Global Games

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Introduction

- Many economic scenarios feature coordination among economic agents
  - Bank runs a la Diamond-Dybvig
  - Currency attack a la Morris-Shin
- Basically, strategic complementarity, as agents want to do something if they do the same thing
  - Typically in the financial market trading game features strategic substitution.
  - If other people by, pushing up the price, so you want to sell.
- Multiple equilibria emerge easily with strategic complementarity
- Global game technique help get a unique equilibrium

Carlsson and van Damme (1993): Setting

- Two players $i = 1, 2$. Binary actions“invest” or “not invest”.
- Normal form

	Invest	Not invest
Invest	θ, θ	$\theta - 1, 0$
Not invest	$0, \theta - 1$	$0, 0$

- If θ is known to players, there are three possibilities
  - $\theta > 1$, Invest is a dominant strategy
  - $\theta \in [0, 1]$ (Invest,Invest) and (NI,NI) are both NE.
  - $\theta < 0$, Not invest is a dominant strategy
- Problem: common knowledge in equilibrium strategies given complete information
  - Perfect guessing each other’s strategies
- Introduce private information about θ to break it

Private Signals

- Nobody knows θ exactly, but observe a private signal

$$x_i = \theta + \varepsilon_i, i = 1, 2$$

  where $\varepsilon \sim N(0, \sigma^2)$, i.i.d across agents
- Bayesian updating. For simplicity. say prior θ is improper prior of θ or noninformative prior (equally likely over real line)
  - Given x_i, the posterior of θ is $N(x_i, \sigma^2)$
  - Given x_i, the posterior of opponent's signal x_j is $N(x_i, 2\sigma^2)$
- Agent i's conditional payoff

$$\mathbb{E}\left[\theta - \mathbb{1}\{j \text{ not invest }\} \mid x_i\right] = \underbrace{x_i}_{\text{fundamental}} - \underbrace{\Pr(j \text{ not invest} \mid x_i)}_{\text{strategic}}$$

- Two distinctive role played by the signal x_i
  - First term: x_i tells me something about the fundamental
  - Second term: x_i tells me something about the distribution of agent $j's$ signal x_j and thus his strategy

Carlsson and van Damme: Strategic component I

- The second term captures the idea of "guessing" each other's strategy, in a simple but powerful way
- Suppose everybody follows a cutoff rule with threshold k

$$\begin{cases} \text{Invest} & x > k \\ \text{Not Invest} & x \leq k \end{cases} \tag{1}$$

- Because $x_j | x_i \sim N(x_i, 2\sigma^2)$, then the prob of agent j not investing is

$$\Pr(x_j \leq k | x_i) = \Phi\left(\frac{k - x_i}{\sqrt{2}\sigma}\right)$$

- So agent i will invest if and only if

$$x_i - \Phi\left(\frac{k - x_i}{\sqrt{2}\sigma}\right) \geq 0$$

Carlsson and van Damme: Strategic component II

- The equilibrium cut-off k : when $x_i = k$, agent i indifferent between invest and not invest

$$k - \Phi\left(\frac{k-k}{\sqrt{2}\sigma}\right) = 0 \Longrightarrow k = \frac{1}{2}$$

- So, the unique equilibrium is that every agent invests if his/her signal $x_i > 1/2$
- Intuition:
  - Symmetry: when receiving $x_i = k$, the probability of j getting signal x_j below k is 0.5
  - Strategic uncertainty (guessing each other) implies the second term to be 0.5
  - The first fundamental term have to be 0.5 for the equilibrium threshold

The role of Dominance Regions

- The assumption of threshold strategy can be relaxed
- Starting from upper and lower dominance regions, unique equilibrium survives after iterated deletion of strictly dominated strategies
  1. Say $x_i \sim U(\theta - \varepsilon, \theta + \varepsilon)$. At $x_i = 1 + \varepsilon$, you know $\theta \geq 1$, invest for sure, **even if j is for sure not investing.**
  2. The same logic applies to j so you know that j invest if $x_j \geq 1 + \varepsilon$
  3. Now $x_i = 1 + 0.99\varepsilon$ so x_j is centered around $1 + 0.99\varepsilon$. This implies you know j invests with probability $0.5 - \delta(\varepsilon) \sim 0.5$
  4. Comparing to step 1, you should invest at $x_i \geq 1 + 0.99\varepsilon$
  5. Symmetrically, j will invest if $x_j \geq 1 + 0.99\varepsilon$
  6. How about $1 + 0.98\varepsilon$? So on so forth
- You can do the same thing starting from the lower end $\theta = -\varepsilon$, with the only difference of "not investing"
- Both sides collide at the equilibrium threshold $k = 0.5$

Discontinuity at the full information case

- Magically, the equilibrium threshold $k = 0.5$ does not depend on how noisy the private signal σ is!
- Say $\sigma \to 0$ so the game seems to converge to the full information case, the equilibrium is still unique
- Interestingly, it implies a negligible probability of getting inside the dominance regions
- Although fundamental uncertainty shrinks, the strategic uncertainty effect remains at 0.5

Public versus private information

- A continuum of agents
  - No investing: 0; Investing: $\theta + I - 1$ where I is the proportion of people investing
- θ is the fundamental, with prior $N(y, \tau^2)$, where y is the public signal.
- Private signal: $x_i = \theta + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2)$
- The posterior of θ given x_i and y is

$$\theta \sim N(\frac{\sigma^2 y + \tau^2 x_i}{\sigma^2 + \tau^2} \equiv \bar{\theta}, \frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2})$$

- Consider a threshold strategy k so that investing iff $\bar{\theta} > k$

Strategic Uncertainty

- Given x_i and y, what is the belief about other agent's signal $x' = \theta + \varepsilon'$?
- The posterior of x' given x_i and y

$$x' \sim N(\bar{\theta}, \frac{2\sigma^2 + \sigma^4}{\sigma^2 + \tau^2})$$

- The posterior prob of $\bar{\theta}' = \frac{\sigma^2 y + \tau^2 x'}{\sigma^2 + \tau^2} > k$ is

$$1 - \Phi\left(\frac{k + \frac{\sigma^2}{\tau^2}(k - y) - \bar{\theta}}{\sqrt{\frac{2\sigma^2\tau^2 + \sigma^4}{\sigma^2 + \tau^2}}}\right) = I$$

Equilibrium Characterization

- At equilibrium threshold $\bar{\theta} = k$, $\bar{\theta} + I - 1 = k + I - 1 = 0$ implies that

$$k = \Phi\left(\frac{k + \frac{\sigma^2}{\tau^2}(k-y) - k}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}\right) = \Phi\left(\frac{\sigma^2/\tau^2}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}(k-y)\right)$$

- Define

$$\gamma(\sigma,\tau) \equiv \frac{\sigma^2}{\tau^4}\left(\frac{\sigma^2+\tau^2}{\sigma^2+2\tau^2}\right)$$

  then the equilibrium threshold k satisfies

$$k = \Phi(\sqrt{\gamma}(k-y))$$

- The only difference from previous example: introduce τ (precision of public signal) and σ (precison of private signal)
- Public signal gives some prior. When $\tau \to \infty$ (improper prior), $k \to \frac{1}{2}$

Multiplicity of equilibria I

- With public signal, not always true that we have a unique equilibrium
- Key equation

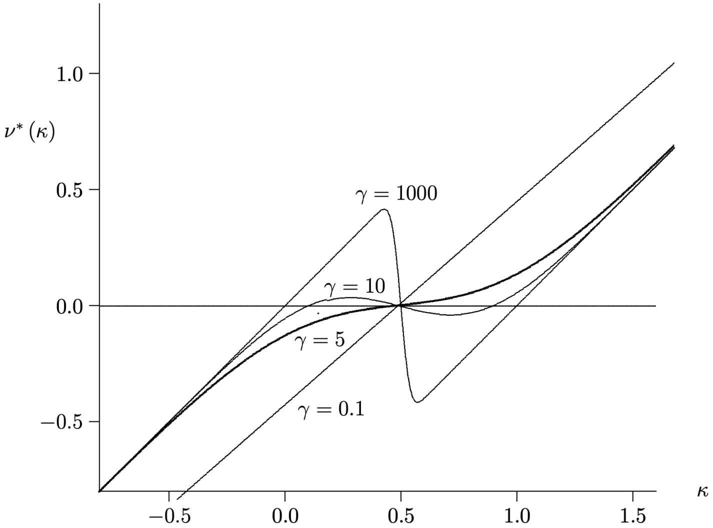
$$v(k) = k - \Phi(\sqrt{\gamma}(k - y)) = 0$$

- Whether the solution is unique depends on whether $\gamma \leq 2\pi$
- Say public signal $y = 0.5$. Then $k = 0.5$ is an equilibrium. Its derivative at that point is

$$v'(k = 0.5) = 1 - \sqrt{\gamma}\phi\left(\sqrt{\gamma}(0.5 - 0.5)\right) = 1 - \sqrt{\gamma}\frac{1}{\sqrt{2\pi}}$$

so if $v'(k = 0.5) < 0$ then there must be at least three equilibria

Multiplicity of equilibria II



Public versus private information

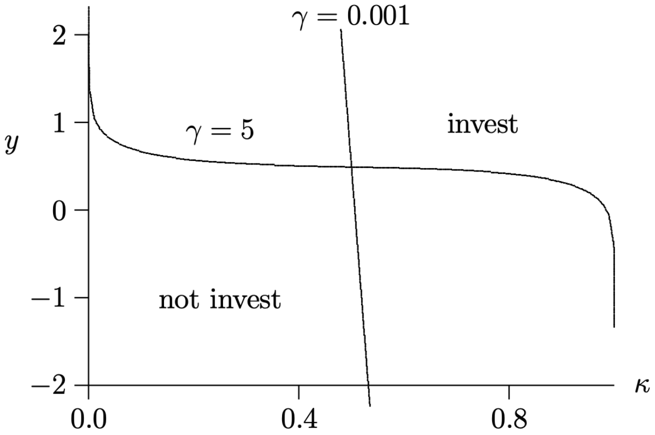


Figure: Investment takes place above and to the right of the line.

The publicity multiplier

- The public signal, as observed by everyone, carry more weight in coordination and thus the equilibrium
- How much a player's private signal must adjust to compensate for a given change in public signal so that he is indifferent between investing and not?
- Without strategic uncertainty effect, $\bar{\theta} = \frac{\sigma^2 y + \tau^2 x}{\sigma^2 + \tau^2} = k$, so

$$\frac{dx}{dy} = -\frac{\sigma^2}{\tau^2}$$

- With strategic effect $\frac{\sigma^2 y + \tau^2 x}{\sigma^2 + \tau^2} = \Phi(\sqrt{\gamma}(\frac{\sigma^2 y + \tau^2 x}{\sigma^2 + \tau^2} - y))$ so

$$\frac{dx}{dy} = -\frac{\sigma^2/\tau^2 + \sqrt{\gamma}\phi(\cdot)}{1 - \sqrt{\gamma}\phi(\cdot)}$$

- The smaller the τ (more precise public signal), the greater the γ, the larger the publicity multiplier
- When the publicity multiplier is too large, the multiplicity of equilibria comes back.

Concluding Remarks

- The common critique of global games is the delicate information structure
  - There is some unobservable fundamental, and everybody gets a private signal about it.
  - How about the financial market which aggregates all the private signal?
- Global games work because it breaks "common knowledge"
- Private signal introduces heterogeneity to individual agents, and that goes a long way to pin down a unique equilibrium.

Implications

- Public signals play a role in coordinating outcomes beyond its mere information content
  - Financial market "overreact" to announcement for Fed. Could be rational
  - Wall street journals or Washington post can affect the agent's belief on others
- Interesting empirical paper: Chwe (1998) on Superbowl advertisement
  - The price of advertisement increases more than linearly in the number of viewers
  - The premium may reflect "coordination value" of consumers' purchase decisions
  - Indeed, the premium mainly concentrate on coordination goods like "Apple Macintosh" or "Beer," but not in "Batteries"

Goldstein-Pauzner (2005,JF)

- A direct application of global games technique in Diamond-Dybvig
- Good application: key insight is the same, but need to deal with realistic complications
- Why? In bank runs setting, naturally strategic complementarity does not hold globally
- Rochet-Vives (2004) assume this away...
- Say liquidation value $I = 0$ and default occurs for sure. Strategic complementarity always holds
- Say n other agents run. My incentives of running is constant (so weakly increasing in n):

$$V(\text{run} \mid n) - V(\text{no run} \mid n) = 0 - 0 = 0$$

- If liquidation value $I > 0$ which is Diamond-Dybvig framework ($I = 1$)
- Then $V(\text{run} \mid n) - V(\text{no run} \mid n) = 1/n$ is decreasing in n
- The more the other people are running, the less the incentives for me to run with them (as all of us have to share the liquidation value)

Key derivations

- Utility $u(\cdot)$, prob λ being early, LT project returns R with prob θ
- Agents observe $\theta_i = \theta + \varepsilon_i, \varepsilon_i \sim U[-\varepsilon, \varepsilon]; r_1$ early payment
- Early type withdraw early; say equilibrium running threshold for late type is θ^*
- Given true fundamental θ and n agents withdrawing early, late type calculates the utility differential $v(\theta, n)$ between stay and run,

$$v(\theta, n) = \begin{cases} \theta u\left(\frac{1-nr_1}{1-n}R\right) - u\left(r_1\right) & \text{if } \lambda \leq n \leq \frac{1}{r_1} \\ 0 - \frac{1}{nr_1}u\left(r_1\right) & \text{if } \frac{1}{r_1} \leq n \leq 1 \end{cases}$$

  - If not enough resource to serve early withdrawals, random rationing
- Only one-sided strategic complementarity
  - Strategic complementarity holds for $n \in \left[\lambda, \frac{1}{r_1}\right]$
  - Strategic complementarity fails for $n \in \left[\frac{1}{r_1}, 1\right]$

key derivations (2)

- Agent θ_i is calculating expected utility differential

$$\Delta\left(\theta_i, \theta^*\right) = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v\left(\theta, n\left(\theta, \theta^*\right)\right) d\theta$$

- Improper prior (agnostic prior) $\theta_i = \theta + \varepsilon_i, \varepsilon_i \sim U[-\varepsilon, \varepsilon] \Rightarrow$ Posterior of $\theta \sim U\left[\theta_i - \varepsilon, \theta_i + \varepsilon\right]$
- $n\left(\theta, \theta^*\right)$: expected # of withdrawals = early + (late with $\theta_i' < \theta^*$), given true fundamental θ

$$n\left(\theta, \theta^*\right) = \begin{cases} 1 & \text{if } \theta < \theta^* - \varepsilon \\ \lambda + (1-\lambda)\frac{\theta^* + \varepsilon - \theta}{2\varepsilon} & \text{if } \theta \in \left[\theta^* - \varepsilon, \theta^* + \varepsilon\right] \\ \lambda & \text{if } \theta > \theta^* + \varepsilon \end{cases}$$

- Equilibrium condition: $\Delta\left(\theta_i = \theta^*, \theta^*\right) = 0$

Results and Drawbacks

- \- They prove the uniqueness of equilibrium, without assuming ad hoc restrictions on strategy space (say, threshold strategy)
- It is a big deal for this paper
- Diamond-Dybvig finds an optimal banking solution for liquidity insurance. But it is subject to bank runs, and without knowing the probability of bank runs, we do not know what is the optimal solution ex ante
- You can put sunspot probabilities to Diamond-Dybvig... but arbitrary
- In Goldstein-Pauzner, endogenous θ^* depends on $r_1 \Rightarrow$ the running probability is a function of solution itself (the payment r_1) $\Rightarrow$ optimal demand-deposit contract
- One unsatisfactory part: how do we interpret the upper dominating threshold?
  - When θ is high enough, even if everyone is running, I want to stay-maybe ok for just one bank
  - If everyone is running on the financial system, probably everything is destroyed