# Theoretical Corporate Finance
# Problem Set 2

**Fall 2023**

1. Consider the variable-investment model (Holmstrom and Tirole(1997)): an entrepreneur initially has cash $A$. For investment $I$, the project yields $RI$ in the case of success and 0 in the case of failure. The probability of success is equal to $p_H \in (0,1)$ if the entrepreneur works and $p_L = 0$ if the entrepreneur shirks. The entrepreneur obtains private benefit $BI$ when shirking and 0 when working. The perunit private benefit $B$ is unknown to all ex ante and is drawn from (common knowledge) uniform distribution $F$ :
$$\Pr(B < \hat{B}) = F(\hat{B}) = \hat{B}/R \text{ for } \hat{B} \leq R,$$
with density $f(\hat{B}) = 1/R$. The entrepreneur borrows $I - A$ and pays back $R_1 = \eta_1 I$ in the case of success. The timing is described in Figure 1.

   (1) For a given contract $(I, n)$, what is the threshold $B^*$, i.e., the value of the private per-unit benefit above which the entrepreneur shirks?

   (2) For a given $B^*$ (or equivalently $\eta_1$, which determines $B^*$ ), what is the debt capacity? For which value of $B^*$ ( or $\eta_1$) is this debt capacity highest?

   (3) Determine the entrepreneur's expected utility for a given $B^*$. Show that the contract that is optimal for the entrepreneur (subject to the investors breaking even) satisfies
$$\frac{1}{2} p_{\mathrm{H}} R < B^* < p_{\mathrm{H}} R.$$

   (4) Interpret this result. Suppose now that the private benefit $B$ is observable and verifiable. Determine the optimal contract between the entrepreneur and the investors (note that the reimbursement can now be made contingent on the level of private benefits: $R_1 = \eta_1(B) I$ ).

2. Consider the Diamond-Dybvig model. There are three dates $\{0,1,2\}$ and a unit mass of ex ante identical investors and a single bank. Each of the investors has an endowment of 1 to invest at date $t = 0$. The type of each investor is revealed at date $t = 1$. A fraction $\alpha \in (0,1)$ are impatient and consume only at

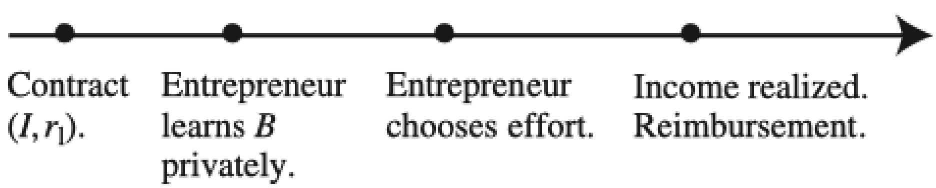


Figure 1: Random Private Benefit

$t=1$. The remaining fraction are patient and indifferent between consuming at either $t=1$ or $t=2$. An individual's realised type is her own private information.

Funds invested for two periods earn a gross return $R>1$ (an illiquid project). Funds invested for only one period earn a gross return of 1 (i.e., the investor just gets their funds back). Each investor has the CRRA utility function

$$u(c)=\frac{c^{1-\sigma}-1}{1-\sigma}, \quad \sigma>1$$

(a) Set up the optimization problem the solution of which gives the efficient amount of risksharing (optimal insurance) between impatient and patient investors.

(b) Suppose the following parameter values: $\alpha=0.5, R=4$ and $\sigma=2$. Using these parameter values, solve the optimization problem for the payments $c_1^*, c_2^*$ to impatient and patient investors.

(c) Explain how the optimal insurance scheme can be implemented by a liquid deposit contract with the bank that pays returns $r_1, r_2$ on dates $t=1$ and $t=2$ respectively. What would the values of $r_1, r_2$ have to be?

(d) Calculate the ex ante expected utility to an investor who enters into this deposit contract. Is this higher or lower than the ex ante expected utility of an investor who just invests and holds the illiquid asset? Explain. How would your answer change (if at all) if the investors were risk neutral (e.g., $u(c)=c$) ? Explain.

(e) Explain the sequential service constraint facing the bank if it offers deposit contracts. Explain why the bank is prone to a run. If the return on the deposit contract paid in the first period $r_1$ is the value calculated in part (c), what is the maximum number of withdrawals $f^*$ beyond which any individual patient investor will find it optimal to withdraw? [the 'tipping point']

3. There are three dates $t=0,1,2$, and at least two firms $i=1,2$. Firm 1 ,the firm of interest, is managed by a risk-neutral entrepreneur, who owns an initial wealth $A$ at date 0 and is protected by limited liability. This firm invests at a variable investment level $I \in[0, \infty)$. The per-unit profitability of investment is random and learned at date 1. The investment yields $RI$ with probability $p+\tau$ and 0 with probability $1-(p+\tau)$. The random variable $\tau$ is drawn from a continuous distribution. The variable $p$ is equal to $p_{\mathrm{H}}$ if the entrepreneur behaves (no private benefit) and $p_{\mathrm{L}}$ if the entrepreneur misbehaves (private benefit $BI$ ). Let

$$\rho_1=\left(p_{\mathrm{H}}+\tau\right) R$$

and

$$\rho_0=\left(p_{\mathrm{H}}+\tau\right)\left(R-\frac{B}{\Delta p}\right) \equiv \rho_1-\Delta \rho$$

denote the random continuation per-unit NPV and pledgeable income when the entrepreneur behaves and the realization of profitability is $\tau$. The distribution on $\tau$ induces a cumulative distribution function $F\left(\rho_0\right)$ on $\left[\underline{\rho}_0, \bar{\rho}_0\right]$.

At date 1, the firm may either continue or resell assets $I$ to firm 2 (or to a competitive market). Firm 2 has a known level $\hat{\rho}_0$ of per-unit pledgeable income per unit of investment (its NPV per unit of investment is in general larger than this).
Firms 1 and 2 do not contract with each other at date 0 . Rather, investors in firm 1 make a take-itor-leave-it offer to firm 2 at date 1 if firm 1's initial contract specifies that assets ought to be reallocated. Assume for simplicity that the contract between firm 1's investors and the entrepreneur can be contingent on the realization of $\rho_0$.

Show that at the optimal contract assets are resold whenever $\rho_0<\rho_0^*$, where

$$\rho_0^*<\hat{\rho}_0,$$

and so the volume of asset reallocations is inefficiently low.