1. Determine whether the following statements are True or False and briefly EXPLAIN. Most points are for you explanations.(5 points each)

   (a) There is a unique pure strategy Nash equilibrium in a two-player game when one player has a dominant strategy.

   False. Even if the player with a dominant strategy plays the pure (dominant) strategy, the other player may be playing a mixed strategy. See, for example, the following game:

	a	b
U	(3, 1)	(0, 1)
D	(0, 0)	(-1, 2)

   While player 1 plays U, player 2 could play both a and b with positive probabilities.

   (b) A strategy that is NEVER be a best response is strictly dominated.

   False. This is true only for two-player game. For more than two-player games, some strategies that are never a best response may nevertheless not be dominated strategies.

   (c) Backward induction, or subgame perfect Nash equilibrium ensures that the Principle of Sequential Rationality is satisfied.

   False. This statement is true only for perfect information games. For some imperfect information games, SPNE may be the same as NE and therefore not specifies optimal choices at some information sets.

   (d) A mixed strategy NE in which both players play totally mixed strategies is a normal form perfect equilibrium.

   True. The totally mixed strategy profile meets the definition pf an ϵ-perfect equilibrium, and its limit is just itself. So, of course, it must be perfect.

   (e) Every finite extensive form game of perfect information has a unique subgame perfect Nash equilibrium.

   False. This is true only if there is no tie in payoffs for any player.

2. For the game given below:

Player 2

		L_2	M_2	N_2	R_2
Player 1	L_1	(1, 4)	(3, 2)	(2, 6)	(2, 7)
	M_1	(0, 0)	(0, 3)	(1, 3)	(1, 0)
	N_1	(2, 0)	(10, -1)	(0, 0)	(0, 2)
	R_1	(4, 7)	(5, 1)	(2,0)	(1, 4)
	S_1	(3, 10)	(7, 4)	(1, 5)	(0, 6)

   (a) Does any player have any strictly dominated strategies? If yes, what are they? (6 points)

   (b) Apply the Iterated Deletion of Strictly Dominated Strategies procedure until there is no strictly dominated strategies in the remaining game. (9 points)

(c) Find all pure strategy NE of the game. (5 points)

(d) Does the game have any mixed strategy NE? If your answer is "Yes," find the mixed strategy NE. (5 points)

Answer:

(a) Yes. Player 1's strategy M_1 is strictly dominated by L_1, and S_1 is strictly dominated by a mixed strategy, for example, by $\sigma_1 = \left(0, 0, \frac{9}{20}, \frac{11}{20}, 0\right)$

(b) After deleting strictly dominated strategy M_1 and S_1, N_2, M_2 both becomes strictly dominated by R_2.

After deleting M_2, N_2, N_1 becomes strictly dominated by R_1. So we have

	L_2	R_2
L_1	(1, 4)	(2, 7)
R_1	(4, 7)	(1, 4)

(c) Two pure strategy NE

$$(L_1, R_2), (R_1, L_2).$$

(d) To find the mixed strategy NE, we consider players' best responses given belief. For player 1, given the belief $(q, 0, 0, 1-q)$, expected payo s from playing L_1, R_1 are

$$u_1(L_1) = q + 2(1-q), \quad u_1(R_1) = 4q + (1-q).$$

To be a mixed NE, it must be true that

$$u_1(L_1) = u_1(R_1), \qquad so \quad q = \frac{1}{4}.$$

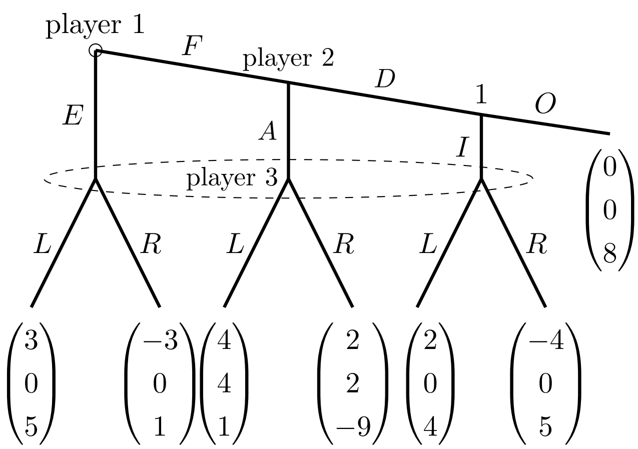
For player 2, given the belief $(p, 0, 0, 1-p, 0)$, expected payoff from playing L_2, R_2 are

$$u_2(L_2) = 4p + 7(1-p), \quad u_2(R_2) = 7p + 4(1-p).$$

This gives p = 1/2. So the mixed strategy NE is:

$$\left(\frac{1}{2}, 0, 0, \frac{1}{2}, 0; \frac{1}{4}, 0, 0, \frac{3}{4}\right)$$

3. For the extensive form game below:



(a) Write down the strategic form, and identify all dominated strategies for the players. (10 points)

(b) Find all pure strategy NEs of the game. (10 points)

(c) Find all normal form perfect equilibria of the game. (5 points)

Answer:

(a) The strategic form

3 plays L (Player 2)

	A	D
EI	(3, 0, 5)	(3, 0, 5)
EO	(3, 0, 5)	(3, 0, 5)
FI	(4, 4, 1)	(2, 0, 4)
FO	(4, 4, 1)	(0, 0, 8)

R (Player 2)

	A	D
EI	(-3, 0, 1)	(-3, 0, 1)
EO	(-3, 0, 1)	(-3, 0, 1)
FI	(2, 2, -9)	(-4, 0, 5)
FO	(2, 2, -9)	(0, 0, 8)

Player 2's D is a weakly dominated strategy.

(b) 4 pure NE:

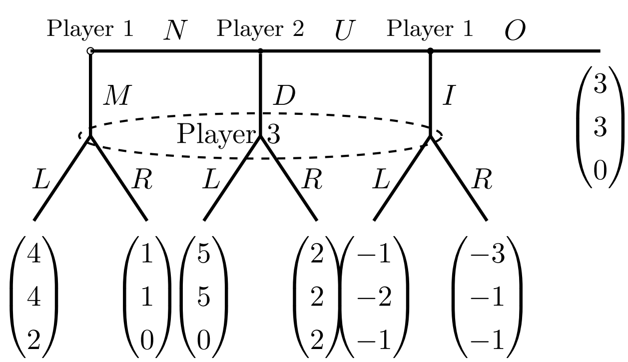
$$(EI, D, L), \quad (EO, D, L), \quad (FI, A, L), \quad (FO, A, L).$$

(c) First, D is a weakly dominated strategy, and the first 2 NE cannot be perfect equilibrium. Next, for any total mixed strategy $(1-\eta, \eta)$ assigned to player 2's pure strategies A, D and $(1-\nu, \nu)$ to 3's pure strategies L, R,

$$\begin{aligned} U_1(FI, \sigma^\epsilon) = &\quad (1-\nu)[4(1-\eta)+2\eta] + \nu[2(1-\eta)-4\eta] > \\ U_1(FO, \sigma^\epsilon) = &\quad 4(1-\nu)(1-\eta) + 2\nu(1-\eta); \end{aligned}$$

implying there exists no ϵ-perfect equilibrium in which FO receives higher probability than FI. Remember: $1-\nu$ is close to 1 while ν is close to zero! Thus, there is only 1 normal form perfect equilibrium, (FI, A, L).

4. For the extensive form game below:



(a) Write down the strategic form of the game. (5 points)

(b) Find all pure strategy NE. (10 points)

(c) Find all pure strategy sequential equilibria, explicitly stating the strategy profiles σ and the associated belief μ for each S.E. (10 points)

Answer:

(a) Strategic form

3 plays L (Player 2 columns, Player 1 rows)

	D	U
MI	(4, 4, 2)	(4, 4, 2)
MO	(4, 4, 2)	(4, 4, 2)
NI	(5, 5, 0)	(-1, -2, -1)
NO	(5, 5, 0)	(3, 3, 0)

3 plays R (Player 2 columns, Player 1 rows)

	D	U
MI	(1, 1, 0)	(1, 1, 0)
MO	(1, 1, 0)	(1, 1, 0)
NI	(2, 2, 2)	(-3, -1, -1)
NO	(2, 2, 2)	(3, 3, 0)

(b) There are 4 pure strategy NE:

$$(MI, U, L), \quad (MO, U, L), \quad (NI, D, R), \quad (NO, U, R).$$

(c) In the first two NE (MI, U, L) and (MO, U, L), player 2's choice is not optimal give player 3 choosing L; choosing D would give player 2 a payoff of 5 instead of 3 or -2.

In the NE (NI, D, R), player 1's choice at her second information set after player 2 choosing U is not optimal since choosing I results in a negative payoff while choosing O ensures a payoff of 3.

So there is only one SE:

$$\sigma : (NO, U, R)$$

$$\mu : \{(\alpha, \beta, 1 - \alpha - \beta) | \alpha \leq \beta, \alpha + \beta \leq 1\}.$$