Advanced Econometrics II:
June 25, 2019

Final Exam

(Tuesday class)

The exam is from 9 AM to 11 PM. Full score of the exam is 110. It has two parts. Part 1 has 2 questions. Part 2 has 4 questions. Please show your derivations for questions in Part 2 (answers without derivations will NOT be given scores). The full score for each question is indicated in each question. Please write down your name, student ID number, and program on both the exam book and answer sheet. Good Luck!

Part 1. Short-answer questions:

1. [10 points] Please briefly describe what the strictly exogenous assumption is. And explain whether the random effect (RE) model needs strict exogeneity to be consistent?

2. [10 points] Please describe the drawbacks of the linear probability model and prove to identify whether this model can hold the assumption of homoskedasticity?

Part 2. Long-answer questions:

1. The data is generated by the classical unit-root process (random-walk process):

$$y_t = y_{t-1} + u_t,$$

with $u_t \sim iid(0, \sigma^2)$ and the fourth moment of u_t is finite.

a) [15 points] Suppose we use the generated data to estimate $y_t = \rho y_{t-1} + u_t$. And $\hat{\sigma}^2 = \frac{1}{n-1}\sum_{t=1}^{n} \hat{u}_t^2$.
Please derive in detail the speed of convergence of $\hat{\sigma}^2 \rightarrow \sigma^2$

b) [15 points] Suppose we use the generated data to estimate $y_t = \alpha + \rho y_{t-1} + u_t$.
And get $\hat{\rho} = \frac{\sum_{t=1}^{n}(y_{t-1}-\bar{y}_{t-1})(y_t-\bar{y}_t)}{\sum_{t=1}^{n}(y_{t-1}-\bar{y}_{t-1})^2}$, where $\bar{y}_t$ and $\bar{y}_{t-1}$ standard for the sample average of the corresponding series.
Please derive the asymptotic distribution of $\hat{\rho}$.

2. Consider the classical Probit model without endogenous variables:

$$P(y = 1 \mid z, q) = \Phi(z_1\delta_1 + \gamma_1 z_2 q)$$

where z_1 and δ_1 are vectors; $z_1\delta_1$ and z_2 and q are scalars; $\mathbf{z} = (z_1, z_2)$; q is independent of $\mathbf{z}$ and distributed as $N(0,1)$; the vector $\mathbf{z}$ is observed.

a) [5 points] If q is observable, please find the marginal effect of z_2 on $E(y \mid \mathbf{z}, q)$.

b) [15 points] If q is not observable, please derive to identify whether:

$$P(y = 1 \mid z) = \Phi[z_1\delta_1/(2 + \gamma_1 z_2)] \ ??$$

3. [20 points] In class, we have constructed the truncated model, but here you are asked to derive the model for censored data. Suppose the observable censored data is generated as follows:

$$y = \begin{cases} a, & y^* \leq a \\ y^*, & y^* > a \end{cases}$$

Where y^* is the latent continuous variable, and $y^* \sim N(\mu, \sigma^2)$.

Please derive: $E(y) = ?$

4. [20 points] Suppose there is a CLT that you can use for the following derivation. Please derive the asymptotic distribution for the multivariate nonparametric kernel density estimator $\hat{f}(x_1, x_2, x_3)$ using 4^{th} order Gaussian kernel function.