High Frequency Evolution of Macro Expectation and Disagreement

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Motivation

- Traditional theories, such as FIRE, suggest that there should be no disagreement among agents (Muth, 1961; Lucas Jr, 1972).
- Empirical evidence shows persistent disagreement among agents (Jonung, 1981).
- Models of information rigidity offer compelling explanations for these observations.
- Central to these models is the important role of information—or “news”.
- The high-frequency nature of this news and the low-frequency survey data are misaligned.
- Goal: develop a framework that can simultaneously integrate high-frequency news with low-frequency survey data.

Literature: Evolution of Expectations and Disagreement

Evolution of Expectations

- Economic agents adjust their expectations in response to newly acquired information (Coibion and Gorodnichenko, 2015).
- This dynamic process of expectation adjustment occurs at a frequency that significantly exceeds that of conventional survey reports.

Evolution of Disagreement

- Lahiri and Sheng (2008) estimate a Bayesian learning model to show three components of disagreement:
  1. prior-mean heterogeneity.
  2. the weights attached to these priors.
  3. diverse interpretations of new information.
- Several econometric issues arise in the estimation of the evolution equation for disagreement (Hsiao and Pesaran, 2008; Lahiri and Sheng, 2008).

Literature: Mixed-Frequency Method

Previous methods (Ghysels and Wright, 2009; Andreou et al., 2013; Chaudhry and Oh, 2020):

- Mixed-Data Sampling Regression (MIDAS)
- Kalman Filter (KF)
- Reinforcement Learning (RL)

Shortcomings of previous methods:

- Estimation efficiency: one may need to estimate many parameters, resulting in low estimation efficiency.
- Empirical performance: due to model complexity and computational limitations, one may encounter a poor out-of-sample performance.
- Interpretability: methods with good empirical performance are difficult to have a reasonable economic explanation.

This Paper

- We develop a novel mixed-frequency framework that enables the simultaneous analysis of high-frequency news and low-frequency expectations.
- By utilizing representative forecasters as proxies for real-world agents, we demonstrate that the evolution of forecast disagreement follows the equation:

$$\sigma\left(\mathbb{F}_{i,t+1}\left[x_{t+1}\right]\right) = \sigma\left(\mathbb{F}_{i,t}\left[x_{t+1}\right]\right) + (\Delta\beta)'\mathbf{r}_{t+1}$$

- We reconstruct the unobserved daily series of both expectations and disagreement regarding macroeconomic growth that span the interval between two quarterly survey releases.

Roadmap

Data and Variables

High-frequency Evolution of Expectations

High-frequency Evolution of Disagreement

Application: Construct Daily Measures of Both Aggregate Growth Expectations and Disagreement

Data

Growth Expectations: quarterly SPF surveys

- One-quarter ahead real GDP growth forecast

$$\mathbb{F}_t\left[x_{t+1}\right] = 100 \times \left[\left(\frac{\mathbb{F}_t\left[X_{t+1}\right]}{\mathbb{F}_t\left[X_t\right]}\right)^4 - 1\right]$$

- Real-time quarterly nowcast

$$\mathbb{F}_{t+1}\left[x_{t+1}\right] = 100 \times \left[\left(\frac{\mathbb{F}_{t+1}\left[X_{t+1}\right]}{X_t}\right)^4 - 1\right]$$

Asset Returns: proxies for new information

- We adopt a broad interpretation to asset prices, encompassing rate changes, spreads, returns, and other value-related metrics of financial assets.

Relation between News and Growth Expectations

- The DGP is defined by $x_t = \rho x_{t-1} + u_t$, where $u_t \sim \mathcal{N}\left(0, \sigma_u^2\right)$ is i.i.d. over time and $\rho > 0$.
- Agent i observes the noise signal $s_t^i = x_t + \epsilon_t^i$, where $\epsilon_t^i \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$ represents forecaster-specific i.i.d. noise.
- According to the Kalman filter, beliefs should be updated as follows

$$\underbrace{\mathbb{F}_{i,t+1}\left[x_{t+1}\right]}_{\text{nowcast}} = \underbrace{\mathbb{F}_{i,t}\left[x_{t+1}\right]}_{\text{forecast}} + \frac{\Sigma}{\Sigma + \sigma_\epsilon^2} \underbrace{\left(s_t^i - \mathbb{F}_{i,t}\left[x_{t+1}\right]\right)}_{\text{new information}}$$

  where Σ is the steady state variance of the prior $f\left(x_{t+1} \mid s_t^i, s_{t-1}^i, \cdots\right)$.
- To help with interpretation of this equation, we transform it into a simpler form:

$$\mathbb{F}_{i,t+1}\left[x_{t+1}\right] = \mathbb{F}_{i,t}\left[x_{t+1}\right] + News_{t+1}$$

  we interpret it as an efficient Bayesian forecaster.

Relation between News and Growth Expectations

- We need a model that allows for heterogeneity without making any specific assumptions:

$$\mathbb{F}_{i,t+1}\left[x_{t+1}\right] = \alpha_i \mathbb{F}_{i,t}\left[x_{t+1}\right] + \boldsymbol{\beta}_i' \mathbf{r}_{t+1}$$
$$FR_{i,t+1} \equiv \mathbb{F}_{i,t+1}\left[x_{t+1}\right] - \mathbb{F}_{i,t}\left[x_{t+1}\right] = (\alpha_i - 1)\mathbb{F}_{i,t}\left[x_{t+1}\right] + \boldsymbol{\beta}_i' \mathbf{r}_{t+1}$$

- Motivated by this expression, we employ the following approximating moment:

$$\mathbb{F}_{t+1}\left[x_{t+1}\right] = \alpha \mathbb{F}_{t}\left[x_{t+1}\right] + \boldsymbol{\beta}' \mathbf{r}_{t+1}$$

  where $\mathbf{r}_{t+1}$ represents a vector of asset returns, since they contain informative content for forecast revisions.

- We conduct several time series regressions to reinforce our use of asset returns as proxies for news, since we observe that certain pairs of assets yield sizable R^2.

Mixed-Frequency Estimation Method

- We encounter the issue with mixed frequencies: forecasts are made quarterly, while the asset returns that represent news are recorded daily.
- A recursive form of the evolution equation:

$$
\begin{aligned}
\mathbb{F}_1[x] &= \alpha\mathbb{F}_0[x] + \beta'\mathbf{r}_1 \\
\mathbb{F}_2[x] &= \alpha\mathbb{F}_1[x] + \beta'\mathbf{r}_2 \\
&\cdots \\
\mathbb{F}_T[x] &= \alpha\mathbb{F}_{T-1}[x] + \beta'\mathbf{r}_T
\end{aligned}
$$

- This recursive approach allows us to clearly establish the relationship between $\mathbb{F}_0[x]$ and $\mathbb{F}_T[x]$ for two consecutive release dates:

$$\mathbb{F}_T[x^p] = \alpha^T\mathbb{F}_0[x^p] + \sum_{k=0}^{T-1}\alpha^k\beta'\mathbf{r}_{T-k}^p$$

Mixed-Frequency Estimation Method

- By setting α to a fixed value α_0, we simplify the equation and therefore allow it to estimate using OLS:

$$\mathbb{F}_T\left[x^p\right] - \alpha_0^T \mathbb{F}_0\left[x^p\right] = \beta'\left(\sum_{k=0}^{T-1} \alpha_0^k \mathbf{r}_{T-k}^p\right)$$

Let $y = \mathbb{F}_T\left[x^p\right] - \alpha_0^T \mathbb{F}_0\left[x^p\right]$ and $\mathbf{X} = \sum_{k=0}^{T-1} \alpha_0^k \mathbf{r}_{T-k}^p$.
Then, the estimator $\hat{\beta}$ is given by $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'y$.

- Our current task is to identify the value of α_0 that minimizes the SSR:

$$\frac{\partial \mathrm{SSR}}{\partial \alpha_0} = 2T\alpha_0^{T-1}\mathbb{F}_0\left[x^p\right]\mathbf{X}\hat{\beta} - 2y'\left(\sum_{k=0}^{T-1} k\alpha_0^{k-1}\mathbf{r}_{T-k}^p\right)\hat{\beta}$$
$$+\hat{\beta}'\left[\left(\sum_{k=0}^{T-1} k\alpha_0^{k-1}\mathbf{r}_{T-k}^p\right)'\mathbf{X} + \mathbf{X}'\left(\sum_{k=0}^{T-1} k\alpha_0^{k-1}\mathbf{r}_{T-k}^p\right)\right]\hat{\beta}$$

Mixed-Frequency Estimation Method

We develop a grid search method to identify the optimal α:

1. We construct a grid over the feasible domain of α and divide it into discrete intervals.
   - Each grid point corresponds to a potential value of α.
   - The domain for α is based on theoretical considerations.
   - We begin by setting a broad sampling range for α and gradually narrowing the interval to enhance accuracy.
2. We employ a rolling window of 40 quarters.
   - The parameters cannot remain constant.
   - The window size should be carefully chosen to balance statistical power and parameter stability.
3. We restrict our analysis to bivariate pairs of assets.
   - A linear combination of two assets may approximate others.
   - We specifically choose those that yield the highest R^2.

Estimation Results

The optimal coefficient α that minimize SSR is 1.

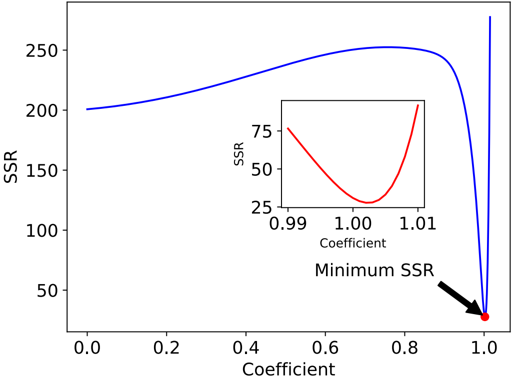

Figure 1: Relationship between SSR and α.

Estimation Results

$$FR^p \equiv \mathbb{F}_T\left[x^p\right] - \mathbb{F}_0\left[x^p\right] = \beta'(\sum_{k=0}^{T-1} \mathbf{r}^p_{T-k}) + (k\mathbb{F}_0\left[x^p\right] + b)$$

	(1)	(2)
β_1 (5YR Fixed-term Index)	-0.235***	-0.362***
	(0.074)	(0.128)
β_2 (Change in AAA-10Y Spread)	0.024*	0.035*
	(0.013)	(0.019)
b (constant)		0.609
		(0.636)
k (past forecast $\mathbb{F}_0\left[x^p\right]$)		-0.086
		(0.253)
R^2	0.224	0.280

Intuition

- The cross-sectional variance is defined as

$$\mathrm{Var}(\mathbb{F}_{i,t}[x_{t+1}]) = \frac{1}{N}\sum_{i=1}^{N}(\mathbb{F}_{i,t}[x_{t+1}] - \mathbb{F}_t[x_{t+1}])^2$$

- By incorporating the evolution equations for both individual and consensus forecasts:

$$\mathrm{Var}(\mathbb{F}_{i,t+1}[x_{t+1}]) = \gamma\mathrm{Var}(\mathbb{F}_{i,t}[x_{t+1}]) + \boldsymbol{\zeta}'\mathbf{r}_{t+1}\mathbf{r}'_{t+1}\boldsymbol{\zeta} + \boldsymbol{\delta}'\mathbf{r}_{t+1}\mathbb{F}_t[x_{t+1}]$$

  where γ is a scalar related to α and α_i, $\boldsymbol{\zeta}$ is a vector of scalars related to $\boldsymbol{\beta}$ and $\boldsymbol{\beta}_i$, and $\boldsymbol{\delta}$ is a vector of scalars related to α, α_i, $\boldsymbol{\beta}$, and $\boldsymbol{\beta}_i$.
- If we relax these constraints and apply the previous method, the results, although computable, do not reflect the true "variance" parameters as needed.

Representative Forecasters

- We posit the existence of two such forecasters:

$$\mathbb{F}_t^H[x_{t+1}] = \mathbb{F}_t[x_{t+1}] + \sigma(\mathbb{F}_{i,t}[x_{t+1}])$$
$$\mathbb{F}_t^L[x_{t+1}] = \mathbb{F}_t[x_{t+1}] - \sigma(\mathbb{F}_{i,t}[x_{t+1}])$$

  we assume that the position of representative forecasts in relation to consensus forecast remains constant over time.

- Since we have

$$\mathbb{F}_{t+1}^H[x_{t+1}] = \alpha^H \mathbb{F}_t^H[x_{t+1}] + (\beta^H)'\mathbf{r}_{t+1}$$
$$\mathbb{F}_{t+1}^L[x_{t+1}] = \alpha^L \mathbb{F}_t^L[x_{t+1}] + (\beta^L)'\mathbf{r}_{t+1}$$

- The relationship between representative forecasters and consensus is

$$2\mathbb{F}_{t+1}[x_{t+1}] = \mathbb{F}_{t+1}^H[x_{t+1}] + \mathbb{F}_{t+1}^L[x_{t+1}]$$
$$2\alpha\mathbb{F}_t[x_{t+1}] + 2\beta'\mathbf{r}_{t+1} = (\alpha^H + \alpha^L)\mathbb{F}_t[x_{t+1}] + \left[(\beta^H)' + (\beta^L)'\right]\mathbf{r}_{t+1}$$
$$+ (\alpha^H - \alpha^L)\sigma(\mathbb{F}_{i,t}[x_{t+1}])$$

Learning the Cross-sectional Variance

- Drawing from the conclusion that $\alpha^H = \alpha^L = \alpha = 1$:

$$[\mathbb{F}_{t+1}[x_{t+1}] + \sigma(\mathbb{F}_{i,t+1}[x_{t+1}])] = [\mathbb{F}_t[x_{t+1}] + \sigma(\mathbb{F}_{i,t}[x_{t+1}])] + (\boldsymbol{\beta}^H)'\mathbf{r}_{t+1}$$
$$\mathbb{F}_{t+1}[x_{t+1}] = \mathbb{F}_t[x_{t+1}] + \boldsymbol{\beta}'\mathbf{r}_{t+1}$$
$$[\mathbb{F}_{t+1}[x_{t+1}] - \sigma(\mathbb{F}_{i,t+1}[x_{t+1}])] = [\mathbb{F}_t[x_{t+1}] - \sigma(\mathbb{F}_{i,t}[x_{t+1}])] + (\boldsymbol{\beta}^L)'\mathbf{r}_{t+1}$$

- It becomes apparent that the cross-sectional standard deviation follows a straightforward evolution equation

$$\sigma(\mathbb{F}_{i,t+1}[x_{t+1}]) = \alpha\sigma(\mathbb{F}_{i,t}[x_{t+1}]) + (\Delta\boldsymbol{\beta})'\mathbf{r}_{t+1}$$

where $\Delta\boldsymbol{\beta} = \boldsymbol{\beta}^H - \boldsymbol{\beta} = \boldsymbol{\beta} - \boldsymbol{\beta}^L$ represent the differential interpretation of news between two representative forecasts and the consensus forecast.

Discussion

- We now turn to a more general case:

$$\mathbb{F}_{i,t}\left[x_{t+1}\right] = \mathbb{F}_{t}\left[x_{t+1}\right] + k_i \cdot \sigma\left(\mathbb{F}_{i,t}\left[x_{t+1}\right]\right)$$

where $k_i \neq 0$ remains constant over time.

- By substituting it into the evolution equation for individuals and consensus:

$$\sigma\left(\mathbb{F}_{i,t+1}\left[x_{t+1}\right]\right) = \alpha_i \sigma\left(\mathbb{F}_{i,t}\left[x_{t+1}\right]\right) + \frac{\alpha_i - \alpha}{k_i}\mathbb{F}_{t}\left[x_{t+1}\right] + \left(\frac{\boldsymbol{\beta}_i - \boldsymbol{\beta}}{k_i}\right)' \mathbf{r}_{t+1}$$

- Notice that this equation holds for any representative forecaster i, substitute $\alpha_i = \alpha$:

$$\sigma\left(\mathbb{F}_{i,t+1}\left[x_{t+1}\right]\right) = \alpha \sigma\left(\mathbb{F}_{i,t}\left[x_{t+1}\right]\right) + \left(\frac{\boldsymbol{\beta}_i - \boldsymbol{\beta}}{k_i}\right)' \mathbf{r}_{t+1}$$

This is equivalent to the evolution equation we derive previously, where $\Delta \boldsymbol{\beta} = (\boldsymbol{\beta}_i - \boldsymbol{\beta})/k_i$.

Estimation Results

The optimal α that minimize SSR for disagreement is also 1.

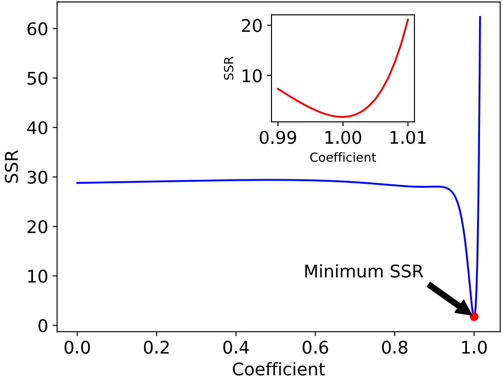

Figure 2: Relationship between SSR and α for disagreement.

Extract Daily Time Series of Mean and Variance

Applying the previous mixed-frequency method, we derive the estimated daily-frequency time series:

$$\begin{bmatrix} \mathbb{F}_1[x^p] \\ \mathbb{F}_2[x^p] \\ \vdots \\ \mathbb{F}_T[x^p] \end{bmatrix} = \mathbf{D} \begin{bmatrix} \mathbb{F}_0[x^p] \\ \mathbb{F}_0[x^p] \\ \vdots \\ \mathbb{F}_0[x^p] \end{bmatrix} + \mathbf{T} \begin{bmatrix} \mathbf{r}_1^p \\ \mathbf{r}_2^p \\ \vdots \\ \mathbf{r}_T^p \end{bmatrix}$$

where $\mathbf{D} = \mathrm{diag}\left(\hat{\alpha}^1, \hat{\alpha}^2, \cdots, \hat{\alpha}^T\right)$ is a scaling matrix that represents the contribution of the initial state $\mathbb{F}_0[x^p]$. $\mathbf{T}$ is a Toeplitz matrix that expresses the impact of asset returns:

$$\mathbf{T} = \begin{bmatrix} \hat{\beta}' & 0 & \cdots & 0 \\ \hat{\alpha}\hat{\beta}' & \hat{\beta}' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\alpha}^{T-1}\hat{\beta}' & \hat{\alpha}^{T-2}\hat{\beta}' & \cdots & \hat{\beta}' \end{bmatrix}$$

Results

Recursive estimation:

- We fit model on previous $T - 1$ quarters and apply to one quarter out-of-sample.

Evaluation:

- We construct the daily series for both cross-sectional mean and variance and achieve impressive R^2 of 93.3% and 84.5% against the actual data from surveys.

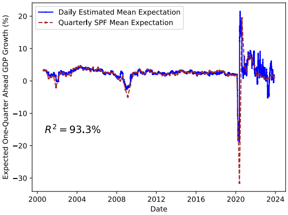

Figure 3: Expectations

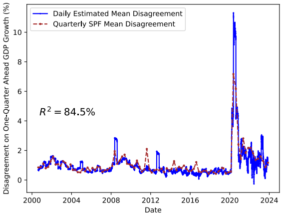

Figure 4: Disagreement

Comparison with RL Approach: ML Interpretability

Both the RL method and our mixed-frequency method demonstrate strong empirical performance, realizing R^2 values of 82.3% and 93.3% for the cross-sectional mean, respectively.

However, several important details warrant discussion:

- The RL method estimates the cross-sectional mean and variance jointly, which yields better estimates of the cross-sectional mean.
- Fixing α at one yields better performance than freely estimating α.
- The parameters that the RL method needs to estimate are identical to those we consider, why we get different estimated results and empirical performance?

Conclusion

Main Findings:

- High-frequency dynamics of macro expectations and the corresponding evolution of disagreement among agents.
- A mixed-frequency framework that integrates high-frequency news with low-frequency survey data.
- Construction of daily time series for both cross-sectional mean and variance.

Future works:

- High-frequency series would enable clean identification in event studies.
- Our mixed-frequency framework can be applied to other macroeconomic variables to enhance the robustness of forecasting models.

Thanks!