Advanced Microeconomics, Spring 2021

Problem set 5, due June 15

1. Suppose Jack and Tom have the endowments $\omega_A = (6, 0)$ and $\omega_B = (0, 6)$. Their preferences are defined by a pair of utility functions. For the following cases, find the utility possibility frontier. State your answer both in precise mathematical notation and in terms of graph.

   Leontief/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

   Leontief/Linear

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

   Cobb-Douglas/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

   Cobb-Douglas/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

   Linear/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J} + x_{2J}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + 2x_{2T}.$$

2. Suppose the economy is endowed with capital and labor, $k = 6$, $l = 6$, and can produce two outputs (b, c) with production functions given below. For each pair of production functions given, derive the transformation function $T(y)$ where $y = (y_1, y_2, y_3, y_4) = (-k, -l, b, c)$.

   Leontief/Cobb-Douglas

$$b(k, l) = \min(k, l/2)$$

$$c(k, l) = k^{1/3} l^{2/3}.$$

Leontief/Linear

$$b(k, l) = \min(k, l/2)$$

$$c(k, l) = k + l.$$

Cobb-Douglas/Cobb-Douglas

$$b(k, l) = k^{1/3} l^{2/3}$$

$$c(k, l) = k^{1/3} l^{2/3}.$$

Cobb-Douglas/Linear

$$b(k, l) = k^{1/3} l^{2/3}$$

$$c(k, l) = k + l.$$

Linear/Linear

$$b(k, l) = k + l$$

$$c(k, l) = k + 2l.$$

3. Suppose there are three people, named Bob, Jack and Tom, whose only purpose in life is to eat chocolate. Their utility functions are

$$u_B(c_B, l_B) = c_B,$$

$$u_J(c_J, l_J) = 2c_J,$$

$$u_T(c_T, l_T) = 3c_T.$$

Each of them is endowed with no chocolate and one unit of labor. There are 3 chocolate factories, whose production functions are

$$f_1(l_1) = l_1,$$

$$f_2(l_2) = 2l_2,$$

$$f_3(l_3) = 3l_3.$$

Bob owns Firm 1, Jack owns Firm 2, and Tom owns Firm 3.

(a) Determine the society's transformation function.

(b) Determine the set of Pareto efficient consumption allocations.

(c) Determine the set of equilibrium consumption allocation(s). For each equilibrium consumption allocation, give the corresponding production allocation and price vector.

4. Consider the following market for used cars. There are N potential sellers and M potential buyers, and $M > N$. Each seller has exactly one used car to sell and is characterized by the quality of the used car he has. Let $\theta \in [0, 1]$ index the quality of used car. If a seller of type θ sells his car for a price of P, his payoff is $u_s(P, \theta)$, and is 0 if he does not sell his car. The payoff for the buyer is $\theta - P$ if he buys a car of quality θ at price P, and is 0 if he does not buy. Information is asymmetric: Sellers know the quality of used cars but buyers do not. However, buyers know the quality of used car is uniformly distributed on [0,1].

   (a) Argue that in a competitive equilibrium under asymmetric information, we must have $E[\theta|P] = P$.

   (b) Find all equilibrium prices when $u_s(P, \theta) = P - \theta/2$.

   (c) Find all equilibrium prices when $u_s(P, \theta) = P - \sqrt{\theta}$. Describe the equilibrium in words. In particular, which cars are traded in equilibrium?