Open Economy Macro: Problem Set 2
2023 Fall

Due on Dec 10th

Question 1. Consider a small open economy inhabited by identical consumers with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\ln c_t - \gamma h_t\right]$$

where c_t denotes consumption, h_t denotes hours worked, and $\beta \in (0,1)$ and $\gamma > 0$ are parameters. The consumption good is a composite made of tradable and nontradable goods via a Leontief aggregator. Formally,

$$c_t = \min\{c_t^{\mathrm{T}}, c_t^{\mathrm{N}}\}$$

where c_t^{T} and c_t^{N} denote, respectively, domestic absorption of tradables and nontradables in period t. To produce his nontraded consumption, each consumer operates a linear technology that uses labor as the sole input:

$$c_t^{\mathrm{N}} = A h_t$$

where A is a parameter. In addition, households can borrow or lend in the international financial market at the rate $r > 0$. Their sequential budget constraint is given by

$$c_t^{\mathrm{T}} + (1+r)d_{t-1} = y^{\mathrm{T}} + d_t$$

where d_t denotes the level of net external debt assumed in period t and maturing in period $t+1$, and $y^{\mathrm{T}} > 0$ denotes a constant endowment of tradable goods. In period 0, households start with outstanding debt equal to $d_{-1} > 0$. Finally, households are subject to a no-Ponzi game constraint of the form

$$\lim_{t\to\infty} \frac{d_t}{(1+r)^t} \leq 0$$

1. Characterize the equilibrium levels of consumption, consumption of nontradables, and hours worked.

2. Suppose that in period 0, foreign lenders unexpectedly decide to forgive an amount $\Delta^d > 0$ of the debt. Assuming that Δ^d is relatively small, characterize the effect of this debt forgiveness shock on consumption, consumption of nontradables, and hours worked.

3. Now suppose $\Delta^d = 0$. Instead, assume that in period 0, the nontraded sector experience a permanent increase in productivity. Specifically, the productivity factor A increases by $\Delta^A > 0$. Characterize the effect of this positive productivity shock on consumption, consumption of nontradables, and hours worked.

Question 2. We generally assume that the consumption aggregator function $A(c^{\mathrm{T}}, c^{\mathrm{N}})$ is increasing, concave, and linearly homogeneous. Thus, the household's demand for nontradables can be given by

$$p = \frac{A_2(c^{\mathrm{T}}, c^{\mathrm{N}})}{A_1(c^{\mathrm{T}}, c^{\mathrm{N}})}$$

where p is the relative price of nontradables in terms of tradables.

1. Show that the assumptions are sufficient to ensure that the demand schedule of nontradables is downward sloping in the space (c^{N}, p), holding c^{T} constant.

2. Show that the aforementioned assumptions about the aggregator $A(c^{\mathrm{T}}, c^{\mathrm{N}})$ are sufficient to guarantee that increases (decreases) in c^{T} shift the demand schedule up and to the right (down and to the left).

3. Assume that the aggregator function takes the Cobb-Douglas form

$$A(c^{\mathrm{T}}, c^{\mathrm{N}}) = \sqrt{c^{\mathrm{T}} c^{\mathrm{N}}}$$

   Find the demand function of nontradables.

4. Now assume the CES form

$$A(c^{\mathrm{T}}, c^{\mathrm{N}}) = \left[a(c^{\mathrm{T}})^{1-\frac{1}{\xi}} + (1-a)(c^{\mathrm{N}})^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}}$$

   Derive the demand function of nontradables. Interpret the parameter ξ.

Question 3. Consider an open economy that lasts for only two periods 1 and 2. Households are endowed with 10 units of tradables in period 1 and 13.2 unites in period 2, i.e. $y_1^{\mathrm{T}} = 10$ and $y_2^{\mathrm{T}} = 13.2$. The country interest rate is 10 percent, or $r = 0.1$. The nominal exchange rate is fixed and equal to 1 in both periods ($\varepsilon_1 = \varepsilon_2 = 1$). Suppose that the foreign currency price of tradable goods is constant and equal to one in both periods, and that the law of one price holds for tradable goods in both periods. Nominal wages are downwardly rigid. Specifically, assume that the nominal wage W, measured in terms of domestic currency, is subject to the constraint

$$W_t \geq W_{t-1}$$

for $t = 1, 2$, with $W_0 = 8.25$. Suppose the economy starts period 1 with no assets or debts carried over from the past ($d_1 = 0$). Households are subject to the no-Ponzi game constraint $d_3 \leq 0$.

Suppose that the household's preferences are defined over tradables and nontradables in period 1 and 2, described by the following utility function:

$$\ln C_1^{\mathrm{T}} + \ln C_1^{\mathrm{N}} + \ln C_2^{\mathrm{T}} + \ln C_2^{\mathrm{N}}$$

where C_i^{T} and C_i^{N} denote consumption of tradables and nontradables in period $i = 1, 2$, respectively. Let p_1 and p_2 denote the relative price of nontradables in terms of tradables in period 1 and 2. Households supply inelastically $\bar{h} = 1$ unit of labor to the market each period. Finally, firms produce nontradable goods using labor as the sole input. The production technology is given by

$$y_t^{\mathrm{N}} = h_t^{\alpha}$$

for $t = 1, 2$, where y_t^{N} and h_t denote, respectively, nontradable output and hours employed in period $t = 1, 2$. The parameter α is equal to 0.75.

1. Compute the equilibrium levels of consumption of tradables, employment, nontradable output, the relative price of nontradables, and the trade balance in period 1 and 2.

2. Suppose now that the country interest rate increases to 32 percent. Calculate the equilibrium levels of consumption of tradables, the trade balance, consumption of nontradables, the level of unemployment, and the relative price of nontradables in period 1 and 2. Provide intuition.

3. Given the situation in the previous question, calculate the minimum devaluation rates in period 1 and 2 consistent with full employment in both periods. To answer this question, assume that the nominal exchange rate in period 0 was also fixed at unity. Explain.

4. Continue to assume that $W_0 = 8.25$ and r is 32 percent. Asume also that the government is not willing to devalue the domestic currency, so that $\varepsilon_1 = \varepsilon_2 = 1$. Instead, the government chooses to apply capital controls in period 1. Specifically, the government imposes a proportional tax τ_1 on borrowed funds. If $\tau_1 < 0$, it serves as a subsidy. Suppose that this tax (subsidy) is rebated (financed) in a lump-sum fashion. Calculate the Ramsey optimal level of τ_1.

Question 4. Consider a small open perfect-foresight economy populated by a large number of identical and infinitely-lived consumers with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t \ln c_t$$

where consumption c_t is a composite good made of tradable and nontradables goods, denoted c_t^{T} and c_t^{N}, respectively, via the aggregator function

$$c_t = \sqrt{c_t^{\mathrm{T}} c_t^{\mathrm{N}}}$$

The sequential budget constraint is given by

$$(1 + \tau_t)(c_t^{\mathrm{T}} + p_t c_t^{\mathrm{N}}) + (1 + r)d_{t-1} = y^{\mathrm{T}} + y^{\mathrm{N}} + d_t + s_t$$

where d_t denote debt acquired in period t and maturing in period $t+1$, τ_t is proportional consumption tax, p_t denotes the relative price of nontradables in terms of tradables, $y^{\mathrm{T}} = 1$

is an endowment of tradable goods, $y^{\mathrm{N}} = 1$ is an endowment of nontradable goods, and s_t denotes a lump-sum transfer received from the government. The interest rate r satisfies $1 + r = \beta^{-1} = 1.04$. Debt is denominated in terms of tradables. Consumers are subject to the no-Ponze game constraint

$$\lim_{j \to \infty} \frac{d_{t+j}}{(1+r)^j} \leq 0$$

Assume that the household's initial debt position is nil, i.e. $d_{-1} = 0$.

The government runs a balanced budget period by period, that is,

$$s_t = \tau_t (c_t^{\mathrm{T}} + p_t c_t^{\mathrm{N}})$$

Suppose that before period 0 the economy was in a steady state with constant consumption of tradables and nontradables and no external debt.

1. Compute the equilibrium paths of c_t^{T}, p_t, the trade balanc, and the current account under two alternative tax policies:

$$\text{policy 1}: \ \tau_t = 0, \ t \geq 0$$
$$\text{policy 2}: \ \tau_t = \begin{cases} 0 & 0 \leq t \leq 11 \\ 0.3 & t \geq 12 \end{cases}$$

2. Compute the welfare cost of policy 2 relative to policy 1, defined as the percentage increase in the consumption stream of a consumer living under policy 2 required to make him as well off as living under policy 1. Formally, the welfare cost of policy 2 relative to policy 1 is given by $\lambda \times 100$, where λ is implicitly given by

$$\sum_{t=0}^{\infty} \beta^t \ln \left[c_t^{p2} (1 + \lambda) \right] = \sum_{t=0}^{\infty} \beta^t \ln c_t^{p1}$$

where c_t^{p1} and c_t^{p2} denote consumption in period t under policy 1 and 2, respectively.