MICROECONOMIC THEORY II

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- In fact, hidden information is probably a better expression for describing this type of asymmetric information.
- Adverse selection is rather a possible consequence of this asymmetric information.

Adverse selection in stock market

"Just as a car buyer can never be sure whether information is being withheld by the seller, in the financial markets a buyer can never be sure whether there is something going on with a stock that is beyond his purview. The person on the other side of the trade might have insider information on the company, or he might know that there is a much larger overhang of potential selling, the demand the buyer sees being a first trickle in what will emerge as a flood of selling.
The adverse selection problem is especially troublesome for market makers, and particularly for market makers in specialized arenas, such as corporate bonds, mortgage securities, and emerging markets."

——A Demon of Our Own Design, Richard Bookstaber

Consequence of Adverse selection in stock market

"Market makers often didn't know who was on the other side of their trade, whether it was a tipped-off hedge fund manager who knew a stock was about to rocket higher (or plunge) or a dumb-as-dirt day trader making a reckless gamble. Because of that ignorance, market makers often would only buy the stock at a low price, or sell at a high price, in order to protect themselves. In response to the chance of getting winged by a well-armed gunslinger, market makers typically widen their quotes, providing a lower bid or higher offer. The result: wider spreads. "

—Dark Pools, Scott Patterson

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- Symmetric information, Zero-profit condition

$$p_i = \pi_i L \quad \forall i.$$

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- Competitive equilibrium price under asymmetric information

$$p^* = E(\pi|\pi \geq h(p^*))L,$$

$$E(\pi|\pi \geq h(p^*)) = \frac{\int_{h(p^*)}^{\bar{\pi}} \pi dF(\pi)}{1-F(h(p^*))}$$

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- Only consumer that is certain to have an accident buy the insurance.


BUY
FOR SALE
2001 GMC SIERRA
160,xxx miles

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  2. Group two: total income Y_2 and

$$u_2 = M + \sum_{i=1}^{n} \frac{3x_i}{2}$$

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$$D(p) = \begin{cases} \frac{Y_2+Y_1}{p}, & p < 1 \\ \frac{Y_2}{p}, & 1 < p < \frac{3}{2} \\ 0 & p > \frac{3}{2} \end{cases}$$

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- Equilibrium

$$p = \begin{cases} 1, & \text{if} \quad Y_2 < N \\ \frac{Y_2}{N} & \text{if} \quad \frac{2Y_2}{3} < N < Y_2 \\ \frac{3}{2} & \text{if} \quad N < \frac{2Y_2}{3} \end{cases}$$

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$$S(p) = \frac{pN}{2}$$

- Average quality supplied

$$\mu = \frac{p}{2}.$$

Asymmetric information (2)

- Total demand

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- NO trade in equilibrium, even if at *any given price* $p \in [0,3]$, there are group one trader willing to sell at a price which group two are willing to pay.

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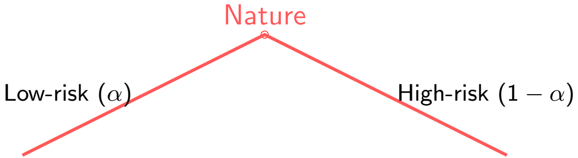
  - ➢ Insurance company (*Receiver*) responds given belief $\beta(B, p)$: accept, reject.

Insurance signaling game



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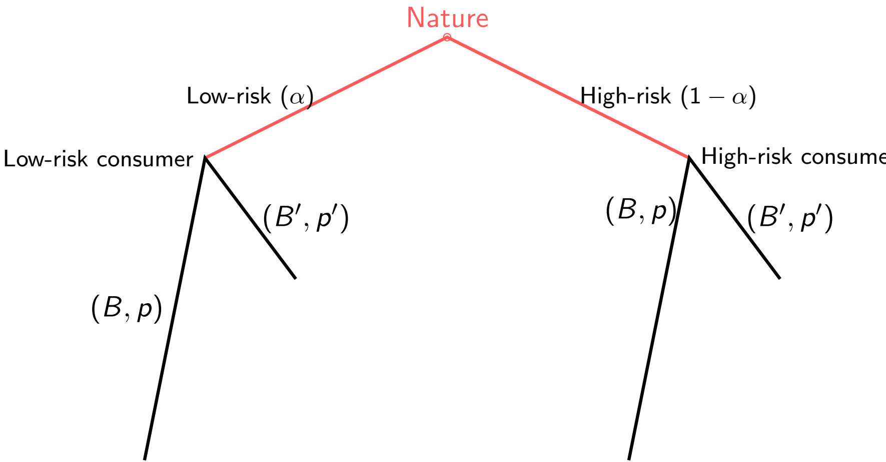

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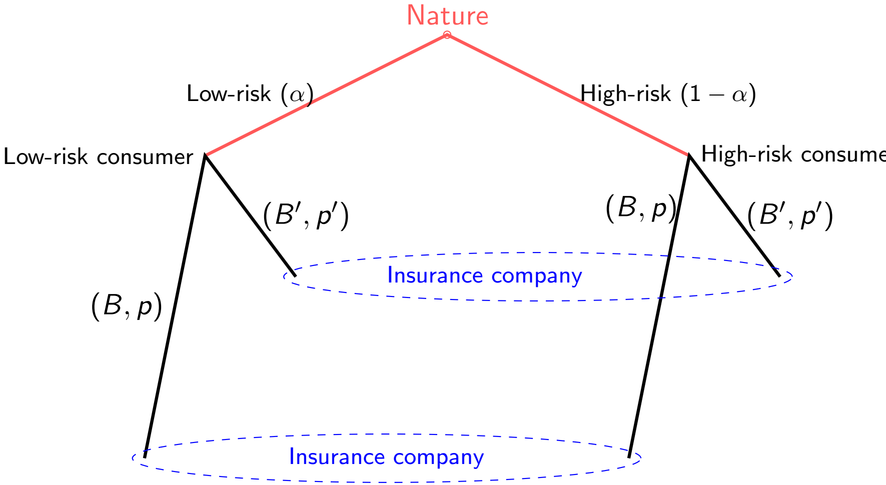

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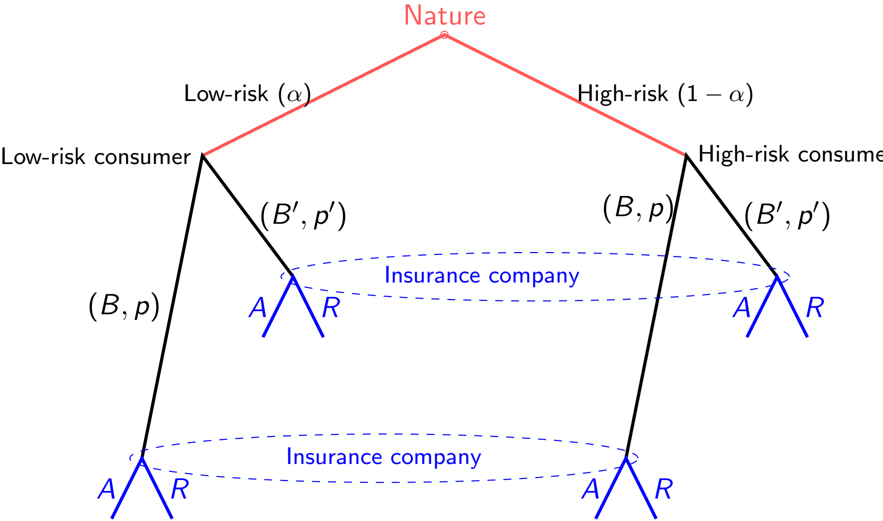

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Consumer optimization review

- Individual's optimal insurance problem:

$$\max_{B} \quad \pi u\left(w - L + B\left(1 - q\right)\right) + \left(1 - \pi\right) u\left(w - Bq\right)$$
$$s.t.\ B \geq 0,\ \ B \leq w/q$$

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- The first-order conditions:

$$\pi u'\left(w-L+B\left(1-q\right)\right)\left(1-q\right)-\left(1-\pi\right)u'\left(w-Bq\right)q-\lambda\leq 0;$$
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- Thus, the optimal B satisfies

$$\frac{\pi u'\left(w-L+B\left(1-q\right)\right)}{\left(1-\pi\right)u'\left(w-Bq\right)} = \frac{q}{1-q}$$

Graphical illustration

Single crossing property

On consumer choices

- Note that $P = Bq$ and $B(1-q) = B - P$, so

$$MRS(B,P) = \frac{\pi u'\left(w - L + B - P\right)}{\pi u'(w - L + B - P) + (1-\pi)\,u'\left(w - P\right)} = \frac{P}{B} = q.$$

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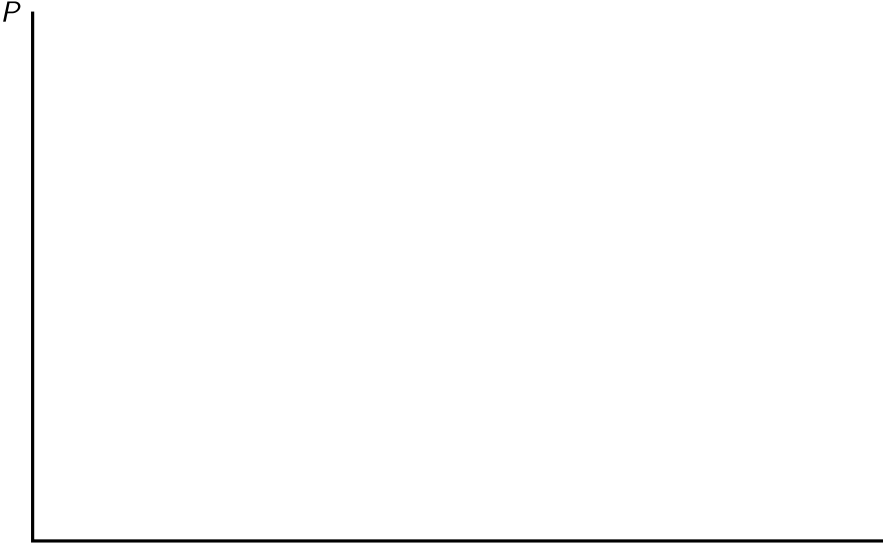

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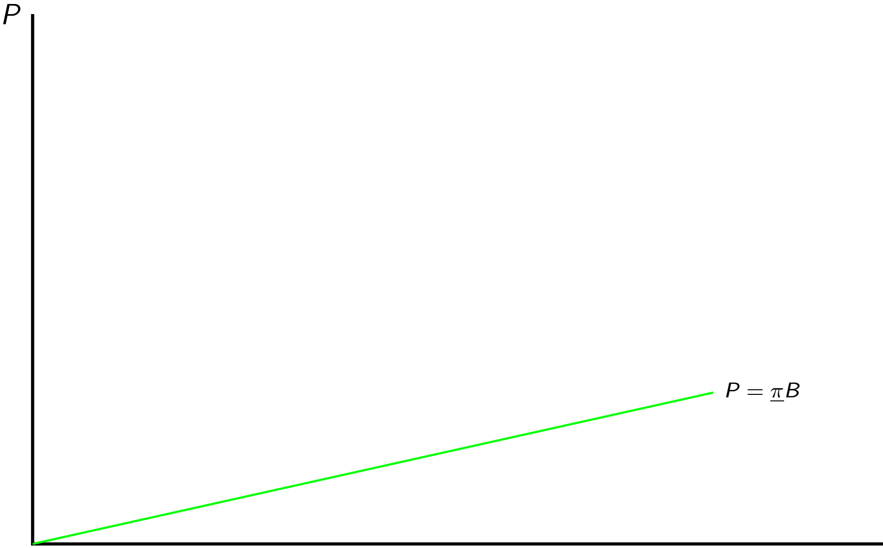

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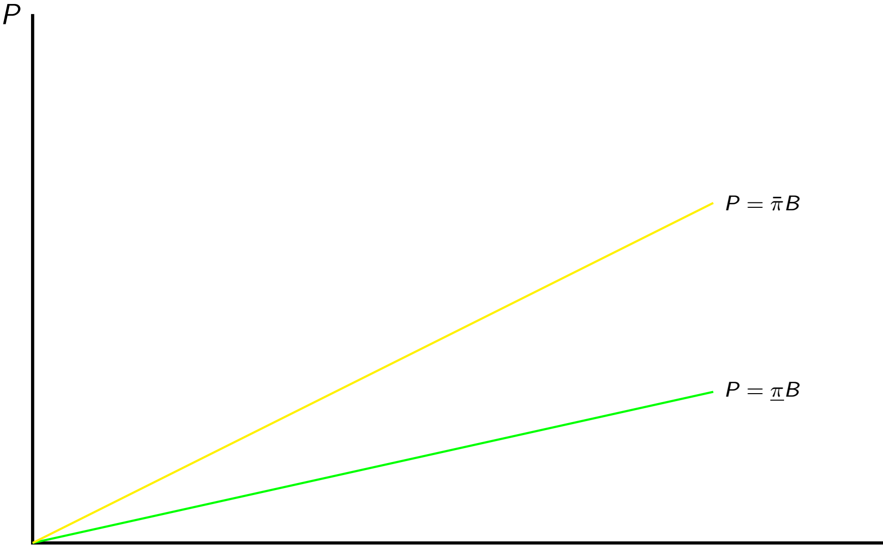
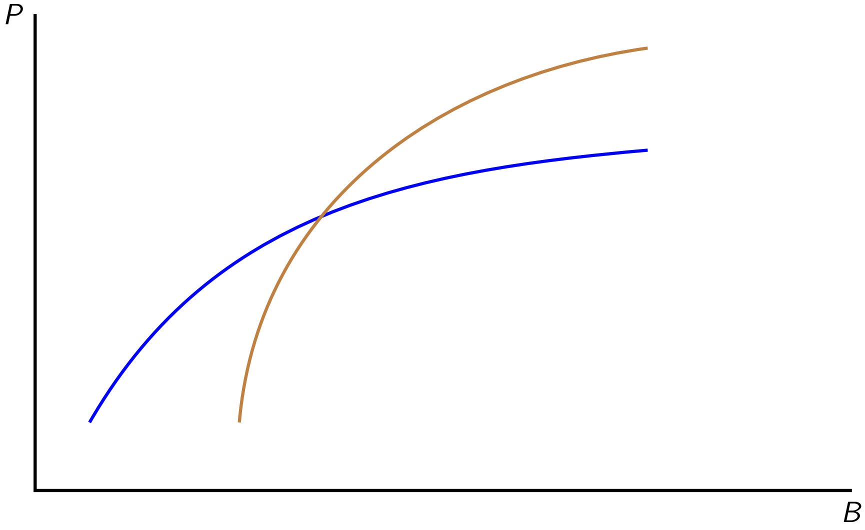


Consumers' preferences for risks



EQUILIBRIUM

On sequential equilibrium

- Lemma 8.1. (Jehle & Reny) Let

$$\tilde{u}_l \equiv \max_{(B,p)} u_l(B,P) \quad s.t. p = \bar{\pi} B \leq w, \qquad u_h^c \equiv u_h(L, \bar{\pi} L).$$

And let $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ be a s.e. with utilities for low-risk and high-risk are, respectively, u_l^* and u_h^*. Then

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  And let $(\psi_l, \psi_h, \sigma(\cdot), \beta(\cdot))$ be a s.e. with utilities for low-risk and high-risk are, respectively, u_l^* and u_h^*. Then

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  - ➢ $u_h^* \geq u_h^c$.

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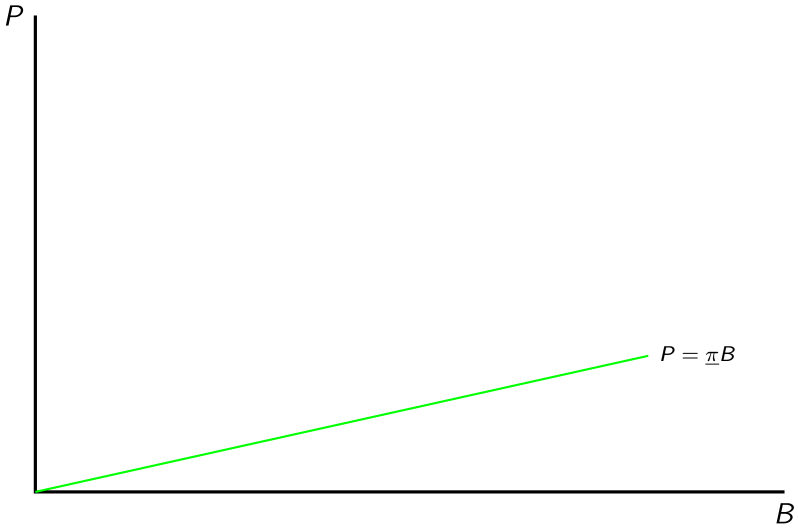
  - $u_h^c \equiv u_h(\psi_h) \geq u_h(\psi_l)$, where $u_h^c \equiv u_h(L, \bar{\pi} L)$ is high-risk's utility in competitive equilibrium with full information

Existence of separating equilibrium



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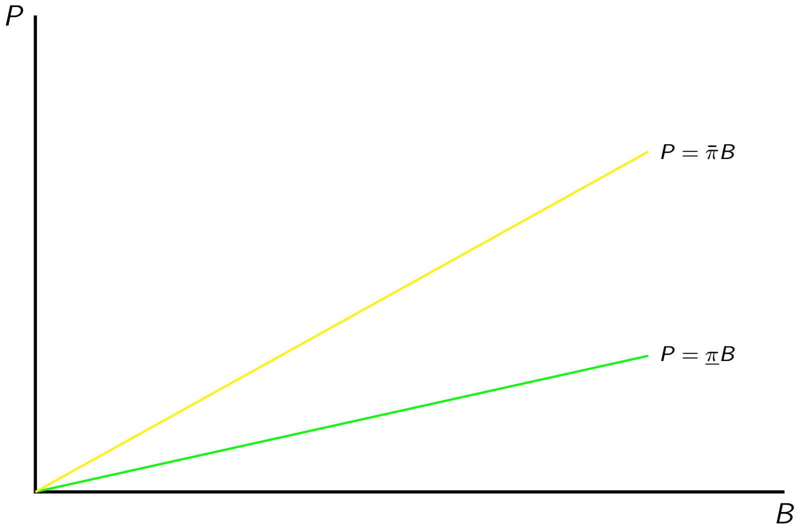

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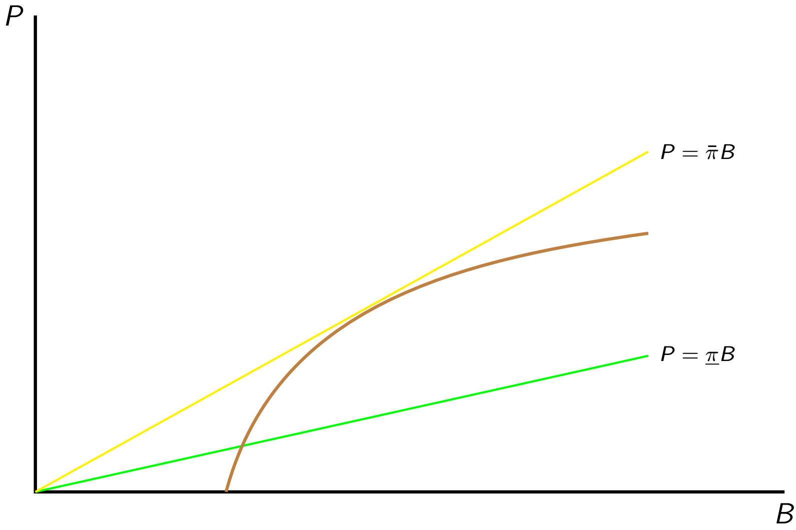

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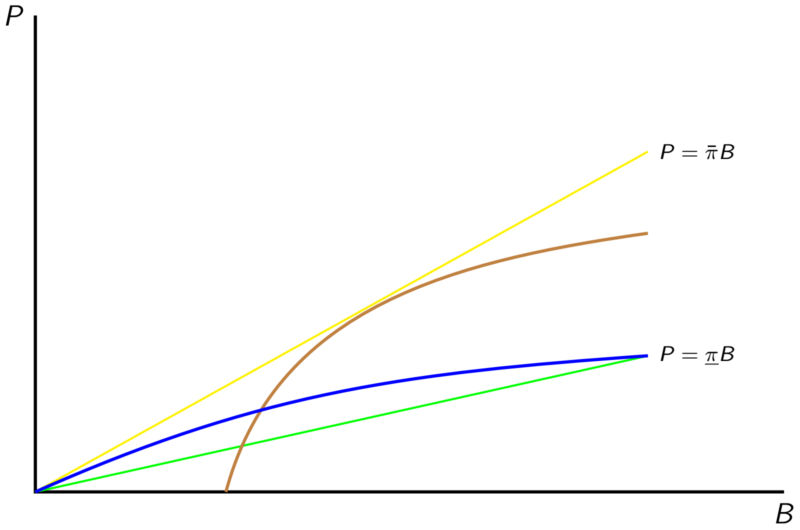

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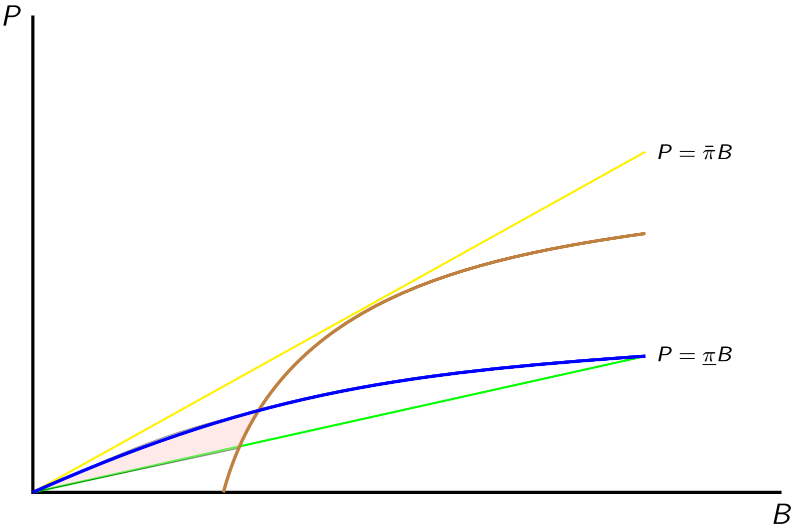

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- Worker's utility

$$u(w, e) = w - \frac{e}{2\theta}$$

  if accepts offer; zero otherwise.

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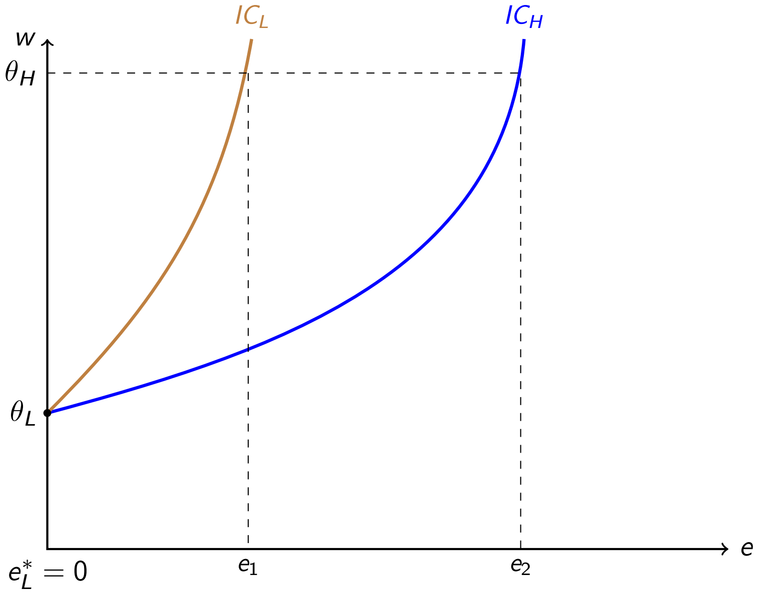

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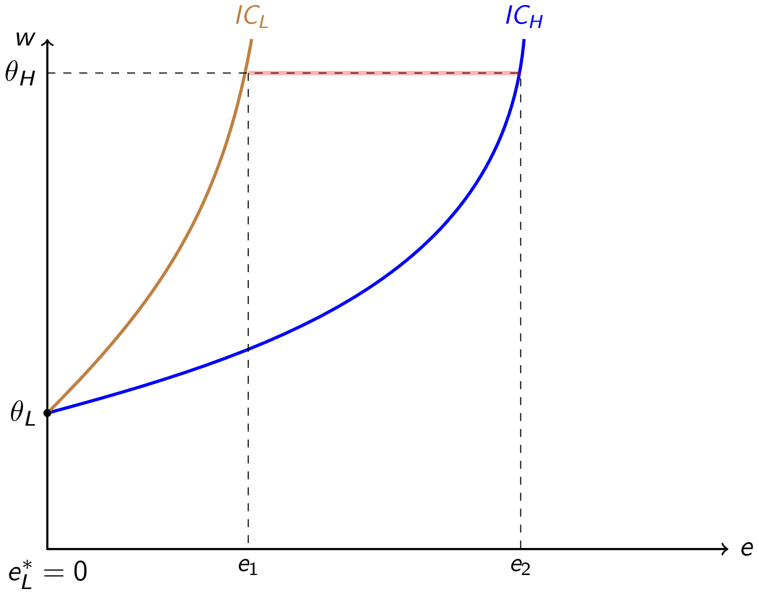

Apply IC to separating equilibrium

- Set of separating equilibria

$$\begin{aligned} e_L^* &= 0, \qquad & e_H^* &\in [e_1, e_2]; \\ w(e_L^*) &= \theta_L \qquad & w(e_H^*) &= \theta_H. \end{aligned}$$

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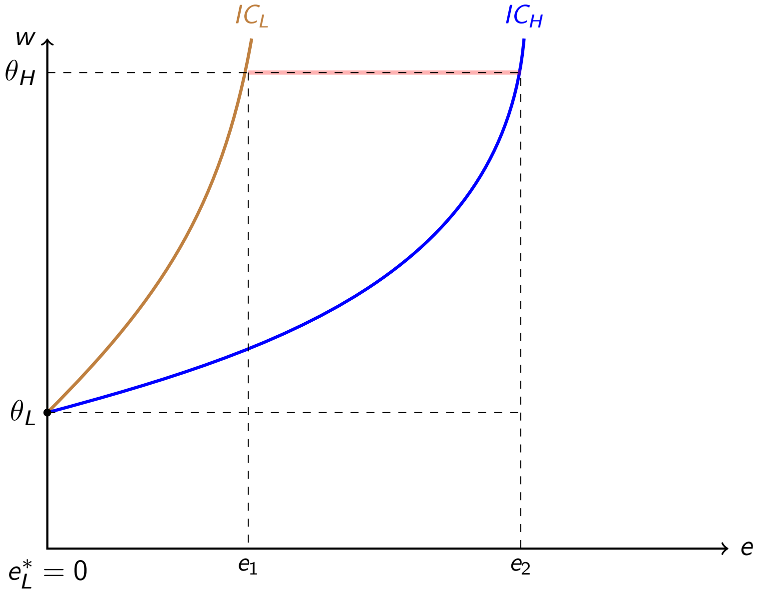

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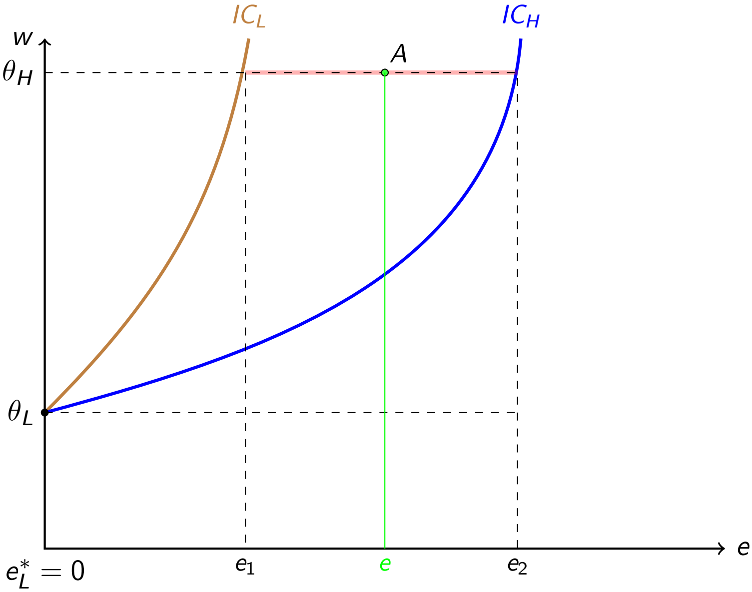

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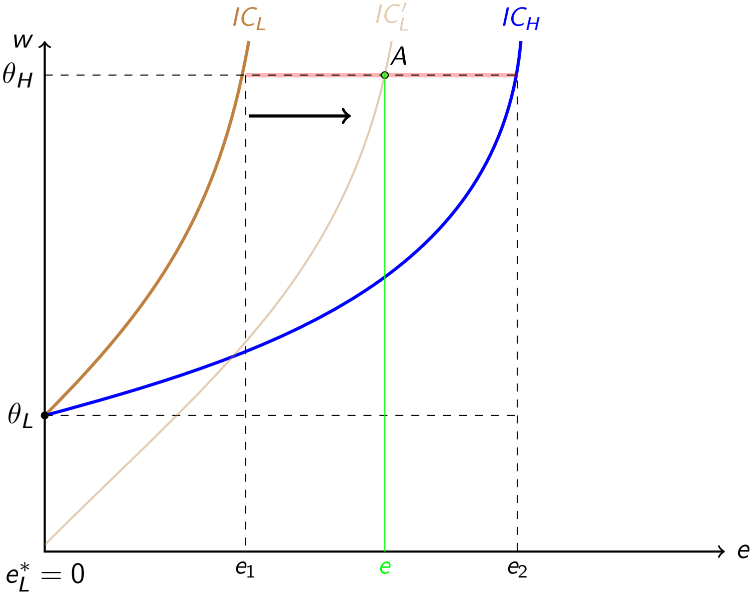

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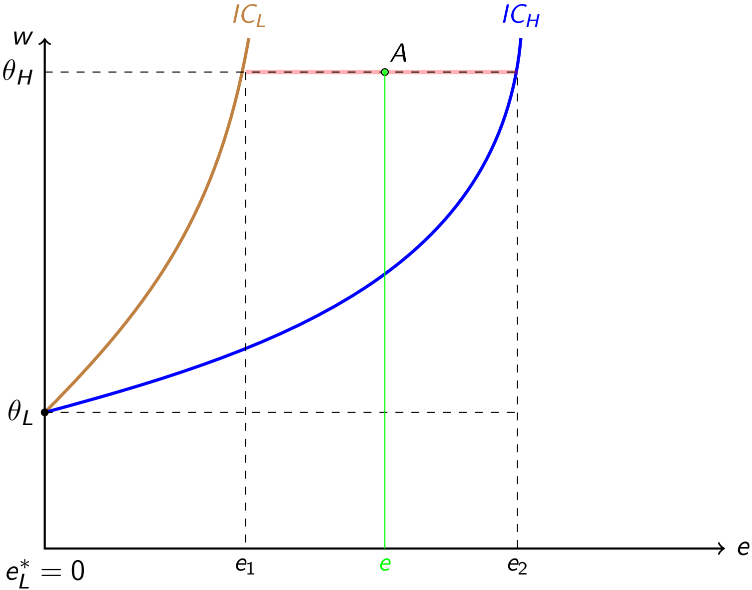

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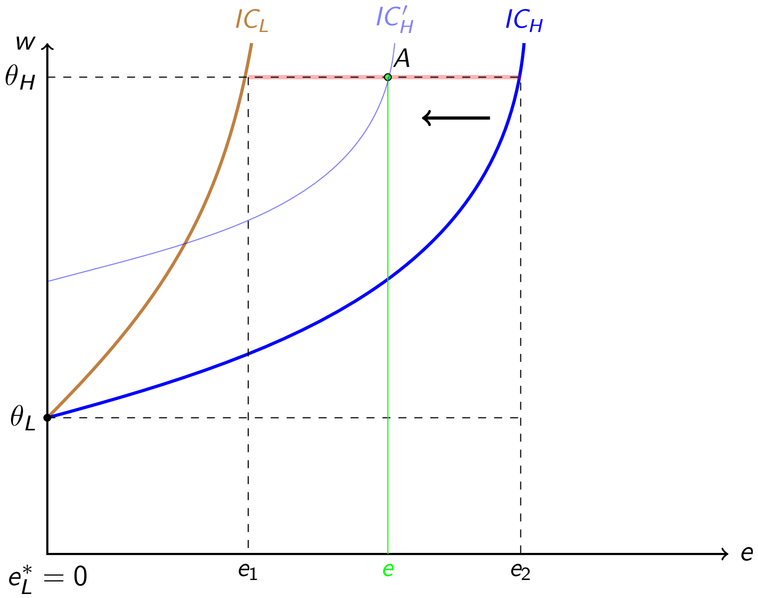

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- θ_L type has no incentive to deviate

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- Thus, off-equilibrium education level can come only from θ_H

$$\Theta^{**}(e) = \{\theta_H\}.$$

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- Thus, the separating equilibrium

$$\{e_L^*(\theta_L), e_H^*(\theta_H)\} = \{0, e_2)\}$$

violates IC.

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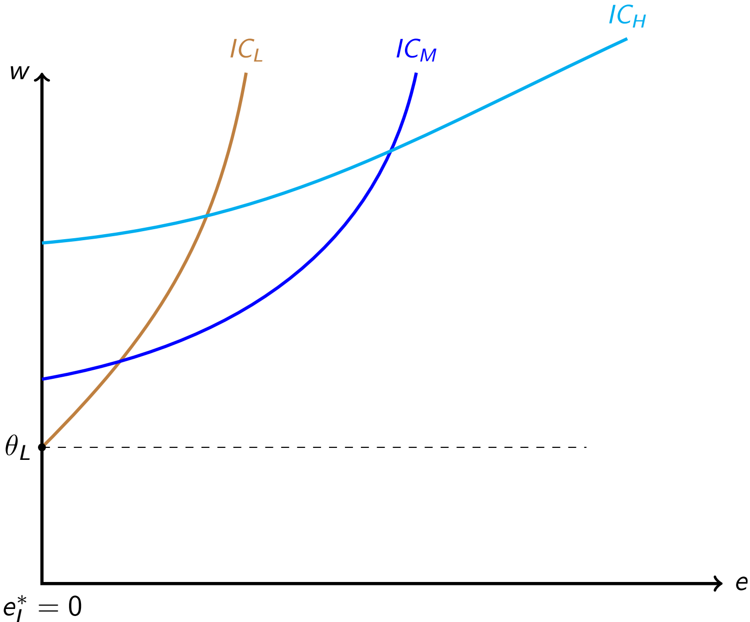

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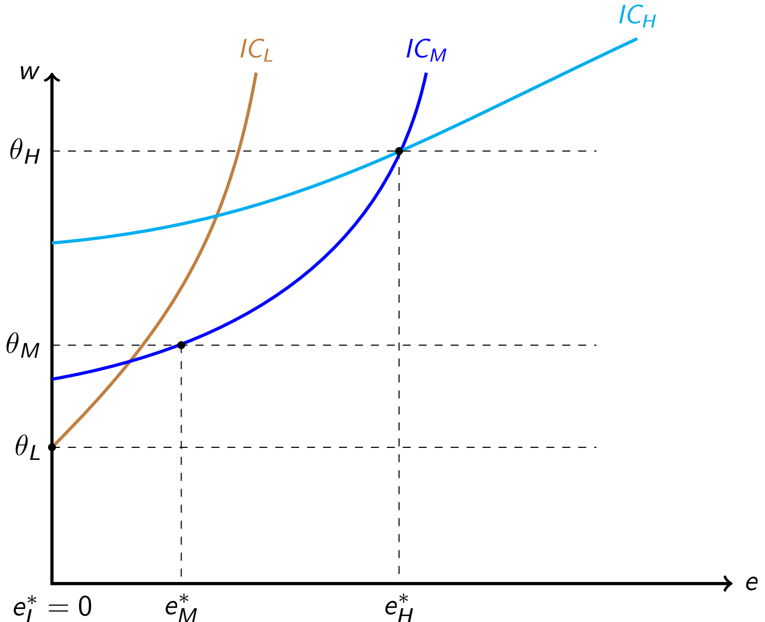

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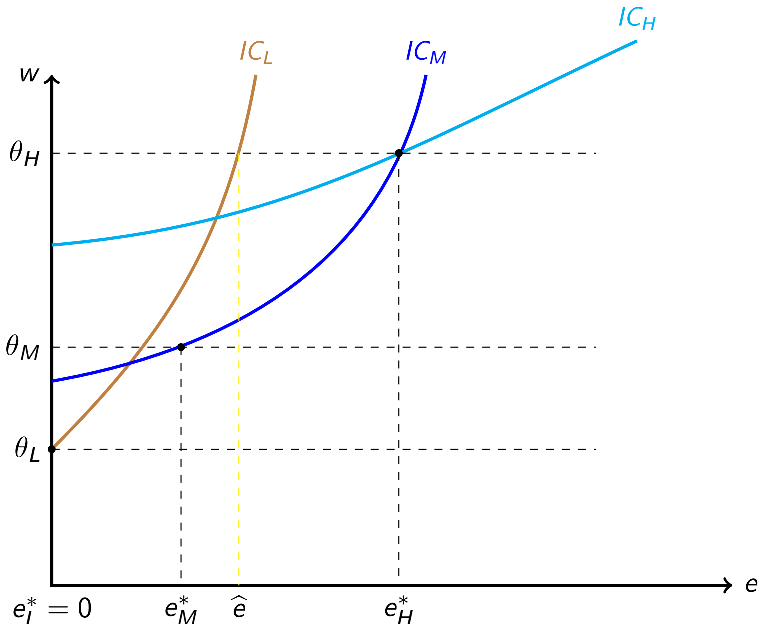

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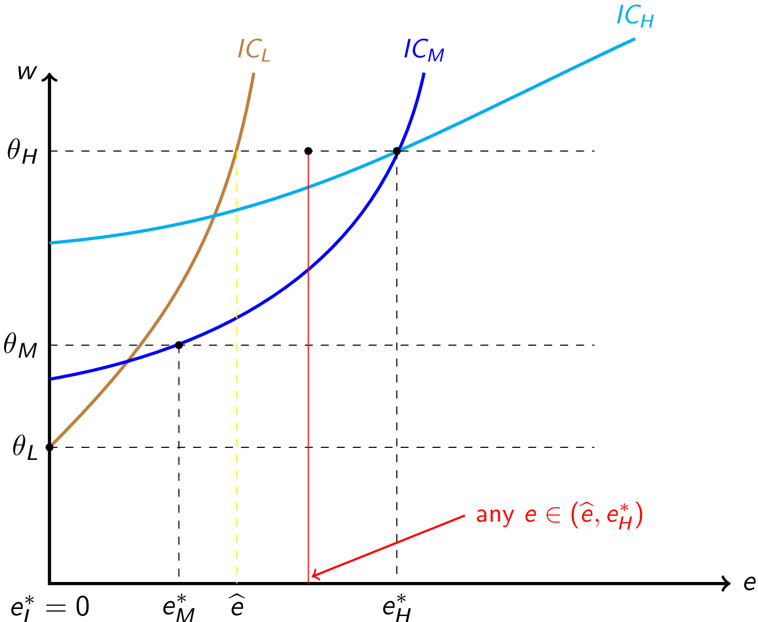

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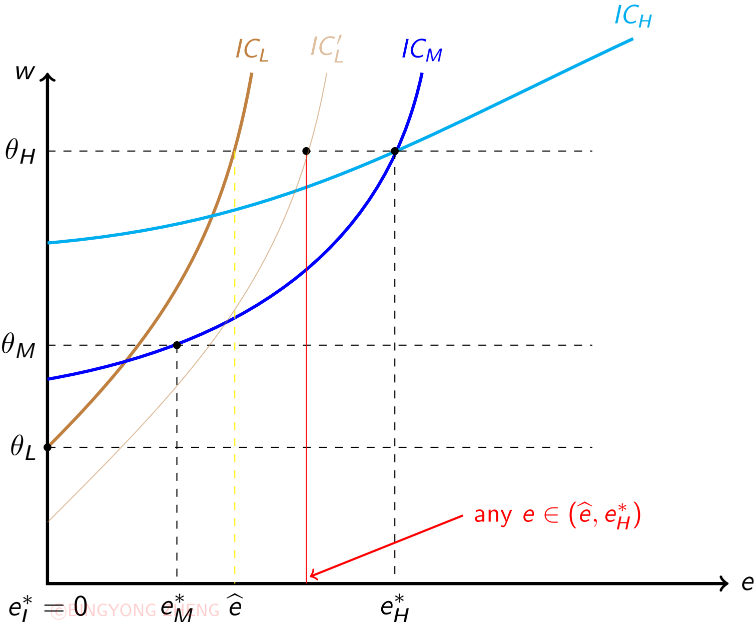

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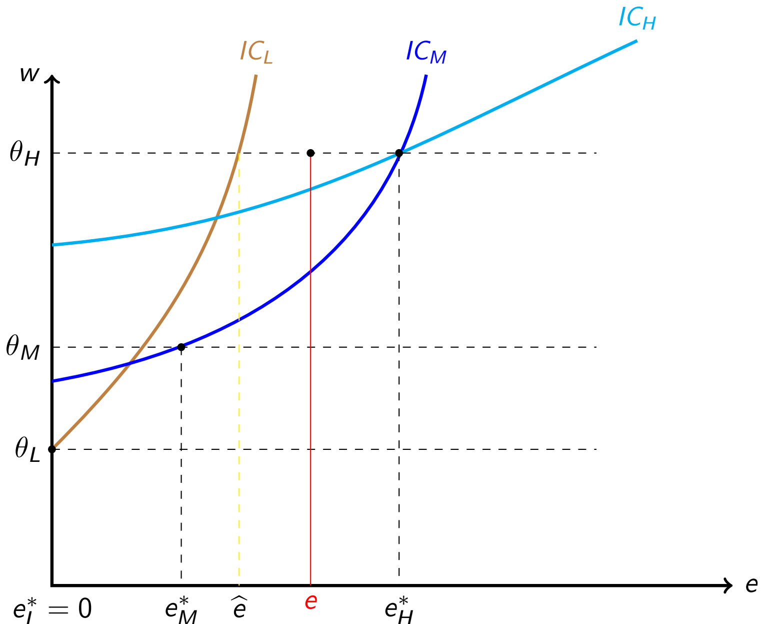

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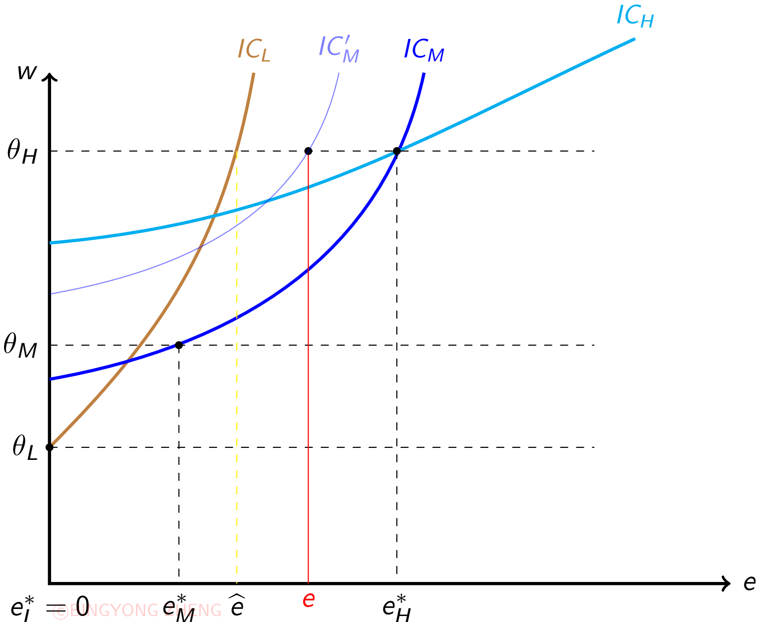

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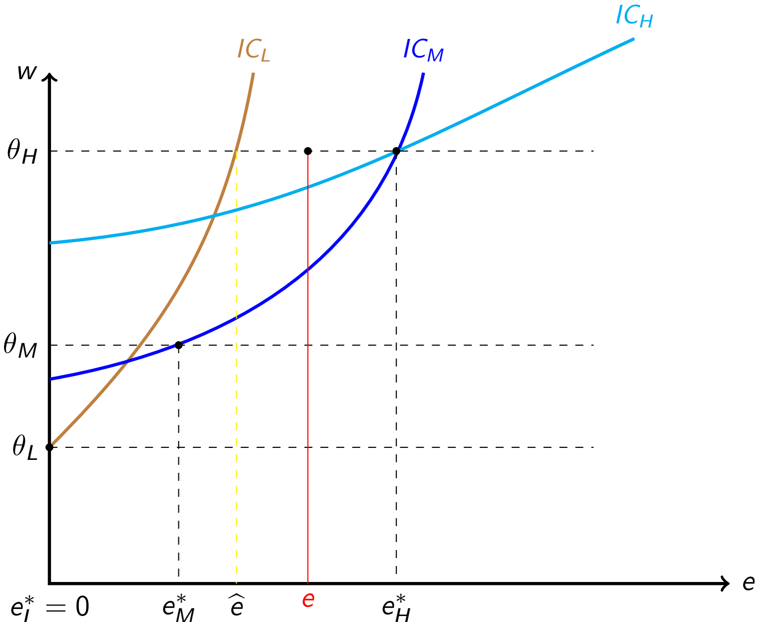

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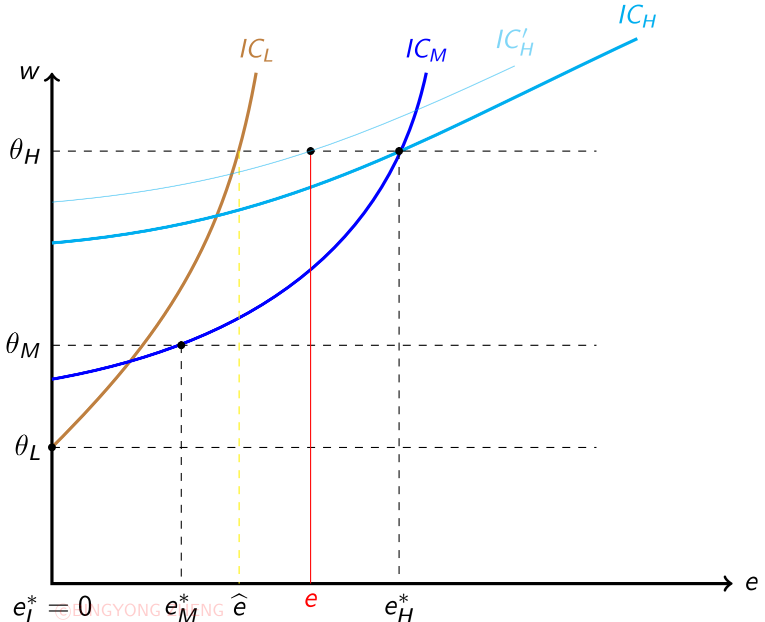

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- θ_L type sending message $e \in (\widehat{e}, e_H^*)$ is equilibrium dominated

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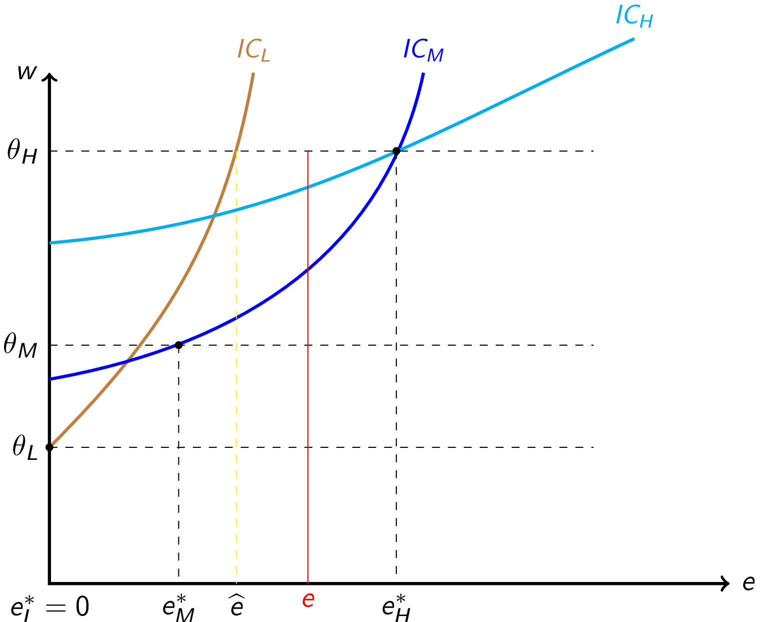
- Hence, observing $e \in (\widehat{e}, e_H^*)$, the firm's belief concentrate on θ_M and θ_H:

$$\Theta^{**} = \{\theta_M, \theta_H\}.$$

IC second step



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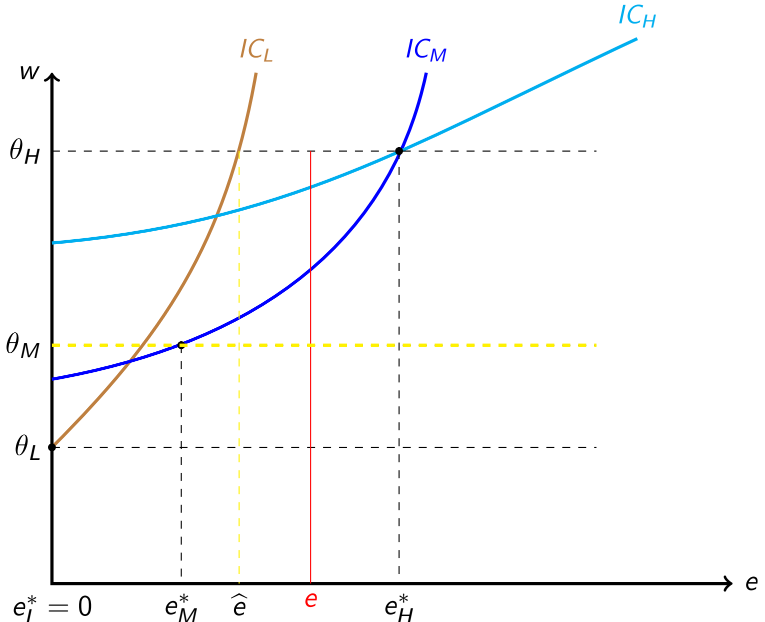

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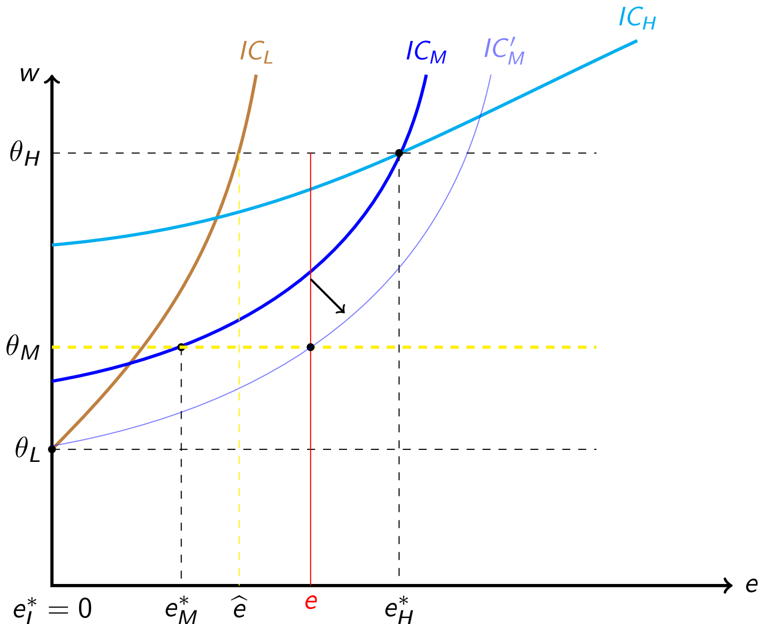

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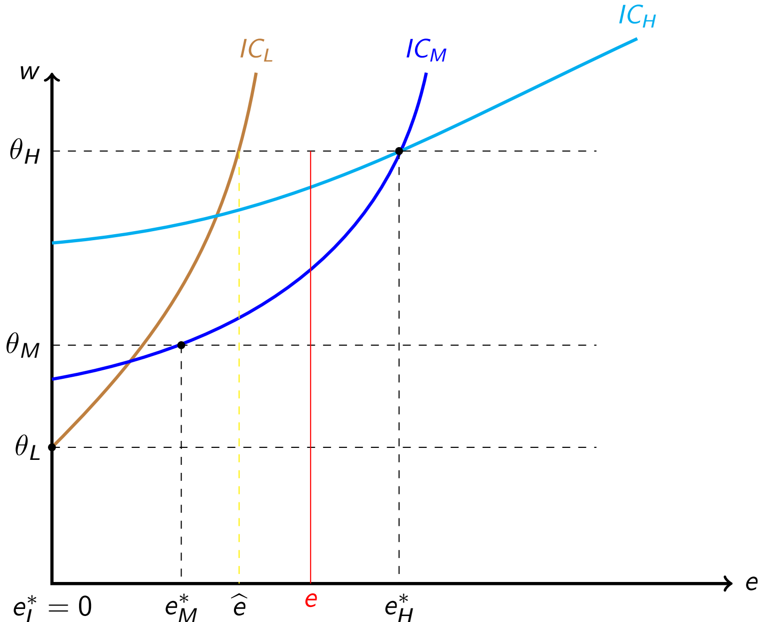

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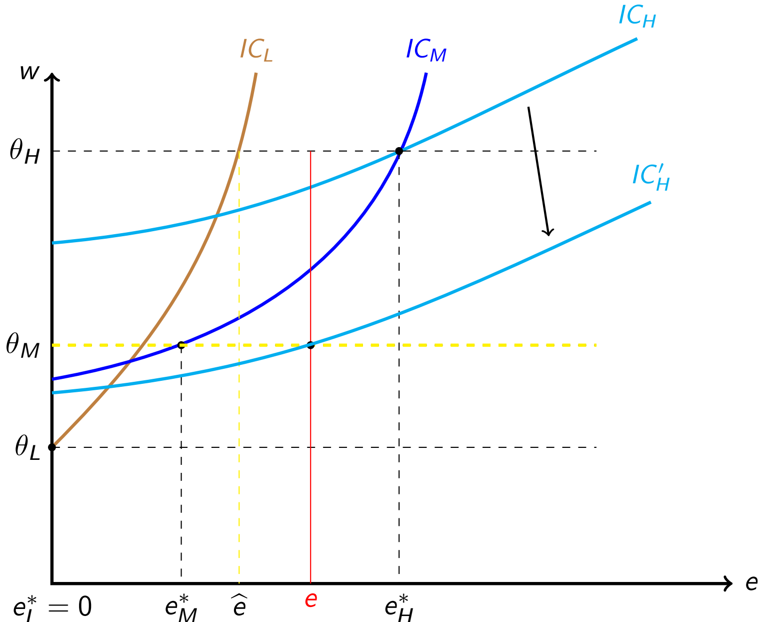

Second step

- Given firm's belief $\Theta^{**} = \{\theta_M, \theta_H\}$, the lowest wage to offer

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- Hence, there is no type of worker $\theta \in \Theta^{**}$ for whom deviation to $e \in (\widehat{e}, e_H^*)$ is profitable.

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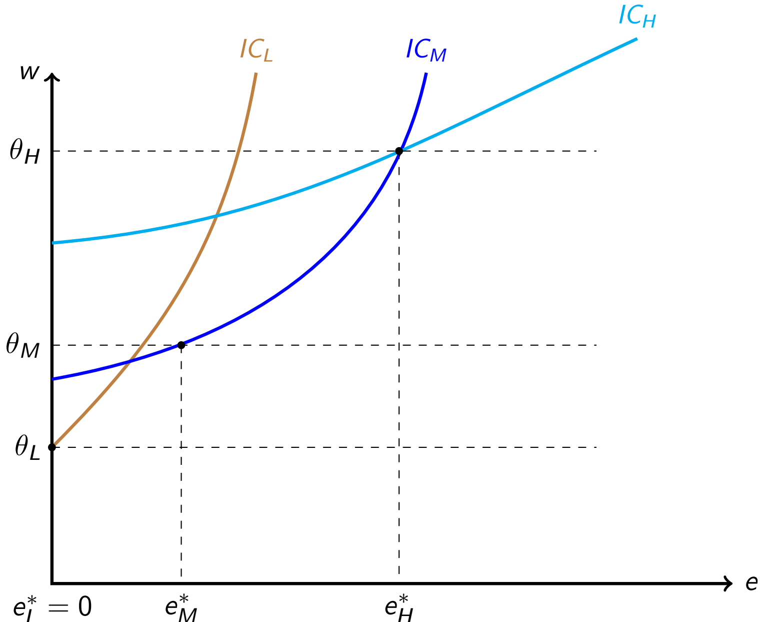
- Also let

$$D^o(\theta_k, \widehat{\Theta}, e') \equiv \{w \in [\theta_L, \theta_H] | u_k(e', w, \theta_k) = u_k^*(\theta_k)\}.$$

D1



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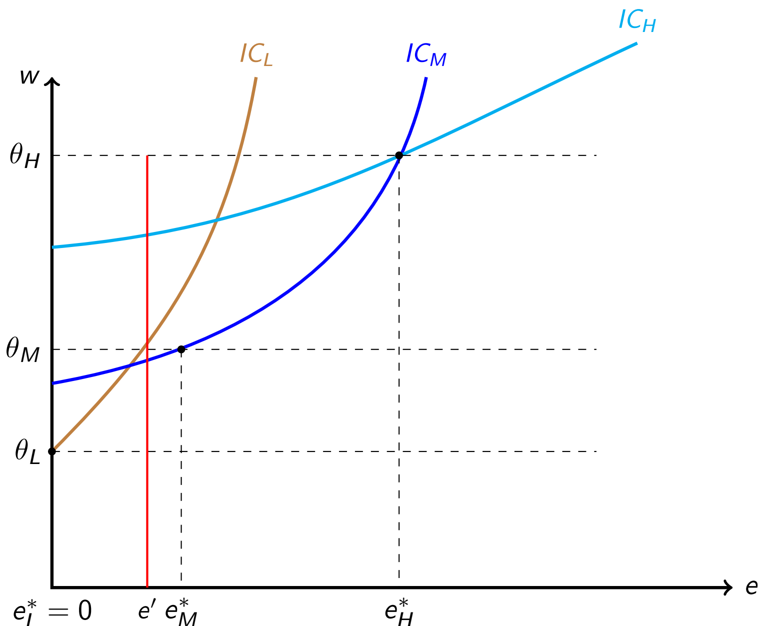

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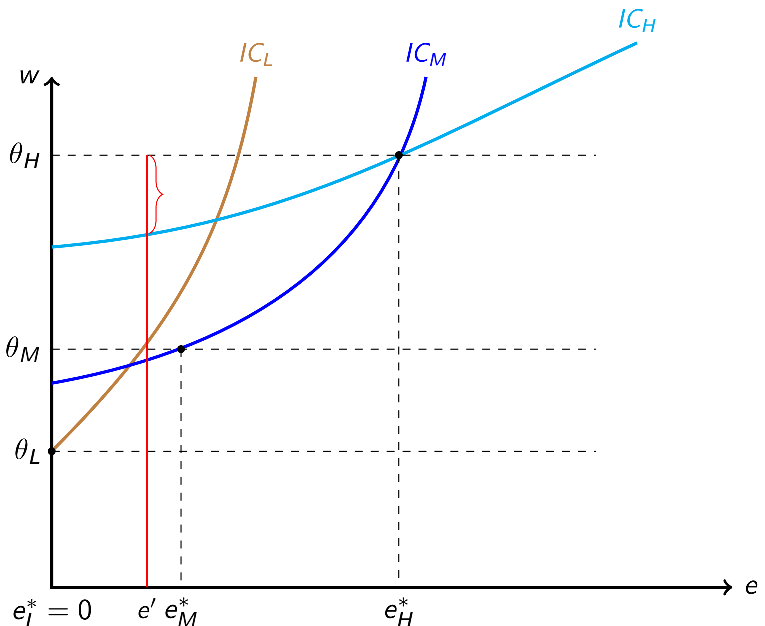

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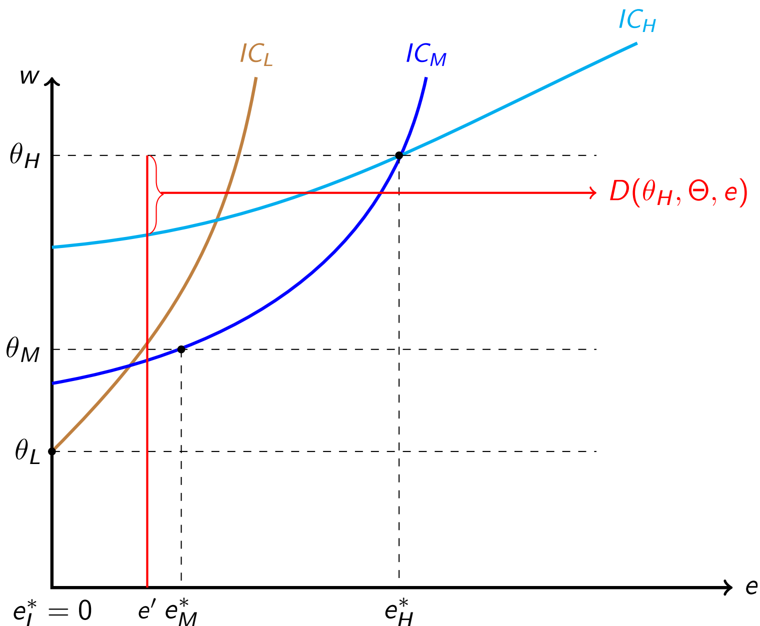

IC_H
IC_L
IC_M
w
θ_H
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θ_M
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e
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e'
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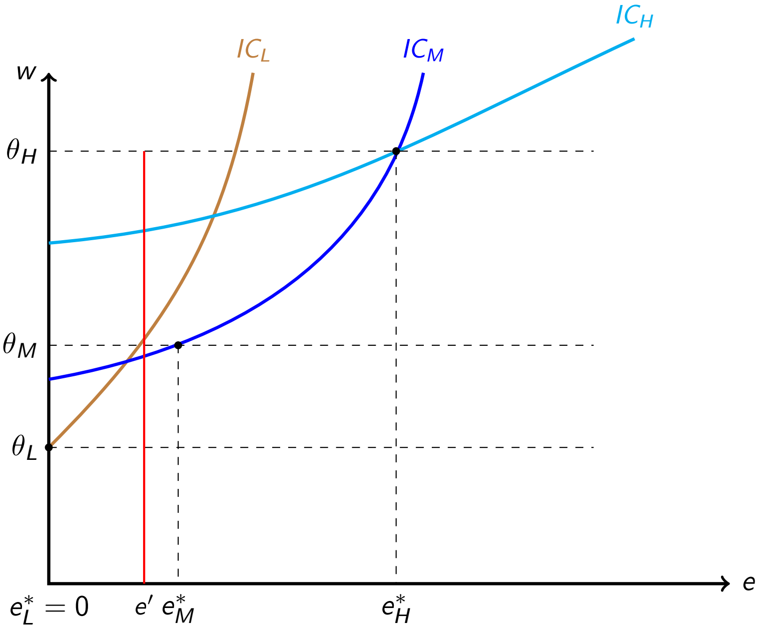

IC_H
IC_L
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θ_H
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e
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e' e_M^*
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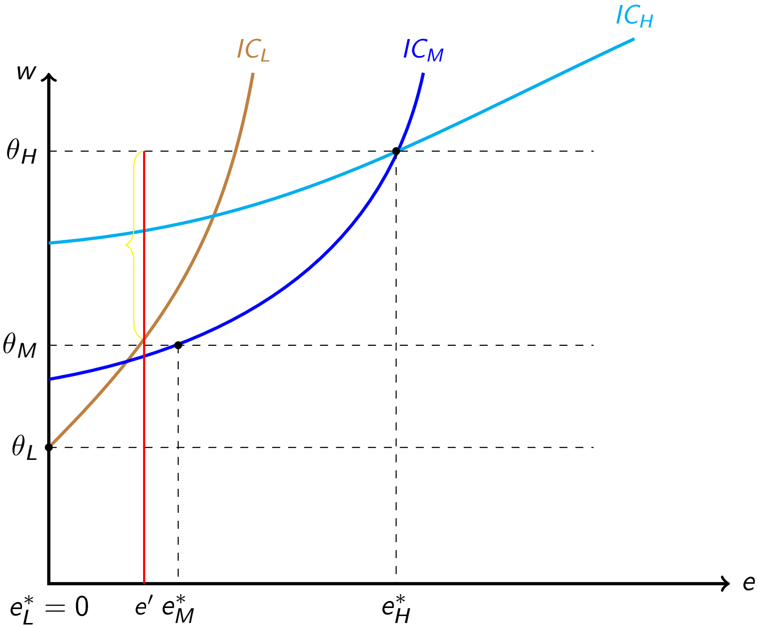

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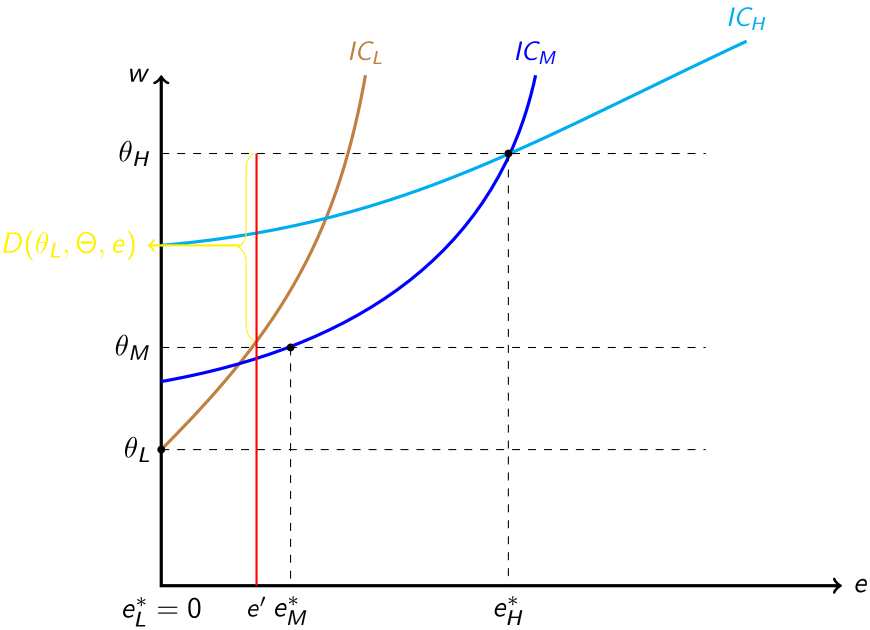

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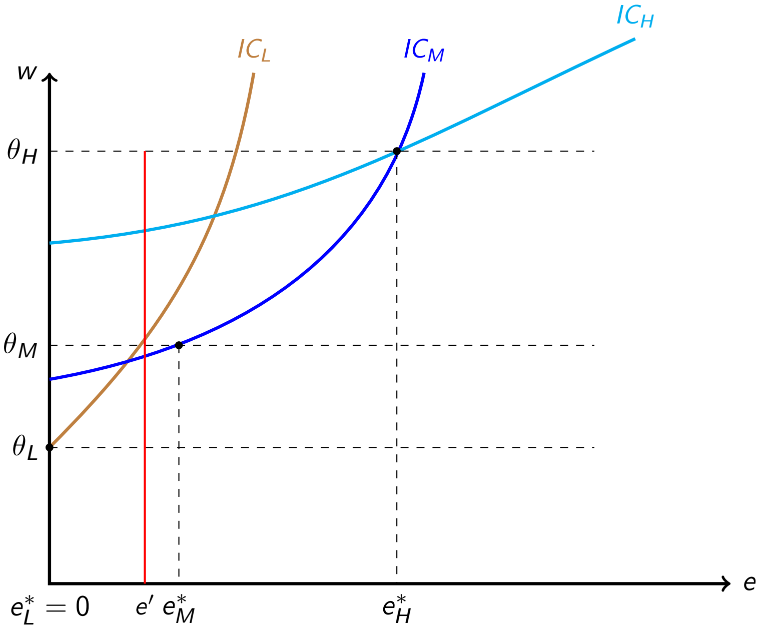
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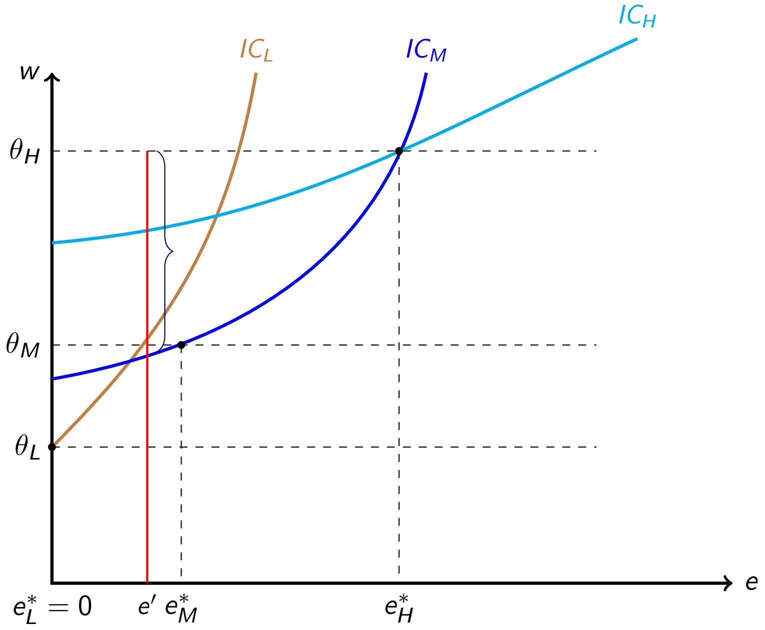

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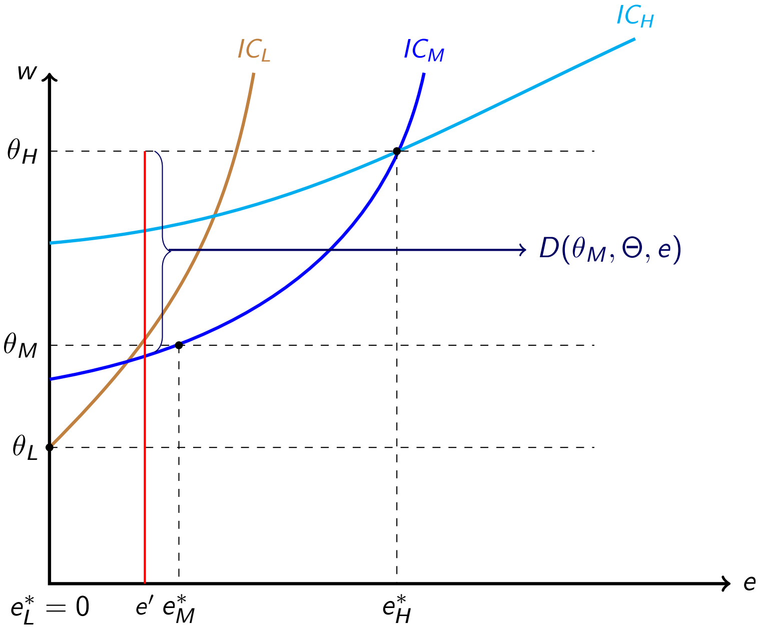

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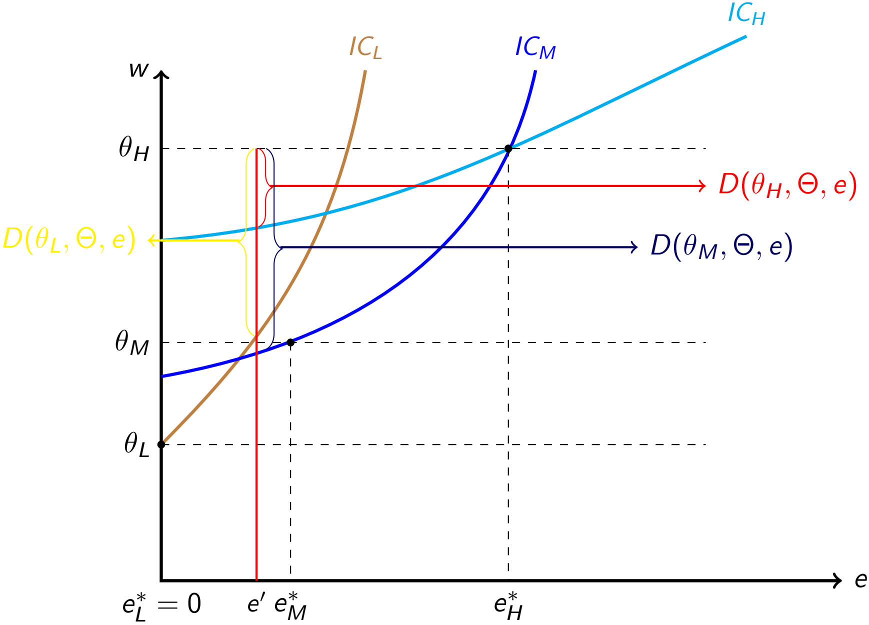
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D1 first step

- We see from the figure

$$D(\theta_H, \widehat{\Theta}, e') \bigcup D^o(\theta_H, \widehat{\Theta}, e') \subset D(\theta_M, \widehat{\Theta}, e').$$

θ_M type has more incentives to deviate to e' than θ_H type

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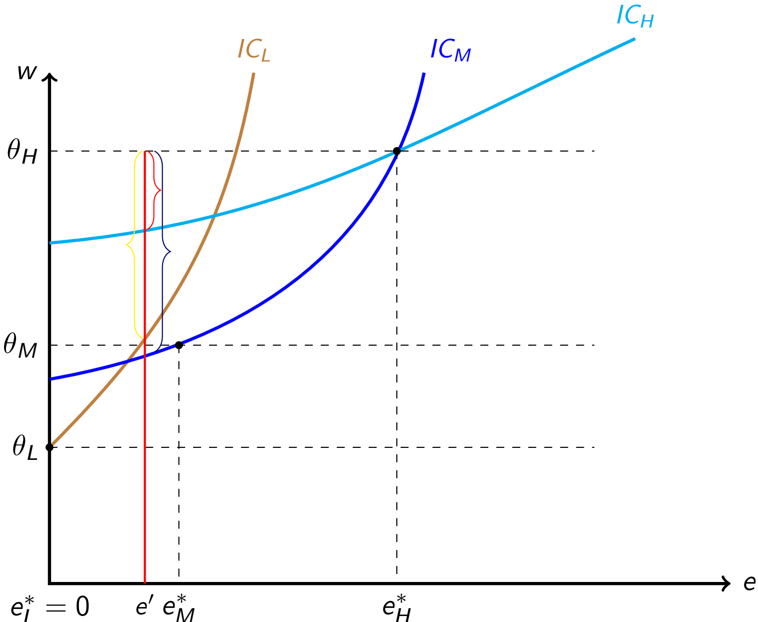
  θ_M type has more incentives to deviate to e' than θ_L type
- Applying the D1 criterion, the θ_M type is the most likely to deviate to e'

$$\Theta^{**}(e') = \{\theta_M\}.$$

D1 second step



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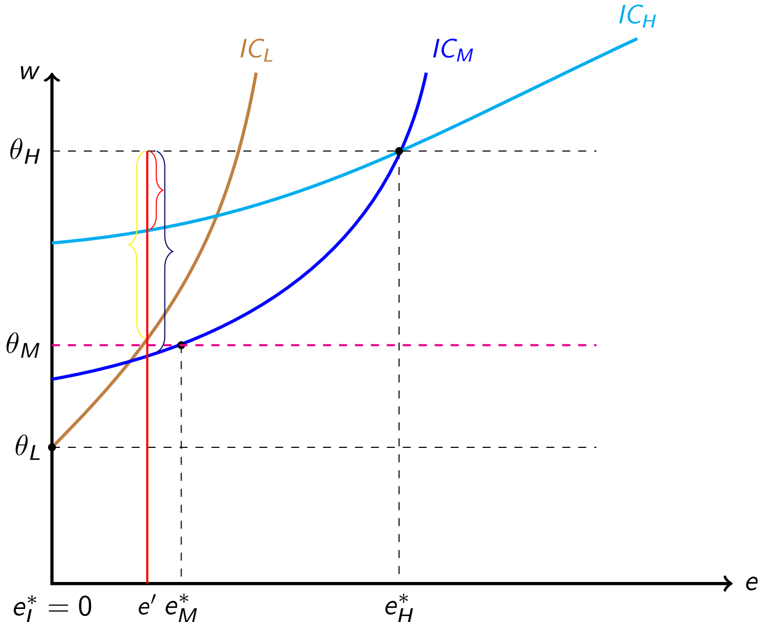

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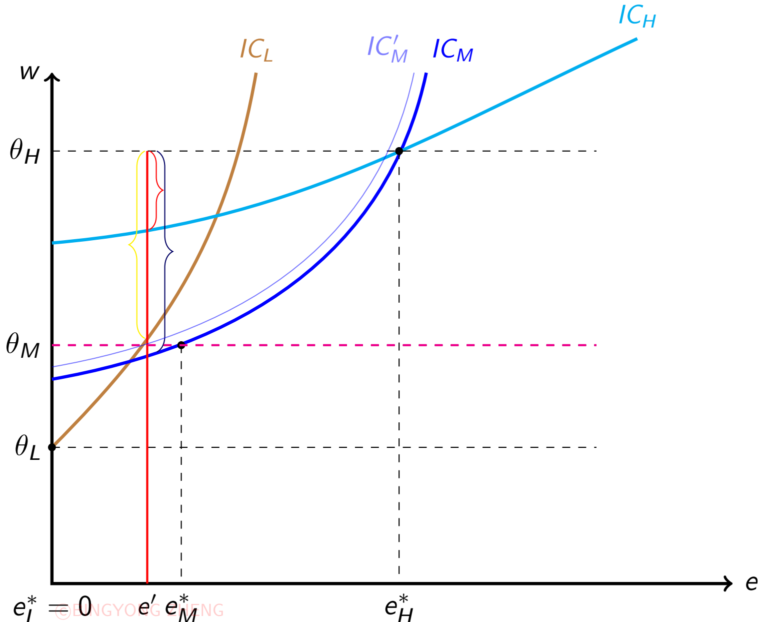

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$$\min_{w \in W^*(\Theta^{**}(e'), e')} u_M(e', w, \theta_M) > u_M^*(\theta_M).$$

Deviating towards e' is profitable!

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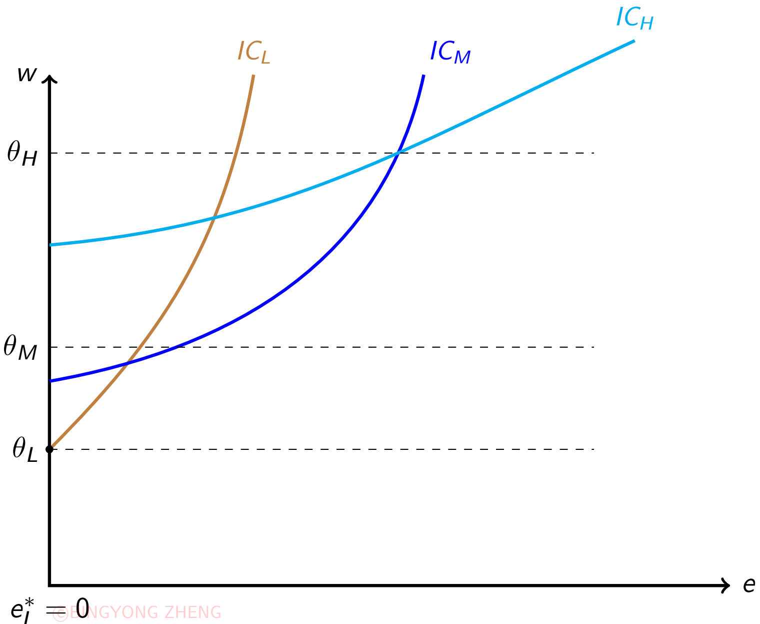
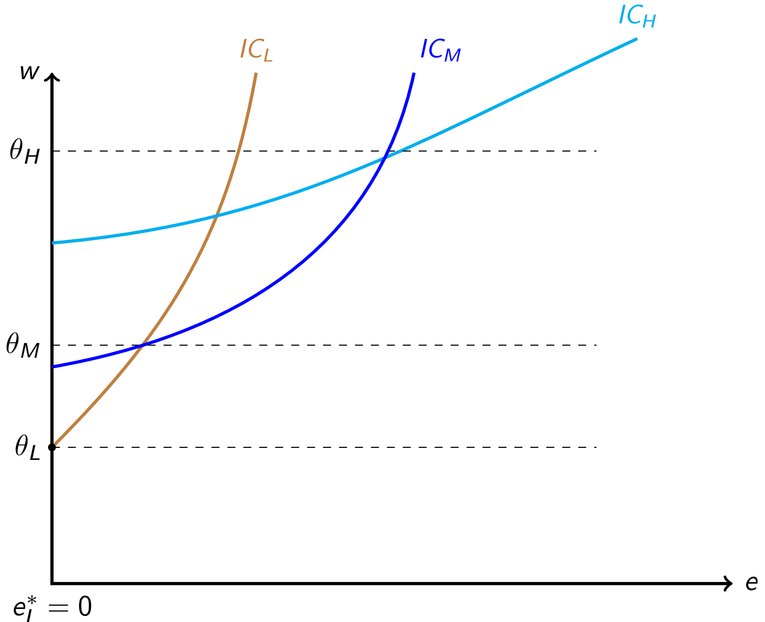
- So the equilibrium (e_L^*, e_M^*, e_H^*) violates the D1 criterion

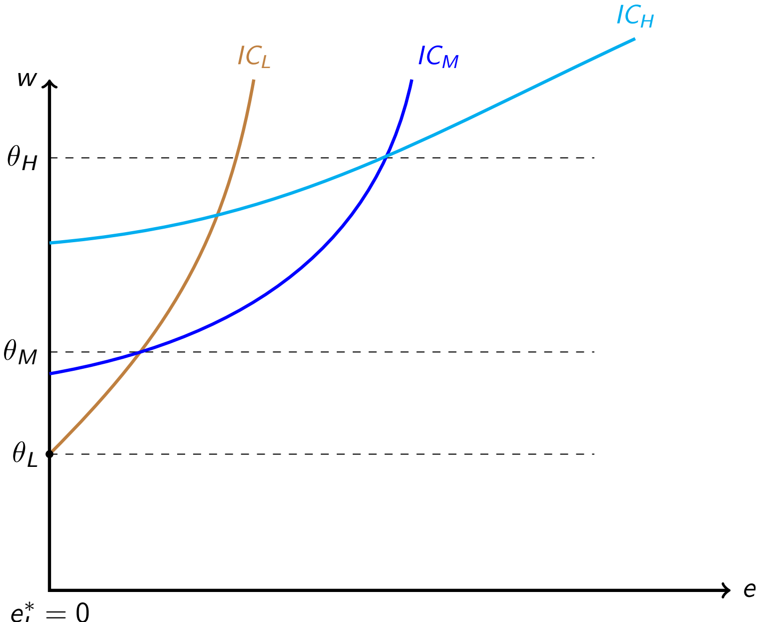
D1 second step continued



D1 SECOND STEP CONTINUED



D1 second step continued



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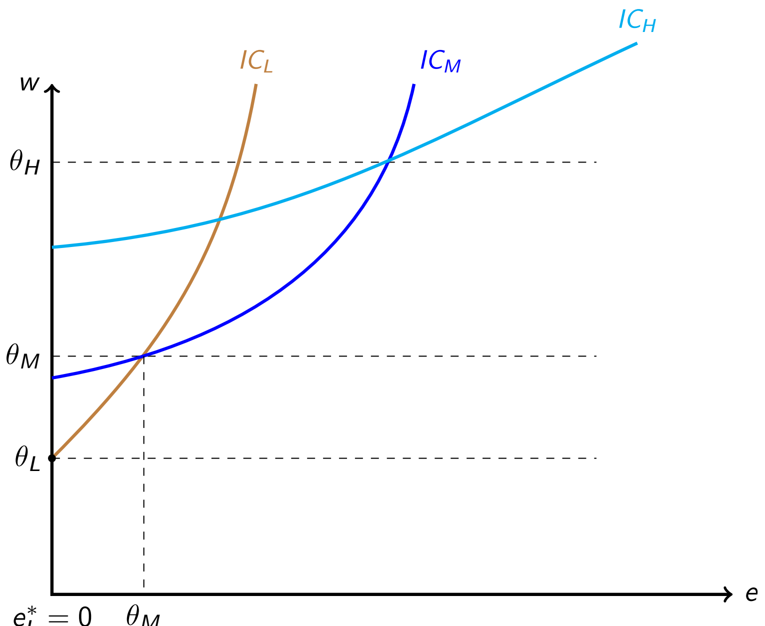

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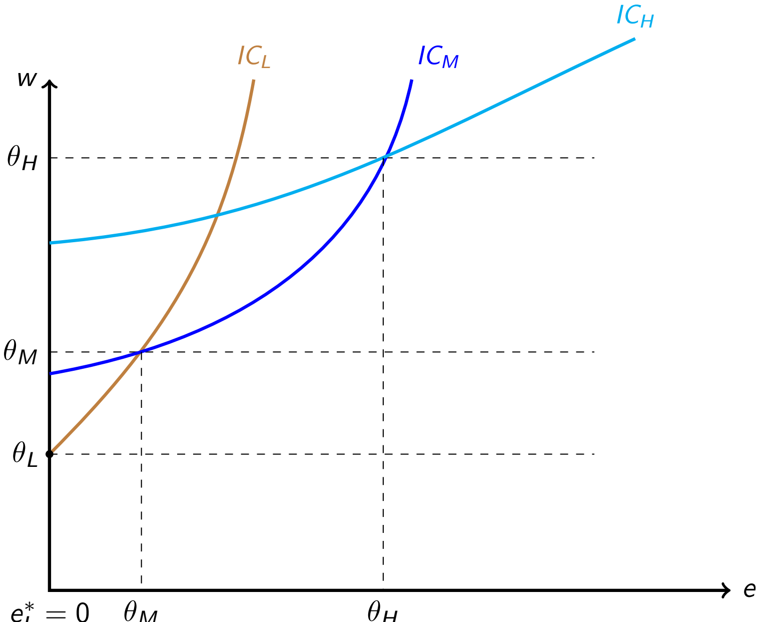

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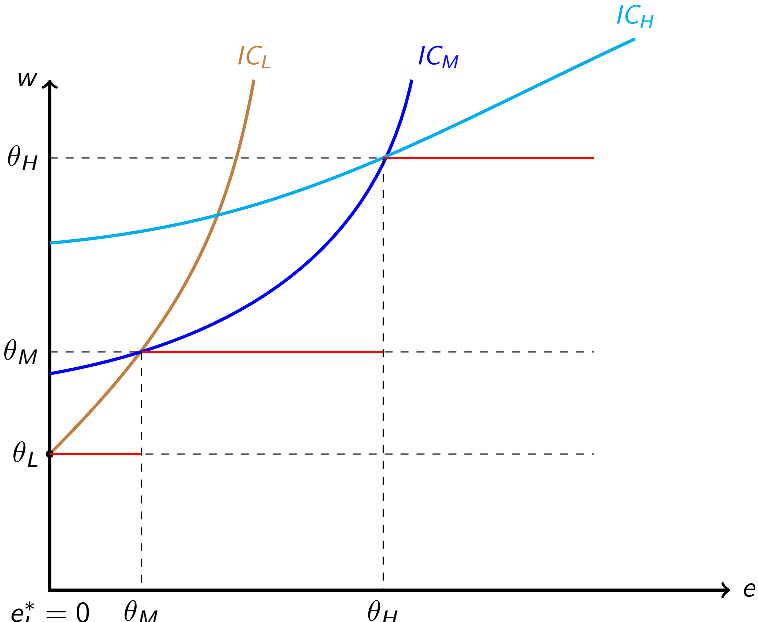
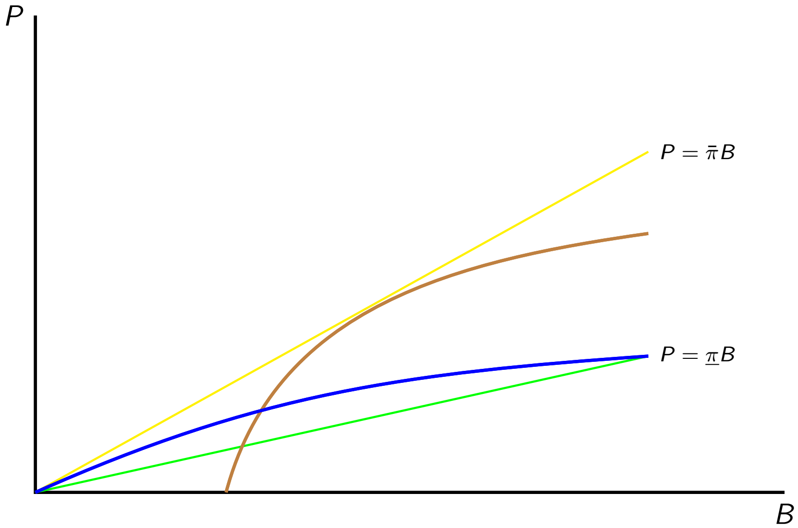


Insurance model: separating equilibrium



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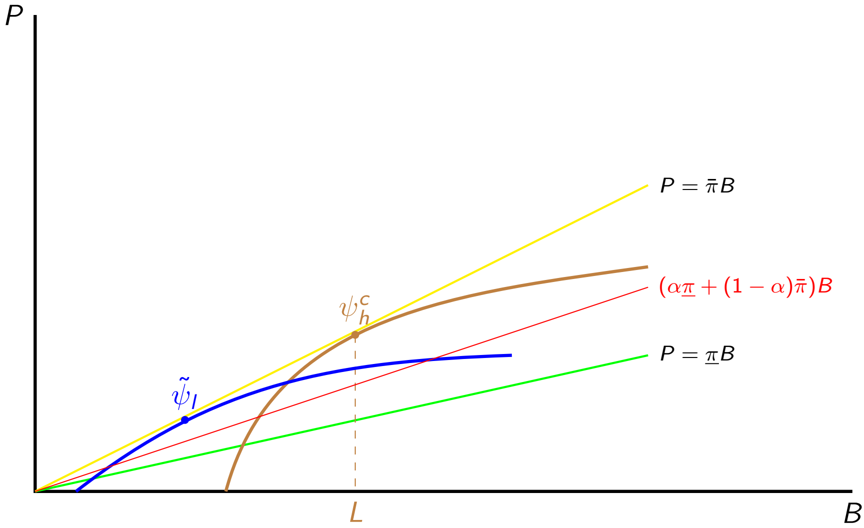

Apply IC to insurance model

- IC to insurance signaling game: Sequential equilibrium $(\psi_l, \psi_h, \sigma, \beta)$ satisfy IC if for all ψ ($\psi \neq \psi_l$ or $\psi \neq \psi_h$),

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- Theorem 8.3. (Jehle& Reny) There is a unique policy pair (ψ_l, ψ_h) that can be supported by a sequential equilibrium satisfying the intuitive criterion. And this equilibrium is the best separating equilibrium for the low-risk consumer.

Screening: competition

Competitive screening

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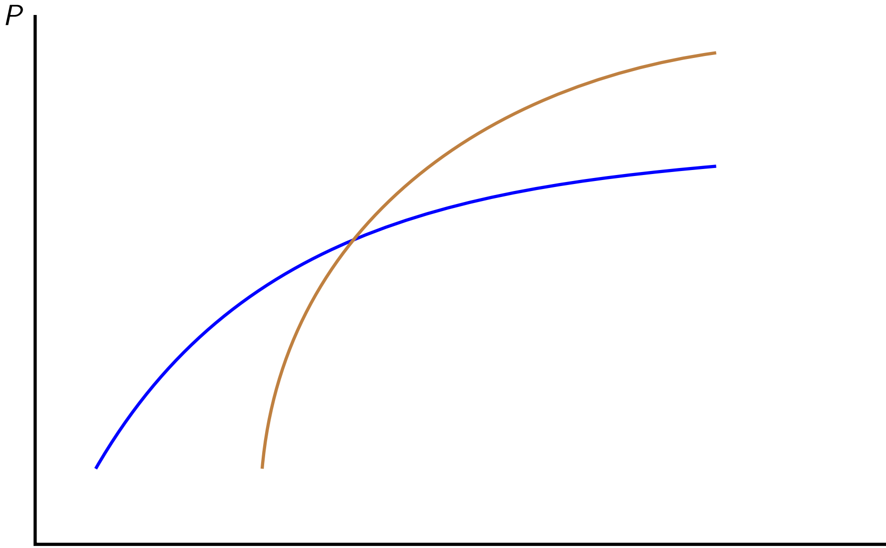
- Lemma 8.2. (Jehle & Reny) Insurance companies earn zero expected profits in equilibrium.

Existence of Pooling equilibrium



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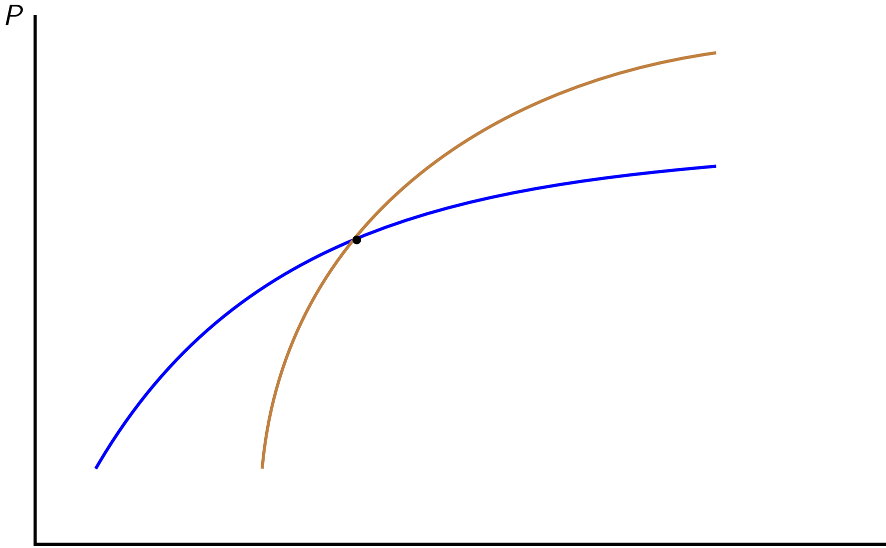

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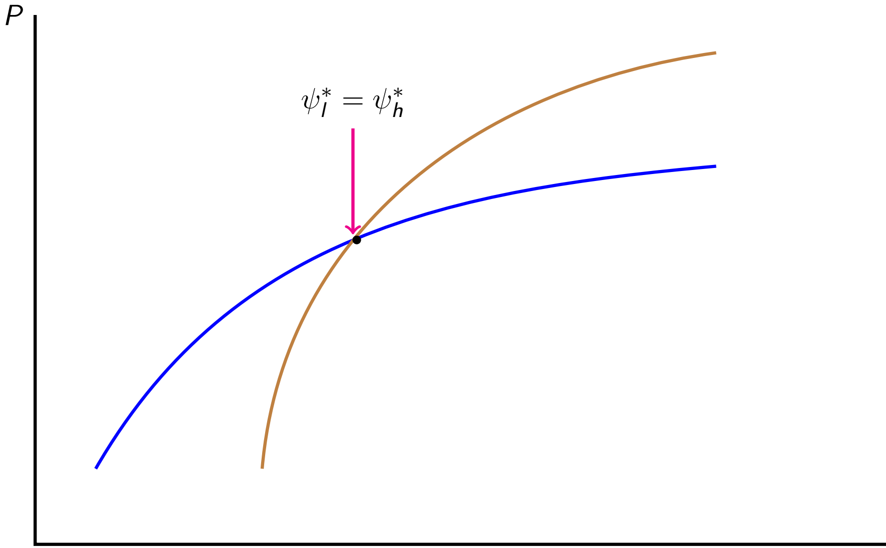

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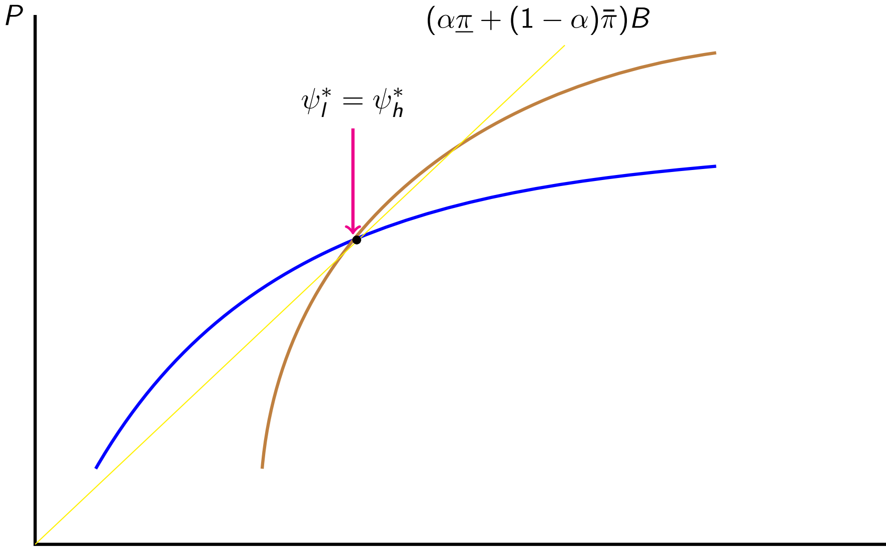

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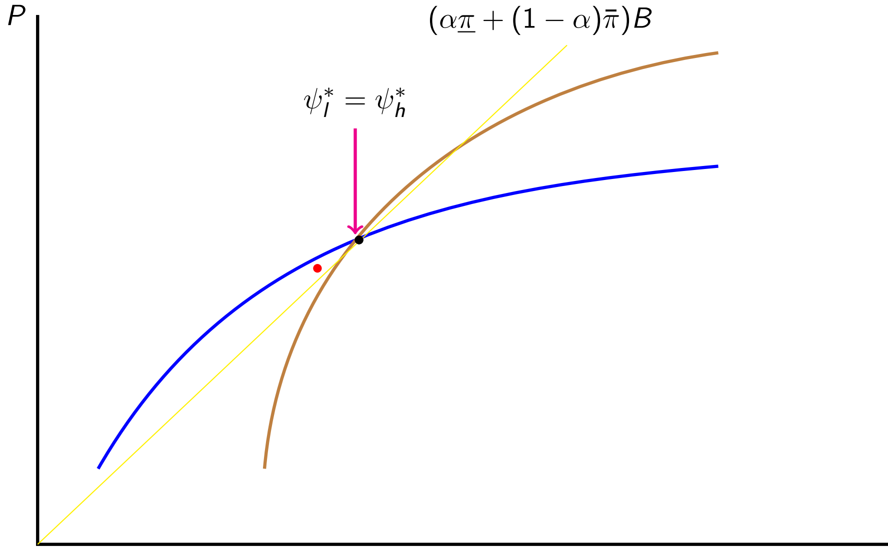

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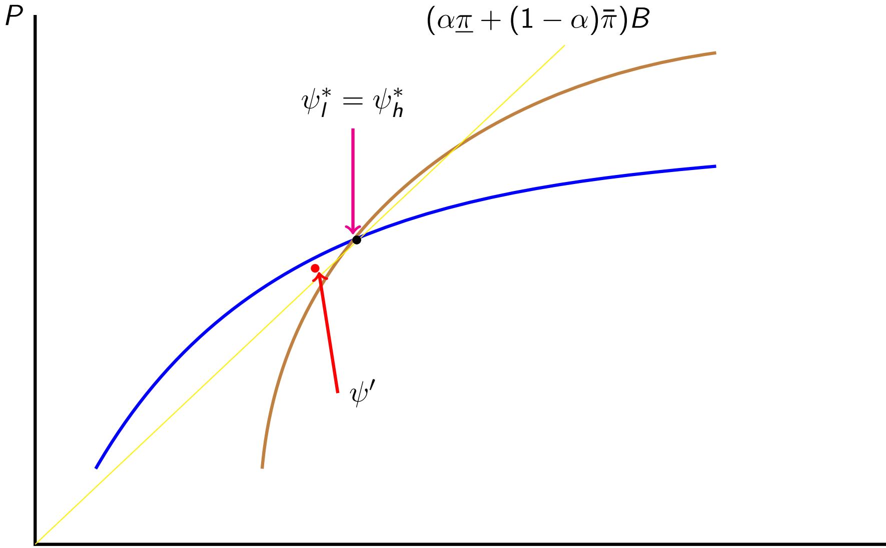

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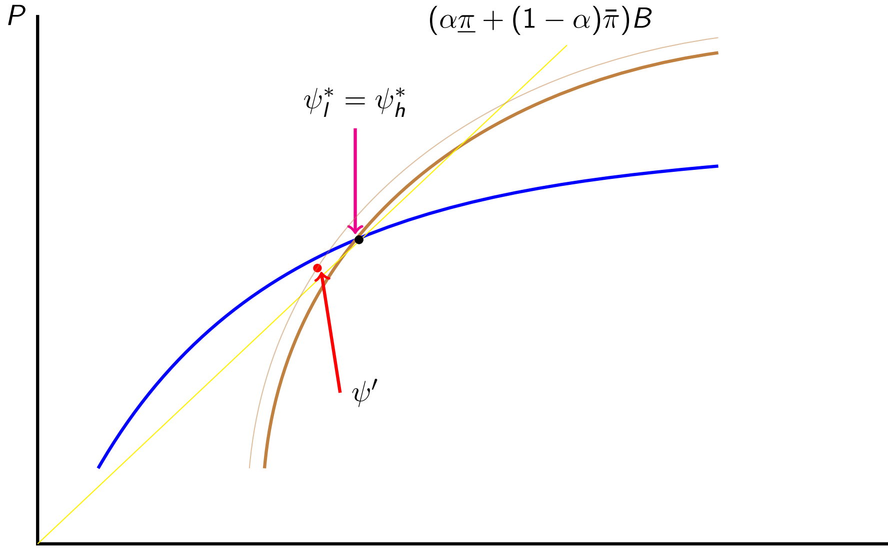
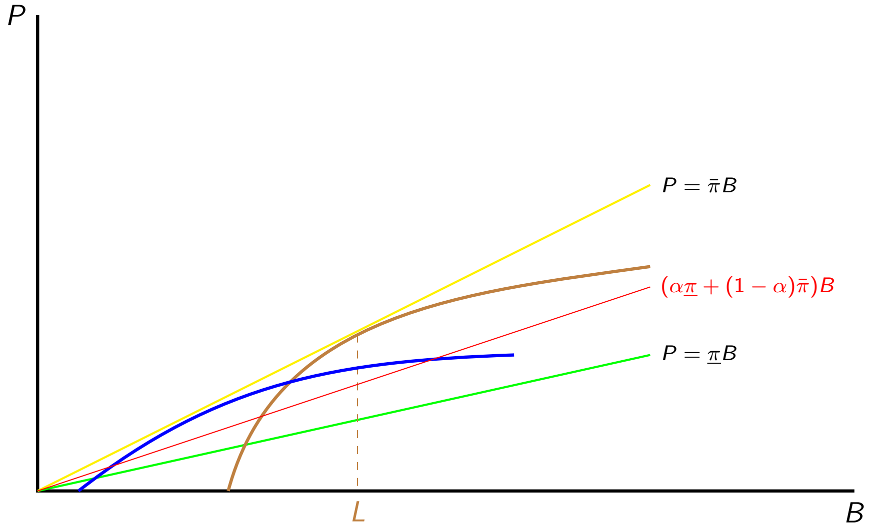


Existence of separating equil. (1)



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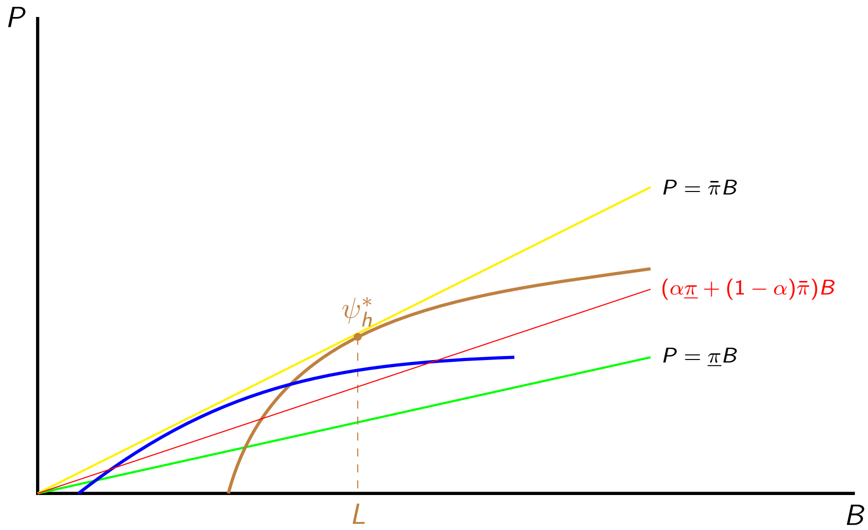

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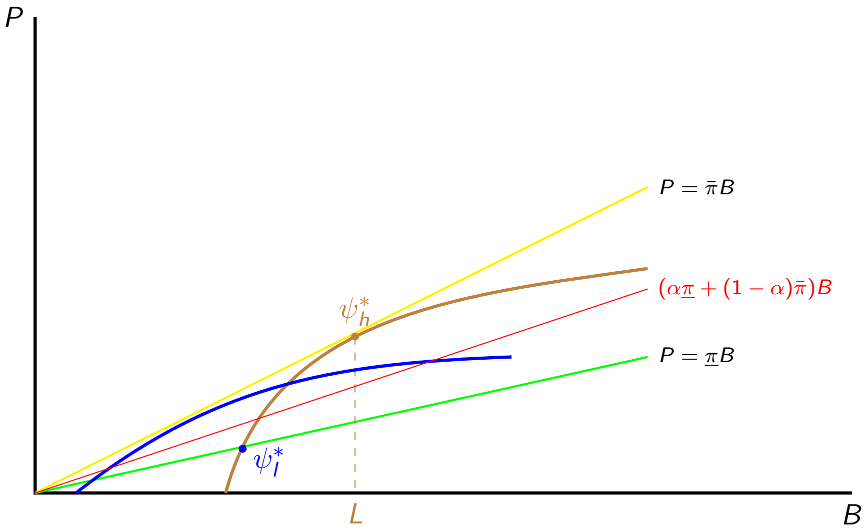

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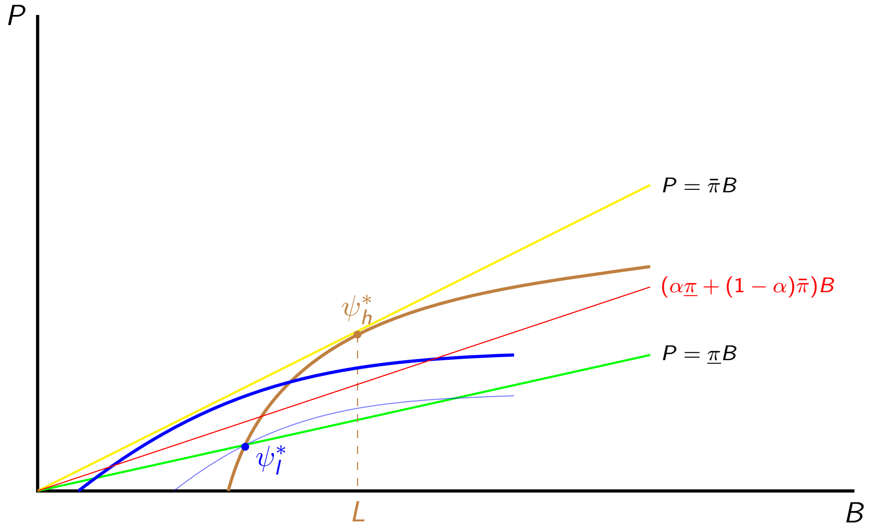

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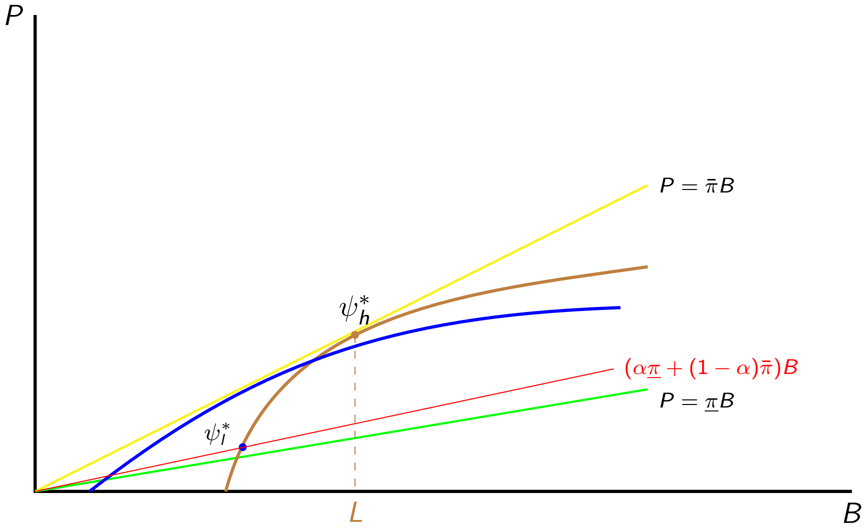

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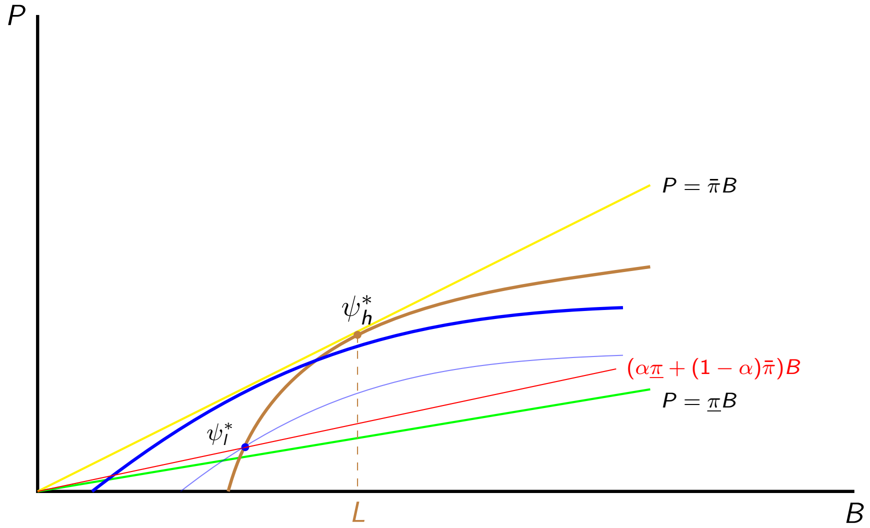

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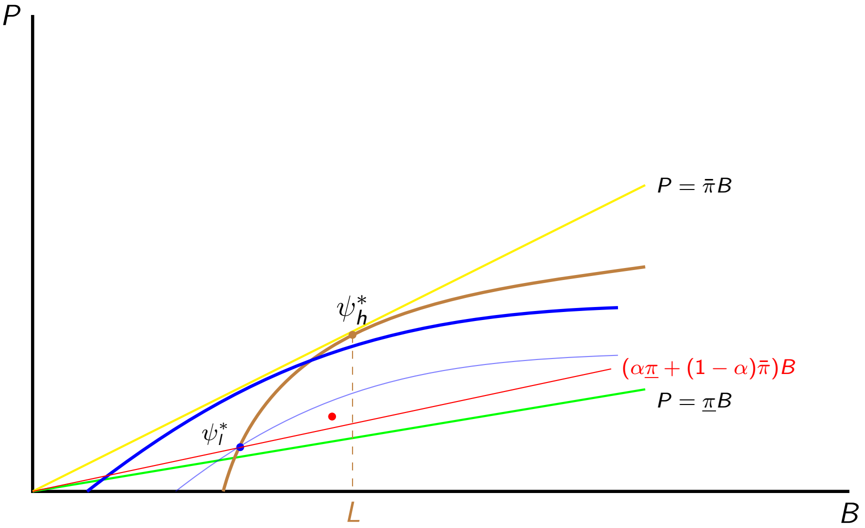

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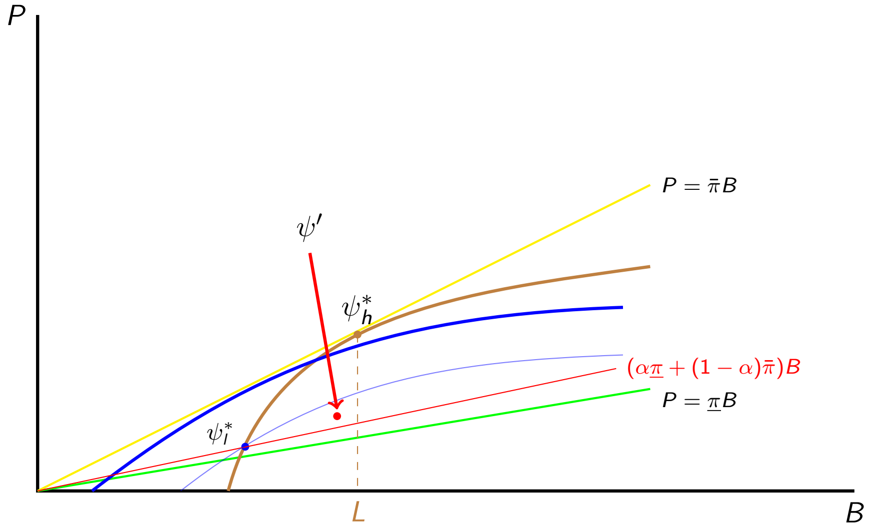

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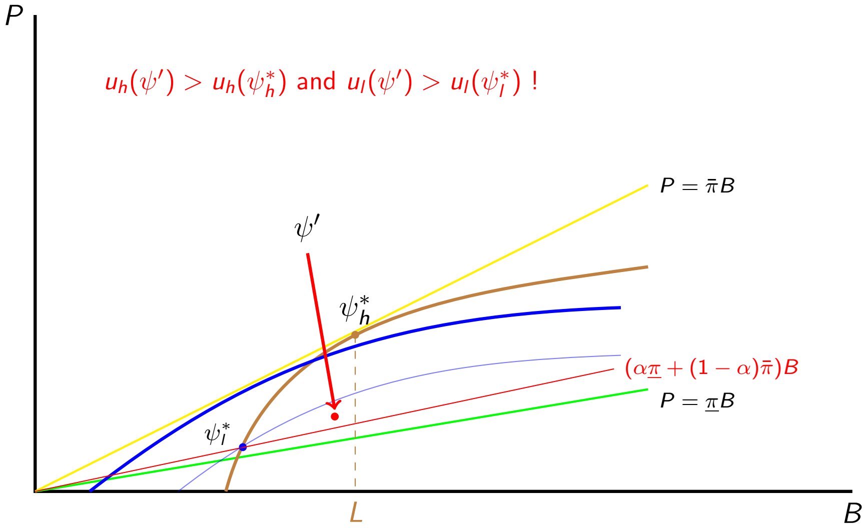

Main result

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- Theorem 8.6. (Jehle & Reny) No pure strategy equilibrium may exist if the proportion of high-risk is too low.

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Insurance: symmetric information

- One insurance company and one consumer.

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  $\pi_l(0)/\pi_l(1)$ is strictly increasing in $l \in \{0, 1, \ldots, L\}$.
- Insurance company chooses policy $(p, B_0, B_1, \ldots, B_L)$ to maximize profit.

$$\max_{e,p,B_l} p - \sum_{l=0}^{L} \pi_l(e) B_l, \qquad \textit{subject to}$$

$$\sum_{l=1}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \bar{u}.$$

Symmetric information optimal contract

- Lagrangian:

$$\mathcal{L} = p - \sum_{l=0}^{L} \pi_l(e) B_l + \lambda \left[\sum_{l=1}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) - \bar{u} \right].$$

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- First order conditions:

$$\frac{\partial \mathcal{L}}{\partial p} = 1 - \lambda \left[\sum_{l=1}^{L} \pi_l(e) u'(w - p - l + B_l) \right] = 0, \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial B_l} = -\pi_l(e) + \lambda \pi_l(e) u'(w - p - l + B_l) = 0, \qquad \forall l \geq 0, \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{l=1}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) - \bar{u} \geq 0. \qquad (3)$$

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- Thus it is optimal to have

$$B_l = l \qquad for l = 0, 1, \ldots, L.$$

Asymmetric information

- Optimization problem

$$\max_{e,p,B_l} p - \sum_{l=0}^{L} \pi_l(e) B_l, \qquad \textit{subject to}$$

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$$\sum_{l=1}^{L} \pi_l(e) u(w - p - l + B_l) - d(e) \geq \sum_{l=1}^{L} \pi_l(e') u(w - p - l + B_l) - d(e').$$

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- If optimal policy to set $e = 0$:
  Similar as the symmetric information case.

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- Optimization problem

$$\max_{e,p,B_l} p - \sum_{l=0}^{L} \pi_l(e) B_l, \qquad \textit{subject to}$$

$$\sum_{l=1}^{L} \pi_l(e) u(w-p-l+B_l) - d(e) \geq \bar{u};$$

$$\sum_{l=1}^{L} \pi_l(e) u(w-p-l+B_l) - d(e) \geq \sum_{l=1}^{L} \pi_l(e') u(w-p-l+B_l) - d(e').$$

- If optimal policy to set $e = 0$:
  Similar as the symmetric information case.
- Optimal policy $e = 1$:

$$\mathcal{L} = p - \sum_{l=0}^{L} \pi_l(1) B_l + \lambda \left[\sum_{l=1}^{L} \pi_l(e) u(w-p-l+B_l) - d(e) - \bar{u} \right]$$

$$\beta \left[\sum_{l=1}^{L} \pi_l(1) u(w-p-l+B_l) - \sum_{l=1}^{L} \pi_l(0) u(w-p-l+B_l) - d(1) + \right.$$

Second best contract

- First order conditions:

$$1-\lambda\left[\sum_{l=1}^{L}\pi_l(1)u'(w-p-l+B_l)\right]-\beta\left[\sum_{l=1}^{L}(\pi_l(1)-\pi_l(0))u'(w-p-l+B_l)\right]$$
$$=0;$$
$$-\pi_l(1)+[\lambda\pi_l(1)+\beta(\pi_l(1)-\pi_l(0))]u'(w-p-l+B_l)=0 \quad \forall l; \qquad (*)$$
$$\sum_{l=1}^{L}\pi_l(1)u(w-p-l+B_l)-d(1)-\bar{u}\geq 0;$$
$$\sum_{l=1}^{L}(\pi_l(1)-\pi_l(0))u(w-p-l+B_l)+d(0)-d(1)\geq 0.$$

Second best contract (continued)

- Equation (*) implies

$$\frac{1}{u'(w - p + B_I - I)} = \lambda + \beta \left[1 - \frac{\pi_I(0)}{\pi_I(1)} \right]. \qquad \text{(CON-OP)}$$

Second best contract (continued)

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$$\frac{1}{u'(w-p+B_I-I)} = \lambda + \beta\left[1 - \frac{\pi_I(0)}{\pi_I(1)}\right]. \qquad \text{(CON-OP)}$$

- Clearly, $\lambda > 0$, $\beta > 0$.

Second best contract (continued)

- Equation (*) implies

$$\frac{1}{u'(w-p+B_l-l)} = \lambda + \beta\left[1 - \frac{\pi_l(0)}{\pi_l(1)}\right]. \qquad \text{(CON-OP)}$$

- Clearly, $\lambda > 0$, $\beta > 0$.
- Thus,

$$l - B_l \quad \text{is strictly increasing in} \quad l = 0, 1, \ldots, L.$$

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- Agent is forced to carry excess responsibility for the outcome and this is the implicit costs involved in contracting under imperfect information.