PROBLEM SET 1 ANSWERS

1. (40) Consider a standard New Keynesian model with price stickiness and working capital channel. Assume that the evolution of the economy is determined by the following three linearized equations.

$$\pi_t = \gamma x_t + \alpha R_t + \beta E_t \pi_{t+1} \tag{1}$$

$$x_t = E_t \left[x_{t+1} - (R_t - \pi_{t+1}) \right] \tag{2}$$

$$R_t = r_\pi \pi_t. \tag{3}$$

where π_t, x_t and R_t denote the inflation, the output gap and the nominal interest rate, respectively. Eq.(1) is the Phillips curve that links the inflation π_t to the output gap x_t and the nominal interest rate R_t. Eq.(2) is the intertemporal Euler equation. Eq.(3) characterizes the monetary policy.

(a).(10) Please substitute out x_t and R_t using Eq.(1), Eq.(2) and Eq.(3) and derive a second-order different equation in π_t.

Answer:

$$(1 - \alpha r_\pi + \gamma r_\pi)\pi_t + (\alpha r_\pi - \beta - 1 - \gamma)\pi_{t+1} + \beta \pi_{t+2} = 0.$$

Including the expectation operator is okay.

(b).(10) Under what condition, the equilibrium with zero inflation ($\pi_t = 0$) is stable? And under what condition, inflation expectations are self-fulfilling?

Answer: The general set of solutions to the equation in (a) is given by, $\pi_t = \alpha_0 \lambda_1^t + \alpha_1 \lambda_2^t$, where λ_1, λ_2 are the roots of the following equation:

$$(1 - \alpha r_\pi + \gamma r_\pi) + (\alpha r_\pi - \beta - 1 - \gamma)\lambda + \beta \lambda^2 = 0.$$

When $|\lambda_1| > 1$ and $|\lambda_2| > 1$, the equilibrium with zero inflation ($\pi_t = 0$) is stable.

When $|\lambda_1| <= 1$ or $|\lambda_2| <= 1$, there are many solutions to the equilibrium conditions, therefore, inflation expectations are self-fulfilling.

(c).(20) Assume $\alpha = 0$. Please derive the necessary and sufficient condition for r_π under which the equilibrium with zero inflation ($\pi_t = 0$) is stable. Please also explain the economic intuition behind your answer.(Hint: $0 < \beta < 1$ and $\gamma > 0$)

Answer: Let $\alpha = 0$. The roots of the following equation:

$$(1 + \gamma r_\pi) + (-\beta - 1 - \gamma)\lambda + \beta \lambda^2 = 0.$$

is given by,

$$\lambda_1 = \frac{\beta + 1 + \gamma + \sqrt{(\beta + 1 + \gamma)^2 - 4\beta(1 + \gamma r_\pi)}}{2\beta}.$$

$$\lambda_2 = \frac{\beta + 1 + \gamma - \sqrt{(\beta + 1 + \gamma)^2 - 4\beta(1 + \gamma r_\pi)}}{2\beta}.$$

When $|\lambda_1| > 1$ and $|\lambda_2| > 1$, the equilibrium with zero inflation ($\pi_t = 0$) is stable.

It is easy to see that, $|\lambda_1| > \frac{\beta+1}{2\beta} > 1$ and $\lambda_2 > 0$. Then, $\lambda_2 > 1$ requires that,

$$\frac{\beta + 1 + \gamma - \sqrt{(\beta + 1 + \gamma)^2 - 4\beta(1 + \gamma r_\pi)}}{2\beta} > 1.$$

The above condition can be reduced to:

$$r_\pi > 1.$$

which is the necessary and sufficient condition under which the equilibrium with zero inflation ($\pi_t = 0$) is stable.

The economic intuition is that, if the inflation deviates above from zero, then the central bank will raise nominal interest rate more ($r_\pi > 1$), so that output gap falls based on Eq.(2). The fall in output gap leads to a decline in the inflation through the Phillips curve, and therefore stabilizes the inflation.

2. (30) Assume that there is a continuum of perfectly competitive firms that produce physical capital, hold it and obtain revenues by renting it out. Note that there are no trades of physical capital among firms. In particular, the representative firm is endowed with capital K_{t-1} at the beginning of period t and obtains capital rents $r_t^k K_{t-1}$ in period t where r_t^k is the capital rent rate. In period t, the representative firm uses part of the capital rents to makes investments I_t and distributes the rest to its shareholders as dividends D_t. The evolution of the physical capital follows,

$$K_t = (1-\delta)K_{t-1} + [1 - \frac{\Omega}{2}(\frac{I_t}{I_{t-1}} - 1)^2]I_t. \tag{4}$$

where δ denotes the capital depreciation rate. Ω denotes the size of the capital adjustment cost.

a. (5) Please write down the expression of the dividends D_t.

Answer:

$$D_t = r_t^k K_{t-1} - I_t.$$

b. (5) Suppose the representative firm maximizes the discounted sum of current and future dividend flows. The discount factor for period t dividend is $\beta^t \Lambda_t$. Please write down the representative firm's objective function in period t.

Answer:

$$\sum_{j=0}^{\infty} \beta^{t+j} \Lambda_{t+j} D_{t+j} = \sum_{j=0}^{\infty} \beta^t \Lambda_{t+j}(r_{t+j}^k K_{t+j-1} - I_{t+j}).$$

Note: It is enough to write down $\sum_{j=0}^{\infty} \beta^{t+j}\Lambda_{t+j}D_{t+j}$. It is also okay to say $\sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} D_{t+j}$

c. (10) Please solve for the firm's investment problem that chooses K_t and I_t to maximize its objective function taking as given the constraint (4), and write down the first order conditions for K_t and I_t.

Answer: Denote μ_t as the Lagrange multiplier on the capital evolution process (4). Then the first order condition on K_t is given by,

$$-\mu_t + \beta E_t[\Lambda_{t+1} r_{t+1}^k + \mu_{t+1}(1-\delta)] = 0.$$

The first order condition on I_t is given by,

$$-\Lambda_t + \mu_t[1 - \frac{\Omega}{2}(\frac{I_t}{I_{t-1}} - 1)^2 - \Omega(\frac{I_t}{I_{t-1}} - 1)\frac{I_t}{I_{t-1}}] + \beta E_t \mu_{t+1}\Omega(\frac{I_{t+1}}{I_t} - 1)(\frac{I_{t+1}}{I_t})^2 = 0.$$

d. (10) Denote μ_t as the Lagrange multiplier on the capital evolution equation Eq.(4) for firm's investment problem. We usually interpret the ratio $q_t \equiv \frac{\mu_t}{\Lambda_t}$ as the market price of physical capital if there are trades of physical capital. Describe the relationship between $q_t \equiv \frac{\mu_t}{\Lambda_t}$ and the capital rents based on your answers in (c). Is current capital price q_t affected by current capital rent r_t^k or expected future capital rent $E_t r^k_{t+1}$? Is the effect positive or negative? Explain the economic intuition behind this relationship.

Answer: Substitute $q_t \equiv \frac{\mu_t}{\Lambda_t}$ into the first order condition for K_t, we have:

$$q_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}[r^k_{t+1} + q_{t+1}(1-\delta)].$$

Taking else equal, the current capital price q_t is positively affected by the expected value of future capital rent $E_t r^k_{t+1}$, not current capital rent. This is because 1) The physical capital that is purchased in the current period can only generate capital rents from the next period. 2) Higher expected value of future capital rents makes physical capital more valuable, and raise the demand and thus the current price of physical capital.