1. In an exchange economy, two consumers, Alan and Beck have utility functions $U^A(X,Y) = X^2 + 2XY + Y^2$, and $U^B(X,Y) = \ln X + 2\ln Y$, respectively. Alan is endowed with 3 units of good X and 3 units of good Y, while Beck is endowed with 15 units of X and 15 units of Y.

   (a) Draw the contract curve in the Edgeworth box. (2 points)

   (b) Solve the general equilibrium, and clearly state the equilibrium price and allocations. (3 points)

Answer:

(a) Note

- When Alan consumers both consumption goods, it is true that

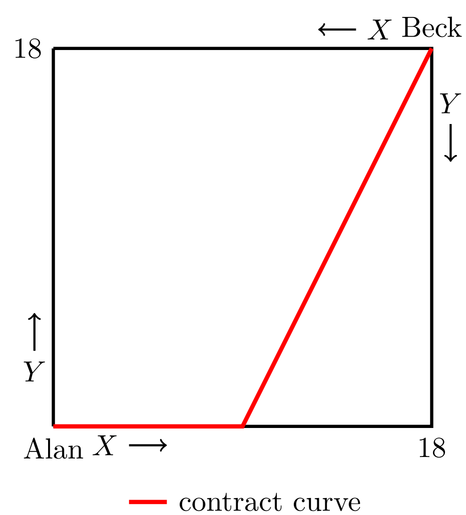
$$MRS^A_{x,y} = 1 = MRS^B_{x,y} = \frac{Y_B}{2X_B} \Longrightarrow$$

$$2X_B = Y_B.$$

  This happens only when $X_B \leq 9$.

- When $X_B > 9$, $Y_B = 18$ and $MRS^B < 1$, and so Alan consumes only Y.

Hence, the contract curve



(b) Given equilibrium price (P_x, P_y), the income of the two consumers are respectively,

$$m_A = 3P_x + 3P_y, \qquad m_B = 15P_x + 15P_y.$$

We can solve Beck's problem to get

$$X_B = \frac{m_B}{3P_x} = 5 + \frac{5P_y}{P_x}, \qquad Y_B = \frac{2m_B}{3P_y} = \frac{10P_x}{P_y} + 10.$$

Let $P_x = 1$. Alan's optimal consumption includes both goods when $P_y = 1$. This is impossible since $P_y = 1$ leads to $Y_B = 20$. So we conclude the only possibility is $P_y > 1$, in which case Alan consumes only X.

Note $Y_B = 18$ only when $P_y = \frac{5}{4}$. Hence, the competitive equilibrium:

$$P_x = 1, \qquad P_y = \frac{5}{4};$$

$$X_A = \frac{27}{4}, \quad Y_A = 0; \qquad X_B = \frac{45}{4}, \quad Y_B = 18.$$