MICROECONOMIC THEORY II

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    - a probability distribution of type, $F(\theta_1, \ldots, \theta_I)$
    - a utility function $\tilde{u}_i : S_i \times \varTheta \rightarrow \mathbb{R}$:

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- Definition: A (pure strategy) Bayesian Nash equilibrium for the Bayesian game $[I, S, u, \mathit{\Theta}, F(\cdot)]$ is a profile of decision rules $(s_1(\cdot), \ldots, s_I(\cdot))$ such that, for all i,

$$\tilde{u}_i(s_i(\cdot), s_{-i}(\cdot)) \geq \tilde{u}_i(s_i'(\cdot), s_{-i}(\cdot))$$

for all $s_i'(\cdot)$

AN EXAMPLE

- Boss and Tim play the game. Tim does not know the payoff of Boss.

Tim

boss		W	S
	M	3, 2	1, 1
	N	4, 3	2, 4

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  - type II ($1 - \mu$) has dominant strategy M.

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  - $M > w > s$ and $w > M/2 > s$


T-7

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ANALYSIS

- The strategic form

player 2

	AA	AN	NA	NN
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AN	$\frac{M}{4}-\frac{s}{2},\ \frac{M}{2}-\frac{s+w}{4}$	$\frac{M-s}{4},\ \frac{M-s}{4}$	$\frac{M}{2}-\frac{s}{4},\ \frac{M-w}{4}$	$\frac{M}{2},\ 0$
NA	$-\frac{w}{2},\ \frac{3M}{4}-\frac{s+w}{4}$	$\frac{M-w}{4},\ \frac{M}{2}-\frac{s}{4}$	$\frac{M-w}{4},\ \frac{M-w}{4}$	$\frac{M}{2},\ 0$
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NN	$0,\ M$	$0,\ \frac{M}{2}$	$0,\ \frac{M}{2}$	$0,\ 0$

- Two pure strategy Bayesian NE:

$$(AA, AN), \qquad (AN, AA).$$

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- Two firms privately observe their own type, and then simultaneously decide to develop/not.

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- Thus, $\hat{\theta}_i$ is determined by the following condition:

$$\hat{\theta}_i^2 - c = \hat{\theta}_i^2 \left(1 - \hat{\theta}_j\right).$$

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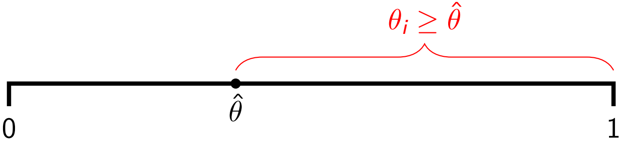

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- The Bayesian NE: For $i = A, B$,

$$s_i(\theta_i) = \begin{cases} 1 & \text{if} \quad \theta_i \geq \hat{\theta} = (c)^{1/3}; \\ 0 & \textit{otherwise} \end{cases}$$

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  - The U.S. spectrum auction in April 2008 raised a total of \$19.8 billion
  - Europe's frenzied 2000 and 2001 auctions reaped nearly \$100 billion

NYSE AS AN AUCTION HOUSE

> *"The trading floor in New York was all about auctioneers. Every stock had a person, called a specialist, who stood by a desk and kept track of buyers and sellers and sometimes stepped in to trade for himself."*

— The Futures, The rise of the speculator and the origins of the world's biggest market

THE REASON FOR AUCTIONING TREASURY BOND

> *"The Federal Reserve opposed securities auctions and helped to finance budget deficits, a main source of inflationary money growth after 1965. Treasury later began auctions."*
>
> *"The Federal Reserve agreed to maintain orderly markets and shared responsibility for success of debt management operations with the Treasury. In practice this responsibility led the Federal Reserve to adopt an "even keel" policy of maintaining interest rates during periods of Treasury borrowing. This permitted money growth to increase especially when budget deficits rose in the late 1960s."*

–A History of Federal Reserve, volume 2, book 1

TWO IMPORTANT CHARACTERISTICS

- Uncertainty about the valuations of the bidders.
  - ➢ This uncertainty leads to the private values assumption about such valuations;
  - ➢ It is modeled as independent random variables from a common distribution.
- "Winner's curse": Because of the uncertainty of the value of the object for sale, a winner of an auction might wonder why all the other bidders' valuation were smaller than hers, and in particular whether this might have happened because of the others' more accurate information about the item's true value.

WINNER'S CURSE IN SECURITY MARKET

> *"Value traders make market resilient by standing ready to trade when prices move away from fundamental values.... Value traders face two serious risks when they trade: adverse selection and the winner's curse. Like dealers, value traders face adverse selection when they supply liquidity to to better-informed traders. They face the winner's curse when they have misestimated instrument values."*

—Trading and Exchange, Market Microstructure for Practioners

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    - ➢ Second-price sealed bid auction.

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4. Risk preferences. Risk neutrality versus risk aversion.

DUTCH AUCTION

Maximum price

English auction

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- u maximized at v,

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- Since $b(0) = 0$, C should be zero,

  $$b(v) = \frac{1}{F^{N-1}(v)}\int_0^v xdF^{N-1}(x)$$

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- In the case of independent private value auctions, the English auction and the Vickrey auction are strategically equivalent.

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- In both auction, the bidder with the highest valuation wins and pays the second highest bidder's value.
- Without independent private values, English and second-price auction would not be strategically equivalent.

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REVENUE EQUIVALENCE BETWEEN FIRST AND SECOND-PRICE AUCTIONS

- k-order statistic of N draws from F_x

$$f_{x_k} = \frac{N!}{(k-1)!(N-k)!}[F(x)]^{N-k}[1-F(x)]^{k-1}f(x).$$

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PROOF OF EQUIVALENCE

- Revenue of first-price auction:

$$
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EV_F &= \int_0^1 N(N-1)\left[\int_0^v xF^{N-2}(x)f(x)dx\right] f(v)dv \\
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- Thus, we conclude

$$EV_F = EV_S = \int_0^1 N(N-1)vF^{N-2}(v)[1-F(v)]f(v)dv.$$

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  - Giving incentive to bidders so that it is in their best interest to bid according to their true valuation

SOME CONCEPTS IN MECHANISM DESIGN

- Definition: A direct selling mechanism is a collection of N probability assignment functions, $p_1(v_1, \ldots, v_N)$, …, $p_N(v_1, \ldots, v_N)$ and N cost functions $c_1(v_1, \ldots, v_N)$, …, $c_N(v_1, \ldots, v_N)$. For each i and all $(v_1, \ldots, v_N)$, $p_i(v_1, \ldots, v_N) \in [0, 1]$ denotes the probability that bidder i receives the object and $c_i(v_1, \ldots, v_N) \in \mathbb{R}$ denotes the payment bidder i must make to the seller.

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- A direct selling mechanism is incentive-compatible if and only if for every v_i, $u_i(r_i, v_i)$ is maximized in r_i at $r_i = v_i$,

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- Efficient rule

$$x_i(\theta) = \begin{cases} 1, & \text{if } \sum_{i=1}^{I} \theta_i \geq c \\ 0 & \textit{otherwise} \end{cases}$$

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- Violates budget-balancedness.