Theory Assignment I

Automata Theory Monsoon 2023, IIIT Hyderabad

August 10, 2023

Total Marks: 35 points Due date: **24/08/23 11:59 pm**

General Instructions: All symbols have the usual meanings (example: $\mathbb{R}$ is the set of reals, $\mathbb{N}$ the set of natural numbers, and so on). FSM stands for finite state machine. DFA stands for deterministic finite automata. NFA stands for non-deterministic finite automata. a^* is the Kleene Star operation.

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1. [2 points] Prove that if a language L has a finite number of strings, then it is regular. **[CO-1,CO-3]**
2. [3 points] Consider a language L over the alphabet $\Sigma = \{0, 1\}$ defined as follows:

   $$L = \{w \mid \text{ There exists atleast one length three substring of } w \text{ which is divisible by 3 } \}.$$

   Prove that L is a regular language by drawing a DFA for it. **[CO-1,CO-2]**

   Example : L will accept string 1101 as it has 110 as a substring whose binary representation corresponds to 6 in decimal which is divisible by 3. L will not accept the string 1010 as its substrings of length 3, 101 (5 in decimal), and 010 (2 in decimal), are not divisible by 3.
3. [4 points] Let L be the language over the alphabet $\{a, b\}$ having an a on the n^{th} position from the end. For example, if $n = 2$ then the string *bbaa* will be accepted, where as *bbba* will not be accepted. Your answers must be in terms of n.

   (a) Give a regular expression that describes this language.

   (b) Construct an NFA for L such that it contains as few states as possible. How many states does such an NFA have (in terms of n)? Explain why a smaller NFA cannot be designed.

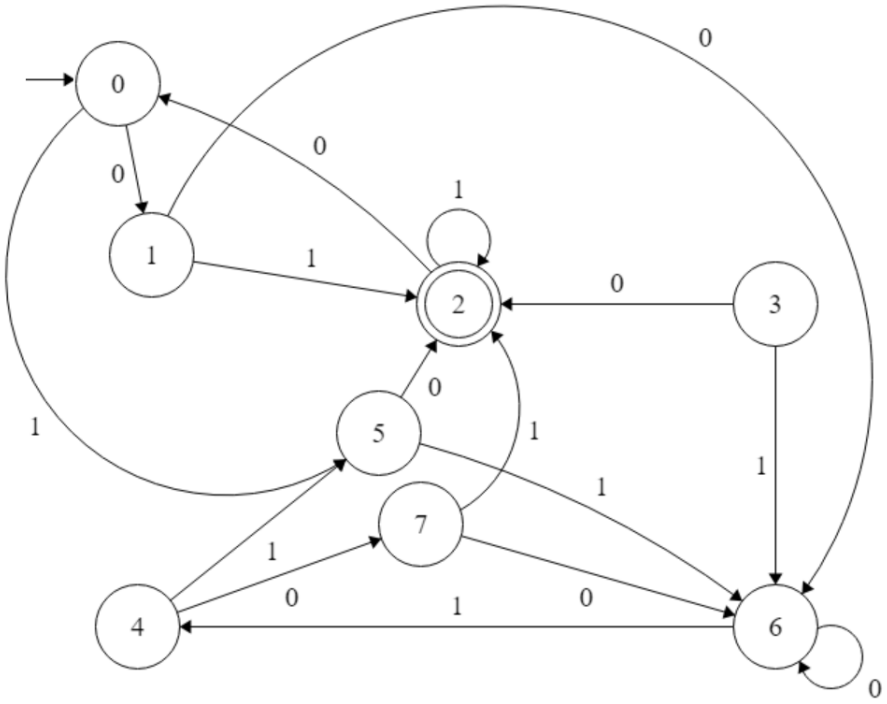
   [CO-1,CO-2,CO-3]
4. [1 point] Write a regular expression for your IIIT email address
   (It should accept student and faculty email addresses, e.g. *shchakra@iiit.ac.in*, *zeeshan.ahmed@research.iiit.ac.in* are valid email addresses) **[CO-1, CO-2]**
5. [4 points] Provide an algorithm for converting regular expression into right linear grammar
   [CO-1, CO-2]
6. [4 points] Prove that the languages A and L are not regular

   - $A = \{\text{bits}(n) \mid n \text{ is a perfect square}\}$. where bits(n) is the binary string representation of the number n. E.g., $bits(6) = 110$.

(b) $L = \{a^p \mid p \text{ is prime}\}$.

[CO-2, CO-3, CO-4]

7. [2 points] Minimize the following DFA: **[CO-1,CO-2]**



8. [3 points] For a symbol a, define $a^+ = \{a, aa, ...\}$. Is the language $L = \{wcw^R | w, c \in \{a, b\}^+\}$ regular? If your answer is yes, write the equivalent regular expression and if no, prove that L is not regular using pumping lemma. **[CO-1, CO-2, CO-3,CO-4]**

9. [3 points] Prove that the following grammars are ambiguous by providing a string with two different leftmost/rightmost derivations, or two different parse trees.

   a) $S \to Sa \mid aS \mid a$

   b) $S \to SS \mid a \mid b$

   c) $S \to S + S \mid S \times S \mid x$

   [CO-2,CO-3]

10. [5 points] Let $\Sigma = \{0, 1\}$. Consider the language $L = \{0^n 1^m | n \leq m \leq 2n\}$. Is L regular or context-free? If you think it is regular, write the regular expression for it and draw the corresponding DFA/NFA. If you think it is context-free, write the grammar rules and draw the corresponding PDA. **[CO-1, CO-2, CO-3, CO-4]**

11. [4 points] Prove that the following language over $\Sigma = \{0, 1\}$ is regular by constructing a DFA

$$L = \{\ w \mid w \equiv 1 \bmod 2 \text{ and } w \equiv 2 \bmod 3\}$$