国际金融作业

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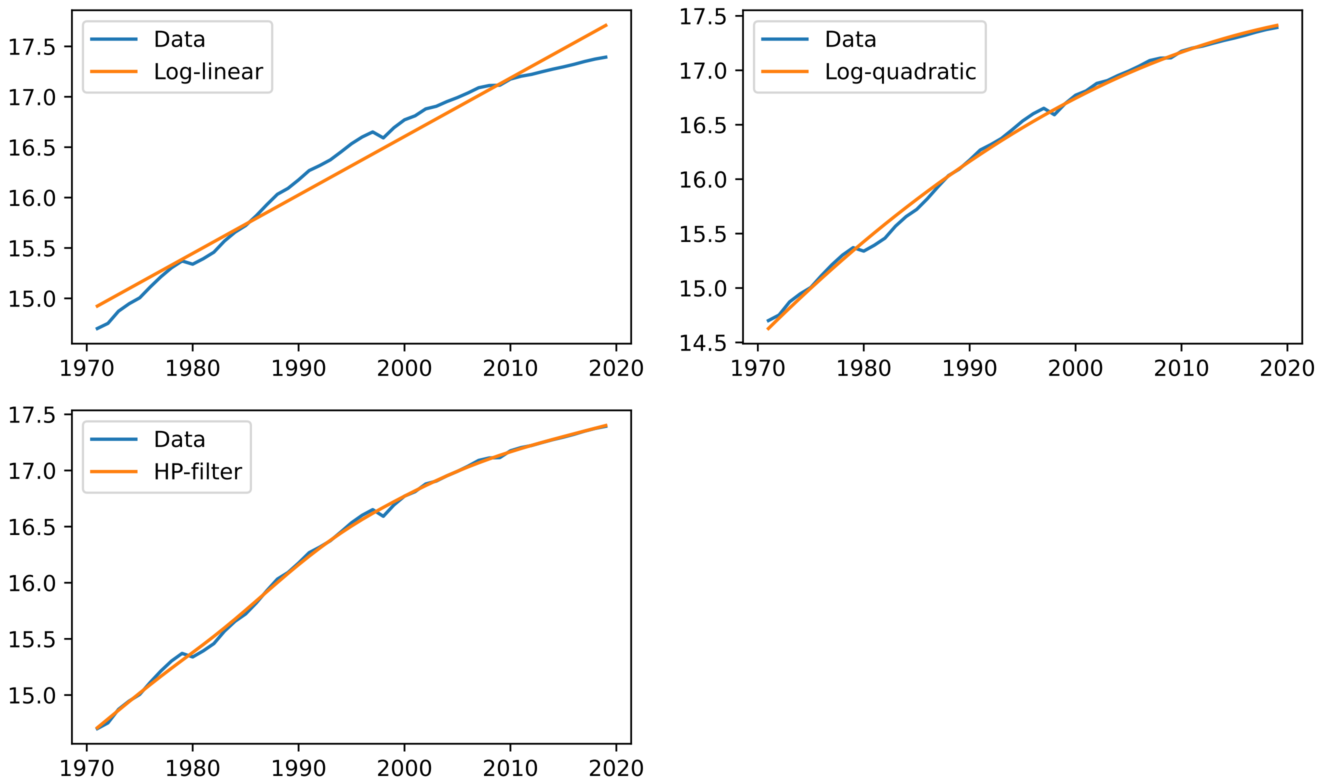
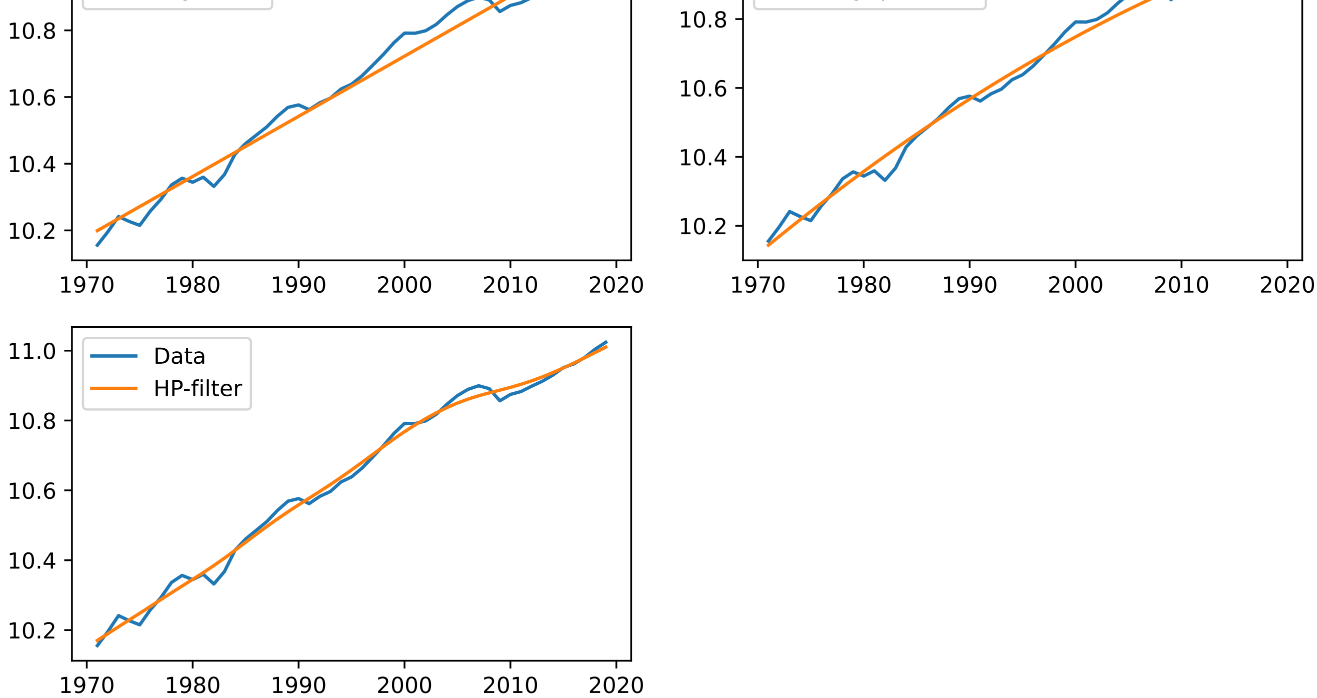
Q1




Data
Log-linear
Data
Log-quadratic
Data
HP-filter
11.0
10.8
10.6
10.4
10.2
1970
1980
1990
2000
2010
2020

Q2. 1. $C_{N,t} + C_{D,t} + i_t + (1+r_{t-1})d_{t-1} = y_t + d_t$

2. $$\max_{\{C_{N,t}, S_t, k_{t+1}, d_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

s.t. $C_{N,t} + S_t + k_{t+1} + (1+r_{t-1})d_{t-1} = F(k_t) + (1-\delta)S_{t-1} + (1-\delta_k)k_t + d_t$

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{N,t}^{1-\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} S_t^{1-\frac{1}{\eta}}\right]^{\frac{1}{1-\frac{1}{\eta}}}$$

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{(1+r)^j} \le 0$$

3. F.O.C. $$\begin{cases} r_t = F'(k_{t+1}) - \delta_k \\ 1 = \beta \dfrac{U'(C_{t+1})}{U'(C_t)} \cdot \dfrac{A_1(C_{N,t+1}, S_{t+1})}{A_1(C_{N,t}, S_t)}(1+r_t) \\ \dfrac{S_t}{C_{N,t}} = \dfrac{\alpha}{1-\alpha}\left(\dfrac{1+r_t}{r_t+\delta}\right)^{\eta} \end{cases}$$

4. $$\frac{C_D}{e} = \frac{\delta}{\delta + \frac{1-\alpha}{\alpha}\left(\frac{r+\delta}{1+r}\right)^{\eta}}$$

5. $$\eta \log\left(\frac{r+\delta}{1+r}\right) = \log\left(\frac{1-\alpha}{\alpha}\right) - \left[\log S_t - \log C_{N,t}\right]$$

$$S_0 = (1-\delta)S_{-1} = C_{D,0}$$

$$C_{D,-1} = \delta S_{-1}$$

$$\Rightarrow \%\Delta S_t = \delta \%\Delta C_{D,t}$$

$$\frac{e_0 - e_{-1}}{e_{-1}} = \frac{1 + \frac{\alpha}{1-\alpha}\left(\frac{1+r}{r+\delta}\right)^{\eta}}{\delta + \frac{\alpha}{1-\alpha}\left(\frac{1+r}{r+\delta}\right)^{\eta}} \cdot 1\% > 1\%$$

6. $$\frac{tb_0 - tb_{-1}}{e_{-1}} = -\frac{e_0 - e_{-1}}{e_{-1}}$$

Q3. 1. $U_c(c,h) = [c^{1-w}(1-h)^{w}]^{1-\sigma}\,\frac{1-w}{c}$

$U_h(c,h) = [c^{1-w}(1-h)^{w}]^{1-\sigma}\left(-\frac{w}{1-h}\right)$

$\frac{k}{h} = \left(\frac{\alpha}{\frac{1}{\beta}-1+\delta}\right)^{\frac{1}{1-\alpha}} = k$

$-\frac{U_h(c,h)}{U_c(c,h)} = (1-\alpha)\left(\frac{k}{h}\right)^{\alpha}$

$\Rightarrow c = \frac{(1-\alpha)(1-w)}{w}k^{\alpha}$

$\therefore r\bar{d} + c + \delta k = k^{\alpha}h^{1-\alpha}$

$h = \frac{r\bar{d} + \frac{(1-\alpha)(1-w)}{w}k^{\alpha}}{\frac{1-\alpha(1-w)}{w}k^{\alpha} - \delta k}$

2. $h = \frac{1}{3}$ $\therefore w = 0.6567.$

3. $U_{cc}(c,h) = [c^{1-w}(1-h)^{w}]^{1-\sigma}\,\frac{(1-\sigma)(1-w)^2-(1-w)}{c^2}$

$U_{ch}(c,h) = [c^{1-w}(1-h)^{w}]^{1-\sigma}\left(-\frac{(1-\sigma)w(1-w)}{c(1-h)}\right)$

$U_{hh}(c,h) = [c^{1-w}(1-h)^{w}]^{1-\sigma}\,\frac{(1-\sigma)w^2-w}{(1-h)^2}$

$\therefore \varepsilon_{cc} = \frac{U_{cc}c}{U_c} = (1-\sigma)(1-w)-1$

$\cancel{\varepsilon_{ch} = \frac{}{U_c}}$

$\varepsilon_{ch} = \frac{U_{ch}h}{U_c} = (1-\sigma)w\left(-\frac{h}{1-h}\right)$

$\varepsilon_{hc} = \frac{U_{hc}c}{U_h} = (1-\sigma)(1-w)$

$\varepsilon_{hh} = \frac{U_{hh}h}{U_h} = [(1-\sigma)w-1]\left(-\frac{h}{1-h}\right)$

Q4. 1. $c_t = c_t^T = c_t^N$

$$\max_{\{d_t, h_t\}} \sum_{t=0}^{\infty} \beta^t [\ln A h_t - \gamma h_t]$$

$$\text{s.t. } A h_t + (1+r) d_{t-1} = y^T + d_t$$

$$\lim_{t\to\infty} \frac{d_t}{(1+r)^t} \le 0$$

$$\begin{cases} \lambda_t - \beta \lambda_{t+1}(1+r) \ge 0 \\ \frac{1}{A h_t} A - \gamma - \lambda_t A \ge 0 \end{cases}$$

$$\frac{1}{h_t} - \gamma = \beta(1+r)\left[\frac{1}{h_{t+1}} - \gamma\right] \Rightarrow \begin{cases} h_t = \frac{1}{\gamma} \\ c_t = c_t^T = c_t^N = \frac{A}{\gamma} \end{cases}$$

$$d_t = [(1+r)^t - 1] \frac{\frac{A}{\gamma} - y^T}{r} + (1+r)^t d_{-1}$$

$$\frac{d_t}{(1+r)^t} = d_{-1} + \left[1 - \frac{1}{(1+r)^t}\right] \frac{\frac{A}{\gamma} - y^T}{r}$$

$$d_{-1} \le \frac{\frac{A}{\gamma} - y^T}{r} \quad \text{and} \quad y^T > \frac{A}{\gamma}$$

2. Only debt decreases.

3. $y^T > \frac{A+\Delta A}{r}$. $h_t = \frac{1}{\gamma}$

$$c_t = c_t^T = c_t^N = \frac{A+\Delta A}{\gamma}$$

Q5. 1. $$\frac{\partial p}{\partial c^N} = \frac{A_{22}(C^T, C^N)A_1(C^T, C^N) - A_2(C^T, C^N)A_{12}(C^T, C^N)}{A_1(C^T, C^N)^2}$$

$$\begin{cases} A_1(C^T, C^N) \geq 0 \\ A_2(C^T, C^N) \geq 0 \\ A_{11}(C^T, C^N) \leq 0 \\ A_{22}(C^T, C^N) \leq 0. \end{cases}$$

$$A_1(\alpha C^T, \alpha C^N) = A_1(C^T, C^N)$$

$$A_{11}(\alpha C^T, \alpha C^N)C^T = A_{12}(\alpha C^T, \alpha C^N)C^N = 0.$$

$$\because A_{11}(C^T, C^N) \leq 0 \quad \therefore A_{12}(C^T, C^N) \geq 0.$$

$$\therefore \frac{\partial p}{\partial c^N} \leq 0.$$

2. $$\frac{\partial p}{\partial c^T} = \frac{A_{21}(C^T, C^N)A_1(C^T, C^N) - A_2(C^T, C^N)A_{11}(C^T, C^N)}{A_1(C^T, C^N)^2}$$

$$\begin{cases} A_1(C^T, C^N) \geq 0 \\ A_2(C^T, C^N) \geq 0 \\ A_{11}(C^T, C^N) \leq 0 \\ A_{22}(C^T, C^N) \leq 0 \\ A_{21}(C^T, C^N) \geq 0. \end{cases} \Rightarrow \frac{\partial P}{\partial c^T} \geq 0.$$

3. $$P = \frac{C^T}{C^N}$$

4. $$P = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{C^T}{C^N}\right)^{\frac{1}{2}}$$