1. Determine whether the following statements are True or False and EXPLAIN. Most marks are for your explanations. (5 points each)

   (a) A player's pure strategy is the same as her choices or actions.

   False. A pure strategy specifies a choice for the player at every information set, including information sets excluded by the player's own previous choices.

   (b) Backward induction ensures that every player makes optimal choice at each of her information set.

   False. The statement is true only for perfect information games. But for games of imperfect information, backward induction may have no refining power, and players can make choices that are not optimal at some information sets. That's why sequential equilibrium is developed for such games.

   (c) Mixed strategy and behavior strategy are equivalent for an extensive form game in which every information set contains a single decision node.

   True. If every information set contains a single decision node, the extensive form game satisfied perfect recall. In this case, mixed strategy and behavior strategy are strategically equivalent.

   (d) No players play dominated strategy in Nash equilibrium.

   False. No player plays strictly dominated strategy in NE, but players can play weakly dominated strategy in NE.

   (e) Sequential equilibrium exists in any finite extensive form game of perfect information.

   True. Every finite extensive form game of perfect information has at least one pure strategy SPNE. But SPNE is also sequential equilibrium for perfect information game.

2. **Answer:**

   (a) The strategic form (10 points)

		Player 2		
		1	2	3
	1	(1, 1)	(0, 0)	(0, 0)
Player 1	2	(0, 0)	(1, 1)	(0, 0)
	3	(0, 0)	(0, 0)	(1, 1)

   (b) Pure strategy NE: (10 points)

$$(1, 1), \qquad (2, 2), \qquad (3, 3).$$

   (c) (5 points) Let the mixed NE be:

$$((p_1, p_2, p_3); (q_1, q_2, q_3))$$

   with $p_1 + p_2 + p_3 = 1$ and $q_1 + q_2 + q_3 = 1$.

Given player 2's mixed strategy σ_2, player 1's payoffs from 3 pure strategies are, respectively,

$$U_1(1, \sigma_2) = q_1, \quad U_1(2, \sigma_2) = q_2, \quad U_1(3, \sigma_2) = q_3.$$

Given σ_1, player 1's payoffs are, respectively,

$$U_2(1, \sigma_1) = p_1, \quad , U_2(2, \sigma_1) = p_2, \quad U_2(3, \sigma_3) = p_3.$$

There are 4 mixed strategy NE:

i. Players randomize between 1, 2 and 3 with equal probabilities:

$$\left(\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right); \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\right)$$

ii. Players randomize only between 1 and 2 and with equal probabilities

$$\left(\left(\frac{1}{2}, \frac{1}{2}, 0\right); \left(\frac{1}{2}, \frac{1}{2}, 0\right)\right)$$

iii. Players randomize only between 1 and 3 and with equal probabilities

$$\left(\left(\frac{1}{2}, 0, \frac{1}{2}\right); \left(\frac{1}{2}, 0, \frac{1}{2}\right)\right)$$

iv. Players randomize only between 2 and 3 and with equal probabilities

$$\left(\left(0, \frac{1}{2}, \frac{1}{2}\right); \left(0, \frac{1}{2}, \frac{1}{2}\right)\right)$$

3. **Answer:**

(a) (10 points) C is strictly dominated. After removing C, c becomes dominated. After removing c, D becomes dominated. After removing D, d becomes dominated. So we end up with

	a	b
A	8, 3	1, 3
B	0, 1	3, 5

(b) Pure strategy NE （10 points)

$$(A, a), \qquad (B, b).$$

(c) (5 points)

Let the mixed strategy profile σ in the reduced game as

$$((p, 1-p), (q, 1-q)).$$

Given σ, player 2's payoff from the two pure strategies are:

$$U_2(a, \sigma) = 3p + (1-p); \qquad U_2(b, \sigma) = 3p + 5(1-p).$$

If $p < 1$, $U_2(a, \sigma) < U_2(b, \sigma)$; so in any mixed NE, $p = 1$.

Given σ, player 1's payoff from the two pure strategies are

$$U_1(A, \sigma) = 8q + (1 - q); \qquad U_1(B, \sigma) = 3(1 - q).$$

For mixed NE to exist,

$$U_1(A, \sigma) = 7q + 1 \geq U_1(B, \sigma) = 3 - 3q \Longrightarrow q \geq \frac{1}{5}.$$

Thus, we conclude that the set of mixed NE of the game is:

$$\left\{ ((1, 0, 0, 0), (q, 1 - q, 0, 0)) | q \in \left[\frac{1}{5}, 1\right) \right\}.$$

It's OK to write $q \geq 1/5$ or $1 \geq q \geq 1/5$; this also includes (A, a), the degenerated mixed NE.

4. **Answer:**

(a) Strategy form:(10 points)

	a	d
L	0, 0, 2	0, 0, 2
M	-1, 0, 2	-1, 0, 2
R	2, 1, -2	0, 0, 2

U

	a	d
L	0, 0, 2	0, 0, 2
M	2, 0, 1	2, 0, 1
R	5, 4, 0	0, 0, 2

V

Two pure strategy NE

$$(L, d, U), \qquad (R, a, V).$$

(b) (5 points) d is a weakly dominated strategy, so the only perfect equilibrium is (R, a, V).

(c) (10 points) For player 2, the optimal choice is a, not d at her information set. Thus, the strategy profile (L, d, U) is not sequential rational, and the unique sequential equilibrium is:

$$\sigma : \ (R, a, V); \qquad \mu : \ (0, 1).$$