Mathematics/Statistics Bootcamp Quiz

1. For what values of $p > 0$ does the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge?

   A $p \in (0, 1)$ B $p \in (1, \infty)$ C $p \in [1, \infty)$ D $p \in (0, 1]$

2. $f(x)$ is a convex function on $\mathbb{R}$, $P = (x_0, f(x_0))$ is a point on $f(x)$, and l is the tangent line of the curve of $f(x)$ at P. Choose the correct statement(s).

   A l must lie above or on the curve of $f(x)$;

   B l must lie below or on the curve of $f(x)$;

   C l can intersect with $f(x)$ at multiple points.

3. Let $c(x_1, x_2, \ldots, x_n)$ be a function of $x_1, x_2, \ldots, x_n$ of which all the partial derivatives exist. Let $g(x_1, x_2, \ldots, x_n) = c(x_1, x_2, \ldots, x_n) \exp(\sum_{i=1}^{n} w_i x_i)$ where $w_1, w_2, \ldots, w_n$ are all constant numbers. Write down the gradient vector of g.

4. If $X = \begin{pmatrix} -2 & 2 \\ -1 & -1 \end{pmatrix}$ and $Y = \begin{pmatrix} 3 & -8 \\ 4 & -1 \end{pmatrix}$, does $XY = YX$?

5. If X is an $n \times n$ projection matrix, show that $I_n - X$ is also a projection matrix.

6. Find the centering matrix C such that the linear transformation CX subtracts the column means of X.

7. Suppose that the random variable X follows a Poisson distribution with mean λ. Suppose that the random variable Y follows a binomial distribution, with number of trials equal to X and probability of success p.

   A Find E[Y].

   B Find V[Y].

8. Male verbal GRE scores are normally distributed, with a mean of 149 and a standard deviation of 9. Female verbal GRE scores are also normally distributed, with a mean of 149 and a standard deviation of 8. 55% of the students who take the GRE are female. What is the probability that a randomly chosen student is female, given that his/her verbal GRE score was 170?

9. Suppose that the random variable X is gamma distributed, with probability density function

$$f(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}.$$

   What is the probability density function of $1/X$? (This is the pdf of the inverse-gamma distribution).

10. One observation is taken on a discrete random variable X with pmf $f(x|\theta)$ where $\theta \in \{1, 2, 3\}$ (see the table below for the values of $f(x|\theta)$). Find the MLE of θ (write down the MLE as a function of the observation x).

x	$f(x\|1)$	$f(x\|2)$	$f(x\|3)$
0	$\frac{1}{3}$	$\frac{1}{4}$	0
1	$\frac{1}{3}$	$\frac{1}{4}$	0
2	0	$\frac{1}{8}$	$\frac{1}{4}$
3	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$
4	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{4}$

11. Let $X_1, X_2, \ldots, X_n$ be i.i.d. samples from $N(\mu, \sigma^2)$ with both μ and σ^2 both unknown. Write down a 95% confidence interval of μ.

12. Which of the following statements about p-values is(are) **incorrect**? (Let H_0 and H_1 denote the null hypothesis and alternative hypothesis.)

    A The p-value is a summary statistic of the observed data;

    B p-values describe how "extreme" H_1 is given the data;

    C smaller p-values suggest stronger evidence that H_1 is true;

    D If the p-value is smaller than 0.05, we should reject H_0.