# 计量期末复习（应试版）

## 一、分位数回归

① 分位数回归目标函数的推导.

your are given the following function:

$$L(y-c)=\begin{cases}\alpha|y-c|, & y-c\geq 0\\ (1-\alpha)|y-c|, & y-c<0\end{cases}$$

where $y$ is a continuous random variable and $\alpha$ is a constant. Please prove to identify whether minimizing the expectation of $L(y-c)$ with respect to $c$ will give us: $c=\text{Quant}_\alpha(y)$ ?

解：$$L(y-c)=\begin{cases}\alpha|y-c|, & y-c\geq 0\\ (1-\alpha)|y-c|, & y-c<0\end{cases}$$

$$\min_c E\, L(y-c)$$

$$=\min E\left[\alpha(y-c)\,1(y-c\geq 0)+(1-\alpha)(y-c)\,1(y-c<0)\right]$$

$$=\min E\left[\alpha(y-c)\,1(y-c\geq 0)-(1-\alpha)(y-c)(1-1(y-c\geq 0))\right]$$

F.O.C: $\frac{\partial(\cdot)}{\partial c}=E\left[-\alpha\,1(y-c\geq 0)+(1-\alpha)(1-1(y-c\geq 0))\right]=0$

$\Rightarrow E\,1(y-c\geq 0)=1-\alpha$

$\Rightarrow P(y-c\geq 0)=1-\alpha$

$\Rightarrow P(y\geq c)=1-\alpha \Rightarrow P(y<c)=\alpha$

$\therefore c=\text{Quant}_\alpha\, y$.

Note：这个题和笔记上的推导类似，关键在于把分段函数写成示性函数 $1(y-c\geq 0)$ 的形式，以及利用 $1(y-c\geq 0)=1-1(y-c<0)$ 化简

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二. 时间序列.

①. ACF 的推导

Ex 1 : Suppose a time series $\{X_t\}_{t=1}^{n}$ has the following representation :

$$X_t = \phi X_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1} \quad , \varepsilon_t \sim WN(0, \sigma^2)$$

where $|\phi| < 1$ and $|\theta| < 1$

a) In class we declare $\hat{\phi}_{OLS}$ is an inconsistent estimator of $\phi$. why does the OLS estimation fail to be consistent?

b). Actually $\hat{\phi}_{OLS} \xrightarrow{P} \rho$ as $n \to \infty$. what is the $\rho$ ?

解: a) $X_{t-1} = \phi X_{t-2} + \varepsilon_{t-1} + \theta \varepsilon_{t-2}$

$\therefore X_{t-1}$ 和 $\varepsilon_{t-1}$ 相关, 原方程由于存在内生性而无法得到一致估计量

b) : ①. $X_t = \phi X_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$

两边同乘 $X_t$, 取期望

$$E(X_t^2) = \phi E X_t X_{t-1} + E \varepsilon_t X_t + \theta E \varepsilon_{t-1} X_t$$

$$\Rightarrow r(0) = \phi r(1) + \sigma^2 + \theta^2 \sigma^2 + \phi \theta \sigma^2$$

两边同乘 $X_{t-1}$, 取期望

$$E(X_t X_{t-1}) = \phi E(X_{t-1}^2) + E \varepsilon_t X_{t-1} + \theta E \varepsilon_{t-1} X_{t-1}$$

$$\Rightarrow r(1) = \phi r(0) + 0 + \theta \sigma^2$$

$$\begin{cases} r(0) = \phi r(1) + \sigma^2 + \theta^2 \sigma^2 + \phi \theta \sigma^2 \\ r(1) = \phi r(0) + \theta \sigma^2 \end{cases}$$

$$\Rightarrow \begin{cases} 1 = \phi \rho(1) + \dfrac{\sigma^2 + \theta^2 \sigma^2 + \phi \theta \sigma^2}{r(1)} \\ \rho(1) = \phi + \dfrac{\theta \sigma^2}{r(0)} \end{cases} \Rightarrow \frac{1 - \phi \rho(1)}{\rho(1) - \phi} = \frac{1 + \phi\theta + \theta^2}{\theta}$$

$$\Rightarrow \rho(1) = \frac{\theta + \phi + \theta^2 \phi + \theta \phi^2}{1 + 2\theta\phi + \theta^2}$$

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两边同乘 $X_{t-2}$ 取期望

$E(X_t X_{t-2}) = \phi X_{t-1} X_{t-2} + \varepsilon_t X_{t-2} + \theta \varepsilon_{t-1} X_{t-2}$

$\Rightarrow r(2) = \phi\, r(1)$

$\Rightarrow \rho(2) = \phi\, \rho(1)$

$\vdots$

$\rho(h) = \phi^{h-1} \rho(h)$

$$\therefore \rho(h) = \begin{cases} \dfrac{\theta+\phi+\theta^2\phi+\theta\phi^2}{1+\theta^2+2\theta\phi} & h=1 \\ \phi^{h-1}\dfrac{\theta+\phi+\theta^2\phi+\theta\phi^2}{1+\theta^2+2\theta\phi} & h>1 \end{cases}$$

Note：求解此类题型的技巧，等式左右两边同乘 $X_t, X_{t-1}, X_{t-2}, \cdots$，取期望，记住几个公式

$r(h) = cov(X_t, X_{t+h}) = E(X_t X_{t+h})$

$r(0) = E(X_t^2) = E(X_{t-1}^2) = E(X_{t-2}^2) = \cdots\cdots$

$ACF(h) = \rho(h) = \dfrac{r(h)}{r(0)}$

同时，有必要掌握笔记上 AR(1) 和 MA(1) 的 ACF 推导，记住 AR(1) 的 ACF 拖尾，MA(1) 的 ACF 截尾.

另一个例子大家可以练习一下，第二次作业第二题.

Ex2: Suppose $\{X_t\}_{t=1}^{n}$ is generated by a classical MA(1) process: $X_t = Z_t + \theta Z_{t-1}$ $Z_t \sim WN(0, \sigma^2)$, $t=1,2,\cdots,n$

If a standard AR(1) process, $X_t = \phi X_{t-1} + Y_t$ is mistakenly fittled to $\{X_t\}$. Please derives the ACF of $\{Y_t\}$.

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②. 判断收敛的速度.(兼论依MSE收敛)

EX1 笔记中的例子:

$y_t = x_t'\beta + u_t$, 考虑 $\hat{r}(1) = \frac{1}{n}\sum_{t=1}^{n} \hat{u}_{t-1}\hat{u}_t$ 的收敛速度.

解: $\hat{u}_t = y_t - x_t'\hat{\beta} = x_t'\beta + u_t - x_t'\hat{\beta}$

$= u_t - x_t'(\hat{\beta}-\beta)$

$\therefore \frac{1}{n}\sum_{t=1}^{n}\hat{u}_{t-1}\hat{u}_t = \frac{1}{n}\sum[u_t u_{t-1} - u_{t-1}x_t'(\hat{\beta}-\beta) - u_t x_{t-1}'(\hat{\beta}-\beta)$

$+ (\hat{\beta}-\beta)x_{t-1}' x_t(\hat{\beta}-\beta)]$

$= A_1 - A_2 - A_3 + A_4$

对于 $A_1$ 来说: $Var(A_1) = E(A_1^2) - (EA_1)^2$

$\Rightarrow Var(\frac{1}{n}\sum u_t u_{t-1}) = \frac{1}{n^2} Var(\sum u_t u_{t-1}) = \frac{1}{n^2}\cdot n\, Var(u_t u_{t-1})$

$= \frac{1}{n}\sigma_u^4) = O(\frac{1}{n})$ $\therefore A_1 = O_p(\frac{1}{\sqrt{n}})$

$A_2 = \frac{1}{n}\sum u_{t-1}x_t'(\hat{\beta}-\beta) = B_2\cdot C_2$

$\because \sqrt{n}(\hat{\beta}-\beta) \sim N$ $\therefore (\hat{\beta}-\beta) = O(\frac{1}{\sqrt{n}})$

$B_2 = \frac{1}{n}\sum u_{t-1}x_t'$

$Var(B_2^2) = \frac{1}{n^2}[n\,Var(x_t u_{t-1}) + \underbrace{2\sum_t\sum_{s>t} cov(x_t u_{t-1}, x_s u_s)}_{=0}]$

$= \frac{1}{n}E(x_t u_{t-1}^2 x_t')$

$= \frac{1}{n}E u_{t-1}^2\, E x_t x_t' = \frac{1}{n}\cdot O(1)\cdot O(1) = O(\frac{1}{n})$

$\therefore B_2 = O_p(\frac{1}{\sqrt{n}})$ $A_2 = O_p(\frac{1}{\sqrt{n}})\cdot O_p(\frac{1}{\sqrt{n}}) = O_p(\frac{1}{n})$

同理 $A_3 = O_p(\frac{1}{n})$

$A_4 = \frac{1}{n}\sum(\hat{\beta}-\beta)x_{t-1}x_t'(\hat{\beta}-\beta)$

其中 $Var(\frac{1}{n}\sum x_{t-1}x_t') = E(\frac{1}{n}\sum x_{t-1}x_t')^2 - (E x_{t-1}x_t')^2 = O(1)$

$\therefore A_4 = O_p(\frac{1}{\sqrt{n}})\cdot O_p(1)\cdot O_p(\frac{1}{\sqrt{n}}) = O_p(\frac{1}{n})$

$\therefore \frac{1}{n}\sum_{t=1}^{n}\hat{u}_{t-1}\cdot\hat{u}_t = O_p(\frac{1}{\sqrt{n}}) - O_p(\frac{1}{n}) - O_p(\frac{1}{n}) + O_p(\frac{1}{n}) = O_p(\frac{1}{\sqrt{n}})$

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EX2 In a classical linear regression model $y_i = X_i'\beta + u_i$ with i.i.d. data, 4-th moment of $u_i$ is finite, we estimate the variance of $u_i$ by $\hat{\sigma}^2 = \frac{1}{n-k}\sum_{i=1}^{n}\hat{u}_i^2$. Please derive that:

(a) $\hat{\beta} - \beta = O_p(n^{-0.5})$

(b) $\hat{\sigma}^2 - \sigma^2 = O_p(n^{-0.5})$

解：(a) $\sqrt{n}(\hat{\beta} - \beta) \sim N(0, (X'X)^{-1}\sigma^2) = O(1)$

$\therefore \hat{\beta} - \beta = O_p(\frac{1}{\sqrt{n}})$

(b). $\hat{\sigma}^2 = \frac{1}{n-k}\sum_{i=1}^{n}\hat{u}_i^2 = \frac{1}{n-k}\sum_{i=1}^{n}(u_i - X'(\hat{\beta}-\beta))^2$

$= \frac{1}{n-k}\sum_{i=1}^{n}(u_i^2 - 2u_iX_i'(\hat{\beta}-\beta) + (\hat{\beta}-\beta)'X_iX_i'(\hat{\beta}-\beta))$

$= \frac{1}{n-k}\sum_{i=1}^{n}u_i^2 - \frac{2}{n-k}(\hat{\beta}-\beta)\sum_{i=1}^{n}X_i'u_i + (\hat{\beta}-\beta)'\frac{1}{n-k}\sum_{i=1}^{n}X_iX_i'(\hat{\beta}-\beta)$

$= A_1 - A_2 + A_3$

$n\to\infty$ $\frac{1}{n-k}\sum_{i=1}^{n}u_i^2 \xrightarrow{P} \frac{1}{n}\sum_{i=1}^{n}u_i^2$

$var(\frac{1}{n}\sum_{i=1}^{n}u_i^2) = \frac{1}{n^2}\sum[\underbrace{E(u_i^4)}_{\text{有限}} - E(u_i^2)^2]$

$= \frac{1}{n^2}\cdot n\,(O(1) + \sigma^4) \Rightarrow \frac{1}{n}\sum_{i=1}^{n}u_i^2 = \sigma^2 + O(\frac{1}{\sqrt{n}})$

由上题可知：

$A_2 = \cdot O(\frac{1}{\sqrt{n}})\cdot O(\frac{1}{\sqrt{n}}) = O_p(\frac{1}{n})$

$A_3 = O(\frac{1}{\sqrt{n}}).O(1).O(\frac{1}{\sqrt{n}}) = O_p(\frac{1}{n})$

$\therefore \hat{\sigma}^2 - \sigma^2 = O_p(\frac{1}{\sqrt{n}})$

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Note：总结一下做这种题的技巧

首先，如果能构造一个统计量服从一个分布，则可以轻松的判断其量级，例如 $\sqrt{n}(\hat{\beta}-\beta) \sim N$

$\therefore \hat{\beta}-\beta = O_p(\frac{1}{\sqrt{n}})$

其次，对于不好判断的，利用 $Var(X)=E(X^2)-E(X)^2$ 判断其方差的量级

这里，不妨记住一些常见的可能会考的量级，考场上可以直接写结论

$\hat{\beta}-\beta = O_p(\frac{1}{\sqrt{n}})$ $\quad \frac{1}{n}\sum_{t=1}^{n} u_{t-1}u_t = O_p(\frac{1}{\sqrt{n}})$

$\frac{1}{n}\sum_{i=1}^{n} u_i^2 = O_p(\frac{1}{\sqrt{n}})+\sigma^2$ $\quad \frac{1}{n}\sum_{t=1}^{n} u_{t-1}x_t = O_p(\frac{1}{\sqrt{n}})$

$\frac{1}{n}\sum_{i=1}^{n} x_i x_i' = O_p(1)$

Ex3：For the classical stationary AR(1) model：$y_t=\rho y_{t-1}+u_t$，where $u_t \sim i.i.d\ (0,\sigma^2)$ and $E y_t^4$ is finite，we have $\sqrt{n}(\hat{\rho}-\rho)=\dfrac{\frac{1}{\sqrt{n}}\sum_{t=1}^{n} y_{t-1}y_t}{\frac{1}{n}\sum y_{t-1}^2}$

please derive that the denominator has $\frac{1}{n}\sum_{t=1}^{n} y_{t-1}^2 \xrightarrow{P} E(y_{t-1}^2) = \sigma_y^2$

bias $=0$

解：$E(\frac{1}{n}\sum y_{t-1}^2) = \frac{1}{n}\cdot n E(y_{t-1}^2) = \sigma_y^2$

$Var(\frac{1}{n}\sum y_{t-1}^2) = \frac{1}{n^2}(\sum Var(y_{t-1}^2) + 2\sum_{t=1}^{n}\sum_{s\neq t}^{n} cov(y_{t-1}^2, y_{s-1}^2))$

$=\frac{1}{n^2}[\cdot n\cdot c + O(n)] = O(\frac{1}{n}) \to 0$

$\therefore \frac{1}{n}\sum_{t=1}^{n} y_{t-1}^2 \xrightarrow{MSE} E y_{t-1}^2 = \sigma_y^2.$

$\therefore \frac{1}{n}\sum_{t=1}^{n} y_{t-1}^2 \xrightarrow{P} E y_{t-1}^2 = \sigma_y^2$

notes：依 MSE 收敛要求 bias $=0$ var $=0$，证明 var $=0$ 时多数要用到前面的判断量级，至于这道题的 cov 的量级，建议直接记结论.

③. 推导分布（利用布朗运动）

EX1：第二次作业第五题

Note：我看不太懂 怂

## 二. 面板数据.

①. FE解(实际上就是去心估计).

EX1: For the classical random-effect model that we have learned in the class: $y_{it} = x_{it}'\beta + \alpha + \varepsilon_{it} + u_i$ where, $x_{it}'$ is strictly exogenous; $Var(\varepsilon_{it}) = \sigma_\varepsilon^2$ and $\sigma_\varepsilon^2$ is unknown. We can estimate the $\sigma^2$ by:

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{n(T-1)-k}\sum_{i=1}^{n}\sum_{t=1}^{T}\ddot{e}_{it}^{(2)} \quad \text{where } \ddot{e}_{it} = e_{it} - \bar{e}_{it}$$

Please prove that $\hat{\sigma}_\varepsilon$ is a consistant estimator of $\sigma_\varepsilon^2$.

解：$y_{it} - \bar{y}_i = (x_{it} - \bar{x}_{it})'\beta + (\varepsilon_{it} - \bar{\varepsilon}_{it})$

$\Rightarrow \ddot{y}_{it} = \ddot{x}_{it}'\beta + \ddot{\varepsilon}_{it}$

$\ddot{e}_{it} = \ddot{y}_{it} - \ddot{x}_{it}' \cdot b = \ddot{\varepsilon}_{it} - \ddot{x}_{it}'(b-\beta)$

$$\therefore \frac{1}{n(T-1)}\sum_{i=1}^{n}\sum_{t=1}^{T}\ddot{e}_{it}^2 = \frac{1}{n(T-1)}\sum_{i=1}^{n}\sum_{t=1}^{T}\left(\ddot{\varepsilon}_{it} - \ddot{x}_{it}'(b-\beta)\right)^2$$

$$= \frac{1}{n(T-1)}\sum_{i=1}^{n}\sum_{t=1}^{T}\left(\ddot{\varepsilon}_{it}^2 + (b-\beta)'\ddot{x}_{it}\ddot{x}_{it}'(b-\beta) - 2(b-\beta)'\ddot{x}_{it}\ddot{\varepsilon}_{it}\right)$$

$$= \frac{T}{T-1}\frac{1}{nT}\sum\sum\left(\ddot{\varepsilon}_{it}^2 + (b-\beta)'\ddot{x}_{it}\ddot{x}_{it}'(b-\beta) - 2(b-\beta)'\ddot{x}_{it}\ddot{\varepsilon}_{it}\right)$$

按照之前学的公式 $\frac{T}{T-1}\left(\sigma_\varepsilon^2 + O_p\left(\frac{1}{\sqrt{nT}}\right) + O_p\left(\frac{1}{nT}\right) - 2O\left(\frac{1}{nT}\right)\right)$

$\xrightarrow{P} \frac{T}{T-1}\sigma_\varepsilon^2$

其中 $\sigma_\varepsilon^2 = \frac{T-1}{T}\sigma_\varepsilon^2$ ∴ 原式 $\to \sigma_\varepsilon^2$

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② . RE2SLS.

Ex1: As what we has discussed in the class, for the random effect model $y_{it} = x_{it}'\beta + \mu_i + \varepsilon_{it}$ with endogenous problem, we can run the following regression with IV:

$$\Omega^{-\frac{1}{2}} y = \Omega^{-\frac{1}{2}} X\beta + v .$$

$Z$ contains all exogenous variables including IVs and it is $nT\times L$, $L>k$. $\Omega^{-\frac{1}{2}}$ is the transformation matrix for RD model. Please derive that as $n\to\infty$, the 2SLS will generate:

$$Var(\sqrt{n}(\hat\beta-\beta)) = \left(\frac{X'\Omega^{-1}Z}{n}\left(\frac{Z'\Omega^{-1}Z}{n}\right)^{-1}\frac{Z'\Omega^{-1}X}{n}\right)^{-1}$$

解：对 $\Omega^{-\frac{1}{2}}y = \Omega^{-\frac{1}{2}}X\beta + v$ 进行 2SLS

First-stage: $\Omega^{-\frac{1}{2}}X = \Omega^{-\frac{1}{2}}Z\delta + \varepsilon$

$$\Rightarrow \hat\delta = (Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}X$$

$$\Rightarrow \widehat{\Omega^{-\frac{1}{2}}X} = \Omega^{-\frac{1}{2}}Z\hat\delta = \Omega^{-\frac{1}{2}}Z(Z'\Omega^{-1}Z')^{-1}Z'\Omega^{-1}X$$

second-stage: $\Omega^{-\frac{1}{2}}y = \widehat{\Omega^{-\frac{1}{2}}X}\beta + \Omega^{-\frac{1}{2}}v$

$$\Rightarrow \hat\beta_{REIV} = [(\widehat{\Omega^{-\frac{1}{2}}X})'(\widehat{\Omega^{-\frac{1}{2}}X})]^{-1}(\widehat{\Omega^{-\frac{1}{2}}X})'(\Omega^{-\frac{1}{2}}y)$$

$$= [(\widehat{\Omega^{-\frac{1}{2}}X})'(\widehat{\Omega^{-\frac{1}{2}}X})]^{-1}[(\widehat{\Omega^{-\frac{1}{2}}X})'(\widehat{\Omega^{-\frac{1}{2}}X}\beta + \Omega^{-\frac{1}{2}}v)]$$

$$= \beta + [(\widehat{\Omega^{-\frac{1}{2}}X})'(\widehat{\Omega^{-\frac{1}{2}}X})]^{-1}[(\widehat{\Omega^{-\frac{1}{2}}X})'\Omega^{-\frac{1}{2}}v]$$

$$\Rightarrow \hat\beta - \beta = (X'\Omega^{-1}Z(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-\frac{1}{2}}\cdot\Omega^{-\frac{1}{2}}Z(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}X)^{-1}$$

$$\cdot(X'\Omega^{-1}Z(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-\frac{1}{2}}\Omega^{-\frac{1}{2}}v)$$

$$= (X'\Omega^{-1}Z(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}X)^{-1}(X'\Omega^{-1}Z(Z'\Omega^{-1}Z)^{-1}$$

$$Z'\Omega^{-1}v$$

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$$Var(\sqrt{n}(\hat\beta-\beta)) = n\cdot(X'\Omega^{-1}Z(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}X)^{-1}X'\Omega^{-1}Z(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}E(VV'|X)\Omega^{-1}Z(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}X(X'\Omega^{-1}Z(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}X)^{-1}$$

$$= n(X'\Omega^{-1}Z(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}X)^{-1}$$

$$= \left(\frac{X'\Omega^{-1}Z}{n}\left(\frac{Z'\Omega^{-1}Z}{n}\right)^{-1}\frac{Z'\Omega^{-1}X}{n}\right)^{-1}$$

note：没啥好说的，和上半学期的两阶段最小二乘很类似，看着复杂但是挺好算的。

③ FE IV. (FE2SLS)

Ex1：As what we have discussed in the class, for the FE model with endogenous problem, we run the following regression with IV：$\ddot{y}_i = \ddot{x}_i\beta + \ddot{\varepsilon}_i$. where $\ddot{y}_i$ is $T\times1$ demean of $y_t$. $\ddot{x}_i$ $\ddot{\varepsilon}_i$ ~~the~~ are the same. $\ddot{z}_i$ is the demean of $T\times L$ martix including all exogenous variables and $L>k$. Please derive that as $n\to+\infty$, the 2SLS will generate：

$$Var(\sqrt{n}(\hat\beta-\beta)) = \sigma^2((E\ddot{x}_i'\ddot{z}_i)(E\ddot{z}_i'\ddot{z}_i)^{-1}(E\ddot{z}_i'\ddot{x}_i))^{-1},$$

解：First stage：$\ddot{x}_i = \ddot{z}_i\delta + u_i \Rightarrow \hat\delta = (\sum_{i=1}^n \ddot{z}_i'\ddot{z}_i)^{-1}\sum_{i=1}^n \ddot{z}_i'\ddot{x}_i$

$$\Rightarrow \hat{\ddot{x}}_i = \ddot{z}_i\hat\delta = \ddot{z}_i(\sum_{i=1}^n \ddot{z}_i'\ddot{z}_i)^{-1}\sum_{i=1}^n \ddot{z}_i'\ddot{x}_i$$

Second stage：$\ddot{y}_i = \hat{\ddot{x}}_i\hat\beta + \ddot{\varepsilon}_i$

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$$\Rightarrow \hat{\beta}_{FEIV} = \left(\sum_{i=1}^{n} \hat{\ddot{x}}_i' \hat{\ddot{x}}_i\right)^{-1} \left(\sum_{i=1}^{n} \hat{\ddot{x}}_i' \ddot{y}_i\right) = \left(\sum_{i=1}^{n} \hat{\ddot{x}}_i' \hat{\ddot{x}}_i\right)^{-1} \left(\sum_{i=1}^{n} \hat{\ddot{x}}_i'\right.$$

$$\left.(\hat{\ddot{x}}_i \beta + \ddot{\varepsilon}_i)\right) = \beta + \left(\sum_{i=1}^{n} \hat{\ddot{x}}_i' \hat{\ddot{x}}_i\right)^{-1} \sum_{i=1}^{n} \hat{x}_i' \ddot{\varepsilon}_i$$

由CLT：$\sqrt{n}(\hat{\beta}_{FEIV} - \beta) \sim N\left(0, \sigma^2 \left(\frac{\sum_{i=1}^{n} \hat{\ddot{x}}_i' \hat{\ddot{x}}_i}{n}\right)^{-1}\right)$

$$\therefore Var(\sqrt{n}(\hat{\beta} - \beta)) = \sigma^2 \left(\frac{\sum_{i=1}^{n} \hat{\ddot{x}}_i' \hat{\ddot{x}}_i}{n}\right)^{-1}$$

$$= n\sigma^2 \left[\sum_{i=1}^{n} \ddot{x}_i' \ddot{z}_i \left(\sum_{i=1}^{n} \ddot{z}_i' \ddot{z}_i\right)^{-1} \sum_{i=1}^{n} \ddot{z}_i' \ddot{z}_i \left(\sum_{i=1}^{n} \ddot{z}_i' \ddot{z}_i\right)^{-1} \sum_{i=1}^{n} \ddot{z}_i' \ddot{x}_i\right]^{-1}$$

$$= n\sigma^2 \left[\left(\sum_{i=1}^{n} \ddot{x}_i' \ddot{z}_i\right)\left(\sum_{i=1}^{n} \ddot{z}_i' \ddot{z}_i\right)^{-1} \cdot \left(\sum_{i=1}^{n} \ddot{z}_i' \ddot{x}_i\right)\right]^{-1}$$

$$= \sigma^2 \left[\left(\frac{\sum_{i=1}^{n} \ddot{x}_i' \ddot{z}_i}{n}\right)\left(\frac{\sum_{i=1}^{n} \ddot{z}_i' \ddot{z}_i}{n}\right)^{-1}\left(\frac{\sum_{i=1}^{n} \ddot{z}_i' \ddot{x}_i}{n}\right)\right]^{-1}$$

当 $n \to \infty$ 时

$$Var(\sqrt{n}(\hat{\beta} - \beta)) = \sigma^2 \left[(E\, \ddot{x}_i' \ddot{z}_i)(E\, \ddot{z}_i' \ddot{z}_i)^{-1} E(\ddot{z}_i' \ddot{x}_i)\right]^{-1}$$

note！思路是一模一样的，所以记这两道题的性价比很高.

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四．离散选择模型．

Ex1: please describe the drawbacks of the linear pobability model and prove to identife whether this model can hold the assumption of homoskedasticity?

解：LPM的缺陷：模型只在均值附近具有解释力。
当X过大或者过小时，概率不在[0,1]区间。

证明，模型一定是异方差的．

$$u_i = \begin{cases} 1 - x_i'\beta & \text{if } y_i = 1 \\ -x_i'\beta & \text{if } y_i = 0 \end{cases} \qquad \begin{aligned} P(y=1) &= x_i'\beta \\ P(y=0) &= 1 - x_i'\beta \end{aligned}$$

$$\therefore Var(u_i|x_i) = (1-x_i'\beta)^2 (x_i'\beta) + (-x_i'\beta)^2 \cdot (1-x_i'\beta)$$
$$= (x_i'\beta)(1-x_i'\beta) \quad \text{异方差.}$$

logit regression model, $P(y_i=1|x_i) = \frac{e^{x_i'\beta}}{1+e^{x_i'\beta}}$

please construct the log likelihood function for the model.

$$P(y_i=1|x_i) = \frac{e^{x_i'\beta}}{1+e^{x_i'\beta}}$$

$$\therefore L(\beta|x_i, y_i) = \log \prod_{i=1}^{n} P(y_i=1|x_i)^{y_i} (1-P(y_i=1|x_i))^{1-y_i}$$

$$= \sum_{i=1}^{n} \left[ y_i \log \frac{e^{x_i'\beta}}{1+e^{x_i'\beta}} + (1-y_i) \log \left(1 - \frac{e^{x_i'\beta}}{1+e^{x_i'\beta}}\right) \right].$$

Date.

五. 非参数估计.

EX1：2020年试卷最后一题，答案我懒得抄了，随便看看吧.