Diamond-Dybvig Model

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Introduction

- Classic Bank runs: fear of insolvency, depositors to demand money back
- Modern bank runs: short-term debt plays the role of deposits
- Basic problem in both cases is liquidity mismatch: short-term liquid liabilities but long-term illiquid assets.
- Liquid liabilities allow for efficient risk-sharing. Investors who may need liquidity prefer to invest in banks rather than hold illiquid assets directly.
- Coordination failure. Implementing efficient risk-sharing with liquid liabilities is one equilibrium. But also another equilibrium where investors panic and run to withdraw deposits.

Diamond-Dybvig (1983)

- Three dates $t \in \{0, 1, 2\}$ and a single consumption good (dollar)
- Measure one of the ex-ante identical consumers with a unit endowment at date 0. Consume either at date 1 or 2.
- Liquidity shocks: Preference:

$$u(c_1, c_2) = \begin{cases} u(c_1) \text{ with prob.} \lambda \\ u(c_1 + c_2) \text{ with prob.} 1 - \lambda \end{cases} \tag{1}$$

- Investment technology:
  - Storage: transform x goods at t to x goods at $t + 1$
  - Long-term investment: one dollar at date 0 yields $R > 1$ dollars at date 2 if the project is completed, $\lambda \leq 1$ dollar if terminated.

First Best planner problem I

- Suppose a planner chooses (c_1, c_2, x, y)

$$\max_{c_1, c_2, x, y} \pi u(c_1) + (1-\pi)u(c_2)$$
$$\pi c_1 \leq x$$
$$(1-\pi)c_2 \leq Ry$$
$$x + y = 1$$

- First order condition:

$$u'(c_1^{FB}) = Ru'(c_2^{FB})$$

- It follows that $c_1^{FB} < c_2^{FB}$, i.e., the first best allocation is incentive compatible.
- Optimal allocation equates MRS with the technological price.

First Best planner problem II

- Define $\eta(c) \equiv -\frac{cu''(c)}{u'(c)}$ as the relative risk aversion coefficient
- **Assumption**: $\eta(c) > 1$ for each $c > 0$
- In this case, $c_1^{FB} > 1$ and $c_2^{FB} < R$.
  - Construct $f(x) = xu'(x)$, which is decreasing in x when $\eta(x) > 1$
  - Therefore $Ru'(R) < u'(1)$

Complete markets allocation

- Assume that the agent can buy contingent claims, c_1 if impatient at price p_1, and c_2 if patient at price p_2

$$\max_{c_1,c_2} \pi u(c_1) + (1-\pi)u(c_2) \tag{2}$$

$$p_1 c_1 + p_2 c_2 \leq 1 \tag{3}$$

- First order condition

$$\frac{u'(c_1)}{u'(c_2)} = \frac{1-\pi}{\pi}\frac{p_1}{p_2}$$

- Firms can transform one unit of $t = 0$ goods into $1/\pi$ units of contingent claims if impatient, or into $\frac{R}{1-\pi}$ units of contingent claims if patient. So competition implies that

$$p_1 = \pi, \quad p_2 = (1-\pi)/R$$

Incomplete markets

- Suppose there are no insurance markets
- The only market is at $t = 1$, where agents can trade $t = 1$ goods against $t = 2$ goods

$$\max_{c_1, c_2, x, y} \pi u(c_1) + (1 - \pi) u(c_2)$$

$$c_1 \leq x + pRy$$

$$c_2 \leq Ry + \frac{x}{p}$$

$$x + y = 1$$

- In any equilibrium consumers invest in both technologies $p = 1/R$
- Optimal allocation: $c_1 = 1, \quad c_2 = R$
- This not necessarily coincides with the first best allocation.

A Bank I

- Consumers get together and create a bank, and put all their endowment in the bank
- The bank contract: each consumer can ask for c_1^{FB} at $t = 1$ or can wait until $t = 2$ and get a pro-rata share of whatever is left.
- Consumers then play a game where actions are “withdraw” or “wait”
- Let f_j be the number of depositors who arrived in line before consumer j and asked to withdraw and
- Let f be the total number of consumers that will eventually ask to withdraw
- The payoff for an impatient consumer is

Withdraw	Wait
$\begin{cases} c_1^{FB} & \text{if } f_j c_1^{FB} < x + \lambda y^{FB} \\ 0 & \text{otherwise} \end{cases}$	0

A Bank II

- The payoff for the patient consumer if he withdraws at $t = 1$ is

$$\begin{cases} c_1^{FB} & \text{if } f_j c_1^{FB} < x^{FB} + \lambda y^{FB} \\ 0 & \text{otherwise} \end{cases}$$

- The payoff for the patient consumer if he waits at $t = 1$ is

$$\max\{R\frac{1 - \pi c_1^{FB} - (f - \pi)\frac{1}{\lambda}c_1^{FB}}{1 - f}, 0\}$$

Symmetric Equilibrium

Good Equilibrium: "withdraw iff impatient" is a Nash Equilibrium, with a payoff equal to the first best allocation.

- Impatient consumers don't want to deviate because they don't care about future consumption
- Patient consumers don't want to deviate because $c_2^{FB} > c_1^{FB}$

Bad Equilibrium: "withdraw no matter what" is also a Nash Equilibrium

- Given that $c_1^{FB} > 1 \geq x^{FB} + \lambda y^{FB}$, if everyone tries to withdraw, then the money will run out.
- Therefore, those who wait will get zero
- This equilibrium produces a very bad allocation.

Suspension of convertibility

- One variant of the contract can rule out the bad equilibrium
- The contract states that you can withdraw c_1^{FB} at $t = 1$ as long as less than π other consumers have withdrawn before you. After that, you are forced to wait
- The payoffs for the impatient consumers are:

Withdraw	Wait
$\begin{cases} c_1^{FB} & \text{if } f_j < \pi \\ 0 & \text{otherwise} \end{cases}$	0

- The payoffs for a patient consumer are

Withdraw	Wait
$\begin{cases} c_1^{FB} & \text{if } f_j < \pi \\ 0 & \text{otherwise} \end{cases}$	c_2^{FB}

- Waiting is dominant for patient consumers, and runs will not take place.