Appendix for Lecture One

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1 More details on log linearization

This section provides more details on how to derive the Phillips curve and the Euler equation by log-linearizing the model.

1.0.1 Phillips curve (the aggregate supply equation)

Recall that the retailer's optimal price-setting condition is given by:

$$\tilde{P}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \lambda_{t+j} \left(\frac{1}{P_{t+j}}\right)^{\frac{-1}{\lambda_f - 1}} Y_{t+j} \lambda_f s_{t+j} P_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \lambda_{t+j} \left(\frac{1}{P_{t+j}}\right)^{\frac{-1}{\lambda_f - 1}} Y_{t+j}} \tag{1}$$

Using the above equation, we have:

$$\tilde{P}_t * E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \lambda_{t+j} \left(\frac{1}{P_{t+j}}\right)^{-\frac{1}{\lambda_f - 1}} Y_{t+j} = E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \lambda_{t+j} \left(\frac{1}{P_{t+j}}\right)^{-\frac{1}{\lambda_f - 1}} Y_{t+j} \lambda_f s_{t+j} P_{t+j}$$

$$\Rightarrow \; E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \left[\hat{\tilde{P}}_t + \hat{\lambda_{t+j}} + \frac{1}{\lambda_f - 1} * \hat{P_{t+j}} + \hat{Y_{t+j}}\right] = E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \left[\hat{\lambda_{t+j}} + \frac{1}{\lambda_f - 1} * \hat{P_{t+j}} + \hat{Y_{t+j}} + \hat{s^n_{t+j}}\right]$$

$$\begin{aligned} \Rightarrow \; (1 - \beta\xi_p)^{-1} \hat{\tilde{P}}_t &= E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j * \hat{s^n_{t+j}} \\ &= \hat{s^n_t} + E_t \sum_{i=0}^{\infty} (\beta\xi_p)^{i+1} \widehat{s^n_{t+i+1}} \\ &= \hat{s^n_t} + (1 - \beta\xi_p)^{-1} \beta\xi_p * E_t \hat{\tilde{P}_{t+1}}. \end{aligned}$$

where $\tilde{P}_t$, s^n_t and P_t denotes the nominal optimal retail price, the nominal wholesale price $s_t P_t$ and the nominal price for aggregate goods. Denote $\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}$ as the real optimal price, so that, $\hat{\tilde{p}}_t \equiv \hat{\tilde{P}}_t - \hat{P}_t$. Then we have:

$$(1 - \beta\xi_p)^{-1} \hat{\tilde{p}}_t = \hat{s}_t + (1 - \beta\xi_p)^{-1} \beta\xi_p E_t \left(\hat{\tilde{p_{t+1}}} + \hat{\pi_{t+1}}\right)$$

Recall that the real optimal price $\tilde{p_t}$ is related to the inflation rate π_t as follows,

$$\tilde{p_t} = \left[\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right]^{-(\lambda_f - 1)}$$

$$\Rightarrow \qquad \hat{\tilde{p_t}} = \frac{\xi_p}{1 - \xi_p} \hat{\pi}_t$$

Then after some algebra:

$$\frac{\xi_p}{(1 - \beta\xi_p)(1 - \xi_p)} \hat{\pi}_t = \hat{s}_t + E_t \frac{\beta\xi_p}{(1 - \beta\xi_p)(1 - \xi_p)} \hat{\pi_{t+1}}$$

$$\Rightarrow \qquad \hat{\pi}_t = \frac{(1 - \beta\xi_p)(1 - \xi_p)}{\xi_p} \hat{s}_t + \beta E_t \hat{\pi_{t+1}}$$

The above equations imply that, under the linear approximation, current inflation π_t in determined by the optimal price $\tilde{p_t}$, which is in turn determined by the marginal production cost s_t and future expected inflation $E_t \hat{\pi_{t+1}}$.

Now, what determines s_t? Recall the wholesale firm's production conditions and the household's labor supply:

$$s_t = (1 - \nu_t) \left(\frac{1}{1 - \gamma} \right)^{1-\gamma} \left(\frac{\bar{w}_t}{\gamma} \right)^{\gamma} (1 - \psi + \psi R_t)$$

$$\bar{w}_t = \frac{W_t}{z_t^{\frac{1}{\gamma}} P_t} = \frac{C_t H_t^{\phi}}{z_t^{\frac{1}{\gamma}}} = \tilde{c}_t H_t^{\phi}$$

$$\Rightarrow \qquad \hat{s}_t = \gamma \left(\phi \hat{H}_t + \hat{\tilde{c}}_t \right) + \frac{\psi R}{1 - \psi + \psi R} \hat{R}_t$$

where $\tilde{c}_t \equiv \frac{C_t}{z_t^{\frac{1}{\gamma}}}$. Recall that:

$$C_t + I_t = p_t^* z_t H_t^{\gamma} I_t^{1-\gamma}$$

$$p_t^* = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right)^{\lambda_f} + \xi_p \frac{\pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*} \right]^{-1} \qquad \Rightarrow \hat{p_t^*} = \xi_p \hat{p_{t-1}^*}$$

After some algebra we have: $\hat{\tilde{C}}_t = \hat{H}_t \equiv \hat{x}_t$. and:

$$\hat{s}_t = \gamma(1 + \phi)\hat{x}_t + \frac{\psi R}{1 - \psi + \psi R} \hat{R}_t$$

1.0.2 Euler equation (the aggregate demand equation)

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\pi_{t+1}}$$

$$\Rightarrow \qquad 1 = E_t \frac{\beta \tilde{c}_t}{\tilde{c_{t+1}}} \mu_{z,t+1}^{-\frac{1}{\gamma}} \frac{R_t}{\pi_{t+1}} \quad where \quad \mu_{z,t+1} = \frac{z_{t+1}}{z_t}$$

$$\Rightarrow \qquad \hat{\tilde{c}}_t = E_t \hat{\tilde{c_{t+1}}} + \frac{1}{\gamma} \hat{\mu_{z,t+1}} + \hat{\pi_{t+1}} - \hat{R}_t$$

$$\Rightarrow \qquad \hat{x}_t = E_t \left[\hat{x_{t+1}} - (\hat{R}_t - \hat{\pi_{t+1}} - \hat{R_t}^*) \right] \quad where \quad R_t^* \equiv E_t \frac{1}{\beta} (\frac{z_{t+1}}{z_t})^{1/\gamma}$$