MICROECONOMIC THEORY II

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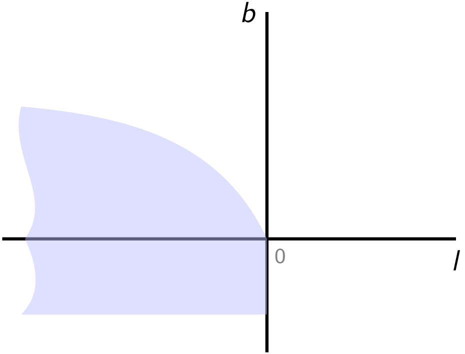

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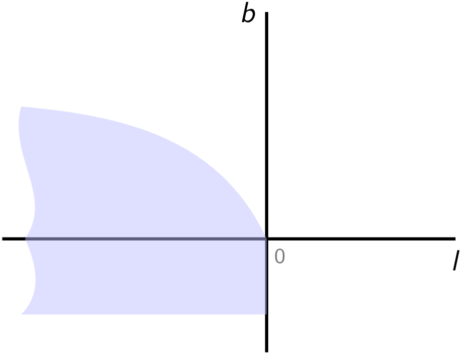

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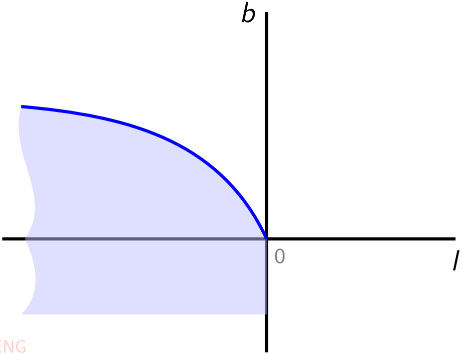

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- $\mathbb{R}^L_- \subseteq Y_j$.

Aggregate production

- y is possible if there exists $(y_1, \dots, y_J)$,

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- Definition: A transformation function (PTF) $T : A \subseteq \mathbb{R}^L \to \mathbb{R}$ represents Y over $A \subseteq \mathbb{R}^L$ if $(\forall y \in A)$,

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- Rule to get $T(y)$: pick one good l (use an output)

$$T(y) = y_l - \max\{y_l' | (y_l', y_{-l}) \in Y\}.$$

Derivatives of Production Transformation Function

- Marginal product of labor:

$$TRS^{12} = \frac{T_1}{T_2} = MP_l^b,$$

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- MRT implies that the economy can get one additional unit of beef by sacrificing MRT^{bc} unit of corn.

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- Derivatives:

$$DT = \left(\frac{5}{2}\left(l - \frac{c^2}{100}\right)^{-1/2}, 1, \frac{c}{20}\left(l - \frac{c^2}{100}\right)^{-1/2}\right)$$

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$$Y_b = \{b = (-k_b, -l_b, b, 0) | b \leq 4(k_b l_b)^{1/2}\},$$
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$$\mathcal{L} = 20 k_c^{\frac{1}{2}} l_c^{\frac{1}{2}} + \lambda[4(k - k_c)^{\frac{1}{2}}(l - l_c)^{\frac{1}{2}} - b]$$

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- From previous conditions:

$$\lambda = 5, \quad , k_c^* = k\left(1 - \frac{b}{4(kl)^{1/2}}\right), \quad l_c^* = l\left(1 - \frac{b}{4(kl)^{1/2}}\right).$$

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  - Note that

$$DT(-k, -l, b, c) = (10k^{\frac{-1}{2}} l^{\frac{1}{2}}, 10k^{\frac{1}{2}} l^{\frac{-1}{2}}, 5, 1),$$

respectively, MP_k^c, MP_l^c, $MRT^{b,c}$

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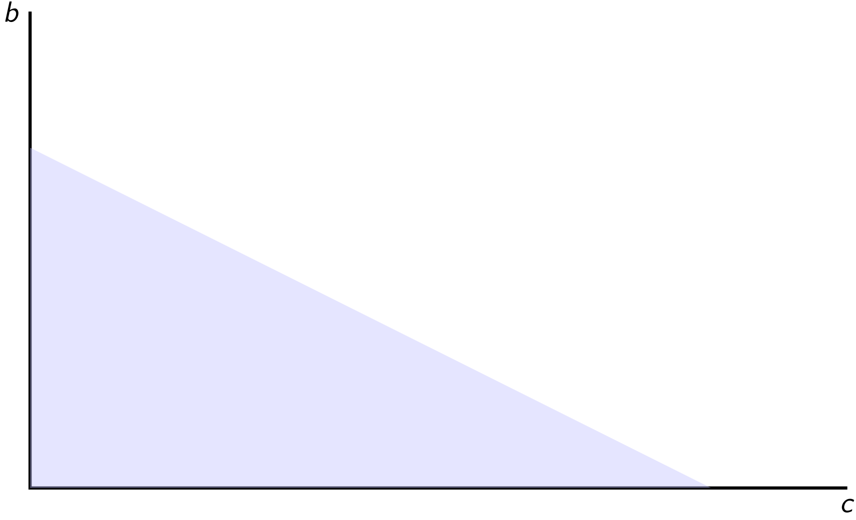
- If the economy is endowed with 5 units of k and l,

$$c + 5b - 100 = 0,$$

constant MRT along the PPF.

Graphical illustration of PPF



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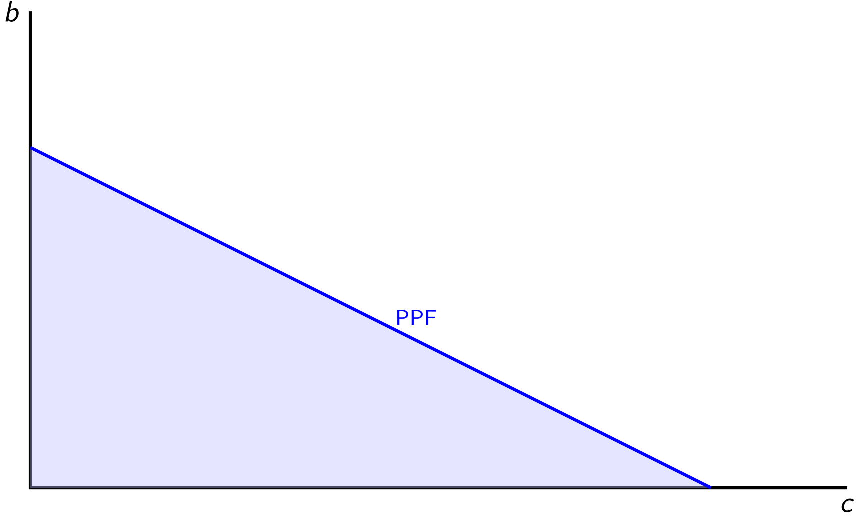

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  - X is feasible iff

$$(\exists\ y), \sum_i X_i \leq \sum_j y_j + \sum_i \omega_i \Leftrightarrow \sum_i X_i - \sum_i \omega_i \in \sum_j Y_j \Leftrightarrow$$

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$$\forall i, X_i' \succeq X_i \quad \text{and} \quad \exists i,\ X_i' \succ_i X_i.$$

Pareto efficiency

- Theorem: Suppose $X \gg 0$ and $(\forall i)$, $\succeq_i$ is represented by a concave u_i which is twice continuously differentiable and strongly monotonic around X_i, and $\sum_j Y_j$ is represented by a convex function T, which is twice continuously differentiable around $(\sum_i X_i - \sum_i \omega_i)$. Then the following are equivalent

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  - X is (Pareto) efficient;
  - $(\exists\ s_1, \ldots, s_I) \in \mathbb{R}^K_{++}$,

$$(\forall i)\ s_i Du_i(X_i) = DT\left(\sum_i X_i - \sum_i \omega_i\right)$$

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  - ➢ Remark: marginal rate of substitution (equal across consumers) equals marginal rate of transformation at PE allocations.

Graphical illustration

Example 1

- Robinson Crusoe economy proudction

$$y = \{y \subset (-l, c) | c \leq 6\sqrt{l}\}$$

$$T(-l, c) = c - 6\sqrt{l} = y_2 - 6(-y_1)^{1/2}$$

$$DT(-l, c) = (3l^{-1/2}, 1) = (3(-y_1)^{-1/2}, 1).$$

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- Efficient allocations:

$$X = \begin{bmatrix} -4 \\ 12 \end{bmatrix} + \begin{bmatrix} 9 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

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- Two-input, two-output production:

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- Efficient allocation

$$X = \begin{bmatrix} -4 \\ -4 \\ 8 \\ 40 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 8 \\ 40 \end{bmatrix}$$

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- Ownership shares θ_{ji}

$$(\forall i,\ j),\ \theta_{ji} \in [0,1],$$

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  Given $\{Y_j\}_j$ and $\{\omega_i, \theta_i, \succeq_i\}_i$, (X^*, y^*, P^*) is an equilibrium if
  - $(\forall\ i)$, X_i^* is the maximal for $\succeq_i$ in $\{X_i | P^* X_i \leq P^* \omega_i + \sum_j \theta_{ji} P^* y_j^*\}$;
  - $(\forall\ j)$, $y_j^* \in arg \max\{P^* y_j | y_j \in Y_j\}$;
  - $\sum_i X_i^* = \sum_i \omega_i + \sum_j y_j^*$.

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    - Feasibility condition: $(\exists\ i)\ X_i' \succ_i X_i^*$,

$$\sum_i X_i' \leq \sum_j y_j' + \sum_i \omega_i.$$

Proof continued

- The first two equations imply

$$(\exists\ i)\ P^*X_i' > P^*\omega_i + \sum_j \theta_{ji} P^* y_j^*$$

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- From the feasibility condition,

$$\sum_i P^*X_i^* \leq \sum_j P^* y_j' + \sum_i P^*\omega_i$$

$$\sum_i P^*X_i' > \sum_j P^* y_j^* + \sum_i P^*\omega_i \implies$$

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- Contradiction, as y^* maximizes profits given P^*

Second Welfare Theorem

- Second welfare Theorem: Suppose that $(\forall j)$, Y_j is convex, $(\forall i)$, $\succeq_i$ is locally non-satiated and convex. Then for every Pareto efficient (X^*, Y^*) such that $X^* \gg 0$, there exists $P^* > 0$ so that (X^*, y^*, P^*) is an equilibrium.

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  - ➢ The problems of distribution and efficiency can be separated
  - ➢ We can redistribute endowments to obtain an ideal distribution
  - ➢ However, price should be used to allocation final consumption, as it reflects the relative scarcity of different resources in the economy.

Example 1

- Blue collar worker $L_b = 150, K_b = 0$;
- White collar worker $L_w = 50, K_w = 50$;
- Production function: food (x), energy (y).

$$x = L_x^{1/2} K_x^{1/2}, \qquad y = L_y^{1/2} K_y^{1/2}.$$

- Production transformation

$$T(-K, -L, x, y) = x - (LK)^{1/2} + y \Longrightarrow$$

$$DT = (\frac{1}{2} K^{-1/2} L^{1/2}, \frac{1}{2} K^{1/2} L^{-1/2}, 1, 1)$$

- Preference

$$U_b(x_b, y_b) = (x_b y_b)^{1/2} \qquad U_w(x_w, y_w) = (x_w y_w)^{1/2}.$$

Production possibility frontier

Solve for equilibrium (1)

- From utility-maximization,

$$x_b = \frac{I_b}{2p_x},\ y_b = \frac{I_b}{2p_y},$$

$$x_w = \frac{I_w}{2p_x},\ y_w = \frac{I_w}{2p_y},$$

- Note

$$I_b = 150w \qquad I_w = 50w + 50r + \pi_x + \pi_y.$$

- From profit-maximization,

$$MRTS^x_{L,K} = MRTS^y_{L,K} = \frac{w}{r},$$

- So

$$wL_x = rK_x \qquad wL_y = rK_y \Longrightarrow$$

$$\frac{L_x}{K_x} = \frac{L_y}{K_y} = \frac{r}{w}.$$

Equilibrium (2)

- In equilibrium, labor market and capital market clear

$$200 = L_x + L_y \qquad 50 = K_x + K_y \Longrightarrow$$

$$\frac{L_x}{K_x} = \frac{L_y}{K_y} = \frac{L_x + L_y}{K_x + K_y} = 4 = \frac{r}{w}.$$

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$$MRT_{x,y} = \frac{MC_x}{MC_y} = \frac{p_x}{p_y} \Longrightarrow MRT_{x,y} = MRS^b_{x,y} = MRS^w_{x,y}.$$

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- In equilibrium, $L_x = 4K_y$, $L_y = 4K_y$

$$MC_x = 4, \qquad MC_y = 4 \Longrightarrow \quad p_x = p_y = 4.$$

Equilibrium (3)

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➢ In this case, the economy's total inputs are
$$L = 100(10+60) = 7000, \qquad K = 100(50+0) = 5000.$$

Example 2 (2)

- Utility-maximization

$$x_B = \frac{3I_B}{4P_x}, \qquad y_B = \frac{I_B}{4P_y};$$

$$x_W = \frac{I_W}{2P_x}, \qquad y_W = \frac{I_W}{2P_y};$$

  with $I_B = 60w$, $I_W = 10w + 50r$.
- Production efficiency
  - ➢ cost-minimization

$$\frac{K_x}{2L_x} = \frac{w}{r} \Longrightarrow L_x = \frac{rK_x}{2w}.$$

$$\frac{K_y}{L_y} = \frac{w}{r} \Longrightarrow L_y = \frac{rK_y}{w}.$$

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  - ➢ Plugging L_x into production function, we can get condition input demand

$$x = 1.89\left(\frac{rK_x}{2w}\right)^{\frac{1}{3}} K_x^{\frac{2}{3}}, \qquad y = 2\left(\frac{rK_y}{w}\right)^{\frac{1}{2}} K_y^{\frac{1}{2}}$$

EXAMPLE 2 (3)

- Production
  - ➢ Conditional input demand:

$$K_x = \frac{2x}{3}\left(\frac{w}{r}\right)^{\frac{1}{3}}, \quad L_x = \frac{x}{3}\left(\frac{r}{w}\right)^{\frac{2}{3}};$$

$$K_y = \frac{y}{2}\left(\frac{w}{r}\right)^{\frac{1}{2}}, \quad L_y = \frac{y}{2}\left(\frac{r}{w}\right)^{\frac{1}{2}}.$$

  - ➢ Given the demand curves, total cost

$$TC_x = wL_x + rK_x = \frac{x}{3}w^{\frac{1}{3}}r^{\frac{2}{3}} + \frac{2x}{3}w^{\frac{1}{3}}r^{\frac{2}{3}} = xw^{\frac{1}{3}}r^{\frac{2}{3}}.$$

$$TC_y = wL_y + rK_y = \frac{y}{2}w^{\frac{1}{2}}r^{\frac{1}{2}} + \frac{y}{2}w^{\frac{1}{2}}r^{\frac{1}{2}} = yw^{\frac{1}{2}}r^{\frac{1}{2}}.$$

  - ➢ Marginal cost

$$MC_x = w^{\frac{1}{3}}r^{\frac{2}{3}}, \quad MC_y = w^{\frac{1}{2}}r^{\frac{1}{2}}.$$

- In equilibrium

$$P_x = MC_x = w^{\frac{1}{3}}r^{\frac{2}{3}}, \qquad P_y = MC_y = w^{\frac{1}{2}}r^{\frac{1}{2}}.$$

Example 2 (3)

- Markets for x, y clears

$$x = 100x_B + 100x_W = \frac{50I_W + 75I_B}{P_x} = \frac{5000w + 2500r}{w^{\frac{1}{3}} r^{\frac{2}{3}}},$$

$$y = 100y_B + 100y_W = \frac{50I_W + 25I_B}{P_y} = \frac{2000w + 2500r}{w^{\frac{1}{2}} r^{\frac{1}{2}}}.$$

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- Market for labor clears

$$\begin{aligned}
7000 &= L_x + L_y = \frac{x}{3}\left(\frac{r}{w}\right)^{\frac{2}{3}} + \frac{y}{2}\left(\frac{r}{w}\right)^{\frac{1}{2}} \\
&= \frac{5000w + 2500r}{3w^{\frac{1}{3}} r^{\frac{2}{3}}} \cdot \left(\frac{r}{w}\right)^{\frac{2}{3}} + \frac{2000w + 2500r}{2w^{\frac{1}{2}} r^{\frac{1}{2}}} \cdot \left(\frac{r}{w}\right)^{\frac{1}{2}} \\
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- This gives

$$\frac{r}{w} = 2.08.$$

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- Plugging $r/w = 2.08$ into the equation for $x,\ y$,

$$x = 6300, \qquad y = 5000.$$

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- We also get $P_x = 1.628$, $P_y = 1.4422$.