

## Lecture 2(b): A two-sector DSGE model with financial frictions and Chinese characteristics

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# Overview of This Lecture

- Institutional background
- Model setup
- Quantitative results
  - ① Impact of RR policy on steady state
  - ② Role of RR policy in stabilizing business cycle fluctuations

This lecture note are based on Chang, Chun & Liu, Zheng & Spiegel, Mark M. & Zhang, Jingyi, 2019. "Reserve requirements and optimal Chinese stabilization policy," Journal of Monetary Economics, Elsevier, vol. 103(C), pages 33-51.

# Institutional background

- State-owned enterprises (SOEs) have superior access to bank loans
  - SOEs enjoy implicit government guarantees on loans.
  - SOEs have lower average productivity (Hsieh and Klenow, 2009; Hsieh and Song, 2015)
- Privately-owned enterprises (POEs) largely rely on informal banking for external finance. (Lu, Guo, Kao and Fung, 2015; Elliott, Kroeber and and Yu, 2015)
- ↑ RR reallocates resources from SOEs to POEs
  - RR taxes formal banking but not informal banking.
  - Increase funding cost of formal bank loans (i.e. SOE loans).
  - Raising RR increases aggregate TFP.
  - Distinguishes RR from conventional interest rate.

# Two sector DSGE model

- Representative household consumes, saves, and supplies labor
- Retail sector: use wholesale goods as inputs; monopolistic competition and sticky prices
- Wholesale sector: intermediate goods produced by SOEs and POEs imperfect substitutes
  - POEs have higher average productivity (Hsieh-Klenow, 2009)
  - External financing for working capital subject to costly state verification: financial accelerator (BGG, 1999)
- Banks provide working capital to firms in both sectors
  - Loans to SOEs are subject to RR, but debt guaranteed by government (on-balance-sheet)
  - Loans to POEs exempt from RR, but no government guarantees (off-balance-sheet)

# Representative household

- Utility function

$$U = \mathbb{E} \sum_{t=0}^{\infty} \beta_t \left[ \ln(C_t) - \psi \frac{H_t^{1+\eta}}{1+\eta} \right],$$

- Budget constraints

$$C_t + I_t + \frac{D_t}{P_t} = w_t H_t + r_t^k K_{t-1} + R_{t-1} \frac{D_{t-1}}{P_t} + T_t$$

- Capital accumulation with adjustment costs (CEE 2005)

$$K_t = (1 - \delta) K_{t-1} + \left[ 1 - \frac{\Omega_k}{2} \left( \frac{I_t}{I_{t-1}} - g_I \right)^2 \right] I_t,$$

# Retail sector

- Final good CES composite of differentiated retail products

$$Y^f = \left[ \int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon} dz \right]^{\epsilon/(\epsilon-1)}$$

- Demand curve facing each retailer

$$Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t^f$$

- Monopolistic competition in retail markets, with quadratic price adjustment costs (Rotemberg, 1982)

$$\frac{\Omega_p}{2} \left( \frac{P_t(z)}{\pi P_{t-1}(z)} - 1 \right)^2 C_t$$

- Optimal price decision → Phillips curve

# Wholesale and intermediate goods

- Wholesale good a CES composite of SOE and POE products

$$M_t = \left( \phi Y_{st}^{\frac{\sigma_m - 1}{\sigma_m}} + (1 - \phi) Y_{pt}^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}}$$

- Intermediate good production function in sector  $j \in \{s, p\}$

$$Y_{jt} = A_{jt} \omega_{jt} K_{jt}^{1-\alpha} [(H_{jt}^e)^{1-\theta} H_{jt}^\theta]^\alpha$$

- where  $\omega_{jt} \sim F_{jt}(\cdot)$  denotes idiosyncratic productivity shocks

# Firms' Financing Activities

- Firms finance working capital with net worth  $N_{j,t-1}$  and external debt  $B_{jt}$  (BGG):

$$\frac{N_{j,t-1} + B_{jt}}{P_t} = w_t H_{jt} + w_{jt}^e H_{jt}^e + r_t^k K_{jt}$$

where  $w_{jt}^e$  is the real wage rate of managerial labor

- Individual firm's return to investment is given by

$$\frac{P_{jt} Y_{jt}}{N_{j,t-1} + B_{jt}} = \tilde{A}_{jt} \omega_{jt}$$

- $\tilde{A}_{jt}$ : the aggregate return to investment in sector  $j$
- $\omega_{jt}$ : the firm-specific idiosyncratic shock in sector  $j$



# Defaults

- Firms default if realized productivity  $\omega_{jt}$  sufficiently low:

$$\omega_{jt} < \bar{\omega}_{jt} \equiv \frac{Z_{jt} B_{jt}}{\tilde{A}_{jt}(N_{j,t-1} + B_{jt})}$$

where  $Z_{j,t}$  is contractual rate of interest

- If firm defaults, liquidated by lender with fraction  $m_j$  of output lost.
- Government covers loan losses on SOE loans (but not POE loans) using lump sum taxes
  - In case of POE defaults, lenders get:

$$(1 - m_j)\omega_{pt}\tilde{A}_{pt}(N_{p,t-1} + B_{pt}) < Z_{pt}B_{pt}.$$

- In case of SOE defaults, lenders get:  $Z_{st}B_{st}$

# Financial contracts

- Consider a loan contract characterized by  $\bar{\omega}_{jt}$  and  $B_{jt}$ .
- The expected nominal income for a firm in sector  $j$  is given by,

$$\begin{aligned} & \int_{\bar{\omega}_{jt}}^{\infty} \tilde{A}_{jt} \omega_{jt} (N_{j,t-1} + B_{jt}) dF(\omega) - (1 - F(\bar{\omega}_{jt})) Z_{jt} B_{jt} \\ &= \tilde{A}_{jt} (N_{j,t-1} + B_{jt}) \left[ \int_{\bar{\omega}_{jt}}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}_{jt})) \bar{\omega}_{jt} \right] \\ &\equiv \tilde{A}_{jt} (N_{j,t-1} + B_{jt}) f(\bar{\omega}_{jt}), \end{aligned}$$

where  $f(\bar{\omega}_{jt})$  is the share of production revenue going to the firm under the loan contract. This is exactly the same as in the standard BGG setup.

# Financial contracts

- Consider a loan contract characterized by  $\bar{\omega}_{jt}$  and  $B_{jt}$ .
- The expected nominal income for the lender to lends to a sector- $j$  firm is given by,

$$\begin{aligned}
 & (1 - F(\bar{\omega}_{jt}))Z_{jt}B_{jt} + \int_0^{\bar{\omega}_{jt}} \{(1 - m_j)\tilde{A}_{jt}\omega(N_{j,t-1} + B_{jt}) \\
 & + l_j[Z_{jt}B_{jt} - (1 - m_j)\tilde{A}_{jt}\omega(N_{j,t-1} + B_{jt})]\}dF(\omega) \\
 & = \tilde{A}_{jt}(N_{j,t-1} + B_{jt})\{[1 - F(\bar{\omega}_{jt})]\bar{\omega}_{jt} + (1 - m_j) \int_0^{\bar{\omega}_{jt}} \omega dF(\omega) \\
 & + l_j \int_0^{\bar{\omega}_{jt}} [\bar{\omega}_{jt} - (1 - m_j)\omega]dF(\omega)\} \\
 & \equiv \tilde{A}_{jt}(N_{j,t-1} + B_{jt})g_j(\bar{\omega}_{jt}),
 \end{aligned}$$

where  $g_j(\bar{\omega}_{jt})$  is the share of production revenue going to the lender.

- $l_j$  is the share of loan loss guaranteed by the government.
- If  $l_j = 0$ ,  $g_j(\bar{\omega}_{jt})$  is exactly the same as in the standard BGG setup.

# Financial contracts

- Tradeoff between social costs of loan defaults versus stabilization effect of loan guarantee:

$$f(\bar{\omega}_{jt}) + g_j(\bar{\omega}_{jt}) = 1 - m_j \int_0^{\bar{\omega}_{jt}} \omega dF(\omega) + l_j \int_0^{\bar{\omega}_{jt}} [\bar{\omega}_{jt} - (1 - m_j)\omega] dF(\omega)$$

where  $l_s = 1$  and  $l_p = 0$  are guarantee ratios on SOE and POE lending respectively

- In case of  $l_p = 0$ , higher default ratio  $F(\bar{\omega}_{jt})$  implies higher liquidation costs and tightens the borrowing constraint.
- In case of  $l_s = 1$ , higher default ratio  $F(\bar{\omega}_{jt})$  implies higher subsidies from government guarantee on loans and relaxes the borrowing constraint.

# Financial contracts

- Denote  $f(\bar{\omega}_{jt})$  and  $g(\bar{\omega}_{jt})$  as profit share of firm and lender, respectively
- Optimal financial contract is a pair  $(\bar{\omega}_{jt}, B_{jt})$  that solves

$$\max \tilde{A}_{jt}(N_{j,t-1} + B_{jt})f(\bar{\omega}_{jt})$$

- subject to the lender's participation constraint

$$\tilde{A}_{jt}(N_{j,t-1} + B_{jt})g_j(\bar{\omega}_{jt}) \geq R_{jt}B_{jt}$$

where  $B_{jt}$  denotes loan amount and  $\bar{\omega}_{jt}$  is cutoff productivity for firm solvency

# Financial intermediaries

- Banks take funds from household at deposit rate  $R_t$
- *On-balance-sheet* loans to SOEs subject to RR
  - RR drives wedge between funding costs  $R_{st}$  and deposit rates  $R_t$

$$(R_{st} - 1)(1 - \tau_t) = (R_t - 1).$$

- Government guarantees imply no default risk:  $Z_{st} = R_{st}$
- *Off-balance-sheet* loans to POEs not subject to RR
  - Funding cost  $R_{pt} = R_t$
  - No government guarantees on POE debt  $\Rightarrow$  default premium (credit spread) over funding cost  $Z_{pt} > R_{pt}$

# Market clearing and equilibrium

- Final goods market clearing

$$\begin{aligned} Y_t^f &= C_t + I_t + G_t + \frac{\Omega_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 C_t \\ &\quad + \sum_{j \in \{s,p\}} \tilde{A}_{jt} \frac{N_{j,t-1} + B_{jt}}{P_t} m_j \int_0^{\bar{\omega}_{jt}} \omega dF(\omega) \end{aligned}$$

- Capital and labor market clearing

$$K_{t-1} = K_{st} + K_{pt}, \quad H_t = H_{st} + H_{pt}$$

- Credit market clearing

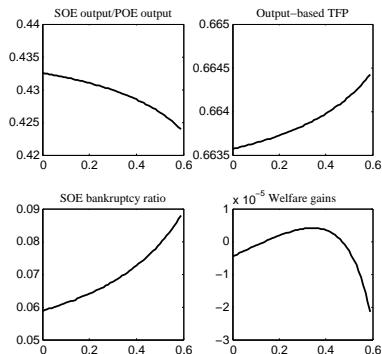
$$B_{st}/(1 - \tau_t) + B_{pt} = D_t$$

# Calibration

- Model solved based on calibrated parameters
- Parameters calibrated to Chinese data where available
  - $\alpha = 0.5$ : labor income share (Zhu, 2012)
  - $\phi = 0.45$ : target SOE share in industrial output of 0.3
  - $\kappa = 1.587$  and  $\omega_m = 0.37$ : match TFP dispersion (Hsieh-Klenow, 2009)
  - Relative TFP of POE  $\bar{A}_p/\bar{A}_s = 1.42$ : Hsieh-Klenow (2009)
  - $\sigma_m = 3$ : substitutability b/n SOE and POE outputs, Chang, et al. (2015)
  - $\xi_s = 0.97$  and  $\xi_p = 0.69$ : match SOE and POE bankruptcy ratios in data
- Other calibration parameters fit to US data
- See [▶ Calibration](#) for details



# Steady state impact of RR increase



- Reallocation from SOE to POE improves TFP
- Higher funding costs increase SOE bankruptcies
- Tradeoff  $\Rightarrow$  interior optimum  $\tau^* = 0.34$  under our calibration

# Monetary policy rules for stabilization

- Three types of shocks (aggregate, SOE-specific and POE-specific TFP shocks)
- Two instruments for monetary policy: deposit rate and RR
  - Consider two types of simple (Taylor-like) policy rules
  - Interest rate rule

$$\ln \left( \frac{R_t}{R} \right) = \psi_{rp} \ln \left( \frac{\pi_t}{\bar{\pi}} \right) + \psi_{ry} \ln \left( \frac{G\tilde{D}P_t}{G\tilde{D}P} \right)$$

- Reserve requirement rule

$$\ln \left( \frac{\tau_t}{\tau} \right) = \psi_{\tau p} \ln \left( \frac{\pi_t}{\bar{\pi}} \right) + \psi_{\tau x} \ln \left( \frac{G\tilde{D}P_t}{G\tilde{D}P} \right)$$

# Compare macro stability and welfare under 4 policy rules

- Benchmark policy: Taylor rule with  $\psi_{rp} = 1.5$  and  $\psi_{ry} = 0.2$  and constant  $\tau = 0.15$
- Optimal interest-rate rule:  $\psi_{rp}$  and  $\psi_{ry}$  set optimally to max welfare, and  $\tau$  kept constant
- Optimal reserve-requirement rule:  $\psi_{\tau p}$  and  $\psi_{\tau y}$  set optimally, Taylor rule coefficients kept at benchmark values
- Jointly optimal rule: Coefficients for both interest rates and reserve requirements set optimally

# Consumption equivalent welfare

- Welfare gains under each alternative policy relative to the benchmark model is measured as the percentage change in permanent consumption that would leave the representative household indifferent between living in an economy under the alternative policy and in the benchmark economy.
- Denote by  $C_t^b$  and  $H_t^b$  the allocations of consumption and hours worked under the benchmark policy regime.
- Denote by  $V^b$  the value of the household's welfare obtained from the equilibrium allocations under the benchmark policy regime.
- Denote by  $V^a$  the value of the household's welfare obtained from the equilibrium allocations under the alternative policy regime.
- The welfare gain under the alternative policy relative to the benchmark is measured by the constant  $\chi$ , which is solved from

$$V^a = E \sum_{j=0}^{\infty} \beta^j \left[ \ln(C_{t+j}^b(1 + \chi)) - \psi \frac{(H_{t+j}^b)^{1+\eta}}{1 + \eta} \right] \equiv V^b + \frac{1}{1 - \beta} \ln(1 + \chi)$$

# Volatilities and welfare: Aggregate TFP shock

Variables	Benchmark	Optimal $\tau$ rule	Optimal $R$ rule	Jointly optimal rule
Policy rule coefficients				
$\psi_{rp}$	1.50	1.50	7.42	5.18
$\psi_{ry}$	0.20	0.20	0.07	-0.12
$\psi_{\tau p}$	0.00	-13.14	0.00	11.67
$\psi_{\tau y}$	0.00	4.81	0.00	15.96
Volatility				
<i>GDP</i>	8.618%	8.155%	5.279%	4.952%
$\pi$	3.409%	3.231%	0.084%	0.136%
<i>C</i>	6.118%	5.950%	4.388%	4.306%
<i>H</i>	2.103%	1.835%	0.599%	0.416%
<i>R</i>	3.412%	3.236%	0.398%	0.349%
$Y_s$	9.091%	6.999%	5.362%	3.415%
$Y_p$	8.132%	8.455%	5.552%	5.982%
Welfare				
Welfare gains	—	0.2423%	1.1799%	1.1801%

Optimal  $\tau$  policy underperforms optimal  $R$  policy in macro-stabilization, with allocative impacts between SOEs and POEs.

Welfare gains of moving from optimal  $R$  policy to joint optimal rule is very small (0.0002%).

# Volatilities and welfare: SOE-specific TFP shock

Variables	Benchmark	Optimal $\tau$ rule	Optimal $R$ rule	Jointly optimal rule
Policy rule coefficients				
$\psi_{rp}$	1.50	1.50	7.72	5.78
$\psi_{ry}$	0.20	0.20	0.32	-0.59
$\psi_{\tau p}$	0.00	-31.81	0.00	71.72
$\psi_{\tau y}$	0.00	-3.99	0.00	-52.78
Volatility				
$GDP$	2.296%	2.192%	1.471%	1.412%
$\pi$	0.908%	0.867%	0.075%	0.170%
$C$	1.572%	1.532%	1.116%	1.027%
$H$	0.664%	0.604%	0.293%	0.311%
$R$	0.911%	0.871%	0.168%	0.203%
$Y_s$	7.993%	7.606%	7.314%	8.407%
$Y_p$	1.479%	1.435%	1.326%	1.785%
Welfare				
Welfare gains	—	0.0126%	0.0648%	0.0734%

Much smaller welfare gains from optimal  $\tau$  policy under SOE-specific TFP shocks.

Welfare gains of moving from optimal  $R$  policy to joint optimal rule are relatively larger under sectoral shocks (0.0086%).

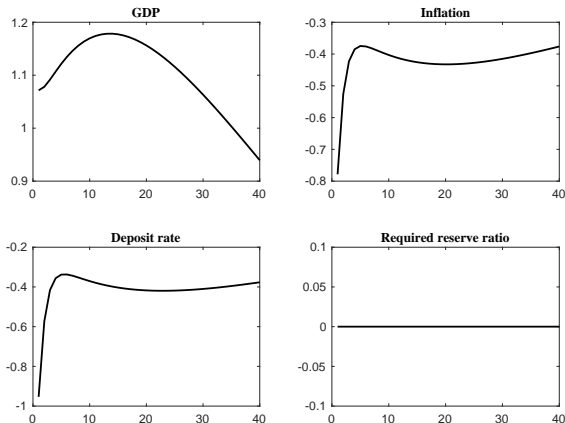
# Volatilities and welfare: POE-specific TFP shock

Variables	Benchmark	Optimal $\tau$ rule	Optimal $R$ rule	Jointly optimal rule
Policy rule coefficients				
$\psi_{rp}$	1.50	1.50	7.54	3.45
$\psi_{ry}$	0.20	0.20	0.17	-0.12
$\psi_{\tau p}$	0.00	-33.04	0.00	3.34
$\psi_{\tau y}$	0.00	-2.68	0.00	22.86
Volatility				
$GDP$	6.324%	5.967%	3.902%	3.518%
$\pi$	2.503%	2.365%	0.111%	0.162%
$C$	4.549%	4.425%	3.323%	3.220%
$H$	1.445%	1.241%	0.377%	0.261%
$R$	2.503%	2.367%	0.285%	0.235%
$Y_s$	4.116%	3.380%	2.774%	4.100%
$Y_p$	8.232%	8.493%	6.575%	6.987%
Welfare				
Welfare gains	—	0.1363%	0.6084%	0.6099%

Welfare gains of moving from optimal  $R$  policy to joint optimal rule are relatively larger under sectoral shocks (0.0015%).

# Aggregate Responses to TFP Shock: Benchmark

Impulse responses to TFP shock

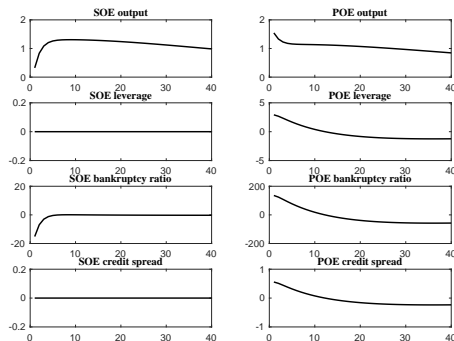




# Sectoral responses to TFP shock: Benchmark

- SOE debt guaranteed by gov't  $\Rightarrow$  no default premium.
- POE debt not guaranteed  $\Rightarrow$  higher default premium.

Impulse responses to TFP shock

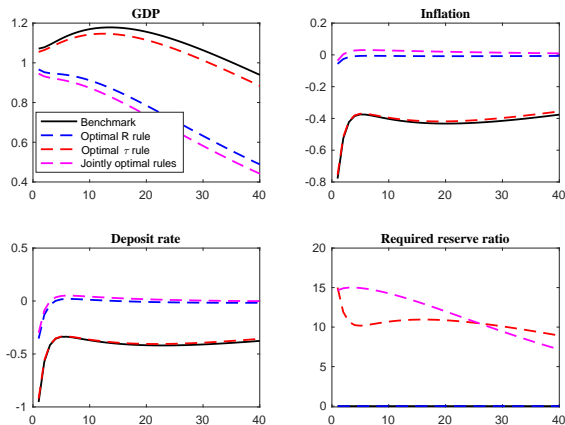


# The financial accelerator mechanism

- POE output more sensitive to agg. macro shocks than SOE output
  - SOE debt guaranteed by gov't  $\Rightarrow$  irresponsive leverage
  - POE debt not guaranteed  $\Rightarrow$  POE leverage  $\uparrow \Rightarrow$  POE default ratio and credit spread  $\uparrow$
- How about financial accelerator effect through net worth? More severe for SOEs in the long run.
  - SOE: leverage do not change  $\Rightarrow$  actual default ratio falls  $\Rightarrow$  net worth  $\uparrow$  relative to POEs.
  - Gov't guarantee  $\Rightarrow$  SOE steady-state leverage higher than POE  $\Rightarrow$  SOE net worth more responsive to agg. macro shocks.

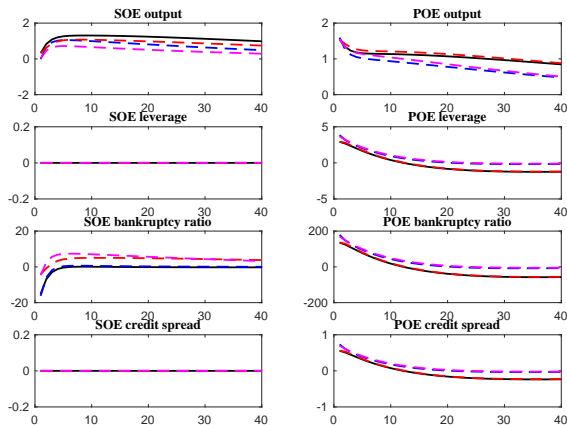
# Aggregate Responses to TFP Shock: Benchmark vs alternative policies

Impulse responses to TFP shock



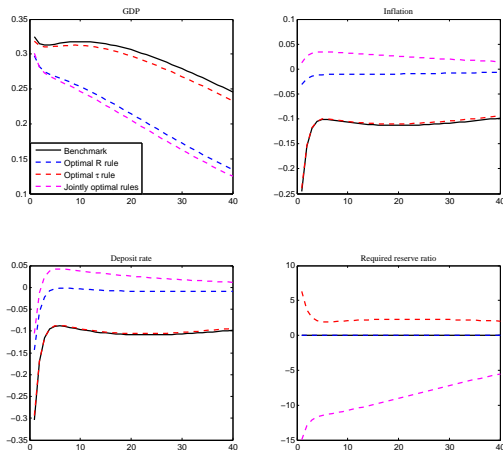
# Sectoral responses to TFP shock: Benchmark vs alternative policies

Impulse responses to TFP shock



# Impulse responses to SOE-specific TFP shock

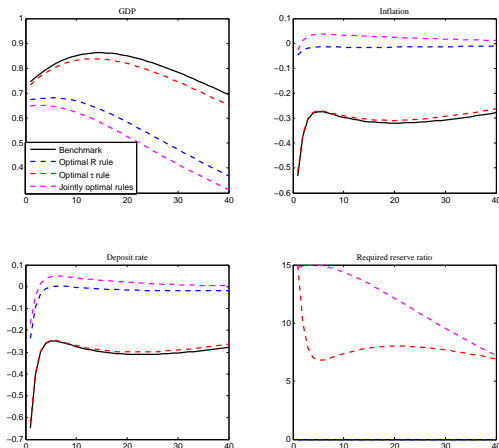
Impulse responses to SOE TFP shock



- Tradeoff:  $\tau \uparrow$  stabilize inflation;  $\tau \downarrow$  improve allocation
- Explains why welfare gains from optimal  $\tau$  policy are much smaller under SOE-specific TFP shocks

# Impulse responses to POE-specific TFP shock

Impulse responses to POE TFP shock



- $\tau \uparrow$  stabilize inflation and improve allocation

# Jointly optimal rule allows for complementary use of policy tools

- Adjust  $R$ -rule to stabilize inflation and GDP
- Adjust  $\tau$ -rule to achieve desired reallocation of resources across sectors
  - Used to correct distortion caused by government guarantee on SOE loans.
- Leads to higher welfare gains than each individually optimal rule  $\Rightarrow$  the two policy instruments are complementary
  - Higher welfare gains in case of sectoral-specific shocks.

# Extension with money growth rule (Chen, et al. 2017)

## POE-specific TFP shocks

Variables	Benchmark	Optimal $\tau$ rule	Optimal money rule	Jointly optimal rule
Policy rule coefficients				
$\psi_{mp}$	-0.65	-0.65	-45.42	-89.88
$\psi_{my}$	0.30	0.30	4.42	19.05
$\psi_{\tau p}$	0.00	-10.38	0.00	-38.79
$\psi_{\tau y}$	0.00	0.09	0.00	13.23
Volatility				
<i>GDP</i>	3.828%	3.808%	3.809%	3.694%
$\pi$	0.180%	0.119%	0.046%	0.050%
<i>C</i>	3.284%	3.275%	3.273%	3.267%
<i>H</i>	0.377%	0.385%	0.353%	0.312%
<i>R</i>	0.084%	0.203%	0.206%	0.237%
$Y_s$	2.848%	2.822%	2.817%	3.459%
$Y_p$	6.549%	6.550%	6.529%	6.861%
Welfare				
Welfare gains	—	0.0032%	0.0032%	0.0039%

- Moving from optimal money growth rule to jointly optimal rules lead to greater welfare gains under sector-specific shocks than under aggregate TFP shocks (not shown)
- Again, optimal RR rules useful for reallocation



# Conclusion

- ① Examine RR policy in DSGE model with BGG financial accelerator and Chinese characteristics
- ② Changes in RR incur tradeoff between allocation efficiency and SOE bailout costs in the steady state
  - RR acts as tax on formal banking and SOE activity
  - Raising RR improves aggregate productivity by diverting capital to more productive POEs
  - But it also raises SOE bailout costs → interior optimal RR
- ③ Transmission mechanism of RR for macro stabilization different from conventional interest rate policy
  - Interest rate increases contracts *general* activity in both sectors
  - But increasing RR contracts *relative* activity of SOEs
  - RR is complementary to interest rate adjustment, especially in times of sectoral shocks.

# Parameter calibration I [▶ Back](#)

Variable	Description	Value
A. Households		
$\beta$	Subjective discount factor	0.995
$\eta$	Inverse Frisch elasticity of labor supply	2
$\psi$	Weight of disutility of working	18
$\delta$	Capital depreciation rate	0.035
$\Omega_k$	Capital adjustment cost	1
B. Retailers		
$\epsilon$	Elasticity of substitution between retail products	10
$\Omega_p$	Price adjustment cost parameter	22
C. Firms		
$g$	Steady state growth rate	1.0125
$k$	Shape parameter in Pareto distribution of idiosyncratic shocks	1.587
$\omega_m$	Scale parameter in Pareto distribution of idiosyncratic shocks	0.37
$A_s$	SOE TFP scale (normalized)	1
$A_p$	POE TFP scale	1.42
$\alpha$	Capital income share	0.5
$\theta$	Share of household labor	0.94
$\psi$	Share parameter for SOE output in intermediate good	0.45
$\sigma_m$	Elasticity of substitution between SOE and POE products	3
C. Financial intermediaries		
$m_s$	SOE monitoring cost	0.15
$m_p$	POE monitoring cost	0.15
$\xi_s$	SOE manager's survival rate	0.97
$\xi_p$	POE manager's survival rate	0.69

# Parameter calibration II

Variable	Description	Value
C. Financial intermediaries		
$m_s$	SOE monitoring cost	0.15
$m_p$	POE monitoring cost	0.15
$\xi_s$	SOE manager's survival rate	0.97
$\xi_p$	POE manager's survival rate	0.69
D. Government policy		
$\pi$	Steady state inflation rate	1.005
$\tau$	Required reserve ratio	0.15
$\psi_{rp}$	Taylor rule coefficient for inflation	1.5
$\psi_{ry}$	Taylor rule coefficient for output	0.2
$\frac{G}{GDP}$	Share of government spending in GDP	0.14
$l_s$	Fraction of SOE debt guaranteed by the government	1
$l_p$	Fraction of SOE debt guaranteed by the government	0
E. Shock process		
$\rho_a$	Persistence of TFP shock	0.95
$\sigma_a$	Standard deviation of TFP shock	0.01