

PROBLEM SET 4

1. (25) The question is related to the open economy setup. Consider a representative household that is endowed with funds W_0 . The household allocates funds between domestic assets B and foreign assets B^* . R and R^* denote the risk-free interest rate on domestic assets and foreign assets, respectively. Assume that investing in foreign assets B^* incurs an adjustment cost $\Omega_b(B^*)^2/2$, where $\Omega_b > 0$.

a. (10) Suppose the household's objective is to maximize the expected income from investing in domestic and foreign assets. Please solve for the optimal choices of domestic assets B and foreign assets B^* .

Answer: The household's problem is given by,

$$\max RB + R^*B^*$$

subject to

$$B + B^* + \Omega_b(B^*)^2/2 = W_0$$

The household's problem can be rewritten as,

$$\max R[W_0 - B^* - \Omega_b(B^*)^2/2] + R^*B^*$$

The optimal condition for B^* is given by,

$$R^* = R(1 + \Omega_b B^*)$$

The optimal choices are given by,

$$B^* = (R^*/R - 1)/\Omega_b, \quad B = W_0 - (R^*/R - 1)/\Omega_b.$$

b. (10) Suppose $\Omega_b = 0$ and the government taxes the return on foreign assets so that the after-tax interest rate on foreign assets becomes $(1 - \tau)R^*$, where $\tau > 0$. In this case, what are the optimal choices of domestic assets B and foreign assets B^* ?

Answer: The household's problem is given by,

$$\max RB + R^*(1 - \tau)B^*$$

subject to

$$B + B^* = W_0$$

The household's problem can be rewritten as,

$$\max R(W_0 - B^*) + R^*(1 - \tau)B^*$$

The first order condition for B^* is given by,

$$R^*(1 - \tau) - R$$

If $R^*(1 - \tau) - R > 0$, the optimal choices are given by,

$$B^* = W_0, \quad B = 0.$$

If $R^*(1 - \tau) - R < 0$, the optimal choices are given by,

$$B^* = 0, \quad B = W_0.$$

If $R^*(1 - \tau) - R = 0$, any combination of (B, B^*) that satisfies $B + B^* = W_0$ is optimal.

c. (5) Given $B^* \in (0, W_0)$, under what condition, will the tax on foreign asset τ and the adjustment cost Ω_b generates the same interest rate spread R^*/R ?

Answer:

$$1 + \Omega_b B^* = 1/(1 - \tau).$$

2. (45) The question is related to the open economy setup. Consider an economy where the private sector holds B_{pt}^* units of foreign assets and the government holds B_{gt}^* units of foreign reserves by the end of period, both earning the risk-free foreign interest rate R_t^* . The foreign assets are denominated with foreign currency. The foreign exchange rate is given by e_t , so that one unit of foreign currency could be exchanged to e_t units of domestic currency. The net export is given by NX_t .

a. (5) Please write down the balance of payment condition of the economy.

Answer: The balance of payment condition requires that the amount of foreign capital outflows equals the current account surplus,

$$e_t(B_{pt}^* + B_{gt}^*) - e_t(B_{p,t-1}^* + B_{g,t-1}^*) = NX_t + e_t(R_{t-1}^* - 1)(B_{p,t-1}^* + B_{g,t-1}^*)$$

b. (5) Under a floating exchange rate regime, how the foreign reserves B_{gt}^* and the foreign exchange rate e_t are determined? Does the government achieve monetary policy independence?

Answer: The foreign reserves B_{gt}^* is zero or constant. The foreign exchange rate e_t is determined by the balance of payment condition. Yes, the government achieves monetary policy independence.

c. (5) Under a fixed exchange rate regime, how the foreign reserves B_{gt}^* and the foreign exchange rate e_t are determined?

Answer: The foreign exchange rate e_t is determined by the policy target. The foreign reserves B_{gt}^* is determined by the balance of payment condition.

d. (15) Consider an unexpected increase in foreign interest rate. Will the foreign asset held by the private sector B_{pt}^* increase or decrease? In the floating exchange rate regime, will the foreign exchange rate e_t increase or decrease? In the fixed exchange rate regime, will the foreign reserves B_{gt}^* increase or decrease? Please explain your answer.

Answer: An unexpected increase in foreign interest rate will raise the return on foreign assets and thus encourage the private sector to hold more foreign assets. So B_{pt}^* will increase.

In the floating exchange rate regime, the increase in private capital outflows will raise the demand for foreign currency. As a result, the foreign exchange rate e_t will increase, which means domestic currency depreciates and foreign currency appreciates.

In the fixed exchange rate regime, based on the balance of payment condition, the government has to reduce foreign reserves B_{gt}^* to offset the increase in private capital outflows to maintain the stability of exchange rate.

e.(15) If the foreign reserves are financed only with money supply, how will the unexpected increase in foreign interest rate affect domestic interest rate and inflation? Under what condition, will the government be able to achieve monetary policy independence and exchange rate stability at the same time? Please explain your answer.

Answer: If the foreign reserves are financed only with money supply, the unexpected increase in foreign interest rate will raise domestic interest rate and reduce inflation. Because the consequent decrease in foreign reserves will lead to a decrease in money supply. The contractionary monetary policy will raise domestic interest rate and reduce inflation by depressing aggregate demand.

The government will be able to achieve monetary policy independence and exchange rate stability at the same time if the private sector cannot freely trade foreign assets (i.e. capital controls are present). If the private sector can freely trade foreign assets, the no-arbitrage condition requires equal returns on foreign assets and on domestic assets. Given fixed exchange rates, domestic interest rates would be pinned down by foreign interest rates.