

# Open Economy Macro: Problem Set 1

## 2023 Fall

**Due on Nov 5th**

**Question 1.** Compute the relevant business cycle statistics for South Korea and the United States using 1) log-linear detrending; 2) log-quadratic detrending; 3) HP filtering with  $\lambda = 100$ . The data should be downloaded from the World Bank's WDI database. The sample period use 1971-2019 annually. Specifically, use the following time series

Variable	Series Name	Series Code
$y$	GDP per capita (constant LCU)	NY.GDP.PCAP.KN
$c$	Households and NPISHs final consumption expenditure (% of GDP)	NE.CON.PRVT.ZS
$i$	Gross capital formation (% of GDP)	NE.GDI.TOTL.ZS
$g$	General government final consumption expenditure (% of GDP)	NE.CON.GOV.T.ZS
$im$	Imports of goods and services (% of GDP)	NE.IMP.GNFS.ZS
$ex$	Exports of goods and services (% of GDP)	NE.EXP.GNFS.ZS

Make a graph showing the natural logarithm of real GDP per capital and the trend for the two countries. Then identify the recessions in the two countries. Are the results the same across the three detrending methods? Also make a table showing the statistics. Attach your code for cleaning data and making tables and plots.

**Question 2.** Consider a SOE model with nondurable and durable consumption goods. Let  $c_{N,t}$  denote consumption of nondurable goods in period  $t$ , and  $c_{D,t}$  denote purchases of durables in period  $t$ . The stock of durable goods  $s_t$  is assumed to evolve over time as

$$s_t = (1 - \delta)s_{t-1} + c_{D,t}$$

where  $\delta \in (0, 1]$  denotes the depreciation rate of durable goods. Households have preferences of the form

$$\sum_{t=0}^{\infty} \beta^t U(c_t)$$

where  $U$  is increasing in consumption and concave. Consumption  $c_t$  is a composite of non-durable consumption and the service flow provided by the stock of consumer durables. Specifically,

$$c_t = \left[ (1 - \alpha)^{\frac{1}{\eta}} c_{N,t}^{\frac{1-\frac{1}{\eta}}{\eta}} + \alpha^{\frac{1}{\eta}} s_t^{\frac{1-\frac{1}{\eta}}{\eta}} \right]^{\frac{1}{1-\frac{1}{\eta}}}$$

where  $\eta > 0$  and  $\alpha \in (0, 1)$ . Households have access to an internationally traded risk-free one-period bond, which pays the interest rate  $r_t$  when held between periods  $t$  and  $t + 1$ .

The relative price of durables in terms of nondurables is one. The household is subject to a borrowing limit that prevents it from engaging in Ponzi schemes. Output  $y_t$  is produced with physical capital  $k_t$  according to a production function of the form

$$y_t = F(k_t)$$

The capital stock evolves over time as

$$k_{t+1} = (1 - \delta_k)k_t + i_t$$

where  $i_t$  denotes investment in period  $t$ , and  $\delta_k$  is the depreciation rate on physical capital.

1. Describe the household's budget constraint
2. State the optimization problem of the household
3. Present the complete set of equilibrium conditions
4. Suppose the interest rate is constant over time and equal to  $r_t = r = \beta^{-1} - 1$ . Assume that up to period  $-1$ , the economy was in a steady state equilibrium in which all variables were constant and  $d = \bar{d}$ , where  $d$  denotes net external debt in the steady state. Find the share of expenditures on durables in consumption expenditures in the steady state in terms of the parameters  $\delta$ ,  $r$ ,  $\alpha$ , and  $\eta$
5. Assume that in period 0, the economy unexpectedly receives a positive income shock as a consequence of the rest of the world forgiving part of the country's net foreign debt. Assume that the positive income shock results in a 1 percent increase in the consumption of nondurables in period 0. Find the percentage increase in purchases of durables and in total consumption expenditures in period 0. Compare your answer to the one you would have obtained if all consumption goods were nondurables
6. Continuing to assume that consumption of nondurables has increased by 1 percent, find the change in trade balance in period 0 expressed as a share of steady-state consumption expenditure. Is the response of the trade balance countercyclical? Compare your findings to those you would have obtained if all consumption goods were nondurables. How much amplification is there due to the presence of durables?

**Question 3.** Modify the GHH period utility function of the SOE-RBC-EDEIR model as follows:

$$U(c, h) = \frac{[c^{1-\omega}(1-h)^\omega]^{1-\sigma} - 1}{1-\sigma}$$

All other features of the model are unchanged.

1. Derive analytically the steady state of the model.
2. Keep the value of all parameters the same as the example in class, except for  $\omega$ . Set  $\omega$  such that  $h = 1/3$  in steady state, i.e. households spend one-third of their time working.

3. Solve the model and produce a table of predicted second moments. You might find it convenient to use the Python code posted on Blackboard, or write your own code.
4. Compare the predictions of the present Cobb-Douglas preference model with those of its GHH preference counterpart.

**Question 4.** Consider a two-period small open economy populated by identical households with preferences given by

$$\ln c_1 + \ln c_2$$

where  $c_1$  and  $c_2$  denote consumption in period 1 and 2, respectively. The household's budget constraints in the two periods are

$$\begin{aligned} c_1 + d_1 + q(k_1 - k) &= F(k) + d_2 \\ c_2 + d_2 &= F(k_1) \end{aligned}$$

where  $d_1$  denotes debt due in period 1 and  $d_2$  denote debt assumed in period 1 and due in period 2,  $k$  denotes an exogenous initial stock of capital,  $k_1$  denotes capital purchased in period 1,  $q$  denotes the relative price of capital in terms of consumption, and  $F(\cdot)$  is an increasing and concave production function. Debt accumulation in period 1 is subject to the following collateral constraint:

$$d_2 \leq \kappa q k_1$$

where  $\kappa < 1$  is a parameter.

1. Derive the first-order conditions associated with the household's optimization problem.
2. Assume that the aggregate stock of capital is fixed. Derive the complete set of equilibrium conditions.
3. Characterize the range of the initial debt positions,  $d_1$ , for which the collateral constraint does not bind in equilibrium.
4. Find sufficient conditions on the initial level of debt,  $d_1$ , for which the economy possesses multiple equilibria (in particular, at least one equilibrium in which the collateral constraint binds and one in which the constraint does not).

**Question 5.** Modify the Eaton-Gersovitz model of sovereign default to allow for a proportional output loss function of the form  $L(y) = a_1 y$ . Calibrate  $a_1$  to match an average output loss due to default of about 7 percent of output per period conditional on the country being in bad financial standing. Use my Matlab code, or write your own code, to produce quantitative predictions of the model including default probability, average debt-output ratio, average and standard deviation of spreads, and correlations of spreads with output and trade balance. Discuss your findings, paying particular attention to how the present output loss specification affects the model's ability to predict country default in bad times.