High-Frequency Expectations from Asset Prices: A Machine Learning Approach

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Contents

Aditya Chaudhry

- Publications
  1. Uncertainty Assessment and False Discovery Rate Control in High-Dimensional Granger Causal Inference
- Working Papers
  1. The Causal Impact of Macroeconomic Uncertainty on Expected Returns
  2. How Much Do Subjective Growth Expectations Matter for Asset Prices?
  3. The Impact of Prices on Analyst Cash Flow Expectations

Primary research interests:

- Joint dynamics of subjective beliefs, asset demand, and asset prices.
- Using new empirical strategies to identify important structural parameters in asset pricing.

Sangmin S. Oh

- Job Market Paper
  1. Social Inflation
- Publications
  1. Cross-sectional Skewness
- Working Papers
  1. Pricing of Climate Risk Insurance: Regulation and Cross-Subsidies
  2. Asset Demand of U.S. Households
  3. Unpacking the Demand for Sustainable Equity Investing
  4. Climate Capitalists

Notebook: https://sangmino.github.io/notebook

Motivation: shortcomings of past approaches

- Investors price assets based on their beliefs about the joint distribution of stochastic discount factor M_{t+1} and the asset's cash flows X_{t+1}.

$$P_t = \mathbb{E}_t \left[M_{t+1} X_{t+1} \right]$$

- Expectations play a central role in asset pricing.
- One of the key drivers of investor expectations is news of macroeconomic events.
- How to test the impact of such news?
  1. Examined the behavior of asset prices around announcement dates.
     Shortcomings: the diversity of information sources.
  2. Utilized surveys that directly measure expectations.
     Shortcomings: the low frequency of survey data.

Aim: construct a daily time series of investor expectations

- Task: recover the unobserved daily series of expectations between two quarterly survey releases dates.
- Previous papers: Kalman filtering (KF) and a mixed frequency data sampling approach (MIDAS)
- In this paper: Reinforcement learning (RL) utilized daily asset prices that reflect investors' updated beliefs about macroeconomic growth.

Why RL? Why asset prices?

Why RL? RL achieves a significant gain in efficiency.

- Observed series (asset prices)
  $y_{t+1} = Hy_t + e_{t+1}, \quad e_{t+1} \sim \mathcal{N}(0, \Sigma)$
- Latent series (macroeconomic growth expectations)
  $x_{t+1} = Fx_t + u_{t+1}, \quad u_{t+1} \sim \mathcal{N}(0, \Phi)$

Update rule for the estimate of x in the Kalman Filter is

$$\hat{x}_{t+1|t} = F\left(\hat{x}_{t|t-1} + \left(\frac{H\Omega_{t|t-1}}{\Sigma + H^2\Omega_{t|t-1}}\right)(y_t - \hat{y}_t)\right)$$

One must estimate the parameters (H, F, Σ, Φ) using ML, while RL avoids this problem by estimating the update function directly: $\hat{x}_{t+1|t} = \hat{x}_{t|t-1} + f\,(y_t)$.

- RL avoids an explicit model of the state dynamics and thus requires estimation of far fewer parameters.

Why asset prices?

1. Data must be available at a daily frequency.
2. Asset prices reflect many variables besides growth expectations.
   - A single asset: cannot extract the component of asset returns driven solely by changes in expectations of macroeconomic growth.
   - With multiple assets: a suitable linear combination can cancel the extraneous sources of return variation.

Task: finding an optimal combination of asset returns that correlates maximally with the change investors' expectations of future macroeconomic growth.

Structure

1. Providing empirical evidence regarding the relationship between asset returns and expectations of macroeconomic growth.
2. Elucidate the differences among RL algorithm, KF and MIDAS regression by presenting a stylized economy with Bayesian agents.
3. Take RL algorithm into real data.
4. Use RL estimated daily series of growth expectations to test the existence of the Fed information effect.

Campbell and Shiller Approximation

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\frac{P_{t+1}}{D_{t+1}} + \frac{D_{t+1}}{D_{t+1}}}{\frac{P_t}{D_t}} \cdot \frac{D_{t+1}}{D_t}$$

$$\frac{P_t}{D_t} = \frac{1}{R_{t+1}} \cdot \left(1 + \frac{P_{t+1}}{D_{t+1}}\right) \cdot \frac{D_{t+1}}{D_t}$$

$$\underbrace{\ln P_t}_{p_t} - \underbrace{\ln D_t}_{d_t} = -\ln R_{t+1} + \ln\left(1 + \frac{P_{t+1}}{D_{t+1}}\right) + \ln\frac{D_{t+1}}{D_t}$$

$$p_t - d_t = -r_{t+1} + \ln\left(1 + e^{p_{t+1}-d_{t+1}}\right) + \Delta d_{t+1}$$

Assume that $pd_t = \frac{1}{t}\sum_{n=1}^{t}(p_i - d_i)$, use Taylor expansion at $p_{t+1} - d_{t+1} = pd_t$, we have

$$\ln\left(1 + e^{p_{t+1}-d_{t+1}}\right) = \ln(1 + e^{pd_t}) + \frac{e^{pd_t}}{1 + e^{pd_t}}(p_{t+1} - d_{t+1} - pd_t)$$

Campbell and Shiller Approximation

$$p_t - d_t = \left[\ln\left(1 + e^{pd_t}\right) + \frac{e^{pd_t}}{1+e^{pd_t}}\left(p_{t+1} - d_{t+1} - pd_t\right)\right] - r_{t+1} + \Delta d_{t+1}$$

$$p_t - d_t = -r_{t+1} + \Delta d_{t+1} + \underbrace{\frac{e^{pd_t}}{1+e^{pd_t}}}_{\rho}\left(p_{t+1} - d_{t+1}\right) + \underbrace{\left[\ln\left(1 + e^{pd_t}\right) - \frac{e^{pd_t}}{1+e^{pd_t}} \cdot pd_t\right]}_{\kappa}$$

$$p_t - d_t = -r_{t+1} + \Delta d_{t+1} + \rho\left(p_{t+1} - d_{t+1}\right) + \kappa$$

$$p_t - d_t = -\sum_{n=0}^{\infty} \rho^n r_{t+i+1} + \sum_{n=0}^{\infty} \rho^n \Delta d_{t+n+1} + \rho^{\infty} pd_{t+\infty} + \kappa \sum_{n=0}^{\infty} \rho^n$$

Campbell and Shiller Approximation

$$p_t - d_t = -\sum_{n=0}^{\infty} \rho^i r_{t+i+1} + \sum_{n=0}^{\infty} \rho^i \Delta d_{t+n+1} + \rho^{\infty} pd_{t+\infty} + \kappa \sum_{n=0}^{\infty} \rho^i$$

$$p_t = \kappa \sum_{n=0}^{\infty} \rho^i - \sum_{n=0}^{\infty} \rho^i r_{t+i+1}$$
$$+ \left[d_t + (d_{t+1} - d_t) + \rho\left(d_{t+2} - d_{t+1}\right) + \cdots\right]$$

Campbell and Shiller Approximation

$$p_t = \frac{\kappa}{1-\rho} - \sum_{n=0}^{\infty} \rho^n r_{t+n+1} + (1-\rho) \sum_{n=0}^{\infty} \rho^n d_{t+n+1}$$

where $\rho = \frac{e^{pd_t}}{1+e^{pd_t}}$, $\kappa = \ln\left(1 + e^{pd_t}\right) - \frac{e^{pd_t}}{1+e^{pd_t}} \cdot pd_t$

State-space model

- GDP growth is persistent

$$\theta_{t+1} = \mu + \delta\theta_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N\left(0, \sigma_\epsilon^2\right)$$

- GDP growth affects each asset's dividend growth

$$d_{t+1}^i - d_t^i = \gamma + \beta^i \theta_{t+1} + \nu_{t+1}^i, \quad \nu_{t+1}^i \sim N\left(0, \sigma_\nu^2\right)$$

- The conditional expected return of asset i depends linearly on another latent factor ζ_t

$$\mathbb{E}_t\left[r_{t+1}^i\right] = \alpha + \phi^i \zeta_t$$

- Latent factor ζ_t is persistent

$$\zeta_{t+1} = \tau + \psi\zeta_t + \xi_{t+1}, \quad \xi_{t+1} \sim N\left(0, \sigma_\xi^2\right)$$

- Innovations to θ_t and ζ_t are correlated: $\mathrm{Corr}\left(\epsilon_t, \zeta_t\right) = \pi$

Solve the state-space model

Campbell and Shiller Approximation

$$p_t^i = \frac{\kappa}{1-\rho} + (1-\rho)\sum_{j=0}^{\infty}\rho^j \mathbb{E}_t\left[d_{t+j+1}^i\right] - \sum_{j=0}^{\infty}\rho^j \mathbb{E}_t\left[r_{t+j+1}^i\right]$$

$$\begin{cases} \theta_{t+1} = \mu + \delta\theta_t + \epsilon_{t+1} \\ d_{t+1}^i - d_t^i = \gamma + \beta^i\theta_{t+1} + \nu_{t+1}^i \end{cases}$$

$$\Downarrow$$

$$\mathbb{E}_t\left[d_{t+j+1}^i\right] = d_t^i + \sum_{n=0}^{j}\left[\gamma + \beta^i\mu\left(\frac{1-\delta^{n+1}}{1-\delta}\right) + \beta^i\delta^{n+1}\theta_t\right]$$

$$\mathbb{E}_t\left[r_{t+1}^i\right] = \alpha + \phi^i\zeta_t \Rightarrow \mathbb{E}_t\left[r_{t+j+1}^i\right] = \begin{cases} \alpha + \phi^i\tau\frac{1-\psi^j}{1-\psi} + \phi^i\psi^j\zeta_t & j > 1 \\ \alpha + \phi^i\zeta_t & j = 1 \end{cases}$$

Model result

Simple function of return r^i_{t+1}

$$r^i_{t+1} = \left[\left(\beta^i + \frac{\delta\beta^i}{1-\rho\delta}\right)\theta_{t+1} - \left(\frac{\delta\beta^i}{1-\rho\delta}\right)\theta_t\right] - \frac{\phi^i}{1-\rho\psi}\left(\zeta_{t+1}-\zeta_t\right) + \nu_{t+1} + \gamma$$

Returns increase with

- contemporaneous growth θ_{t+1}
- shock to the dividend process

and decrease with

- previous period's growth θ_t
- change in ζ_{t+1}

Asset prices should be useful to understand changes in investor expectations (GDP growth expectation)

Asset Prices → Growth Expectations

Examine whether asset returns
can explain innovations in the average growth forecast.

1. Forecast innovation: the difference between the nowcast and the lag-one-period forecast for period t.
2. Run time-series regressions of innovations in mean growth expectations on asset returns (bivariate pairs of assets).
3. The CRSP U.S. Treasury five-year fixed-term index and the CRSP value-weighted portfolio ($R^2 = 38.3\%$).

Thus, asset returns contain useful information about forecast innovations empirically.

Incorporating Bayesian Agents

Instantiate 20 Bayesian agents who observe realized returns and form expectations of the latent growth process (cannot observe growth).

- prior-mean heterogeneity: the mean of each agent's prior belief regarding $\theta_t \sim N(\theta_0, 0.5\theta_0)$ at the start of the quarter.
- learning heterogeneity: each agents draws his value of the parameters from a normal distribution centered at the baseline parameter value with variance parameterized by a fixed signal-to-noise ratio (different parameters in state and observation equation).

Learning the Cross-sectional Moments

The expression for the optimal Kalman gain implies the following relationship:

$$\underbrace{\mu_{i,t}}_{\mathbb{E}_t^i[\theta_{t+1}]} = c_{0,t}^i + c_{1,t}^i \mu_{i,t-1} + \left(\mathbf{c}_{2,t}^i\right)' \mathbf{r}_t$$

Averaging across all agents, we get the cross-sectional mean of growth expectations at period t:

$$\mu_t \equiv \frac{1}{N}\sum_{i=1}^{N} \mu_{i,t} = \frac{1}{N}\sum_{i=1}^{N} c_{0,t}^i + \frac{1}{N}\sum_{i=1}^{N} c_{1,t}^i \mu_{i,t-1} + \left(\frac{1}{N}\sum_{i=1}^{N} \mathbf{c}_{2,t}^i\right)' \mathbf{r}_t$$

Use the following approximating moment:

$$\begin{aligned} \mu_t &= c_0 + c_1 \mu_{t-1} + \mathbf{c}_2' \mathbf{r}_t \approx c_1 \mu_{t-1} + \mathbf{c}_2' \mathbf{r}_t \\ &= c_1 \mu_{t-1} + \mathbf{c}_2' \left[\mathbf{1}\gamma + \mathbf{a}\theta_t + \mathbf{b}\theta_{t-1} + \mathbf{c}\left(\zeta_t - \zeta_{t-1}\right) + \boldsymbol{\nu}_t\right] \end{aligned}$$

The Kalman Filtering (KF) Approach

State equation:

$$
\begin{aligned}
\theta_{t+1} &= \mu + \delta\theta_t + \epsilon_{t+1} \\
\zeta_{t+1} &= \tau + \psi\zeta_t + \xi_{t+1} \\
\mu_{t+1} &= \mathbf{c}_2'(\mathbf{1}\gamma + \mathbf{a}\mu + \mathbf{c}\tau) + \mathbf{c}_2'(\mathbf{a}\delta + \mathbf{b})\theta_t + \mathbf{c}_2'\mathbf{c}(\psi - 1)\zeta_t + c_1\mu_t
\end{aligned}
$$

Observation equation:

$$
\mathbf{c}_2'\mathbf{r}_t = \mu_t - c_1\mu_{t-1}
$$

- $3m + 11$ parameters to be estimated.
  (m is the number of assets used)

The Mixed Data Sampling (MIDAS) Approach

$$y_t = \alpha^\tau + \rho^\tau y_{t-1} + \sum_{i=1}^{m} \beta_i^\tau \underbrace{\gamma^\tau(L) r_\tau^i}_{\sum_{d=\tau-l+1}^{\tau} \gamma_d^\tau r_d^i} + \epsilon_t$$

- Use a maximal lag of $l = 90$ days.
- y_t is μ_t, the quarterly observed cross-sectional mean survey expectation.
- Each MIDAS regression involves estimating $m + 4$ parameters.

The Reinforcement Learning (RL) Approach

- agent's state = current expectation + asset returns

$$\varphi(s_t) = \begin{pmatrix} \hat{\mu}_{t-1} \\ \hat{\sigma}^2_{t-1} \\ \mathbf{r}'_t \end{pmatrix} \in \mathbb{R}^{m+2}, \quad \varphi(s_1) = \begin{pmatrix} \mu_0 \\ \sigma^2_0 \\ \mathbf{r}'_1 \end{pmatrix}$$

- policy: function of the current state that yields the agent's new growth expectation.

$$g_{\boldsymbol{\lambda}}(s_t) \equiv \begin{pmatrix} \mu_t \\ \sigma_t \end{pmatrix} = \begin{pmatrix} c_1\mu_{t-1} + \mathbf{c}'_2\mathbf{r}_t \\ \sqrt{c_3\sigma^2_{t-1} + \mathbf{c}'_4\mathbf{r}_t\mathbf{r}'_t\mathbf{c}_4 + \mathbf{c}'_5\mathbf{r}_t\mu_{t-1}} \end{pmatrix} \in \mathbb{R}^2$$

- action: agent's updated growth expectation.
- rewards:

$$r_t(s^t) = \begin{cases} 0 & \text{if } t < T \\ -\left\| \begin{pmatrix} \hat{\mu}_{T|T-1} \\ \hat{\sigma}_{T|T-1} \end{pmatrix} - \begin{pmatrix} \mu_T \\ \sigma_T \end{pmatrix} \right\| & \text{if } t = T \end{cases}$$

A Three-Way Comparison

- The interpretation of the output of each method.
  - RL and KF approaches yield daily estimates of the current latent cross-sectional mean expectation. $\left(\mathbb{E}\left[\mu_t \mid \mathcal{F}_t^E\right]\right)$
  - MIDAS produces a prediction of the end-of-quarter cross-sectional mean expectation. $\left(\mathbb{E}\left[\mu_T \mid \mathcal{F}_t^E\right]\right)$
  - RL and KF approaches prove better suited to our setting than the MIDAS approach.
- The bias-variance tradeoff each method incurs.
  - parameters
    KF: $3m + 11$, RL: $m + 1$, MIDAS: $60(m + 4)$
  - RL approach proves far more efficient than the other two methods.

Performance of RL

Policy function:

$$g_{\boldsymbol{\lambda}}(s_t) \equiv \begin{pmatrix} \mu_t \\ \sigma_t \end{pmatrix} = \begin{pmatrix} \dfrac{c_1 \mu_{t-1} + \mathbf{c}_2' \mathbf{r}_t}{\sqrt{c_3 \sigma_{t-1}^2 + \mathbf{c}_4' \mathbf{r}_t \mathbf{r}_t' \mathbf{c}_4 + \mathbf{c}_5' \mathbf{r}_t \mu_{t-1}}} \end{pmatrix} \in \mathbb{R}^2$$

Table 1: Recursive Out-of-Sample Estimation Results

	RL Approach	Naive	MIDAS	KF
RMSE	0.449	0.588	0.916	39.103
R^2	0.823	0.647	0.392	0.0237

Origins of RL's Outperformance

Core difficulty: obtaining a daily law of motion for expectations given quarterly training data.

- RL vs. KF
  - KF: imposing parametric assumptions and using ML.
  - RL: directly estimating the Kalman gain using a linear learning rule (bias-efficiency trade-off).
- RL vs. MIDAS
  - MIDAS: applies a non-monotonic weighting scheme to 90 days of lagged asset returns.
  - RL: uses only asset returns since the start of the last survey release, weighting them uniformly (treatment of lagged asset returns proves more useful).

Hyper-parameters: step size and noise in behavioral policy

- step size:
  too small → one may get stuck in a local maximum
  too large → algorithm may have trouble converging
- noise in the behavioral policy:
  too little exploration → a suboptimal policy
  too much exploration → prevent the algorithm from making proper gradient updates to the weights

A proper hyper-parameter optimization procedure

1. Divide the sample into a training subsample and a pseudo-testing subsample.
2. Train a model at each grid point on the training subsample and test on the pseudo-testing subsample.
3. Choose the set of hyper-parameters that performs best in the pseudo-testing subsample.

Testing the "Fed Information Effect"

Fed Information Effect:
Hawkish surprises for interest rates correspond to increases in real GDP growth expectations.

$$\mathbb{E}_{t+15}\left[g_Q\right] - \mathbb{E}_{t-15}\left[g_Q\right] = \beta_0 + \underbrace{\beta_1}_{positive} \text{Shock}_t + \epsilon_t$$

An omitted variable:
economic news released between day $t-15$ and day $t-1$

$$\Delta CX\ Mean_t = \beta_0 + \underbrace{\beta_1}_{negative} \text{Shock}_t + \epsilon_t$$

1. Author
2. Background
3. Model of the economy
4. Empirical framework
5. **Conclusion**

Attribution

- The first serious application of reinforcement learning in the growing literature that uses machine learning methods in finance.
- Present reinforcement learning as a more efficient improvement over traditional filtering methods.
- Obtain a daily series of expectations for any macroeconomic variable with a low-frequency panel of forecasts.

How to apply ”Machine Learning Approach” in finance?

- Compare to traditional methods.
- A model of the economy.
- Economic intuition.
- Use the result to test something.

Thanks!