1. Two players, 1 and 2, simultaneously chooses a positive integer up to 3, that is, $s_i \in \{1, 2, 3\}$. Let $i, j \in \{1, 2\}$ and $i \neq j$. If $s_i + s_j \leq 4$ and $s_i \neq s_j$, each player receives the numbers of dollars she names, i.e., s_i dollars. If $s_i = s_j$ or if $s_i + s_j > 4$, then each player receives 0.

   (a) Write down the strategical form of the game; (3 points)

   (b) Identify all strictly dominated and weakly dominated strategies of the players. (1 points)

   (c) Identify all NE of this game. (1 points)

   Answer: The strategic form

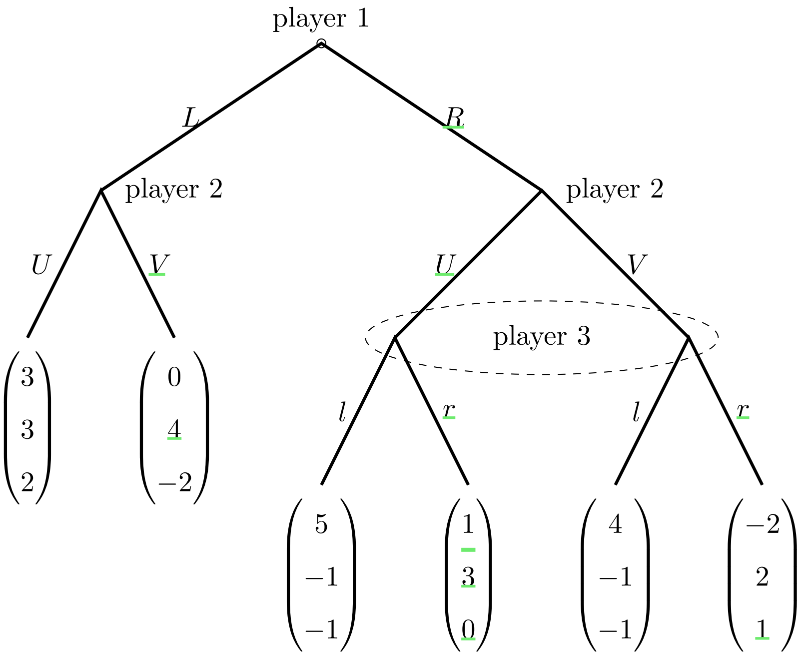
		Player 2		
		1	2	3
Player 1	1	(0, 0)	$(\underline{1}, 2)$	$(\underline{1},\underline{3})$
	2	$(2, \underline{1})$	(0, 0)	(0, 0)
	3	$(\underline{3}, \underline{1})$	(0, 0)	(0, 0)

   "2" is a strictly dominated strategy for player i by a mixed strategy, for example, by $\sigma_i = (\frac{1}{4}, 0, \frac{3}{4})$, $i = 1, 2$.

   There are 2 pure strategy NE, $(3, 1)$ and $(1, 3)$, and 1 mixed NE:

$$\left(\frac{1}{4}, 0, \frac{3}{4}; \frac{1}{4}, 0, \frac{3}{4}\right).$$

1. For the extensive-form game given below



   (a) Write down the normal-form of the game.

   (b) Find all pure strategy NE of the game.

   (c) Find all subgame perfect NE of the game.

Answer:

   (a) The normal-form

	UU	UV	VU	VV
L	3, 3, 2	3, 3, 2	0, 4, -2	0, 4, -2
R	5, -1, -1	4, -1, -1	5, -1, -1	4, -1, -1

l

	UU	UV	VU	VV
L	3, 3, 2	3, 3, 2	0, 4, -2	0, 4, -2
R	1, 3, 0	-2, 2, 1	1, 3, 0	-2, 2, 1

r

   (b) Pure NE

$$(R, VU, r), \quad (L, VV, r)$$

(c) SPNE

$$(R, VU, r).$$

To see this, note that in the subgame after L, player 2's optimal choice is V. In the subgame after R, the NE is (U, r).

	l	r
U	-1, -1	3, 0
V	-1, -1	2, 1

2. Two players, 1 and 2, simultaneously choose a number between 0 and 3, that is, $s_i \in \{0, 1, 2, 3\}$. If the sum of numbers they choose is less than or equal to 3, $s_1 + s_2 \leq 3$, each player i gets s_i dollars. However, if the sum they report is greater than 3, $s_1 + s_2 > 3$, each player gets 0 dollars. Identify all pure NE.

   Answer. The strategic form

		player 2			
		0	1	2	3
	0	0, 0	0, 1	0, 2	0, 3
Player 1	1	1, 0	1, 1	1, 2	0, 0
	2	2, 0	2, 1	0, 0	0, 0
	3	3, 0	0, 0	0, 0	0, 0

Four pure NE:

$$(3, 0), (2, 1), (1, 2), (0, 3), (3, 3).$$

1. Consider the Bayesian game with two players 1 and 2. The set of actions for player 1 is $\{U, D\}$, the set of actions for player 2 is $\{L, M, R\}$. They may play one of the two games given below:

2

		L	M	R
1	U	3, 2	3, 0	3, 3
	D	6, 6	0, 0	0, 9

G1

2

		L	M	R
1	U	3, 2	3, 3	3, 0
	D	6, 6	0, 9	0, 0

G2

   (a) Suppose both players are fully informed as to which game they are playing, find the NE. (2 points)

   (b) Suppose now that G1 and G2 may be played with probability 0.5. Player 1 knows whether they are playing G1 or G2, but player 2 does not. Find the BNE of the Bayesian game. (3 points)

Answer:

(a) Unique NE if G1 is played (U,R). Unique NE if G2 is played (U, M).

(b) The strategic form

2

	L	M	R
UU	3, 2	3, 1.5	3, 1.5
UD	4.5, 4	1.5, 4.5	1.5, 1.5
DU	4.5, 4	1.5, 1.5	1.5, 4.5
DD	6, 6	0, 4.5	0, 4.5

The unique BNE is (DD, L)

1. In an exchange economy, two consumers, Alan and Beck have utility functions $U^A(X,Y) = X^2 + 2XY + Y^2$, and $U^B(X,Y) = \ln X + 2\ln Y$, respectively. Alan is endowed with 3 units of good X and 3 units of good Y, while Beck is endowed with 15 units of X and 15 units of Y.

   (a) Draw the contract curve in the Edgeworth box. (2 points)

   (b) Solve the general equilibrium, and clearly state the equilibrium price and allocations. (3 points)

Answer:

(a) Note

- When Alan consumers both consumption goods, it is true that

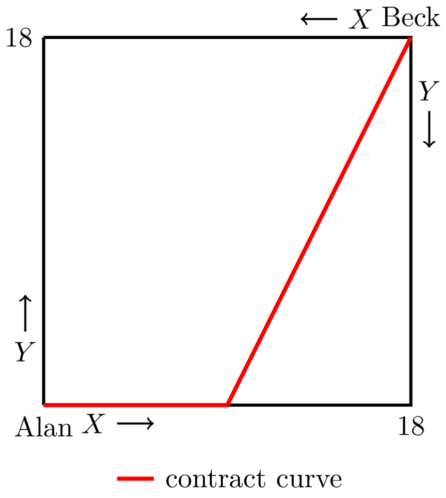
$$MRS^A_{x,y} = 1 = MRS^B_{x,y} = \frac{Y_B}{2X_B} \Longrightarrow$$

$$2X_B = Y_B.$$

  This happens only when $X_B \leq 9$.

- When $X_B > 9$, $Y_B = 18$ and $MRS^B < 1$, and so Alan consumes only Y.

Hence, the contract curve



(b) Given equilibrium price (P_x, P_y), the income of the two consumers are respectively,

$$m_A = 3P_x + 3P_y, \qquad m_B = 15P_x + 15P_y.$$

We can solve Beck's problem to get

$$X_B = \frac{m_B}{3P_x} = 5 + \frac{5P_y}{P_x}, \qquad Y_B = \frac{2m_B}{3P_y} = \frac{10P_x}{P_y} + 10.$$

Let $P_x = 1$. Alan's optimal consumption includes both goods when $P_y = 1$. This is impossible since $P_y = 1$ leads to $Y_B = 20$. So we conclude the only possibility is $P_y > 1$, in which case Alan consumes only X.

Note $Y_B = 18$ only when $P_y = \frac{5}{4}$. Hence, the competitive equilibrium:

$$P_x = 1, \qquad P_y = \frac{5}{4};$$

$$X_A = \frac{27}{4}, \quad Y_A = 0; \qquad X_B = \frac{45}{4}, \quad Y_B = 18.$$

Answer:

1. Utility-maximization gives Alfred's demand:

$$x_{1A} = \frac{10P_1}{2P_1} = 5, \quad x_{2A} = \frac{10P_1}{2P_2} = \frac{5P_1}{P_2}$$

Bob's demand:

$$x_{1B} = \frac{10P_1 + 10P_2}{P_1 + P_2} = 10, \qquad x_{2B} = \frac{10P_1 + 10P_2}{P_1 + P_2} = 10.$$

Carl's demand:

$$x_{1C} = \frac{10P_2^2}{P_1(P_2 + P_2)}, \qquad x_{2C} = \frac{10P_1}{(P_1 + P_2)}.$$

The excess demand function is

$$Z(P) = \begin{bmatrix} \frac{10P_2^2}{P_1(P_1+P_2)} - 5 \\ \frac{5P_1}{P_2} + \frac{10P_1}{P_1+P_2} - 10. \end{bmatrix}$$

2. $Z(tP) = \begin{bmatrix} \frac{10t^2P_2^2}{tP_1(tP_1+tP_2)} - 5 \\ \frac{5tP_1}{tP_2} + \frac{10tP_1}{tP_1+tP_2} - 10. \end{bmatrix} = \begin{bmatrix} \frac{10P_2^2}{P_1(P_1+P_2)} - 5 \\ \frac{5P_1}{P_2} + \frac{10P_1}{P_1+P_2} - 10. \end{bmatrix} = Z(P)$

Thus, $Z(P)$ is homogeneous of degree zero.

3. Walras' Law

$$PZ(P) = \frac{10P_2^2}{P_1 + P_2} - 5P_1 + 5P_1 + \frac{10P_1P_2}{P_1 + P_2} - 10P_2 = 0.$$

$Z(P)$ satisfy Walras' Law.

4. The equilibrium price P^* clears the market,

$$Z_1(P) = 0 \Longrightarrow 10P_2^2 - 5P_1^2 + 5P_1P_2.$$

Normalize $P_1 = 1$, so $P^* = (1, 1)$. The equilibrium allocation is

$$X^* = \begin{bmatrix} 5 & 10 & 5 \\ 5 & 10 & 5 \end{bmatrix}$$