

Advanced Microeconomics, Spring 2024

Problem set 5, due June 13

1. Suppose Jack and Tom have the endowments $\omega_A = (6, 0)$ and $\omega_B = (0, 6)$. Their preferences are defined by a pair of utility functions. For the following cases, find the utility possibility frontier. State your answer both in precise mathematical notation and in terms of graph.

Leontief/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

Leontief/Linear

$$U_J(x_{1J}, x_{2J}) = \min(x_{1J}, x_{2J}/2)$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

Cobb-Douglas/Cobb-Douglas

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T}^{1/3} x_{2T}^{2/3}.$$

Cobb-Douglas/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J}^{1/3} x_{2J}^{2/3}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + x_{2T}.$$

Linear/Linear

$$U_J(x_{1J}, x_{2J}) = x_{1J} + x_{2J}$$

$$U_T(x_{1T}, x_{2T}) = x_{1T} + 2x_{2T}.$$

2. Suppose the economy is endowed with capital and labor, $k = 6$, $l = 6$, and can produce two outputs (b, c) with production functions given below. For each pair of production functions given, derive the transformation function $T(y)$ where $y = (y_1, y_2, y_3, y_4) = (-k, -l, b, c)$.

Leontief/Cobb-Douglas

$$b(k, l) = \min(k, l/2)$$

$$c(k, l) = k^{1/3} l^{2/3}.$$

Leontief/Linear

$$b(k, l) = \min(k, l/2)$$

$$c(k, l) = k + l.$$

Cobb-Douglas/Cobb-Douglas

$$b(k, l) = k^{1/3}l^{2/3}$$

$$c(k, l) = k^{1/3}l^{2/3}.$$

Cobb-Douglas/Linear

$$b(k, l) = k^{1/3}l^{2/3}$$

$$c(k, l) = k + l.$$

Linear/Linear

$$b(k, l) = k + l$$

$$c(k, l) = k + 2l.$$

3. Suppose there are three people, named Bob, Jack and Tom, whose only purpose in life is to eat chocolate. Their utility functions are

$$u_B(c_B, l_B) = c_B,$$

$$u_J(c_J, l_J) = 2c_J,$$

$$u_T(c_T, l_T) = 3c_T.$$

Each of them is endowed with no chocolate and one unit of labor. There are 3 chocolate factories, whose production functions are

$$f_1(l_1) = l_1,$$

$$f_2(l_2) = 2l_2,$$

$$f_3(l_3) = 3l_3.$$

Bob owns Firm 1, Jack owns Firm 2, and Tom owns Firm 3.

- (a) Determine the society's transformation function.
- (b) Determine the set of Pareto efficient consumption allocations.

- (c) Determine the set of equilibrium consumption allocation(s). For each equilibrium consumption allocation, give the corresponding production allocation and price vector.
4. Consider the following market for used cars. There are N potential sellers and M potential buyers, and $M > N$. Each seller has exactly one used car to sell and is characterized by the quality of the used car he has. Let $\theta \in [0, 1]$ index the quality of used car. If a seller of type θ sells his car for a price of P , his payoff is $u_s(P, \theta)$, and is 0 if he does not sell his car. The payoff for the buyer is $\theta - P$ if he buys a car of quality θ at price P , and is 0 if he does not buy. Information is asymmetric: Sellers know the quality of used cars but buyers do not. However, buyers know the quality of used car is uniformly distributed on $[0, 1]$.
- (a) Argue that in a competitive equilibrium under asymmetric information, we must have $E[\theta|P] = P$.
- (b) Find all equilibrium prices when $u_s(P, \theta) = P - \theta/2$.
- (c) Find all equilibrium prices when $u_s(P, \theta) = P - \sqrt{\theta}$. Describe the equilibrium in words. In particular, which cars are traded in equilibrium?