

Trend-Cycle Decomposition

To characterize business cycle facts we decompose a time series, y_t , into a

- cyclical component, y_t^c , and a
- secular (or trend) component, y_t^s

$$y_t = y_t^c + y_t^s$$

There are various methods to extract the cyclical component. Here is a list of commonly used ones:

- log-linear detrending
- log-quadratic detrending
- HP filtering
- Time differences
- Band pass filtering

Log-linear detrending

Let

$$y_t \equiv \ln Y_t$$

denote the natural logarithm of a time series Y_t , such as real GDP per capita, where t denotes time. Then write

$$y_t = a + bt + \epsilon_t$$

where

$$\text{cycle: } y_t^c = \epsilon_t$$

$$\text{secular trend: } y_t^s = a + bt$$

The parameters a and b can be estimated via ordinary least squares (OLS).

This is, for example, how King, Plosser, and Rebelo (JME, 1988) define the cyclical component of U.S. time series in their seminal real-business-cycle paper. They further impose constant spending shares in the long run, that is, they impose that b is the same for output, consumption, and investment.

Log-quadratic detrending

$$y_t = a + bt + ct^2 + \epsilon_t$$

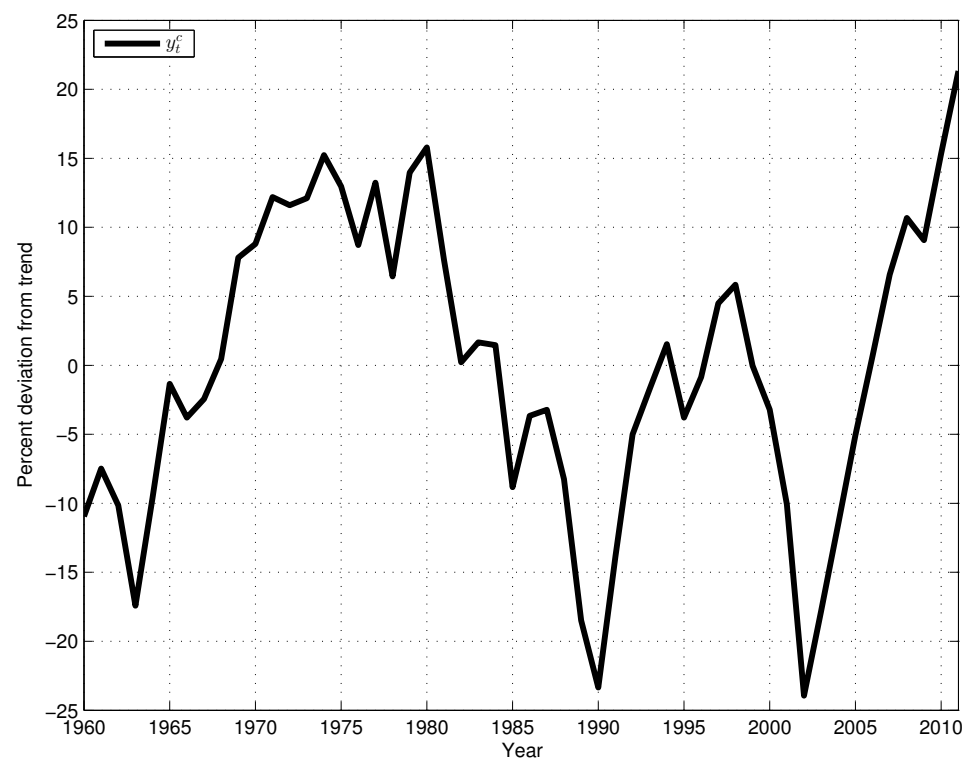
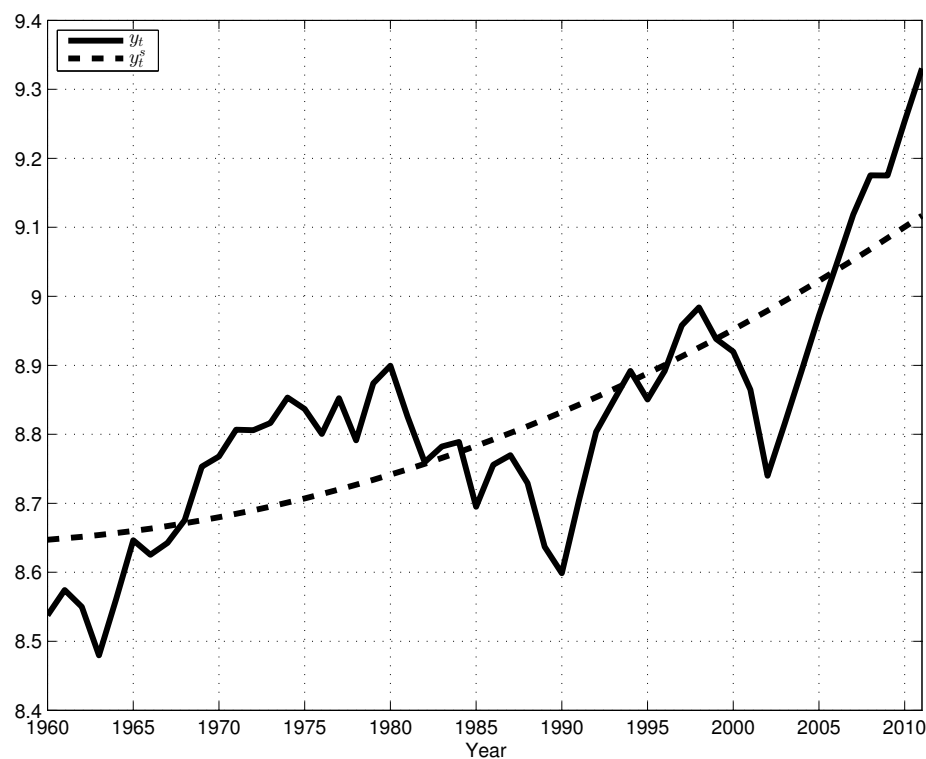
$$\text{cycle: } y_t^c = \epsilon_t$$

$$\text{secular trend: } y_t^s = a + bt + ct^2$$

Again, the parameters a , b , and c can be estimated via OLS.

This is, for example, how Mendoza (AER, 1991) defines the cyclical component of Canadian time series in his small-open-economy real-business-cycle model.

Log-quadratic detrending: Application to Argentine real GDP per capita, annual data, 1960-2011



Business Cycle Facts with Annual Data

- Data Source: Annual data from the World Development Indicators (WDI), starting in 1960 and ending in 2011.
- Restriction: To be included in the present sample, a country must have at least 30 consecutive observations of the following six variables:
 - y_t (log of) real GDP per capita
 - c_t (log of) real private consumption per capita
 - g_t (log of) real government consumption per capita
 - i_t (log of) real investment per capita
 - x_t (log of) real exports per capita
 - m_t (log of) real imports per capita
- There are 120 countries satisfying this requirement in the WDI data set. Of those 120 countries, 94 have 30 consecutive years of data on the current account. Thus, statistics regarding the current account are based only on a sample of 94 countries.

Business-Cycle Facts with Quadratic Detrending

We first present business-cycle facts based on quadratically detrended versions of the logs of output, consumption, government expenditure, investment, exports, and imports (y_t , c_t , g_t , i_t , x_t , and m_t).

Trade Balance, $TB_t \equiv X_t - M_t$ can take negative values, so its log doesn't always exist. So instead, we detrend it by first scaling it by the trend component of output to obtain $tb_t \equiv \frac{X_t - M_t}{\exp(y_t^s)}$ and then removing a quadratic trend. In this way, the deviations from trend are measured in percent of trend output. Same with the Current Account, denoted CA_t ; thus, $ca_t \equiv \frac{CA_t}{\exp(y_t^s)}$.

For each country, we compute standard deviations, contemporaneous correlations with output, and serial correlations of all variables. We then compute a population-weighted average of each statistic.

Business Cycles Around the World

Ten Facts

High Global Volatility

Business-Cycle		
Statistic	World	U.S.
σ_y	6.2%	2.9%

Fact 1: The cross-country average standard deviation of output is twice as large as its U.S. counterpart.

Excess Consumption Volatility

Business-Cycle Statistic	United States	World Average
$\frac{\sigma_c}{\sigma_y}$	1.02	1.05

Fact 2: On average, across countries, private consumption (including durables) is more volatile than output.

Global Ranking of Volatilities

Business-Cycle Statistic	World Average
$\frac{\sigma_m}{\sigma_y}$	3.23
$\frac{\sigma_i}{\sigma_y}$	3.14
$\frac{\sigma_x}{\sigma_y}$	3.07
$\frac{\sigma_g}{\sigma_y}$	2.26
$\frac{\sigma_c}{\sigma_y}$	1.05

Fact 3: The ranking of cross-country average standard deviations from top to bottom is imports, investment, exports, government spending, consumption, and output.

Cyclicalities

Business-Cycle Statistic	World Average
$\text{corr}(c, y)$	0.69
$\text{corr}(i, y)$	0.66
$\text{corr}(x, y)$	0.19
$\text{corr}(m, y)$	0.24
$\text{corr}(tb, y)$	-0.18
$\text{corr}(ca, y)$	-0.28
$\text{corr}(g/y, y)$	-0.02

Definition: A variable is said to be procyclical, countercyclical, or acyclical if its correlation with output is, respectively, positive, negative, or close to zero.

Fact 4:

Consumption, investment, exports, and imports are procyclical.

Fact 5:

The trade balance and the current account are countercyclical.

Fact 6:

The share of government consumption in output is acyclical.

Persistence

Business-Cycle Statistic	World Average
$\text{corr}(y_t, y_{t-1})$	0.71
$\text{corr}(c_t, c_{t-1})$	0.66
$\text{corr}(g_t, g_{t-1})$	0.76
$\text{corr}(i_t, i_{t-1})$	0.56
$\text{corr}(x_t, x_{t-1})$	0.68
$\text{corr}(m_t, m_{t-1})$	0.61

Fact 7: All components of demand (c, g, i, x) and supply (y, m) are positively serially correlated.

Business Cycles in Rich, Emerging, and Poor Countries

Are business cycles different in rich, poor, and emerging countries?

To answer this question, the first thing to do is to determine which countries are rich, poor, or emerging (or middle income). This, in turn, requires coming up with a measure of income per capita that is comparable across countries. We use the geometric average of PPP-converted GDP per capita in U.S. dollars of 2005 over the period 1990-2009.

Loosely speaking, PPP-converted GDP in a given country is the value of all goods and services produced in that country evaluated at U.S. prices. By evaluating production of goods and services in different countries at the same prices, PPP conversion makes cross-country comparisons more sensible.

A good source for PPP-converted GDP data is the World Bank's *International Comparison Program (ICP)*.

PPP-Converted GDP — An example

Suppose that in a given year country X produces: 3 hair cuts and 1 ton of grain.

Suppose the unit prices of these items in country X are 1 and 200 dollars, respectively.

Hence the nonconverted measure of GDP in country X is 203 dollars.

Suppose that grain trades freely in international commodity markets, so its price is the same across countries. By contrast, haircuts are not traded internationally, so their prices can vary across countries.

Specifically, suppose that in the United States a hair cut costs 20 dollars and one ton of grain 200 dollars.

Then, the PPP-converted measure of GDP in country X is 260 dollars.

Country X is 28% ($260/203 - 1$) richer when GDP is measured at PPP prices than when it is measured at domestic prices. The reason is that nontradable services are more expensive in the U.S. than in country X.

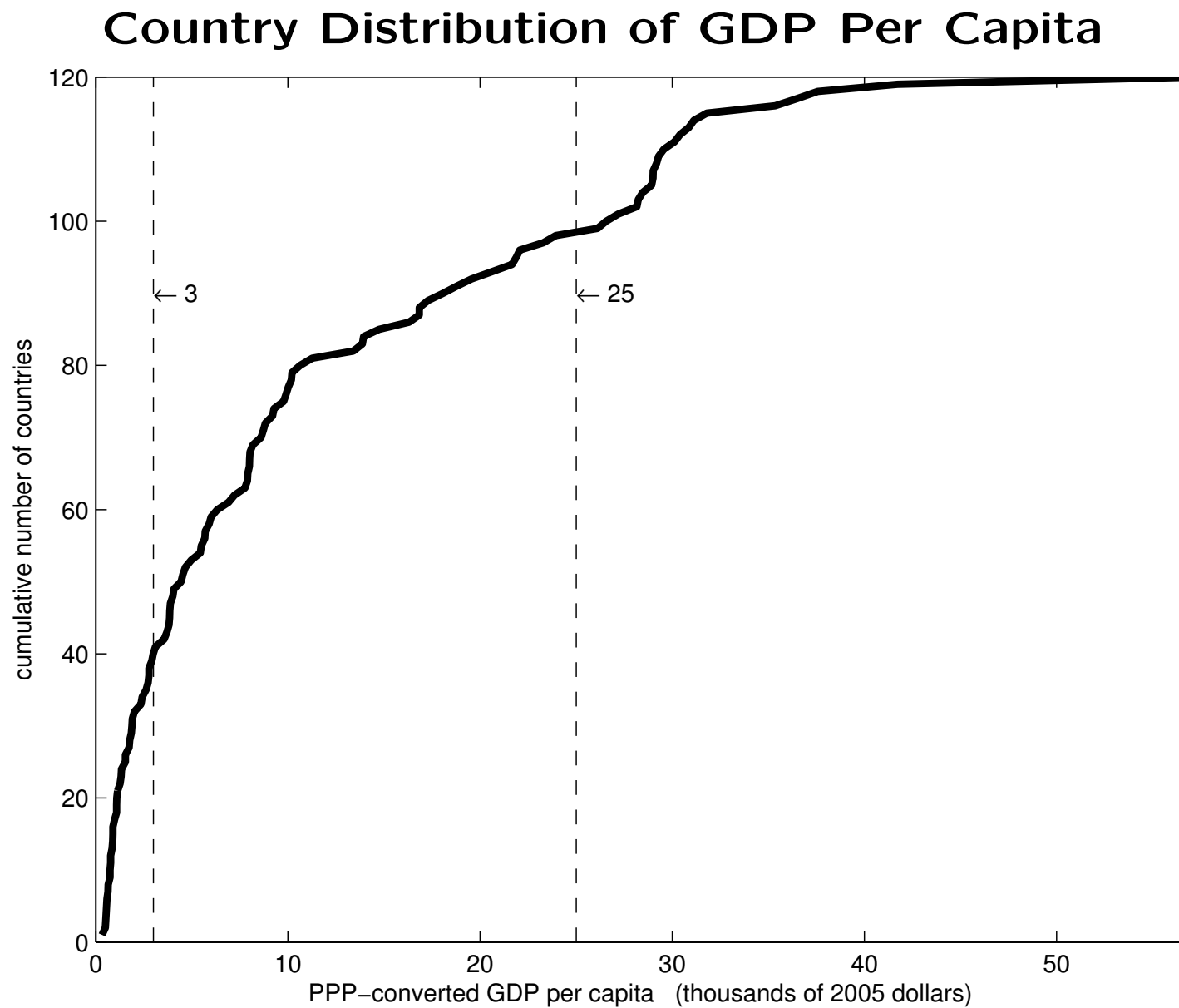
This hypothetical result is indeed typical for poor and emerging countries, where labor-intensive services are far cheaper than in the U.S.

The Country Distribution of GDP Per Capita

The next figure displays the distribution of PPP-converted GDP per capita across countries. The horizontal axis measures the average PPP-converted GDP per capita in U.S. dollars of 2005 over the period 1990 to 2009. The vertical axis measures the number of countries with GDP less than or equal to the associated level on the horizontal axis (recall that the total number of countries is 120).

GDP per capita is far from evenly distributed across countries. This is reflected in the fact that the plotted line is quite steep at low levels of output. Another indication of the uneven distribution of income across countries is that the median (PPP-converted) GDP per capita is almost half as large as the mean (\$6,615 versus \$11,254). Put differently, eighty countries (or $2/3$) have per capita incomes below the mean.

Q: How would the figure look if income was evenly distributed across countries?



Defining Three Income Groups

Let's divide the sample of 120 countries into three groups: Poor, emerging, and rich countries. These groups are defined as all countries with average PPP converted GDP per capita within the ranges:

less than \$3,000,
\$3,000 to \$25,000,
more than \$25,000,

respectively.

This results in 40 poor ($1/3$), 58 emerging ($1/2$), and 22 ($1/6$) rich countries.

Poor Countries

- Benin, Bhutan, Burkina Faso, Burundi, Central African Republic, Comoros, Gambia, Guyana, Honduras, Lesotho, Malawi, Mali, Mauritania, Mongolia, Niger, Papua New Guinea, Rwanda, Senegal, Sierra Leone, Togo, Zambia, Zimbabwe.
- Cameroon, Congo, Côte d'Ivoire, Ghana, Kenya, Madagascar, Mozambique, Nepal, Sri Lanka, Sudan, Uganda.
- Bangladesh, China, Ethiopia, India, Indonesia, Pakistan, Philippines.

Emerging Countries

- Albania, Antigua and Barbuda, Bahrain, Barbados, Bolivia, Botswana, Bulgaria, Chile, Costa Rica, Cuba, Cyprus, Dominica, Dominican Republic, Ecuador, El Salvador, Fiji, Gabon, Greece, Grenada, Guatemala, Hungary, Israel, Jordan, Malta, Mauritius, Namibia, New Zealand, Panama, Paraguay, Portugal, Puerto Rico, Seychelles, St. Kitts and Nevis, St. Lucia, St. Vincent and the Grenadines, Suriname, Swaziland, Tonga, Trinidad and Tobago, Tunisia, Uruguay.
- Algeria, Argentina, Colombia, Iran, Malaysia, Morocco, Peru, South Africa, South Korea, Spain, Syria, Thailand, Turkey, Venezuela.
- Brazil, Egypt, Mexico.

Rich Countries

- Austria, Belgium, Denmark, Finland, Hong Kong, Iceland, Ireland, Luxembourg, Macao, Netherlands, Norway, Singapore, Sweden, Switzerland.
- Australia, Canada, France, Italy, United Kingdom.
- Germany, Japan, United States.

Note. Subgroups by size are marked with dashes.

Three differences in business cycles across rich, poor, and emerging countries

Excess Volatility in Poor and Emerging Countries

Business-Cycle			
Statistic	Poor	Emerging	Rich
σ_y	6.1%	8.7%	3.3%

Fact 8: Business Cycles in poor and emerging countries are about **twice** as volatile as business cycles in rich countries.

Less Consumption Smoothing in Poor and Emerging than in Rich Countries

Business-Cycle			
Statistic	Poor	Emerging	Rich
σ_c/σ_y	1.12	0.98	0.87

Fact 9: The relative consumption volatility is higher in poor and emerging countries than in rich countries.

The Countercyclicity of Government Spending Increases With Income

Business-Cycle			
Statistic	Poor	Emerging	Rich
$\text{corr}(g/y, y)$	0.08	-0.08	-0.39

Fact 10: The share of government consumption is countercyclical in rich countries, but acyclical in emerging and poor countries.

To recap: Three important differences between poor and emerging and rich country business cycles:

- Poor and emerging countries are twice as volatile as rich countries (Fact 8).
- Poor and emerging countries have a higher relative consumption volatility than rich countries (Fact 9).
- In rich countries, the share of government consumption is more countercyclical than in emerging or poor countries. (Fact 10).

Country Size and Business Cycles

A Demographic Definition of Country Size

Countries are sorted into three size categories: small, medium, and large. These three categories are defined according to their population in 2009, as follows:

Small: population less than 20 million.

Medium: population between 20 and 80 million.

Large: population larger than 80 million.

The sample contains 77 small countries, 30 medium size countries, and 13 large countries.

Volatility of Output Controlling for Country Size and Income

	σ_y		
	Poor	Emerging	Rich
All	6.1%	8.7%	3.3%
Small	8.2%	9.5%	4.3%
Medium	9.5%	9.0%	3.1%
Large	5.6%	7.9%	3.3%

Fact 8 is robust to controlling for country size: poor and emerging economies are at least twice as volatile as rich economies.

Relative Volatility of Consumption, σ_c/σ_y , Controlling for Country Size and Income

	σ_c/σ_y		
	Poor	Emerging	Rich
All	1.1	0.98	0.87
Small	1.4	0.97	0.92
Medium	1.1	0.93	0.93
Large	1.1	1.1	0.84

Fact 9 is robust to controlling for country size: in poor and emerging countries consumption is relatively more volatile than in rich countries.

Are the ten business cycle facts robust to alternative detrending methods?

Consider two commonly used alternative detrending methods:

- (a) Hodrick-Prescott (HP) filtering.
- (b) First-differencing.

HP-Filtered Business Cycles

(a) The Hodrick and Prescott (1997) Filter

Given a time series y_t , for $t = 1, 2, \dots, T$, pick y_t^c and y_t^s to

$$\min_{\{y_t^c, y_t^s\}_{t=1}^T} \left\{ \sum_{t=1}^T (y_t^c)^2 + \lambda \sum_{t=2}^{T-1} \left[(y_{t+1}^s - y_t^s) - (y_t^s - y_{t-1}^s) \right]^2 \right\}$$

subject to $y_t^s + y_t^c = y_t$

where λ is a parameter.

As $\lambda \rightarrow \infty$, changes in the growth rate of y_t^s become infinitely costly, and the HP trend component converges to a linear trend.

As $\lambda \rightarrow 0$, the cycle disappears ($y^c = 0$), and the secular trend is the time series itself ($y_t^s = y_t$).

The Size of λ Matters

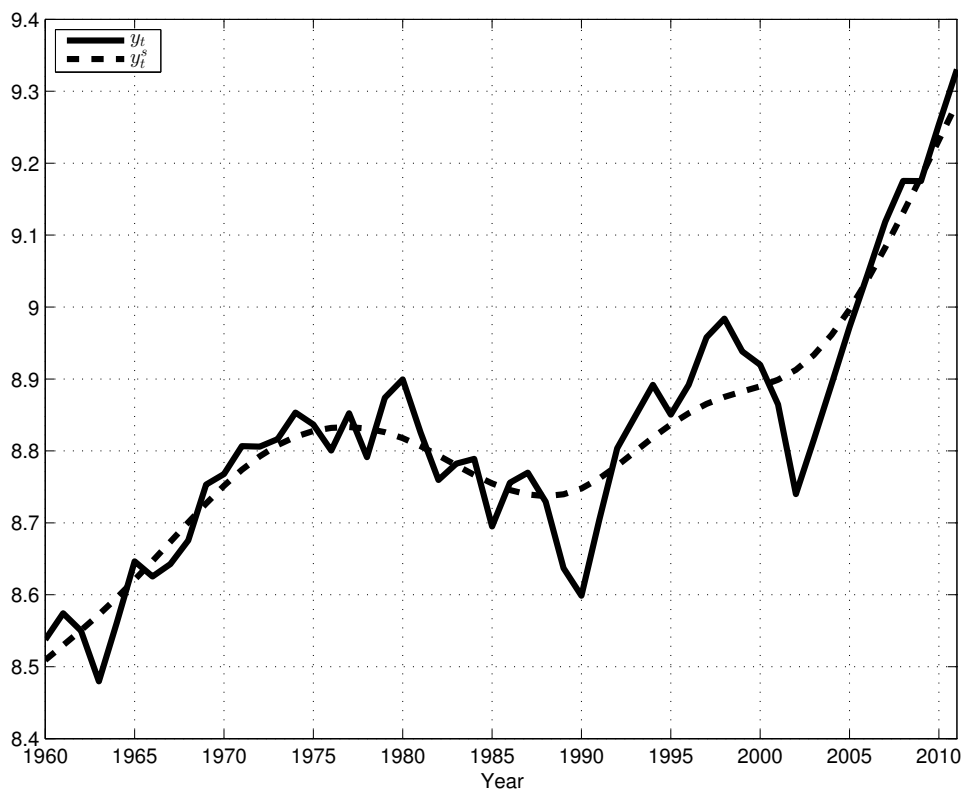
Annual data: $\lambda = 100$. But Ravn and Uhlig (2001) suggest $\lambda = 6.25$

Example: Argentina

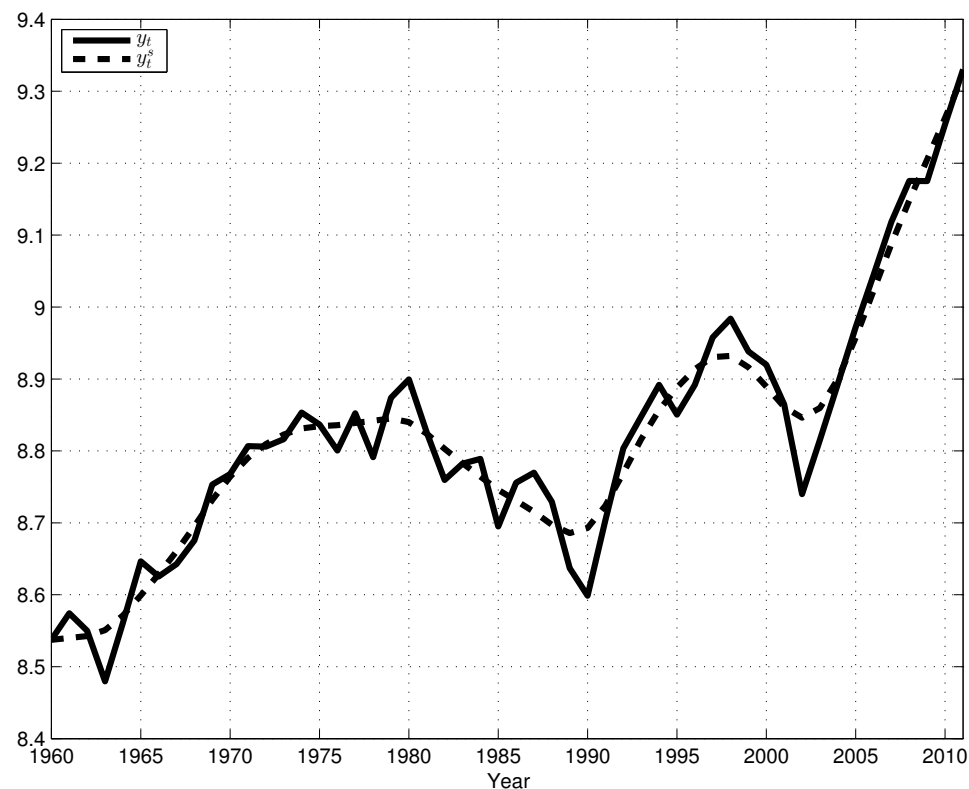
λ		
	100	6.25
σ_y	5.7%	3.6%

HP Filtered Trend of Argentine Output

($\lambda = 100$)



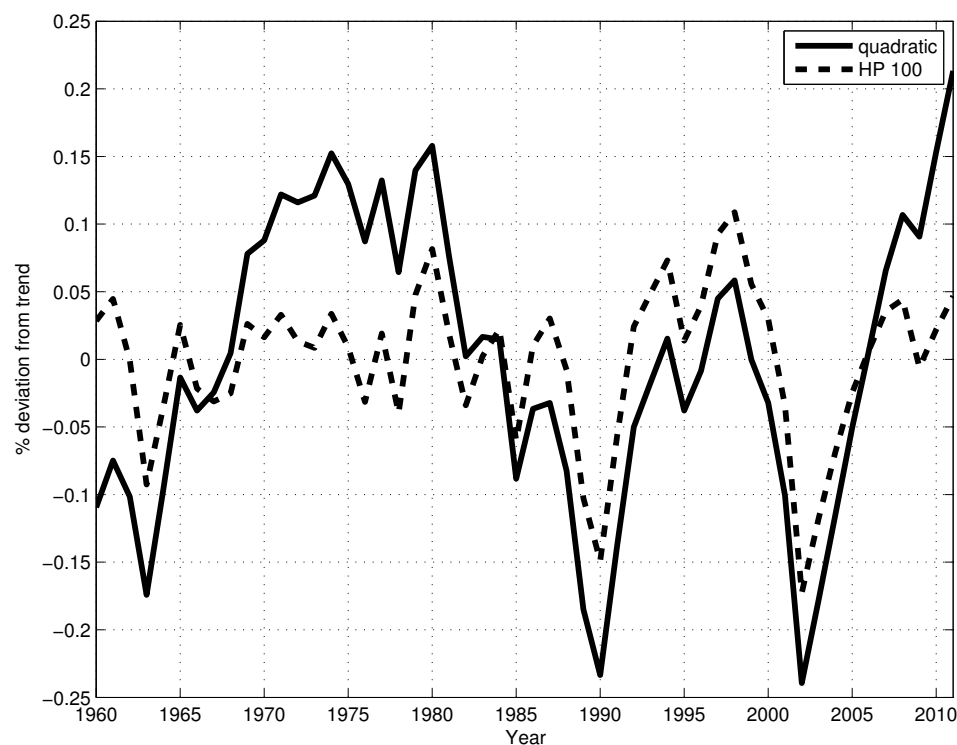
($\lambda = 6.25$)



HP-6.25 attributes bulk of the 1989 crisis and of the 2001 crisis to trend. But both were cyclical rather than secular for both were followed by rapid recovery. Thus, we will use $\lambda = 100$ for remainder of section.

Cyclical Component of Argentine Output:

HP 100 Filter Versus Quadratic Trend



High Global Volatility

Detrending Method	σ_y	
	World Average	USA
QT	6.2%	2.9%
HP	3.8%	2.0%

Fact 1: The cross-country average volatility of output is twice as large as its U.S. counterpart.

- Fact 1 continues to hold.
- Under HP filtering volatility falls by about 2/3.

Excess Volatility of Poor and Emerging Countries

Detrending Method	σ_y		
	Poor	Emerging	Rich
QT	6.1%	8.7%	3.3%
HP	4.1%	4.0%	2.0%

Fact 8: Business Cycles in rich countries are about half as volatile as business cycles in emerging or poor countries.

- Fact 8 continues to hold under HP filtering

Less Consumption Smoothing In Poor and Emerging Countries

Detrending Method	σ_c/σ_y		
	Poor	Emerging	Rich
QT	1.12	0.98	0.87
HP	1.09	1.23	0.87

Fact 9: The relative consumption volatility is higher in poor and emerging countries than in rich countries.

- Fact 9 continues to hold under HP Filtering

The countercyclicality of Government Spending Increases With Income

Detrending Method	corr($g/y, y$)		
	Poor	Emerging	Rich
QT	0.08	-0.08	-0.39
HP	0.02	-0.06	-0.56

Fact 10: The share of government consumption is countercyclical in rich countries, but acyclical in emerging and poor countries.

- Fact 10 continues to hold under HP filtering

Countercyclicity of the Trade-Balance-to-Output Ratio

$\text{corr}(tby, y)$	QT	HP
All	-0.15	-0.18
Poor	-0.11	-0.08
Emerging	-0.21	-0.34
Rich	-0.26	-0.37

Fact 5: The trade balance is negatively correlated with output.

- Fact 5 continues to hold under HP filtering

Summary of Comparison

- Quadratic detrending and HP(100) filtering result in largely the same business cycle facts.
- The main difference is that HP filtering implies that the volatilities of output and the components aggregate demand are about 2/3 of those implied by quadratic detrending.

(b) First-differenced data

$$\Delta y_t \equiv \ln Y_t - \ln Y_{t-1}$$

Statistic	All Countries	Poor Countries	Emerging Countries	Rich Countries
Standard Deviations				
$\sigma_{\Delta y}$	4.39	4.94	4.08	2.38
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.14	1.14	1.34	0.85
$\sigma_{\Delta g}/\sigma_{\Delta y}$	2.14	2.28	2.39	1.17
$\sigma_{\Delta i}/\sigma_{\Delta y}$	3.81	3.80	4.06	3.49
$\sigma_{\Delta x}/\sigma_{\Delta y}$	3.37	3.22	3.98	3.22
$\sigma_{\Delta m}/\sigma_{\Delta y}$	3.60	3.50	3.84	3.76
$\sigma_{tb/y}$	2.34	2.12	3.80	1.25
$\sigma_{ca/y}$	2.16	2.06	3.08	1.39
Correlations with Δy				
Δy	1.00	1.00	1.00	1.00
Δc	0.60	0.54	0.64	0.79
g/y	-0.10	-0.02	-0.18	-0.32
Δi	0.64	0.59	0.66	0.83
Δx	0.21	0.18	0.15	0.42
Δm	0.33	0.26	0.40	0.57
tb/y	-0.10	-0.08	-0.20	-0.07
ca/y	-0.07	-0.06	-0.12	-0.07
Serial Correlations				
Δy	0.29	0.28	0.29	0.32
Δc	0.02	-0.03	0.02	0.27
Δg	0.18	0.14	0.11	0.48
Δi	0.01	-0.01	0.03	0.08
Δx	0.07	0.08	-0.00	0.10
Δm	0.04	0.08	-0.02	-0.04
tb/y	0.61	0.59	0.62	0.69
ca/y	0.57	0.55	0.52	0.71

First-differenced Business Cycles

Note. The variables Δy , Δc , Δg , Δi , Δx , and Δm denote, respectively the log differences of output, consumption, government consumption, investment, exports, and imports. The variables g/y , tb/y , and ca/y are quadratically detrended in levels. All variables are expressed in percent.

Summary of Comparison

- QT, HP, and first-differencing result in largely the same business cycle facts!
- In particular, Facts 8 and 9 continue to hold: Emerging countries are about twice as volatile as rich countries, and emerging countries have a larger relative consumption volatility.

Business Cycle Facts with Quarterly Data

Main problem of quarterly data: not many long time series.

Include countries that have quarterly data on y , c , g , i , x , and m for at least 30 years. This requirement reduces the number of countries from 120 to **28!**

poor countries: 0 (out of 40)

emerging countries: 11 (out of 58)

rich countries: 17 (out of 22)

The sample period is 1980Q1 to 2012Q4. The data is online in file `usg_data_quarterly.xls` on the book's Web site. We collected this data from national statistical agencies, the OECD, IFS, and Eurostat. (It would be great if in the future some international organization, like the IMF, would provide a quarterly data base for more countries, especially, poor and emerging ones that goes back to the 1980s or even further!)

Business Cycles in Emerging and Rich Countries, Quarterly Data, 1980Q1-2012Q4

Statistic	Log-Quadratic Time Trend			HP Filter			First Differences		
	All	Emerging	Rich	All	Emerging	Rich	All	Emerging	Rich
Standard Deviations									
σ_y	3.26	4.27	2.74	1.80	2.60	1.38	1.12	1.70	0.81
σ_c/σ_y	0.99	1.23	0.87	1.01	1.32	0.85	1.18	1.48	1.03
σ_g/σ_y	1.46	2.07	1.15	1.30	2.02	0.93	2.07	3.33	1.41
σ_i/σ_y	3.44	3.67	3.31	3.73	3.88	3.65	4.32	4.95	3.99
σ_x/σ_y	3.77	3.97	3.67	4.01	3.80	4.11	4.38	4.65	4.25
σ_m/σ_y	3.52	3.55	3.51	4.44	3.65	4.84	4.60	4.26	4.77
σ_{tb}/σ_y	1.80	2.93	1.21	1.09	1.95	0.64	1.80	2.93	1.21
Correlations with y									
y	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
c	0.83	0.72	0.88	0.78	0.78	0.78	0.61	0.62	0.61
g/y	-0.43	-0.11	-0.59	-0.58	-0.22	-0.78	-0.16	-0.17	-0.15
i	0.86	0.82	0.88	0.84	0.77	0.87	0.65	0.57	0.70
x	0.17	-0.00	0.26	0.43	-0.05	0.67	0.33	0.04	0.48
m	0.60	0.48	0.66	0.68	0.52	0.76	0.44	0.37	0.47
tb/y	-0.44	-0.52	-0.41	-0.39	-0.56	-0.31	-0.02	-0.11	0.02
tb	-0.44	-0.51	-0.40	-0.39	-0.56	-0.31			

Notes: Moments are averaged across countries using population weights. Rich Countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Sweden, Switzerland, United Kingdom, United States. Emerging Countries: Argentina, Israel, South Korea, Mexico, New Zealand, Peru, Portugal, South Africa, Spain, Turkey, and Uruguay.

Observations on the table

In quarterly data, as in annual data,

- Business cycles in emerging countries are about twice as volatile as business cycles in rich countries. (Fact 8)
- There is less consumption smoothing in emerging countries than in rich countries. (Fact 9)
- The countercyclicality of government spending increases with income. (Fact 10)
- The trade balance is negatively correlated with output. (Fact 5)