Lecture 3: GK model
–A Model of Financial Intermediation

Jingyi Zhang

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Overview of This Lecture

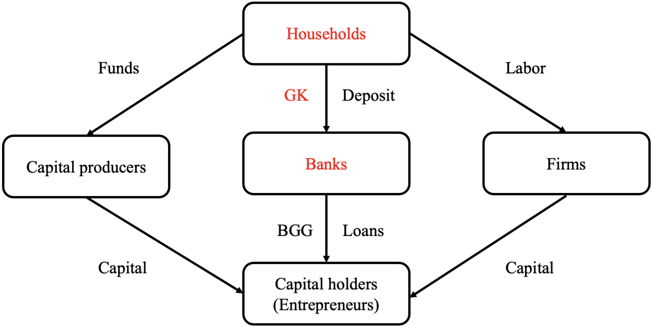
- GK model versus BGG model
- Model
- Financial Accelerator Effect
- Credit Policies

This lecture notes is based on paper Gertler, Mark & Kiyotaki, Nobuhiro, 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis," Handbook of Monetary Economics, in: Benjamin M. Friedman & Michael Woodford (ed.), Handbook of Monetary Economics, edition 1, volume 3, chapter 11, pages 547-599, Elsevier.

Motivation

- Most of the previous literature focus on models with frictionless capital markets or frictions on non financial firms.
- GK model focuses on a real business cycle model with frictions on financial intermediation.
- How frictions on financial intermediation may affect real activity?
- How various credit policies may work given crisis scenario?

Differences between GK and BGG



- GK model: agency problem between deposits and banks;
  BGG model: agency problem between entrepreneurs and banks.
- GK model: investment banks;
  BGG model: commercial banks.

Environment I

- The economy consists of a unit continuum of islands.

  a) Each island has a continuum of banks and entrepreneurs

  b) Banks only make loans to local entrepreneurs (actually, it's not loans. it's equity funds. And banks can collect deposits from all households.)

  c) Capital is not mobile

  d) At the beginning of each period, each island receives a $0-1$ shock on whether there is investment opportunity. A fraction π^i of islands are endowed with investment opportunities, and the other fraction π^n of islands are not. We have

$$\pi^i + \pi^n = 1.$$

Environment II

- The amount of capital stock supplied by capital producers on each type of islands by the end of period t is given by,

$$K_t^i = [I_t + (1-\delta)\pi_i \psi_t K_{t-1}], \quad K_t^n = (1-\delta)\pi_n \psi_t K_{t-1},$$

where I_t denotes the new investments made by the islands with investment opportunities. K_t denotes the total amount of capital stock by the end of period t. ψ_t denotes aggregate capital quality shock to allow for variation in capital price (same across islands).

- Capitals are acquired by the entrepreneurs using funds supplied by the banks, denoted by S_t^h. Capital market clearing requires,

$$S_t^i = Q_t^i [I_t + (1-\delta)\pi_i \psi_t K_{t-1}], \quad S_t^n = Q_t^n (1-\delta)\pi_n \psi_t K_{t-1},$$

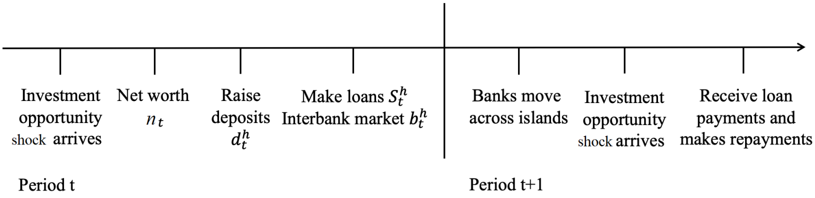
where Q_t^h denotes the price of capital in type h islands.

- The accumulation of aggregate capital stock follows,

$$K_t = I_t + (1-\delta)\psi_t K_{t-1},$$

Banks I

- Timeline of bank



Note: 1) The bank's net worth n_t depends on the period-t realization of investment opportunity in the island that the bank stays in the period $t-1$.
2) GK assumes that deposits are collected before the realization of investment opportunities. According to the constraint that ensures that bankers do not divert, the supply of deposits to a bank depends on its net worth. Here, we assume that deposits are collected after the realization of investment opportunities, which should not affect the model equilibrium.

Banks II

- Banks' flow of funds constraint: $S_t^h = n_t + b_t^h + d_t^h, \quad h \in \{i, n\}$.

- Bank's return on loans (again, bank loans in form of equity)

$$R_{t+1}^h = \psi_{t+1} \frac{r_{t+1}^k + (1-\delta) Q_{t+1}^{h'}}{Q_t^h},$$

where ψ_{t+1} denotes aggregate capital quality shock that allows for variation in capital price (same across islands); h' denotes what type the island becomes in the next period $t+1$.

Note: R_{t+1}^h is affected by both capital price in period t and capital price in period $t+1$.

- The banker's next-period net worth n_{t+1} is then given by,

$$n_{t+1} = R_{t+1}^h S_t^h - R_{bt} b_t^h - R_t d_t^h.$$

Banks III

- Each banker pay dividends when it exits (with probability $1-\sigma$) and becomes a worker. The banker's objective function is the expected discounted net worth when it exits:

$$V_t^h = \max E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1}\beta^i \frac{\lambda_{t+i}}{\lambda_t} n_{t+i},$$

Incentive Constraint (Bank's financial constraint):

$$V_t^h \geq \theta \left(S_t^h - w b_t^h \right),$$

where λ_t denotes the Lagrange multiplier of the household's budget constraint.

- Agency problem between bankers and depositors (imperfect contract enforcement): the banker may exit the business and transfer a fraction θ of divertible assets to its family.
- If $w > 0$, the interbank market (the wholesale market) is less frictional than the deposit market (the retail market).

How to solve the banker's problem?

- **Step 1:** Guess $V_t^h = \gamma_t^h n_t$.
In other words, we assume that the banker's optimized objective value V_t^h is proportional to his or her initial net worth n_t. And γ_t^h denotes the banker's return to equity and is assumed to be independent of the banker's net worth.

The bank's problem can be summarized as following

$$\begin{aligned} V_t^h &= \max_{S_t^h, d_t, b_t^h} E_t \sum_{i=1}^{\infty} (1-\sigma)\, \sigma^{i-1} \beta^i \frac{\lambda_{t+i}}{\lambda_t} n_{t+i} \\ s.t. \quad V_t^h &\geq \theta \left(S_t^h - w b_t^h\right) \\ S_t^h &= n_t + b_t^h + d_t^h \\ n_{t+1} &= R_{t+1}^h S_t^h - R_{bt} b_t^h - R_t d_t^h \end{aligned}$$

How to solve the banker's problem?

- We can see that maximizing the expected discounted net worth when exiting is equivalent to maximizing the expected net worth in the next period $t+1$:

$$\begin{aligned} V_t^h &= \max E_t \left(1-\sigma\right) \beta \frac{\lambda_{t+1}}{\lambda_t} n_{t+1} + \sigma \beta \frac{\lambda_{t+1}}{\lambda_t} \times V_{t+1}^{h'} \\ &= \max E_t \left(1-\sigma+\sigma\gamma_{t+1}^{h'}\right) \beta \frac{\lambda_{t+1}}{\lambda_t} n_{t+1}, \end{aligned}$$

where h' denotes the type of island banker allocates in next period.

Therefore, the banker's problem becomes:

$$\begin{aligned} \max \quad E_t n_{t+1}^h &= E_t R_{t+1}^h S_t^h - R_{b_t} b_t^h - R_t d_t^h \\ &= E_t R_{t+1}^h S_t^h - R_{b_t} b_t^h - R_t \left(S_t^h - b_t^h - n_t\right) \\ &= E_t \left(R_{t+1}^h - R_t\right) S_t^h - \left(R_{b_t} - R_t\right) b_t^h + R_t n_t \\ \lambda_{b_t}^h : \ \gamma_t^h n_t &\geq \theta \left(S_t^h - w b_t^h\right) \end{aligned}$$

How to solve the banker's problem?

- **Step 2:** Take γ_t^h as given when we solve the banker's optimal decisions.

  The first order conditions of the above problem gives the banker's optimal decisions:

$$S_t^h(E_t R_{t+1}^h - R_t - \theta\lambda_{b_t}^h) = 0,$$
$$b_t^h(R_{b_t} - R_t - \theta w \lambda_{b_t}^h) = 0,$$

  which imply that
  1) If the banker's balance sheet constraint binds ($\lambda_{b_t}^h > 0$), the return to bank assets (inefficiently) exceeds the cost of bank deposits.
  2) The cost of interbank borrowing is higher than the cost of bank deposits if the banker's balance sheet constraint binds ($\lambda_{b_t}^h > 0$) and the wholesale interbank market operates more efficiently than the retail deposit market ($w > 0$).

When wholesale financial market is less frictional ($w > 0$)

$$\frac{E_t R^h_{t+1} - R_t}{R_{b_t} - R_t} = \frac{1}{w} \;\Rightarrow E_t R^h_{t+1} \textit{ equals across islands.}$$

- Capital prices Q^h_t and return on bank loans R^h_{t+1} equal across islands.

  Recall: $E_t R^h_{t+1} = E_t \psi_{t+1} \frac{r^k_{t+1} + (1-\delta) Q^{h'}_{t+1}}{Q^h_t}$ equals across islands.

  Because: ψ_{t+1}, r^k_{t+1} equals across islands, $Q^{h'}_{t+1}$ depends on h', the type of island in period $t+1$, which is unknown in period t and is not specific to h, the type of island in period t.

  Thus: $E_t \psi_{t+1}$, $E_t r^k_{t+1}$ and $E_t Q^{h'}_{t+1}$ equals across islands $\Rightarrow$ Q^h_t equals across islands $\Rightarrow$ Using the same logic for $t+1$, then Q^h_{t+1} equals across islands.

  Then, $R^h_{t+1} = \psi_{t+1} \frac{r^k_{t+1} + (1-\delta) Q^{h'}_{t+1}}{Q^h_t}$ equals across islands.

  Rewrite: $Q^h_t \equiv Q^k_t$ and $R^h_{t+1} \equiv R^k_{t+1} = \psi_{t+1} \frac{r^k_{t+1} + (1-\delta) Q^k_{t+1}}{Q^k_t}$.

When wholesale financial market is less frictional ($w > 0$)

- The return to bank equity γ_t^h equals across islands.

  Recall: $V_t^h = \max E_t(1-\sigma+\sigma\gamma_{t+1}^{h'})\frac{\lambda_{t+1}}{\lambda_t}n_{t+1}$

$$\gamma_t^h n_t = E_t(1-\sigma+\sigma\gamma_{t+1}^{h'})\beta\frac{\lambda_{t+1}}{\lambda_t}[(R_{t+1}^k - R_t)(S_t^h - wb_t^h) + R_t n_t]$$

$$\Rightarrow \gamma_t^h = E_t(1-\sigma+\sigma\gamma_{t+1}^{h'})\beta\frac{\lambda_{t+1}}{\lambda_t}[(R_{t+1}^k - R_t)\frac{\gamma_t^h}{\theta} + R_t]$$

$$\Rightarrow \gamma_t^h \textit{ equals cross } h$$

  Rewrite: $\gamma_t^h = \gamma_t$. We can solve for the evolution equation of the return to bank equity γ_t:

$$\gamma_t = E_t(1-\sigma+\sigma\gamma_{t+1})\beta\frac{\lambda_{t+1}}{\lambda_t}[(R_{t+1}^k - R_t)\frac{\gamma_t}{\theta} + R_t] \qquad (1)$$

  Notes: 1) an increase in the expected external financial premium $(R_{t+1}^k - R_t)$ leads to an increase in the return to bank equity γ_t.
  2) an increase in the return to bank equity γ_t itself leads to higher bank leverage $(\frac{S_t}{n_t})$ by relaxing the balance sheet constraint, and further raises the return to bank equity.

When wholesale financial market is less frictional ($w > 0$)

- Capital prices Q_t^h and return on bank loans R_{t+1}^h equal across islands.
- The return to bank equity γ_t^h equals across islands.
- Bankers in all islands are either constrained together or unconstrained together. (because $\lambda_{b_t}^h$ equals across islands.)

Symmetric Frictions ($w = 0$)

- Banks on non-investing island have no desire to lend on interbank market ($b_t^h = 0$).
  - **Proof:** If $b_t^h > 0$, then the return to interbank loans equals the cost of bank deposits ($R_{bt} - R_t = 0$), and must be no higher than the expected return to capital ($R_{bt} \leq E_t R_{t+1}^k$).
  - In other words, banks on investing island are only willing to borrow interbank loans at the risk-free interest rate / deposit rate R_t.
  - Banks on non-investing island would rather allocate funds to capital, or obtain less deposits from household, than lending on interbank market at R_t.
- Resource misallocation across islands.

Symmetric Frictions ($w = 0$)

- Resource misallocation across islands.
  - **Intuition:** With more investment installed, investing islands have higher capital supply and therefore lower capital prices ($Q_t^i < Q_t^n$) (because demand for capital are constrained by bank net worth).
  - $\Rightarrow$ higher expected return to capital ($E_t R_{t+1}^i > E_t R_{t+1}^n$) in investing islands $\Rightarrow$ resource misallocation across islands: investing islands are underinvested.
- $E_t R_{t+1}^i > E_t R_{t+1}^n$ also implies,
  - higher return to bank equity ($\gamma_t^i > \gamma_t^n$) and higher bank leverage ($\frac{S_t^i}{N_t^i} > \frac{S_t^n}{N_t^n}$) on investing islands

$$\gamma_t^h = E_t(1 - \sigma + \sigma\gamma_{t+1}^{h'})\beta\frac{\lambda_{t+1}}{\lambda_t}[(R_{t+1}^h - R_t)\frac{\gamma_t^h}{\theta} + R_t]$$

  - banks on investing island face tighter constraint ($\lambda_{bt}^i > \lambda_{bt}^n$)

$$E_t R_{t+1}^h - R_t = \theta\lambda_{bt}^h$$

A Negative Capital Quality Shock $\psi_t \downarrow$

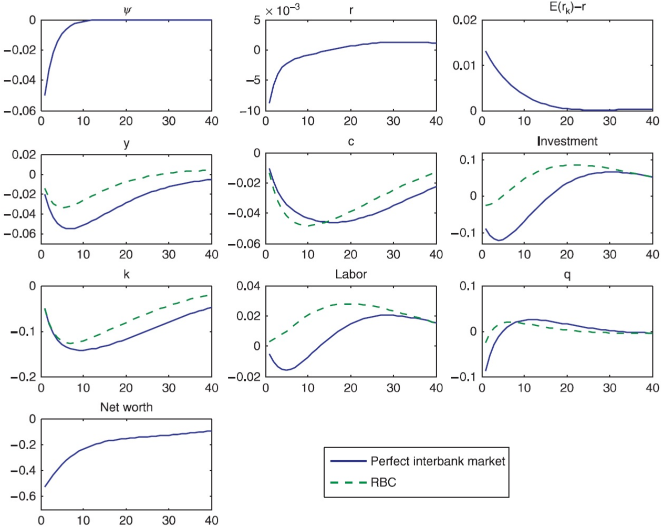


Figure 1 Crisis experiment: Perfect interbank market.

A Negative Capital Quality Shock $\psi_t \downarrow$

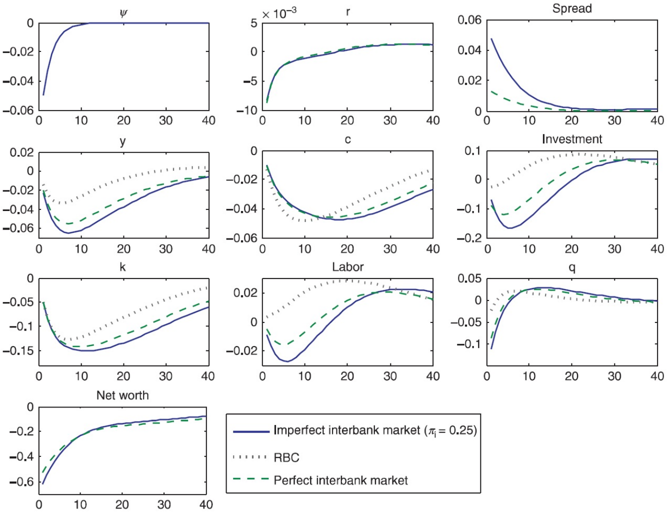


Figure 2 Crisis experiment: Imperfect interbank market.

A Negative Capital Quality Shock $\psi_t \downarrow$

- RBC economy: Modest downturn in output and consumption, and high expected return to capital induces increase in investment and employment
- Perfect interbank market ($w = 1$): Output, investment and employment decline more than in the RBC case
- Imperfect interbank market ($w = 0$): Overall deterioration is further magnified and sharp rise in credit spread

A Negative Capital Quality Shock $\psi_t \downarrow$: Mechanism

- Perfect interbank market $(w = 1)$
  - The realized return to capital R_t^k falls, leading to a decline in bankers' net worth (N_t). The bankers' financial constraint implies that banks' borrowing capacity are more constrained and their supply for firm loans also falls (S_t). The shrinkage in credit supply reduces the demand for physical capital and therefore the capital price Q_t^k. The fall in capital price reduces bankers' net worth N_t further, accelerating the decline in capital demand and capital price.
  - Note the credit spread $E_t R_{t+1}^k - R_t$ rises dramatically, implying tighter financial constraint (high λ_{bt}). The increase in $E_t R_{t+1}^k$ is driven by the magnified drop in the investment (low capital stock implies high capital rents).
  - As banks are constrained in taking high leverage, bank net worth cannot recover very fast, contributing to the slow recovery of the whole economy.

A Negative Capital Quality Shock $\psi_t \downarrow$: Mechanism

- Imperfect interbank market ($w = 0$)
  - Banks in investing islands take higher leverage and have tighter financial constraint. This makes banks in investing islands more responsive to the shock than banks in non-investing islands.
  - Resource misallocation problem becomes more severe. The larger loss from resource mis-allocation cross islands amplifies the output decline.

Directed Lending to Entrepreneurs I

- Assume central bank issue government debt to the household D_{gt} and lend funds S_{gt} to the entrepreneurs at the market rate $(S_{gt} = D_{gt})$. The household's budget constraint is given by,

$$(D_{pt} + D_{gt}) + C_t = W_t H_t + R_{t-1}(D_{p,t-1} + D_{gt-1}) + Transfers.$$

- If bank financial constraint binds $\lambda_{bt} > 0 \implies$ Overall supply of credit S_t rises,

$$\left.\begin{array}{c} S_t = S_{gt} + S_{pt} \\ S_{pt} = \dfrac{\gamma_t}{\theta} N_t \end{array}\right\} \implies S_t = \frac{\gamma_t}{\theta} N_t + S_{gt}.$$

- If bank financial constraint does not bind $\lambda_{bt} = 0$, then the increase in government credit S_{gt} completely crowds out the private credit S_{pt}, with the total amount of credit S_t unchanged and is given by, $E_t R^k_{t+1} = R_t$.

Directed Lending to Entrepreneurs II

- Suppose that the central bank chooses to intermediate the fraction ϕ_t of total credit: $S_{gt} = \phi_t S_t$. The policy tool ϕ_t follows a simple rule that responses to the credit spread in the investing islands,

$$\phi_t = \nu_g[(E_t(R^i_{t+1}) - R_t) - (\bar{R}^i - \bar{R})].$$

- Comment:
  - The directed lending to entrepreneurs expand the total credit supply directly.
  - Directed lending may also expand the private credit supply S_{pt} by raising capital price, raising bank net worth and relaxing bank financial constraints.
  - However, central banks may be less inefficient in monitoring loans than private banks.

Directed Lending to Entrepreneurs III

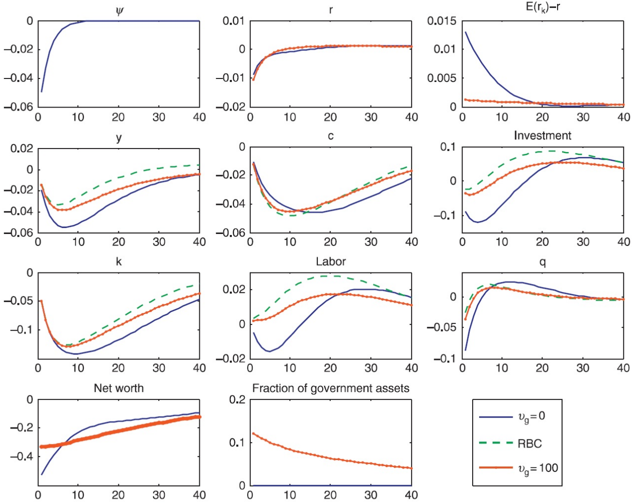


Figure 3 Lending facilities: Perfect interbank market.

Directed Lending to Entrepreneurs IV

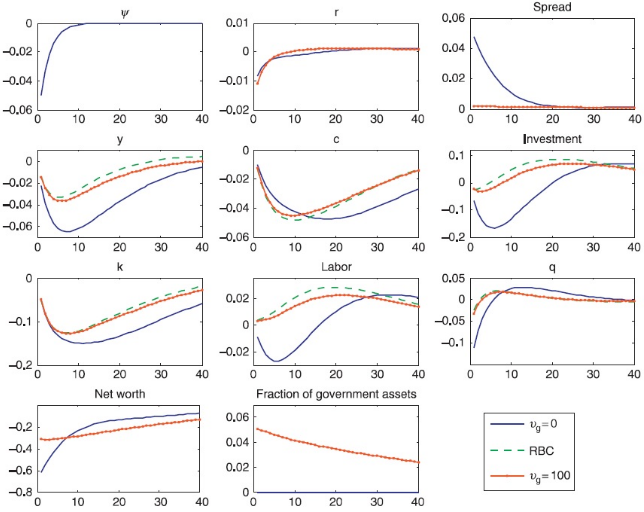


Figure 4 Lending facilities: Imperfect interbank market.

Directed Lending to Entrepreneurs V

- RBC economy: Modest downturn in output and consumption, and high return to capital induces increase in investment and employment
- Perfect interbank market ($w = 1$): reduces the rise in the credit spread and reduces the overall drop in investment and output.
- Imperfect interbank market ($w = 0$): Policy effectively dampens the overall deterioration and is in fact much more effective as compared to the perfect interbank market case.

Why the policy intervention is even more effective in imperfect interbank case ?

- The policy intervention reduces the overall drop in investment by around 10% but the fraction of government credit in total credit is only 5% in the case of imperfect interbank market ($w = 0$), but over 10% in the case of perfect interbank market ($w = 1$).
- **Reason:** In the case ($w = 0$), only entrepreneurs on investing islands obtain government credit. Because entrepreneurs on investing islands have higher expected return to capital and offer higher interest rate on government credit. This interst rate is too high for entrepreneurs on non-investing islands to be profitable.
- In the case ($w = 1$), all entrepreneurs have the same expected return to capital and obtain government credit.
- Central bank provides less credit with higher interst rates in the case ($w = 0$) than the case ($w = 1$). However, these credits are granted to islands with very tight financial constraints (investing islands), and generate a stimulating effect as large as the case ($w = 1$).

Discount Window Lending to Banks I

- Assume that banks in island h obtain deposits d_t^h, interbank loans b_t^h and discount window loans m_t^h from the central bank, to finance its loans to the entrepreneurs S_t^h. The bank's flow of funds constraint is given by,

$$S_t^h \geq d_t^h + b_t^h + m_t^h + n_t.$$

The bank's divertible assets become, $S_t^h - wb_t^h - w_g m_t^h$, where $(1 > w_g > w \geq 0)$, m_t^h, implying that the central bank is better able to enforce bank repayment than depositors (households) and private banks.

This relaxes the bank's financial constraint,

$$V_t^h \geq \theta(S_t^h - wb_t^h - w_g m_t^h).$$

Discount Window Lending to Banks II

- The bank's problem is then given by,

$$
\begin{aligned}
\max \quad E_t n^h_{t+1} &= E_t R^h_{t+1} S^h_t - R_{bt} b^h_t - R_t d^h_t - R_{mt} m^h_t \\
&= E_t R^h_{t+1} S^h_t - R_{b_t} b^h_t - R_t \left(S^h_t - b^h_t - n_t - m^h_t \right) - R_{mt} m^h_t \\
&= E_t \left(R^h_{t+1} - R_t \right) S^h_t - \left(R_{b_t} - R_t \right) b^h_t - \left(R_{mt} - R_t \right) m^h_t + R_t n_t \\
\lambda^h_{b_t} : \; \gamma^h_t n_t &\geq \theta \left(S^h_t - w b^h_t - w_g m^h_t \right)
\end{aligned}
$$

Bank's optimization conditions are given by,

$$
\begin{cases}
R^h_{t+1} - R_t = \theta \lambda^h_{bt} \\
R_{bt} - R_t = \theta w \lambda^h_{bt} \\
R_{mt} - R_t = \theta w_g \lambda^h_{bt}
\end{cases}
$$

Discount Window Lending to Banks III

- If the balance sheet constraint is binding ($\lambda_{bt}^{h} > 0 : R_{t+1}^{h} > R_{mt} > R_{bt} > R_t$)

  Banks are willing to pay a premium for discount window loans because it relaxes their financial constraint.

  Increasing discount window loans also increase banks' total supply of credit, which is given by,

  $$S_t = \frac{\gamma_t}{\theta} N_t + w_g M_t$$

  (from $\gamma_t N_t = \theta(S_t - wB_t - w_g M_t)$, $B_t = 0$.)

Discount Window Lending to Banks IV

- In the case with imperfect interbank market ($w = 0$), only banks on the investing islands are willing to borrow from discount window lending such that,

$$R_{mt} - R_t = w_g(E_t R^i_{t+1} - R_t)$$

- Recall that, $E_t R^i_{t+1} > E_t R^n_{t+1}$, so that,

$$R_{mt} - R_t > w_g(E_t R^n_{t+1} - R_t)$$

  Therefore, in the case with imperfect interbank market, interest rate on discount window loans would be too high for banks on non-investing islands.

Comments on two types of credit policies

- **Directed lending to entrepreneurs**
  - Direct lending to entrepreneurs helps expand the credit supply by providing government credit directly.
  - However, central banks may be less inefficient in monitoring loans than private banks, generating concerns on credit risks.
- **Discount window lending to banks**
  - Discout window lending to banks helps expand the credit supply by relaxing the bank balance sheet constraint.
  - Banks still bear all the credit risk on firm loans. This is an advantage for discount window lending because central banks do not have monitor firms directly. However, central bank still have to monitor the banks. And it can be costly if M_t is really large.