Advanced Microeconomics, Spring 2024

Problem set 1, due March 21

1. Consider the following simplified version of poker. There are two players and a deck of three cards–A, K and Q of spades. Each player is randomly dealt one hand. Each player sees her own card but not that of her opponent and can either bet \$10 or "fold". If one or bother players fold, no money changes hands. If both players bet, the player with the higher card wins, so that she wins \$10 in net terms.

   A pure strategy for a player is a rule that indicates whether to bet or to fold conditional on the card held.

   (a) Informally but carefully ARGUE that some of each player's strategies are weakly dominated. (Specify which ones, that is, and why they are weakly dominated.)

   (b) Informally but carefully DERIVE the result of continuing the process of iterated elimination of weakly dominated strategies as far as possible.

   (c) Suppose the deck of cards is all 13 spades–A, K,. . ., 2. Each player still gets a single card, dealt at random. Answer (b) for this new situation.

2. For the 3-player game shown below:

Player 3 plays L

		Player 2	
		x	y
Player 1	A	(0, 0, 3)	(0, 0, 3)
	B	(-4, 0, 1)	(-4, 0, 1)
	C	(1, 1, -2)	(-4, 0, 1)
	D	(1, 1, -2)	(0, 0, 3)

Player 3 plays R

	Player 2	
	x	y
A	(0, 0, 3)	(0, 0, 3)
B	(3, 0, 1)	(3, 0, 1)
C	(5, 4, 0)	(3, 0, 1)
D	(5, 4, 0)	(0, 0, 3)

   (a) Find all pure strategy NE;

   (b) Among the NE from (a), identify those involving players playing dominated strategies;

   (c) Show there exists NE which involves no players playing dominated strategy but nevertheless is not normal-form perfect.

3. (Mixed strategy) Show that the two-player game illustrated below has a unique equilibrium in pure strategies and that there is no additional equilibrium in mixed strategies.

	L	M	R
U	$(1,-2)$	$(-2,1)$	$(0,0)$
M	$(-2,1)$	$(1,-2)$	$(0,0)$
D	$(0,0)$	$(0,0)$	$(1,1)$

4. Consider the following majority voting game. There are three congressmen. Their names are A, B and C. There are three possible alternatives to vote for: X, Y, and Z. The alternative that gets the majority of votes wins. In case that all alternatives receive the same vote, then each one is chosen with equal probability. The payoffs of the congressmen depend exclusively on which alternative is chosen through voting. Their payoffs are as follows:

$$U_A(X) = U_B(Y) = U_C(Z) = 2;$$
$$U_A(Y) = U_B(Z) = U_C(X) = 1;$$
$$U_A(Z) = U_B(X) = U_C(Y) = 0.$$

Determine whether there are any Nash equilibria in pure strategies for this game. (You may find more than one Nash equilibrium).

5. (Hotelling) Suppose that we have two firms located on a line of length 1. The unit costs of the good for each store is c. Consumers incur a transportation cost of tx^2 for a length of x. Consumers have unit demands, and are uniformly distributed along the line. Firm 1 is located at point $a \geq 0$ and firm 2 at point $1-b$, where $b \geq 0$ and without loss of generality, $1-a-b \geq 0$ (firms 1 is to the left of firms 2; $a = b = 0$ corresponds to maximal differentiation and $a + b = 1$ corresponds to minimal differentiation, i.e. perfect substitutes). Assumes that the market is covered and firms sell positive quantities.

(a) Show that given a and b, firm 1 will charge

$$p_1 = c + t(1-a-b)(1 + \frac{a-b}{3})$$

whereas firm 2 will charge

$$p_2 = c + t(1-a-b)(1 + \frac{b-a}{3}).$$

(b) Find firms' market shares of the market.

(c) Now consider a first stage to this game where the two firms choose their locations, knowing that prices will be chosen in the second stage as in (a). Where do they locate?

(d) What would be socially optimal locations of the two firms? Compare with the market outcome.