Theory of Corporate Finance

Dynamic Contracting

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Motivation

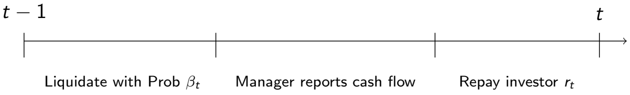
- Some positive NPV project may not be funded due to agency frictions.
  - The inefficiency can be alleviated if the financing relationship becomes long-term.
- Some financing contract in practice depends on firm performance history, e.g., credit line.
- Empirically, many managerial incentive pay or rents are back-loaded.
- Dynamic contracting models potentially rationalize these firm financing and compensation patterns, as well as reducing the constraints or frictions.

Setup

- Firm produces cash flows $x_t \in \{\ell, h\}$ in period t, which are iid.
- Running the firm in each period incurs a cost $c > 0$
  - It can be interpreted as a working capital requirement.
- Assume: $h > c > \ell \geq 0, \quad p = Prob(x_t = h), \quad \mu = E(x_t) > c$.
- All agents are risk-neutral with zero discounting.
- Conflict of interest:
  - Manager privately observes x_t and can divert $h - \ell$.

Contracting and timing

- The timing of events in period t is:



- Given the history of reported cash flows, the investor chooses:
  - to liquidate the firm with probability $\beta_t \in [0, 1]$.
  - to collect the repayment amount r_t.
- Upon liquidation, the investor (principal) and the manager (agent) both get *zero*.
- Question: what happens if the model has only one-period? can the firm be financed?

Two-period case: Bolton and Scharfstein (1990)

- Now suppose there are two periods, or $t = 0, 1, 2$.
- The contract has two components.
- The firm is liquidated with probability β_i, where $i \in \{\ell, h\}$ denotes the first period report, in the beginning of the second period.
- The firm repays (r_i, r_j^i) to the investor, where $i, j \in \{\ell, h\}$, denotes the first and the second period report respectively.

Second-period Incentives

- Consider the incentive compatibility in the second period, given the first-period report is $i \in \{\ell, h\}$.
- When the second-period cash flow is h, the IC constraint is

$$h - r_h^i \geq h - r_\ell^i$$

- When the second-period cash flow is ℓ, the IC constraint is

$$\ell - r_\ell^i \geq \ell - r_h^i$$

- Jointly, they imply that $r_\ell^i = r_h^i$, which is denoted as r^i.

First-period Incentives

- Go back to the IC constraint in the first period.
- When the first-period cash flow is h, the IC constraint is

$$h - r_h + \beta_h(\mu - r^h) \geq h - r_\ell + \beta_\ell(\mu - r^\ell) \qquad \text{(IC)}$$

- We ignore the other IC constraint, which can be verified as slack after obtaining the optimal contract.

Other Constraints

- Limited liability at the first and the second period require the following conditions:

$$r_i \leq i, \quad r^i \leq i - r_i + \ell \tag{LL}$$

- The participation constraint for the agent is:

$$p[h - r_h + \beta_h(\mu - r^h)] + (1-p)[\ell - r_\ell + \beta_\ell(\mu - r^\ell)] \geq 0 \tag{IR}$$

Investor's Problem

- The investor who designs the contract has the bargaining power.
- Any optimal contract maximizes the investor value by choosing a policy that solves:

$$\max_{\beta_i, r_i, r^i} \; -c + p[r_h + \beta_h(r^h - c)] + (1-p)[r_\ell + \beta_\ell(r^\ell - c)]$$

$$\text{s.t.} \quad (\text{IC}), (\text{LL}), (\text{IR})$$

Policy Characterization

- We characterize the investor's problem by showing a sequence of results.

Lemma 1

The constraint (LL) *implies that* (IR) *holds.*

Proof.

$$
\begin{aligned}
i - r_i + \beta_i(\mu - r^i) &\geq i - r_i + \beta_i(\ell - r^i) \\
&\geq i - r_i - \beta_i(i - r_i) \\
&\geq 0
\end{aligned}
$$

where the second inequality is implied by (LL). □

Lemma 2

The constraint (IC) must be binding.

Proof.

Suppose not. Then we can drop (IC) in the program, which implies that the policy solves

$$\max_{\beta_i, r_i, r^i} r_i + \beta_i(r^i - c)$$

subject to (LL). An optimal solution is $\{r_i = i, r^i = \ell\}$. However, if we plug it back into (IC), then

$$h - \ell + \beta_\ell(\mu - \ell) \leq h - h + \beta_h(\mu - \ell)$$

which is a contradiction. $\square$

Policy Characterization

Lemma 3

Optimality requires that $\beta_h^* = 1$.

Proof.

From the binding (IC), we know

$$r_h + \beta_h r^h = (\beta_h - \beta_\ell)\mu + r_\ell + \beta_\ell r^\ell$$

Simplify the objective function of the investor's problem to be:

$$-c + r_\ell + \beta_\ell[r^\ell - p\mu - (1-p)c] + p\beta_h(\mu - c) \tag{1}$$

which implies that $\beta_h^* = 1$. □

Lemma 4

Optimality requires that $\beta_\ell^* = 0$*, and* $r_\ell^* = \ell$*.*

Proof.

From $\beta_h^* = 1$ and (1), we can simplify the objective to maximize

$$f(r_\ell, \beta_\ell) = r_\ell + \beta_\ell[r^\ell - p\mu - (1-p)c] \tag{2}$$

Suppose $\beta_\ell^* > 0$. Then from (LL) we know

$$f(r_\ell^*, \beta_\ell^*) \leq \ell + \ell - p\mu - (1-p)c < \ell = f(\ell, 0)$$

which is a contradiction. Hence, $\beta_\ell^* = 0$, which further implies that $r_\ell^* = \ell$ by (2). □

Policy Characterization

- Note that the optimal contract here is not unique in terms of the repayments r_h and r^h.
- From (IC) we know

$$r_h + r^h = \mu + \ell$$

  So one optimal contract is $r_h = \mu, r^h = \ell$, for instance.
- From (1), the investor gets expected payoff of:

$$\ell - c + p(\mu - c)$$

  So the firm gets funding initially if $c < \mu - \frac{\mu - \ell}{1+p}$.

Taking Stock

- There exists a parameter space where the firm is financed initially and continues to get funding if its performance is good in the first period, while it is never funded in the static setting.
- The threat to stop funding if the first-period performance is poor reduces the agent's incentive to lie, alleviating the friction.
- The feature that future investment and funding depend on past performance is not driven by the correlation in performances.

Taking Stock

- Is the contract renegotiation-proof? In particular, is there room to renegotiate if the first-period performance is bad?
- What if the agent's outside option increases?
- How to interpret/implement the optimal contract? (e.g., equity, debt, credit-line etc.)
- Can the model be equivalently set up using effort or the typical MH approach?