Empirical Asset Pricing: Capital Asset Pricing Model

Jiangyuan LI
SHUFE SOF

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CAPM

- Instead of predicting returns at the aggregate index level. There is a large literature that focuses on cross-sectional stock return predictability.

- The starting point of all of these is CAPM.

  1. Markowits initiates (MV analysis).

  2. Tobin picks up (Two fund separation).

  3. Sharpe concludes (CAPM).

Mean-variance and Expected Utility Theory

- Markowits describes an investor who's utiliy function depends on two components:
  1. Mean.
  2. Variance.
- The MV investor should solve the following problem:

$$\max_{w} \left[E(w'R) - \frac{\gamma}{2} Var(w'R) \right],$$

$$\text{s.t.} \quad w' 1_N = 1.$$

- Is the mean and variance too artificial?
- No.

Mean-variance and Expected Utility Theory

- Let's revisit the constant absolute risk aversion utility function:

$$U(w'R) = \frac{-e^{\gamma(w'R)}}{\gamma}.$$

- Assume $R \sim N(\mu, \Sigma)$, then $w'R \sim N(w'\mu, w'\Sigma w)$,

$$E[U(w'R)] = \frac{-e^{-\gamma w'\mu + \gamma^2 w'\Sigma w/2}}{\gamma},$$

  the additional second-order term comes from the compensation of convexity difference between linear function and exponential function.

Mean-variance and Expected Utility Theory

- Because $U(x) = \frac{-e^{-\gamma x}}{\gamma}$ is a monotonic increasing function, when a mean-variance investor solves the $\max_w [E(w'R) - \frac{\gamma}{2} Var(w'R)]$, it is equivalent to $\max_w E[U(w'R)]$.
- So mean-variance analysis is in line with expected utility theory.
- Again, Markowits puts forward a question more general than his original idea.
- In general, Mean-Variance analysis tries to solve a “demand” problem in capital market.
- Even there are plenty unsloved questions in portfolio choice (how to get right mean? how to incorporate transaction cost ?), but at least we have a guidance at least.

Two fund separation

- Tobin begins to think about another side of the capital market the supply side.
- Even though there are so many different type of assets in the world, Tobin abstracts two main features from all of them in theory:
    1. Risky asset.
    2. Risk-free asset.
- Is there an absolute definition of risk-free?
- No, Fed Reserve has changed the definition of risk-free rate three times.
- Risk-free asset is more like a conceptual tool instead of a real asset.

Two fund separation

- It is very easy to prove that the collection of these risky asset could consist a feasible investment set.
- Tobin argues that when a MV invstor solves the maxmization in Markowits' setting, it is equivalent to find a point at the boundary of feasible investment set, or formally efficient boundary.
- When we add another constraint with additional asset, the risk-free asset:

  $$w'R = 1$$

  where $w' = [w_r f, w_1, ..., w_N]$, $w_r f$.
  The MV investor's optimal choice suddenly lies on the tangent point on efficient boundary of the line through risk-free asset.

Two fund separation

- The choice is actually a combination of risk-free asset and tangent portfolio, which means MV investor should separate his wealth into two funds.

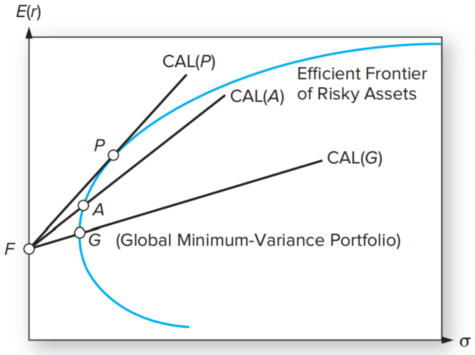


Figure 7.13 Capital allocation lines with various portfolios from the efficient set

CAPM: From Partial Equlibrium to General Equlibrium

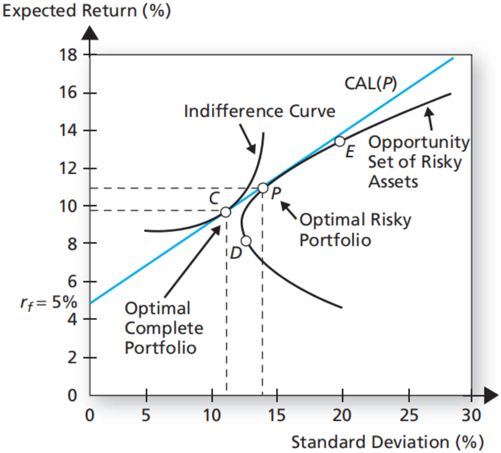
- Markowits and Tobin together help us to solve the demand side of the investment, i.e., given the feasible investment set, what kind of a investment portfolio a MV investor would like to choose.
- Shapre points out another side of the issue, supply side.
- So that, he pushes the literature from Partial Equlibrium to General Equlibrium.

CAPM: From Partial Equlibrium to General Equlibrium

- If everyone in the economy holds an efficient(tangent) portfolio according to Tobin, then how should securities be priced so that they are actually bought 100% in equilibrium?
- In equilibrium the supply of financial assets equals demand, the market portfolio consisting of all existing financial assets must coincide with the tangency portfolio.
- In theoritical paper, you would opten see the assumption like:
  1. There is a infinite elastic short-term bond (risk-free) asset market.
  2. There is a unit of risky asset.

General Equlibrium: CAPM

- Everyone's optimal choice on risky asset should happen to be tangent portfolio, otherwise we have not reached the equilibrium or Pareto Optimum.



Revisit Tangent Portfolio

- Recall that tangent portfolio is actually a linear combination of all the risky asset on the plain.
- What should be the epxected return of a single risky asset?
- The porportion of the expected market return according to its risk contribution.

Deriveing CAPM

- One risk-free asset Rf and N risky assets with mean μ and covariance Σ.
- The portfolio on risky asset is w, $R_p = R_f + w'(R - R_f)$.
    1. Contribution to mean return: $\frac{dE[R_p]}{dw_i} = E[R_i - R_f]$.
    2. Contribution to variance: $\frac{dVar[R_p]}{dw_i} = 2Cov[R_i, R_p]$.
- When portfolio is efficient, the ratio of these two effect should be same for all assets.

$$\frac{dE[R_p]}{dw_i} = k \frac{dVar[R_p]}{dw_i},$$

like, $k = \gamma/2$.

Deriveing CAPM

- So that any two risky assets should have the same "risk-return contribution ratio",

$$\frac{E[R_i] - R_f}{Cov(R_i, R_p)} = \frac{E[R_j] - R_f}{Cov(R_j, R_p)},$$

- Just like any other portfolio, market portfolio also holds

$$\frac{E[R_i] - R_f}{Cov(R_i, R_p)} = \frac{E[R_M] - R_f}{Var(R_M)},$$

- Finally,

$$E[R_i] - R_f = \frac{Cov(R_i, R_M)}{Var(R_M)}(E(R_M) - R_f) = \beta_i(R_M - R_f),$$

  where $\beta_i = Cov(R_i, R_M) / Var(R_M)$ is the regression coefficient of asset *i* on market portfolio.

Beyond CAPM

- After Sharpe puts forward the CAPM, it gradually becomes the golden standard theory of financial economics.
- We would all expect the performance of this elegent theory in practice.
- The results are quite annoying.
- It works in the data before 1960s, and fails afterwards.

Test CAPM

- The single factor structure suggests a very straightforward way to test:

$$R_{i,t} - R_{f,t} = \alpha + \beta(R_{M,t} - R_{f,t}).$$

  or formally, we could test the slope of Secuity Market Line.

- However, the SML seems to flat or even reversed.

Frazzini and Pedersen (2014): Bet against CAPM!

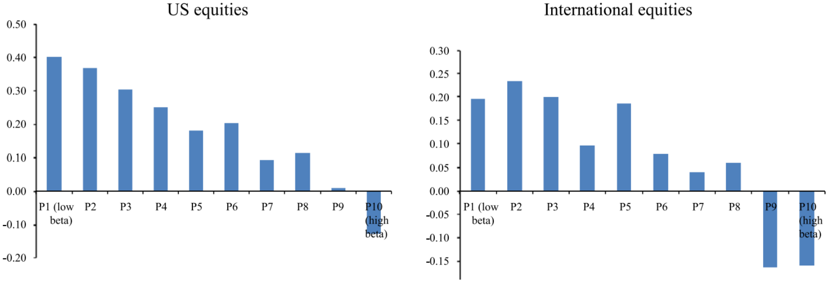
- If we simply sort a portfolio by their beta, and get the long-short portfolio, its beta is not beta-neutral.

- Frazzini and Pedersen (2014) construct a beta-neutral portfolio:

$$R_{t+1}^{BAB} = \frac{1}{\beta_L} R_{L,t+1} - \frac{1}{\beta_H} R_{H,t+1}.$$

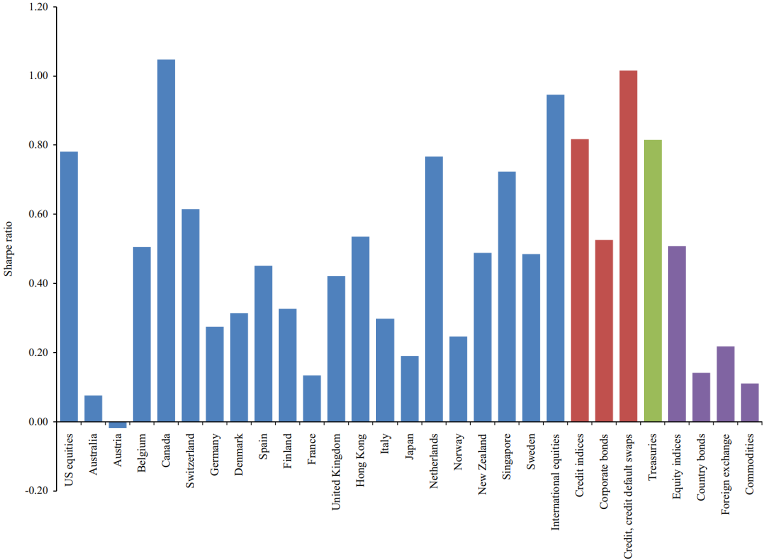
Frazzini and Pedersen (2014): Bet against CAPM in US!!

- High beta, low return!



Frazzini and Pedersen (2014): Bet against CAPM everywhere!!!

- High beta, low return everywhere!!!



Cederburg and O'Doherty (2016): Estimate Carefully!

- CAPM works! However, one needs to use the right estimation.
- After adjust the unconditional α to conditional α, CAPM works again!
- We need to seriously consider the moment condition in CAPM (GMM): Instrument variable to identify alpha.

Yu and Yuan (2011): Sentiment Matters

- CAPM holds when investors trade rationlly.
- When sentiment is high, CAPM doesn’t work.

Yu and Yuan (2011): Sentiment Matters

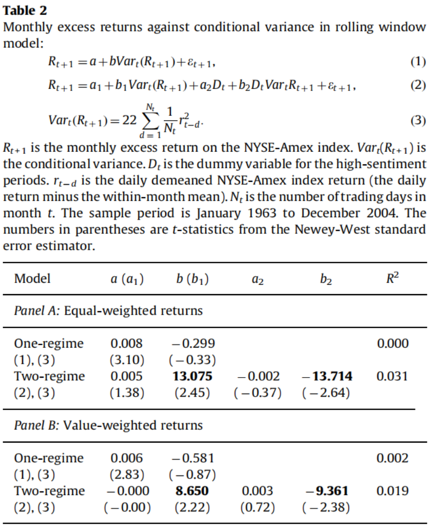


Table 2

Monthly excess returns against conditional variance in rolling window model:

$$R_{t+1} = a + bVar_t(R_{t+1}) + \varepsilon_{t+1}, \tag{1}$$

$$R_{t+1} = a_1 + b_1 Var_t(R_{t+1}) + a_2 D_t + b_2 D_t Var_t R_{t+1} + \varepsilon_{t+1}, \tag{2}$$

$$Var_t(R_{t+1}) = 22 \sum_{d=1}^{N_t} \frac{1}{N_t} r_{t-d}^2. \tag{3}$$

R_{t+1} is the monthly excess return on the NYSE-Amex index. $Var_t(R_{t+1})$ is the conditional variance. D_t is the dummy variable for the high-sentiment periods. r_{t-d} is the daily demeaned NYSE-Amex index return (the daily return minus the within-month mean). N_t is the number of trading days in month t. The sample period is January 1963 to December 2004. The numbers in parentheses are t-statistics from the Newey-West standard error estimator.

Model	a (a_1)	b (b_1)	a_2	b_2	R^2
Panel A: Equal-weighted returns					
One-regime	0.008	−0.299			0.000
(1), (3)	(3.10)	(−0.33)			
Two-regime	0.005	**13.075**	−0.002	−**13.714**	0.031
(2), (3)	(1.38)	(2.45)	(−0.37)	(−2.64)	
Panel B: Value-weighted returns					
One-regime	0.006	−0.581			0.002
(1), (3)	(2.83)	(−0.87)			
Two-regime	−0.000	**8.650**	0.003	−**9.361**	0.019
(2), (3)	(−0.00)	(2.22)	(0.72)	(−2.38)	

Savor and Wilson (2014): Some Dates Matter

- CAPM's holds only on FOMC meeting.

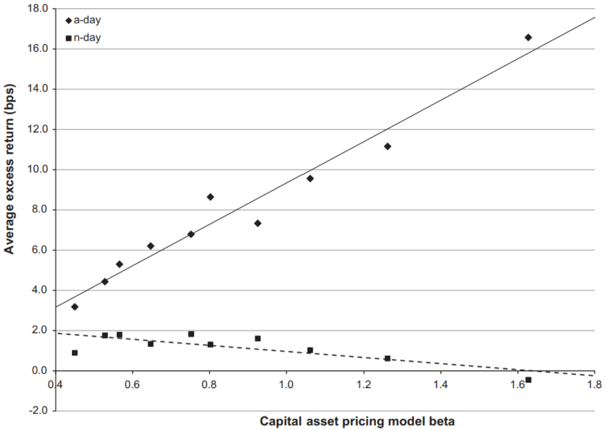


Fig. 1. Average excess returns for ten beta-sorted portfolios. This figure plots average daily excess returns in basis points (bps) against market betas for ten beta-sorted portfolios of all NYSE, Amex, and Nasdaq stocks separately for announcement days or a-days (days on which inflation, employment, or Federal Open Market Committee interest rate decisions are scheduled to be announced) and non-announcement days or n-days (all other days). The implied ordinary least squares estimates of the securities market line for each type of day are also plotted. The sample covers the 1964–2011 period. For each test portfolio, the same estimate of its full-sample beta is used for both types of day.

Cieslak, Morse, Vissing-Jorgensen (2019): Macro Condition Matters

- CAPM's validation varys along FMOC cycle.

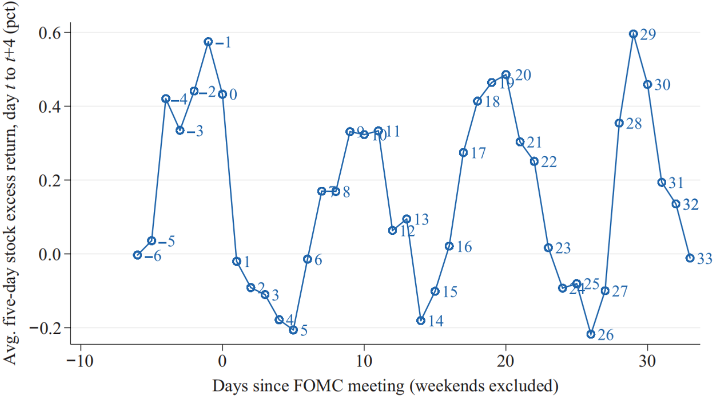
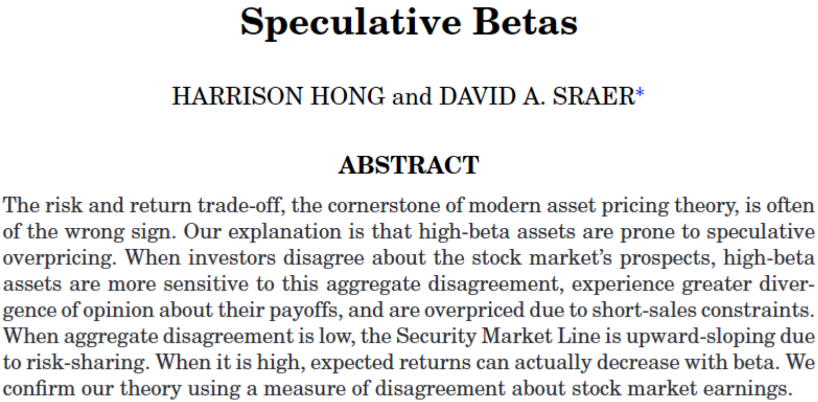


Figure 1. Stock returns over the FOMC cycle, 1994 to 2016. The plot is based on data covering 184 FOMC cycles (eight scheduled FOMC meetings per year). The numbers along the line indicate the value on the horizontal axis. The five-day (forward) returns computed for any of days −6 through −1 of the FOMC cycle are not used in the right part of the graph, so points to the right do not use any data for days −2 and later. (Color figure can be viewed at wileyonlinelibrary.com)

Hong and Sraer (2016)

- Speculate on high beta stocks!



Speculative Betas

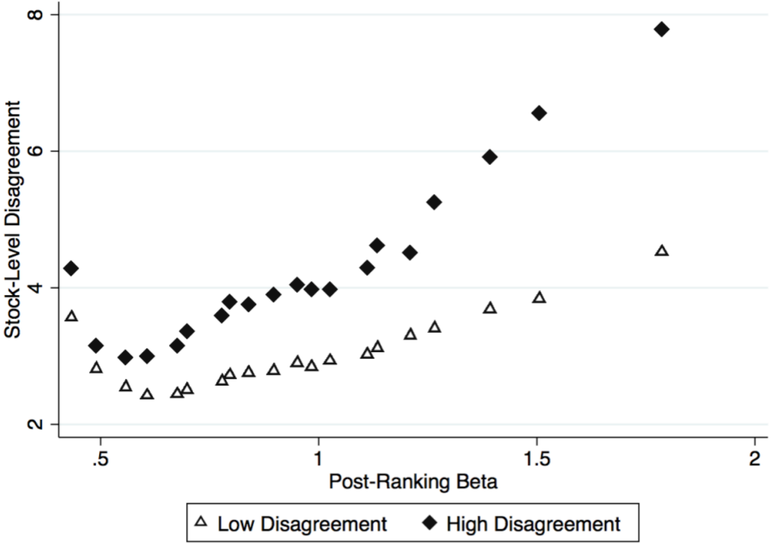
HARRISON HONG and DAVID A. SRAER*

ABSTRACT

The risk and return trade-off, the cornerstone of modern asset pricing theory, is often of the wrong sign. Our explanation is that high-beta assets are prone to speculative overpricing. When investors disagree about the stock market's prospects, high-beta assets are more sensitive to this aggregate disagreement, experience greater divergence of opinion about their payoffs, and are overpriced due to short-sales constraints. When aggregate disagreement is low, the Security Market Line is upward-sloping due to risk-sharing. When it is high, expected returns can actually decrease with beta. We confirm our theory using a measure of disagreement about stock market earnings.

Hong and Sraer (2016)

- Disagreement matters!



Failure of CAPM?

- Could we simply conclude that CAPM is a theoretically elegant, but practically clumsy model?

- It depends how you would like to depict the issue.

- Let's employ the Sharpe's interview response as conclusion:

  The fundamental idea remains that there's no reason to expect reward just for bearing risk. Otherwise, you'd make a lot of money in Las Vegas. If there's reward for risk, it's got to be special. There's got to be some economics behind it or else the world is a very crazy place. I don't think differently about those basic ideas at all.

  -Sharpe, 1998, Dow Jones Asset Manager.

Conclusion: Beyond CAPM, Anomaly

- CAPM is still the most insightful theory for financial economics.
- We could derive a single factor model from CAPM.
- The single factor model can't explain all the return behaviors.
- Fama and French name the empirical fact that can't be explained by CAPM as Anomaly.
- Most salient ones are Size, Value and Momentum factors.
- Let's carry on multifactor models in the next class.