Lecture 2(a): DSGE model with Financial Accelerator

Jingyi Zhang

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Overview of This Lecture

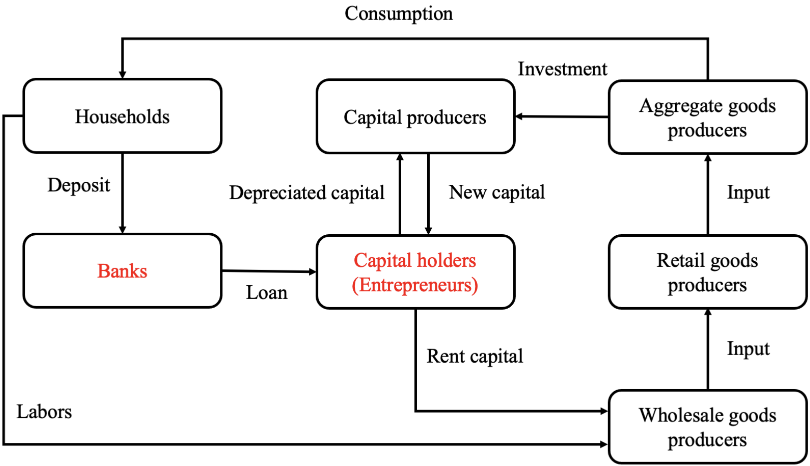
- Building block of the Financial-Accelerator Model (BGG)
- Optimal financial contract
- Mechanism of financial accelerator
- Importance of risk shocks

This lecture note are based on the following two papers:

Lawrence J. Christiano & Roberto Motto & Massimo Rostagno, 2014. Risk Shocks, American Economic Review, American Economic Association, vol. 104(1), pages 27-65, January.

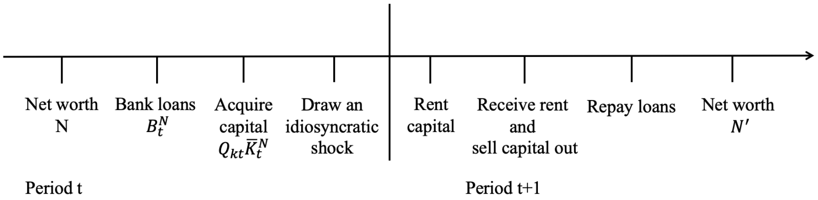
Bernanke, Ben S. & Gertler, Mark & Gilchrist, Simon, 1999. The financial accelerator in a quantitative business cycle framework, Handbook of Macroeconomics, in: J. B. Taylor & M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 21, pages 1341-1393, Elsevier.

Framework of the Financial-Accelerator Model (BGG)



Entrepreneurs (Capital Holders) I

- Timeline of entrepreneurs



Entrepreneurs (Capital Holders) II

- Budget constraint in period t,

$$N + B_t^N = Q_{kt} \bar{K}_t^N.$$

- Idiosyncratic shock converts one unit of capital into ω units of capital.
    - the entrepreneur's capital holding: $\bar{K}_t^N \Rightarrow \omega \bar{K}_t^N$.
    - ω is independently distributed across entrepreneurs and time. $F(\omega)$ denotes the cumulative probability function of the idiosyncratic shock ω and satisfies $E(\omega) = 1$.

Entrepreneurs (Capital Holders) III

- In period $t+1$, the entrepreneur rent a fraction u_{t+1} the capital it holds $\omega \bar{K}_t^N$ to firms at the real rental rate r_{t+1}^k subject to capital utilization cost $a(u_{t+1})$.
- By the end of period $t+1$, the entrepreneur sells the residual capital out at the capital price $Q_{k,t+1}$.
- The entrepreneur's revenue from capital holdings is given by,

$$\left[u_{t+1} r_{t+1}^k P_{t+1} - a\left(u_{t+1}\right) P_{t+1}\right] \omega \bar{K}_t^N + \left(1-\delta\right) Q_{k,t+1} \omega \bar{K}_t^N$$
$$= \omega \bar{K}_t^N Q_{kt} \tilde{R}_{t+1}^k,$$

where $\tilde{R}_{t+1}^k$ denotes the aggregate return to capital.

$$\tilde{R}_{t+1}^k \equiv \frac{\left[u_{t+1} r_{t+1}^k - a\left(u_{t+1}\right)\right] P_{t+1} + \left(1-\delta\right) Q_{k,t+1}}{Q_{kt}}$$

Optimal Financial Contract

- Assume that when the borrower (entrepreneur) defaults, the lender (bank) must pay a cost to observe the borrower's realized returns. In the process of liquidation, a μ of realized payoffs: $\omega \tilde{R}^k_{t+1} \bar{K}^N_t Q_{kt}$ is lost as the bankruptcy cost.

- To cover the bankruptcy costs, the lenders charges a state-contingent interest rate Z^N_{t+1}.

- There exists $\bar{\omega}^N_{t+1}$ such that entrepreneurs that draw $\omega < \bar{\omega}^N_{t+1}$ choose to default, which is given by,

$$\bar{\omega}^N_{t+1} \tilde{R}^k_{t+1} Q_{kt} \bar{K}^N_t = B^N_t Z^N_{t+1},$$

where $Z^N_{t+1} > R_t$ and R_t denotes the risk-free nominal interest rate from period t to period $t+1$.

Entrepreneurs' Expected Payoff

- Only entrepreneurs that draw $\omega \geq \bar{\omega}_{t+1}^N$ are able to repay the loans and obtain non-negative payoffs. Entrepreneurs that draw $\omega < \bar{\omega}_{t+1}^N$ claims bankruptcy and their net worth becomes zero.

- The entrepreneur's expected payoff is therefore,

$$E_t \int_{\bar{\omega}_{t+1}^N}^{\infty} \left[\omega \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N - B_t^N Z_{t+1}^N \right] \mathrm{d}F(\omega)$$

$$= E_t \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N \left\{ \int_{\bar{\omega}_{t+1}^N}^{\infty} \omega \, \mathrm{d}F(\omega) - \bar{\omega}_{t+1}^N \left[1 - F\left(\bar{\omega}_{t+1}^N\right) \right] \right\}$$

$$= E_t \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N g\left(\bar{\omega}_{t+1}^N\right)$$

where $g'\left(\bar{\omega}_{t+1}^N\right) = -\left[1 - F\left(\bar{\omega}_{t+1}^N\right)\right] < 0$.

(Contractual loan interest rate $Z \uparrow, \bar{\omega} \uparrow, g(\bar{\omega}) \downarrow$)

Lender's Expected Payoff

- Banks obtain full repayments $Z_{t+1}^N B_t^N$ for entrepreneurs that repay. For entrepreneurs that default, bank obtain $(1-\mu)\omega \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N$ after paying the liquidation cost.

- The lender's expected payoff is therefore,

$$
\begin{aligned}
& E_t \left[1 - F\left(\bar{\omega}_{t+1}^N\right)\right] Z_{t+1}^N B_t^N + (1-\mu) \int_0^{\bar{\omega}_{t+1}^N} \omega \, \mathrm{d}F(\omega) \, \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N \\
= {} & E_t \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N \left\{ \bar{\omega}_{t+1}^N \left[1 - F\left(\bar{\omega}_{t+1}^N\right)\right] + (1-\mu) \int_0^{\bar{\omega}_{t+1}^N} \omega \, \mathrm{d}F(\omega) \right\} \\
= {} & E_t \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N h\left(\bar{\omega}_{t+1}^N\right)
\end{aligned}
$$

where $h'\left(\bar{\omega}_{t+1}^N\right) = \left[1 - F\left(\bar{\omega}_{t+1}^N\right)\right] - \mu \bar{\omega}_{t+1}^N f\left(\bar{\omega}_{t+1}^N\right) > 0$, if μ is small.

$(\, Z \uparrow, \bar{\omega} \uparrow, h(\bar{\omega}) \uparrow)$

Optimal financial contract

- Financial contract, featured by $\{B_t^N, Z_{t+1}^N\}$, determines how total capital returns are distributed between borrowers and lenders. Note:

$$g\left(\bar{\omega}_{t+1}^N\right) + h\left(\bar{\omega}_{t+1}^N\right) = 1 - \mu \int_0^{\bar{\omega}_{t+1}^N} \omega \,\mathrm{d}F(\omega)$$

- For simplicity, we assume that banks are perfectly competitive, so that it will grant loans if and only its expected payoff is able to cover its funding cost. The bank's participation constraint is given by,

$$\tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N h\left(\bar{\omega}_{t+1}^N\right) \geq R_t B_t^N. \tag{1}$$

  where R_t denotes the deposit rate (i.e. the bank's funding cost).
- Also assume that banks offer risk-free deposits and have no equity. In this case, the lending interst rate Z_{t+1}^N have be state-contingent to ensure that (1) holds for each state of nature in period $t+1$
- The entrepreneur takes its initial endowment N given and chooses a financial contract to maximize its expected payoff,

$$E_t \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N g(\bar{\omega}_{t+1}^N)$$

Lender's Participation Constraint

- The bank's participation constraint is given by,

$$\tilde{R}^k_{t+1} Q_{kt} \bar{K}^N_t h\left(\bar{\omega}^N_{t+1}\right) \geq R_t B^N_t.$$

- The above participation constraint is relaxed if:
  - External financial premium $\frac{\tilde{R}^k_{t+1}}{R_t}$ rises $\Rightarrow$ Total revenues of capital holdings $\tilde{R}^k_{t+1} Q_{kt} \bar{K}^N_t$ rise up
  - Lending rate Z^N_{t+1} rises $\Rightarrow$ The bankers obtains a larger share.
- Relaxed participation constraint allows for higher debt ratio $\frac{B^N_t}{Q_{kt} \bar{K}^N_t}$.

Entrepreneur's Optimization Problem I

- The entrepreneur takes its initial endowment N as given and choose the optimal financial contract to maximize its expected payoff,

$$\max_{\bar{K}_t^N, B_t^N, Z_{t+1}^N, \bar{\omega}_{t+1}^N} \quad E_t \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N g(\bar{\omega}_{t+1}^N)$$

$$s.t. \ \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N h(\bar{\omega}_{t+1}^N) \geqslant R_t B_t^N$$

$$Q_{kt} \bar{K}_t^N = B_t^N + N.$$

$$\bar{\omega}_{t+1}^N \tilde{R}_{t+1}^k Q_{kt} \bar{K}_t^N = B_t^N Z_{t+1}^N.$$

Define $L_t^N \equiv \frac{Q_{kt} \bar{K}_t^N}{N}$. The problem can be rewritten as follows:

$$\max_{L_t^N, \bar{\omega}_{t+1}^N} \quad E_t \tilde{R}_{t+1}^k L_t^N g(\bar{\omega}_{t+1}^N)$$

$$s.t. \ \tilde{R}_{t+1}^k L_t^N h(\bar{\omega}_{t+1}^N) \geqslant R_t \left(L_t^N - 1 \right)$$

$$\Rightarrow L_t^N \leq \frac{R_t}{R_t - \tilde{R}_{t+1}^k h(\bar{\omega}_{t+1}^N)}.$$

Entrepreneur's Optimization Problem II

- The maximization problem is then equivalent to:

$$
\begin{aligned}
& max \;\; E_t \tilde{R}^k_{t+1} g(\bar{\omega}^N_{t+1}) \frac{R_t}{R_t - \tilde{R}^k_{t+1} h(\bar{\omega}^N_{t+1})} \\
& \bar{\omega}^N_{t+1} : E_t \tilde{R}^k_{t+1} g'(\bar{\omega}^N_{t+1}) \frac{R_t}{R_t - \tilde{R}^k_{t+1} h(\bar{\omega}^N_{t+1})} \\
& \quad + \tilde{R}^k_{t+1} g(\bar{\omega}^N_{t+1}) \frac{R_t}{[R_t - \tilde{R}^k_{t+1} h(\bar{\omega}^N_{t+1})]^2} \tilde{R}^k_{t+1} h'(\bar{\omega}^N_{t+1}) = 0 \\
& \Rightarrow: E_t \tilde{R}^k_{t+1} g'(\bar{\omega}^N_{t+1}) L^N_t + \tilde{R}^k_{t+1} g(\bar{\omega}^N_{t+1}) (L^N_t)^2 \frac{\tilde{R}^k_{t+1}}{R_t} h'(\bar{\omega}^N_{t+1}) = 0 \\
& \Rightarrow: E_t \tilde{R}^k_{t+1} \{ g'(\bar{\omega}^N_{t+1}) (L^N_t)^{-1} + g(\bar{\omega}^N_{t+1}) \frac{\tilde{R}^k_{t+1}}{R_t} h'(\bar{\omega}^N_{t+1}) \} = 0 \\
& \Rightarrow: E_t \tilde{R}^k_{t+1} \{ g'(\bar{\omega}^N_{t+1}) [1 - \frac{\tilde{R}^k_{t+1}}{R_t} h(\bar{\omega}^N_{t+1})] + g(\bar{\omega}^N_{t+1}) \frac{\tilde{R}^k_{t+1}}{R_t} h'(\bar{\omega}^N_{t+1}) \} = 0
\end{aligned}
$$

Entrepreneur's Optimization Problem III

$$E_t \tilde{R}^k_{t+1}\{g'(\bar{\omega}_{t+1})[1-\frac{\tilde{R}^k_{t+1}}{R_t}h(\bar{\omega}_{t+1})]+g(\bar{\omega}_{t+1})\frac{\tilde{R}^k_{t+1}}{R_t}h'(\bar{\omega}_{t+1})\}=0$$

Note:

- BGG(1999) prove that $\frac{dL_t^N}{d\left(\tilde{R}^k_{t+1}/R_t\right)} > 0$: Entrepreneurs takes higher leverage L_t^N if external financial premium $\tilde{R}^k_{t+1}/R_t$ rises.
- $\bar{\omega}^N_{t+1} \equiv \bar{\omega}_{t+1}$ and $L_t^N \equiv L_t$ equal across entrepreneurs.
- No expectation operation in the lender's incentive constraint. Because the loan contract is state-contingent so that the lender's incentive constraint always holds.

Aggregation in the Entrepreneurs' Sector

$$\bar{K}_t = \int \bar{K}_t^N f_t(N)dN, \quad N_t = \int N f_t(N)dN, \quad B_t = \int B_t^N f_t(N)dN$$

$$L_t^N = \frac{Q_t^k \bar{K}_t^N}{N} \Rightarrow L_t = \frac{Q_t^k \bar{K}_t}{N_t}$$

$$N_t = \gamma \tilde{R}_t^k Q_{t-1}^k \bar{K}_{t-1} g(\bar{\omega}_t) + \omega_t^e$$

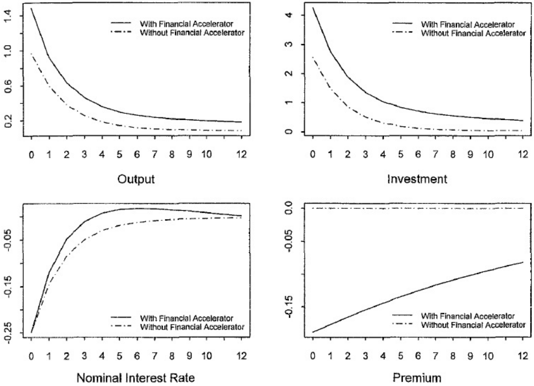
- $1-\gamma$: entrepreneur's exit rate (to ensure they do not accumulate enough wealth to be able to self-finance)
  - If $\gamma = 1$, total net worth N_t grows at a rate $\tilde{R}_t^k L_{t-1} g(\bar{\omega}_t) > 1$
  - $\Rightarrow$ Total capital holding $\bar{K}_t$ is also growing
  - $\Rightarrow$ Over time, return on capital $\tilde{R}_t^k$ falls to $E_t \tilde{R}_t^k = R_t$.
  - $E_t \tilde{R}_t^k = R_t$ is the optimal capital holding condition if entrepreneurs are able to self-finance.
- ω_t^e : entrepreneur's labor income (to ensure new entrepreneurs have start up funds)

Financial Accelerator I

- **Asset Price**:

$$Q_t^k \uparrow \Rightarrow \tilde{R}_t^k \uparrow \Rightarrow N_t \uparrow \Rightarrow demand \quad for \quad K_t \uparrow \Rightarrow Q_t^k \uparrow\uparrow, E_t R_{t+1}^k \downarrow$$

- **BGG(1999)**, Figure 3, Page 1371-1372



Financial Accelerator II

- **BGG(1999), Figure 3**
  - The above figures shows the impulse response to an expansionary monetary policy shock (a fall in the interest rate R_t) with and without the financial accelerator.
  - The horizontal axes show the quarters after the impact period of the shock.
  - The units on the vertical axes are percent deviations from the steady state levels.
  - Premium means the "external financial premium", defined as $E_t R^k_{t+1}/R_t$.

Financial Accelerator III

- Implications for monetary policy
  - Price stickiness in the NK model suggests that unexpected inflation leads to resource misallocation $\Rightarrow$ Price stability becomes a goal of monetary authorities since 1970s.
  - In the BGG model, asset price and capital investment work together to amplify macro fluctuations inefficiently $\Rightarrow$ Financial stability (or asset price stability) becomes another concern since 2008 financial crisis.
  - There is a tradeoff between financial stability and price stability, especially when inflation and asset prices go in opposite directions.

Risk Shocks

- Assume that the idiosyncratic shock follows a time-varying log-normal distribution: $\ln\omega \sim N(-\frac{1}{2}\sigma_t^2, \sigma_t^2)$ where σ_t denotes the period-t standard deviation of $\ln\omega$).

  Note: this risk shock measures the cross-sectional uncertainty, as opposed to time-series uncertainty.[1]

- Methodology: Bayesian estimation (A very useful tool to identify what shocks to drive business cycle fluctuations).[2]

[1] If you are interested in time-series uncertainty, please read Jesus Fernandez-Villaverde, Pablo Guerron-Quintana, Juan F. Rubio-Ramirez and Martin Uribe, 2011. "Risk Matters: The Real Effects of Volatility Shocks," American Economic Review, American Economic Association, vol. 101(6), pages 2530-2561, October.

[2] If you are interested in Bayesian estimation, please carefully read Section 2 in Christiano et al.(2014).

Importance of Risk Shock (Christiano et al., 2014)

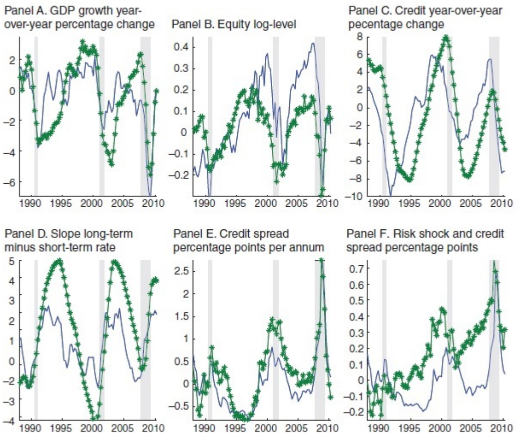


FIGURE 1. THE ROLE OF THE RISK SHOCK IN SELECTED VARIABLES

Notes: All data are demeaned. With the exception of panels B and F, the solid line is the data. The solid line in panel B differs from the actual data by a small, estimated measurement error. The starred line in panels A–E is the result of feeding only the estimated anticipated and unanticipated components of the risk shock to the model. Panel F displays the credit spread (solid line) and the risk shock, σ, (the latter expressed in percent deviation from steady state). Shaded areas indicate NBER recession dates.

Importance of Risk Shock (Christiano et al., 2014)

TABLE 5—VARIANCE DECOMPOSITION AT BUSINESS CYCLE FREQUENCY (*Percent*)

Shock variable	*Risk* σ_t	*Equity* γ_t	*M.E.I.* $\zeta_{I,t}$	*Technol.* ε_t, $\mu_{z,t}$,	*Markup* $\lambda_{f,t}$,	*M.P.* ϵ_t	*Demand* $\zeta_{c,t}$	*Exog.Spend.* g_t
GDP	62|16|38	0	13	2	12	2	4	3
drop all fin. var	1|0|1	0	44	12	22	3	11	8
CEE	[–]	[–]	[39]	[18]	[31]	[4]	[3]	[5]
Consumption	16|3|12	0	11	3	19	2	46	3
drop all fin. var	0|0|0	0	2	15	26	3	51	2
CEE	[–]	[–]	[6]	[12]	[9]	[1]	[67]	[5]
Investment	73|18|46	0	21	0	4	1	1	0
drop all fin. var	2|0|2	0	85	2	7	2	2	0
CEE	[–]	[–]	[57]	[10]	[24]	[3]	[5]	[0]
Equity	69|23|35	2	23	0	1	2	0	0
Credit spread	95|39|42	1	3	0	0	0	0	0
Credit	64|12|46	10	17	2	4	1	1	0
Slope	56|12|38	0	17	3	8	6	2	0

Notes: For each variable indicated in the first column, variance decompositions are generated by the baseline model evaluated at the mode of the posterior distribution. Results in the row marked *drop all fin. var* are generated by the baseline model evaluated at the mode of the posterior distribution when our four financial variables are dropped. Results in the rows marked *CEE* are generated by the *CEE* model (i.e., the model without financial frictions), evaluated at the mode of the posterior distribution computed based on our eight standard macroeconomic variables. Numbers in each row may not add up to 100 due to rounding. The table does not display results for shocks (such as π_t^* and $\mu_{\Upsilon,t}$) whose contribution is less than 1/2 of 1 percent. To save space, we also dropped results for the term premium shock. With one exception it contributes roughly zero to the variance of all variables. In the exceptional case, the term premium shock accounts for 7 percent of the variance of *Slope*, the slope of the term structure. Data on equity is also explained by measurement error, which is estimated to contribute 3 percent in the baseline model. The contribution of the risk shock, σ_t, is presented in the following way: the first entry is the contribution of the entire shock, the second entry is the contribution of ξ_0, and the third entry is the contribution of $\xi_1, \ldots, \xi_8$. The latter two contributions do not sum up to the first entry as they ignore the correlation between the ξs. Business cycle frequency is measured as a periodic component with cycles of 8–32 quarters, obtained using the model spectrum.

Why risk shock is important in the data, compared with other shocks?

- Assume that the evolution capital stock follows

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + [1 - S(\xi_{I,t} I_t / I_{t-1})] I_t,$$

where $\xi_{I,t}$ is a shock to the marginal efficiency of investment in producing capital (in short, MEI shock).

Note: If $\xi_{I,t}$ goes up, it becomes more costly to increase investment, and therefore less efficient to produce capital using investment. Consequently, the supply of capital falls, leading an increase in the capital price and a fall in the output. This is a supply-side shock to the capital market.

Why risk shock is important in the data, compared with other shocks?

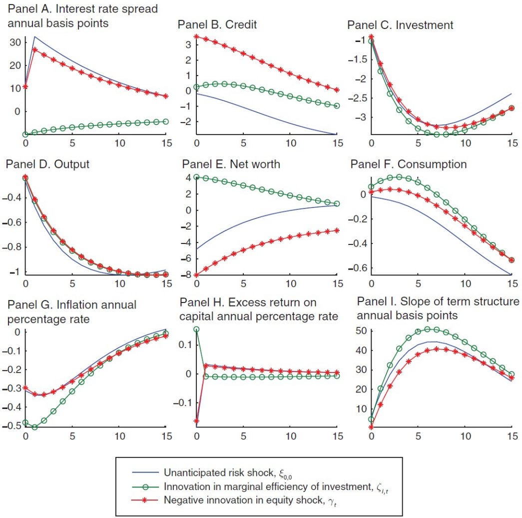


FIGURE 4. DYNAMIC RESPONSES TO THREE SHOCKS

Why risk shock is important in the data, compared with other shocks?

- **Impulse responses to three shocks (Figure 4):**
  - Impulse responses of an increase in risk shock σ_t versus an increase in $\xi_{I,t}$ (Figure 4: interest spread $Z_t - R_t$, credit B_t and excess return on capital $\tilde{R}^k_t - R_{t-1}$).
  - The former shock leads to a decline in the demand for capital, while the latter shock leads to a decline in the supply for capital.
  - Both shocks have contractionary output effects but take opposite effects on capital prices, and therefore the demand for credit and the credit spreads.
  - Impulse responses of an increase in risk shock σ_t versus an decrease in entrepreneur's net worth γ_t (Figure 4: credit B_t)

Why risk shock is important in the data, compared with other shocks?

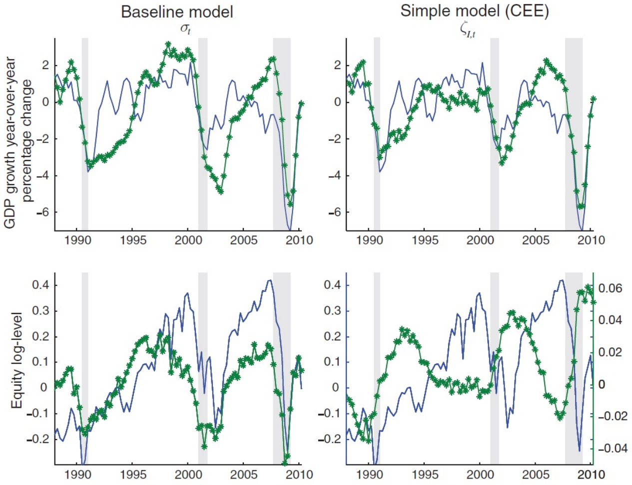


FIGURE 5. HISTORICAL DECOMPOSITIONS IN TWO MODELS

Why risk shock is important in the data, compared with other shocks?

- **Historical decomposition in baseline model and in simple model (Figure 5):**
  - The left column of graphs shows what output and equity would have been according to the baseline model at its posterior mode if only the estimated risk shocks had been active in our sample.
  - The right column of graphs shows what output and equity would have been according to the CEE model at its posterior mode if only the marginal efficiency of investment shocks had been active.[3]
  - Each type of shock accounts well for the dynamics of output growth. However, only the risk shock can also account well for the dynamics of equity.

[3] In the CEE model, we proxy equity by the real price of capital.