Module 2 Assignment – The LP Model

Stephen Tweel

9/7/25

1) Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

   a. <u>Clearly define the decision variables</u>
      a. The decision variables are trying to identify what quantity of each kind of backpack, the Collegiate and the Mini, the company should make per week.
         c = the number of Collegiate bags to produce per week
         m = the number of Mini bags to produce per week
   b. <u>What is the objective function?</u>
      a. The objective function is to maximize profit based on the decision variables. The function will look like:
         Maximize Z = 32c + 24m
   c. <u>What are the constraints?</u>
      a. There are several constraints to consider:
         First, there is a maximum number of each backpack that forecasts say consumers will purchase is 1000 Collegiates, and 1200 Minis, so making more than these will not increase profits.
         c <= 1000
         m <= 1200

Next, Back Savers receives 5000 sq. ft. of nylon per week, so we cannot exceed 5000 sq. ft of material needed for the number of backpacks we produce.

$3c + 2m <= 5000$

Also, there are time and labor requirements. There are 35 laborers working 40 hrs per week, or 1400 total labor hours per week (84000 minutes), to produce these backpacks, with the Collegiate taking 45 minutes to produce, and the Mini taking 40. We can't exceed their working hours to produce more backpacks, or hire more workers at the moment.

$45c + 40m <= 84000$

One last constraint is the negativity constraint – we can't make negative numbers of backpacks, but a computer doesn't know that until you tell it:
$c >= 0$
$m >= 0$

d. <u>Write down the full mathematical formulation for this LP problem.</u>
Maximize $Z = 32c + 24m$ (maximize profits)

$0 <= c <= 1000$ (max forecasted demand for this item)
$0 <= m <= 1200$ (max forecasted demand for this item)

$3c + 2m <= 5000$ (max material available to make the backpacks)

$45c + 40m <= 84000$ (max time and labor available to make the backpacks)

2) The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of

available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. <u>Define the decision variables</u>
   The decision variables are trying to maximize the number of Large, Medium, and Small units produced, within capacity, storage, demand, and equal usage constraints.

   Maximize Z = 420(L1 + L2 + L3) + 360(M1 + M2 + M3) + 300(S1 + S2 + S3)
   This makes sure we are accounting for the number of units that can be made at each plant are accounted for separately but with similar variable names.

b. <u>Formulate a linear programming model for this problem.</u>

Maximize Z = 420(L1 + L2 + L3) + 360(M1 + M2 + M3) + 300(S1 + S2 + S3)

Constraints: We cannot make more than the excess capacity of each plant for each item.

Capacity: L1 + M1 + S1 <= 750 (plant 1)
L2 + M2 + S2 <= 900 (plant 2)
L3 + M3 + S3 <= 450 (plant 3)

Daily Max Storage: Even if we can make more, we cannot store more items than we can store in a day, we have nowhere else to put them.

20L1 + 15M1 + 12S1 <= 13000 (plant 1)
20L2 + 15M2 + 12S2 <= 12000 (plant 2)
20L3 + 15M3 + 12S3 <= 5000 (plant 3)

Forecast Demand: If we make more than the forecasted demand, no one will buy the product, so we won't increase profits.

$L1 + L2 + L3 <= 900$	large item demand
$M1 + M2 + M3 <= 1200$	medium item demand
$S1 + S2 + S3 <= 750$	small item demand

Same Percentage: (using p as a variable for percentage of free capacity)

$750 * p = L1 + M1 + S1$	(plant 1 capacity %)
$900 * p = L2 + M2 + S2$	(plant 2 capacity %)
$450 * p = L3 + M3 + S3$	(plant 3 capacity %)

Can't produce negative items:

$L1 >= 0$
$L2 >= 0$
$L3 >= 0$
$M1 >= 0$
$M2 >= 0$
$M3 >= 0$
$S1 >= 0$
$S2 >= 0$
$S3 >= 0$