Theory of Corporate Finance

Optimal Contracting and Security Design

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Towards Modern Corporate Theory

- Berle and Means (1932): Separation of ownership and control.
- In reality, managers or entrepreneurs pursue their own interests rather than the shareholders.
  - Shirking, stealing, and manipulating earnings...
  - Consuming perks (non-pecuniary consumption: fancy office, private jet, etc.), empire building (conglomerate mergers in 80s).
  - These actions benefit the manager but do not necessarily create value for shareholders or may even hurt them.
- How can we align the incentives of investors and the manager?
- Key friction that invalidates MM: some decisions are unobservable and information is costly.

What is Financial Contracting?

- First generation of agency theory of capital structure takes corporate securities as given.
  - How do they arise? are these securities optimal?
- Second generation views securities as contracts and derive the optimal securities from primitive frictions.
  - Does the optimal contract look like debt, equity, credit lines, convertibles, or some other corporate securities?
  - Provide useful perspectives and insights on cash flow rights, liquidation rights, and control rights.

Contract Theory

- Principal-Agent Theory
  - Adverse selection: the agent (manager) has private information over profit, cash flow, etc. that leads to conflicts of interests.
  - Moral hazard: the agent takes an unobservable action which affects some contractible performance (profit, cash flow, revenue etc.).
- Incomplete Contract (Grossman-Hart-Moore paradigm)
  - Contracts are "incomplete"
  - Some variables are observable but unverifiable (courts cannot officially observe or do not know how to define), so cannot be written in contracts
  - Emphasize on allocation of control rights
- Here, we focus on principal-agent theory or complete contracting.

Mechanism Design

- **Mechanism Design:** in an economic environment where agents have private information (types), how should a principal design a mechanism to achieve certain objectives (e.g., optimality or efficiency)?
- An agent has private information of type $\theta \in \Theta$. The principal designs a mechanism aiming to implement a desired allocation $f(\theta)$.
  - Auction: θ = valuation of a good, $f(\theta)$ = sell or not? price?
- In general, a mechanism denoted as (M, g) specifies:
  - message space M;
  - and outcome function $g(m)$, where $m \in M$.
- Agent sends a message (report) $m = \sigma(\theta)$, based on his information θ. The outcome function then assigns $g(m)$ as the allocation.
  - The message space M can be very general.

Revelation Principle

- **Revelation principle** (Myerson 1979): wlog., we can focus on direct revelation mechanism that requires the agent to report their type, so $M = \Theta$, and implements outcome $f = g \circ \sigma$
  - Any "equilibrium" of the mechanism (M, g) can be replicated by a **truthful** equilibrium of a **direct** revelation mechanism (Θ, f).
  - That is, in such equilibrium, the agent reports type truthfully $\sigma(\theta) = \theta$, and allocation $f(\theta) = g \circ \sigma(\theta) = g(\theta)$ is achieved.
- It's a powerful tool! Simplifies the search for optimal mechanisms
- Wide range of applications:
  - auctions, matching markets, compensation contracts...(Mas-Colell, Whinston, and Green Ch. 23 and Jehle and Reny Ch. 9)

Townsend (1979): Motivation

- The earlier literature posited, rather than derived, specific financial structures.
- Townsend's contribution was the first to obtain a financial structure from an optimization problem, and therefore from primitive assumptions.
- Though neoclassic economic theories (Arrow & Debreu) are built on complete markets, in practice, many contracts are not state contingent.
  - Debt contracts are not contingent on firm profits before bankruptcy.
  - Insurance contracts typically have deductions.
- How to rationalize the popularity of debt contracts in financing?
- Key idea: some contracting party may not have information of which events (states of nature) occurred.

Townsend (1979): setup

- A project requires outside financing I; generates cash flow $x \in [0, \infty)$ with density $\pi(\cdot)$.
  - The borrower privately observes cash flows x.
  - Investors do not know x and therefore cannot enforce repayment.
- Both parties are risk-neutral; borrower has limited liability.
- Credit market is competitive; investors will lend as long as they break even in expectation.
- Given borrower's report, investors can choose whether to audit the project performance or not.
  - If audit, investors need to pay a cost $c > 0$, and they will find out the true cash flow.
- This model is typically referred to as costly state verification (CSV).
  - It's a fundamental building block for debt instruments.

Formulating the Contracting Problem

- Trade-off facing investors:
  - Auditing is needed to recoup investment. Otherwise, the manager would always claim "zero cash-flow".
  - But auditing is costly.
- The borrower privately observes cash flows x and reports $\hat{x}$.
- A contract consists of two components.
  - It specifies the **audit decision** $a(\hat{x}) \in \{0, 1\}$;
  - If $\hat{x}$ is audited, then the contract specifies the **repayment** to be $r_1(x, \hat{x})$; Otherwise, the **repayment** is $r_0(\hat{x})$.

Formulating the Contracting Problem

- Assume that the credit market is competitive, implying that the borrower's expected payoff is maximized.
- So the optimal contract solves:

$$\max_{\{a,r_0,r_1\}} \int_0^\infty \{[1-a(x)][x-r_0(x)]+a(x)[x-r_1(x,x)]\}\pi(x)dx$$

subject to

- Incentive compatibility (IC): for any x and $\hat{x}$

$$[1-a(x)][x-r_0(x)]+a(x)[x-r_1(x,x)]$$

$$\geq (1-a(\hat{x}))[x-r_0(\hat{x})]+a(\hat{x})[x-r_1(x,\hat{x})]$$

- Investors' participation (IR):

$$\int_0^\infty \{[1-a(x)]r_0(x)+a(x)[r_1(x,x)-c]\}\pi(x)dx \geq I$$

- Limited liability (LL): $r_0(x), r_1(x,x) \leq x$

Implications of Incentive Compatibility

- Two regions: no-audit region R_0 and audit region R_1
  - $R_0 \cap R_1 = \emptyset$ and $R_0 \cup R_1 = [0, \infty)$
- Result 1: on R_0, repayment $r_0(x)$ must be constant.
  - why?
  - So let $r_0(x) = D$ on R_0 .
  - IC requires information insensitivity on R_0, which implies a contract with a flat region.
- Result 2: on R_1, repayment $r_1(x, x)$ must be smaller than D.
  - Otherwise, for $x \in R_1$ and $r_1(x, x) > D$, the manager will report $\hat{x} \in R_0$ and make a smaller repayment.
- Result 3: obviously, $0 \in R_1$.
  - Otherwise, always reports no income, and financing breaks down.

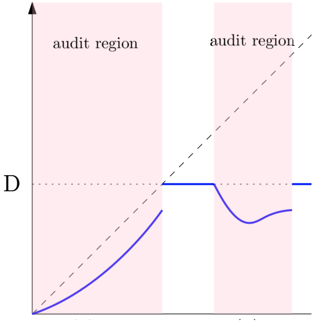
Standard Debt Contract

- **Standard debt contract:** face value D, no audit if D is repaid, and audit with no reward.
  - $a(x) = 0$ for $x \geq D$ and $a(x) = 1$ for $x < D$;
  - borrower payoff $= \max\{x - D, 0\}$;
  - auditing is similar to a bankruptcy procedure.
- In the following, we show that a standard debt contract is optimal.
  - Proof strategy: for any feasible contract (satisfying IC, IR and LL), there exists a standard debt contract that does at least as well for the borrower.
- Result 4: the optimal contract in fact **minimizes the auditing cost**.
  - IR binds, implying that investors' expected payoff equals I. So,

$$\text{Borrower expected payoff} = \underbrace{\int_0^\infty x\pi(x)dx - I}_{\text{project NPV}} - \underbrace{c\int_0^\infty a(x)\pi(x)dx}_{\text{expected auditing cost}}$$

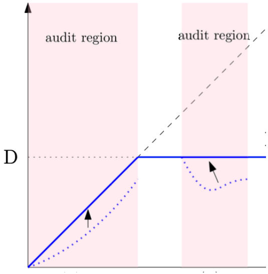
  - Maximizing borrower expected payoff is equivalent to minimizing the expected auditing cost.

Step 0: Start with any feasible contract



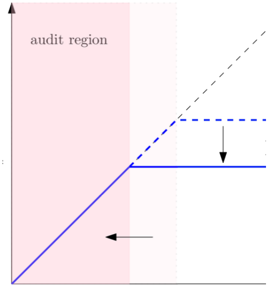
- Take any feasible contract. In the picture, we have
  - $0 \in R_1$;
  - on R_0, the repayment $r_0(x) =$ D, a constant;
  - on R_1, the repayment $r_1(x,x) \leq min\{D,x\}$.

Step 1: Construct a debt contract that reduces auditing cost and pays out more to investors



- Raise repayment $r_1(x, x)$ on R_1 to $\min\{D, x\}$.
- Revome the upper auditing region.
- The IR constraint will be slack after these adjustments.

Step 2: Further save auditing cost by reducing the audit region and adjusting the debt level



- Adjust the debt face value to D^* until investors' participation constraint binds.

$$\underbrace{\int_0^{D^*} (x-c)\,\pi(x)dx}_{\text{audit region}} + \underbrace{(1-\Pi(D^*))\,D^*}_{\text{non-audit region}} = I$$

- All steps together imply that the final debt contract dominates the initial contract.

Renegotiation

- The optimal contract requires commitment and is not renegotiation-proof.
  - The auditing region is committed by the contract but may not be efficient ex-post.
  - Auditing afterwards on R_1 is a deadweight loss, because the borrower reports truthfully.
  - Especially, in the case of low-cash-flow report, the threat to audit may not be credible.
- The borrower and investors are tempted to renegotiate to reduce the auditing cost.
- But anticipating the absence of audit after renegotiation undermines the borrower's incentive to truthfully report.

Moral Hazard and Contracting

- Another typical agency issue in corporate finance models is **moral hazard**.
  - The agent can take some hidden action that potentially benefits himself but hurt efficiency.
- Motivation: **credit rationing** widely exists and inhere in the very nature of loan market. Why?
  - A borrower can not obtain a loan even if he is willing to pay the interest that the lender asks for, perhaps a even higher rate.
  - Moral hazard is a possible friction that explains credit rationing.
  - **Optimal contracting** is one approach to reduce this friction, rationalizing certain securities or financial policy.

Moral Hazard and Contracting: Set-up

- Here, we use the setup in Ch3 of Tirole's book.
- An agent is risk-neutral, has wealth A, and is protected by limited liabitlity.
- A principal (investor) is risk-neutral and has **deep pockets**.
- A project that needs investment I, and produces cash-flow R.
  - Assume $x \in \{x_S, x_F\}$, and denote $\Delta x \equiv x_S - x_F > 0$.
- The project cash-flow is affected by the agent's effort choice e.
  - Assume $e \in \{e_H, e_L\}$, and that the agent gets **priviate benefit** $B > 0$ if shirking (i.e., $e = e_L$)
  - and $\Pr(x_S|e_H) = p_H$, $\Pr(x_S|e_L) = p_L$, with $\Delta p \equiv p_H - p_L > 0$.

Moral Hazard and Contracting: Set-up

- Timing:
  - At $t = 0$, the agent and the investor sign a contract that specifies each party's payoff.
  - At $t = 1$, the agent chooses effort level e.
  - At $t = 2$, the project delivers cash-flow x.
- The project has positive NPV *only if the agent works hard*:

$$p_H x_S + (1 - p_H) x_F - I > 0$$

- If the agent shirks, the project has negative NPV even with the private benefit:

$$p_L x_S + (1 - p_L) x_F - I + B < 0$$

- These assumptions ensure that we *only need* to consider inducing high effort in any optimal contract.

Moral Hazard and Contracting: Set-up

- **First best**: effort is observable.
  - Contract stipulates e_H directly and produces the project NPV.
  - How the pie is split does not affect output (MM).
- **Moral hazard** happens when effort is not observable.
  - Not feasible to write "you exert effort e_H" in the contract.
  - Agent may shirk that increase his own payoff but reduce the overall surplus of the relationship.
- Cash flow is **verifiable**, so, contract can be contingent on cash-flow.
- A contract (w_S, w_F) specifies how cash flows are splitted.
  - The agent receives w and the investor receives $x - w$; appropriate split provides incentives for the agent to work hard.

Incentive Compatibility

- Once the financing has been secured, the agent faces the following trade-off:
  - By shirking, he obtains private benefit but reduces the total pie.
- The agent volentarilly chooses to work hard if:

$$\underbrace{p_H w_S + (1 - p_H) w_F}_{\text{expected pay by working}} \geq \underbrace{p_L w_S + (1 - p_L) w_F}_{\text{expected pay by shirking}} + \underbrace{B}_{\text{private benefit}}$$

  - This is called the **incentive compatibility** (IC) constraint.
- We can simplify this constraint to be:

$$IC \Leftrightarrow \Delta p (w_S - w_F) \geq B$$

- IC requires large enough spread in agent's payoffs.

Incentive Compatibility

- IC implies that the following split of expected cash-flow at time 0.
- The expected cash-flow paid to the agent:

$$U_A = w_F + p_H(w_S - w_F) \geq w_F + p_H \frac{B}{\Delta p} \geq p_H \frac{B}{\Delta p}$$

- The last inequality is given by limited liability (LL) that says $w_F \geq 0$.
- The expected cash-flow that allocates to the investor:

$$U_I = \underbrace{p_H \Delta x + x_F}_{\text{Expeced cash-flow}} - U_A \leq p_H\left(\Delta x - \frac{B}{\Delta p}\right) + x_F \equiv \mathscr{P}$$

Endogenous Financial Constraint

- Agency friction imposes an upper limit of financing, or a financial constraint.
- IC implies $\mathscr{P}$ is the largest amount of expected cash-flow that can be committed to the investor.
  - It's called the project's **pledgeable income**.
  - With incentive provision and limited liability, agent earns at least $p_H \frac{B}{\Delta p}$ of the expected cash-flow as the **agency rent**.
- Project can only be financed if the investor participates: $\mathscr{P} \geq I - A$
  - Otherwise, forgo investments for projects with $\mathscr{P} < I - A$
- Alternatively, define $\bar{A} \equiv I - \mathscr{P} = I - p_H(\Delta x - \frac{B}{\Delta p}) - x_F$
- Investor's participation constraint $\Leftrightarrow A \geq \bar{A}$
  - Only the agent with wealth $A \geq \bar{A}$ can be financed; $A < \bar{A}$ will be credit rationed.

The Optimal Contract

- The above implications are all derived from the constraints.
- The optimal contract (w_S^*, w_F^*) depends on the barginning power or who designs the contract.
- Suppose $A \geq \bar{A}$ and the agent has barginning power (i.e., the agent designs contract).
- The optimal contract maximizes the agent's expected total payoff:

$$\max_{(w_S, w_F)} p_H w_S + (1 - p_H) w_F - A$$

subject to (IC), (LL), and (IR_I)

- Optimality$\Rightarrow$ $p_H(w_S^* - w_F^*) + w_F^* = p_H(x_S - x_F) + x_F - (I - A)$.
- **HW**: specify an optimal contract. Is it unique?

Features of Optimal Contract

- Optimal contract provides incentives by rewarding good outcomes.
- Two factors may make the firm credit constrained.
- First, low cash-on-hand (A).
- Second, high agency cost:
  - Large private benefit of shirking B.
  - Small likelihood ratio $\frac{\Delta p}{p_H}$: effort choice is less informative.