MICROECONOMIC THEORY II

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ADVERSE SELECTION (不对称信息的直接后果)

- Adverse selection describes a principal-agent problem in which the agent has private information about a parameter of his optimization problem.
- It is also referred to as hidden information, or hidden knowledge.
- In fact, hidden information is probably a better expression for describing this type of asymmetric information.
- Adverse selection is rather a possible consequence of this asymmetric information.

Adverse selection in stock market

"Just as a car buyer can never be sure whether information is being withheld by the seller, in the financial markets a buyer can never be sure whether there is something going on with a stock that is beyond his purview. The person on the other side of the trade might have insider information on the company, or he might know that there is a much larger overhang of potential selling, the demand the buyer sees being a first trickle in what will emerge as a flood of selling.
The adverse selection problem is especially troublesome for market makers, and particularly for market makers in specialized arenas, such as corporate bonds, mortgage securities, and emerging markets."

——A Demon of Our Own Design, Richard Bookstaber

Consequence of Adverse selection in stock market

"Market makers often didn't know who was on the other side of their trade, whether it was a tipped-off hedge fund manager who knew a stock was about to rocket higher (or plunge) or a dumb-as-dirt day trader making a reckless gamble. Because of that ignorance, market makers often would only buy the stock at a low price, or sell at a high price, in order to protect themselves. In response to the chance of getting winged by a well-armed gunslinger, market makers typically widen their quotes, providing a lower bid or higher offer. The result: wider spreads. "

—Dark Pools, Scott Patterson

INSURANCE MARKET (消费者风险厌恶)

大 风险中性

- Model

  每个消费者发生事故概率

  - ➢ Consumer: initial wealth w, accident occurs with $\pi_i \in [0,1]$ in which L dollar loss
  - ➢ Insurance companies: identical and offer full insurance at price p

    价格竞争 保险公司

- Symmetric information, Zero-profit condition

$$p_i = \pi_i L \quad \forall i.$$

Asymmetric information

- Assume

$$\pi \in [\underline{\pi}, \bar{\pi}]$$

- Consumer purchase policy iff

消费者不买保险的期望效用

保费p

$$u(w - p) \geq \pi u(w - L) + (1 - \pi) u(w)$$

- So consumer purchase policy iff accident probability

保公仅知消费者发生事故的平均概率

$$\pi \geq \frac{u(w) - u(w - p)}{u(w) - u(w - L)} \equiv h(p)$$

- Competitive equilibrium price under asymmetric information

$$p^* = E(\pi | \pi \geq h(p^*)) L,$$

$$E(\pi | \pi \geq h(p^*)) = \frac{\int_{h(p^*)}^{\bar{\pi}} \pi dF(\pi)}{1 - F(h(p^*))}$$

Numerical example

- Suppose $\pi \sim U(0,1)$
- In this case

$$E(\pi|\pi \geq h(p)) = \frac{1 + h(p)}{2}$$

- Let $g(p) = E(\pi|\pi \geq h(p))L$, there is a unique equilibrium p^*

$$p^* = \frac{1 + h(p^*)}{2} L.$$

竞争均衡只有1个

- That is $P^* = L$.
- Only consumer that is certain to have an accident buy the insurance.


BUY
FOR SALE
2001 GMC SIERRA
160,xxx miles

旧车价格 旧车质量

- Price of automobile p, quality $\mu(p)$
- Two groups of traders
  1. Group one: total income Y_1 and has N used cars 向第二组卖旧车

     x_i 旧车的质量

$$u_1 = M + \sum_{i=1}^{n} x_i$$

x_i quality of ith automobile

  2. Group two: total income Y_2 and 向第一组买旧车

$$u_2 = M + \sum_{i=1}^{n} \frac{3x_i}{2}$$

Symmetric information

- Symmetric information: both groups only knows

$$x_i \sim U(0,2)$$ 每辆旧车的平均质量为1

and expected quality

$$\mu = 1.$$

- Supply

$$S(p) = \begin{cases} N, & p > 1 \\ 0 & p < 1 \end{cases}$$

(把all旧车拿出来卖)
(不卖)

- Demand

$$D(p) = \begin{cases} \frac{Y_2+Y_1}{p}, & p < 1 \\ \frac{Y_2}{p}, & 1 < p < \frac{3}{2} \\ 0 & p > \frac{3}{2} \end{cases}$$

$p=1$ 均 1可买可不买

$p=\frac{3}{2}$ 2可买可不买

- Equilibrium

$$p = \begin{cases} 1, & \text{if } \; Y_2 < N \\ \frac{Y_2}{N} & \text{if } \; \frac{2Y_2}{3} < N < Y_2 \\ \frac{3}{2} & \text{if } \; N < \frac{2Y_2}{3} \end{cases}$$

旧车数量非常大

第二组所有人都买到旧车 $p<\frac{3}{2}$

旧车数量非常少

Asymmetric information

- Group one knows quality, group two does not
- Demand of group one

每辆车的质量

$$D_1(p) = \begin{cases} \frac{Y_1}{p}, & \mu > p \\ 0 & \mu < p \end{cases}$$

，想买旧车

- Demand of group two

市场中平均车的质量

$$D_2(p) = \begin{cases} \frac{Y_2}{p}, & \frac{3\mu}{2} > p \\ 0 & \frac{3\mu}{2} < p \end{cases}$$

p 市场中最高质量

$\frac{p}{2}$ 平均质量 → 没有人会买旧车

- Supply

平均质量

$$S(p) = \frac{pN}{2}$$

- Average quality supplied

$$\mu = \frac{p}{2}.$$

Asymmetric information (2)

- Total demand

$$D(p,\mu) = \begin{cases} \frac{Y_1+Y_2}{p}, & \mu > p \\ \frac{Y_2}{p} & \mu < p < \frac{3\mu}{2} \\ 0 & \frac{3\mu}{2} < p \end{cases}$$

- Note that with price p,

$$\mu = \frac{p}{2}.$$

- NO trade in equilibrium, even if at *any given price* $p \in [0, 3]$, there are group one trader willing to sell at a price which group two are willing to pay.

SIGNALING 信号

信息多 → 信息少
提供信息

→ 减弱由不对称信息造成的后果

- Consumers can credibly communicate how risky they are to insurance companies, i.e., by purchasing different types of policies.
- The signaling game

  P(低风险) 在低风险发生事故概率

  发生风险 type high / low

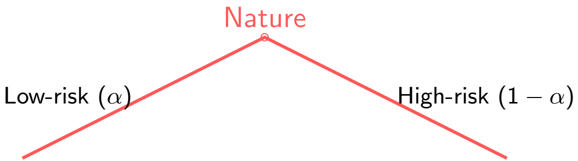
  - ➢ Nature choose risk type $t_i \in T \equiv \{h, l\}$
  - ➢ Prior belief: $Prob(\underline{\pi}) = \alpha, \; Prob(\bar{\pi}) = 1 - \alpha$.

    发生事故后赔B

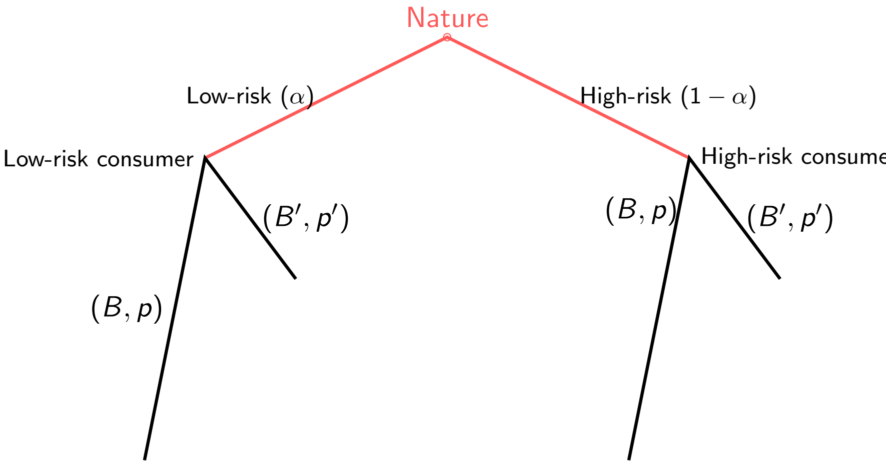
  - ➢ Consumer (*Sender*) chooses message $m_i \in M \equiv \{(B, p)\}$. 保费 p
  - ➢ Insurance company (*Receiver*) responds given belief $\beta(B, p)$: accept, reject.

    p=0 → 高风险

Insurance signaling game



Insurance signaling game



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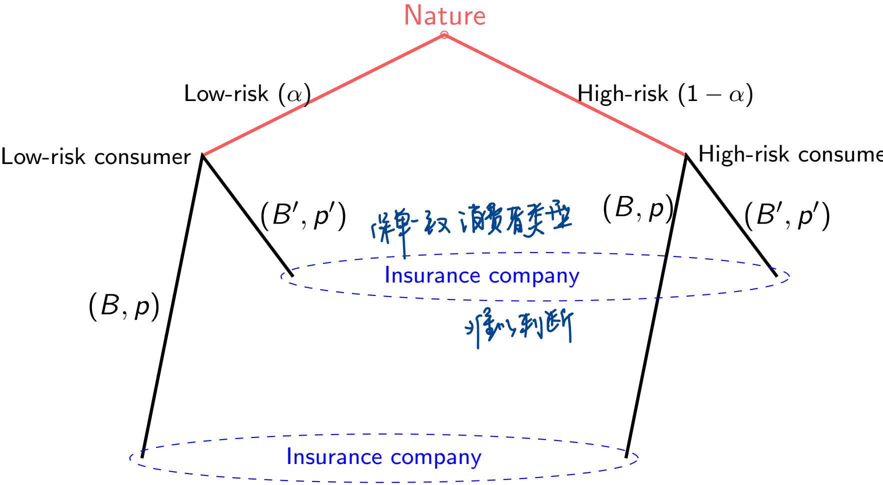

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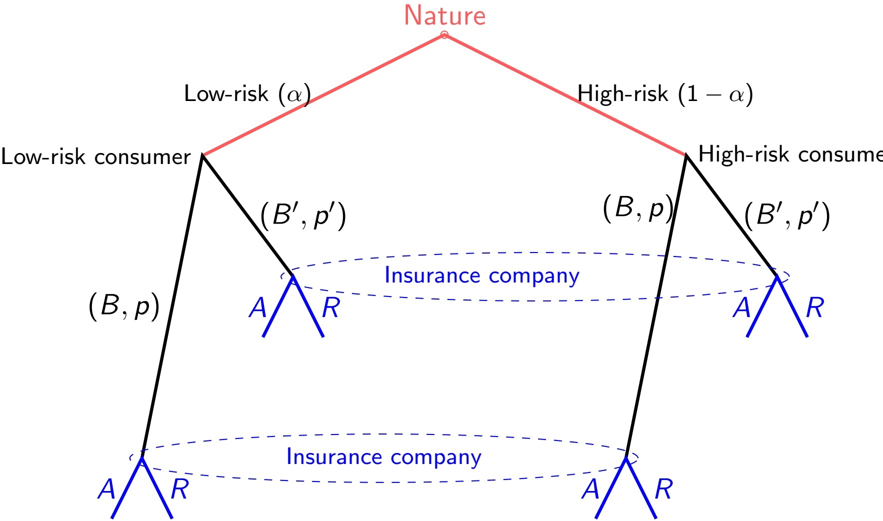


Sequential equilibrium

保单
对低风险
消费者

- A pure strategy for the low-risk consumer is a policy $\psi_l(B_l, p_l)$, and a pure strategy for the high-risk is $\psi_h(B_h, p_h)$.
- Belief: $\beta(B, p)$–the consumer who proposes (B, p) is low-risk type

  纯策略 对保险公司 (接受/拒绝)

  信念
- Signaling game pure strategy sequential equilibrium: $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ is a pure strategy sequential equilibrium of the insurance signaling game if
  - Given $\sigma(\cdot)$, ψ_l, ψ_h maximize low-risk, high-risk's expected utility respectively; $\sigma(\cdot)$ maximizes insurance company's expected profit given belief
  - Belief satisfy Bayes rule,
    - $\beta(\psi) \in [0, 1]$
    - If $\psi_l \neq \psi_h$, then $\beta(\psi_l) = 1$, $\beta(\psi_h) = 0$
    - If $\psi_l = \psi_h$, then $\beta(\psi_l) = \beta(\psi_h) = \alpha$

Consumer optimization review

- Individual's optimal insurance problem: $p=Bq$

$$\max_{B} \quad \pi u\left(w-L+B\left(1-q\right)\right)+\left(1-\pi\right)u\left(w-Bq\right)$$

单位保费

$$s.t.\ B\geq 0,\ \ B\leq w/q$$

- Lagrangian function

$$\mathcal{L}=\pi u\left(w-L+B\left(1-q\right)\right)+\left(1-\pi\right)u\left(w-Bq\right)+\lambda\left(w/q-B\right).$$

- The first-order conditions:

$$\pi u'\left(w-L+B\left(1-q\right)\right)\left(1-q\right)-\left(1-\pi\right)u'\left(w-Bq\right)q-\lambda\leq 0;$$

$$\lambda\left(B-w/q\right)=0,\lambda\geq 0,B\geq 0.$$

- Thus, the optimal B satisfies $u(\cdot)$ 凹(风险厌恶) $u''<0$ u'递减

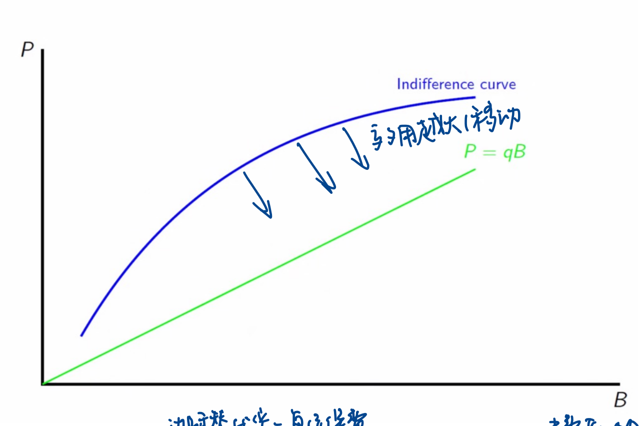
$$\frac{\pi u'\left(w-L+B\left(1-q\right)\right)}{\left(1-\pi\right)u'\left(w-Bq\right)}=\frac{q}{1-q}$$

单位保费恰好等于发生事故的概率

$q=\pi$, $B=L$ (保险公平,全额保险

$q>\pi$, $B<L$

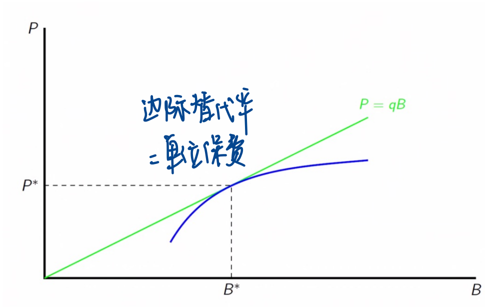
GRAPHICAL ILLUSTRATION

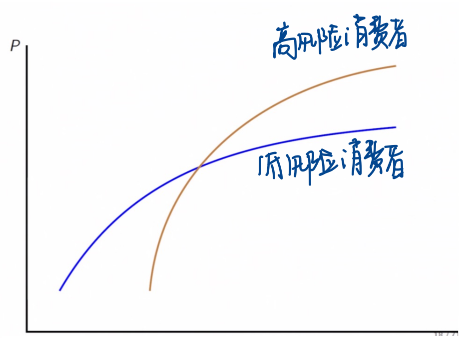


边际替代率=单位保费

消费者B个越好
P↓↓

Single crossing property





On consumer choices

- Note that $P = Bq$ and $B(1-q) = B - P$, so

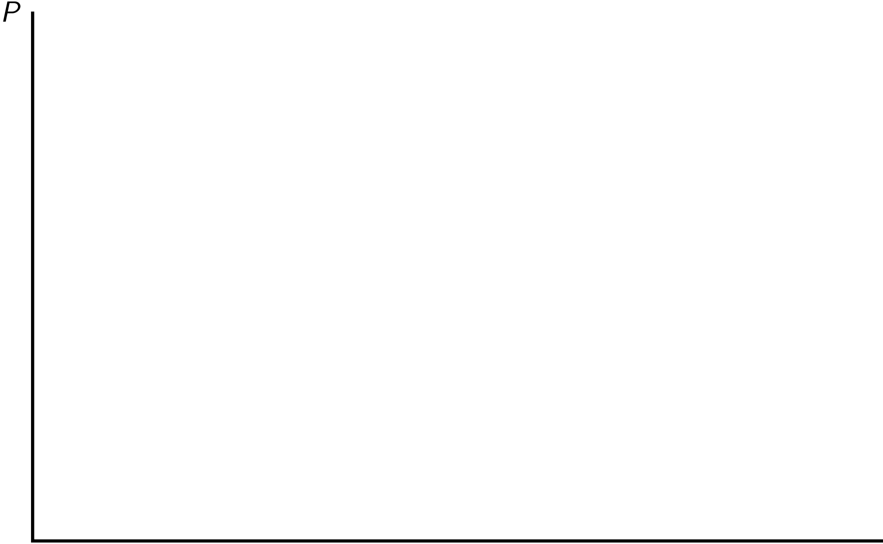
$$MRS(B,P) = \frac{\pi u'(w - L + B - P)}{\pi u'(w - L + B - P) + (1-\pi)\, u'(w - P)} = \frac{P}{B} = q.$$

- Single crossing property

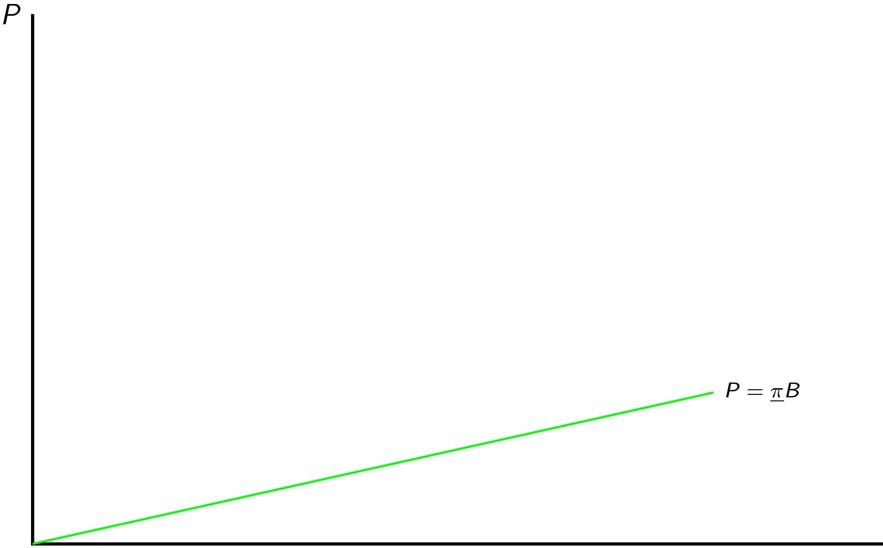
$$MRS_l(B,P) < MRS_h(B,P)$$

- Hence:
  - $u_l(B,p)$ and $u_h(B,p)$ are continuous, differentiable, strictly concave in (B,p)
  - $MRS_l(B,p)$ ($MRS_h(B,p)$) is greater than, equal to or less than $\underline{\pi}$ ($\bar{\pi}$) as B is less than, equal to or greater than L.
  - $MRS_l(B,p) < MRS_h(B,p)$

Insurance company's problem



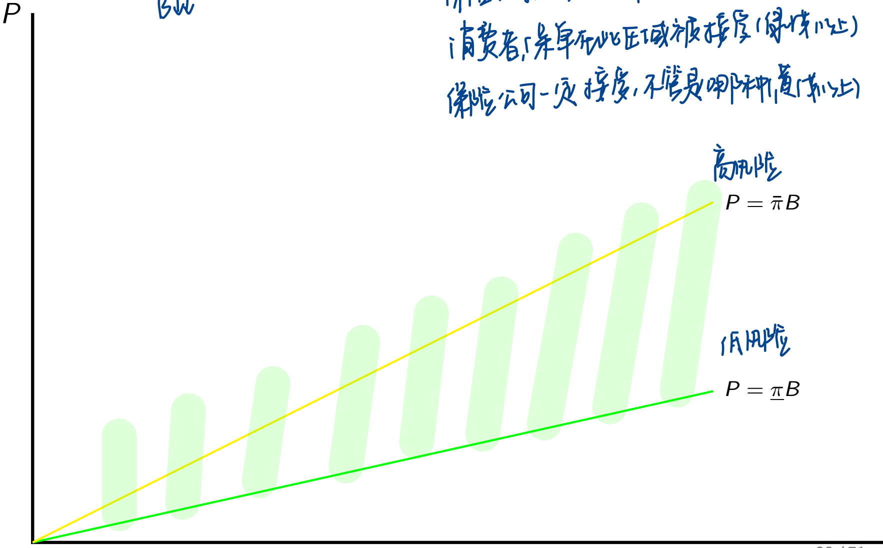
Insurance company's problem



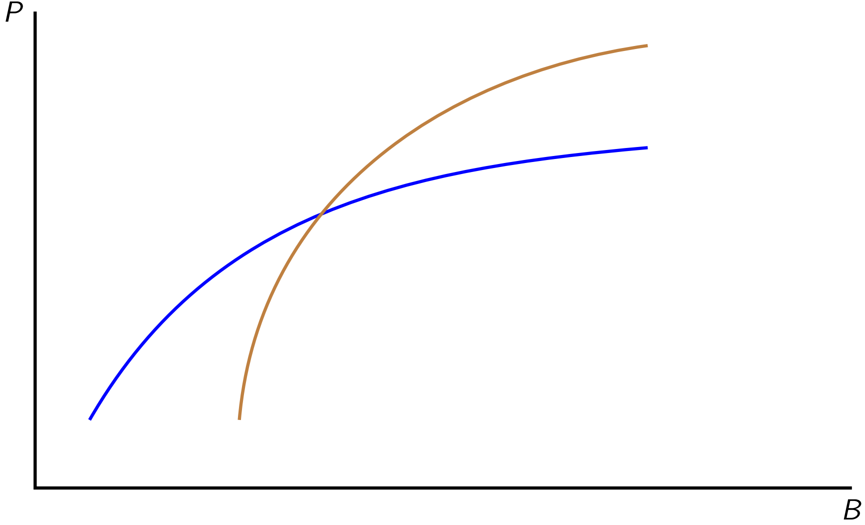
Insurance company's problem

保险公司 P↑越好
B↓

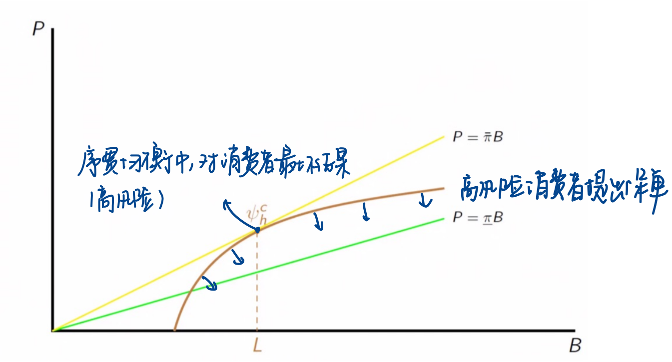
保险公司知提出保单的是低风险
消费者:保单在此区域被接受(绿线以上)
保险公司一定接受,不管是哪种,黄(黄以上)



Consumers' preferences for risks



Equilibrium



On sequential equilibrium

- Lemma 8.1. (Jehle & Reny) Let

$$\tilde{u}_l \equiv \max_{(B,p)} u_l(B,P) \quad s.t. p = \bar{\pi}B \leq w, \qquad u_h^c \equiv u_h(L, \bar{\pi}L).$$

And let $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ be a s.e. with utilities for low-risk and high-risk are, respectively, u_l^* and u_h^*. Then

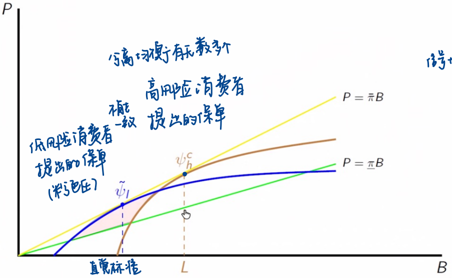
- $u_l^* \geq \tilde{u}_l$;
- $u_h^* \geq u_h^c$.

Separating equilibrium 分离均衡

保险公司见保单知高/低风险

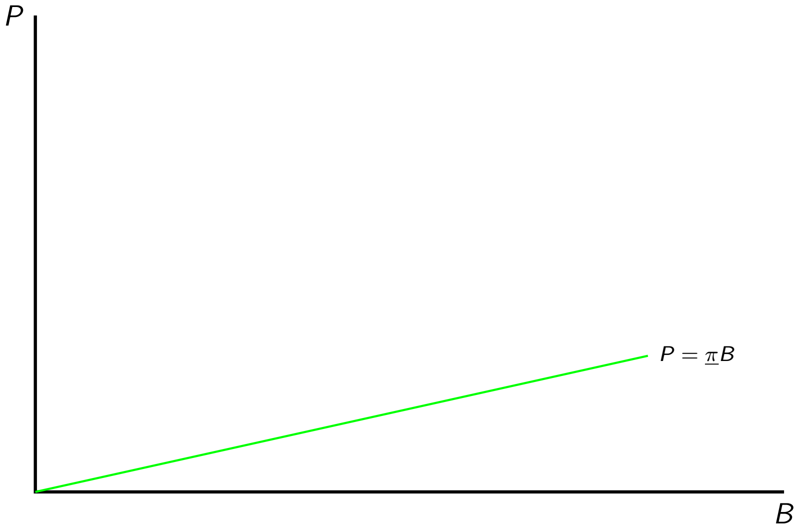
- An equilibrium is a separating equilibrium if the different types of consumers propose different policies.
- Theorem 8.1. (Jehle & Reny) In separating equilibrium,
  - ➢ $\psi_l \neq \psi_h = (L, \bar{\pi}L)$
  - ➢ $p_l \geq \underline{\pi}B_l$
  - ➢ $u_l(\psi_l) \geq \tilde{u}_l \equiv \max_{(B,p)} u_l(B,p)$ $s.t.$ $p = \underline{\pi}B \leq w$
  - ➢ $u_h^c \equiv u_h(\psi_h) \geq u_h(\psi_l)$, where $u_h^c \equiv u_h(L, \bar{\pi}L)$ is high-risk's utility in competitive equilibrium with full information

分离均衡
永远存在

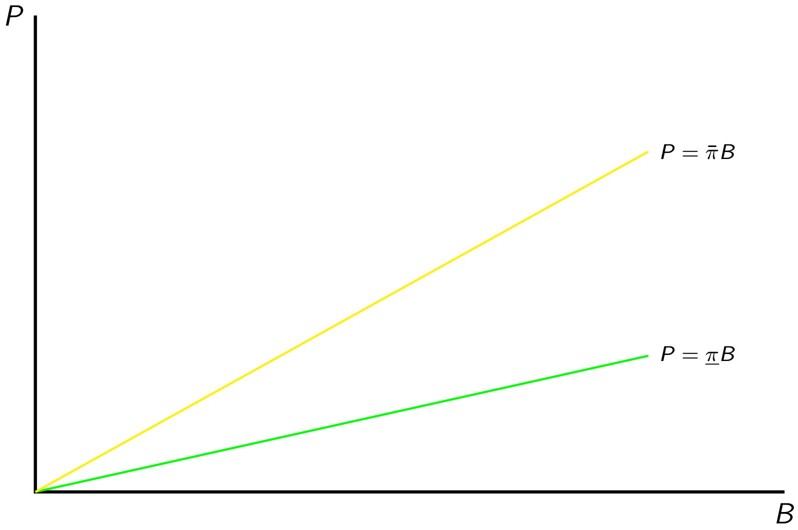


信号博弈→序贯均衡
的信念不一定合理

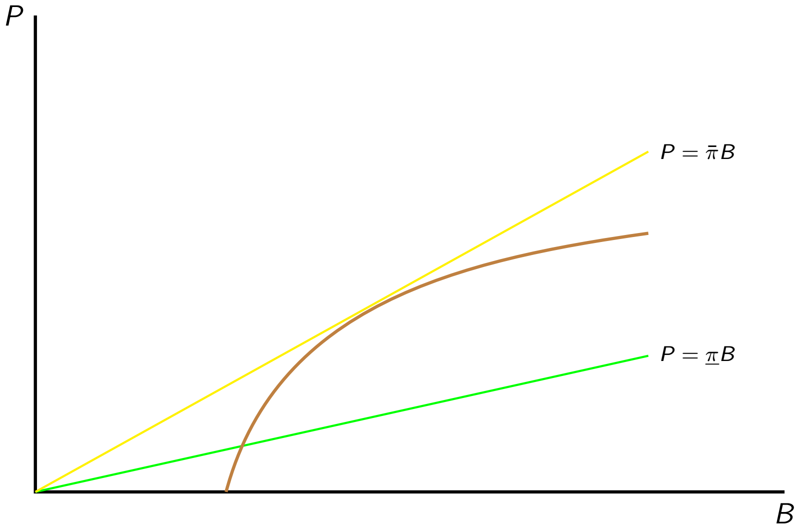
Existence of separating equilibrium



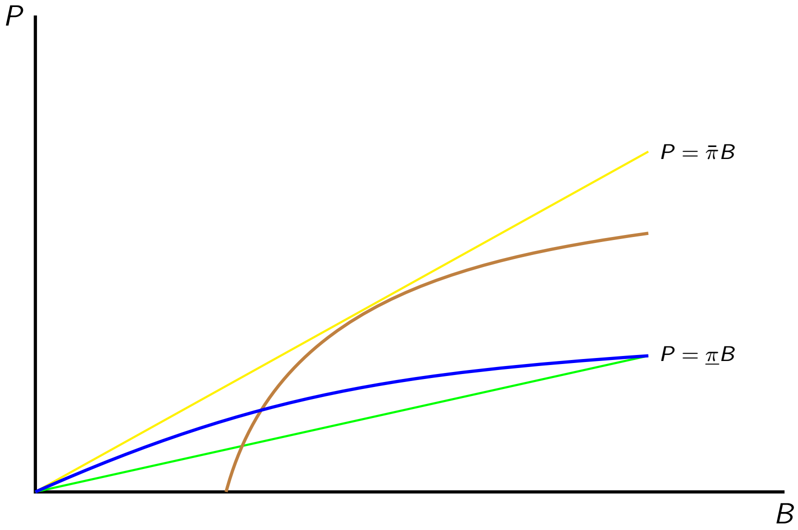
Existence of separating equilibrium



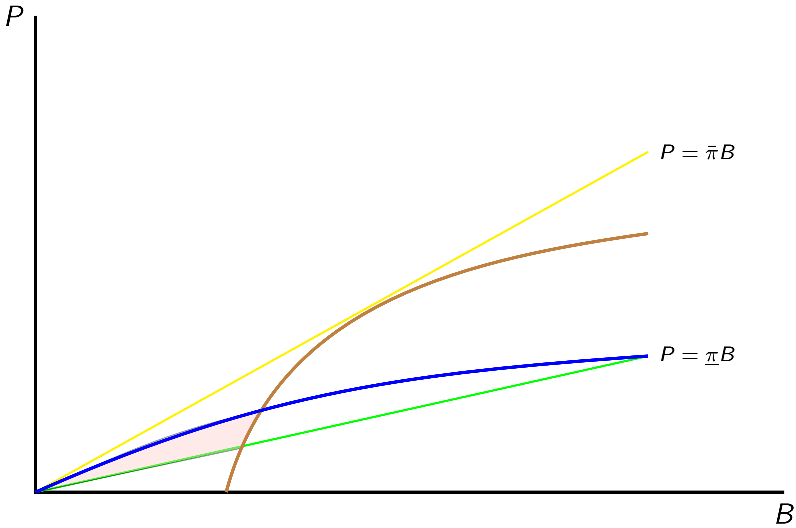
Existence of separating equilibrium



Existence of separating equilibrium



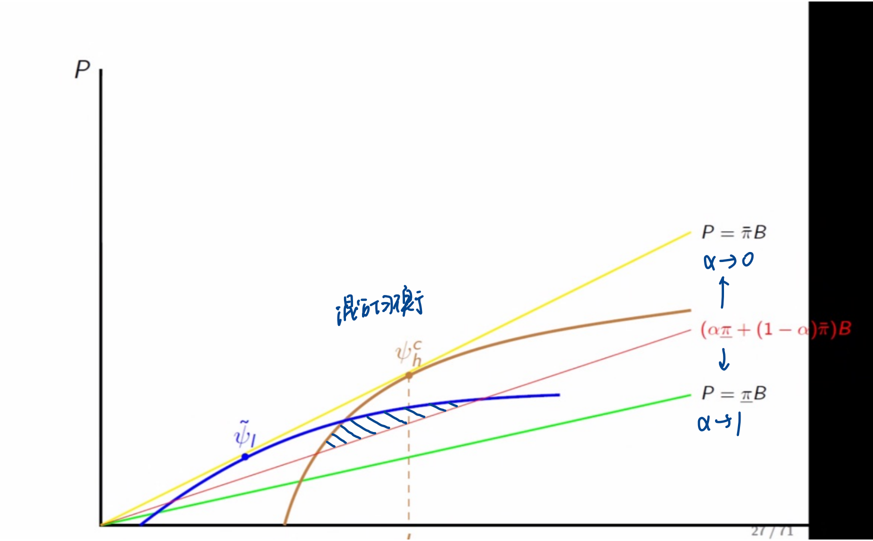
Existence of separating equilibrium 均衡的存在性



Pooling equilibria

- An equilibrium is pooling equilibrium if both high-risk and low-risk propose the same policy.
- Theorem 8.2. (Jehle &Reny) $\psi = (B, p)$ is the outcome in some pooling equilibrium if and only if
  - $u_l(B, p) \geq \tilde{u}_l$, $u_h(B, p) \geq u_h^c$
  - $p \geq (\alpha\underline{\pi} + (1-\alpha)\bar{\pi})B$

Pooling equilibria



Existence of pooling equilibra

animation by animate[2012/12/06]

JOB MARKET SIGNALING GAME 劳动力市场的信号博弈

type

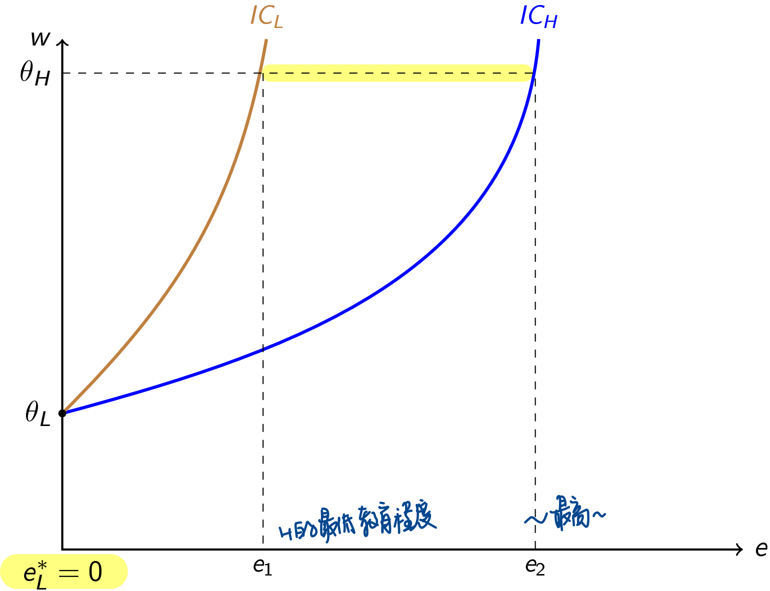
- Sequential-move game between firm and worker 高能力/低能力 工人
- Nature selects the type of a worker, θ_H or θ_L, $\theta_H > \theta_L$.
- Worker know own type θ, chooses $e \geq 0$. 教育程度(信号作用)
- Observing e but not θ, the firm offers wage $w(e)$.
- Worker's utility

$$u(w, e) = w - \frac{e}{2\theta}$$

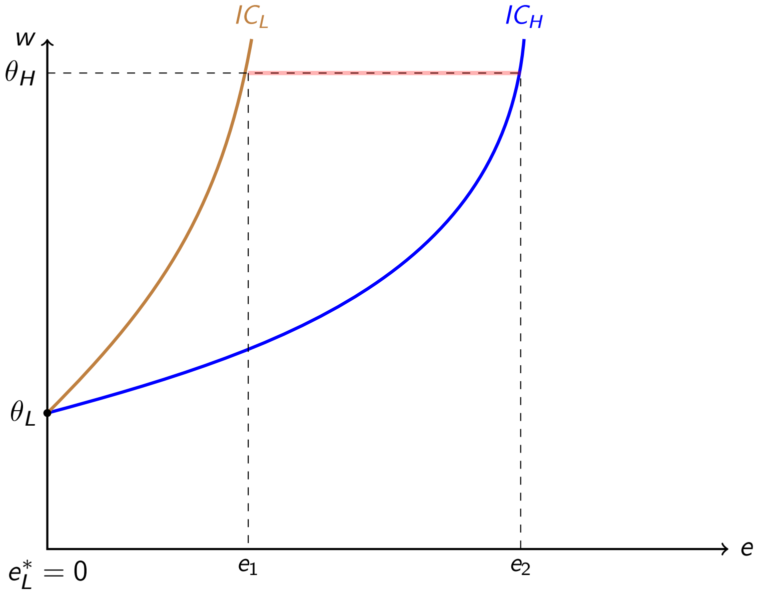
if accepts offer; zero otherwise.

SEPARATING EQUILIBRIUM 分离均衡

低能力的人在完全竞争市场中所得效用



Separating equilibrium



Apply IC to separating equilibrium

- Set of separating equilibria

$$\begin{aligned} e_L^* &= 0, & e_H^* &\in [e_1, e_2]; \\ w(e_L^*) &= \theta_L & w(e_H^*) &= \theta_H. \end{aligned}$$

Apply IC to separating equilibrium

- Set of separating equilibria

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- Take separating equilbrium $(e_L^* = 0, e_H^* = e_2)$;

Apply IC to separating equilibrium

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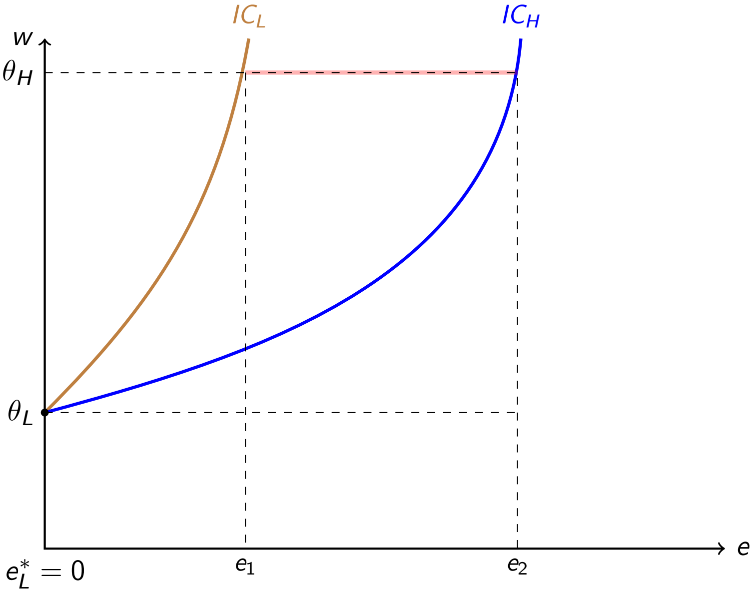
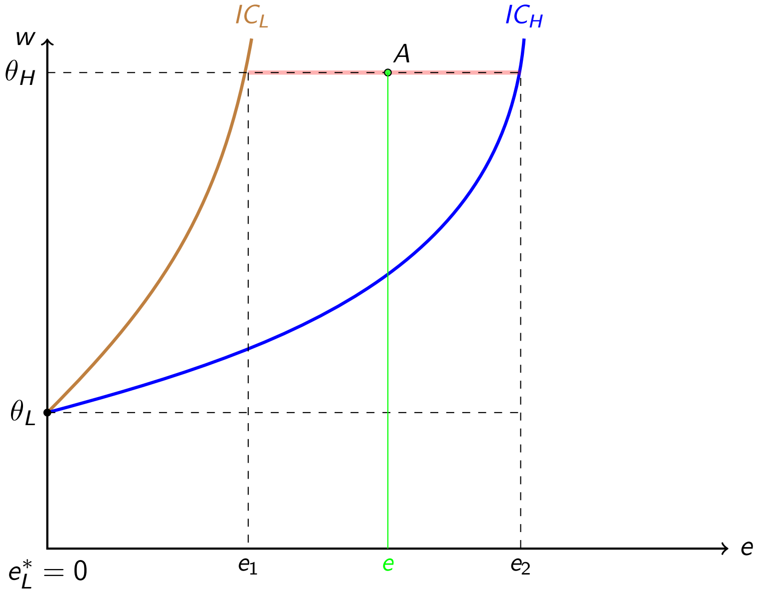
$$\begin{aligned} e_L^* &= 0, \qquad & e_H^* &\in [e_1, e_2]; \\ w(e_L^*) &= \theta_L \qquad & w(e_H^*) &= \theta_H. \end{aligned}$$

- Take separating equilbrium $(e_L^* = 0, e_H^* = e_2)$;
- Consider an off-the-equilibrium message $e \in (e_1, e_2)$.

First step



First step



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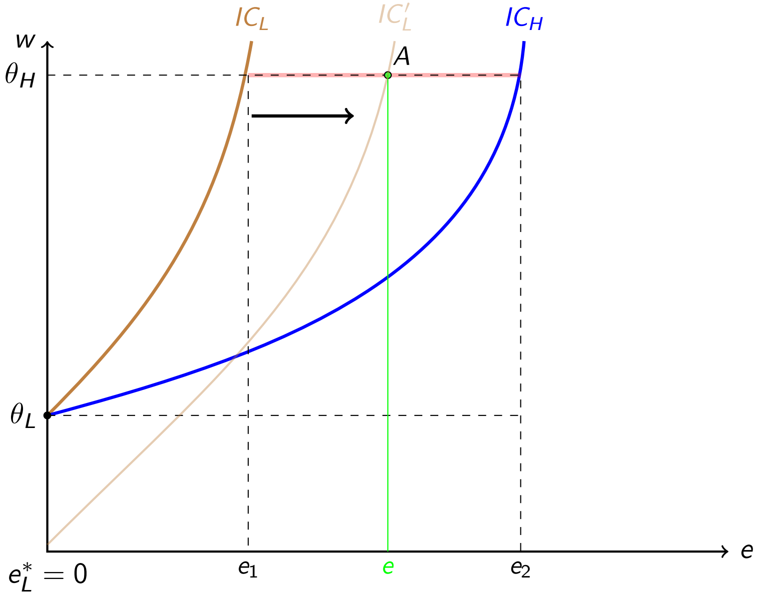

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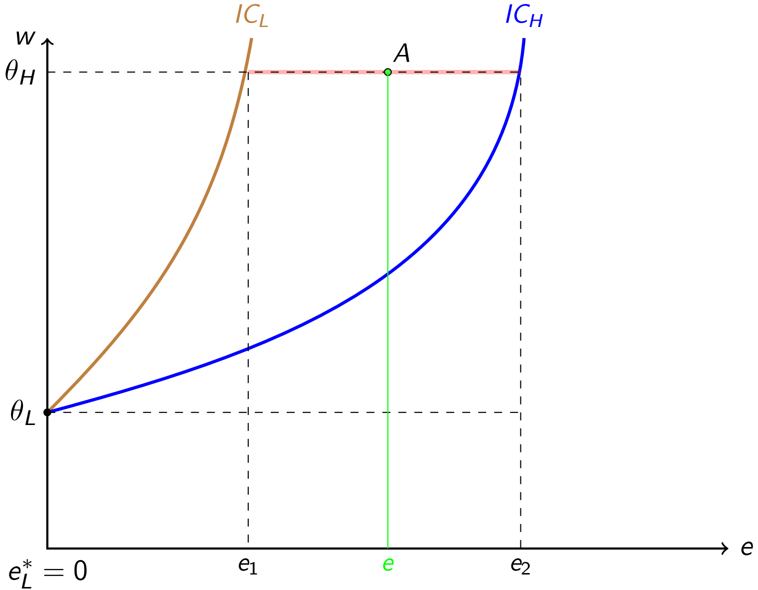
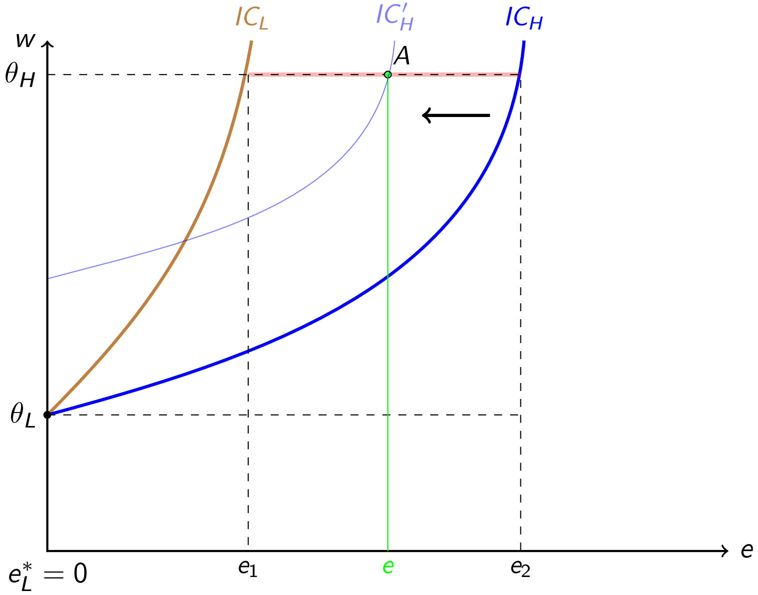


First step



First step

- θ_L type has no incentive to deviate

$$\underbrace{u_L^*(\theta_L)}_{\text{Equilibrium payoff}} > \underbrace{\max_{w \in W^*(\Theta, m)} u_L(e, w, \theta_L)}_{\text{Max payoff from deviating to } e}.$$

First step

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$$\underbrace{u_L^*(\theta_L)}_{\text{Equilibrium payoff}} \quad > \quad \underbrace{\max_{w \in W^*(\Theta, m)} u_L(e, w, \theta_L)}_{\text{Max payoff from deviating to } e} \quad .$$

- But θ_H type has incentive to deviate

$$\underbrace{u_H^*(\theta_H)}_{\text{Equilibrium payoff}} \quad < \quad \underbrace{\max_{w \in W^*(\Theta, m)} u_H(e, w, \theta_H)}_{\text{Max payoff from deviating to } e} \quad .$$

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- Thus, off-equilibrium education level can come only from θ_H

$$\Theta^{**}(e) = \{\theta_H\}.$$

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Second step

- Given e only comes from θ_H, best response for firm to offer $w(e) = \theta_H$;

Second step

- Given e only comes from θ_H, best response for firm to offer $w(e) = \theta_H$;
- For θ_H type

$$\underbrace{\min_{w \in W^*(\Theta^{**}(e), e)} u_H(e, w, \theta_H)}_{\theta_H - c(e, \theta_H)} > \underbrace{u_H^*(\theta_H)}_{\theta_H - c(e_2, \theta_H)} .$$

Second step

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- For θ_H type

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- Lowest payoff θ_H type obtains from deviating to e is higher than the equilibrium payoff;

Second step

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$$\underbrace{\min_{w \in W^*(\Theta^{**}(e), e)} u_H(e, w, \theta_H)}_{\theta_H - c(e, \theta_H)} > \underbrace{u_H^*(\theta_H)}_{\theta_H - c(e_2, \theta_H)} .$$

- Lowest payoff θ_H type obtains from deviating to e is higher than the equilibrium payoff;
- Thus, the separating equilibrium

$$\{e_L^*(\theta_L), e_H^*(\theta_H)\} = \{0, e_2)\}$$

violates IC.

Three types

- Now suppose there are three types: θ_L, θ_M and θ_H;

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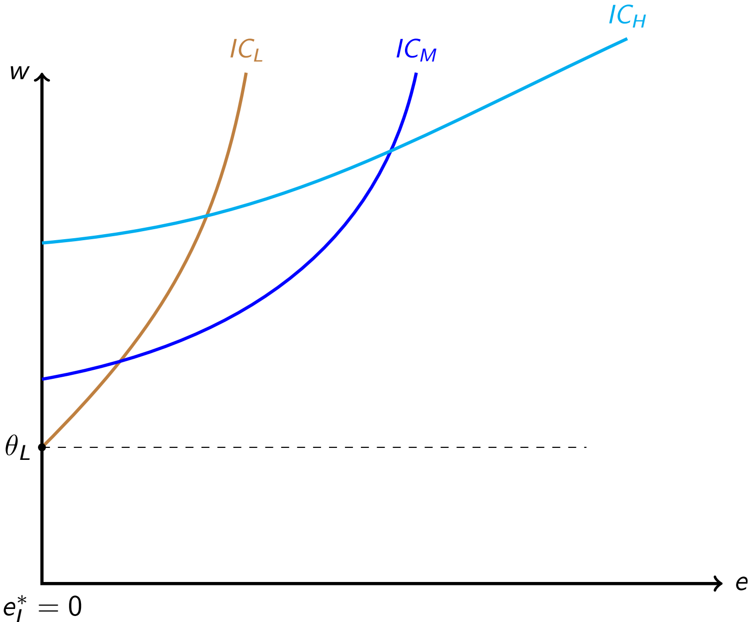
- Take one off-the-equilibrium message $e \in (\widehat{e}, e_H^*)$.

IC first step



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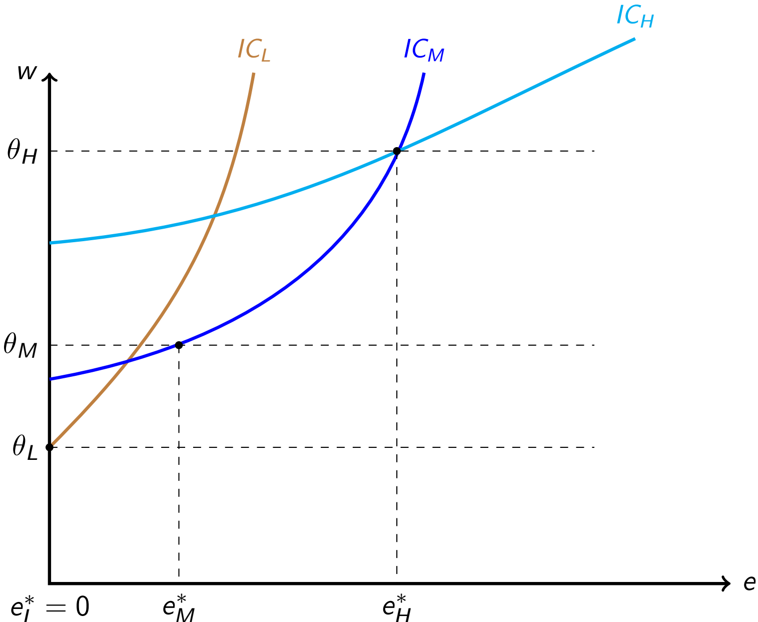
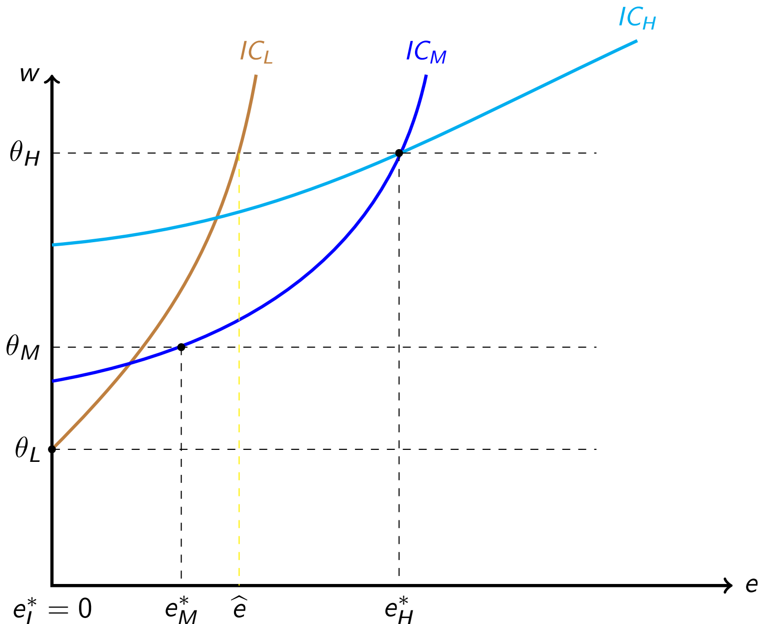


IC FIRST STEP



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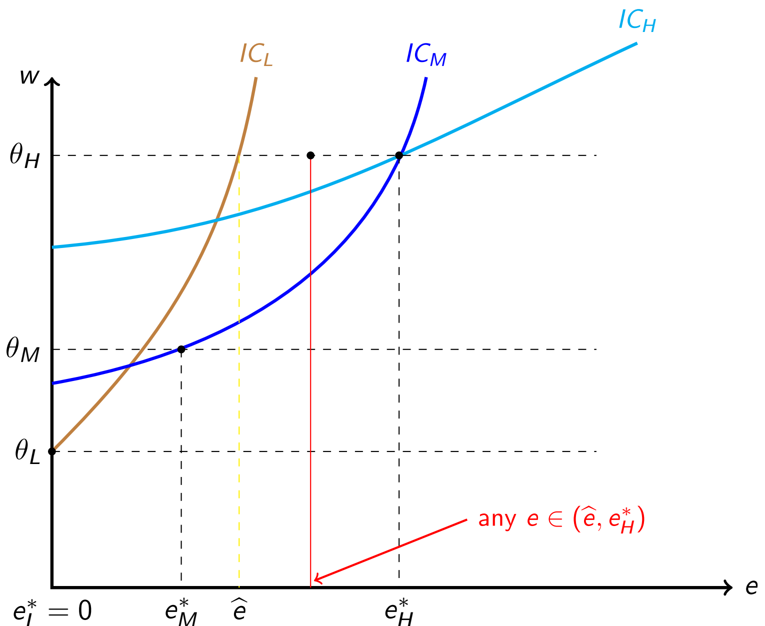

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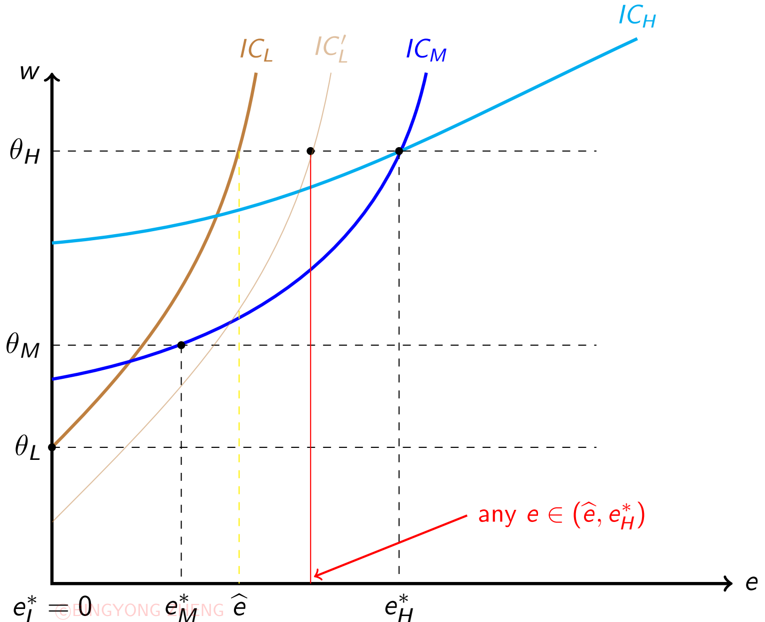

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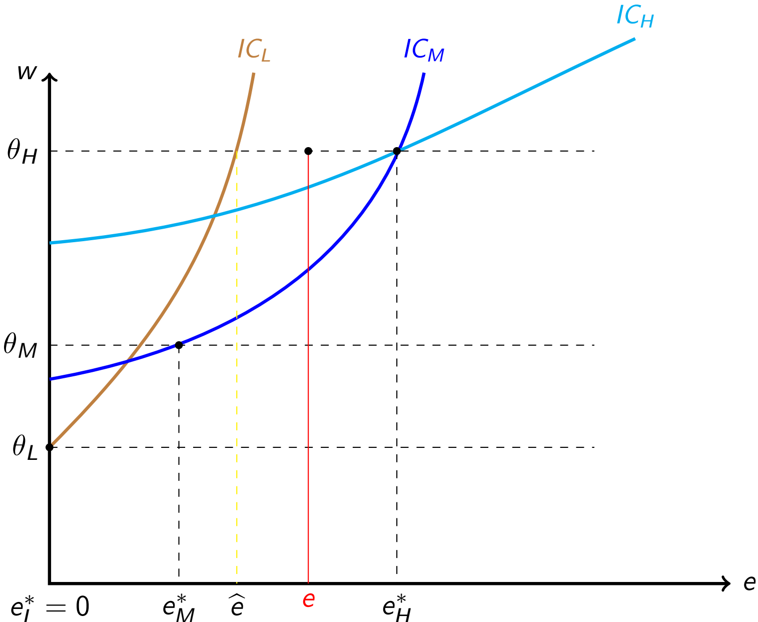

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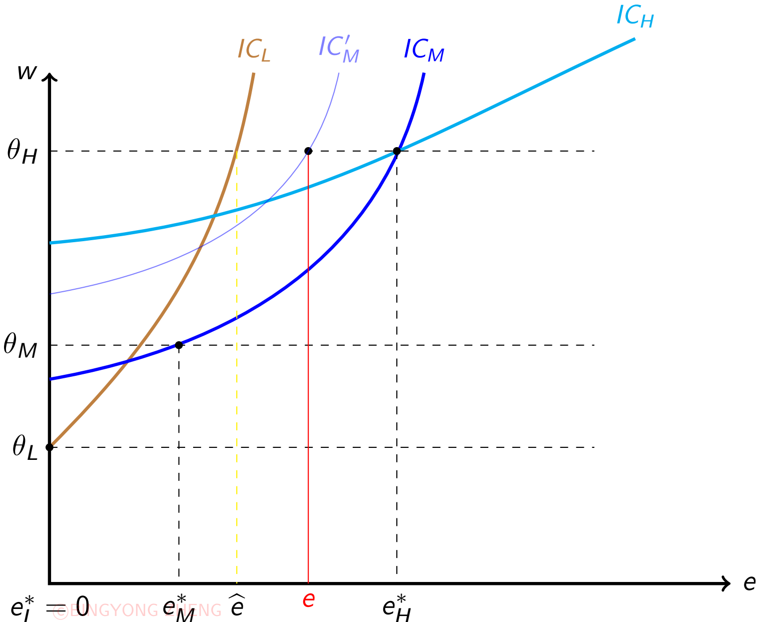

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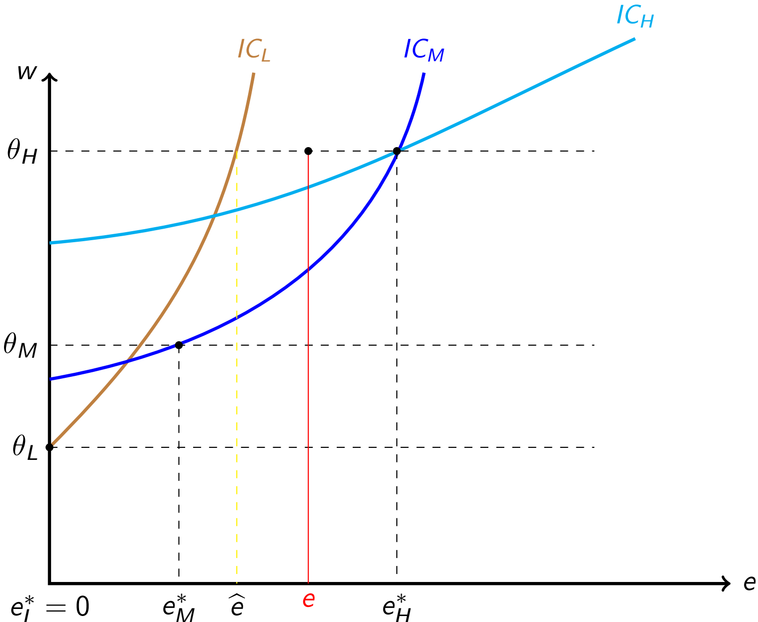

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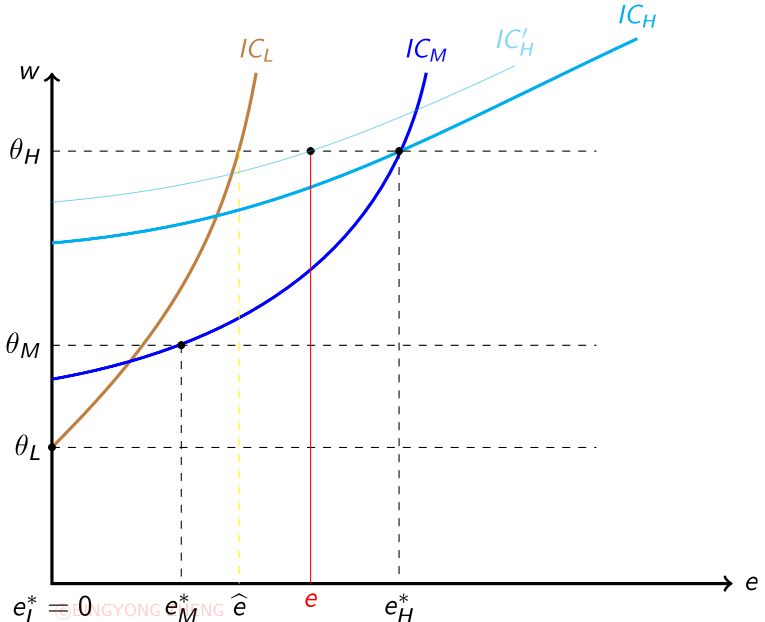

First step

- θ_L type sending message $e \in (\widehat{e}, e_H^*)$ is equilibrium dominated

$$\underbrace{u_L^*(\theta_L)}_{\text{Equilibrium payoff}} > \underbrace{\max_{w \in W^*(\Theta, m)} u_L(e, w, \theta_L)}_{\text{Max payoff from deviating to } e} .$$

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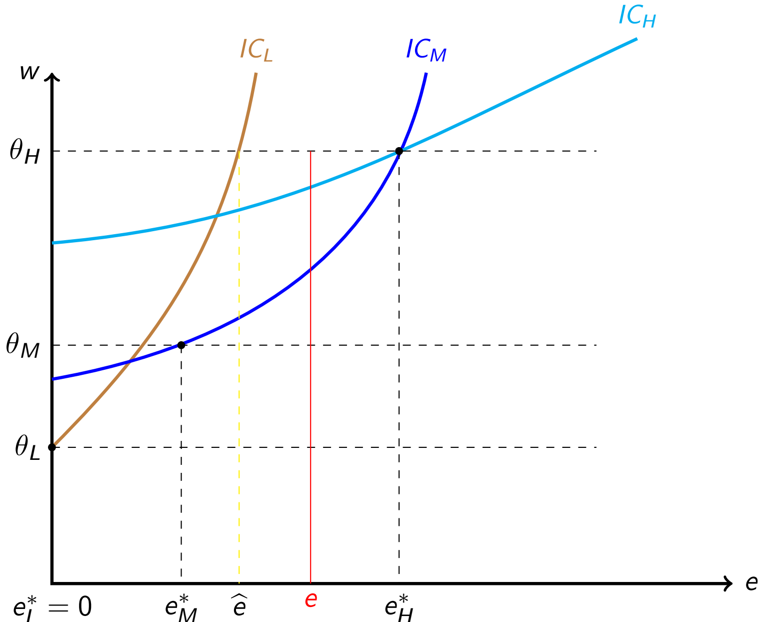
- Hence, observing $e \in (\widehat{e}, e_H^*)$, the firm's belief concentrate on θ_M and θ_H:

$$\Theta^{**} = \{\theta_M, \theta_H\}.$$

IC second step



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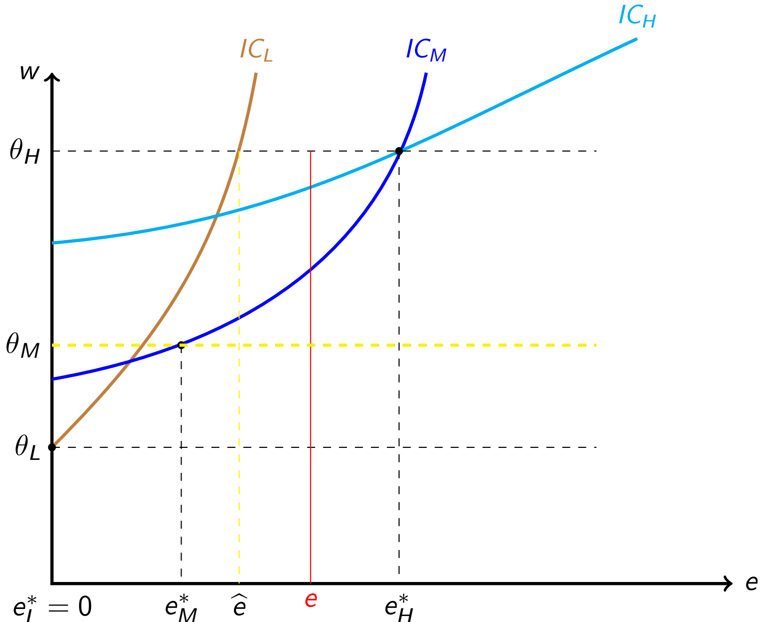
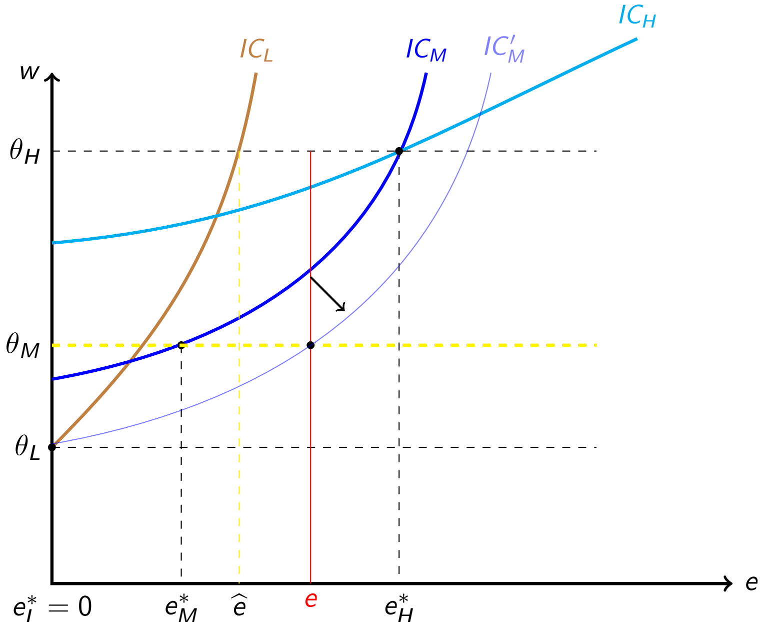


IC SECOND STEP



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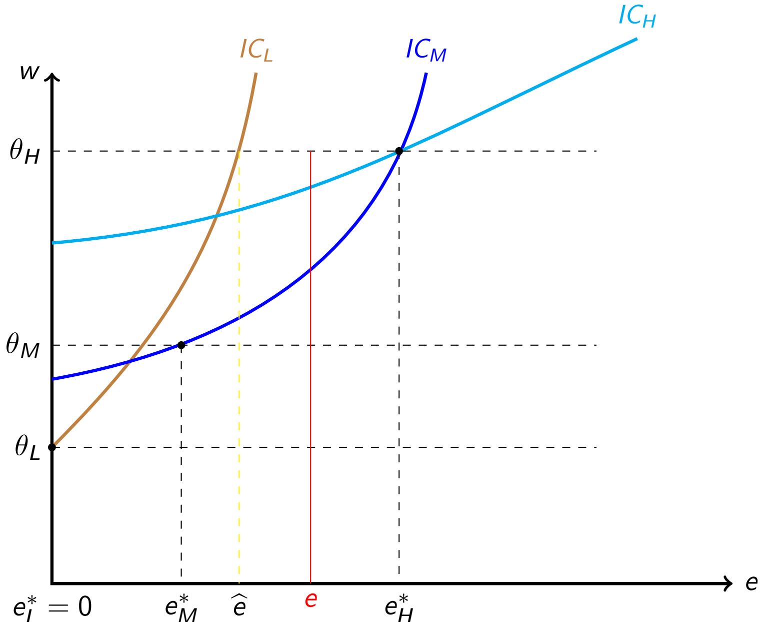

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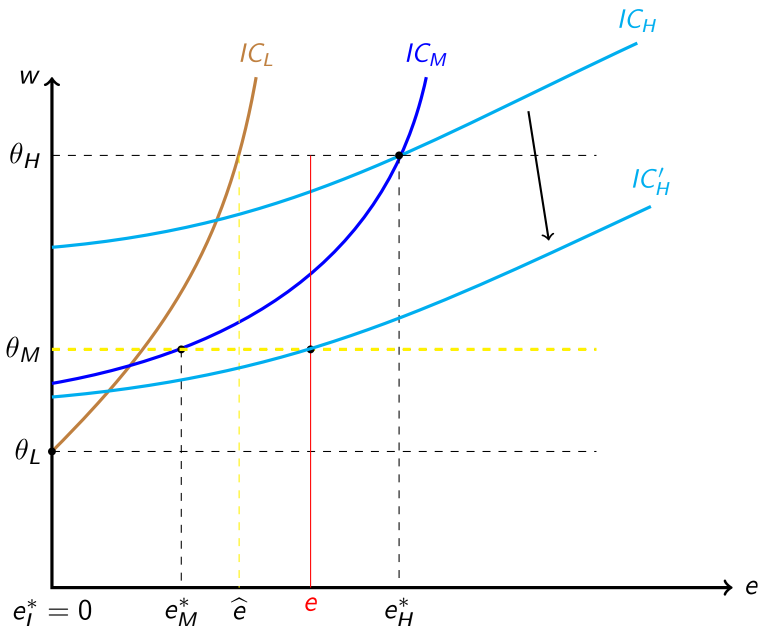

Second step

- Given firm's belief $\Theta^{**} = \{\theta_M, \theta_H\}$, the lowest wage to offer

$$\theta_M = \min\{w | w \in W^*(\Theta^{**}(e), e)\}.$$

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- Hence, there is no type of worker $\theta \in \Theta^{**}$ for whom deviation to $e \in (\widehat{e}, e_H^*)$ is profitable.

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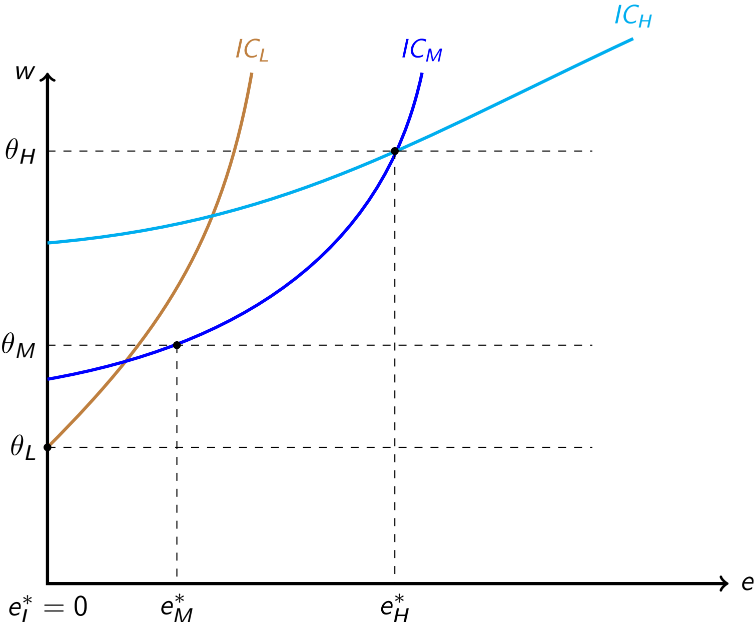
- Also let

$$D^o(\theta_k, \widehat{\Theta}, e') \equiv \{w \in [\theta_L, \theta_H] | u_k(e', w, \theta_k) = u_k^*(\theta_k)\}.$$

D1



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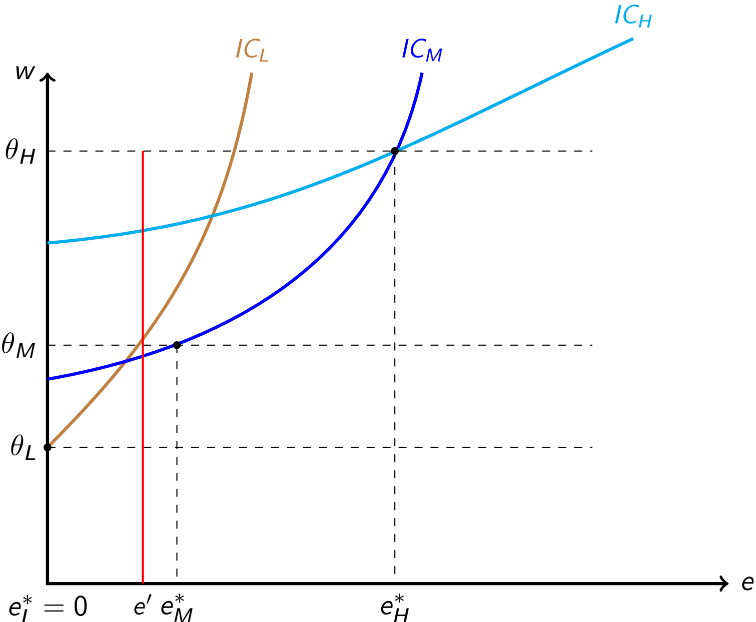

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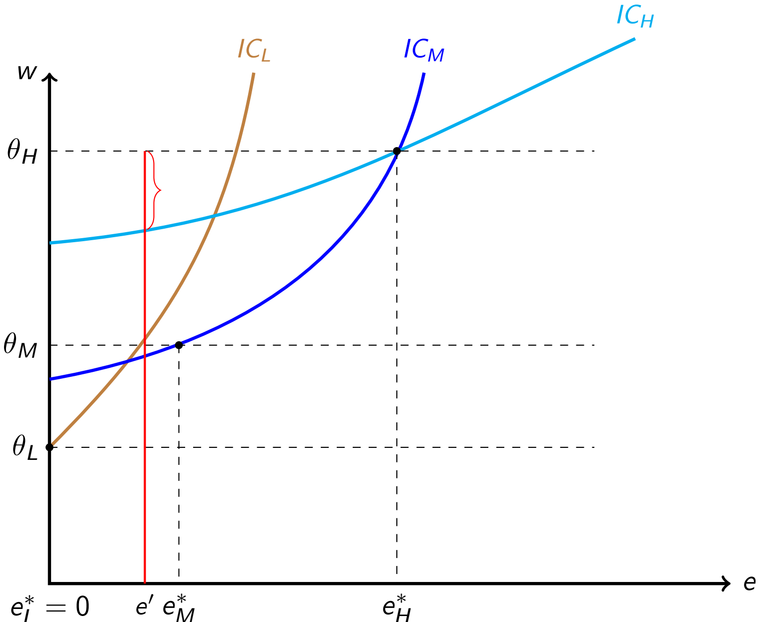

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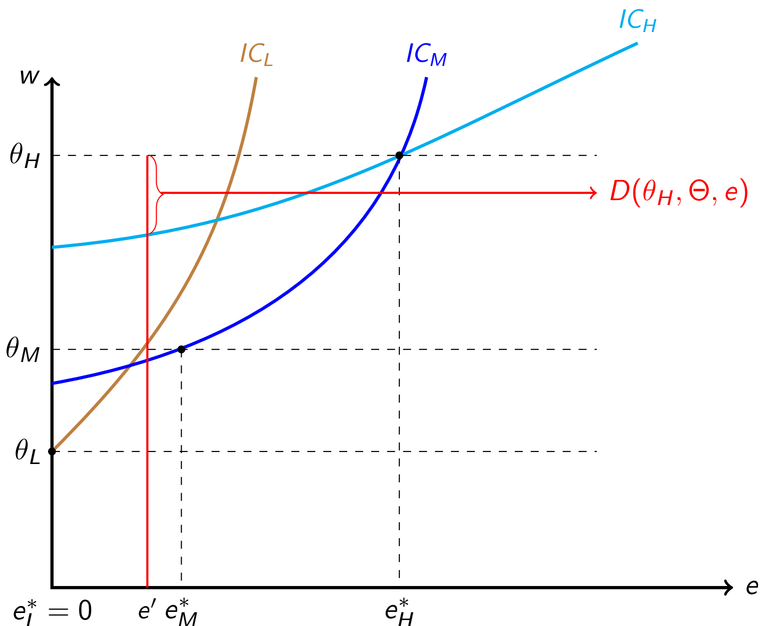

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IC_M
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θ_H
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θ_L
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e' e_M^*
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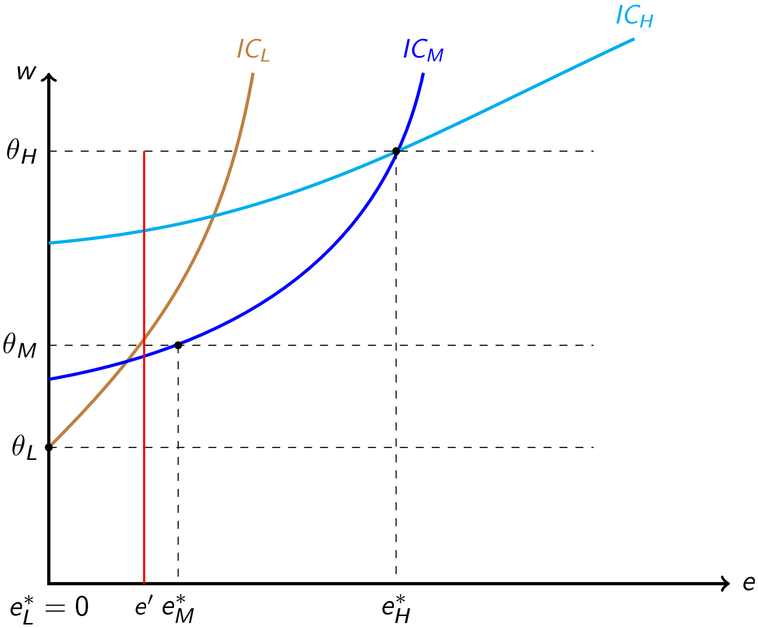

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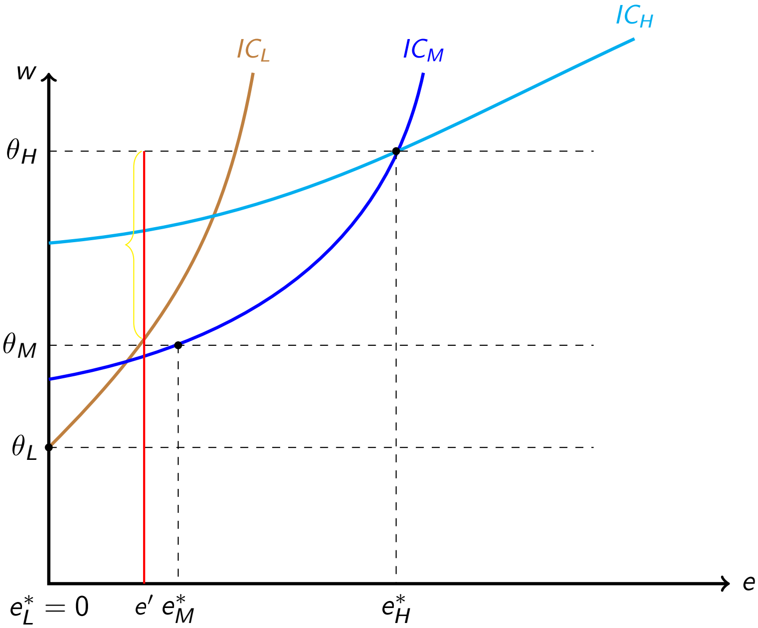
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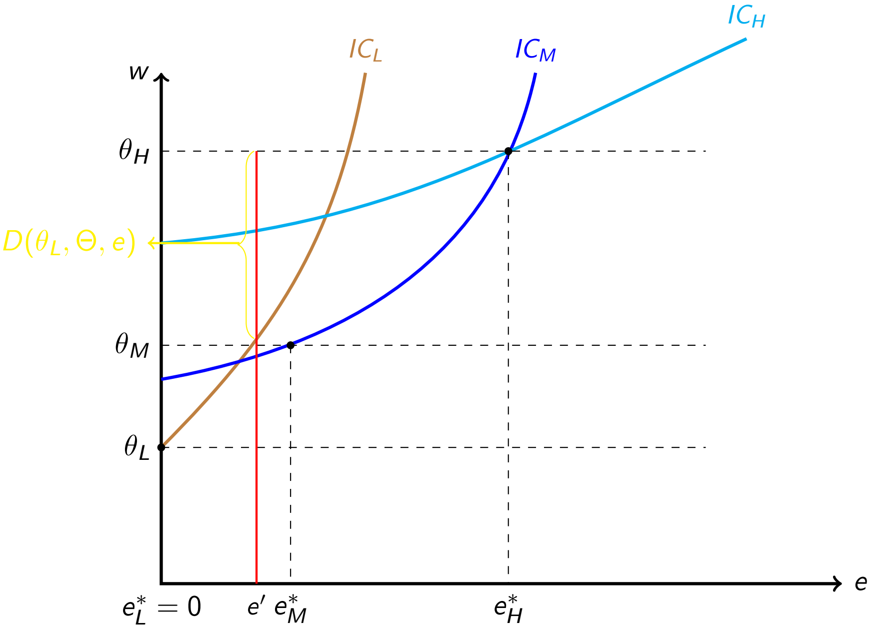

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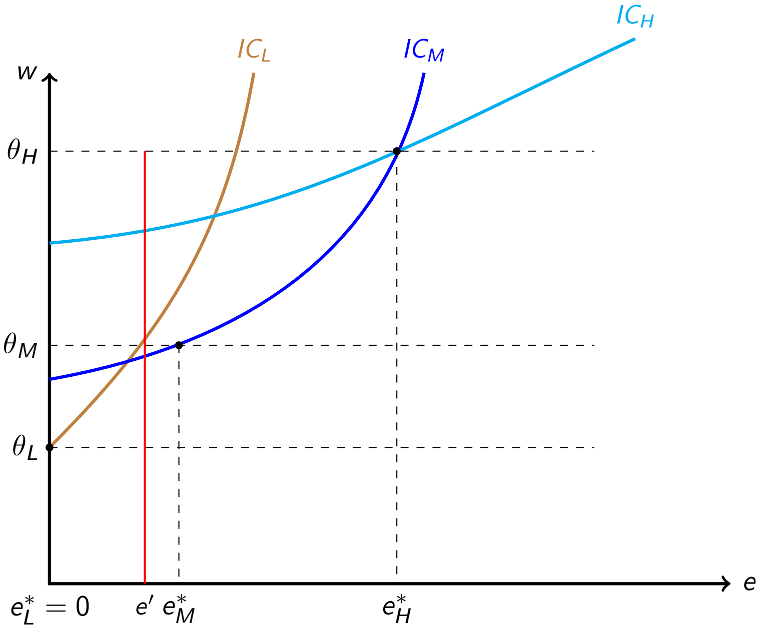

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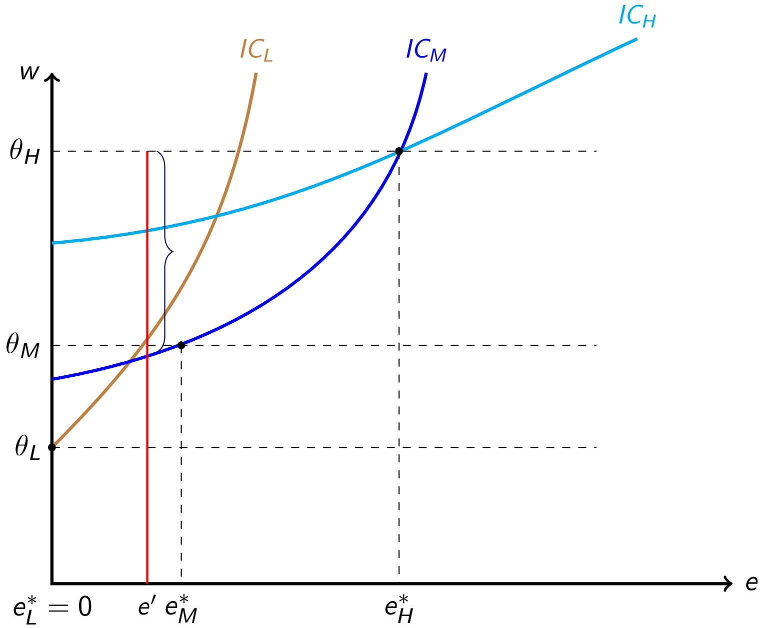

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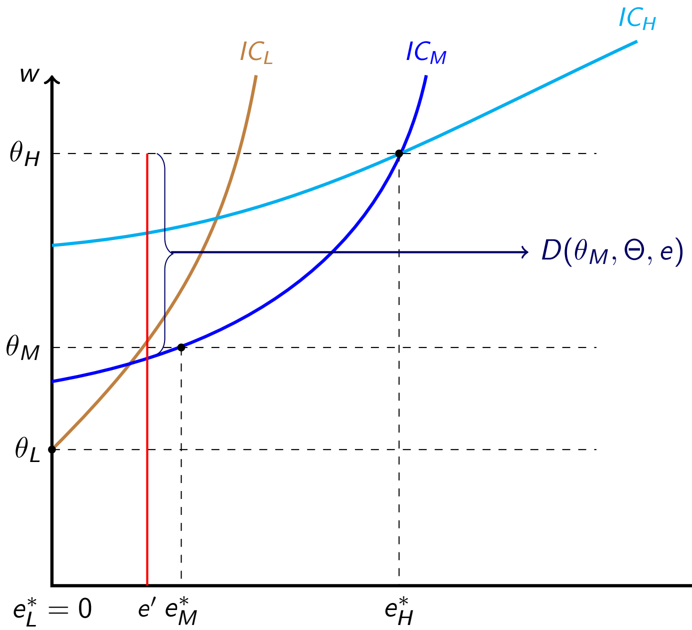

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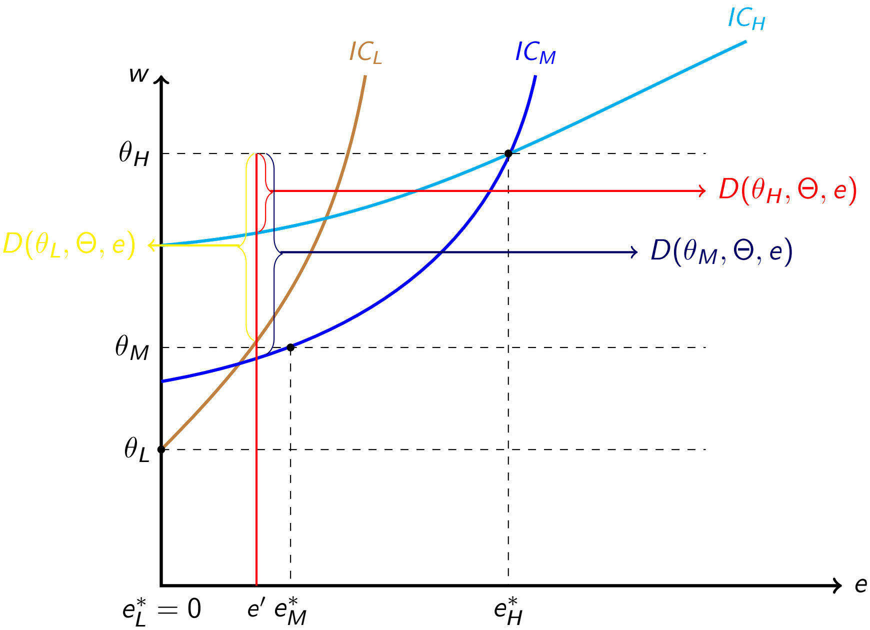

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D1 first step

- We see from the figure

$$D(\theta_H, \widehat{\Theta}, e') \bigcup D^o(\theta_H, \widehat{\Theta}, e') \subset D(\theta_M, \widehat{\Theta}, e').$$

θ_M type has more incentives to deviate to e' than θ_H type

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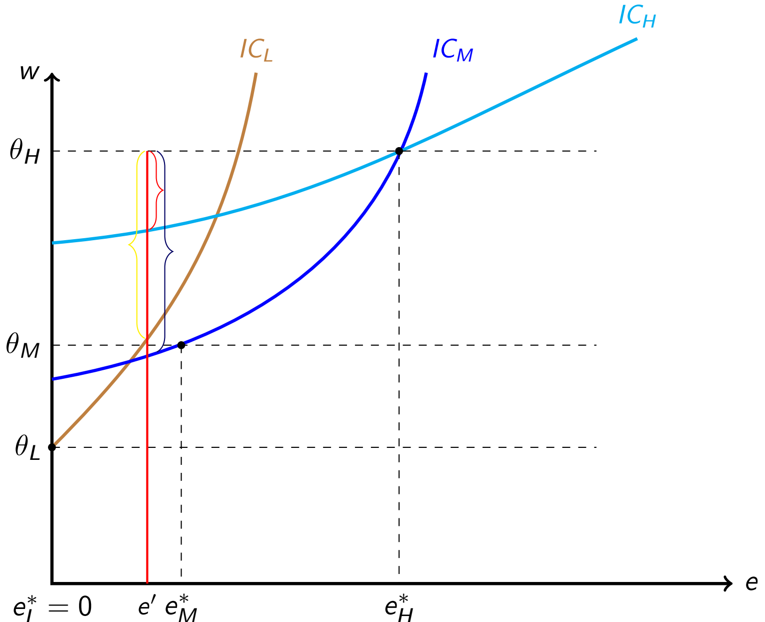
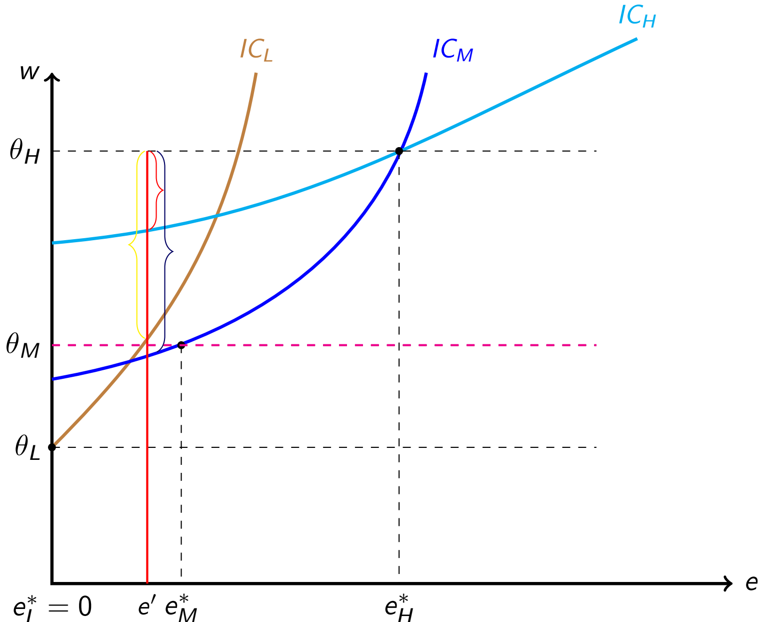
  θ_M type has more incentives to deviate to e' than θ_L type
- Applying the D1 criterion, the θ_M type is the most likely to deviate to e'

$$\Theta^{**}(e') = \{\theta_M\}.$$

D1 SECOND STEP



D1 second step



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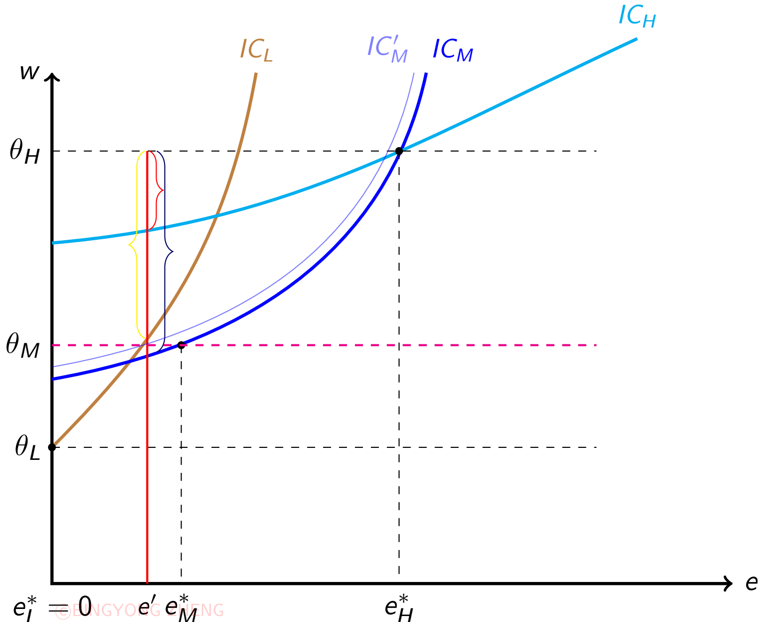

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Deviating towards e' is profitable!

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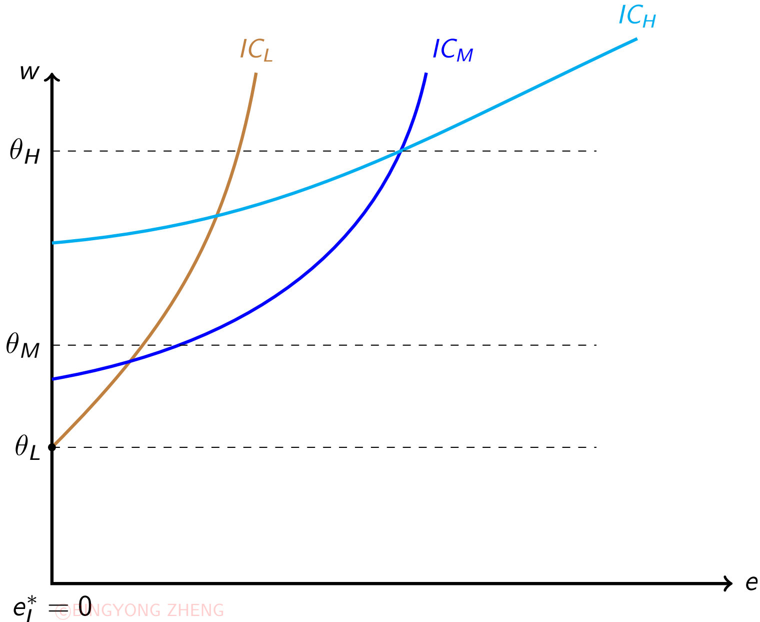
  Deviating towards e' is profitable!

- So the equilibrium (e_L^*, e_M^*, e_H^*) violates the D1 criterion

D1 second step continued



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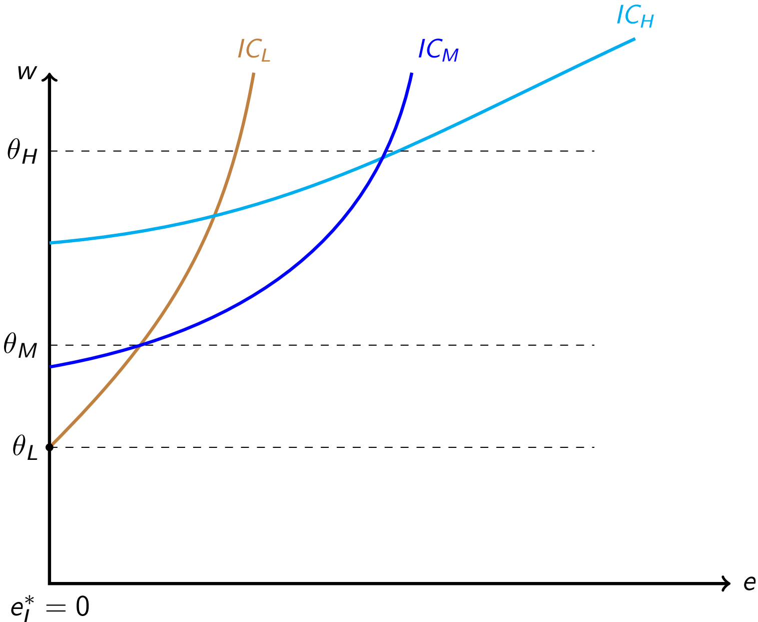

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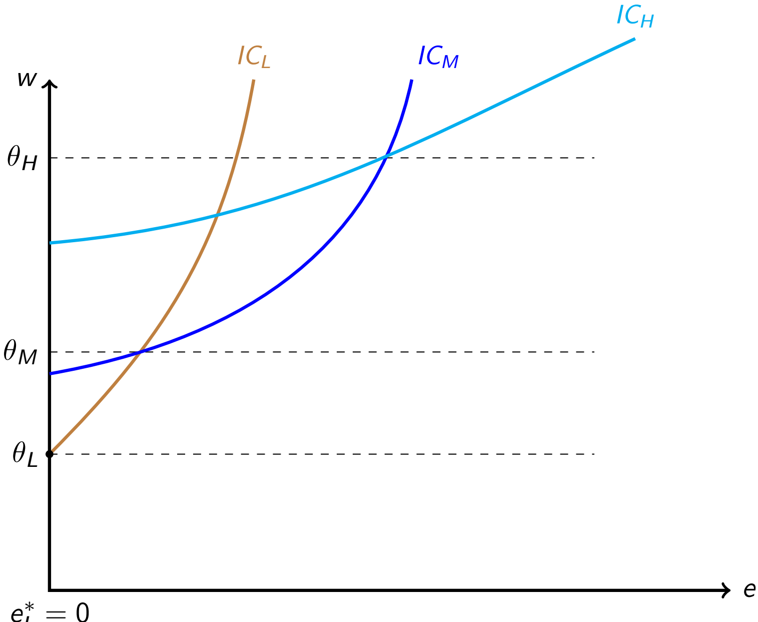

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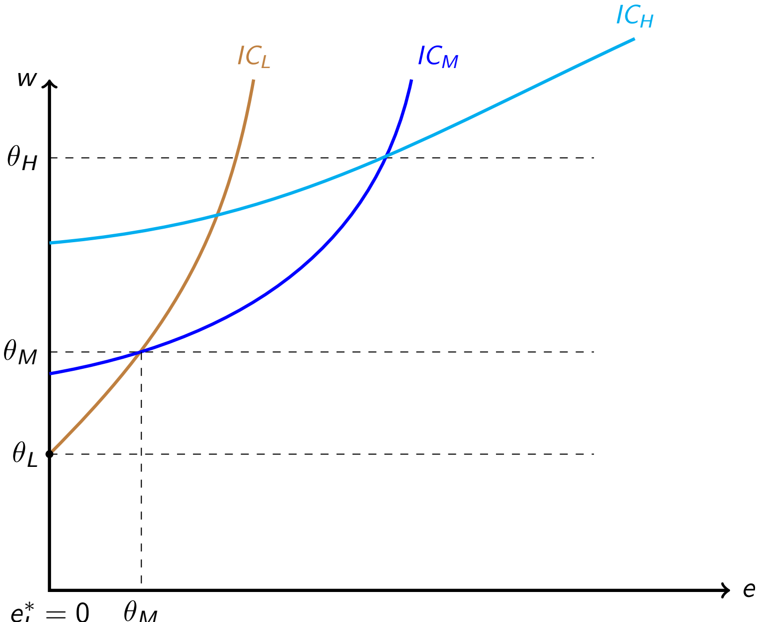

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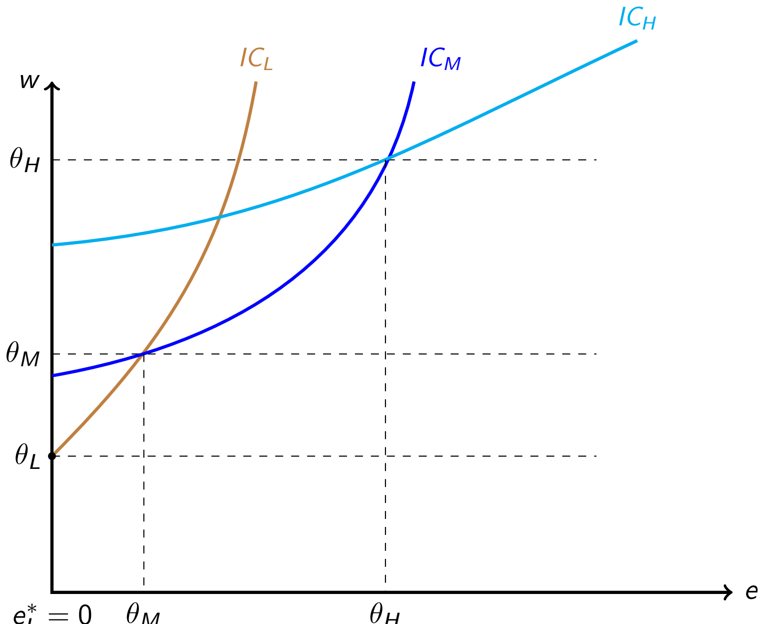

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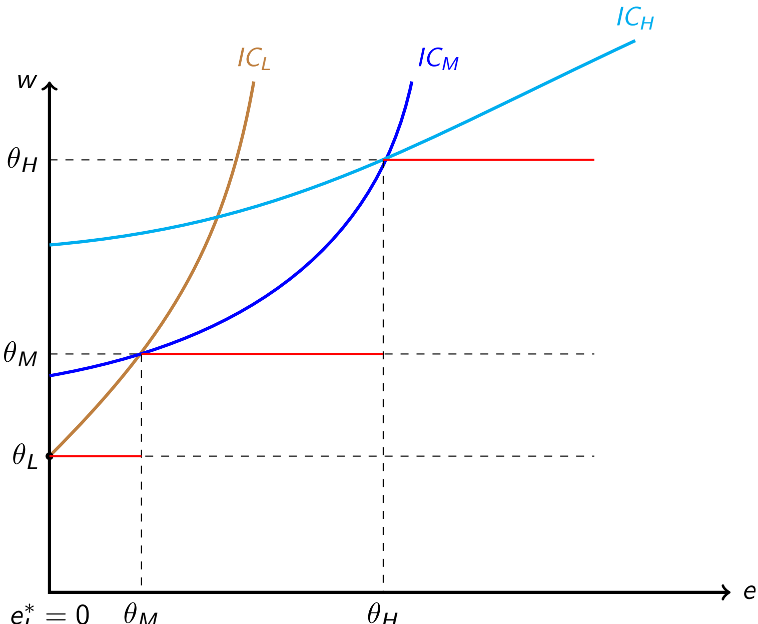
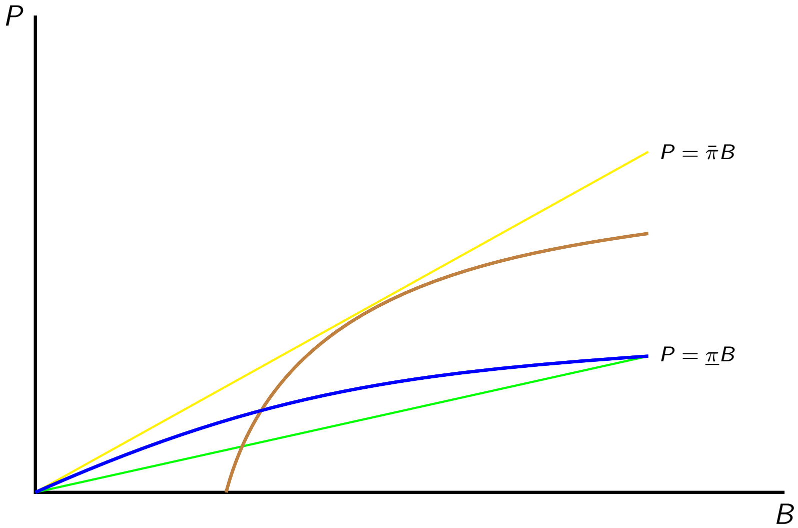
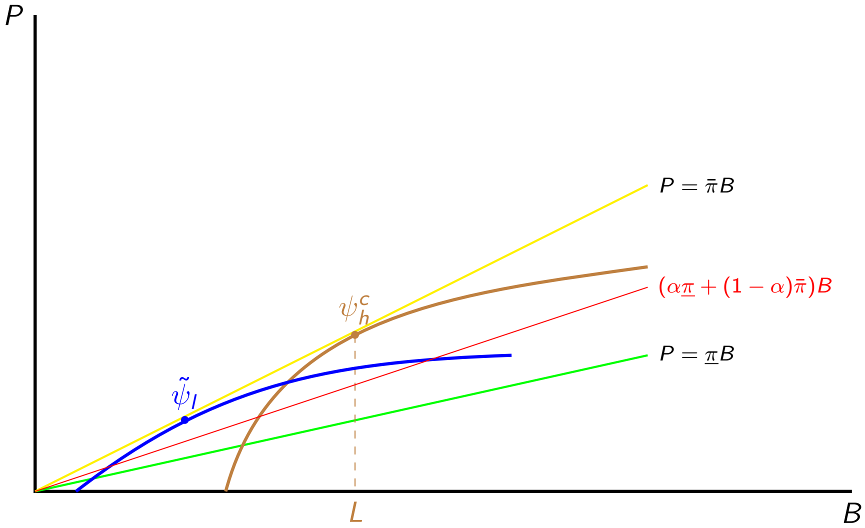


Insurance model: separating equilibrium



Insurance model: pooling equilibrium



Apply IC to insurance model

- IC to insurance signaling game: Sequential equilibrium $(\psi_l, \psi_h, \sigma, \beta)$ satisfy IC if for all ψ ($\psi \neq \psi_l$ or $\psi \neq \psi_h$),

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  - ➢ $u_h(\psi) > u_h^*$ and $u_l(\psi) < u_l^* \Longrightarrow \beta(\psi) = 0$.
- Theorem 8.3. (Jehle& Reny) There is a unique policy pair (ψ_l, ψ_h) that can be supported by a sequential equilibrium satisfying the intuitive criterion. And this equilibrium is the best separating equilibrium for the low-risk consumer.

Screening: competition

animation by animate[2012/12/06]

Competitive screening

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- Cream skimming occurs when one insurance company takes strategic advantage of the set of policies offered by the other by offering a policy that would attract away only the low-risk consumers from the competing company.

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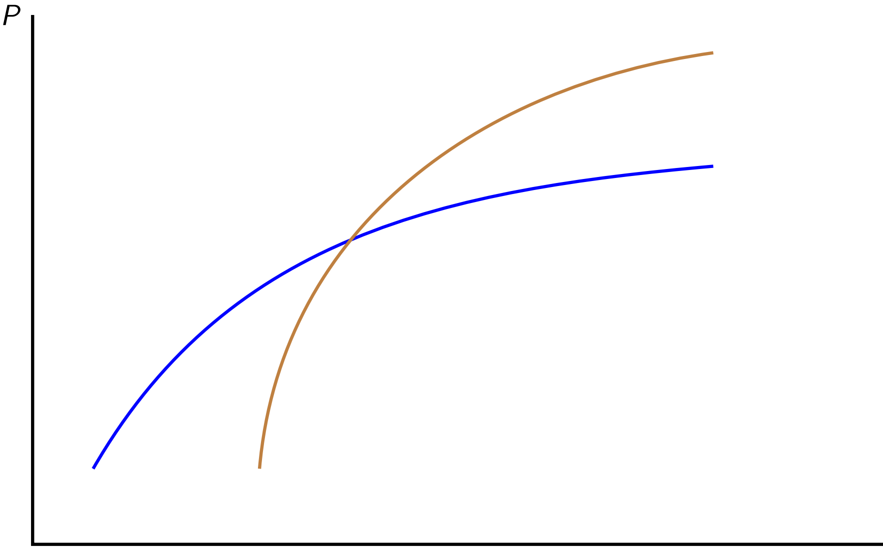
- Model: assume two insurance companies the engage in Bertrand competition;
- Firms offer policies to screen consumers: high-risk type chooses one policy and low-risk choose another;
- Cream skimming occurs when one insurance company takes strategic advantage of the set of policies offered by the other by offering a policy that would attract away only the low-risk consumers from the competing company.
- Lemma 8.2. (Jehle & Reny) Insurance companies earn zero expected profits in equilibrium.

Existence of Pooling equilibrium



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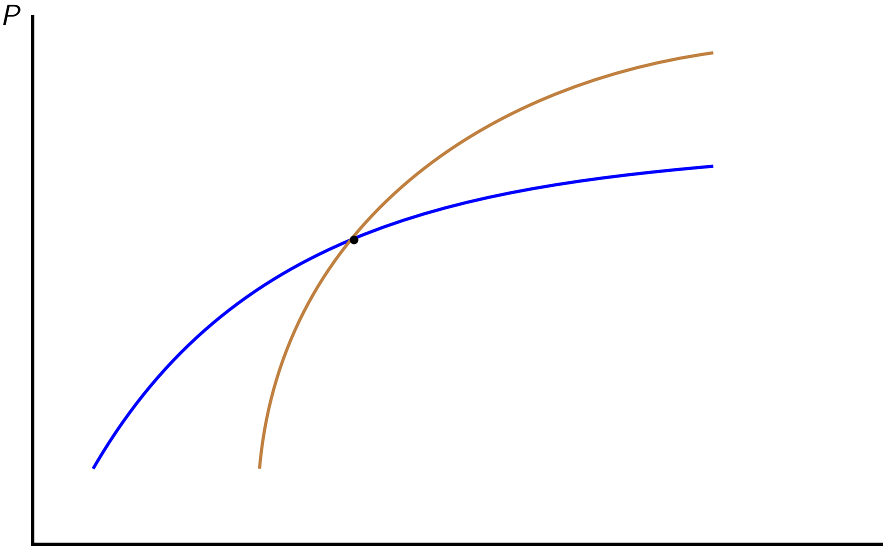

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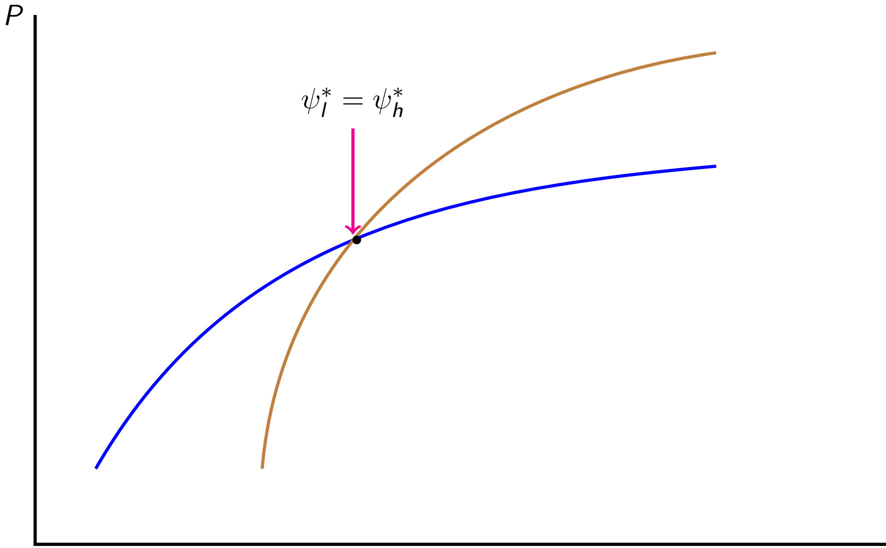

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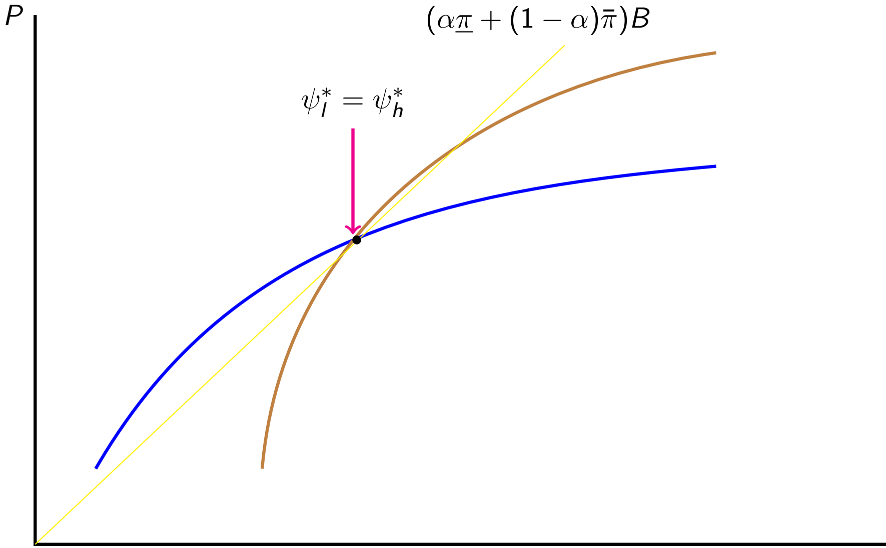

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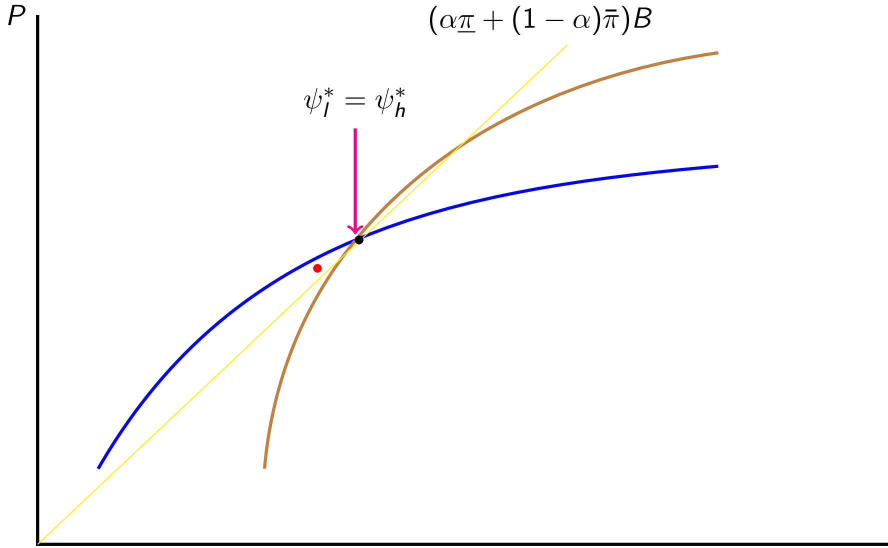

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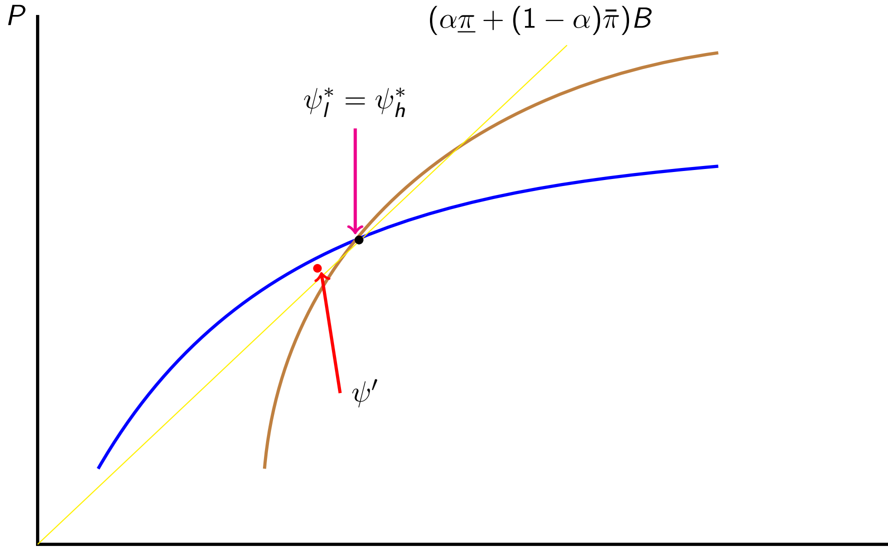

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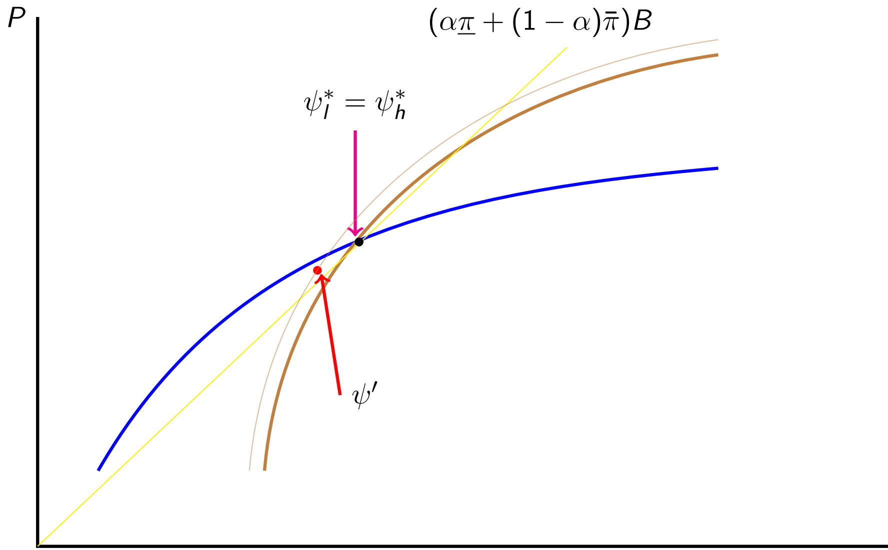

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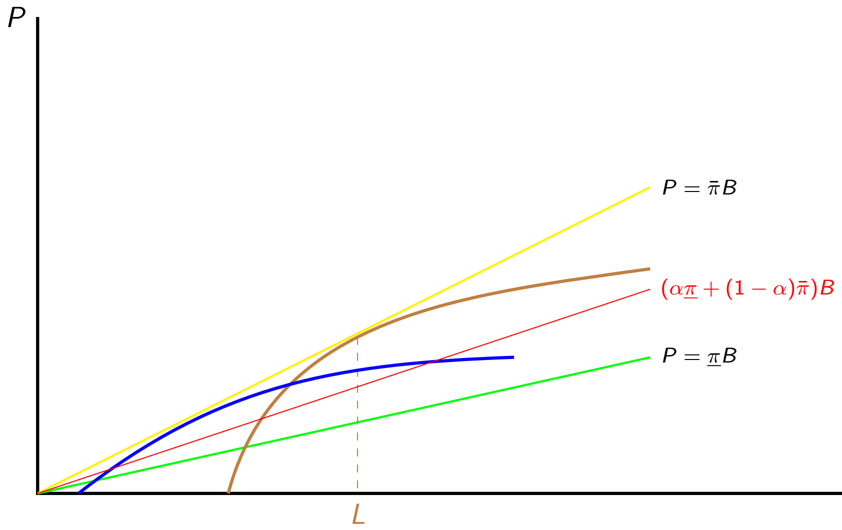

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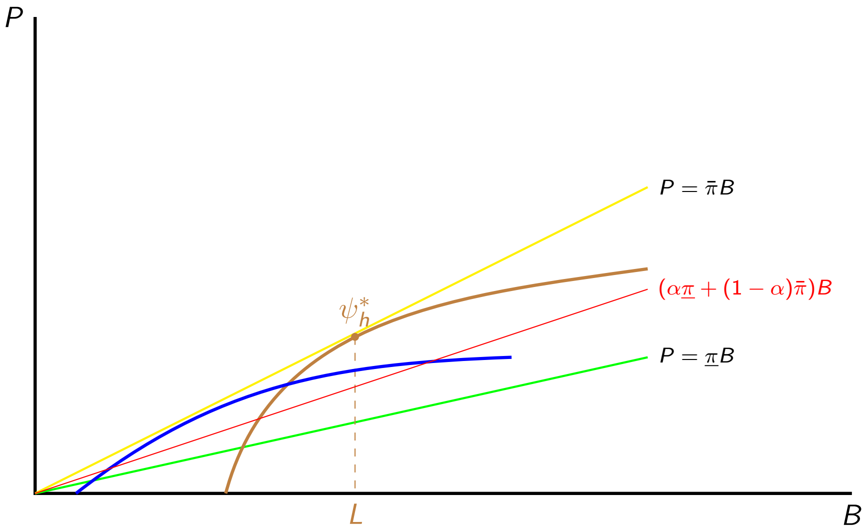

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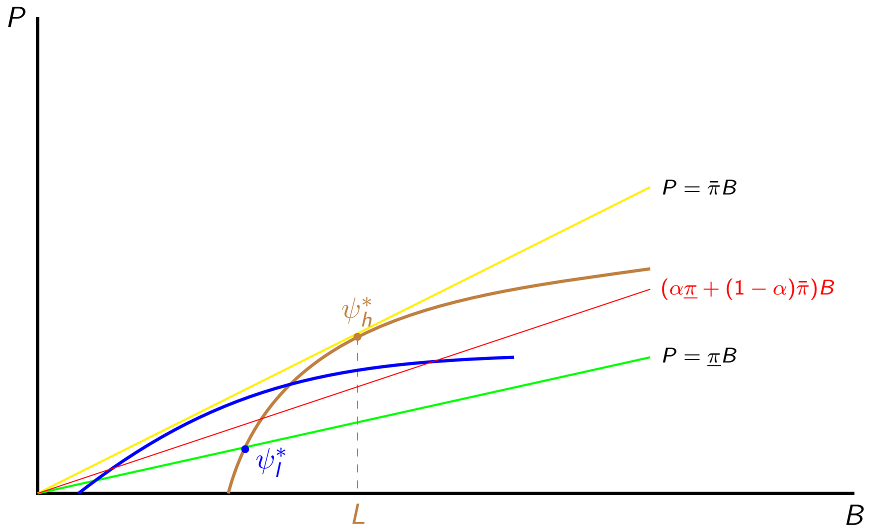

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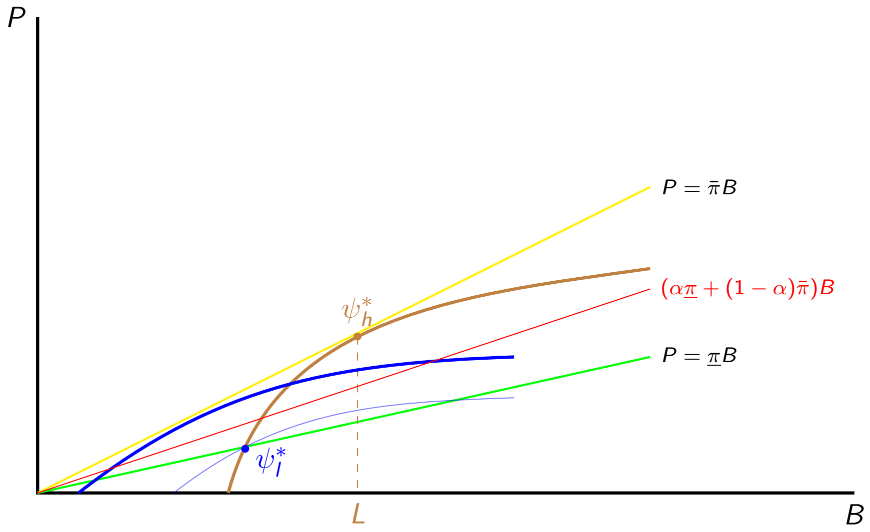

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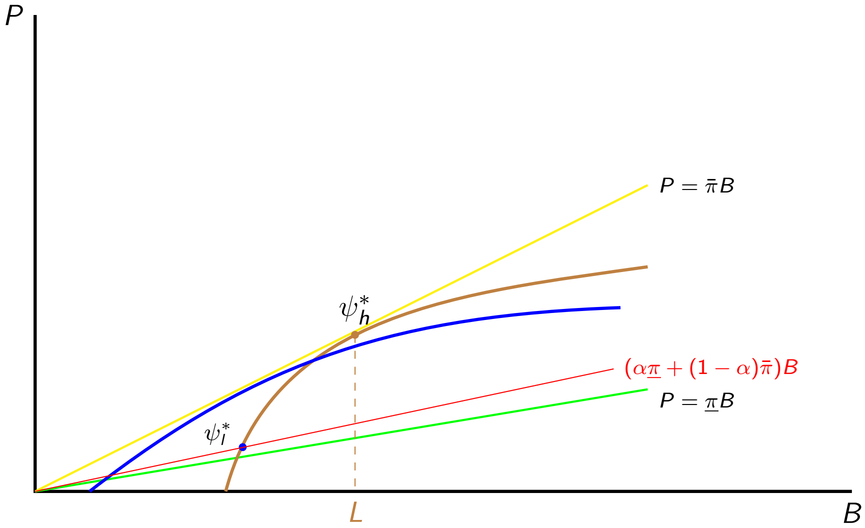

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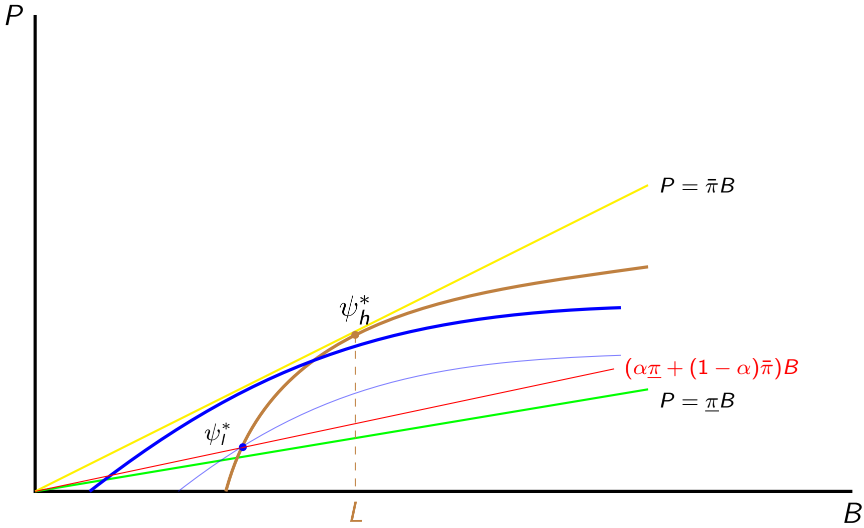

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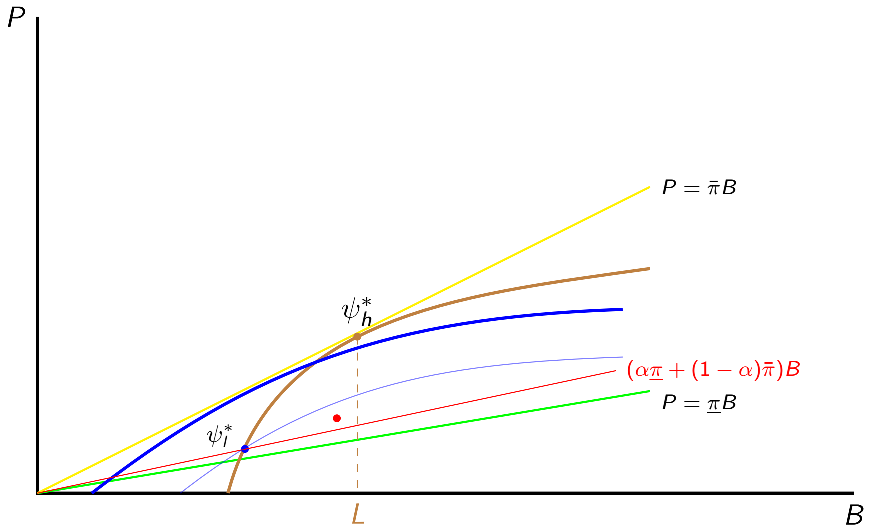

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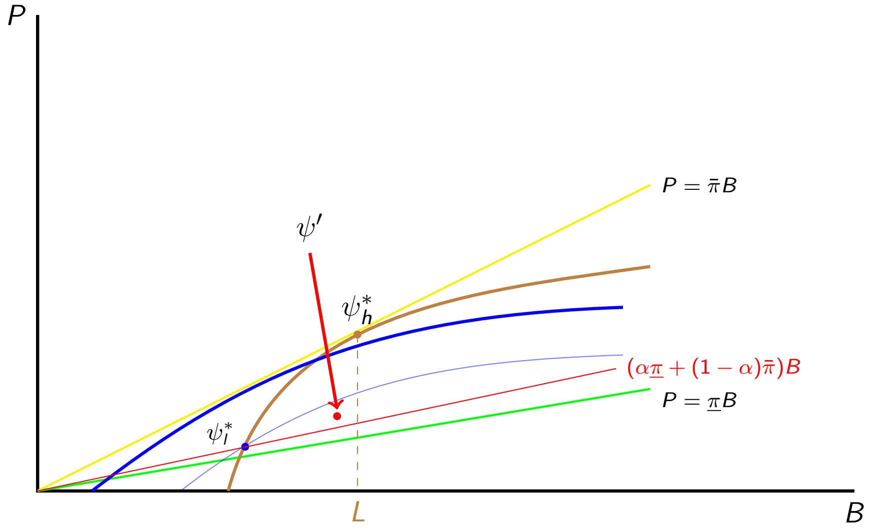

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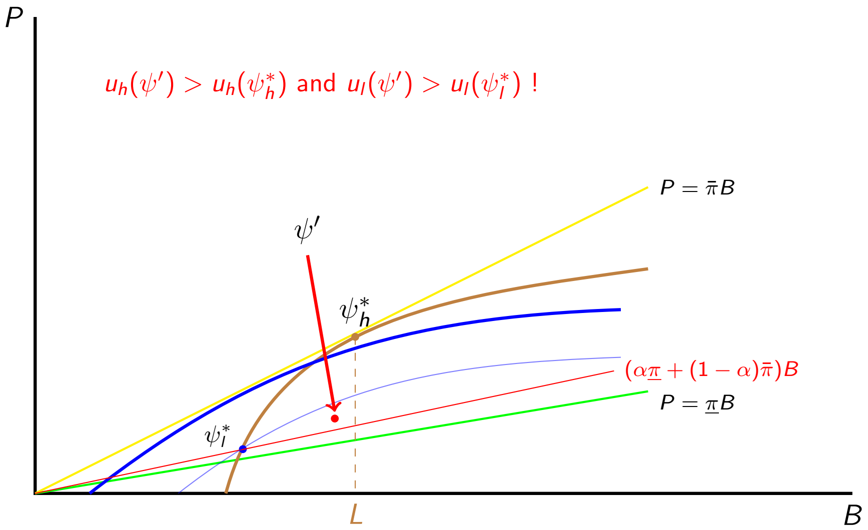

Main result

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- Theorem 8.6. (Jehle & Reny) No pure strategy equilibrium may exist if the proportion of high-risk is too low.

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- Insurance company chooses policy $(p, B_0, B_1, \ldots, B_L)$ to maximize profit.

$$\max_{e,p,B_l} p - \sum_{l=0}^{L} \pi_l(e) B_l, \qquad \textit{subject to}$$

$$\sum_{l=1}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \bar{u}.$$

Symmetric information optimal contract

- Lagrangian:

$$\mathcal{L} = p - \sum_{l=0}^{L} \pi_l(e) B_l + \lambda \left[\sum_{l=1}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) - \bar{u} \right].$$

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$$\frac{\partial \mathcal{L}}{\partial p} = 1 - \lambda \left[\sum_{l=1}^{L} \pi_l(e) u'(w - p - l + B_l) \right] = 0, \qquad (1)$$

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- Thus it is optimal to have

$$B_l = l \qquad for l = 0, 1, \ldots, L.$$

Asymmetric information

- Optimization problem

$$\max_{e,p,B_l} p - \sum_{l=0}^{L} \pi_l(e) B_l, \qquad \textit{subject to}$$

$$\sum_{l=1}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \bar{u};$$

$$\sum_{l=1}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \sum_{l=1}^{L} \pi_l(e') u(w - p - l + B_l) - d(e').$$

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- Optimal policy $e = 1$:

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$$\beta \left[\sum_{l=1}^{L} \pi_l(1) u(w - p - l + B_l) - \sum_{l=1}^{L} \pi_l(0) u(w - p - l + B_l) - d(1) + \right.$$

Second best contract

- First order conditions:

$$1-\lambda\left[\sum_{l=1}^{L}\pi_l(1)u'(w-p-l+B_l)\right]-\beta\left[\sum_{l=1}^{L}(\pi_l(1)-\pi_l(0))u'(w-p-l+B_l)\right]$$
$$=0;$$
$$-\pi_l(1)+[\lambda\pi_l(1)+\beta(\pi_l(1)-\pi_l(0))]u'(w-p-l+B_l)=0 \quad \forall l; \qquad (*)$$
$$\sum_{l=1}^{L}\pi_l(1)u(w-p-l+B_l)-d(1)-\bar{u}\geq 0;$$
$$\sum_{l=1}^{L}(\pi_l(1)-\pi_l(0))u(w-p-l+B_l)+d(0)-d(1)\geq 0.$$

Second best contract (continued)

- Equation (*) implies

$$\frac{1}{u'(w - p + B_I - I)} = \lambda + \beta \left[1 - \frac{\pi_I(0)}{\pi_I(1)} \right]. \qquad \text{(CON-OP)}$$

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- Thus,

$$l - B_l \quad \text{is strictly increasing in} \quad l = 0, 1, \ldots, L.$$

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- Agent is forced to carry excess responsibility for the outcome and this is the implicit costs involved in contracting under imperfect information.