Problem Set 1

Due on September 21

Problem 1. Consider a new model of preferences, the *PI-model*. The primitives of this model are two binary relations, P and I, defined on X, where P is interpreted as the "strictly better than" relation, and I is interpreted as the "indifference" relation. We impose three conditions on P and I in this model: (1) for any $x \in X$, xIx and $x\bar{P}x$; (2) for any $x, y \in X$ with $x \neq y$, *exactly* one of the following three is true: xPy, yPx and xIy; (3) both P and I are transitive. Based on the construction in this model, prove the following results.

(a) I is symmetric.

(b) If xPy and yIz, then xPz. If xIy and yPz, then xPz.

(c) The PI-model is equivalent to the $\succeq$-model.

Problem 2. Let C be a choice correspondence defined on the domain $\mathscr{D}$. Assume that for any $A, B \in \mathscr{D}$ with $A \cap B \neq \emptyset$, $A \cap B \in \mathscr{D}$. Show that if C satisfies Sen's properties α and β, then C satisfies the weak axiom of revealed preference.

Problem 3. Let $\succeq$ be a preference relation defined on a *finite* set X, and $\succ$ is the asymmetric component of $\succeq$. Notice that $\succeq$ is not assumed to be rational. We say $\succ$ is *acyclic* if there does not exist a list $(x_1, x_2, ..., x_{n-1}, x_n)$ such that $x_k \in X$ for each $k \in \{1, 2, ..., n\}$, $n \geq 2$, and $x_1 \succ x_2 \succ ... \succ x_{n-1} \succ x_n \succ x_1$. For any $A \subseteq X$, let

$$C_{\succ}(A) = \left\{x \in A : \text{there does not exist } \ y \in A \ \text{ such that } \ y \succ x\right\}.$$

Prove the following results.

(a) $C_{\succ}(A) \neq \phi$ for all non-empty $A \subseteq X$ if and only if $\succ$ is acyclic.

(b) Assume $\succ$ is acyclic. $C_{\succ}$ satisfies Sen's property α, but may not satisfy property β.

Problem 4. Show that if a choice correspondence C (defined on the domain $\mathscr{D}$) can be rationalized, then it satisfies the *path-invariance* property: for any $B_1, B_2 \in \mathscr{D}$ such that $B_1 \cup B_2 \in \mathscr{D}$ and $C(B_1) \cup C(B_2) \in \mathscr{D}$, we have

$$C(B_1 \cup B_2) = C\big(C(B_1) \cup C(B_2)\big).$$