Empirical Asset Pricing: Anomaly

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Failure of CAPM

- After the establishment of CAPM, scholars seem to believe that CAPM should be the right model to capture the patterns in financial markets.
- What actually make the case more embarrassing is that we actually could find a great portion of evidence to prove the validation of CAPM.
- The validation of CAPM becomes the rare case, in the data.
- The main results seem to prove that the CAPM works before 1960s.
- When the data quality and empirical method improve in CRSP ear, CAPM disappears.

Fama and French (1996)

Multifactor Explanations of Asset Pricing Anomalies

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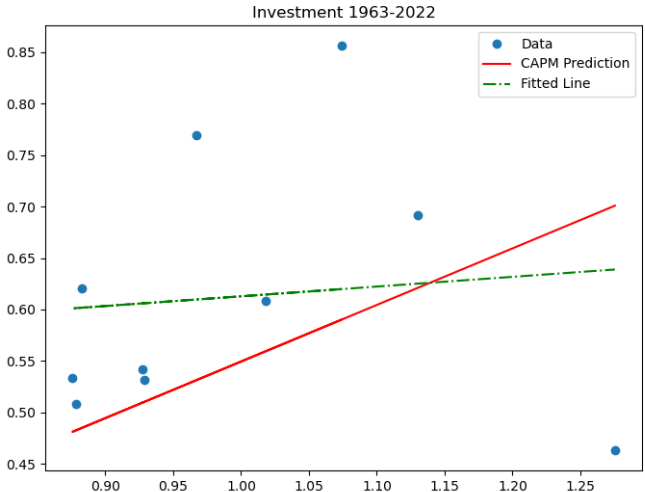
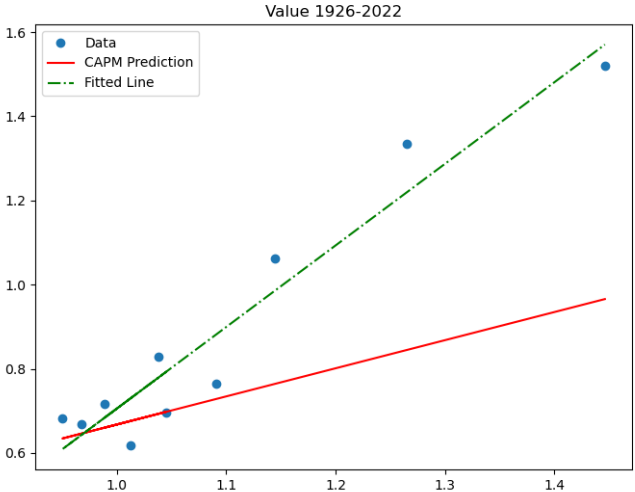
ABSTRACT

Previous work shows that average returns on common stocks are related to firm characteristics like size, earnings/price, cash flow/price, book-to-market equity, past sales growth, long-term past return, and short-term past return. Because these patterns in average returns apparently are not explained by the CAPM, they are called anomalies. We find that, except for the continuation of short-term returns, the anomalies largely disappear in a three-factor model. Our results are consistent with rational ICAPM or APT asset pricing, but we also consider irrational pricing and data problems as possible explanations.

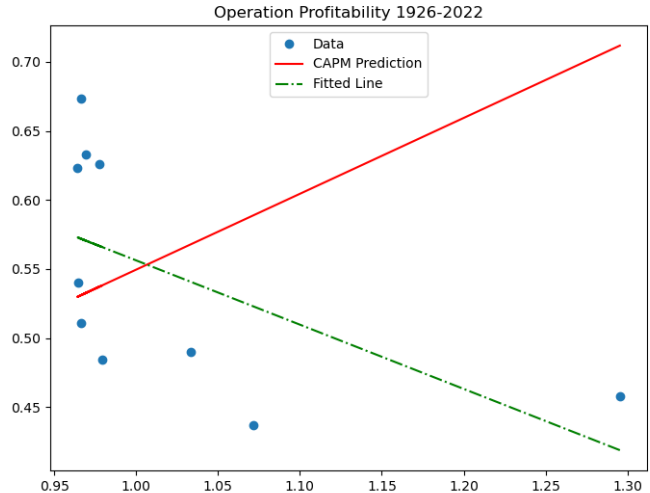
Investment Portfolio against CAPM prediction



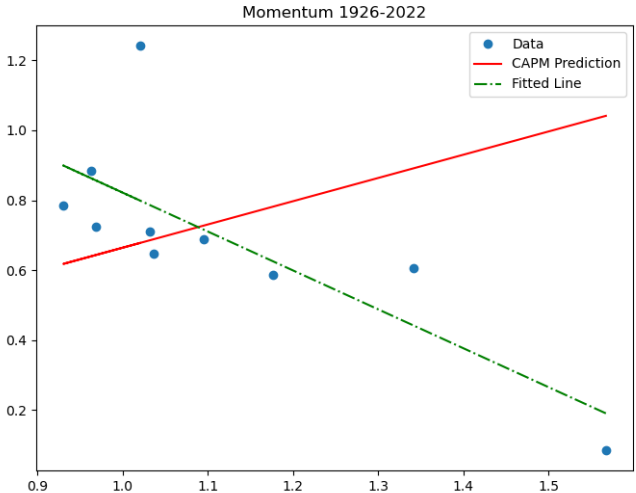
Value Portfolio against CAPM prediction



Operation Profitability Portfolio against CAPM prediction



Momentum Portfolio against CAPM prediction



Why we could use multifactor model?

- The logic of CAPM comes from a three step analysis.
    1. Mean-variance analysis.
    2. Two fund separation theorem.
    3. General equilibrium analysis.
- The natural question is, if I just want to run a regression, do I really need that much?
- Stephen Ross tells us: Not Really.
- He introduces the Arbitrage Pricing Theory.

Arbitrage and Mispricing

- Arbitrage: Simultaneous purchase and sale of equivalent securities and make profit from the discrepancies.
- We name the discrepancies as mispricing.
- A basic principle of capital market theory is that equilibrium market prices should rule out the mispricing. Otherwise, the equilibrium has not been reached.

One Price Law

- Some times, we name the non arbitrage as One Price Law.
- The Law of One Price states that if two assets are equivalent in all economically relevant respects, then they should have the same market price.
- Actually, one price law doesn't hold automatically. It holds because the existence of arbitrageurs.
- If they observe a violation of the law, they will engage in arbitrage activity simultaneously buying the asset where it is cheap and selling where it is expensive.
- In the process, they will bid up the price where it is low and force it down where it is high until the arbitrage opportunity is eliminated.

One Price Law

- Arbitrage could sometimes be extremely difficult, which is just the power of the market.
- Does non arbitrage means there is no arbitrage activities at all? No.
- I would like to use the following example to present the existence and difficulty of arbitrage .
- What is the probability for you to pick up a wallet on the street of Shanghai today?
- Nearly 0.
- What is the probability if there is someone who picks up a wallet in Shanghai today?
- Nearly 1.

Arbitrage Pricing Theory

- Just like CAPM, Arbitrage Pricing Theory predict a security market line which connects expected return and risk. Of course, in a very different way.
- APT requires three assumptions:
  1. Security returns can be described by a factor model.
  2. There are sufficient securities to diversify away idiosyncratic risk.
  3. Well-functioning security markets do not allow for the persistence of arbitrage opportunities.

Arbitrage in APT and Practice

- In the definition of APT, an arbitrage opportunity arises when an investor can earn risk-free profits without making a net investment.

- Practitioners often use the terms arbitrage and arbitrageurs more loosely than our strict definition.

- Arbitrageur often refers to a professional searching for mispriced securities in specific areas such as merger-target stocks, rather than to one who seeks strict (risk-free) arbitrage opportunities.

- Such activity is sometimes called risk arbitrage to distinguish it from pure arbitrage.

Arbitrage Pricing Theory

- Let's recall the portfolio premium equation, and assume a single factor structure:

$$R_P = E(R_P) + \beta_P F + \epsilon_P.$$

where

$$\begin{aligned} \beta_P &= \sum_{i=1}^{n} w_i \beta_i, \\ E(R_P) &= \sum_{i=1}^{n} w_i E(R_i) \\ \sigma_P^2 &= \beta_P^2 \sigma_P^2 + \sigma^2(\epsilon_P) \end{aligned}$$

the above equation holds because we treat the expectation as constant.

- When the portfolio are well diversified,

$$\lim_{n \to \infty} \sigma^2(\epsilon_P) = 0$$

Arbitrage Pricing Theory

- Let's assume excess return structure follows:

$$R_P = \mu_P + \beta_P F.$$

$$E(R_P) = \mu_P + \beta_P E(F).$$

- Now let's execute the arbitrage. How could we construct a portfolio with 0 beta without initial investment?

- Condition 1, no initial investment:

$$w'\iota = 0.$$

where

$$\iota' = [1, 1, ..., 1]$$

Arbitrage Pricing Theory

- Condition 2, no risk:

$$w'\beta = 0.$$

- Expected return of the arbitrage portfolio:

$$R_a = w'R_P = w'\mu.$$

- Let's apply the Non-Arbitrage Condition:

$$R_a = w'R_P = w'\mu = 0.$$

Arbitrage Pricing Theory

- In this step, I need some linear algebra knowledge from you.
- What's the meaning of $w'\iota = 0$ in geometry?
- Othoganal, $\perp$.
- 1st Conclusion: $w \perp \iota$ and $w \perp \beta$.
- 2nd Conclusion: $w \perp \text{Plane}\{\iota, \beta\}$.
- Reminder: we also have one non-arbitrage condition

$$w'\mu = 0.$$

- Now, please answer the question: where is the μ?.

Arbitrage Pricing Theory

- μ must lie in the Plane$\{\iota, \beta\}$.
- Accordingly, we could write the alpha as liner combination of ι and β:

$$\mu = \gamma_1 \iota + \gamma_2 \beta.$$

- We have two special assets, risk-free asset and market asset.
- Risk-free asset: $\mu = R_f$, $\beta = 0$:

$$\gamma_1 = R_f.$$

- Market asset: $\beta = 1$ and $\gamma_1 = R_f$.

$$\gamma_2 = \mu_M - R_f.$$

Arbitrage Pricing Theory

- Let's plug in the γ_1 and γ_2:

$$\mu_i = R_f + \beta_i(\mu_m - R_f).$$

- Still remember CAMP? How do the two look like?

- We are able to derive the factor model without any utility function, or two-fund separation.

- It is relatively easy to extend to multifactor case,

$$E(R_i) = \mu_i - R_f = \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + ... + \beta_{iK}\lambda_K.$$

where the λ is the factor K's risk premium, β is the risk exposure.

Sort Portfolio

- The typical routine to get a factor or anomaly is the so called “portfolio sort”:

  1. Sort stocks on a characteristic into five(quintile) or ten(decile) portfolios and compute average returns.
  2. Construct a long-short strategy that buys the top portfolio shorts the bottom portfolio.
  3. The factor constructed in last step is regard as a factor to explain average returns.

- By forming a long-short strategy, we tease out some of the passive exposures, for instance, to the market that would arise from a long-only portfolio.

Fama-Fench Two-Way Sort

- The single sort method works as long as market betas or other variables and the characteristic you sort on are not highly correlated.
- However, this is not always true.
- We usually turn to another “controlling variables” method: double sort.
- In most cases, the first order sorting variables is the variable that you want.
- The second order sorting variables is the variable that you want to control.

Fama-Fench Two-Way Sort

- SMB(B/M) = 1/3 (Small Value + Small Neutral + Small Growth) - 1/3 (Big Value + Big Neutral + Big Growth).
- SMB(OP) = 1/3 (Small Robust + Small Neutral + Small Weak) - 1/3 (Big Robust + Big Neutral + Big Weak).
- SMB(INV) = 1/3 (Small Conservative + Small Neutral + Small Aggressive) - 1/3 (Big Conservative + Big Neutral + Big Aggressive).
- SMB = 1/3 (SMB(B/M) + SMB(OP) + SMB(INV))

Conditonal Sort and Unconditional Sort

- When we apply the two-way sort, which means a stock could be in two dimensional groups and on two labels.
- There are two traditional methods:
  1. Conditional sort: after the fisrt stage sort, the second stage sort is within each group in the fisrt stage.
  2. Unconditional sort: after first stage sort, the second stage sort is implimented like there is no first stage.
- When two variables are independent or zero correlated, the conditional and unconditional sort will generate the same results.

Size

- The size anomaly goes back to Banz (1981). Size anomaly is the first major challenge to the CAPM. Small stocks earn positive CAPM alphas.

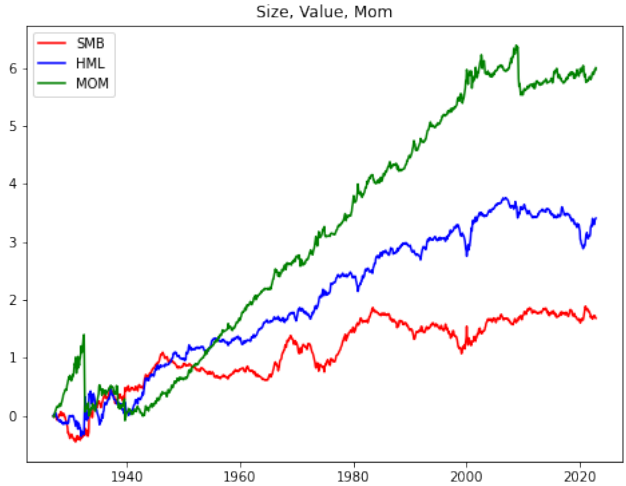
- The underlying idea is to explore the return spread between small companise and big companise.

- The size factor has not done particularly well since its discovery in early 1980s.

- Annualized return of the SMB factor is -0.2% during 2010:01-2020:06. Overall stockmarket: 12.5% during the same period, SMB return of 2.7% from 1926.07-2009.12.

- The size effect has also been linked to the January effect, tax-loss selling, and window dressing.

Size, Value, Momentum



Value

- We mainly employ on the book-to-market ratio to capture valuation.
- HML factor = Long value stocks (high BM), short growth stocks (low BM).
- The HML factor performs very poorly in the last decade. Annualized returns during 2010:01-2020:06 for HML were -5.6% compared to +5.1% for 1926.07-2009.12.
- The value premium is larger for small firms.

Israel and Moskowitz (2013): Value Premium Condition on Size

	Smallest				Largest	
	Size 1	Size 2	Size 3	Size 4	Size 5	Size 1-Size 5
VALUE						
Returns						
5-1 spread	11.22	7.28	5.42	4.24	3.70	7.13
	(3.87)	(3.88)	(2.96)	(1.93)	(1.90)	(2.10)
Long side	16.45	14.26	13.48	12.36	11.01	6.07
	(4.58)	(4.34)	(4.17)	(3.67)	(3.86)	(2.74)
Percent long side	146.5	195.9	248.8	291.3	297.6	
Long=short (*t*-statistic)	(2.81)	(3.50)	(3.66)	(3.81)	(4.01)	
Alphas						
5-1 spread	12.99	6.38	4.63	1.54	2.19	10.58
	(4.52)	(3.41)	(2.53)	(0.74)	(1.14)	(3.21)
Long side	6.15	4.15	3.26	1.73	1.97	4.31
	(2.78)	(2.38)	(2.05)	(1.04)	(1.21)	(1.97)
Percent long side	47.4	65.1	70.4	112.9	89.9	
Long=short (*t*-statistic)	(0.15)	(0.67)	(0.89)	(1.18)	(1.15)	

Momentum

- We sort on the price change between months $t-12$ and $t-2$. Long the winners, short the losers, to the the momentum portfolio.
- The month $t-1$ is excluded for short-term reversals from $t-1$ to t.
- Momentum returns and CAPM alphas are prettey large on average.
- Momentum returns are not strongly related to size.
- Momentum (UMD factor) had annualized returns of 4.1% during 2010:01-2020:06, only half of the 8.4% return from 1927:01-2009:12.

Israel and Moskowitz (2013): Momentum Premium Condition on Size

MOMENTUM						
Returns						
5-1 spread	10.87	12.99	11.53	10.79	7.49	3.42
	(4.50)	(6.22)	(4.76)	(4.08)	(2.95)	(1.56)
Long side	18.76	17.17	15.61	14.98	10.98	7.79
	(5.59)	(5.71)	(5.89)	(6.09)	(4.95)	(3.35)
Percent long side	172.6	132.1	135.3	138.9	146.6	
Long=short (*t*-statistic)	(3.72)	(3.29)	(3.36)	(3.45)	(2.81)	
Alphas						
5-1 spread	13.12	15.30	14.48	14.19	10.24	2.88
	(5.59)	(7.66)	(6.32)	(5.72)	(4.23)	(1.31)
Long side	9.30	7.89	7.26	7.17	3.92	5.37
	(4.47)	(5.13)	(5.71)	(6.24)	(3.83)	(2.40)
Percent long side	70.9	51.6	50.1	50.5	38.3	
Long=short (*t*-statistic)	(1.34)	(0.17)	(0.02)	(0.09)	(1.78)	

Investment

- Investment is usually refer to capital expenditures in financial accounting.
- However, in recent literature, financial economists use change in assets as investment.
- Asset growth strongly predicts returns in the cross-section. Firms with high asset growth get low average returns.
- Fama and French name their investment factor as CMA. Long conservative (low investment) stocks and short aggressive (high investment) stocks.

Cooper, Gulen, and Schill (2008)

Panel B.2: Value-Weighted Portfolio Average Monthly Raw Returns

	Asset Growth Deciles											
YEAR	1(Low)	2	3	4	5	6	7	8	9	10(High)	Spread (10-1)	*t*(spread)
−5	0.0121	0.0123	0.0117	0.0129	0.0142	0.0146	0.0165	0.0207	0.0243	0.0271	0.0150	7.55
−4	0.0114	0.0109	0.0119	0.0131	0.0128	0.0146	0.0172	0.0202	0.0288	0.0307	0.0193	9.15
−3	0.0064	0.0085	0.0100	0.0123	0.0143	0.0151	0.0157	0.0212	0.0279	0.0357	0.0292	10.92
−2	0.0062	0.0083	0.0090	0.0116	0.0135	0.0149	0.017	0.0206	0.0266	0.0396	0.0334	12.86
−1	0.0223	0.0175	0.0153	0.0146	0.0147	0.0141	0.0153	0.0177	0.0192	0.0230	0.0007	0.28
1	0.0148	0.0124	0.0122	0.0116	0.0100	0.0100	0.0102	0.0092	0.0077	0.0043	−0.0105	−5.04
2	0.0133	0.0126	0.0125	0.0101	0.0109	0.0102	0.0098	0.0097	0.0097	0.0065	−0.0068	−3.39
3	0.0169	0.0137	0.0141	0.0126	0.0102	0.0112	0.0116	0.0105	0.0116	0.0116	−0.0053	−2.82
4	0.0132	0.0107	0.012	0.0109	0.0114	0.0103	0.0103	0.0123	0.0111	0.0120	−0.0012	−0.61
5	0.0128	0.0133	0.0121	0.0123	0.0103	0.01	0.0107	0.0113	0.013	0.0126	−0.0002	−0.11
	Cumulative Return											
[−5,−1]	1.0449	0.9918	1.0078	1.2375	1.3631	1.4788	1.7985	2.5321	3.9221	6.4272	5.3822	4.78
[1, 5]	1.2879	1.1133	1.1305	1.0038	0.931	0.8934	0.9352	0.9056	0.9458	0.7911	−0.4967	−4.25

Profitability

- The defintions of profitability are a little bit different in main papers.
- Novy-Marx (2013): gross profits/assets.
- Hou, Xue, and Zhang (2015): ROE.
- Fama and French (2015): operating profitability (minus interest expense).

Novy-Marx (2013)

Table 2
Excess returns to portfolios sorted on profitability.

This table shows monthly value-weighted average excess returns to portfolios sorted on gross profits-to-assets [(REVT − COGS)/AT], employing NYSE breakpoints, and results of time series regressions of these portfolios' returns on the Fama and French factors [the market factor (MKT), the size factor small-minus-large (SMB), and the value factor high-minus-low (HML)], with test-statistics (in square brackets). It also shows time series average portfolio characteristics [portfolio gross profits-to-assets (GP/A), book-to-market (B/M), average firm size (ME, in millions of dollars), and number of firms (n)]. Panel B provides similar results for portfolios sorted on book-to-market. The sample excludes financial firms (those with one-digit standard industrial classification codes of six) and covers July 1963 to December 2010.

		Alphas and three-factor loadings				Portfolio characteristics			
Portfolio	r^e	α	MKT	SMB	HML	GP/A	B/M	ME	n
Panel A: Portfolios sorted on gross profits-to-assets									
Low	0.31	−0.18	0.94	0.04	0.15	0.10	1.10	748	771
	[1.65]	[−2.54]	[57.7]	[1.57]	[5.87]				
2	0.41	−0.11	1.03	−0.07	0.20	0.20	0.98	1,100	598
	[2.08]	[−1.65]	[67.5]	[−3.13]	[8.51]				
3	0.52	0.02	1.02	−0.00	0.12	0.30	1.00	1,114	670
	[2.60]	[0.27]	[69.9]	[−0.21]	[5.42]				
4	0.41	0.05	1.01	0.04	−0.24	0.42	0.53	1,114	779
	[1.94]	[0.83]	[70.6]	[1.90]	[−11.2]				
High	0.62	0.34	0.92	−0.04	−0.29	0.68	0.33	1,096	938
	[3.12]	[5.01]	[58.3]	[−2.03]	[−12.3]				
High–low	0.31	0.52	−0.03	−0.08	−0.44				
	[2.49]	[4.49]	[−0.99]	[−2.15]	[−10.8]				
Panel B: Portfolios sorted on book-to-market									
Low	0.39	0.13	0.98	−0.09	−0.39	0.43	0.25	1,914	965
	[1.88]	[2.90]	[90.1]	[−5.62]	[−23.9]				
2	0.45	−0.02	0.99	0.05	0.04	0.31	0.54	1,145	696
	[2.33]	[−0.29]	[78.1]	[2.61]	[2.23]				
3	0.56	0.03	0.96	0.04	0.22	0.26	0.79	849	640
	[2.99]	[0.53]	[63.5]	[2.09]	[9.71]				
4	0.67	−0.00	0.96	0.10	0.53	0.21	1.12	641	655
	[3.58]	[−0.03]	[74.8]	[5.66]	[27.1]				
High	0.80	0.07	1.01	0.25	0.51	0.21	5.47	367	703
	[3.88]	[1.04]	[60.7]	[10.7]	[20.5]				
High–low	0.41	−0.06	0.03	0.34	0.91				
	[2.95]	[−0.71]	[1.44]	[12.0]	[30.0]				

Novy-Marx (2013)

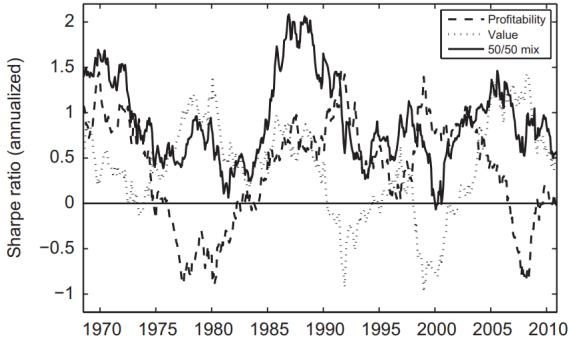


Fig. 1. Performance over time of profitability and value strategies. The figure shows the trailing five-year Sharpe ratios of profitability and value strategies (dashed and dotted lines, respectively) and a 50/50 mix of the two (solid line). The strategies are long–short extreme value-weighted quintiles from sorts on gross profits-to-assets and book-to-market, respectively, and correspond to the strategies considered in Table 2. The sample excludes financial firms and covers June 1963 to December 2010.

Conclusion: Factor and Anomaly

- We usually use factor and anomaly without any strond definition clarification.
- However, it seem that we define factor as these very special anomaly that could explain a lot of other anomalies.
- Above are the main factors.
- Let's stop here right now, there is a factor zoo in ahead of us.