Princeton University Press, 2017

slides

chapter 4

the open economy

real-business-cycle model

Motivation:

In the previous chapter, we built a model of the open economy driven by productivity shocks and argued that it can capture the observed countercyclicality of the trade balance. We also established that two features of the model are important for making this prediction possible. First, productivity shocks must be sufficiently persistent. Second, capital adjustment costs must not be too strong. In this chapter, we ask more questions about the ability of that model to explain observed business cycles. In particular, we ask whether it can explain the sign and magnitude of business-cycle indicators, such as the standard deviation, serial correlation, and correlation with output of output, consumption, investment, the trade balance, and the current account.

The Small Open Economy RBC Model

To make the models studied in chapters 2 and 3 more empirically realistic and to give them a better chance to account for observed business-cycle regularities add:

1. endogenous labor supply and demand
2. uncertainty in the technology shock process
3. capital depreciation.

The resulting theoretical framework is known as the Small Open Economy Real-Business-Cycle model, or, succinctly, the SOE-RBC model.

The Household's Maximization Problem

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{4.1}$$

subject to

$$c_t + i_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = y_t + d_t \tag{4.2}$$

$$y_t = A_t F(k_t, h_t) \tag{4.3}$$

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{4.4}$$

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{\prod_{s=0}^{j}(1 + r_s)} \leq 0 \tag{4.5}$$

Capital adjustment cost, $\Phi(0) = \Phi'(0) = 0$; $\Phi''(0) > 0$

Additions/differences to the model analyzed in Chapter 3

- endogenous labor supply, $U(c_t, h_t)$
- endogenous labor demand, $F(k_t, h_t)$
- uncertainty, A_t is stochastic
- the interest rate is no longer constant, $r_t \neq r$
- depreciation, δ no longer 0

Household's Optimality Conditions

$$c_t + k_{t+1} - (1-\delta)k_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = A_t F(k_t, h_t) + d_t \tag{4.6}$$

$$\lambda_t = \beta(1 + r_t)E_t\lambda_{t+1} \tag{4.7}$$

$$U_c(c_t, h_t) = \lambda_t \tag{4.8}$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \tag{4.9}$$

$$1 + \Phi'(k_{t+1} - k_t) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \tag{4.10}$$

Inducing Stationarity: External debt-Elastic Interest Rate (EDEIR)

$$r_t = r^* + p(\widetilde{d}_t) \tag{4.14}$$

r^* = constant world interest rate
$p(\tilde{d}_t)$ = country interest-rate premium
$\tilde{d}_t$ = cross-sectional average of debt

In equilibrium cross-sectional average of debt must equal individual debt

$$\widetilde{d}_t = d_t \tag{4.15}$$

Evolution of Total Factor Productivity, AR(1) process

$$\ln A_{t+1} = \rho \ln A_t + \widetilde{\eta}\epsilon_{t+1} \tag{4.12}$$

The Trade Balance

$$tb_t = y_t - c_t - i_t - \Phi(k_{t+1} - k_t) \tag{4.20}$$

The Current Account

$$ca_t = tb_t - r_{t-1}d_{t-1} \tag{4.21}$$

Equilibrium Conditions

$$-\frac{U_h(c_t,h_t)}{U_c(c_t,h_t)} = A_t F_h(k_t,h_t) \tag{4.11}$$

$$c_t + k_{t+1} - (1-\delta)k_t + \Phi(k_{t+1}-k_t) + [1+r^* + p(d_{t-1})]d_{t-1} = A_t F(k_t,h_t) + d_t \tag{4.16}$$

$$U_c(c_t,h_t) = \beta(1+r^*+p(d_t))E_t U_c(c_{t+1},h_{t+1}) \tag{4.17}$$

$$1 = \beta E_t \left\{ \frac{U_c(c_{t+1},h_{t+1})}{U_c(c_t,h_t)} \frac{[A_{t+1}F_k(k_{t+1},h_{t+1}) + 1 - \delta + \Phi'(k_{t+2}-k_{t+1})]}{1+\Phi'(k_{t+1}-k_t)} \right\} \tag{4.18}$$

This is a system of non-linear stochastic difference equations. It does not have a closed form solution. We will use numerical techniques to find a first-order accurate approximate solution around the nonstochastic steady state. This is a local approximation. (Later we will also consider a global solution method.)

The system is a second-order difference equation as it features k_t, k_{t+1} and k_{t+2}. By defining auxiliry variables, it can be trnsformed into a first-order system. To this end, introduce the auxiliary variable k_t^f, and equation $k_t^f = k_{t+1}$, and replace k_{t+2} by k_{t+1}^f. Note that k_t^f is in the information set of period t. The transformed system is a set of stochastic first-order difference equations in the 5 unknowns: c_t, h_t, d_{t-1}, k_t, and k_t^f.

First-Order Accurate Approximaiton of the Equilibrium Conditions

Let y_t be a vector containing rthe endogenous nonpredetermined variables of the model, c_t h_t, and k^f_t, and x_t a vector containing the predetermined endogenous variables, k_t and d_{t-1}, and exogenous, A_t, variables of the model Then the solution of the model can be written as

$$y_t = g(x_t, \sigma) \quad \text{and} \quad x_{t+1} = h(x_t, \sigma) + \sigma\eta\epsilon_{t+1},$$

where σ is a scalar such that if $\sigma = 0$, the system becomes deterministic. The method consists in applying a Taylor expansion of this system with respect to y_t, x_t, and σ around the deterministic steady state of y_t and x_t and $\sigma = 1$.

The first-order accurate solution is of the form

$$\widehat{y}_t = g_x\widehat{x}_t \quad \text{and} \quad \widehat{x}_{t+1} = h_x\widehat{x}_t + \eta\epsilon_{t+1},$$

where $\widehat{x}_t$ and $\widehat{y}_t$ are (log) deviations of x_t and y_t from their steady-state values. The appendix shows how to obtain this approximation.

Taking stock:

An important element of the implementation of the first-order accurate solution of the model is finding numerical values for the derivatives of the functions $g(.,.)$ and $h(.,.)$ at the non-stochastic steady state.

Our approach is to use the Symbolic Math Toolbox of Matlab to do most of the work. This has several advantages. One is that the room for error is much smaller and the other is that it eliminates any tedious linearization by hand.

To allow a Symbolic Math toolbox to implement the linearization it is convenient to specify functional forms for the utility, production, country premium, and adjustment cost functions. What will matter, given that we perform a first-order approximation to the equilibrium conditions, is at most the first and second derivatives of those functions.

Functional Forms

Period utility function (CRRA(GHH))

$$U(c,h) = \frac{G(c,h)^{1-\sigma} - 1}{1-\sigma} \quad \text{with} \quad G(c,h) = c - \frac{h^{\omega}}{\omega}; \quad \omega > 1, \sigma > 0$$

Debt-elastic interest rate

$$p(d) = \psi \left(e^{d-\bar{d}} - 1 \right); \quad \psi > 0$$

Production function

$$F(k,h) = k^{\alpha} h^{1-\alpha}; \quad \alpha \in (0,1)$$

Adjustment cost function

$$\Phi(x) = \frac{\phi}{2} x^2; \quad \phi > 0$$

6 structural parameters: σ, ω, ψ, $\bar{d}$, α, ϕ

What's Special about GHH Preferences?

Recall the period utility function is of the form

$$U(c,h) = \frac{G(c,h)^{1-\sigma} - 1}{1-\sigma} \quad \text{with} \quad G(c,h) = c - \frac{h^\omega}{\omega}$$

The marginal rate of substitution between consumption and labor is independent of consumption:

$$-\frac{U_h(c,h)}{U_c(c,h)} = \frac{G(c,h)^{-\sigma} G_h(c,h)}{G(c,h)^{-\sigma} G_c(c,h)} = h^{\omega-1}.$$

Implicaiton: The labor supply, $-Uh/Uc = w$ is independent of c_t, i.e., depends on the wage rate only. Put differently, GHH preferences kill the wealth effect on labor supply.

Deterministic Steady State

The steady state is the quadruple (d, k, c, h) satisfying

$$-\frac{U_h(c,h)}{U_c(c,h)} = AF_h(k,h) \tag{4.11'}$$

$$c + \delta k + (r^* + p(d))d = AF(k,h) \tag{4.16'}$$

$$1 = \beta(1 + r^* + p(d)) \tag{4.17'}$$

$$1 = \beta\left[AF_k(k,h) + 1 - \delta\right] \tag{4.18'}$$

$$A = 1.$$

Using the assumed functional forms the steady state becomes

$$h^{\omega-1} = (1-\alpha)(k/h)^{\alpha} \tag{4.11''}$$

$$c + \delta k + (r^* + \psi(e^{d-\bar{d}} - 1))d = (k/h)^{\alpha} h \tag{4.16''}$$

$$1 = \beta(1 + r^* + \psi(e^{d-\bar{d}} - 1)) \tag{4.17''}$$

$$1 = \beta\left[\alpha(k/h)^{\alpha-1} + 1 - \delta\right] \tag{4.18''}$$

This is a system of 4 equations in 4 unknown endogenous variables, (c, d, h, k) and 7 unknown parameters, $\omega, \alpha, \delta, r^*, \psi, \bar{d}, \beta$. The model has 4 additional structural parameters, σ, ϕ, ρ, $\tilde{\eta}$, which do not enter the steady state but which also need to be assigned values to. In sum, there are 11 structural parameters to be calibrated. They are:

$$\left[\ \omega \quad \alpha \quad \delta \quad r^* \quad \beta \quad \sigma \quad \phi \quad \rho \quad \tilde{\eta} \quad \bar{d} \quad \psi\ \right]$$

The Role of Persistence and Capital Adjustment Costs

In chapter 3, we showed that

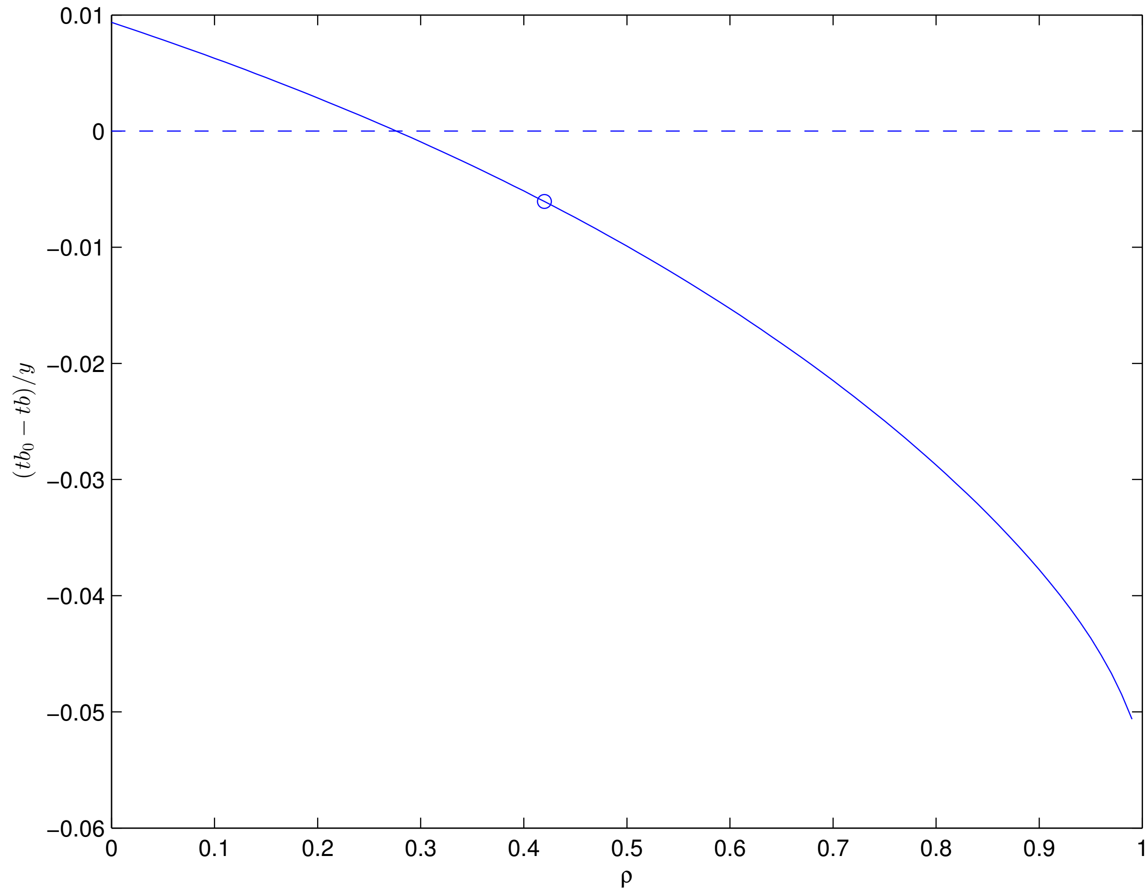
• the more persistent productivity shocks are, the more likely it will be that a positive productivity shock will cause a deterioration of the trade balance.

• the more pronounced are capital adjustment costs, the smaller will be the initial trade balance deterioration in response to a positive productivity shock.

• the more persistent the technology shock is, the higher the volatility of consumption relative to output will be.

The next three figures show that these analytical results do indeed hold in the fully-fledged stochastic dynamic open economy real-business-cycle model.

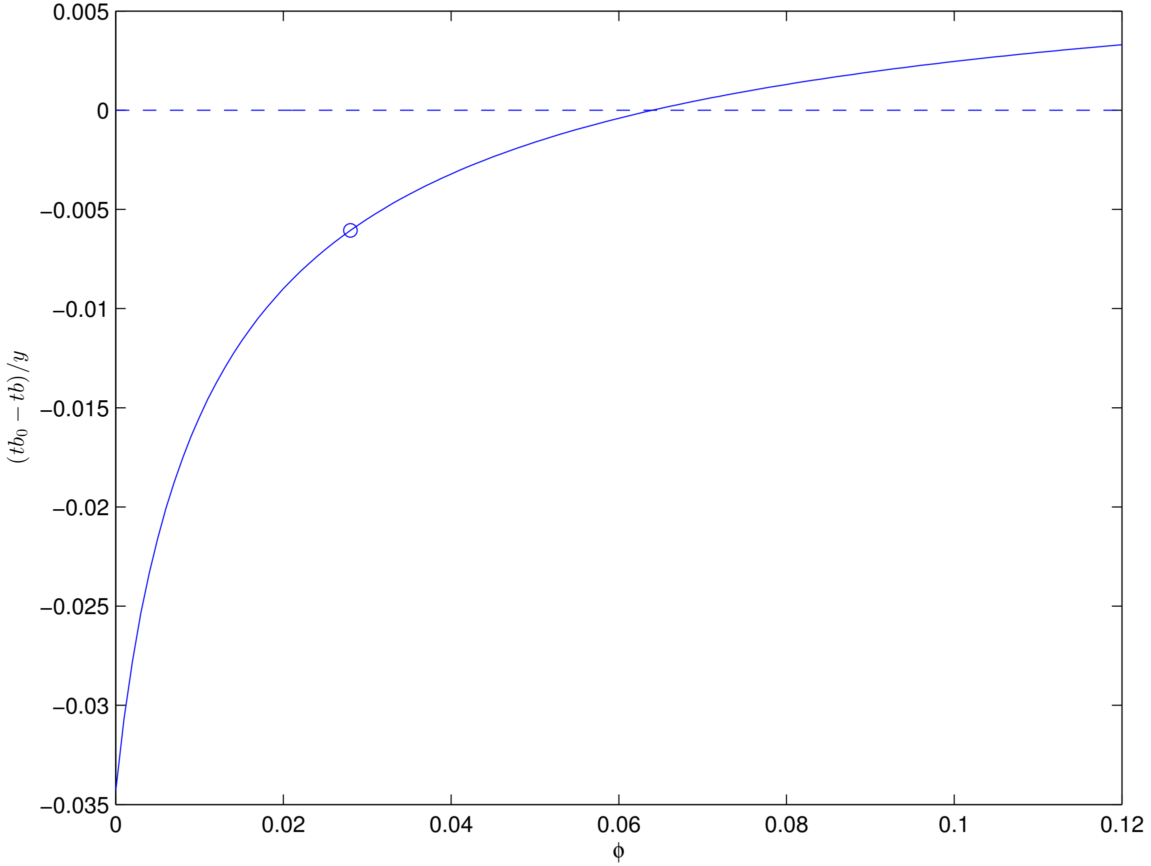
Impact response of the trade balance as a function of the persistence of the technology shock



The figure shows the impact response of the trade balance to a one percent positive innovation in productivity predicted by the EDEIR model presented in Chapter 4. The response of the trade balance is measured in units of steady-state output. All parameters other than ρ take the values shown in Table 4.1. The open circle indicates the baseline value of ρ.

Comments: The figure shows that the more persistent the productivity shock is the smaller the impact response of the trade balance will be. For $\rho > 0.3$, the response of the trade balance is negative, confirming the analytical results of chapters 2 and 3.

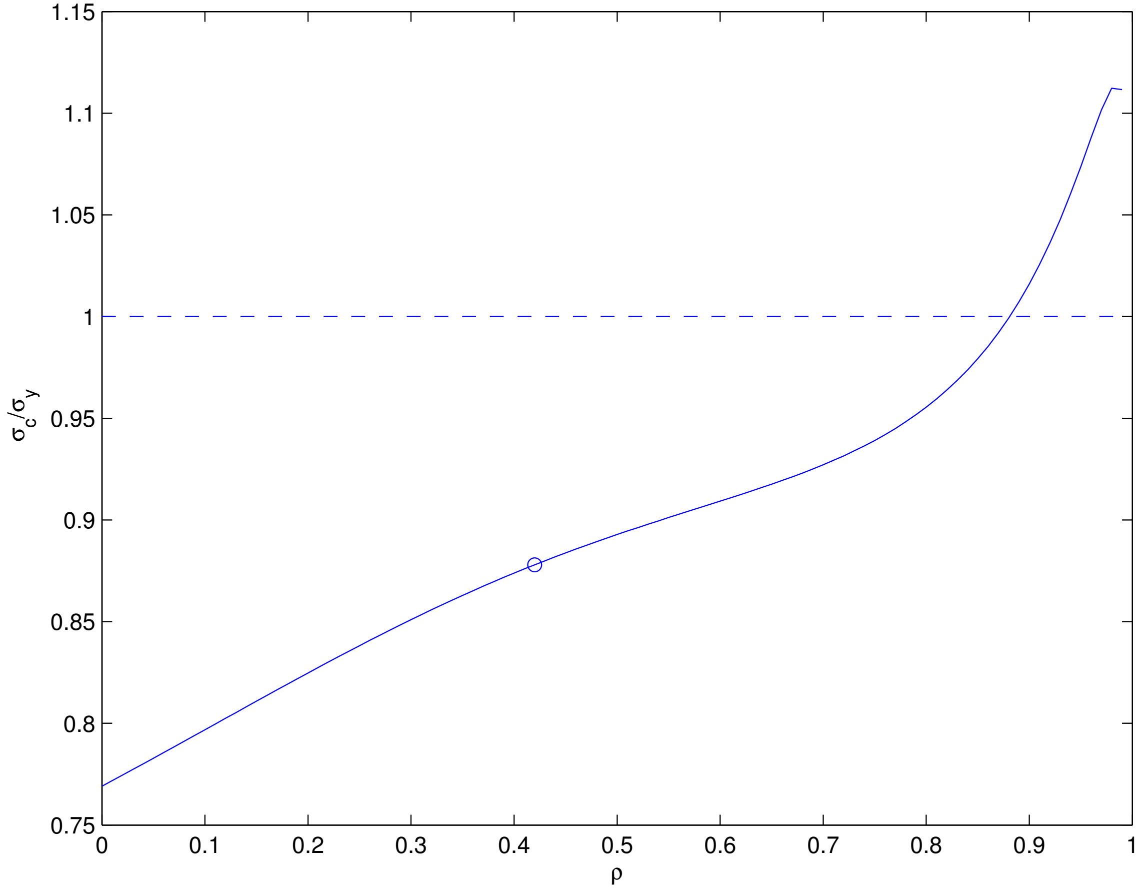
Impact response of the trade balance as a function of capital adjustment costs



Notes. The figure shows the impact response of the trade balance in response to a one percent positive innovation in productivity as a function of the size of capital adjustment costs, ϕ, predicted by the EDEIR model presented in Chapter 4. The response of the trade balance is measured in units of steady-state output. All parameters other than ρ take the values shown in Table 4.1. The open circle indicates the baseline ϕ value.

Comments: The figure shows that the higher capital adjustment costs are the larger the impact response of the trade balance will be. For $\phi > 0.06$, the response of the trade balance turns positive, confirming the analytical results of chapters 2 and 3.

Relative volatility of consumption as a function of the persistence of the stationary technology shock



Notes. The relative standard deviation shown is that implied by the EDEIR model presented in Chapter 4. All parameters other than ρ take the values shown in Table 4.1. The open circle indicates the baseline value of ρ.

Comments: The figure shows that the more persistent stationary productivity shocks are, the higher the standard deviation of consumption relative to the standard deviation of output will be, just as derived analytically in the permanent income model of Chapter 2.

We now turn an analysis of second moments predicted by the SOE-RBC model and compare them to the Canadian data.

Some Empirical Regularities of the Canadian Economy

Why Canada? Because it is the (open) economy on which we based the calibration of the model, following Mendoza (1991).

Variable	Canadian Data		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	2.8	0.61	1
c	2.5	0.7	0.59
i	9.8	0.31	0.64
h	2	0.54	0.8
$\frac{tb}{y}$	1.9	0.66	-0.13

Source: Mendoza AER, 1991. Annual data. Log-quadratically detrended.

Comments

- Volatility ranking: $\sigma_{tb/y} < \sigma_c < \sigma_y < \sigma_i$.
- Consumption, investment, and hours are procyclical.
- The trade-balance-to-output ratios is countercyclical.
- All variables considered are positively serially correlated.
- Similar stylized facts emerge from other small developed countries (see, e.g., chapter 1).

Empirical and Theoretical Second Moments

	Canadian Data 1946 to 1985			Canadian Data 1960 to 2011			Model		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}
y	2.8	0.6	1	3.7	0.9	1	$\boxed{3.1}$	$\boxed{0.6}$	1
c	2.5	0.7	0.6	2.2	0.7	0.6	2.7	0.8	0.8
i	9.8	0.3	0.6	10.3	0.7	0.8	$\boxed{9.0}$	0.1	0.7
h	2.0	0.5	0.8	3.6	0.7	0.8	$\boxed{2.1}$	0.6	1
$\frac{tb}{y}$	1.9	0.7	-0.1	1.7	0.8	0.1	$\boxed{1.8}$	0.5	-0.04
$\frac{ca}{y}$							1.4	0.3	0.05

Comments:

- σ_h, σ_i, σ_y, $\sigma_{tb/y}$, and $\rho_{y_t,y_{t-1}}$ were targeted by calibration, so no real test here.
- model correctly places σ_c below σ_y and σ_i and above σ_h and $\sigma_{tb/y}$.
- model correctly makes tb/y countercyclical.
- model overestimates the correlations of hours and consumption with output.

Why is corr($\mathrm{h_t, y_t}$) Exactly Equal to One?

Recall the equilibrium condition in the labor market:

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = A_t F_h(k_t, h_t)$$

with GHH preferences and Cobb-Douglas technology, this becomes

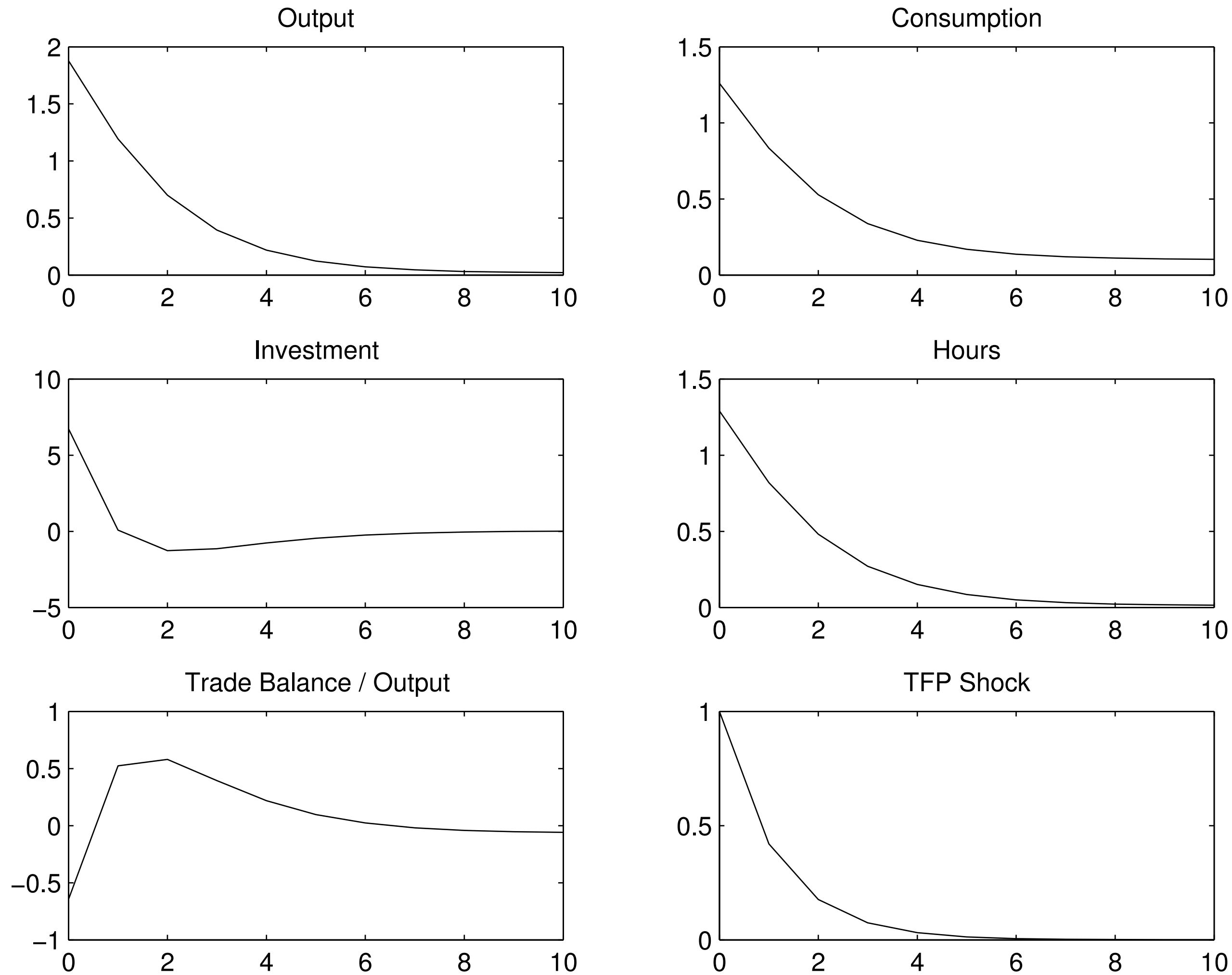
$$\begin{aligned} h_t^{\omega-1} &= A_t(1-\alpha)k_t^{\alpha-1}h_t^{\alpha} \\ &= (1-\alpha)\frac{y_t}{h_t} \end{aligned}$$

Log-linearizing,

$$\omega \widehat{h}_t = \widehat{y}_t,$$

which says taht up to first order, hours and output are perfectly correlated.

Response to a Positive Technology Shock



Source: Schmitt-Grohé and Uribe (JIE, 2003)

Comments:

- Output, consumption, investment, and hours expand.
- The trade balance deteriorates.

4.9 The Complete Asset Markets (CAM) Model

$$E_t r_{t+1} b_{t+1} = b_t + y_t - c_t - i_t - \Phi(k_{t+1} - k_t),$$

$$\lim_{j \to \infty} E_t q_{t+j} b_{t+j} \geq 0,$$

$$q_t = r_1 r_2 \dots r_t,$$

$$\lambda_t r_{t+1} = \beta \lambda_{t+1}.$$

$$\lambda_t^* r_{t+1} = \beta \lambda_{t+1}^*.$$

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\lambda_{t+1}^*}{\lambda_t^*}.$$

$$\lambda_t = \xi \lambda_t^*,$$

$$\lambda_t = \psi_4,$$

Calibration: Set ψ_4 so that steady-state consumption equals steady-state consumption in the model with Uzawa preferences.

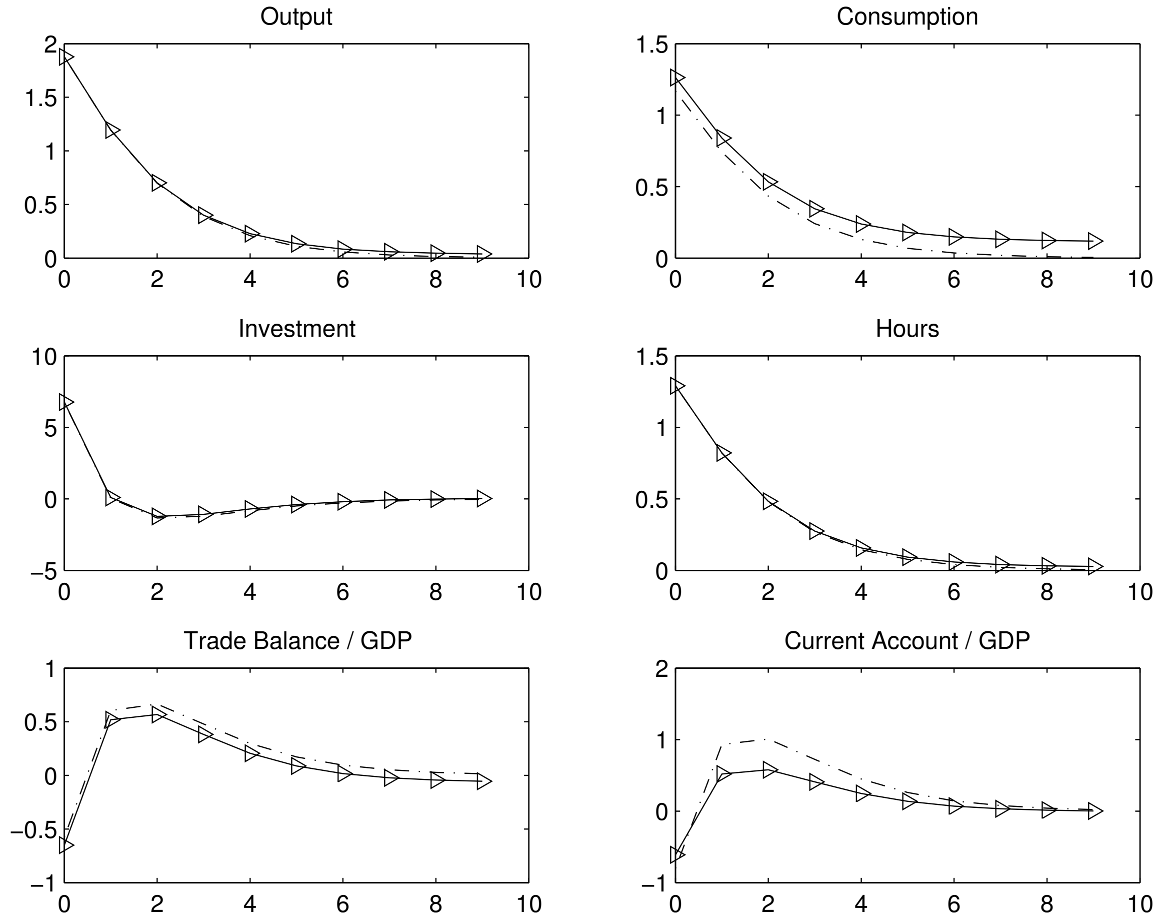
The SOE-RBC Model With Complete Asset Markets: Predicted Second Moments

	σ_{x_t}		$\rho_{x_t,x_{t-1}}$		ρ_{x_t,GDP_t}	
variable	CAM	EDEIR	CAM	EDEIR	CAM	EDEIR
y	3.1	3.1	0.61	0.62	1.00	1.00
c	1.9	2.71	0.61	0.78	1.00	0.84
i	9.1	9.0	0.07	0.07	0.66	0.67
h	2.1	2.1	0.61	0.62	1.00	1.00
$\frac{tb}{y}$	1.6	1.78	0.39	0.51	0.13	-0.04
$\frac{ca}{y}$	3.1	1.45	-0.07	0.32	-0.49	0.05

Note. Standard deviations are measured in percentage points. The columns labeled CAM are produced with the Matlab program `cam_run.m` available at

`http://www.columbia.edu/~mu2166/closing.htm`.

Impulse Response to a Unit Technology Shock
One-Bond Versus Complete Asset Market Models



Dash-diamond, EDEIR model. Dash-dotted, complete-asset-market model.

4.10 Alternative Ways to Induce Stationarity

4.10.1 The Internal Debt-Elastic Interest Rate (IDEIR) Model

$$r_t = r + p(d_t),$$

The Euler equation becomes

$$\lambda_t = \beta[1 + r + p(d_t) + p'(d_t)d_t]E_t\lambda_{t+1}$$

$$p(d) = \psi_2\left(e^{d-\bar{d}} - 1\right),$$

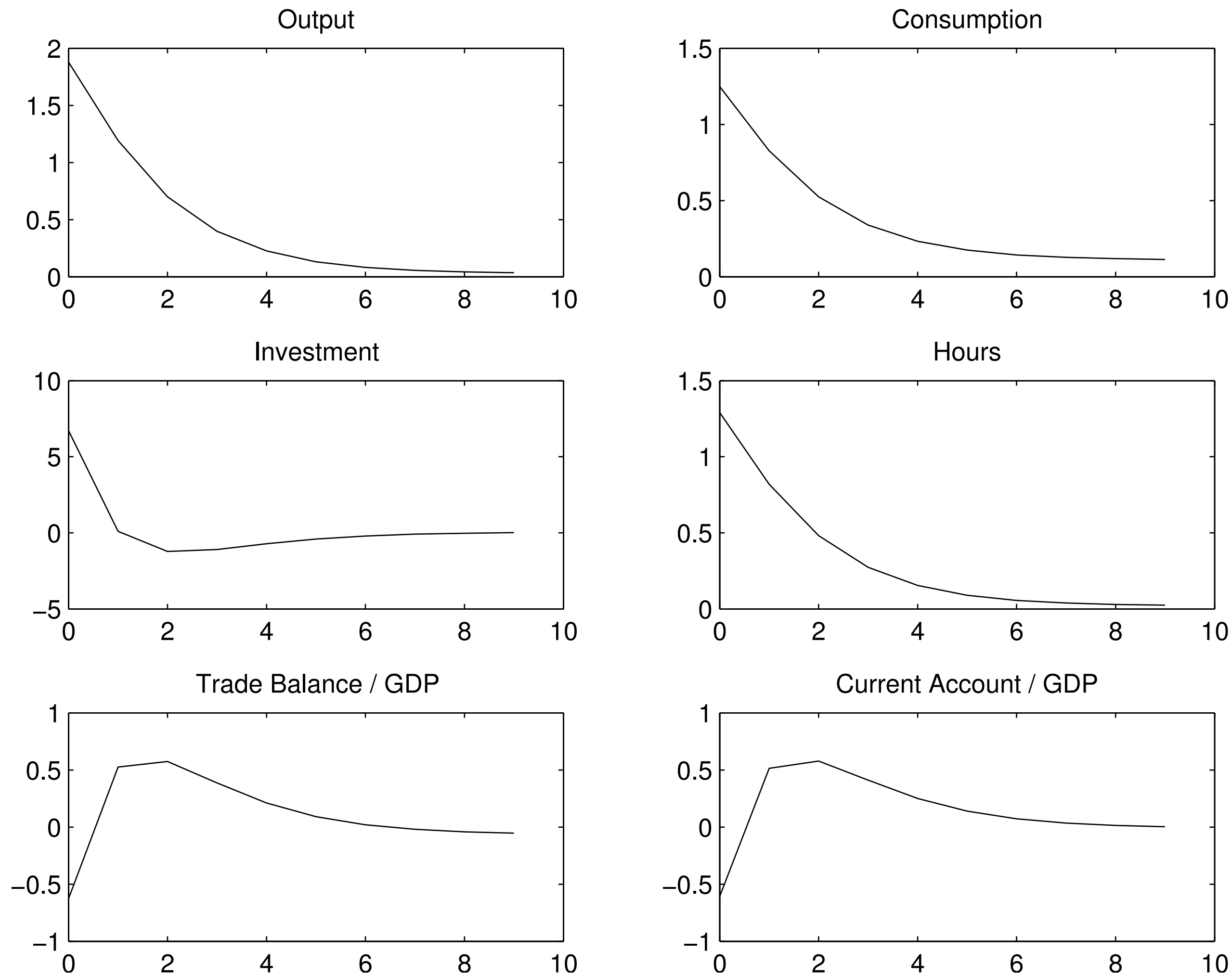
Calibration: Same as in the external case. Note that the steady-state value of debt is no longer equal to $\bar{d}$. Instead, d solves

$$(1 + d)e^{d-\bar{d}} = 1 \Rightarrow d = 0.4045212.$$

Internal Debt-Elastic Interest-Rate

Variable	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	3.1	0.62	1
c	2.5	0.76	0.89
i	9	0.068	0.68
h	2.1	0.62	1
tb/y	1.6	0.43	-0.036
ca/y	1.4	0.31	0.041

Internal Debt-Elastic Interest Rate Premium Response to a Positive Technology Shock



Comment: The economy with internal debt-elastic interest rate premium behaves very simularly to the economies featuring other stationarity inducing devices.

The portfolio adjustment cost (PAC) model

- HHs face assets-holding costs $\Psi(d_t)$, and the budget constraint becomes

$$\begin{aligned} d_t =& (1+r^*)d_{t-1} - A_t F(k_t, h_t) + c_t + k_{t+1} \\ & - (1-\delta)k_t + \Phi(k_{t+1} - k_t) + \Psi(d_t) \end{aligned}$$

The portfolio adjustment cost (PAC) model

- The FOC becomes

$$\lambda_t = \beta \frac{1 + r^*}{1 - \Psi'(d_t)} \mathsf{E}_t \lambda_{t+1}$$

so the effective interest rate r_t is debt elastic

$$1 + r_t = \frac{1 + r^*}{1 - \Psi'(d_t)}$$

Decentralization of the model

– HHs borrow or lend $\tilde{d}_t$ with banks with r_t

$$\begin{aligned}\tilde{d}_t =&(1 + r_{t-1})\tilde{d}_{t-1} - A_t F(k_t, h_t) + c_t + k_{t+1} \\ &- (1 - \delta)k_t + \Phi(k_{t+1} - k_t) - \Pi_t\end{aligned}$$

Decentralization of the model

- Banks can borrow d_t from abroad at rate r^* and maximize profits $\Pi_{t+1} = (1 + r_t)\tilde{d}_t - (1 + r^*)d_t$
- Banks suffer operational costs $\Psi(d_t)$, and subject to the constraint $\tilde{d}_t = d_t - \Psi(d_t)$
- By solving the maximization problem, we can also get

$$1 + r_t = \frac{1 + r^*}{1 - \Psi'(d_t)}$$

4.10.3 The External Discount Factor (EDF) Model

$$\theta_{t+1} = \beta(\tilde{c}_t, \tilde{h}_t)\theta_t \qquad t \geq 0,$$

$$\theta_0 = 1,$$

where $\tilde{c}_t$ and $\tilde{h}_t$ denote per capita consumption and hours worked.

$$\lambda_t = \beta(\tilde{c}_t, \tilde{h}_t)(1 + r_t)E_t\lambda_{t+1}$$

$$\lambda_t = U_c(c_t, h_t)$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t)$$

$$\begin{aligned} \lambda_t[1 + \Phi'_t] &= \beta(\tilde{c}_t, \tilde{h}_t)E_t\lambda_{t+1}[A_{t+1}F_k(k_{t+1}, h_{t+1}) \\ &+ 1 - \delta + \Phi'_{t+1}] \end{aligned}$$

In Equilibrium

$$c_t = \tilde{c}_t \text{ and } h_t = \tilde{h}_t$$

Other ways

- The internal discount factor (IDF) model
- The complete asset market (CAM) model
- The perpetual youth (PY) model