Advanced Microeconomics, Spring 2024

Problem set 3, due on May 8

1. Consider the N-player version of the (Rubinstein's) bargaining game presented in class. At dates 1, $N+1$, $2N+1$, ..., player 1 offers a division $(x_1, ..., x_N)$ of the pie with $x_i \geq 0$ for all i, and $\sum_{i=1}^{N} x_i \leq 1$. At dates 2, $N+2$, $2N+2$, ..., player 2 offers a division, and so on. When player i offers a division, the other players simultaneously accept or veto the division. If all accept, the pie is divided; if at least one vetoes, player $i+1$ $(mod\ N)$ offers a division in the following period. Assuming that the players have common discount factor δ, show that, for all i, player i offering a division

$$\left(\frac{v}{1+\cdots+\delta^{N-1}}, \frac{v\delta}{1+\cdots+\delta^{N-1}}, ..., \frac{v\delta^{N-1}}{1+\cdots+\delta^{N-1}}\right)$$

for players i, $i+1$, ...,$i+N-1$ $(mod\ N)$ at each date $(kN+i)$ and the other players' accepting any offer equal or higher than those amounts is a subgame-perfect Nash equilibrium.

2. Two persons A and B can play one of the following games: G1:

G1 (A = row player, B = column player)

	L	R
U	4, 4	0, 0
D	0, 0	1, 1

G2 (A = row player, B = column player)

	L	R
U	-1, -1	0, 0
D	0, 0	4, 4

   (a) Suppose both A and B know they play game G1. Find all NE of the game.

   (b) Now suppose they play G1 and G2 with equal probabilities, which is common knowledge among them. In addition, A knows which game they are playing but B does not know if they are as in G1 or as in G2. Model the game as a Bayesian game and find pure strategy Bayesian NE of the game.

3. Suppose that Michael and John are playing the following game of incomplete information. Michael(the row player) is perfectly aware of the payoffs but John (the column player) does not know if they are as in G1 or as in G2.

John

	L	R
U	1, 1	0, 0
D	0, 0	0, 0

Michael

G1

John

	L	R
U	0, 0	0, 0
D	0, 0	2, 2

Michael

G2

a) Model this situation as a Bayesian game.

b) Assuming that it is common knowledge that payoffs as in G1 or as in G2 with equal probabilities, find *all* Bayesian-Nash equilibria of the game.

4. There are two individuals $i = 1, 2$ who need to decide simultaneously whether to contribute to a public good or not ("C" or "NC"). Each player derives a benefit of 1 if at least one contributes and 0 if none does. A player's cost of contributing is c_i. While the benefit is common knowledge, each agent's cost c_i is known only to himself. However, it is common knowledge that for $i = 1, 2$, c_i is independently drawn from a uniform distribution on [0,2]. Identify the Bayesian Nash equilibria of this game.

5. In a first-price, all-pay auction, the bidders simultaneously submit sealed bids. The highest bid wins the objects and every bidder pays the seller the amount of his bid. The all-pay auction is a useful model of lobbying activity. In such models, different interest groups spend money (their bids) in order to influence government policy and the one spending the most (the highest bidder) is able to tilt policy in its favored direction (winning the auction). Since money spent on lobbying is a sunk cost borne by all groups regardless of which group is suceessful in obtaining its preferred policy, such situations have a natural all-pay aspects.

   Consider the independent private values model with symmetric bidders whose values are each distributed according to the distribution function F, with density f.

   (a) Find the unique symmetric equilibrium bidding function. Interpret.

   (b) Do bidders bid higher or lower than in a first-price auction?

   (c) Find an expression for the seller's expected revenue.

   (d) Show that the seller's expected revenue is the same as in a first-price auction.