Credit, Complexity and Systemic Risk
Case 1 (2024): Migration Default Risk

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The first case of the Credit Risk was one engaging into the Migration Default risk based on a paper from JP Morgan on this topic.

Initially we are given the one period migration and default transition matrix depicting the probability of risk migration from each grade ranging from AAA to D (default). Before we analyse the questions, we would like to go through the framework that proved essential for the calculation and computation of the questions that follow which is common for both of them.

The first step was to compute the bins that determine the range of each grade . Using the one factor model as given by

$$X_i = \sqrt{\rho}\, Y + \sqrt{1-\rho}\, \varepsilon_i$$

Where ρ denotes the fraction of covariance between our factors. Y is the systemic factor and ε is the idiosyncratic factor whose importance and special nature will be discussed in question 2 further down. After the computations we get the following table

Table 1: The probabilities of migration and the migration bins

Rating	AAA	AA	A	BBB	BB	B	CCC	D (default)
AAA	91.115%	8.179%	0.607%	0.072%	0.024%	0.003%	0.000%	0.000%
AA	0.844%	89.626%	8.954%	0.437%	0.064%	0.036%	0.018%	0.021%
A	0.055%	2.595%	91.138%	5.509%	0.499%	0.107%	0.045%	0.052%
BBB	0.031%	0.147%	4.289%	90.584%	3.898%	0.708%	0.175%	0.168%
BB	0.007%	0.044%	0.446%	6.741%	83.274%	7.667%	0.895%	0.926%
B	0.008%	0.031%	0.150%	0.490%	5.373%	82.531%	7.894%	3.523%
CCC	0.000%	0.015%	0.023%	0.091%	0.388%	7.630%	83.035%	8.818%

Rating	AAA		AA		A		BBB		BB		B		CCC		D	
AAA	-1.348	inf	-2.454	-1.348	-3.093	-2.454	-3.460	-3.093	-4.013	-3.460	-inf	-4.013	-inf	-inf	-inf	-inf
AA	2.389	inf	-1.309	2.389	-2.527	-1.309	-2.991	-2.527	-3.175	-2.991	-3.360	-3.175	-3.527	-3.360	-inf	-3.527
A	3.264	inf	1.935	3.264	-1.537	1.935	-2.456	-1.537	-2.872	-2.456	-3.099	-2.872	-3.279	-3.099	-inf	-3.279
BBB	3.423	inf	2.915	3.423	1.699	2.915	-1.650	1.699	-2.308	-1.650	-2.704	-2.308	-2.933	-2.704	-inf	-2.933
BB	3.808	inf	3.285	3.808	2.578	3.285	1.707	2.578	-1.159	1.707	-1.678	-1.159	-1.778	-1.678	-inf	-1.778
B	3.775	inf	3.360	3.775	2.896	3.360	2.468	2.896	1.445	2.468	-1.279	1.445	-2.023	-1.279	-inf	-2.023
CCC	inf	inf	3.615	inf	3.367	3.615	3.014	3.367	2.564	3.014	1.395	2.564	-1.352	1.395	-inf	-1.352

After computing the bins, we can now utilise our one factor model to deduct the X value that will in turn provide the information on weather or not there was a migration of risk to another grade in that one time period step. To do this we calculate the X value and we check into which of the bins it is contained numeric wise. For example, a value of X = 1 for an initial AAA grade will fall into the AAA bin of that row and thus would not migrate. We run 500000 scenarios for each migration step and used this to deduct the portfolio expected value as follows:

Question 1)

For question 1 we have 1 single issuer meaning that we can adjust the weights of our portfolios as given without needing to control for the difference in idiosyncratic error that different issuers would impose. Thus, after computing our migration scenarios, we estimated the expected portfolio value as follows:

$$E_i = 1500 * w_i * \sum p_k * v_k * \ (in\ millions) \begin{cases} p_k\ denotes\ the\ chance\ of\ a\ ceratin\ migration\ grade \\ w_i\ denotes\ the\ weight\ of\ the\ current\ initial\ grade \\ v_k\ denotes\ the\ value\ (shocked\ or\ base) of\ the\ grade \end{cases}$$

So p_k essentially denotes the probability that our grade will migrate to another certain grade or remain unchanged when v_k denotes the value of that grade taken as shock value if there was a migration event or taken on the base values if no migration happened.

Of course this was an example of 10 scenarios while in our code we run half a million.

Initial_State	Corresponding_State
BBB	B
BBB	A
BBB	BB
BBB	BBB
BBB	BBB
BBB	BBB
BBB	BBB
BBB	D
BBB	BBB
BBB	BBB

For example, on the 10-prediction scenario on the left

The chance of migrating to B from BBB is 1 /10 same as for BB, A and D while the chance to not migrate is 6/10

Meaning that the p_k would be those values respectively and the v_k would be the base values for the BBB -> BBB cases and the shock values for the other cases with an actual migration.

The Ei denotes the current starting grade because our portfolio 1 had different weights for different starting allocation grades. The total portfolio value will be

$$E = \sum_{i=1}^{3} E_i$$

Table 2: The results for the first case

Single Issuer per rating	Rho	Expected Value	90% VaR	99.5% VaR	90% ES	99.5% ES
Portfolio I	0%	134487.84	134607.00	135710.54	134643.53	135744.49
Portfolio I	33%	134471.25	134591.64	135695.05	134627.93	135729.22
Portfolio I	66%	134441.08	134562.11	135665.28	134598.86	135698.51
Portfolio I	100%	134437.70	134557.68	135660.81	134594.06	135693.80
Portfolio II	0%	108989.57	109846.47	118909.25	110225.39	119896.29
Portfolio II	33%	108929.97	109803.53	115296.91	110179.62	119659.01
Portfolio II	66%	108911.18	109773.67	115126.76	110150.48	118462.90
Portfolio II	100%	108839.24	109710.44	115199.10	110086.62	119544.77

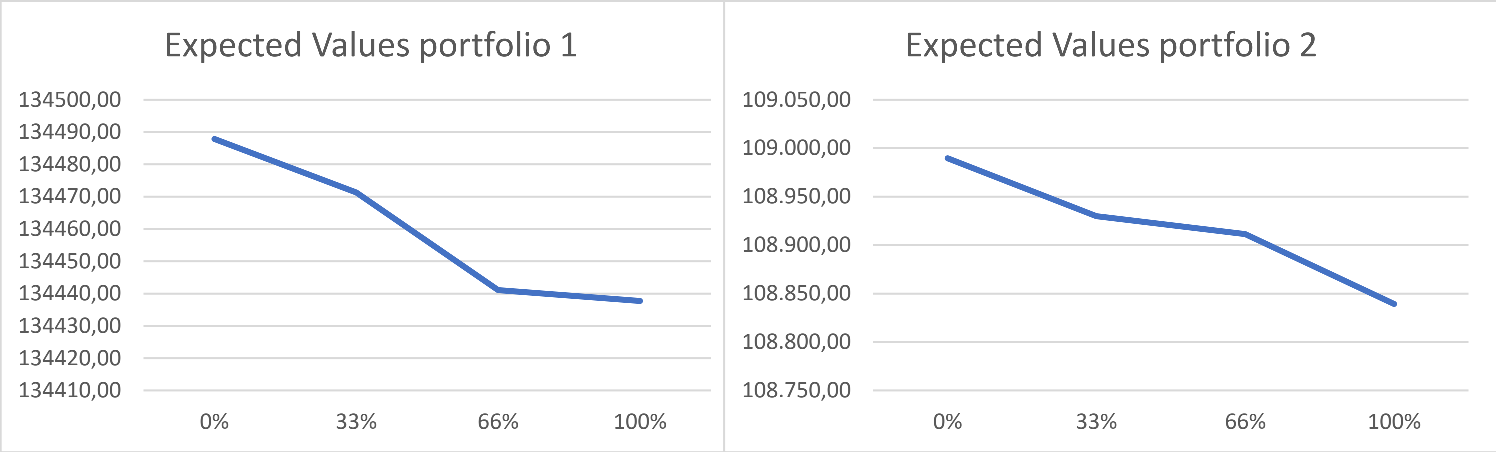

Figure 1: Graphical Representation of Expected value against different ρ values

Question 2)

Following an identical path as question one we computed the case for the second question. This time however the allocation of the internal weight for each of the starting grade positions was allocated and computed differently. With 100 different issuers even though our systemic factor Y remains unchanged the idiosyncratic factor ε changes amongst them. Thus, we generated a new factor ε 100 times applying to it 1/100 of the individual weight of the total starting grade allocation.

We then got the mean of those ε and derived one common ε factor representing the mean idiosyncratic term for all issuers and computed the equation as in 1.

Table 3: The results for the second case

Single Issuer per rating	Rho	Expected Value	90% VaR	99.5% VaR	90% ES	99.5% ES
Portfolio I	0%	134413.77	134534.65	135637.59	134571.16	135670.52
Portfolio I	33%	134330.03	134451.07	135553.33	1344878.97	135584.98
Portfolio I	66%	134434.88	134554.14	135657.24	134590.78	135690.51
Portfolio I	100%	134411.20	134531.10	135634.02	134567.56	135668.09
Portfolio II	0%	108944.29	109811.61	115305.97	110187.49	119658.64
Portfolio II	33%	108934.53	109806.51	118865.78	110182.15	119831.41
Portfolio II	66%	108998.31	109856.31	115352.69	110231.61	119690.06
Portfolio II	100%	109061.36	109926.69	115425.96	110299.69	119851.98

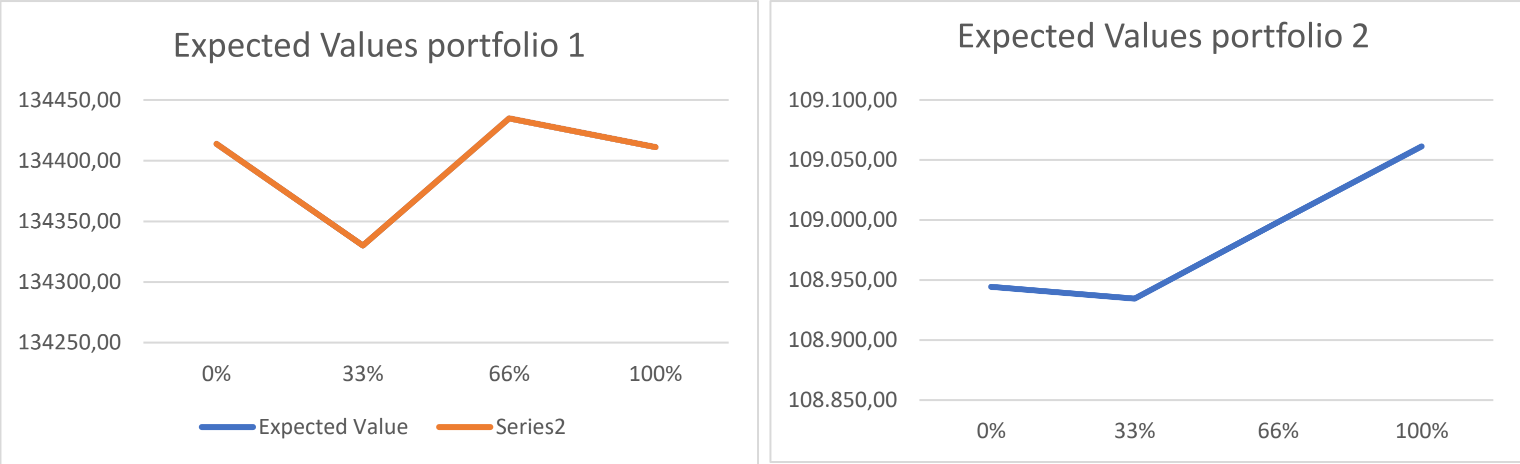

Figure 2: Graphical Representation of Expected value against different ρ values

Question 3)

a) In both cases the results between portfolio 1 and 2 are in agreement and the denote a much higher expected value for our portfolio when we start from higher grades of allocation compared the “junk” portfolio II. As however our expected value is smaller so is our Value at Risk and our Expected shortfall.

b) We can see from the provided graphs that in general the expected value and the other metrics tend to decrease as the p factor increases when we have a single issuer. The case changes drastically however when we have multiple (100) issuers as the increase in ρ in this case causes an increase in the expected value of our portfolio. An irregularity is observed in portfolio 1 of the second question were the change in ρ causes a fluctuation on the behaviour of the curve instead of monotone rise or fall. The second portfolio also shows a change of direction with a minimum at 33% and a rise after that.

c) In general, the (one factor) Merton model assumes a continuous distribution and most often, a binomial distribution. This does not be the case, as other underlying distributions can be used. If we would like to tailor our model to accommodate more extreme losses, then a student-t distribution would be more appropriate. This would allow our model to accommodate for heavier tails in the extreme cases of the distribution.