Open Macroeconomics

Small Open Economy Models

Xiaohan Wang

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An Open Endowment Economy

- a large number of infinitely-lived households
- consumption c_t
- exogenous and stochastic endowment of goods y_t
- borrow from abroad

A representative consumer's problem is

$$
\begin{aligned}
\max_{\{c_t, d_t\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \\
\text{subject to} \quad & c_t + (1+r)d_{t-1} = y_t + d_t \\
& \lim_{j \to \infty} \mathbb{E}_t \frac{d_{t+j}}{(1+r)^j} \leq 0
\end{aligned}
$$

r is an exogenous interest rate

d_t is the outstanding debt at the end of period t

the second constraint is No-Ponzi constraint, and holds with equality at optimal the Euler equation is

$$U'(c_t) = \beta(1+r)\mathbb{E}_t U'(c_{t+1}) \tag{1}$$

the intertemporal resource constraint is

$$(1+r)d_{t-1} = \sum_{j=0}^{\infty} \frac{\mathbb{E}_t(y_{t+j} - c_{t+j})}{(1+r)^j} \tag{2}$$

$$tb_t \equiv y_t - c_t \tag{3}$$

tb is trade balance

Two simplifying assumptions

$$\beta(1+r) = 1$$
$$U(c) = -\frac{1}{2}(c-\bar{c})^2$$

using (1), (2) and the two assumptions we can get

$$c_t = \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{\mathbb{E}_t(y_{t+j})}{(1+r)^j} - rd_{t-1}$$

define **nonfinancial permanent income** y_t^p as

$$y_t^p \equiv \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{\mathbb{E}_t(y_{t+j})}{(1+r)^j}$$

then we have

$$c_t + r d_{t-1} = y_t^p$$

Intuition: consumption is chosen given permanent income

$$d_t - d_{t-1} = y_t^p - y_t$$

Intuition: the economy borrows (lends) if permanent income is higher (lower) than current income recall that current account *ca* satisfies

$$ca_t \equiv tb_t - rd_{t-1}$$

$$ca_t = -(d_t - d_{t-1})$$

the **intertemporal approach to the balance of payments**

$$ca_t = y_t - y_t^p \tag{4}$$

$$tb_t = y_t - y_t^p + rd_{t-1} \tag{5}$$

to get countercyclical ca_t and tb_t, y_t^p should increase by more than y_t when y_t increases

Case 1: y_t follows AR(1) process

$$y_t = \rho y_{t-1} + \epsilon_t, \quad \rho \leq 1$$

permanent income can be expressed as

$$\begin{aligned} y_t^p &= \frac{r}{1+r-\rho} y_t \\ y_t - y_t^p &= \frac{1-\rho}{1+r-\rho} y_t \end{aligned}$$

so permanent income $\leq$ current income consumption adjustment

$$c_t = \frac{r}{1 + r - \rho} y_t - r d_{t-1}$$

if $\rho = 0$, consumption increases less than current endowment

if $\rho \to 1$, consumption adjusts one-for-one with current income external debt adjustment

$$d_t = d_{t-1} - \frac{1-\rho}{1+r-\rho} y_t$$

if $\rho = 0$, debt decreases almost one-for-one with income

if $\rho \to 1$, debt is almost unchanged current account adjustment

$$ca_t = \frac{1-\rho}{1+r-\rho} y_t$$

if $\rho = 0$, the change in current account is close to the change in current income

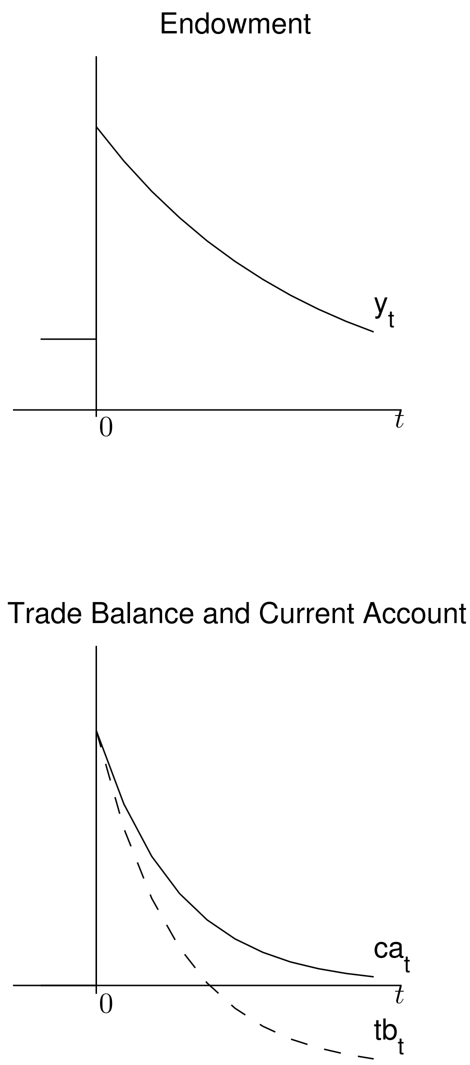
if $\rho \to 1$, current account is unchanged and equal to zero trade balance adjustment

$$tb_t = \frac{1-\rho}{1+r-\rho} y_t + r d_{t-1}$$

if $\rho = 0$, the increase in income is exported resulting in trade balance improvement

if $\rho \to 1$, none of changes in income is exported and trade balance is unchanged

Response to a positive and persistent endowment shock: AR(1) process



Summary

the AR(1) process fails to generate countercyclical current account and trade balance

the main reason is that given a positive income shock, future income is expected to be lower than current income

so we need an income process such that increase in current income creates expectations of even higher income in the future

Case 2: A nonstationary income process

$$\Delta y_t \equiv y_t - y_{t-1}$$
$$\Delta y_t = \rho \Delta y_{t-1} + \epsilon_t$$

in general, we have

$$y_t - y_t^p = -\sum_{j=0}^{\infty} \frac{\mathbb{E}_t(\Delta y_{t+j})}{(1+r)^j}$$

given the income process, we have

$$
\begin{aligned}
y_t - y_t^p &= -\frac{\rho}{1+r-\rho}\Delta y_t \\
ca_t = y_t - y_t^p &= -\frac{\rho}{1+r-\rho}\Delta y_t \\
tb_t = y_t - y_t^p + rd_{t-1} &= -\frac{\rho}{1+r-\rho}\Delta y_t + rd_{t-1}
\end{aligned}
$$

if $\rho > 0$, the economy can generate countercyclical response of current account and trade balance this income process can also generate excess consumption volatility, define

$$\Delta c_t \equiv c_t - c_{t-1}$$

we can express it as

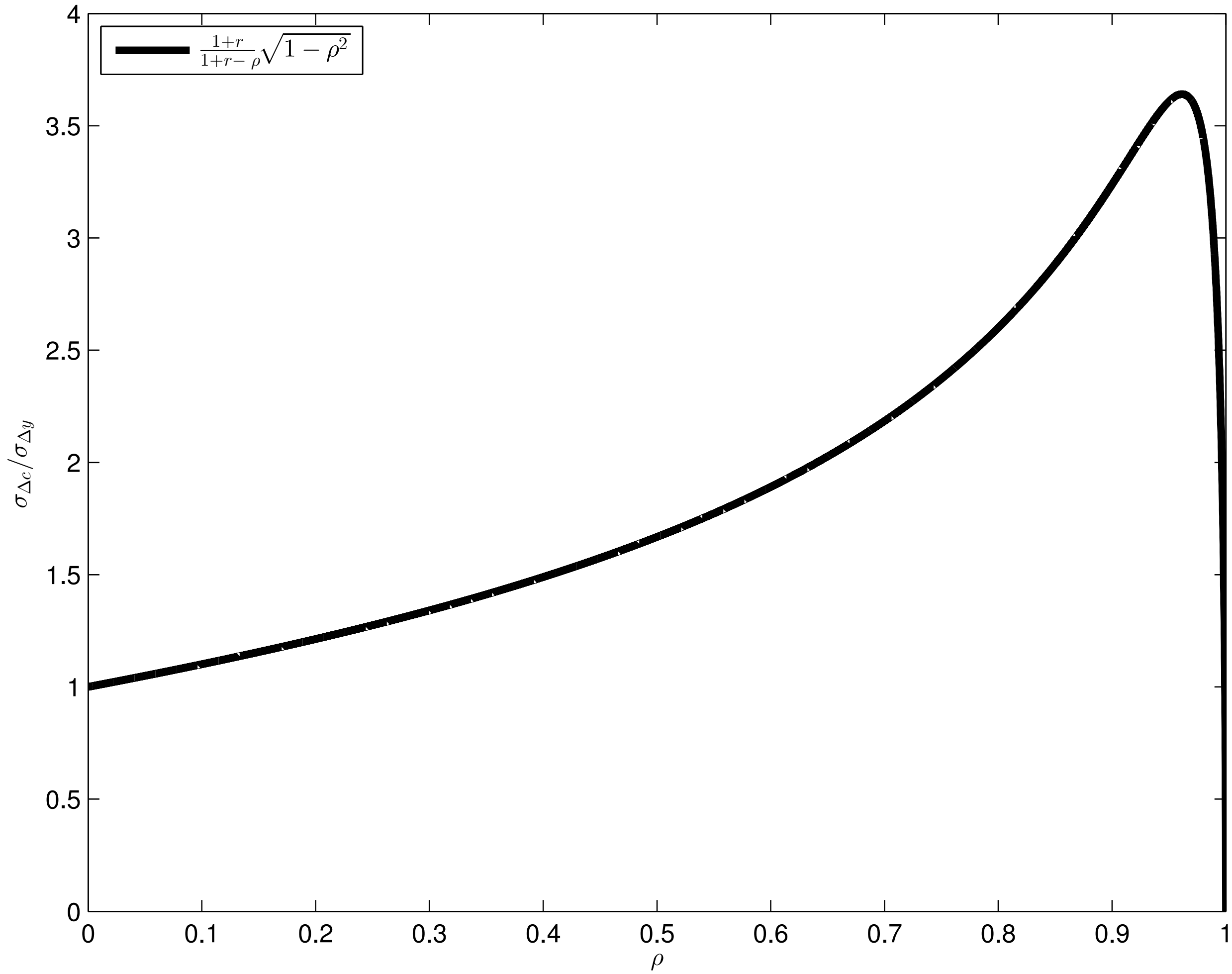
$$\Delta c_t = \frac{1+r}{1+r-\rho}\epsilon_t$$

we have

$$\frac{\sigma_{\Delta c_t}}{\sigma_{\Delta y_t}} = \frac{1+r}{1+r-\rho}\sqrt{1-\rho^2}$$

$\sigma_{\Delta c_t}$ and $\sigma_{\Delta y_t}$ are the standard deviations of consumption and income changes

Excess Volatility of Consumption Changes and the Persistence of Output Changes



Note. This figure plots equation (2.30) as a function of ρ for an interest rate of 4 percent ($r = 0.04$).

An Open Economy with Capital

$$
\begin{aligned}
\max_{\{c_t, d_t, i_t, y_t, k_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t U(c_t) \\
\text{subject to} \quad & c_t + i_t + (1+r)d_{t-1} = y_t + d_t \\
& y_t = A_t F(k_t) \\
& k_{t+1} = k_t + i_t \\
& \lim_{j \to \infty} \frac{d_{t+j}}{(1+r)^j} \leq 0
\end{aligned}
$$

A_t exogenous and deterministic

the optimality conditions are

$$r = A_{t+1} F'(k_{t+1}) \tag{6}$$

$$c_t + rd_{t-1} = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{A_{t+j} F(k_{t+j}) - (k_{t+j+1} - k_{t+j})}{(1+r)^j} \tag{7}$$

equation (6) can be expressed as

$$k_{t+1} = \kappa\left(\frac{A_{t+1}}{r}\right)$$

trade balance and current account are

$$tb_t = y_t - c_t - i_t$$

$$ca_t = tb_t - rd_{t-1}$$

A steady-state equilibrium

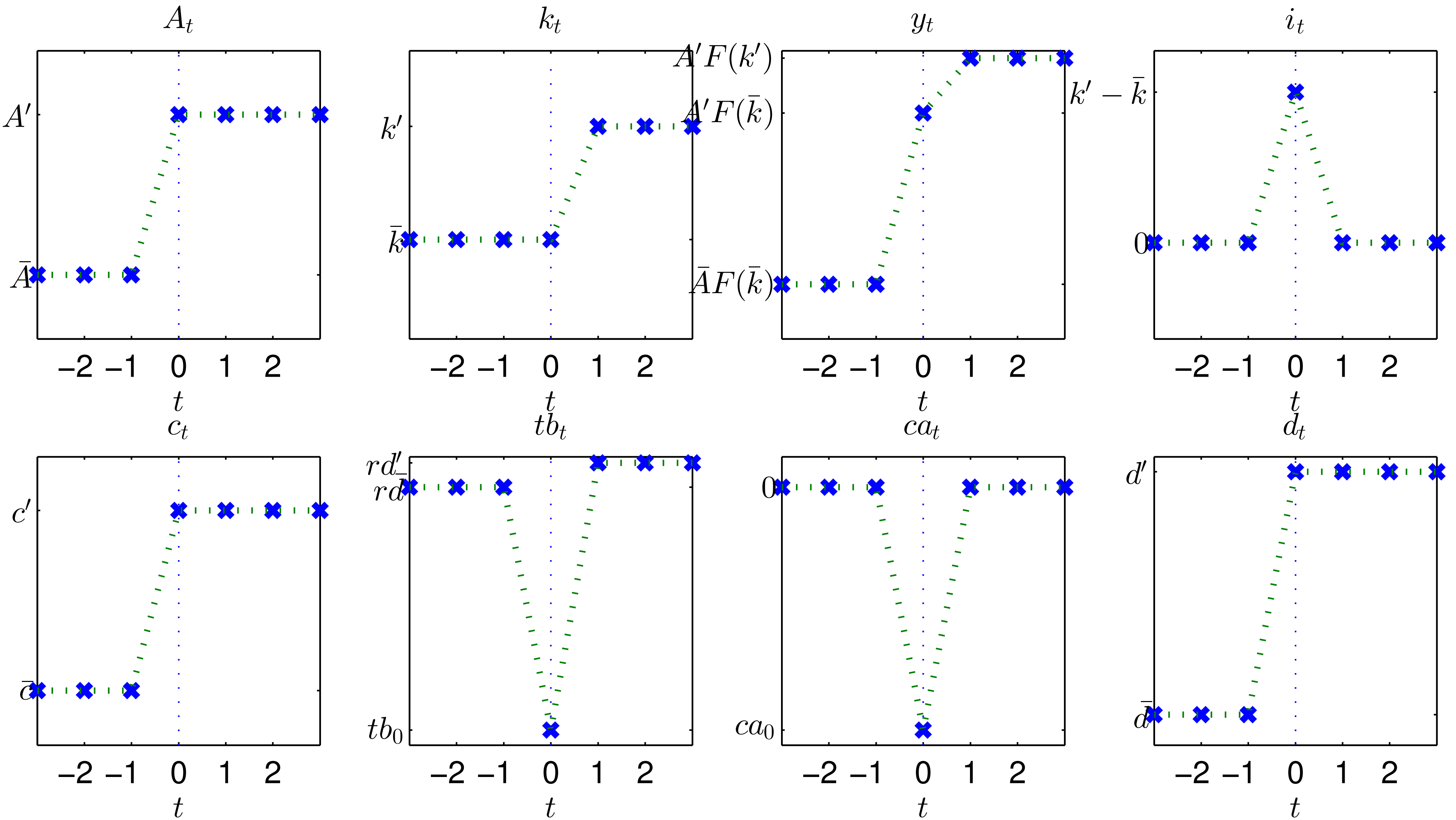
$$
\begin{aligned}
A_t &= \bar{A} \\
k_t &= \bar{k} = \kappa(\frac{\bar{A}}{r}) \\
y_t &= \bar{y} = \bar{A}F(\bar{k}) \\
i_t &= 0 \\
c_t &= \bar{c} = -rd_{-1} + \bar{A}F(\bar{k}) \\
d_t &= d_{-1} \\
tb_t &= \bar{tb} = rd_{-1} \\
ca_t &= -(d_t - d_{t-1}) = 0
\end{aligned}
$$

Adjustment to a permanent shock at $t = 0$

$$A_t = \begin{cases} \overline{A} & t < 0 \\ A' > \overline{A} & t \geq 0 \end{cases}$$

before $t = 0$, A_t was expected to be $\bar{A}$ forever. At $t = 0$, it is learned that A_t increases to A' for all $t \geq 0$

Summary of Adjustment to Permanent Productivity Shock



Two ingredients are important to generate countercyclical trade balance and current account

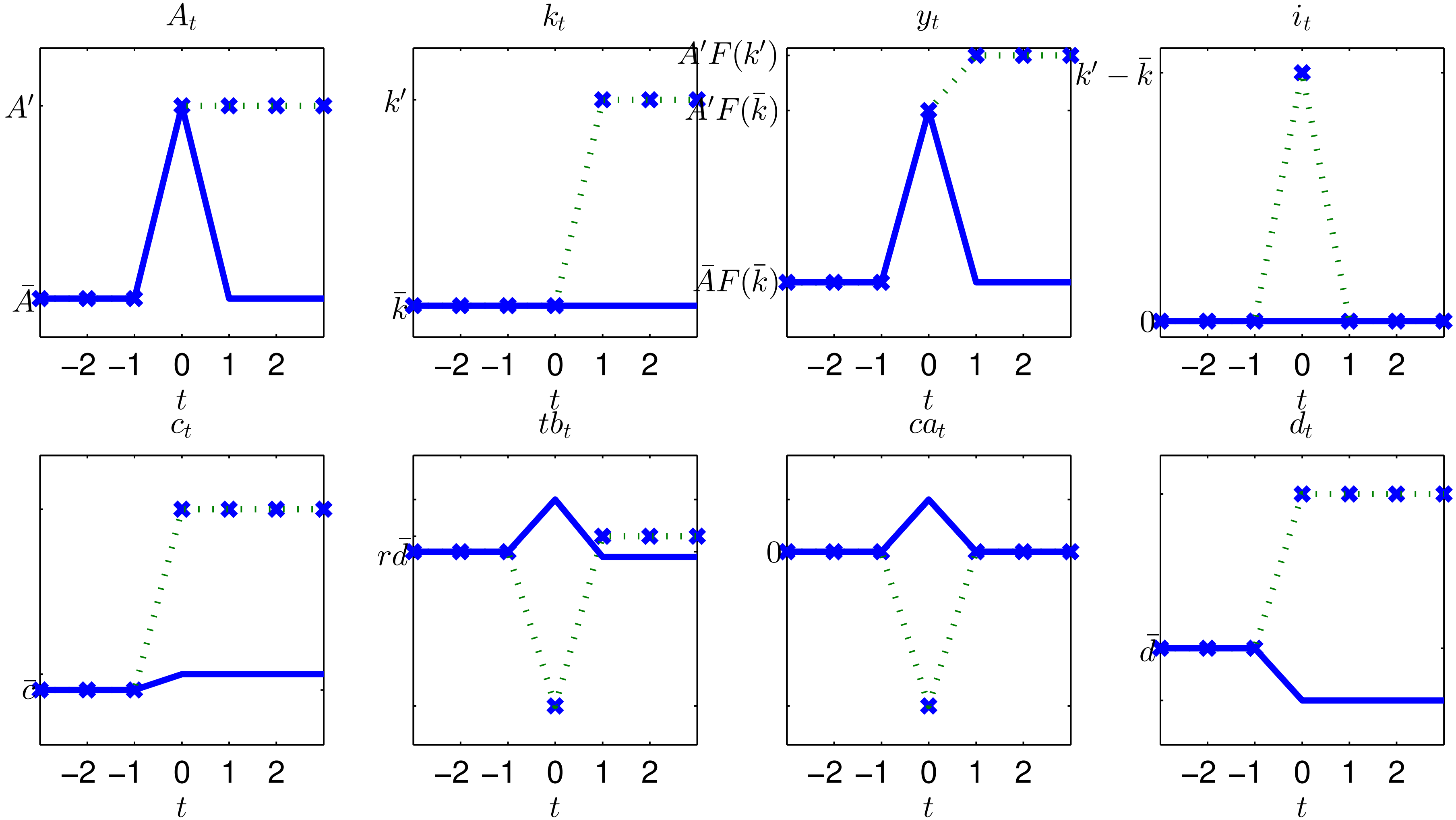
1. persistent productivity shock
2. demand for investment

Adjustment to a temporary shock at $t = 0$

$$A_t = \begin{cases} \overline{A} & t < 0 \\ A' > \overline{A} & t = 0 \\ \overline{A} & t > 0 \end{cases}$$

before $t = 0$, A_t was expected to be $\bar{A}$ forever. At $t = 0$, it is learned that A_t increases to A', but after that, A_t moves back to $\bar{A}$ for all $t > 0$

Adjustment to Temporary and Permanent Productivity Increases



▬▬: Temporary Productivity Increase x. . . x. . . x. . . : Permanent Productivity Increase

Principle I: The more persistent productivity shocks are, the more likely an initial deterioration of the trade balance will be.

A SOE model with Capital Adjustment Cost

$$\max_{\{c_t, d_t, i_t, k_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t U(c_t)$$

$$\text{s.t.} \quad c_t + i_t + (1+r)d_{t-1} + \frac{i_t^2}{2k_t} = A_t F(k_t) + d_t$$

$$k_{t+1} = k_t + i_t$$

$$\lim_{j\to\infty} \frac{d_{t+j}}{(1+r)^j} \leq 0$$

$\frac{i_t^2}{2k_t}$ is the cost function for capital adjustment let $\beta^t \lambda_t$ and $\beta^t \lambda_t q_t$ be the two multipliers, the optimal conditions are

$$\lambda_t = \beta(1+r)\lambda_{t+1}$$

$$q_t = 1 + \frac{i_t}{k_t}$$

$$\lambda_t q_t = \beta \lambda_{t+1} \left[q_{t+1} + A_{t+1} F'(k_{t+1}) + \frac{1}{2} \left(\frac{i_{t+1}}{k_{t+1}} \right)^2 \right]$$

q_t is known as *Tobin's q*

assume $\beta(1+r) = 1$ again, we have

$$(1+r)q_t = q_{t+1} + A_{t+1}F'(k_{t+1}) + \frac{1}{2}\left(\frac{i_{t+1}}{k_{t+1}}\right)^2$$

which captures the tradeoff between investment in production (RHS) and in the international financial markets (LHS)

Consider how the economy converges to steady state

- (q_t, k_t) moves following

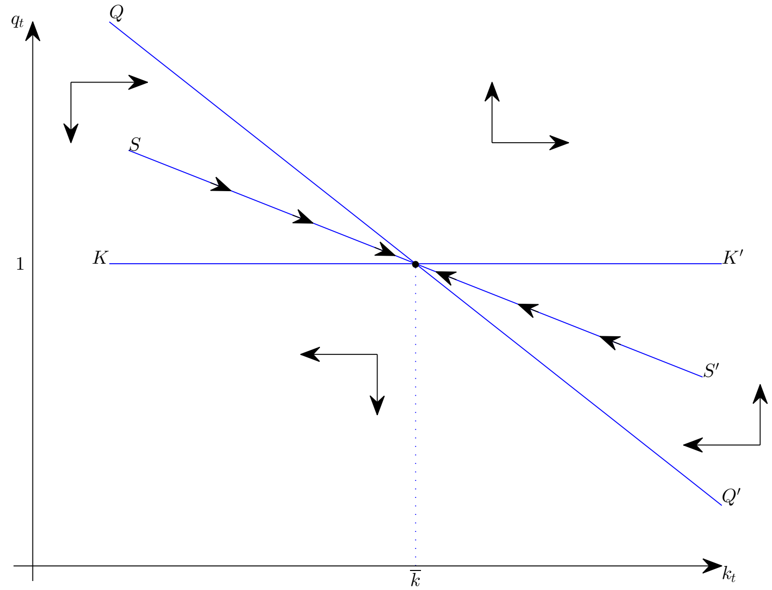
$$
\begin{aligned}
k_{t+1} &= q_t k_t \\
q_t &= \frac{1}{1+r}\left[A_{t+1}F'(q_t k_t) + \frac{1}{2}\left(q_{t+1}-1\right)^2 + q_{t+1}\right]
\end{aligned}
$$

- in steady state, we have $k_{t+1} = k_t$ and $q_{t+1} = q_t$, which implies

$$
\begin{aligned}
q_t &= 1 \\
rq_t &= \bar{A}F'(q_t k_t) + \frac{1}{2}\left(q_t - 1\right)^2
\end{aligned}
$$

note that the second term on the RHS can be ignored for linear approximation

This yields the phase diagram:



• The intersection of $\overline{KK'}$ and $\overline{QQ'}$ is the steady state pair $(k, q) = (\bar{k}, 1)$

• The locus $\overline{SS'}$ is the saddle path.

• Given the initial capital stock, k_0, Tobin's q, q_0, jumps to the saddle path, and (k_t, q_t) converge monotonically to $(\bar{k}, 1)$.

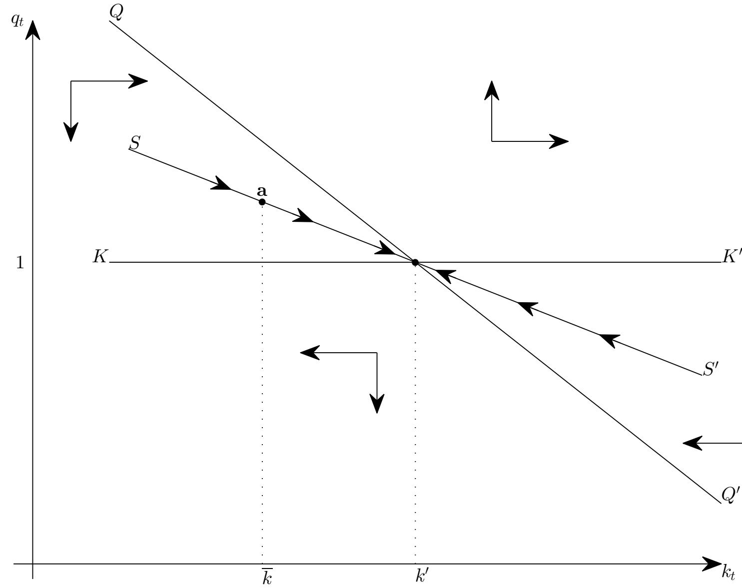
Experiment 1: Adjustment to a temporary productivity shock. $\rightarrow$ identical to the economy without capital adjustment costs, as there is no reason to adjust the capital stock. (results as in Section 3.4).

Experiment 2: Adjustment to a permanent productivity shock. In period 0 it is learned that A_t increases from $\bar{A}$ to $A' > \bar{A}$ for all $t \geq 0$. Prior to period 0, A_t was expected to be $\bar{A}$ forever.

$$A_t = \begin{cases} \bar{A} & \text{for } t \leq -1 \\ A' > \bar{A} & \text{for } t \geq 0 \end{cases} .$$

How can we capture this in the phase diagram? The $\overline{KK'}$ locus does not change. But the $\overline{QQ'}$ locus changes. The new locus is implicitly given by $rq_t = A'F'(q_t k_t) + (q_t - 1)^2/2$. This means that the $\overline{QQ'}$ locus shifts up and to the right. The new steady state is $(k_t, q_t) = (k', 1)$, where k' solves $r = A'F'(k')$. The initial capital stock is $k_0 = \bar{k}$, hence $k_0 < k'$.

The dynamics of the capital stock can be read of the graph below.



In period 0 the economy jumps to point a, where $q_0 > 1$ and $k_0 = \bar{k}$. That is, capital converges monotonically to k' from below and Tobin's q converges monotonically to 1 from above. Investment is positive during the entire transition, but, importantly, $i_0 < k' - \bar{k}$. It follows that domestic absorption increases by less on impact in the presence of capital adjustment costs. And thus, the deterioration of the trade balance in response to a positive permanent productivity shock is smaller on impact. We summarize these results as follows:

Principle II: The more pronounced are capital adjustment costs, the smaller will be the initial trade balance deterioration in response to a positive and persistent productivity shock.