



DeepLearning.AI

# Math for Machine Learning

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## Linear algebra - Week 3

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations



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# Vectors and Linear Transformations

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## **Machine Learning motivation**

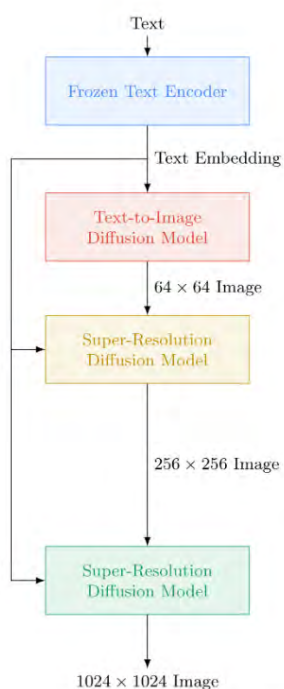
# Neural Networks - AI generated images



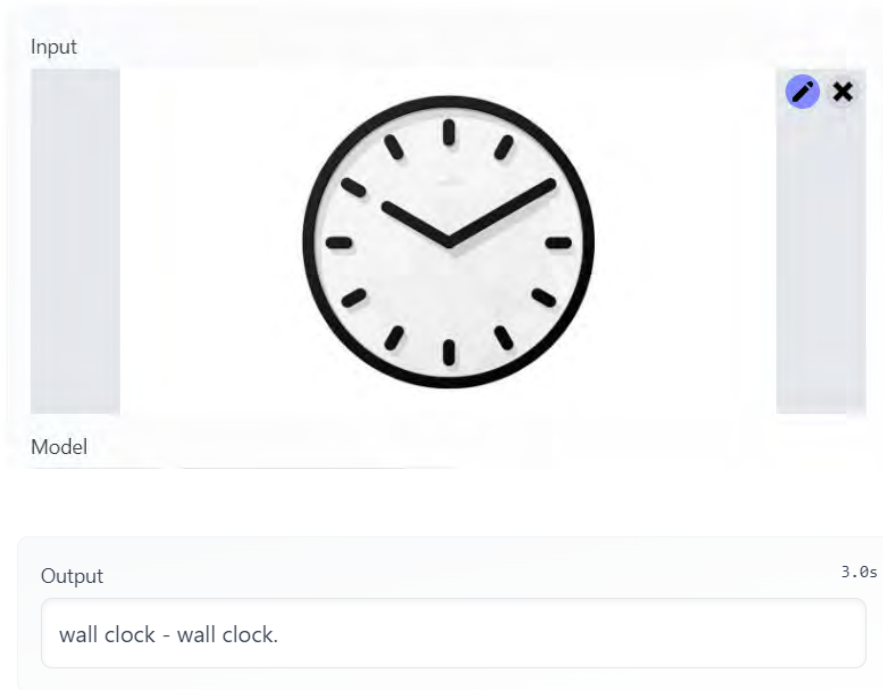
AI-generated human faces.

- Generative learning: Generating realistic looking images.

# Text-to-image and image-to-text generation



"A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck."





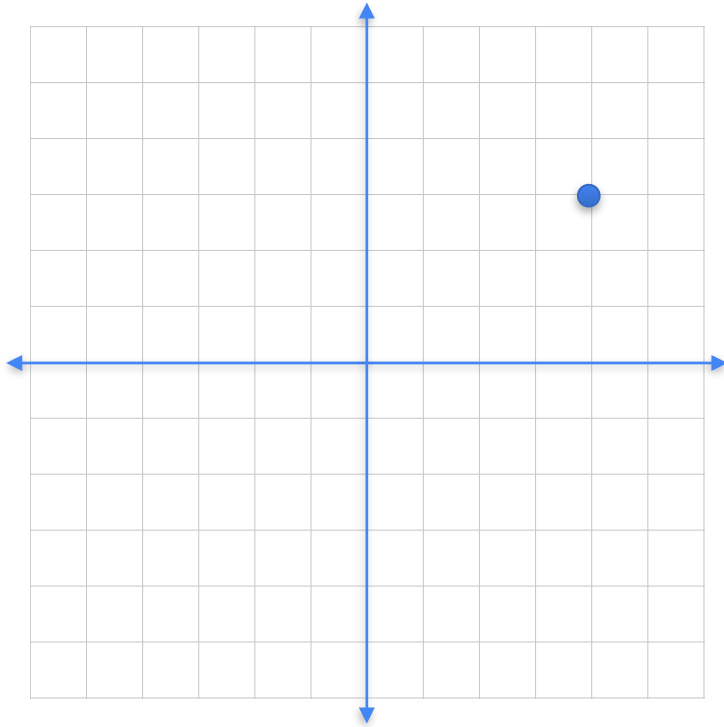
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# Vectors and Linear Transformations

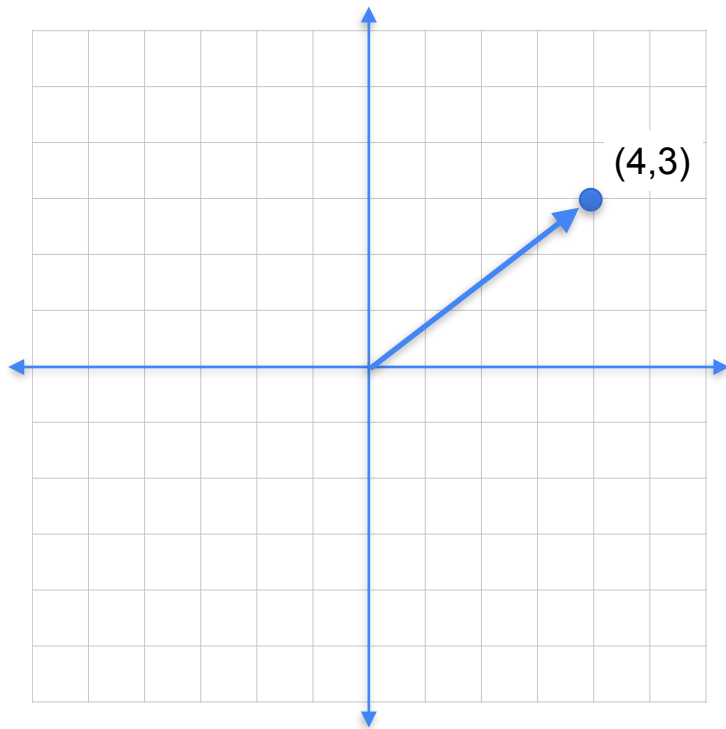
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## **Vectors and their properties**

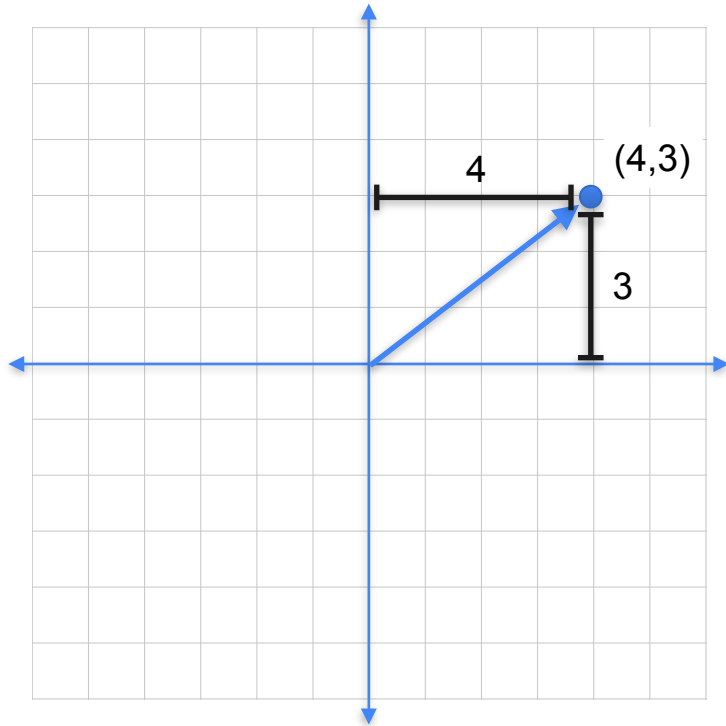
# Vectors



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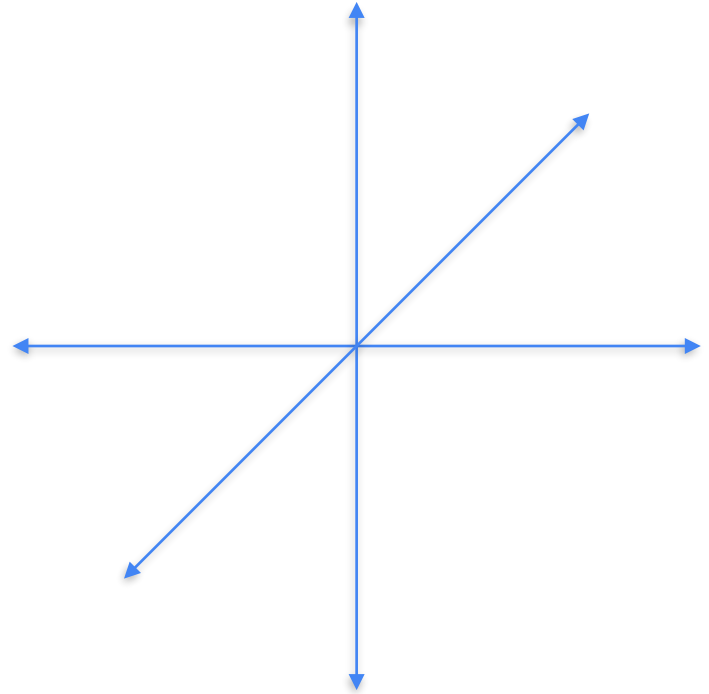
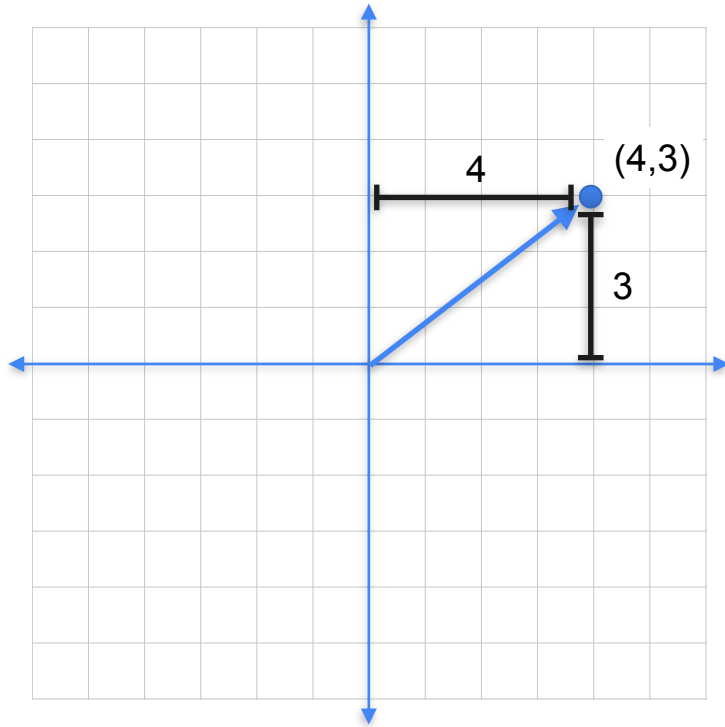


# Vectors

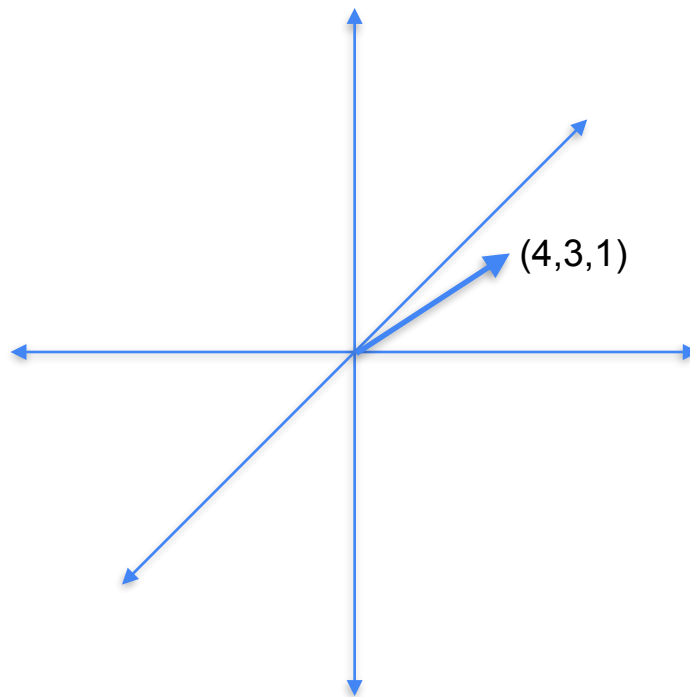
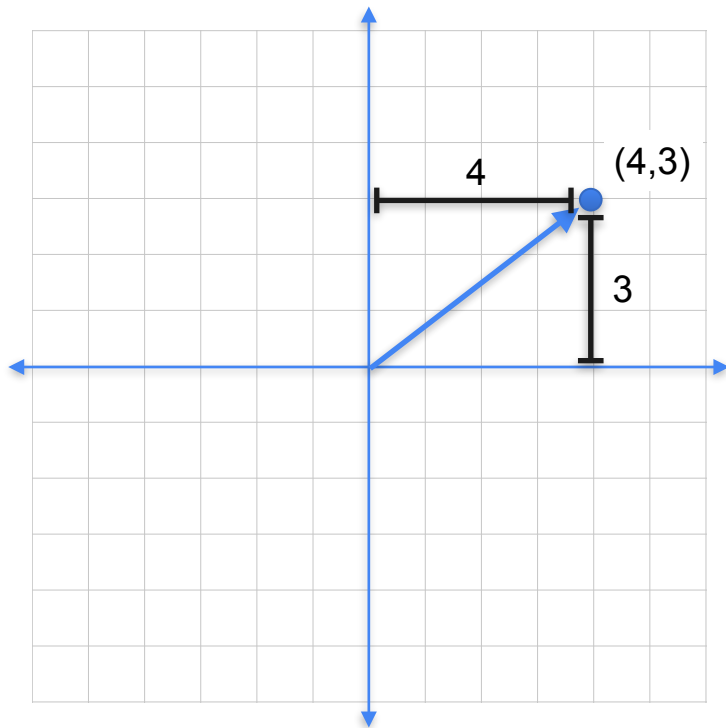




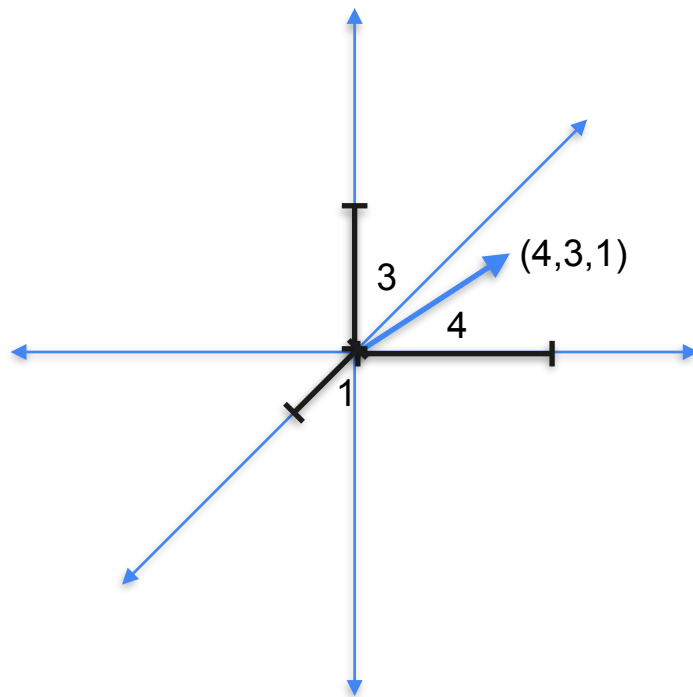
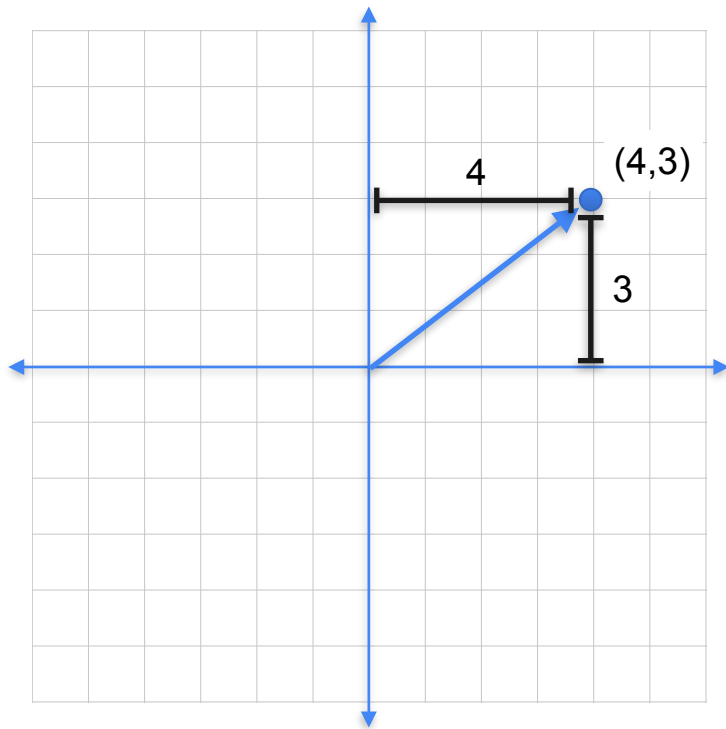
# Vectors



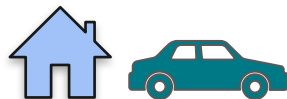
# Vectors



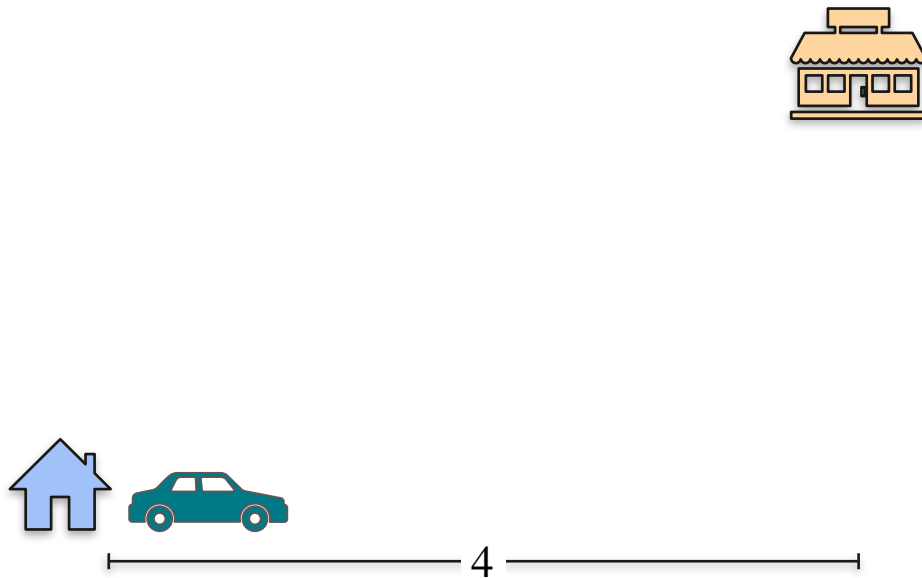
# Vectors



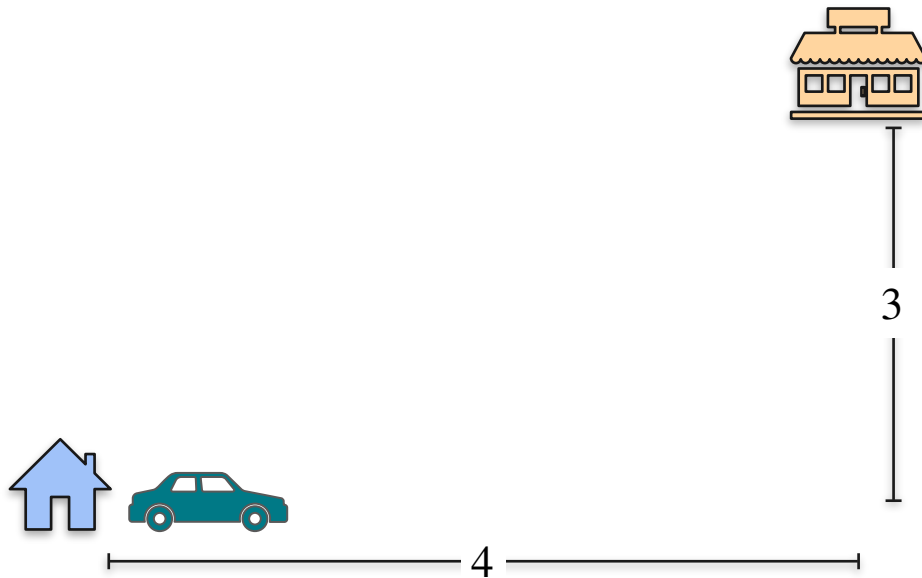
# How to get from point A to point B?



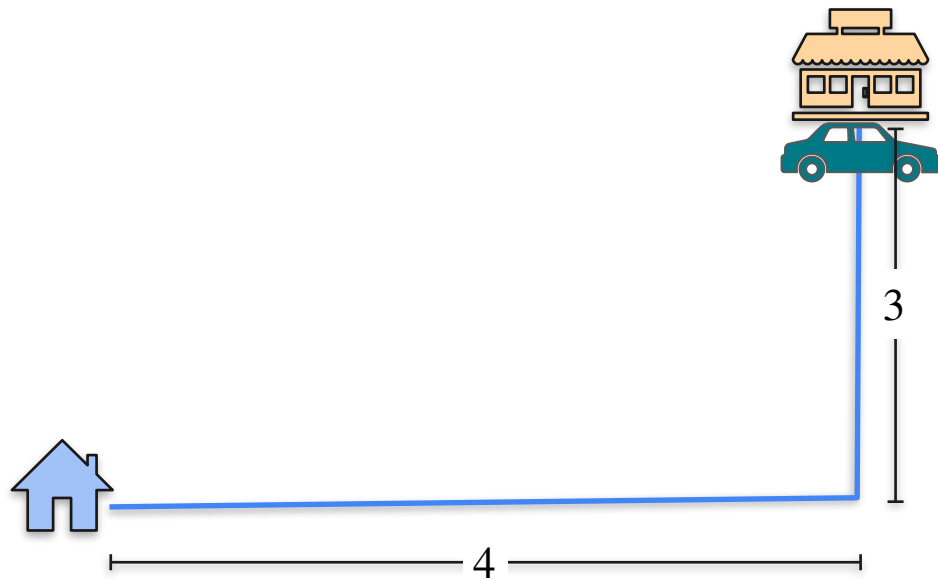
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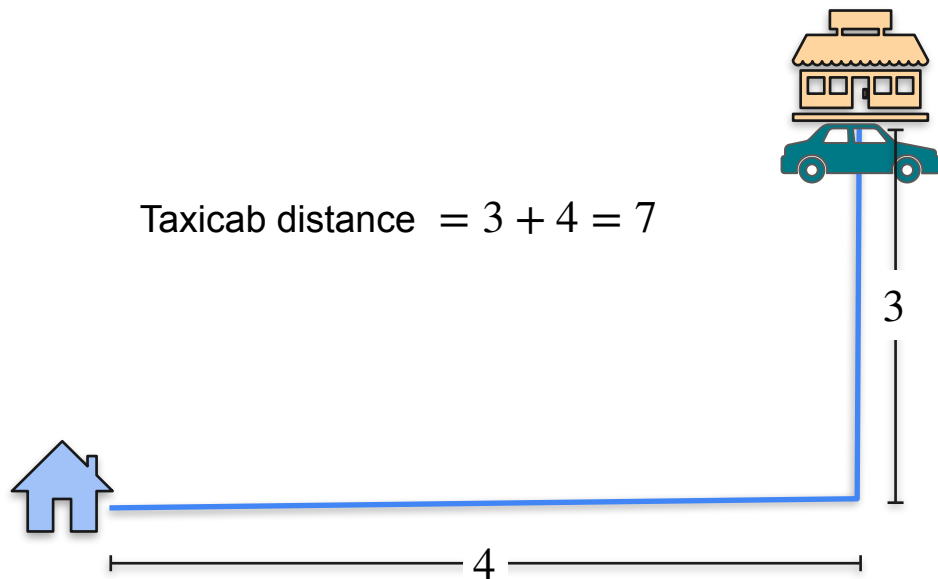
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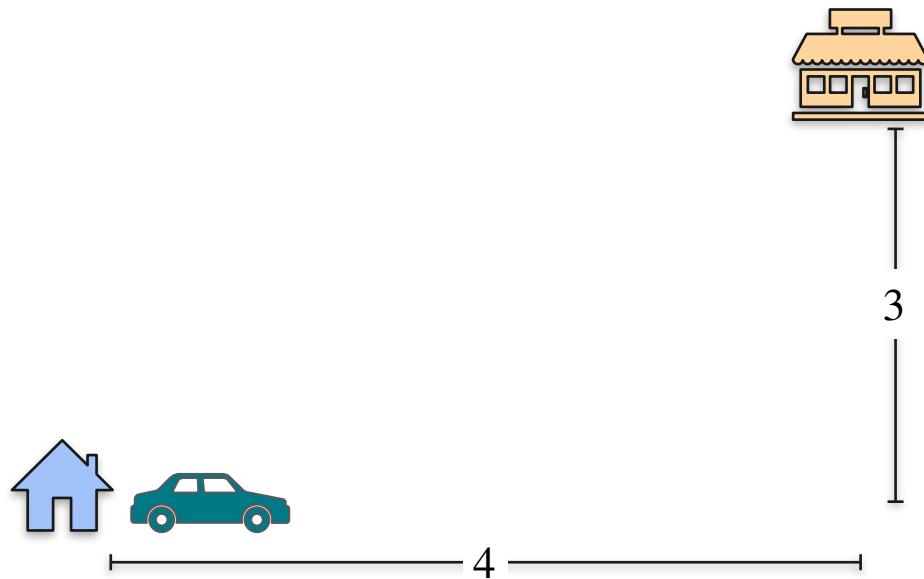


# How to get from point A to point B?

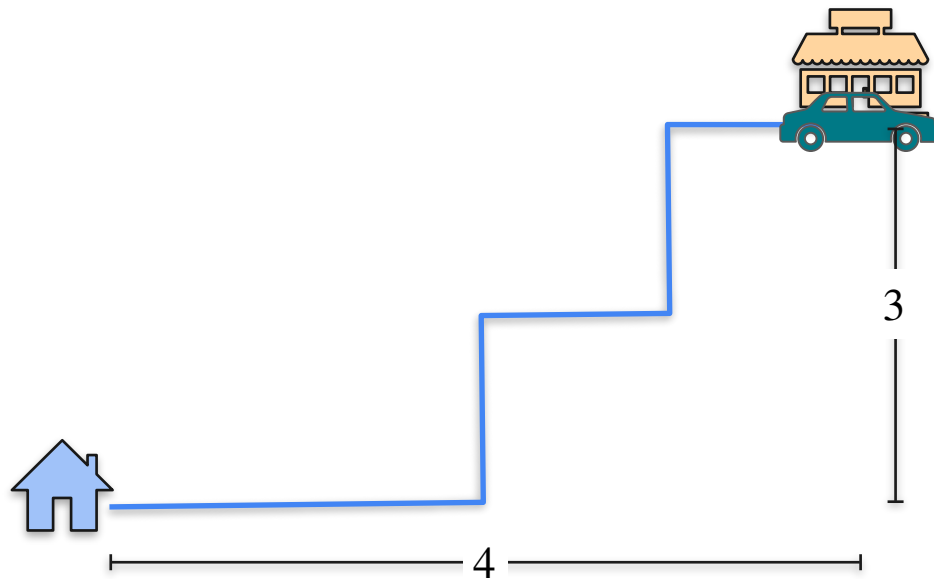




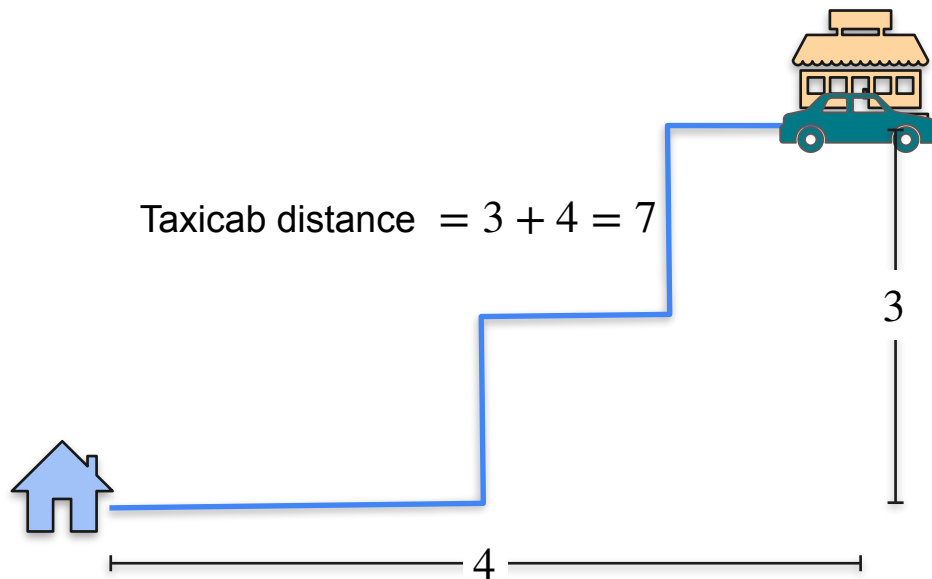
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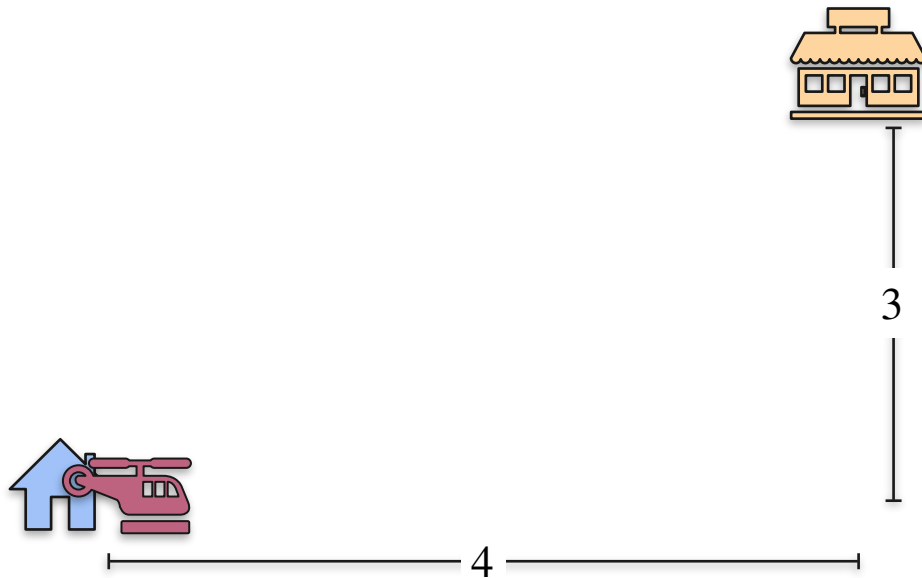
# How to get from point A to point B?



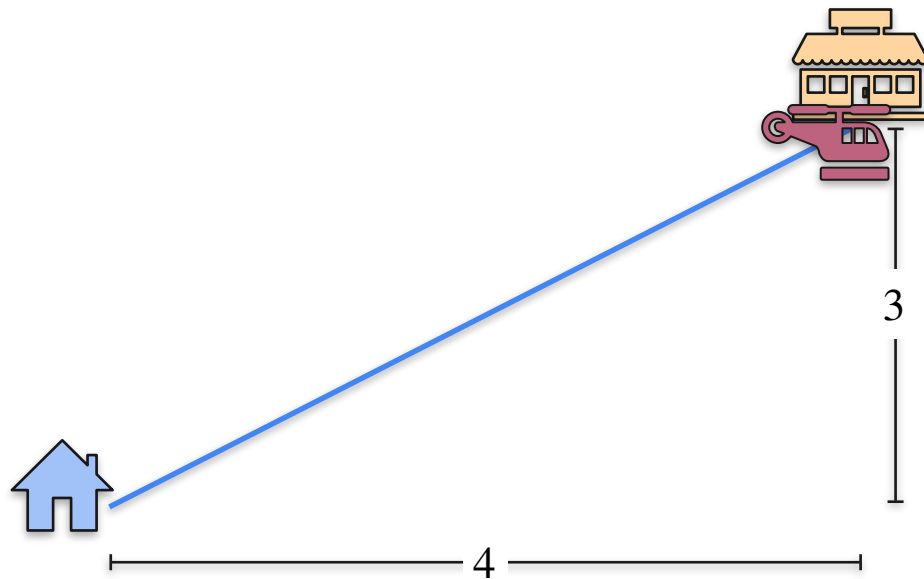
# How to get from point A to point B?



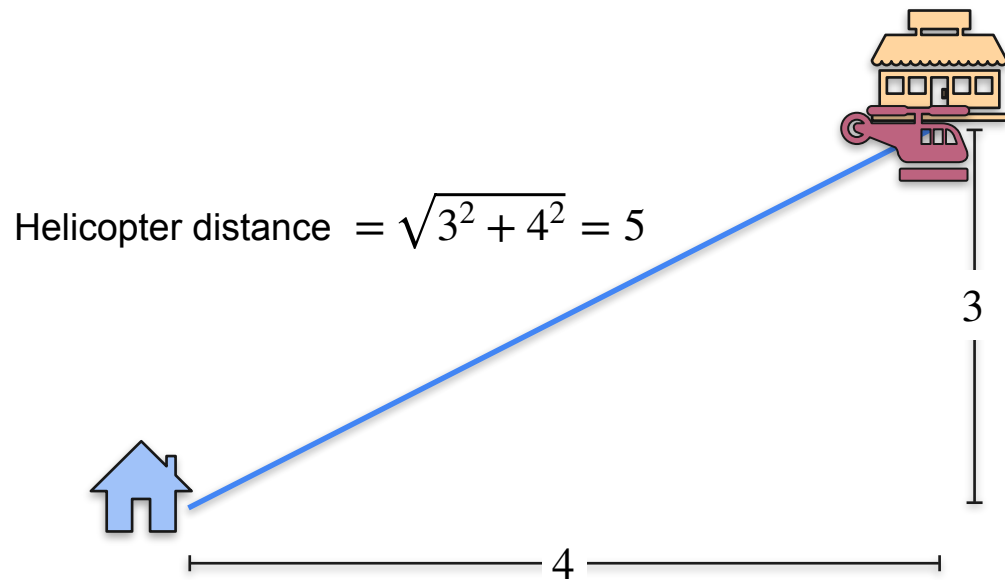
# How to get from point A to point B?



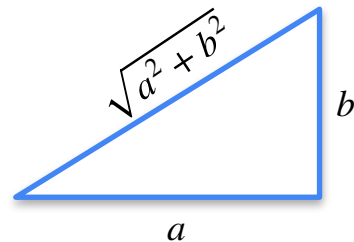
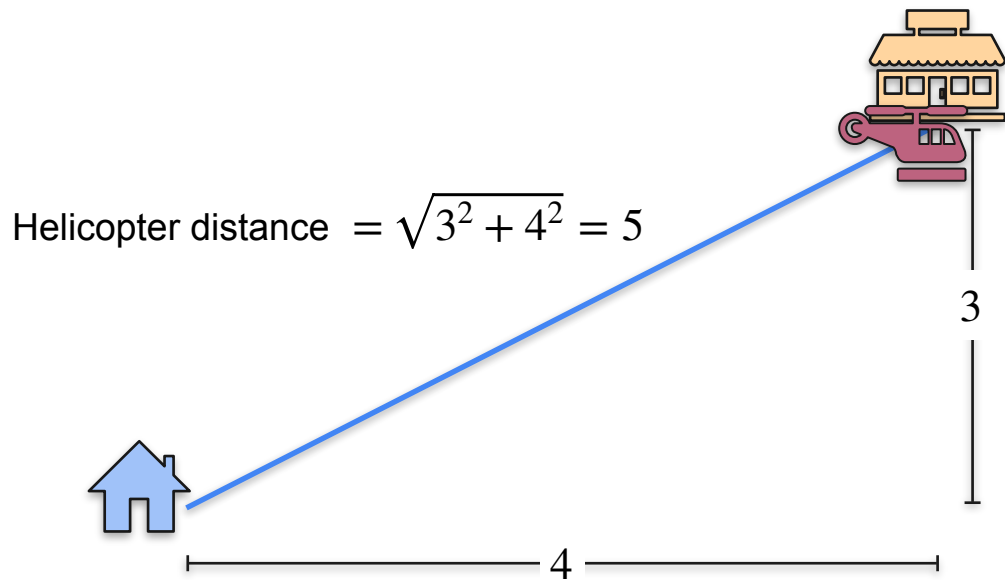
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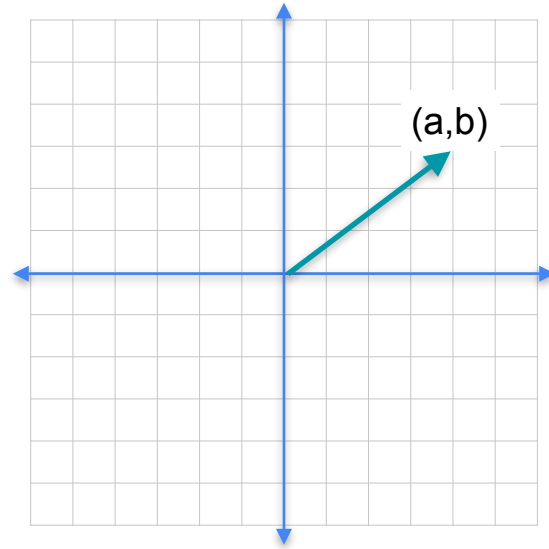


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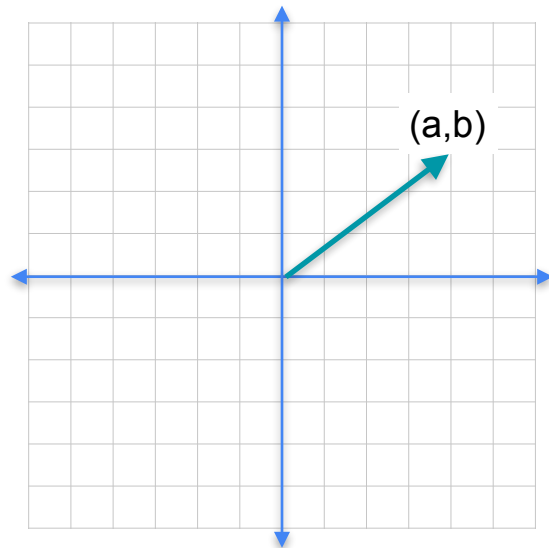
Pythagorean Theorem

# Norms



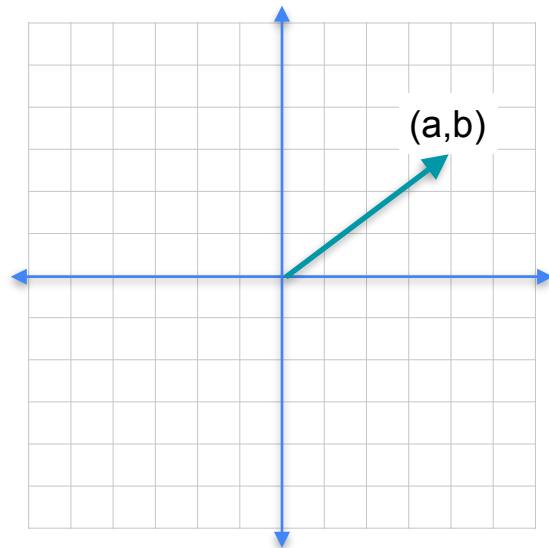


# Norms



$$\text{L1-norm} = |(a,b)|_1 = |a| + |b|$$

# Norms

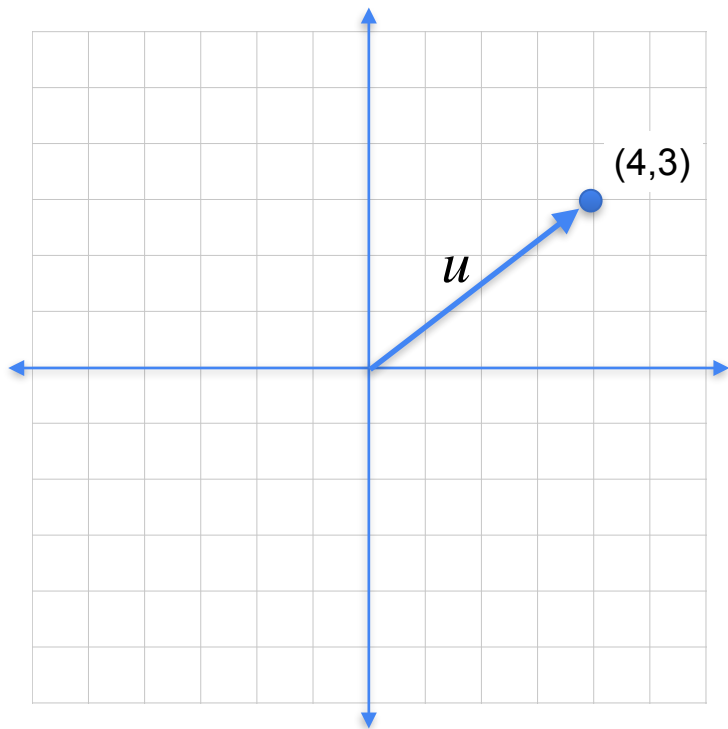


$$\text{L1-norm} = |(a,b)|_1 = |a| + |b|$$

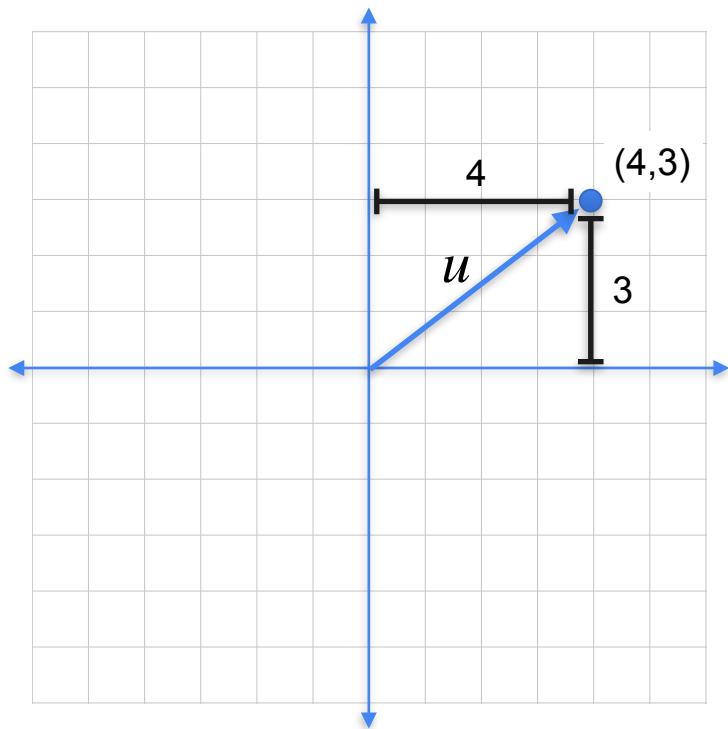


$$\text{L2-norm} = |(a,b)|_2 = \sqrt{a^2 + b^2}$$

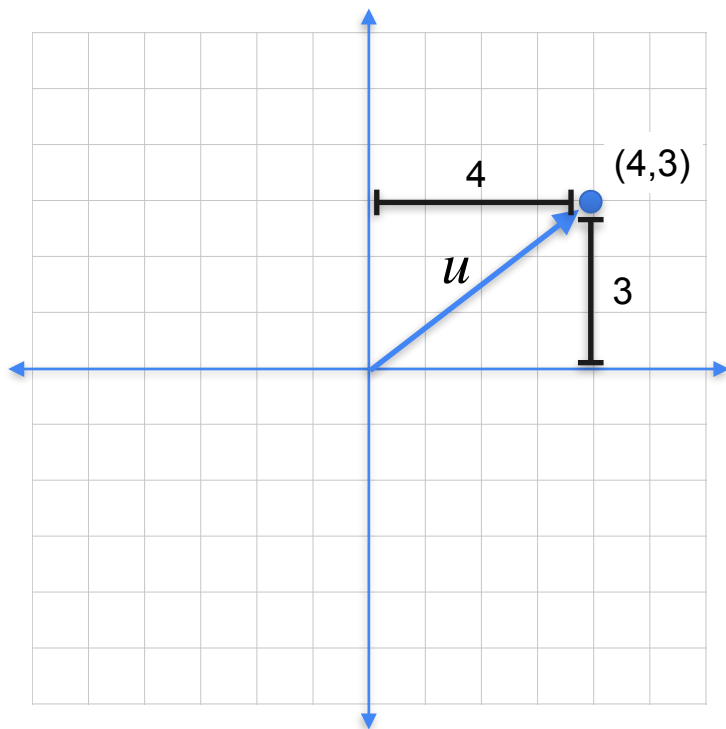
# Norm of a vector



# Norm of a vector

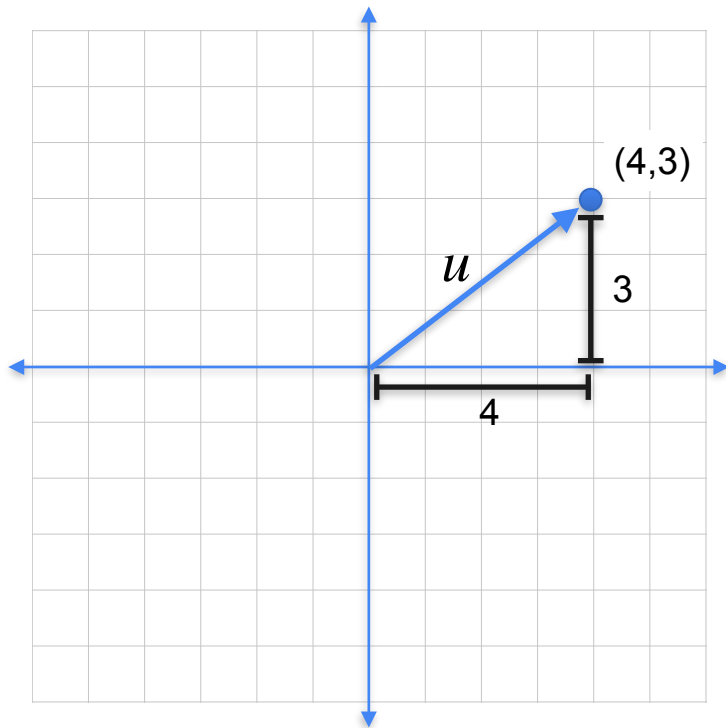


# Norm of a vector

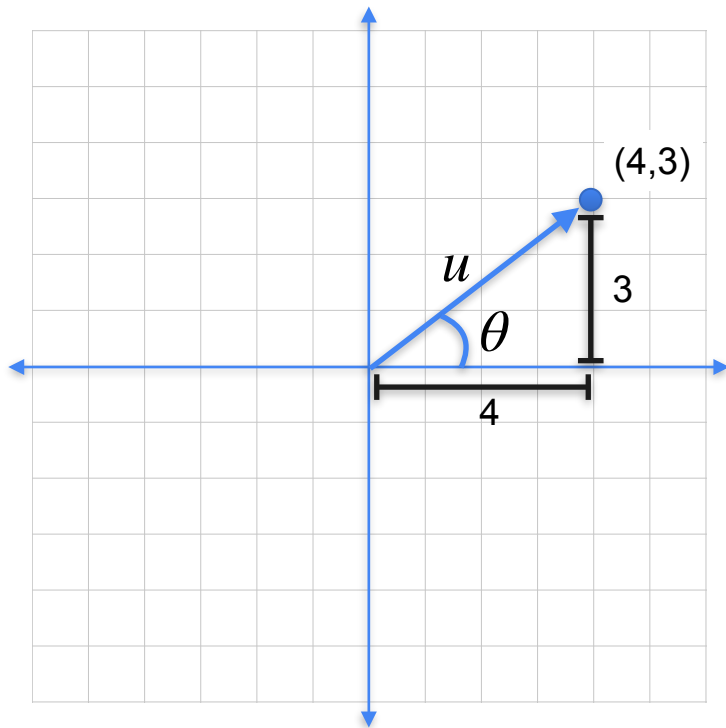


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

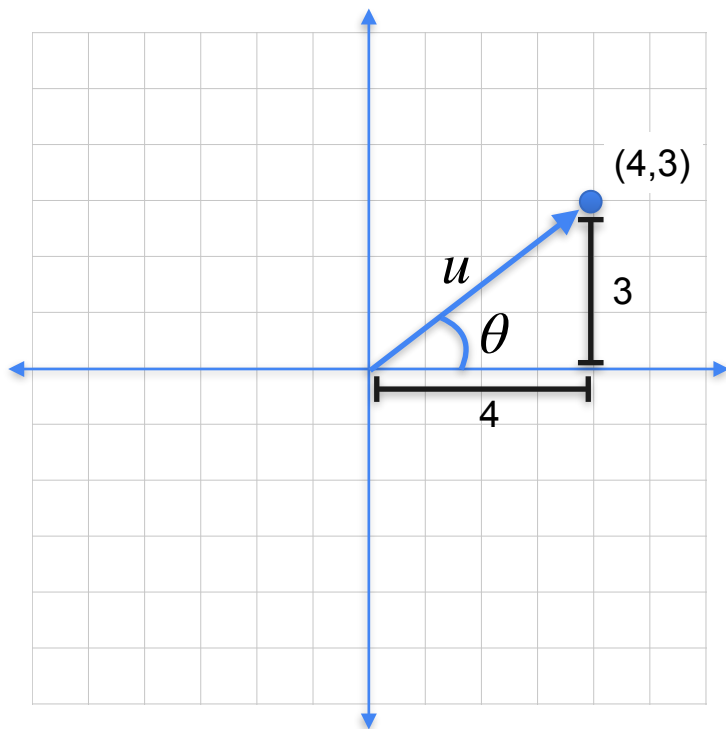
# Direction of a vector



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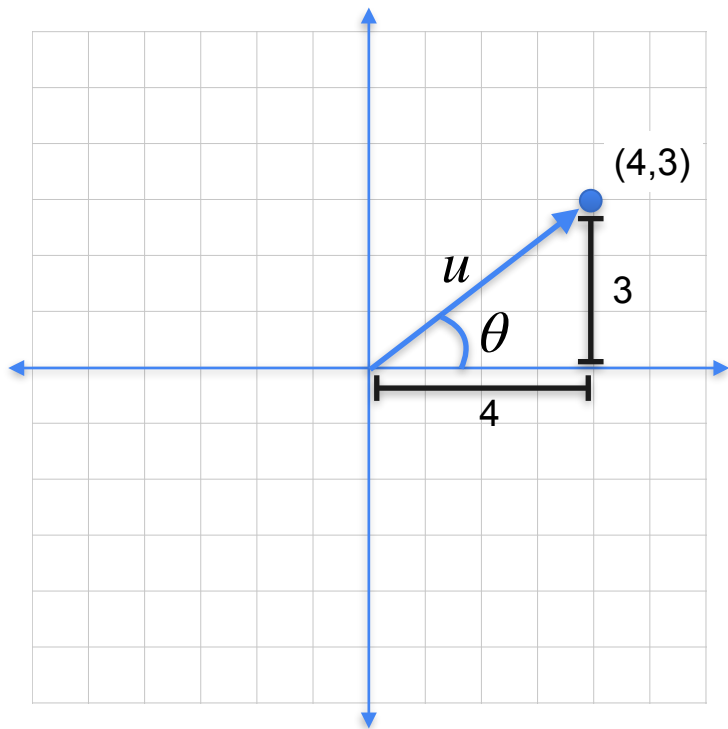
# Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$



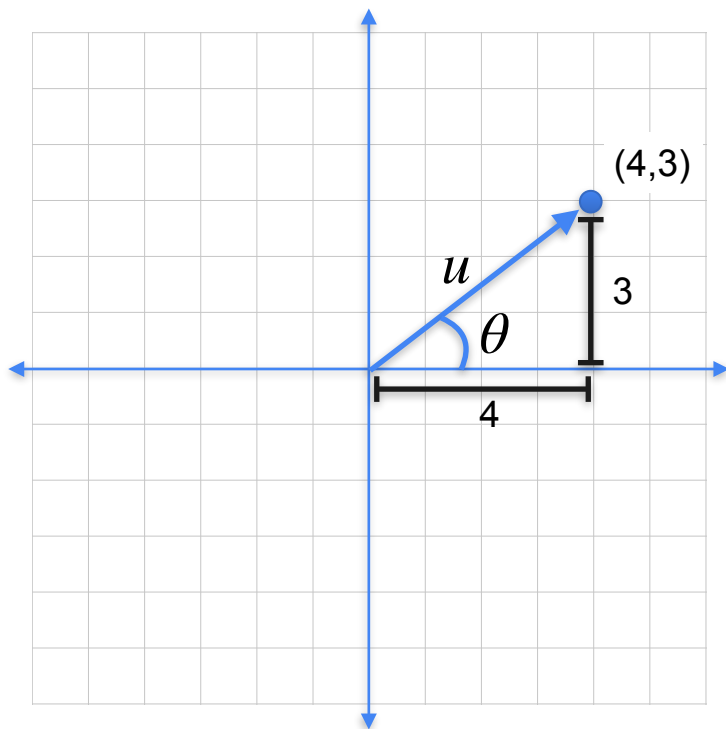
# Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64$$

# Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$



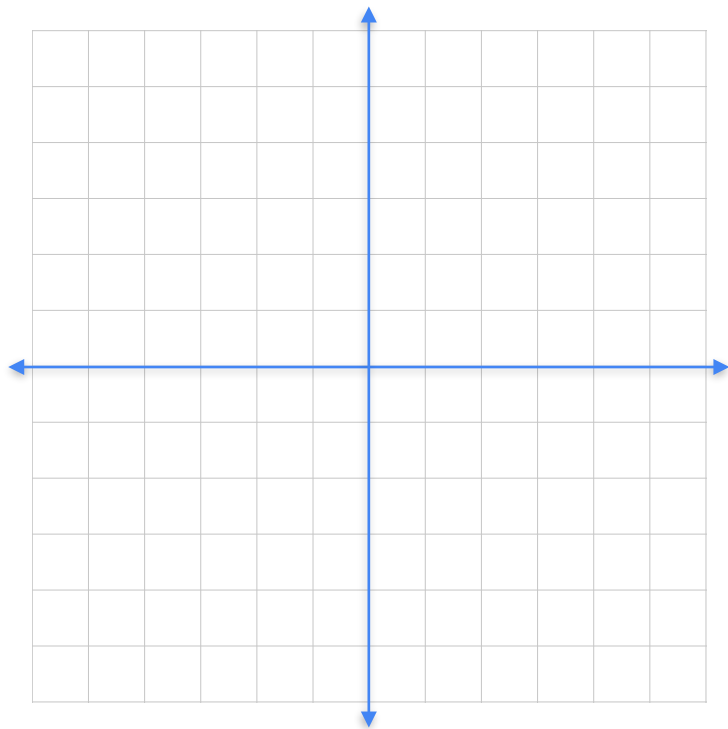
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# Vectors and Linear Transformations

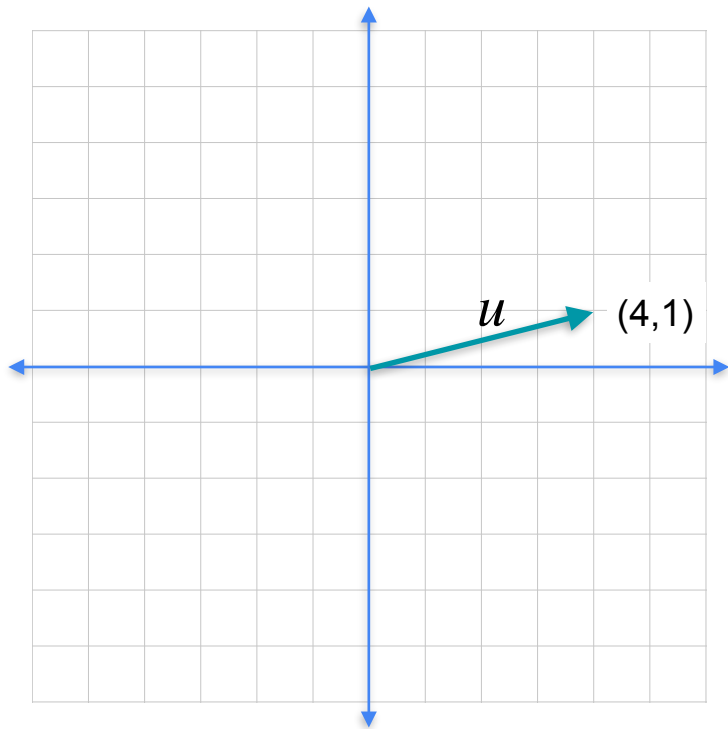
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## **Sum and difference of vectors**

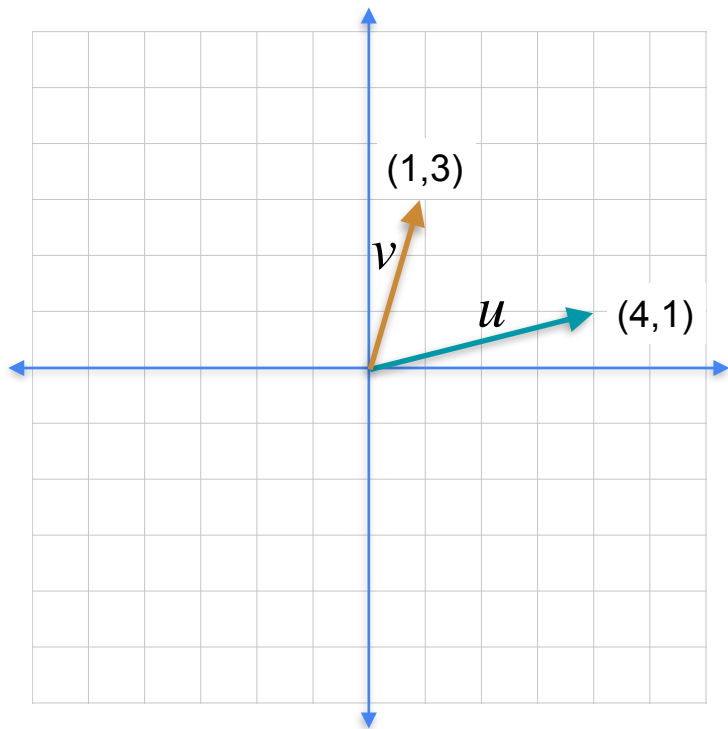
# Sum of vectors



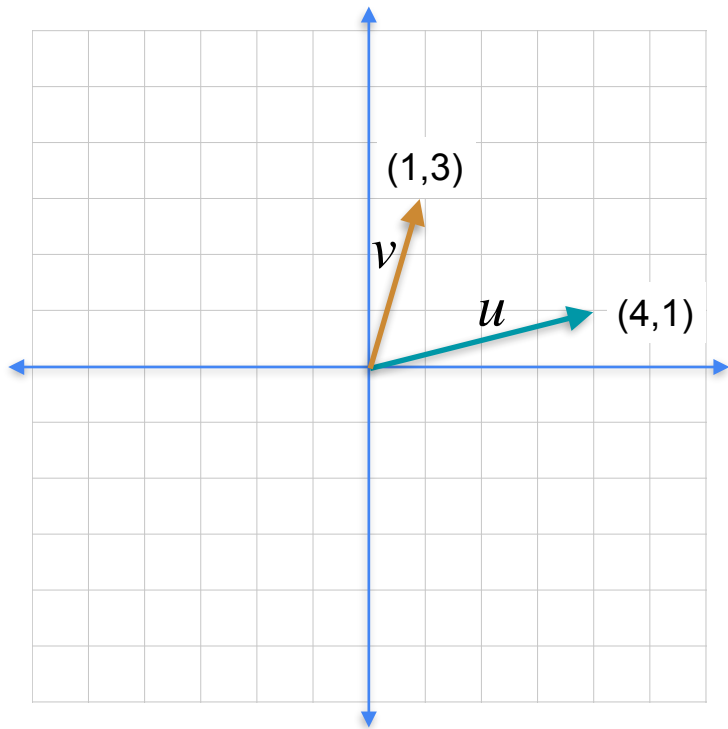
# Sum of vectors



# Sum of vectors

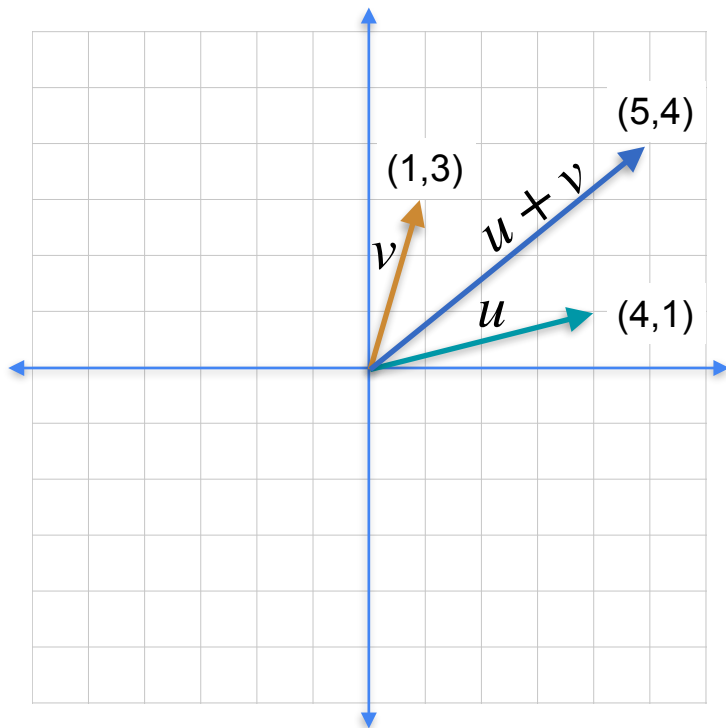


# Sum of vectors



$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

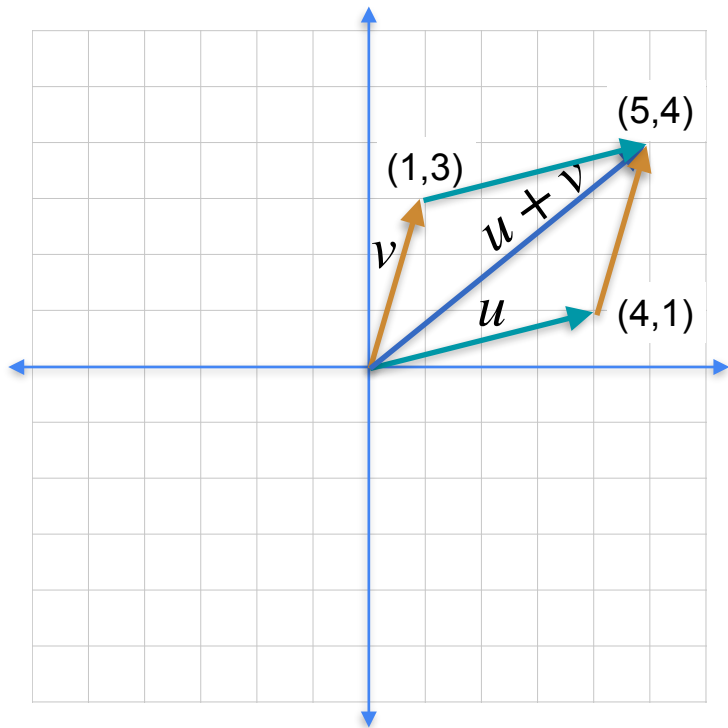
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$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

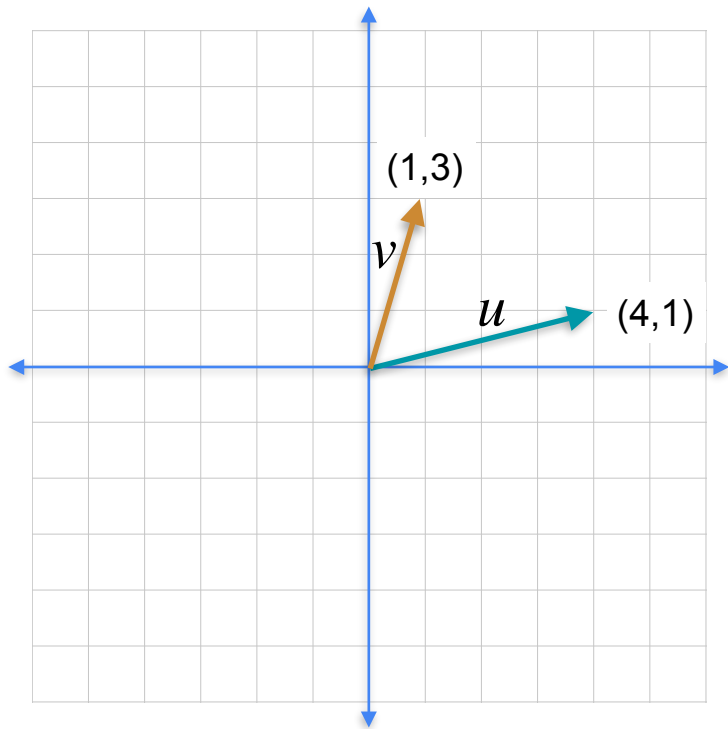


# Sum of vectors

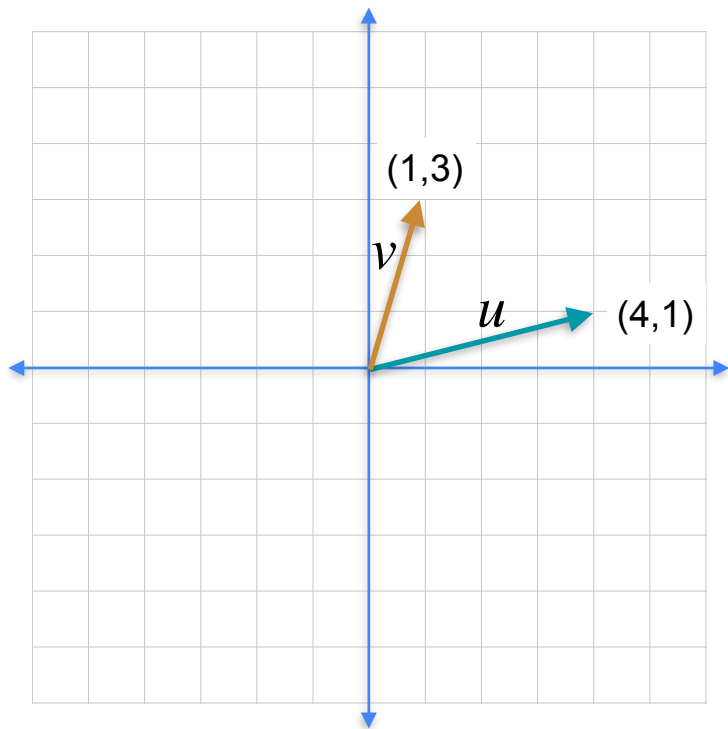


$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

# Difference of vectors

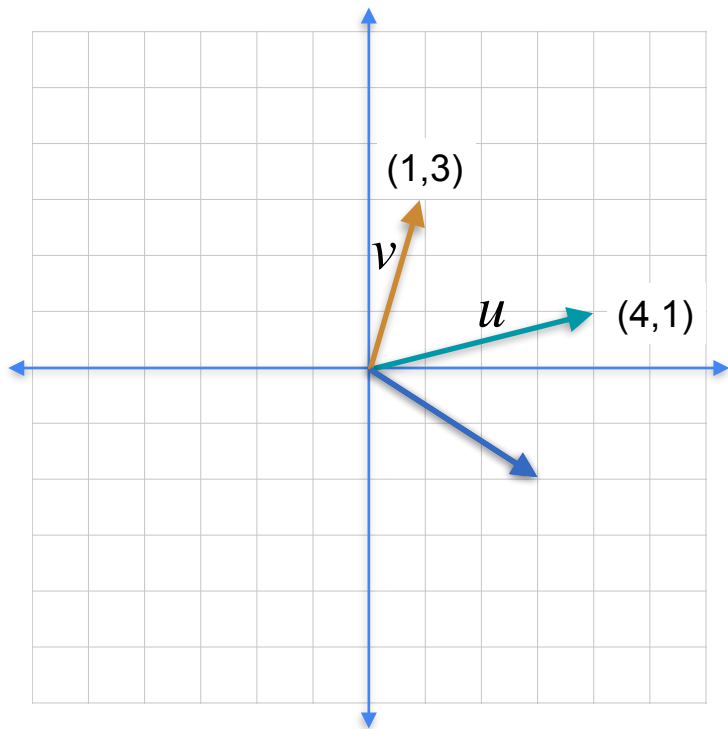


# Difference of vectors



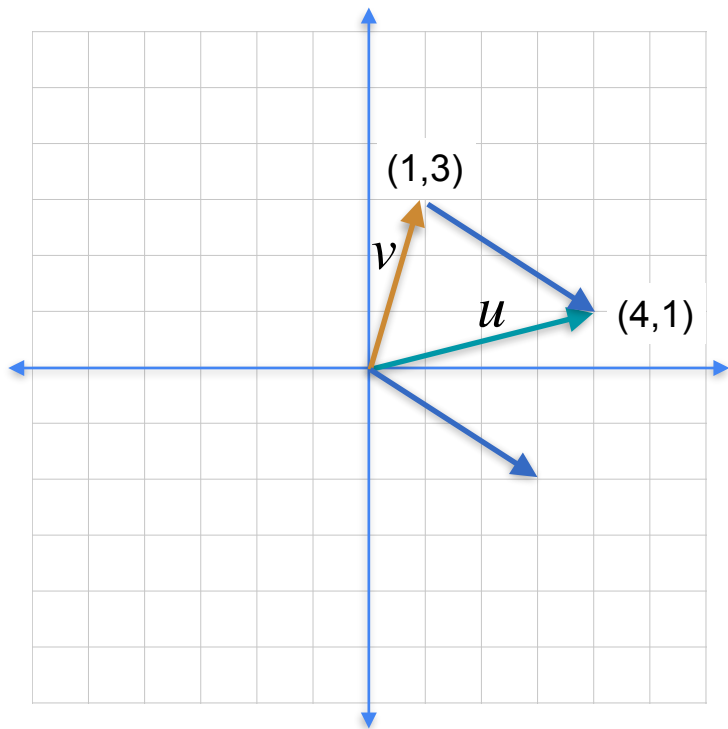
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

# Difference of vectors



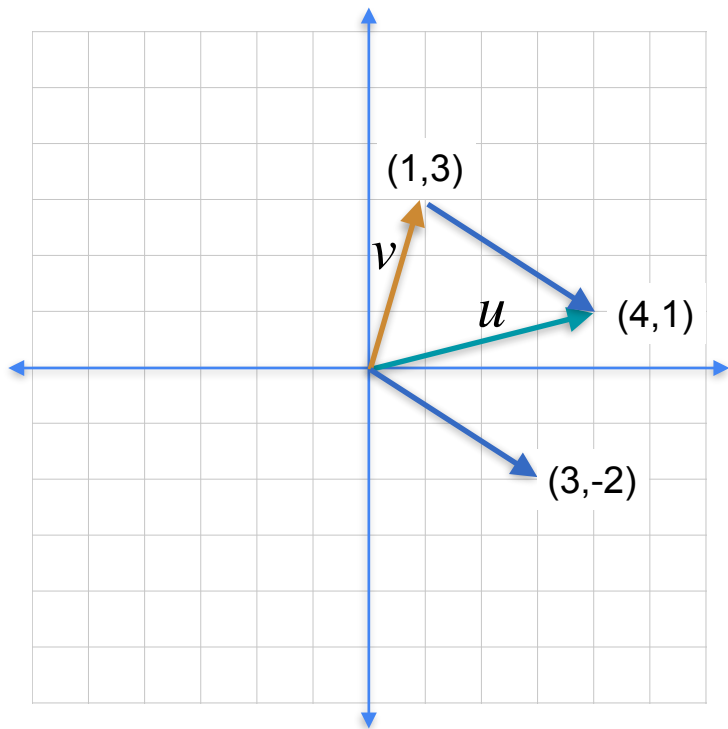
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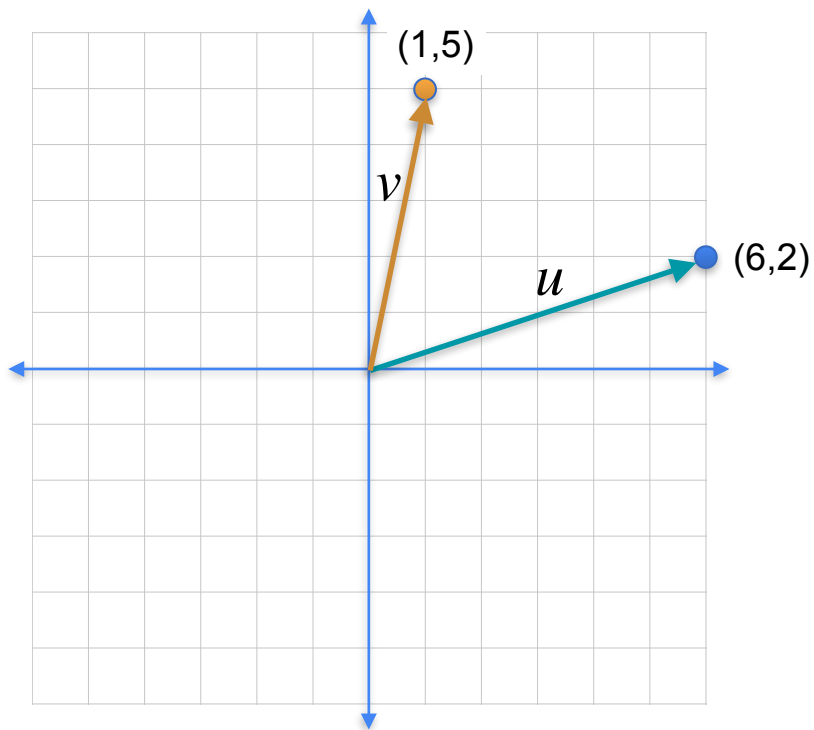
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# Vectors and Linear Transformations

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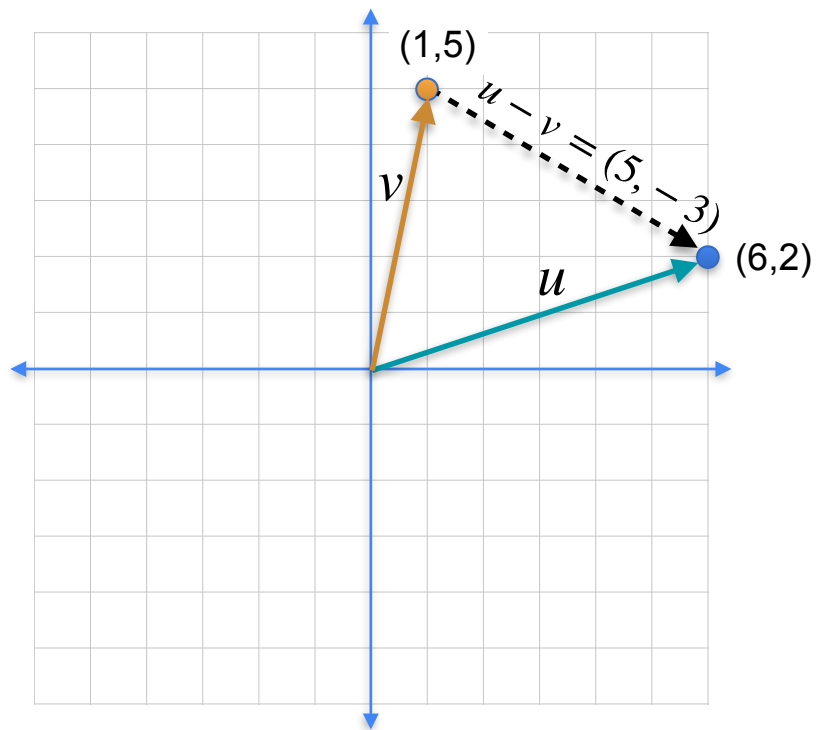
## **Distance between vectors**

# Distances

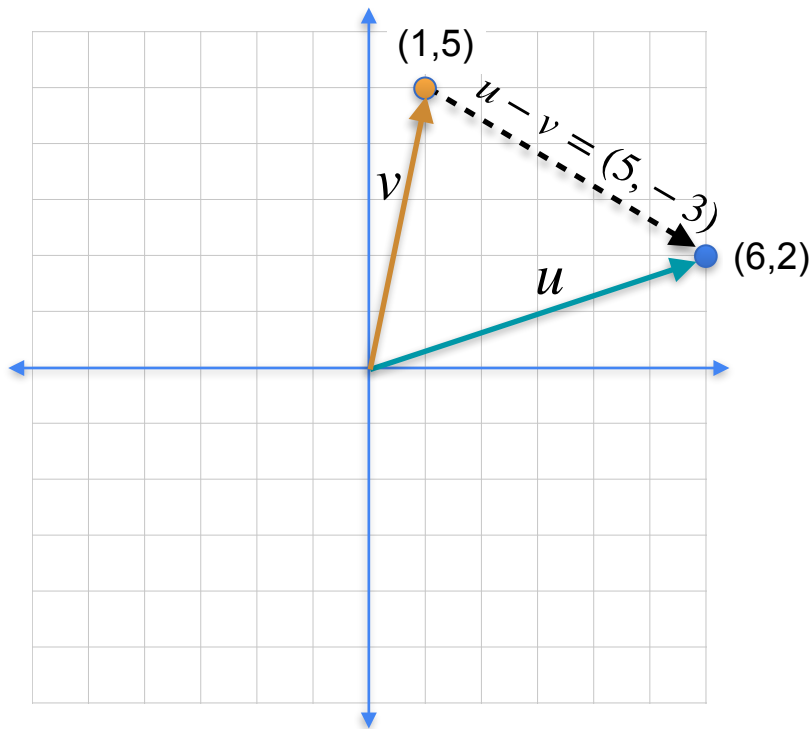




# Distances



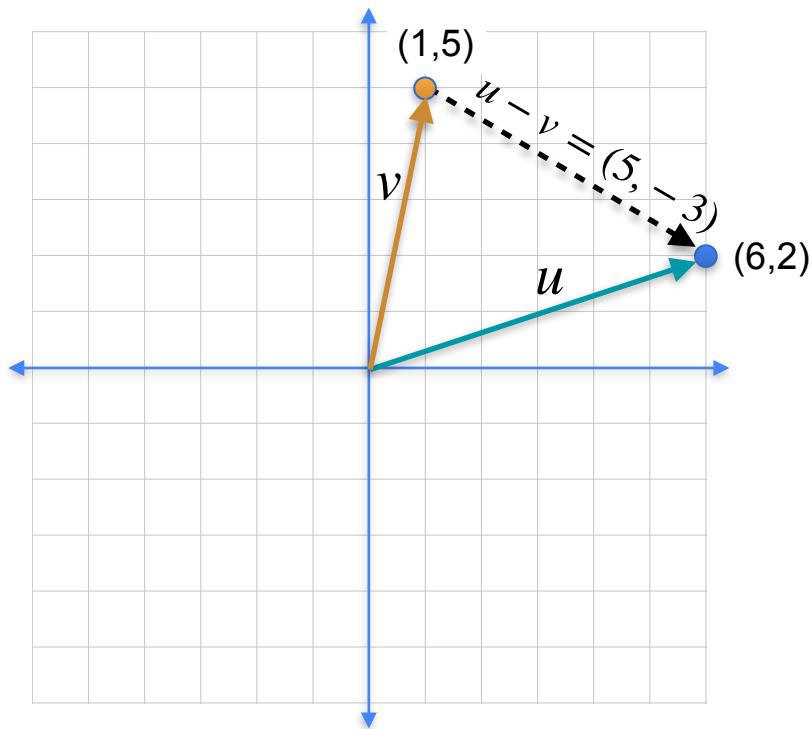
# Distances



L1-distance

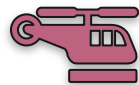
$$|u - v|_1 = |5| + |-3| = 8$$

# Distances



L1-distance

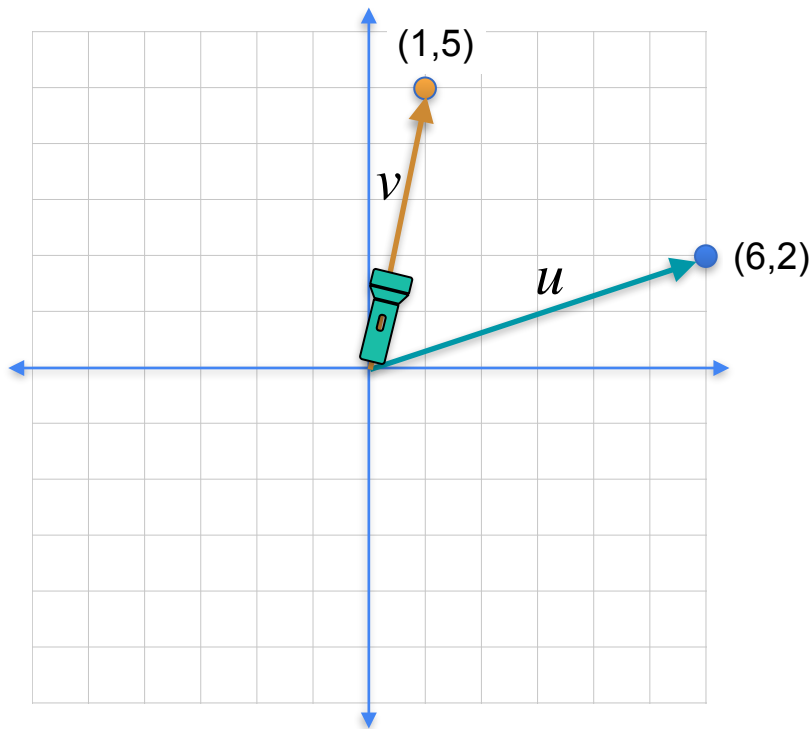
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

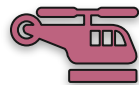
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

# Distances



L1-distance

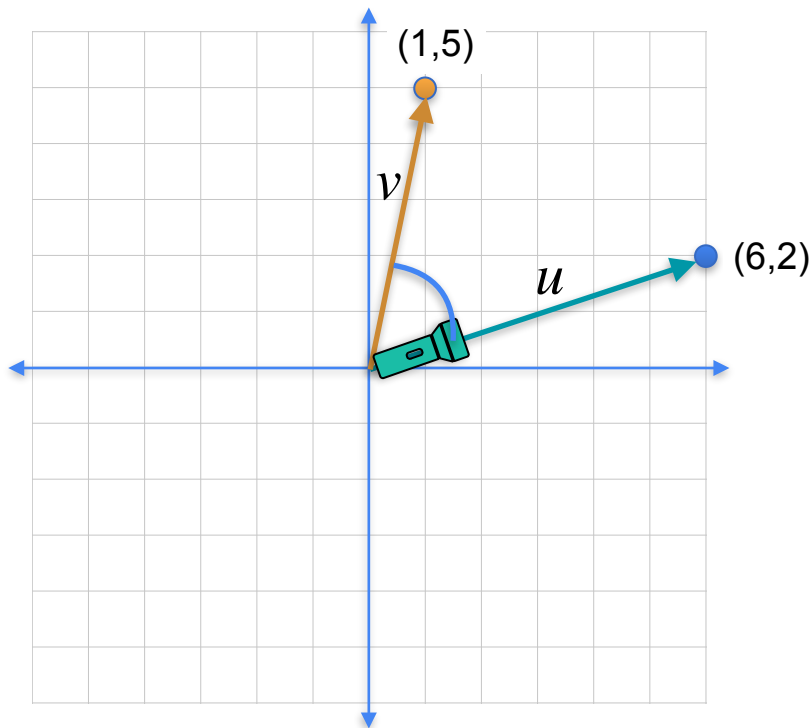
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L2-distance

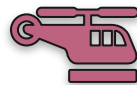
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# Distances



L1-distance

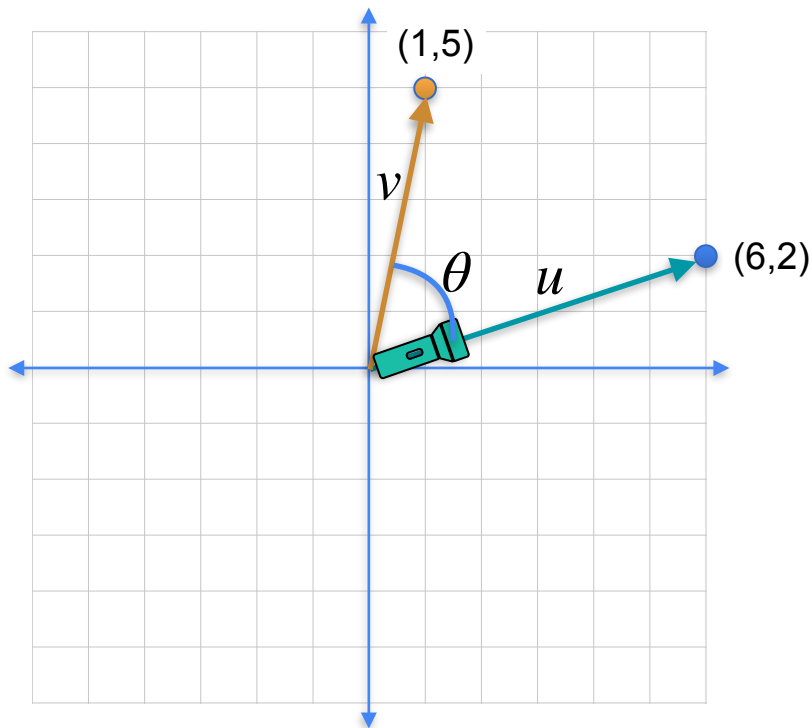
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L2-distance

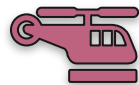
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# Distances



L1-distance

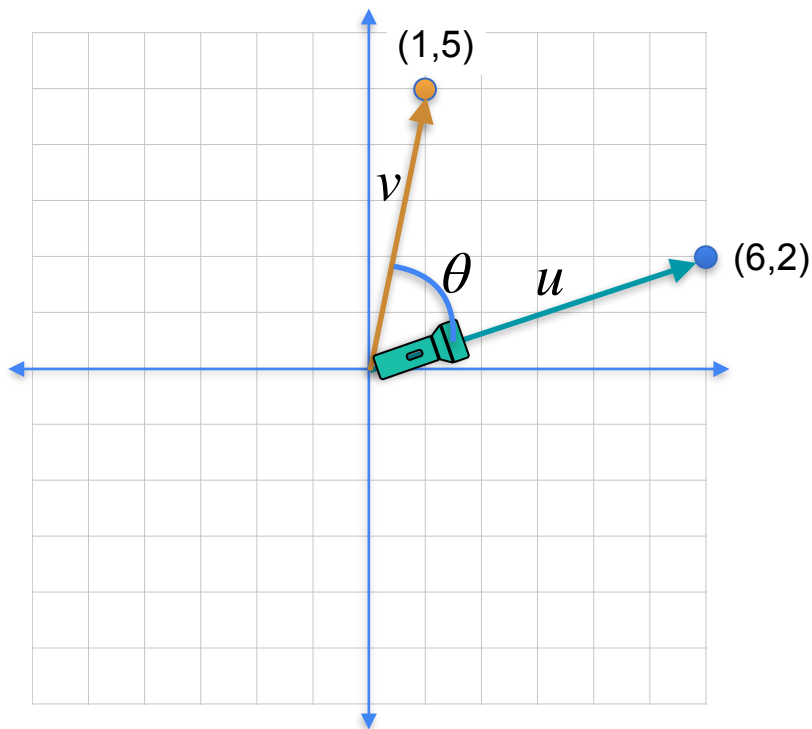
$$|u - v|_1 = |5| + |-3| = 8$$





L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

# Distances



  $|u - v|_1 = |5| + |-3| = 8$   
L1-distance

  $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$   
L2-distance

  $\cos(\theta)$   
Cosine distance



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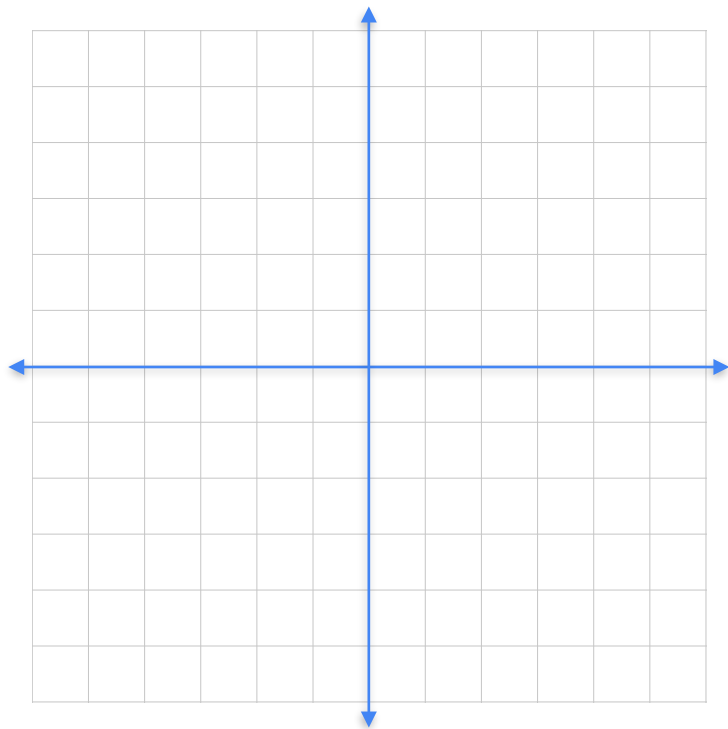
# Vectors and Linear Transformations

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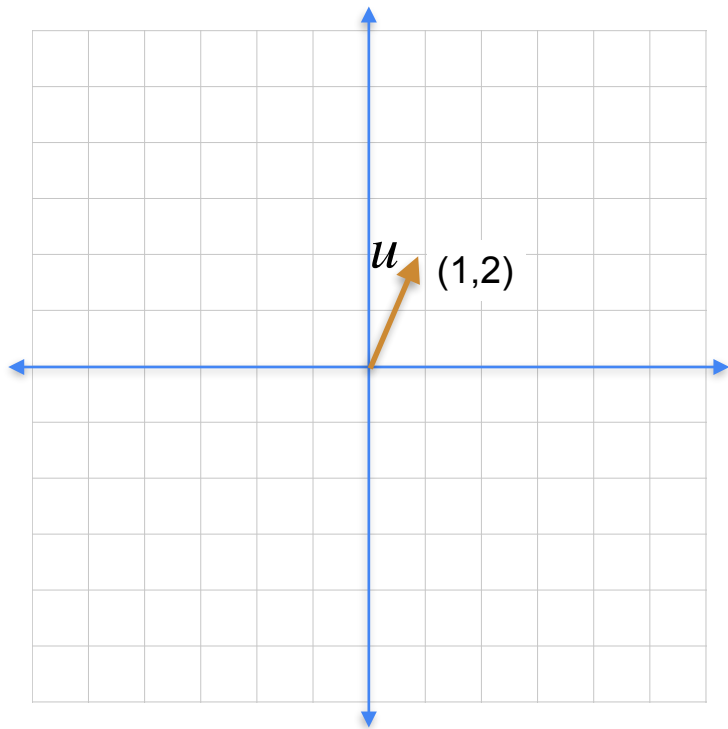
**Multiplying a vector by a scalar**



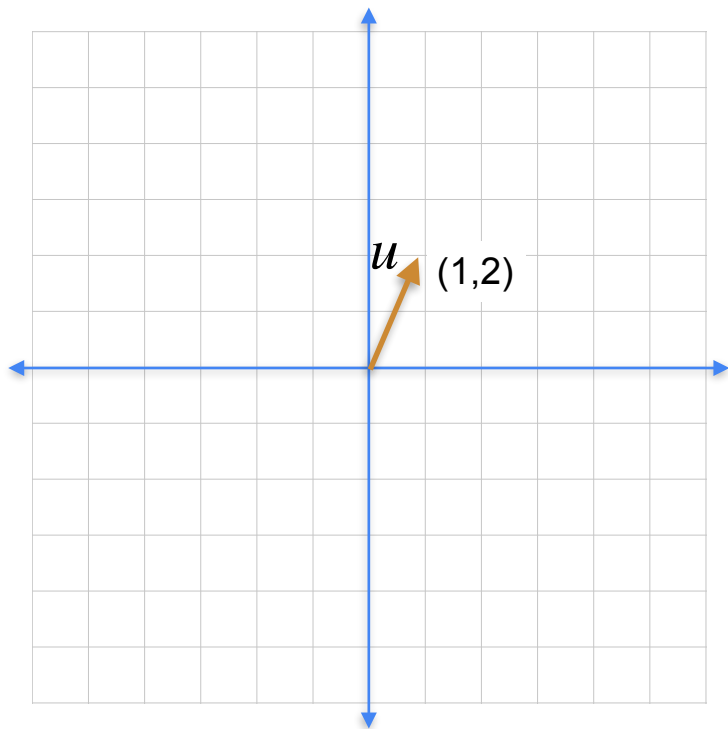
# Multiplying a vector by a scalar



# Multiplying a vector by a scalar

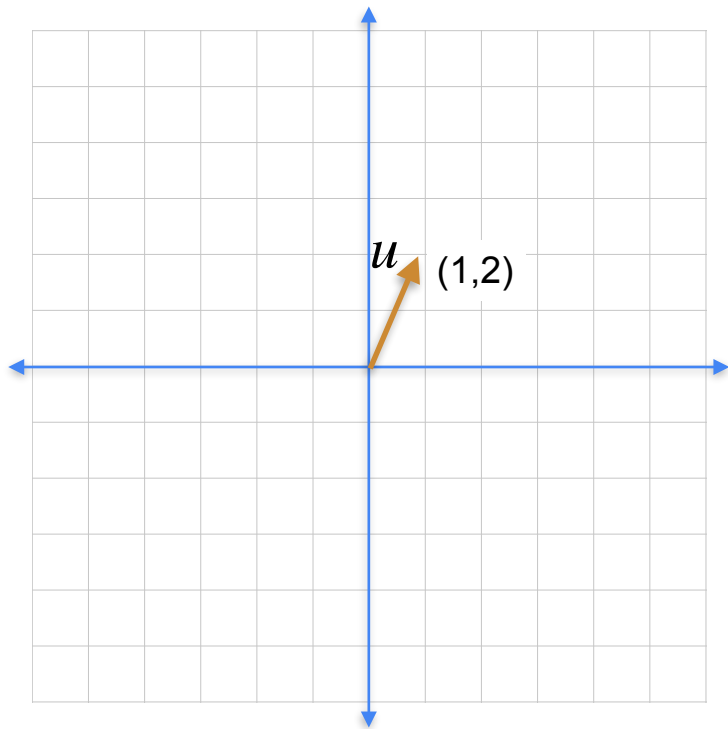


# Multiplying a vector by a scalar



$$u = (1,2)$$

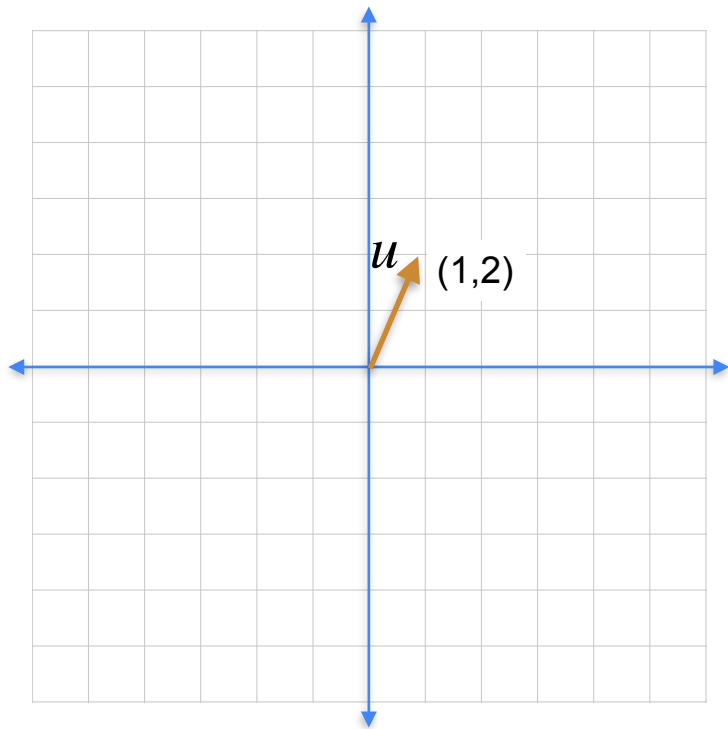
# Multiplying a vector by a scalar



$$u = (1,2)$$

$$\lambda = 3$$

# Multiplying a vector by a scalar

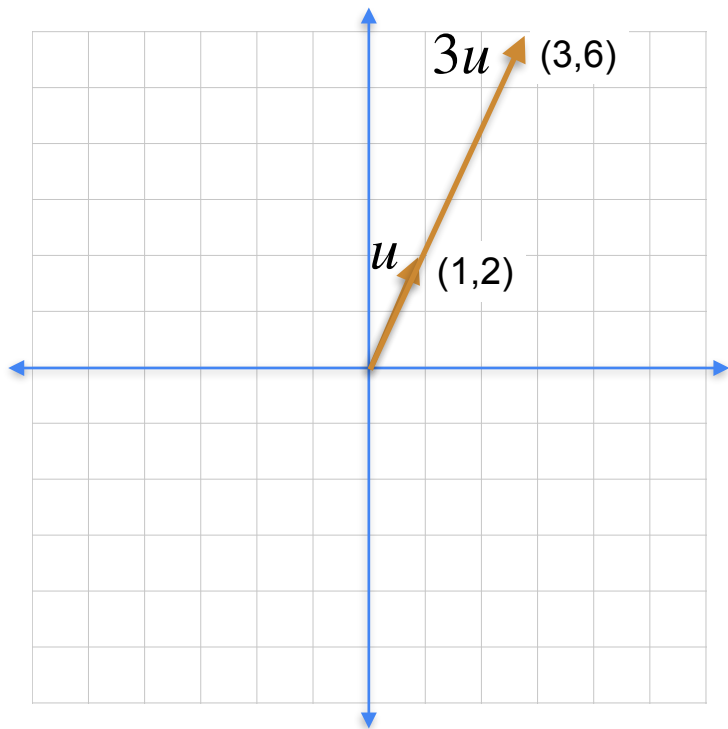


$$u = (1,2)$$

$$\lambda = 3$$

$$\lambda u = (3,6)$$

# Multiplying a vector by a scalar

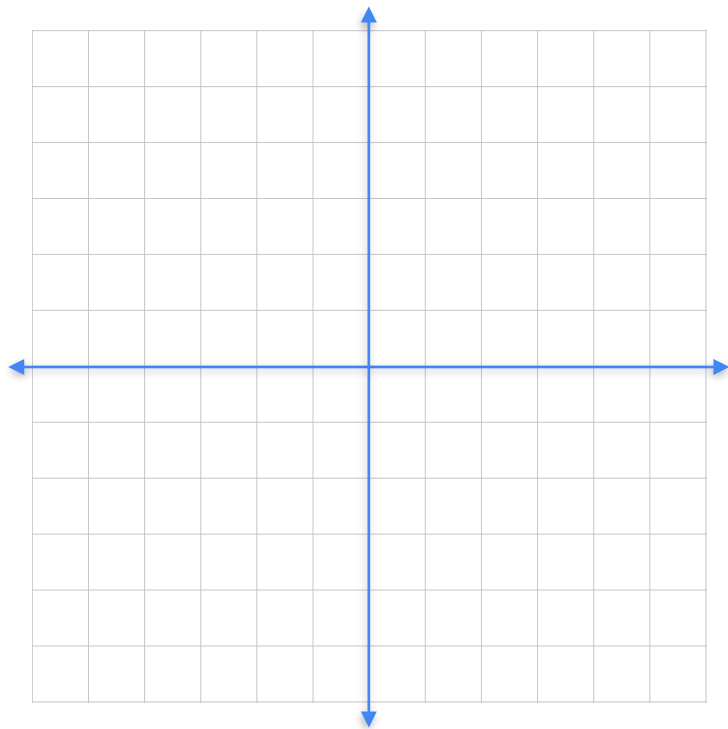


$$u = (1,2)$$

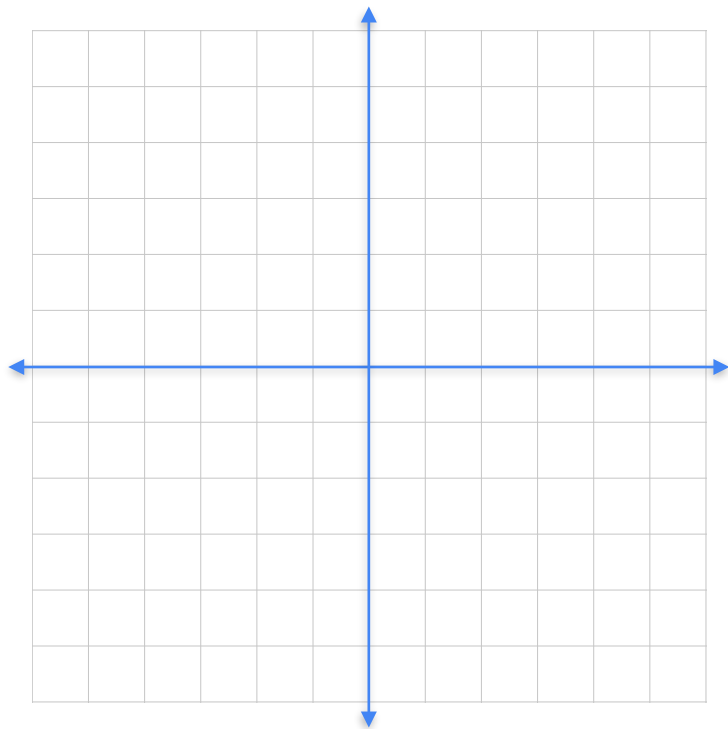
$$\lambda = 3$$

$$\lambda u = (3,6)$$

# If the scalar is negative



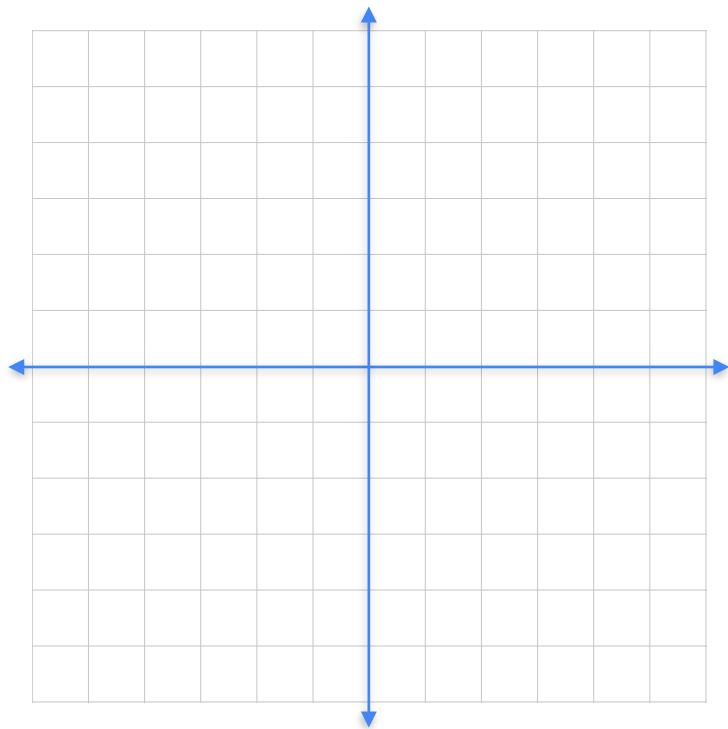
# If the scalar is negative



$$u = (1,2)$$



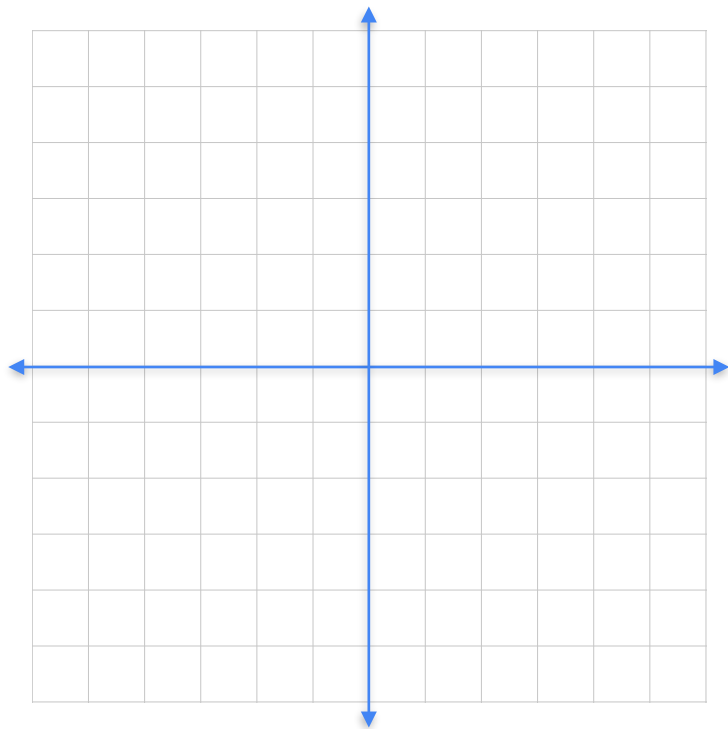
# If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

# If the scalar is negative

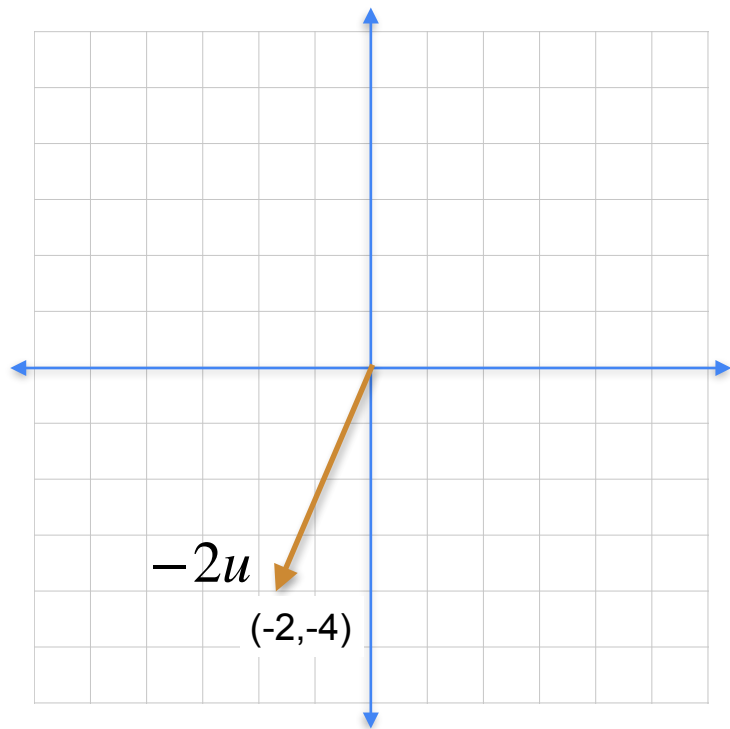


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

# If the scalar is negative

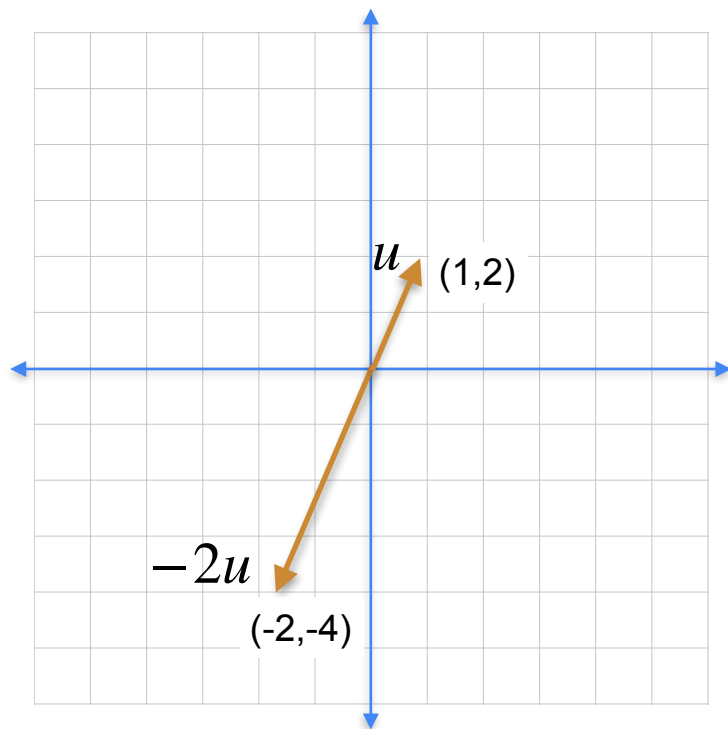


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

# If the scalar is negative



$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$



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# Vectors and Linear Transformations

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## **The dot product**

# A shortcut for linear operations

# A shortcut for linear operations

## **Quantities**

2 apples

4 bananas

1 cherry

# A shortcut for linear operations

## **Quantities**

2 apples  
4 bananas  
1 cherry

## **Prices**

apples: \$3  
bananas: \$5  
cherries: \$2



# A shortcut for linear operations

## **Quantities**

2 apples  
4 bananas  
1 cherry

## **Prices**

apples: \$3  
bananas: \$5  
cherries: \$2

## **Total price**




# A shortcut for linear operations

## Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

## Prices

apples: \$3

bananas: \$5

cherries: \$2

## Total price




# A shortcut for linear operations

## Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

## Prices

apples: \$3

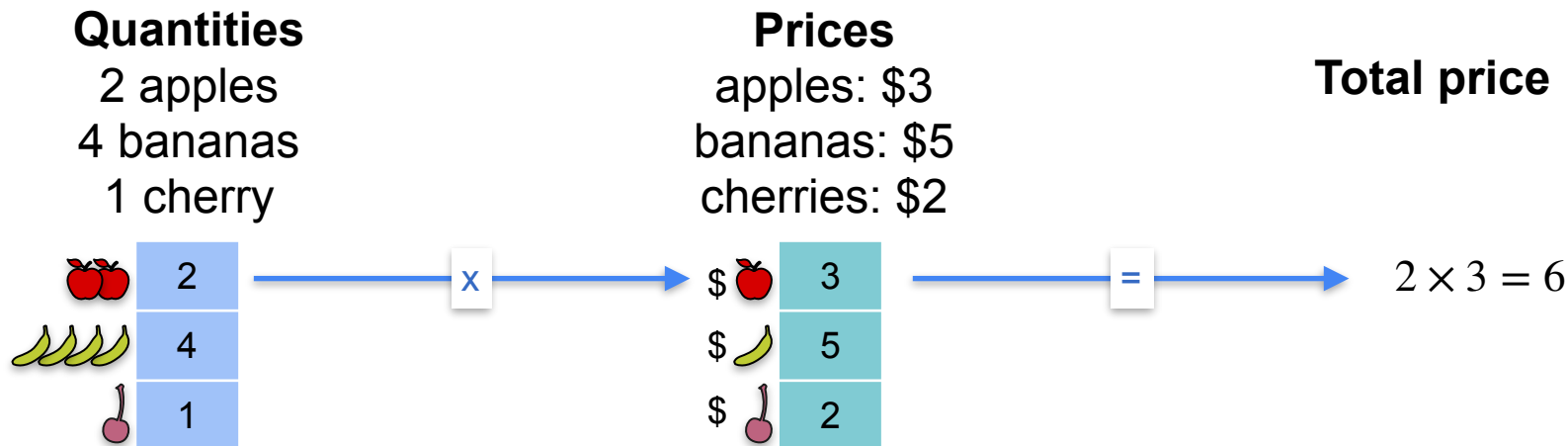
bananas: \$5

cherries: \$2

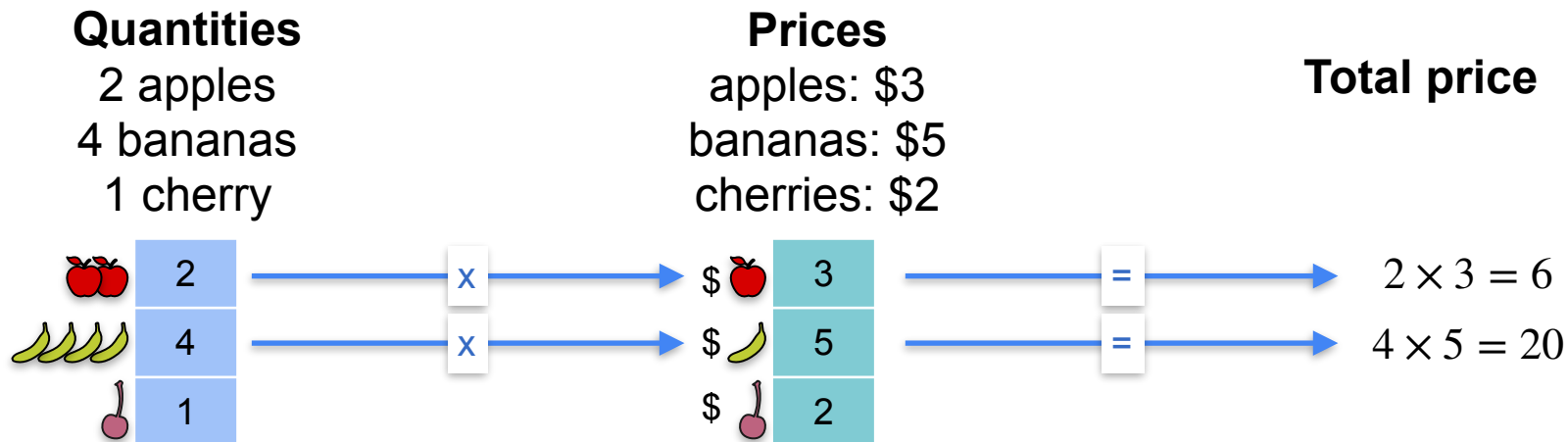
\$ 	3
\$ 	5
\$ 	2

**Total price**

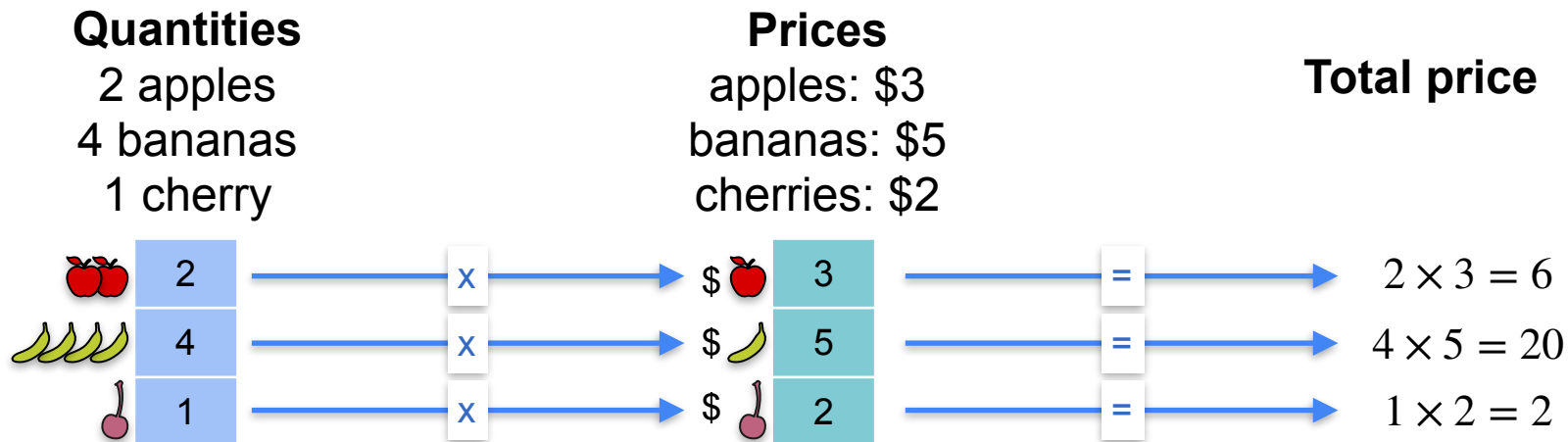
# A shortcut for linear operations



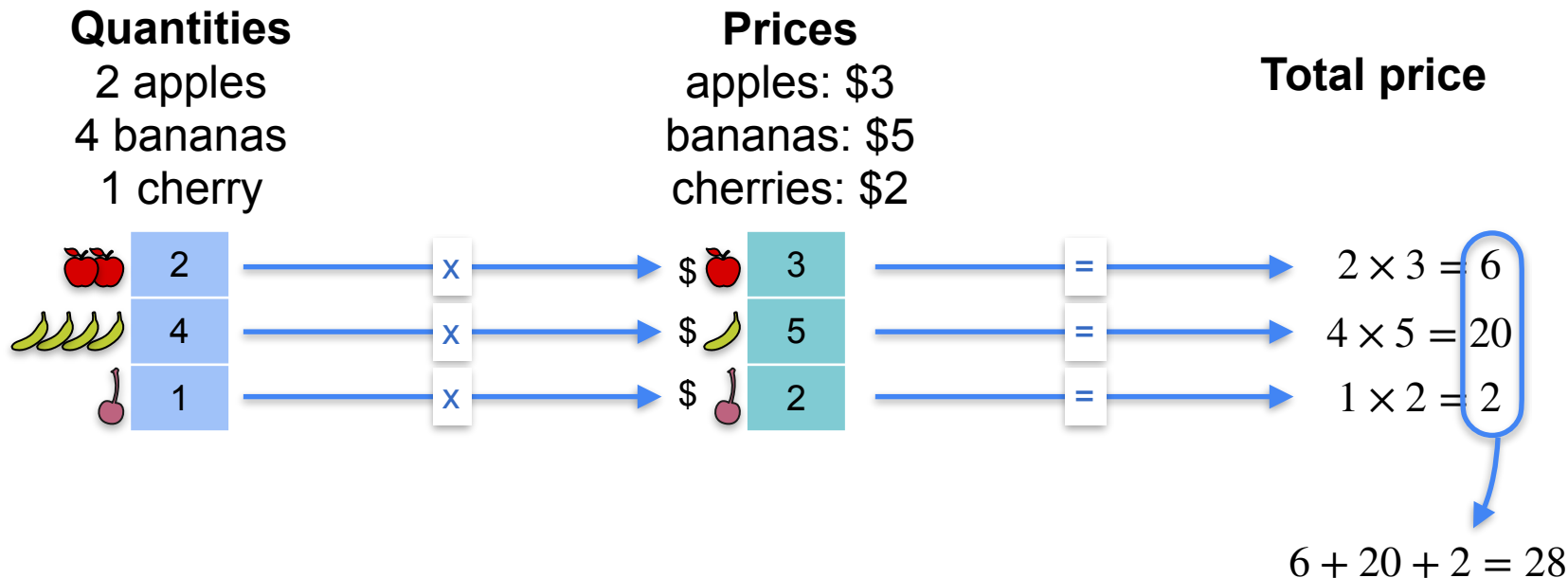
# A shortcut for linear operations



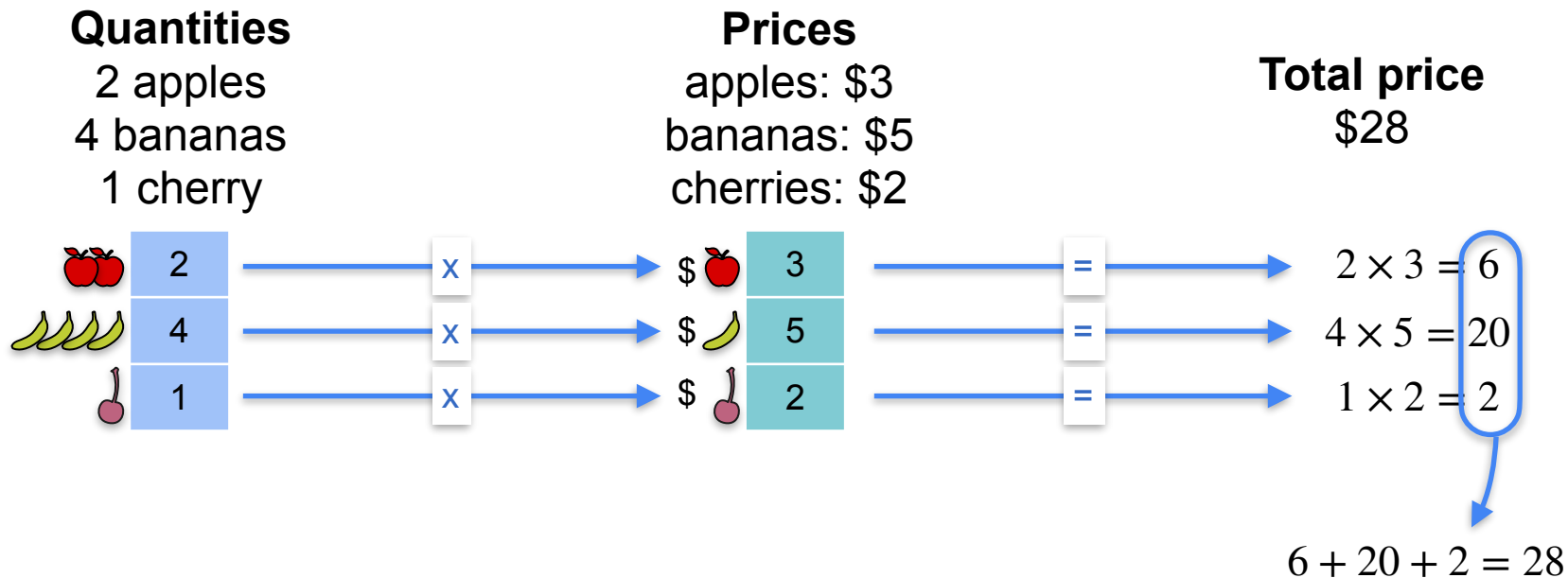
# A shortcut for linear operations



# A shortcut for linear operations




# A shortcut for linear operations











# The dot product



The diagram illustrates the dot product of two vectors using fruit prices. The first vector (blue boxes) represents quantities: 2 apples, 4 bananas, and 1 cherry. The second vector (teal boxes) represents prices: \$3 per apple, \$5 per banana, and \$2 per cherry. The dot product is calculated as the sum of the products of corresponding elements:  $2 \times \$3 + 4 \times \$5 + 1 \times \$2 = \$28$ .

	2
	4
	1




·

\$ 	3
\$ 	5
\$ 	2




=

\$28

# The dot product

	2
	4
	1

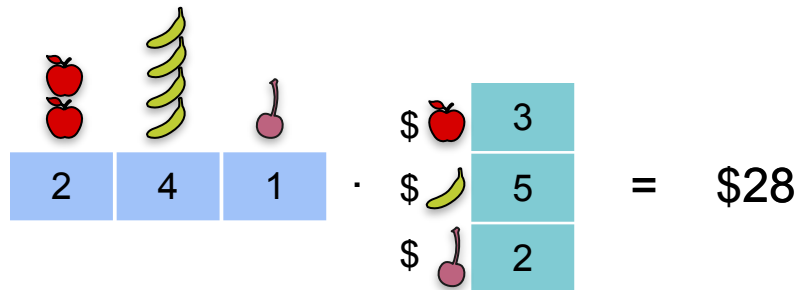
 $\cdot$ 

\$ 	3
\$ 	5
\$ 	2

 $= \$28$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

# The dot product



The diagram illustrates a dot product calculation for fruit prices. On the left, a vector of quantities is shown in blue boxes: 2 (with 2 apples above), 4 (with 4 bananas above), and 1 (with 1 cherry above). This is followed by a dot operator. To the right, a vector of prices is shown in teal boxes: 3 (with \$ and 1 apple to the left), 5 (with \$ and 1 banana to the left), and 2 (with \$ and 1 cherry to the left). An equals sign follows, leading to the result \$28.

2	4	1	·	\$ 🍎	3	= \$28
				\$ 🍌	5	
				\$ 🍒	2	

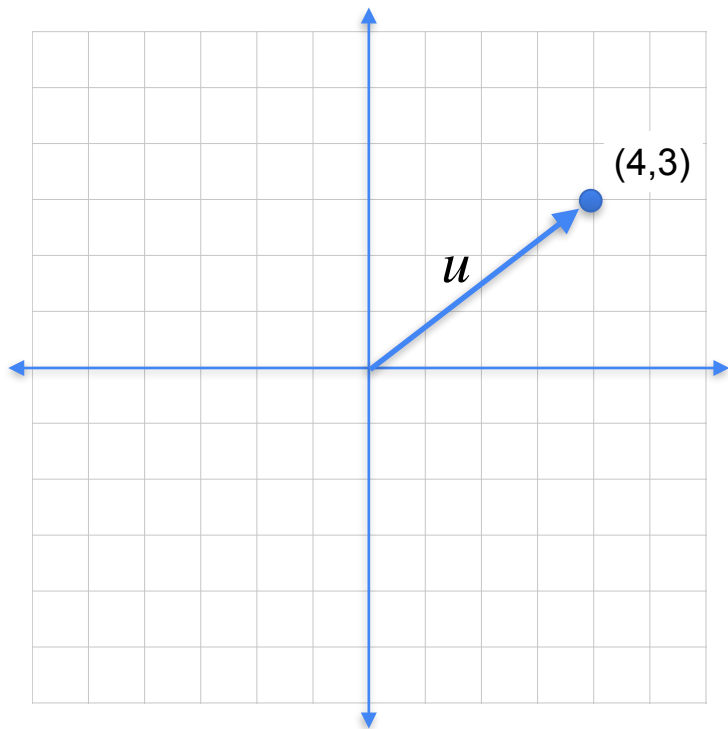
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

# The dot product

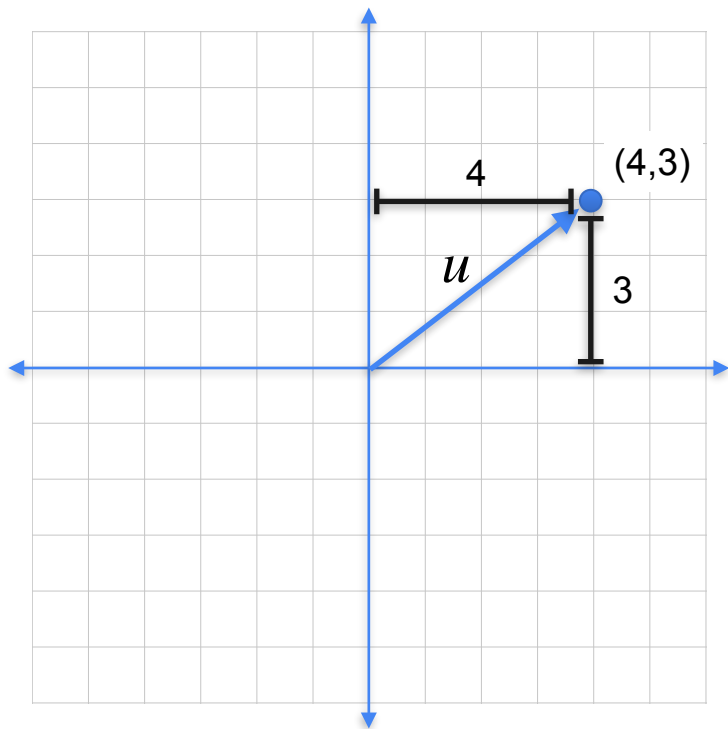
$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

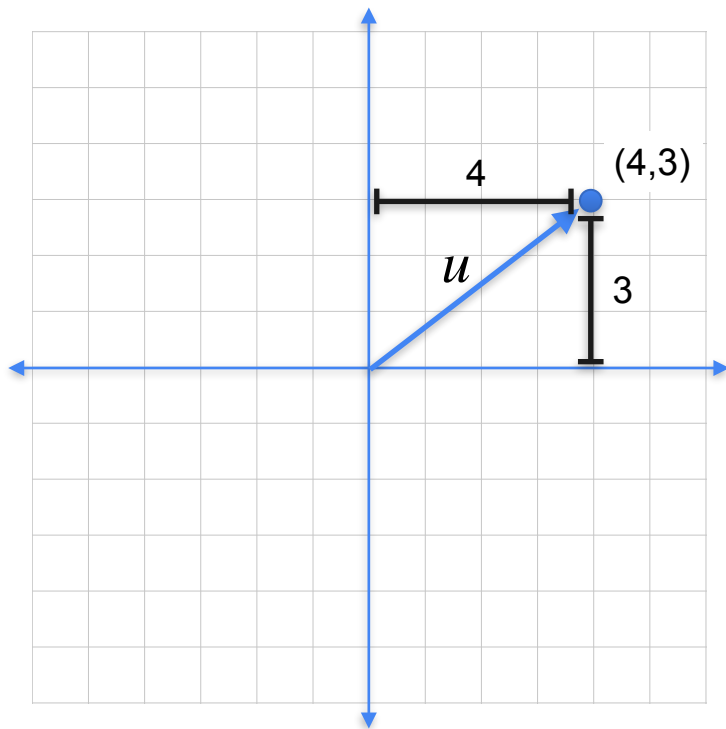
# Norm of a vector using dot product



# Norm of a vector using dot product

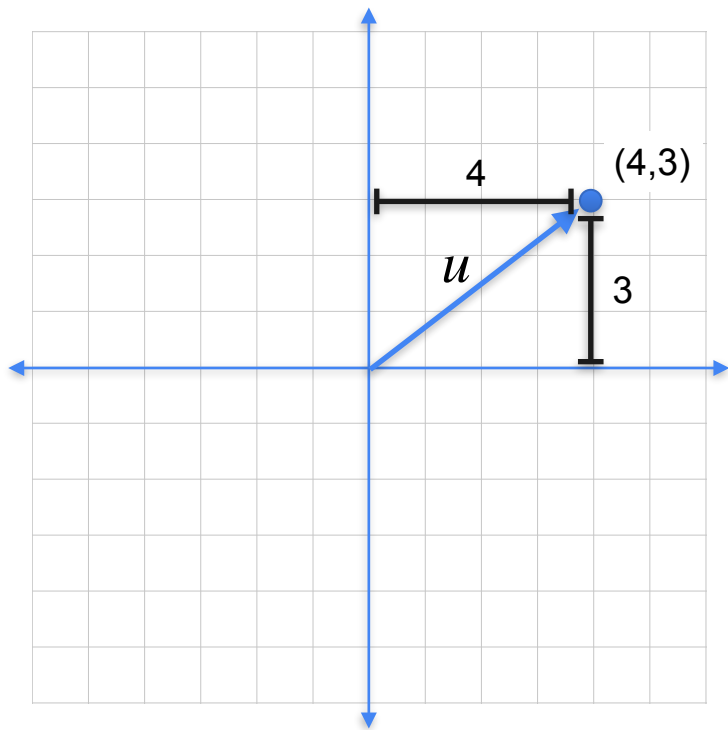


# Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

# Norm of a vector using dot product

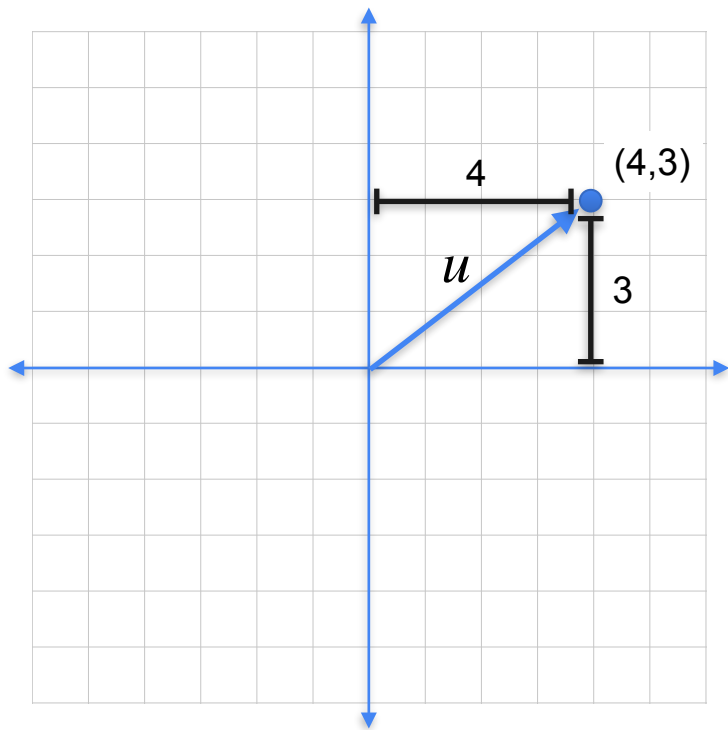


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$



# Norm of a vector using dot product

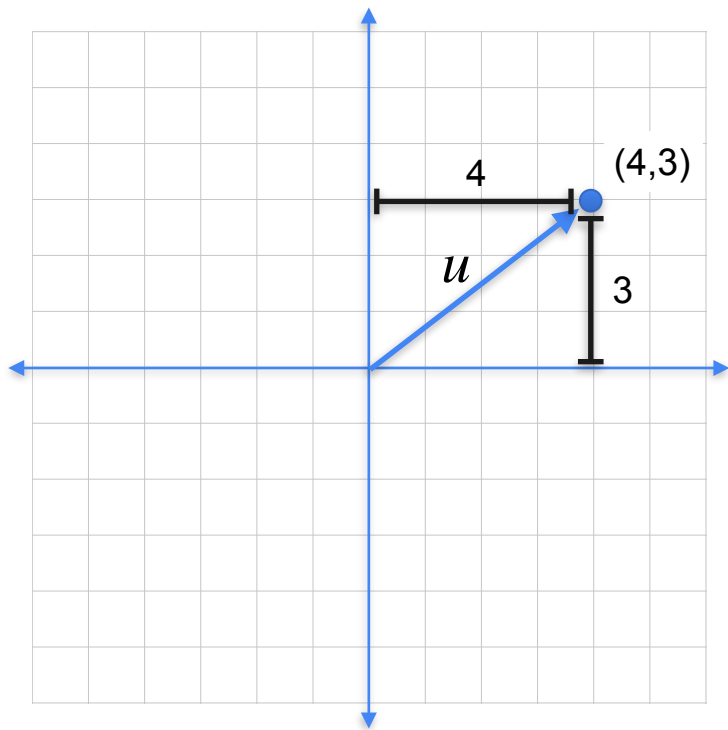


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

# Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

$$|u|_2 = \sqrt{\langle u, u \rangle}$$



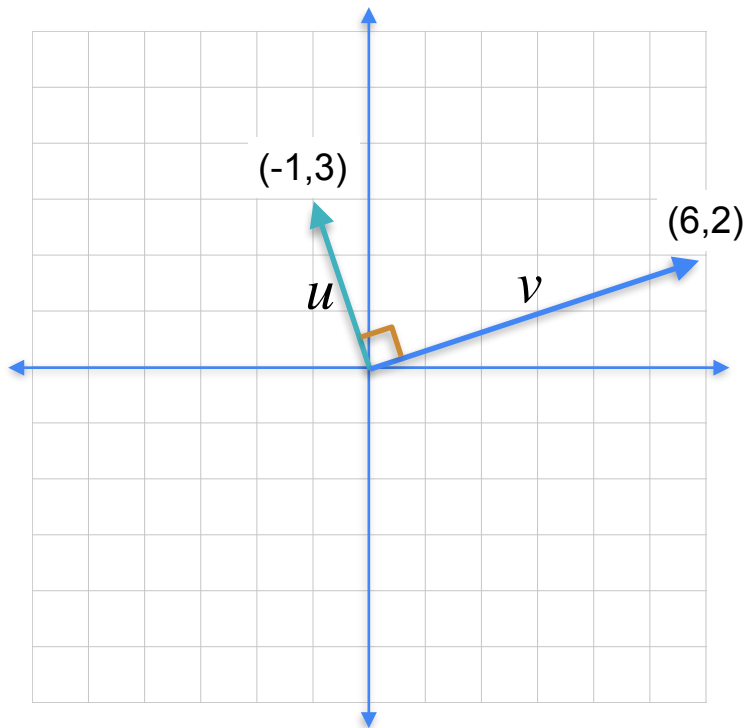
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# Vectors and Linear Transformations

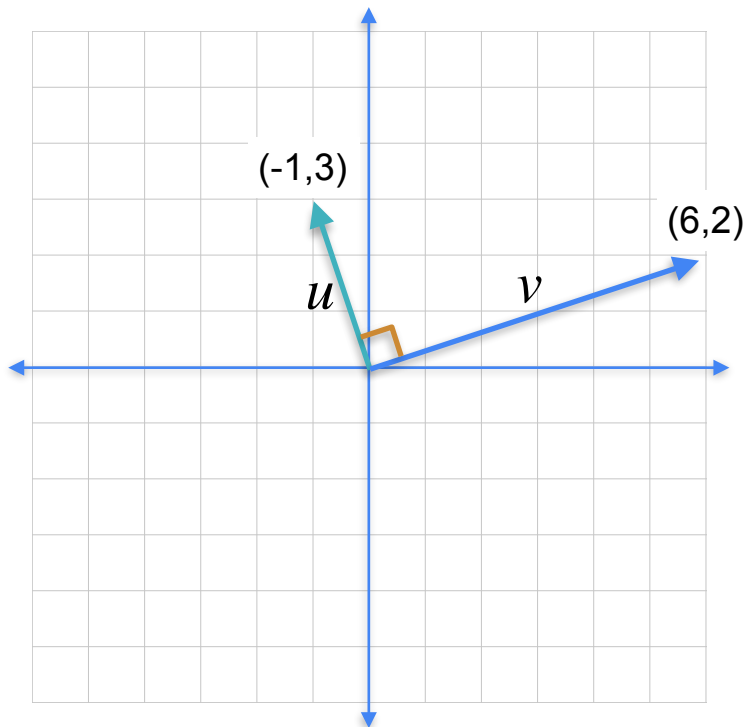
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## **Geometric dot product**

# Orthogonal vectors have dot product 0



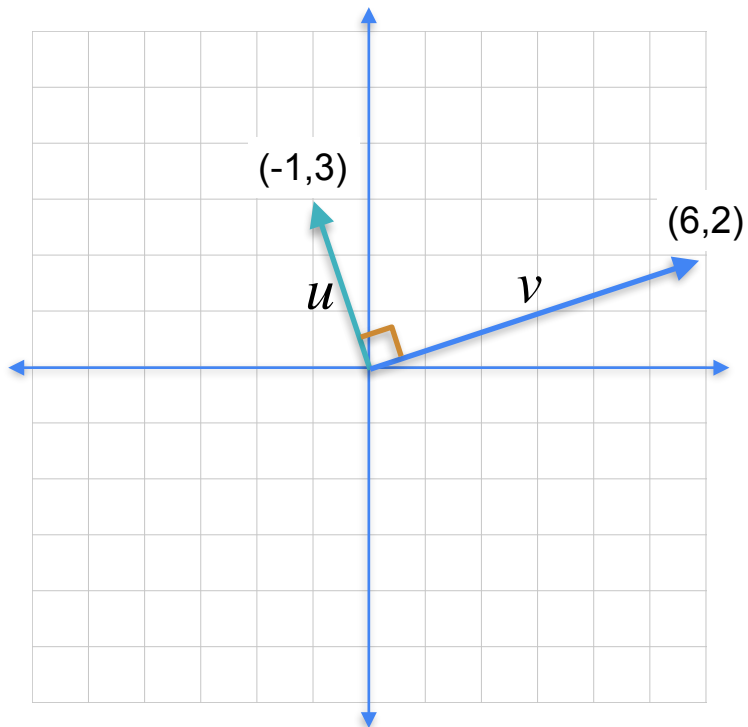
# Orthogonal vectors have dot product 0



6

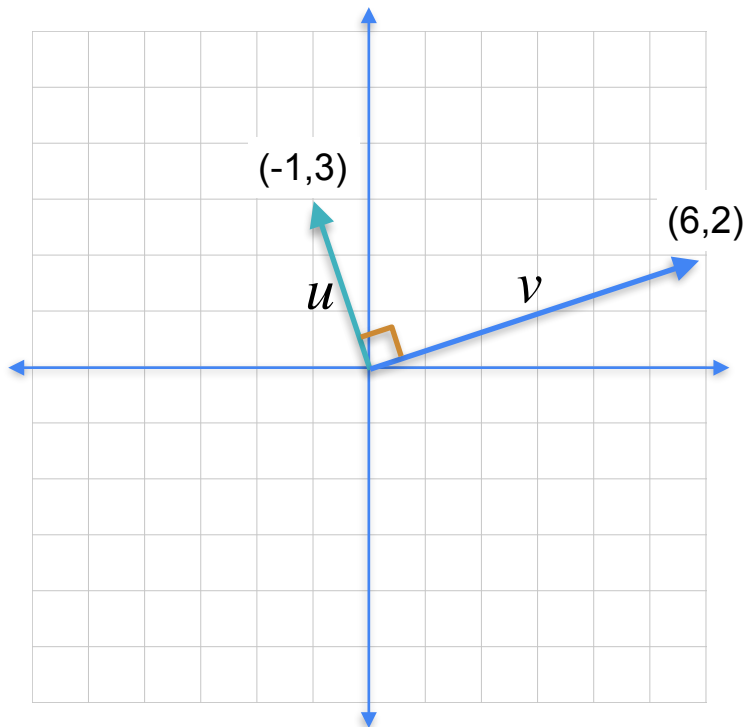
2

# Orthogonal vectors have dot product 0



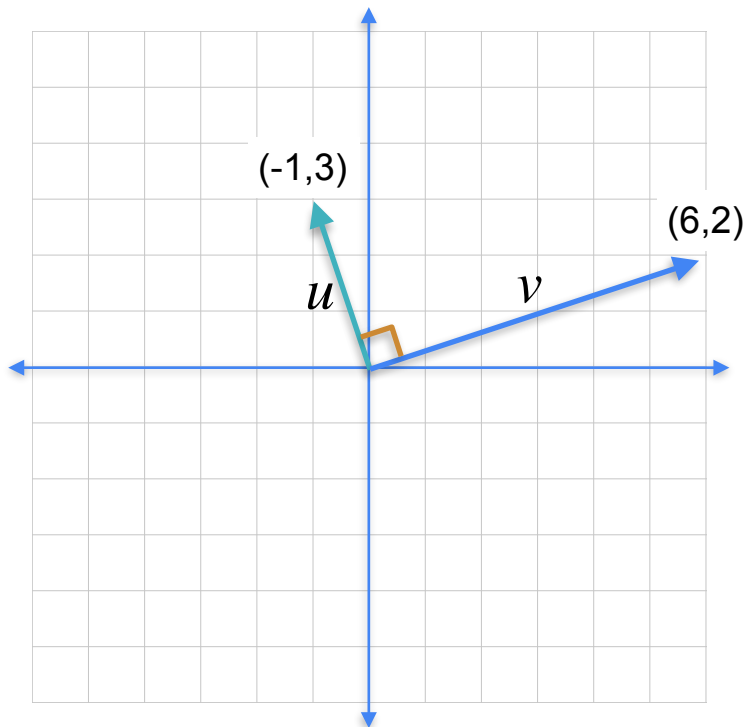
6	2	-1
		3

# Orthogonal vectors have dot product 0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

# Orthogonal vectors have dot product 0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\langle u, v \rangle = 0$$

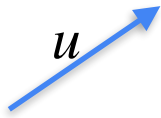


# The dot product

# The dot product

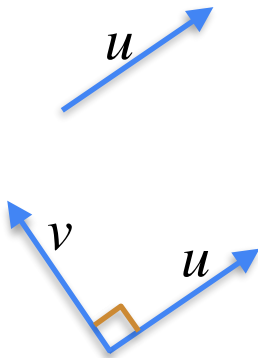


# The dot product



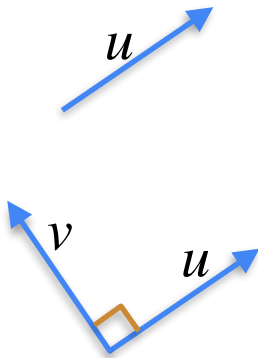
$$\langle u, u \rangle = |u|^2$$

# The dot product



$$\langle u, u \rangle = |u|^2$$

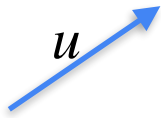
# The dot product



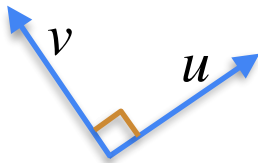
$$\langle u, u \rangle = |u|^2$$

$$\langle u, v \rangle = 0$$

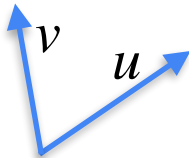
# The dot product



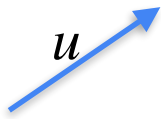
$$\langle u, u \rangle = |u|^2$$



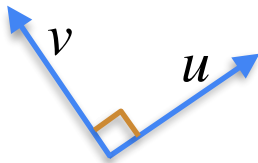
$$\langle u, v \rangle = 0$$



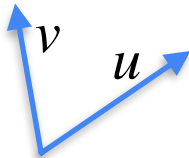
# The dot product



$$\langle u, u \rangle = |u|^2$$



$$\langle u, v \rangle = 0$$



$$\langle u, v \rangle = ?$$

# The dot product



# The dot product

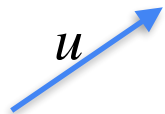


# The dot product



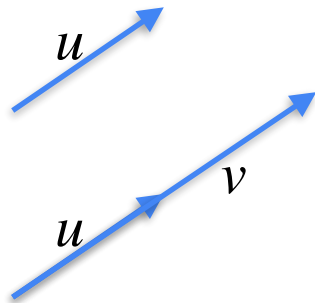
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

# The dot product



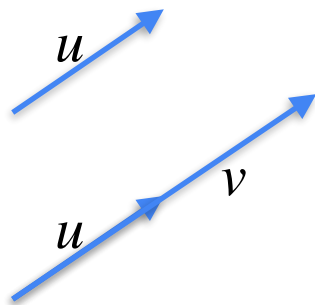
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

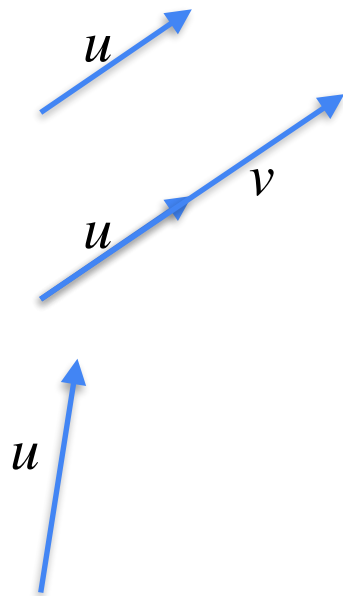
# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

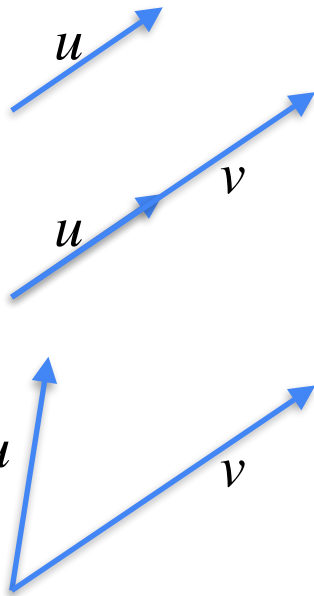
# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

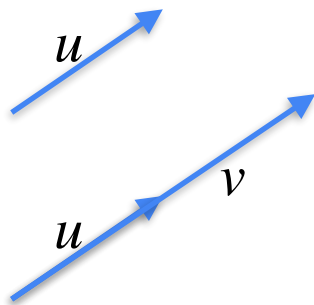
# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

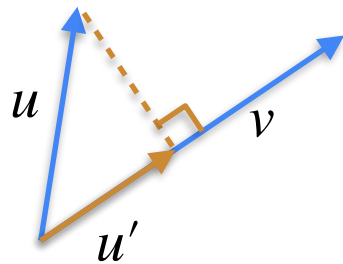
$$\langle u, v \rangle = |u| \cdot |v|$$

# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

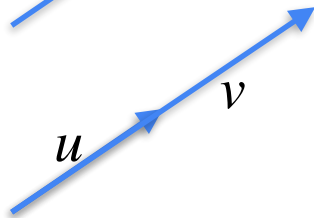




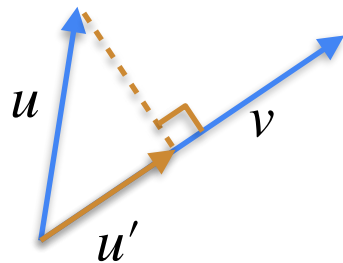
# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

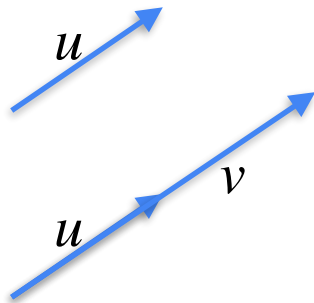


$$\langle u, v \rangle = |u| \cdot |v|$$



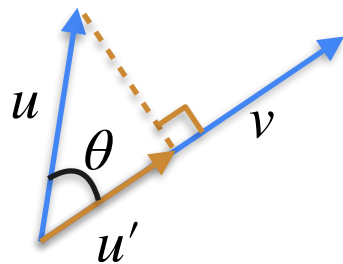
$$\langle u, v \rangle = |u'| \cdot |v|$$

# The dot product



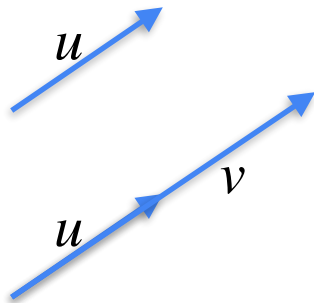
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$



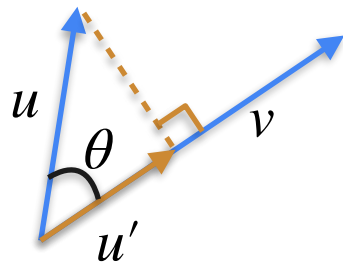
$$\langle u, v \rangle = |u'| \cdot |v|$$

# The dot product



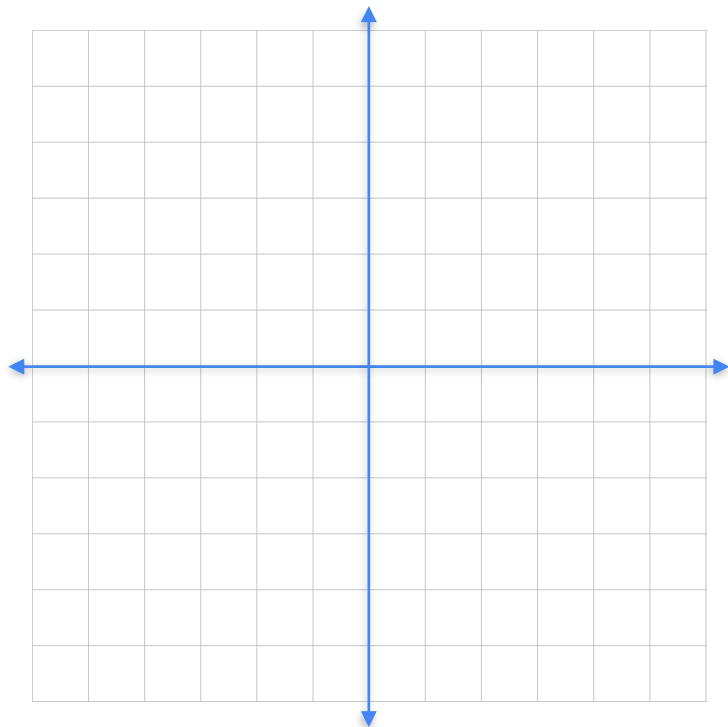
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

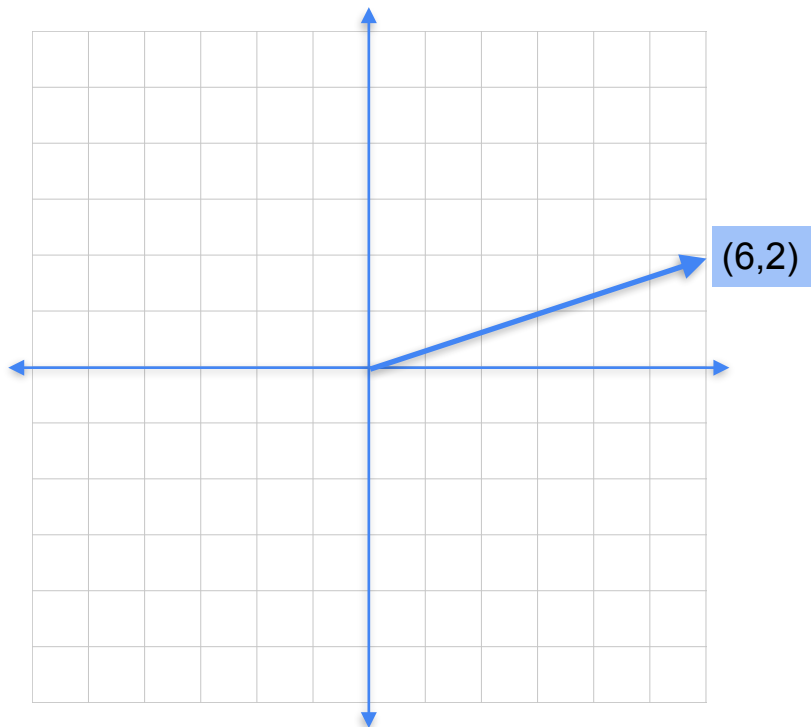


$$\begin{aligned}\langle u, v \rangle &= |u'| \cdot |v| \\ &= |u| |v| \cos(\theta)\end{aligned}$$

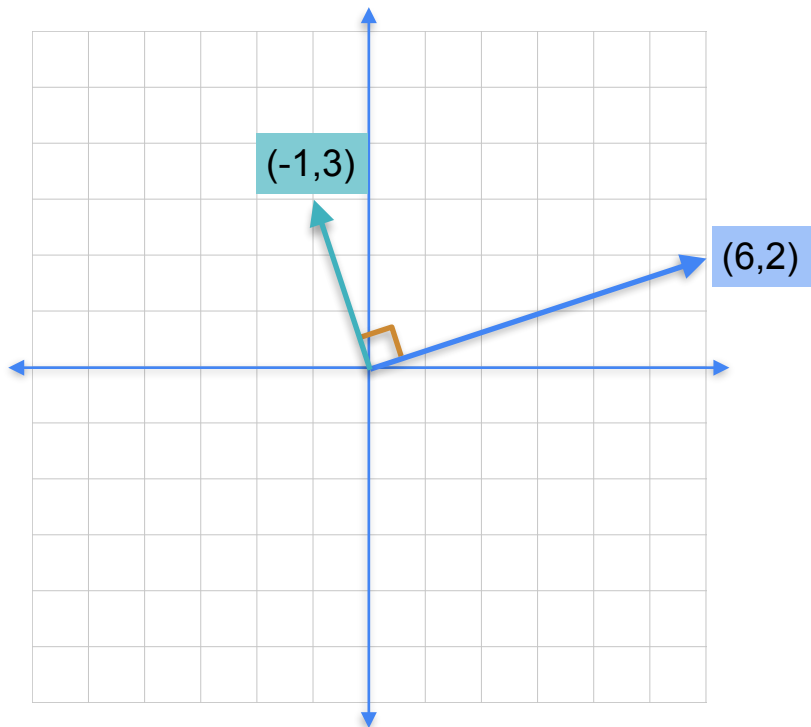
# Geometric dot product



# Geometric dot product

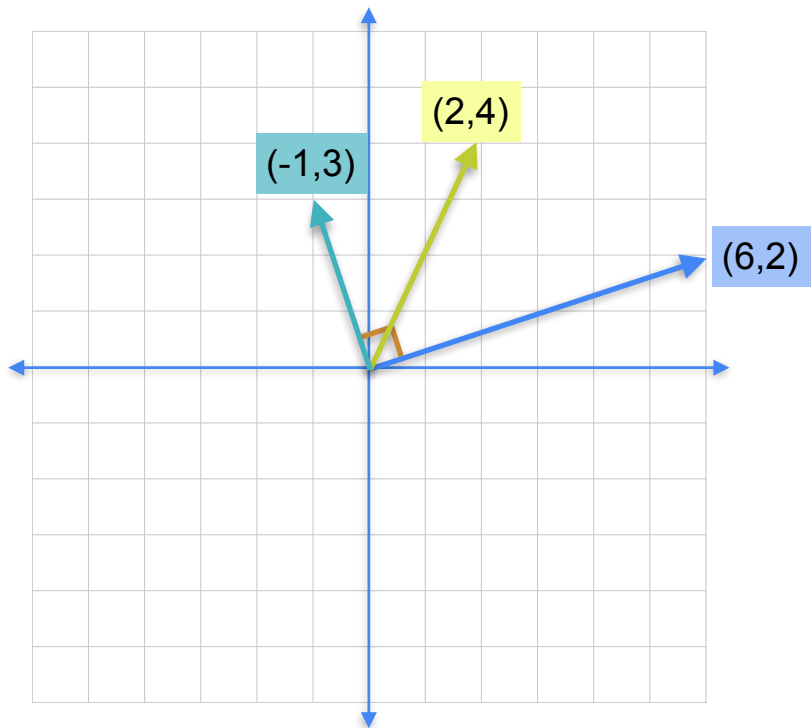


# Geometric dot product



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

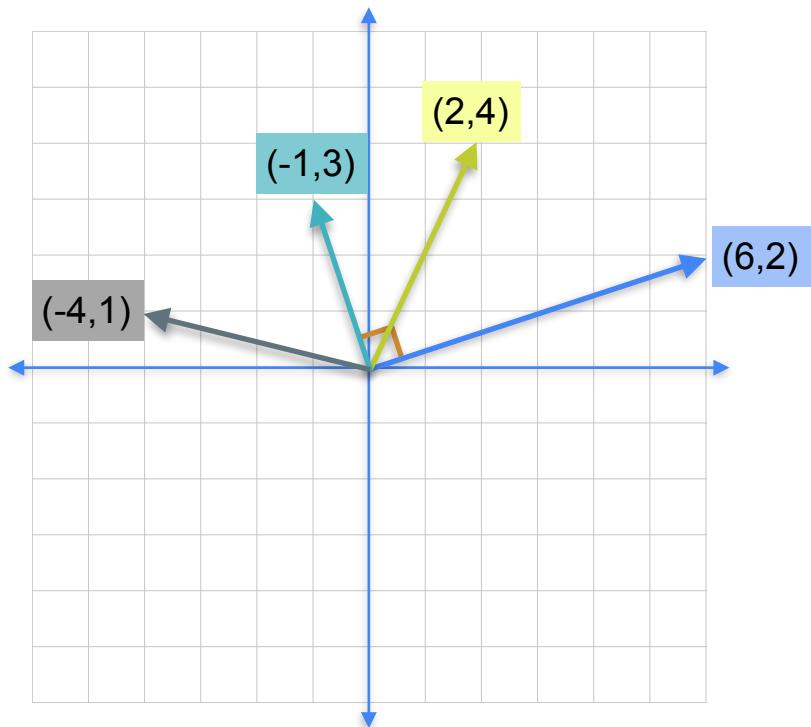
# Geometric dot product



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

# Geometric dot product



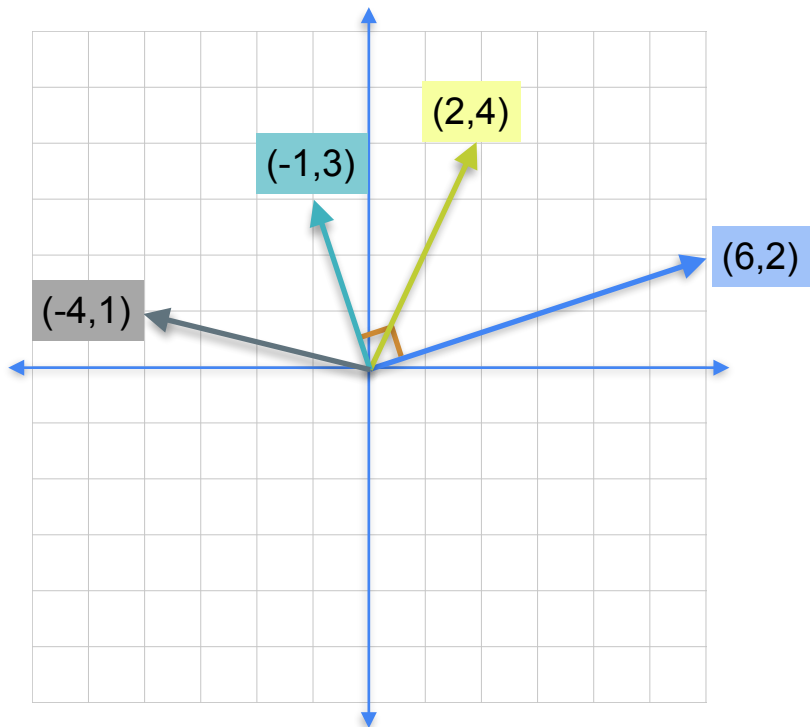
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$



# Geometric dot product

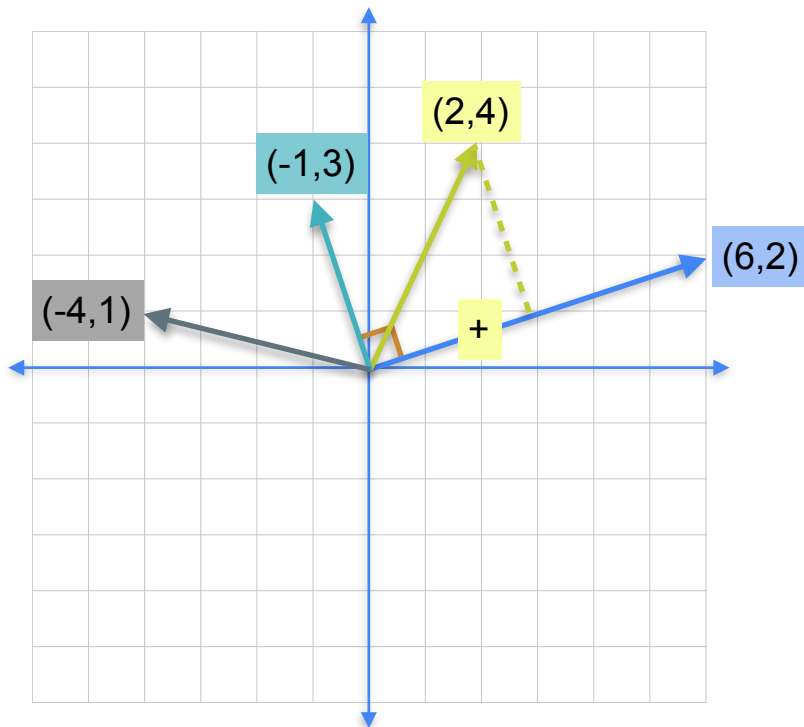


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

# Geometric dot product

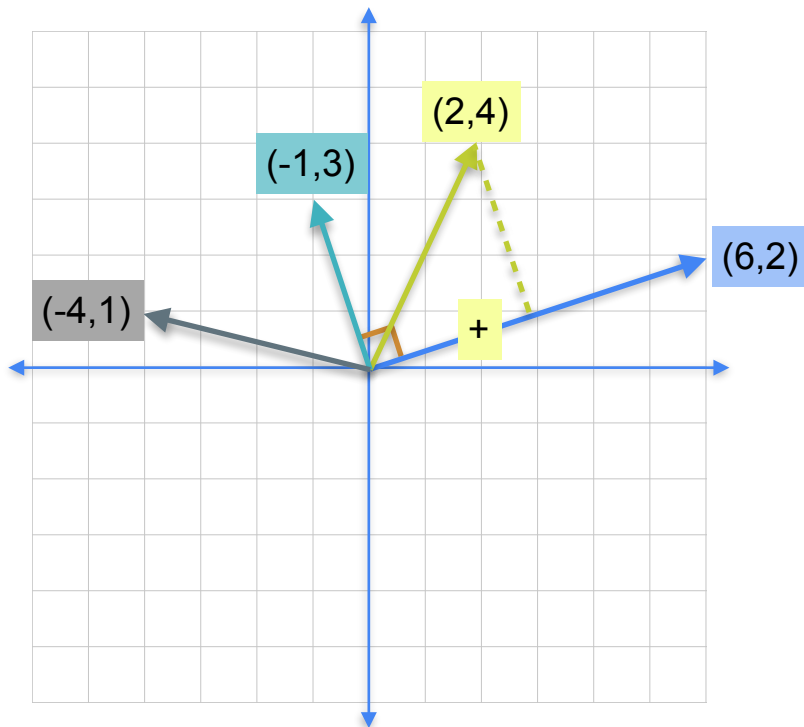


$$\begin{matrix} 6 & 2 \end{matrix} \begin{matrix} 2 \\ 4 \end{matrix} = 20 \quad \text{Positive}$$

$$\begin{matrix} 6 & 2 \end{matrix} \begin{matrix} -1 \\ 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \end{matrix} \begin{matrix} -4 \\ 1 \end{matrix} = -22$$

# Geometric dot product

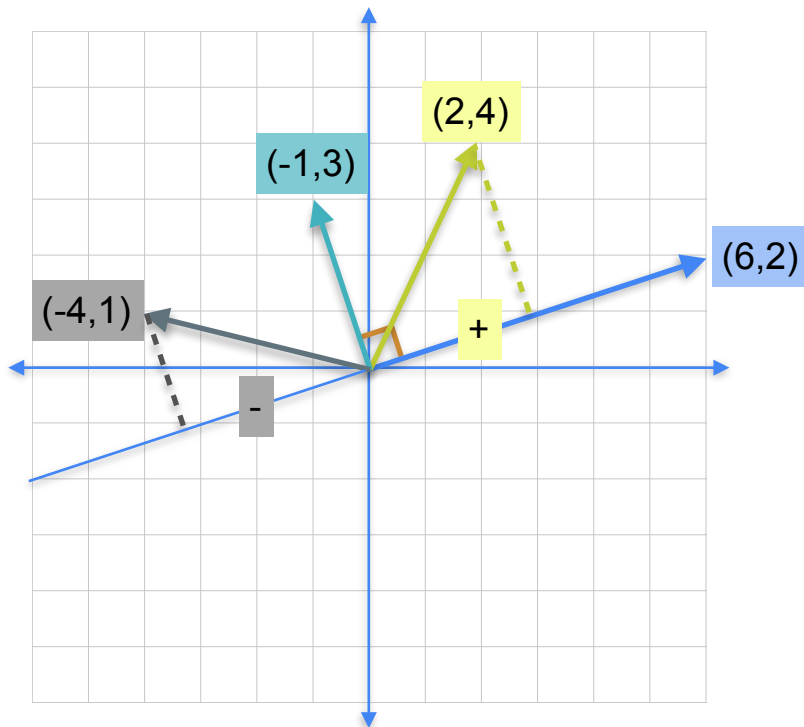


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22 \quad \text{Negative}$$

# Geometric dot product

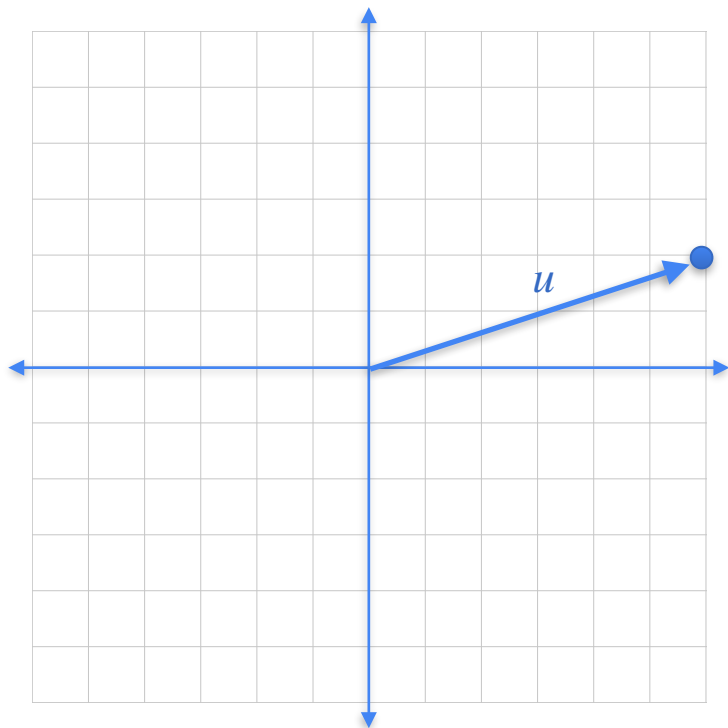


$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

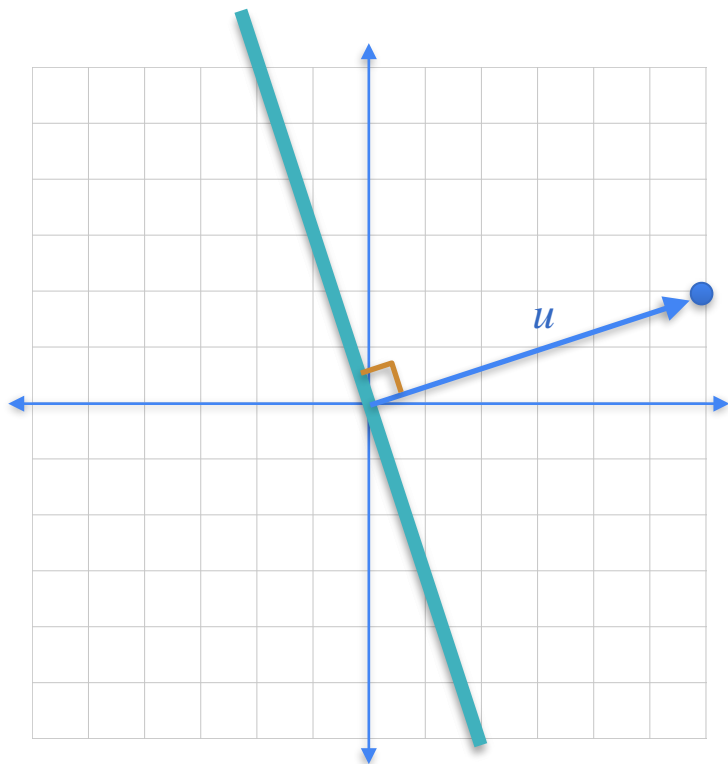
$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22 \quad \text{Negative}$$

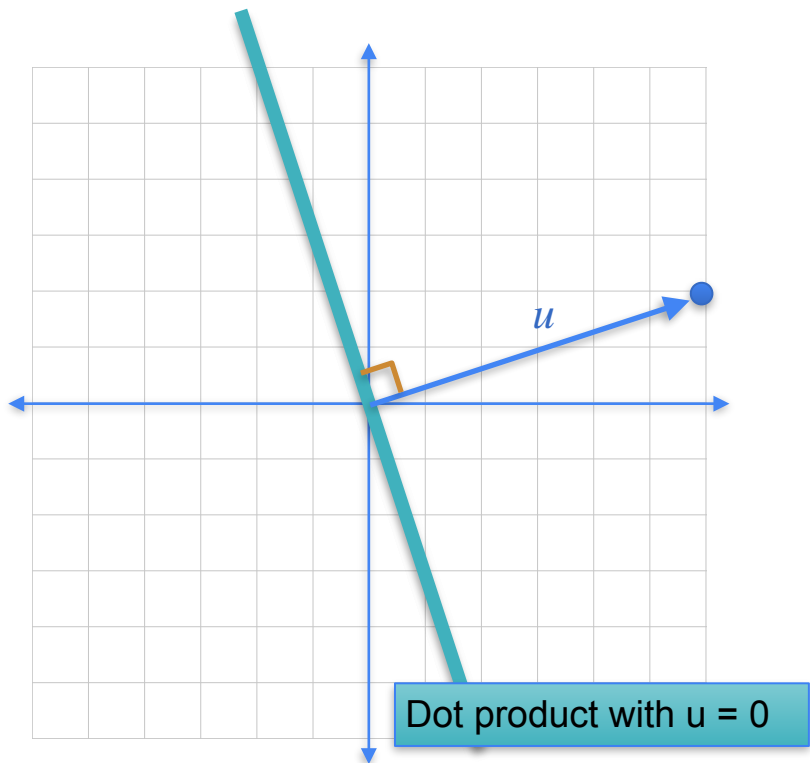
# Geometric dot product



# Geometric dot product

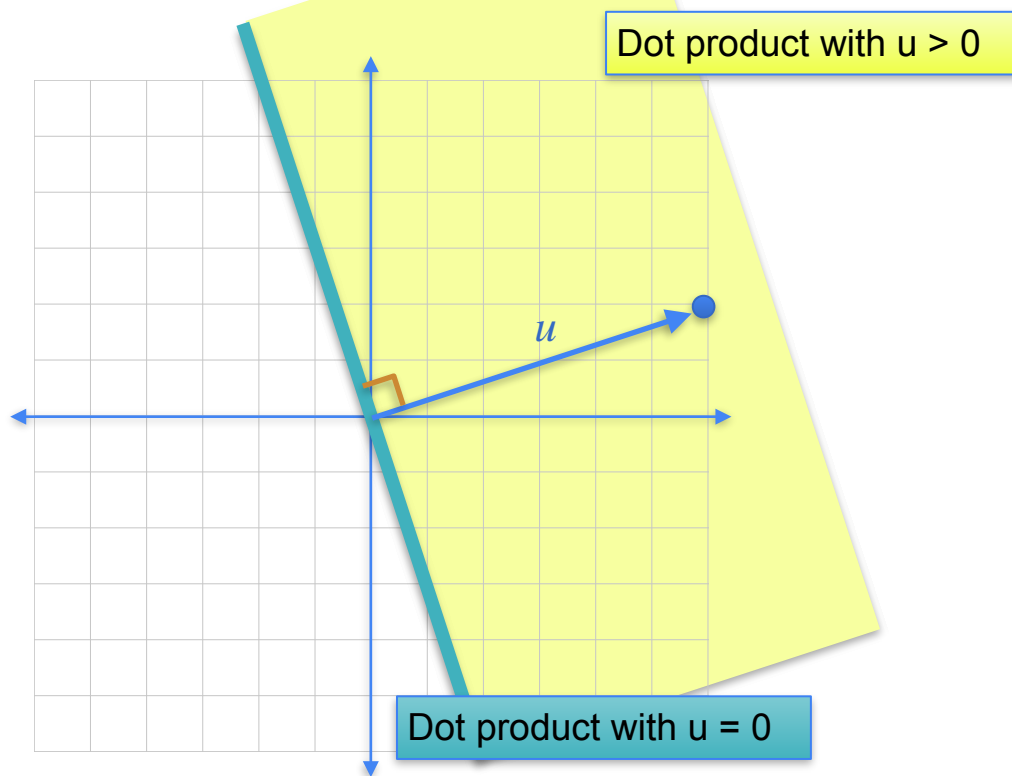


# Geometric dot product



$$\langle u, v \rangle = 0$$

# Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$



# Geometric dot product

Dot product with  $u > 0$

$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$

Dot product with  $u < 0$

Dot product with  $u = 0$



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# Vectors and Linear Transformations

---

**Multiplying a matrix by a  
vector**

# Equations as dot product

$$2a + 4b + c = 28$$

The diagram illustrates the equation  $2a + 4b + c = 28$  using fruit icons and boxes. On the left, there are three blue boxes containing the numbers 2, 4, and 1. Above the box with 2 are two red apples. Above the box with 4 are four yellow bananas. Above the box with 1 is one red cherry. To the right of these boxes is a dot. Further right is a vertical stack of three light blue boxes containing the letters a, b, and c. To the left of the 'a' box is a red apple icon with a dollar sign. To the left of the 'b' box is a yellow banana icon with a dollar sign. To the left of the 'c' box is a red cherry icon with a dollar sign. To the right of this stack is an equals sign, followed by a dollar sign and an orange box containing the number 28.

# Equations as dot product

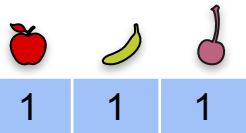
$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$

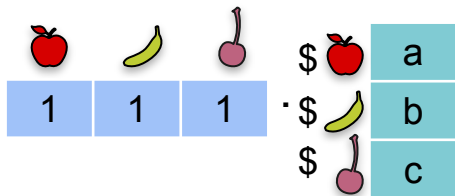


$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation  $a + b + c = 10$ . The row vector  $[1, 1, 1]$  is represented by three blue boxes containing the numbers 1, 1, and 1. The column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  is represented by three teal boxes containing the variables  $a$ ,  $b$ , and  $c$ . The dot product is shown as  $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 10$ , where the dollar sign and the number 10 are in an orange box.

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation  $a + b + c = 10$ . The row vector  $[1, 1, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 10. The fruit icons (apple, banana, cherry) are used to represent the variables  $a$ ,  $b$ , and  $c$  respectively.

$$a + 2b + c = 15$$

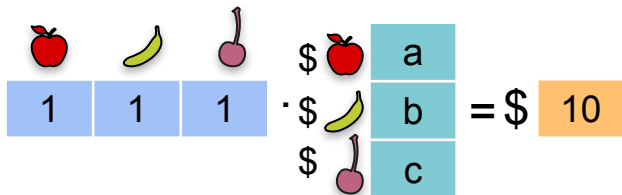
Diagram illustrating the dot product for the equation  $a + 2b + c = 15$ . The row vector  $[1, 2, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 15. The fruit icons (apple, banana, cherry) are used to represent the variables  $a$ ,  $b$ , and  $c$  respectively.

$$a + b + 2c = 12$$

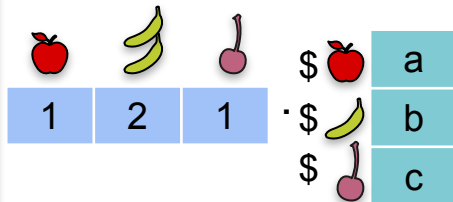


# Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$



$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation  $a + b + c = 10$ . The row vector  $[1, 1, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 10.

$$a + 2b + c = 15$$

Diagram illustrating the dot product for the equation  $a + 2b + c = 15$ . The row vector  $[1, 2, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 15.

$$a + b + 2c = 12$$

# Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation  $a + b + c = 10$ . The vector  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  is multiplied by the vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to yield the scalar value 10.

$$a + 2b + c = 15$$

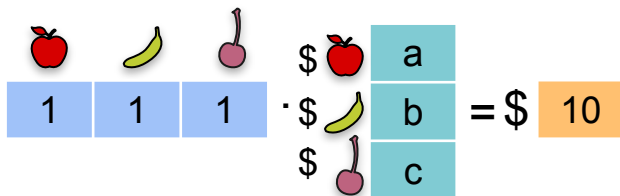
Diagram illustrating the dot product for the equation  $a + 2b + c = 15$ . The vector  $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$  is multiplied by the vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to yield the scalar value 15.

$$a + b + 2c = 12$$

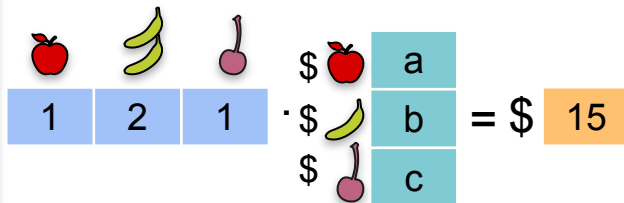
Diagram illustrating the dot product for the equation  $a + b + 2c = 12$ . The vector  $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$  is multiplied by the vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to yield the scalar value 12.

# Equations as dot product

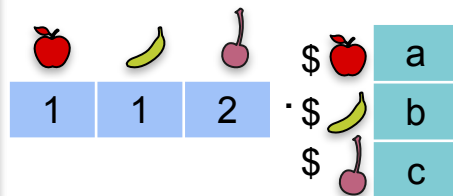
$$a + b + c = 10$$



$$a + 2b + c = 15$$



$$a + b + 2c = 12$$



# Equations as dot product

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 15$

$$a + b + 2c = 12$$

$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 12$

# Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation  $a + b + c = 10$ . The row vector  $[1, 1, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 10.

$$a + 2b + c = 15$$

Diagram illustrating the dot product for the equation  $a + 2b + c = 15$ . The row vector  $[1, 2, 1]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 15.

$$a + b + 2c = 12$$

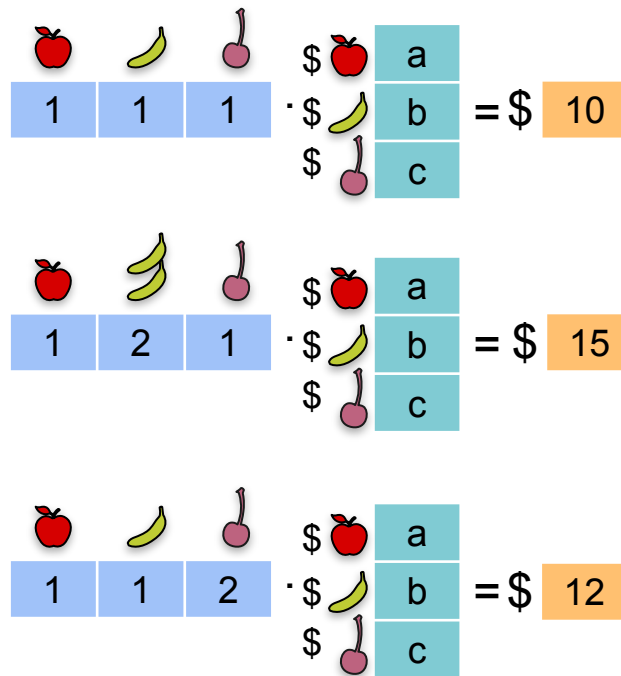
Diagram illustrating the dot product for the equation  $a + b + 2c = 12$ . The row vector  $[1, 1, 2]$  is multiplied by the column vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  to equal 12.

# Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

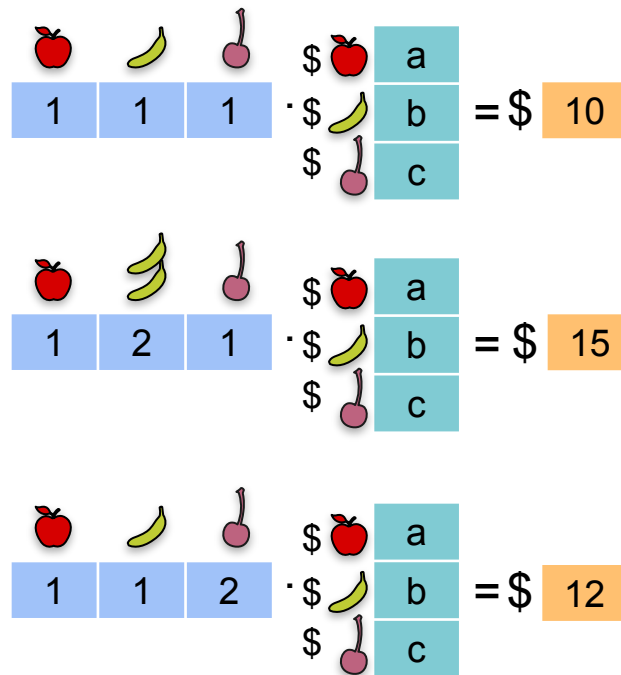


# Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$





# Equations as dot product







## System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## Matrix product

					
1	1	1	\$ 	a	10
1	2	1	\$ 	b	15
1	1	2	\$ 	c	12

The matrix product is represented as a 3x3 matrix of coefficients (1, 1, 1; 1, 2, 1; 1, 1, 2) multiplied by a column vector of variables (a, b, c) to equal a column vector of constants (10, 15, 12). The variables are represented by fruit icons: apple for a, banana for b, and cherry for c.

# Equations as dot product

## System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## Matrix product

1	1	1	a	=	10
1	2	1	b		15
1	1	2	c		12





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# Vectors and Linear Transformations

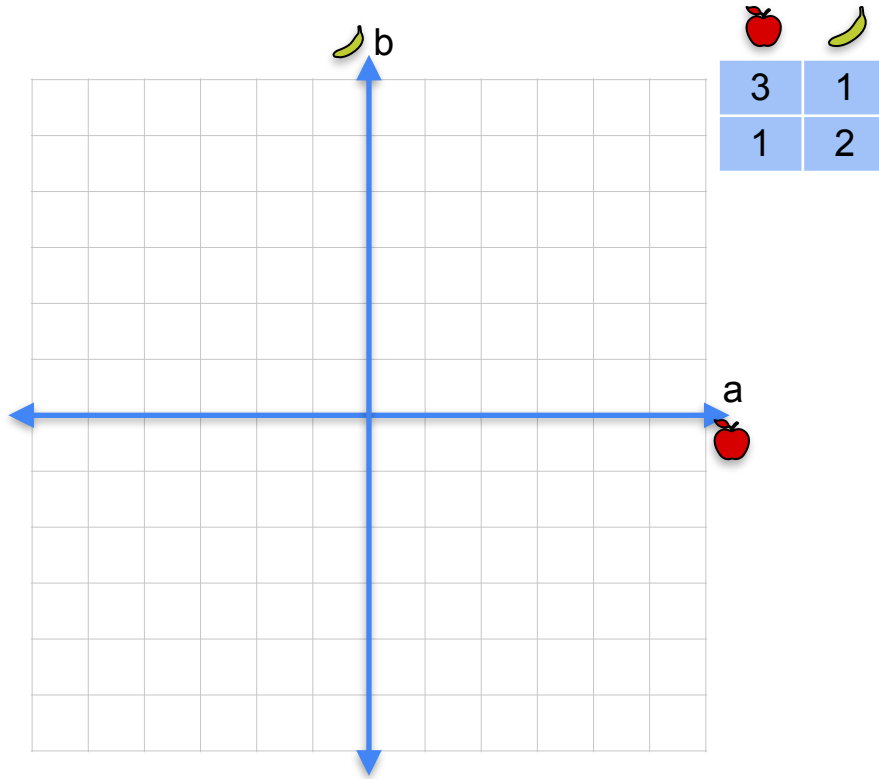
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**Matrices as linear  
transformations**

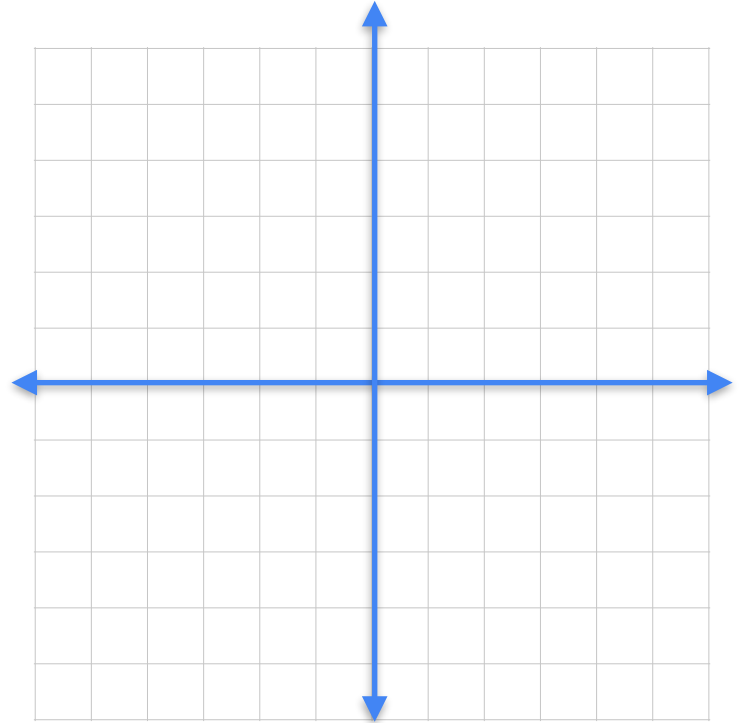
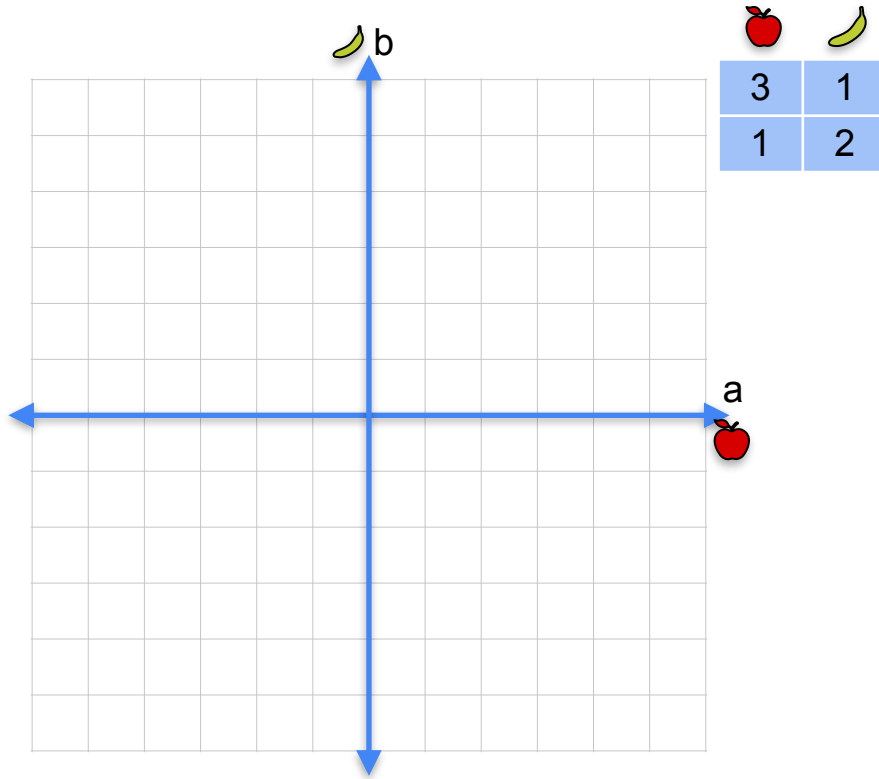
# Matrices as linear transformations

	
3	1
1	2

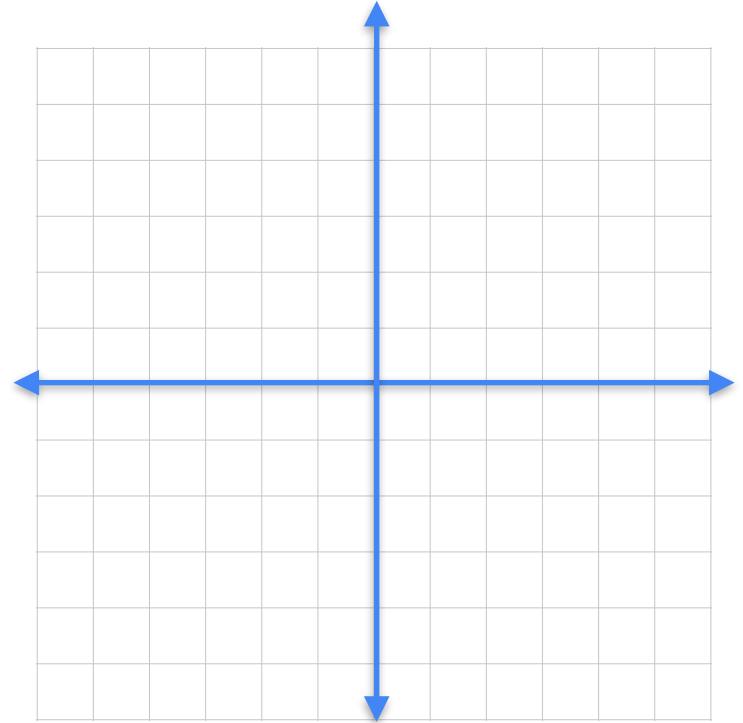
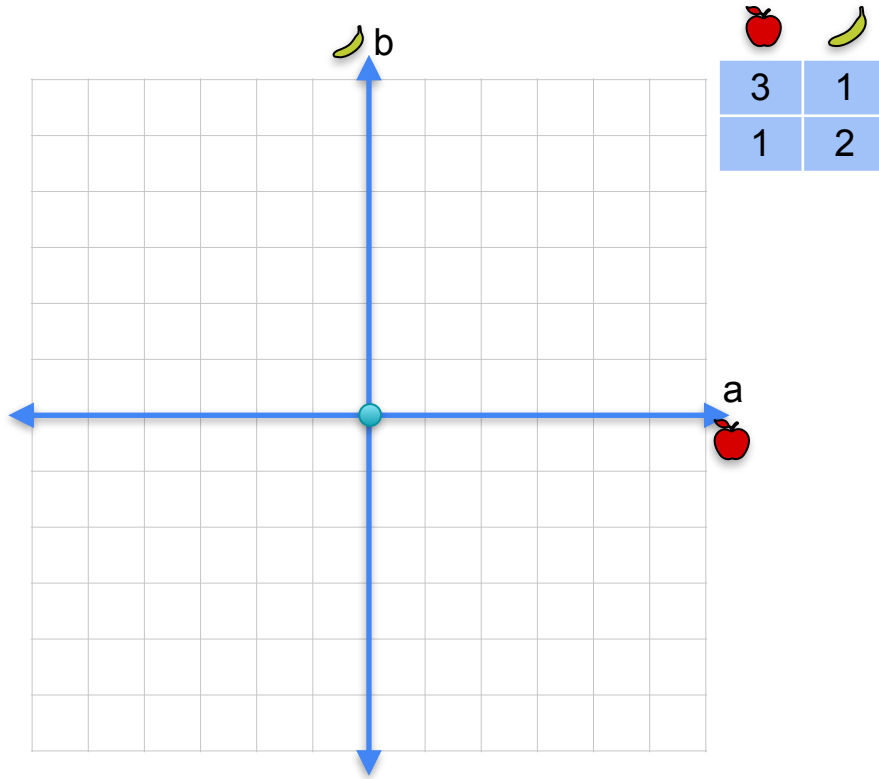
# Matrices as linear transformations



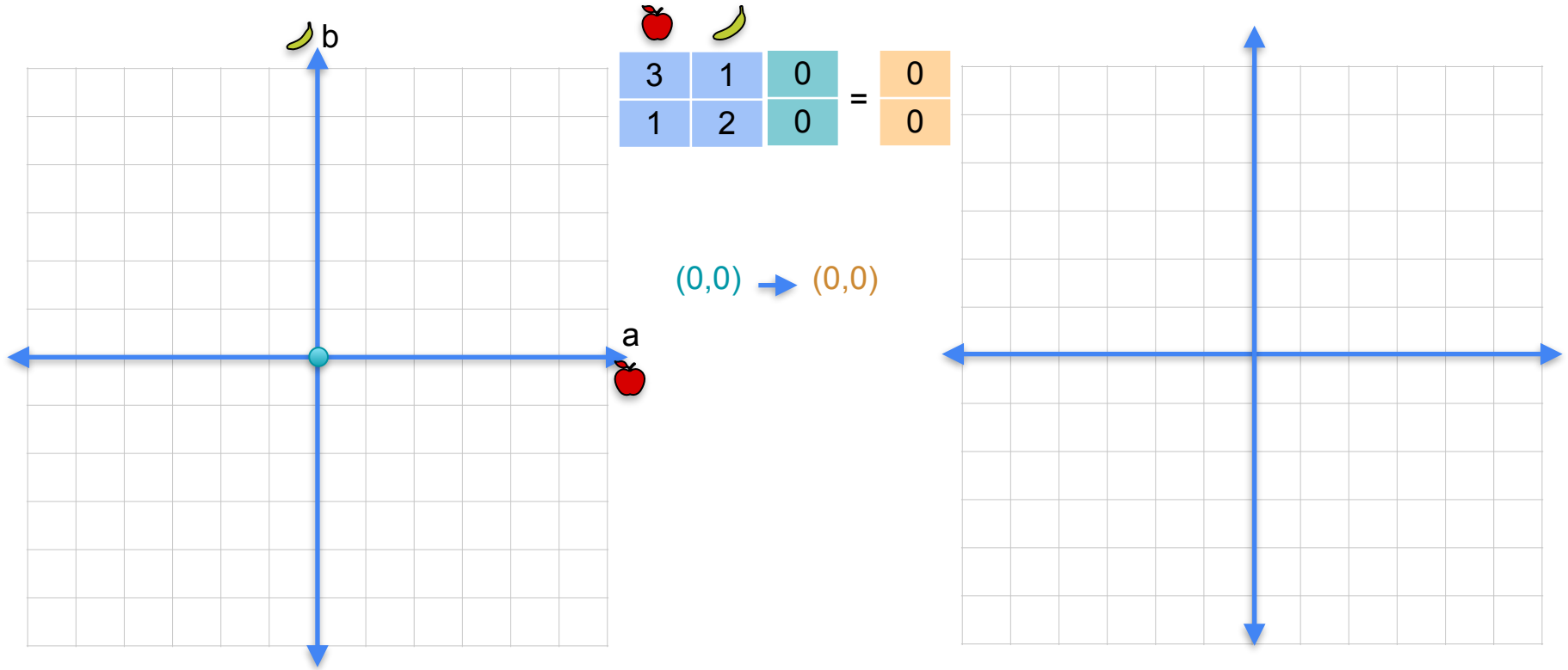
# Matrices as linear transformations



# Matrices as linear transformations

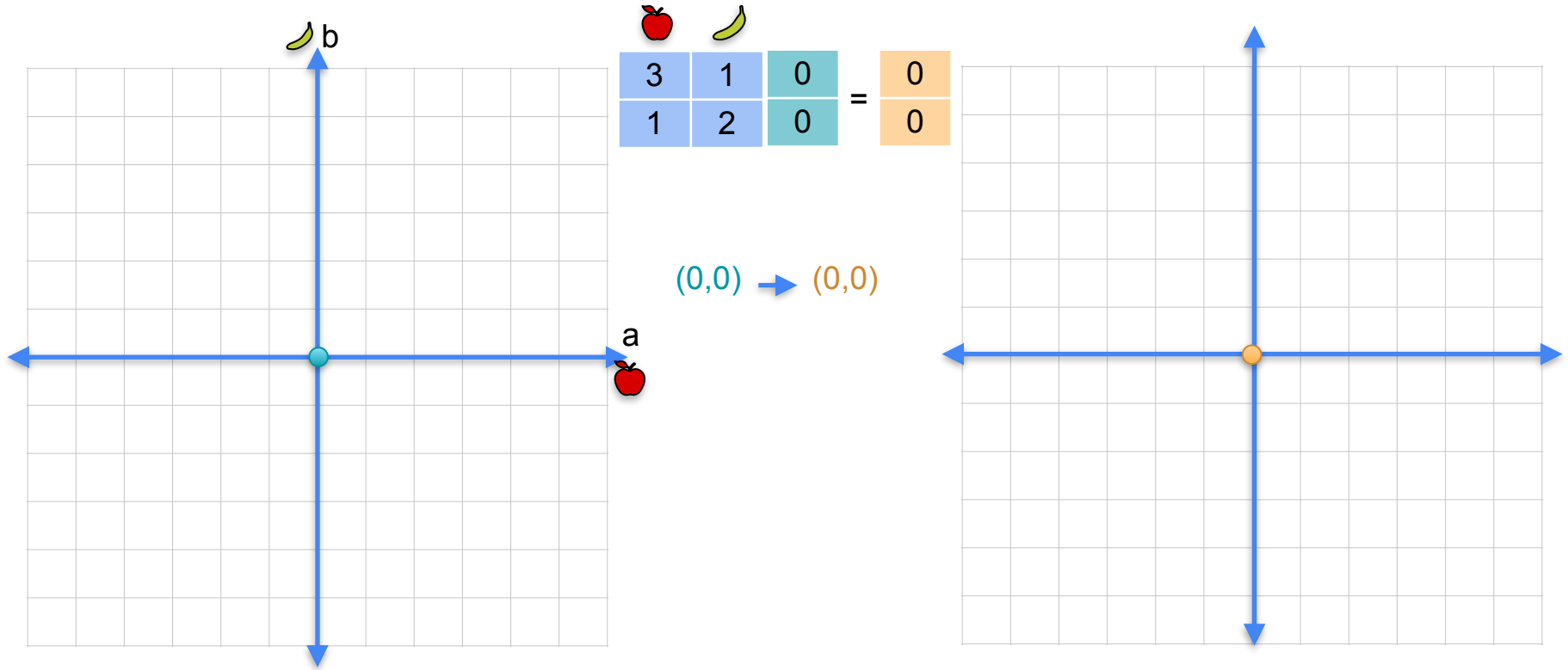


# Matrices as linear transformations

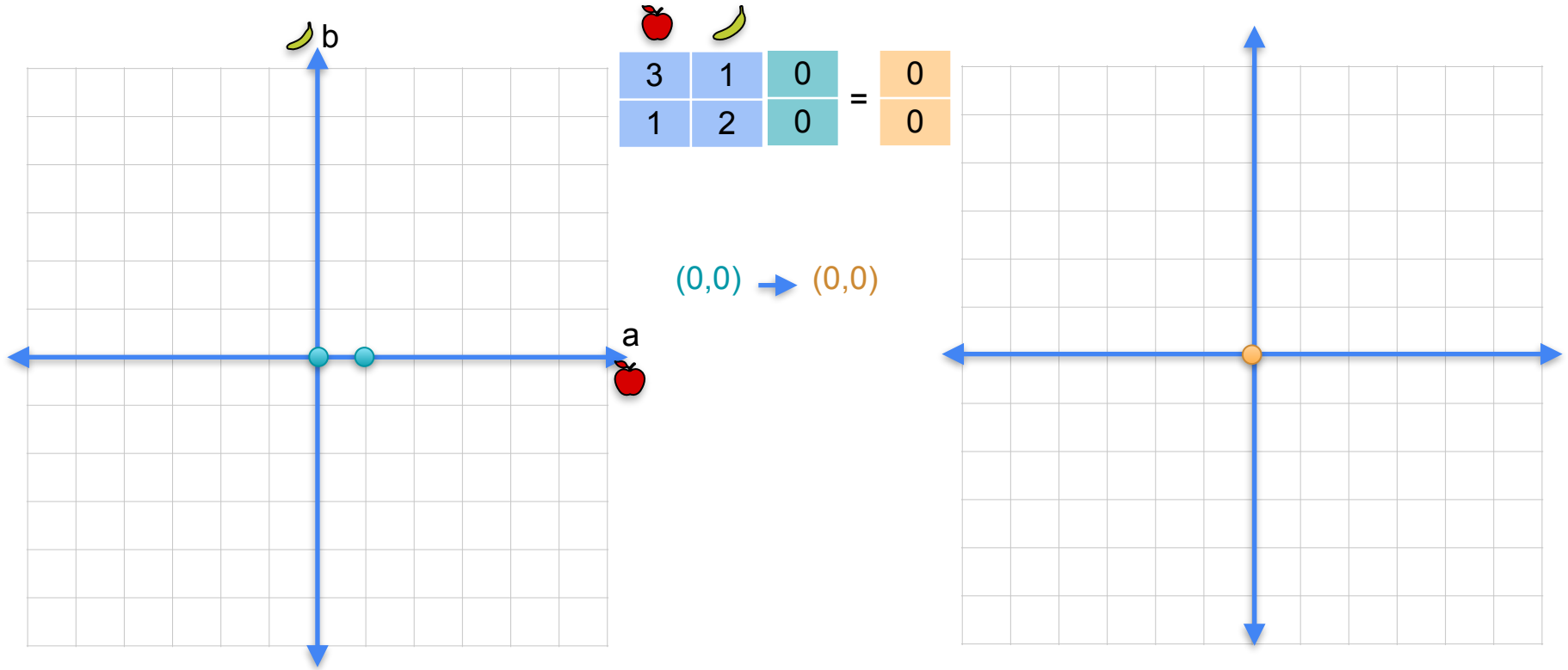




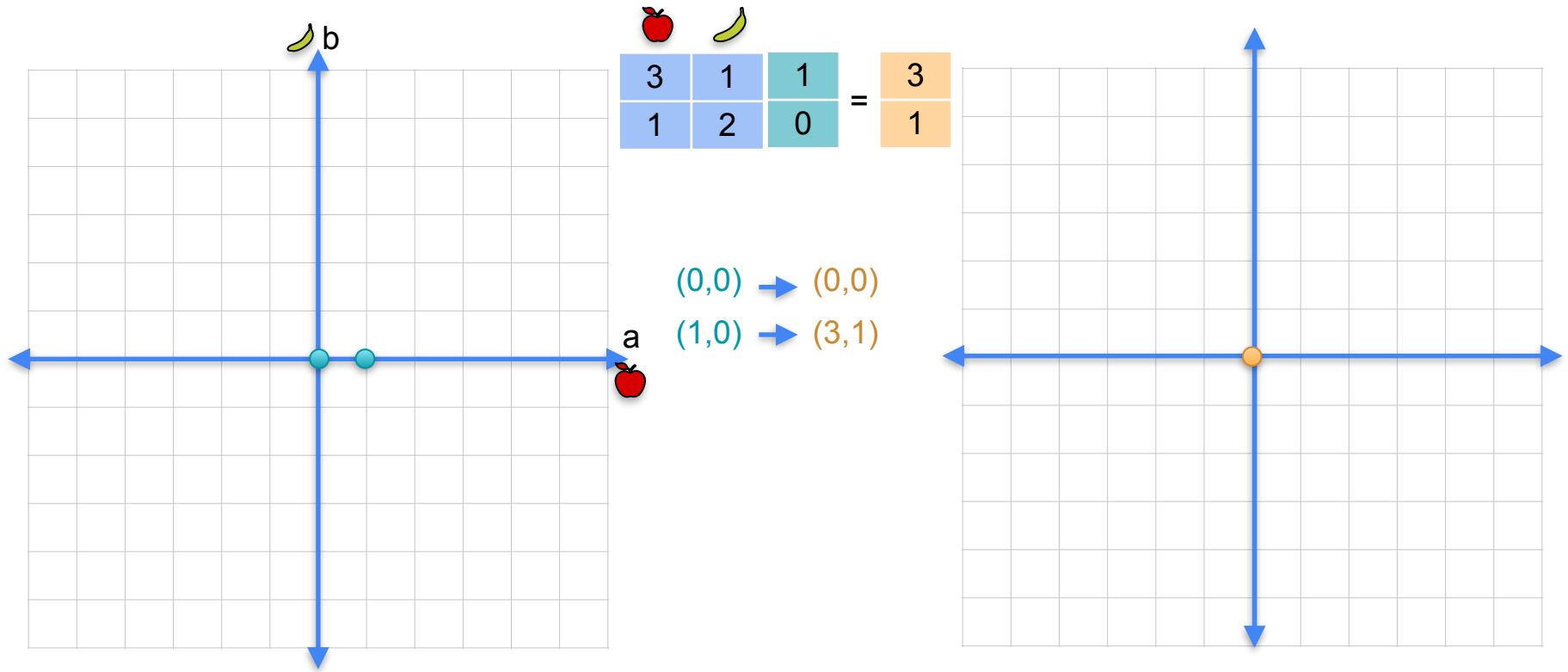
# Matrices as linear transformations



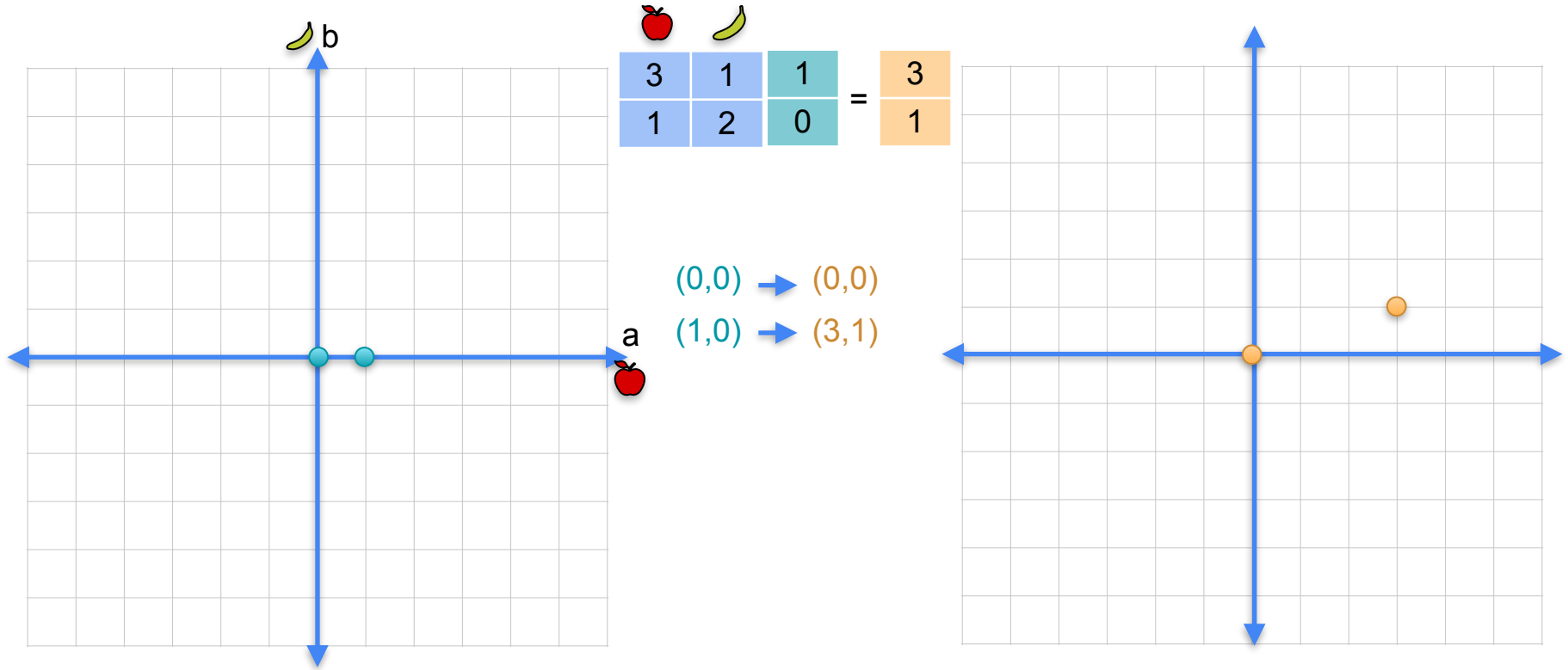
# Matrices as linear transformations



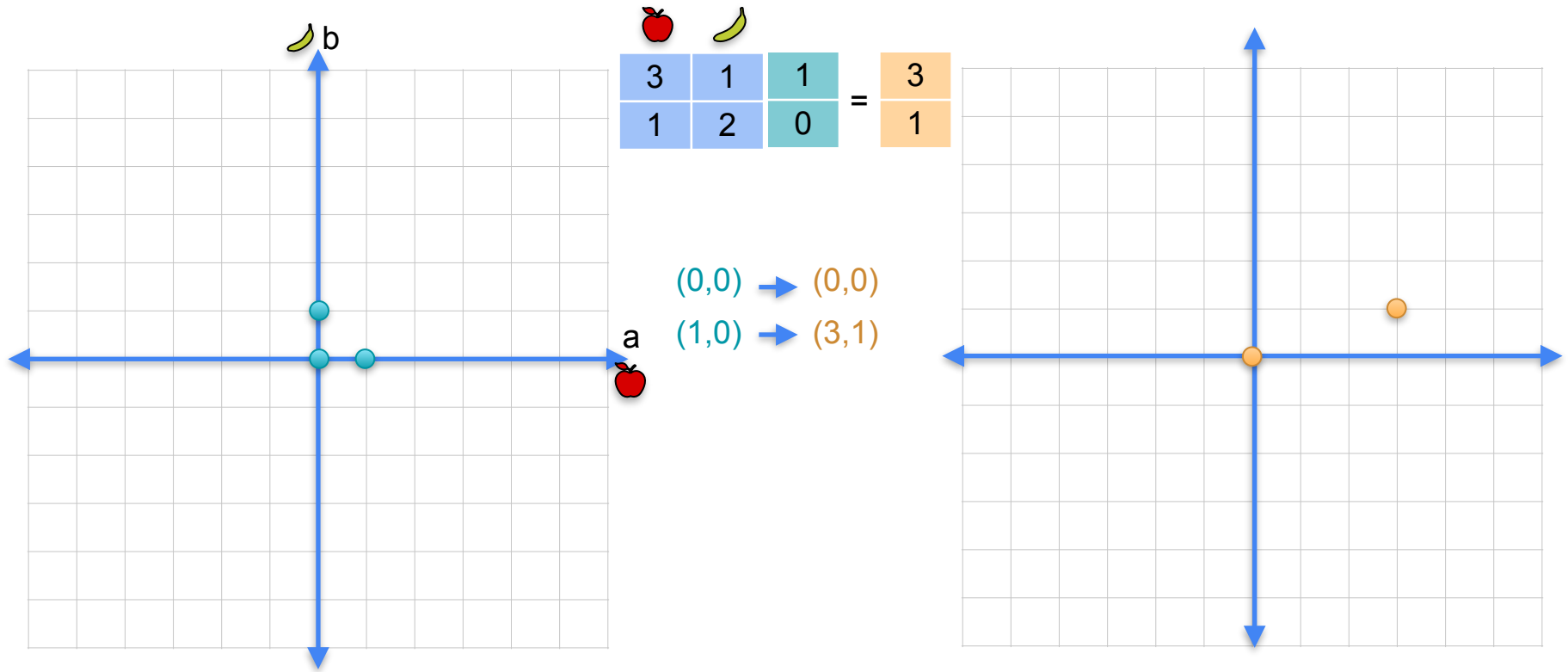
# Matrices as linear transformations



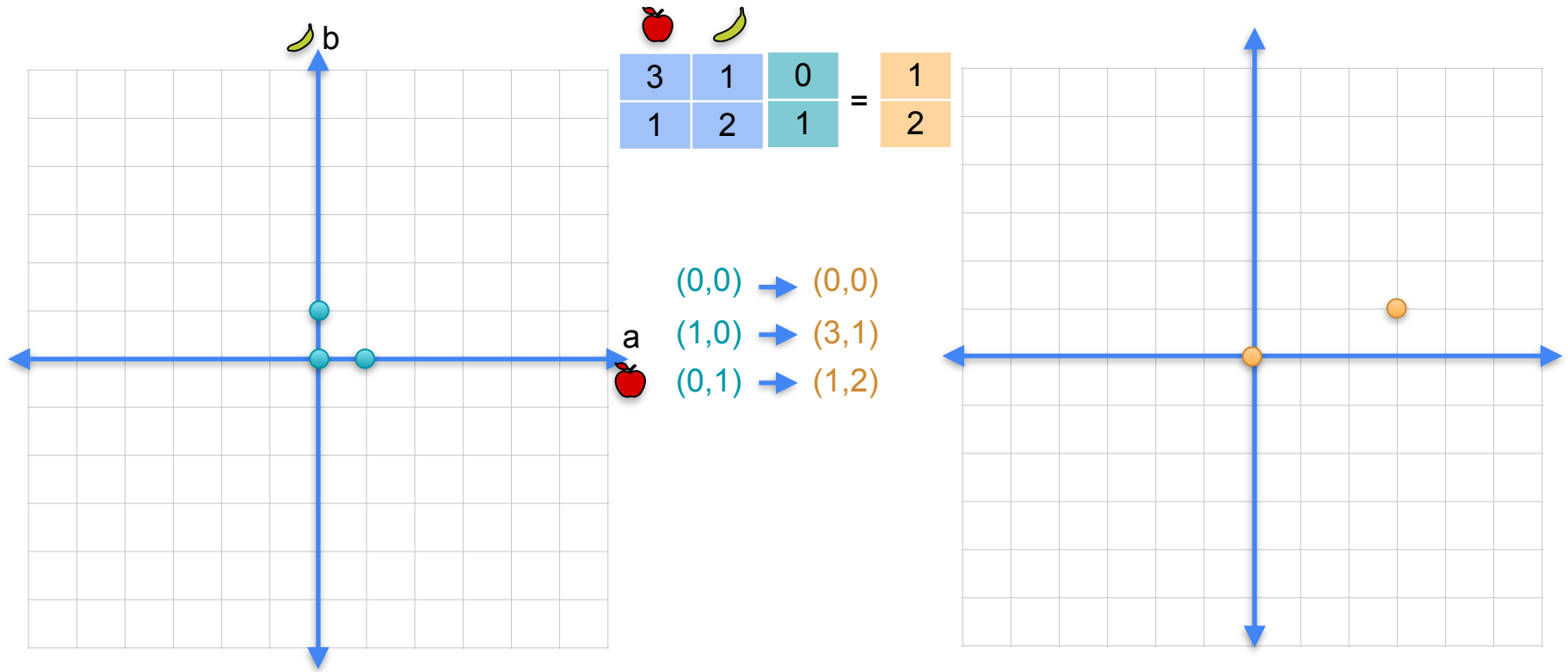
# Matrices as linear transformations



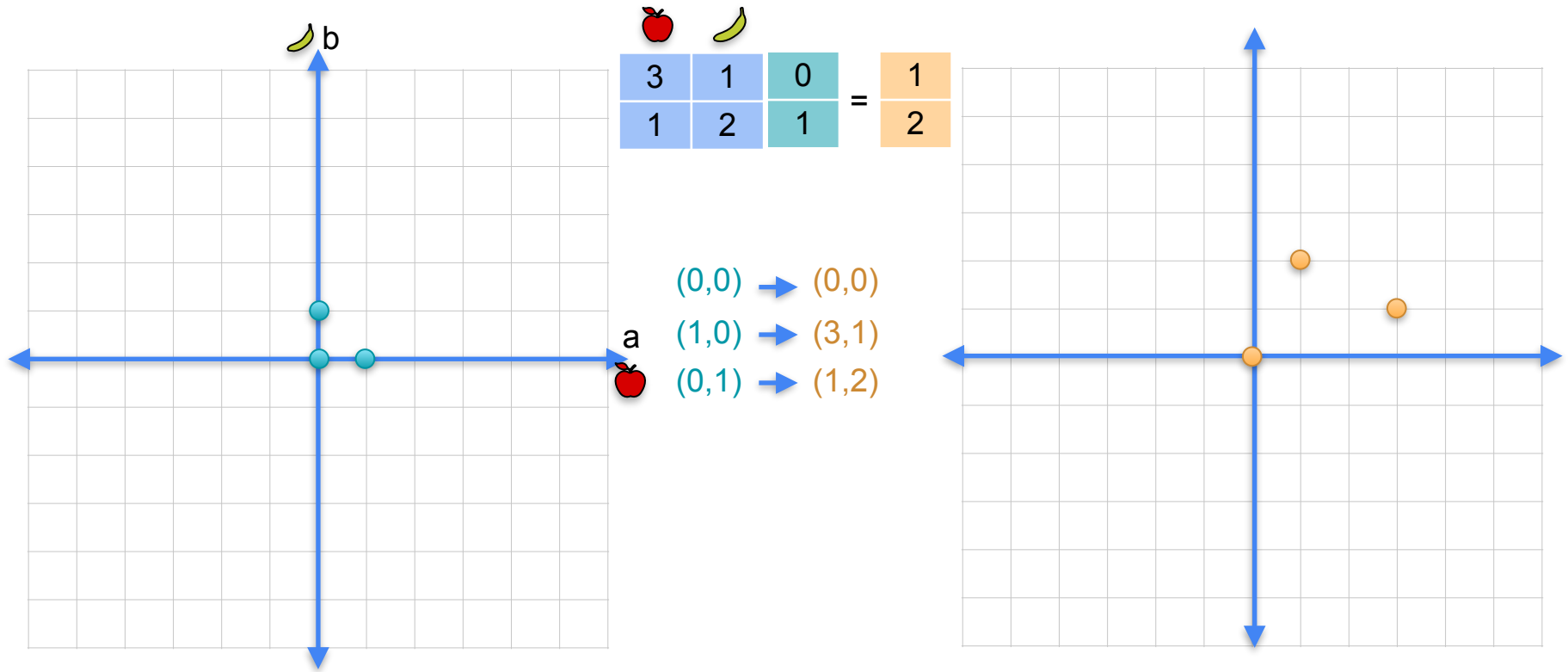
# Matrices as linear transformations



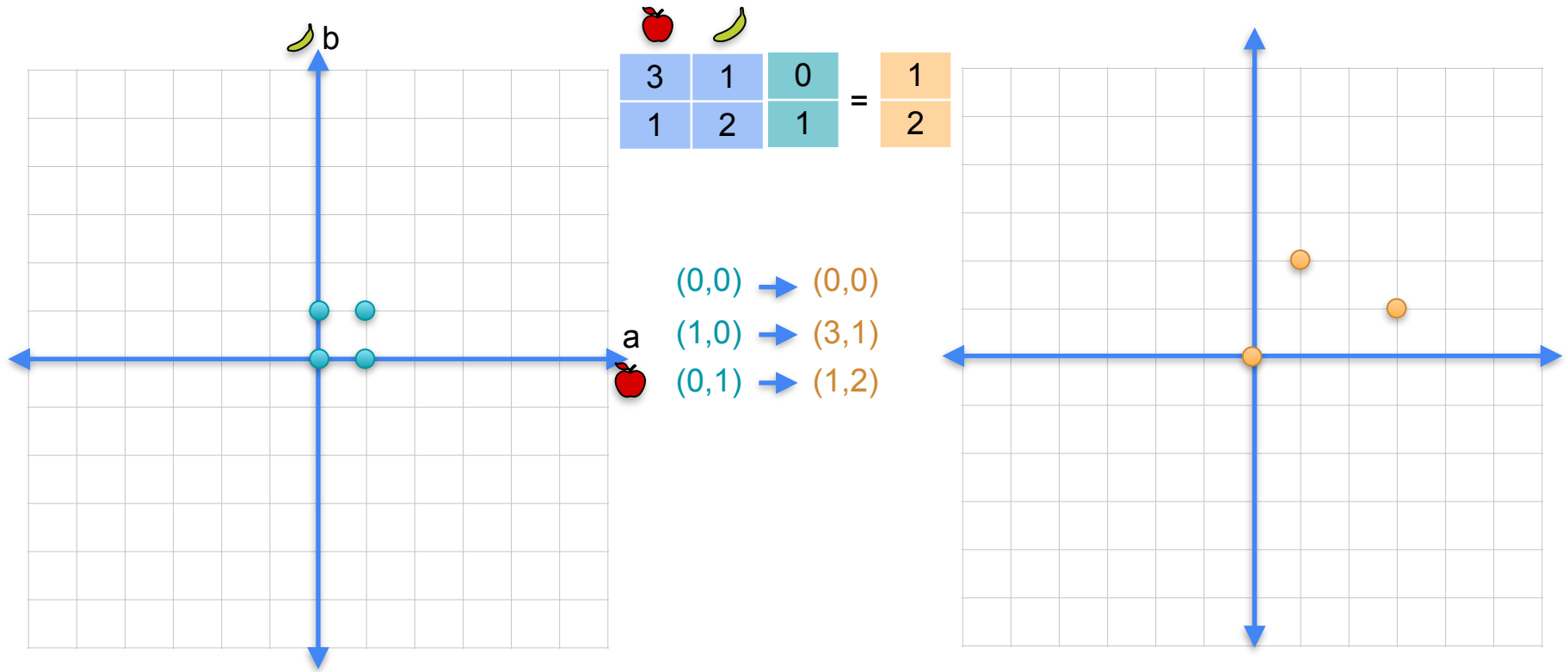
# Matrices as linear transformations



# Matrices as linear transformations

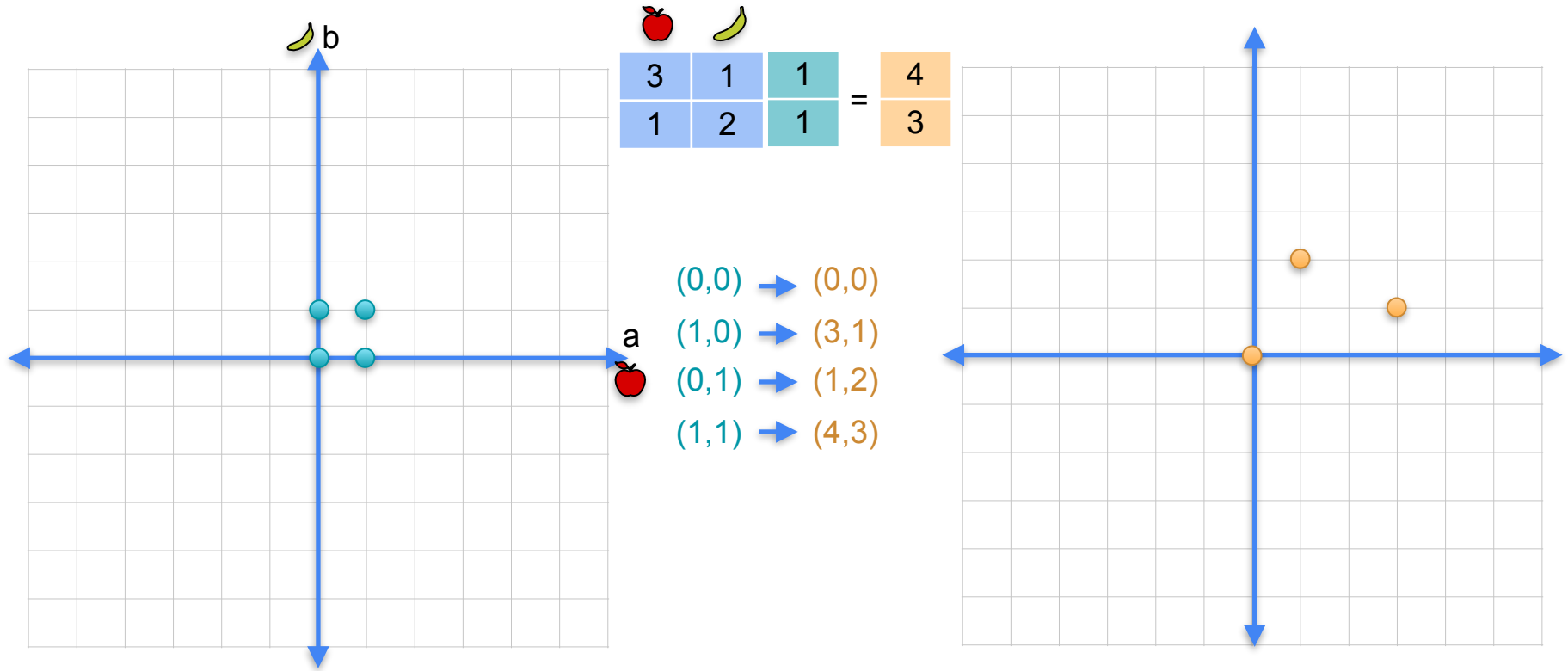


# Matrices as linear transformations

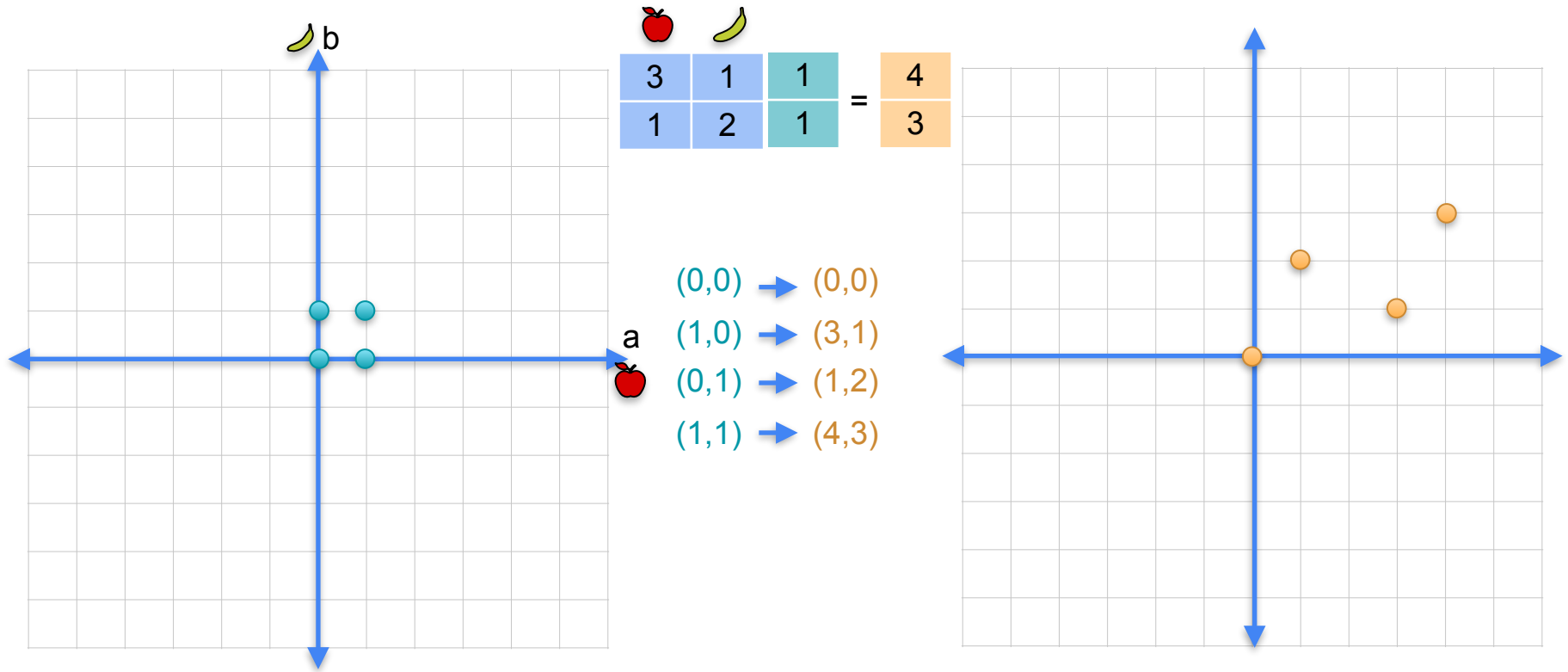




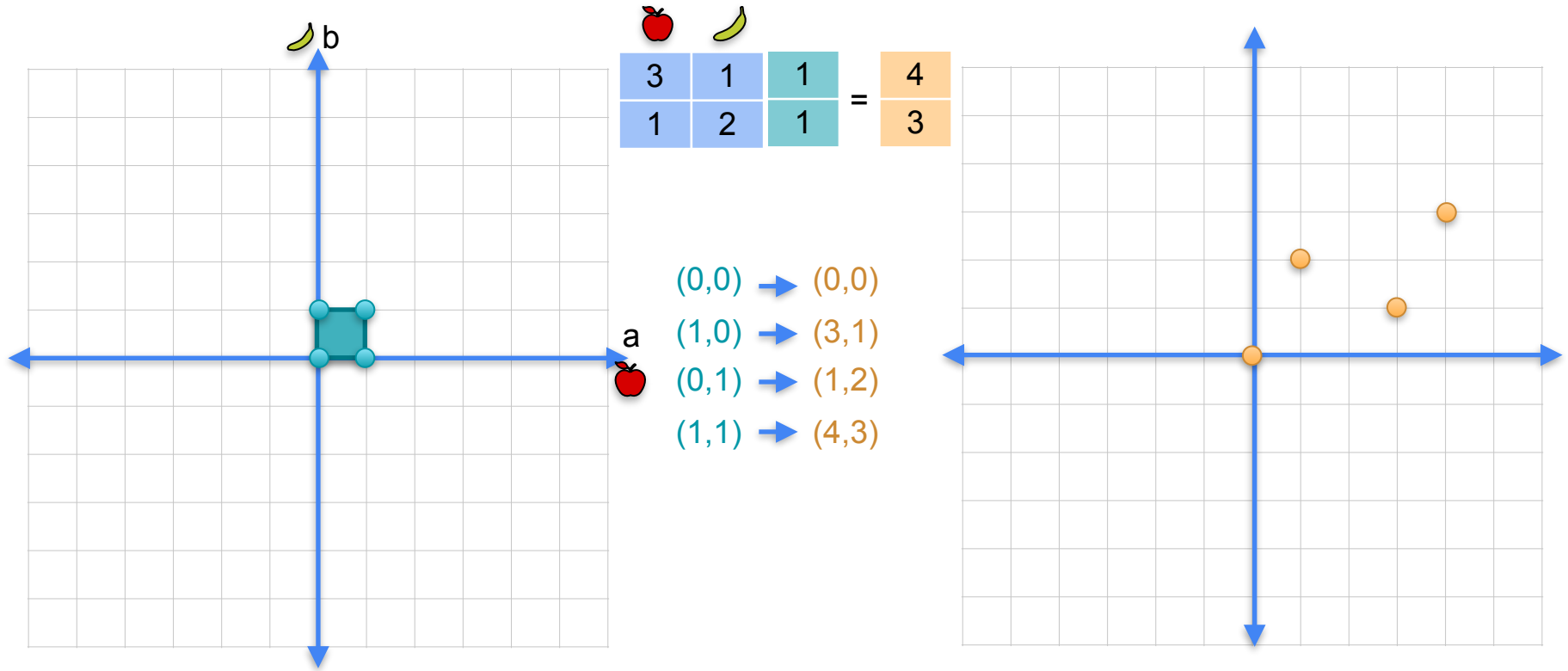
# Matrices as linear transformations



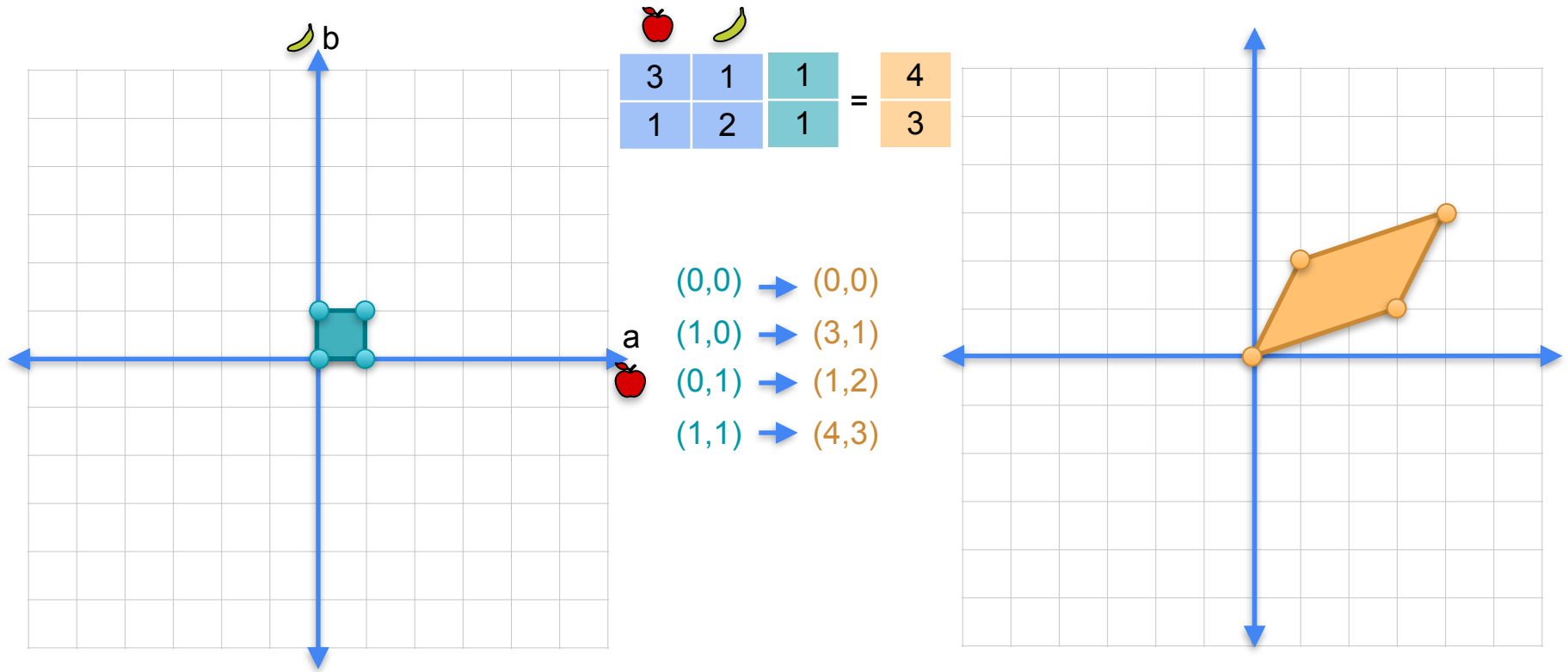
# Matrices as linear transformations



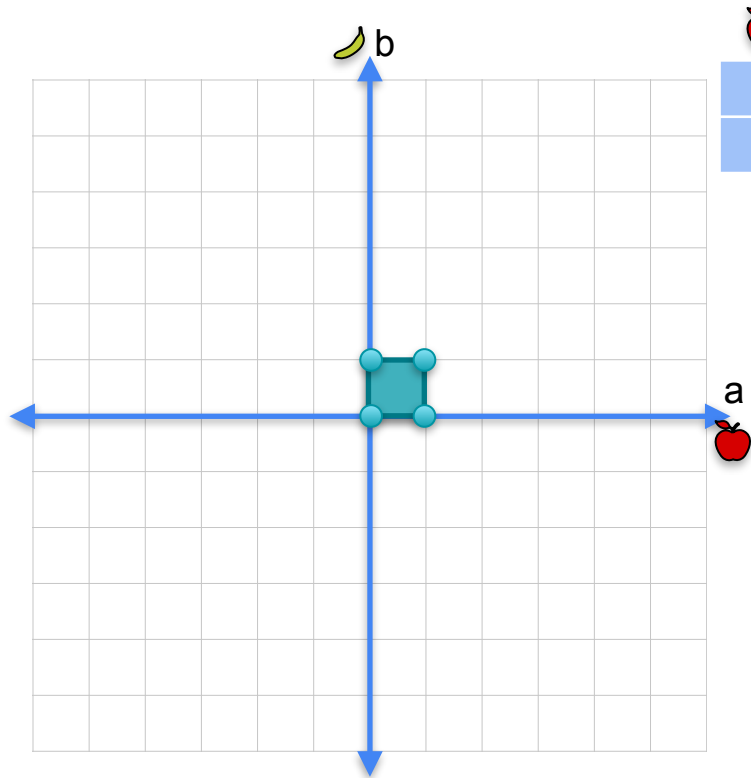
# Matrices as linear transformations



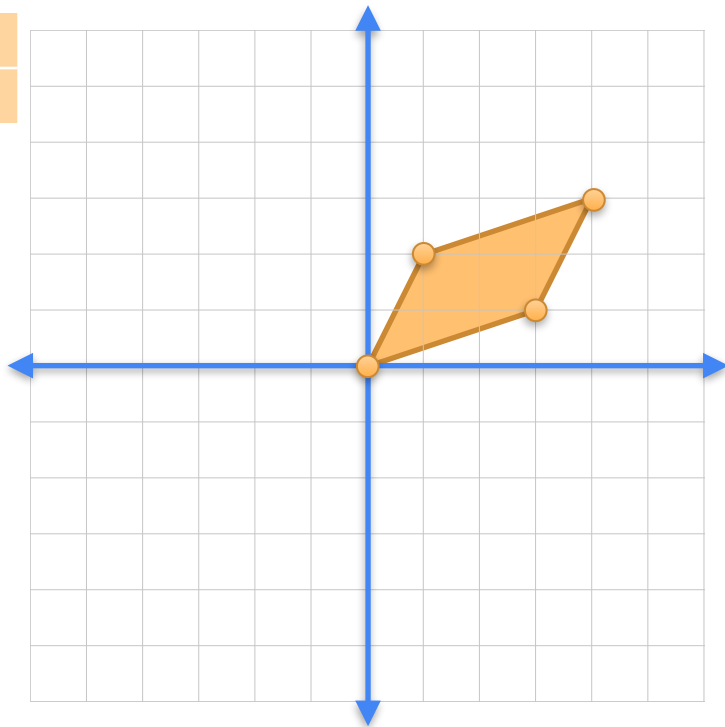
# Matrices as linear transformations



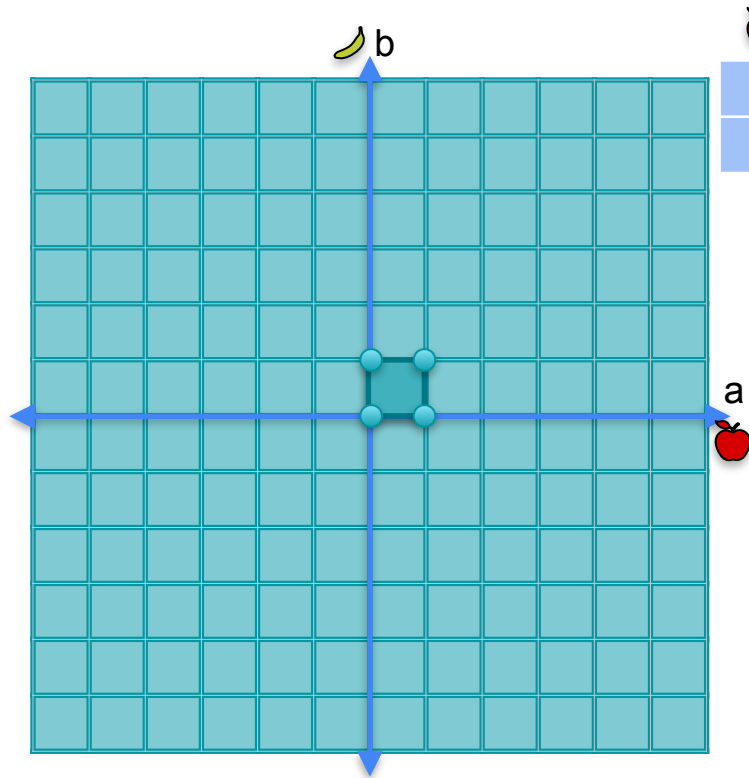
# Matrices as linear transformations



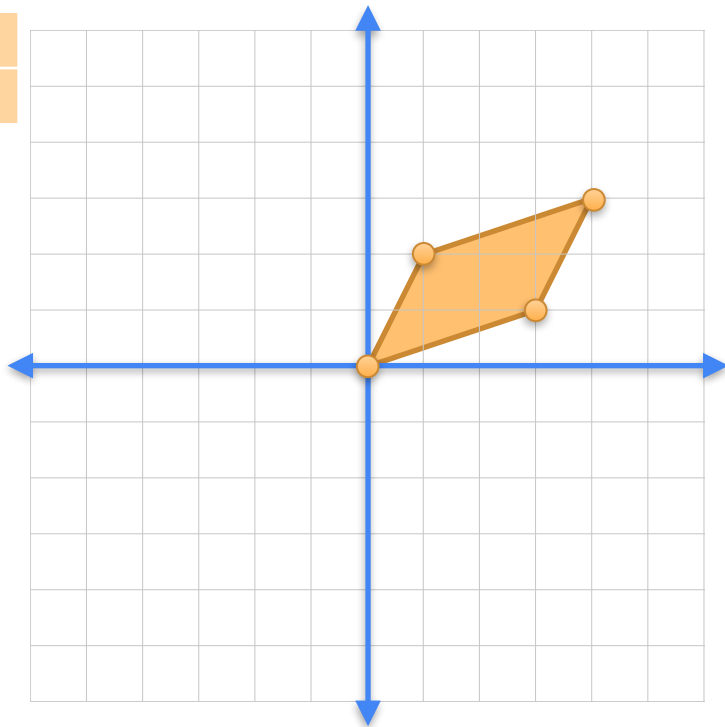
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



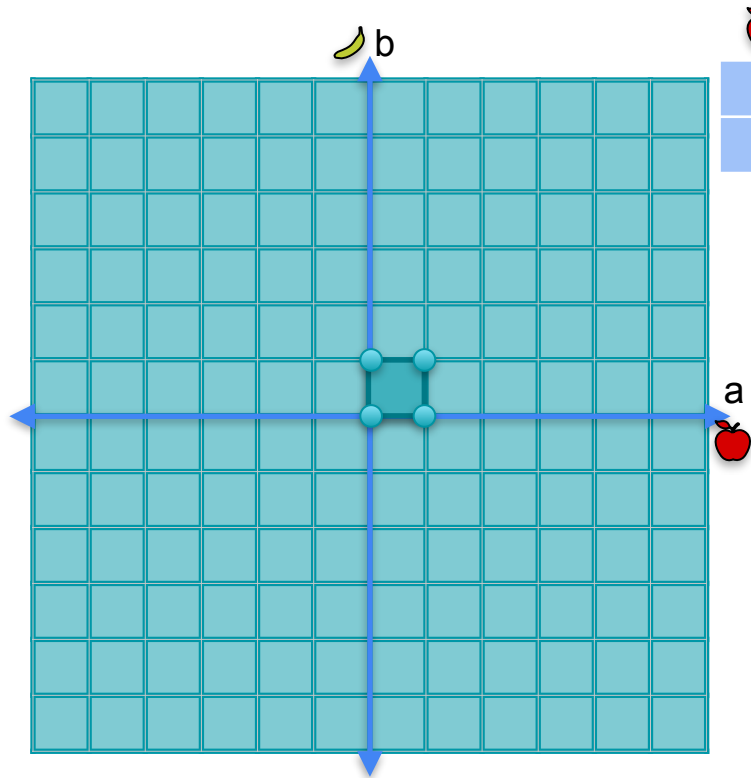
# Matrices as linear transformations



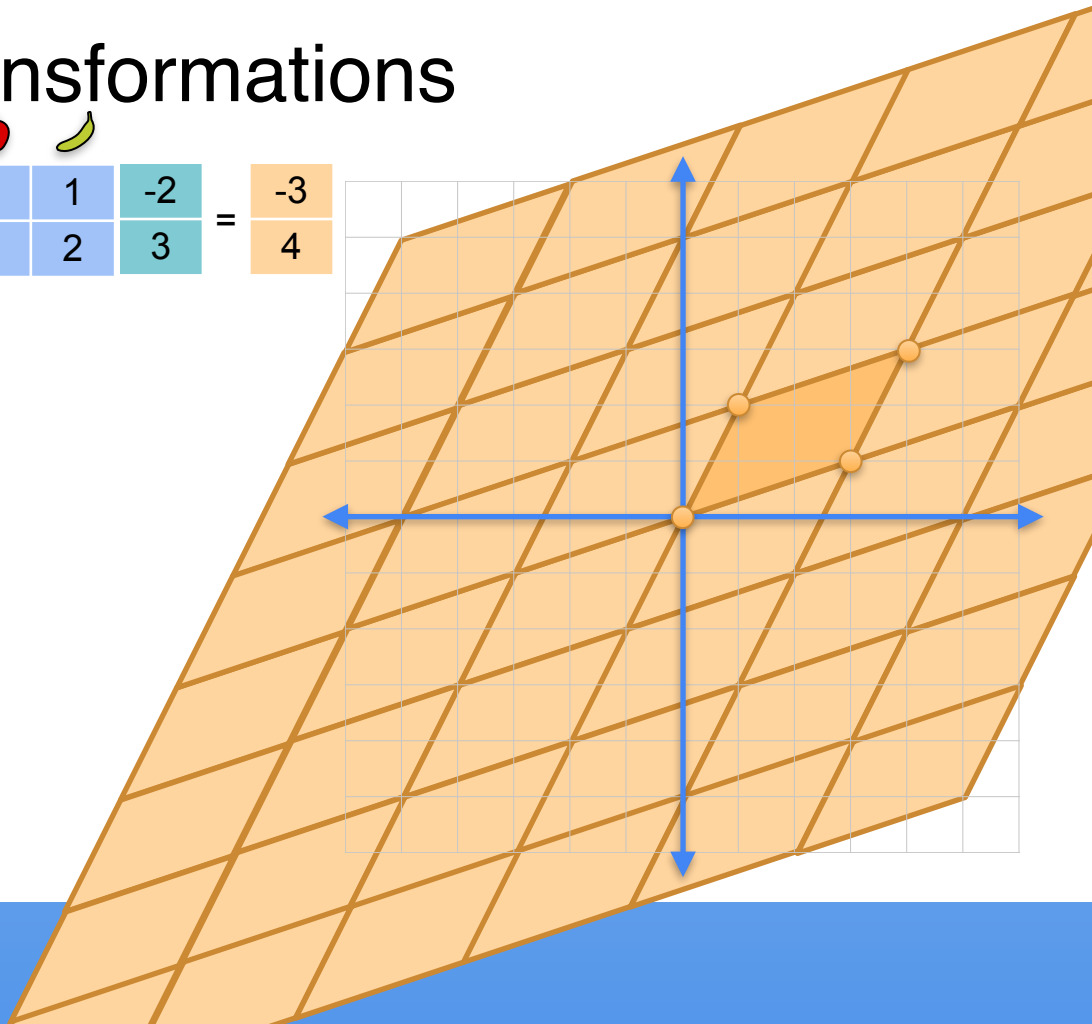
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



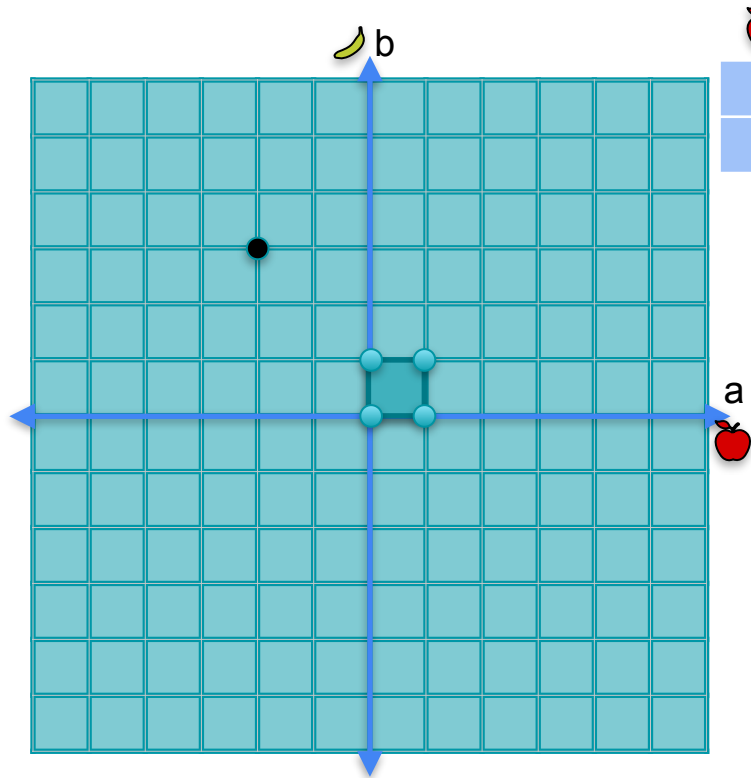
# Matrices as linear transformations



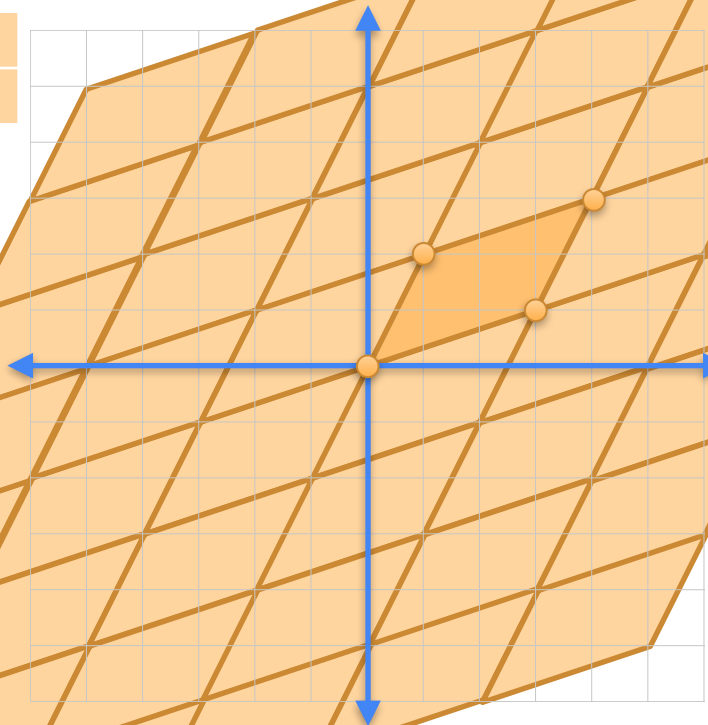
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



# Matrices as linear transformations

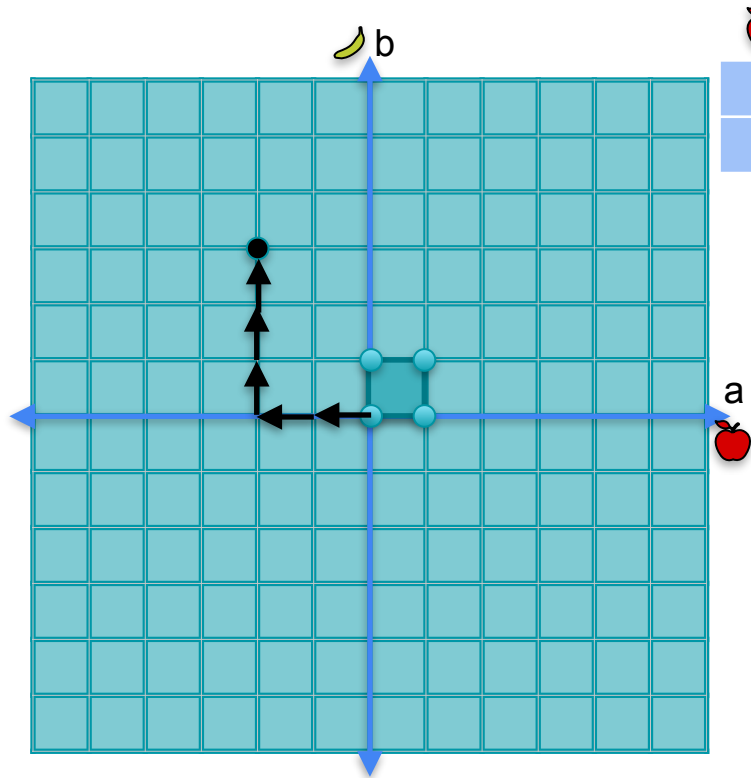


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

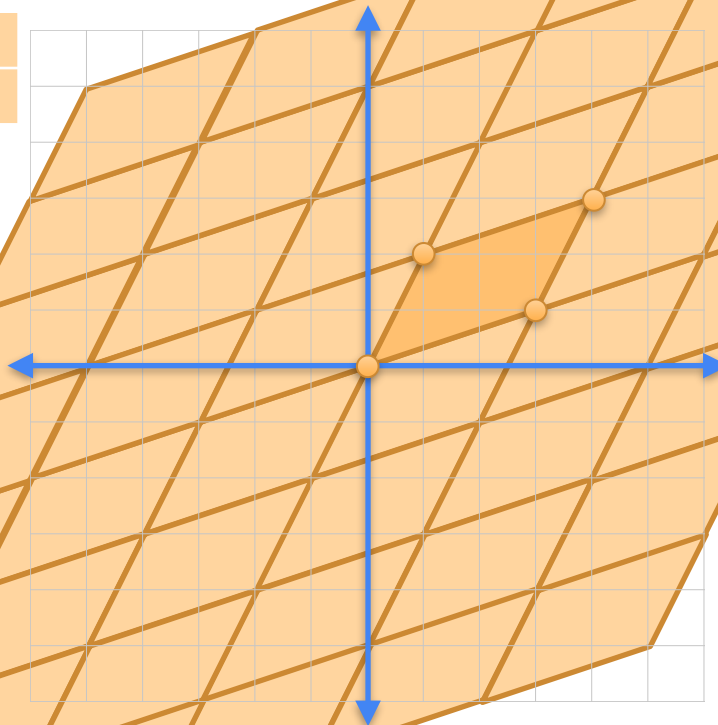




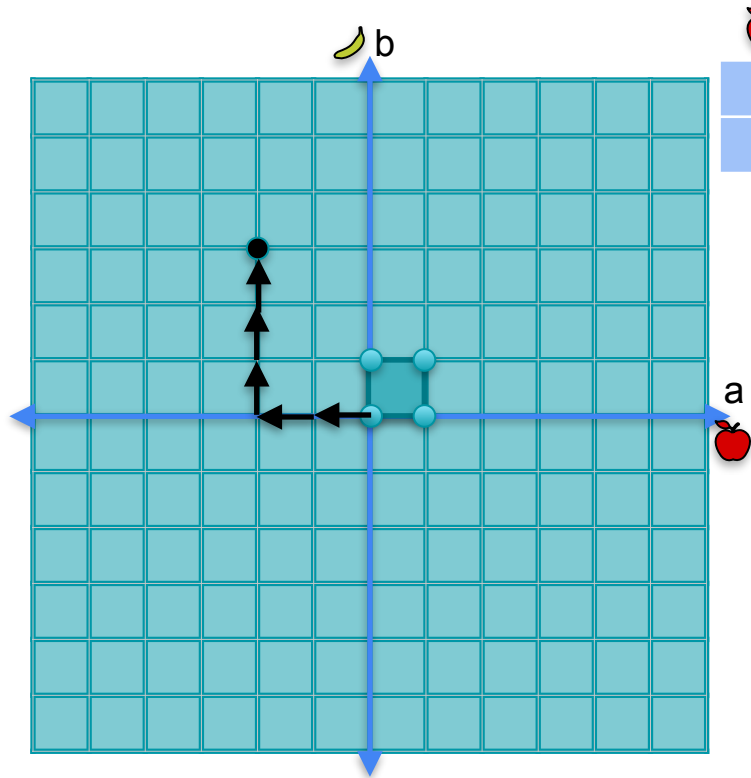
# Matrices as linear transformations



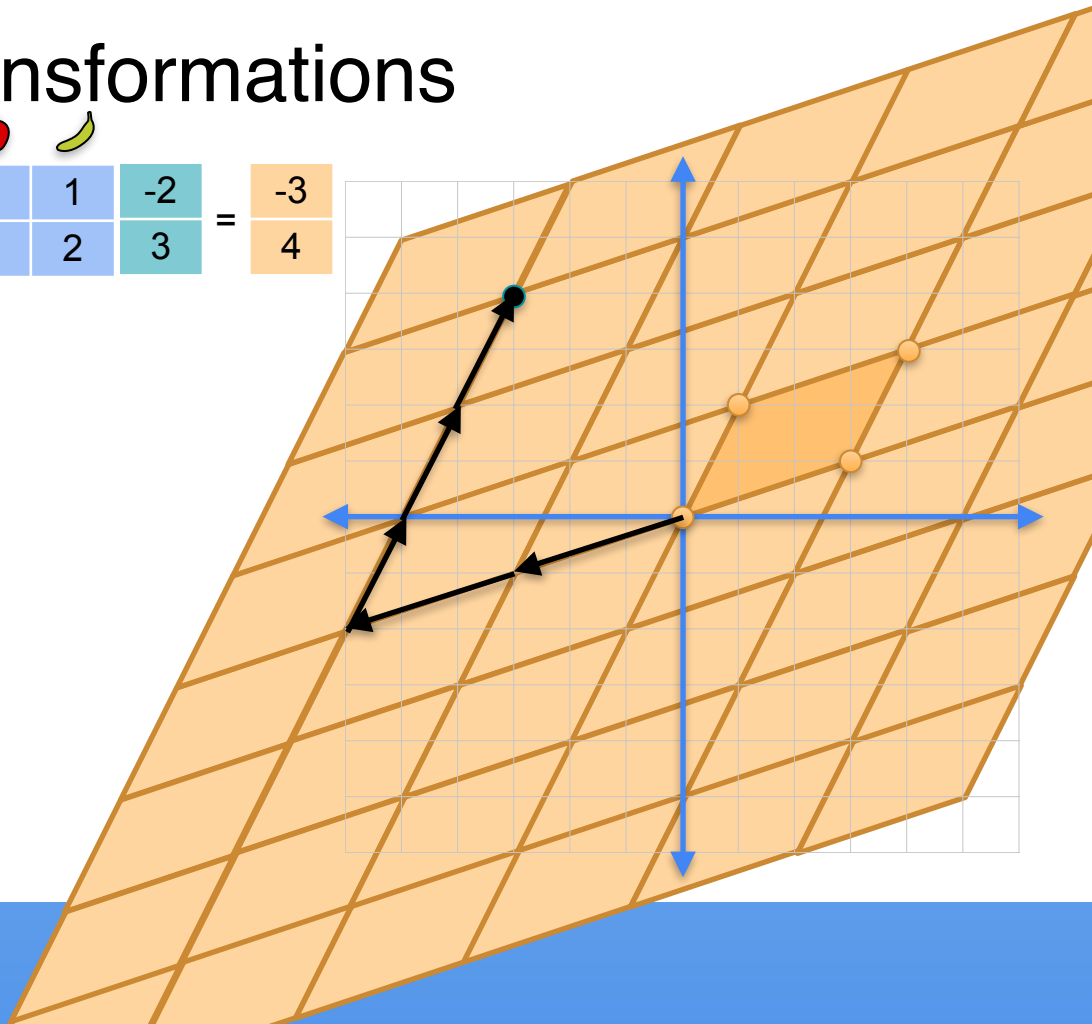
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



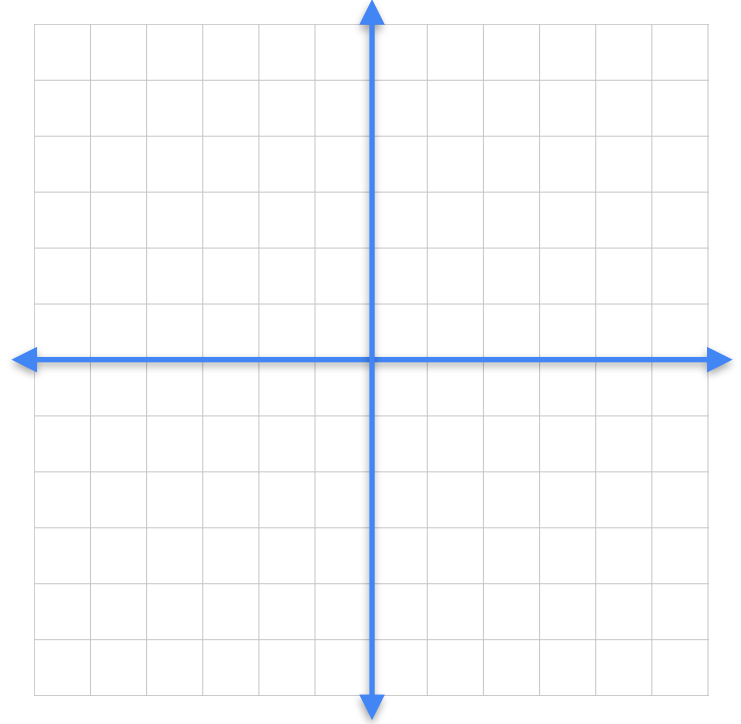
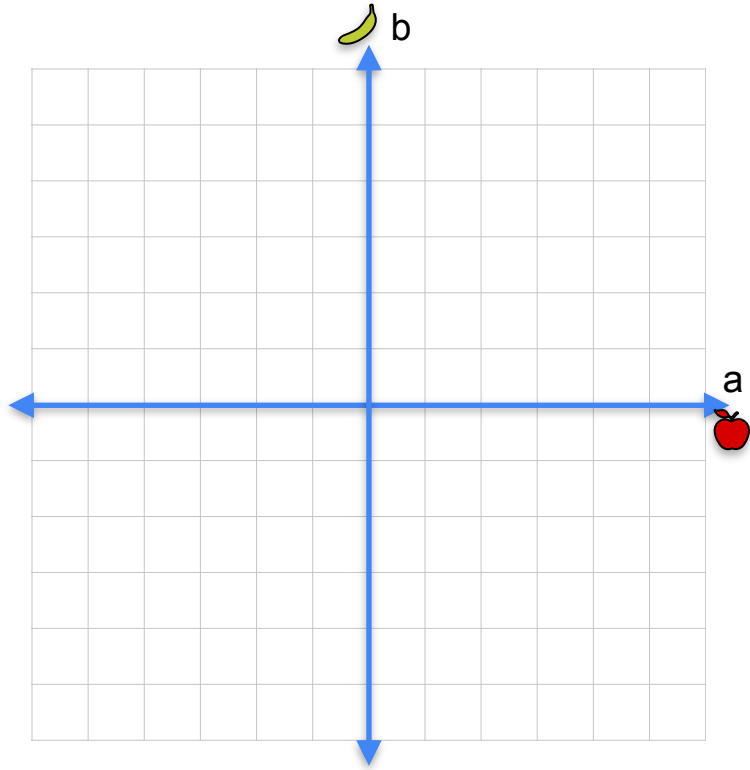
# Matrices as linear transformations



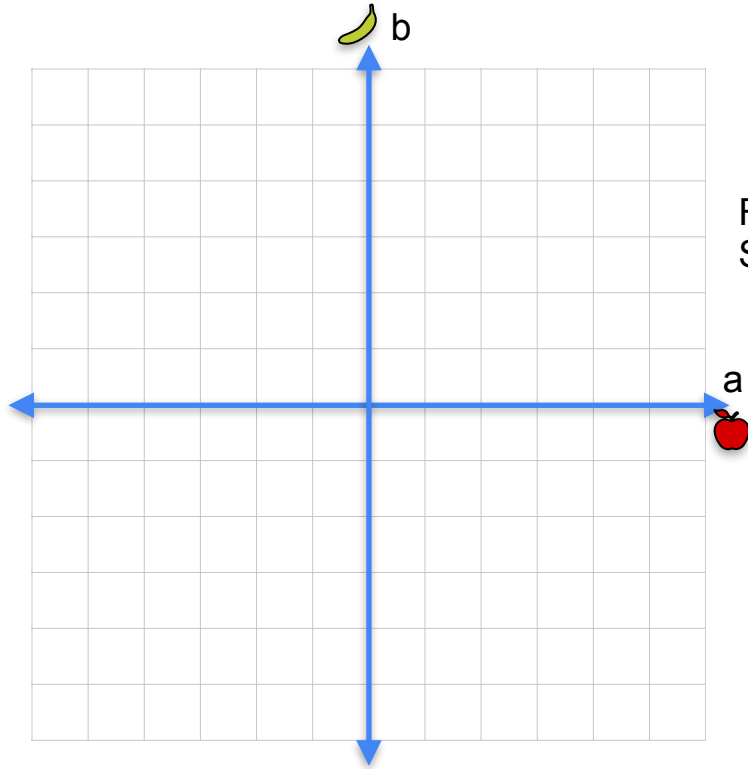
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



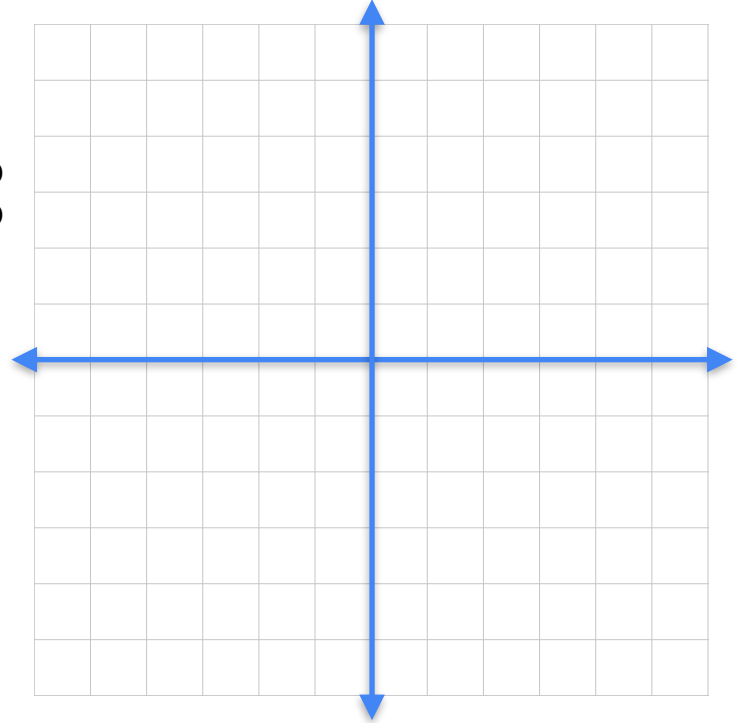
# Systems of equations as linear transformations



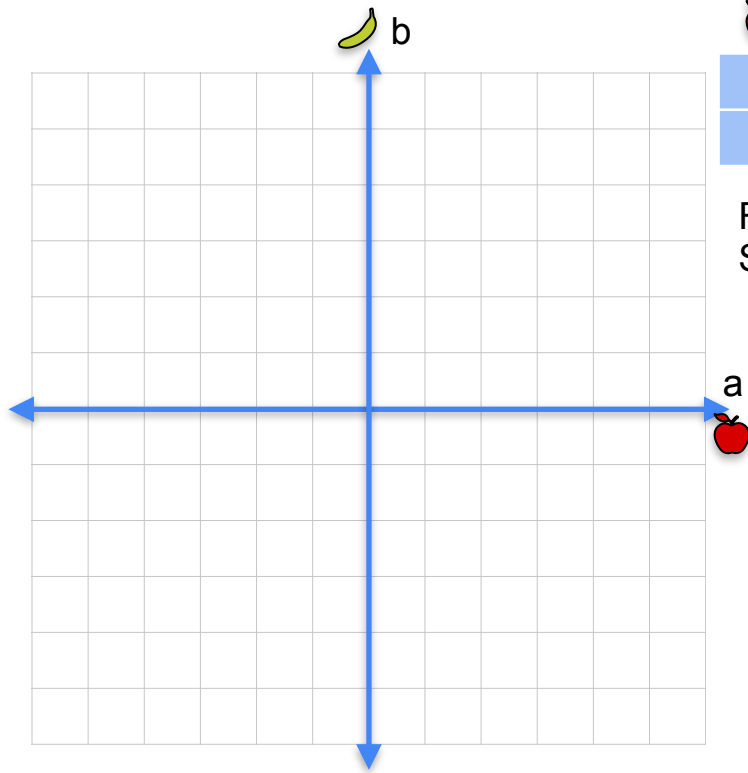
# Systems of equations as linear transformations



First day:  $3a + b$   
Second day:  $a + 2b$



# Systems of equations as linear transformations

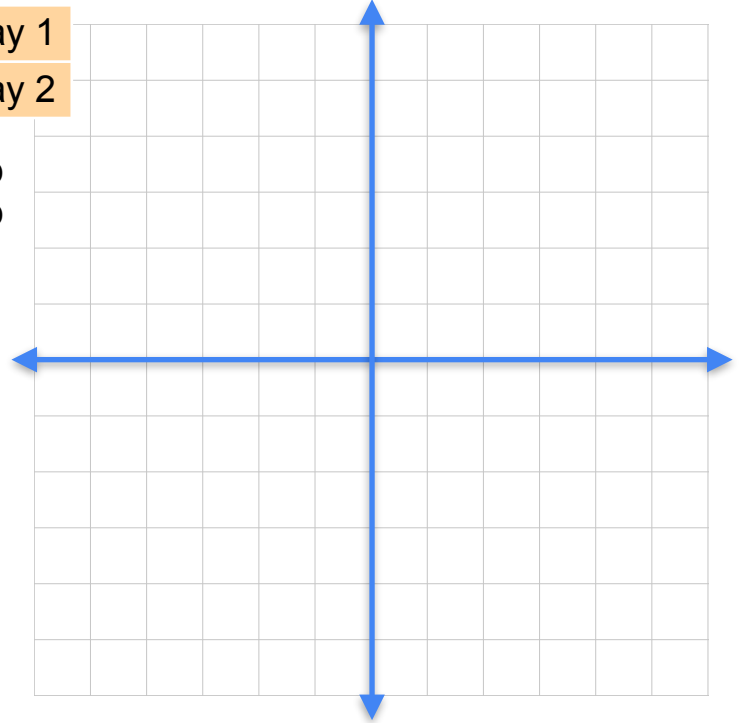


3	1	a
1	2	b

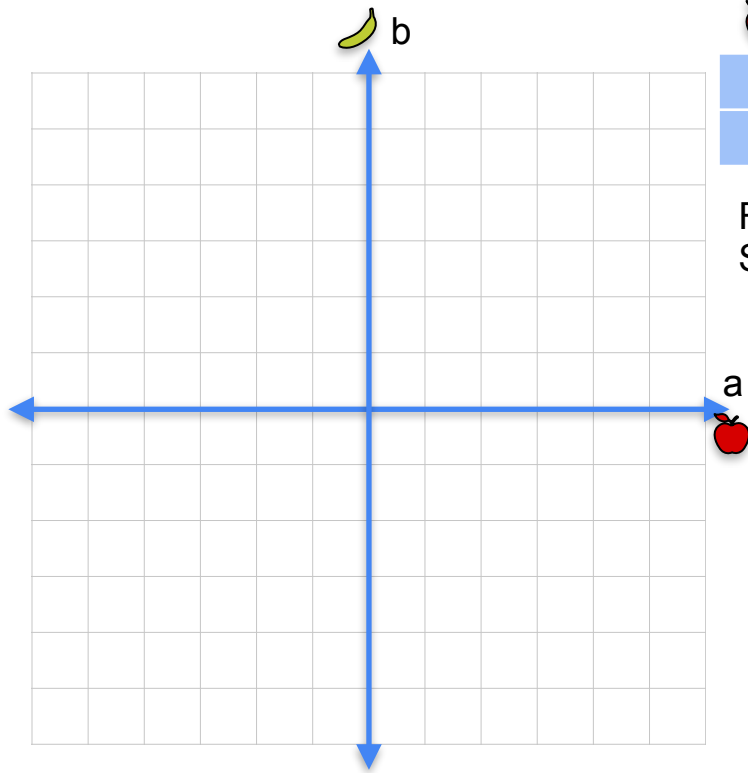
 = 

Day 1
Day 2

First day:  $3a + b$   
Second day:  $a + 2b$

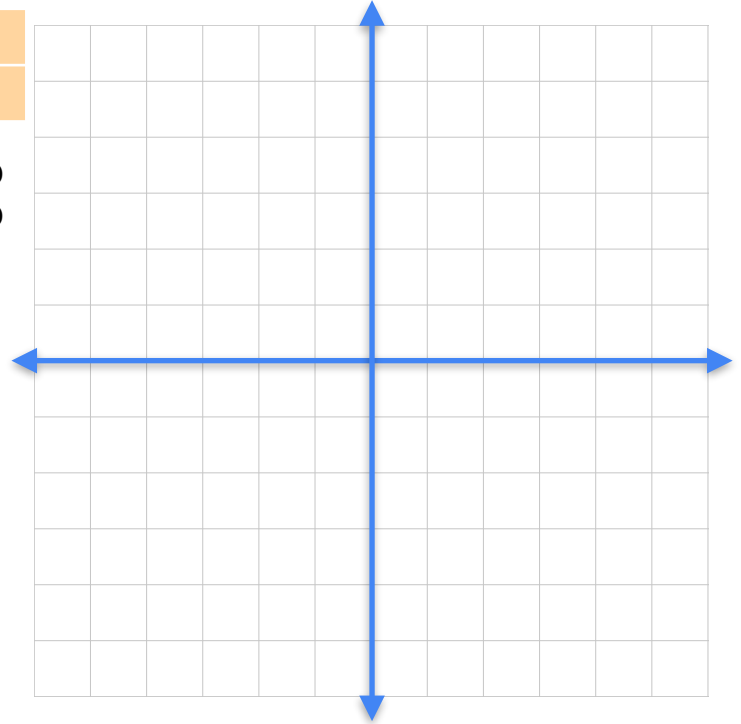


# Systems of equations as linear transformations



3	1	1	=	4
1	2	1	=	3

First day:  $3a + b$   
Second day:  $a + 2b$



# Systems of equations as linear transformations

 b



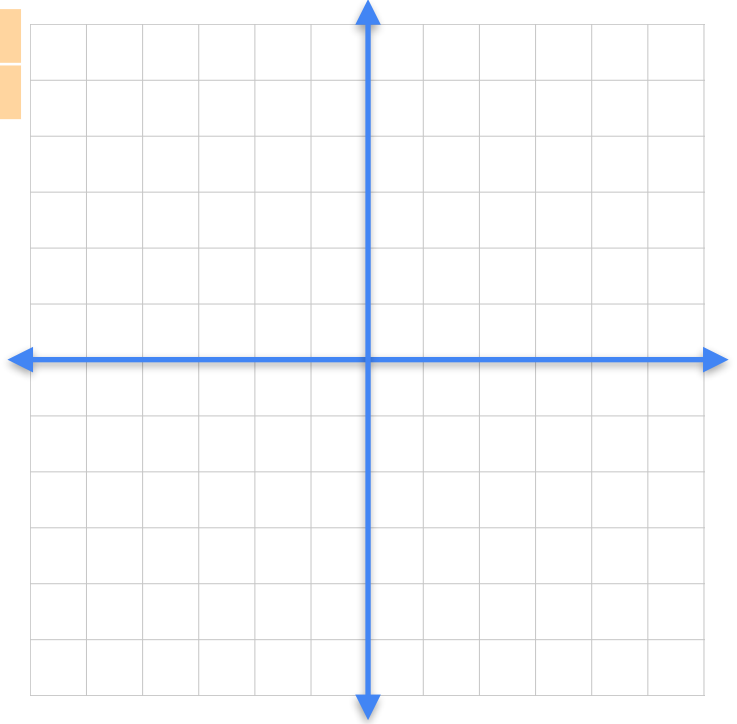
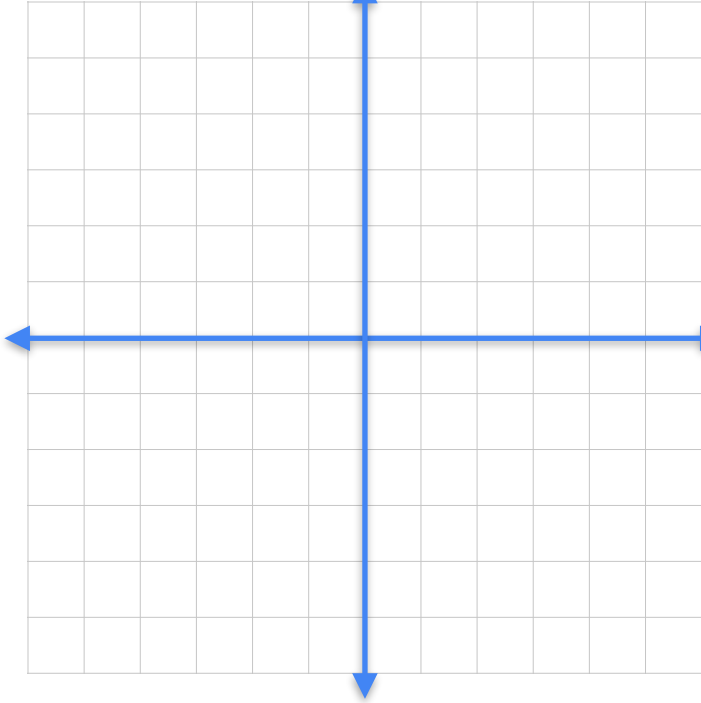
3	1	1	=	4
1	2	1	=	3

First day:  $3a + b$

Second day:  $a + 2b$

a

$(1,1) \rightarrow (4,3)$



# Systems of equations as linear transformations


 b



3	1	1	=	4
1	2	1	=	3

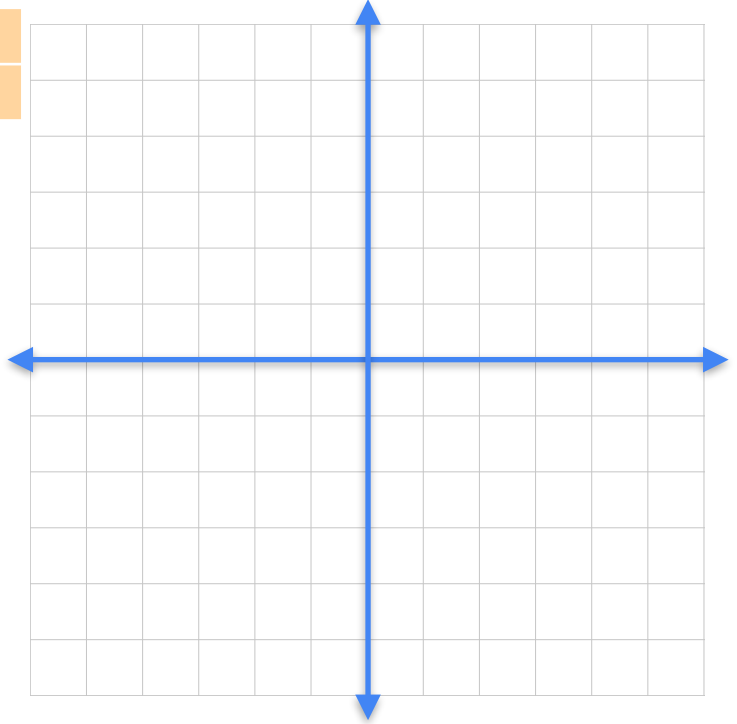
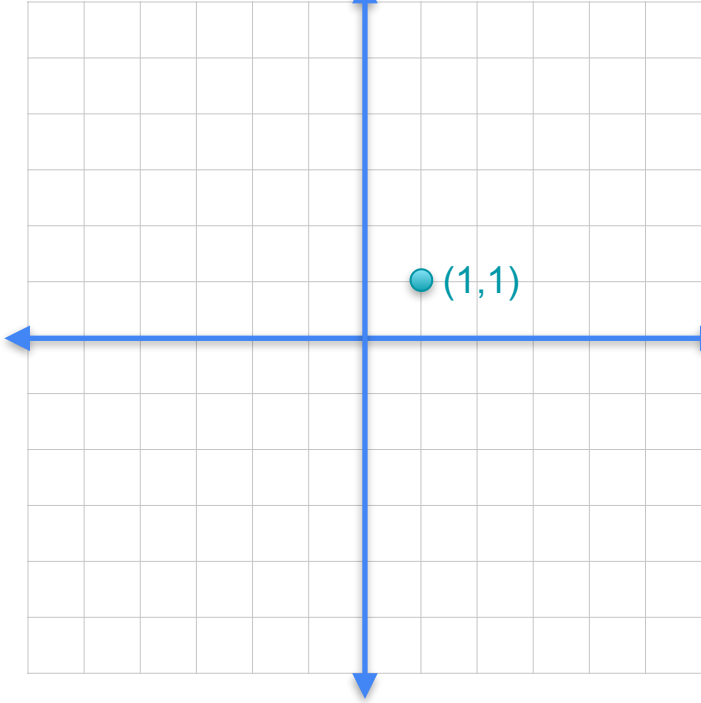
First day:  $3a + b$

Second day:  $a + 2b$

 (1,1)

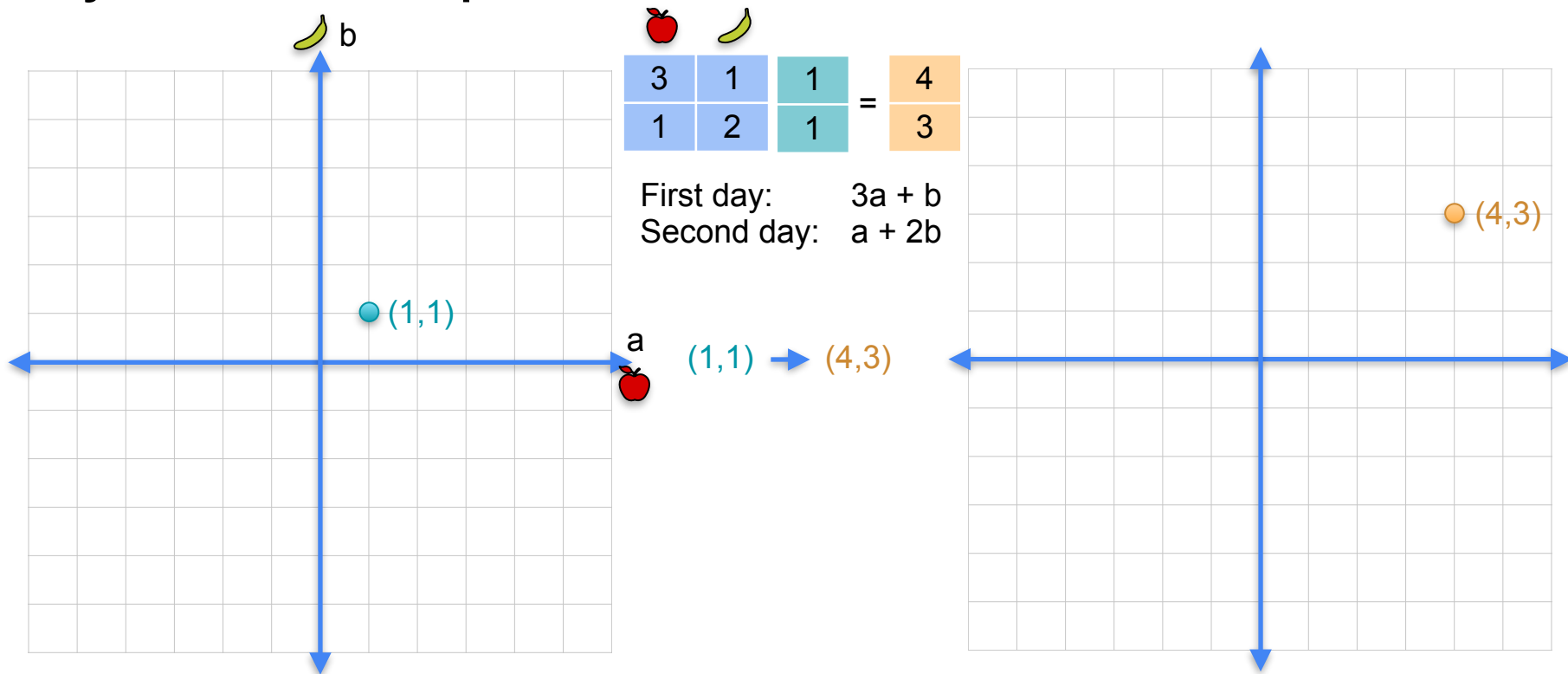
a

(1,1) → (4,3)

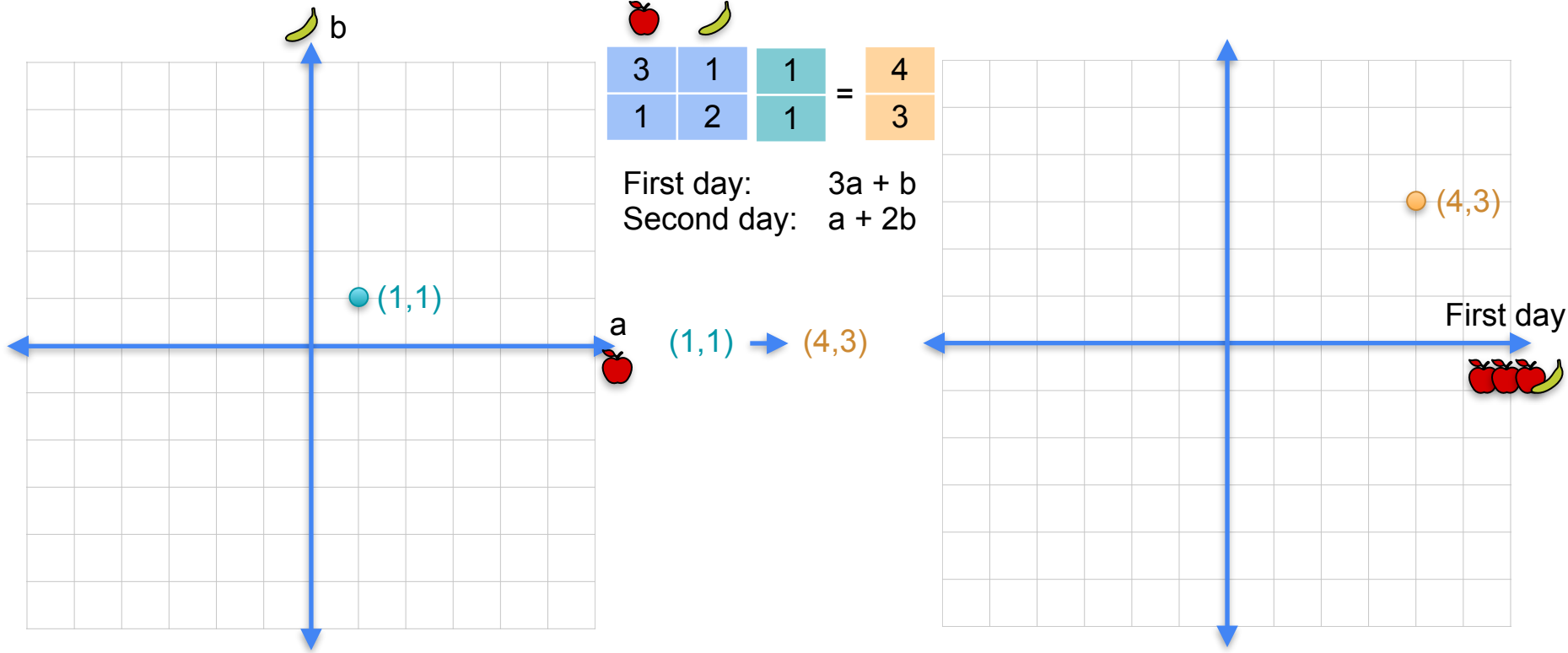




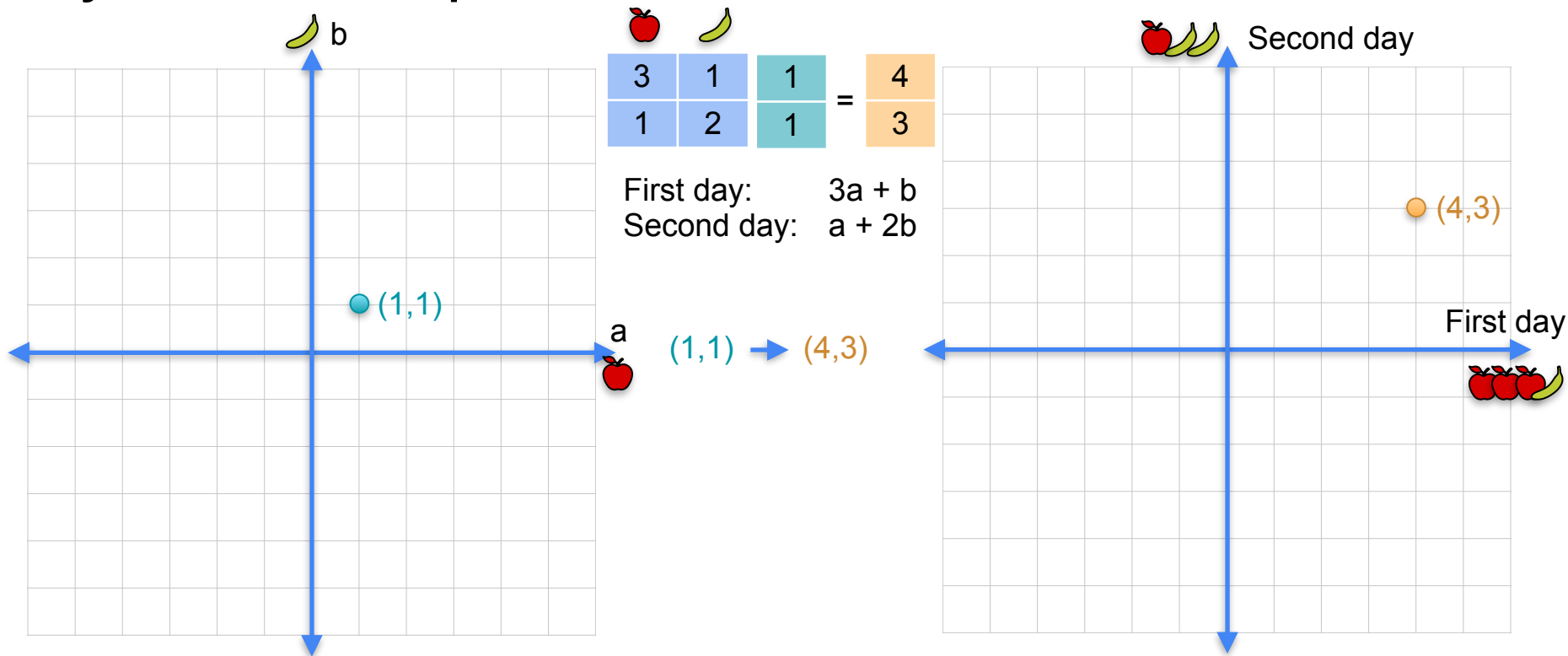
# Systems of equations as linear transformations



# Systems of equations as linear transformations



# Systems of equations as linear transformations





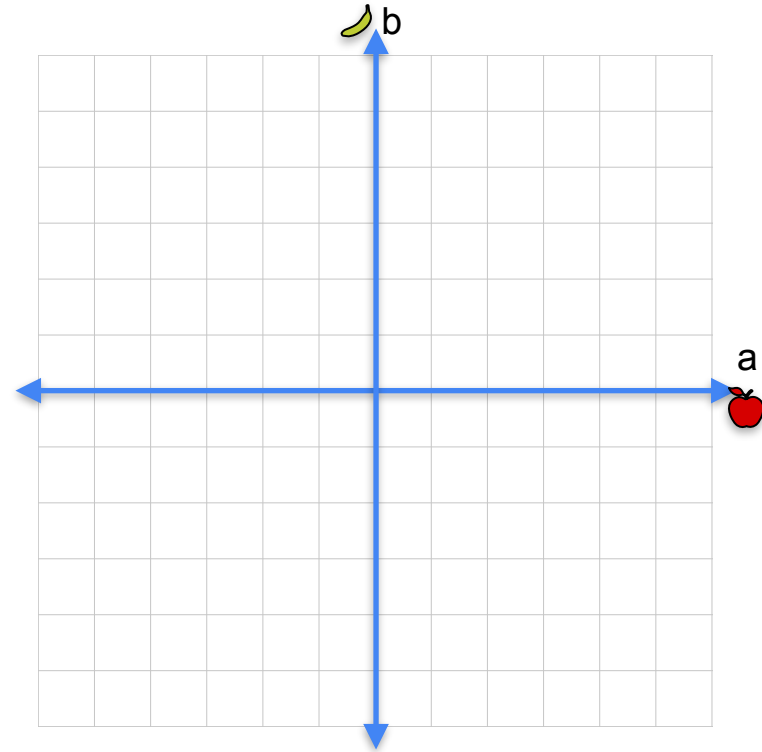
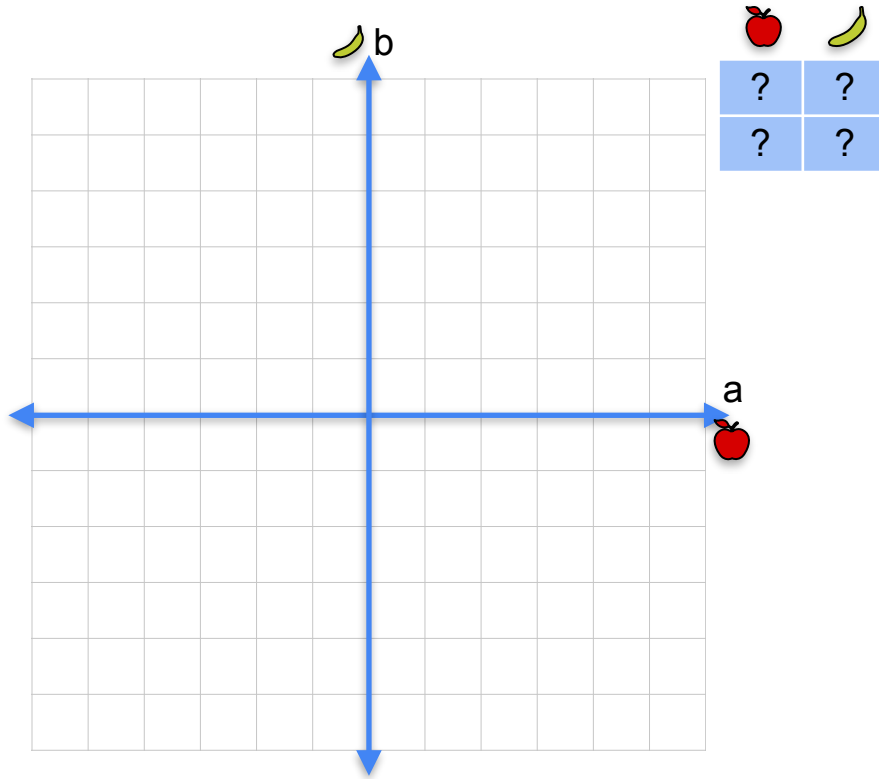
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# Vectors and Linear Transformations

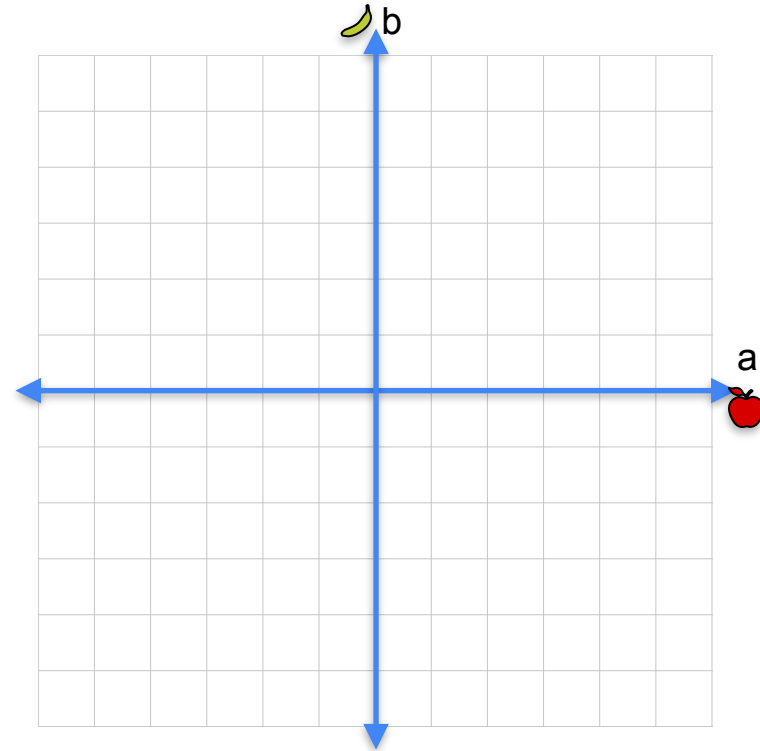
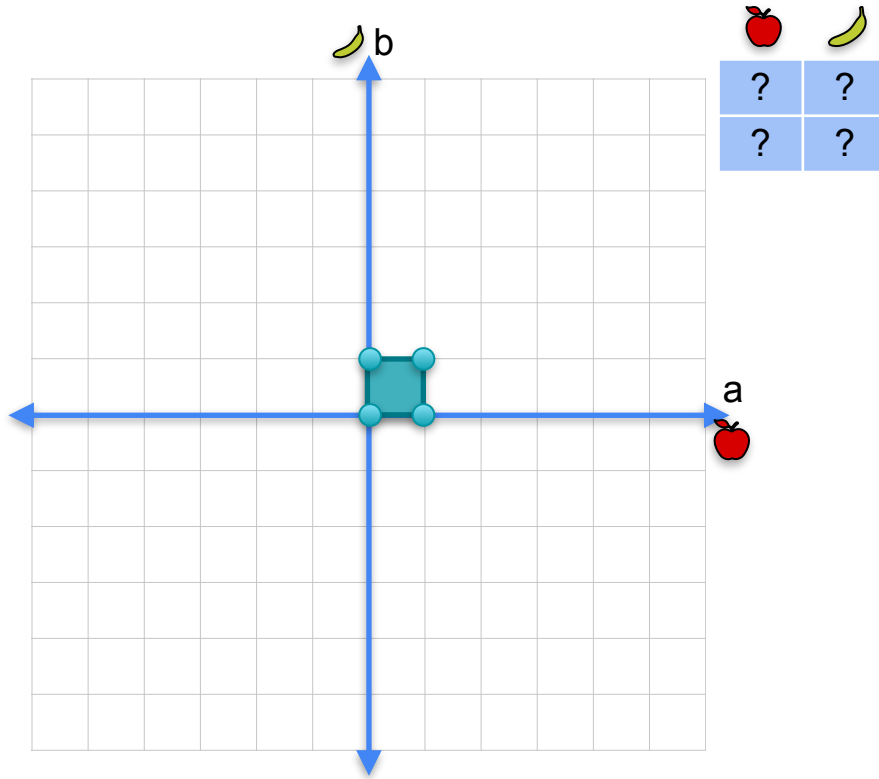
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**Linear transformations as  
matrices**

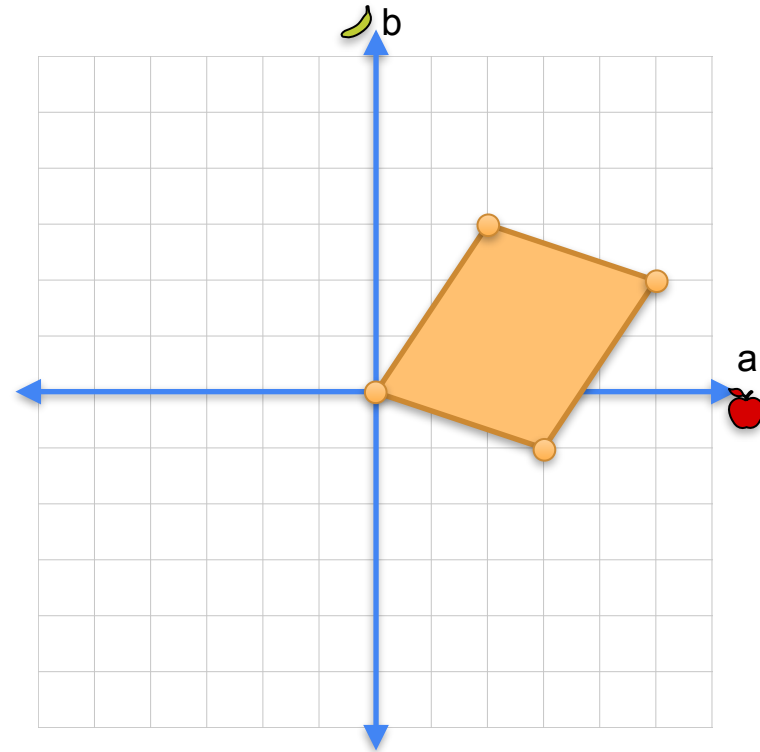
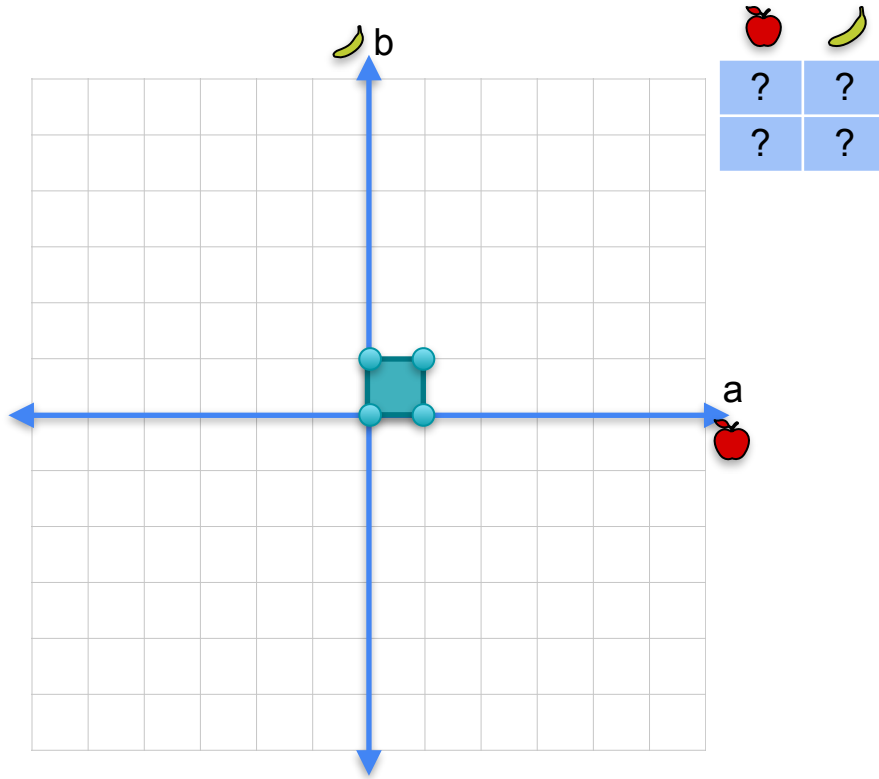
# Linear transformations as matrices



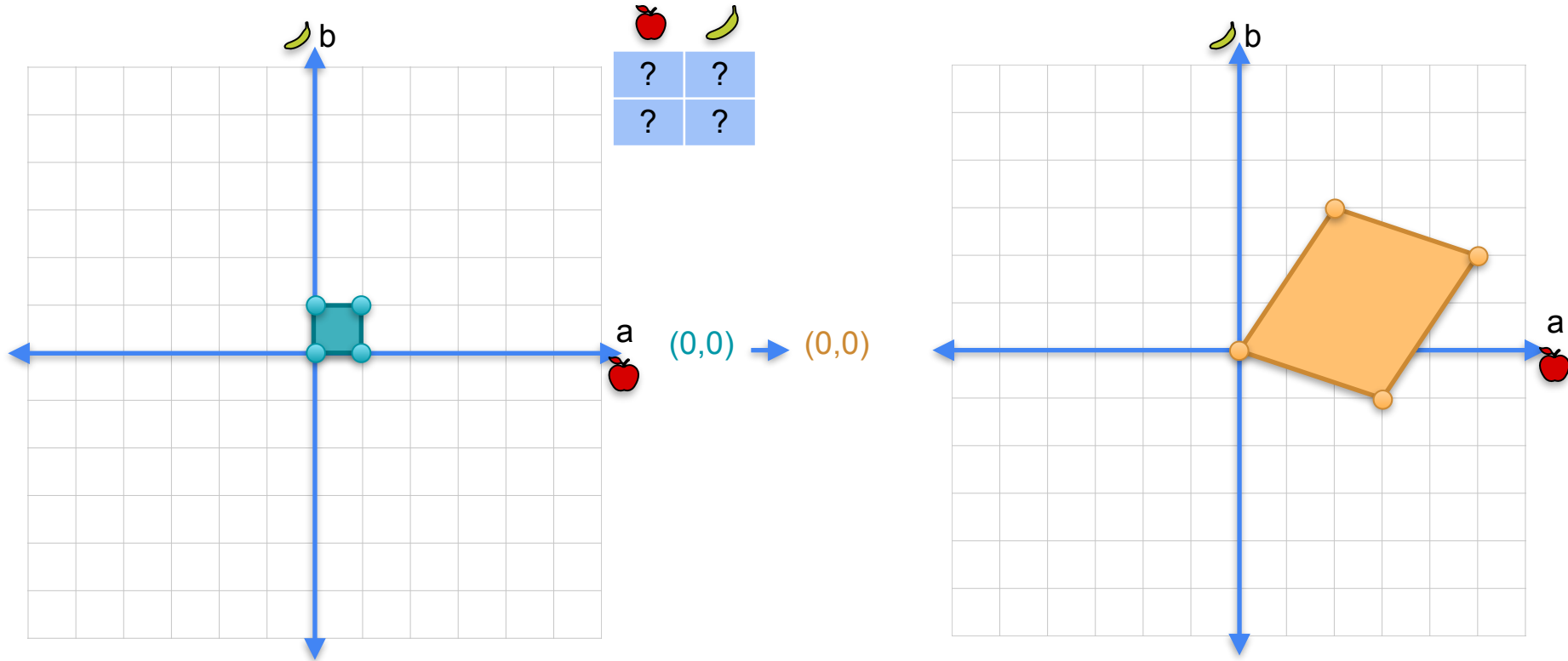
# Linear transformations as matrices



# Linear transformations as matrices

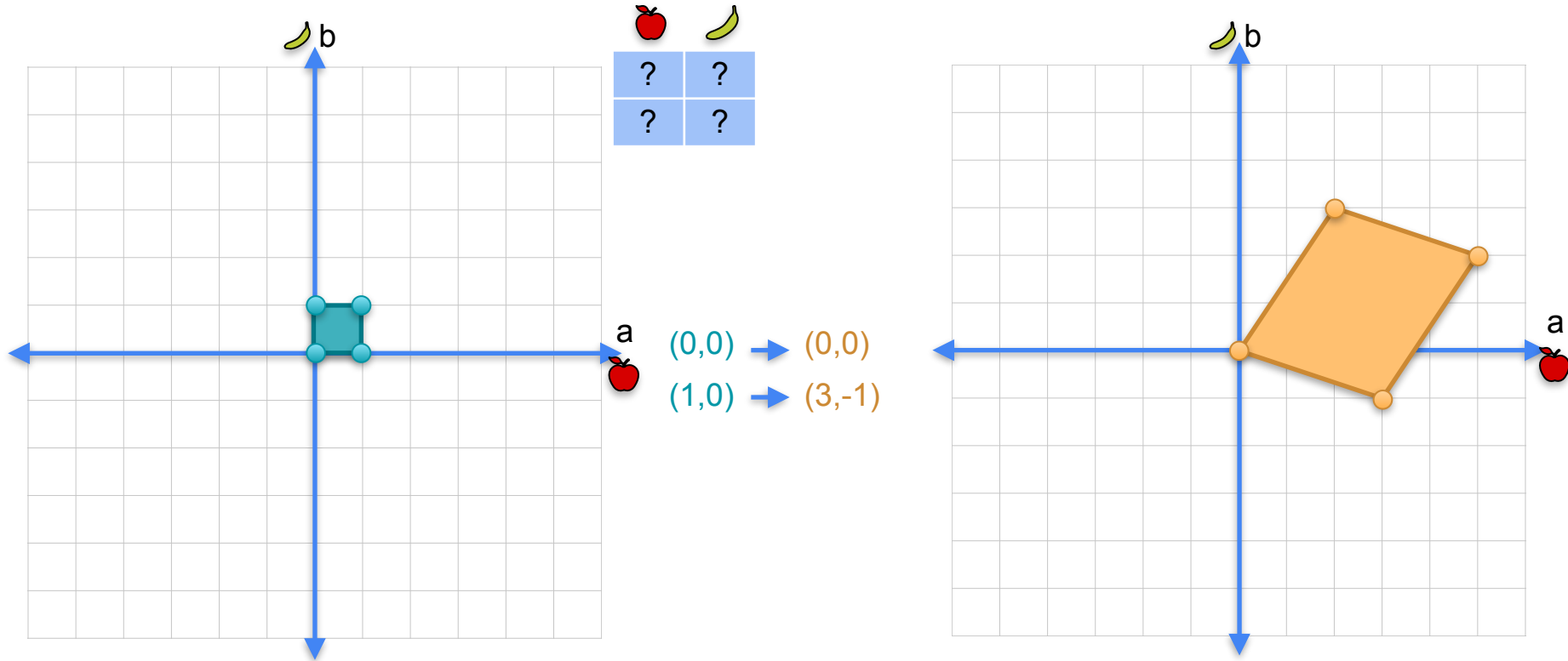


# Linear transformations as matrices

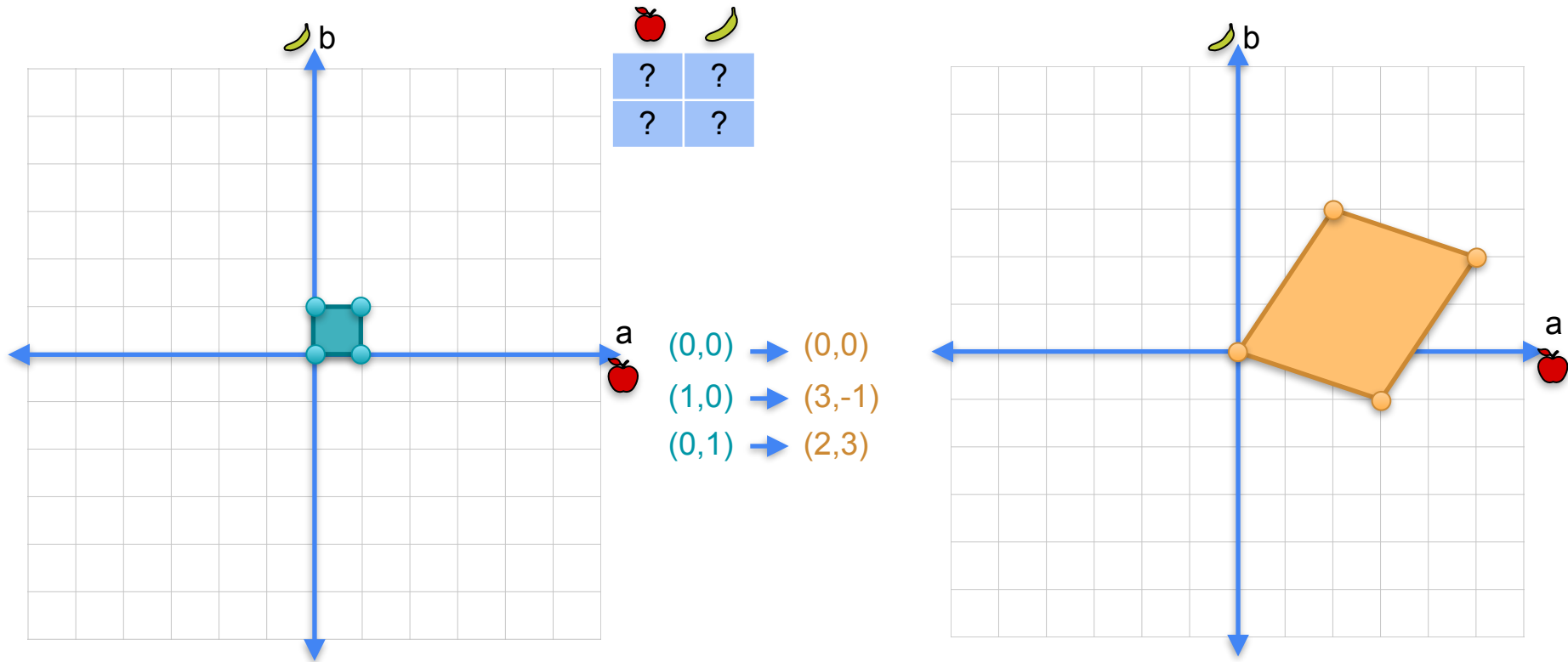




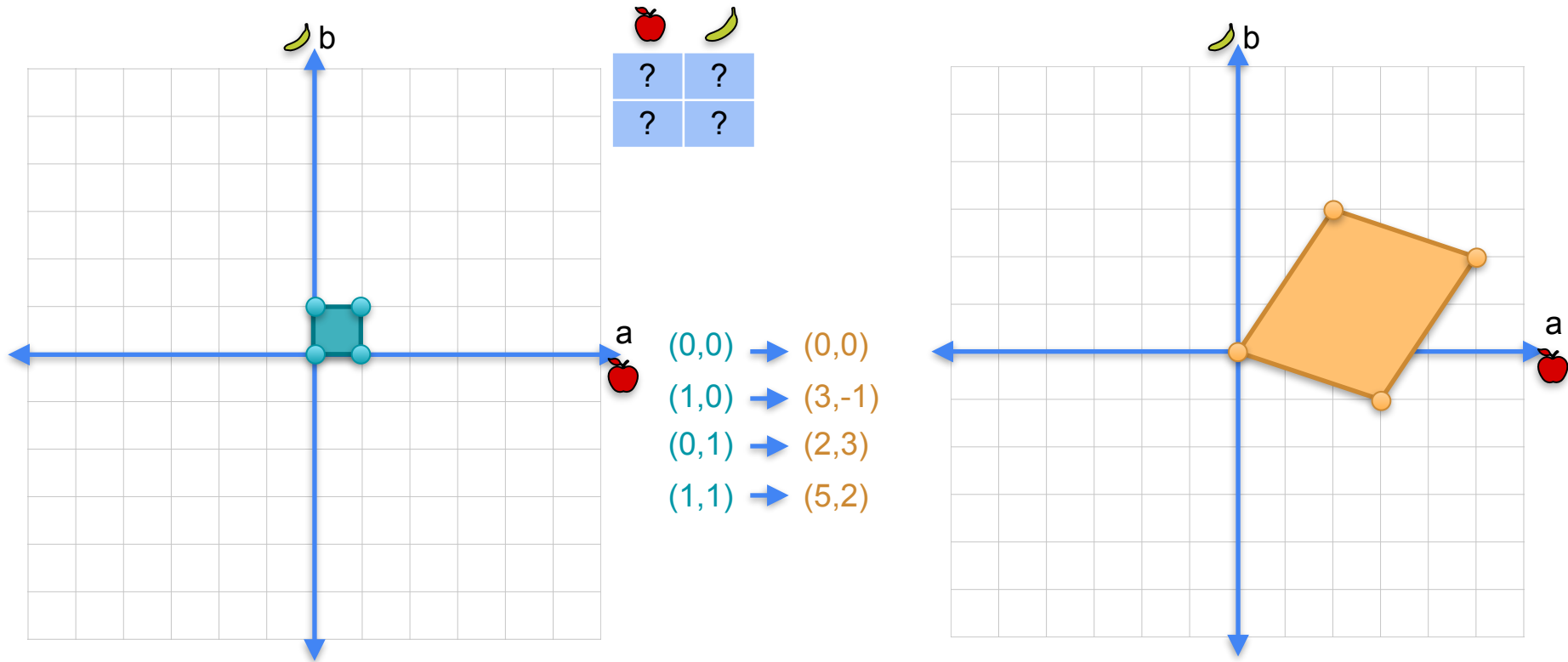
# Linear transformations as matrices



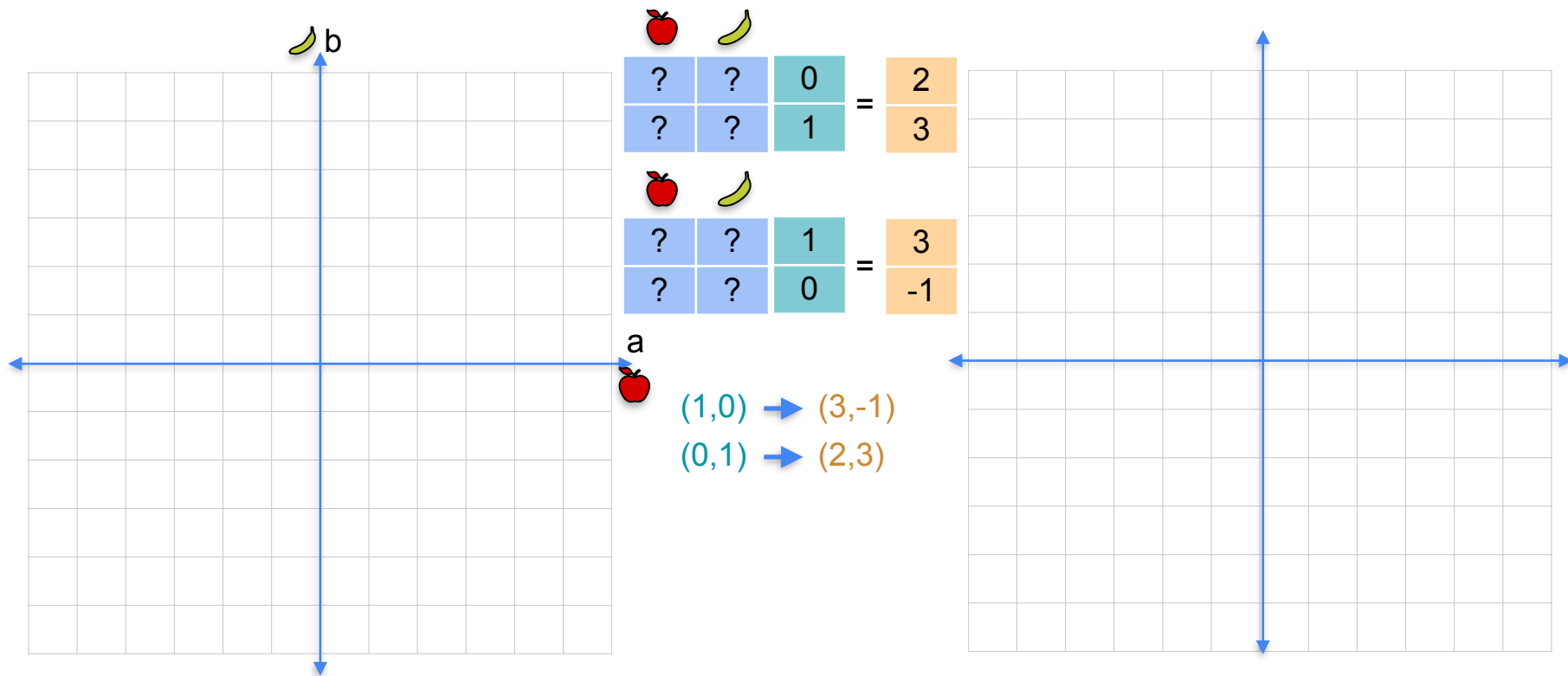
# Linear transformations as matrices



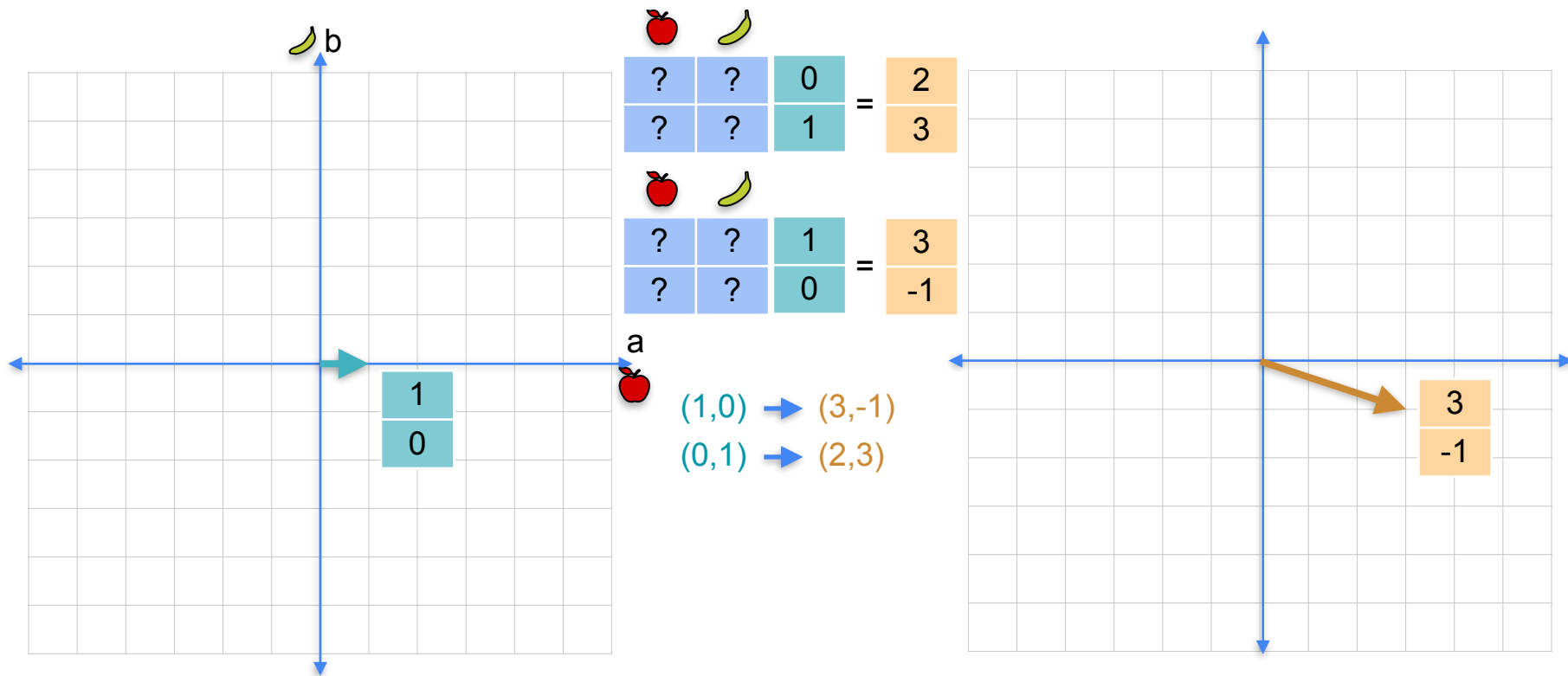
# Linear transformations as matrices



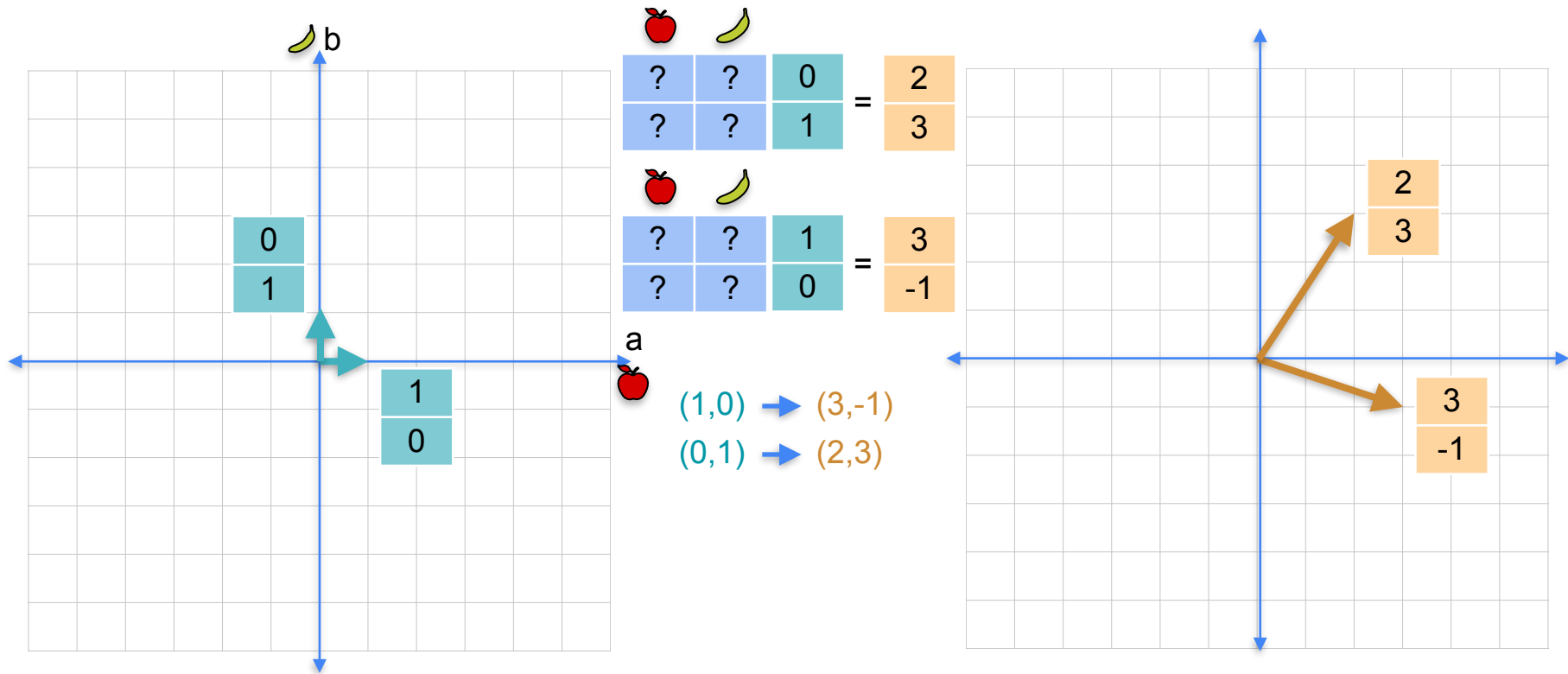
# Linear transformations as matrices



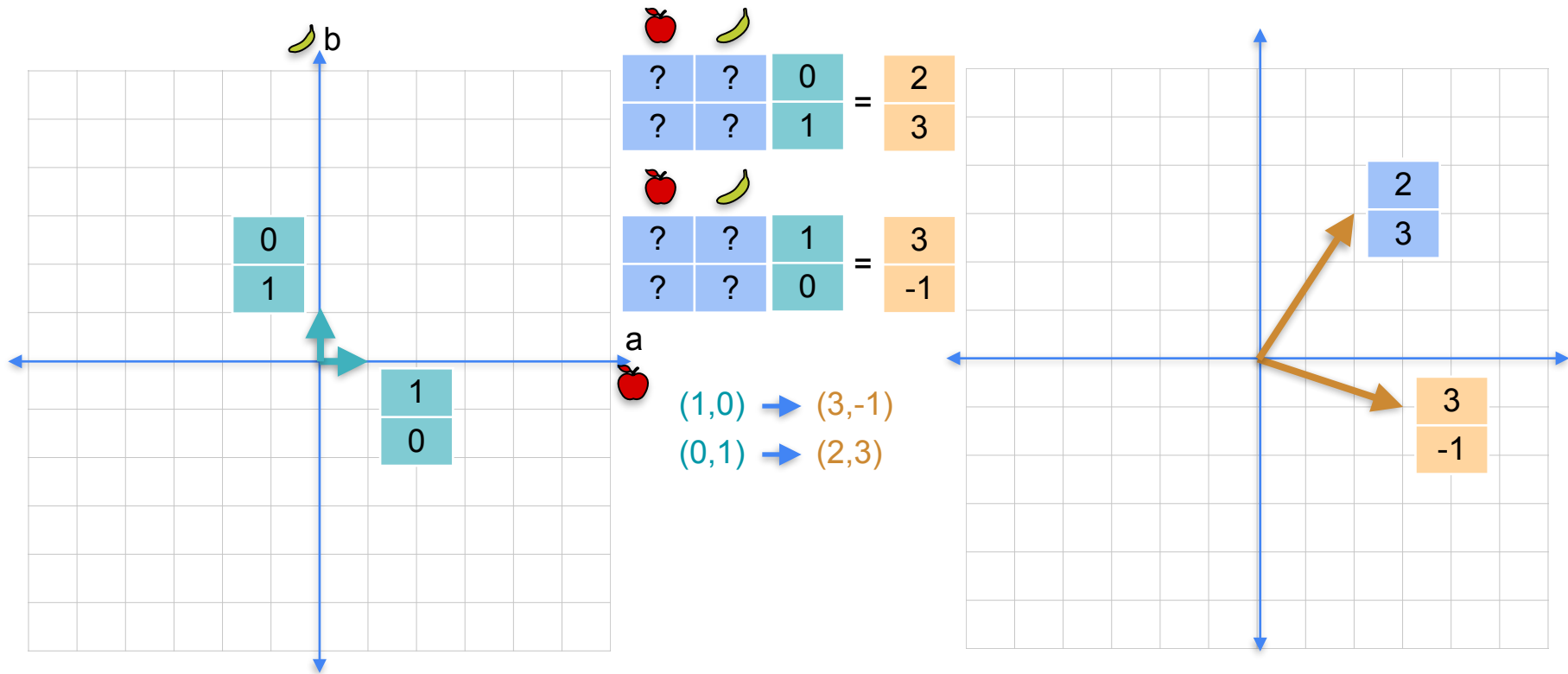
# Linear transformations as matrices



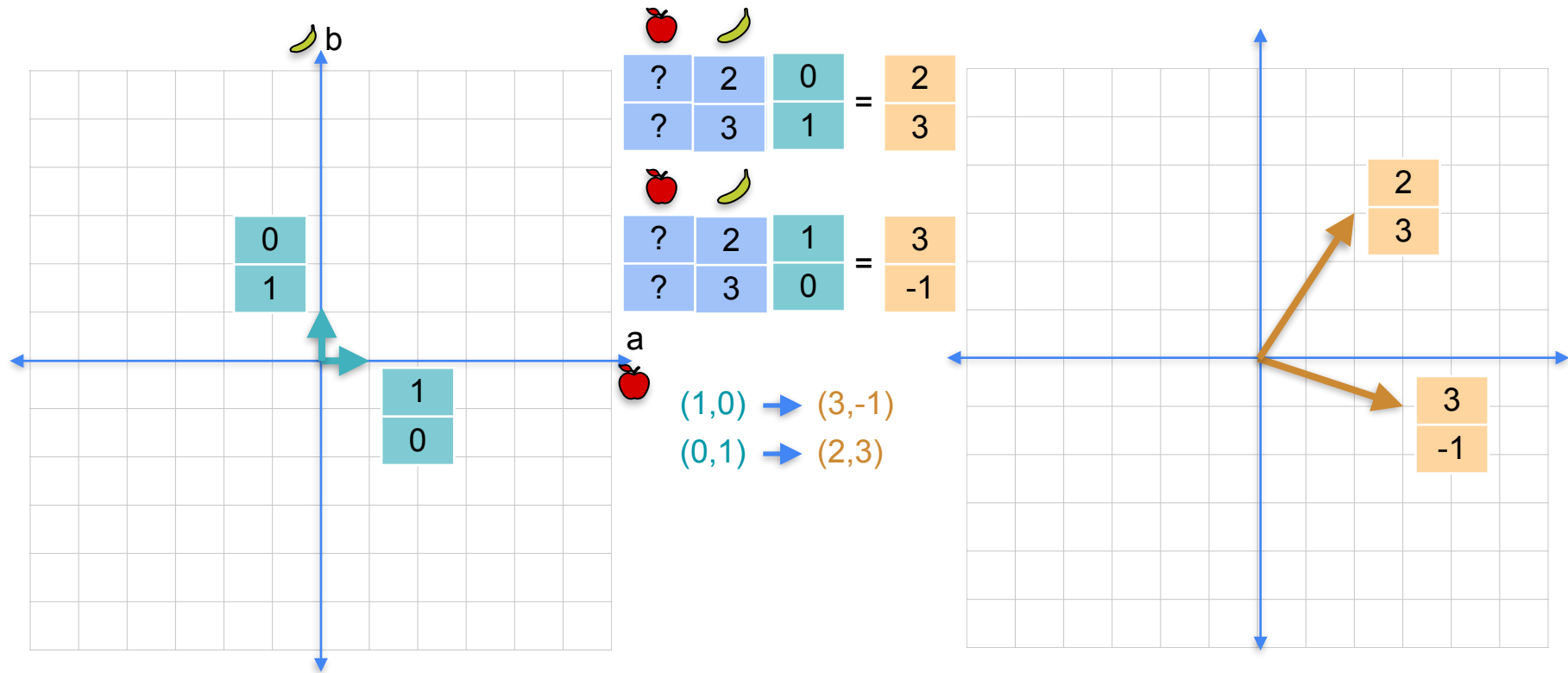
# Linear transformations as matrices



# Linear transformations as matrices

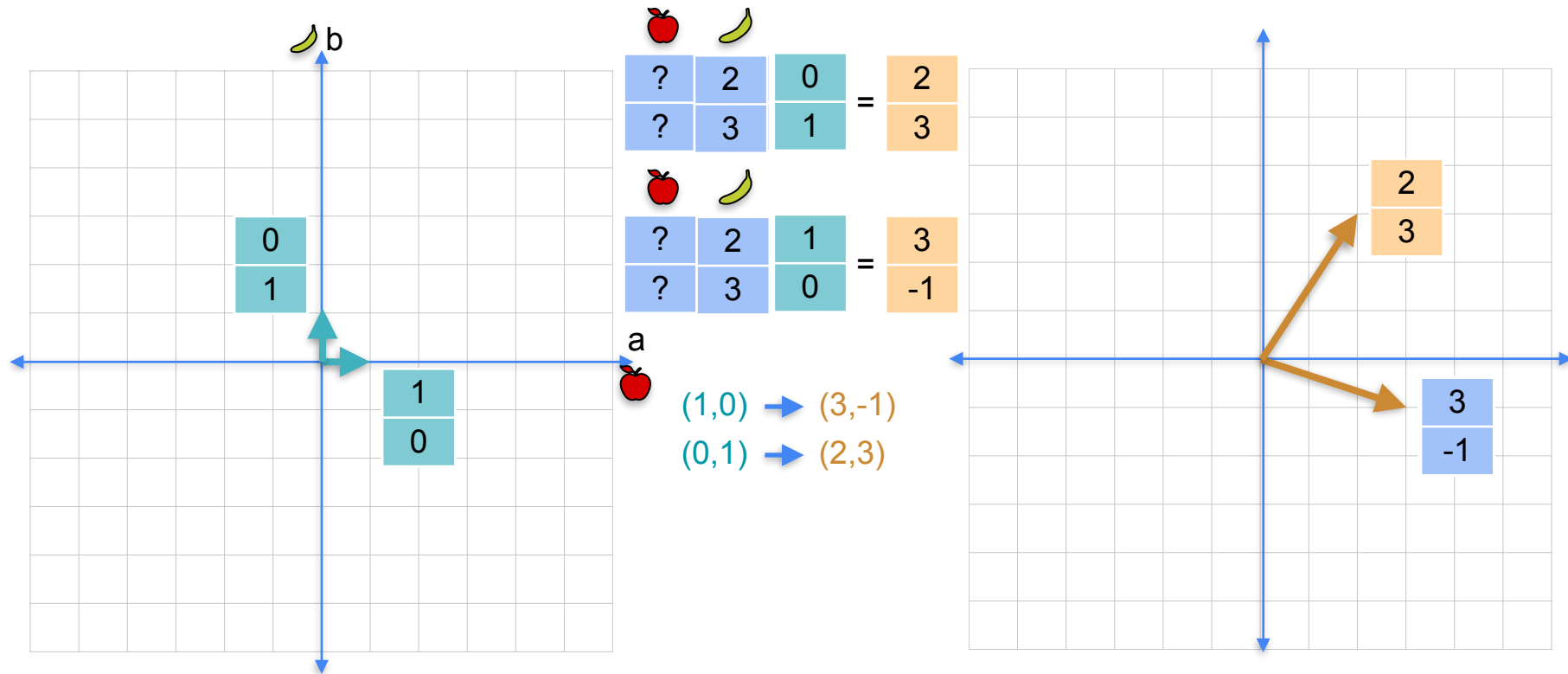


# Linear transformations as matrices

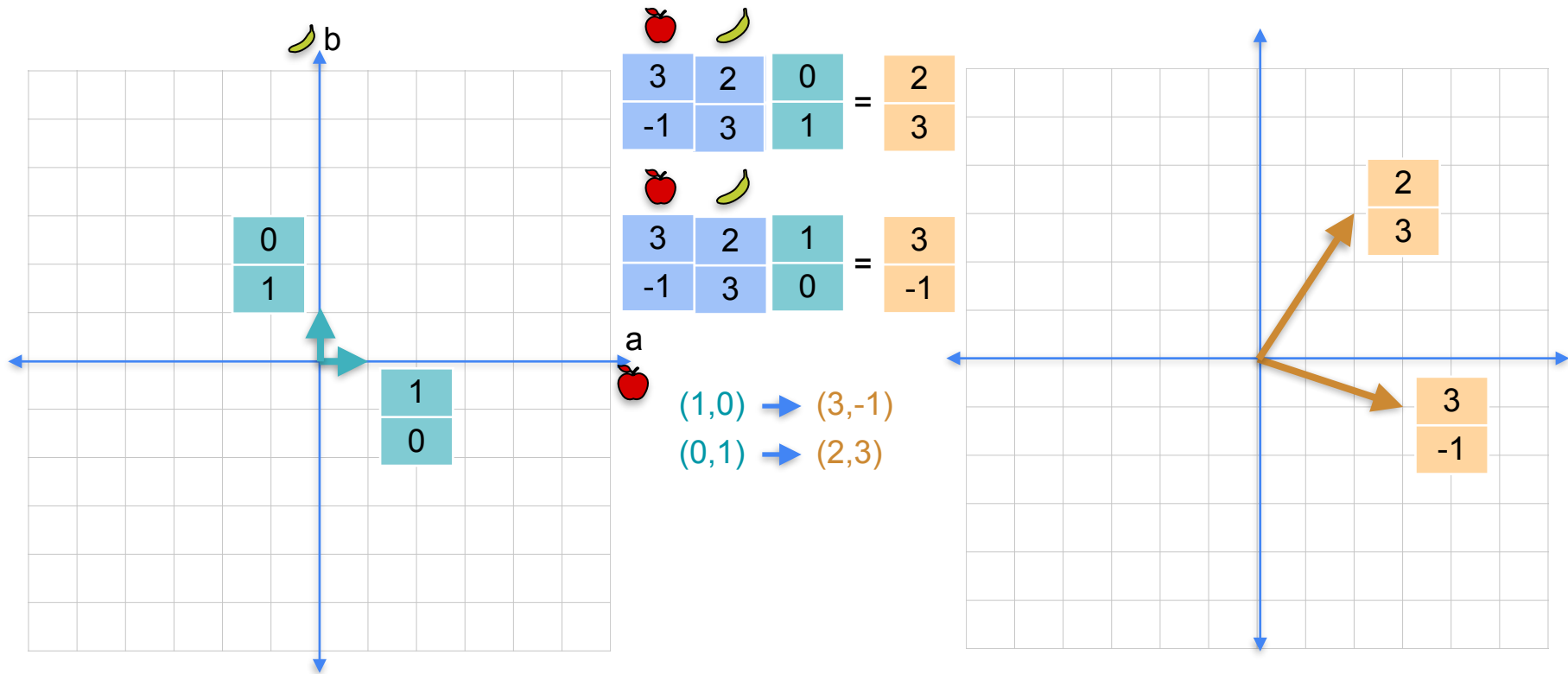




# Linear transformations as matrices



# Linear transformations as matrices





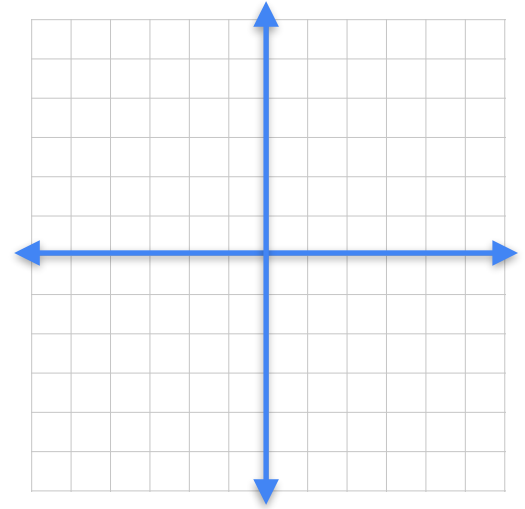
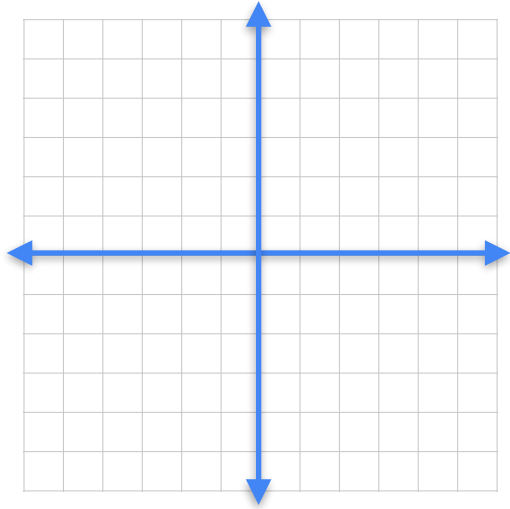
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# Vectors and Linear Transformations

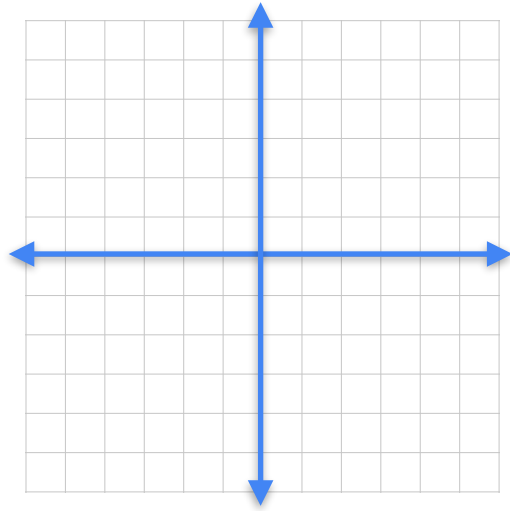
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## **Matrix multiplication**

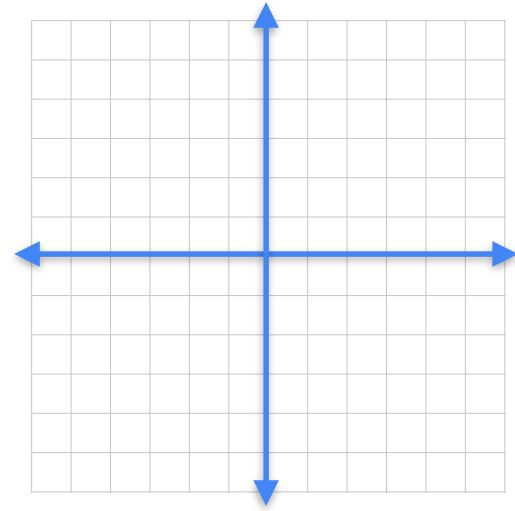
# Combining linear transformations



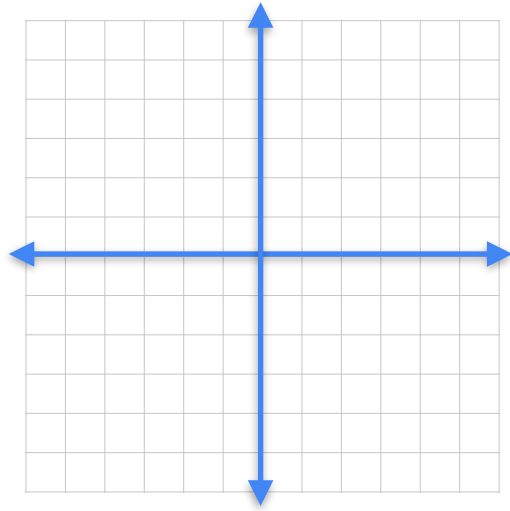
# Combining linear transformations



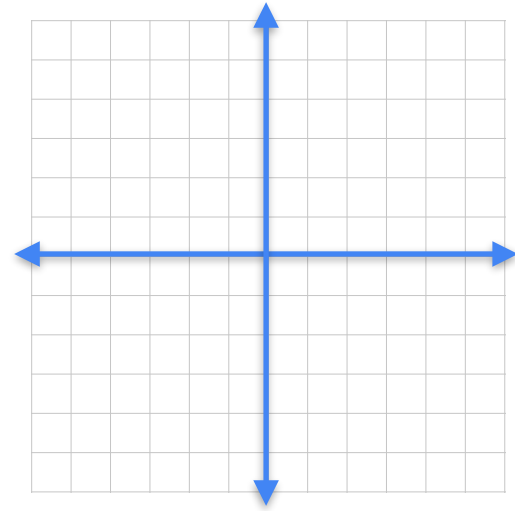
3	1
1	2



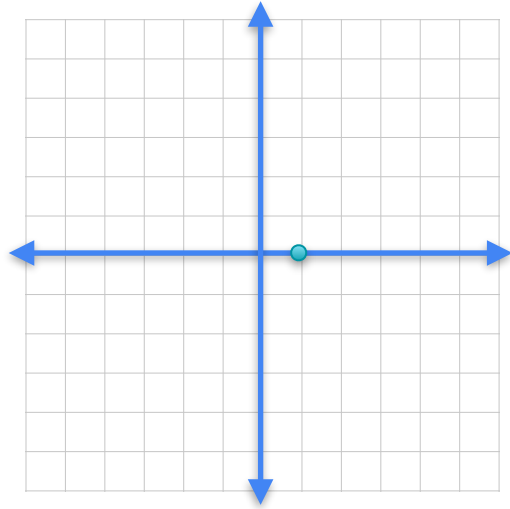
# Combining linear transformations



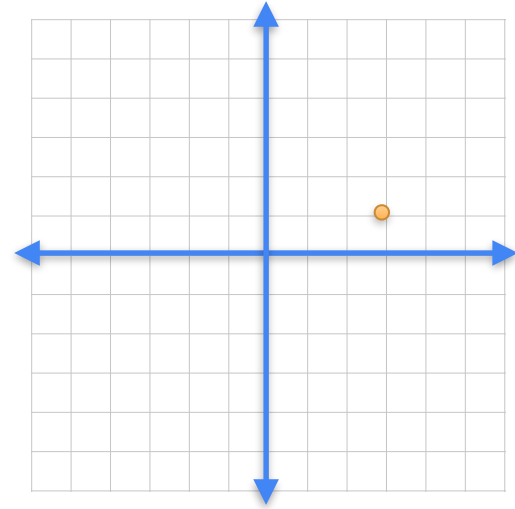
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



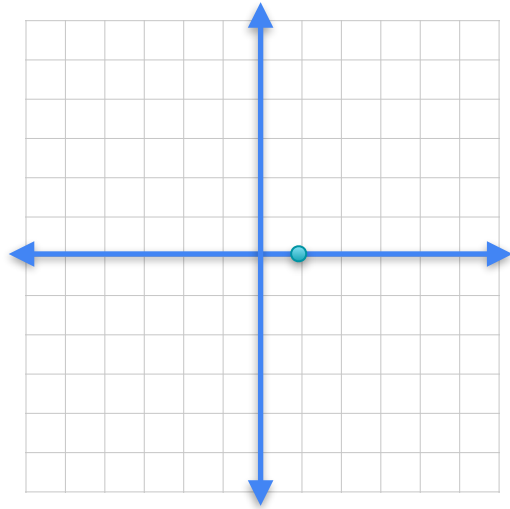
# Combining linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

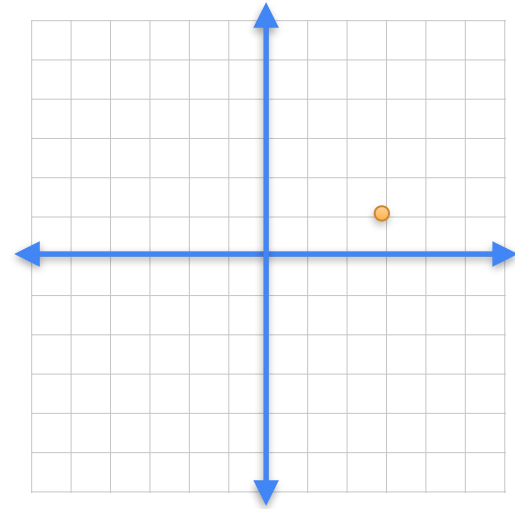


# Combining linear transformations



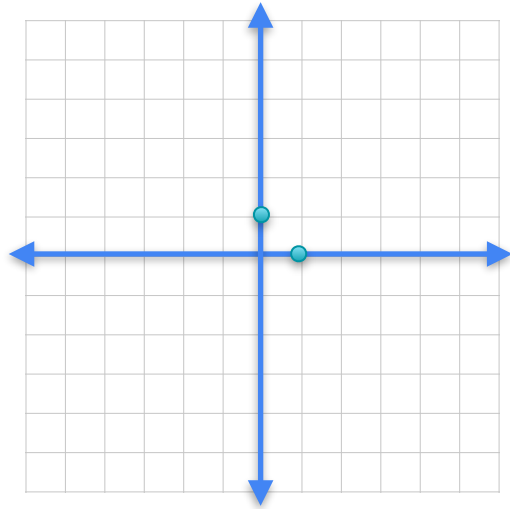
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



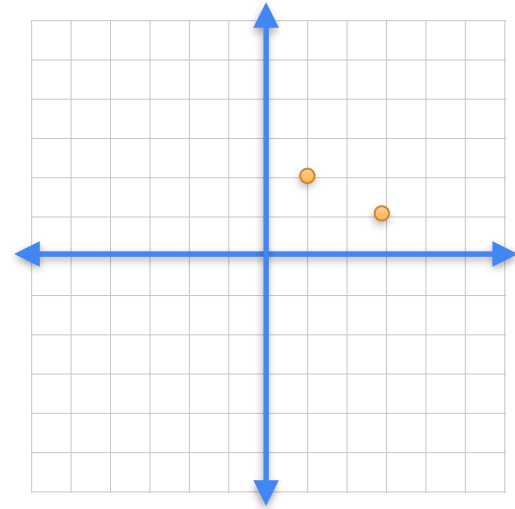


# Combining linear transformations

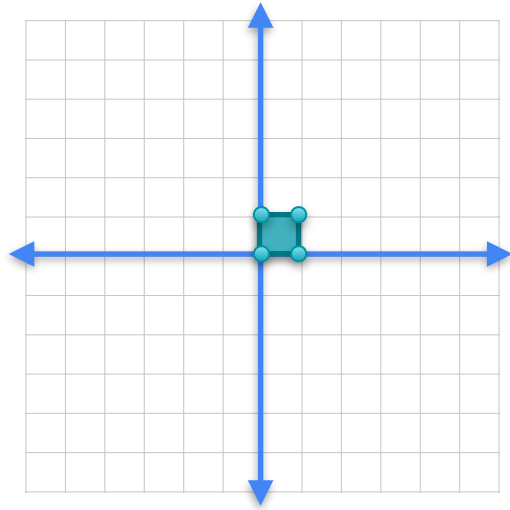


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

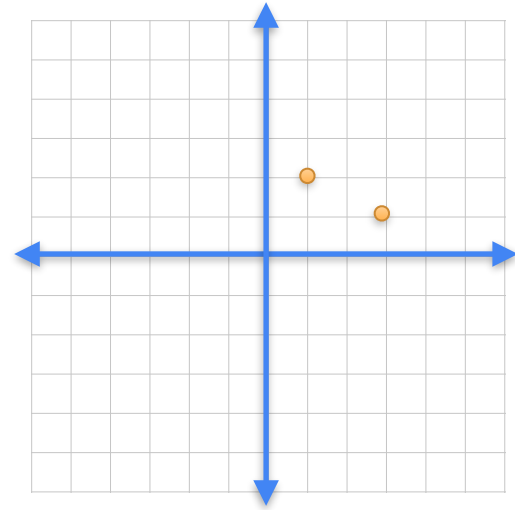


# Combining linear transformations

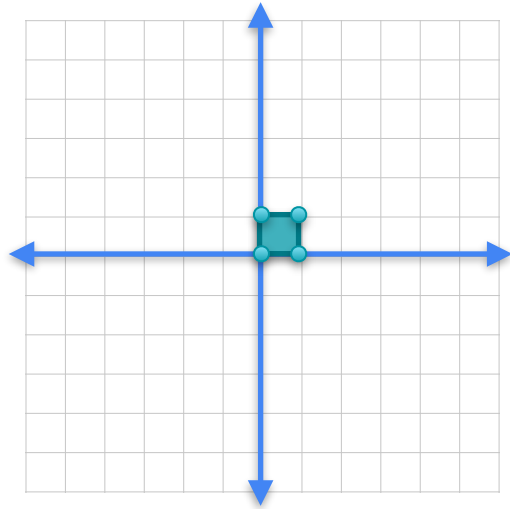


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

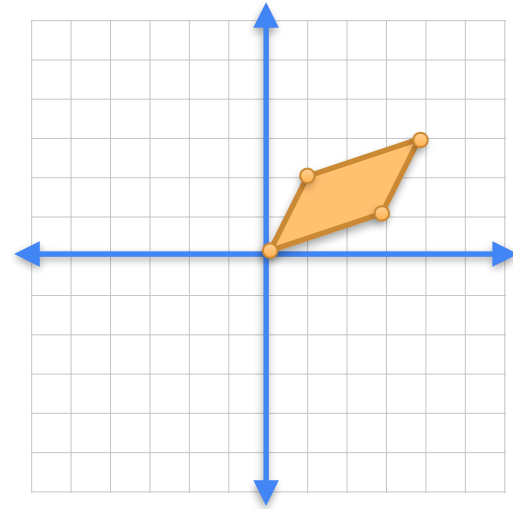


# Combining linear transformations

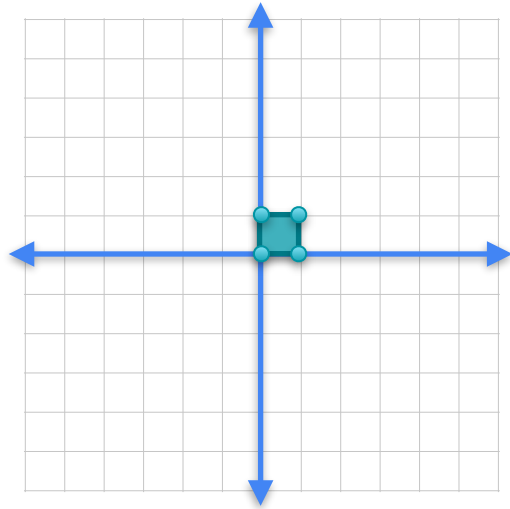


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

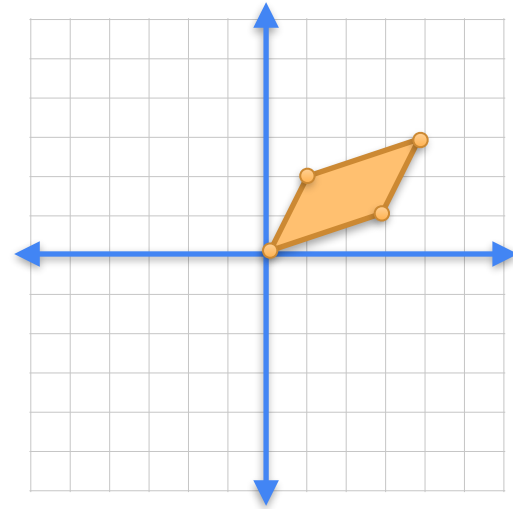


# Combining linear transformations

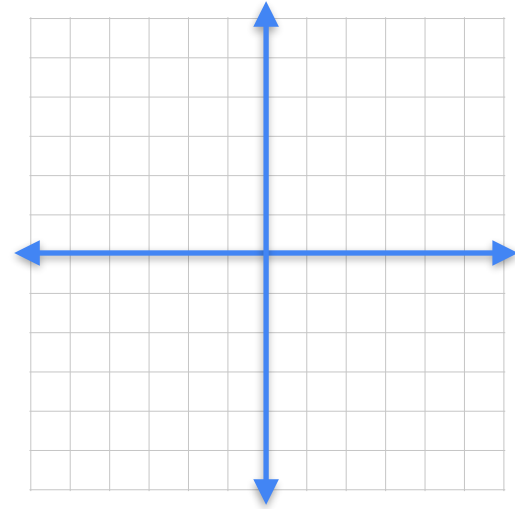
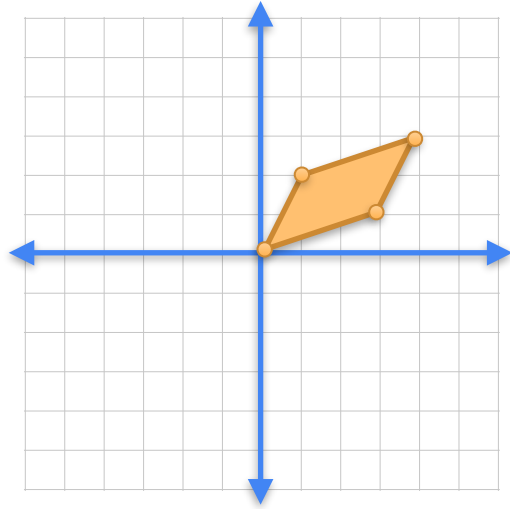


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

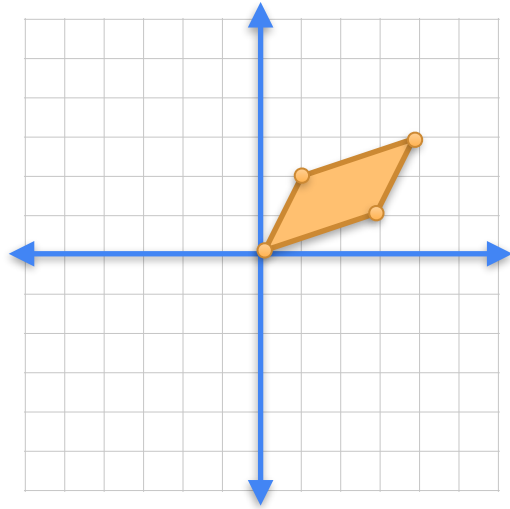
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



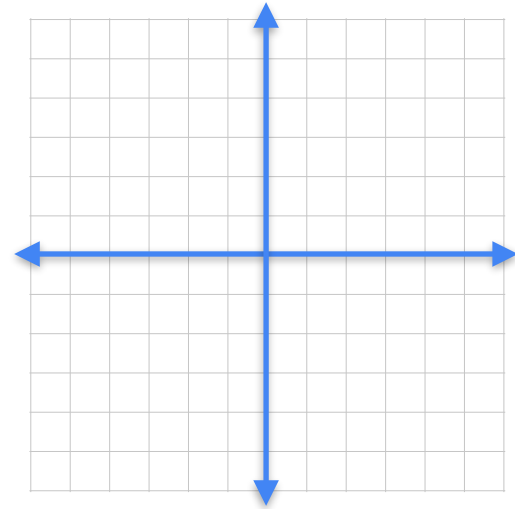
# Combining linear transformations



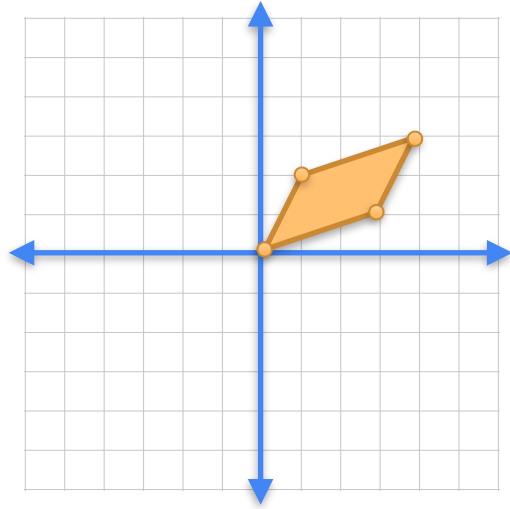
# Combining linear transformations



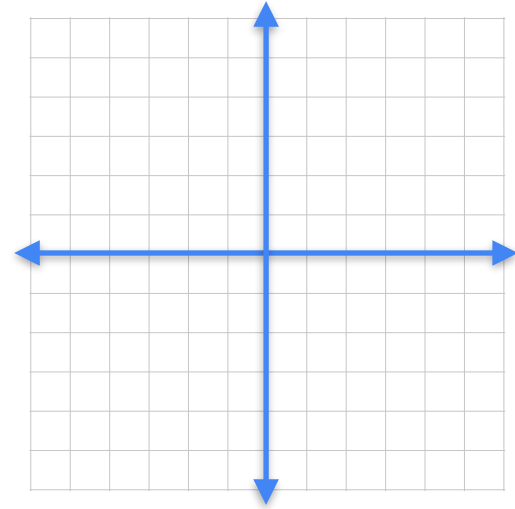
2	-1
0	2



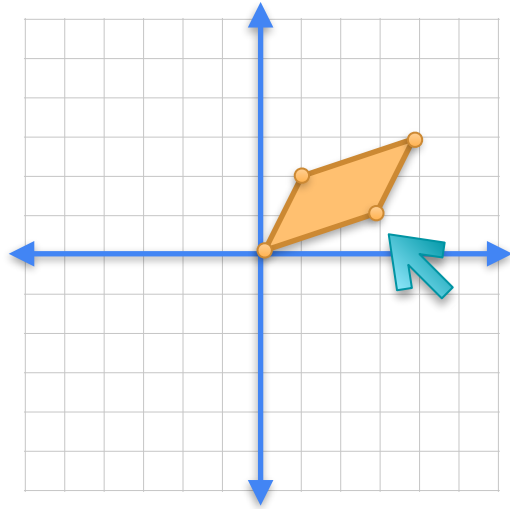
# Combining linear transformations



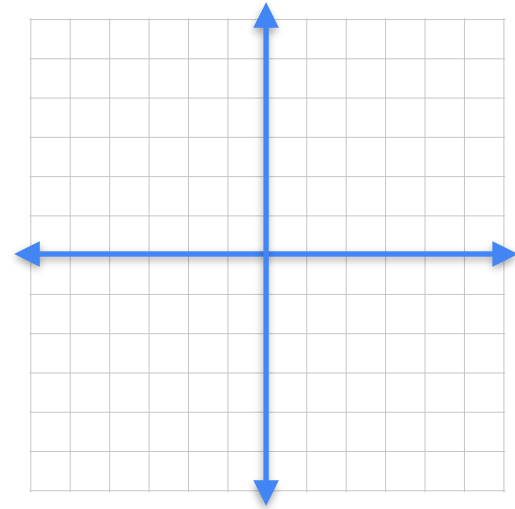
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



# Combining linear transformations

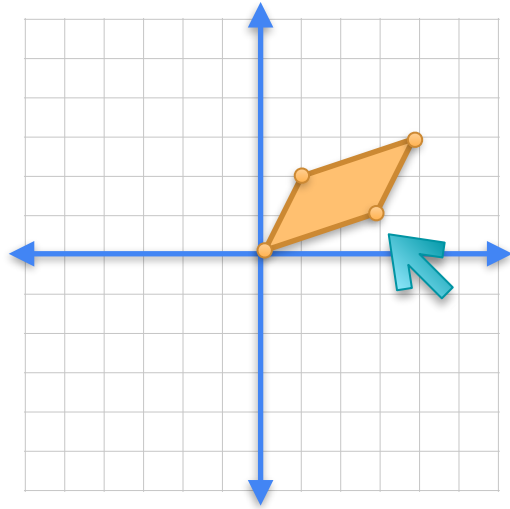


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

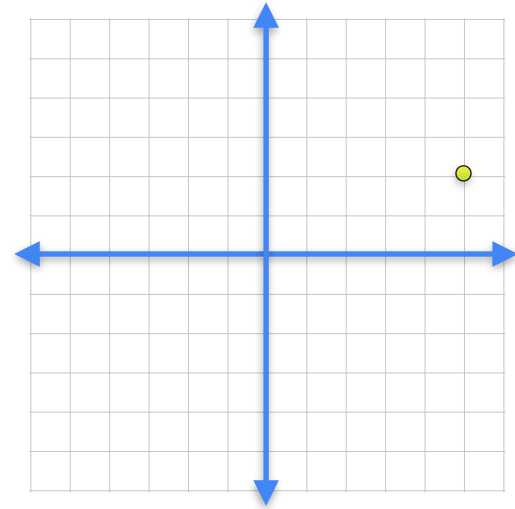




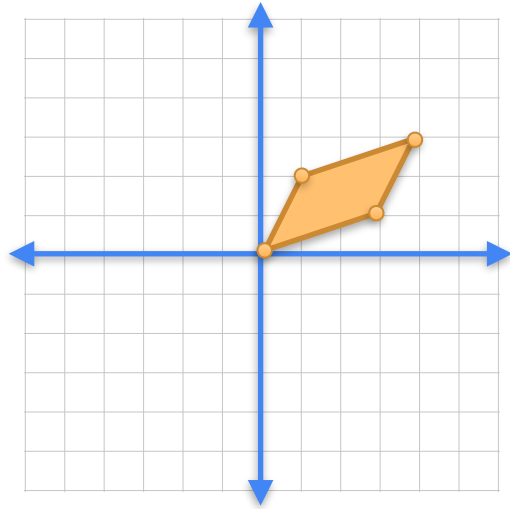
# Combining linear transformations



$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

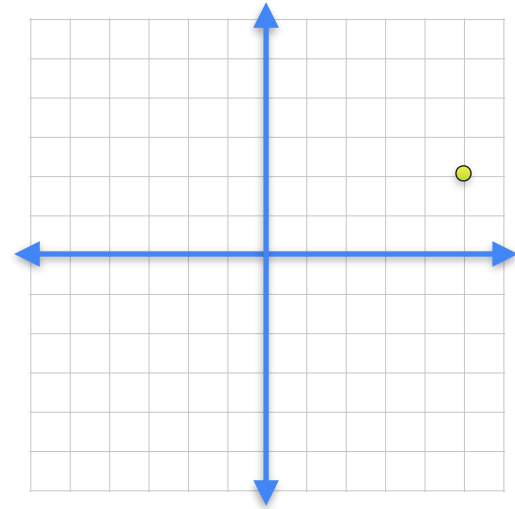


# Combining linear transformations

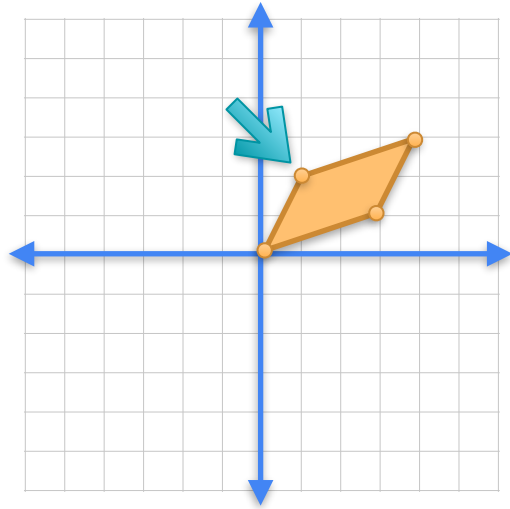


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

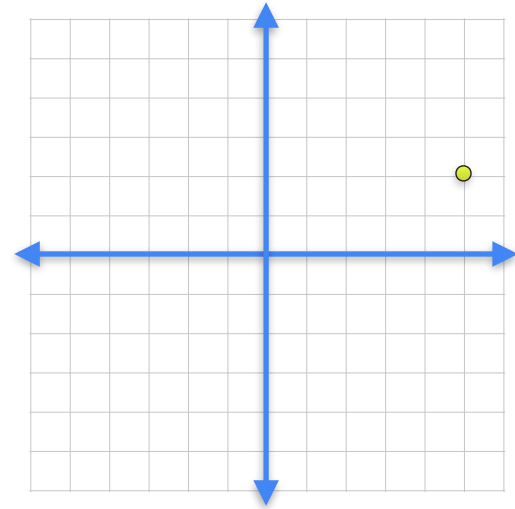


# Combining linear transformations

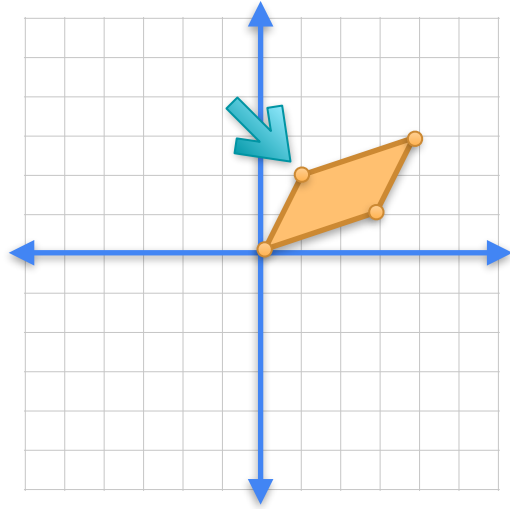


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

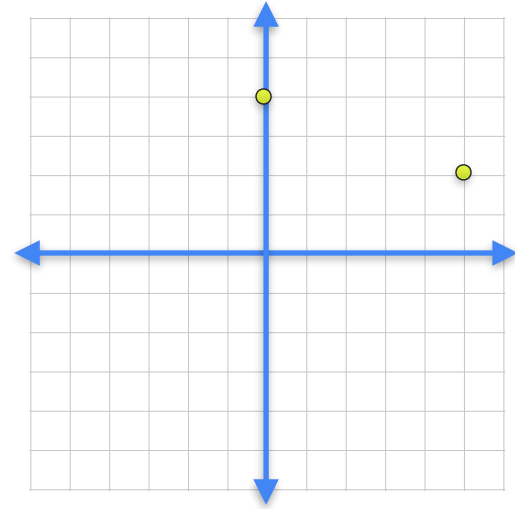


# Combining linear transformations

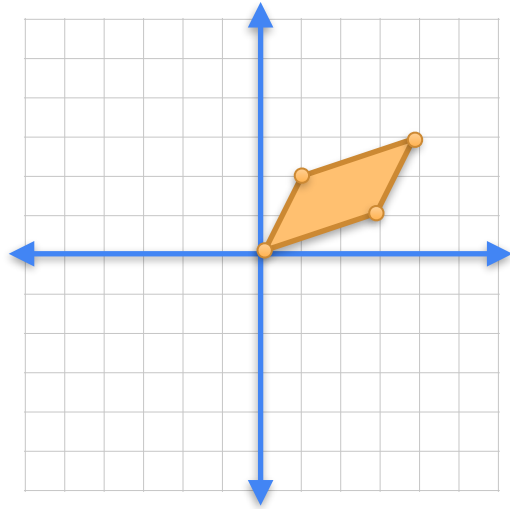


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

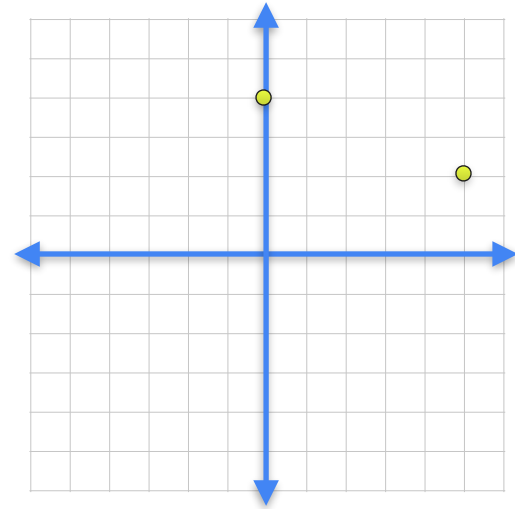


# Combining linear transformations

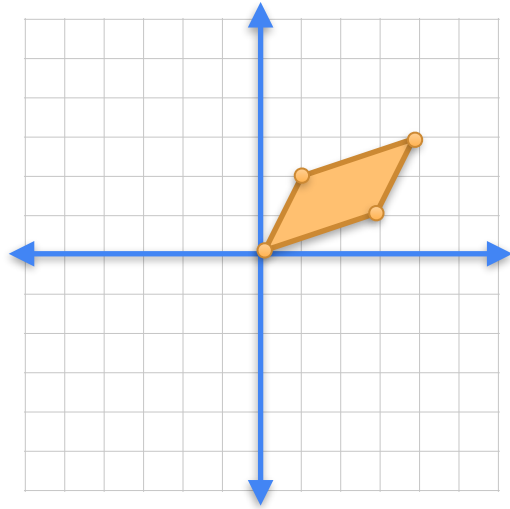


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

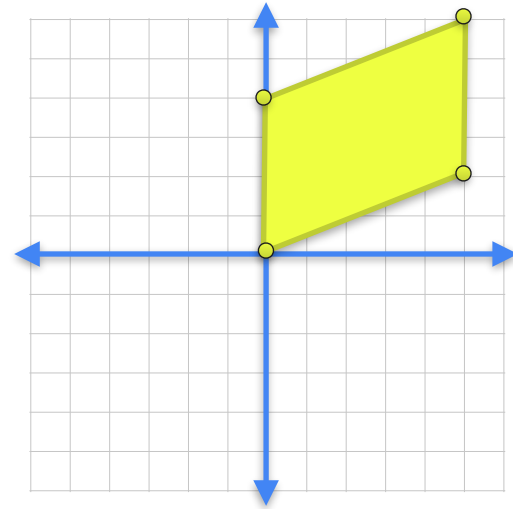


# Combining linear transformations

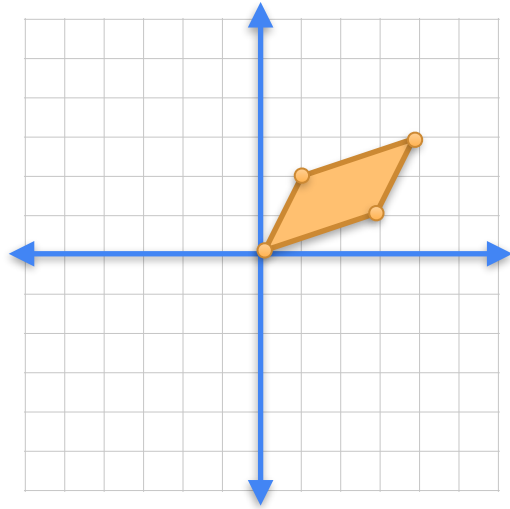


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

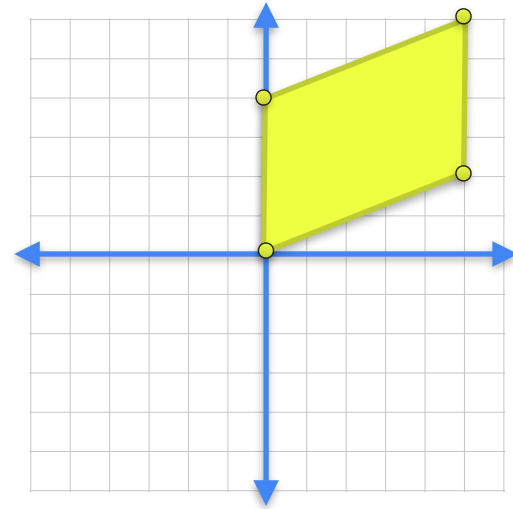


# Combining linear transformations

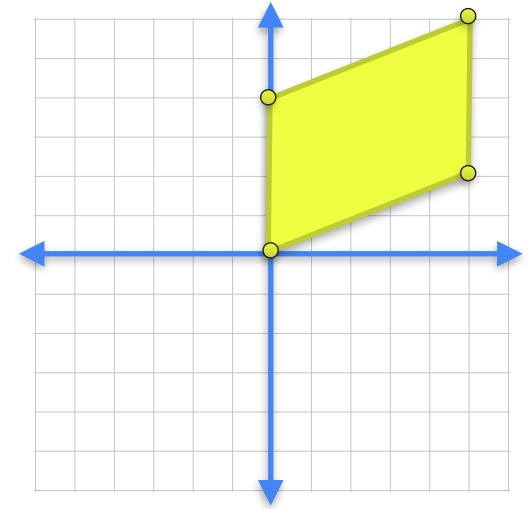
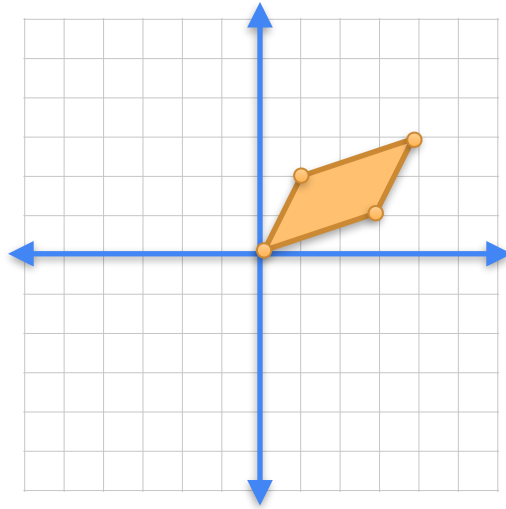
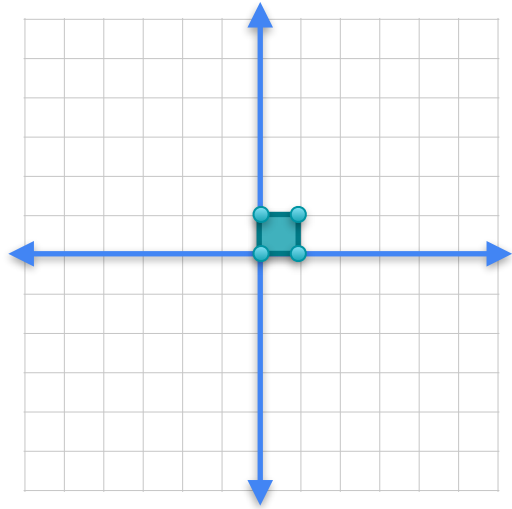


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

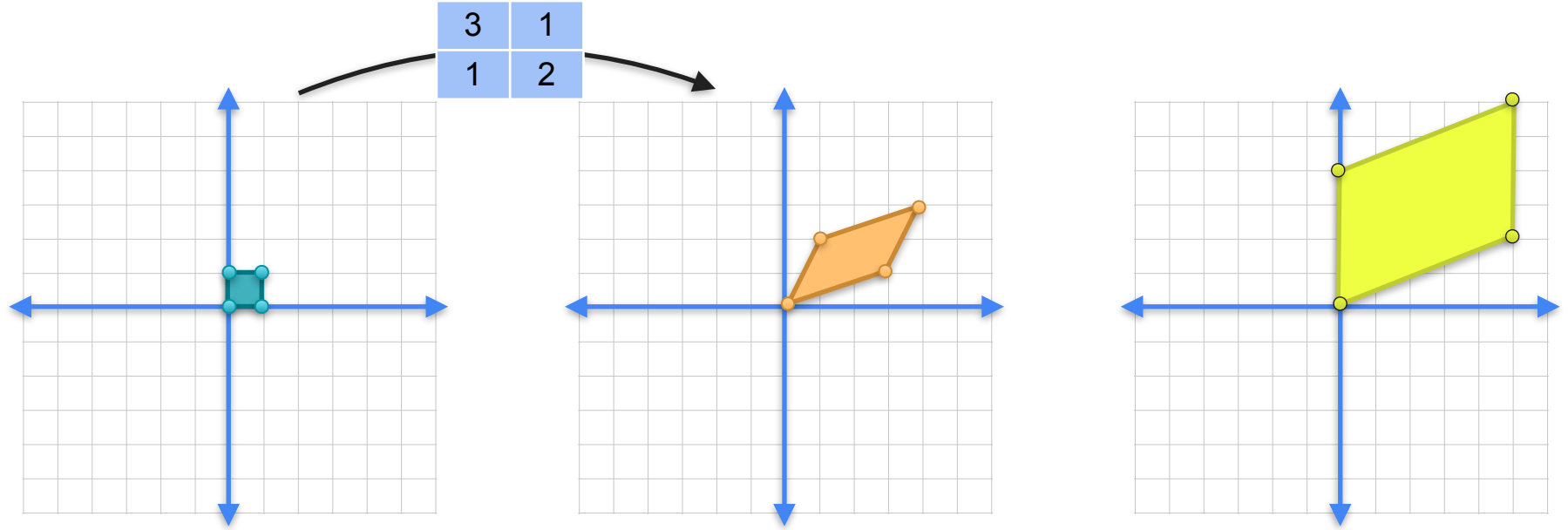


# Combining linear transformations

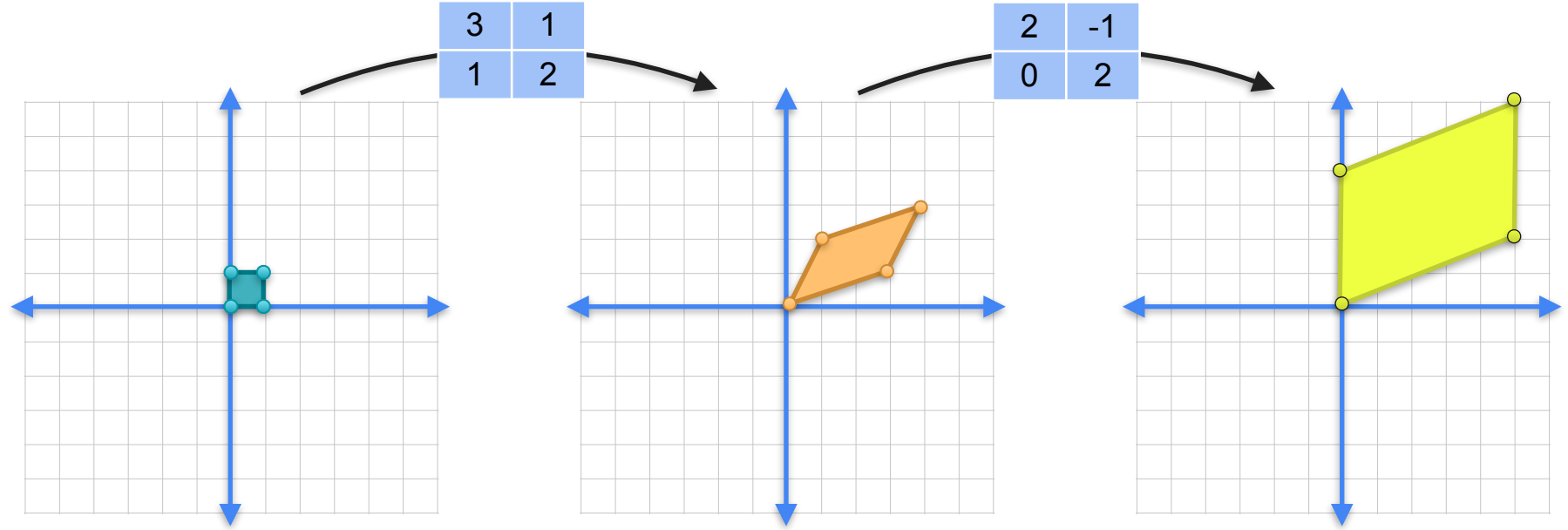




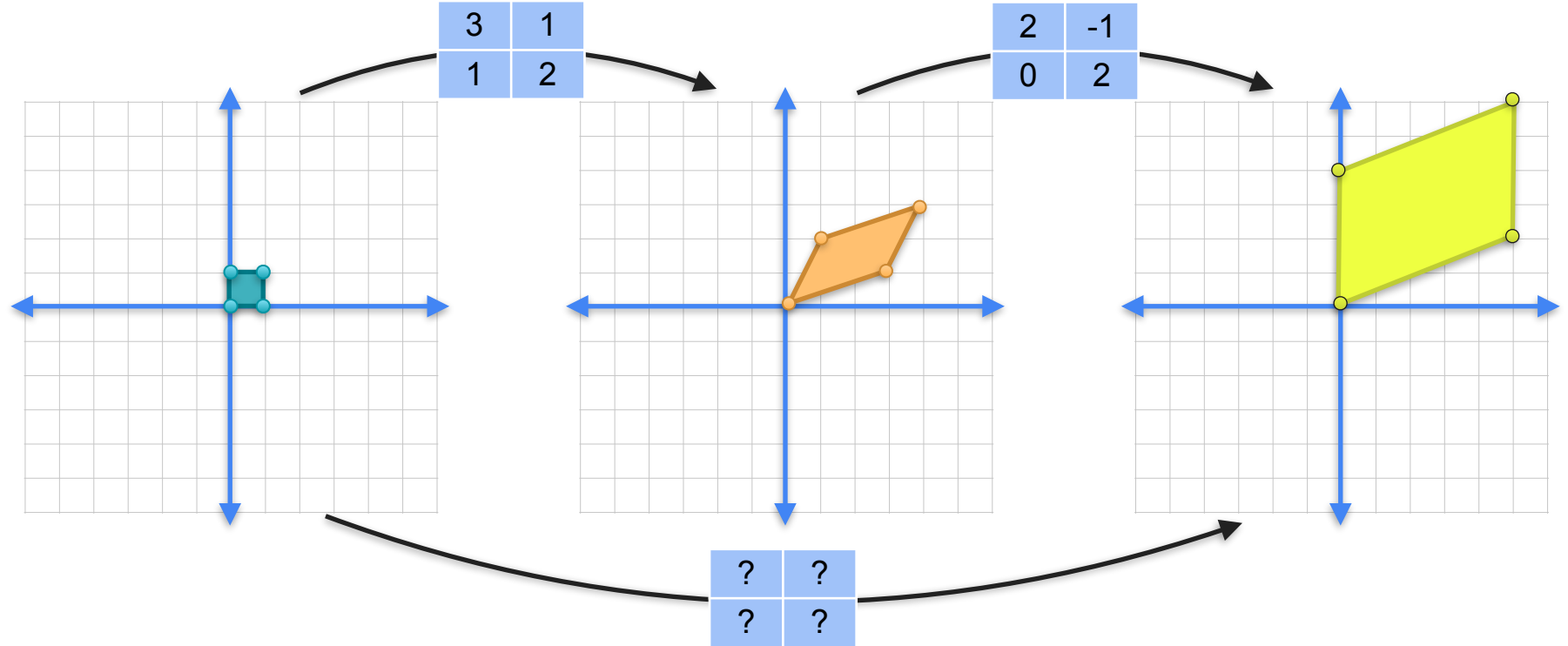
# Combining linear transformations



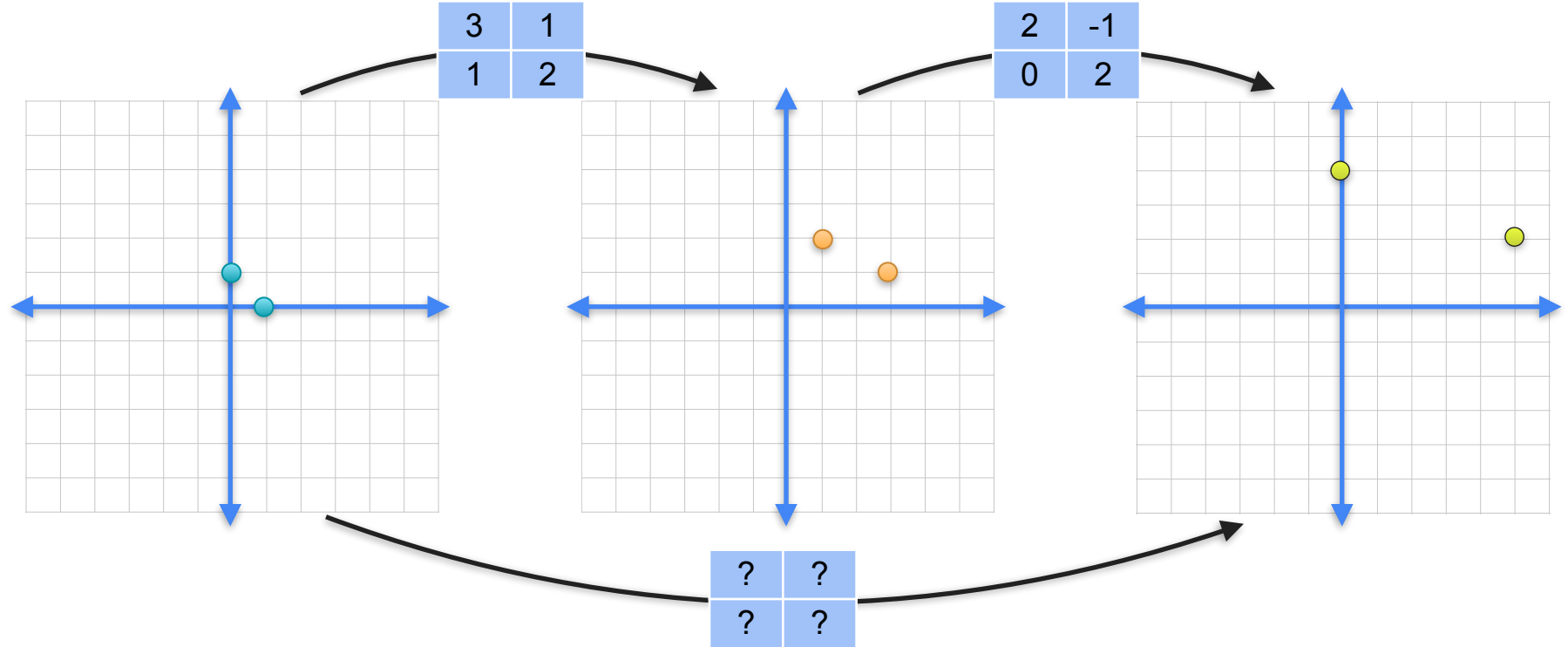
# Combining linear transformations



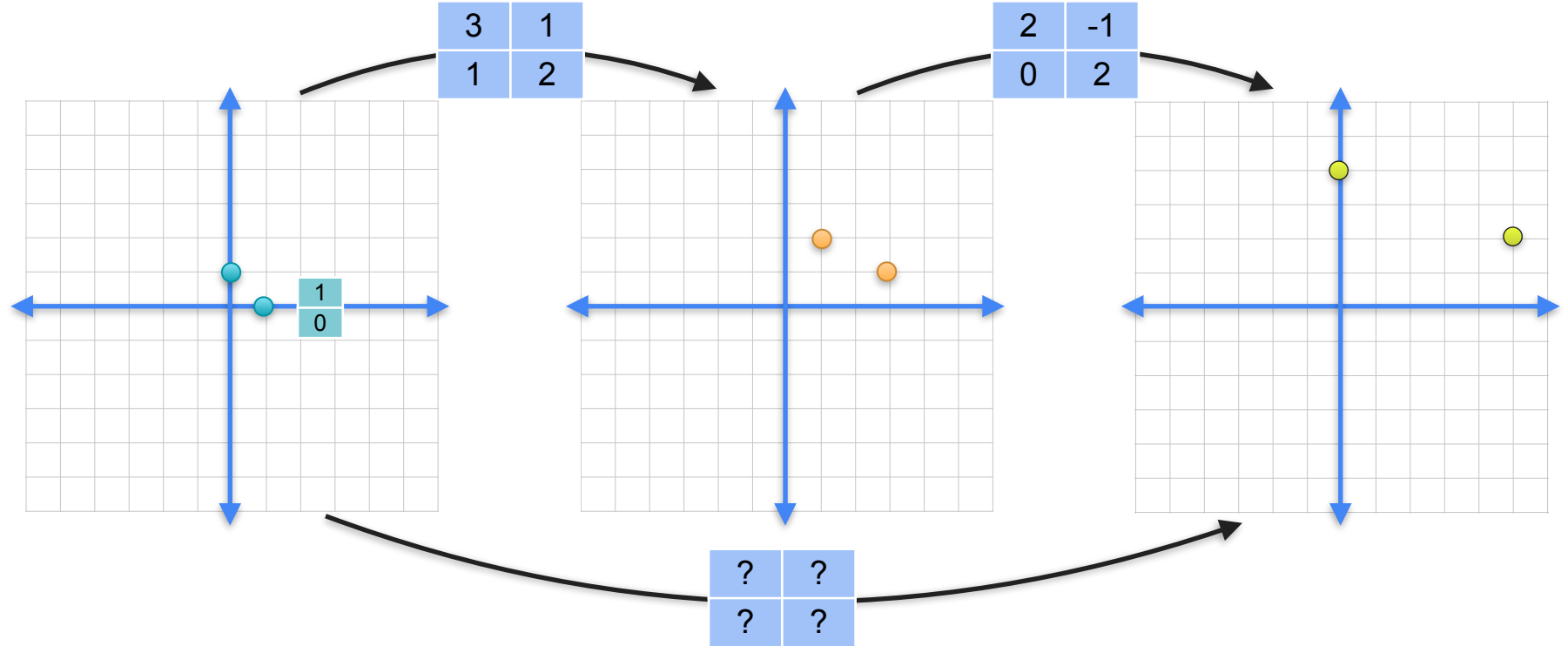
# Combining linear transformations



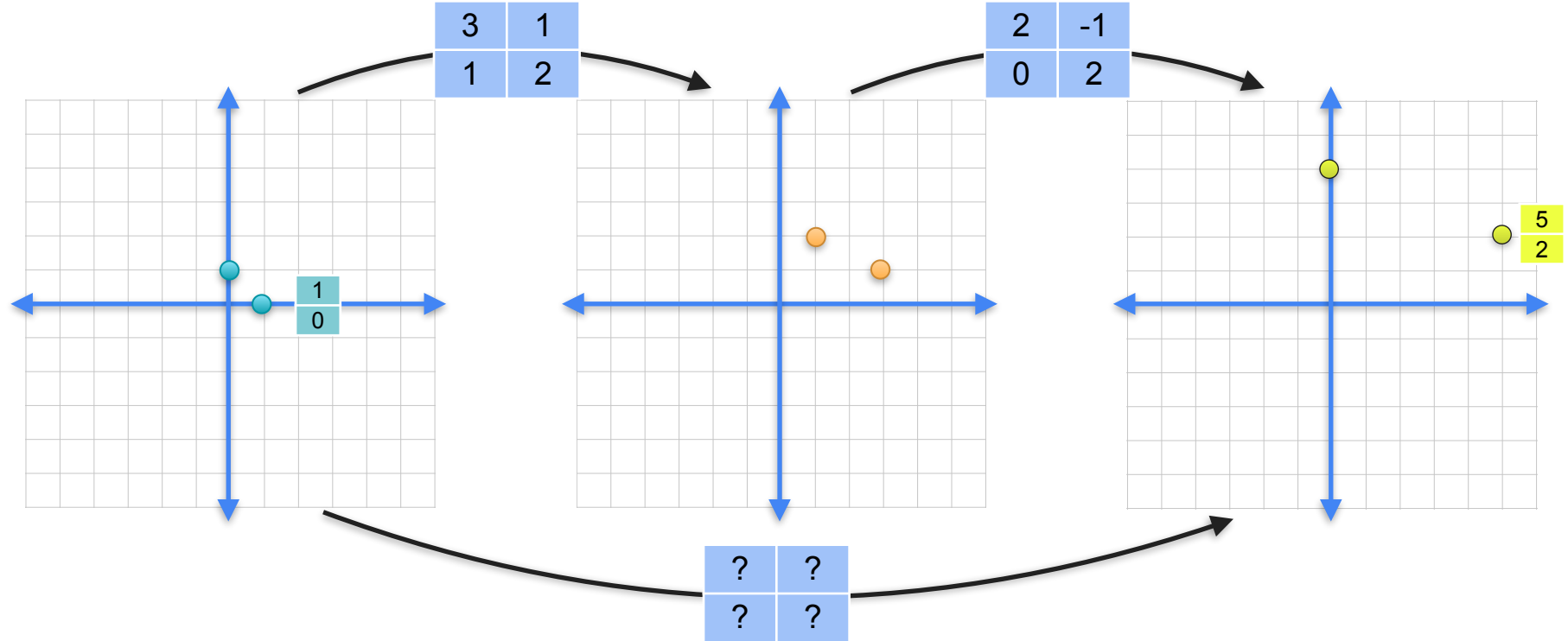
# Combining linear transformations



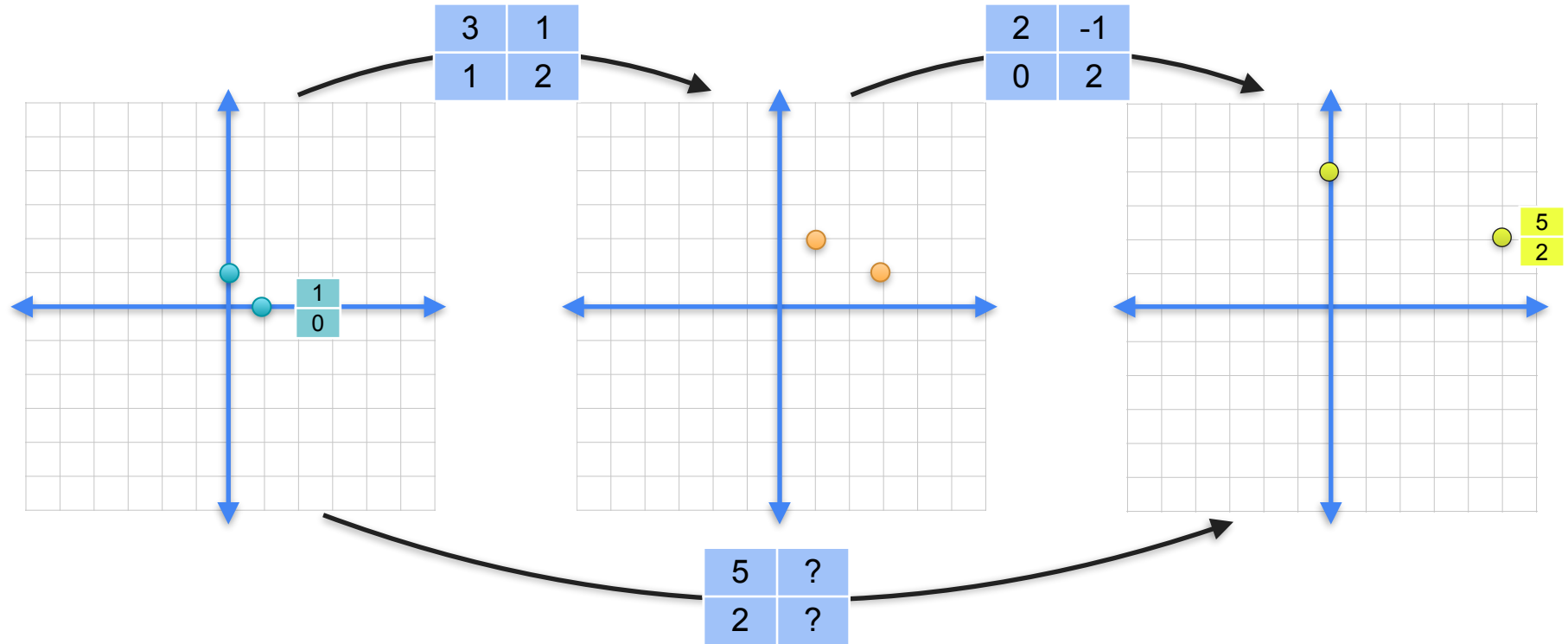
# Combining linear transformations



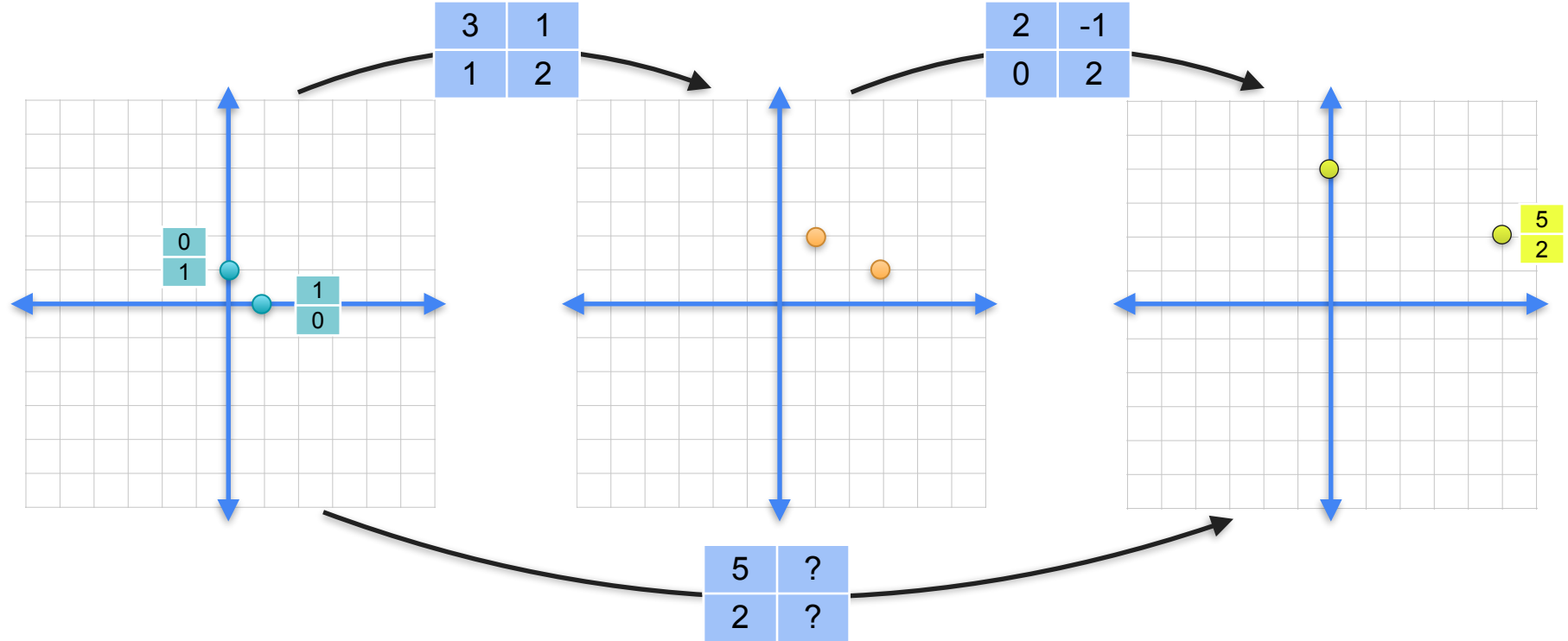
# Combining linear transformations



# Combining linear transformations

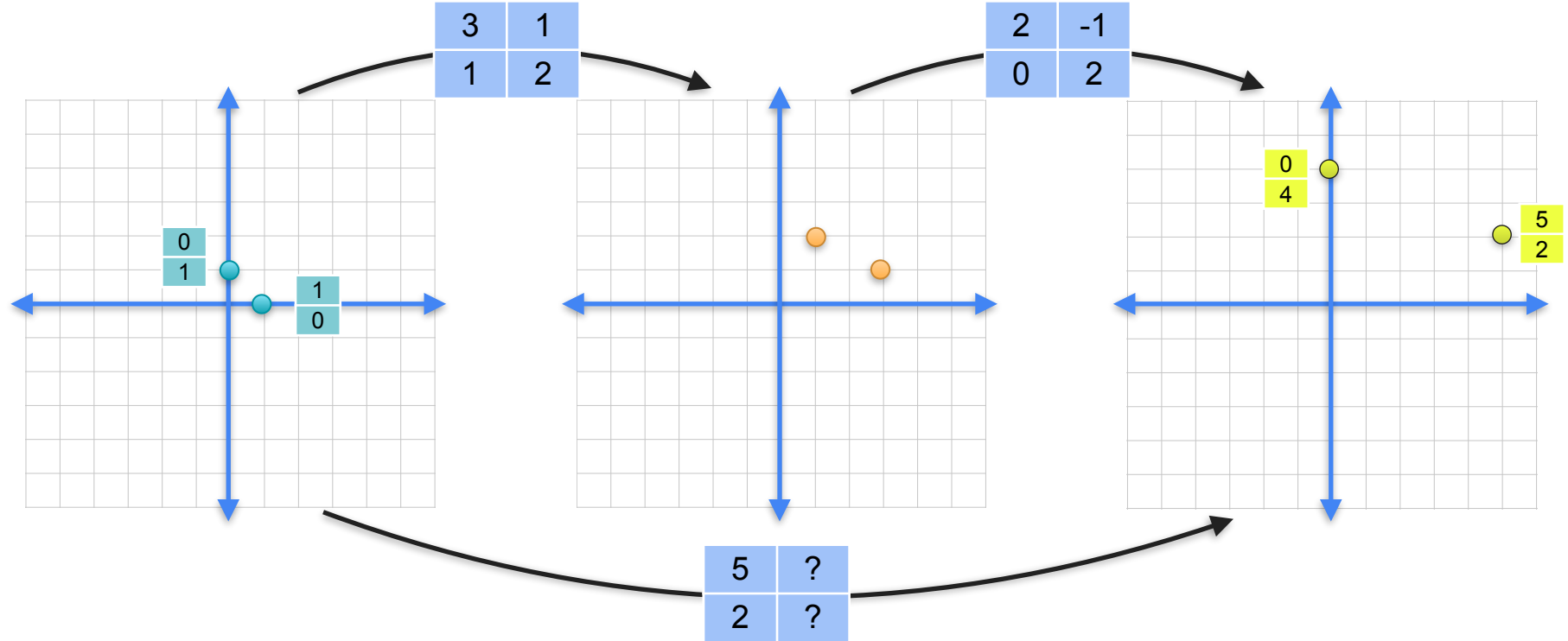


# Combining linear transformations

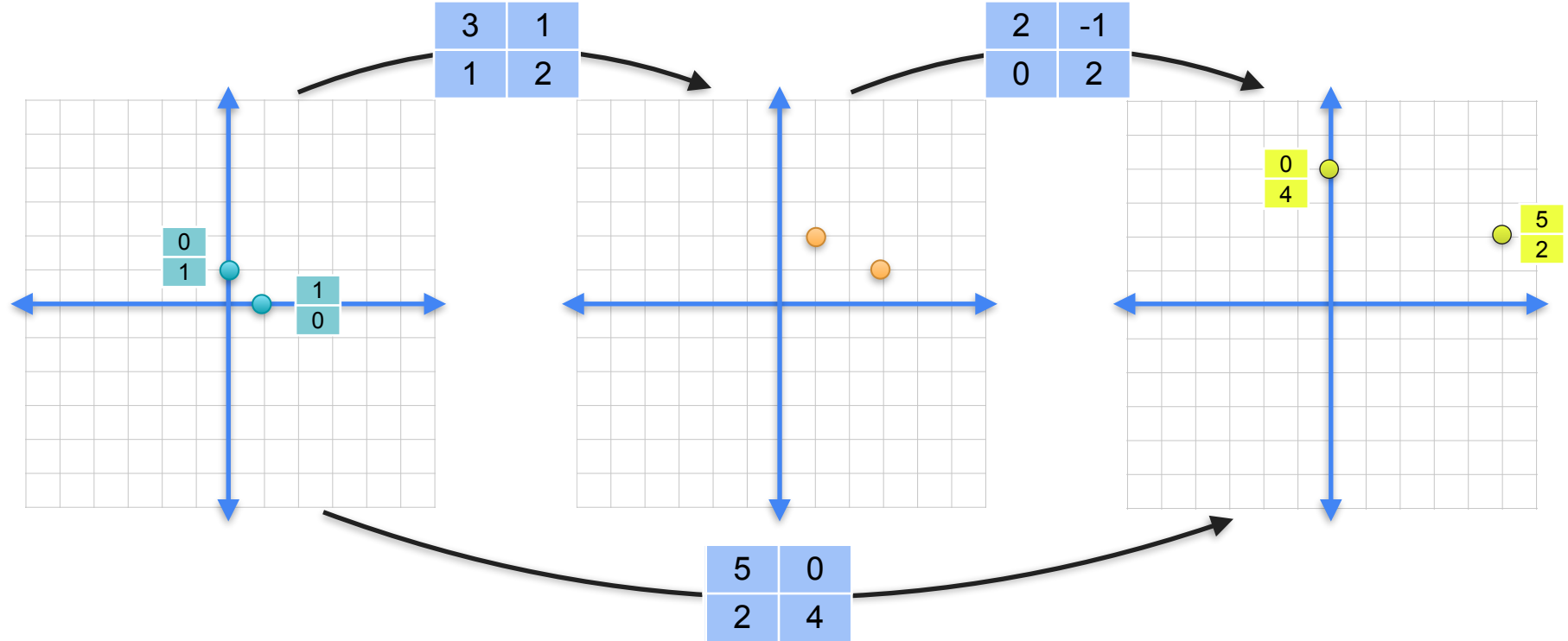




# Combining linear transformations



# Combining linear transformations



# Combining linear transformations

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

# Combining linear transformations

First

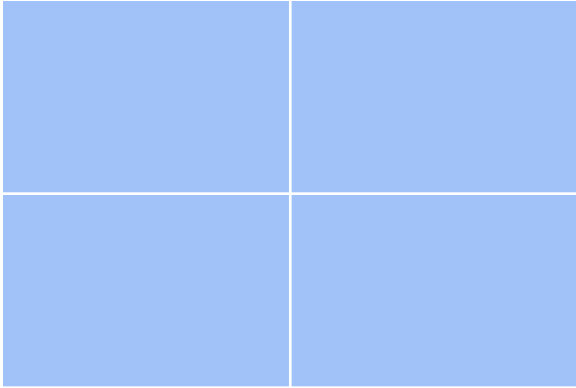
↓

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

# Combining linear transformations

$$\begin{array}{c} \text{Second} \\ \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \text{First} \\ \downarrow \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

# Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$


# Multiplying matrices

The diagram illustrates the multiplication of two 2x2 matrices. The first matrix (teal) is  $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$  and the second matrix (orange) is  $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ . The result is shown as a 2x2 grid of four 2x1 sub-matrices, each representing a dot product of a row from the first matrix and a column from the second matrix.

Matrix 1 (Teal):  $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$

Matrix 2 (Orange):  $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

Resulting 2x2 Grid (Blue background):

- Top-left:  $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$
- Top-right:  $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$
- Bottom-left:  $\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$
- Bottom-right:  $\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

# Multiplying matrices

The diagram illustrates the multiplication of two 2x2 matrices. The first matrix (teal) is  $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$  and the second matrix (orange) is  $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ . The result is a 2x2 block matrix where each element is a 2x2 sub-matrix.

The result matrix is composed of four blocks:

- Top-left block: The scalar value 5.
- Top-right block: A 2x2 matrix formed by the first row of the first matrix and the second column of the second matrix:  $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$ .
- Bottom-left block: A 2x2 matrix formed by the second row of the first matrix and the first column of the second matrix:  $\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$ .
- Bottom-right block: A 2x2 matrix formed by the second row of the first matrix and the second column of the second matrix:  $\begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$ .



# Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$$

# Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & \begin{bmatrix} 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$$

# Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$



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# Vectors and Linear Transformations

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## **The identity matrix**

# The identity matrix

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

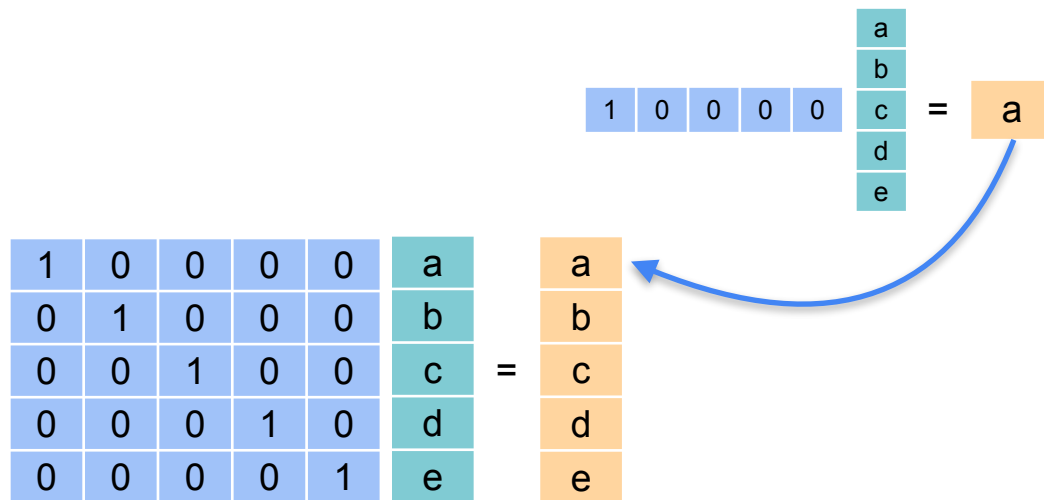
# The identity matrix

1	0	0	0	0	a
0	1	0	0	0	b
0	0	1	0	0	c
0	0	0	1	0	d
0	0	0	0	1	e

# The identity matrix

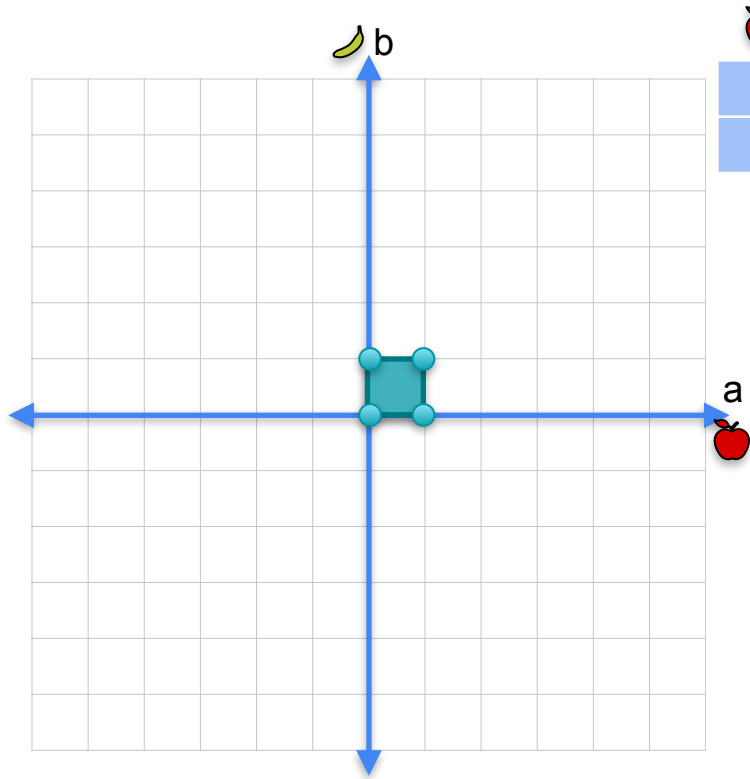
1	0	0	0	0	a	a
0	1	0	0	0	b	b
0	0	1	0	0	c	c
0	0	0	1	0	d	d
0	0	0	0	1	e	e

# The identity matrix

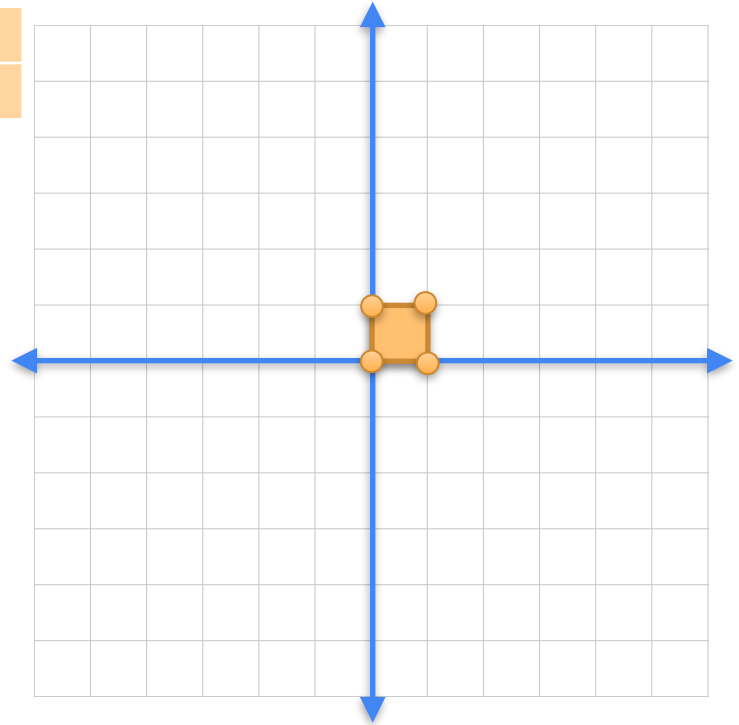




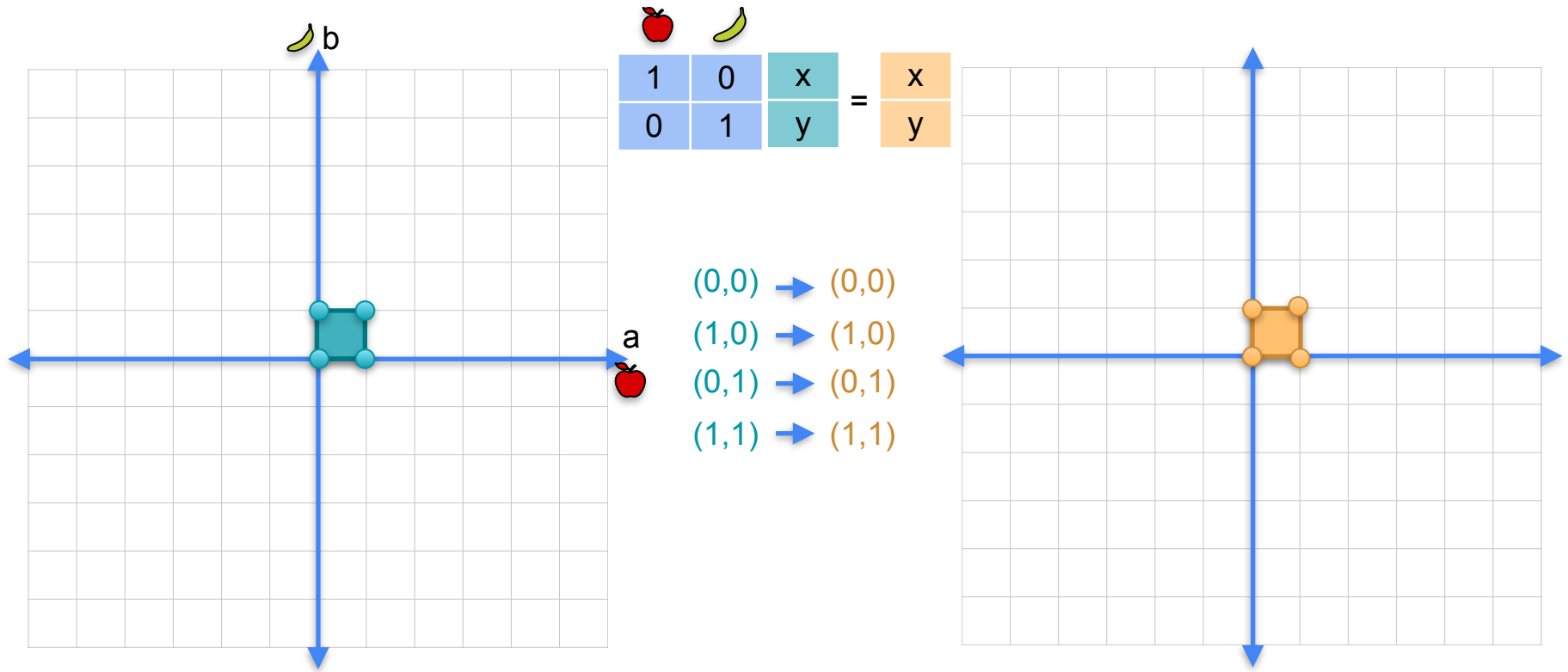
# The identity matrix



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{matrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$



# The identity matrix





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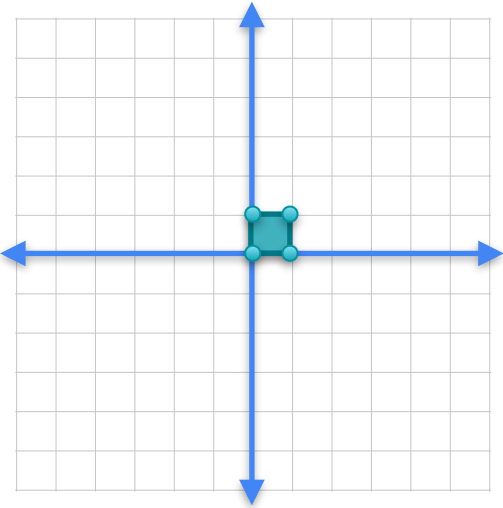
# Vectors and Linear Transformations

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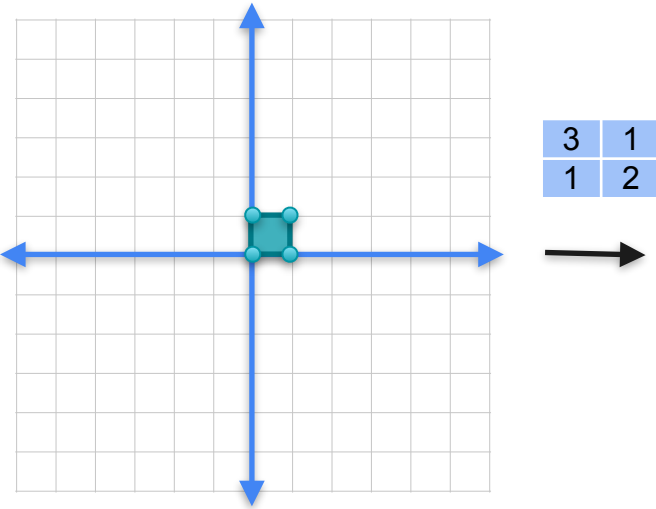
## **Matrix inverse**

# Matrix inverses

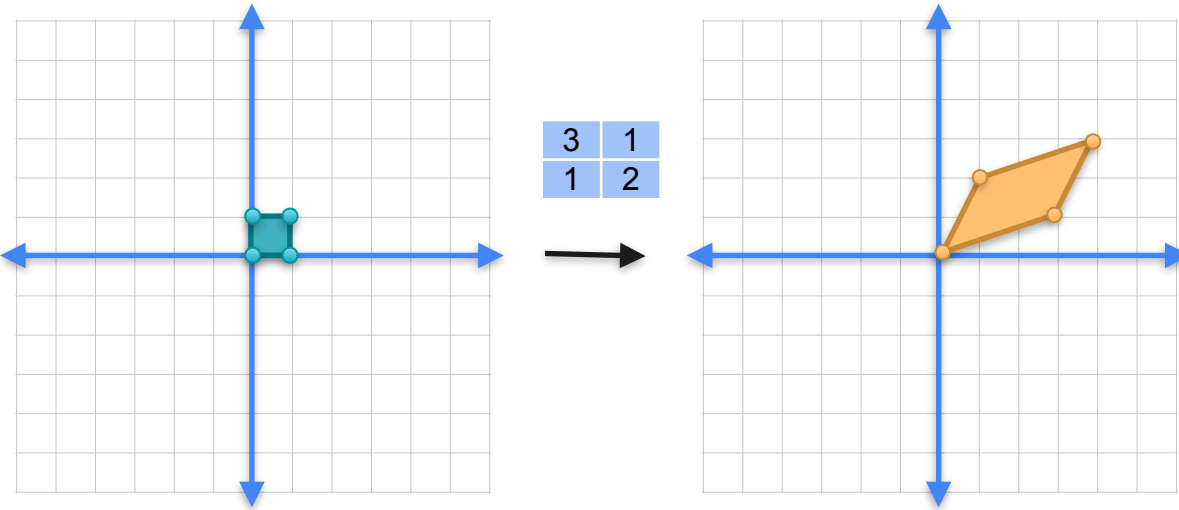
# Matrix inverses



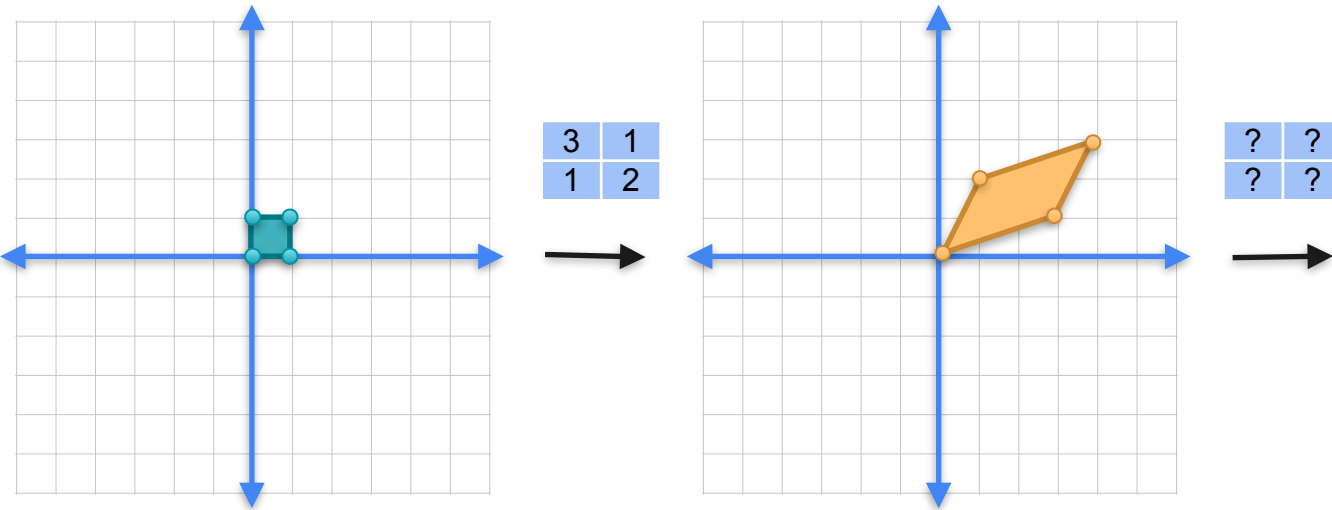
# Matrix inverses



# Matrix inverses

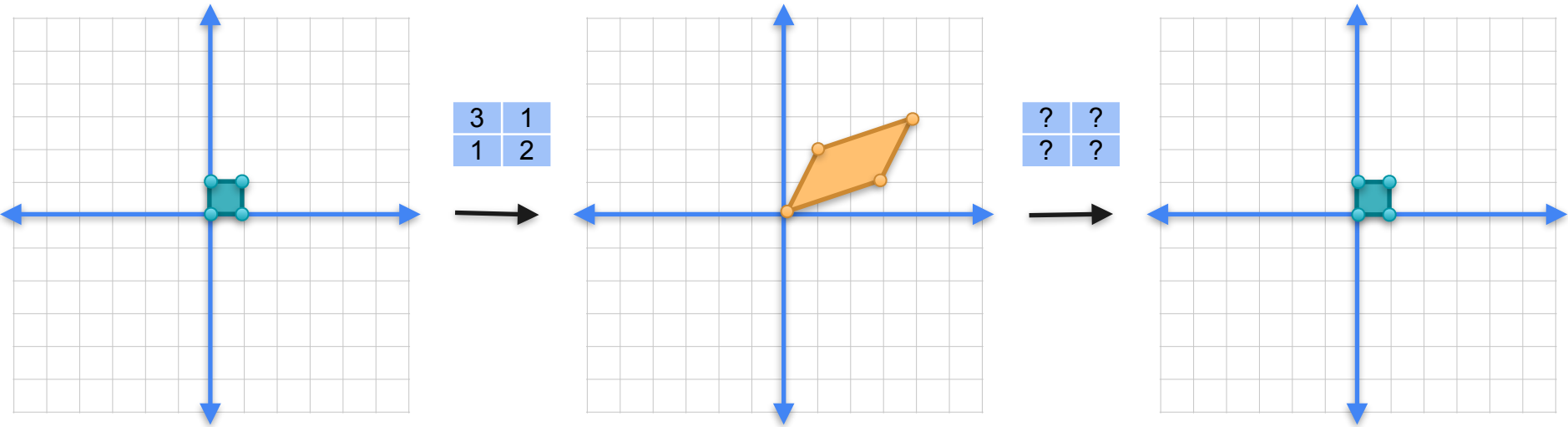


# Matrix inverses

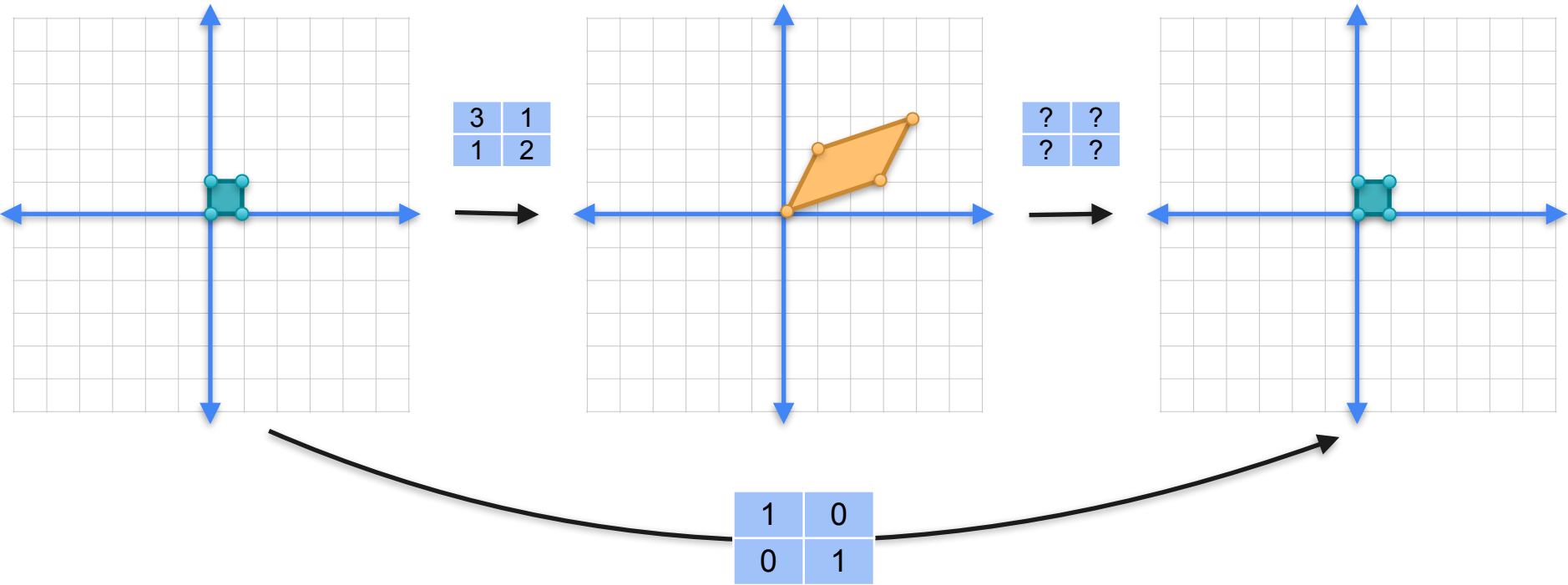




# Matrix inverses



# Matrix inverses



# Multiplying matrices

# Multiplying matrices

a	b
c	d

# Multiplying matrices


$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

# Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

# Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

# Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
  
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix}$$

The diagram illustrates the relationship between a matrix and its inverse. A blue arrow points from the matrix  $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$  to the matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  in the first equation, indicating that  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is the inverse of  $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ .



# How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

# How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1 \\ \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0 \\ \begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0 \\ \begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \end{array}$$

# How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1$$

$$3a + 1b = 1$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$1a + 2b = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0$$

$$3c + 1d = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

$$1c + 2d = 1$$

# How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1$$

$$3a + 1b = 1$$

$$a = \frac{2}{5}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$1a + 2b = 0$$

$$b = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0$$

$$3c + 1d = 0$$

$$c = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

$$1c + 2d = 1$$

$$d = \frac{3}{5}$$

# Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

5	2
1	2

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} \begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1 \\ \begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0 \\ \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0 \\ \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1 \end{array}$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\bullet 5a + 2c = 1$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\bullet 5b + 2d = 0$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\bullet b + 2d = 1$$



# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

$$\bullet d = 5/8$$

# Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

1	1
2	2

# Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says  $a+c=1$ , and equation 3 says  $2a+2c=0$ .



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# Vectors and Linear Transformations

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**Which matrices have an  
inverse?**

# Which matrices have inverses?



# Which matrices have inverses?

$$5^{-1} = 0.2$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$



# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

3	1
1	2

<sup>-1</sup> = 

0.4	-0.2
-0.2	0.6

Non-singular matrix  
Invertible

5	2
1	2

<sup>-1</sup> = 

0.25	-0.25
-0.125	0.625

Non-singular matrix  
Invertible

1	1
2	2

 = 

?	?
?	?

Singular matrix  
Non-invertible

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix  
Non-invertible

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 8$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix  
Non-invertible

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

3	1
1	2

<sup>-1</sup> = 

0.4	-0.2
-0.2	0.6

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

5	2
1	2

<sup>-1</sup> = 

0.25	-0.25
-0.125	0.625

Non-singular matrix  
Invertible

$$\text{Det} = 8$$

1	1
2	2

 = 

?	?
?	?

Singular matrix  
Non-invertible

$$\text{Det} = 0$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 8$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix  
Non-invertible

$$\text{Det} = 0$$

Non-zero determinants



# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix  
Invertible

$$\text{Det} = 8$$

Non-zero determinants

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix  
Non-invertible

$$\text{Det} = 0$$

Zero determinant



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# Vectors and Linear Transformations

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**Neural networks and  
matrices**



# Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

# Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

**Scores:**

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

# Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

## Scores:

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

## Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

# Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

## Scores:

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

## Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

## Rule:

If the number of points of the sentence is bigger than \_\_\_\_, then the email is spam.

# Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

## Scores:

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

## Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

## Rule:

If the number of points of the sentence is bigger than \_\_\_\_, then the email is spam.

## Goal: Find the best points and threshold

Lottery: \_\_\_\_ point

Win: \_\_\_\_ point

Threshold: \_\_\_\_ points



# Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

## Solution:

Lottery: 1 point

Win: 1 point

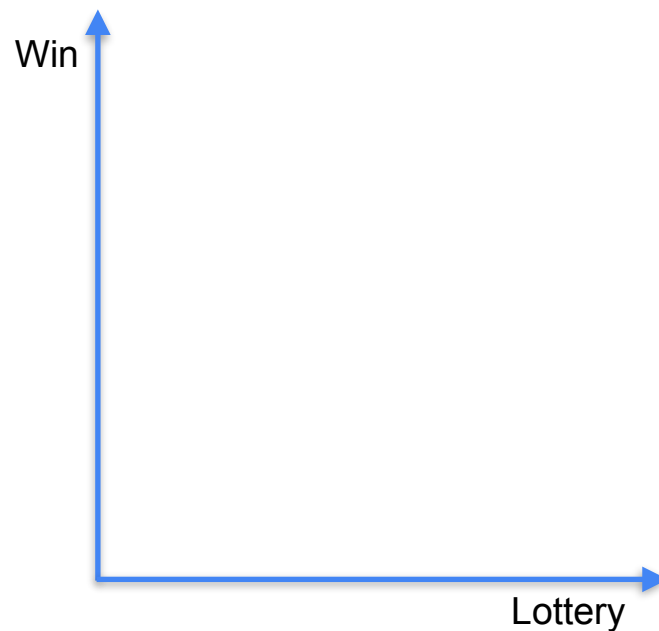
Threshold: 1.5 points

# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

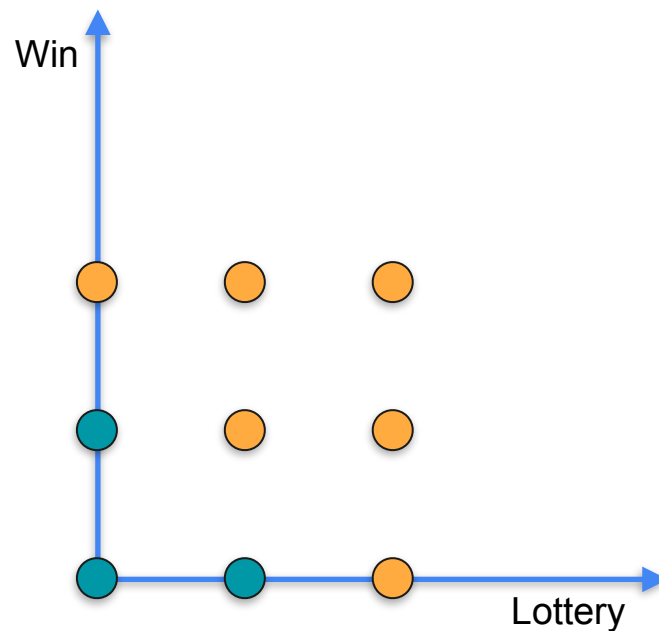
# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

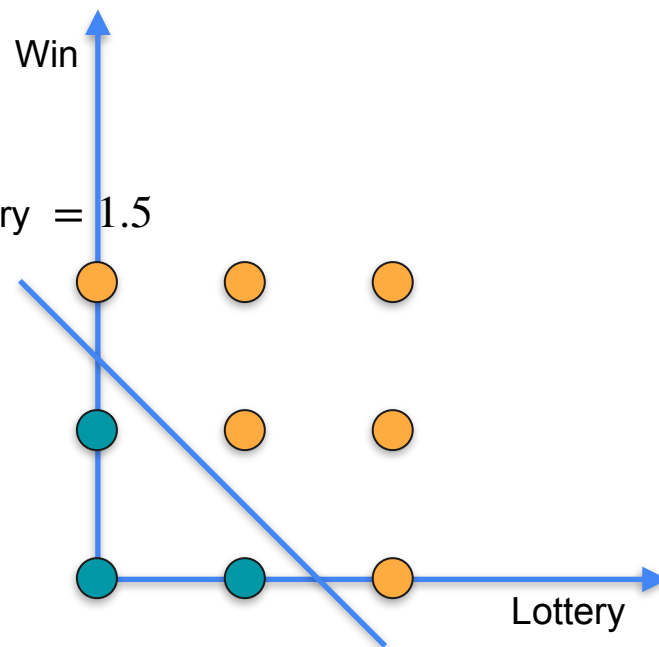


# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Line:

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$

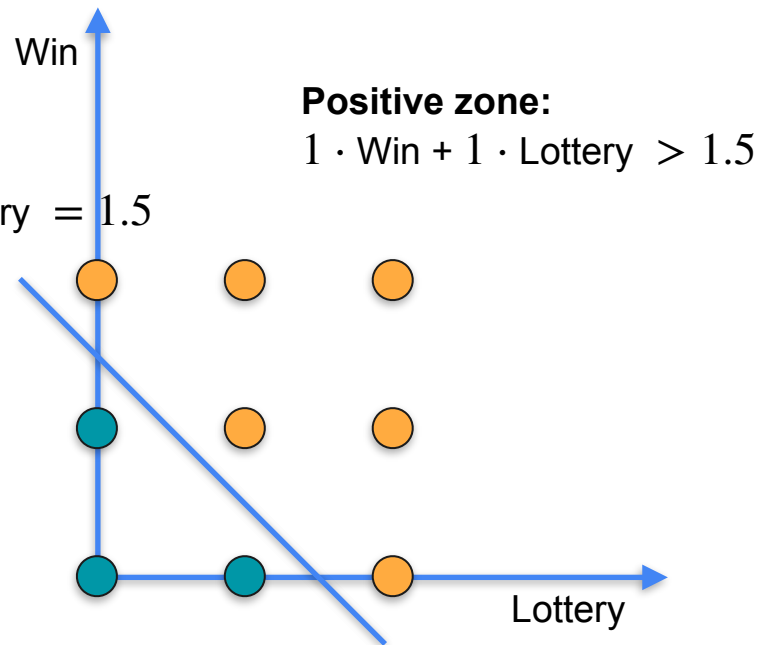


# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

**Line:**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$



# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

**Line:**

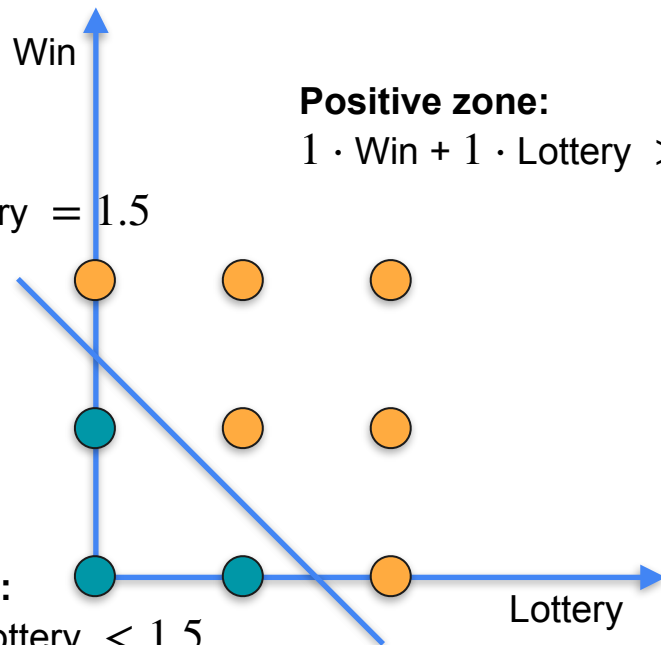
$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$

**Negative zone:**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} < 1.5$$

**Positive zone:**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} > 1.5$$



# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?



# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2	1
---	---

Model
1
1

Check: > 1.5?

# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2

1

Model
1
1

= 3

Check: > 1.5?

# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2

1

Model
1
1

= 3

Check: > 1.5?



Spam

# Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check:  $> 1.5$ ?

# Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0	1
---	---

Model
1
1

Check: > 1.5?

# Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0

1

Model
1
1

= 1

Check: > 1.5?

# Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0

1

Model
1
1

= 1

Check: > 1.5?



Not spam

# Matrix multiplication

Spam	Lottery	Win	<div>Model</div>
Yes	1	1	
Yes	2	1	
No	0	0	
Yes	0	2	
No	0	1	
No	1	0	
Yes	2	2	
Yes	2	0	
Yes	1	2	
			1
			1



# Matrix multiplication

Spam	Lottery	Win	Model	=	Prod
Yes	1	1			2
Yes	2	1	1		3
No	0	0	1		0
Yes	0	2			2
No	0	1			1
No	1	0			1
Yes	2	2			4
Yes	2	0			2
Yes	1	2			3

# Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?



# Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

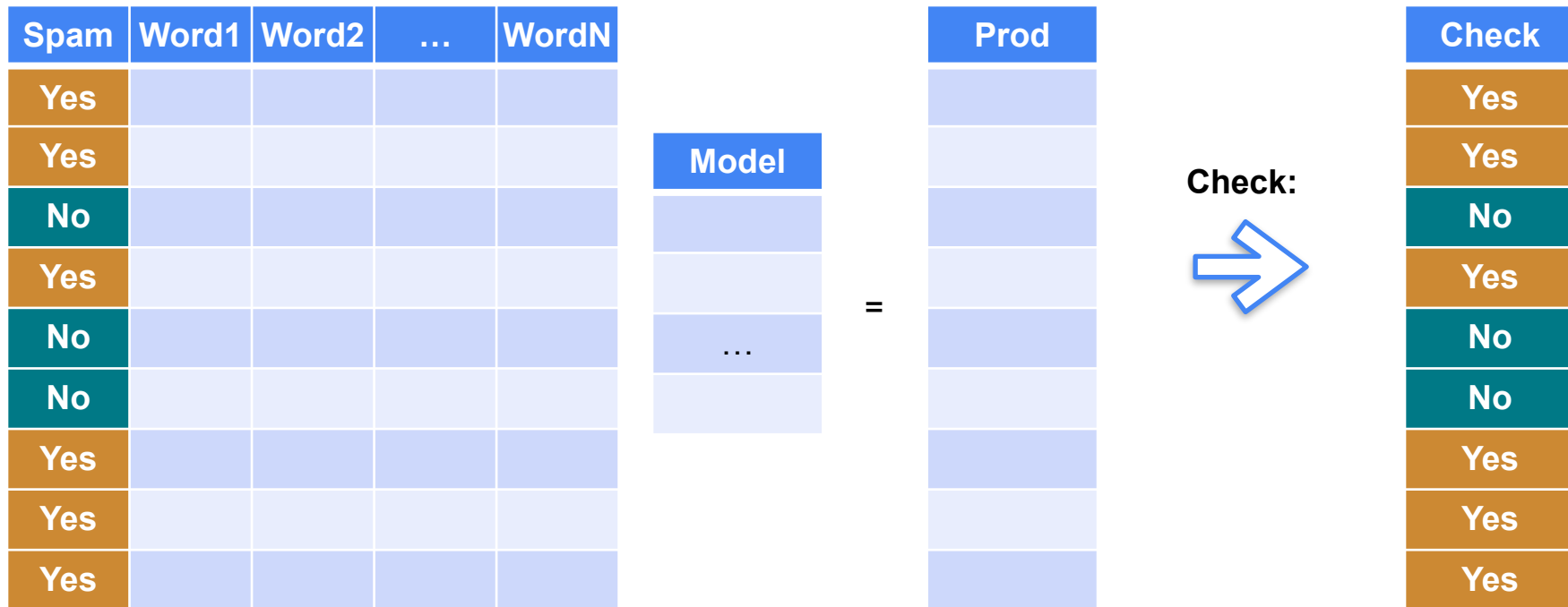
Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?



Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

# Perceptrons



# Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

# Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

 Threshold

Model
1
1

Check:  $> 1.5$ ?

# Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

**Check**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Model
1
1

**Check: > 1.5?**

# Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

**Check**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Model
1
1

**Check: > 1.5?**



# Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

**Check**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$
$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Model
1
1
-1.5

**Check: > 1.5?**

# Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

**Check**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Bias

Model
1
1
-1.5

**Check: > 0?**

Bias

# The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

# The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

# The AND operator

AND	x	y		Dot prod
No	0	0		0
No	1	0		1
No	0	1		1
Yes	1	1		2

Model
1
1

=

Dot prod
0
1
1
2

# The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

Check: >1.5?



# The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

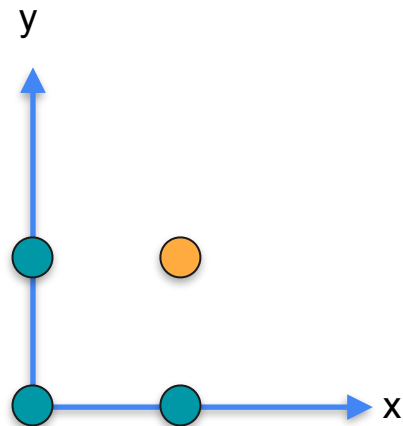
Check: >1.5?



Check
No
No
No
Yes

# The AND operator

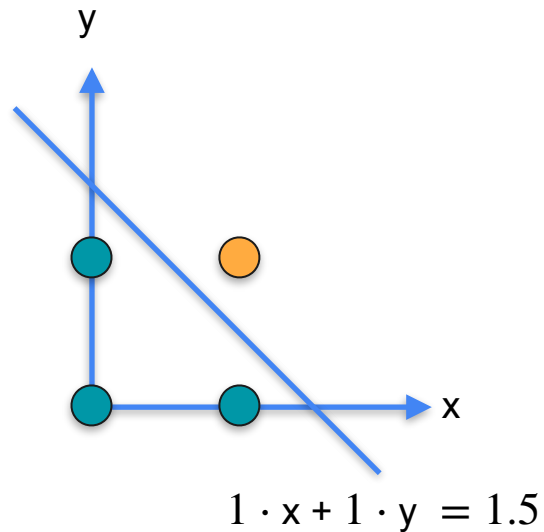
AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1



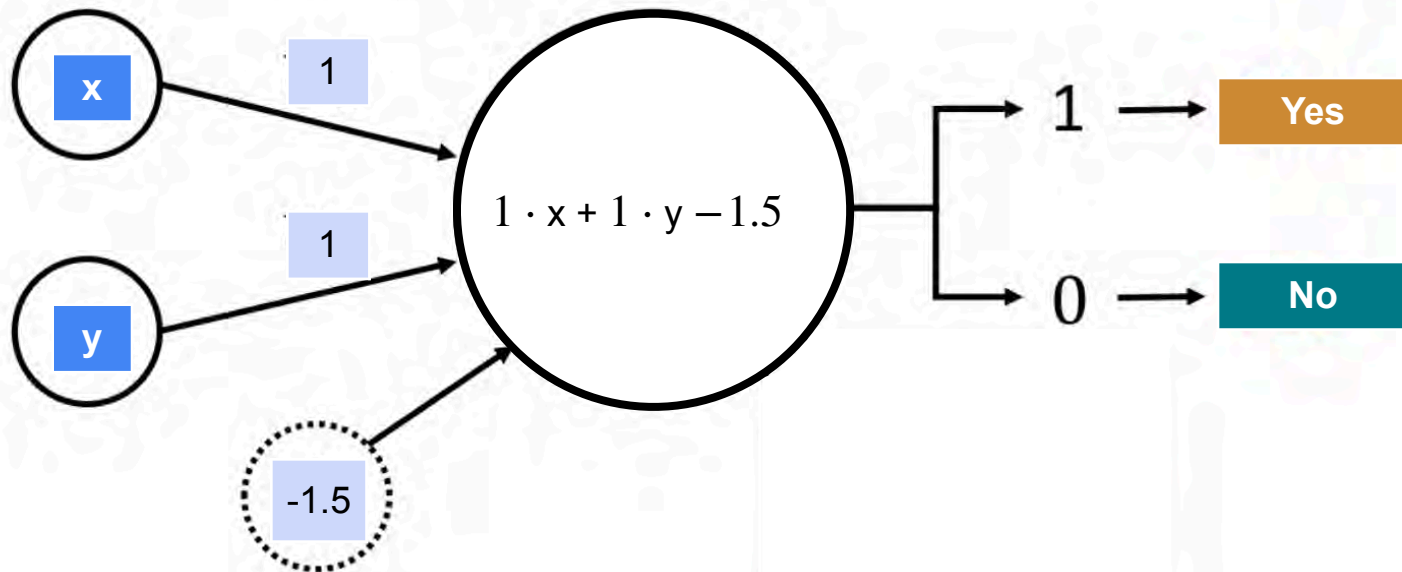


# The AND operator

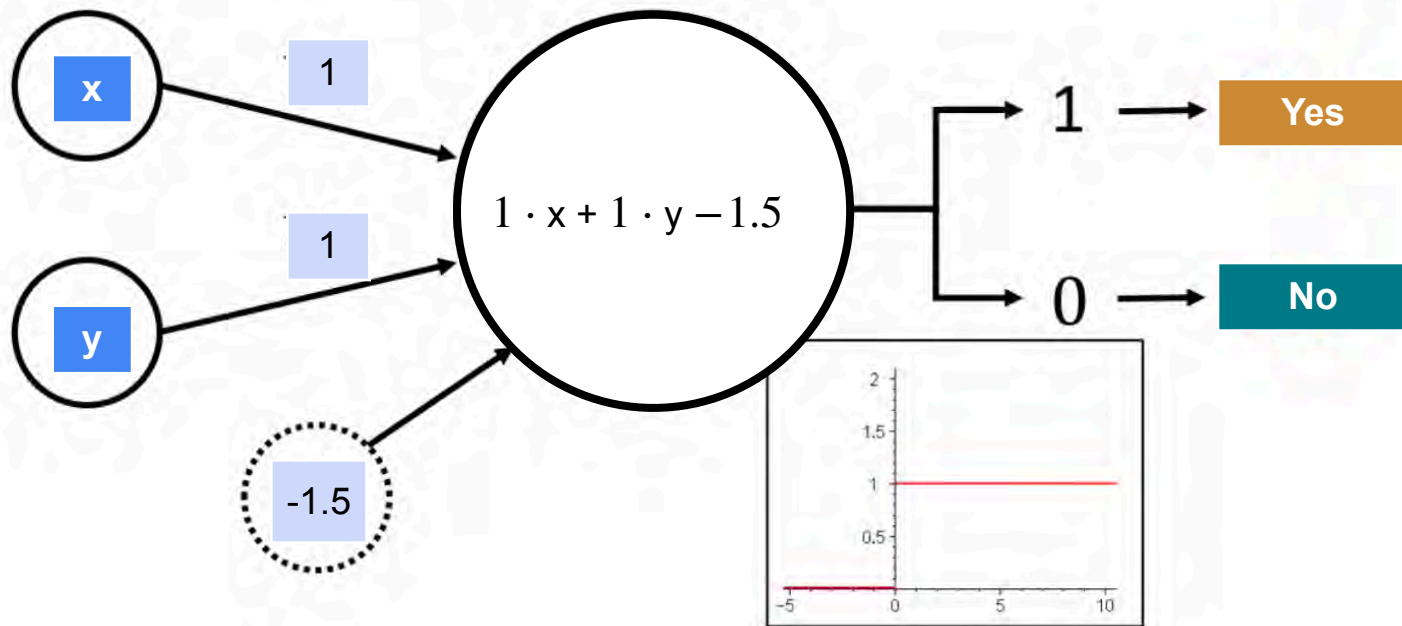
AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

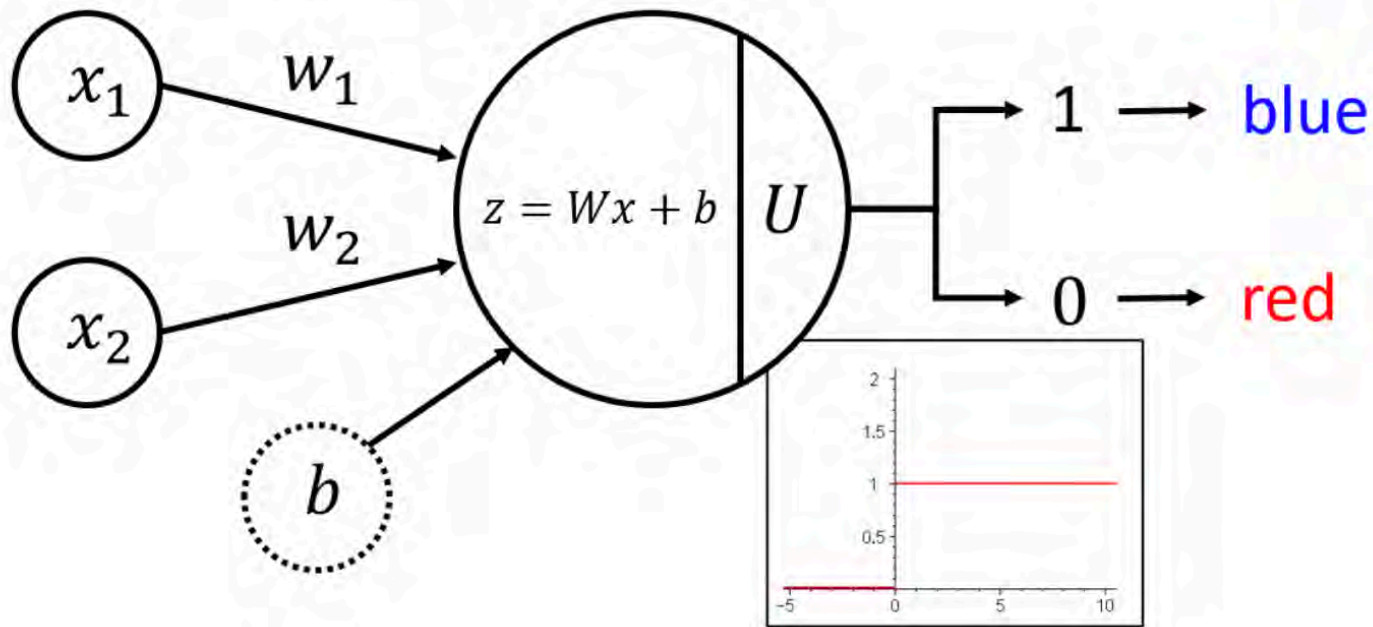


# The perceptron



# The perceptron







DeepLearning.AI

# Vectors and Linear Transformations

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## Conclusion