1. Determine whether the following statements are True or False and briefly EXPLAIN. Most points are for you explanations.(5 points each)

   (a) In a two-player game, if one player has a dominant strategy, then there exists only pure strategy NE and no mixed strategy NE.

   False. Even if the player with a dominant strategy plays the pure (dominant) strategy, the other player may be playing a mixed strategy. See, for example, the following game:

	a	b
u	3, 1	0, 1
d	0, 0	-1, 2

   While player 1 plays *u*, player 2 could play both *a* and *b* with positive probabilities.

   (b) A strategy that is NEVER be a best response is strictly dominated.

   False. This is true only for two-player game. For more than two-player games, some strategies that are never a best response may nevertheless not be dominated strategies.

   (c) In subgame perfect Nash equilibrium, a player's equilibrium strategy specifies optimal choices at every point in the game tree.

   Answer: False. This statement is true only for perfect information games. For some imperfect information games, SPNE may be the same as NE and therefore not specifies optimal choices at some information sets.

   (d) In a simultaneous-move game, every NE is also a sequential equilibrium.

   Answer: True. In a simultaneous-move game each player has only one information set. By definition of NE, each player is making optimal choice at the information set. So every NE is also sequential equilibrium.

   (e) An extensive-form game is of perfect recall if every information set has contains only one decision node.

   False. A game is of perfect recall if no player ever forget any information he once knew.

   Alternatively, a game is of perfect information if every information set contains only one decision node.

2. For the game given below:

   Player 2

		L_2	M_2	N_2	R_2
Player 1	L_1	(1, 4)	(3, 2)	(2, 6)	(2, 7)
	M_1	(0, 0)	(0, 3)	(1, 3)	(1, 0)
	N_1	(2, 0)	(10, -1)	(0, 0)	(0, 2)
	R_1	(4, 7)	(5, 1)	(2,0)	(1, 4)

   (a) Does any player have any strictly dominated strategies? If yes, what are they? (5 points)

   (b) Apply the Iterated Deletion of Strictly Dominated Strategies procedure until there is no strictly dominated strategies in the remaining game. (10 points)

   (c) Find all pure strategy NE of the game. (5 points)

   (d) Does the game have any mixed strategy NE? If your answer is "Yes," find the mixed strategy NE. (5 points)

   Answer

   (a) Yes. Player 1's strategy M_1 is strictly dominated by L_1.

(b) After deleting strictly dominated strategy M_1, N_2, M_2 both becomes strictly dominated by R_2. After deleting M_2, N_2, N_1 becomes strictly dominated by R_1.

	L_2	R_2
L_1	(1, 4)	(2, 7)
R_1	(4, 7)	(1, 4)

(c) Two pure strategy NE

$$(L_1, R_2), \qquad (R_1, L_2).$$

To find the mixed strategy NE, we consider players' best responses given belief. For player 1, given the belief $(q, 0, 0, 1-q)$, expected payoffs from playing L_1, R_1 are

$$u_1(L_1) = q + 2(1-q), \qquad u_1(R_1) = 4q + (1-q).$$

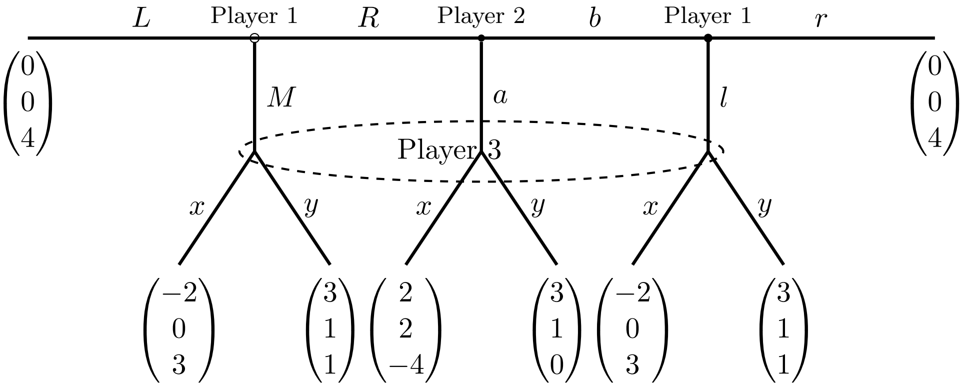
(d) To be a mixed NE, it must be true that $u_1(L-1) = u_1(R_1)$, so $q = 1/4$.
For player 2, given the belief $(p, 0, 0, 1-p)$, expected payoffs from playing L_2, R_2 are

$$u_2(L_2) = 4p + 7(1-p), \qquad u_2(R_2) = 7p + 4(1-p)$$

This gives $p = 1/2$. So the mixed strategy NE is:

$$\left(\frac{1}{2}, 0, 0, \frac{1}{2}; \frac{1}{4}, 0, 0, \frac{3}{4}\right).$$

3. For the extensive form game below:



(a) Determine pure strategy NE of the game. (10 points)

(b) Determine pure strategy normal-form perfect equilibria of the game. (5 points)

(c) Determine pure strategy sequential equilibria of this game, explicitly stating the strategy profiles σ and the associated belief μ for each S.E. (10 points)

Answer

(a) The Normal-form

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player 1		a	b
	Ll	0, 0, 4	0, 0, 4
	Lr	0, 0, 4	0, 0, 4
	Ml	-2, 0, 3	-2, 0, 3
	Mr	-2, 0, 3	-2, 0, 3
	Rl	2, 2, -4	-2, 0, 3
	Rr	2, 2, -4	0, 0, 4

2

	a	b
Ll	0, 0, 4	0, 0, 4
Lr	0, 0, 4	0, 0, 4
Ml	3, 1, 1	3, 1, 1
Mr	3, 1, 1	3, 1, 1
Rl	3, 1, 0	3, 1, 1
Rr	3, 1, 0	0, 0, 4

player 3 plays x y

Pure strategy NE:

$$(Ll, b, x), (Lr, b, x), (Rl, a, y), (Rr, a, y).$$

(b) Only one normal-form perfect equilibrium: (Rl, a, y).

$(Ll, b, x), (Lr, b, x)$ involve dominated strategy. Note that for any totally mixed strategies of player 2, 3, $\sigma^{\epsilon}_{-1} = ((1-\epsilon, \epsilon); (\eta, 1-\eta))$,

$$u_1(Rl, \sigma^{\epsilon}_{-1}) > u_1(Rr, \sigma^{\epsilon}_{-1}),$$

thereby precluding (Rr, a, y) as a limit of any sequence of ϵ-perfect equilibria.

(c) S.E.

$$\sigma : (Rl, a, y); \mu : (0, 1, 0).$$

4. Two firms, $i = 1, 2$, both become aware of consumers' demand for a new product, and need to decide secretly whether to conduct R&D to develop the new product to satisfy consumer demand. The fixed cost to develop the new product for firm i is θ_i and is privately known only to the firm itself. However, it is common knowledge that θ_i is independently drawn from a uniform distribution on $[0, 90]$. Suppose both firms know that the potential demand is given by $P = 20 - Q$. The cost of producing the product if it is developed is the same for both firms and $C(q_i) = 2q_i$, $i = 1, 2$.

   Firm i will obtain the the monopoly profit minus the R&D cost if it alone develops the product, $\pi_i = \Pi^m - \theta_i$. If both firms conduct R&D to develop the product, then they engage in Cournot competition after that. In that case, the profit for each firm would be $\pi_i = \Pi^d - \theta_i$ where Π^d stands for duopoly profit. A firm receives zero profit if it does not develop the market.

   (a) Determine the monopoly profit Π^m and duopoly profit Π^d. (15 points)

   (b) Determine the Bayesian Nash equilibrium of this game. (10 points) **Answer:**

   i. Solve monopolist's profit-maximization problem:

$$\max_q \Pi^m = (20 - q)q - 2q.$$

   This gives

$$q = 9, \quad P = 11, \quad \Pi^m = 81.$$

   With duopoly, for each firm i, the profit-maximization problem is:

$$\max_{q_i} \Pi^d = (20 - q_i - q_j)q_i - 2q_i.$$

   Solving for the Cournot equilibrium gives: for $i = 1, 2$,

$$q_i = 6, \quad P = 8, \Pi^d = 36.$$

   ii. Denote by θ^*_i the cutoff point for firm i for $i = 1, 2$. Firm i chooses"develop" if $\theta_i \leq \theta^*_i$, but chooses "not" otherwise.

   For player i, "enter" is best response if

$$(\Pi^d - \theta_i)F(\theta^*_j) + (\Pi^m - \theta_i)[1 - F(\theta^*_j)] \geq 0 \Longrightarrow$$

$$\theta_i \leq \Pi^d + (\Pi^m - \Pi^d)[1 - F(\theta^*_j)] = \Pi^d + (\Pi^m - \Pi^d)(1 - \frac{\theta^*_j}{90}).$$

   Thus, we have, for $i = 1, 2$,

$$\theta^*_i = \theta^* = 54.$$

   The BNE is: $i = 1, 2$, firm i chooses "develop" if $\theta_i \leq 54$; chooses "not" otherwise.