Advanced Microeconomics, Spring 2024

Problem set 1, due March 21

1. Consider the following simplified version of poker. There are two players and a deck of three cards–A, K and Q of spades. Each player is randomly dealt one hand. Each player sees her own card but not that of her opponent and can either bet \$10 or "fold". If one or bother players fold, no money changes hands. If both players bet, the player with the higher card wins, so that she wins \$10 in net terms.

   A pure strategy for a player is a rule that indicates whether to bet or to fold conditional on the card held.

   (a) Informally but carefully ARGUE that some of each player's strategies are weakly dominated. (Specify which ones, that is, and why they are weakly dominated.)

   (b) Informally but carefully DERIVE the result of continuing the process of iterated elimination of weakly dominated strategies as far as possible.

   (c) Suppose the deck of cards is all 13 spades–A, K,. . ., 2. Each player still gets a single card, dealt at random. Answer (b) for this new situation.

2. For the 3-player game shown below:

Player 2

Player 1

	x	y
A	(0, 0, 3)	(0, 0, 3)
B	(-4, 0, 1)	(-4, 0, 1)
C	(1, 1, -2)	(-4, 0, 1)
D	(1, 1, -2)	(0, 0, 3)

Player 3 plays L

Player 2

	x	y
A	(0, 0, 3)	(0, 0, 3)
B	(3, 0, 1)	(3, 0, 1)
C	(5, 4, 0)	(3, 0, 1)
D	(5, 4, 0)	(0, 0, 3)

Player 3 plays R

   (a) Find all pure strategy NE;

   (b) Among the NE from (a), identify those involving players playing dominated strategies;

   (c) Show there exists NE which involves no players playing dominated strategy but nevertheless is not normal-form perfect.

3. (Mixed strategy) Show that the two-player game illustrated below has a unique equilibrium in pure strategies and that there is no additional equilibrium in mixed strategies.

	L	M	R
U	$(1,-2)$	$(-2,1)$	$(0,0)$
M	$(-2,1)$	$(1,-2)$	$(0,0)$
D	$(0,0)$	$(0,0)$	$(1,1)$

4. Consider the following majority voting game. There are three congressmen. Their names are A, B and C. There are three possible alternatives to vote for: X, Y, and Z. The alternative that gets the majority of votes wins. In case that all alternatives receive the same vote, then each one is chosen with equal probability. The payoffs of the congressmen depend exclusively on which alternative is chosen through voting. Their payoffs are as follows:

$$U_A(X) = U_B(Y) = U_C(Z) = 2;$$
$$U_A(Y) = U_B(Z) = U_C(X) = 1;$$
$$U_A(Z) = U_B(X) = U_C(Y) = 0.$$

   Determine whether there are any Nash equilibria in pure strategies for this game. (You may find more than one Nash equilibrium).

5. (Hotelling) Suppose that we have two firms located on a line of length 1. The unit costs of the good for each store is c. Consumers incur a transportation cost of tx^2 for a length of x. Consumers have unit demands, and are uniformly distributed along the line. Firm 1 is located at point $a \geq 0$ and firm 2 at point $1-b$, where $b \geq 0$ and without loss of generality, $1-a-b \geq 0$ (firms 1 is to the left of firms 2; $a = b = 0$ corresponds to maximal differentiation and $a + b = 1$ corresponds to minimal differentiation, i.e. perfect substitutes). Assumes that the market is covered and firms sell positive quantities.

   (a) Show that given a and b, firm 1 will charge

$$p_1 = c + t(1-a-b)(1+\frac{a-b}{3})$$

   whereas firm 2 will charge

$$p_2 = c + t(1-a-b)(1+\frac{b-a}{3}).$$

   (b) Find firms' market shares of the market.

   (c) Now consider a first stage to this game where the two firms choose their locations, knowing that prices will be chosen in the second stage as in (a). Where do they locate?

(d) What would be socially optimal locations of the two firms? Compare with the market outcome.

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Problem set 2, due April 11

1. Consider the two-player game whose extensive form representation (excluding payoffs) is depicted in Figure 1.
   (a) Show that for any behavior strategy that player 1 might play, there is a realization equivalent mixed strategy; that is, a mixed strategy that generates the same probability distribution over the terminal nodes for any mixed strategy choice by player 2.
   (b) Show that the converse is also true: For any mixed strategy that player 1 might play, there is a realization equivalent behavior strategy.
   (c) Suppose that we change the game by merging the information sets at player 1's second round of moves (so that all four nodes are now in a single information set). Which of the two results in (b) and (c) still holds?

2. For the game given below:

Player 2

Player 1		B1	B2	B3	B4
	A1	(0, 6)	(3, 1)	(2, 0)	(3, 7)
	A2	(1, 0)	(9, 4)	(0, 12)	(1, 1)
	A3	(0, 0)	(10, -1)	(2,-3)	(0, 1)
	A4	(7, 3)	(0, 0)	(5, 1)	(-1, 2)
	A5	(2, 8)	(-2, 1)	(3, 1)	(1, 0)

   (a) Does any player have any dominated strategies? If yes, what are they?
   (b) Find all pure strategy NE of the game.
   (c) Does the game have any mixed strategy NE? If yes, please find the mixed strategy NE.

3. For the game depicted in Figure 2:
   (a) Determine all SPNE of this game;
   (b) Determine all (pure strategy) sequential equilibrium of this game.

4. For the game depicted in Figure 3:
   (a) Determine all pure strategy normal-form perfect equilibrium of this game;

(b) Determine all pure strategy sequential equilibrium of this game.

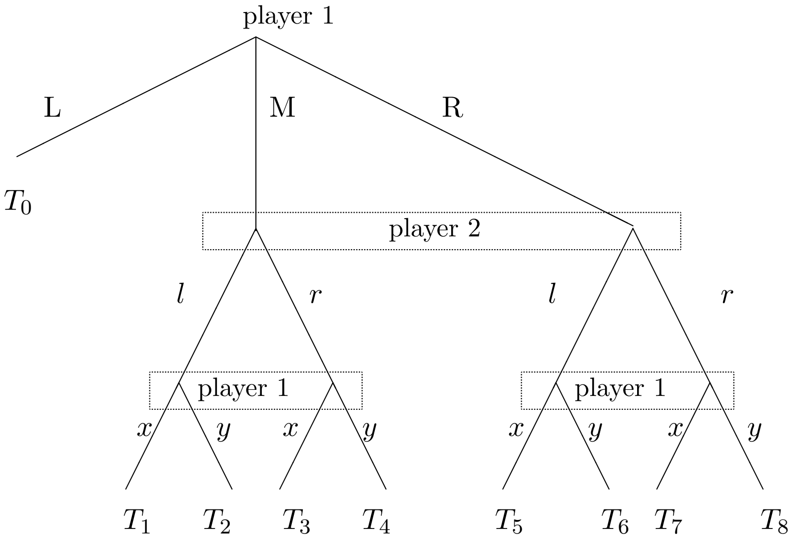

Figure 1

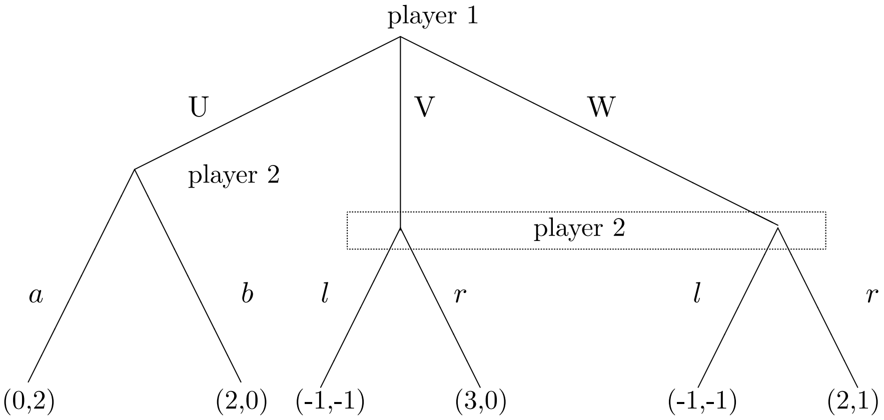

Figure 2

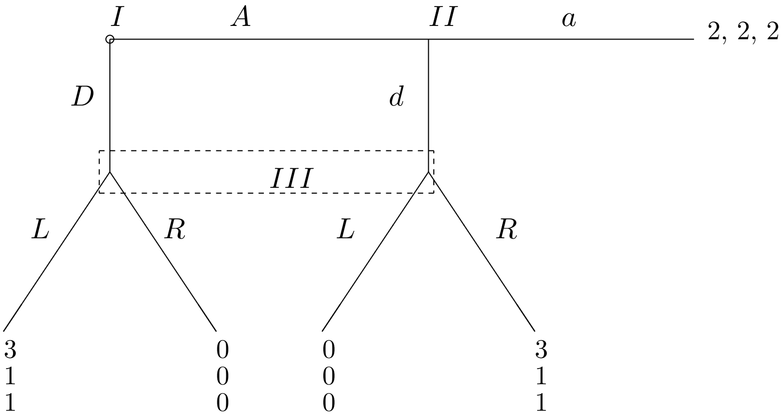

Figure 3

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Problem set 3, due on May 8

1. Consider the N-player version of the (Rubinstein's) bargaining game presented in class. At dates 1, $N+1$, $2N+1$, ..., player 1 offers a division $(x_1, ..., x_N)$ of the pie with $x_i \geq 0$ for all i, and $\sum_{i=1}^{N} x_i \leq 1$. At dates 2, $N+2$, $2N+2$, ..., player 2 offers a division, and so on. When player i offers a division, the other players simultaneously accept or veto the division. If all accept, the pie is divided; if at least one vetoes, player $i+1$ $(mod\ N)$ offers a division in the following period. Assuming that the players have common discount factor δ, show that, for all i, player i offering a division

$$\left(\frac{v}{1+\cdots+\delta^{N-1}}, \frac{v\delta}{1+\cdots+\delta^{N-1}}, ..., \frac{v\delta^{N-1}}{1+\cdots+\delta^{N-1}}\right)$$

for players i, $i+1$, ...,$i+N-1$ $(mod\ N)$ at each date $(kN+i)$ and the other players' accepting any offer equal or higher than those amounts is a subgame-perfect Nash equilibrium.

2. Two persons A and B can play one of the following games: G1:

B

		L	R
A	U	4, 4	0, 0
	D	0, 0	1, 1

G1

B

		L	R
A	U	-1, -1	0, 0
	D	0, 0	4, 4

G2

   (a) Suppose both A and B know they play game G1. Find all NE of the game.

   (b) Now suppose they play G1 and G2 with equal probabilities, which is common knowledge among them. In addition, A knows which game they are playing but B does not know if they are as in G1 or as in G2. Model the game as a Bayesian game and find pure strategy Bayesian NE of the game.

3. Suppose that Michael and John are playing the following game of incomplete information. Michael(the row player) is perfectly aware of the payoffs but John (the column player) does not know if they are as in G1 or as in G2.

John

Michael	L	R
U	1, 1	0, 0
D	0, 0	0, 0

G1

John

Michael	L	R
U	0, 0	0, 0
D	0, 0	2, 2

G2

a) Model this situation as a Bayesian game.

b) Assuming that it is common knowledge that payoffs as in G1 or as in G2 with equal probabilities, find *all* Bayesian-Nash equilibria of the game.

4. There are two individuals $i = 1, 2$ who need to decide simultaneously whether to contribute to a public good or not ("C" or "NC"). Each player derives a benefit of 1 if at least one contributes and 0 if none does. A player's cost of contributing is c_i. While the benefit is common knowledge, each agent's cost c_i is known only to himself. However, it is common knowledge that for $i = 1, 2$, c_i is independently drawn from a uniform distribution on [0,2]. Identify the Bayesian Nash equilibria of this game.

5. In a first-price, all-pay auction, the bidders simultaneously submit sealed bids. The highest bid wins the objects and every bidder pays the seller the amount of his bid. The all-pay auction is a useful model of lobbying activity. In such models, different interest groups spend money (their bids) in order to influence government policy and the one spending the most (the highest bidder) is able to tilt policy in its favored direction (winning the auction). Since money spent on lobbying is a sunk cost borne by all groups regardless of which group is suceessful in obtaining its preferred policy, such situations have a natural all-pay aspects.

   Consider the independent private values model with symmetric bidders whose values are each distributed according to the distribution function F, with density f.

   (a) Find the unique symmetric equilibrium bidding function. Interpret.

   (b) Do bidders bid higher or lower than in a first-price auction?

   (c) Find an expression for the seller's expected revenue.

   (d) Show that the seller's expected revenue is the same as in a first-price auction.

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Problem set 4, due on May 30

1. There are two individuals in the economy, Mike and Harry. Mike is endowed with 90 units of good X and 10 units of good Y, while Harry is endowed with 10 units of good X and 90 units of good Y. Their utility functions are, respectively, $U^M(X,Y) = (X-20)(Y-10)$, and $U^H(X,Y) = 10(X-10)^{\frac{1}{2}}(Y-20)^{\frac{1}{2}}$.

   (a) Find Mike and Harry's demand functions, that is, X^M, Y^M, X^H, Y^H as a function of P_X, P_Y.

   (b) Find the excess demand function Z_X and Z_Y, and show that the Walras' law holds.

   (c) Solve the competitive equilibrium.

2. Suppose Jack and Tom have the endowment $\omega_J = (4,1)$ and $\omega_T = (1,4)$. Then consider the following six cases. In each case, state, in precise mathematical terms, the set of Pareto-efficient allocations, the core, the set of equilibrium price vectors and the corresponding set of equilibrium allocations. (Let good 1 be the numeraire, i.e. fix $P_1 = 1$)

   Leontief/Leontief

   $$U_J(x_{1J}, x_{2J}) = \min(x_{1J}/2, x_{2J})$$

   $$U_T(x_{1T}, x_{2T}) = \min(x_{1T}, x_{2T}/2).$$

   Leontief/Cobb-Douglas

   $$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

   Leontief/Linear

   $$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T} + 2x_{2T}.$$

   Cobb-Douglas/Cobb-Douglas

   $$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

Cobb-Douglas/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

Linear/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J} + 3x_{2J}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

3. Compute the aggregate excess demand function Z in each of the following 6 examples, given that the endowments are $\omega_J = (5, 0)$ and $\omega_T = (0, 5)$. Show that your excess demand function Z is homogeneous of degree 0, satisfying Walras's Law.

Leontief/Leontief

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = \min(x_{1T}, x_{2T}/2)/2.$$

Leontief/Leontief

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J})$$

$$U_T(x_{1T}, x_{2T}) = \min(x_{1T}, x_{2T}).$$

Leontief/Leontief

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + 2x_{2T}.$$

Leontief/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

Cobb-Douglas/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

Cobb-Douglas/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/2} x_{2J}^{1/2}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{4/5} x_{2T}^{1/5}.$$

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Problem set 5, due June 13

1. Suppose Jack and Tom have the endowments $\omega_A = (6, 0)$ and $\omega_B = (0, 6)$. Their preferences are defined by a pair of utility functions. For the following cases, find the utility possibility frontier. State your answer both in precise mathematical notation and in terms of graph.

   Leontief/Cobb-Douglas

   $$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

   Leontief/Linear

   $$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

   Cobb-Douglas/Cobb-Douglas

   $$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

   Cobb-Douglas/Linear

   $$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

   Linear/Linear

   $$U_J(x_{1J}, x_{2J}) = x_{1J} + x_{2J}$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T} + 2x_{2T}.$$

2. Suppose the economy is endowed with capital and labor, $k = 6$, $l = 6$, and can produce two outputs (b, c) with production functions given below. For each pair of production functions given, derive the transformation function $T(y)$ where $y = (y_1, y_2, y_3, y_4) = (-k, -l, b, c)$.

   Leontief/Cobb-Douglas

   $$b(k, l) = \min(k, l/2)$$

   $$c(k, l) = k^{1/3} l^{2/3}.$$

Leontief/Linear

$$b(k,l) = \min(k, l/2)$$

$$c(k,l) = k + l.$$

Cobb-Douglas/Cobb-Douglas

$$b(k,l) = k^{1/3}l^{2/3}$$

$$c(k,l) = k^{1/3}l^{2/3}.$$

Cobb-Douglas/Linear

$$b(k,l) = k^{1/3}l^{2/3}$$

$$c(k,l) = k + l.$$

Linear/Linear

$$b(k,l) = k + l$$

$$c(k,l) = k + 2l.$$

3. Suppose there are three people, named Bob, Jack and Tom, whose only purpose in life is to eat chocolate. Their utility functions are

$$u_B(c_B, l_B) = c_B,$$

$$u_J(c_J, l_J) = 2c_J,$$

$$u_T(c_T, l_T) = 3c_T.$$

Each of them is endowed with no chocolate and one unit of labor. There are 3 chocolate factories, whose production functions are

$$f_1(l_1) = l_1,$$

$$f_2(l_2) = 2l_2,$$

$$f_3(l_3) = 3l_3.$$

Bob owns Firm 1, Jack owns Firm 2, and Tom owns Firm 3.

   (a) Determine the society's transformation function.

   (b) Determine the set of Pareto efficient consumption allocations.

(c) Determine the set of equilibrium consumption allocation(s). For each equilibrium consumption allocation, give the corresponding production allocation and price vector.

4. Consider the following market for used cars. There are N potential sellers and M potential buyers, and $M > N$. Each seller has exactly one used car to sell and is characterized by the quality of the used car he has. Let $\theta \in [0, 1]$ index the quality of used car. If a seller of type θ sells his car for a price of P, his payoff is $u_s(P, \theta)$, and is 0 if he does not sell his car. The payoff for the buyer is $\theta - P$ if he buys a car of quality θ at price P, and is 0 if he does not buy. Information is asymmetric: Sellers know the quality of used cars but buyers do not. However, buyers know the quality of used car is uniformly distributed on [0,1].

   (a) Argue that in a competitive equilibrium under asymmetric information, we must have $E[\theta|P] = P$.

   (b) Find all equilibrium prices when $u_s(P, \theta) = P - \theta/2$.

   (c) Find all equilibrium prices when $u_s(P, \theta) = P - \sqrt{\theta}$. Describe the equilibrium in words. In particular, which cars are traded in equilibrium?