Advanced Microeconomics, Spring 2021

Problem set 4, due June 4

1. There are two individuals in the economy, Mike and Harry. Mike is endowed with 90 units of good X and 10 units of good Y, while Harry is endowed with 10 units of good X and 90 units of good Y. Their utility functions are, respectively, $U^M(X,Y) = (X-20)(Y-10)$, and $U^H(X,Y) = 10(X-10)^{\frac{1}{2}}(Y-20)^{\frac{1}{2}}$.

   (a) Find Mike and Harry's demand functions, that is, X^M, Y^M, X^H, Y^H as a function of P_X, P_Y.

   (b) Find the excess demand function Z_X and Z_Y, and show that the Walras' law holds.

   (c) Solve the competitive equilibrium.

2. Suppose Jack and Tom have the endowment $\omega_J = (4,1)$ and $\omega_T = (1,4)$. Then consider the following six cases. In each case, state, in precise mathematical terms, the set of Pareto-efficient allocations, the core, the set of equilibrium price vectors and the corresponding set of equilibrium allocations. (Let good 1 be the numeraire, i.e. fix $P_1 = 1$)

   Leontief/Leontief

   $$U_J(x_{1J}, x_{2J}) = \min(x_{1J}/2, x_{2J})$$

   $$U_T(x_{1T}, x_{2T}) = \min(x_{1T}, x_{2T}/2).$$

   Leontief/Cobb-Douglas

   $$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

   Leontief/Linear

   $$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T} + 2x_{2T}.$$

   Cobb-Douglas/Cobb-Douglas

   $$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

   $$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

Cobb-Douglas/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

Linear/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J} + 3x_{2J}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

3. Compute the aggregate excess demand function Z in each of the following 6 examples, given that the endowments are $\omega_J = (5, 0)$ and $\omega_T = (0, 5)$. Show that your excess demand function Z is homogeneous of degree 0, satisfying Walras's Law.

   Leontief/Leontief

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = \min(x_{1T}, x_{2T}/2)/2.$$

   Leontief/Leontief

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J})$$

$$U_T(x_{1T}, x_{2T}) = \min(x_{1T}, x_{2T}).$$

   Leontief/Leontief

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + 2x_{2T}.$$

   Leontief/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

   Cobb-Douglas/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

   Cobb-Douglas/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/2} x_{2J}^{1/2}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{4/5} x_{2T}^{1/5}.$$