Open Economy Macro: Problem Set 1
2024 Fall

Due on Nov 6th

Question 1. Compute the relevant business cycle statistics for South Korea and the United States using 1) log-linear detrending; 2) log-quadratic detrending; 3) HP filtering with $\lambda = 100$. The data should be downloaded from the World Bank's WDI database. The sample period use 1971-2019 annually. Specifically, use the following time series

Variable	Series Name	Series Code
y	GDP per capita (constant LCU)	NY.GDP.PCAP.KN
c	Households and NPISHs final consumption expenditure (% of GDP)	NE.CON.PRVT.ZS
i	Gross capital formation (% of GDP)	NE.GDI.TOTL.ZS
g	General government final consumption expenditure (% of GDP)	NE.CON.GOVT.ZS
im	Imports of goods and services (% of GDP)	NE.IMP.GNFS.ZS
ex	Exports of goods and services (% of GDP)	NE.EXP.GNFS.ZS

Make a graph showing the natural logarithm of real GDP per capital and the trend for the two countries. Then identify the recessions in the two countries. Are the results the same across the three detrending methods? Also make a table showing the statistics. Attach your code for cleaning data and making tables and plots.

Question 2. Consider a SOE model with nondurable and durable consumption goods. Let $c_{N,t}$ denote consumption of nondurable goods in period t, and $c_{D,t}$ denote purchases of durables in period t. The stock of durable goods s_t is assumed to evolve over time as

$$s_t = (1-\delta)s_{t-1} + c_{D,t}$$

where $\delta \in (0,1]$ denotes the depreciation rate of durable goods. Households have preferences of the form

$$\sum_{t=0}^{\infty} \beta^t U(c_t)$$

where U is increasing in consumption and concave. Consumption c_t is a composite of non-durable consumption and the service flow provided by the stock of consumer durables. Specifically,

$$c_t = \left[(1-\alpha)^{\frac{1}{\eta}} c_{N,t}^{1-\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} s_t^{1-\frac{1}{\eta}}\right]^{\frac{1}{1-\frac{1}{\eta}}}$$

where $\eta > 0$ and $\alpha \in (0,1)$. Households have access to an internationally traded risk-free one-period bond, which pays the interest rate r_t when held between periods t and $t+1$.

The relative price of durables in terms of nondurables is one. The household is subject to a borrowing limit that prevents it from engaging in Ponzi schemes. Output y_t is produced with physical capital k_t according to a production function of the form

$$y_t = F(k_t)$$

The capital stock evolves over time as

$$k_{t+1} = (1-\delta_k)k_t + i_t$$

where i_t denotes investment in period t, and δ_k is the depreciation rate on physical capital.

1. Describe the household's budget constraint

2. State the optimization problem of the household

3. Present the complete set of equilibrium conditions

4. Suppose the interest rate is constant over time and equal to $r_t = r = \beta^{-1} - 1$. Assume that up to period -1, the economy was in a steady state equilibrium in which all variables were constant and $d = \bar{d}$, where d denotes net external debt in the steady state. Find the share of expenditures on durables in consumption expenditures in the steady state in terms of the parameters δ, r, α, and η

5. Assume that in period 0, the economy unexpectedly receives a positive income shock as a consequence of the rest of the world forgiving part of the country's net foreign debt. Assume that the positive income shock results in a 1 percent increase in the consumption of nondurables in period 0. Find the percentage increase in purchases of durables and in total consumption expenditures in period 0. Compare your answer to the one you would have obtained if all consumption goods were nondurables

6. Continuing to assume that consumption of nondurables has increased by 1 percent, find the change in trade balance in period 0 expressed as a share of steady-state consumption expenditure. Is the response of the trade balance countercyclical? Compare your findings to those you would have obtained if all consumption goods were nondurables. How much amplification is there due to the presence of durables?

Question 3. Modify the GHH period utility function of the SOE-RBC-EDEIR model as follows:

$$U(c,h) = \frac{\left[c^{1-\omega}(1-h)^{\omega}\right]^{1-\sigma} - 1}{1-\sigma}$$

All other features of the model are unchanged.

1. Derive analytically the steady state of the model.

2. Keep the value of all parameters the same as the example in class, except for ω. Set ω such that $h = 1/3$ in steady state, i.e. households spend one-third of their time working.

3. Solve the model and produce a table of predicted second moments. You might find it convenient to use the Python code posted on Blackboard, or write your own code.

4. Compare the predictions of the present Cobb-Douglas preference model with those of its GHH preference counterpart.

Question 4. Consider a small open economy inhabited by identical consumers with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\ln c_t - \gamma h_t\right]$$

where c_t denotes consumption, h_t denotes hours worked, and $\beta \in (0,1)$ and $\gamma > 0$ are parameters. The consumption good is a composite made of tradable and nontradable goods via a Leontief aggregator. Formally,

$$c_t = \min\{c_t^{\mathrm{T}}, c_t^{\mathrm{N}}\}$$

where c_t^{T} and c_t^{N} denote, respectively, domestic absorption of tradables and nontradables in period t. To produce his nontraded consumption, each consumer operates a linear technology that uses labor as the sole input:

$$c_t^{\mathrm{N}} = A h_t$$

where $A > 0$ is a parameter. In addition, households can borrow or lend in the international financial market at the rate $r > 0$. Their sequential budget constraint is given by

$$c_t^{\mathrm{T}} + (1+r)d_{t-1} = y^{\mathrm{T}} + d_t$$

where d_t denotes the level of net external debt assumed in period t and maturing in period $t+1$, and $y^{\mathrm{T}} > 0$ denotes a constant endowment of tradable goods. In period 0, households start with outstanding debt equal to $d_{-1} > 0$. Finally, households are subject to a no-Ponzi game constraint of the form

$$\lim_{t\to\infty} \frac{d_t}{(1+r)^t} \le 0$$

1. Characterize the equilibrium levels of consumption, consumption of nontradables, and hours worked.

2. Suppose that in period 0, foreign lenders unexpectedly decide to forgive an amount $\Delta^d > 0$ of the debt. Assuming that Δ^d is relatively small, characterize the effect of this debt forgiveness shock on consumption, consumption of nontradables, and hours worked.

3. Now suppose $\Delta^d = 0$. Instead, assume that in period 0, the nontraded sector experience a permanent increase in productivity. Specifically, the productivity factor A increases by $\Delta^A > 0$. Characterize the effect of this positive productivity shock on consumption, consumption of nontradables, and hours worked.

Question 5. We generally assume that the consumption aggregator function $A(c^{\mathrm{T}}, c^{\mathrm{N}})$ is increasing, concave, and linearly homogeneous. Thus, the household's demand for nontradables can be given by

$$p = \frac{A_2(c^{\mathrm{T}}, c^{\mathrm{N}})}{A_1(c^{\mathrm{T}}, c^{\mathrm{N}})}$$

where p is the relative price of nontradables in terms of tradables.

1. Show that the assumptions are sufficient to ensure that the demand schedule of nontradables is downward sloping in the space (c^{N}, p), holding c^{T} constant.

2. Show that the aforementioned assumptions about the aggregator $A(c^{\mathrm{T}}, c^{\mathrm{N}})$ are sufficient to guarantee that increases (decreases) in c^{T} shift the demand schedule up and to the right (down and to the left).

3. Assume that the aggregator function takes the Cobb-Douglas form

$$A(c^{\mathrm{T}}, c^{\mathrm{N}}) = \sqrt{c^{\mathrm{T}} c^{\mathrm{N}}}$$

   Find the demand function of nontradables.

4. Now assume the CES form

$$A(c^{\mathrm{T}}, c^{\mathrm{N}}) = \left[a(c^{\mathrm{T}})^{1-\frac{1}{\xi}} + (1-a)(c^{\mathrm{N}})^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}}$$

   Derive the demand function of nontradables. Interpret the parameter ξ.