Collateral, Asset Prices, and Risk Management

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Kiyotaki-Moore (1997)

Credit Cycles

- Kiyotaki and Moore (1997) is the seminal paper on incorporating financial frictions via the so-called "limited enforcement" approach
- Basic idea:
    - Debt contracts are not perfectly enforceable - if the borrower defaults, the lender cannot force the borrower to continue to work, have negative consumption, etc.
    - The lender may be able to recover some of the borrower's assets, but because of limited enforceability, these assets are worth less to the lender than to the borrower
    - The lender will restrict the amount of credit a borrower can access
    - So we have a collateral constraint - amount borrower can borrow is a function of value of its assets

Model

- Two groups of agents, risk-neutral, infinite horizon
- capital stock k_t with endogenous price q_t; and consumption good
- **Farmers** are productive agents with unit mass
    - output $y_{t+1} = (a + c)k_t$
    - a is tradable output, c is only for farmer's consumption
    - discount rate $\beta < 1$
- **Gatherers** with unit mass, they are less productive in using k
    - $y_{t+1} = G(k_t^G)$
    - discount rate $\beta^G > \beta$
- Fixed aggregate supply of capital $\bar{K}$, hence in equilibrium $k_t + k_t^G = \bar{K}$
- Riskless one-period bond with zero net supply, at price R_t

Credit market

- Farmers will want to borrow in a steady state
  - because they are impatient and more productive
  - if they are patient, they will save to eliminate borrowing constraint in a steady state
- Debt b_t is one period of riskfree debt collateralized by capital
- **Key friction**: farmer can walk away from any debt but lender can seize the collateral
- Collateral constraint

$$R_t b_t \leq q_{t+1} k_t$$

- Farmers will want to borrow more but they are constrained
- gatherers are lenders on the margin, and their preferences pin down the interest rate

$$R_t = R = 1/\beta^G$$

Demand for credit and capital

- Farmers' intertemporal budget equation

$$ak_{t-1} + q_t k_{t-1} + b_t = q_t k_t + Rb_{t-1}$$

- Farmers will max out their borrowing constraint so the collateral constraint binds

$$Rb_t = q_{t+1} k_t$$

- Then farmer's period- t demand for capital

$$k_t = \frac{(a + q_t)\, k_{t-1} - Rb_{t-1}}{q_t - q_{t+1}/R}$$

  - Each unit of capital has an effective price of $q_t - q_{t+1}/R$, and the farmer has net worth of $(a + q_t)\, k_{t-1} - Rb_{t-1}$
  - Why $q_t - q_{t+1}/R$? Capital has a price of q_t, but he can borrow $b_t = q_{t+1}/R$

Gatherer's demand for capital and equilibrium prices

- Gatherers are not credit constrained $\Rightarrow$ they determine capital prices!
- Their demand for k_t^G is given by FOC

$$\beta^G \left[G'\left(k_t^G\right) + q_{t+1}\right] = \left[G'\left(k_t^G\right) + q_{t+1}\right]/R = q_t$$

  - $G'\left(k_t^G\right)$ gives marginal output. Assumption: $G'' < 0 : G'\left(k_t^G\right)$ is higher for lower k_t^G
  - $q_t - q_{t+1}/R$ so-called user cost of capital
- Market clearing for capital $k_t + k_t^G = \bar{K}$ implying

$$\frac{1}{R} G'\left(\bar{K} - k_t\right) = q_t - \frac{q_{t+1}}{R} \Rightarrow q_t = \sum_{s=0}^{\infty} R^{-s} \left(\frac{1}{R} G'\left(\bar{K} - k_{t+s}\right)\right)$$

- If $k_{t+s} \uparrow$ for all s, i.e. productive farmers have more capital $\Rightarrow G' \uparrow \Rightarrow$ gatherers demand of capital $\uparrow \Rightarrow q_t \uparrow$

Steady state equilibrium

- Borrowing is determined by the collateral constraint

$$b^* = \frac{q^* k^*}{R}$$

- Farmer's demand is determined by their budget constraint

$$k^* = \frac{(a + q^*)\, k^* - Rb^*}{q^*(1 - 1/R)} \Rightarrow q^* = \frac{aR}{R - 1}$$

- Use gatherer's demand schedule to solve for steady state capital level k^*

$$\frac{1}{R} G'\left(\bar{K} - k^*\right) = q^* - \frac{q^*}{R} = a$$

- Frictionless economy benchmark k^{FB} solves

$$\max_k (a + c)k + G(\bar{K} - k) \Rightarrow a + c = G'\left(\bar{K} - k^{FB}\right)$$

  - Difference: c is not tradable/pledgeable

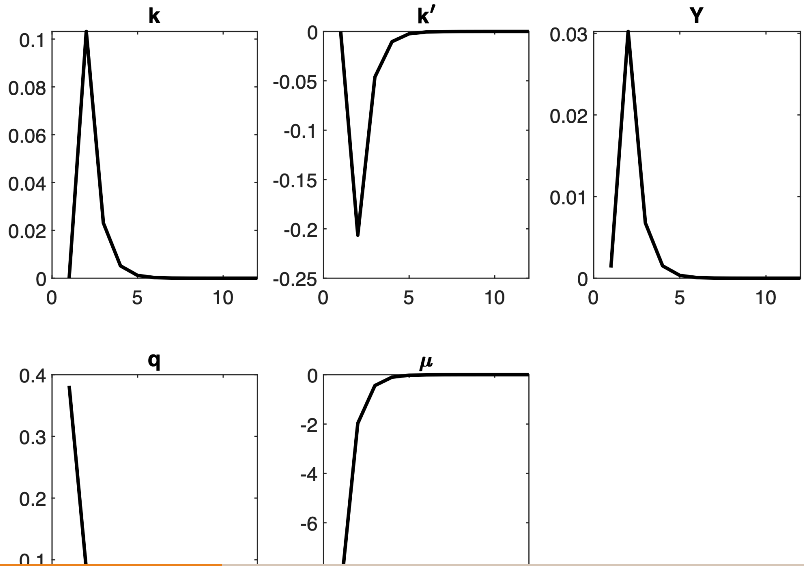
Impluse responses

- Consider unexpected one-time shock that increases production by δ percent, i.e., a becomes $a(1+\delta)$
- Specifically, $k_{t-1} = k^*$, $b_{t-1} = b^*$. At the beginning of t, realized output $a(1+\delta)$. Need to solve for $\{q_{t+s}, k_{t+s}\}$ for $s \geq 0$
- Recall we have derived

$$k_t = \frac{(\tilde{a} + q_t)\, k_{t-1} - Rb_{t-1}}{q_t - q_{t+1}/R} = \frac{(\tilde{a} + q_t)\, k_{t-1} - Rb_{t-1}}{G'\left(\bar{K} - k_t\right)/R}$$

- $\underbrace{\frac{1}{R} G'\left(\bar{K} - k_t\right) k_t}_{\text{quantity effect}} = a(1+\delta)k^* + \underbrace{\left(q_t - q^*\right) k^*}_{\text{collateral price effect}}$
- Ignore the price feedback effect, i.e., hold $q_{t+s} = q^*$ always. There is long-lasting effect through "quantity effect"
  - $k_t \uparrow$ by IRS $\eta \Rightarrow t+1$ output $ak_t \uparrow$ by $\eta \Rightarrow k_{t+1} \uparrow$ by $\eta^2 \ldots$
- Collateral price effect can be much larger. $q_t = \sum_{s=0}^{\infty} R^{-s}\left(\frac{1}{R} G'\left(\bar{K} - k_{t+s}\right)\right)$ so its IRS is about $1/\eta R$
  - Today's higher q_t allows for more borrowing $k_t \uparrow$, so on so forth

Impulse Response



Rampini and Viswanathan (2010)

Model

- Environment
  - Discrete time, infinite horizon
  - investor/ owner
- Owner/borrower ("firm", "entrepreneur")
  - Preferences: risk neutral, impatient $\beta < R^{-1}$, subject to limited liability
  - Endowment: the borrower has limited funds $w > 0$
- Investor has deep pockets
- Technology
  - Capital k invested in current period
  - Payoff ("cash flow") next period $Af(k)$
  - Strict concavity $f_k(k) > 0$ and $f_{kk}(k) < 0$; also: $\lim_{k \to 0} f_k(k) = +\infty; \lim_{k \to \infty} f_k(k) = 0$
  - Capital is durable and depreciates at rate $\delta \in (0, 1]$
- Collateral constraints:
  - Need to collateralize loan repayment with a tangible asset.

Noeclassical Investment: Investor's problem

- Investor's objective
  - Maximize value: the present discounted value of dividends
- Investor's problem - recursive formulation
  - Choose current dividend d and invest capital k to solve

$$\max_{\{d,w',k\}} d + R^{-1} v\left(w'\right)$$

subject to budget constraints (but no limited liability constraints)

$$w \geq d + k$$

$$Af(k) + k(1-\delta) \geq w'$$

- Investment Euler Equation

$$1 = R^{-1}(Af_k(k) + (1-\delta))$$

- User cost of capital

$$u \equiv r + \delta$$

Limited enforcement implies collateral constraints

- **Enforcement constraint**
  - Ensure that borrower prefers to repay instead of absconding; heuristically,

$$\underbrace{v\left(w'\right)}_{\text{value when repaying}} \geq \underbrace{v(Af(k)+(1-\theta)k(1-\delta))}_{\text{value when defaulting}}$$

  and since $v(\cdot)$ is strictly increasing

$$w' \geq Af(k)+(1-\theta)k(1-\delta)$$

  and using budget constraint to substitute for w' given borrowing b

$$\underbrace{Af(k)+k(1-\delta)-Rb}_{\text{payoff when repaying}} = w' \geq \underbrace{Af(k)+(1-\theta)k(1-\delta)}_{\text{payoff when defaulting}}$$

- **Collateral constraint**

$$\theta k(1-\delta) \geq Rb$$

Dynamic financing problem with collateral constraints

- **Firm's problem**

$$v(w) \equiv \max_{\{d,k,b,w'\}} d + \beta v(w')$$

subject to budget constraints and collateral constraint

$$\begin{aligned} w + b &\geq d + k && (\mu) \\ Af(k) + k(1-\delta) &\geq w' + Rb && (\beta\mu') \\ \theta k(1-\delta) &\geq Rb && (\beta\lambda') \end{aligned}$$

and limited liability $d \geq 0$

- Net worth next period $w' = Af(k) + k(1-\delta) - Rb$

Investment Euler Equation

- First-order conditions (multipliers μ, $\beta\mu'$, and $\beta\lambda'$)

$$1 \leq \mu, v_w\left(w'\right) = \mu'$$

$$\mu = \beta\mu'\left[Af_k(k) + (1-\delta)\right] + \beta\lambda'\theta(1-\delta), \mu = \beta\mu' R + \beta\lambda' R$$

- Also: envelope condition $v_w(w) = \mu$
- Investment Euler Equation

$$1 = \beta\frac{\mu'}{\mu}\frac{Af_k(k) + (1-\theta)(1-\delta)}{1 - R^{-1}\theta(1-\delta)}$$

Collateral and capital structure

- "**Minimal down payment**" (per unit of capital)

$$\wp \equiv 1 - \underbrace{R^{-1}\theta(1-\delta)}_{\text{PV of } \theta \times \text{ resale value of capital}}$$

- **Capital structure**
  - In the deterministic case, collateral constraints always bind
  - Debt per unit of capital

$$R^{-1}\theta(1-\delta)$$

  - Internal funds per unit of capital

$$\wp = 1 - R^{-1}\theta(1-\delta)$$

Investment policy

- Investment Euler Equation for dividend paying firm

$$1 = \beta \frac{Af_k(k) + (1-\theta)(1-\delta)}{\wp}$$

- Dividend-paying firm: capital $\bar{k}$ solves equation above
    - Comparing FOCs can show $\bar{k} < k^*$ (underinvestment)
- Non-dividend paying firm: $k = \frac{1}{\wp} w$ (invest all net worth and lever as much as possible)

Dividend policy

- Threshold policy
- Pay out dividends today $(d' > 0)$ if $w \geq \bar{w}$
- Can we show threshold is optimal?
  - Suppose pay dividends at w but not at $w^+ > w$
  - At w, invest $\bar{k}$
  - If not paying dividends at w^+, must invest more; can **IEE** hold?

Value of Internal Funds

- Value of internal funds μ
  - Premium on internal funds (unless firm pays dividends) since $\mu \geq 1$
- User cost $u(w)$
  - User cost such that $u(w) = R\beta \frac{\mu'}{\mu} A f_k(k)$ where

$$u(w) \equiv r + \delta + \underbrace{R\beta \frac{\lambda}{\mu}(1-\theta)(1-\delta)}_{\text{internal funds require premium}} > u$$

Net worth Accumulation and Firm Growth

- Dividend policy and net worth accumulation
  - Dividend policy is threshold policy
  - For $w \geq \bar{w}$, pay dividends $d = w - \bar{w}$
  - For $w < \bar{w}$, pay no dividends and reinvest everything (''retain all earnings'')
- Investment policy and firm growth
  - For $w \geq \bar{w}$, keep capital constant at $\bar{k}$ (no growth)
  - For $w < \bar{w}$, invest everything $k = \frac{1}{\wp} w$ resulting in net worth $w' > w$ next period
- Firm age
  - Young firms ($w < \bar{w}$) do not pay dividends, reinvest everything, grow
  - Mature firms ($w \geq \bar{w}$) pay dividends and do not grow

Dynamic Debt Capacity Management

- **Technology**
  - Capital k invested in current period yields stochastic payoff ("cash flow") in state s' next period

$$A\left(s'\right) f(k)$$

    where $A' \equiv A\left(s'\right)$ is realized "total factor productivity" (TFP)
  - Strict concavity $f_k(k) > 0$; $f_{kk}(k) < 0$; also: $\lim_{k \to 0} f_k(k) = +\infty$; $\lim_{k \to \infty} f_k(k) = 0$
  - Capital is durable and depreciates at rate δ
    - Depreciated capital $k(1-\delta)$ remains next period
- **Collateral constraints**
  - Need to collateralize all promises to pay with tangible assets
  - Can pledge up to fraction $\theta < 1$ of value of depreciated capital

Firm's dynamic debt capacity management problem

- State-contingent borrowing $b' \equiv b(s')$
  - Collateral constraint for state-contingent borrowing b'

$$\theta k(1-\delta) \geq Rb'$$

- Firm's debt capacity use problem

$$\max_{\{d,w',k,b'\}} d + \beta \sum_{s' \in \mathcal{S}} \Pi(s,s')\, v(w',s')$$

subject to budget constraints and collateral constraints, $\forall s' \in \mathcal{S}$,

$$w + \underbrace{\sum_{s' \in \mathcal{S}} \Pi(s,s')\, b'}_{\text{total borrowing}} \geq d + k$$

$$A' f(k) + k(1-\delta) \geq Rb' + w'$$

$$\theta k(1-\delta) \geq Rb'$$

and limited liability $d \geq 0$

Dynamic debt capacity choice–Optimal conditions

- First-order conditions (multipliers $\mu, \Pi(s, s')\beta\mu(s')$, and $\Pi(s, s')\beta\lambda(s')$)

$$1 \le \mu, \quad v_w(w', s') = \mu'$$
$$\sum_{s' \in \mathcal{S}} \Pi(s, s')\beta\mu'\left[A' f_k(k) + (1-\theta)(1-\delta)\right], \quad \mu = \beta\mu' R + \beta\lambda' R$$

- Investment Euler equation

$$1 = \sum_{s' \in \mathcal{S}} \Pi(s, s')\beta\frac{\mu'}{\mu}\frac{A' f_k(k) + (1-\theta)(1-\delta)}{\wp}$$

- Firms do not exhaust debt capacity against all states
    - Debt capacity use/leverage: $\theta(1-\delta) \ge R\sum_{s' \in \mathcal{S}} \Pi(s, s')\, b'/k$
    - Recall: equality in deterministic case

Corporate Risk Management

- Financial constraints give a rationale for corporate risk management
  - If firms' net worth matters, then firms are as if risk averse
  - Collateral constraints link financing and risk management
  - More constrained firms hedge less and often not at all
  - Financing vs. risk management trade-off
    - Limited enforcement: need to collateralize promises to financiers and counterparties
    - Collateral constraints link financing and risk management
    - More constrained firms hedge less as financing needs dominate hedging concerns

Corporate Risk Management Problem

- Equivalent risk management formulation
  - Collateral constraint for state-contingent borrowing b'

$$\theta k(1-\delta) \geq Rb'$$

  - Equivalently, borrow as much as possible and hedge

$$h' \equiv \theta k(1-\delta) - Rb' \geq 0$$

- Firm's risk management problem

$$\max_{\{d, w', k, h'\}} d + \beta \sum_{s' \in \mathcal{S}} \Pi(s, s') \, v(w', s')$$

subject to budget constraints and short sale constraints, $\forall s' \in \mathcal{S}$,

$$w \geq d + \wp k + \underbrace{R^{-1} \sum_{s' \in \mathcal{S}} \Pi(s, s') \, h'}_{\text{cost of hedging portfolio}}$$

$$A' f(k) + (1-\theta) k (1-\delta) + h' \geq w'$$

$$h' \geq 0$$

Financing vs. Risk Management Trade-off

- **Investment Euler equation**

$$1 = \sum_{s' \in \mathcal{S}} \Pi(s, s')\, \beta \frac{\mu'}{\mu} \frac{A' f_k(k) + (1-\theta)(1-\delta)}{\wp}$$
$$\geq \Pi(s, s')\, \beta \frac{\mu'}{\mu} \frac{A' f_k(k) + (1-\theta)(1-\delta)}{\wp}$$

  - As $w \to 0$, capital $k \to 0$ and marginal product $f_k(k) \to \infty$
  - Therefore, marginal value of net worth in state-s' (relative to current period) $\mu'/\mu \to 0$
  - Using first-order condition for hedging

$$\lambda'/\mu = (\beta R)^{-1} - \mu'/\mu > 0$$

    so severely constrained firms do not hedge at all
- Financing vs. risk management trade-off
  - Hedging uses up net worth which is better used to purchase additional capital/downsize less
  - IID case: if firms hedge, they hedge states with low net worth due to low cash flows