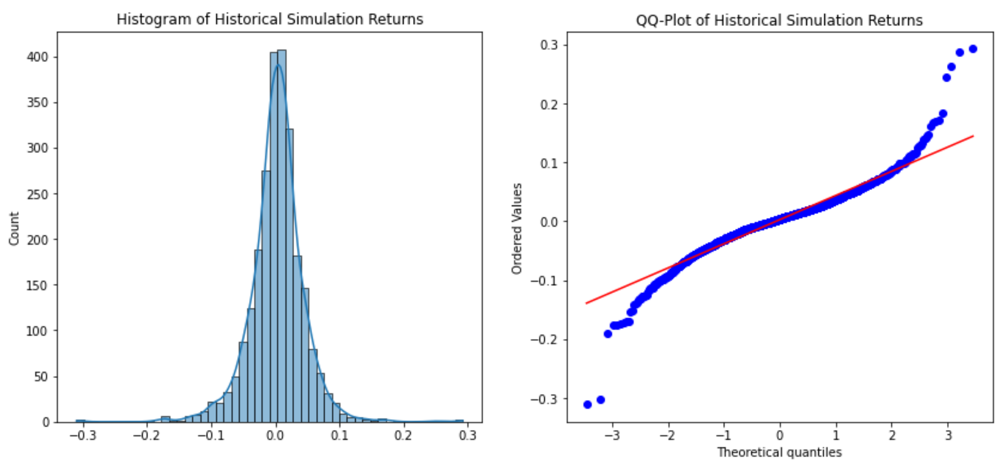



**Figure 6:** Histogram of returns and QQ-plot of Historical Simulation method (10yr)

# 6 CCC-GARCH Model

The CCC-GARCH model captures time-varying volatility by fitting a GARCH(1,1) model to each asset's returns. The portfolio VaR is then calculated by aggregating the conditional volatilities. The GARCH(1,1) model is given by:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

where $\sigma_t^2$ is the conditional variance, $\epsilon_t$ is the residual return, and $\omega$, $\alpha$, and $\beta$ are model parameters. We calculated VaR and ES using this model and backtested the results.

**Table 1:** Estimated Parameters for GARCH(1,1) Model for Each Asset

| **Asset** | **Omega** | **Alpha** | **Beta** |
|---|---|---|---|
| AAPL | 0.1603 | 0.1157 | 0.8336 |
| GOOGL | 0.1547 | 0.1077 | 0.8462 |
| SPY | 0.0378 | 0.2020 | 0.7704 |
| TLT | 0.0151 | 0.0717 | 0.9109 |
| GLD | 0.0145 | 0.0431 | 0.9385 |

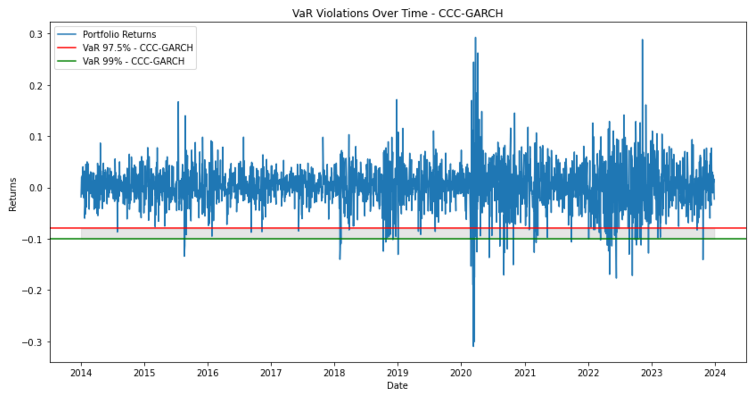



**Figure 7:** VaR violations for CCC-GARCH method.

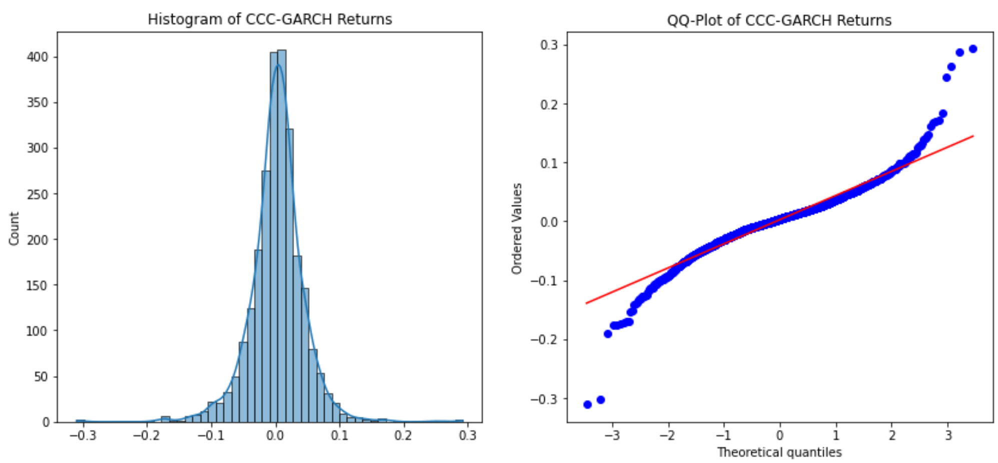



**Figure 8:** Histogram of returns and QQ-plot of CCC-GARCH(1,1) method

# 7 Filtered Historical Simulation with EWMA

The Filtered Historical Simulation combines historical returns with volatility weighting using Exponentially Weighted Moving Average (EWMA). The EWMA model is given by:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\epsilon_{t-1}^2$$

where $\lambda$ is the smoothing parameter. In this analysis, we used $\lambda = 0.94$, which is a common choice in financial applications. We calculated VaR and ES using this method and backtested the results.

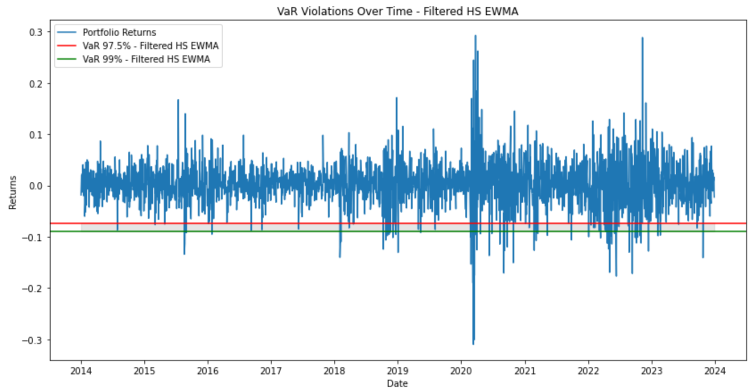



**Figure 9:** VaR violations for EWMA method.

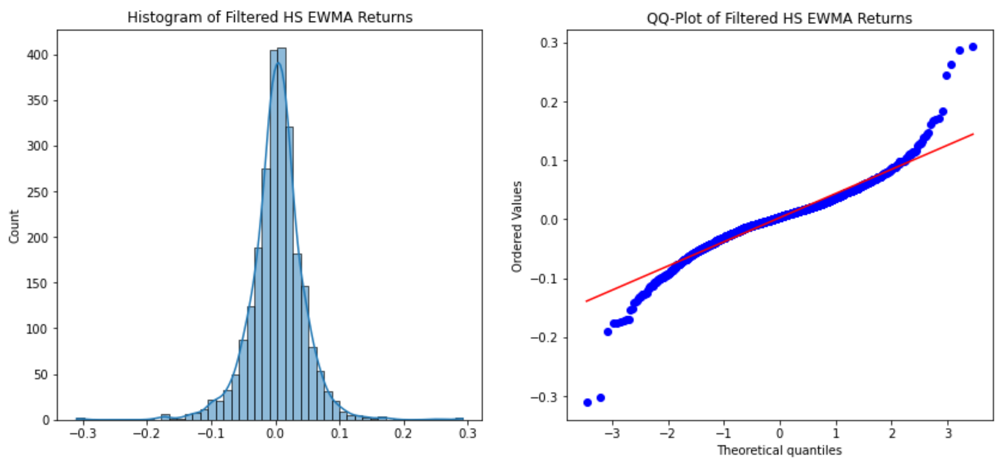


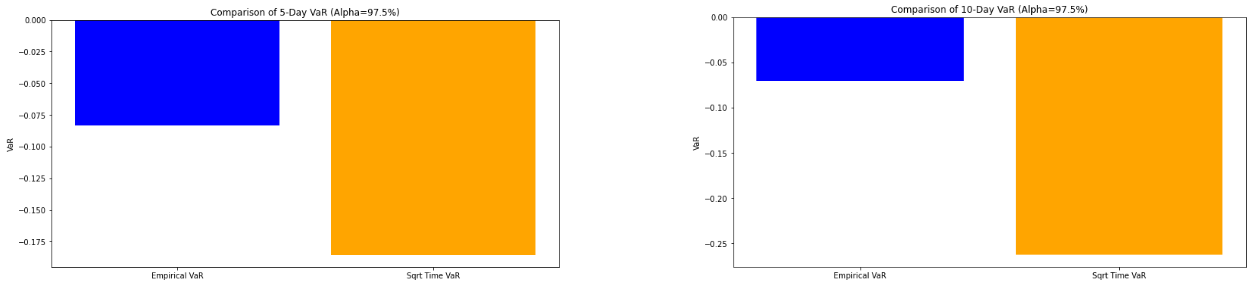

**Figure 10:** Histogram of returns and QQ-plot of EWMA method

# 8 Empirical Multi-Day VaR

We calculated empirical 5-day and 10-day VaRs using non-overlapping historical data and compared them to the one-day VaR scaled by the square root of time. This comparison assessed the adequacy of the square root of time rule for our portfolio.

**Table 2:** Comparison of Empirical and Sqrt Time VaR

| VaR | Empirical VaR | Sqrt Time VaR |
|---|---|---|
| 5-day VaR 97.5% | 0.083 | 0.1857 |
| 10-day VaR 97.5% | 0.0705 | 0.2626 |




**(a)** 5-day VaR Comparison **(b)** 10-day VaR Comparison

**Figure 11:** Comparison of Empirical VaR and Sqrt Time VaR

# 9 Backtesting

We backtested our VaR systems by comparing the expected and actual number of VaR violations per year and the average discrepancy between these two over all years. We also compared the expected shortfalls with average (per year) shortfalls. Additionally, we plotted the VaR violations against time to investigate whether VaR violations occurred in clusters or were evenly spread out over time.

**Table 3:** Backtesting Results: Actual vs. Expected VaR Violations

| Method | Actual Violations 97.5% | Expected Violations 97.5% | Actual Violations 99% | Expected Violations 99% |
|---|---|---|---|---|
| Variance-Covariance | 73 | 62.88 | 44 | 25.15 |
| Student-t (df=3) | 31 | 62.88 | 13 | 25.15 |
| Student-t (df=4) | 31 | 62.88 | 13 | 25.15 |
| Student-t (df=5) | 31 | 62.88 | 13 | 25.15 |
| Student-t (df=6) | 31 | 62.88 | 13 | 25.15 |
| Historical Simulation 5y | 39 | 62.88 | 13 | 25.15 |
| Historical Simulation 10y | 62 | 62.88 | 25 | 25.15 |
| CCC-GARCH | 79 | 62.88 | 41 | 25.15 |
| Filtered HS EWMA | 98 | 62.88 | 63 | 25.15 |

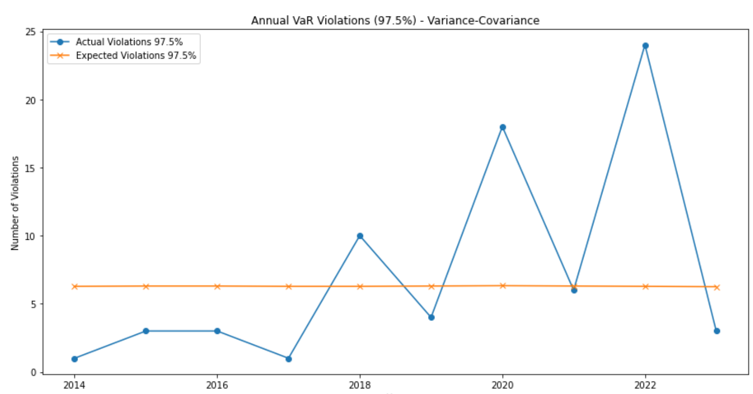
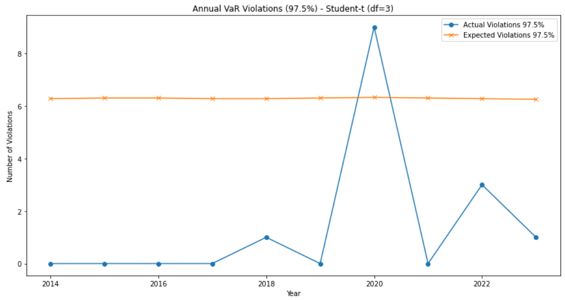
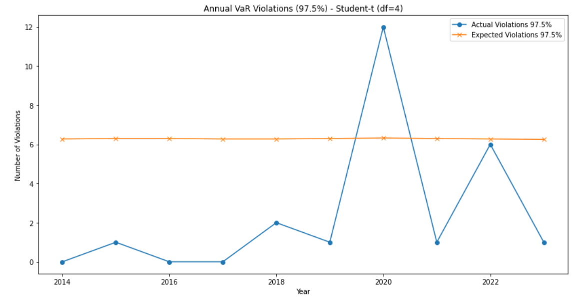



**Figure 12:** Annual VaR violations for Variance-Covariance method.

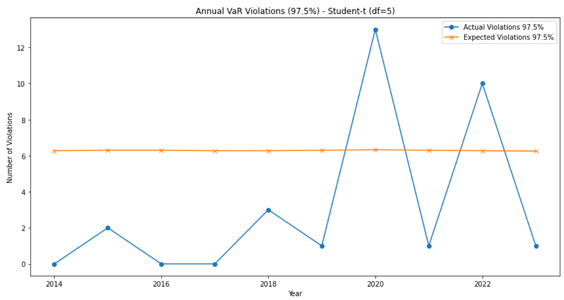
From the plots above it becomes obvious that in resent years markets have become more volatile. Consistently, across all methods VaR violations from 2018-2019 onwards, are higher than the expected violations. Which can be expected since that was the Covid-19 period followed by war events in Eastern Europe and Middle East. Again, when looking at the clustering plots, events are concentrated in the years following the Covid-19 period and beyond .



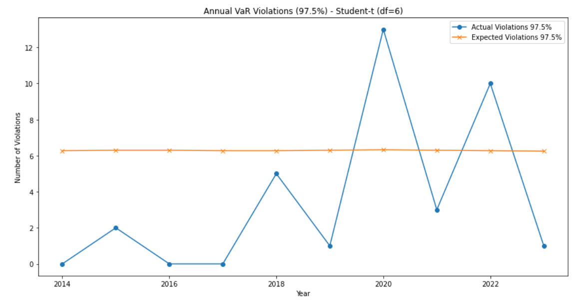

**(a)** Student-t (df=3)




**(b)** Student-t (df=4)

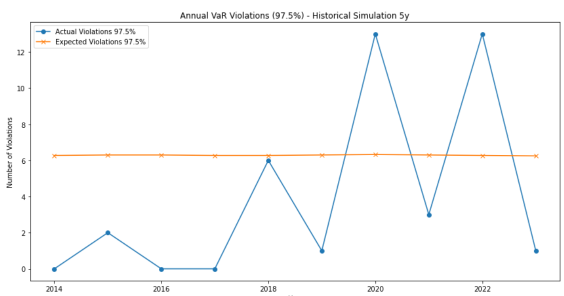



**(c)** Student-t (df=5)

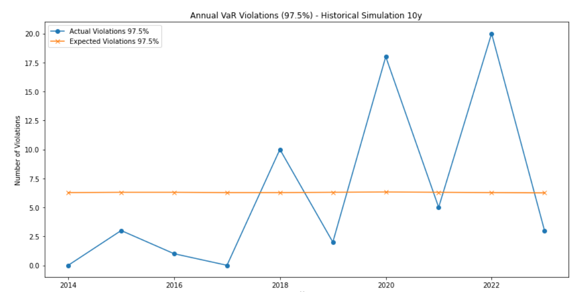



**(d)** Student-t (df=6)

**Figure 13:** Annual VaR violations for Student-t distribution with different degrees of freedom.




**(a)** Historical Simulation 5y




**(b)** Historical Simulation 10y

**Figure 14:** Annual VaR violations for Historical Simulation (5yr & 10yr)

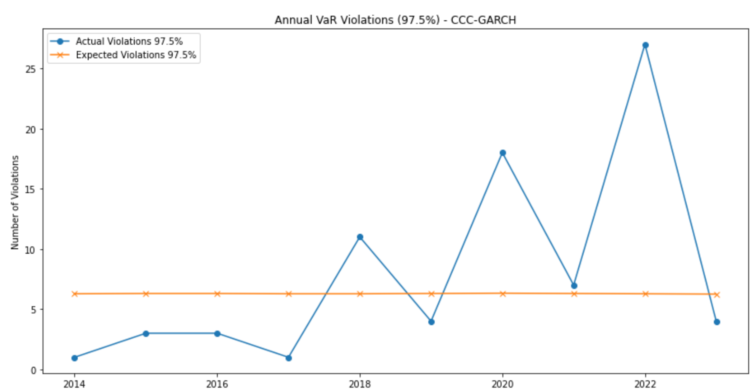



**Figure 15:** Annual VaR violations for CCC-GARCH(1,1) method.

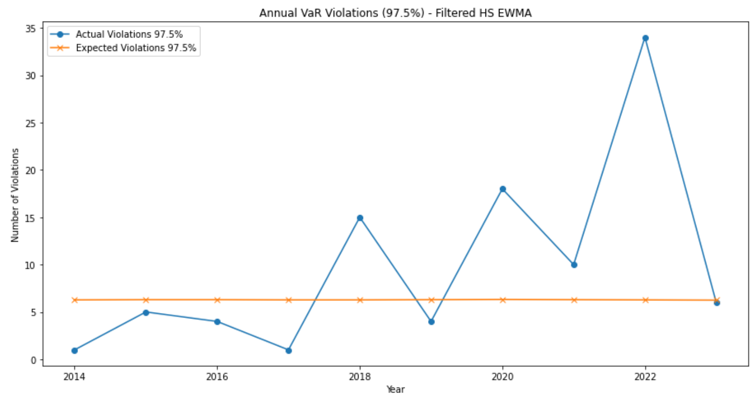



**Figure 16:** Annual VaR violations for EWMA method.

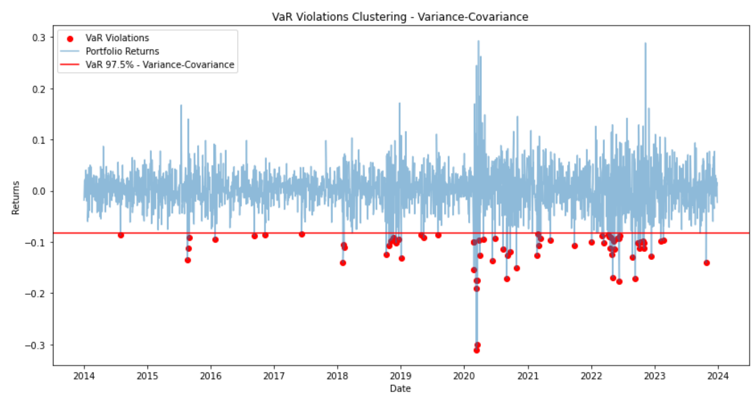
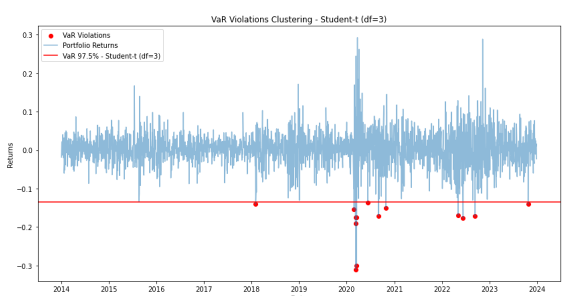
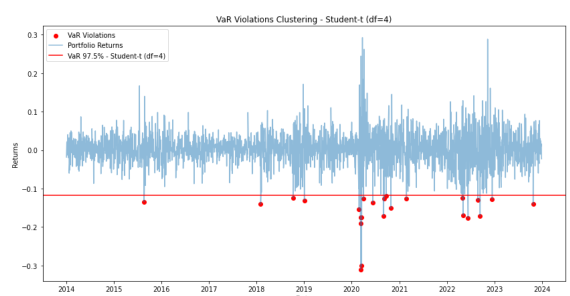


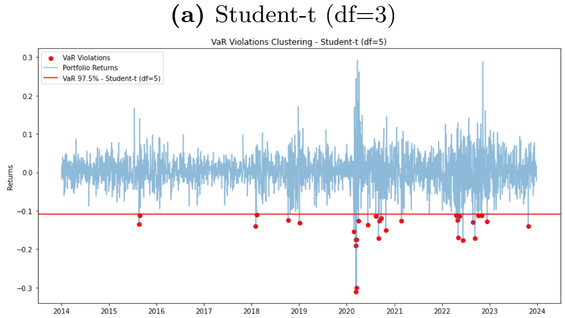

**Figure 17:** Clustering of VaR violations for Variance-Covariance method.



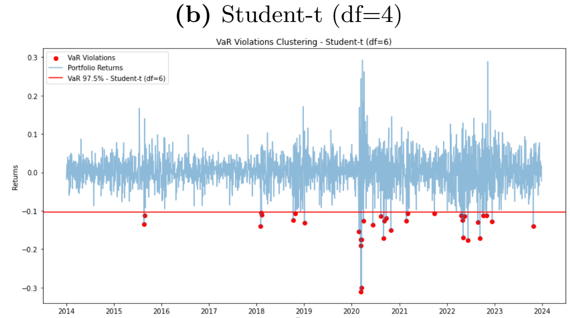

**(a)** Student-t (df=3)




**(b)** Student-t (df=4)

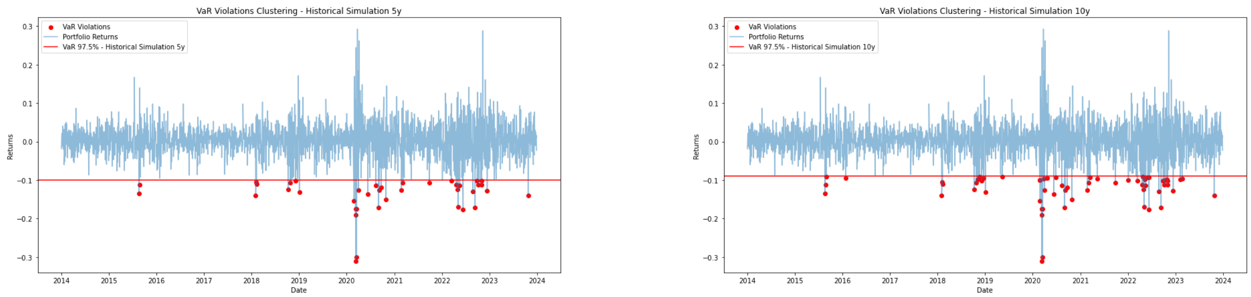



**(c)** Student-t (df=5)




**(d)** Student-t (df=6)

**Figure 18:** Clustering of VaR violations for Student-t distribution with different degrees of freedom.




**(a)** Historical Simulation 5y

**(b)** Historical Simulation 10y

**Figure 19:** Clustering of VaR violations for Historical Simulation (5yr & 10yr)

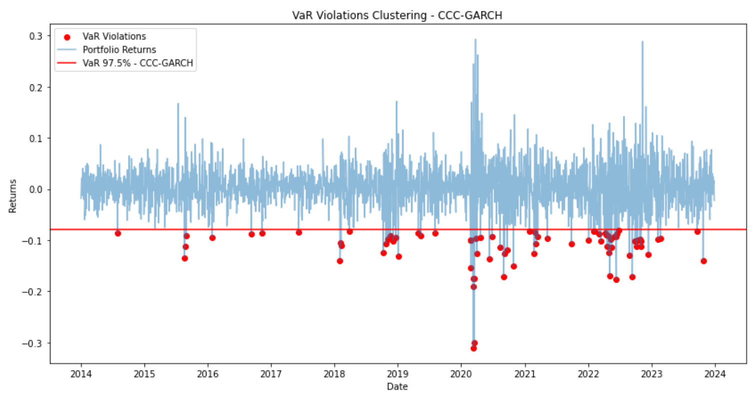



**Figure 20:** Clustering of VaR violations for CCC-GARCH(1,1) method.

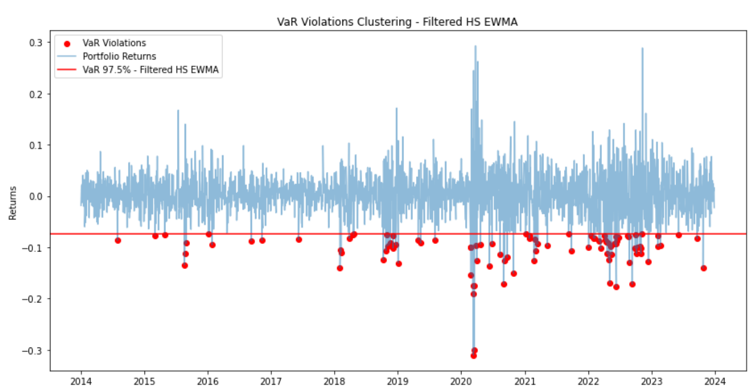



**Figure 21:** Clustering of VaR violations for EWMA method.

# 10 Results and Discussion

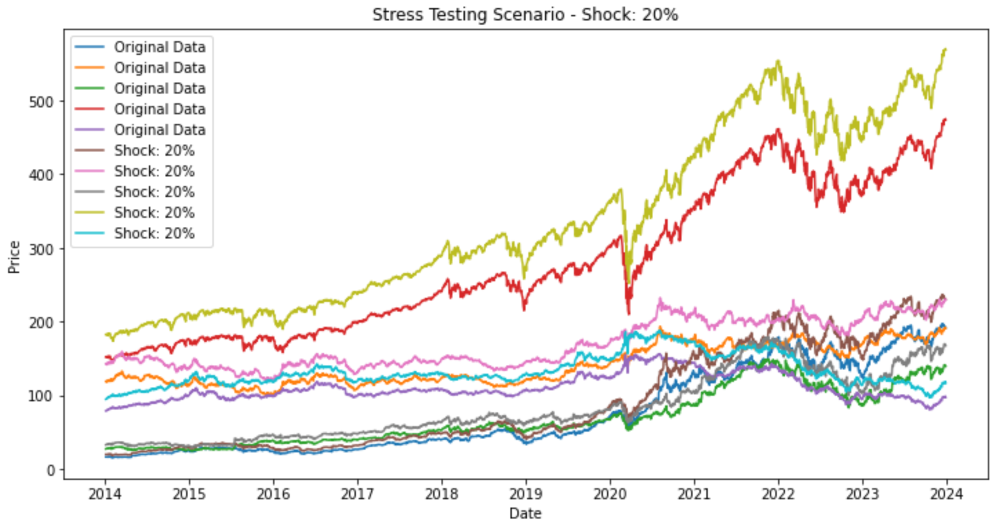
Table 4 summarizes the VaR and ES calculations for each method. The results show that methods accounting for fat tails (Student-t) and time-varying volatility (GARCH) provide more conservative VaR estimates. Backtesting results, detailed in Table 3, indicate that the Variance-Covariance method tends to underestimate risk, while the CCC-GARCH and Filtered HS EWMA methods offer better performance in capturing actual market risks.

**Table 4:** Summary of VaR and ES Calculations

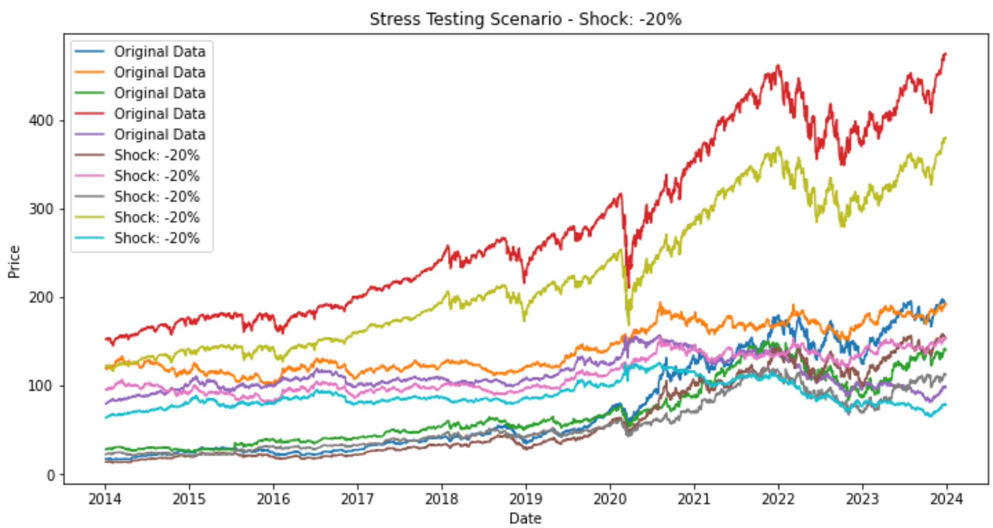
| Method | VaR 97.5% | VaR 99% | ES 97.5% | ES 99% |
|---|---|---|---|---|
| Variance-Covariance | 0.0830 | 0.0986 | 0.1180 | 0.1354 |
| Student-t (df=3) | 0.1110 | 0.1420 | 0.1562 | 0.1968 |
| Student-t (df=4) | 0.1060 | 0.1350 | 0.1490 | 0.1861 |
| Student-t (df=5) | 0.1010 | 0.1280 | 0.1410 | 0.1750 |
| Student-t (df=6) | 0.0960 | 0.1220 | 0.1340 | 0.1650 |
| Historical Simulation 5y | 0.1010 | 0.1370 | 0.1400 | 0.1870 |
| Historical Simulation 10y | 0.0900 | 0.1140 | 0.1240 | 0.1580 |
| CCC-GARCH | 0.0800 | 0.1000 | 0.1150 | 0.1380 |
| Filtered HS EWMA | 0.0730 | 0.0900 | 0.1080 | 0.1230 |

# 11 Stress Testing

Stress testing was conducted by applying equity shocks of ±20% and ±40% to the portfolio. The resulting stressed values are shown in the following figures. This analysis helps to understand the portfolio's vulnerability to extreme market movements. In the next set of plots, we introduce an interest rate shock of ±1% and ±3%. Finally, we introduce exchange rate shocks of ±5% and ±10%. Below are the results and how each asset is affected.

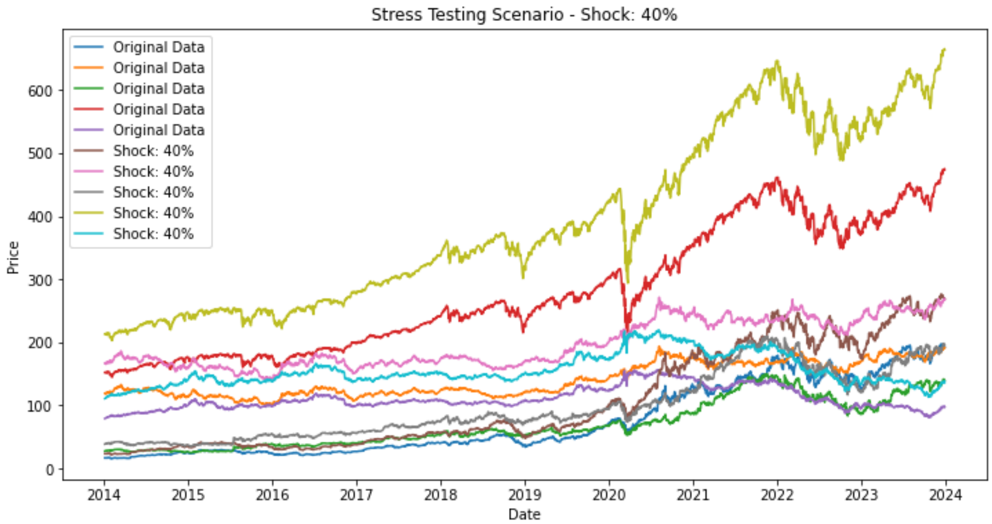



**(a)** Stress Testing Scenario +20%

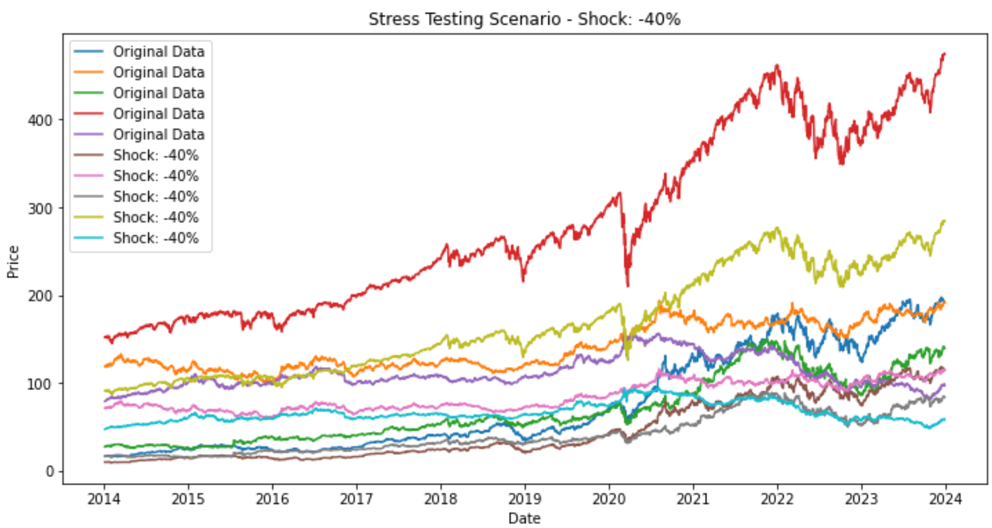



**(b)** Stress Testing Scenario -20%

**Figure 22:** Stress Testing Scenarios with +20% and -20% equity shocks




**(a)** Stress Testing Scenario +40%

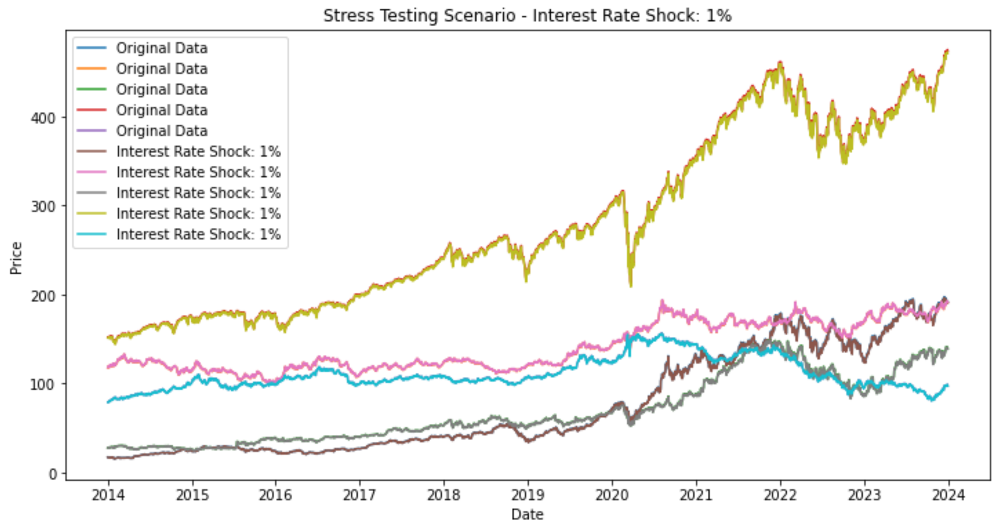



**(b)** Stress Testing Scenario -40%

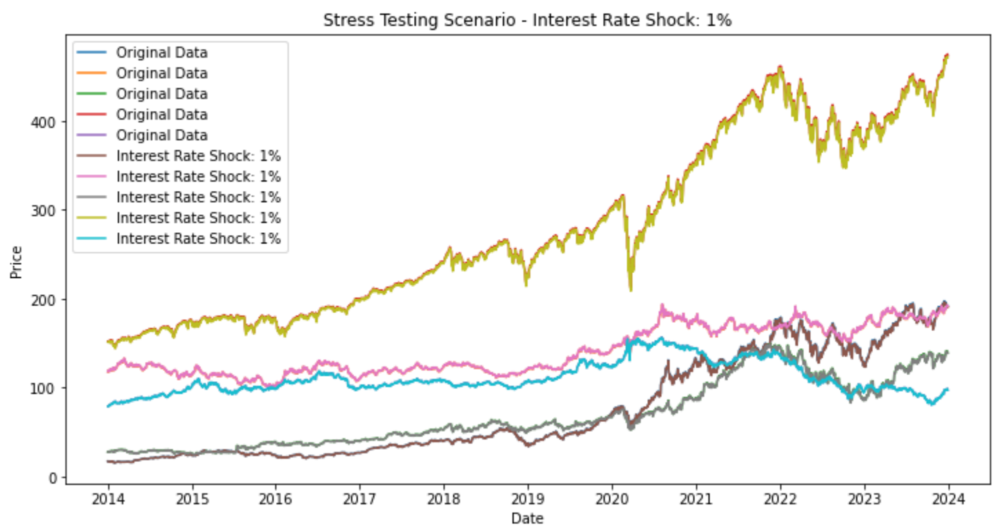
**Figure 23:** Stress Testing Scenarios with +40% and -40% equity shocks

# 12 Conclusion

This report evaluated various VaR methods, including Variance-Covariance, Student-t, Historical Simulation, CCC-GARCH, and Filtered Historical Simulation with EWMA. The results indicate that methods accounting for fat tails and time-varying volatility provide more conservative risk estimates. Backtesting demonstrated that the Variance-Covariance method tends to underestimate risk, while the CCC-GARCH and Filtered HS EWMA methods better capture actual market risks. Stress testing revealed the portfolio's sensitivity to extreme market movements. Overall, a combination of methods may offer the most comprehensive risk assessment for a diversified portfolio.

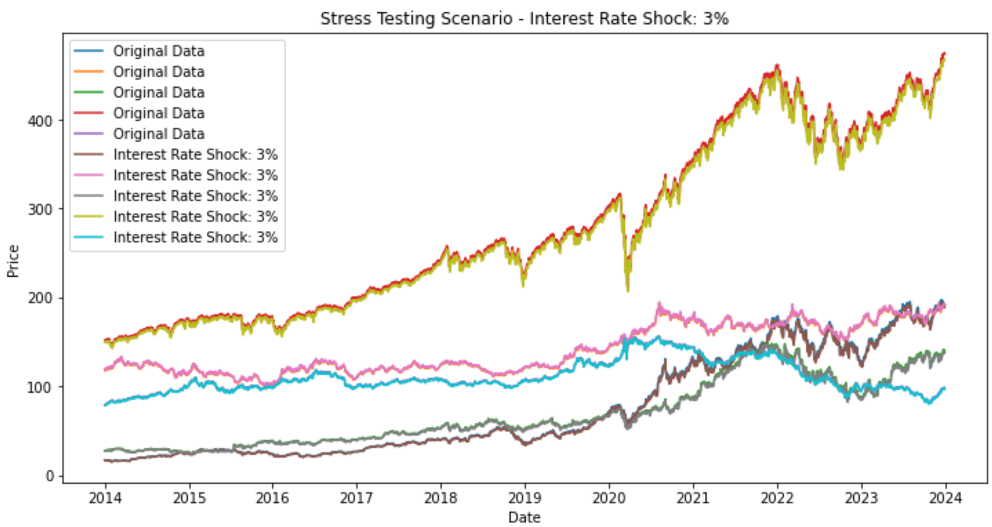



**(a)** Stress Testing Scenario +1%

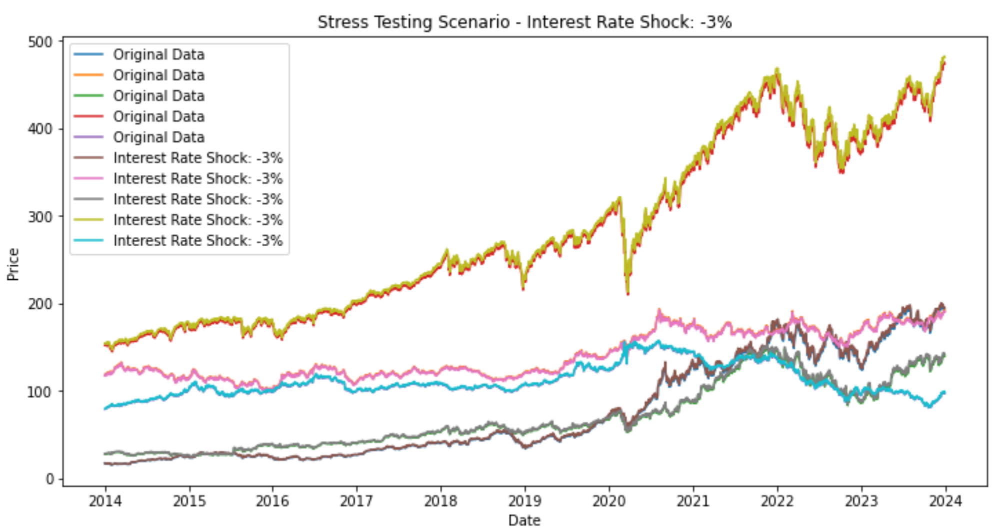



**(b)** Stress Testing Scenario -1%

**Figure 24:** Stress Testing Scenarios with +1% and -1% interest rate shocks

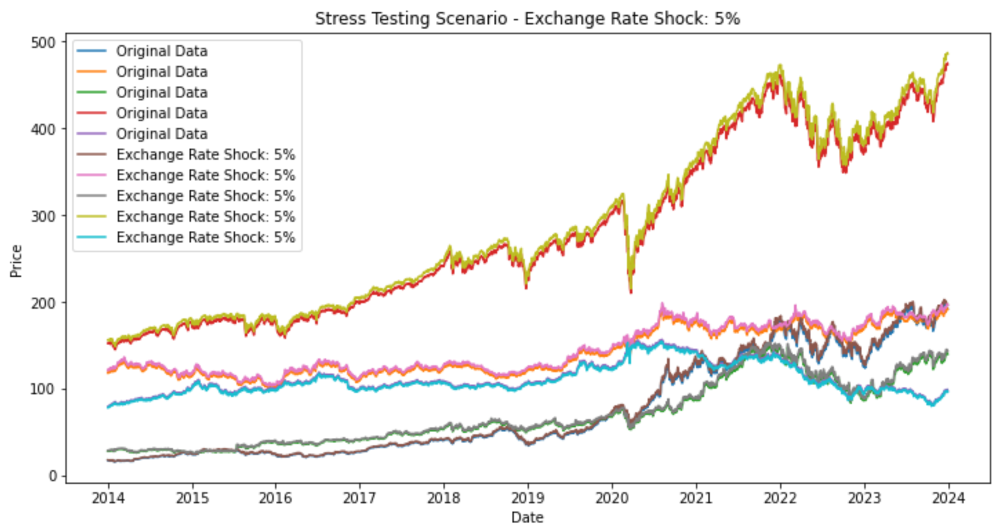



**(a)** Stress Testing Scenario +3%

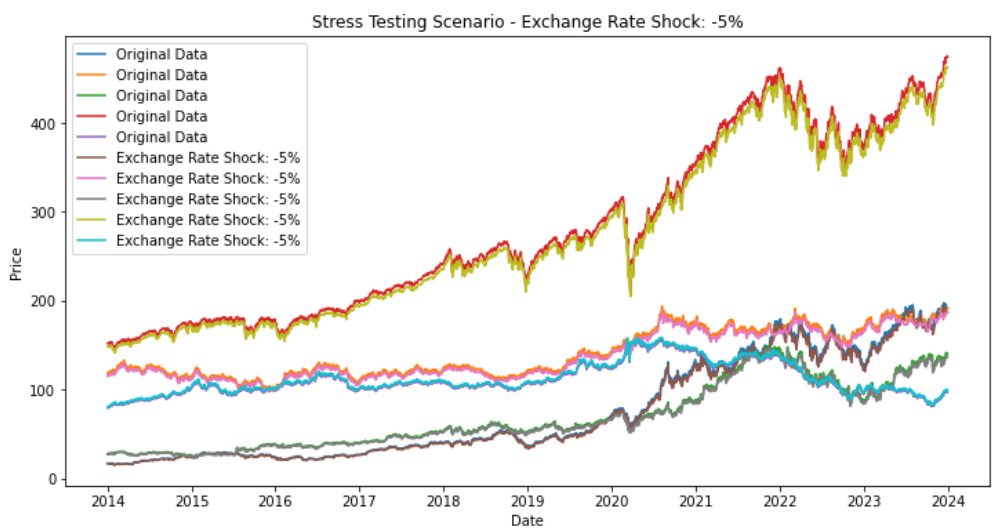



**(b)** Stress Testing Scenario -3%

**Figure 25:** Stress Testing Scenarios with +3% and -3% interest rate shocks

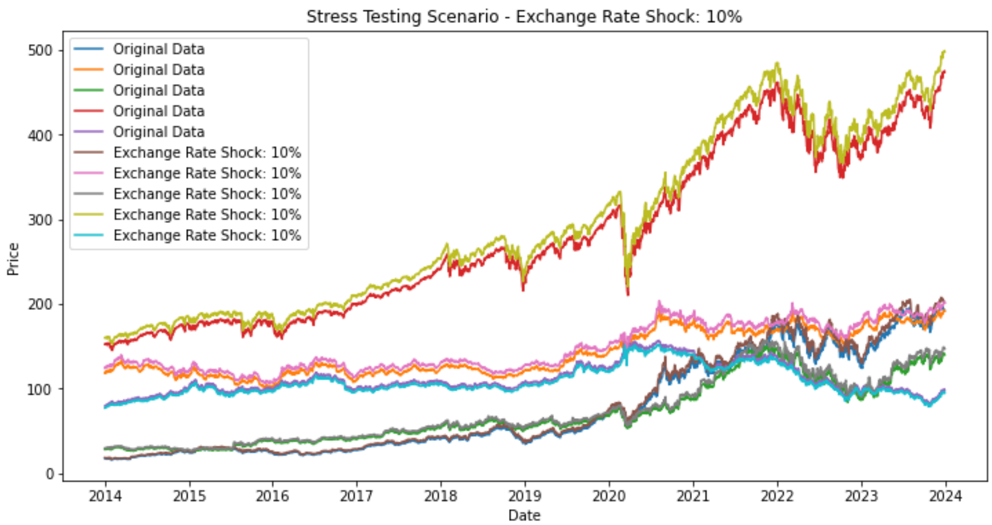



**(a)** Stress Testing Scenario +5%

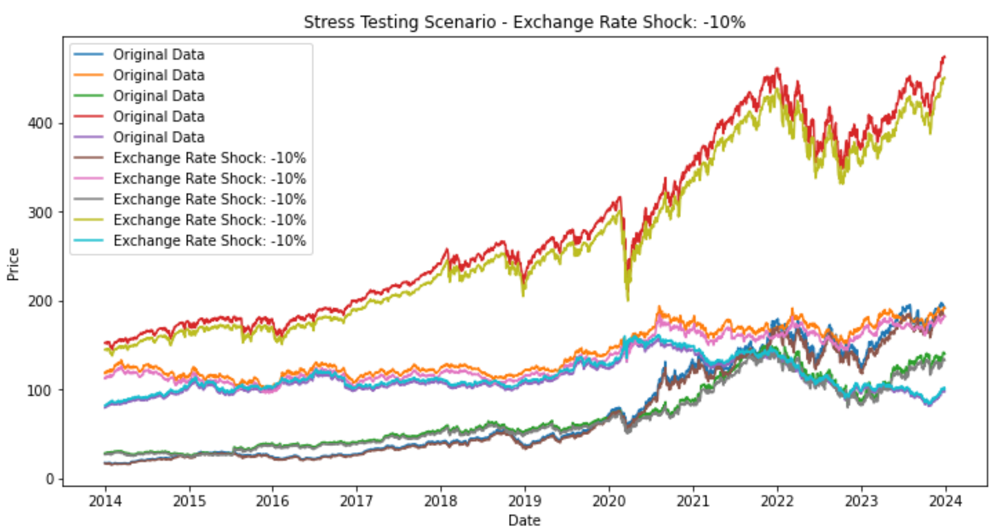



**(b)** Stress Testing Scenario -5%

**Figure 26:** Stress Testing Scenarios with +5% and -5% exchange rate shocks




**(a)** Stress Testing Scenario +10%




**(b)** Stress Testing Scenario -10%

**Figure 27:** Stress Testing Scenarios with +10% and -10% exchange rate shocks