MICROECONOMIC THEORY II

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- Adverse selection is rather a possible consequence of this asymmetric information.

Insider trading: Ivan Boesky

Michael Milken

The problem for market-makers

Consider a market-maker trading stocks of a given company by offering a buy price of $b = \$54.5$ and a sell price of $a = \$55.50$. If the buy and sell order flow generated by liquidity takers was approximately balanced, then the market-maker would earn $s = \$1.00$ per round-trip trade. If, however, an insider knows that the given company is about to announce a drop in profits, they will revise their private valuations of the stock downwards, say to \$50.00. If the market-maker continues to offer the same quotes, then he or she would experience a huge influx of sell orders from insiders, who regard selling the stock at \$54.50 to be extremely attractive. The market-maker would therefore quickly accumulate a large net buy position by purchasing more and more stocks at the price \$54.50, which will likely be worth much less soon after, generating a huge loss. This is adverse selection.

—Trades, Quotes and Prices, Financial Markets under the Microscope

Santa Fe Insider Trading

"One famous example involved Santa Fe, an oil company that was a takeover target by the Kuwaitis in 1981. At the time, the stock was at $25 and the option traders on the floor filled an order for one thousand 35 calls at $1/16. Shortly afterwards, the stock jumped from $25 to $ 45 and the options went from $1/16 to $10. The floor traders had a virtual overnight loss of about $1 million. Although they eventually got their money back, it took years. If you're a market maker and you're broke, waiting to get your capital back is not pleasant. You live in fear that you're going to be the one selling the option to an informed source."
—-The New Market Wizard

Stock market: cost of ignorance

While market makers have far more information than mom-and-pop investors, they're often outgunned by more sophisticated traders, hedge fund aces or Warren Buffett types. Making a market for such traders can be hazardous. They may know something the market maker doesn't, such as a likelihood that Intel is going to come out with blowout earnings or Sears is going to put up horrendous sales numbers. Stepping in front of such orders can mean big losses.

In response to the chance of getting winged by a well – armed gunslinger, market makers typically widen their quotes, providing a lower bid or higher offer.

The result: wider spreads.

Jeff Yass on insider trading and bid/ask spread

"The more successful the SEC is in catching people trading on inside information–and lately they seem to be catching everyone–the tighter the bid/ask spreads will be. Every trade we do involves some risk premium for the possibility that the other side of the trade represents informed activity.' Therefore, if everyone believes that the SEC is going to catch all inside traders, then the market will price away that extra risk premium. In essence, it's really the average investor who ends up paying for insider trading through the wider bid/ask spreads."
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  - Insurance companies: identical and offer full insurance at price p
- Symmetric information, Zero-profit condition

$$p_i = \pi_i L \quad \forall i.$$

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- Competitive equilibrium price under asymmetric information

$$p^* = E(\pi | \pi \geq h(p^*))L,$$

$$E(\pi | \pi \geq h(p^*)) = \frac{\int_{h(p^*)}^{\bar{\pi}} \pi dF(\pi)}{1 - F(h(p^*))}$$

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- Only consumer that is certain to have an accident buy the insurance.


BUY
FOR
SALE
2001 GMC SIERRA
160,xxx miles

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  2. Group two: total income Y_2 and

$$u_2 = M + \sum_{i=1}^{n} \frac{3x_i}{2}$$

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- Equilibrium

$$p = \begin{cases} 1, & \text{if } Y_2 < N \\ \frac{Y_2}{N} & \text{if } \frac{2Y_2}{3} < N < Y_2 \\ \frac{3}{2} & \text{if } N < \frac{2Y_2}{3} \end{cases}$$

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- Average quality supplied

$$\mu = \frac{p}{2}.$$

Asymmetric information (2)

- Total demand

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- NO trade in equilibrium, even if at *any given price* $p \in [0,3]$, there are group one trader willing to sell at a price which group two are willing to pay.

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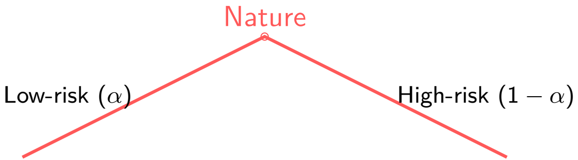
  - ➢ Insurance company (*Receiver*) responds given belief $\beta(B, p)$: accept, reject.

Insurance signaling game



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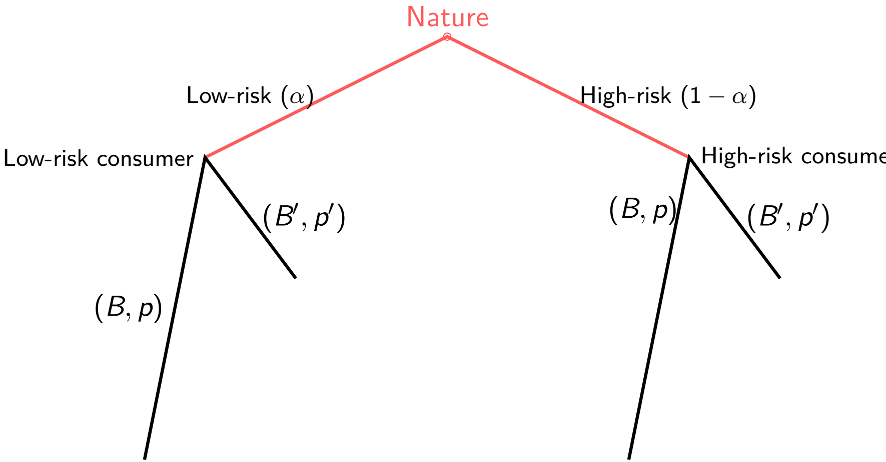

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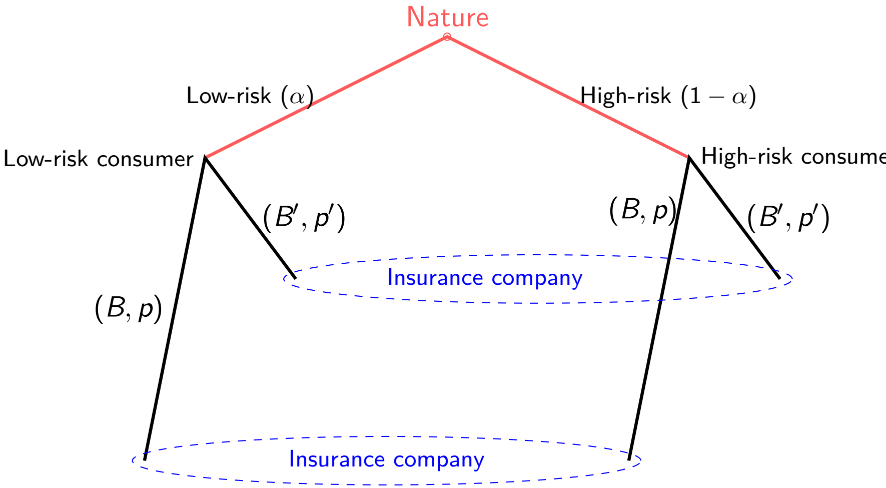

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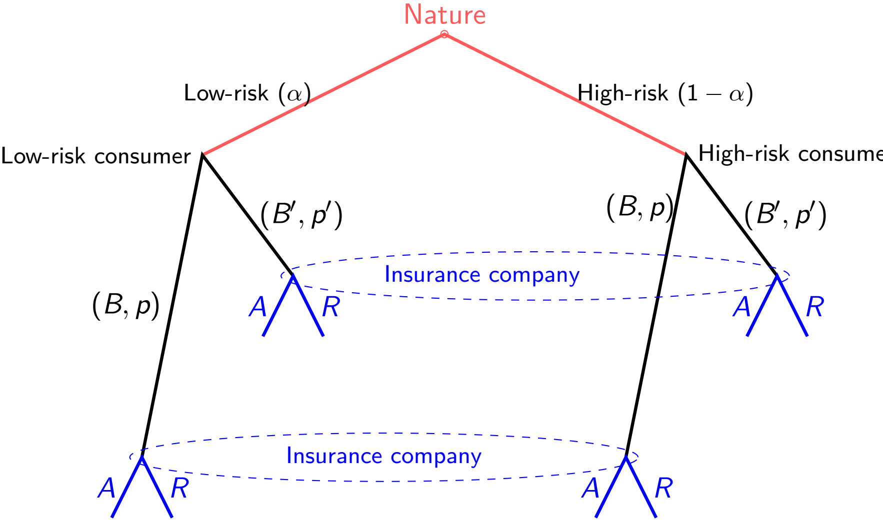

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Consumer optimization review

- Individual's optimal insurance problem:

$$\max_B \quad \pi u(w - L + B(1-q)) + (1-\pi) u(w - Bq)$$
$$s.t.\ B \geq 0,\ B \leq w/q$$

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- The first-order conditions:

$$\pi u'\left(w - L + B\left(1-q\right)\right)\left(1-q\right) - \left(1-\pi\right) u'\left(w - Bq\right) q - \lambda \leq 0;$$
$$\lambda\left(B - w/q\right) = 0, \lambda \geq 0, B \geq 0.$$

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$$\lambda\left(B-w/q\right)=0,\lambda\geq 0,B\geq 0.$$

- Thus, the optimal B satisfies

$$\frac{\pi u'\left(w-L+B\left(1-q\right)\right)}{\left(1-\pi\right)u'\left(w-Bq\right)}=\frac{q}{1-q}$$

Graphical illustration

Single crossing property

On consumer choices

- Note that $P = Bq$ and $B(1-q) = B - P$, so

$$MRS(B,P) = \frac{\pi u'\left(w - L + B - P\right)}{\pi u'(w - L + B - P) + (1-\pi)\, u'\left(w - P\right)} = \frac{P}{B} = q.$$

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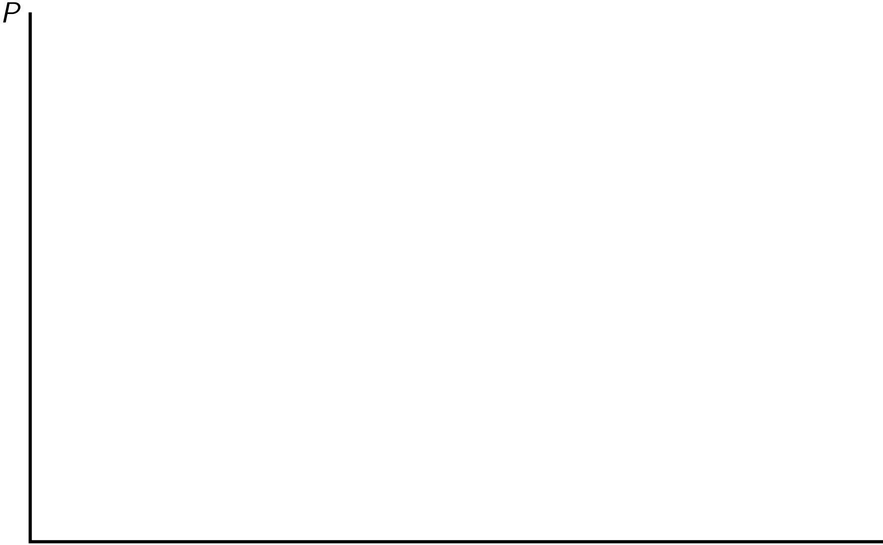
Insurance company's problem



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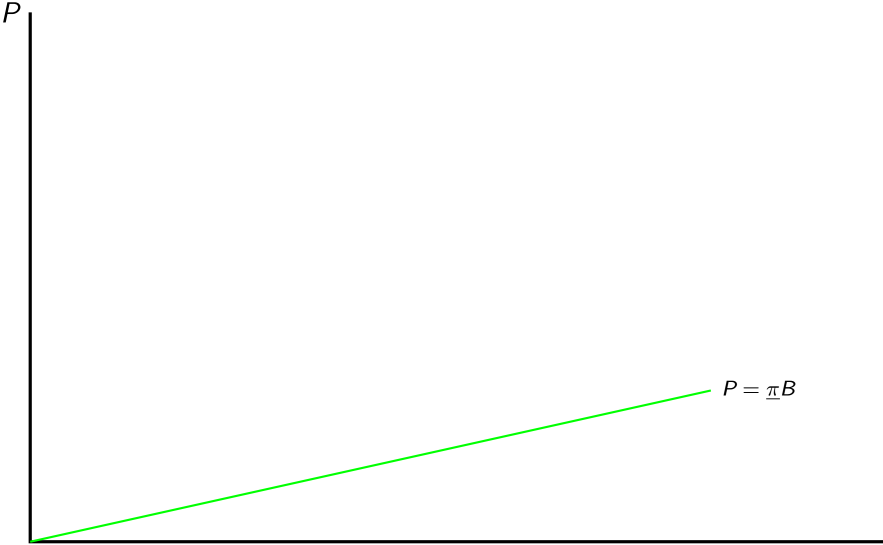

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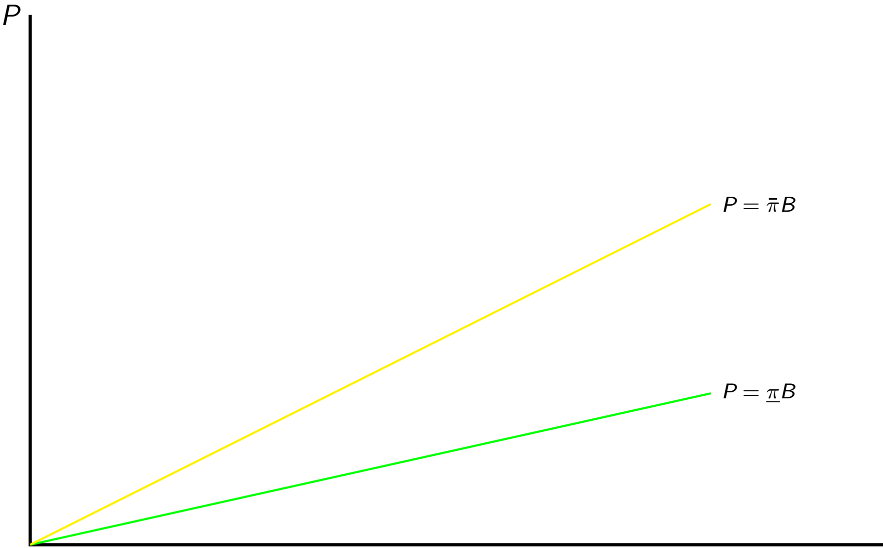
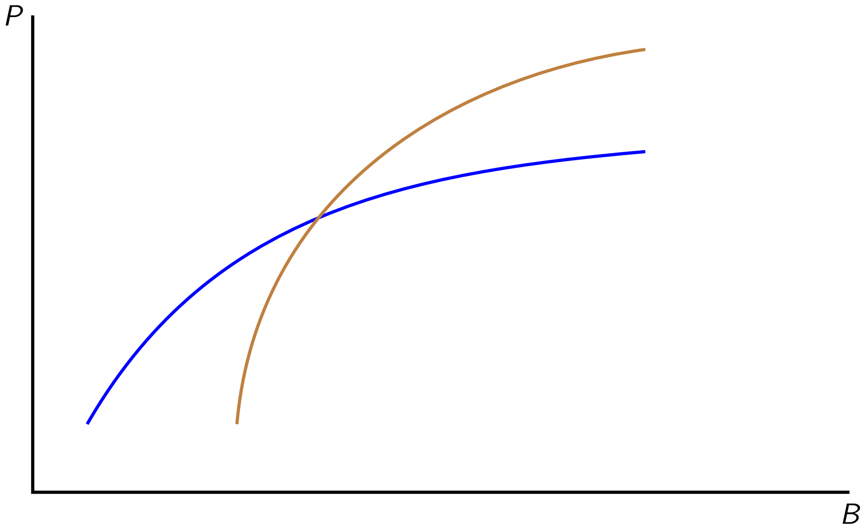


Consumers' preferences for risks



Equilibrium

- Lemma 8.1. (Jehle & Reny) Let

$$\tilde{u}_l \equiv \max_{(B,p)} u_l(B,P) \quad s.t. p = \bar{\pi} B \leq w, \qquad u_h^c \equiv u_h(L, \bar{\pi} L).$$

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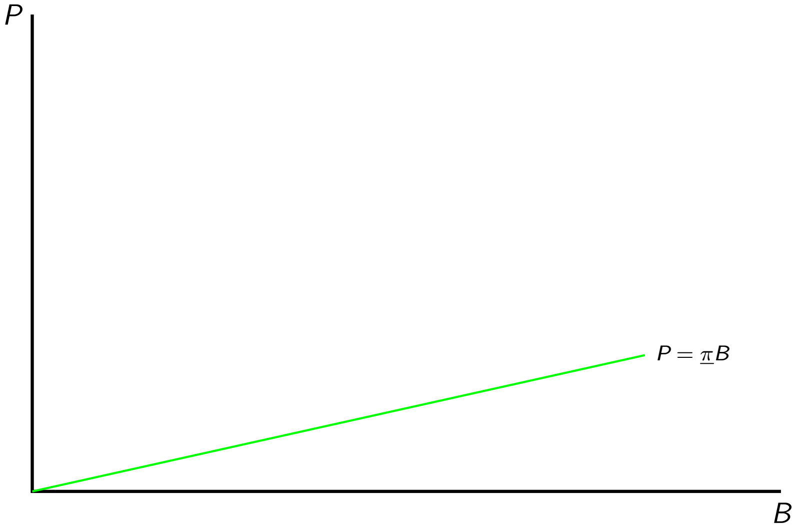
Existence of separating equilibrium



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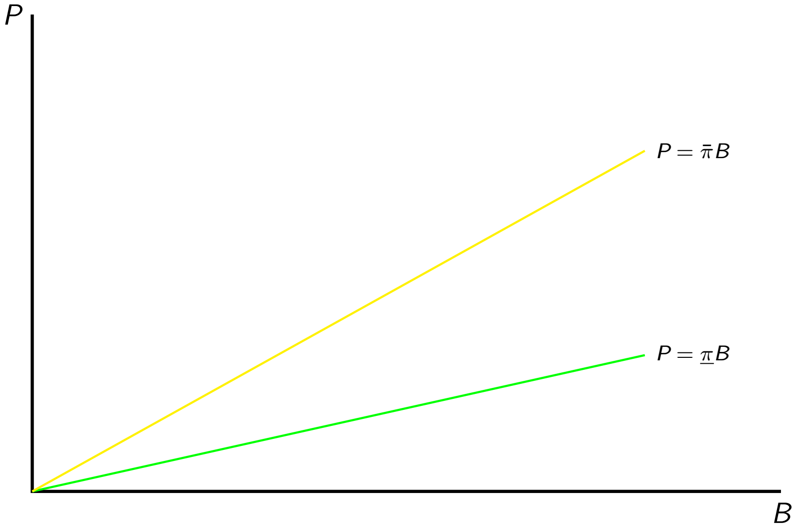

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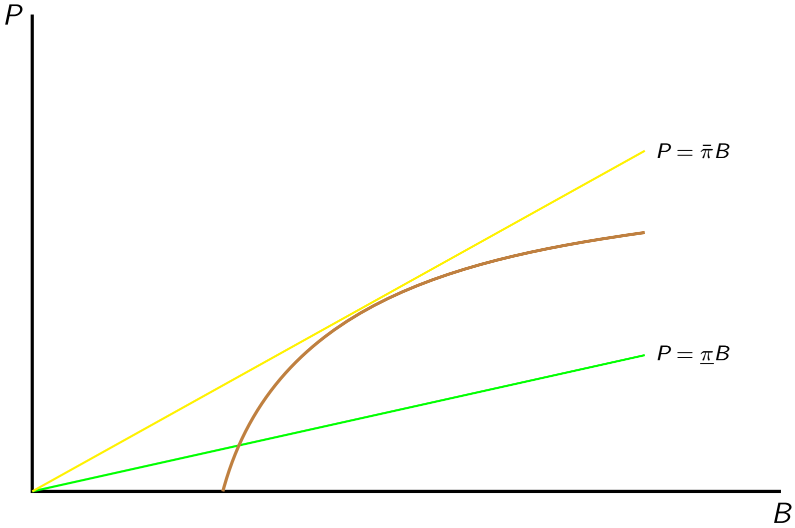

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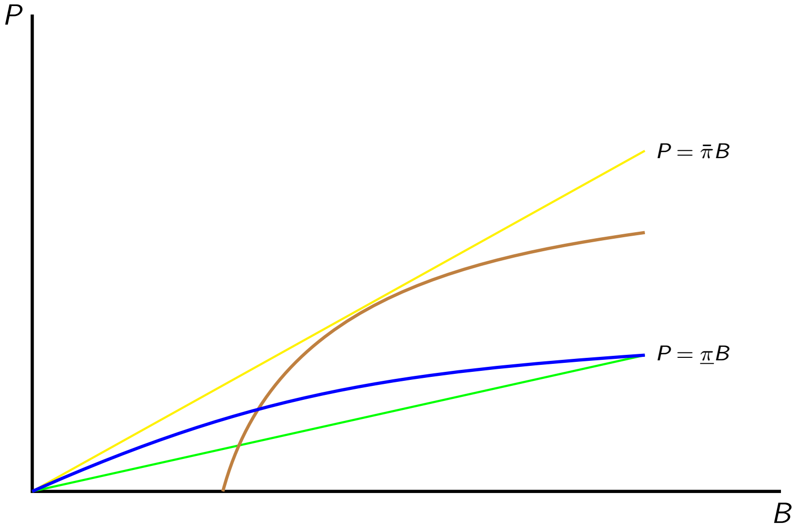

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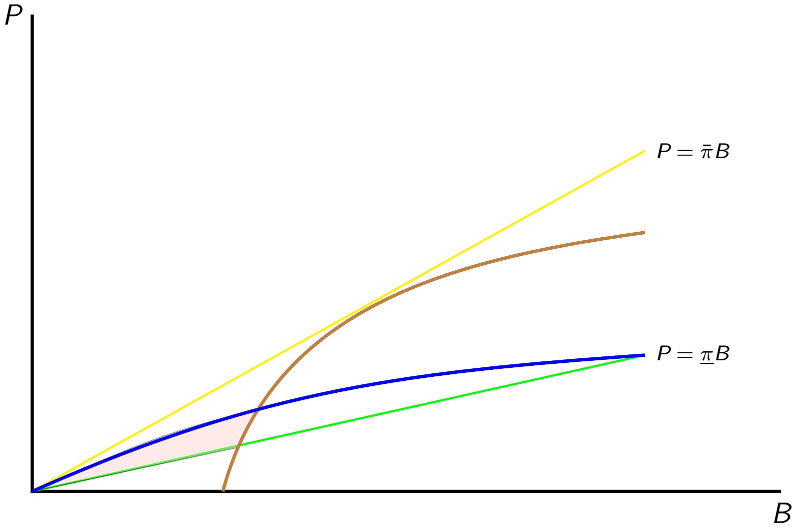

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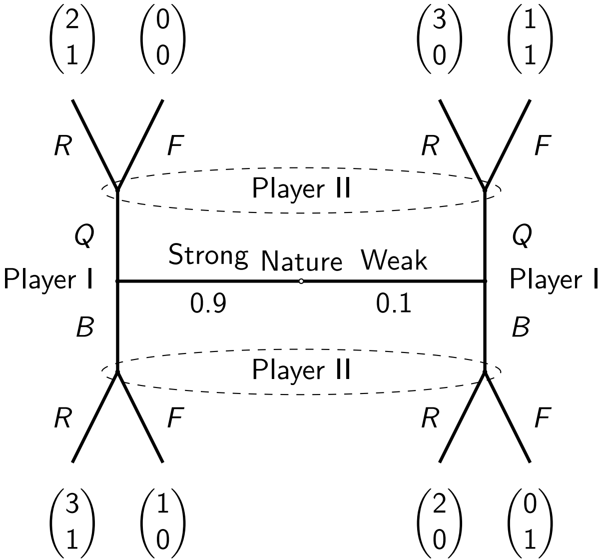
Intuitive criterion

- Briefly, a sequential equilibrium satisfies Intuitive Criterion if no type of sender could obtain a payoff higher than his equilibrium payoff were he to choose a nonequilibrium message and the receiver responds with an optimal reply to the belief that imputes zero probability to Nature's choice of those types that can not gain from such a deviation regardless of the receiver's responses.

Beer-Quiche game studied by Cho and Kreps



Beer-Quiche (2)

- The strategic form

II

	FF	FR	RF	RR
I BB	0.9, 0.1	0.9, 0.1	2.9, 0.9	2.9, 0.9
I BQ	1, 0.1	1.2, 0	2.8, 1	3, 0.9
I QB	0, 0.1	1.8, 1	0.2, 0	2, 0.9
I QQ	0.1, 0.1	2.1, 0.9	0.1, 0.1	2.1, 0.9

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- If the strategy profile is (QB, FF), player I sends signal Q if weak, and sends signal B if strong.
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- The rest payoffs can be obtained similarly

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- Once we impose this reasonable restriction, the equilibrium does not survive!

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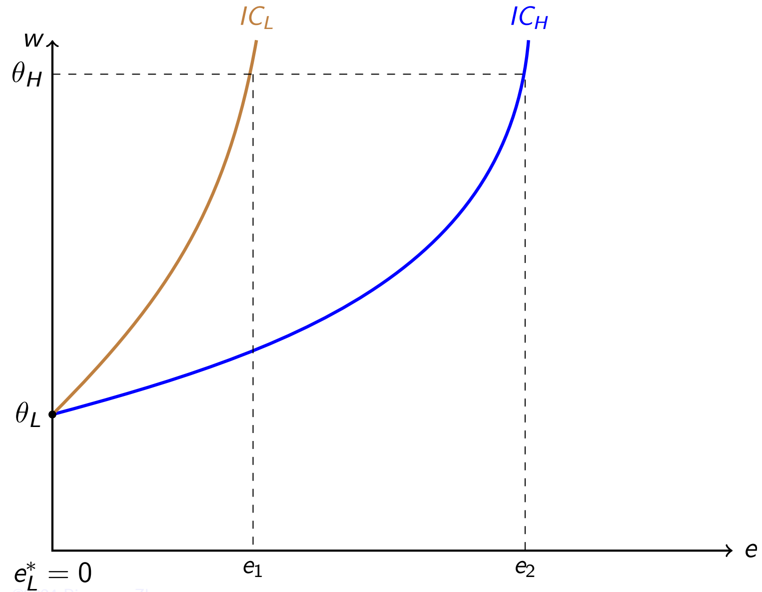
- Worker's utility

$$u(w, e) = w - \frac{e}{2\theta}$$

if accepts offer; zero otherwise.

Separating equilibrium



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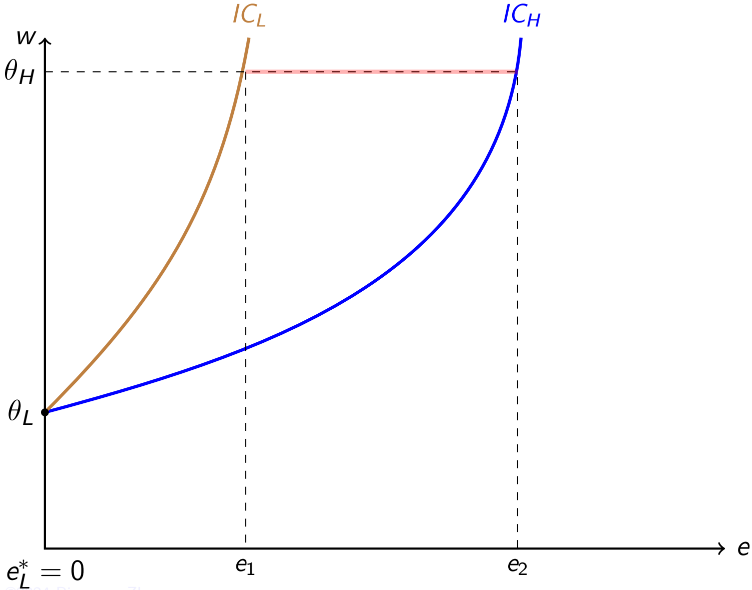

Apply IC to separating equilibrium

- Set of separating equilibria

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\begin{aligned}
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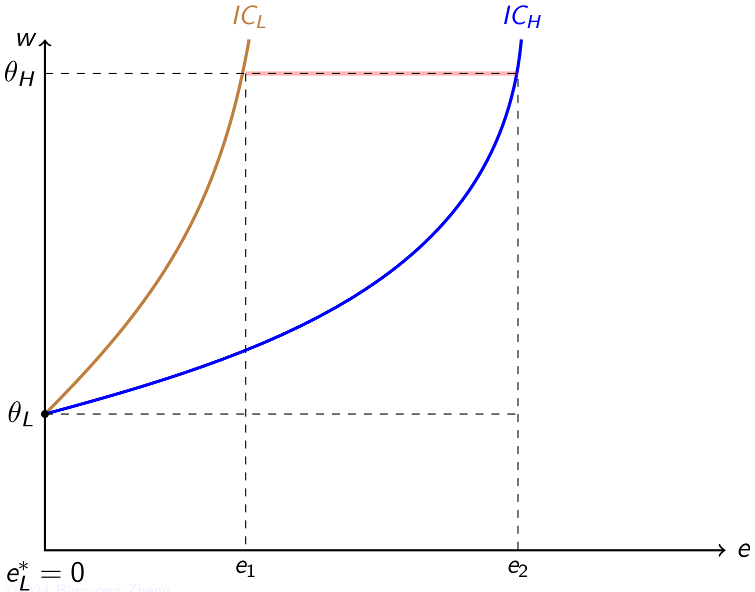
First step



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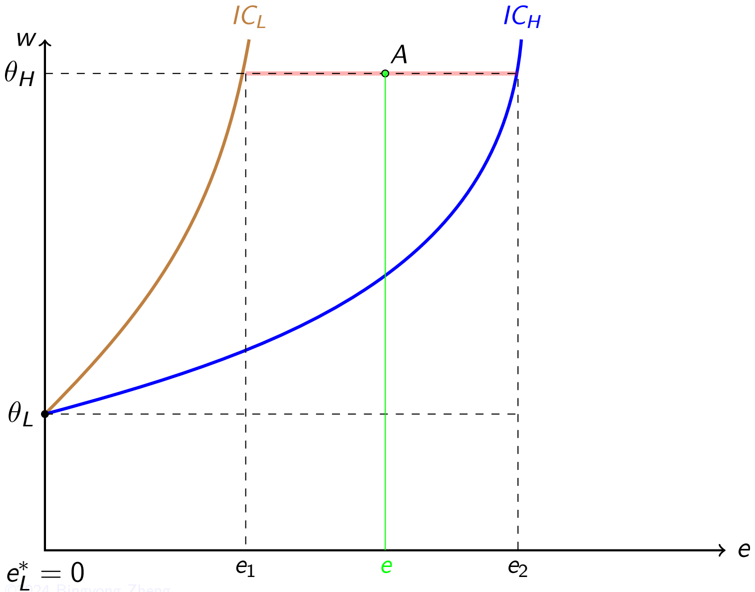

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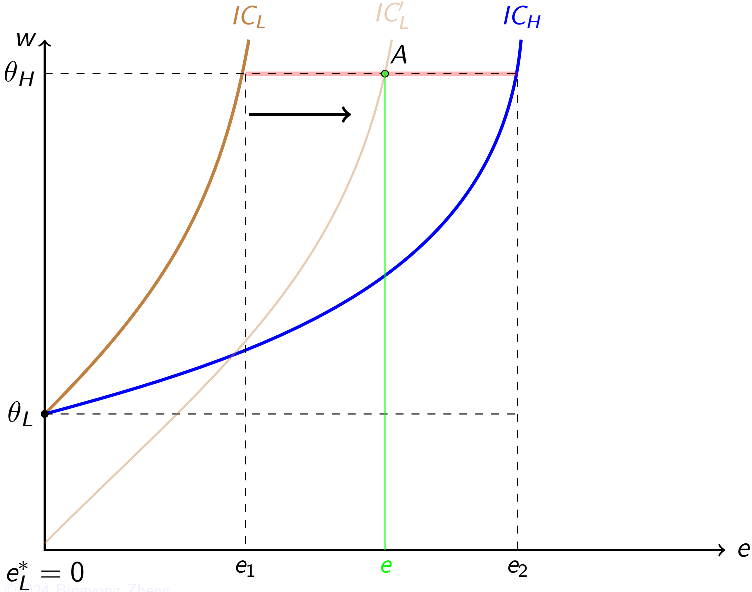

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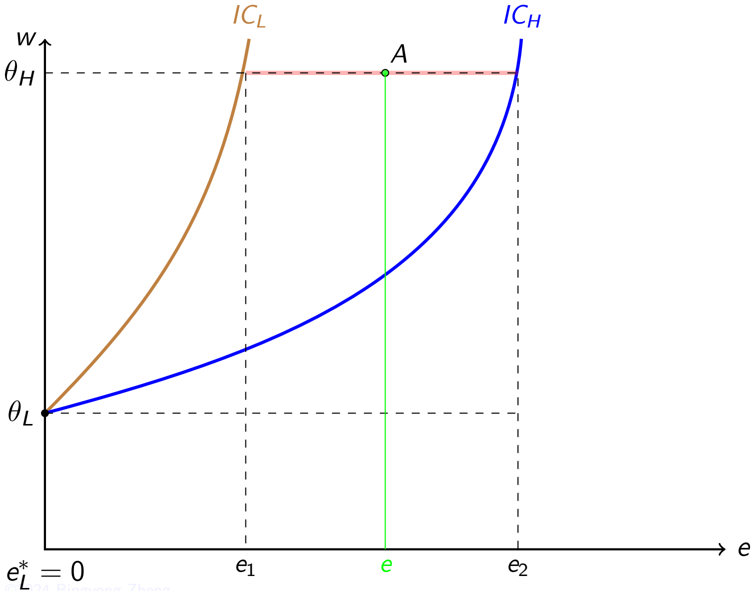

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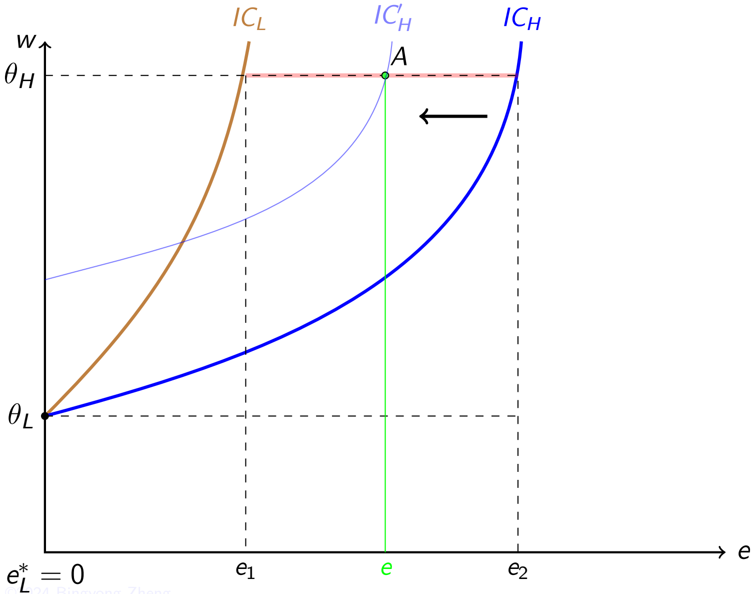

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- Thus, off-equilibrium education level can come only from θ_H

$$\Theta^{**}(e) = \{\theta_H\}.$$

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- Thus, the separating equilibrium

$$\{e_L^*(\theta_L), e_H^*(\theta_H)\} = \{0, e_2)\}$$

violates IC.

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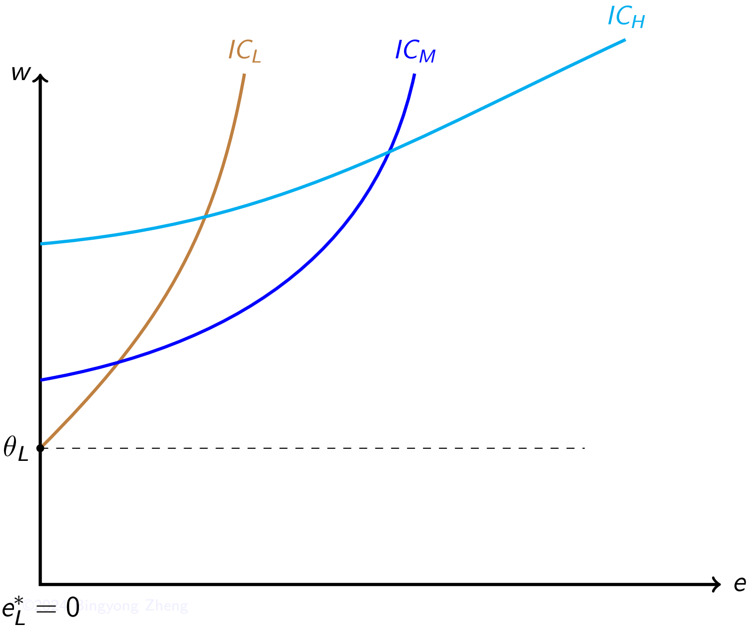

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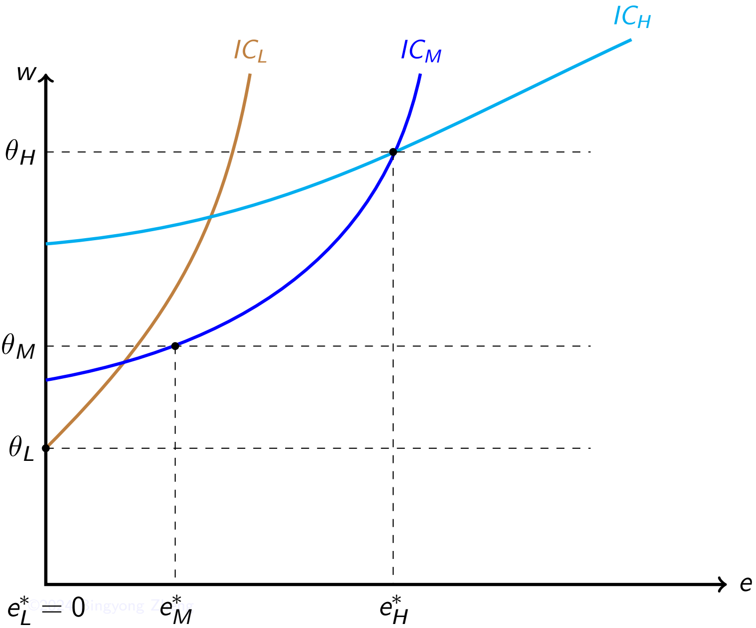

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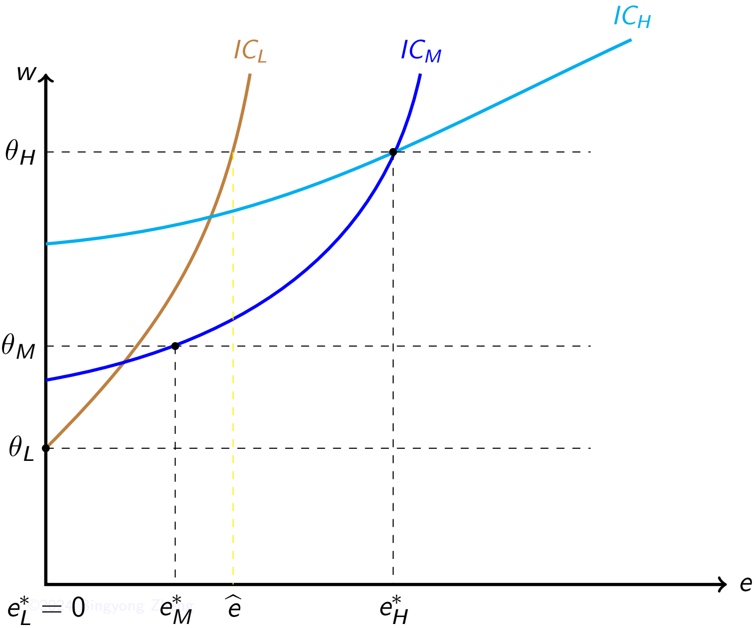

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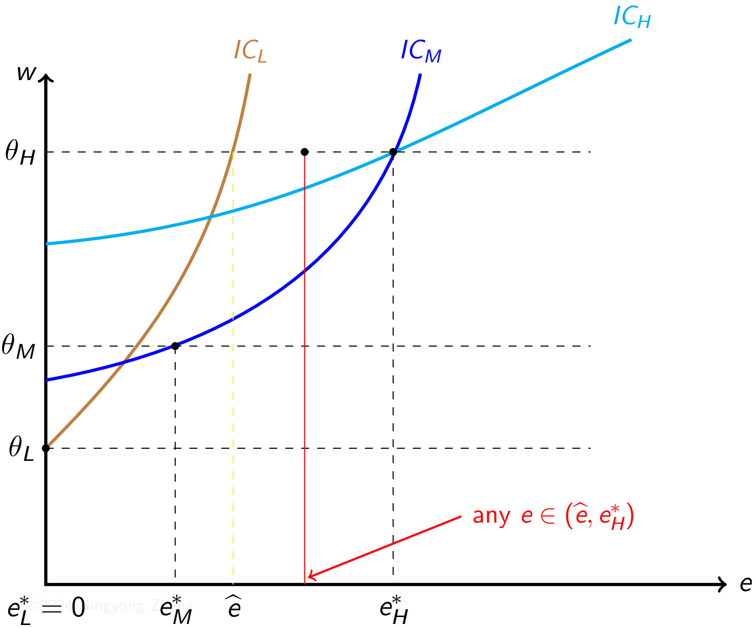

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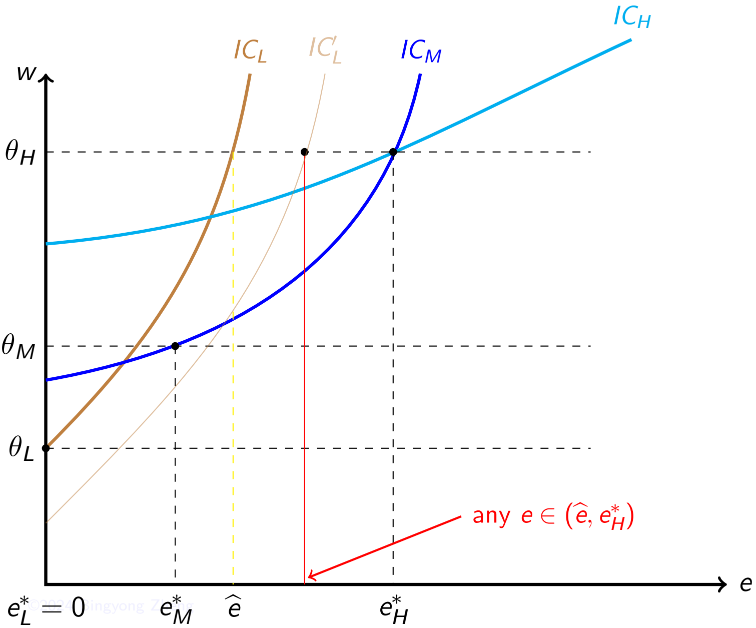

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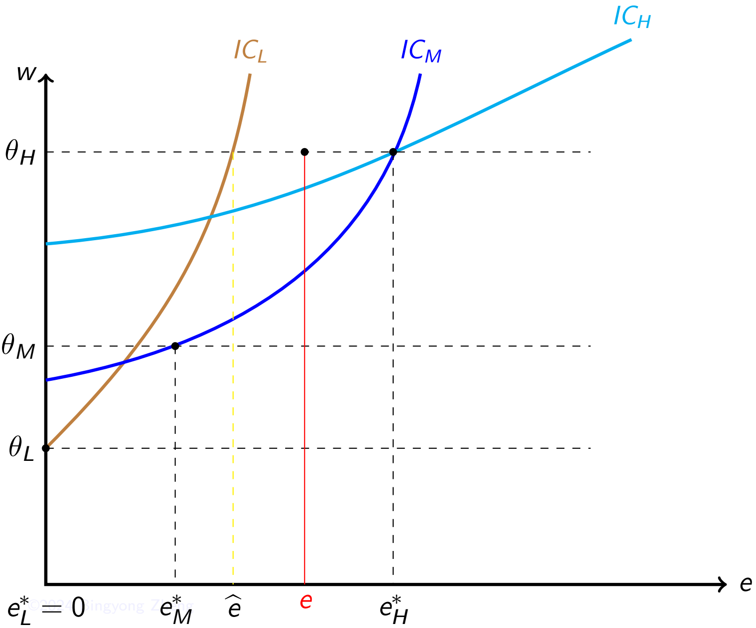

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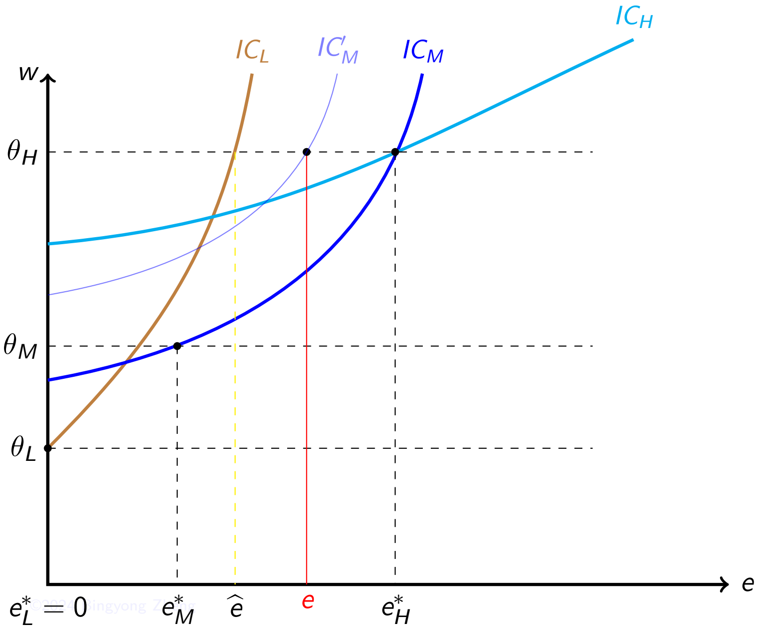

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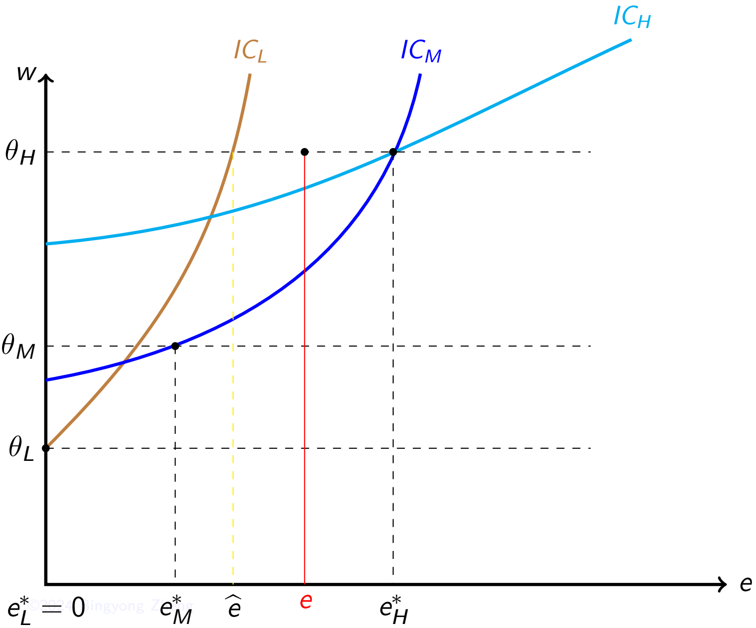

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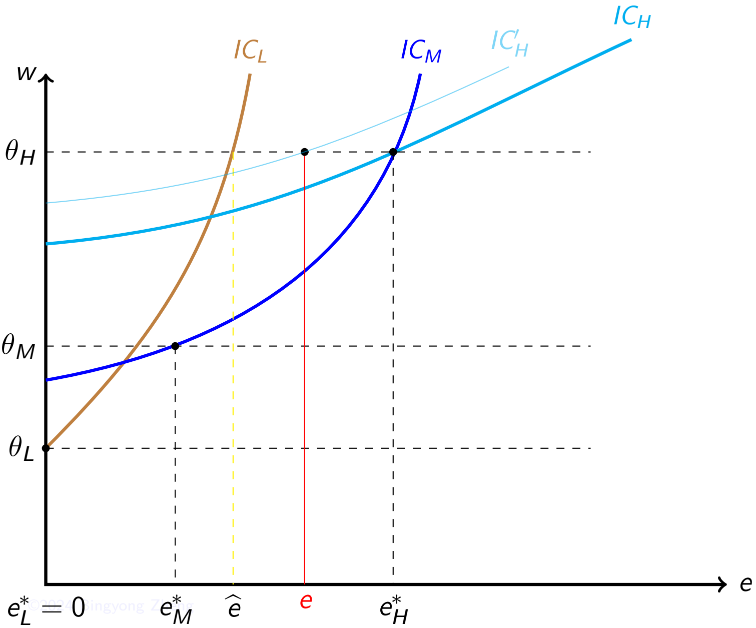

First step

- θ_L type sending message $e \in (\widehat{e}, e_H^*)$ is equilibrium dominated

$$\underbrace{u_L^*(\theta_L)}_{\text{Equilibrium payoff}} > \underbrace{\max_{w \in W^*(\Theta, m)} u_L(e, w, \theta_L)}_{\text{Max payoff from deviating to } e}.$$

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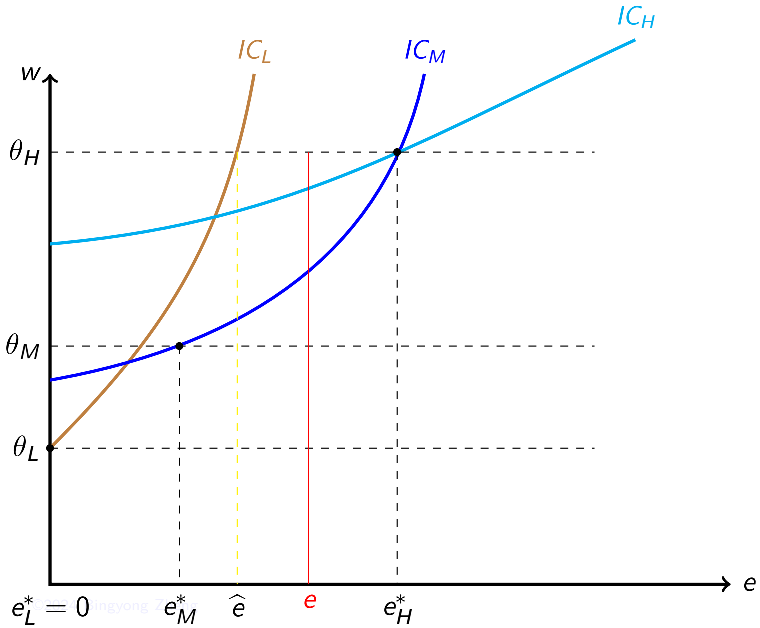
- Hence, observing $e \in (\widehat{e}, e_H^*)$, the firm's belief concentrate on θ_M and θ_H:

$$\Theta^{**} = \{\theta_M, \theta_H\}.$$

IC second step



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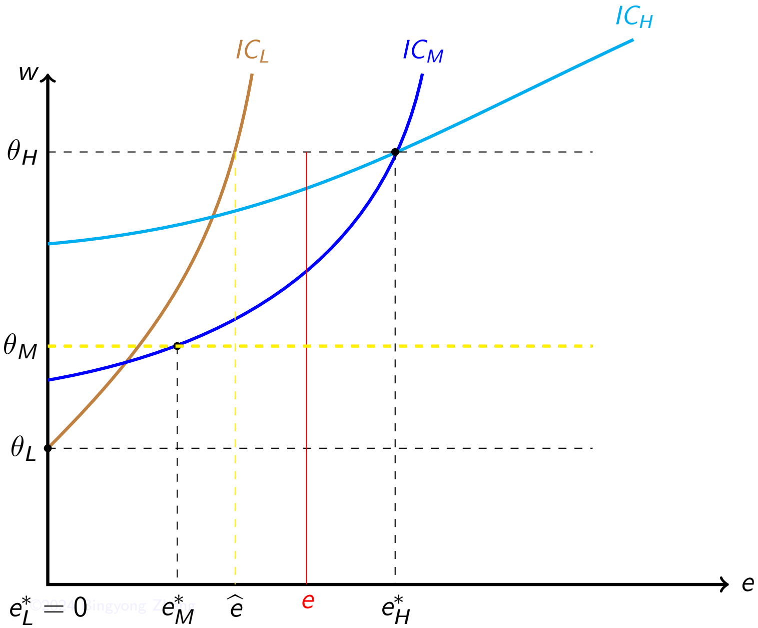

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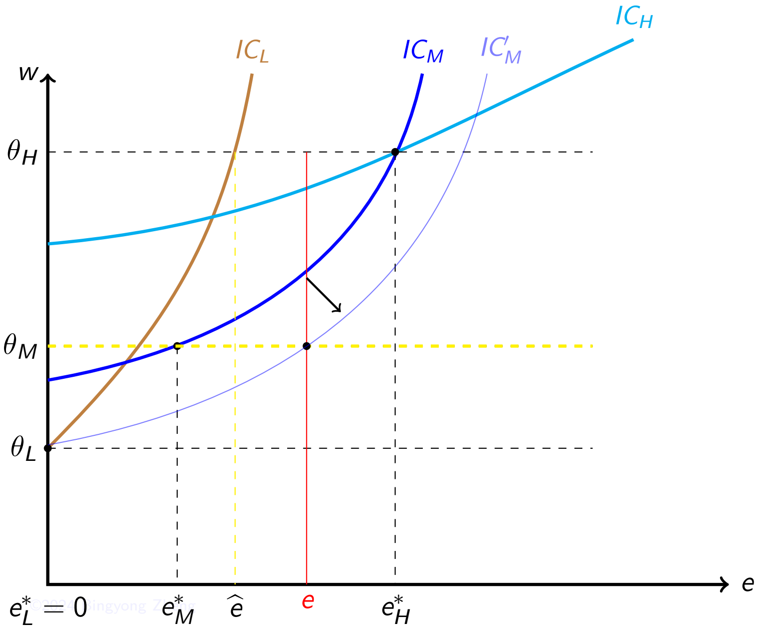

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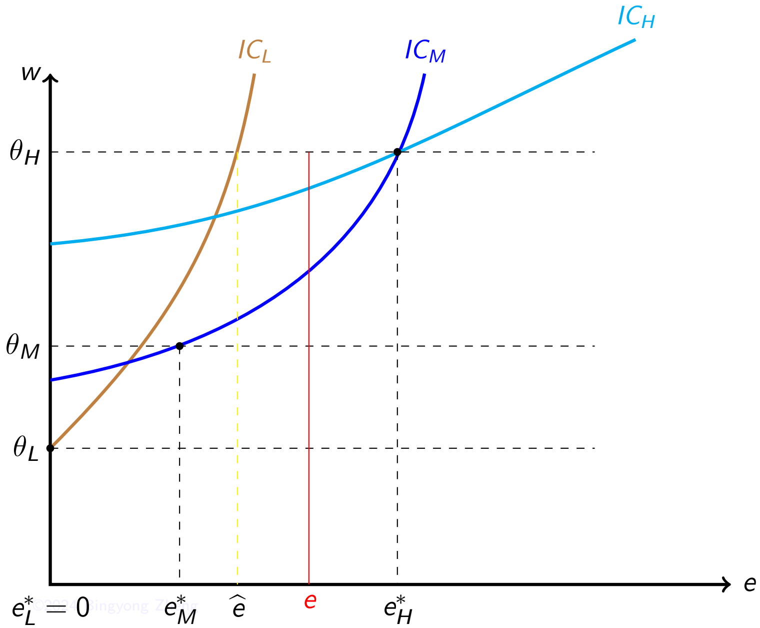

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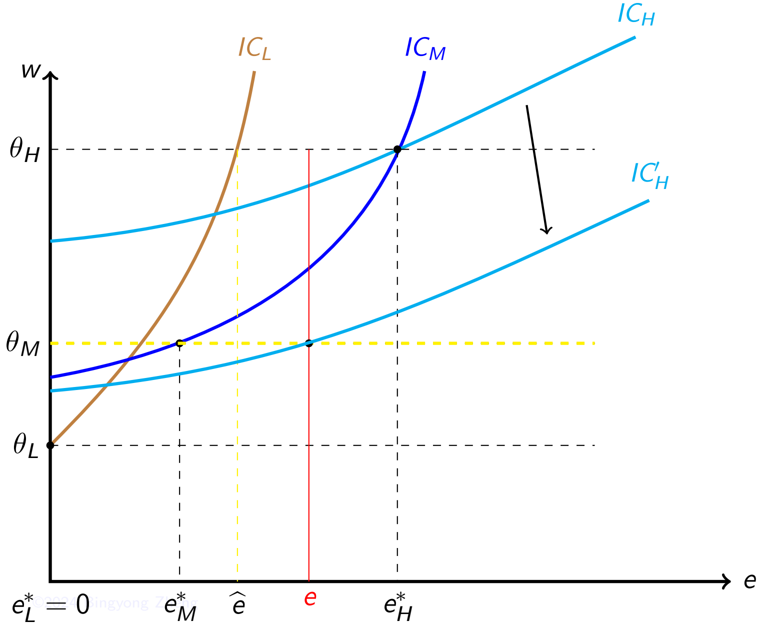

Second step

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$$\min_{w \in W^*(\Theta^{**}(e), e)} u_H(e, w, \theta_H) < u_H^*(\theta_H).$$

- Hence, there is no type of worker $\theta \in \Theta^{**}$ for whom deviation to $e \in (\widehat{e}, e_H^*)$ is profitable.

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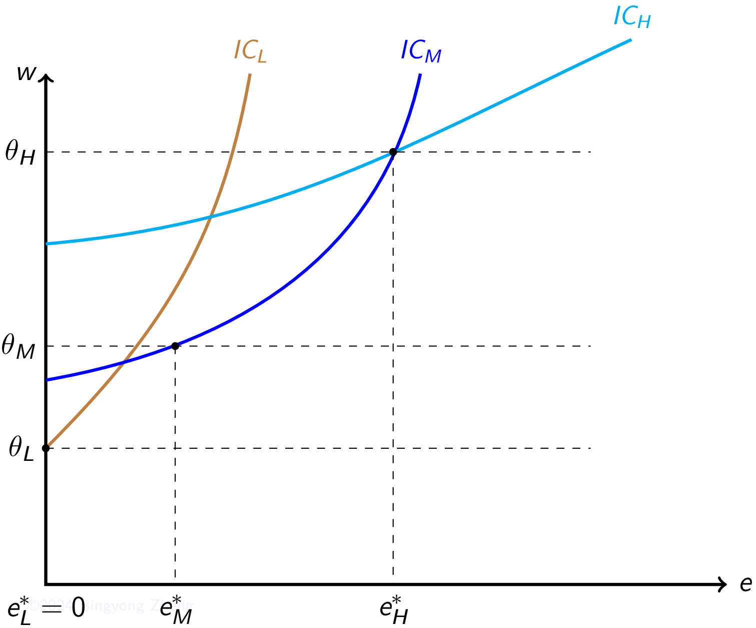
D1



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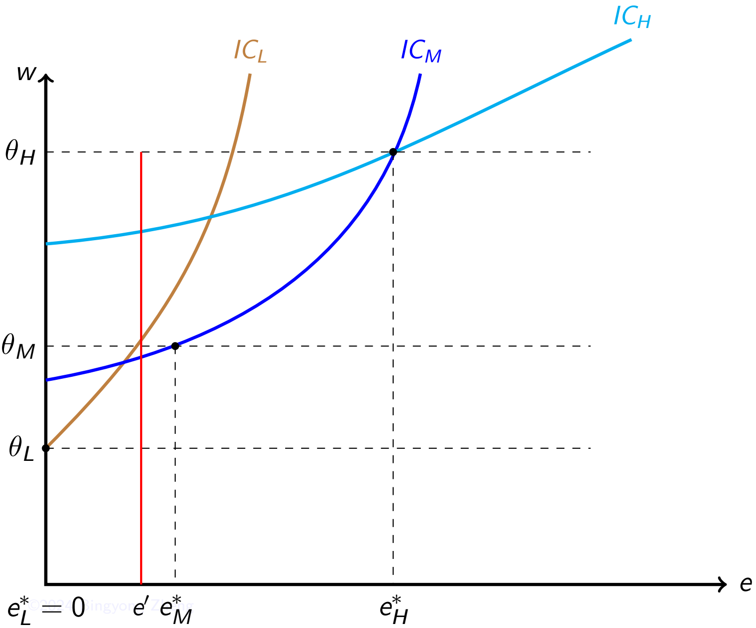

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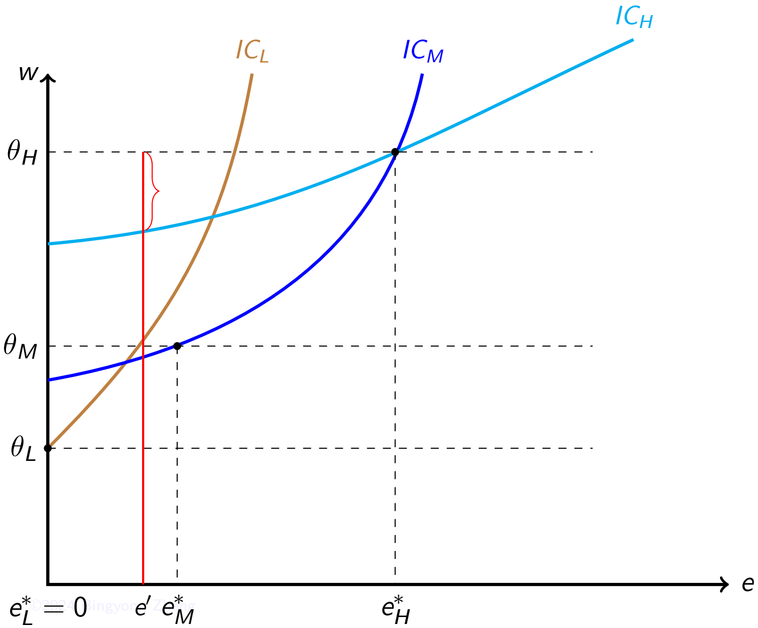

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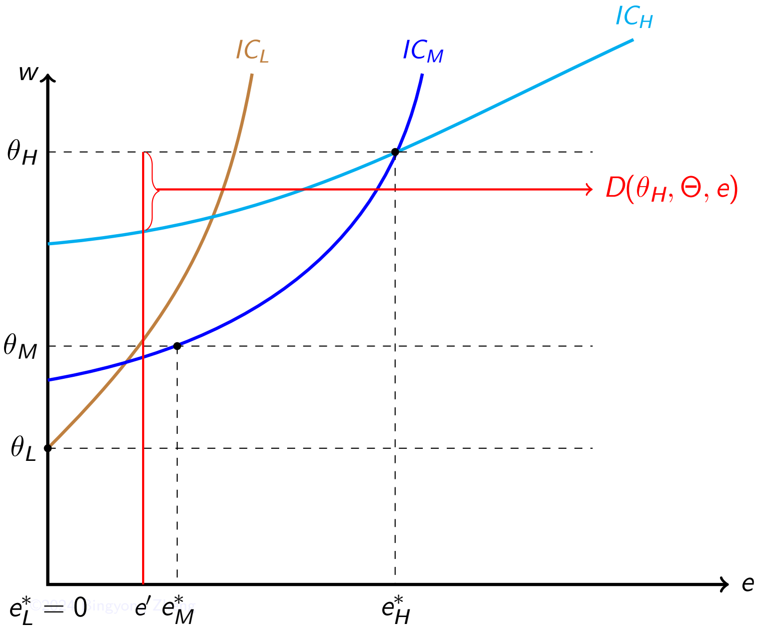
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IC_L
IC_M
w
θ_H
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e'
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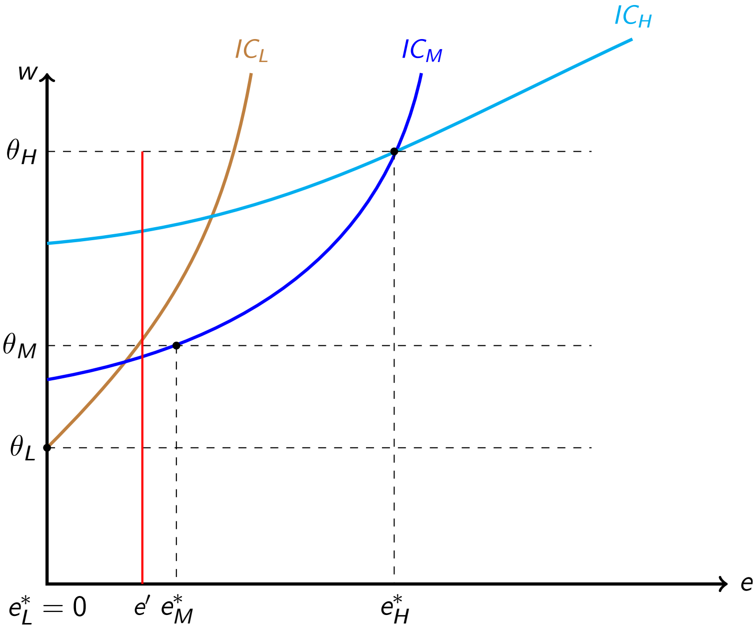

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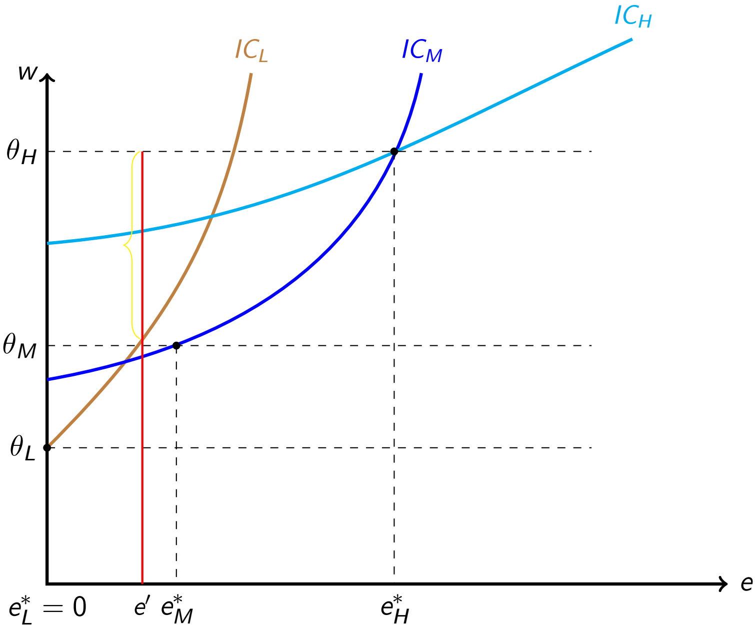
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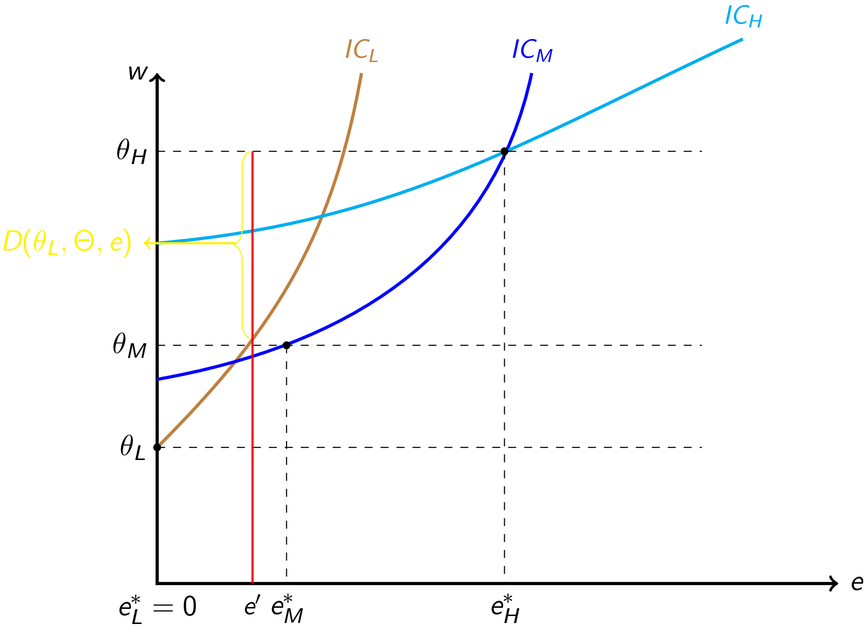
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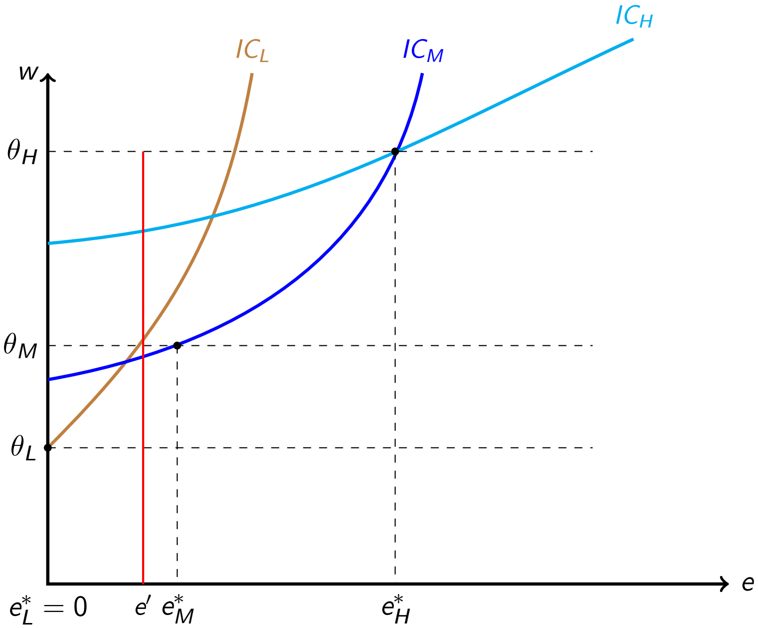

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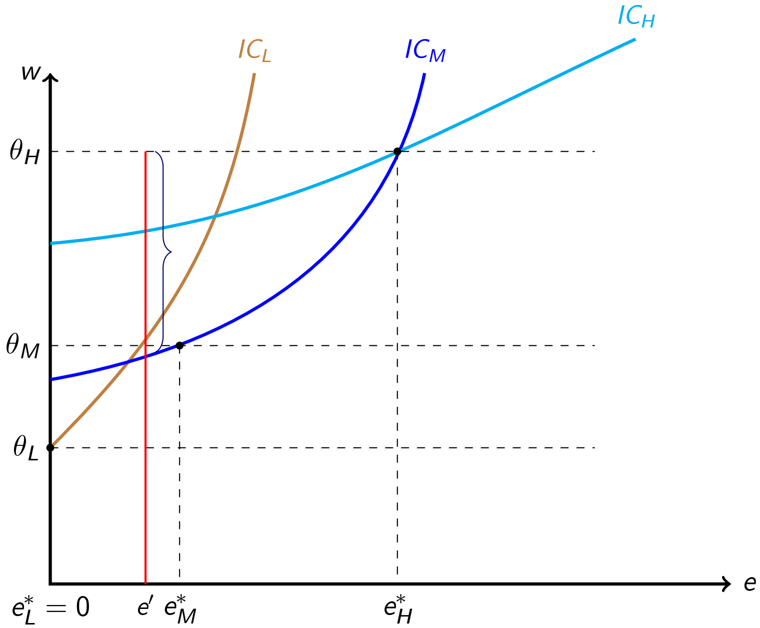
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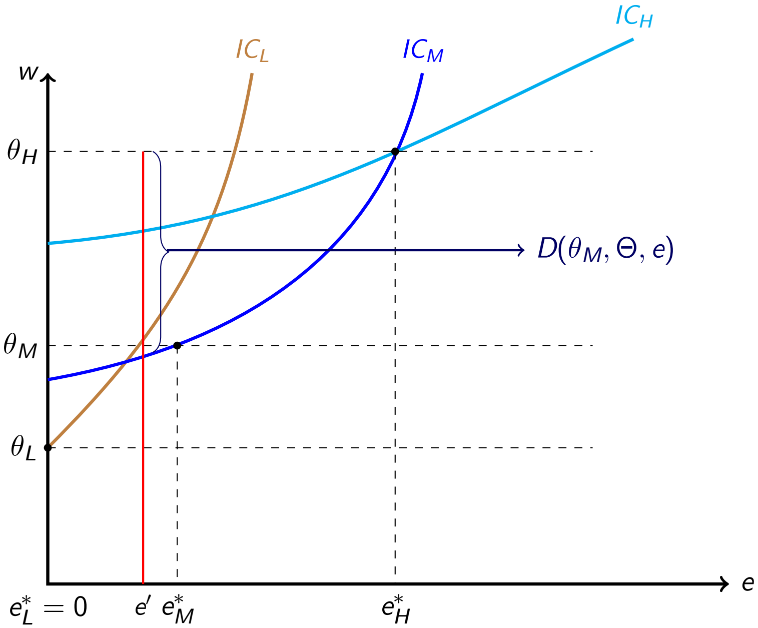
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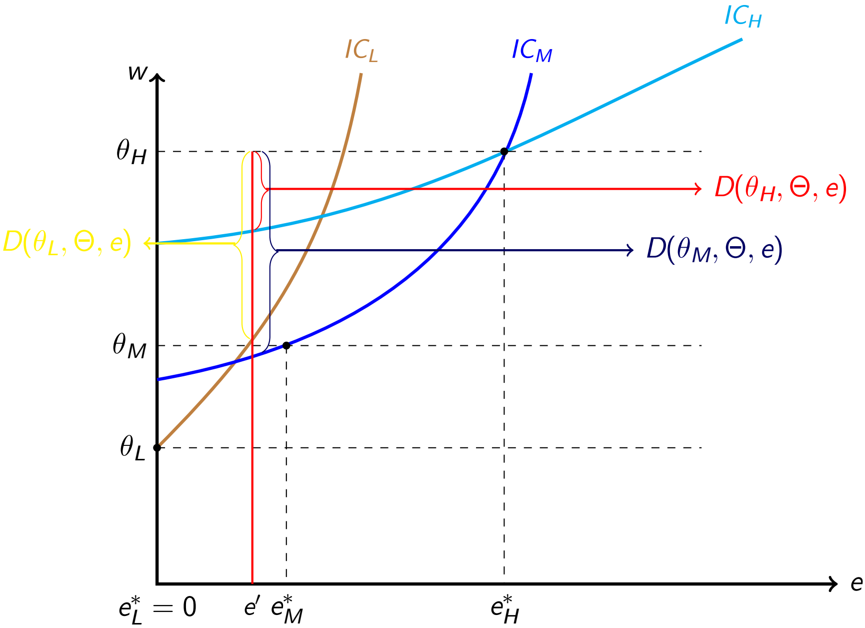
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D1 first step

- We see from the figure

$$D(\theta_H, \widehat{\Theta}, e') \bigcup D^o(\theta_H, \widehat{\Theta}, e') \subset D(\theta_M, \widehat{\Theta}, e').$$

θ_M type has more incentives to deviate to e' than θ_H type

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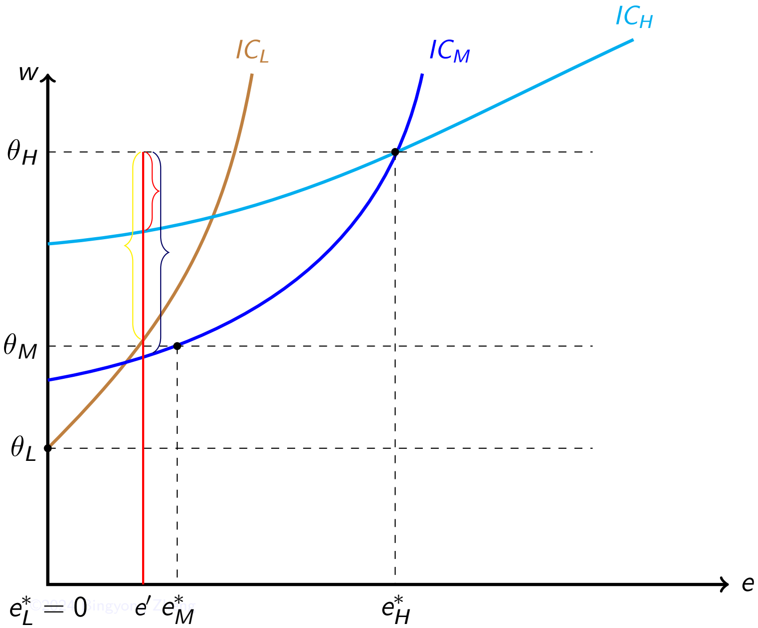
θ_M type has more incentives to deviate to e' than θ_L type
- Applying the D1 criterion, the θ_M type is the most likely to deviate to e'
$$\Theta^{**}(e') = \{\theta_M\}.$$

D1 SECOND STEP



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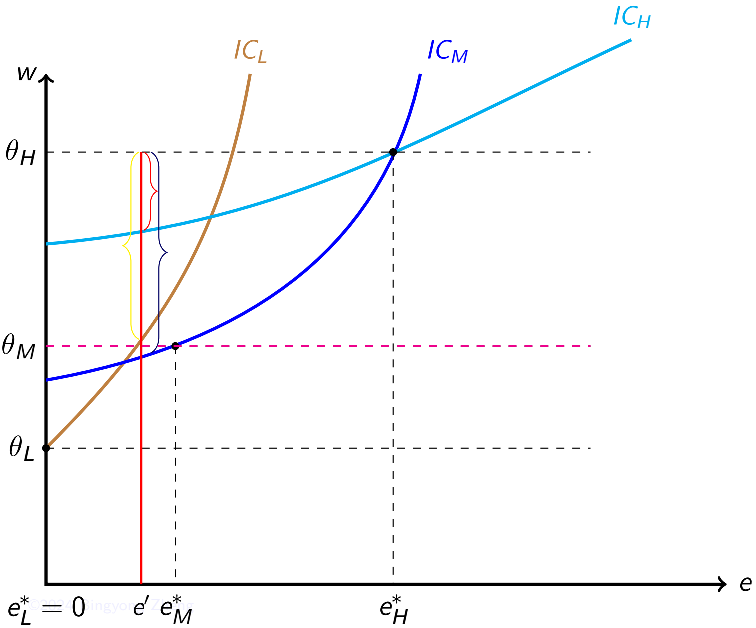

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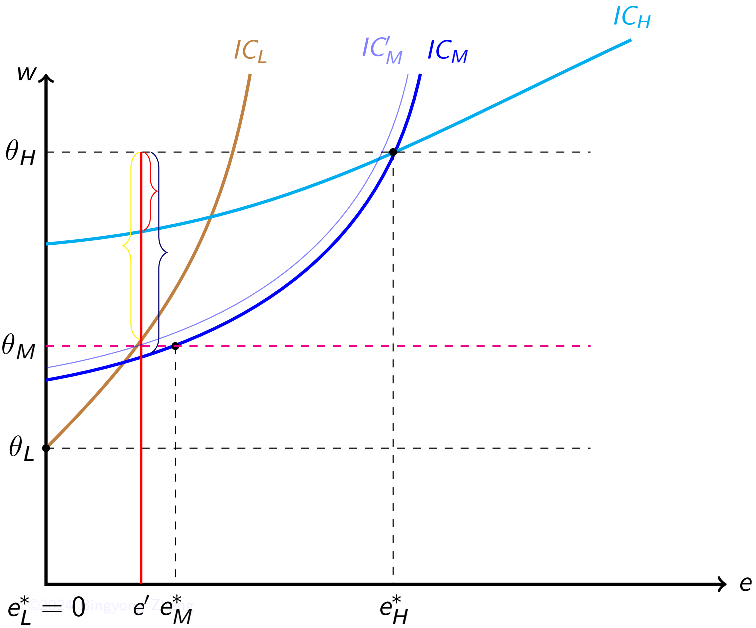

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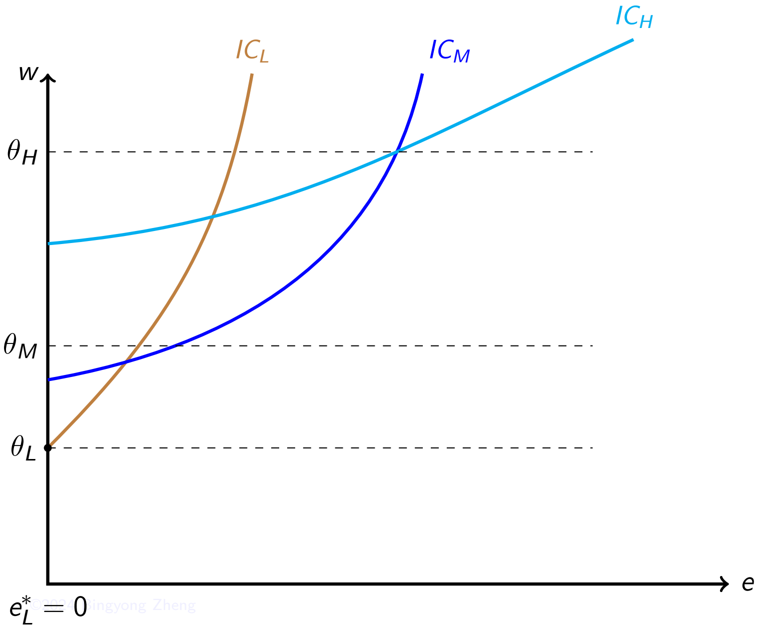
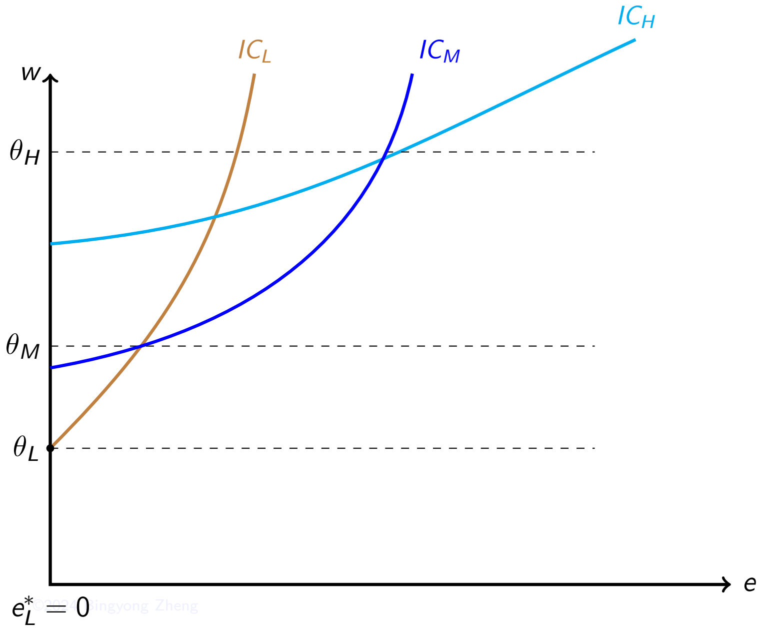
  Deviating towards e' is profitable!
- So the equilibrium (e_L^*, e_M^*, e_H^*) violates the D1 criterion

D1 SECOND STEP CONTINUED



D1 second step continued



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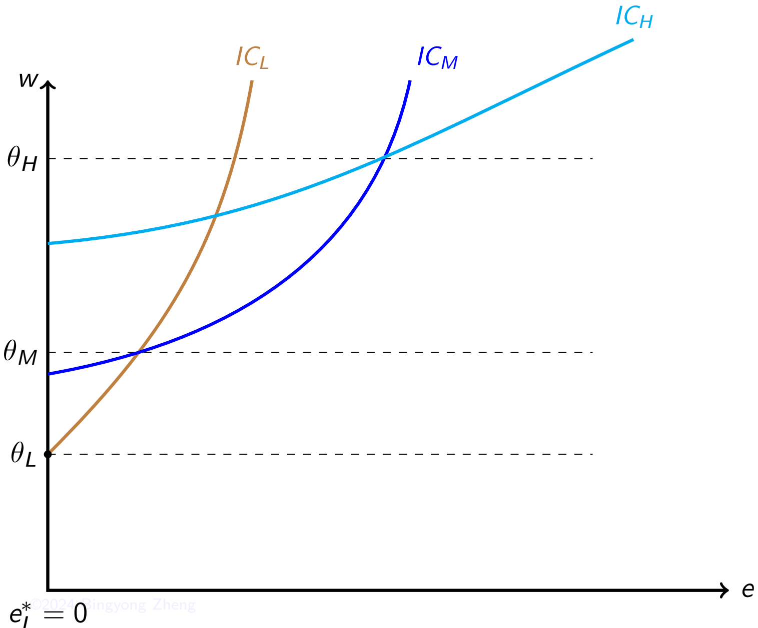

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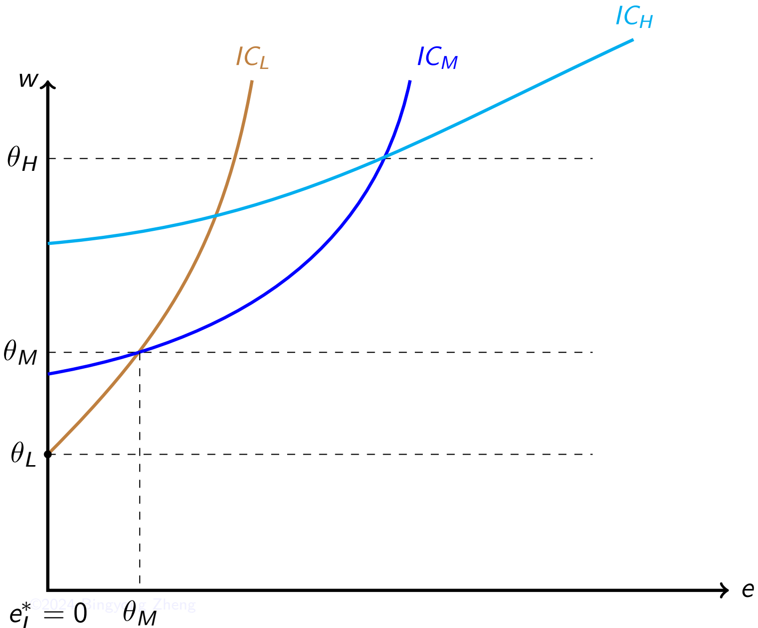

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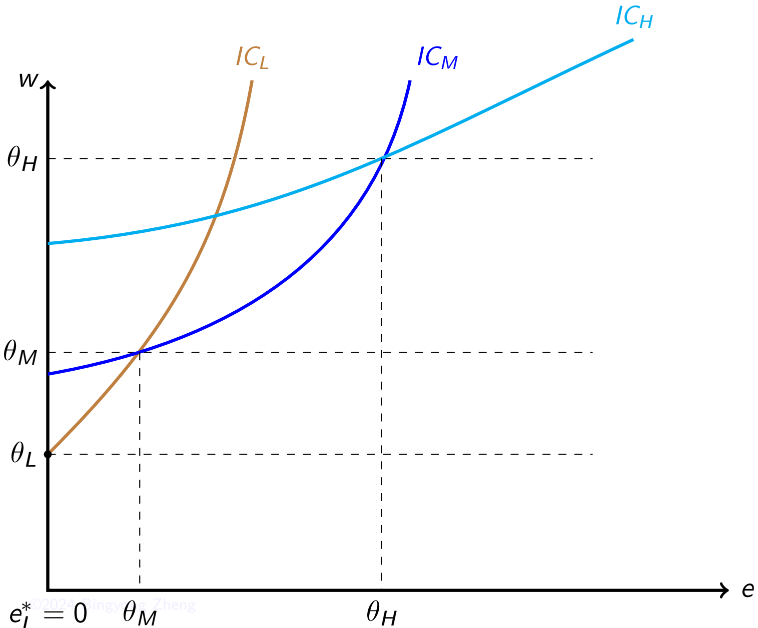

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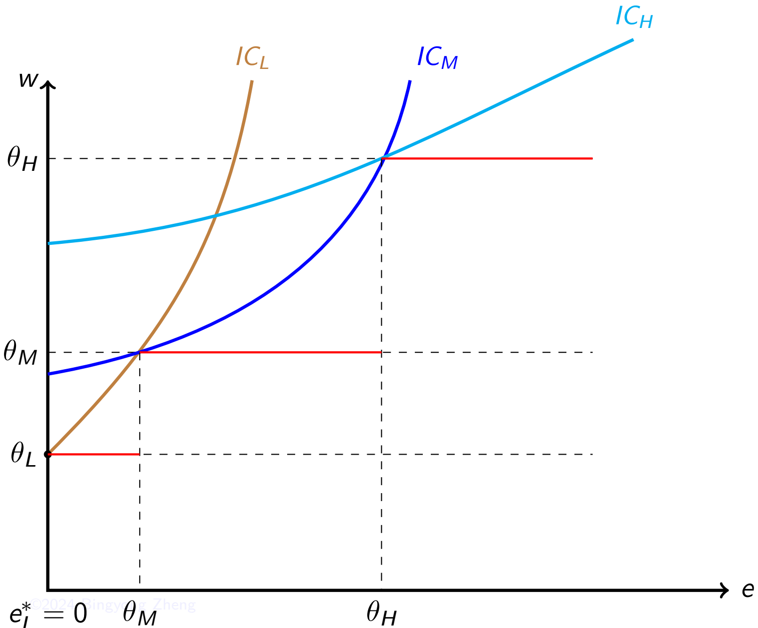
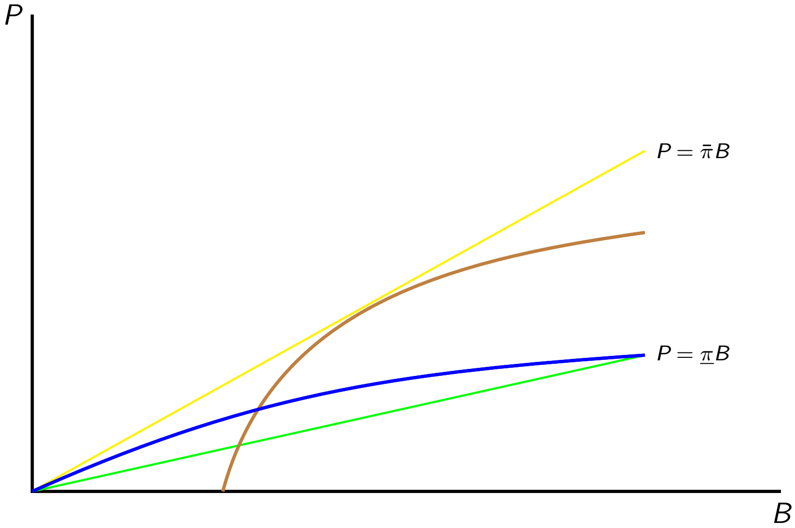
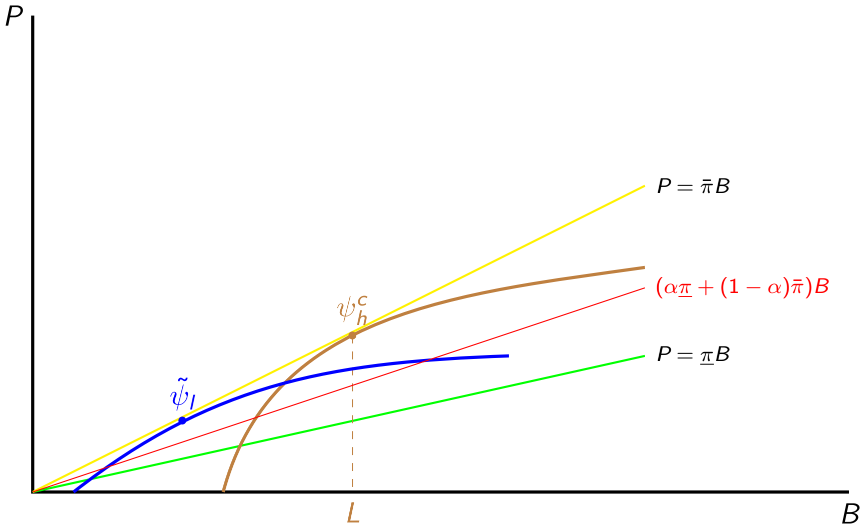


Insurance model: separating equilibrium



Insurance model: pooling equilibrium



Apply IC to insurance model

- IC to insurance signaling game: Sequential equilibrium $(\psi_l, \psi_h, \sigma, \beta)$ satisfy IC if for all ψ ($\psi \neq \psi_l$ or $\psi \neq \psi_h$),

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- Theorem 8.3. (Jehle& Reny) There is a unique policy pair (ψ_l, ψ_h) that can be supported by a sequential equilibrium satisfying the intuitive criterion. And this equilibrium is the best separating equilibrium for the low-risk consumer.

Screening: competition

Competitive screening

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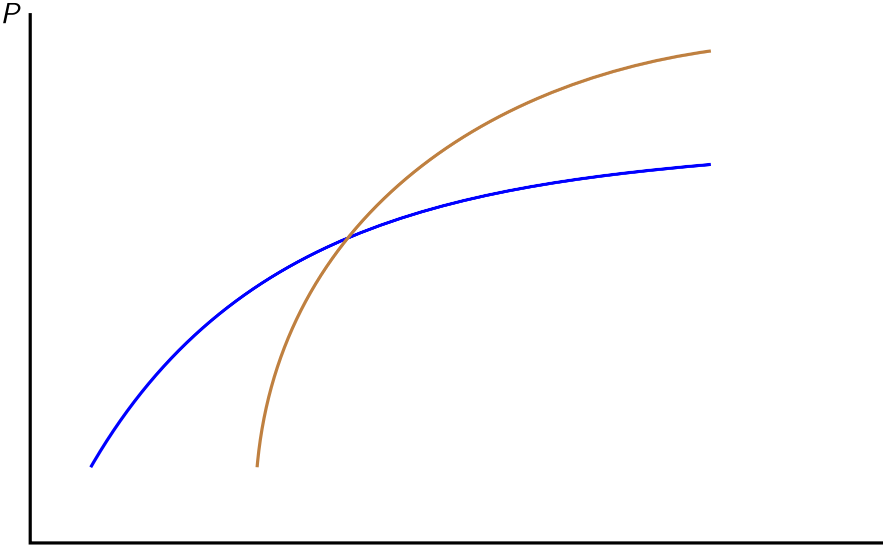
- Lemma 8.2. (Jehle & Reny) Insurance companies earn zero expected profits in equilibrium.

Existence of Pooling equilibrium



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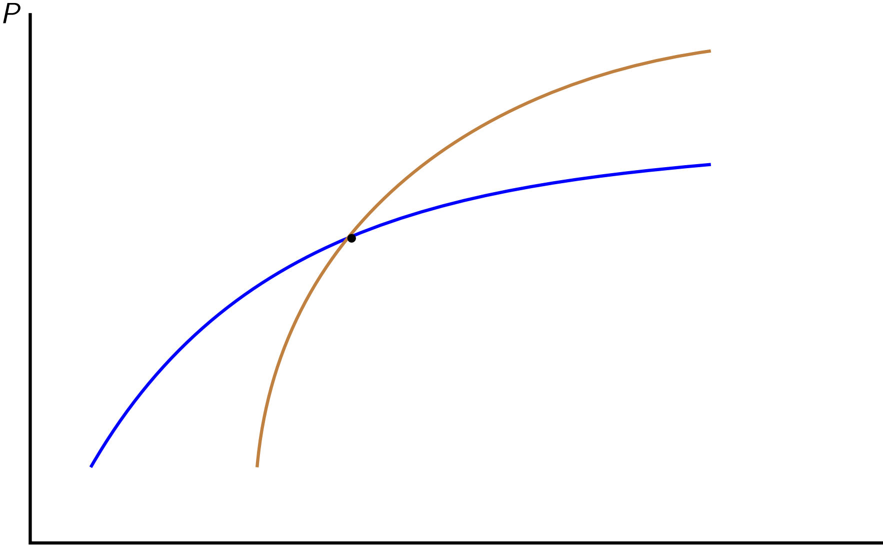

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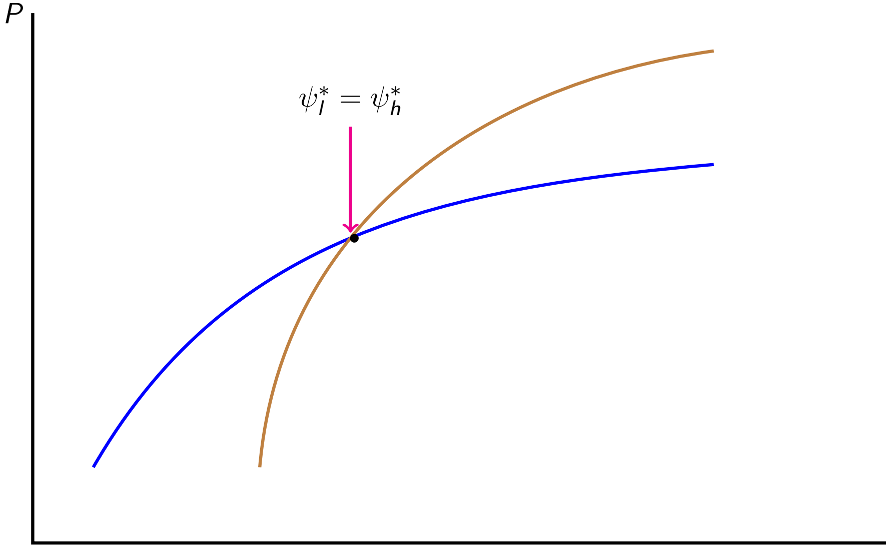

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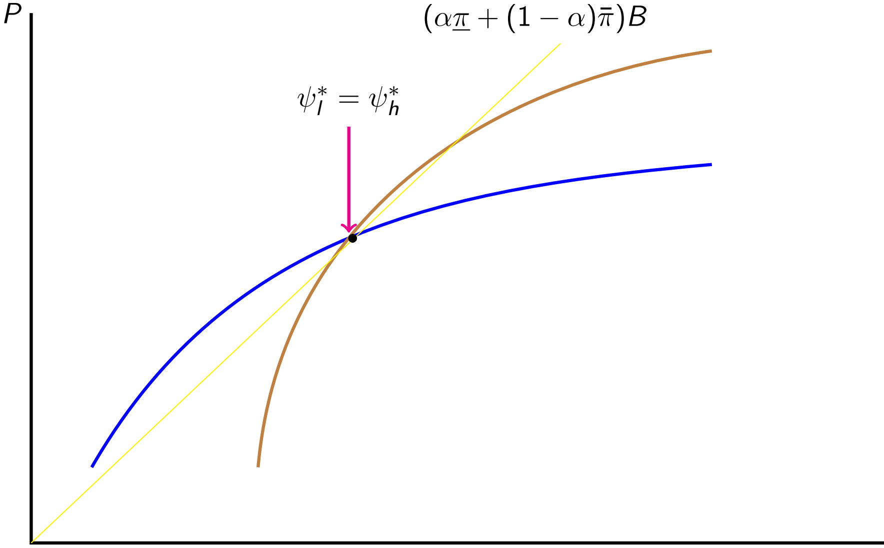

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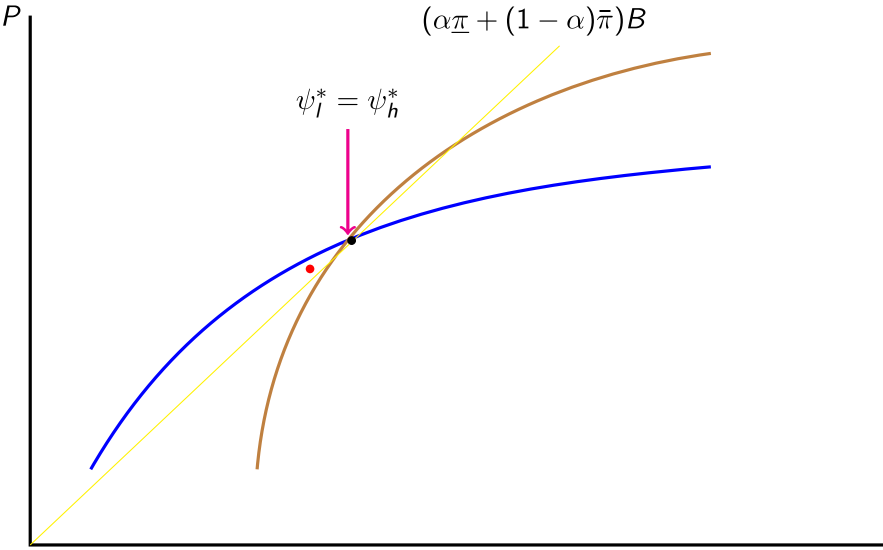

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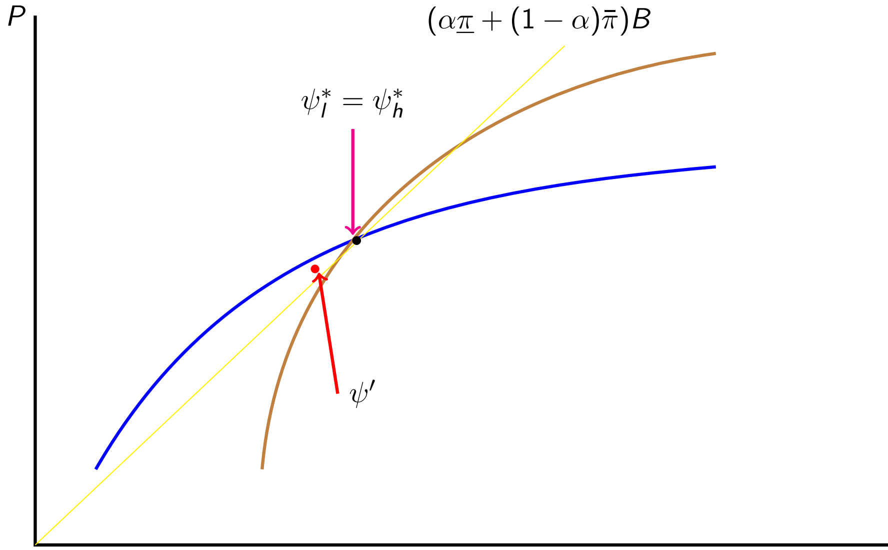

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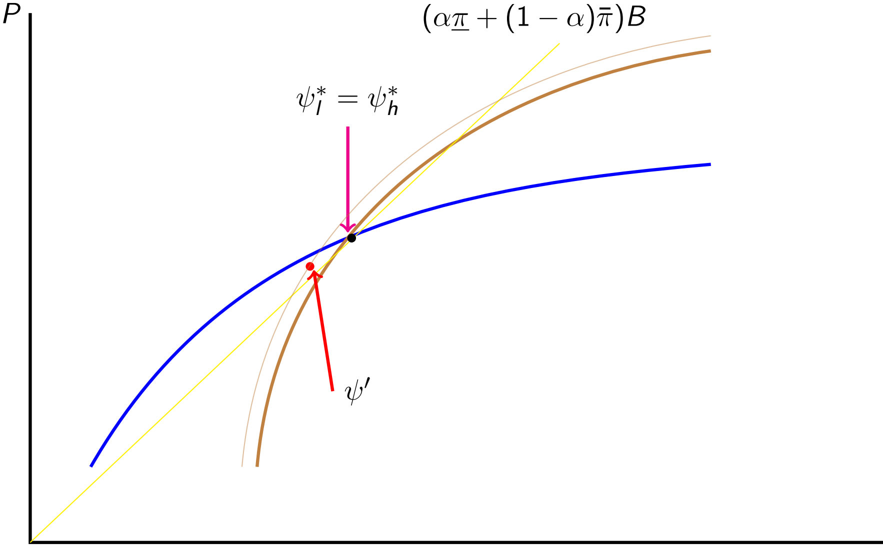

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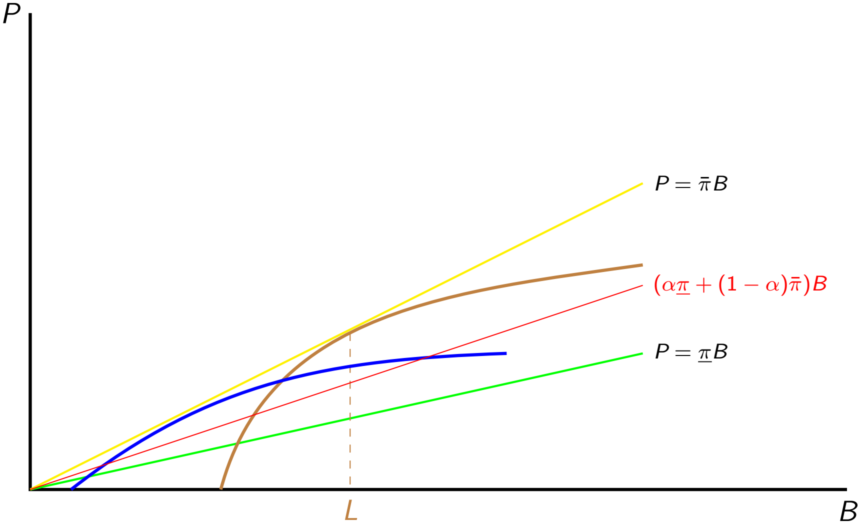

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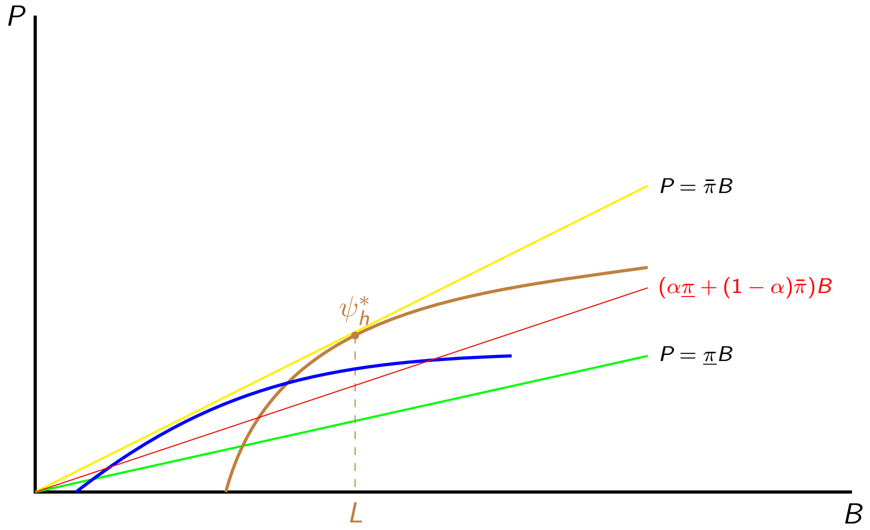

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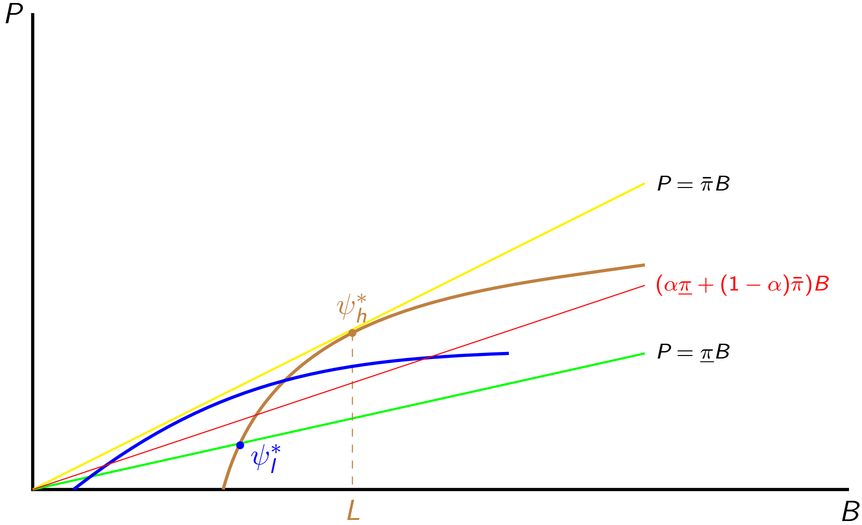

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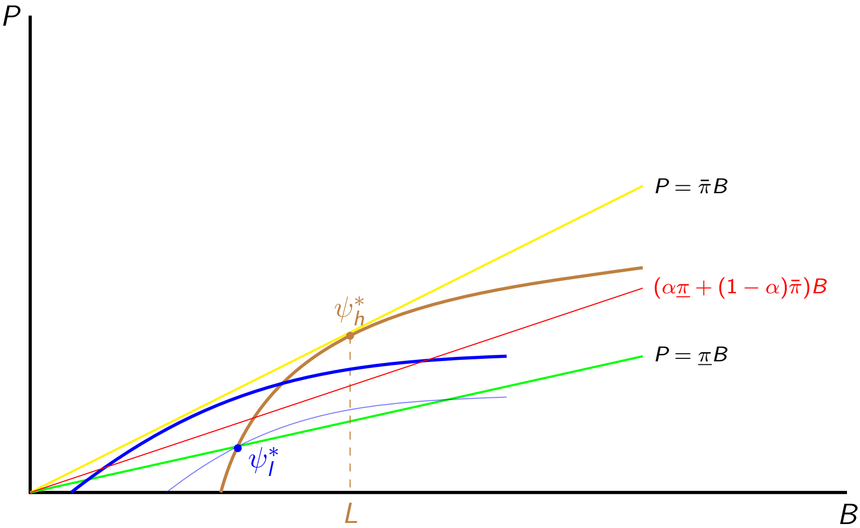

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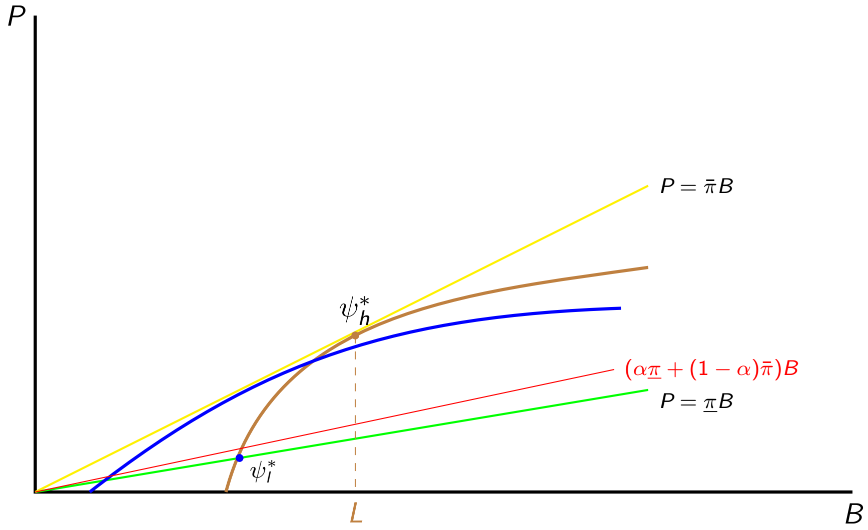

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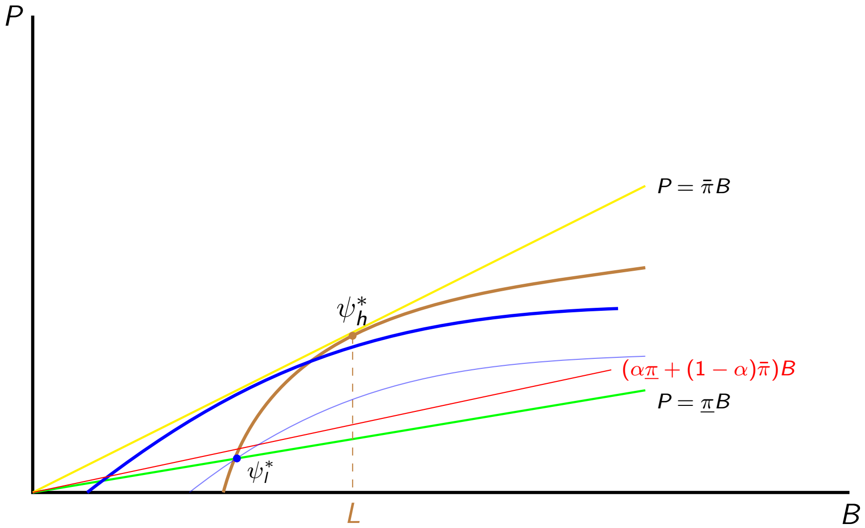

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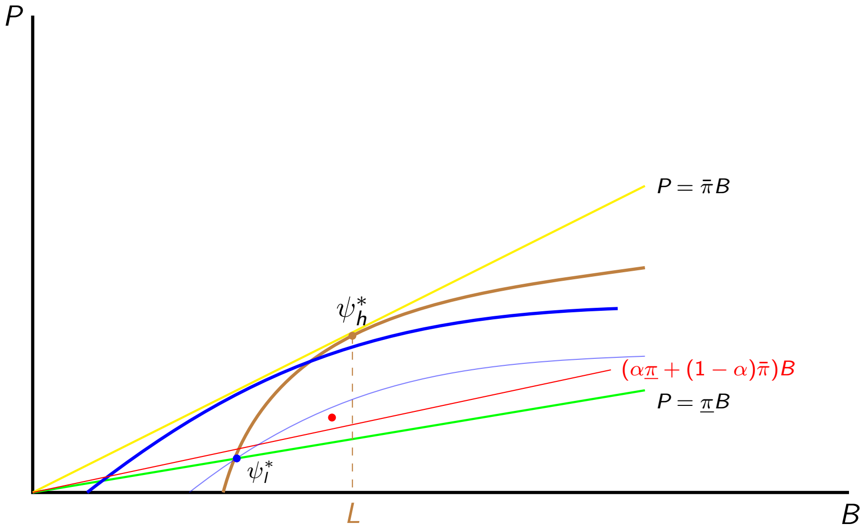

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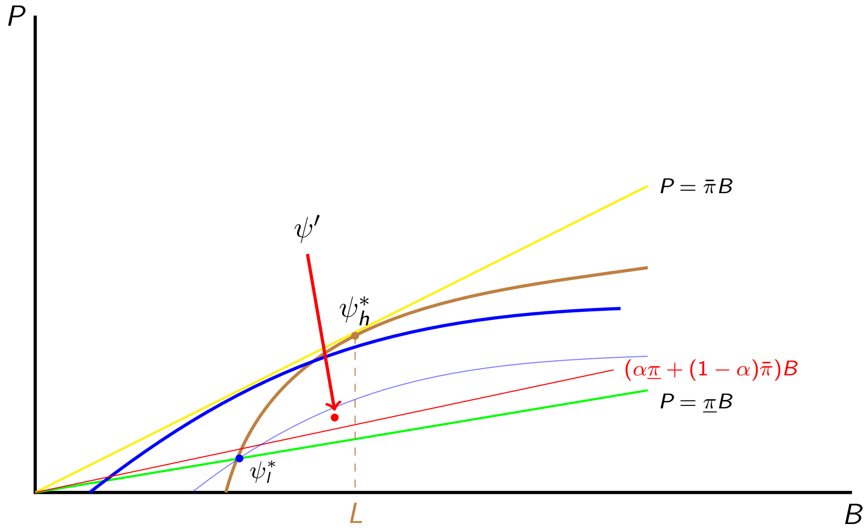

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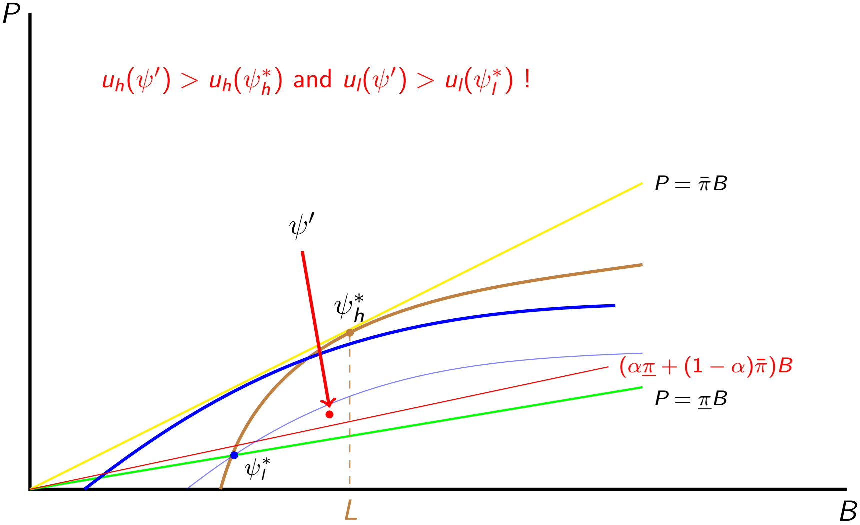

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- Theorem 8.6. (Jehle & Reny) No pure strategy equilibrium may exist if the proportion of high-risk is too low.

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- Insurance company chooses policy $(p, B_0, B_1, \ldots, B_L)$ to maximize profit.

$$\max_{e,p,B_l} p - \sum_{l=0}^{L} \pi_l(e) B_l, \qquad \textit{subject to}$$

$$\sum_{l=0}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \bar{u}.$$

- Lagrangian:

$$\mathcal{L} = p - \sum_{l=0}^{L} \pi_l(e) B_l + \lambda \left[\sum_{l=0}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) - \bar{u} \right].$$

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- First order conditions:

$$\frac{\partial \mathcal{L}}{\partial p} = 1 - \lambda \left[\sum_{l=0}^{L} \pi_l(e) u'(w - p - l + B_l) \right] = 0, \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial B_l} = -\pi_l(e) + \lambda \pi_l(e) u'(w - p - l + B_l) = 0, \qquad \forall l \geq 0, \quad (2)$$

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- Thus it is optimal to have

$$B_l = l \qquad for l = 0, 1, \ldots, L.$$

- Optimization problem

$$\max_{e,p,B_l} p - \sum_{l=0}^{L} \pi_l(e) B_l, \qquad \textit{subject to}$$

$$\sum_{l=0}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \bar{u};$$

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- If optimal policy to set $e = 0$:
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- Optimal policy $e = 1$:

$$\mathcal{L} = p - \sum_{l=0}^{L} \pi_l(1) B_l + \lambda \left[\sum_{l=0}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) - \bar{u} \right]$$

$$\beta \left[\sum_{l=0}^{L} \pi_l(1) u(w - p - l + B_l) - \sum_{l=0}^{L} \pi_l(0) u(w - p - l + B_l) - d(1) + d(\right.$$

Second best contract

- First order conditions:

$$1-\lambda\left[\sum_{l=0}^{L}\pi_l(1)u'(w-p-l+B_l)\right]-\beta\left[\sum_{l=0}^{L}(\pi_l(1)-\pi_l(0))u'(w-p-l+B_l)\right]$$
$$=0;$$
$$-\pi_l(1)+[\lambda\pi_l(1)+\beta(\pi_l(1)-\pi_l(0))]u'(w-p-l+B_l)=0 \quad \forall l; \qquad (*)$$
$$\sum_{l=0}^{L}\pi_l(1)u(w-p-l+B_l)-d(1)-\bar{u}\geq 0;$$
$$\sum_{l=0}^{L}(\pi_l(1)-\pi_l(0))u(w-p-l+B_l)+d(0)-d(1)\geq 0.$$

- Equation (*) implies

$$\frac{1}{u'(w-p+B_I-I)} = \lambda + \beta\left[1 - \frac{\pi_I(0)}{\pi_I(1)}\right]. \qquad \text{(CON-OP)}$$

Second best contract (continued)

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- Thus,

$$l - B_l \quad \text{is strictly increasing in} \quad l = 0, 1, \ldots, L.$$

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- Agent is forced to carry excess responsibility for the outcome and this is the implicit costs involved in contracting under imperfect information.

Contract with continuous effort

- The agent's preference

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- Linear contract

$$w = w_0 + \beta y.$$

- The Agent's expected utility:

$$
\begin{aligned}
EV &= \int -exp\{-r[w_0 + \beta(e+\varepsilon) - C(e)]\} f(\varepsilon) d\varepsilon \\
&= -exp\{-r[w_0 + \beta e - C(e^2)]\} exp\left\{\frac{r^2\beta^2\sigma^2}{2}\right\} \\
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- The Certainty Equivalent for the Agent from exerting effort e^* is

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- Solving for β gives

$$\beta = \frac{1}{1 + rc\sigma^2}.$$