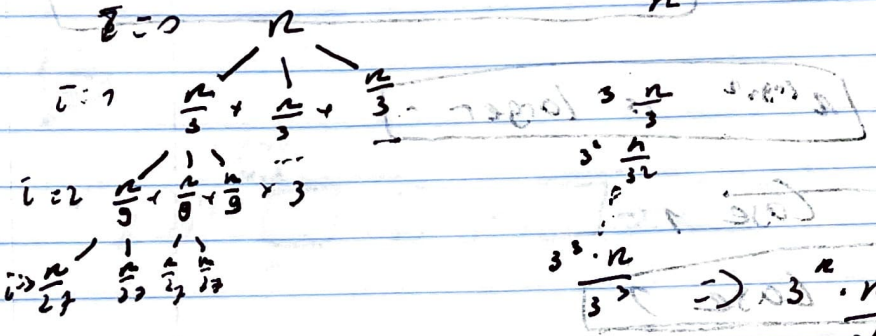
Assignment 2

Question 1

$T(n) = 3T(\frac{n}{3}) + n$



$3 \cdot \frac{n}{3} = n$

$3^2 \cdot \frac{n}{3^2} = n$

$\frac{3^k \cdot n}{3^k} \Rightarrow 3^k \cdot \frac{n}{3^k} = n$

$T(\frac{n}{3^i}) = 3^i \frac{n}{3^i}$

$\frac{n}{3^i} = 1 \Rightarrow n = 3^i \Rightarrow i = \log_3 n$

$3^{\log_3 n} = n$

Thus $T(n) = O(n \log_3 n)$

~~Thus $T(n) = n \log_3(n)$~~

Verification

$T(n) = 3T(n/3) + n$

$T(n) = 3(3T(\frac{n}{9}) + \frac{n}{3}) + n$

$= 3(3(3T(\frac{n}{27}) + \frac{n}{9}) + \frac{n}{3}) + n$

$27T(\frac{n}{27}) + 3n$

$T(n/3) = 3T(\frac{n}{9}) + \frac{n}{3}$

$T(\frac{n}{9}) = 3T(\frac{n}{27}) + \frac{n}{9}$

$3^k T(\frac{n}{3^k}) + kn$

Lets say $T(1) = 1$

$\frac{n}{3^k} = 1 \Rightarrow n = 3^k \Rightarrow k = \log_3 n \Rightarrow$

$\Rightarrow 3^{\log_3 n} + \log_3(n) n = n + \lg_3(n) n = n \log_3(n)$

Verified that $T(n) = O(n \log_3 n)$

Question 8

Masters method

Note: $T(n) = aT(\frac{n}{b}) + f(n)$ $a \geq 1$ $b > 1$

- $T(n) = 2T(\frac{n}{4}) + 1$

  $a = 2$ $b = 4$ $f(n) = 1$

  $n^{\log_4 2}$ vs 1 — $n^{\log_4 2}$ is larger

  ~~$T(n) = \Theta(n \log_4 2)$~~ case 1

  $T(n) = \Theta(n^{\log_4 2})$ case 1

- $T(n) = 2T(n/4) + \sqrt{n}$

  $n^{\log_4 2}$ vs $n^{\frac{1}{2}}$ — grow at the same rate

  $T(n) = \Theta(n^{\log_4 2} \log(n))$ | case 2

- $T(n) = 2T(\frac{n}{4}) + n$

  $n^{\log_4 2}$ vs n — n is larger

  $T(n) = \Theta(f(n)) = \Theta(n)$ case 3

- $T(n) = 2T(\frac{n}{4}) + n^2$

  $n^{\log_4 2}$ vs n^2 — n^2 is larger

  $T(n) = \Theta(n^2)$ case 3