SOE of SUFE

Advanced Microeconomics II

Spring, 2021

Assignment 4

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4.1. There are two individuals in the economy, Mike and Harry. Mike is endowed with 90 units of good X and 10 units of good Y, while Harry is endowed with 10 units of good X and 90 units of good Y. Their utility functions are, respectively, $U^M(X,Y) = (X-20)(Y-10)$, and $U^H(X,Y) = 10(X-10)^{\frac{1}{2}}(Y-20)^{\frac{1}{2}}$.

(a) Find Mike and Harry's demand functions, that is, X^M, Y^M, X^H, Y^H as a function of P_X, P_Y.

(b) Find the excess demand function Z_X and Z_Y, and show that the Walras' law holds.

(c) Solve the competitive equilibrium.

Solution.

(a) Solving Mike's UMP, we have

$$\frac{Y^M - 10}{X^M - 20} = \frac{P_X}{P_Y}; \tag{1}$$

$$P_X X^M + P_Y Y^M = 90P_X + 10P_Y. \tag{2}$$

Combining (1) and (2), we obtain

$$X^M = 55, \qquad Y^M = 35\frac{P_X}{P_Y} + 10. \tag{3}$$

Solving Harry's UMP, we have

$$\frac{5(X^H-10)^{-\frac{1}{2}}(Y^H-20)^{\frac{1}{2}}}{5(X^H-10)^{\frac{1}{2}}(Y^H-20)^{-\frac{1}{2}}} = \frac{P_X}{P_Y}; \tag{4}$$

$$P_X X^H + P_Y Y^H = 10P_X + 90P_Y. \tag{5}$$

Combining (4) and (5), we obtain

$$X^H = 35\frac{P_Y}{P_X} + 10, \qquad Y^H = 55. \tag{6}$$

(3) and (6) express the demand functions.

(b) The excess demand functions:

$$Z_X = X^M + X^H - 100 = 35\frac{P_Y}{P_X} - 35; \tag{7}$$

$$Z_Y = Y^M + Y^H - 100 = 35\frac{P_X}{P_Y} - 35. \tag{8}$$

It is clear that Walras' law holds:

$$P_X Z_X + P_Y Z_Y = 35P_Y - 35P_X + 35P_X - 35P_Y = 0. \tag{9}$$

(c) Let $P_X = 1$. Then, solving $Z_X = Z_Y = 0$, we obtain $P_Y = 1$. With (3) and (6), we can find the C.E. as shown in Figure 1:

$$\Big\{(X^M, Y^M), (X^H, Y^H); (P_X, P_Y)\Big\} = \Big\{(55, 45), (45, 55); (1, 1)\Big\}. \tag{10}$$

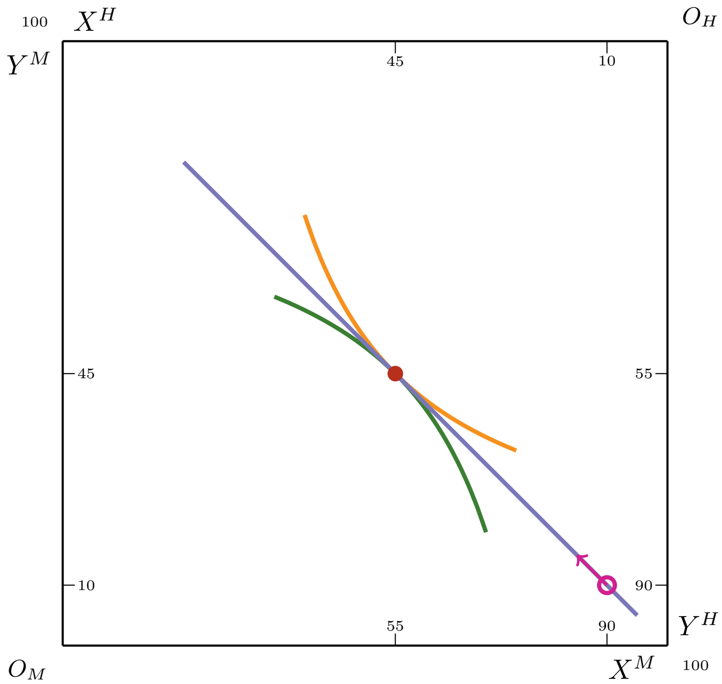

Figure 1: The Edgeworth Box with Mike and Harry

4.2. Suppose Jack and Tom have the endowment $\omega_J = (4, 1)$ and $\omega_T = (1, 4)$. Then consider the following six cases. In each case, state, in precise mathematical terms, the set of Pareto-efficient allocations, the core, the set of equilibrium price vectors and the corresponding set of equilibrium allocations (Let good 1 be the numeraire, i.e. fix $P_1 = 1$).

(a) Leontief/Leontief

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}/2, x_{2J})$$

$$U_T(x_{1T}, x_{2T}) = \min(x_{1T}, x_{2T}/2).$$

(b) Leontief/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

(c) Leontief/Linear

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + 2x_{2T}.$$

(d) Cobb-Douglas/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$
$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

(e) Cobb-Douglas/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$
$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

(f) Linear/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J} + 3x_{2J}$$
$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

Solution.

(a) The Edgeworth box can be drawn as Figure 2:

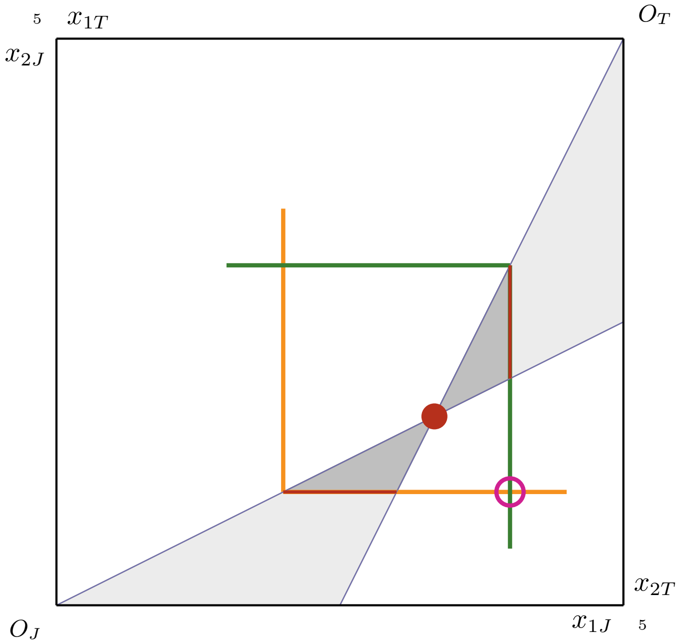

Figure 2: The Edgeworth Box - Leontief/Leontief

Define

$$C_J(\psi, \phi) = \left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} 2x_{2J} \le x_{1J} \le \dfrac{x_{2J}+5}{2},\ \psi \le x_{2J} \le \phi \\ x_{1T} = 5 - x_{1J},\ x_{2T} = 5 - x_{2J} \end{array} \right\},$$

$$C_T(\psi, \phi) = \left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} 2x_{1T} \le x_{2T} \le \dfrac{x_{1T}+5}{2},\ \psi \le x_{1T} \le \phi \\ x_{1J} = 5 - x_{1T},\ x_{2J} = 5 - x_{2T} \end{array} \right\}.$$

The set of P.E. allocations is (represented by the gray regions)

$$C_J(0, \frac{5}{3}) \cup C_T(0, \frac{5}{3}).$$

The core is (represented by the DARK gray regions)

$$C_J(1, \frac{5}{3}) \cup C_T(1, \frac{5}{3}).$$

The sets of equilibrium price vectors and the corresponding sets of equilibrium allocations (marked by the red) are

i.

$$\left\{(1, P_2) : P_2 = 0\right\},$$

and

$$C_T(1,1) = \left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = 4,\ x_{2J} \in [2,3], \\ x_{1T} = 1,\ x_{2T} = 5 - x_{2J} \end{array} \right\};$$

ii.

$$\left\{(1, P_2) : P_2 = 1\right\},$$

and

$$C_J(1, \frac{5}{3}) \cap C_T(1, \frac{5}{3}) = \left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = \frac{10}{3},\ x_{2J} = \frac{5}{3}, \\ x_{1T} = \frac{5}{3},\ x_{2T} = \frac{10}{3} \end{array} \right\};$$

iii. ‡

$$\left\{(1, P_2) : P_2 \to +\infty\right\},$$

and

$$C_J(1,1) = \left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} \in [2,3],\ x_{2J} = 1, \\ x_{1T} = 5 - x_{1J},\ x_{2T} = 4 \end{array} \right\}.$$

(b) The Edgeworth box can be drawn as Figure 3:

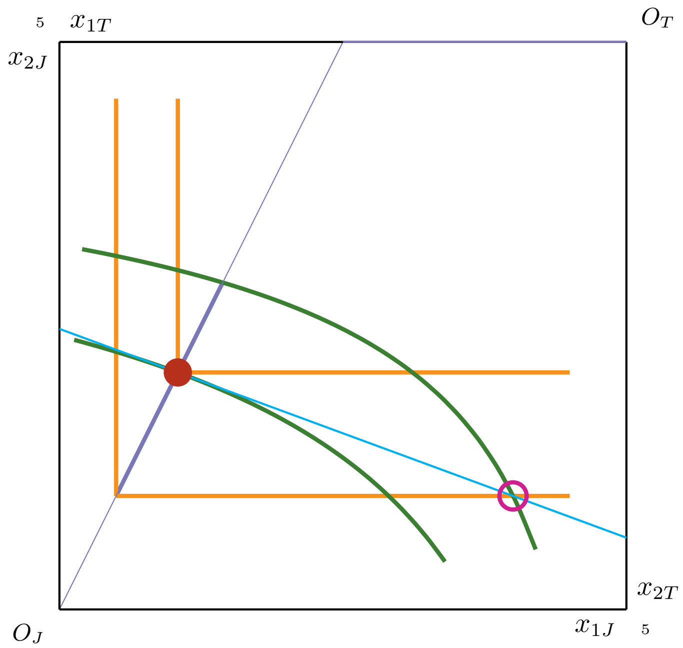

Figure 3: The Edgeworth Box - Leontief/Cobb-Douglas

‡Equivalently, let $P_1 = 0$ and $P_2 > 0$.

The set of P.E. allocations is (represented by the purple line segments, consisting of two parts)

$$\left\{\begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} \in [0,5],\ x_{2J} = \min\{2x_{1J}, 5\}, \\ x_{1T} = 5 - x_{1J},\ x_{2T} = 5 - x_{2J} \end{array}\right\}.$$

The core is (represented by the DARK purple line segment)

$$\left\{\begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} \in [0.5, \mu], \\ (5-2\mu)\sqrt{5-\mu} = 4, \\ x_{2J} = 2x_{1J}, \\ x_{1T} = 5 - x_{1J}, \\ x_{2T} = 5 - x_{2J} \end{array}\right\}.$$

The set of equilibrium price vectors is a singleton:

$$\left\{(1, P_2) : P_2 = \frac{21+\sqrt{505}}{16}\right\};$$

and the corresponding set of equilibrium allocations is a singleton as well:

$$\left\{\begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = \dfrac{35-\sqrt{505}}{12},\ x_{2J} = \dfrac{35-\sqrt{505}}{6}, \\ x_{1T} = \dfrac{25+\sqrt{505}}{12},\ x_{2T} = \dfrac{-5+\sqrt{505}}{6} \end{array}\right\}.$$

(c) The Edgeworth box can be drawn as Figure 4:

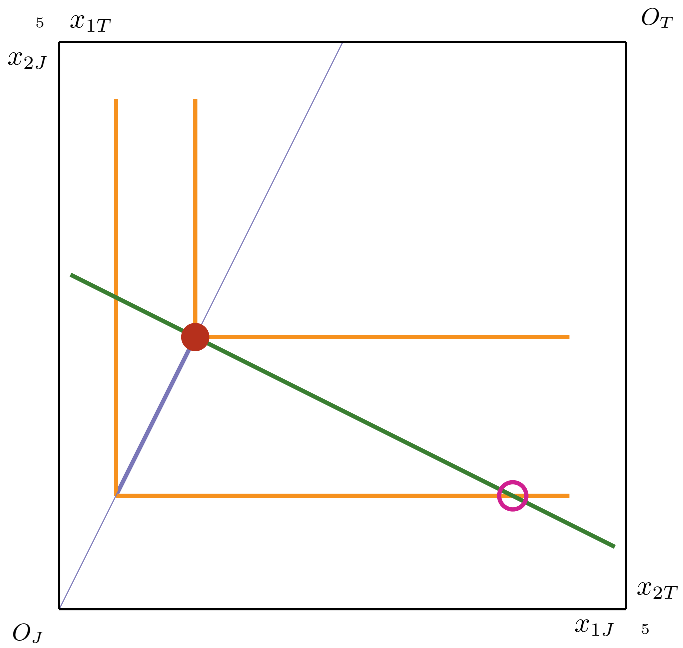

Figure 4: The Edgeworth Box - Leontief/Linear

The set of P.E. allocations is (represented by the purple line segment)

$$\left\{\begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} \in [0,2.5],\ x_{2J} = 2x_{1J}, \\ x_{1T} = 5 - x_{1J},\ x_{2T} = 5 - x_{2J} \end{array}\right\}.$$

The core is (represented by the DARK purple line segment)

$$\left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} \in [0.5, 1.2],\ x_{2J} = 2x_{1J}, \\ x_{1T} = 5 - x_{1J},\ x_{2T} = 5 - x_{2J} \end{array} \right\}.$$

The set of equilibrium price vectors is a singleton:

$$\left\{ (1, P_2) : P_2 = 2 \right\};$$

and the corresponding set of equilibrium allocations is a singleton as well:

$$\left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = 1.2,\ x_{2J} = 2.4, \\ x_{1T} = 3.8,\ x_{2T} = 2.6 \end{array} \right\}.$$

(d) The Edgeworth box can be drawn as Figure 5:

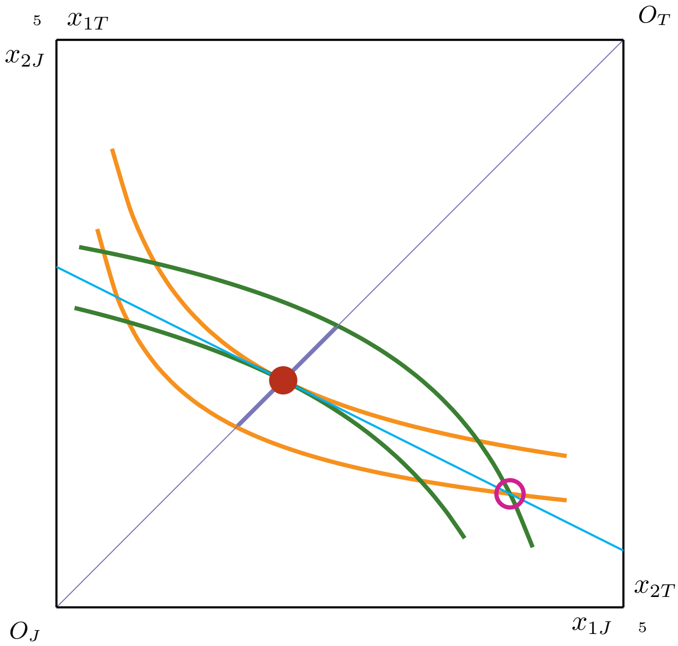

Figure 5: The Edgeworth Box - Cobb-Douglas/Cobb-Douglas

The set of P.E. allocations is (represented by the purple line segment)

$$\left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = x_{2J} \in [0, 5], \\ x_{1T} = 5 - x_{1J}, \\ x_{2T} = 5 - x_{2J} \end{array} \right\}.$$

The core is (represented by the DARK purple line segment)

$$\left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = x_{2J} \in [\sqrt[3]{4}, 5 - \sqrt[3]{16}], \\ x_{1T} = 5 - x_{1J}, \\ x_{2T} = 5 - x_{2J} \end{array} \right\}.$$

The set of equilibrium price vectors is a singleton:

$$\left\{(1, P_2) : P_2 = 2\right\};$$

and the corresponding set of equilibrium allocations is a singleton as well:

$$\left\{\begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = x_{2J} = 2, \\ x_{1T} = x_{2T} = 3 \end{array}\right\}.$$

(e) The Edgeworth box can be drawn as Figure 6:

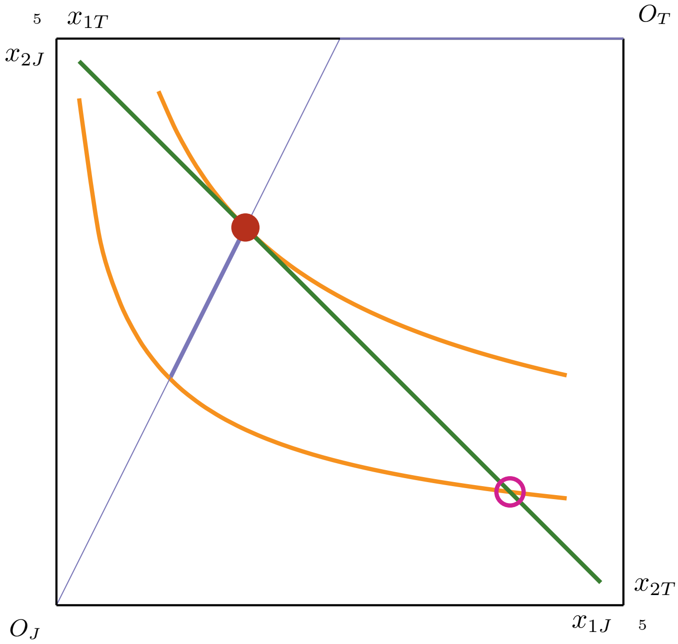

Figure 6: The Edgeworth Box - Cobb-Douglas/Linear

The set of P.E. allocations is (represented by the purple line segments, consisting of two parts)

$$\left\{\begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} \in [0, 5],\ x_{2J} = \min\{2x_{1J}, 5\}, \\ x_{1T} = 5 - x_{1J},\ x_{2T} = 5 - x_{2J} \end{array}\right\}.$$

The core is (represented by the DARK purple line segment)

$$\left\{\begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} \in \left[1, \frac{5}{3}\right],\ x_{2J} = 2x_{1J} \\ x_{1T} = 5 - x_{1J},\ x_{2T} = 5 - x_{2J} \end{array}\right\}.$$

The set of equilibrium price vectors is a singleton:

$$\left\{(1, P_2) : P_2 = 1\right\};$$

and the corresponding set of equilibrium allocations is a singleton as well:

$$\left\{\begin{bmatrix} x_{1J} & x_{1T} \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = \frac{5}{3},\ x_{2J} = \frac{10}{3}, \\ x_{1T} = \frac{10}{3},\ x_{2T} = \frac{5}{3} \end{array}\right\}.$$

(f) The Edgeworth box can be drawn as Figure 7:

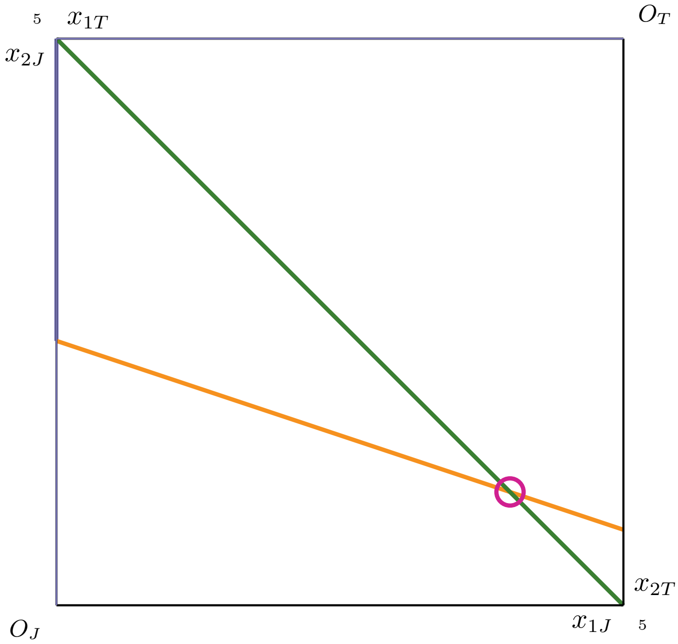

Figure 7: The Edgeworth Box - Linear/Linear

The set of P.E. allocations is (represented by the purple line segments, consisting of two parts)

$$\left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} \in [0,5], \\ x_{2J} = 5, \\ x_{1T} = 5 - x_{1J}, \\ x_{2T} = 0 \end{array} \right\} \bigcup \left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = 0, \\ x_{2J} \in [0,5], \\ x_{1T} = 5, \\ x_{2T} = 5 - x_{2J} \end{array} \right\}.$$

The core is (represented by the DARK purple line segment)

$$\left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = 0, \ x_{2J} \in \left[\frac{7}{3}, 5\right], \\ \\ x_{1T} = 5, \ x_{2T} = 5 - x_{2J} \end{array} \right\}.$$

The set of equilibrium price vectors is

$$\left\{ (1, P_2) : P_2 \in [1,3] \right\};$$

and the corresponding set of equilibrium allocations is the same as the core, i.e.

$$\left\{ \begin{bmatrix} x_{1J} & x_{1T} \\ \\ x_{2J} & x_{2T} \end{bmatrix} : \begin{array}{c} x_{1J} = 0, \ x_{2J} \in \left[\frac{7}{3}, 5\right], \\ \\ x_{1T} = 5, \ x_{2T} = 5 - x_{2J} \end{array} \right\}.$$

4.3. Compute the aggregate excess demand function Z in each of the following 6 examples, given that the endowments are $\omega_J = (5,0)$ and $\omega_T = (0,5)$. Show that your excess demand function Z is homogeneous of degree 0, satisfying Walras's Law.

(a) Leontief/Leontief

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = \min(x_{1T}, x_{2T}/2)/2.$$

(b) Leontief/Leontief

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J})$$

$$U_T(x_{1T}, x_{2T}) = \min(x_{1T}, x_{2T}).$$

(c) Leontief/Linear

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + 2x_{2T}.$$

(d) Leontief/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

(e) Cobb-Douglas/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

(f) Cobb-Douglas/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/2} x_{2J}^{1/2}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{4/5} x_{2T}^{1/5}.$$

Proof. Note that Walras' Law will be just applied or satisfied when solving the UMPs with these locally nonsatiated preferences.

(a) Let $x_{2J} = 2x_{1J}, x_{2T} = 2x_{1T}, P_1 x_{1J} + P_2 x_{2J} = 5P_1, P_1 x_{1T} + P_2 x_{2T} = 5P_2$. Then

$$x_{1J} = \frac{5P_1}{P_1 + 2P_2}, \; x_{1T} = \frac{5P_2}{P_1 + 2P_2}, \; x_{2J} = \frac{10P_1}{P_1 + 2P_2}, \; x_{2T} = \frac{10P_2}{P_1 + 2P_2}.$$

Thus

$$Z_1(P) = \frac{5P_1}{P_1 + 2P_2} + \frac{5P_2}{P_1 + 2P_2} - 5 = \frac{-5P_2}{P_1 + 2P_2},$$
$$Z_2(P) = \frac{10P_1}{P_1 + 2P_2} + \frac{10P_2}{P_1 + 2P_2} - 5 = \frac{5P_1}{P_1 + 2P_2}.$$

It is clear that $PZ(P) = 0$ where

$$Z(P) = \begin{bmatrix} Z_1(P) \\ Z_2(P) \end{bmatrix}$$

is homogeneous of degree 0, i.e. $Z(\alpha P) = Z(P), \forall \alpha > 0$.

(b) Let $x_{2J} = x_{1J}, x_{2T} = x_{1T}, P_1 x_{1J} + P_2 x_{2J} = 5P_1, P_1 x_{1T} + P_2 x_{2T} = 5P_2$. Then

$$x_{1J} = \frac{5P_1}{P_1 + P_2},\ x_{1T} = \frac{5P_2}{P_1 + P_2},\ x_{2J} = \frac{5P_1}{P_1 + P_2},\ x_{2T} = \frac{5P_2}{P_1 + P_2}.$$

Thus

$$Z_1(P) = \frac{5P_1}{P_1 + P_2} + \frac{5P_2}{P_1 + P_2} - 5 = 0,$$
$$Z_2(P) = \frac{5P_1}{P_1 + P_2} + \frac{5P_2}{P_1 + P_2} - 5 = 0.$$

It is clear that $PZ(P) = 0$ where

$$Z(P) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

is homogeneous of degree 0.

(c) Let $x_{2J} = 2x_{1J}, P_1 x_{1J} + P_2 x_{2J} = 5P_1$. Then

$$x_{1J} = \frac{5P_1}{P_1 + 2P_2},\ x_{2J} = \frac{10P_1}{P_1 + 2P_2}.$$

Solving Tom's UMP

$$\max\ x_{1T} + 2x_{2T} \quad \text{s.t. } P_1 x_{1T} + P_2 x_{2T} = 5P_2,$$

we can find:

i. $x_{1T} = 0, x_{2T} = 5$ if $\frac{P_1}{P_2} > \frac{1}{2}$;
ii. $x_{1T} = \frac{5P_2}{P_1}, x_{2T} = 0$ if $\frac{P_1}{P_2} < \frac{1}{2}$;
iii. $(x_{1T}, x_{2T}) \in \big\{(x_{1T}, x_{2T}) : P_1 x_{1T} + P_2 x_{2T} = 5P_2,\ x_{1T} \in [0, 5]\big\}$ if $\frac{P_1}{P_2} = \frac{1}{2}$.

Therefore,

i. when $\frac{P_1}{P_2} > \frac{1}{2}$,

$$Z_1(P) = \frac{5P_1}{P_1 + 2P_2} - 5 = \frac{-10P_2}{P_1 + 2P_2}, \quad Z_2(P) = \frac{10P_1}{P_1 + 2P_2};$$

ii. when $\frac{P_1}{P_2} < \frac{1}{2}$,

$$Z_1(P) = \frac{5P_1}{P_1 + 2P_2} + \frac{5P_2}{P_1} - 5 = \frac{-5P_1P_2 + 10P_2^2}{P_1^2 + 2P_1P_2},$$
$$Z_2(P) = \frac{10P_1}{P_1 + 2P_2} - 5 = \frac{5P_1 - 10P_2}{P_1 + 2P_2};$$

iii. when $\frac{P_1}{P_2} = \frac{1}{2}$,

$$Z_1(P) = \frac{5P_1}{P_1 + 2P_2} + x_{1T} - 5 = x_{1T} - 4 = 6 - 2x_{2T},$$
$$Z_2(P) = \frac{10P_1}{P_1 + 2P_2} + x_{2T} - 5 = x_{2T} - 3, \quad x_{2T} \in [0, 5].$$

It is clear that, for any case of P, $PZ(P) = 0$ where

$$Z(P) = \begin{bmatrix} Z_1(P) \\ Z_2(P) \end{bmatrix}$$

is homogeneous of degree 0.

(d) Let $x_{2J} = 2x_{1J}, P_1 x_{1J} + P_2 x_{2J} = 5P_1$. Then

$$x_{1J} = \frac{5P_1}{P_1 + 2P_2},\ x_{2J} = \frac{10P_1}{P_1 + 2P_2}.$$

According to C-D function's property, we have the UMP solution:

$$x_{1T} = \frac{5P_2}{3P_1},\ x_{2T} = \frac{10P_2}{3P_2} = \frac{10}{3}.$$

Thus

$$Z_1(P) = \frac{5P_1}{P_1 + 2P_2} + \frac{5P_2}{3P_1} - 5 = \frac{-25P_1P_2 + 10P_2^2}{3P_1^2 + 6P_1P_2},$$
$$Z_2(P) = \frac{10P_1}{P_1 + 2P_2} + \frac{10}{3} - 5 = \frac{25P_1 - 10P_2}{3P_1 + 6P_2}.$$

It is clear that $PZ(P) = 0$ where

$$Z(P) = \begin{bmatrix} Z_1(P) \\ Z_2(P) \end{bmatrix}$$

is homogeneous of degree 0.

(e) According to C-D function's property, we have the UMP solutions:

$$x_{1J} = \frac{5P_1}{3P_1} = \frac{5}{3},\ x_{2J} = \frac{10P_1}{3P_2},\ x_{1T} = \frac{5P_2}{3P_1},\ x_{2T} = \frac{10P_2}{3P_2} = \frac{10}{3}.$$

Thus

$$Z_1(P) = \frac{5}{3} + \frac{5P_2}{3P_1} - 5 = \frac{-10P_1 + 5P_2}{3P_1},$$
$$Z_2(P) = \frac{10P_1}{3P_2} + \frac{10}{3} - 5 = \frac{10P_1 - 5P_2}{3P_2}.$$

It is clear that $PZ(P) = 0$ where

$$Z(P) = \begin{bmatrix} Z_1(P) \\ Z_2(P) \end{bmatrix}$$

is homogeneous of degree 0.

(f) According to C-D function's property, we have the UMP solutions:

$$x_{1J} = \frac{5P_1}{2P_1} = \frac{5}{2},\ x_{2J} = \frac{5P_1}{2P_2},\ x_{1T} = \frac{20P_2}{5P_1} = \frac{4P_2}{P_1},\ x_{2T} = \frac{5P_2}{5P_2} = 1.$$

Thus

$$Z_1(P) = \frac{5}{2} + \frac{4P_2}{P_1} - 5 = \frac{-5P_1 + 8P_2}{2P_1},$$
$$Z_2(P) = \frac{5P_1}{2P_2} + 1 - 5 = \frac{5P_1 - 8P_2}{2P_2}.$$

It is clear that $PZ(P) = 0$ where

$$Z(P) = \begin{bmatrix} Z_1(P) \\ Z_2(P) \end{bmatrix}$$

is homogeneous of degree 0.

□