Diamond-Dybvig Model

Fangyuan

Shanghai University of Finance and Economics

October 24, 2023

Why do banks exist?

- Banks provide screening and monitoring functions.
  - Banks as "delegated monitors" (Diamond, 1984) and all subsequent relationship lending literature
  - Banks are cheaper, more effective than secondary markets at overcoming information and incentive problems between investors and firms (or individual borrowers).
- Banks provide liquidity insurance to risk averse depositors
  - Demand deposits and vulnerability to runs when more than the "expected" fraction of early depositors withdraw prematurely (Bryant, 1980; Diamond and Dybvig, 1983)
  - Banks are cheaper, more effective than secondary markets at providing insurance to individuals and firms against idiosyncratic shocks to consumption or production or other needs for cash.

Why do depositors run?

- Bank runs as panic, sunspot, multiple equilibria
  - Diamond and Dybvig (1983)
- Business cycle, essential crises, linked to fundamentals
  - Jacklin and Bhattacharya (1988)
- Common feature.
  - Banks issue liquidity liabilities: demandable deposits
  - Banks invest in illiquid assets.

Diamond-Dybvig (1983)–The Environment

- Three dates $t \in \{0, 1, 2\}$ and a single consumption good (dollar)
- Measure one of the ex-ante identical consumers with a unit endowment at date 0. Consume either at date 1 or 2.
- Liquidity shocks: Preference:

$$u(c_1, c_2) = u(c_1 + \theta c_2)$$

- If $\theta = 1$ the consumer is "late" or "patient", otherwise, he is "early" or "impatient". $\pi = \Pr(\pi = 0)$
- Investment technology:
  - Storage: transform x goods at t to x goods at $t + 1$
  - Long-term investment: one dollar at date 0 yields $R > 1$ dollars at date 2 if the project is completed, $\lambda \leq 1$ dollar if terminated.

  Note: original paper has $\lambda = 1$, so the storage technology is redundant $\rightarrow$ the ex-ante choice of technology is not the main issue.

First Best planner problem I

- Suppose a planner chooses (c_1, c_2, x, y)

$$\max_{c_1, c_2, x, y} \pi u(c_1) + (1-\pi) u(c_2)$$

$$\pi c_1 \leq x$$

$$(1-\pi) c_2 \leq Ry$$

$$x + y = 1$$

- First order condition:

$$\frac{u'(c_1)}{u'(c_2)} = R$$

- It follows that $c_1^{FB} < c_2^{FB}$, i.e., the first best allocation is incentive compatible.
- Optimal allocation equates MRS with the technological price.

Complete markets allocation

- Assume that the agent can buy contingent claims, c_1 if impatient at price p_1, and c_2 if patient at price p_2

$$\max_{c_1,c_2} \pi u(c_1) + (1-\pi)u(c_2) \tag{1}$$

$$p_1 c_1 + p_2 c_2 \leq 1 \tag{2}$$

- First order condition

$$\frac{u'(c_1)}{u'(c_2)} = \frac{1-\pi}{\pi}\frac{p_1}{p_2}$$

- Firms can transform one unit of $t=0$ goods into $1/\pi$ units of contingent claims if impatient, or into $\frac{R}{1-\pi}$ units of contingent claims if patient. So competition implies that

$$p_1 = \pi, \quad p_2 = (1-\pi)/R$$

- Therefore,

$$\frac{u'(c_1)}{u'(c_2)} = R$$

Incomplete markets I

- Suppose there are no insurance markets
- The only market is at $t = 1$, where agents can trade $t = 1$ goods against $t = 2$ goods
- Let the price of $t = 2$ goods at $t = 1$ be p

$$\max_{c_1, c_2, x, y} \pi u(c_1) + (1 - \pi) u(c_2)$$

$$c_1 \leq x + pRy$$

$$c_2 \leq Ry + \frac{x}{p}$$

$$x + y = 1$$

- In any equilibrium consumers invest in both technologies, it must be that

$$p = \frac{1}{R}$$

Incomplete markets II

- If $p > \frac{1}{R}$, then it is always preferable to invest in the long-term technology at $t = 0$, but then all early consumers will be trying to sell at $t = 1$ and nobody would buy
- If $p < \frac{1}{R}$, then it is always preferable to invest in storage $t = 0$, but then all late consumer will be trying to pay p for $t = 2$ goods, so $p < \frac{1}{R}$ cannot be equilibrium price.
- Therfore the conusmer's problem reduces to

$$\max_{c_1,c_2} \pi u(c_1) + (1-\pi)u(c_2)$$
$$c_1 \leq 1$$
$$c_2 \leq R$$

- The allocation:

$$c_1 = 1, \quad c_2 = R$$

Incomplete markets III

- This not necessarily coincides with the first best allocation.
- Allocations coincide for the special case of log preferences:
  - General optimality condition:
    $$\frac{u'(c_1)}{u'(c_2)} = R$$
  - Special case of log utility
    $$\frac{c_2}{c_1} = R$$
    so the allocation $c_1, c_2 = R$ satisfies optimality
- Assume that consumer is more risk averse than log, i.e.,

$$-\frac{cu''(c)}{u'(c)} > 1$$

Incomplete markets IV

- In this case, $c_1^{FB} > 1$ and $c_2^{FB} < R$. Proof:

$$
\begin{aligned}
Ru'(R) =& u'(1) + \int_1^R \frac{\partial c u'(c)}{\partial c} dc \\
=& u'(1) + \int_1^R [cu''(c) + u'(c)] dc \\
=& u'(1) + \int_1^R u'(c)[\frac{cu''(c)}{u'(c)} + 1] dc \\
<& u'(1)
\end{aligned}
$$

- Numerical Example: suppose $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $R = 2, \pi = 0.25, \sigma = 2$

A Bank I

- Consumers get together and create a bank,
- They each deposit their endowment with the bank, which invest the first best amount in each technology.
- The bank contract: each consumer can ask for c_1^{FB} at $t = 1$ or can wait until $t = 2$ and get a pro-rata share of whatever is left.
- The bank commits to satisfying **sequential service**: satisfy consumer's withdrawals in the order in which they arrive (which, assume, is random)
- If the stored goods are not enough to satisfy a consumer who demands payment, then the bank must liquidate the long-term investment until that, too, runs out.
- Consumers then play a game where actions are "withdraw" or "wait"

A Bank II

- Let f_j be the number of depositors who arrived in line before consumer j and asked to withdraw and
- Let f be the total number of consumers that will eventually ask to withdraw
- The payoff for an impatient consumer is

Withdraw	Wait
$\begin{cases} c_1^{FB} & \text{if } f_j c_1^{FB} < x + \lambda y^{FB} \\ 0 & \text{otherwise} \end{cases}$	0

A Bank III

- The payoff for the patient consumer if he withdraws at $t = 1$ is

$$\begin{cases} c_1^{FB} & \text{if } f_j c_1^{FB} < x^{FB} + \lambda y^{FB} \\ 0 & \text{otherwise} \end{cases}$$

- The payoff for the patient consumer if he waits at $t = 1$ is

$$\max\left\{ R\frac{1 - \pi c_1^{FB} - (f - \pi)\frac{1}{\lambda}c_1^{FB}}{1 - f}, 0 \right\}$$

Symmetric Equilibrium

Good Equilibrium: "withdraw iff impatient" is a Nash Equilibrium, with a payoff equal to the first best allocation.

- Impatient consumers don't want to deviate because they don't care about future consumption
- Patient consumers don't want to deviate because $c_2^{FB} > c_1^{FB}$

Bad Equilibrium: "withdraw no matter what" is also a Nash Equilibrium

- Given that $c_1^{FB} > 1 \geq x^{FB} + \lambda y^{FB}$, if everyone tries to withdraw, then the money will run out.
- Therefore, those who wait will get zero
- This equilibrium produces a very bad allocation.

Suspension of convertibility

- One variant of the contract can rule out the bad equilibrium
- The contract states that you can withdraw c_1^{FB} at $t = 1$ as long as less than π other consumers have withdrawn before you. After that, you are forced to wait
- The payoffs for the impatient consumers are:

$$\begin{array}{cc} \text{Withdraw} & \text{Wait} \\ \hline \begin{cases} c_1^{FB} & \text{if } f_j < \pi \\ 0 & \text{otherwise} \end{cases} & 0 \end{array}$$

- The payoffs for a patient consumer are

$$\begin{array}{cc} \text{Withdraw} & \text{Wait} \\ \hline \begin{cases} c_1^{FB} & \text{if } f_j < \pi \\ 0 & \text{otherwise} \end{cases} & c_2^{FB} \end{array}$$

- Waiting is **dominant** for patient consumers, and runs will not take place.

Jacklin's Critique

Jacklin (1987) makes three related points

- Under the Diamond and Dybvig assumptions, you don't really need a bank, equity is good enough
- The Diamond and Dybvig (1983) assumptions are very special. Under more general preferences
  1. you can do more with an bank than with equity
  2. A bank may or may not be able to achieve the first best
- If there is a market, a bank that is useful cannot survive

Who needs a bank?

- Suppose instead of a "bank" consumers set up a "firm"
- The firm invests the first-best amount and issues shares. Each consumer gets one share
- It declares (and commits to) the following dividend policy: a dividend of $d_1 = \pi c_1^{FB}$ will be paid at $t = 1$ and a dividend of $d_2 = (1 - \pi)c_2^{FB}$ will be paid at $t = 2$
- Consumers can trade shares in the firm at $t = 1$ (they trade "ex-dividend" - after paying dividends) at a price of p goods per share.
- Supply of shares (impatient consumers sell): $S = \lambda$

Who needs a bank? (cont.)

- Demand for shares (patient consumers perhaps buy):

$$D = \begin{cases} \frac{(1-\pi)d_1}{p} & \text{if } p < d_2 \\ 0 & \text{otherwise} \end{cases}$$

- Market clearing:

$$\pi = \frac{(1-\lambda)d_1}{p}$$

$$p = \frac{(1-\pi)d_1}{\pi} = (1-\pi)c_1^{\text{FB}}$$

- This is sometimes known as "cash-in-the-market" pricing
- Buyers are at a corner solution. They would want to buy more but have no more money.

Who needs a bank? (cont.)

- Consumption attained by early consumers

$$c_1 = d_1 + p = c_1^{FB}$$

- Consumption attained by late consumers

$$\begin{aligned} c_2 &= d_2 \left(1 + \frac{d_1}{p}\right) \\ &= (1-\pi)c_2^{\text{FB}} \left(1 + \frac{\pi c_1^{\text{FB}}}{(1-\pi)c_1^{\text{FB}}}\right) \\ &= c_2^{\text{FB}} \end{aligned}$$

- No need for demand deposits, no risk of bank runs!