DeepLearning.AI

Math for Machine Learning

Probability and Statistics

DeepLearning.AI

Math for Machine Learning

Probability and Statistics - Week 2

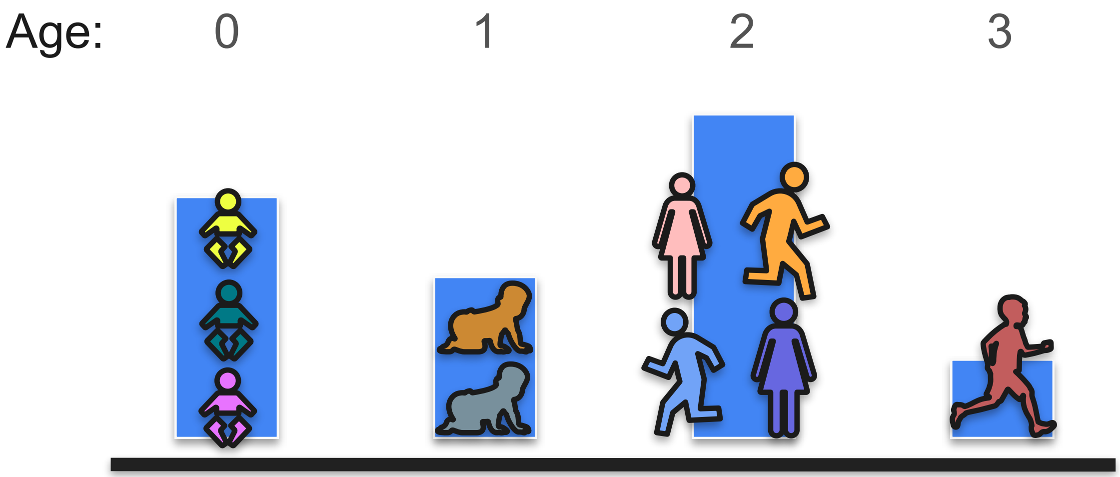
W2 Lesson 1

DeepLearning.AI

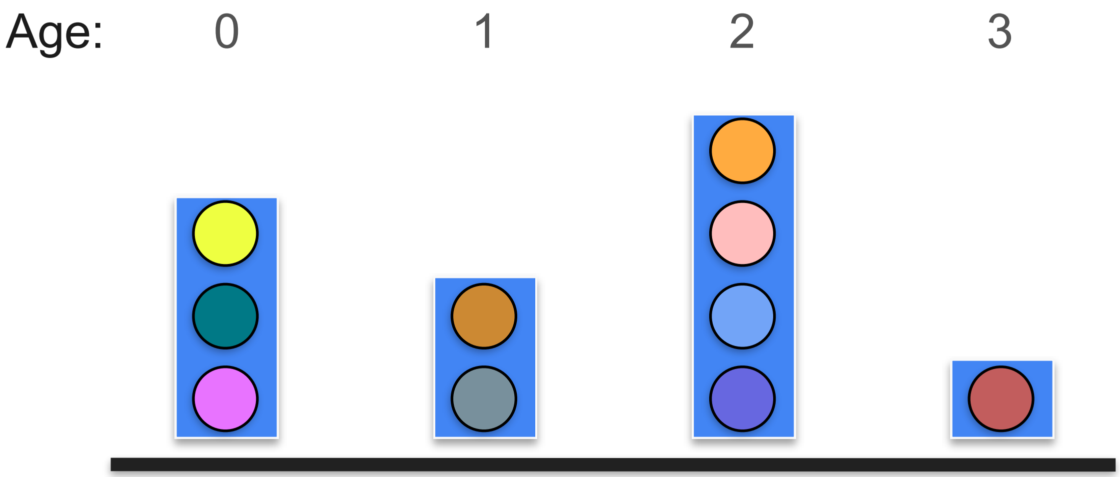
Describing Distributions

Measures of Central Tendency

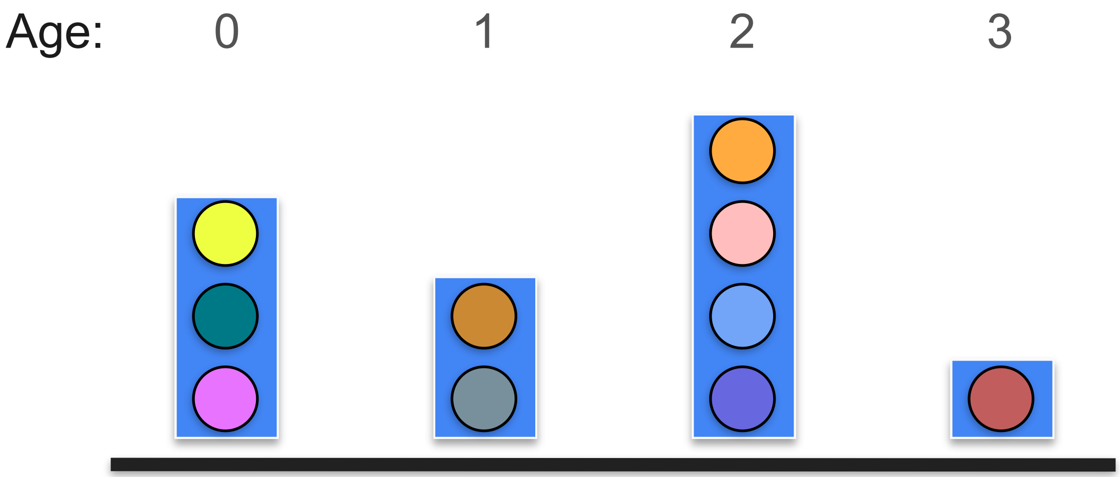
Mean: Example



Mean: Example

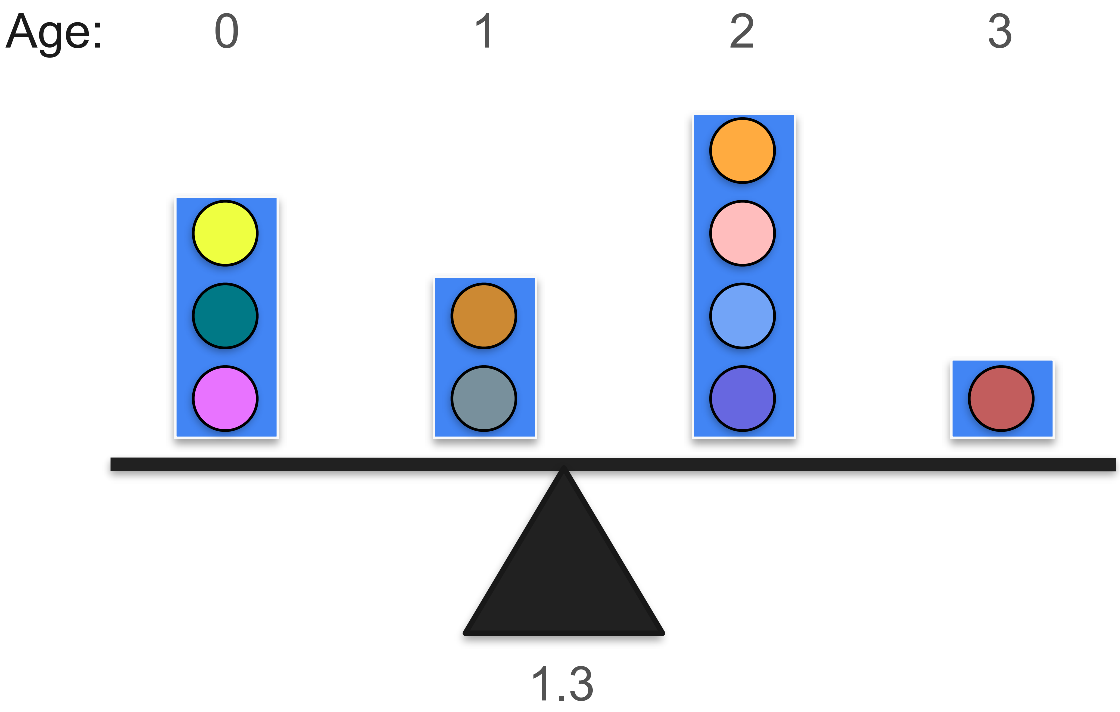


Mean: Example

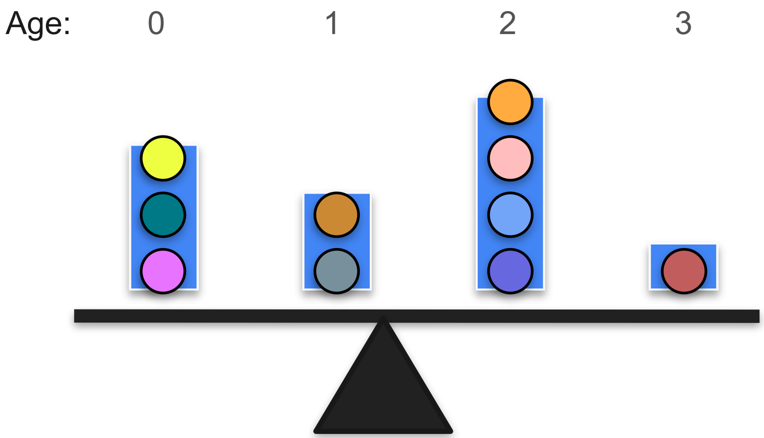


1.3

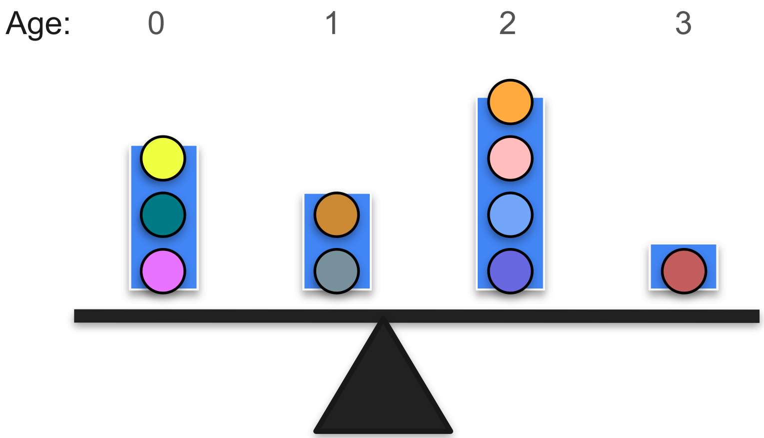
Mean: Example



Mean: Example

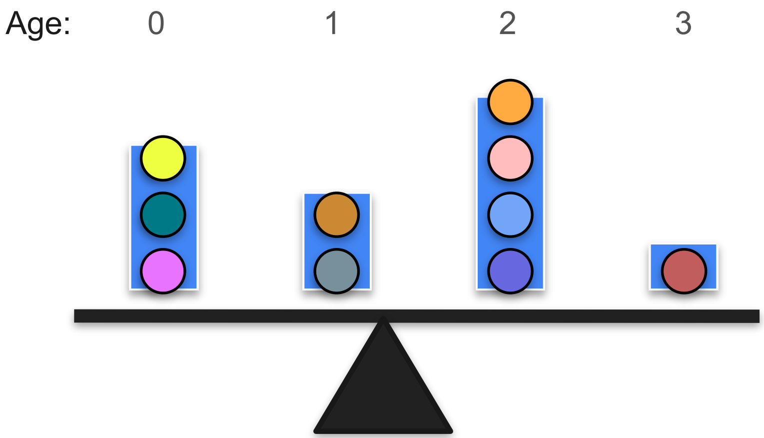


Mean: Example



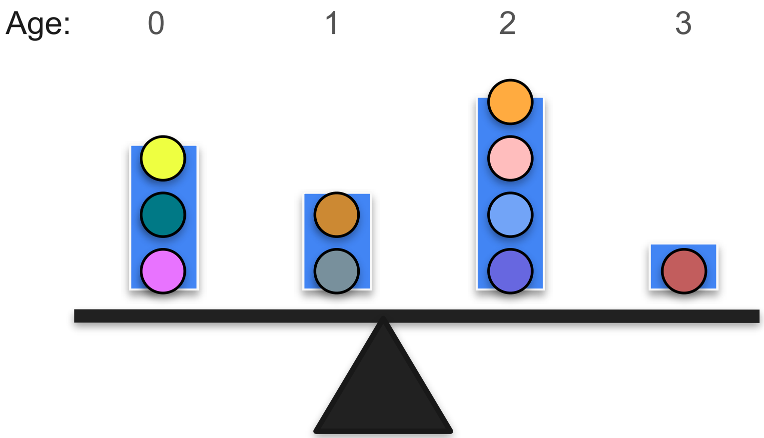
$$0 + 0 + 0$$

Mean: Example



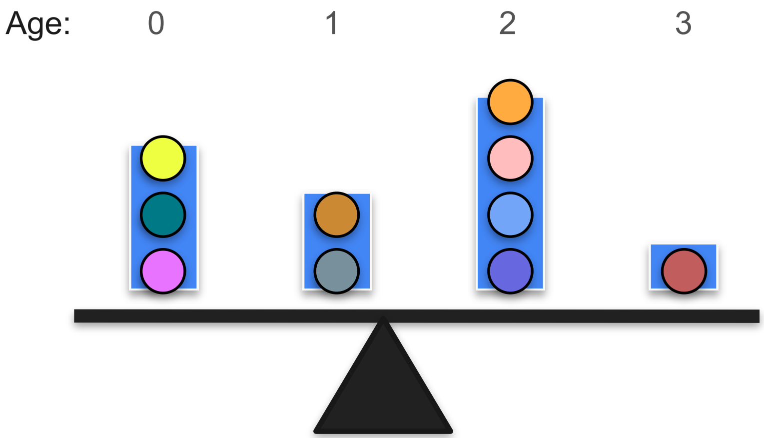
$$0 + 0 + 0 \ \ + 1 + 1$$

Mean: Example



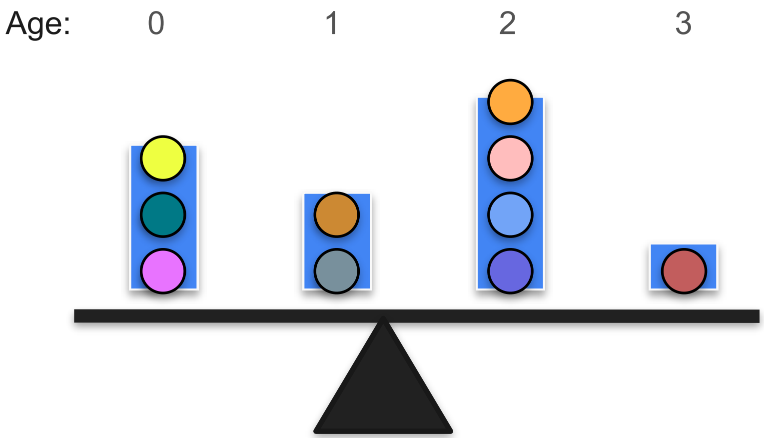
$$0 + 0 + 0 \;\; + 1 + 1 \;\; + 2 + 2 + 2 + 2$$

Mean: Example



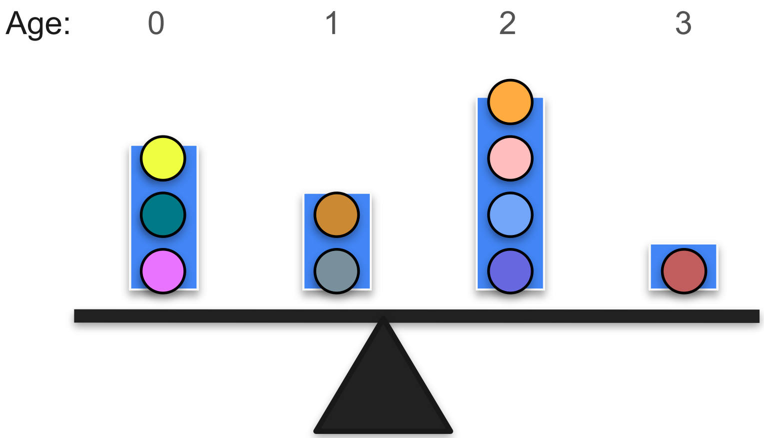
$$0+0+0+1+1+2+2+2+2+3$$

Mean: Example



$$\frac{0+0+0\ +1+1\ +2+2+2+2\ +3}{10}$$

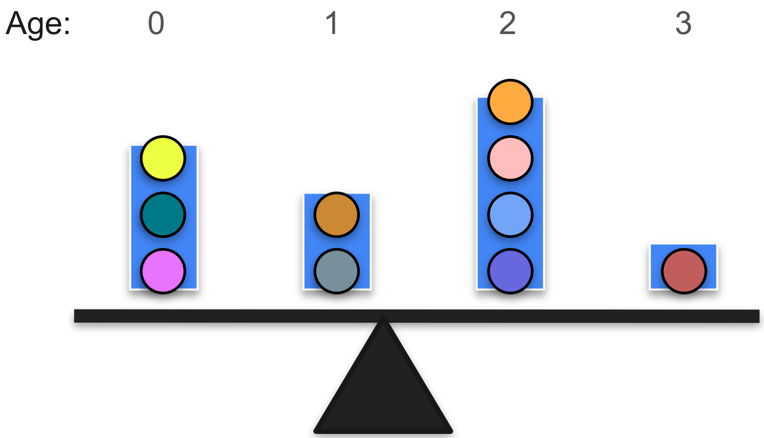
Mean: Example



$$\frac{0+0+0\ +1+1\ +2+2+2+2\ +3}{10}$$

$$= \frac{13}{10}$$

Mean: Example

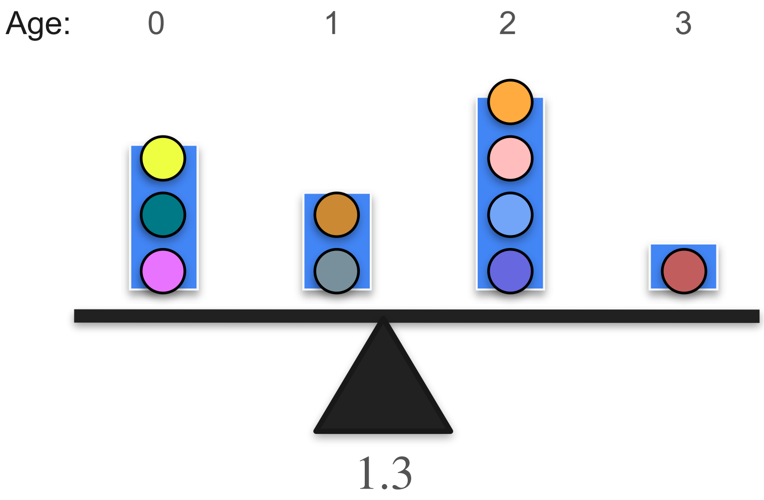


$$\frac{0+0+0\ +1+1\ +2+2+2+2\ +3}{10}$$

$$=\frac{13}{10}$$

$$=1.3$$

Mean: Example

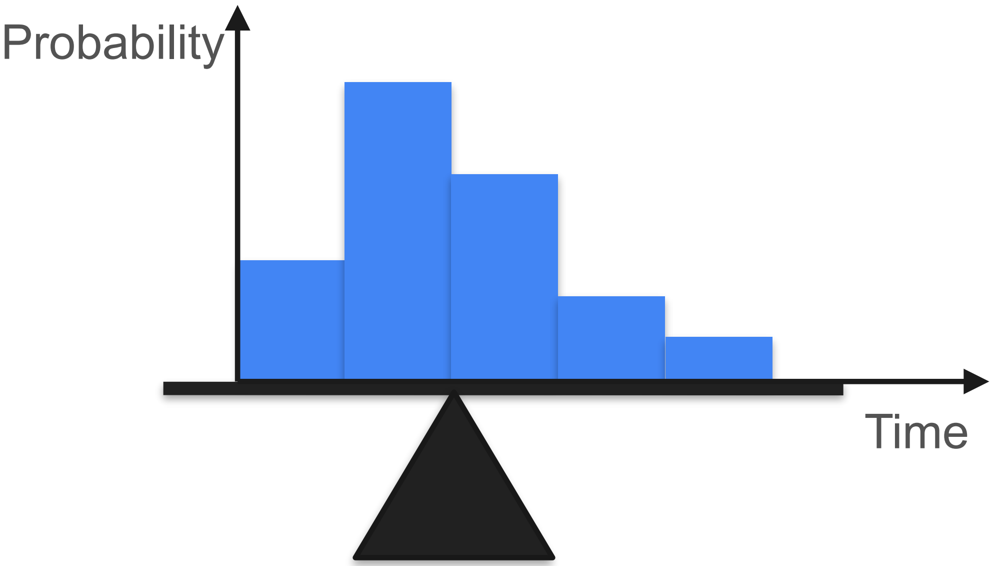


$$\frac{0+0+0+1+1+2+2+2+2+3}{10}$$

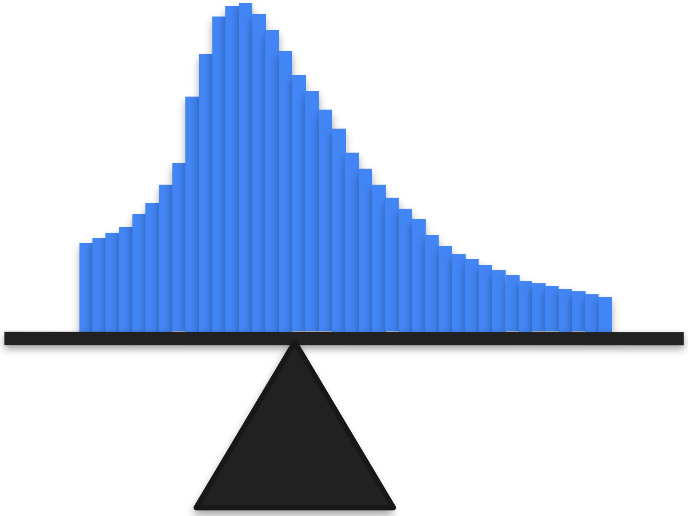
$$=\frac{13}{10}$$

$$=1.3$$

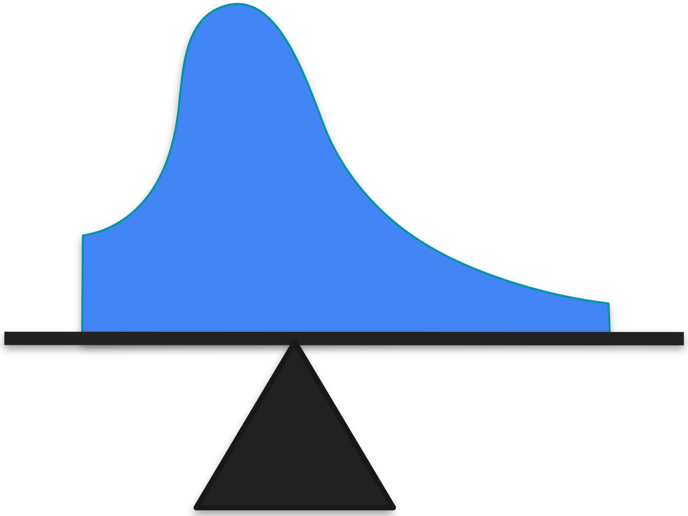
Mean



Mean



Mean



Median: Motivation

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Michael Jordan

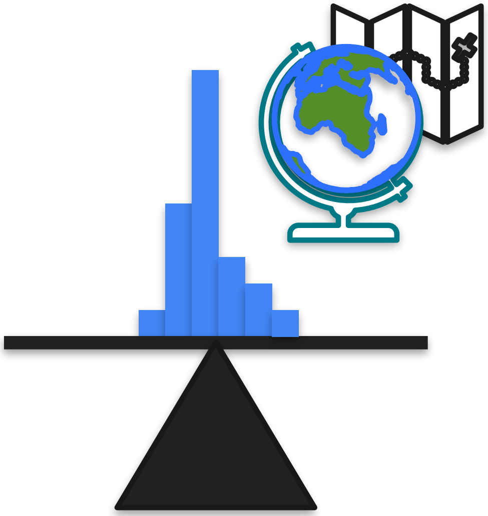
Outliers

Outliers

Outliers

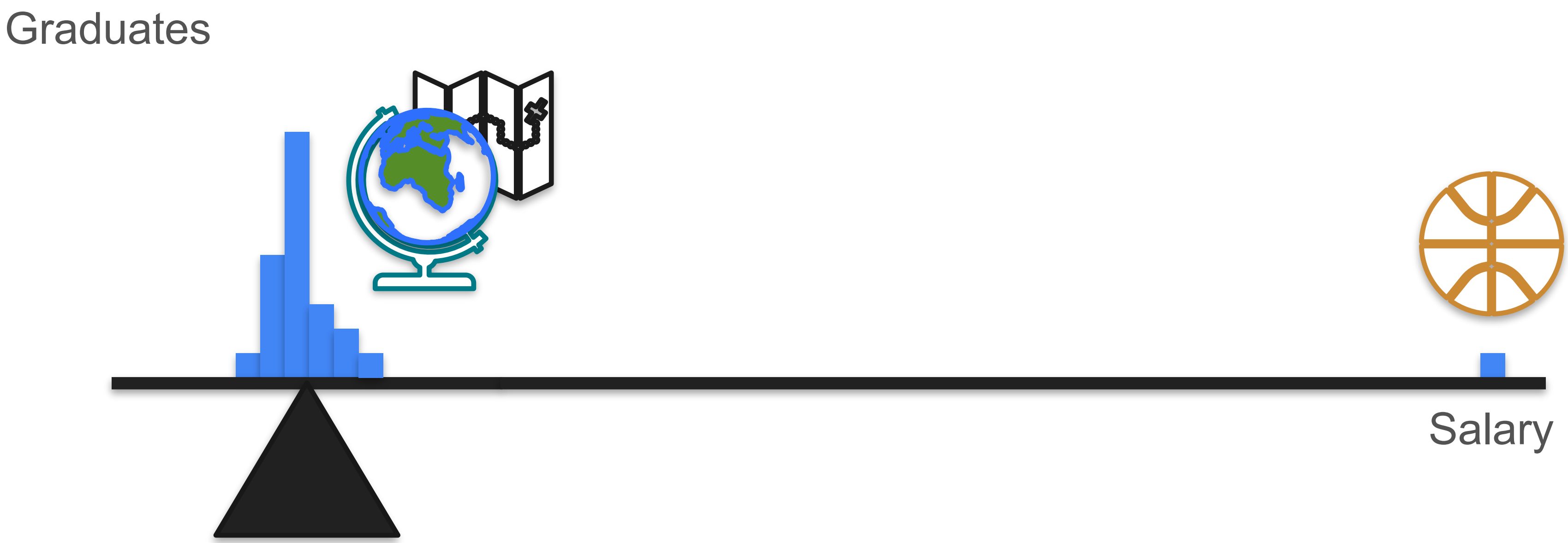
Outliers

Graduates

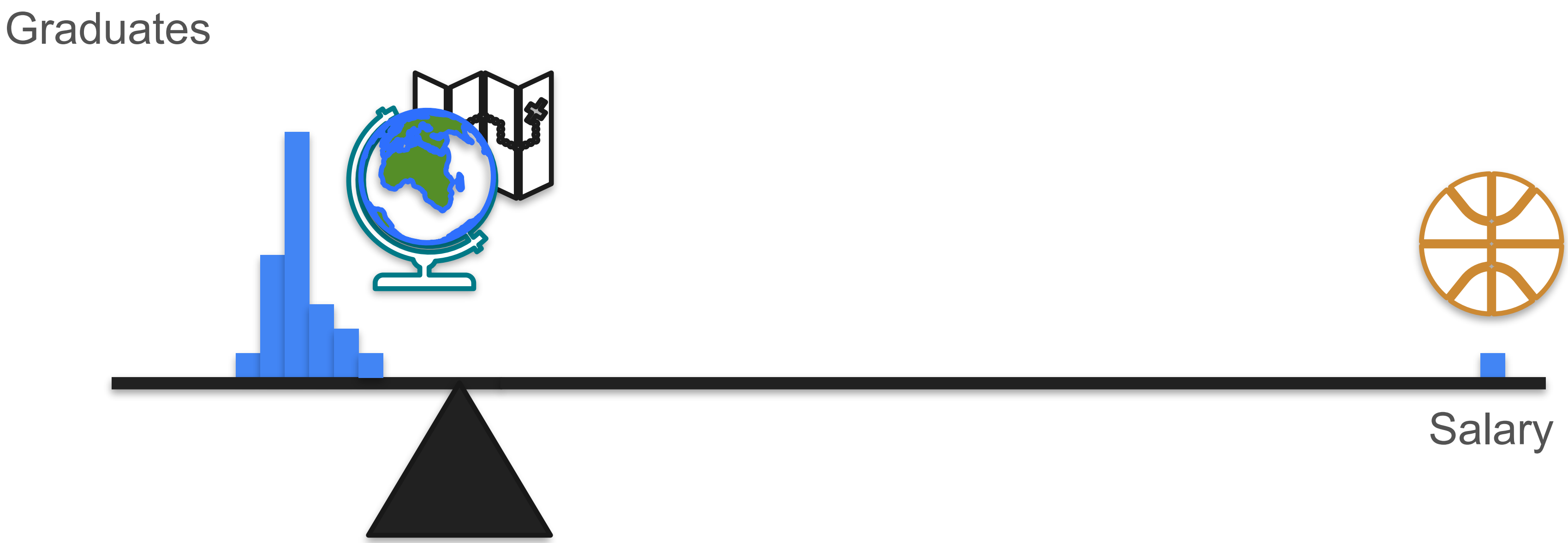


Salary

Outliers



Outliers



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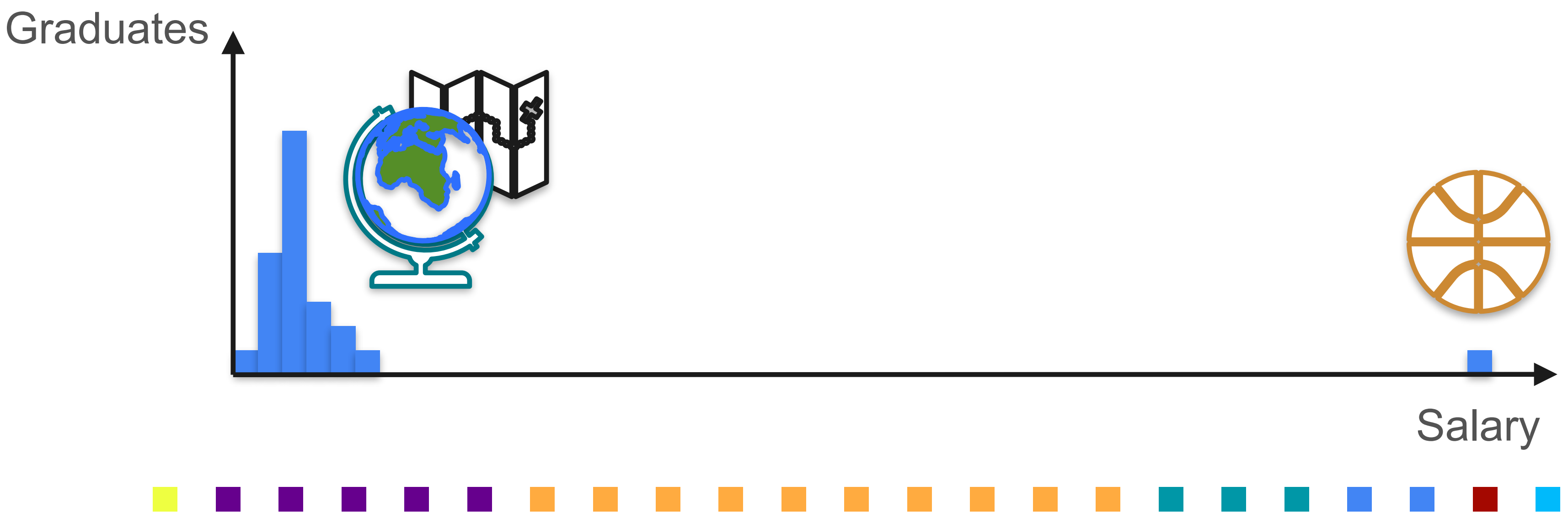
Outliers

Median



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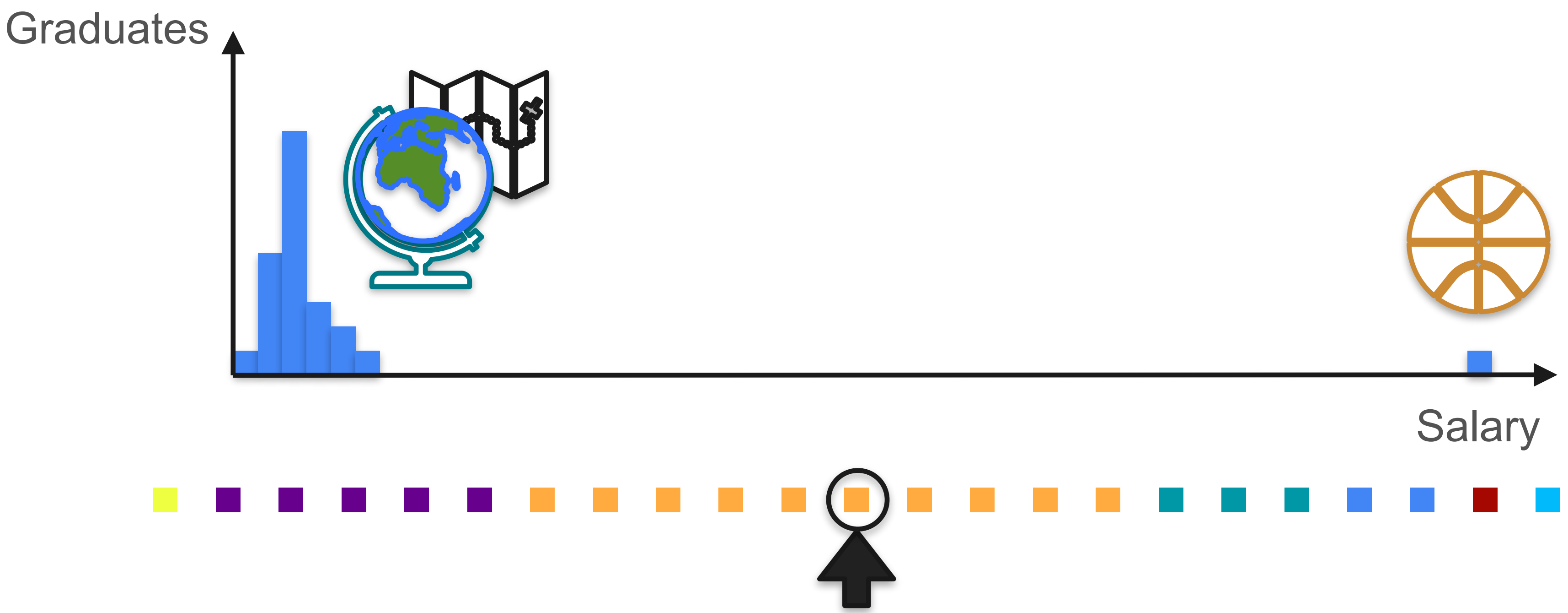
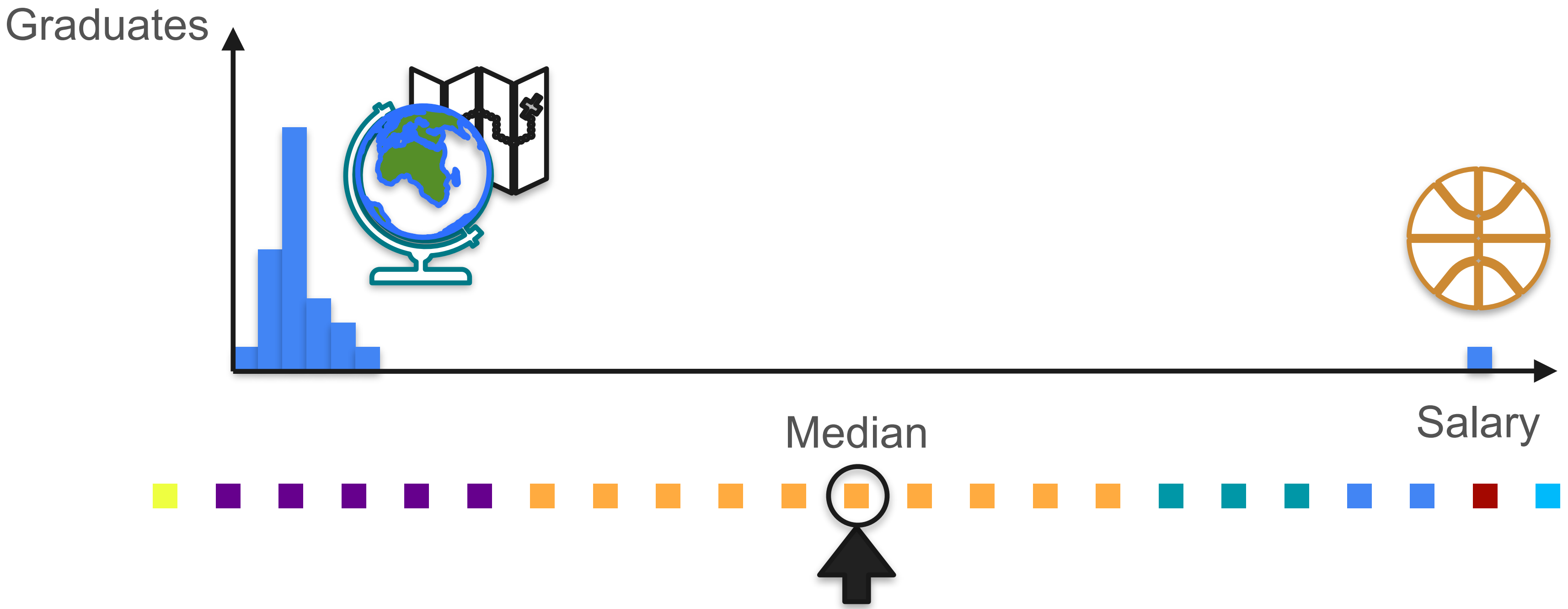
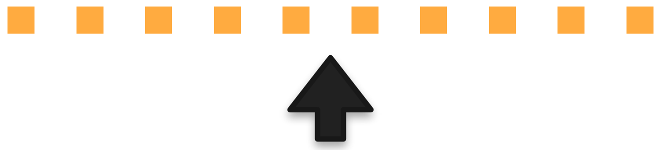
Median



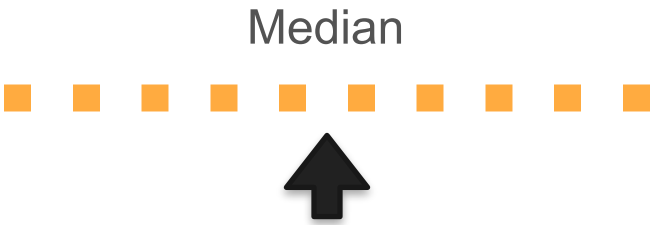

Graduates
Salary
Median

Median

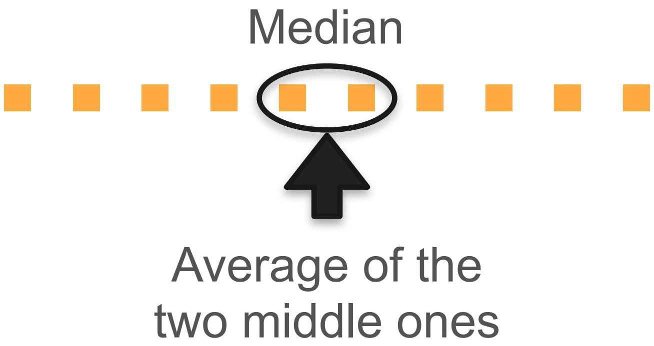
Median



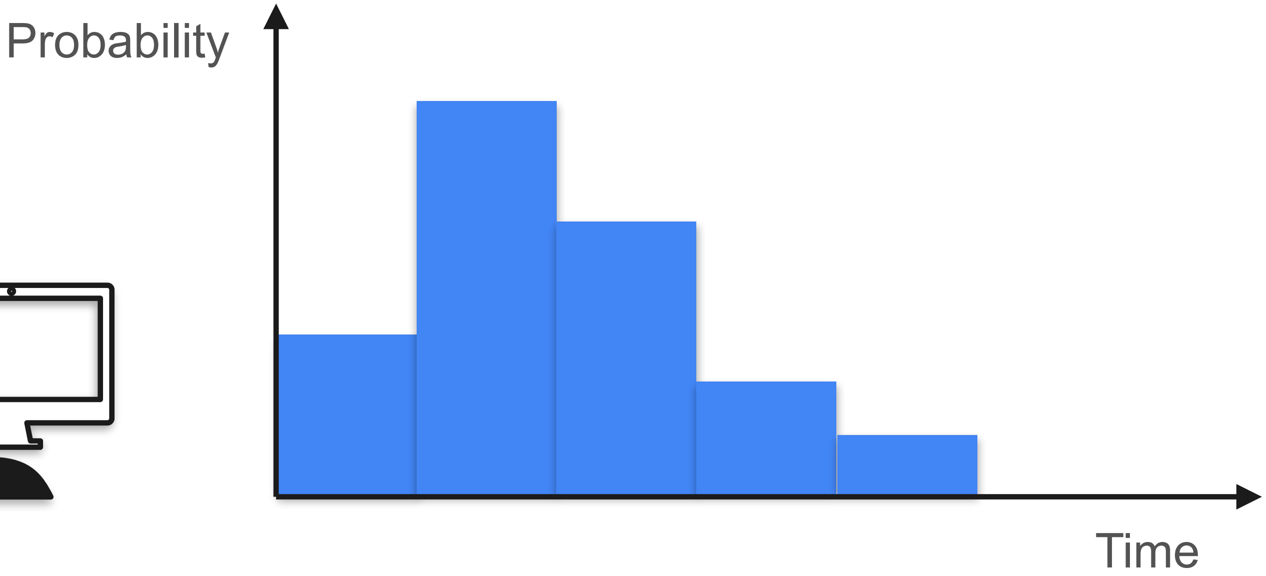
Median



Median



Mode



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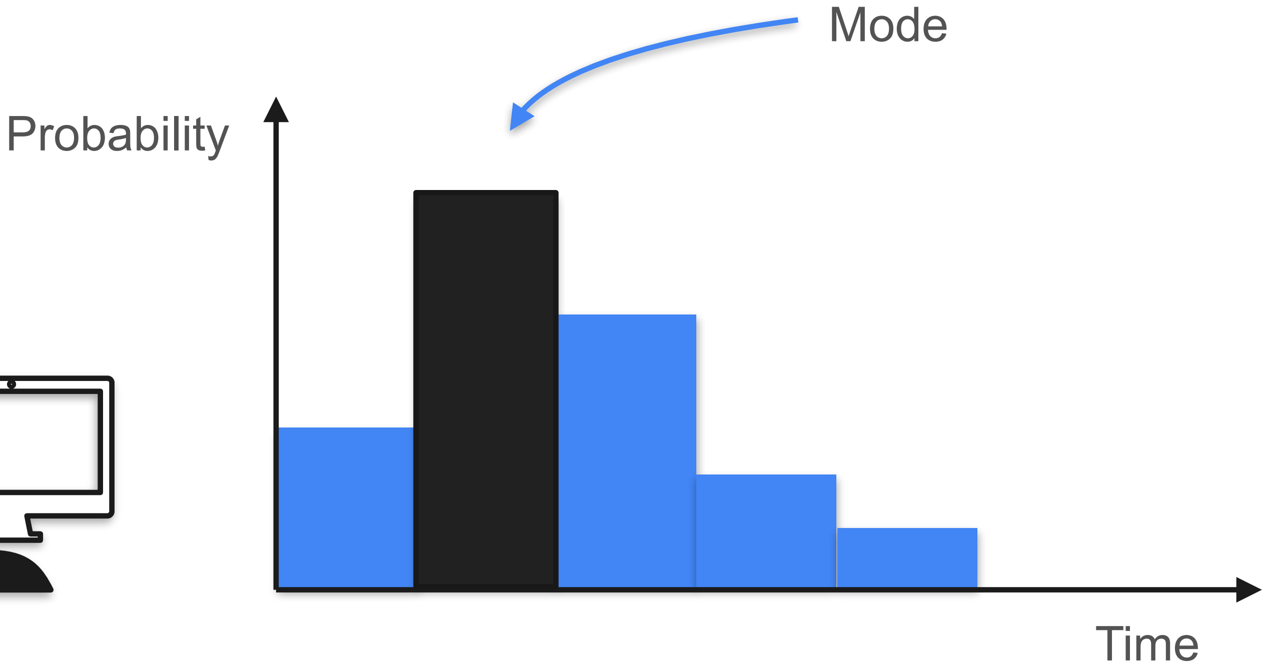

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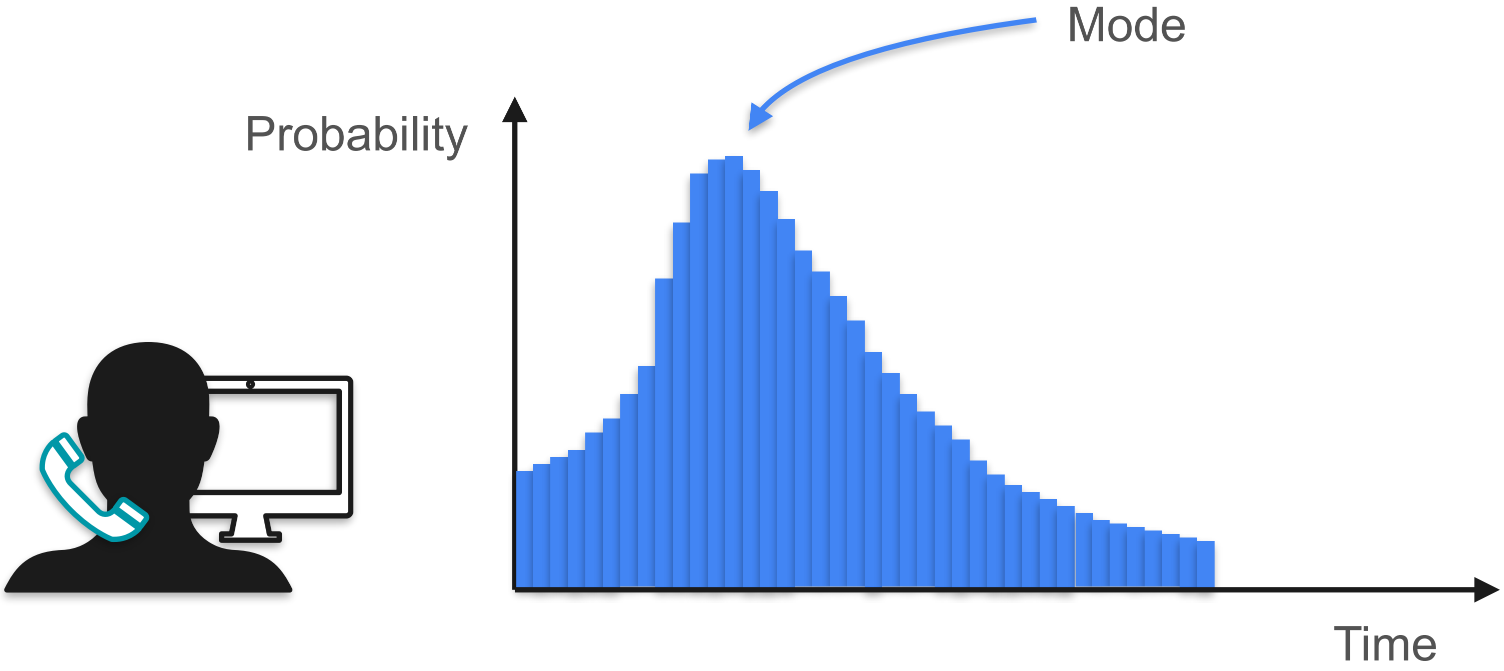

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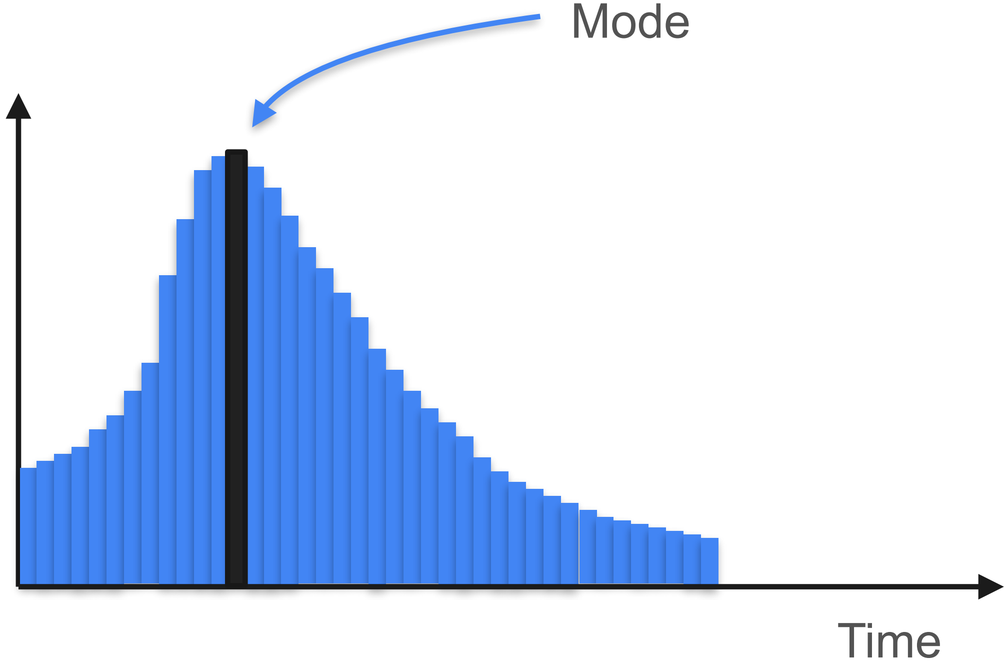

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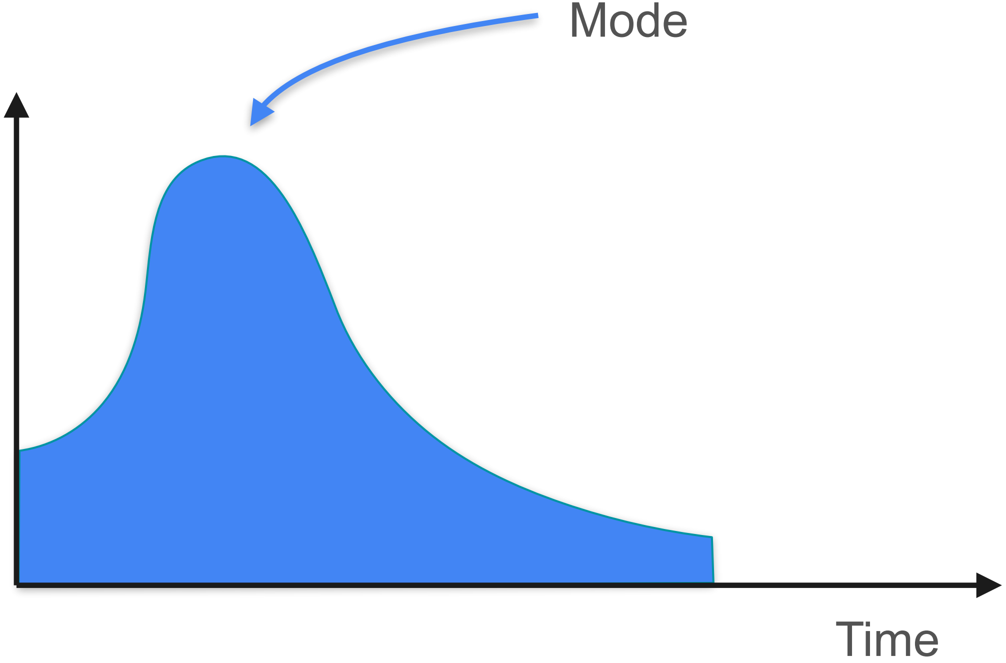
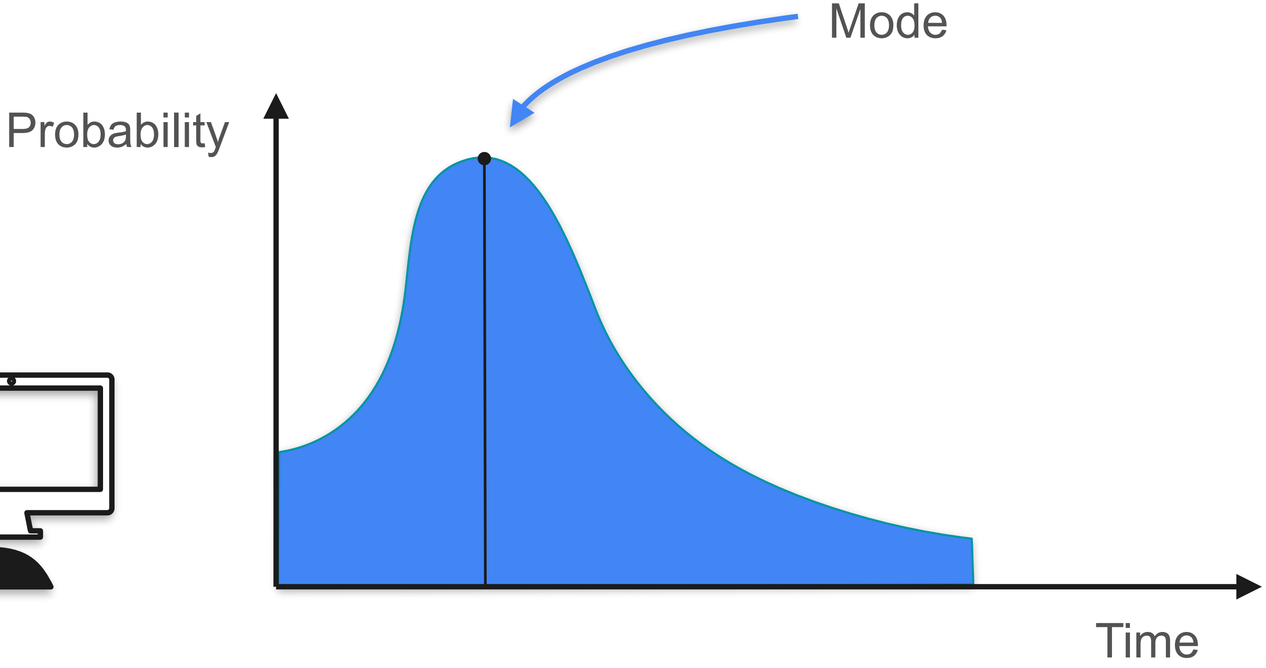
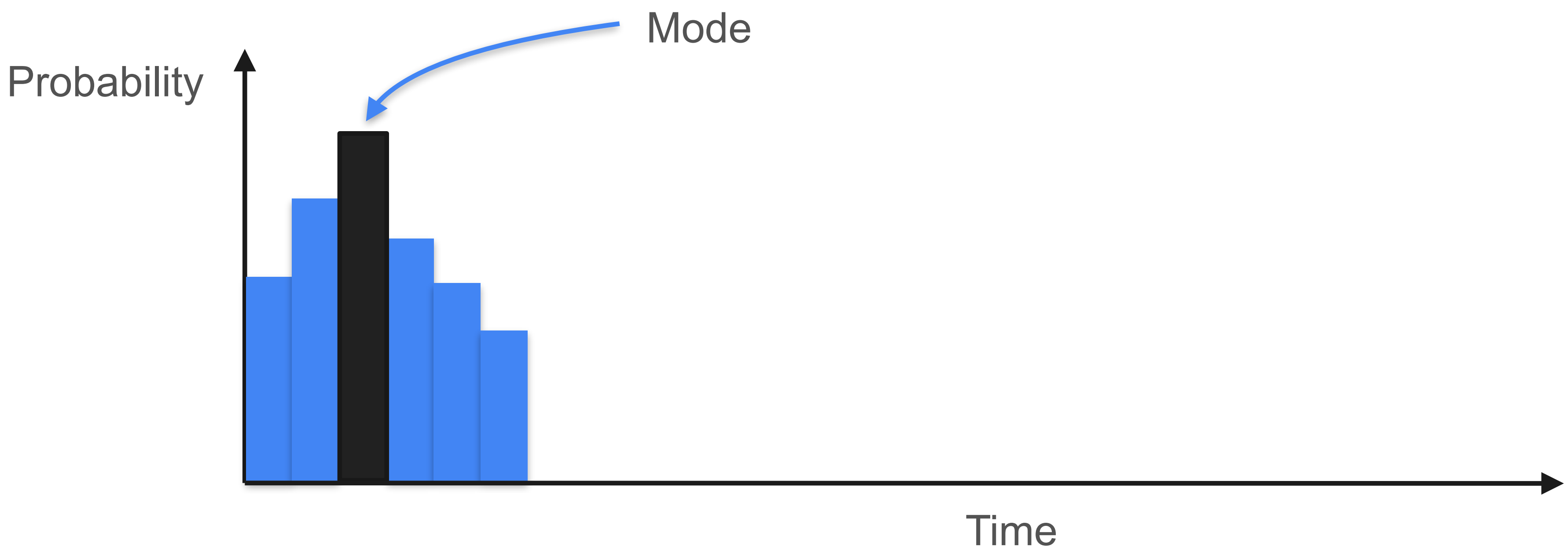


Mode



Mode: Multimodal Distribution



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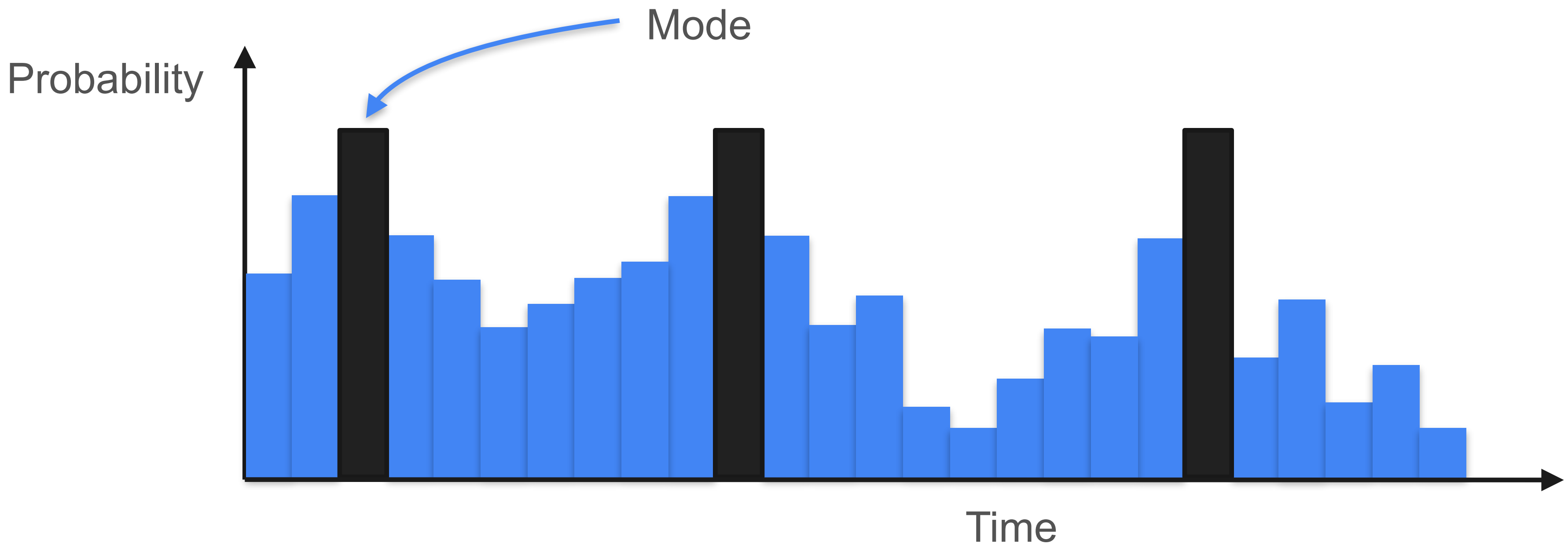

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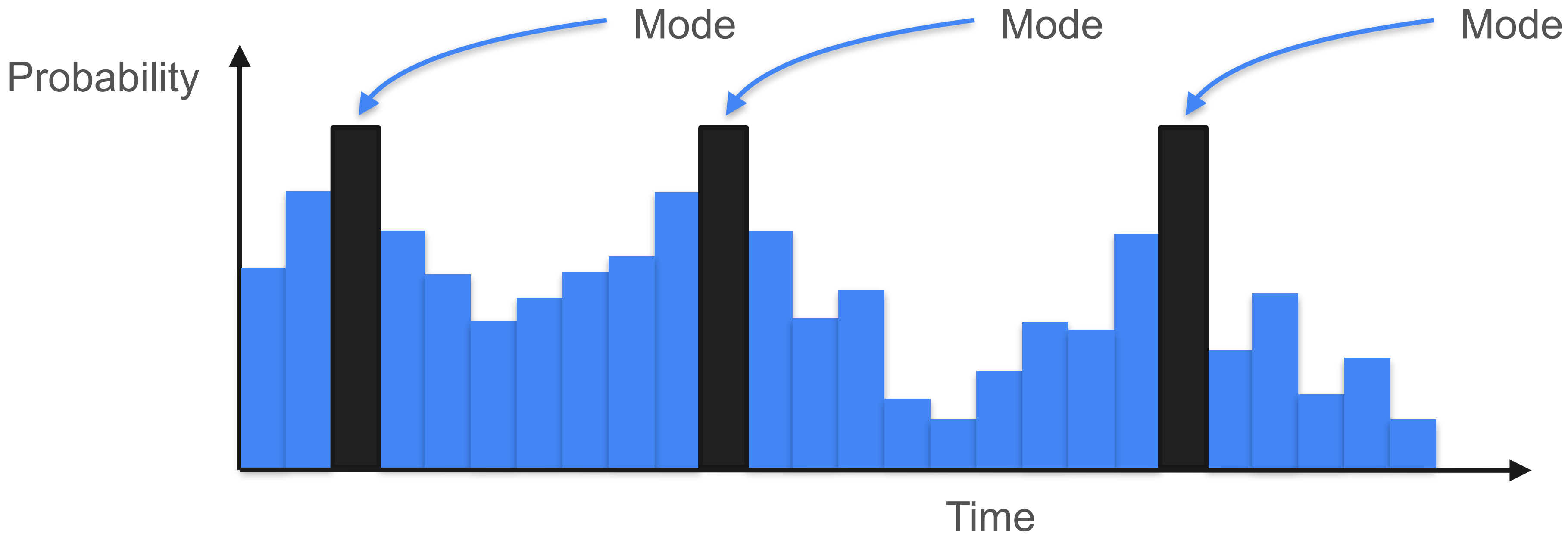
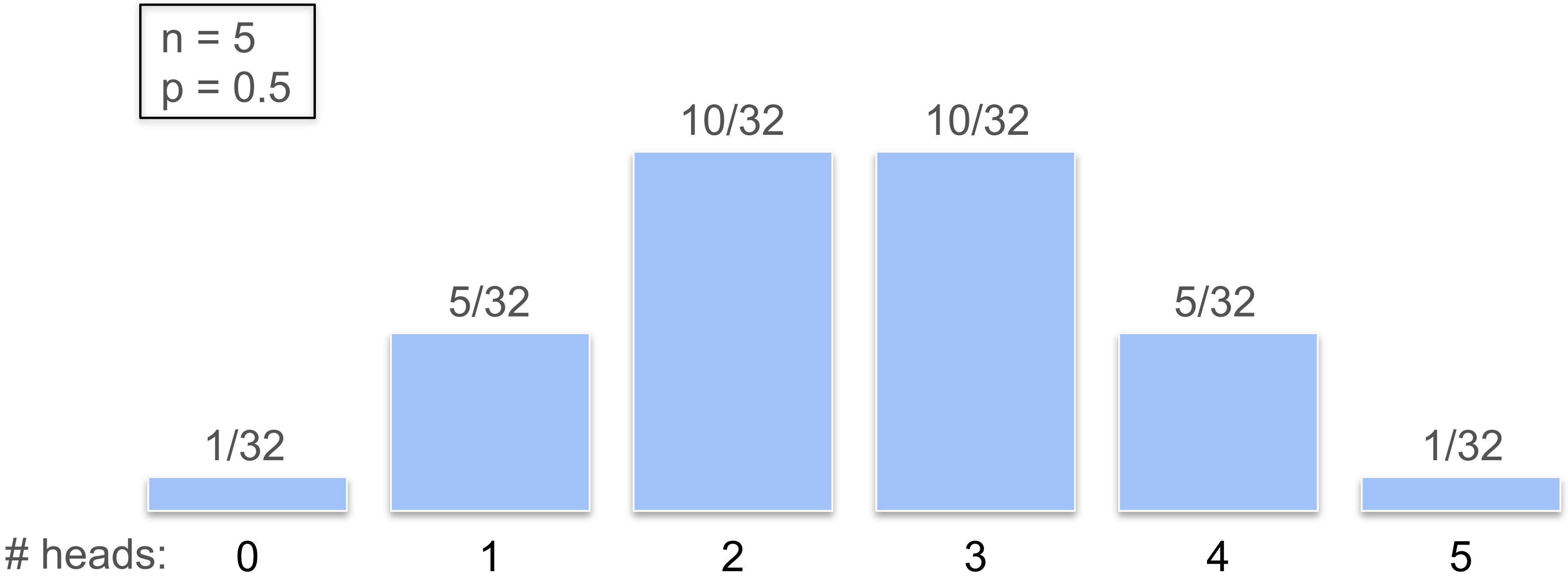


Mean, Median and Mode in Binomial Distribution



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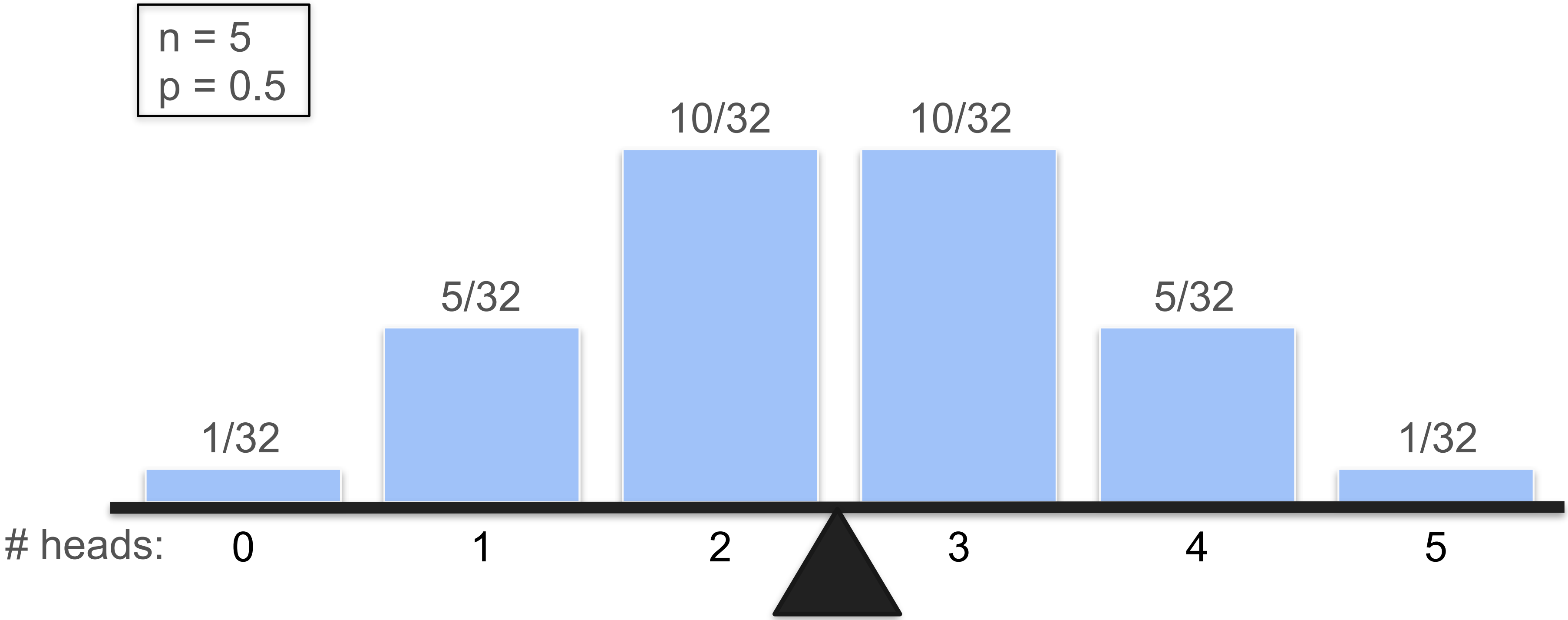

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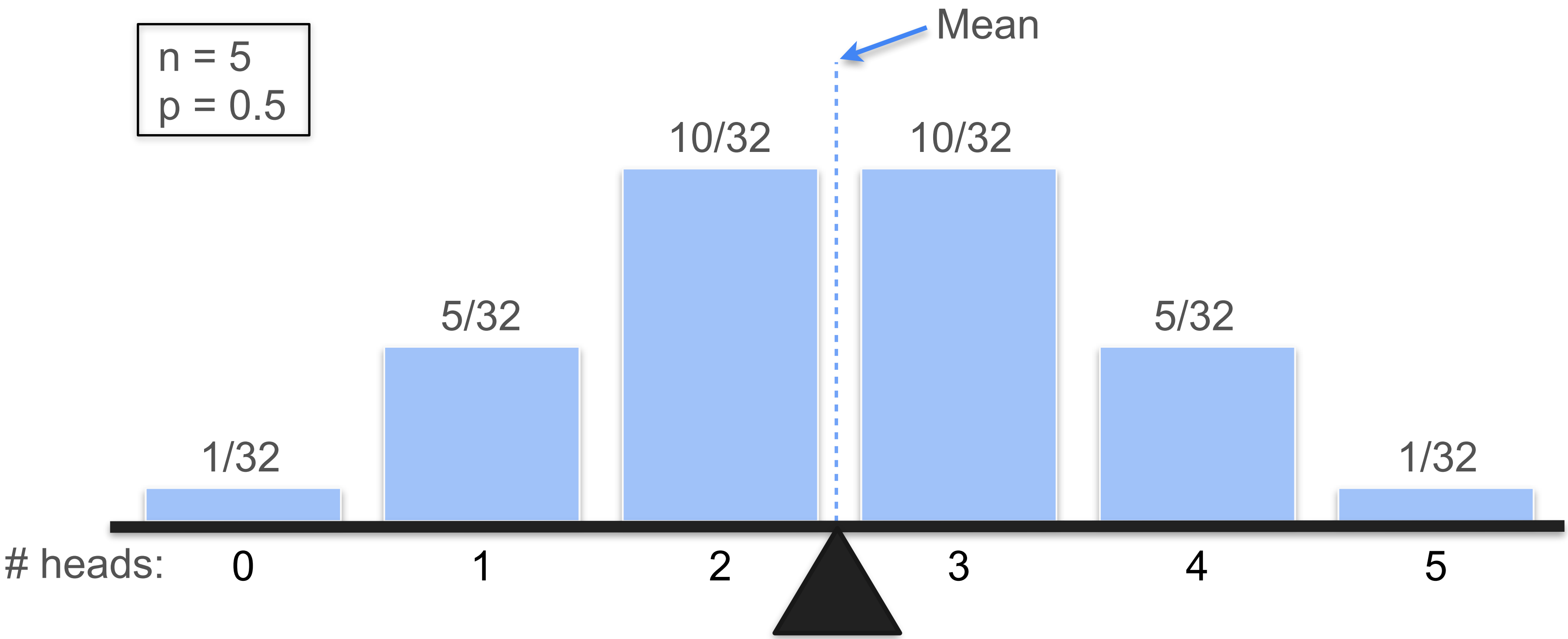
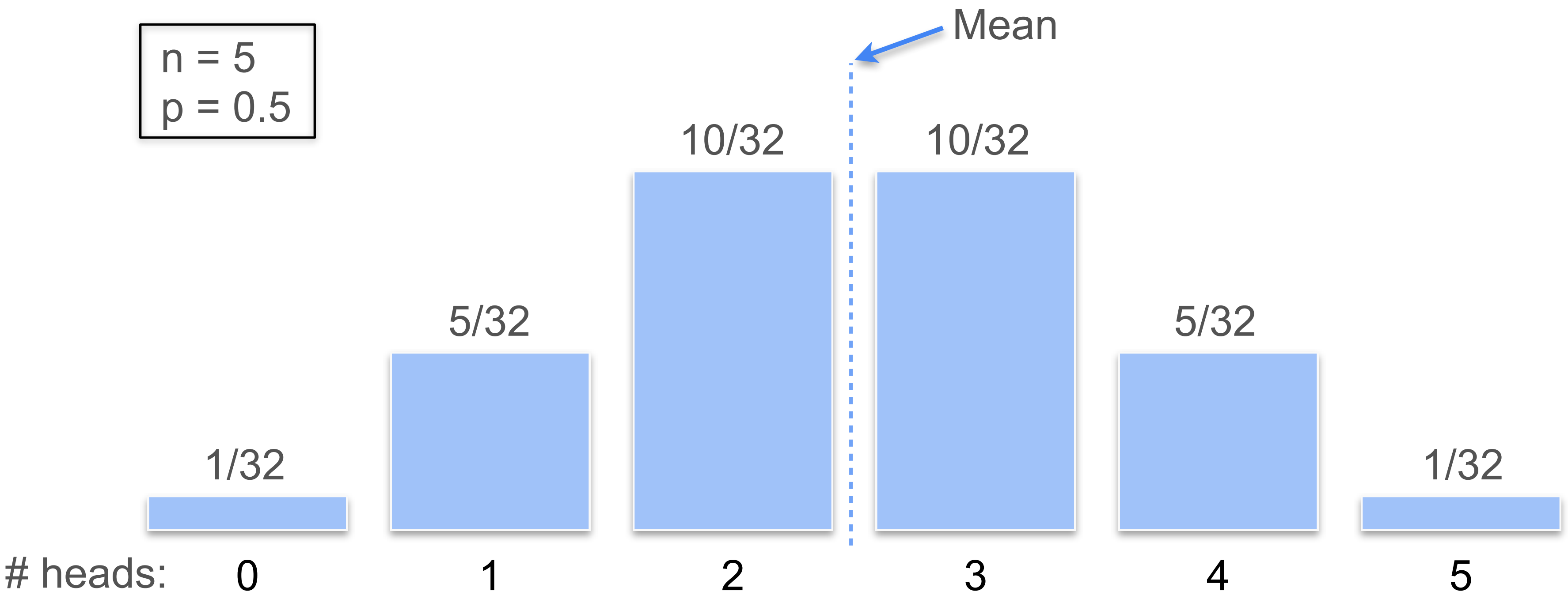


Mean, Median and Mode in Binomial Distribution

n = 5
p = 0.5



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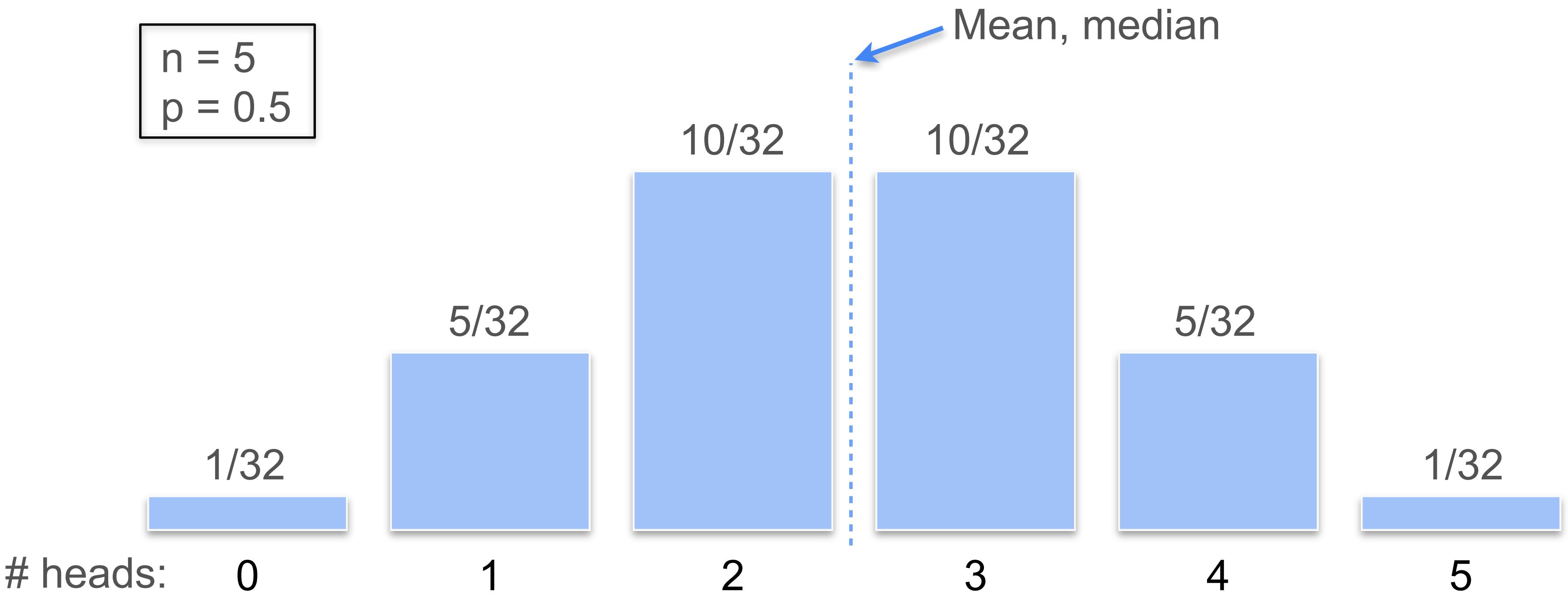

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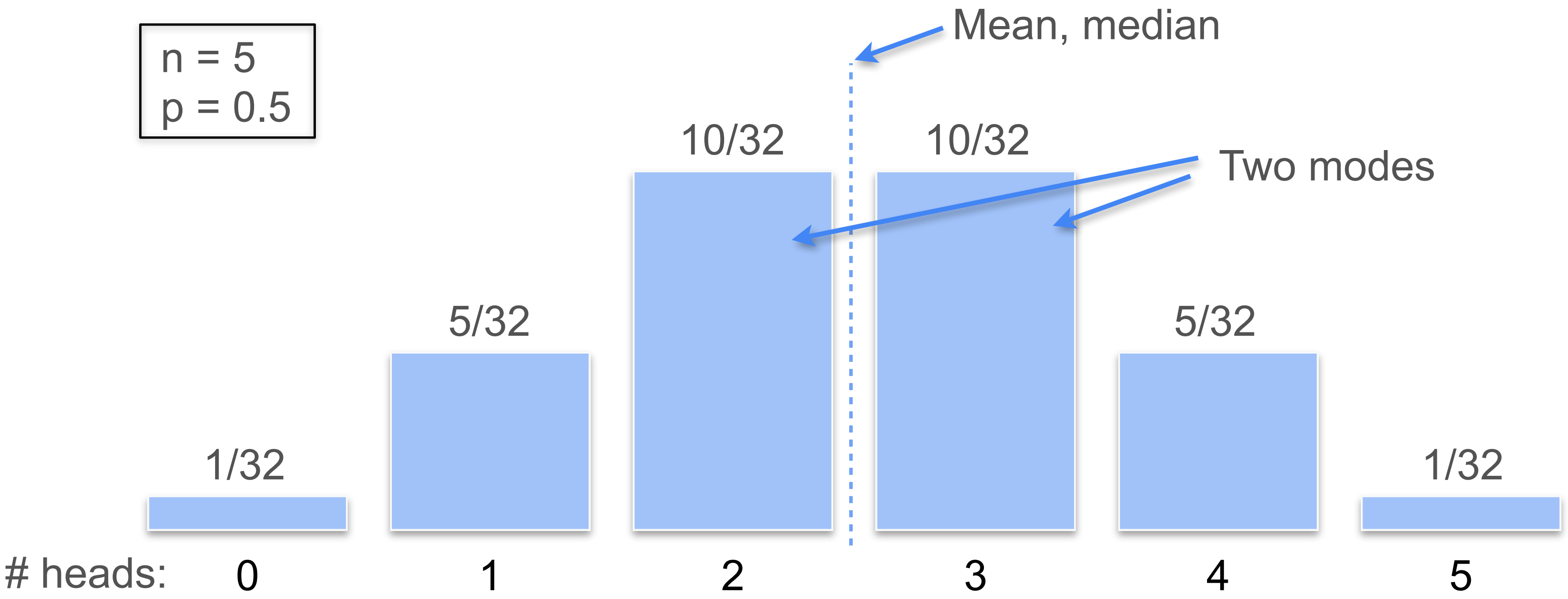
Mean, Median and Mode in Binomial Distribution



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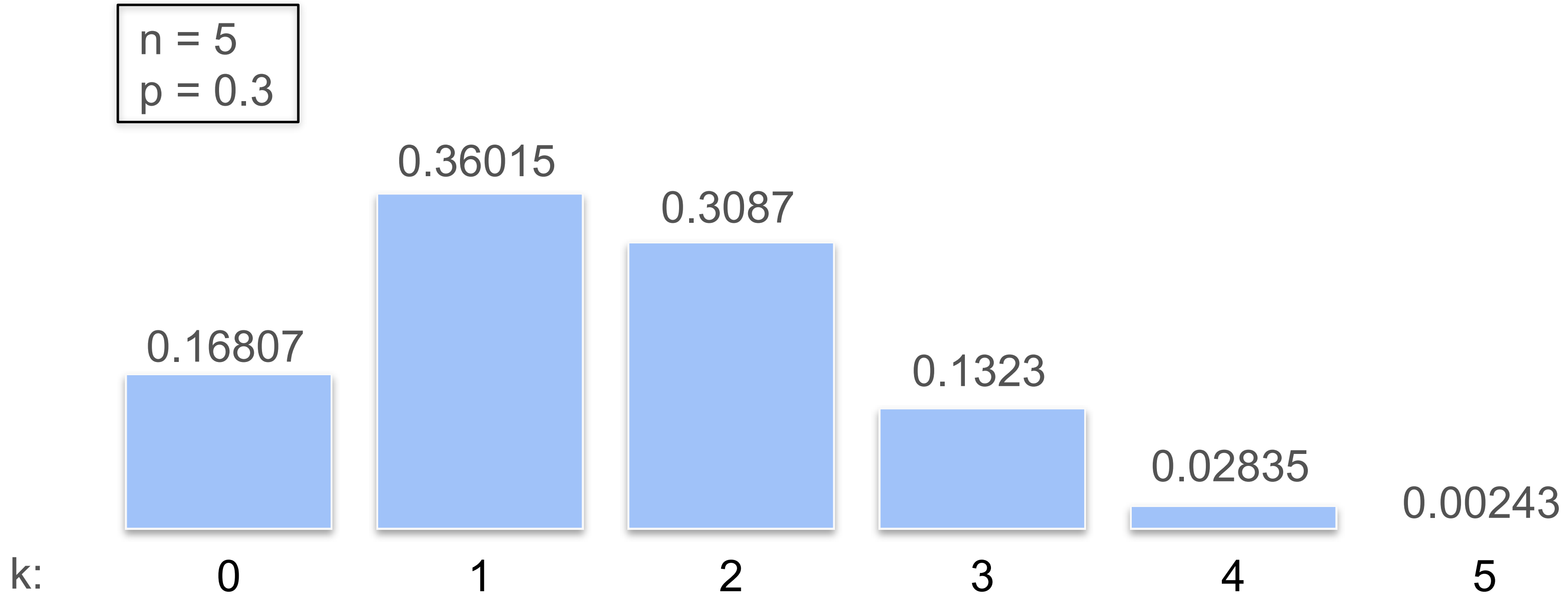

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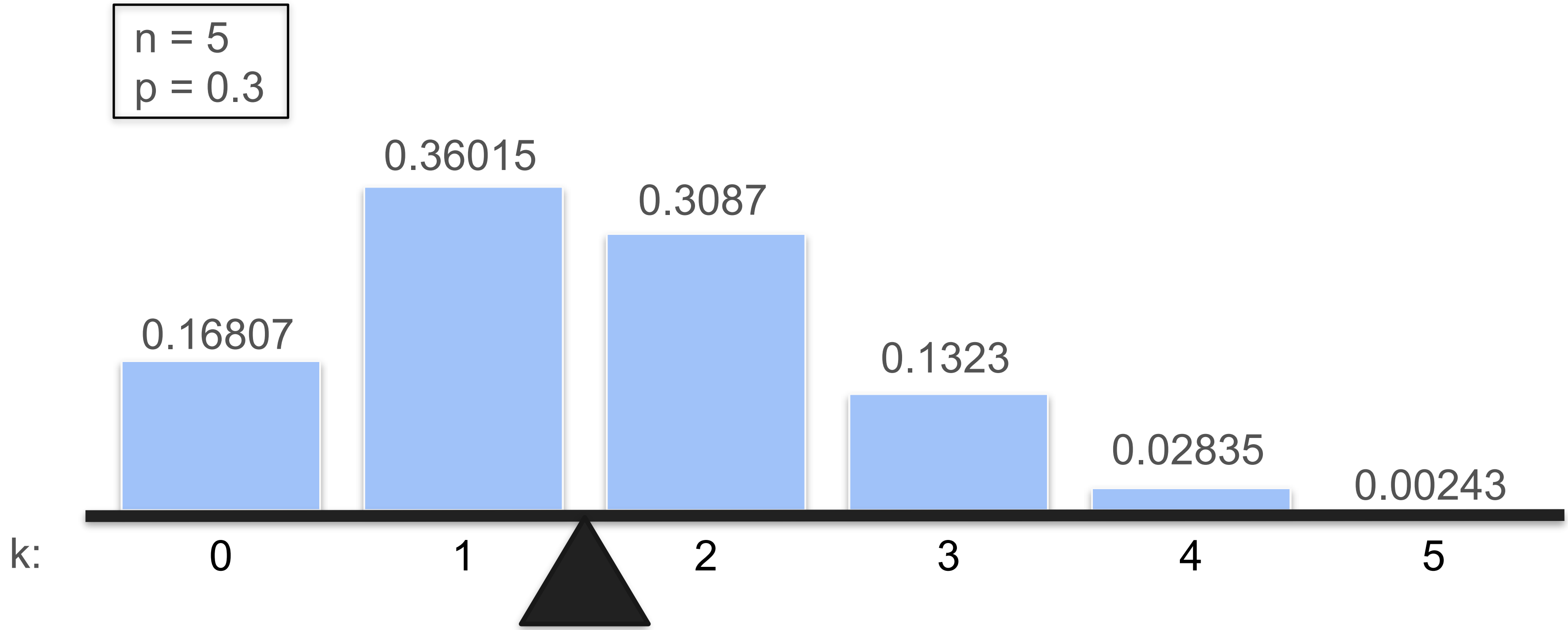

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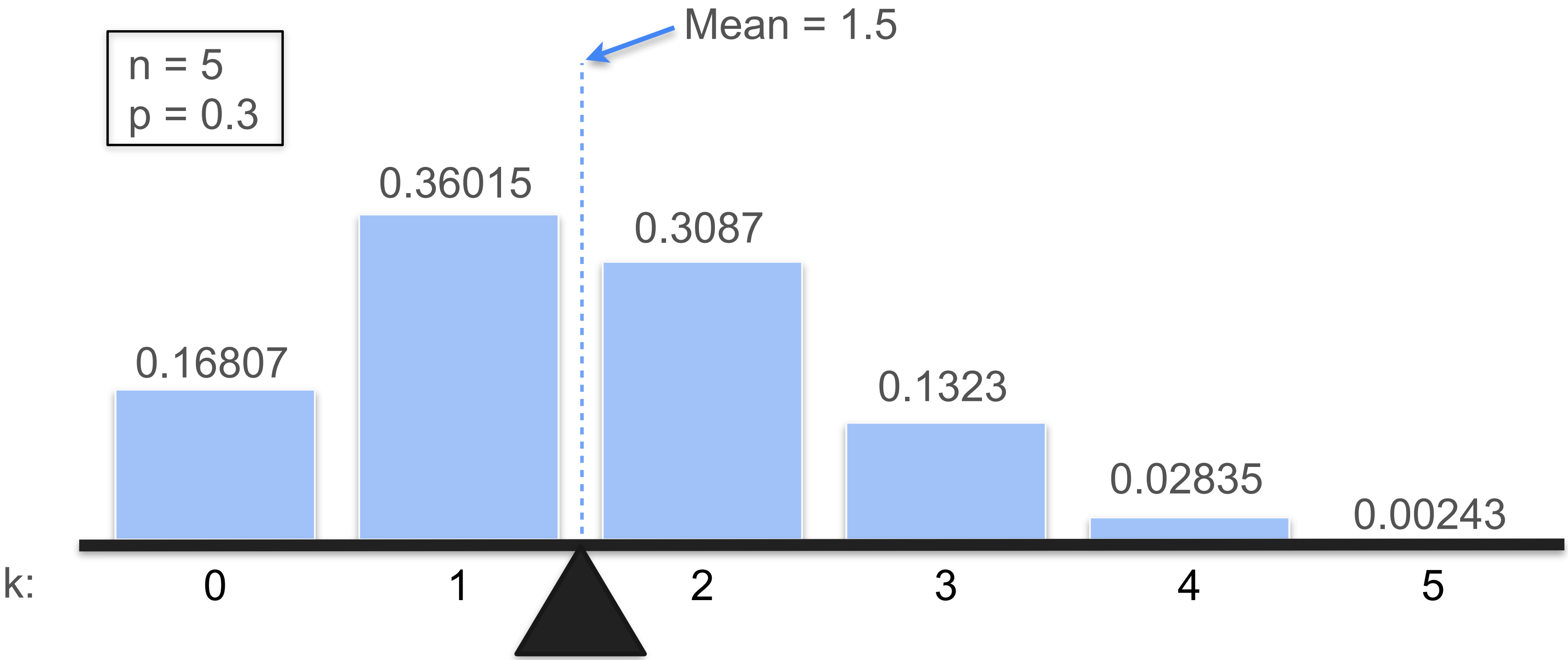

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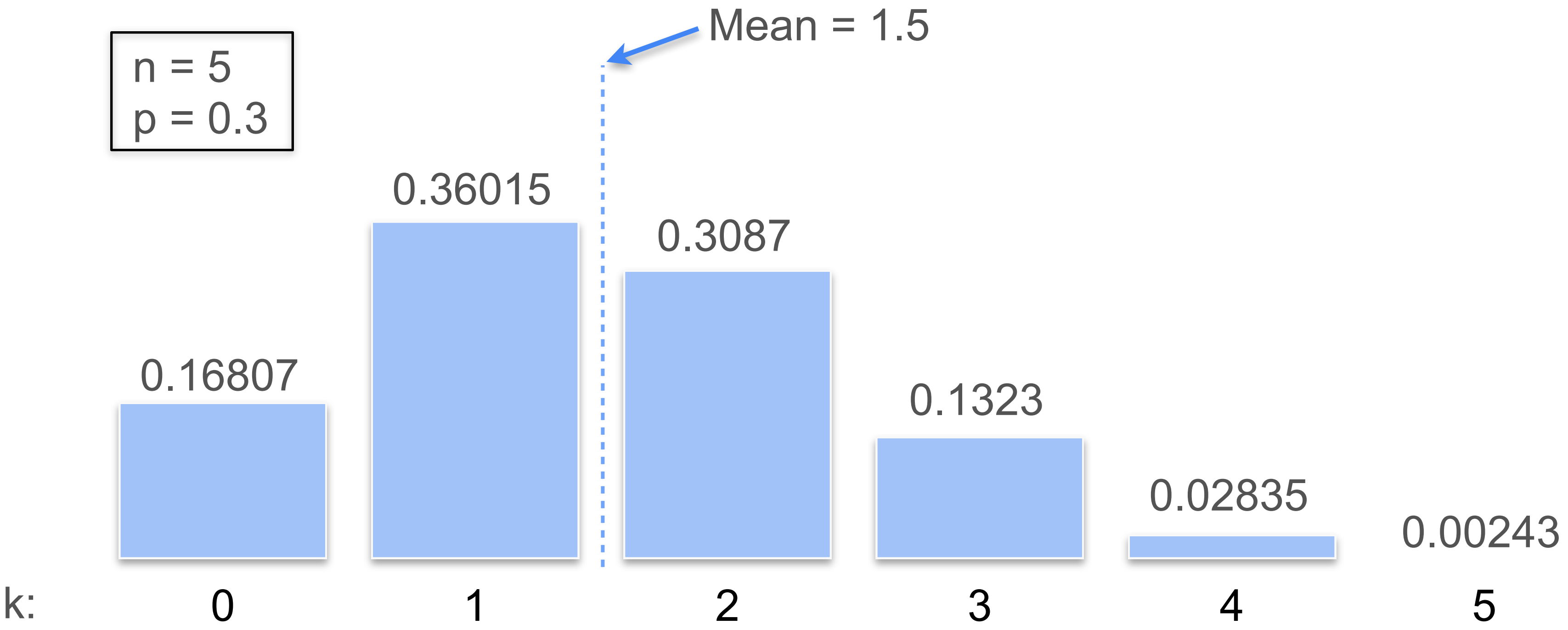

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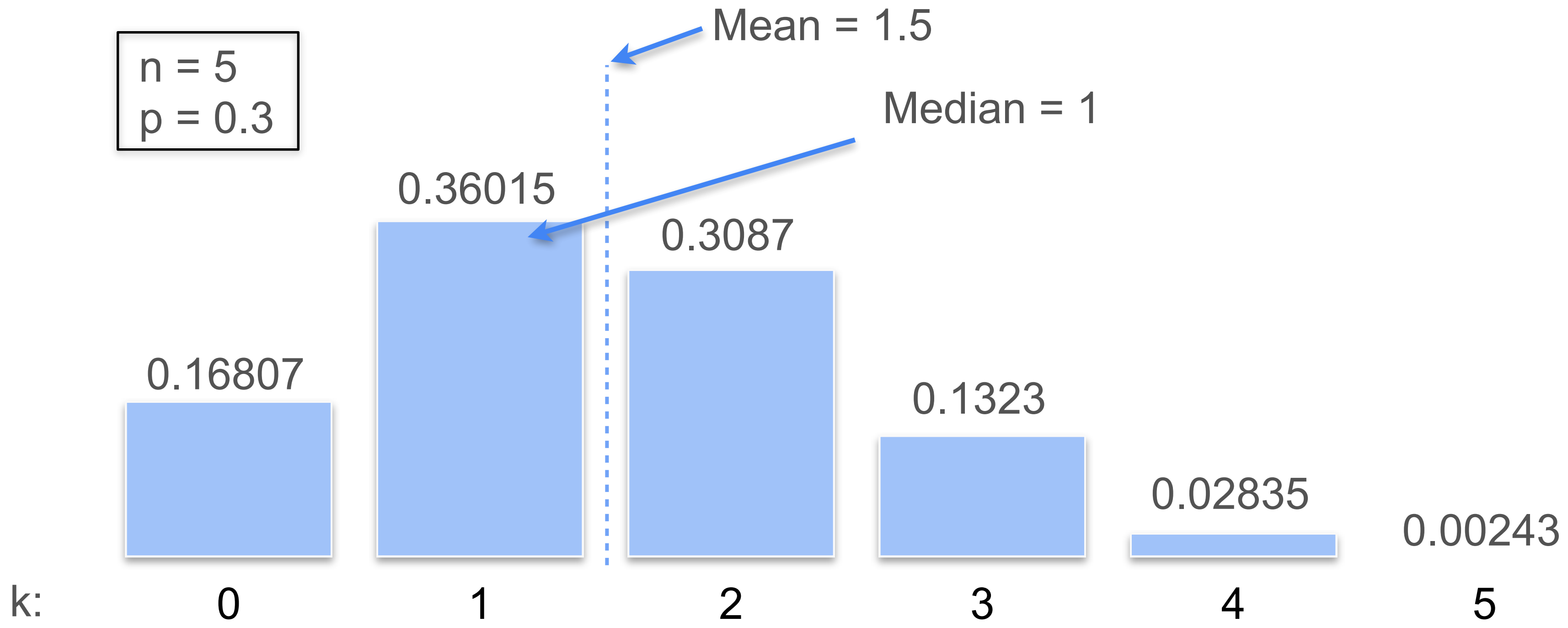
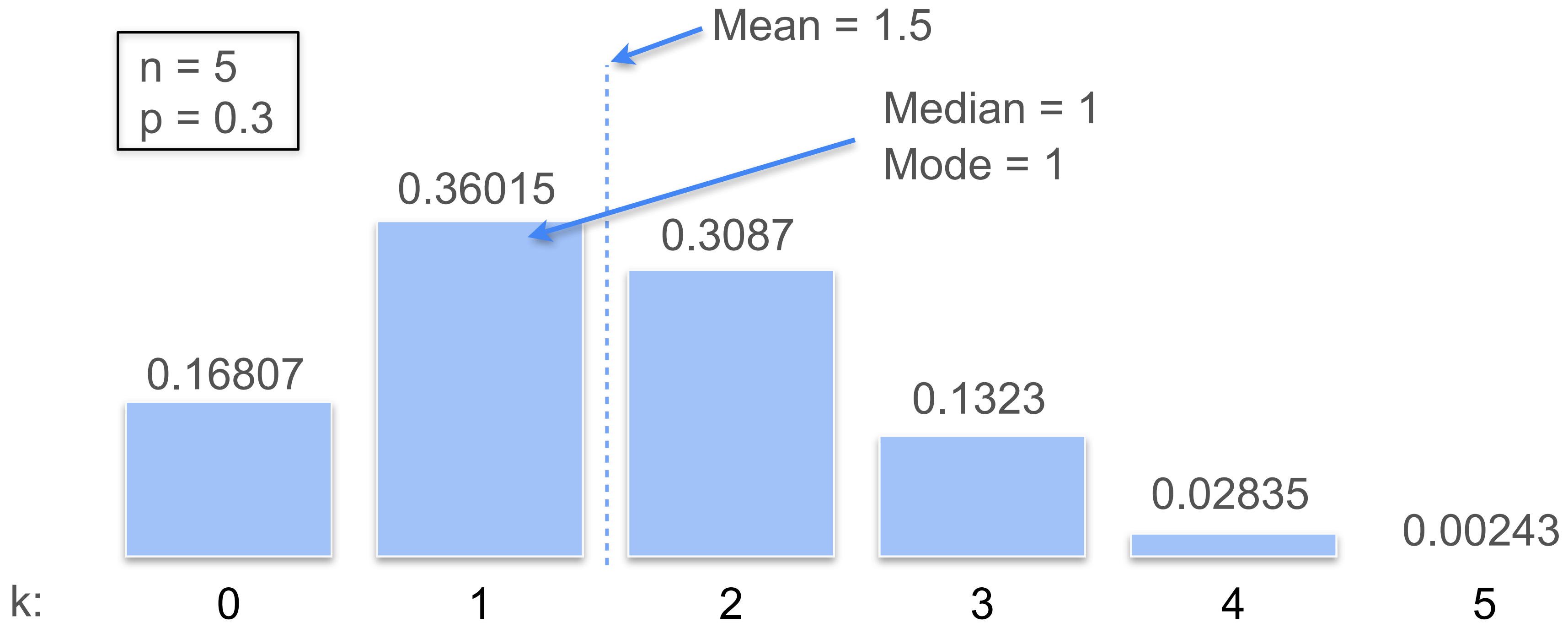
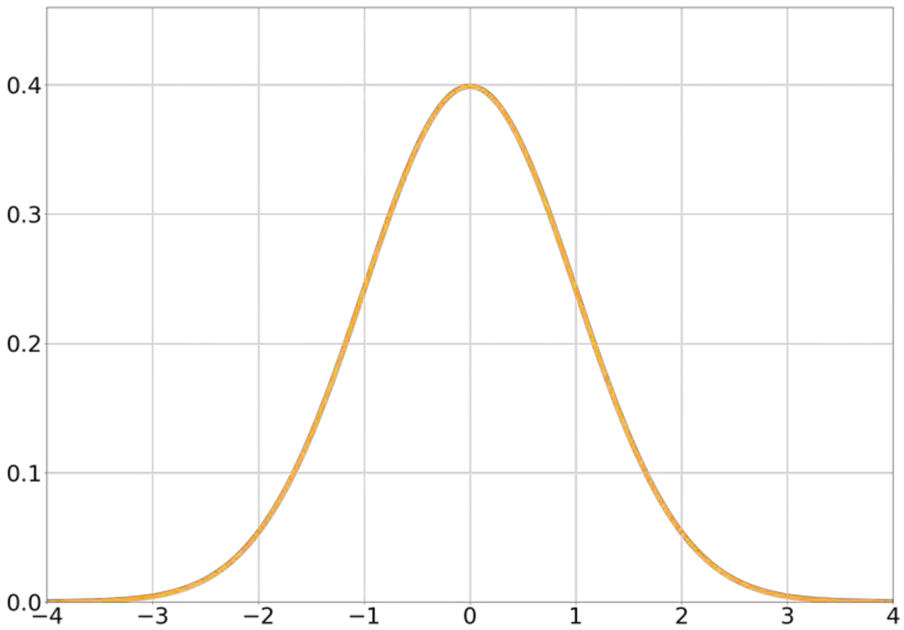


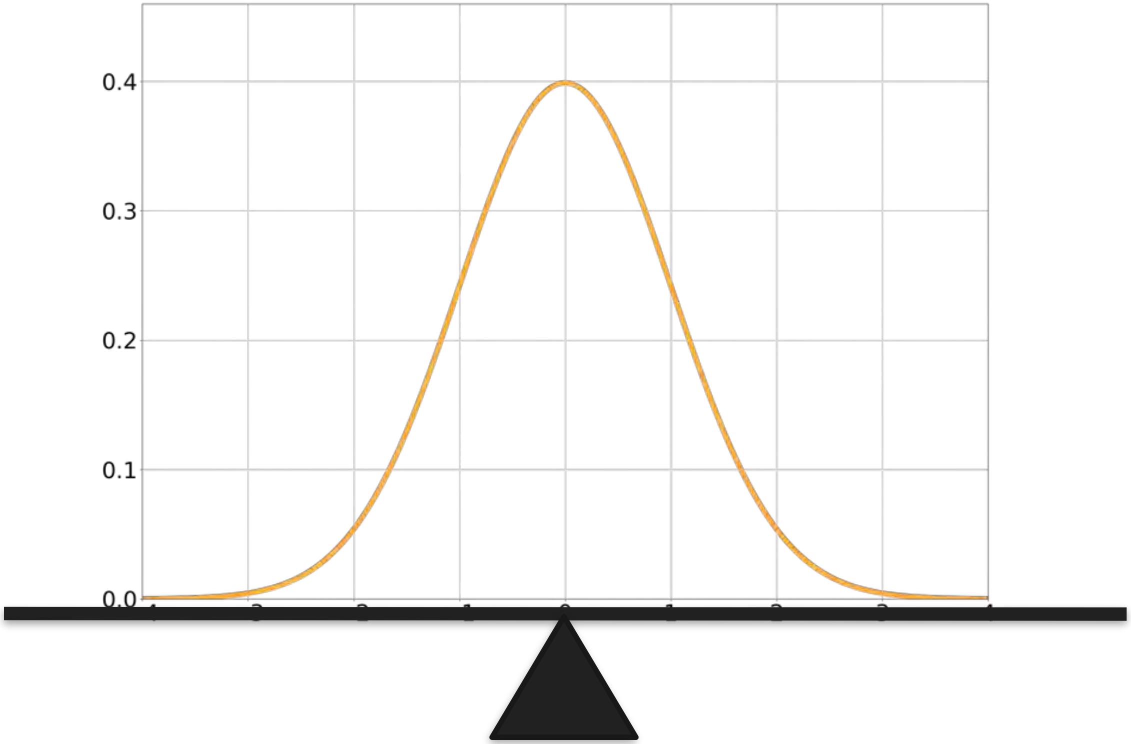
Mean, Median and Mode in Binomial Distribution



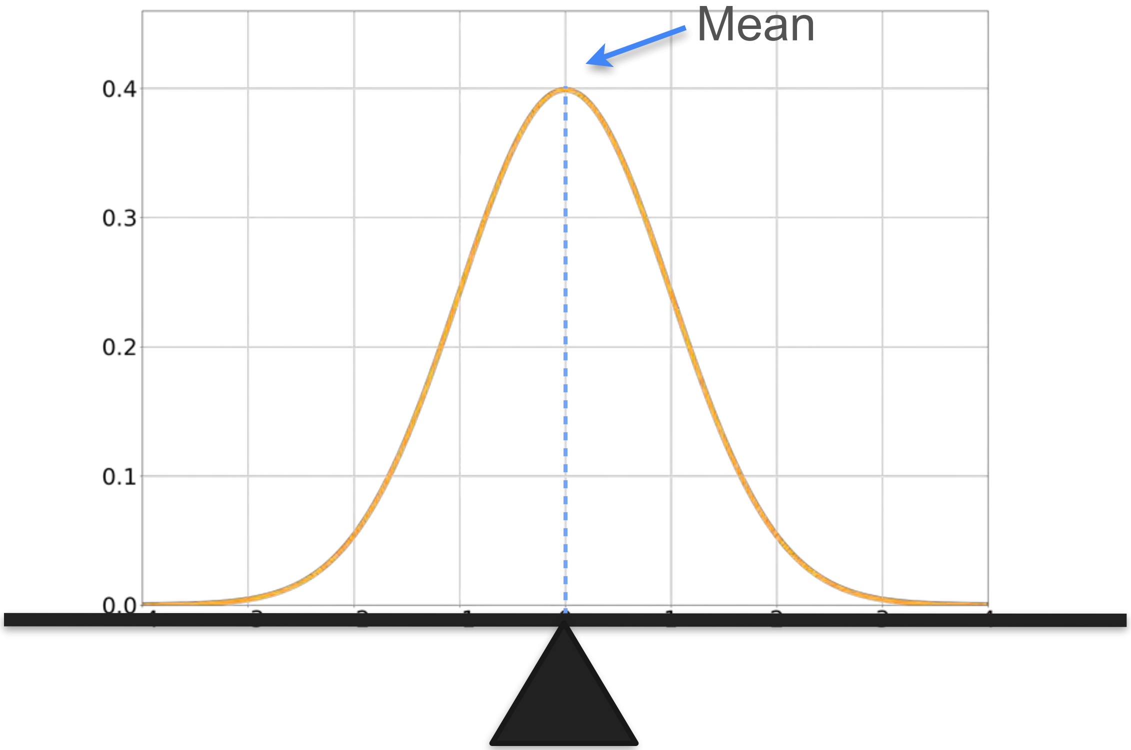
Mean, Median and Mode in Normal Distribution



Mean, Median and Mode in Normal Distribution



Mean, Median and Mode in Normal Distribution



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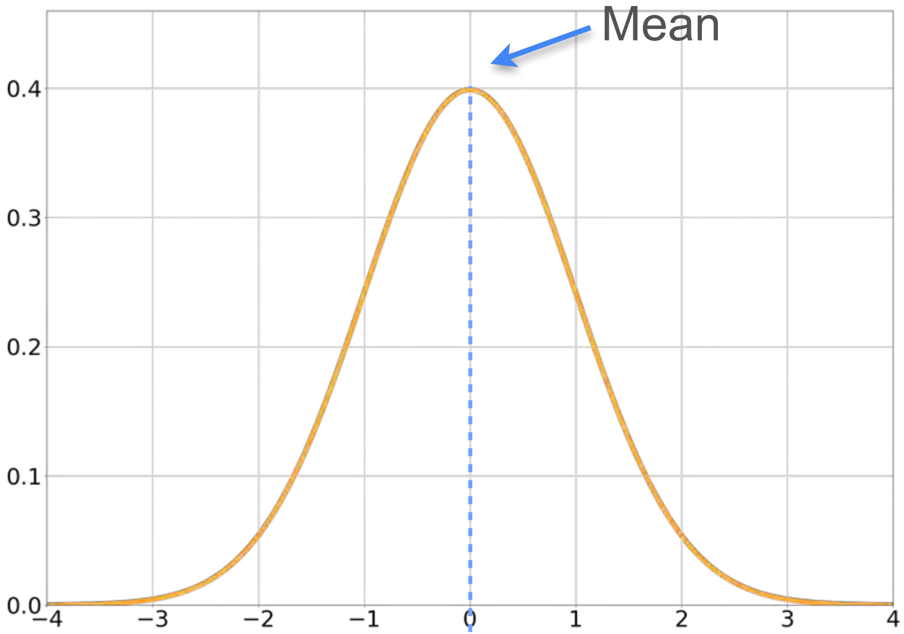

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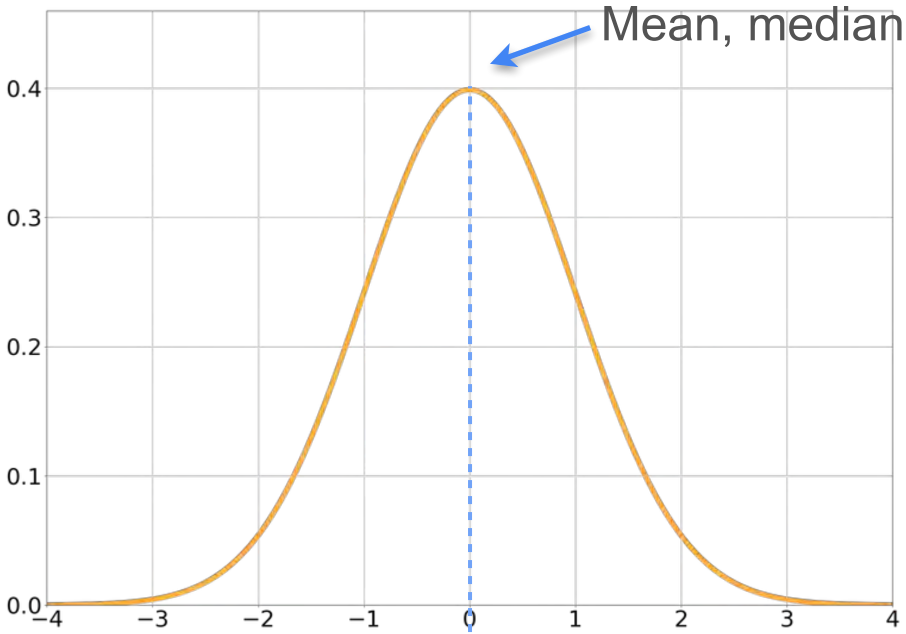

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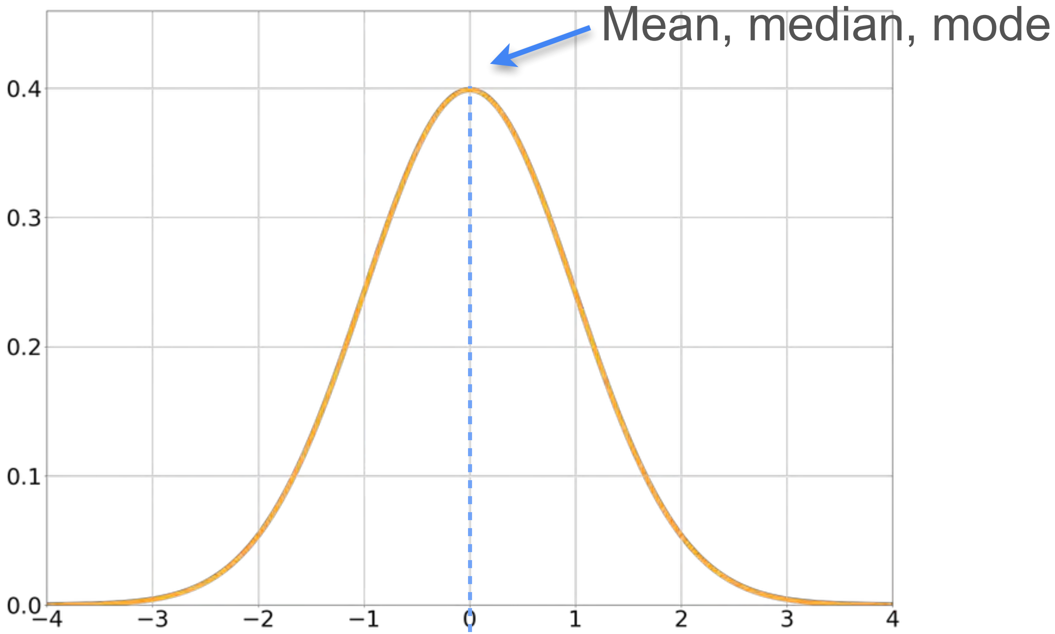

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Describing Distributions

Expected Value

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Game cost:

$6

Do you play the game?

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Game cost:

$4

Do you play the game?

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Game cost:

$4

Do you play the game?

What is the maximum amount of money you would pay to play this game?

Expected Value: Motivation Example 1

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Long term:

Expected Value: Motivation Example 1

You play a game with a friend

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You win nothing

Long term: $0.5 \cdot \$10$

Expected Value: Motivation Example 1

You play a game with a friend

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Long term: $0.5 \cdot \$10 \ +0.5 \cdot \$0$

Expected Value: Motivation Example 1

You play a game with a friend

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Long term: $0.5 \cdot \$10 \ +0.5 \cdot \$0 = \$5$

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You play a game with a friend

You win 10 dollars

You win nothing

Long term: $0.5 \cdot \$10 \ +0.5 \cdot \$0 = \$5$ You expect to win $\$5$ on average

$\mathbb{E}[X] = 5$

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Game cost:

$5

Long term: $0.5 \cdot \$10 \;\; + 0.5 \cdot \$0 = \$5$ ⇨ You expect to win $\$5$ on average

$\mathbb{E}[X] = 5$

Expected Value: Motivation Example 1

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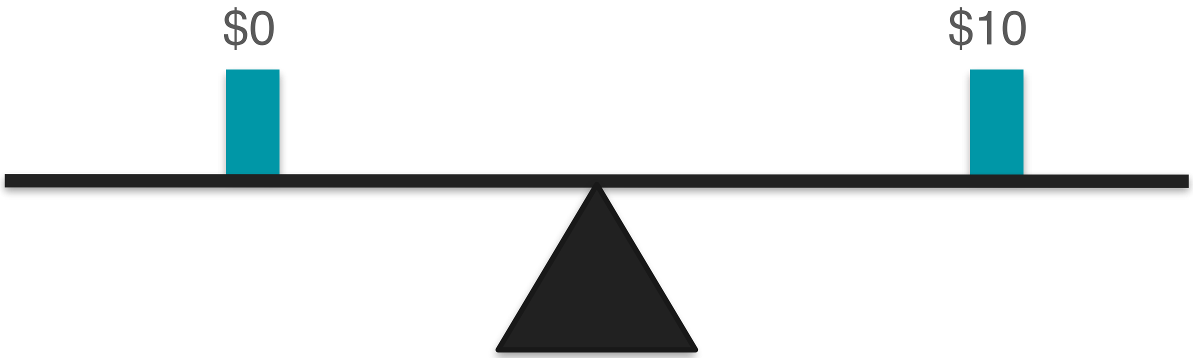
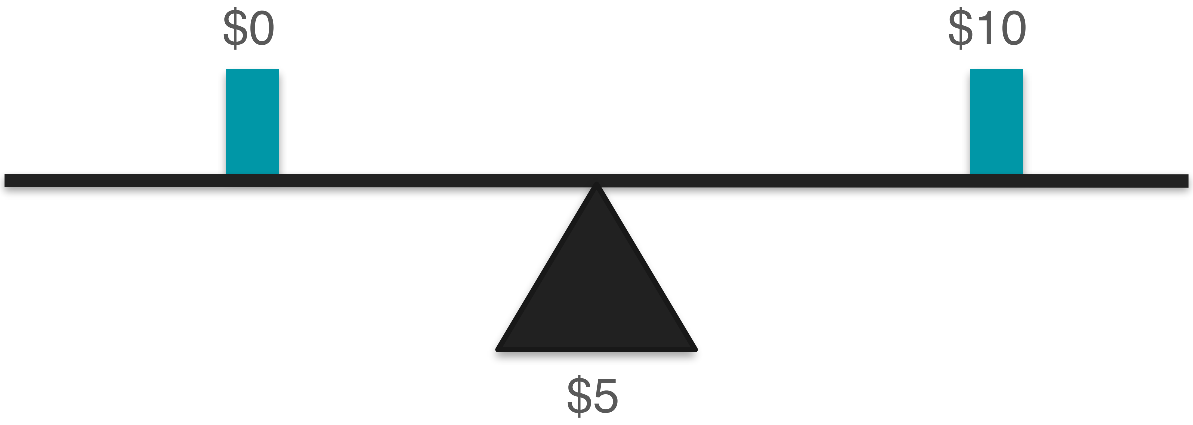
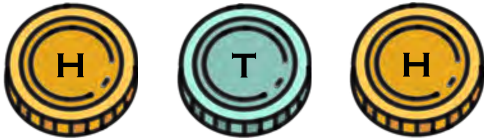


Expected Value: Motivation Example 1



Expected Value: Motivation Example 2

Play another game

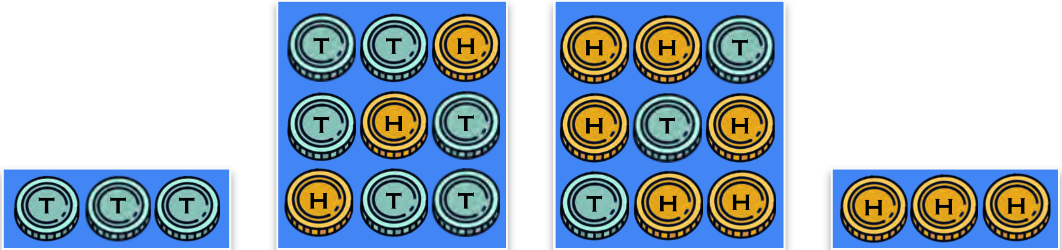


Flip three coins. For each heads you win $1

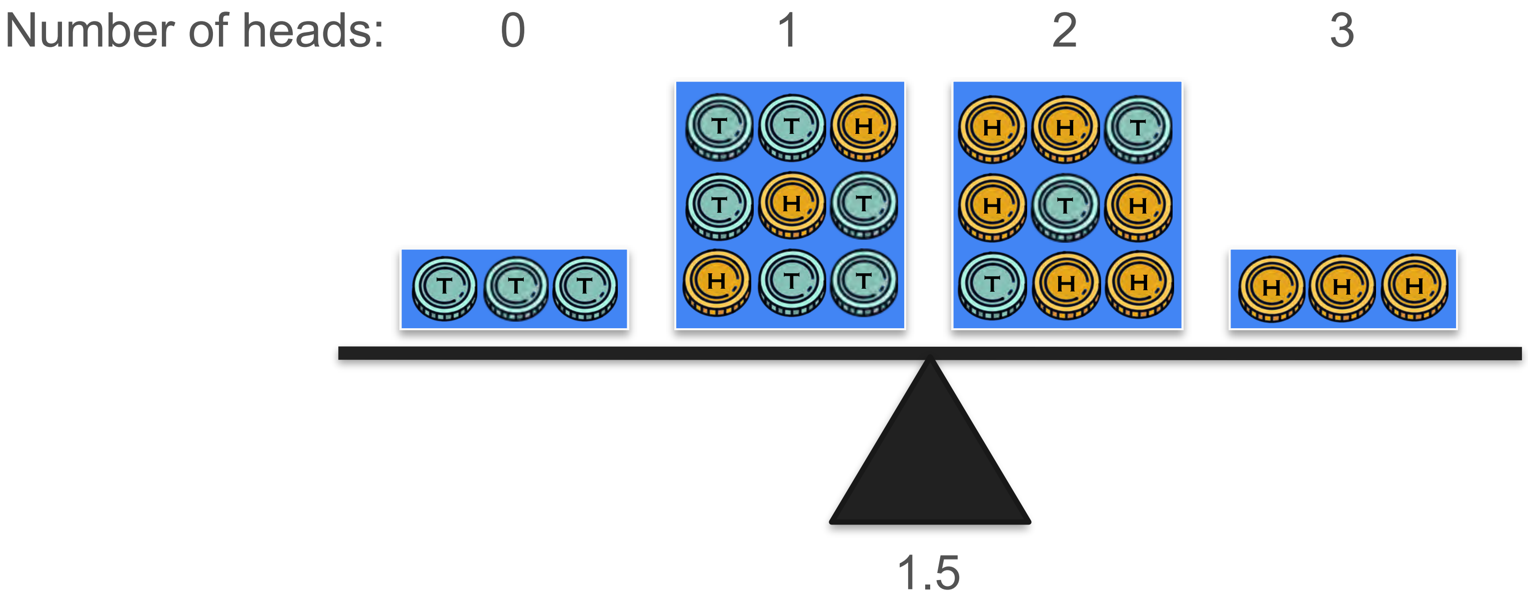
What is the maximum amount of money you would pay to play this game?

Expected Value: Motivation Example 2

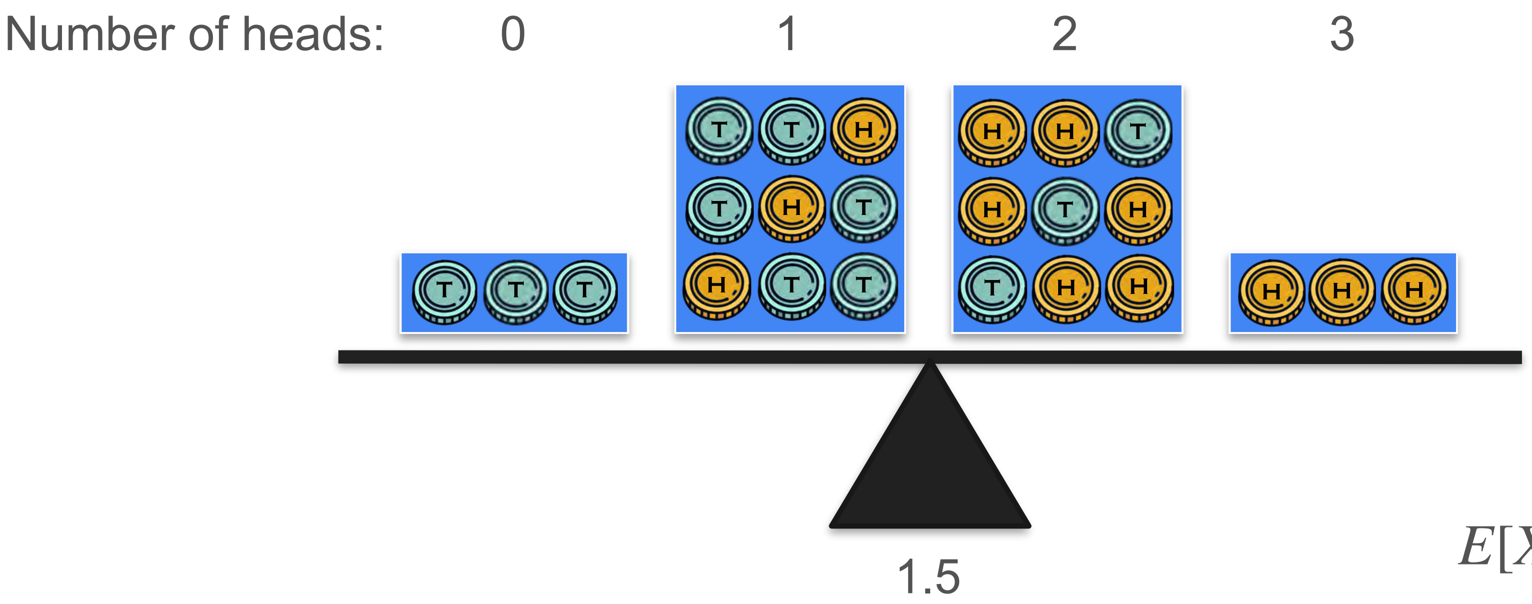
Number of heads: 0 1 2 3



Expected Value: Motivation Example 2



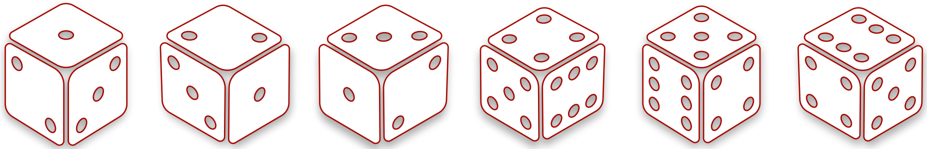
Expected Value: Motivation Example 2



$$E[X] = 1.5$$

Expected Value: Motivation Example 3

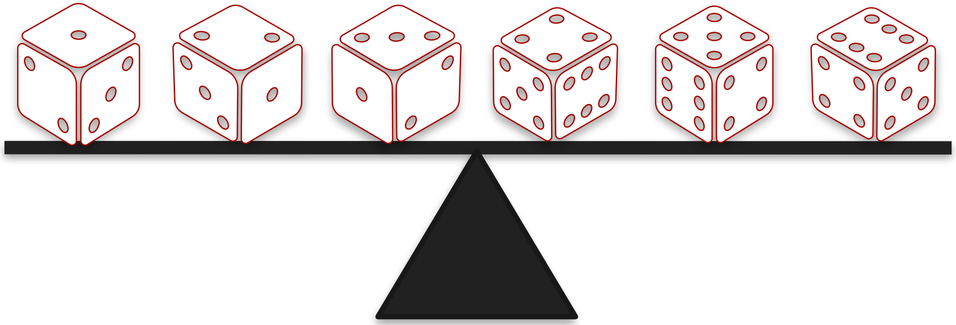
Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6



$$\frac{1+2+3+4+5+6}{6} = 3.5$$

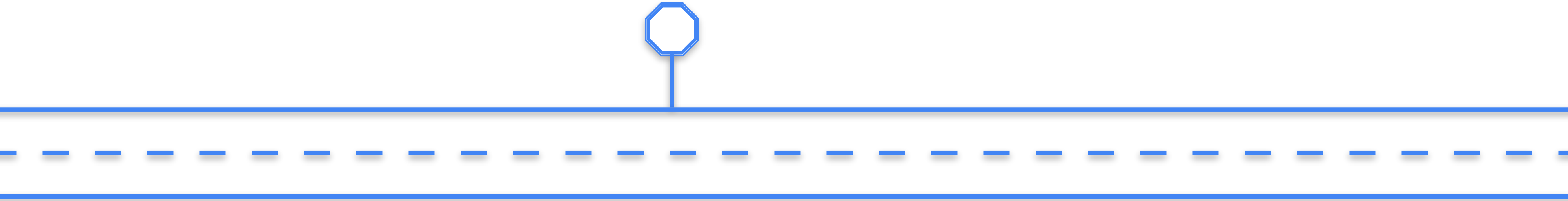
Expected Value: Motivation Example 3

Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6

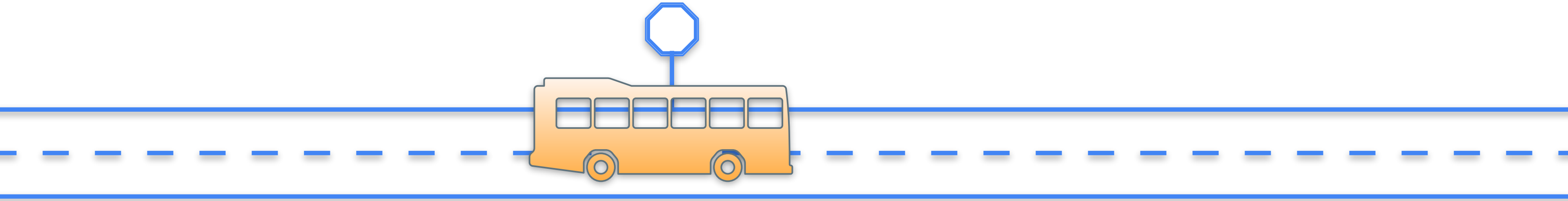


3.5

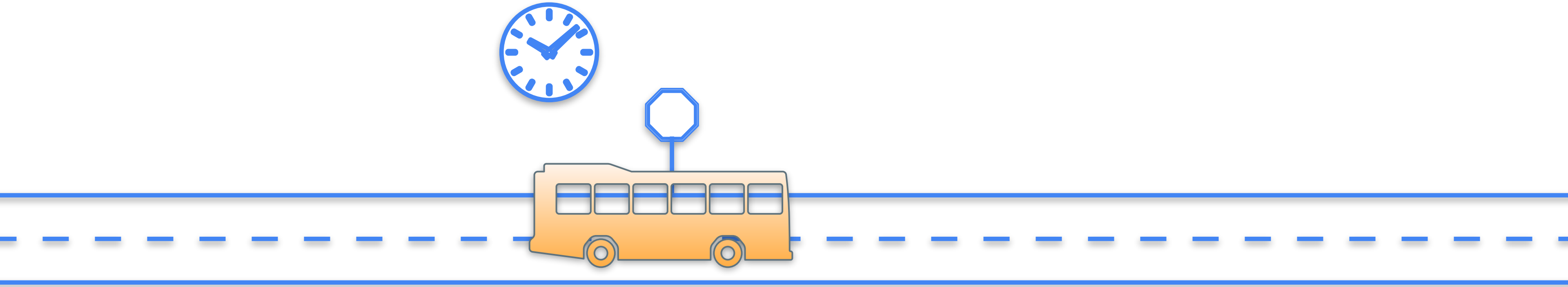
Expected Value



Expected Value

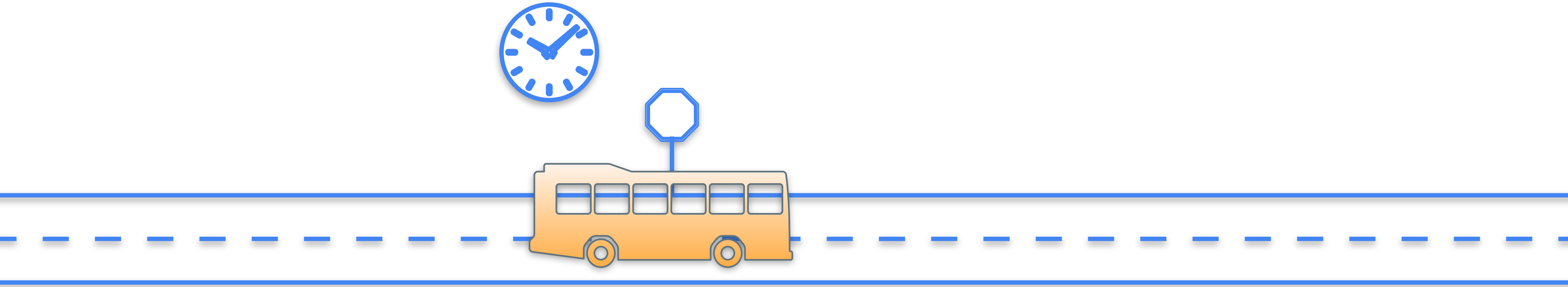


Expected Value



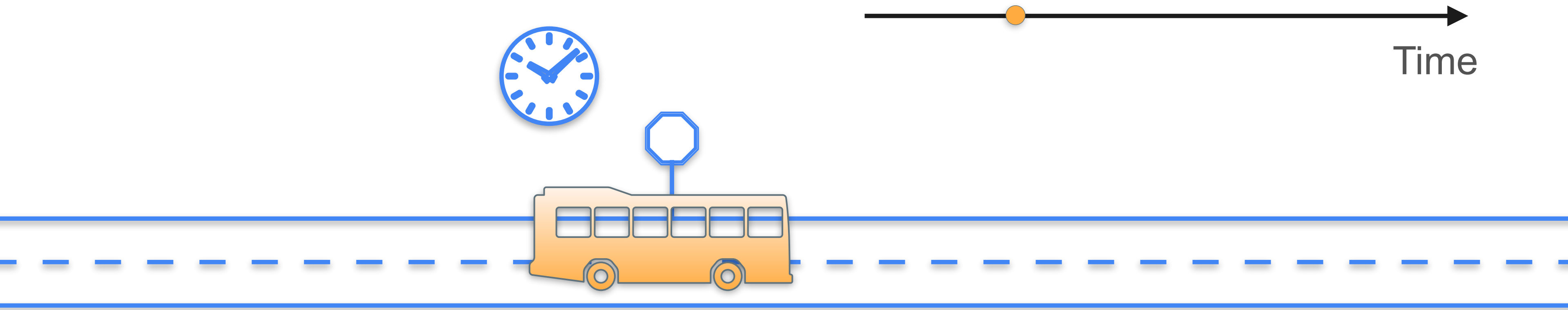
Expected Value

Waiting Time
15 min



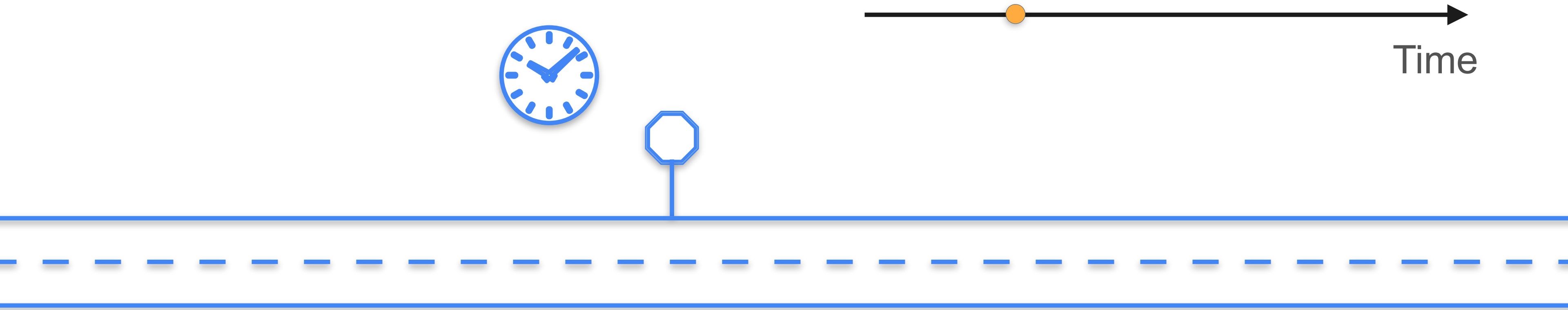
Expected Value

Waiting Time
15 min



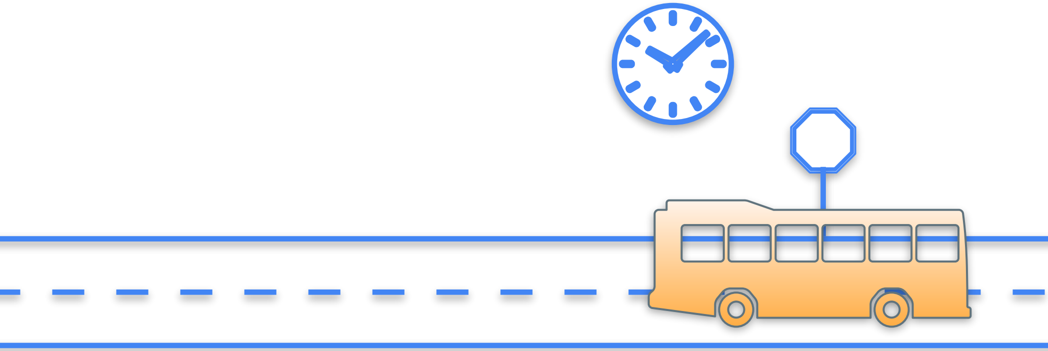
Expected Value

Waiting Time
15 min



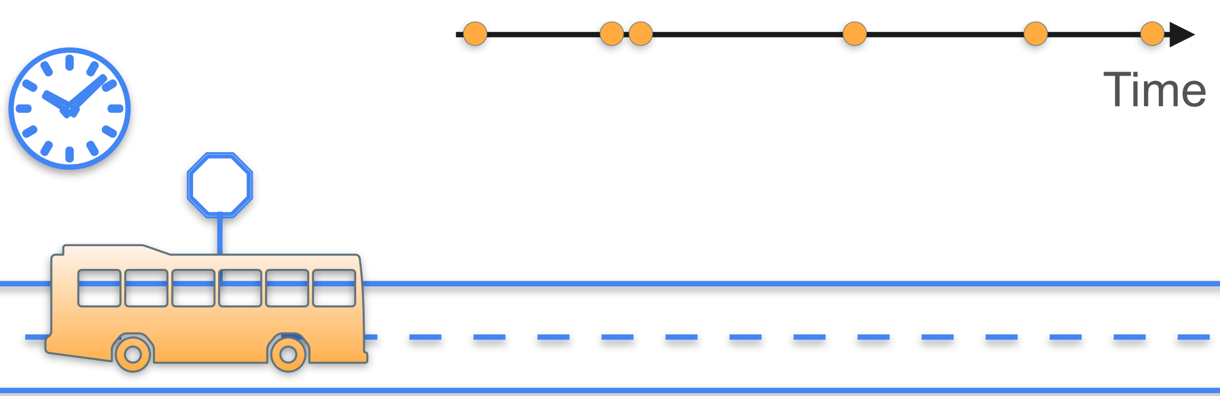
Expected Value

Waiting Time
15 min



Expected Value

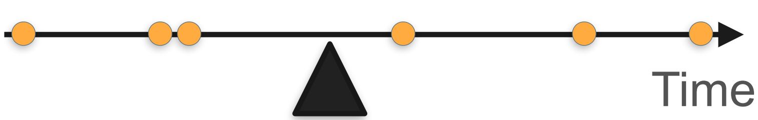
Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min

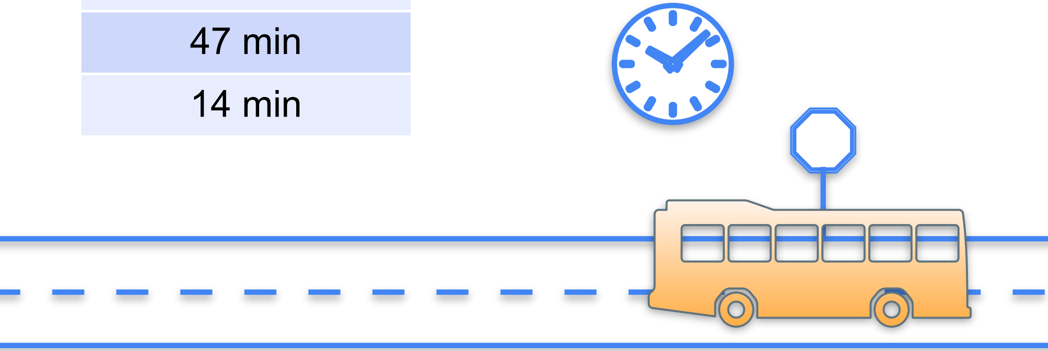


Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min

Average = 27.833

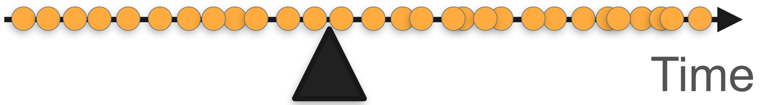


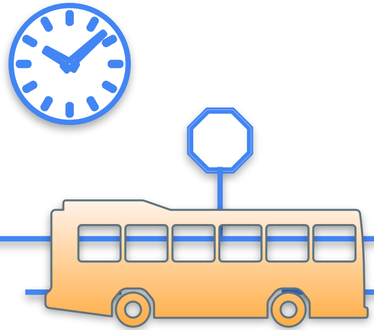


Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min
37 min
8 min
29 min
55 min
…

Average = 27.833

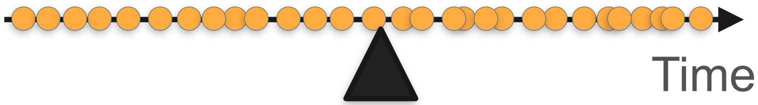


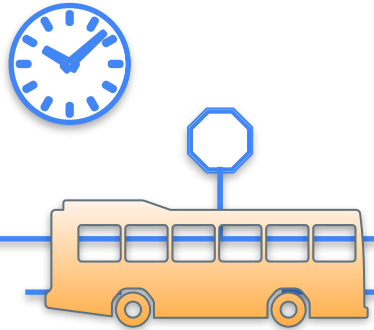


Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min
37 min
8 min
29 min
55 min
…

Average = 30

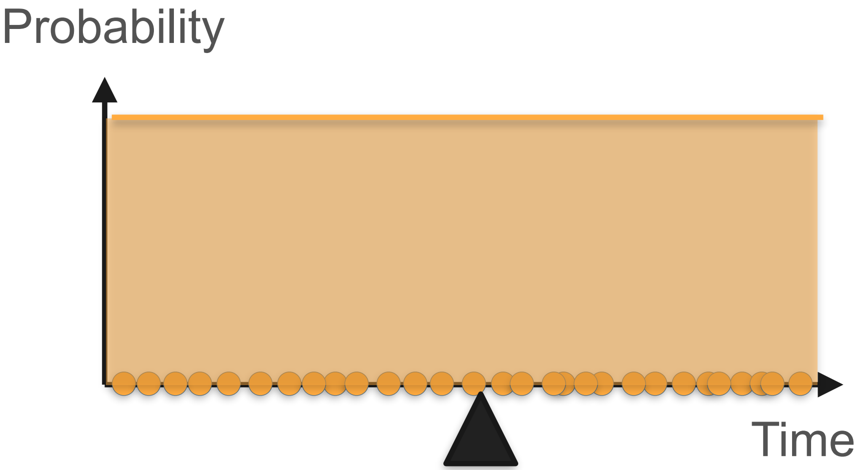


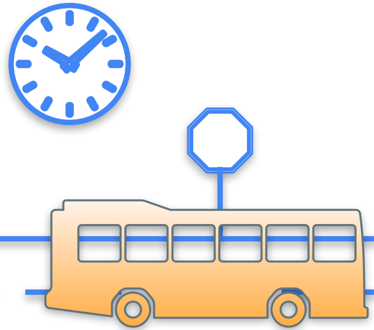


Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min
37 min
8 min
29 min
55 min
…

Average = 30

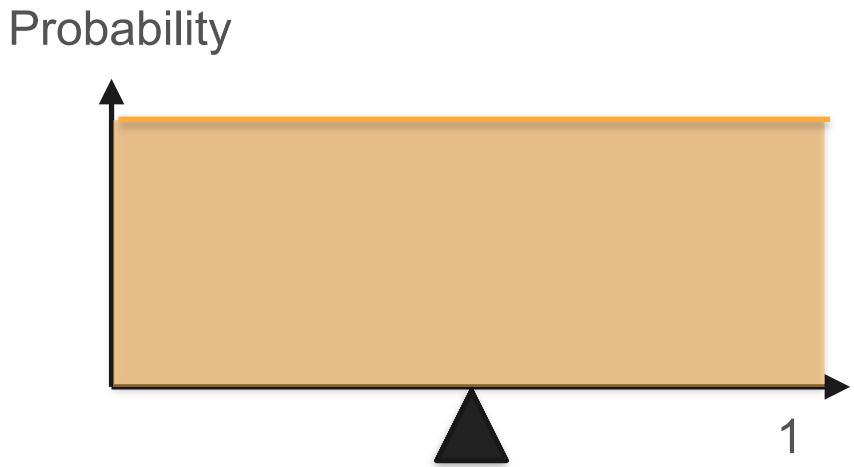




DeepLearning.AI

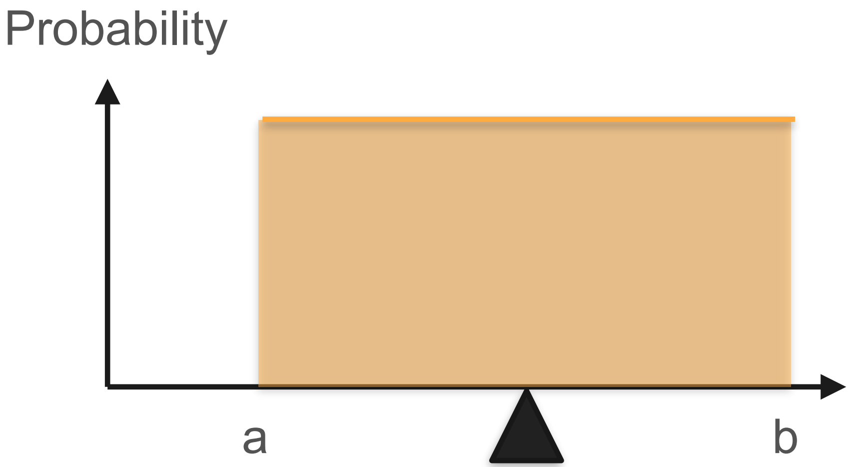
Expected Value: Uniform Distribution

Average = 0.5

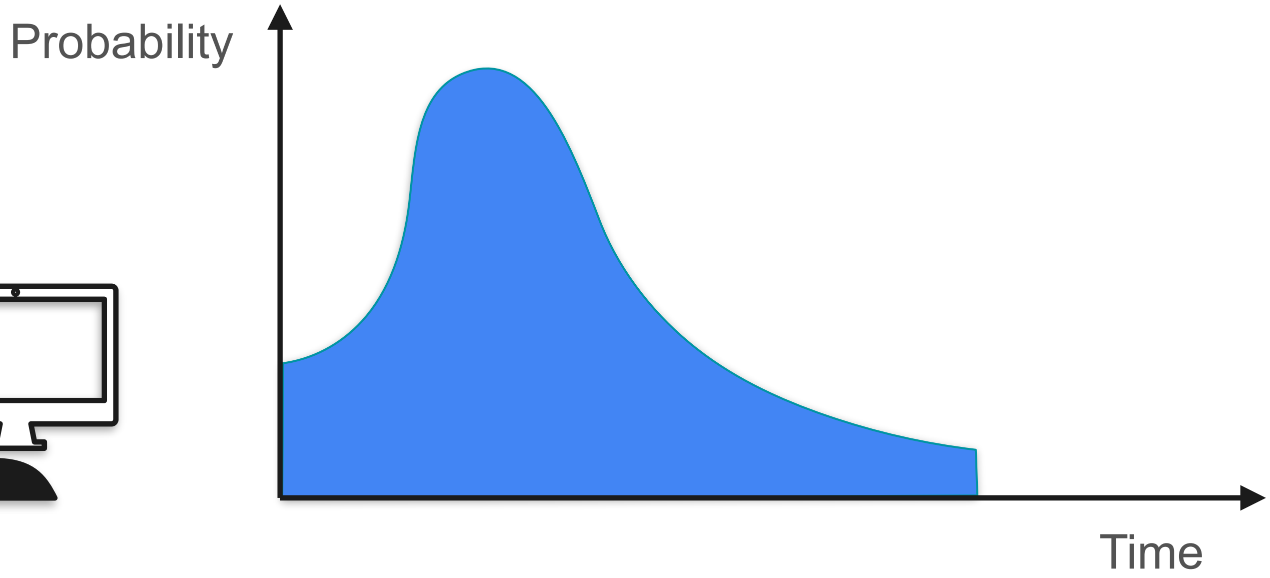


Expected Value: Uniform Distribution

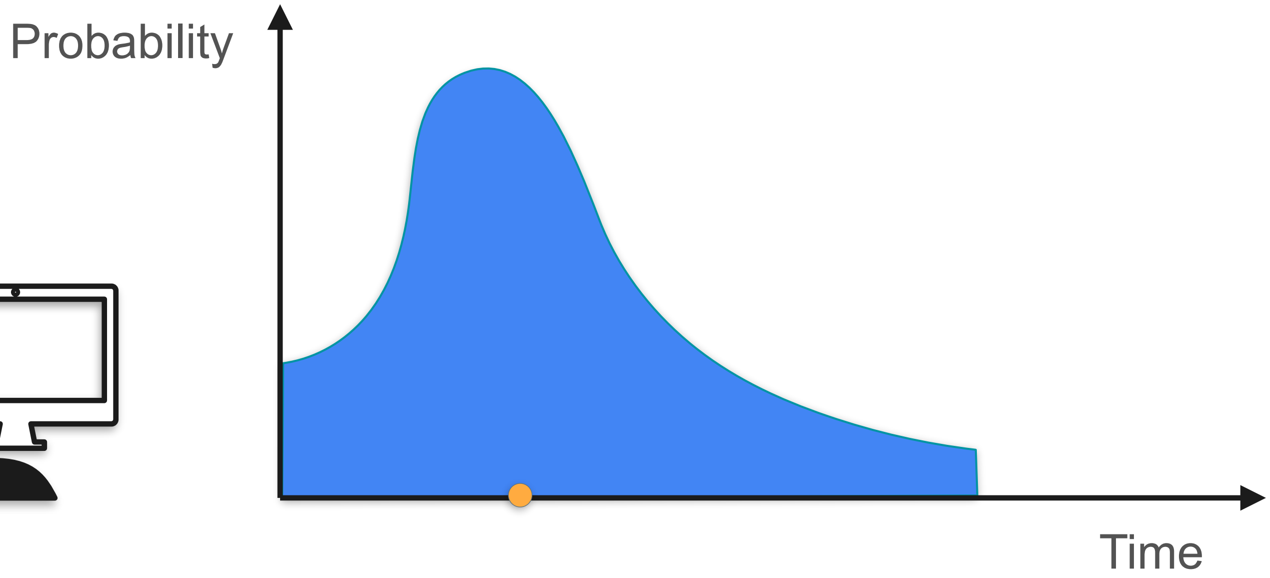
$$\text{Average} = \frac{a+b}{2}$$



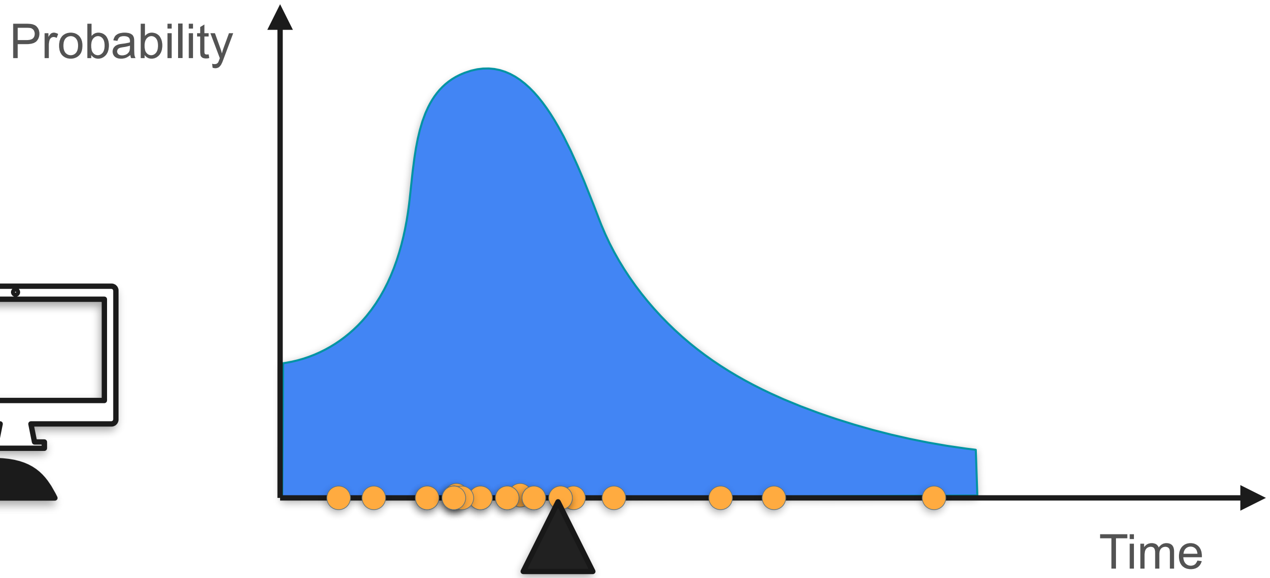
Expected Value



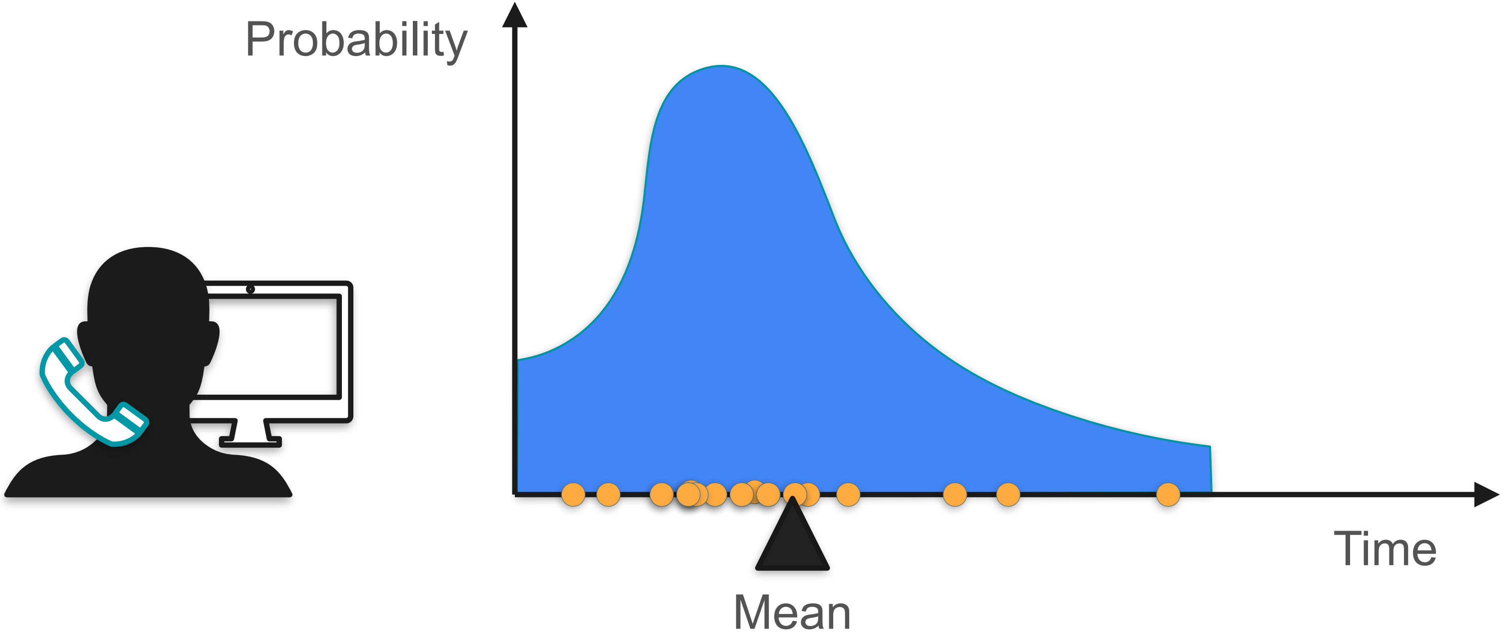
Expected Value



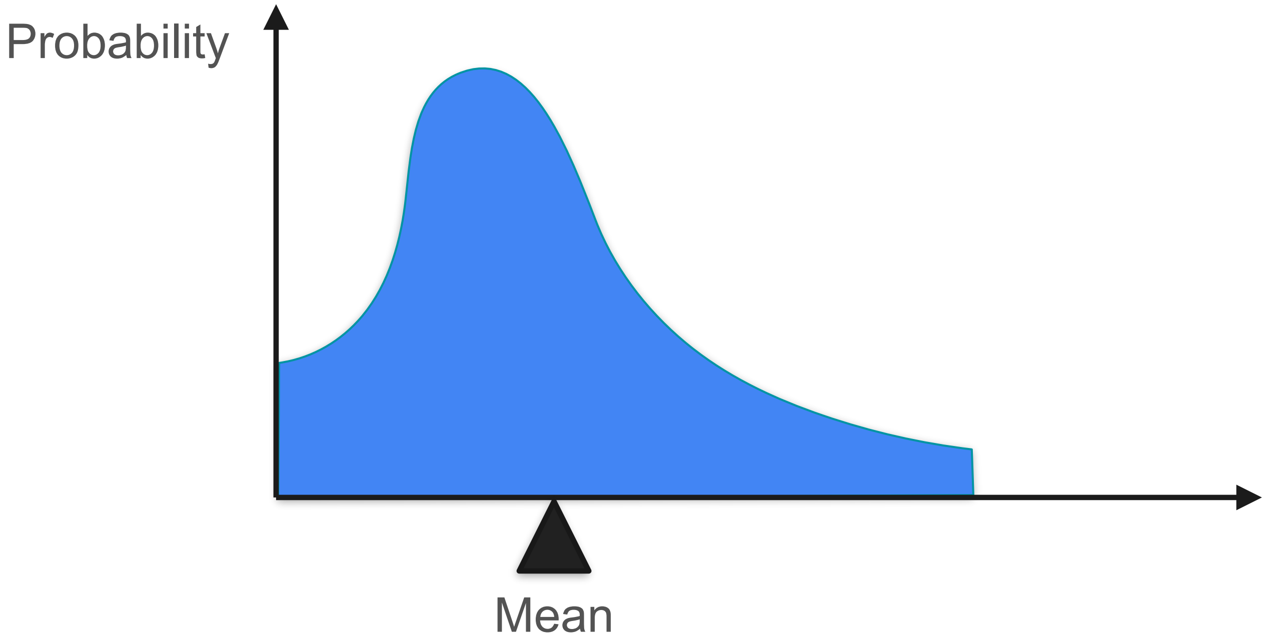
Expected Value



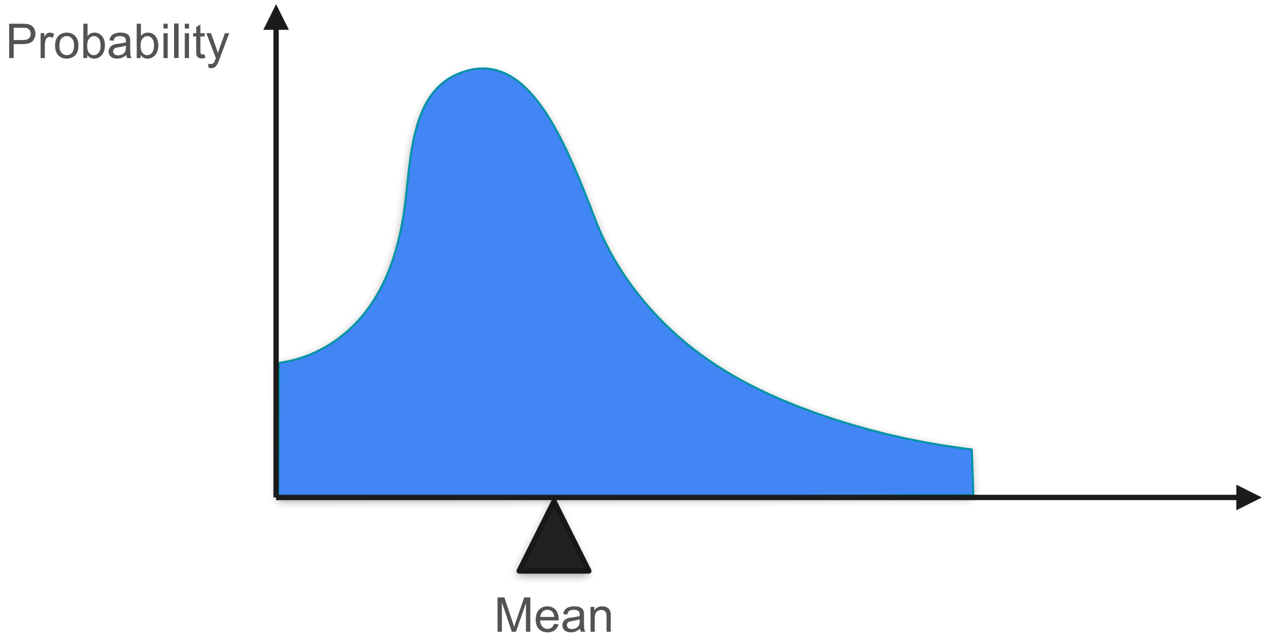
Expected Value



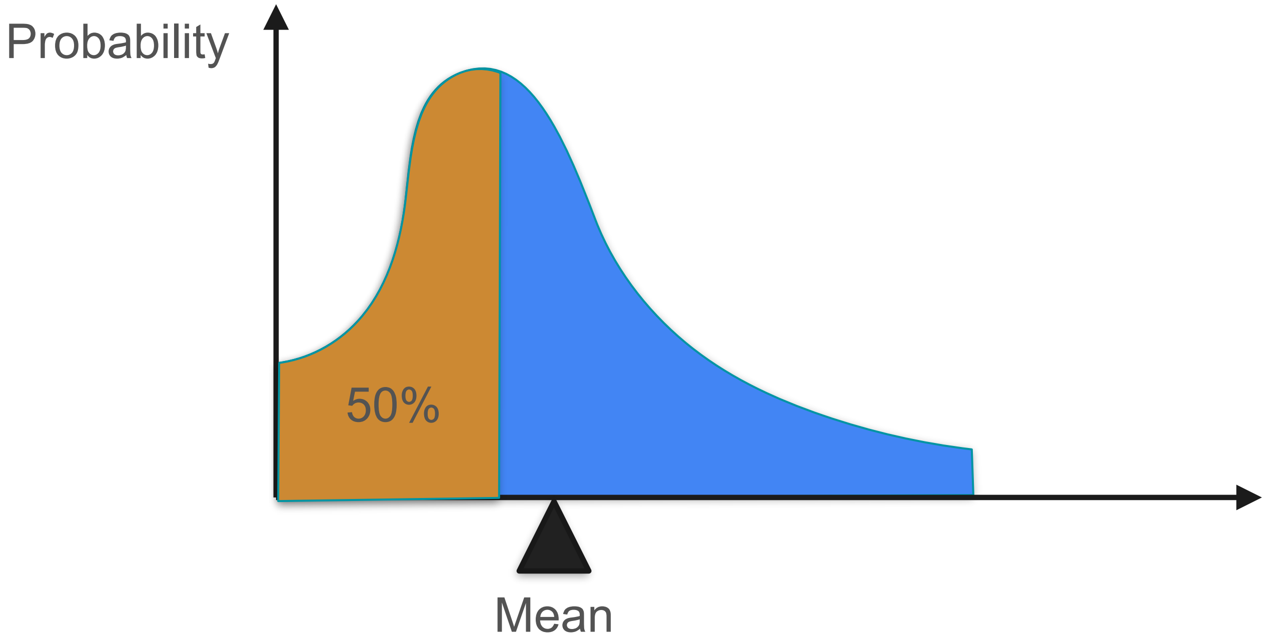
Expected Value: General Case



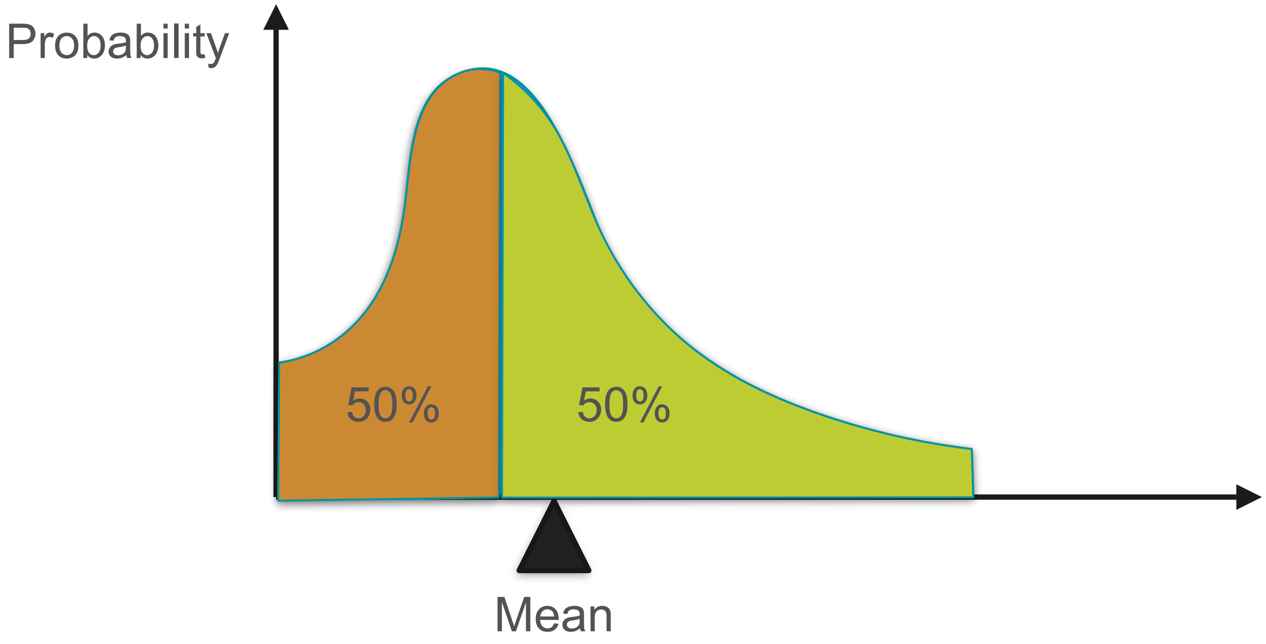
Expected Value: Common Misconception



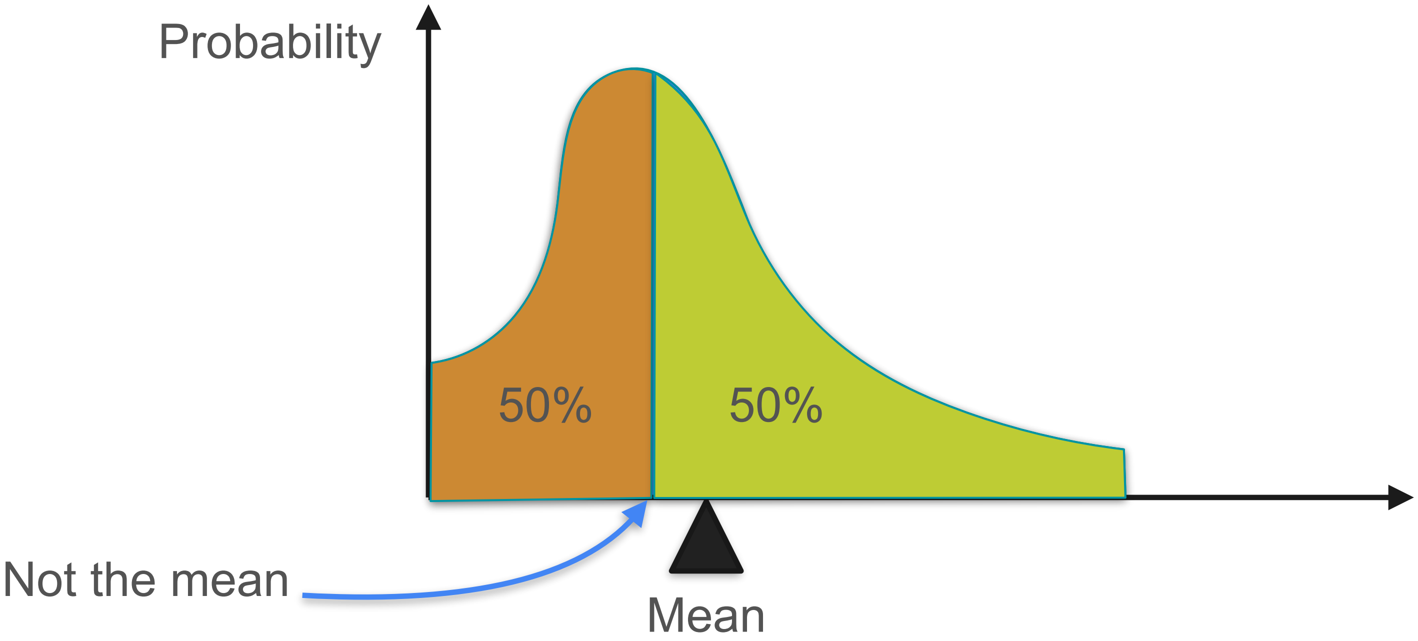
Expected Value: Common Misconception



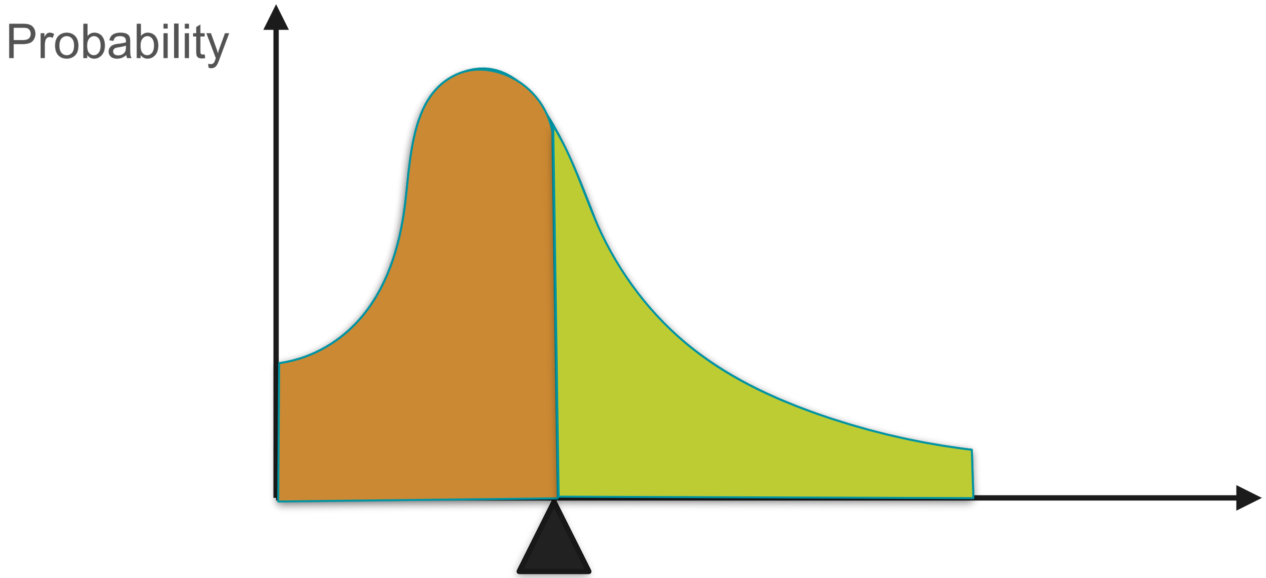
Expected Value: Common Misconception



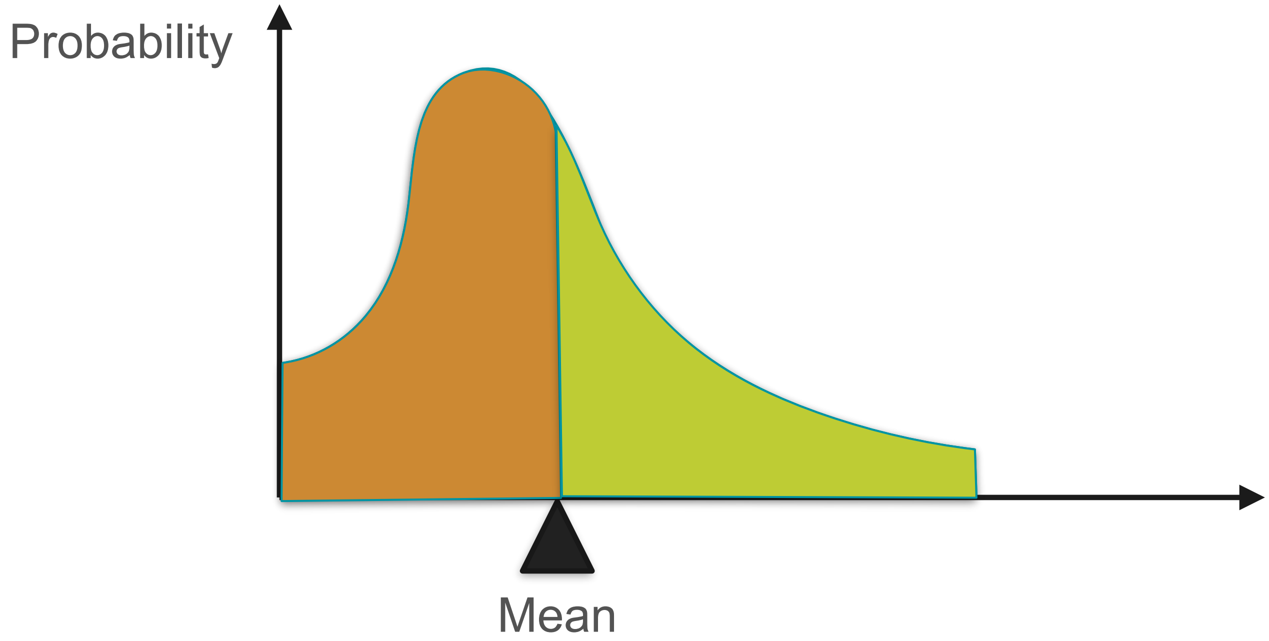
Expected Value: Common Misconception



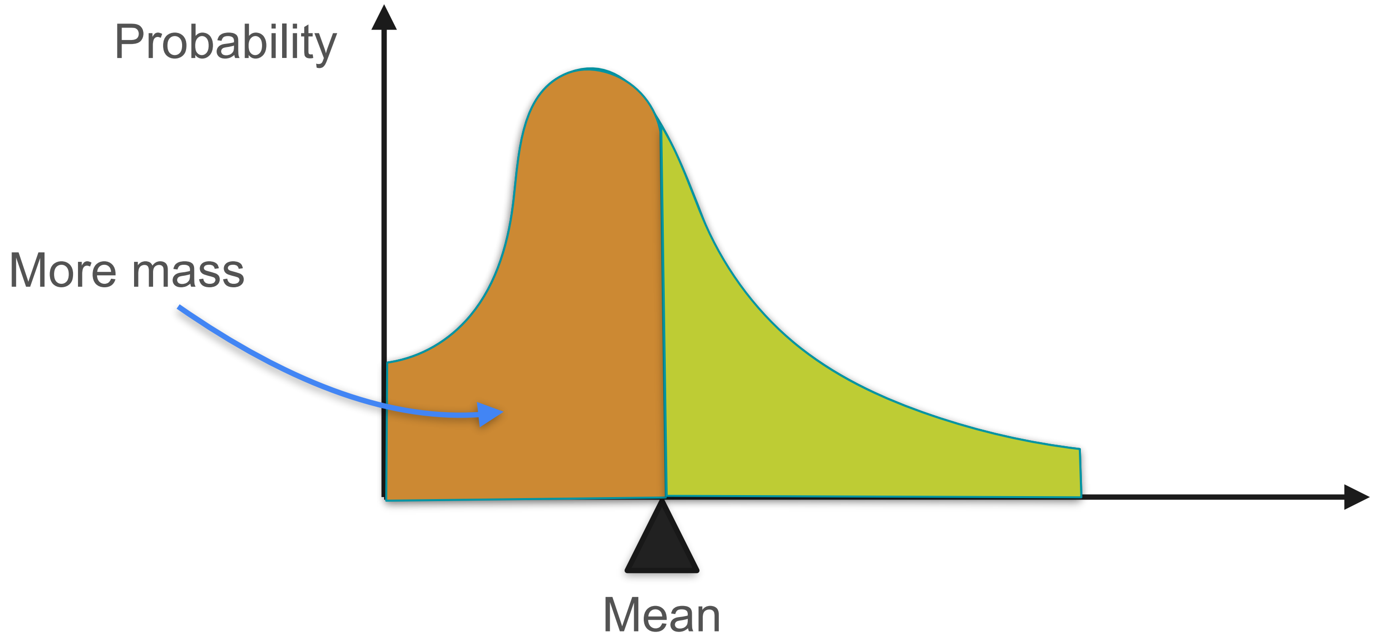
Expected Value: Common Misconception



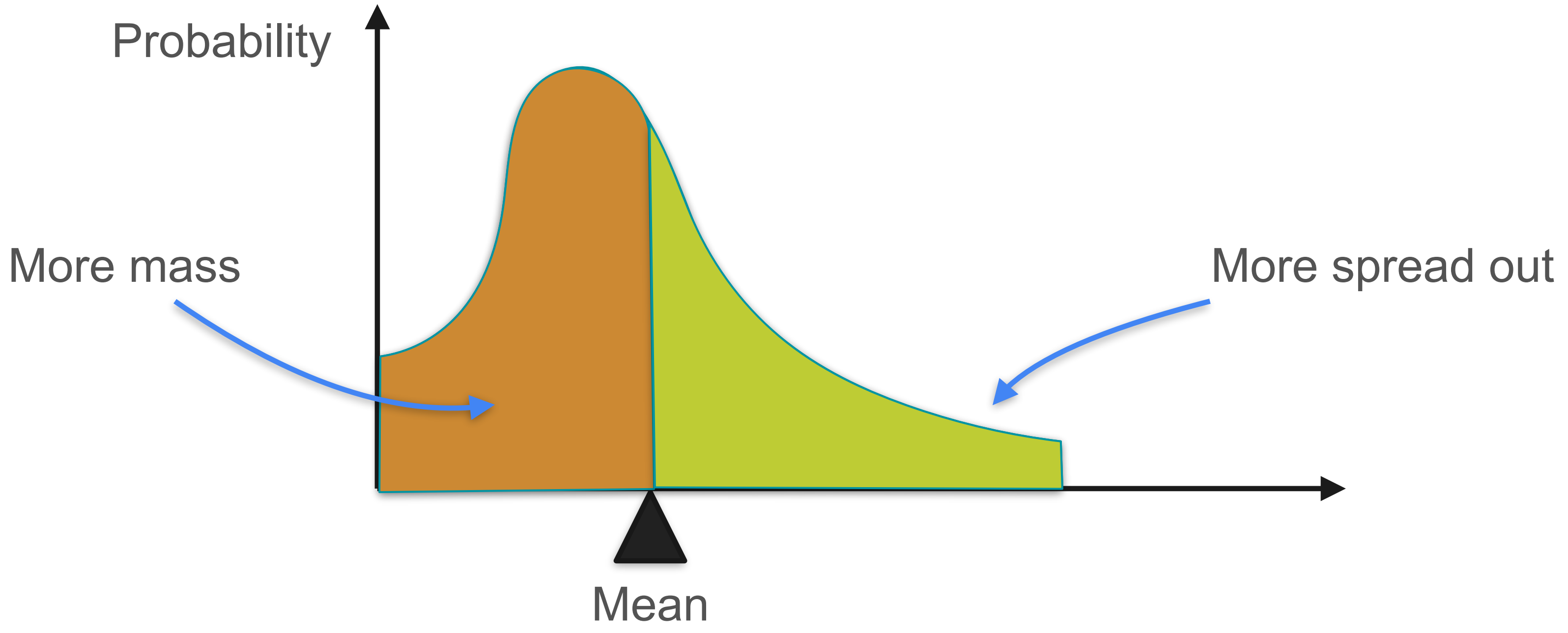
Expected Value: Common Misconception



Expected Value: Common Misconception



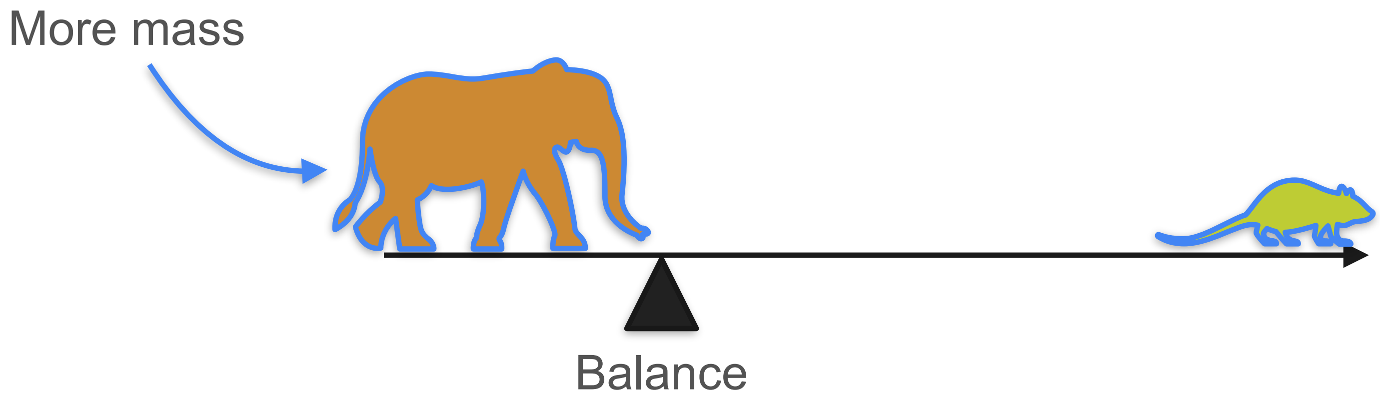
Expected Value: Common Misconception



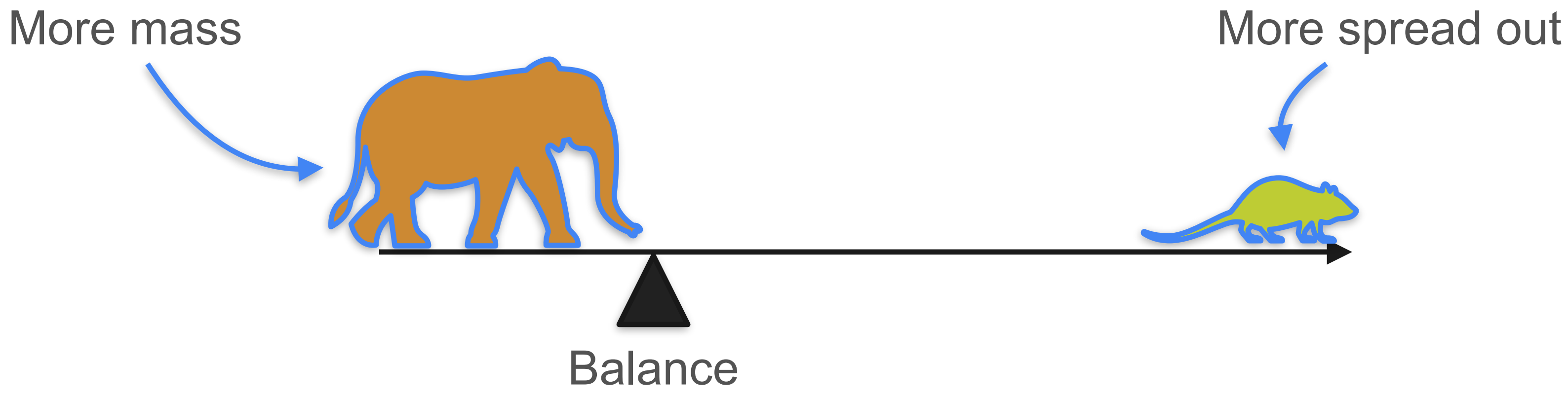
Expected Value: Common Misconception

Expected Value: Common Misconception

Expected Value: Common Misconception



Expected Value: Common Misconception

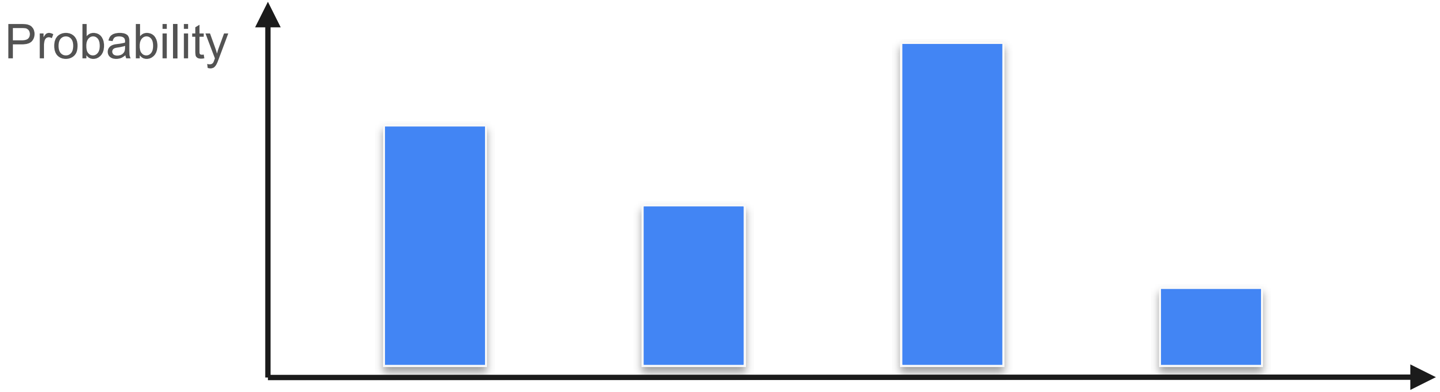


DeepLearning.AI

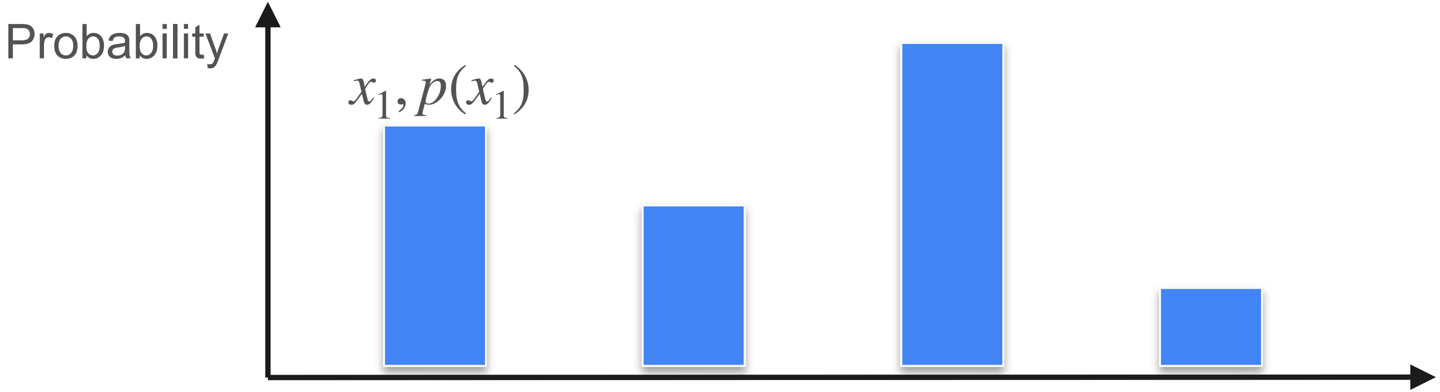
Describing Distributions

Expected value of a function

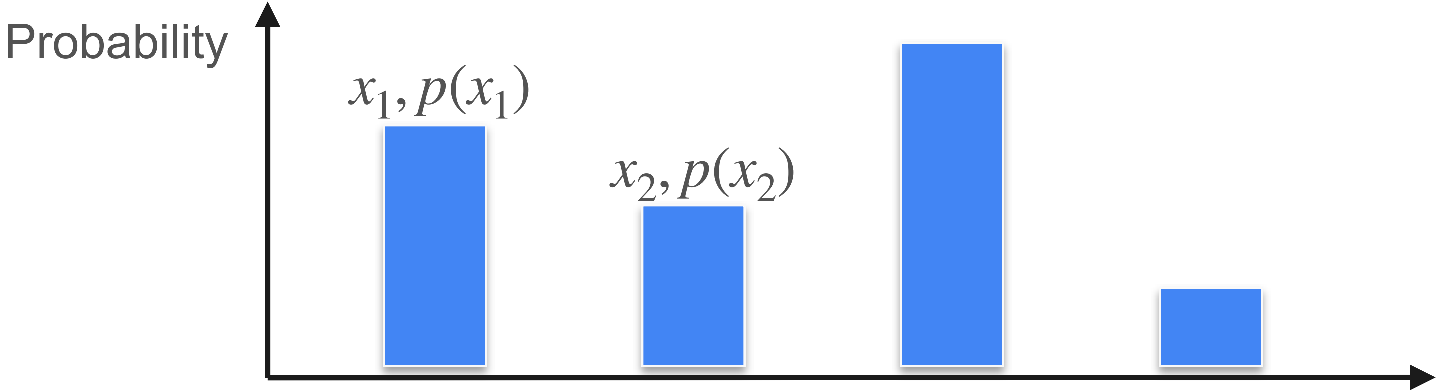
Expected Value of a Function



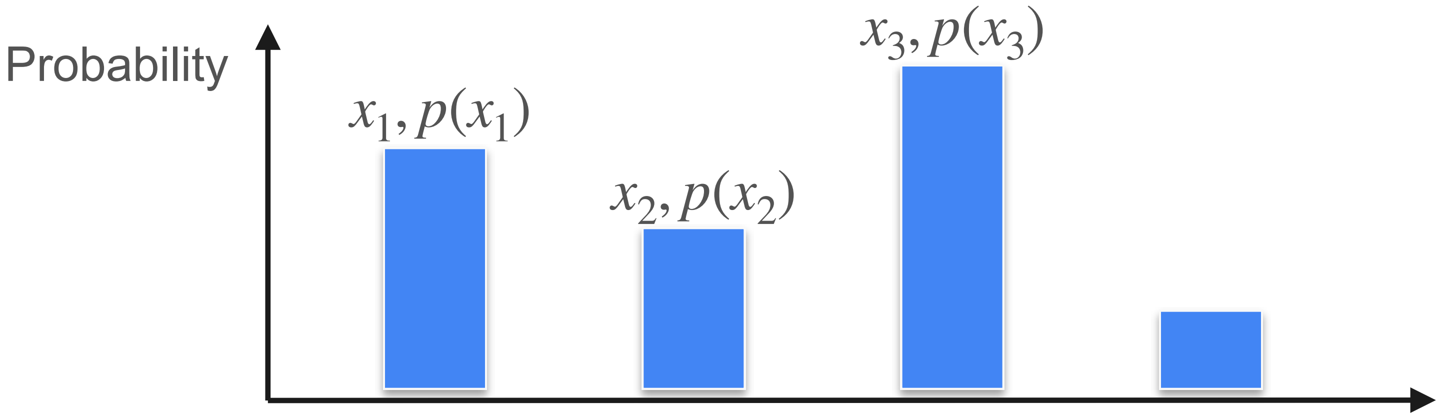
Expected Value of a Function



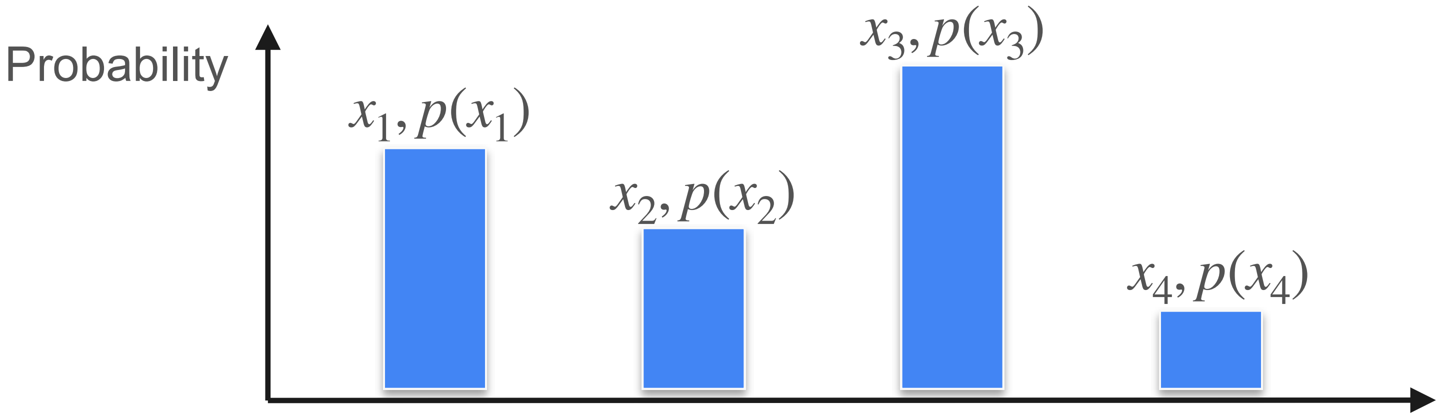
Expected Value of a Function



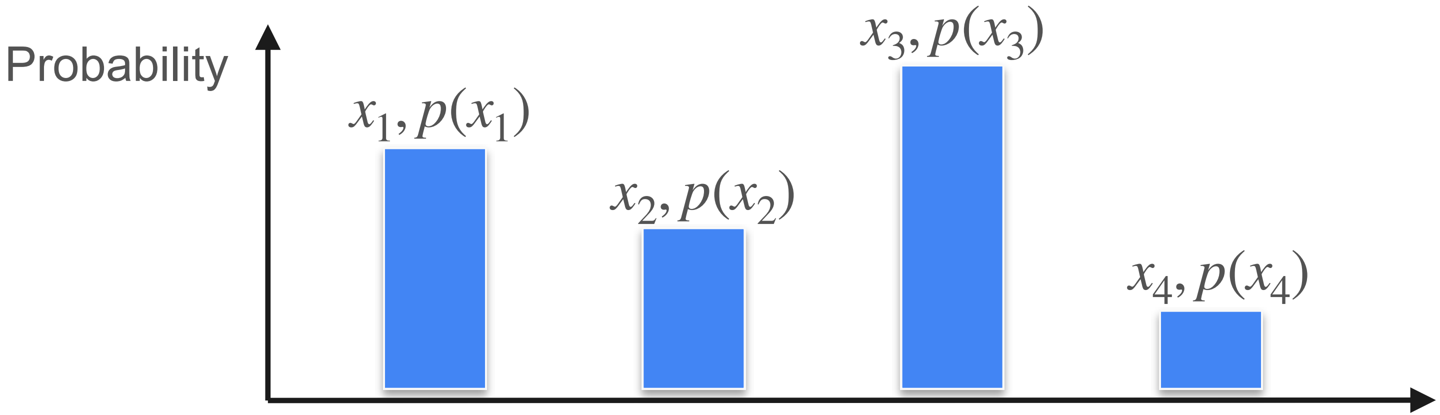
Expected Value of a Function



Expected Value of a Function

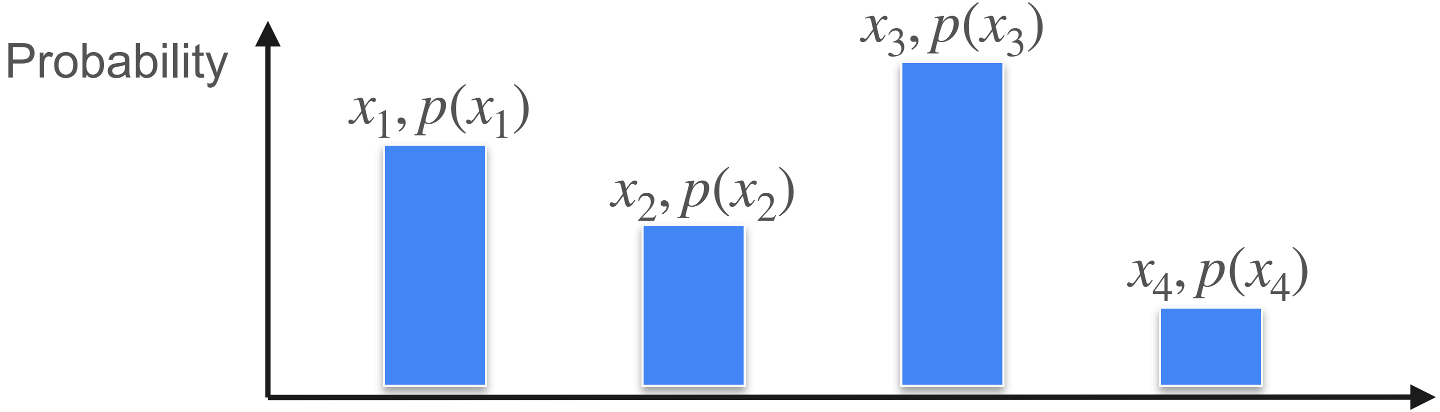


Expected Value of a Function



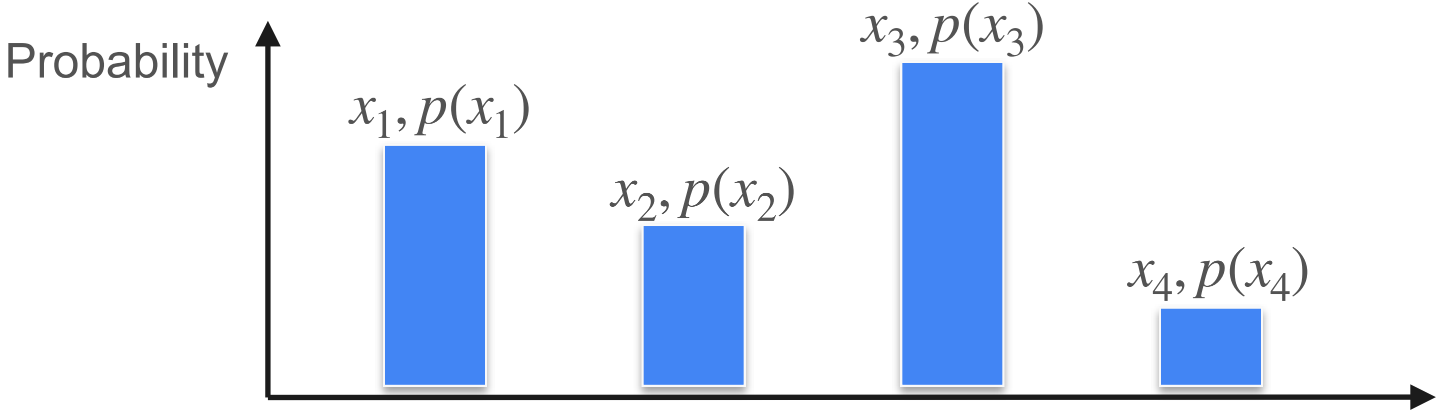
$$\mathbb{E}[X] =$$

Expected Value of a Function



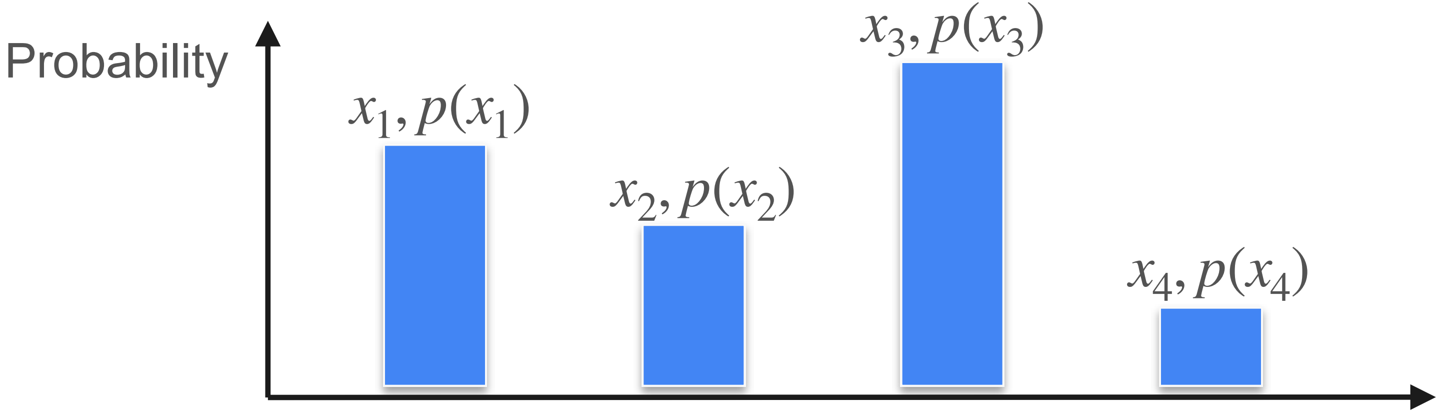
$$\mathbb{E}[X] = x_1 p(x_1)$$

Expected Value of a Function



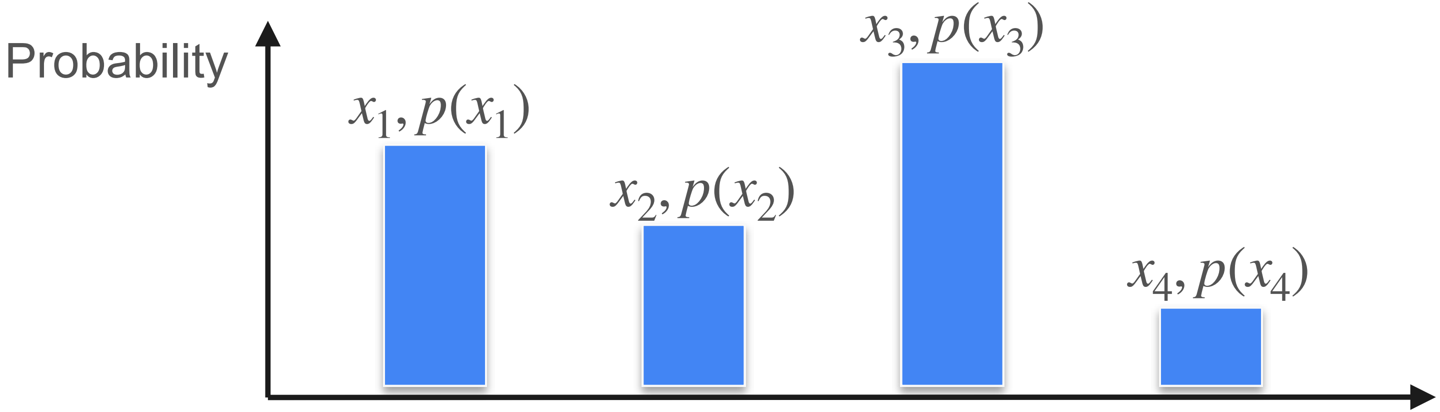
$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2)$$

Expected Value of a Function



$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

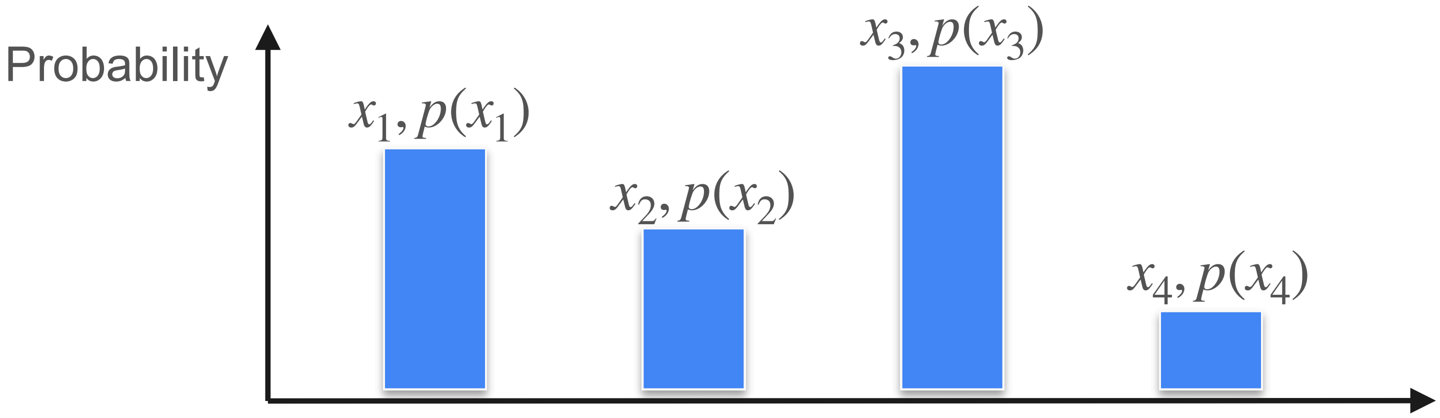
Expected Value of a Function



$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

$$E[f(X)] =$$

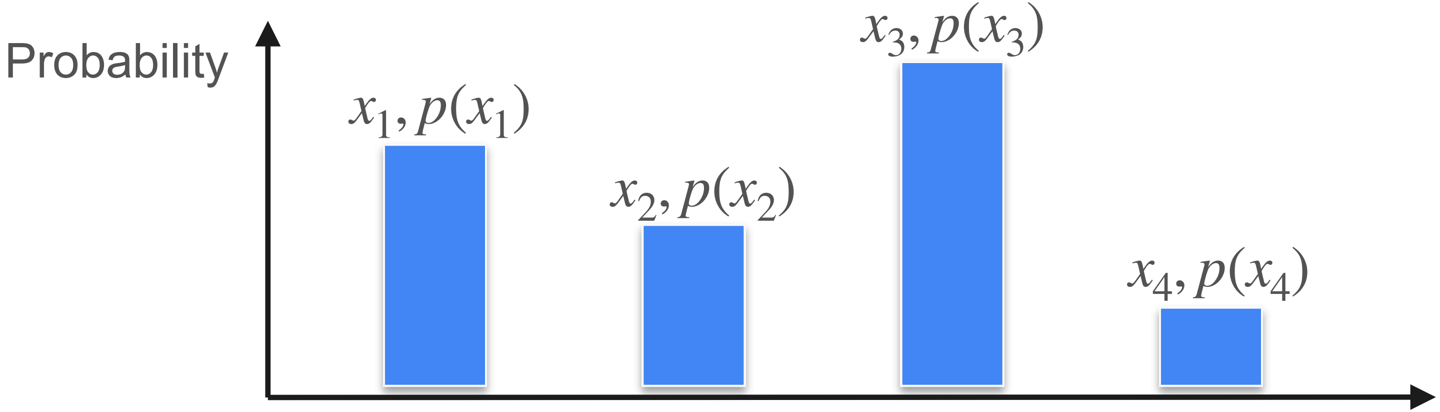
Expected Value of a Function



$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

$$E[f(X)] = f(x_1) p(x_1)$$

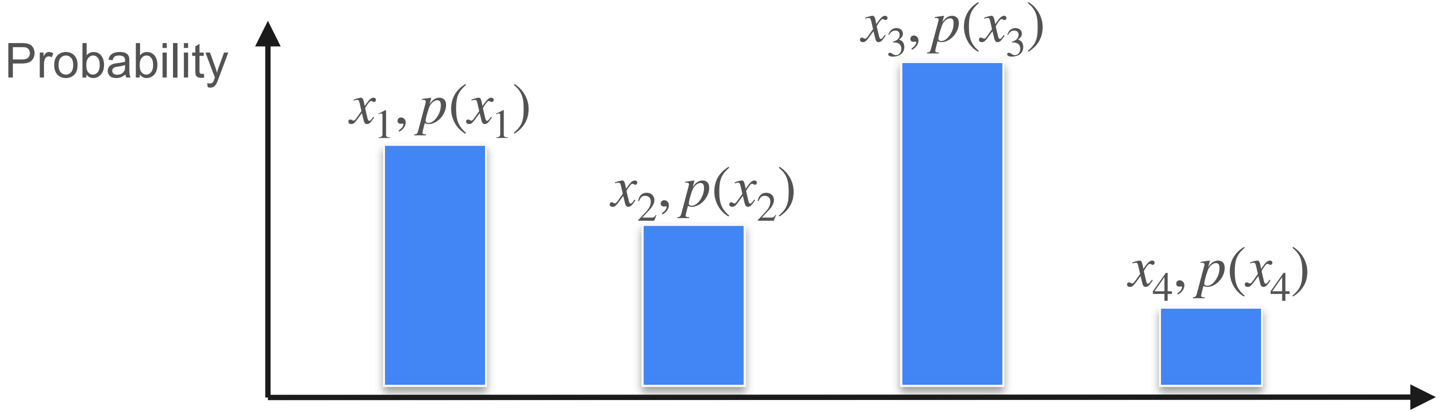
Expected Value of a Function



$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

$$E[f(X)] = f(x_1)p(x_1) + f(x_2)p(x_2)$$

Expected Value of a Function

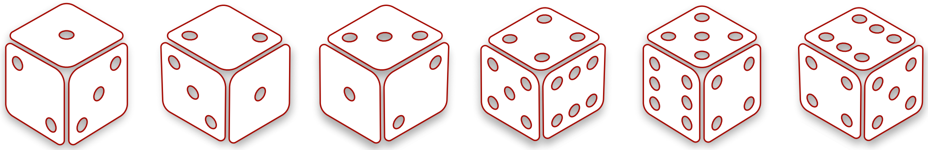


$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

$$E[f(X)] = f(x_1)p(x_1) + f(x_2)p(x_2) + f(x_3)p(x_3) + f(x_4)p(x_4)$$

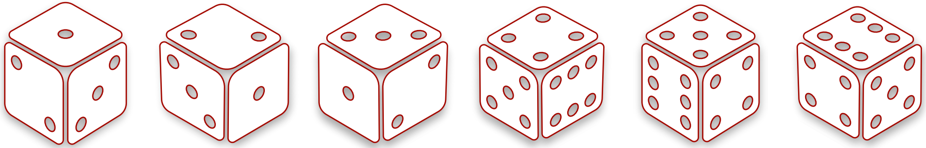
Expected Value of a Function

Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6

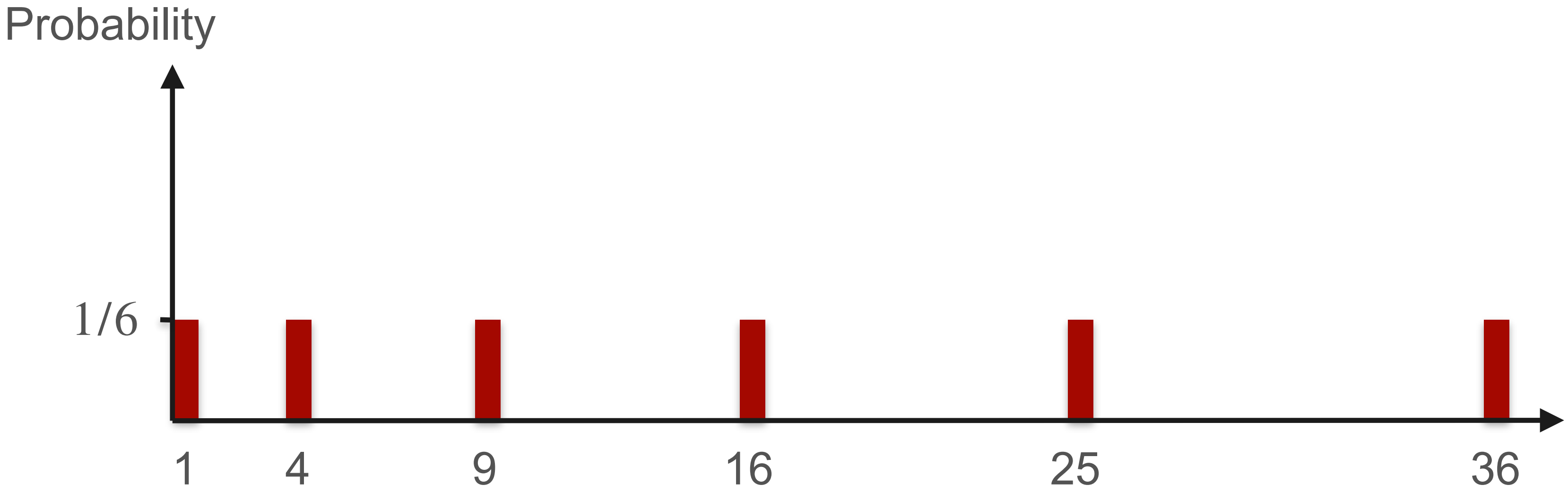


Expected Value of a Function

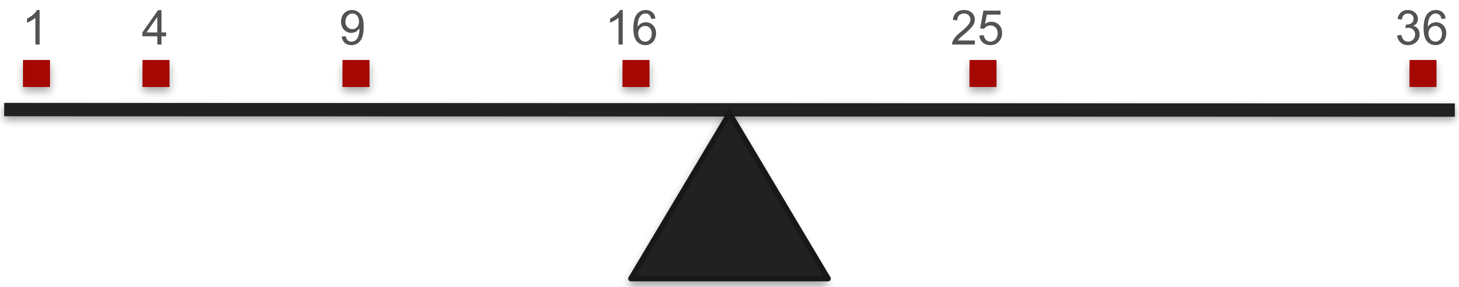
Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6
Square:	1	4	9	16	25	36



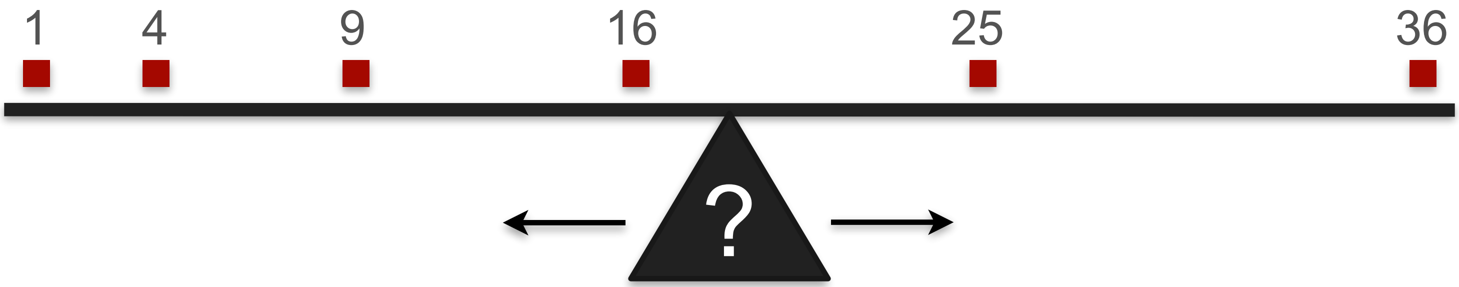
Expected Value of a Function



Expected Value of a Function

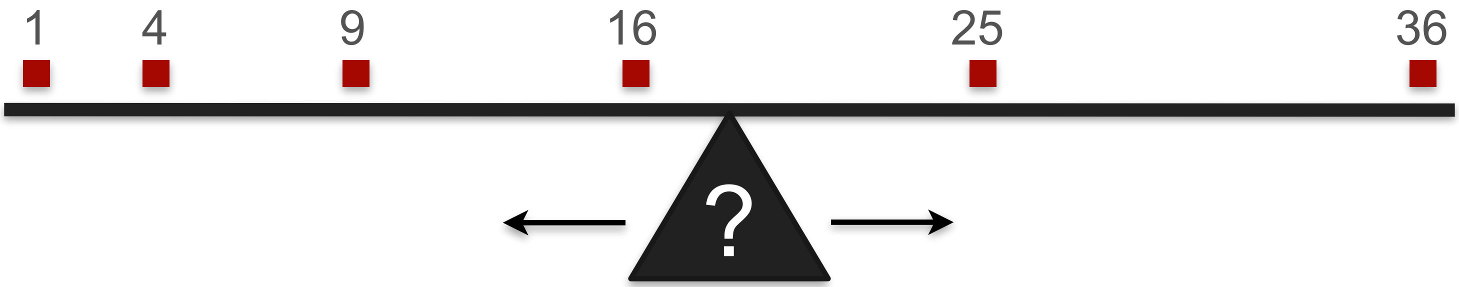


Expected Value of a Function



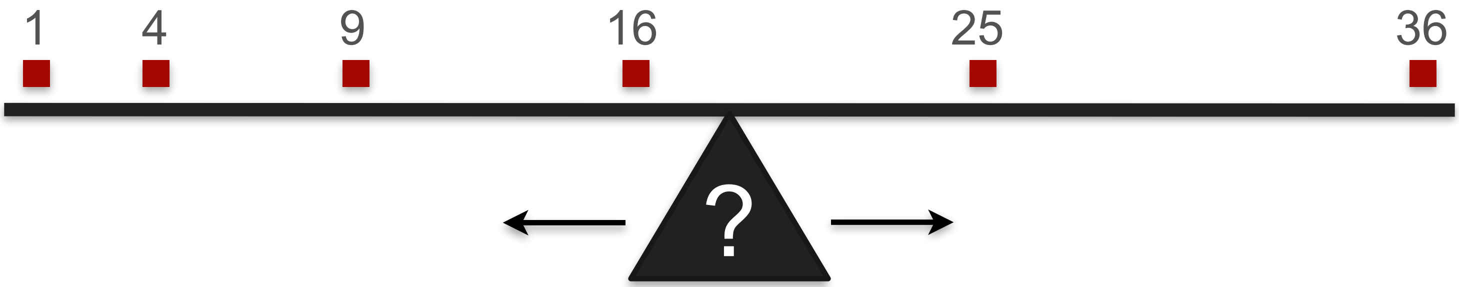
Expected Value of a Function

$$\frac{1+4+9+16+25+36}{6}$$



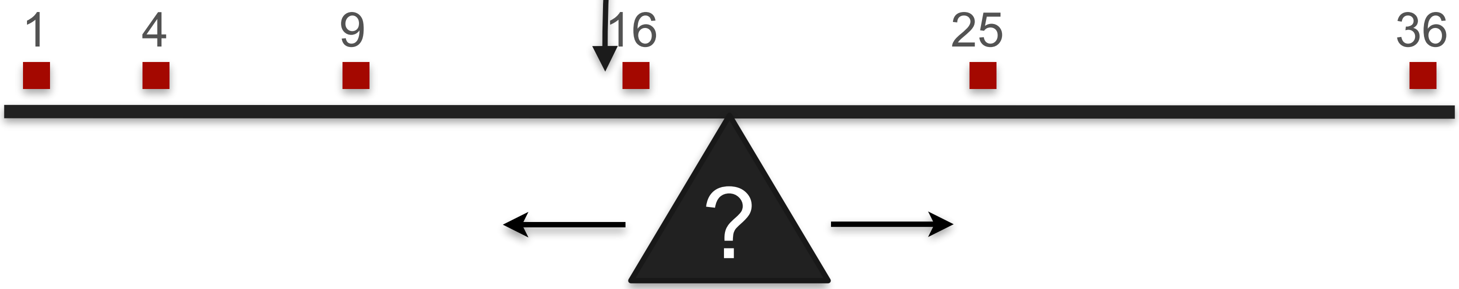
Expected Value of a Function

$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6}$$



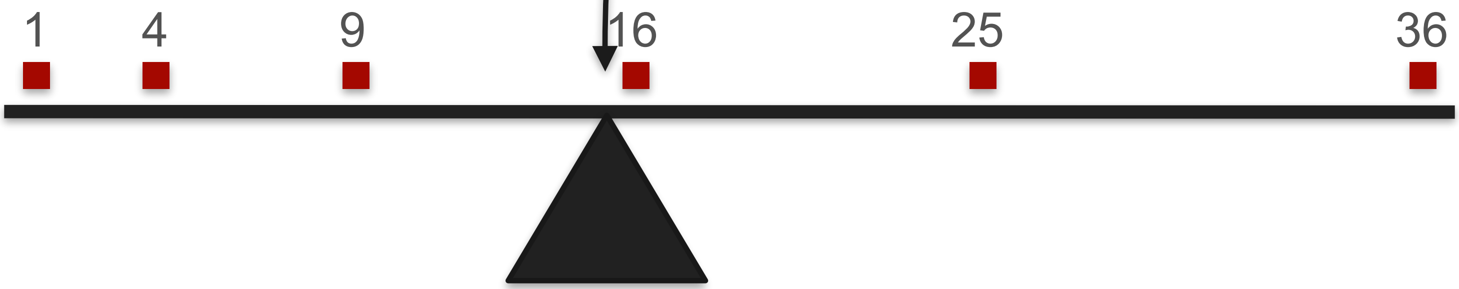
Expected Value of a Function

$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6}$$



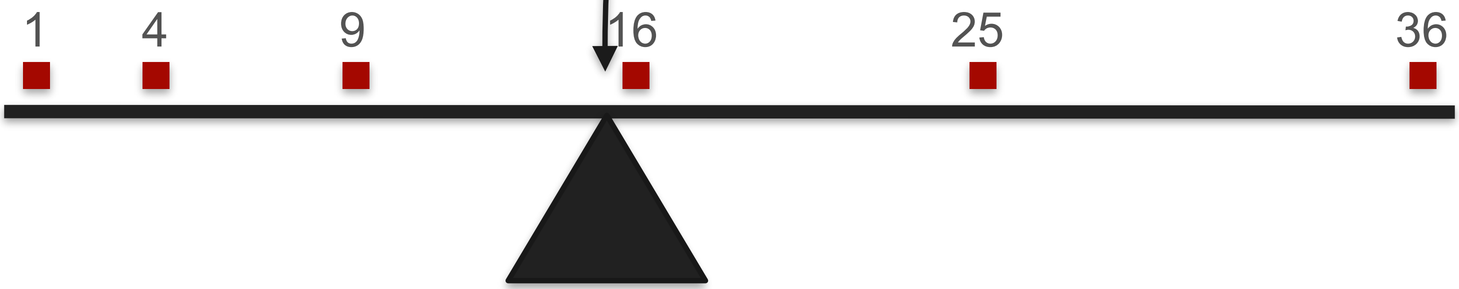
Expected Value of a Function

$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6}$$



Expected Value of a Function

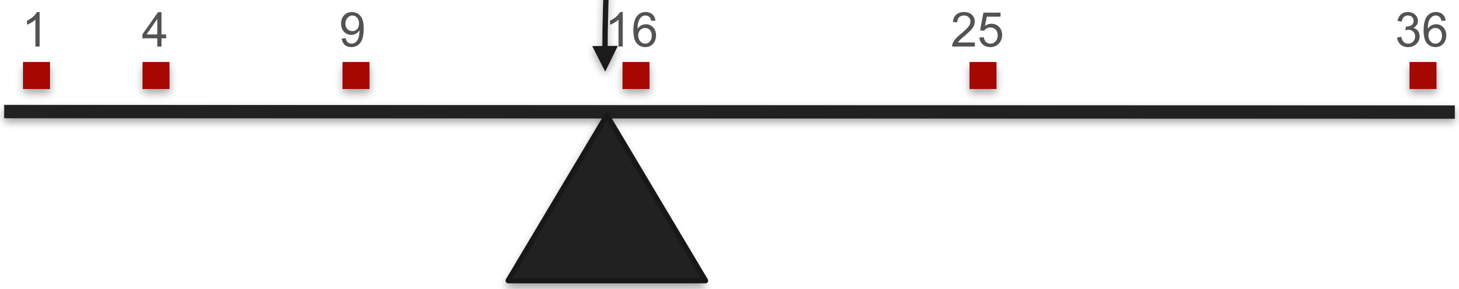
$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6} = \frac{1^2}{6} + \frac{2^2}{6} + \frac{3^2}{6} + \frac{4^2}{6} + \frac{5^2}{6} + \frac{6^2}{6}$$



Expected Value of a Function

$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6} = \frac{1^2}{6} + \frac{2^2}{6} + \frac{3^2}{6} + \frac{4^2}{6} + \frac{5^2}{6} + \frac{6^2}{6}$$

$$= \mathbb{E}\left[X^2\right]$$



Describing Distributions

Sum of expectations

Sum of Expectations

You play a game:

Sum of Expectations

You play a game:

Flip a coin. If heads you win \$ 1, else you win nothing.

Sum of Expectations

You play a game:

Flip a coin. If heads you win $ 1, else you win nothing.

Win $1

Win nothing

Sum of Expectations

You play a game:

Flip a coin. If heads you win $ 1, else you win nothing.

Then roll a die. You win the amount you roll.

Win $1

Win nothing

Sum of Expectations

You play a game:

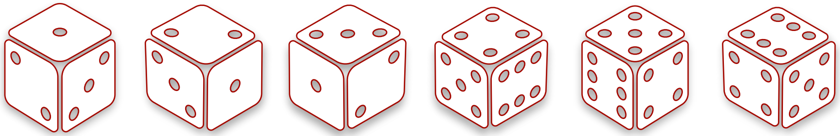
Flip a coin. If heads you win $ 1, else you win nothing.

Then roll a die. You win the amount you roll.

Win $1

Win nothing

Win	$1	$2	$3	$4	$5	$6



Sum of Expectations

You play a game:

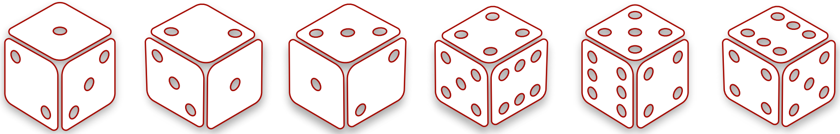
Flip a coin. If heads you win $ 1, else you win nothing.

Then roll a die. You win the amount you roll.

Win $1

Win nothing

Win $1 $2 $3 $4 $5 $6



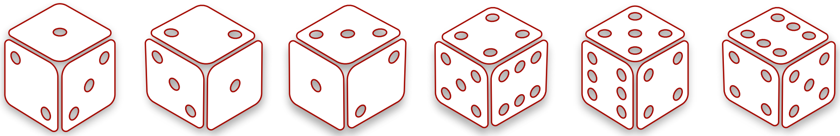
What are your expected winnings for the game?

Sum of Expectations

Win \$1

Win nothing

Win	\$1	\$2	\$3	\$4	\$5	\$6



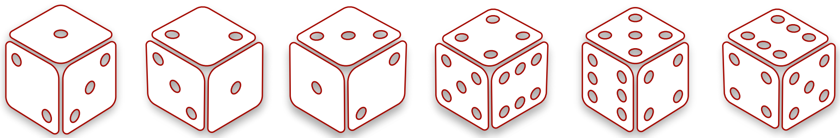
Sum of Expectations

Win $1

Win nothing

$$\mathbb{E}\left[X_{coin}\right] = \$0.5$$

Win $1 $2 $3 $4 $5 $6

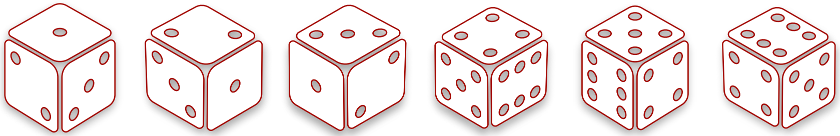


Sum of Expectations

Win \$1

Win nothing

Win \$1 \$2 \$3 \$4 \$5 \$6



$$\mathbb{E}\left[X_{coin}\right] = \$0.5$$

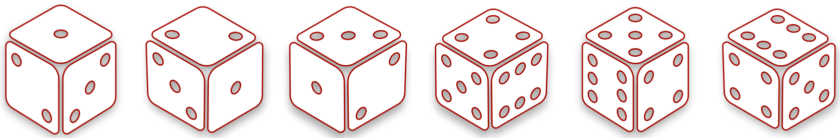
$$\mathbb{E}\left[X_{dice}\right] = \$3.5$$

Sum of Expectations

Win $1

Win nothing

Win $1 $2 $3 $4 $5 $6



$$\mathbb{E}\left[X_{coin}\right] = \$0.5$$

$$\mathbb{E}\left[X_{dice}\right] = \$3.5$$

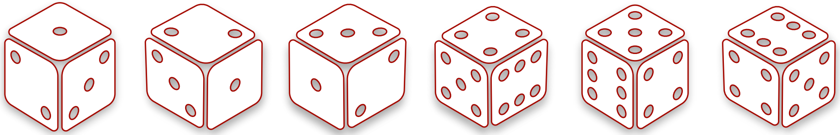
$$\mathbb{E}[X] = \mathbb{E}\left[X_{coin}\right] + \mathbb{E}\left[X_{dice}\right] = \$0.5 + \$3.5 = \$4$$

Sum of Expectations

Win \$1

Win nothing

Win \$1 \$2 \$3 \$4 \$5 \$6



$$\mathbb{E}\left[X_{coin}\right] = \$0.5 \qquad \mathbb{E}\left[X_{dice}\right] = \$3.5$$

$$\mathbb{E}[X] = \mathbb{E}\left[X_{coin}\right] + \mathbb{E}\left[X_{dice}\right] = \$0.5 + \$3.5 = \$4$$

In general: $\mathbb{E}\left[X_1 + X_2\right] = \mathbb{E}\left[X_1\right] + \mathbb{E}\left[X_2\right]$

Sum of Expectations

8 billion people

Sum of Expectations

8 billion people

Sum of Expectations

8 billion people

Sum of Expectations

Expected number of correct assignments?

8 billion people

Sum of Expectations

1

Expected number of correct assignments?

8 billion people

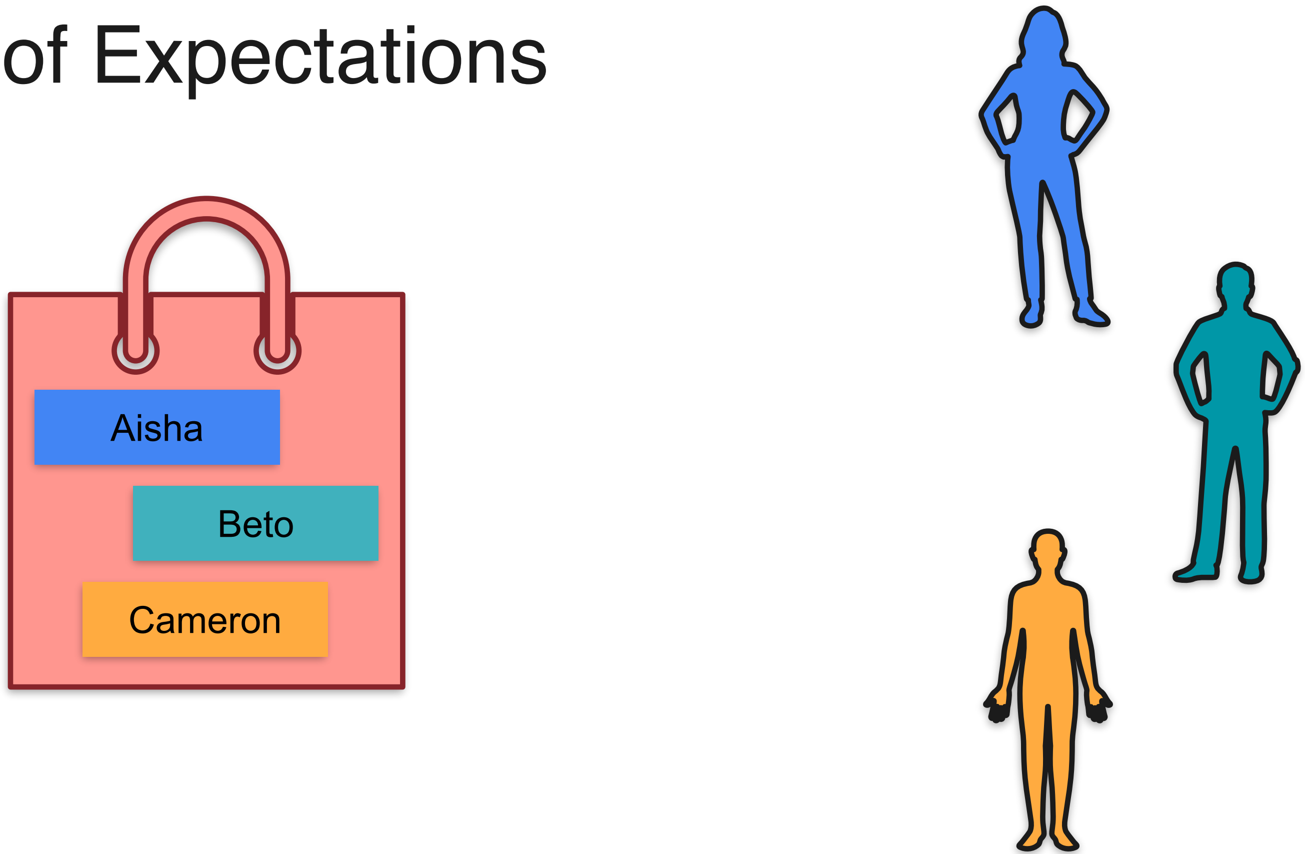
Sum of Expectations

1

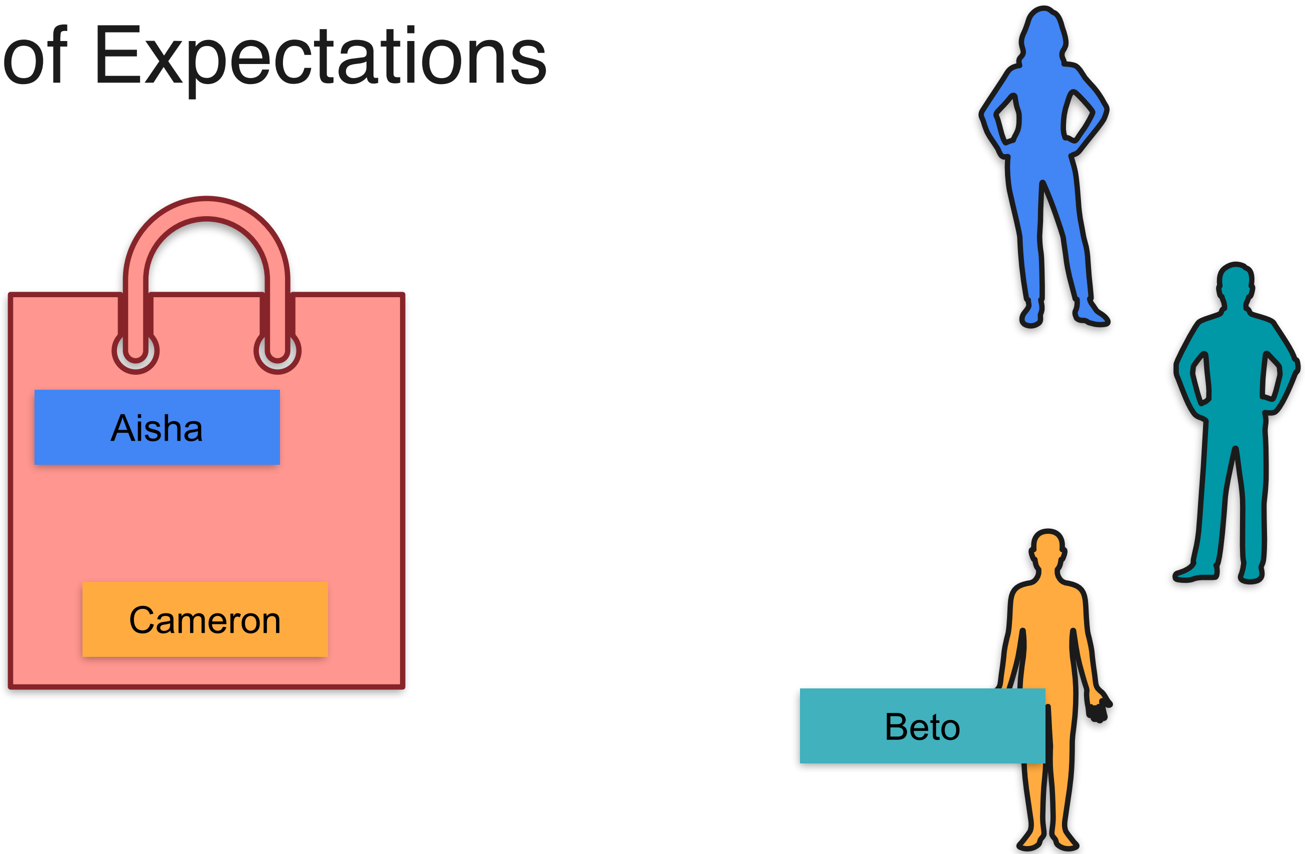
Expected number of correct assignments?

8 billion people

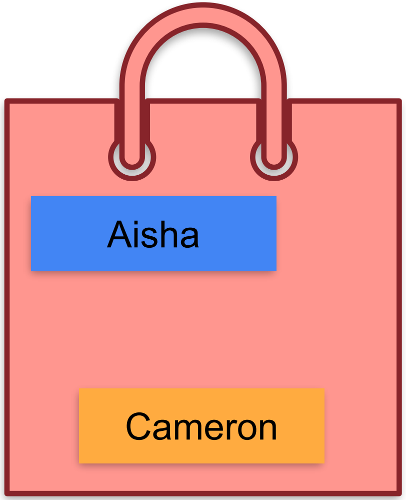
Sum of Expectations

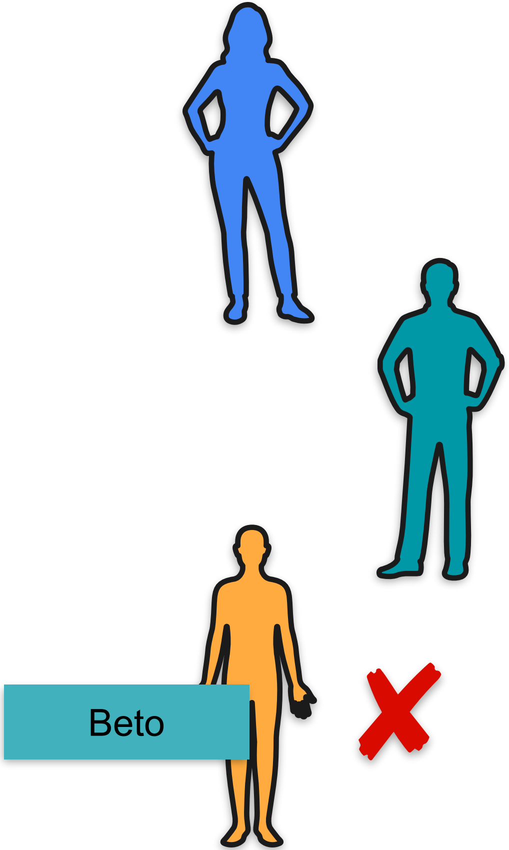


Sum of Expectations

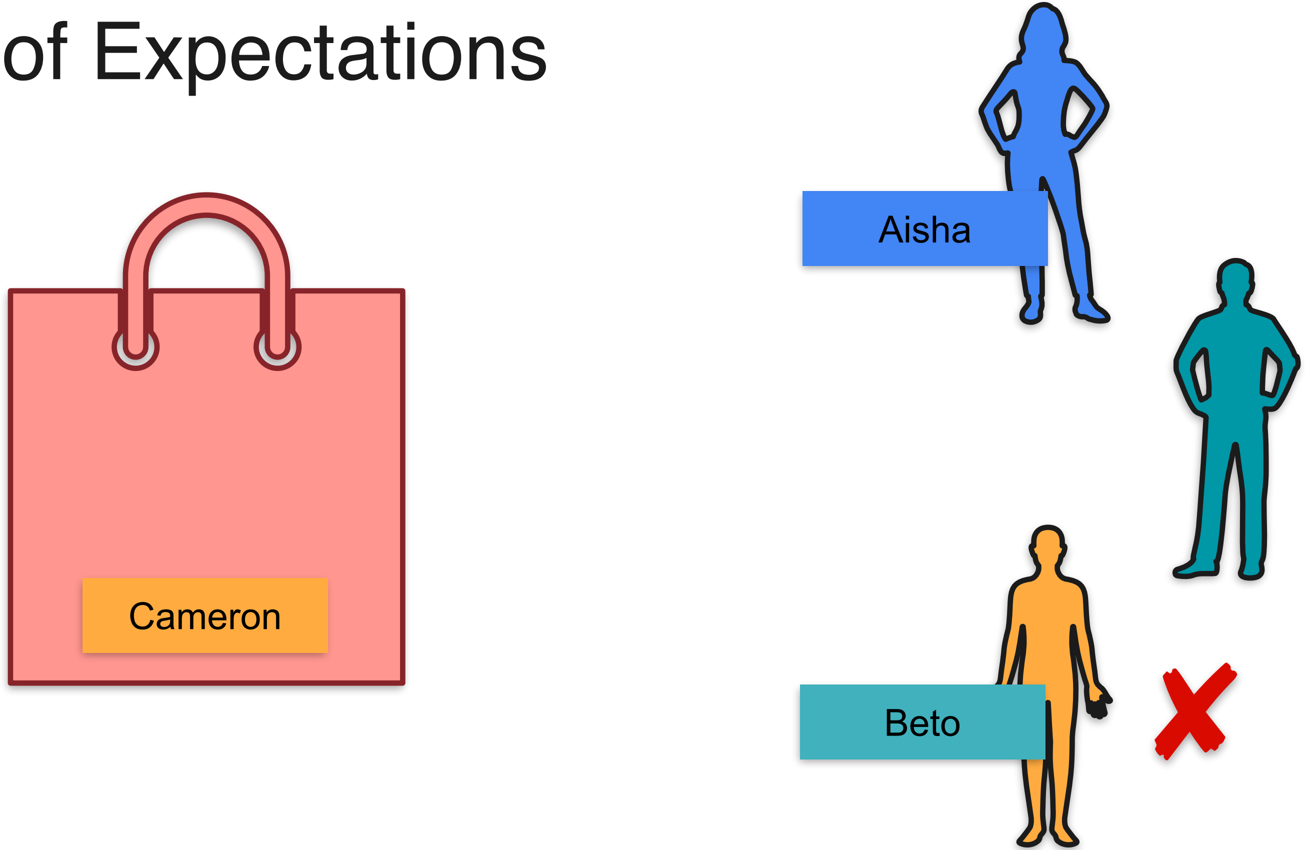


Sum of Expectations

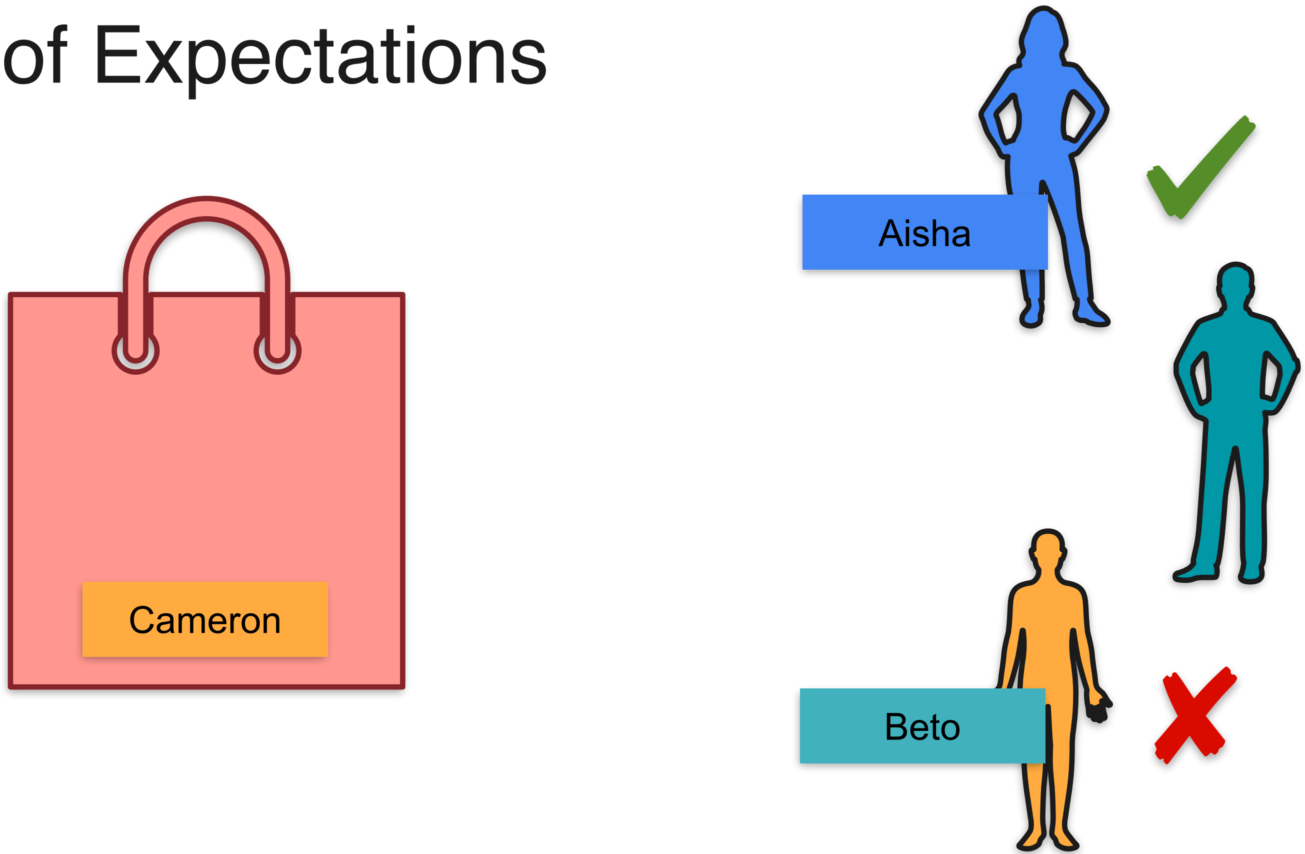




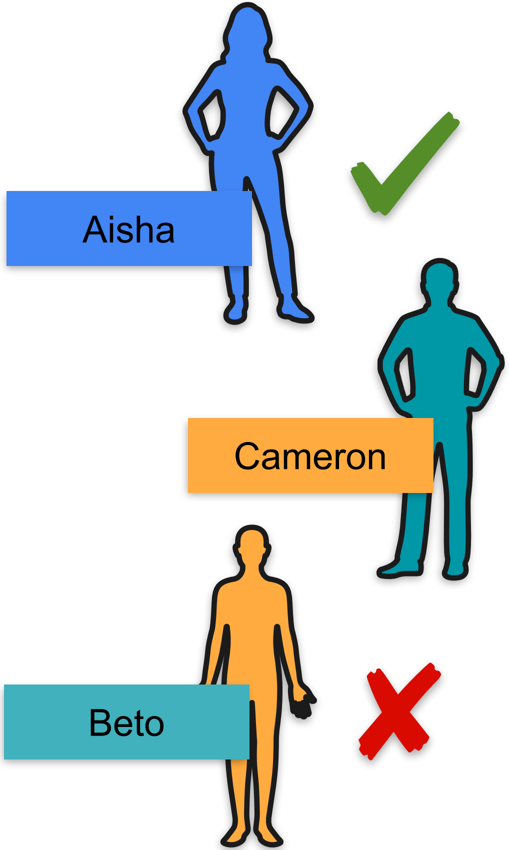
Sum of Expectations



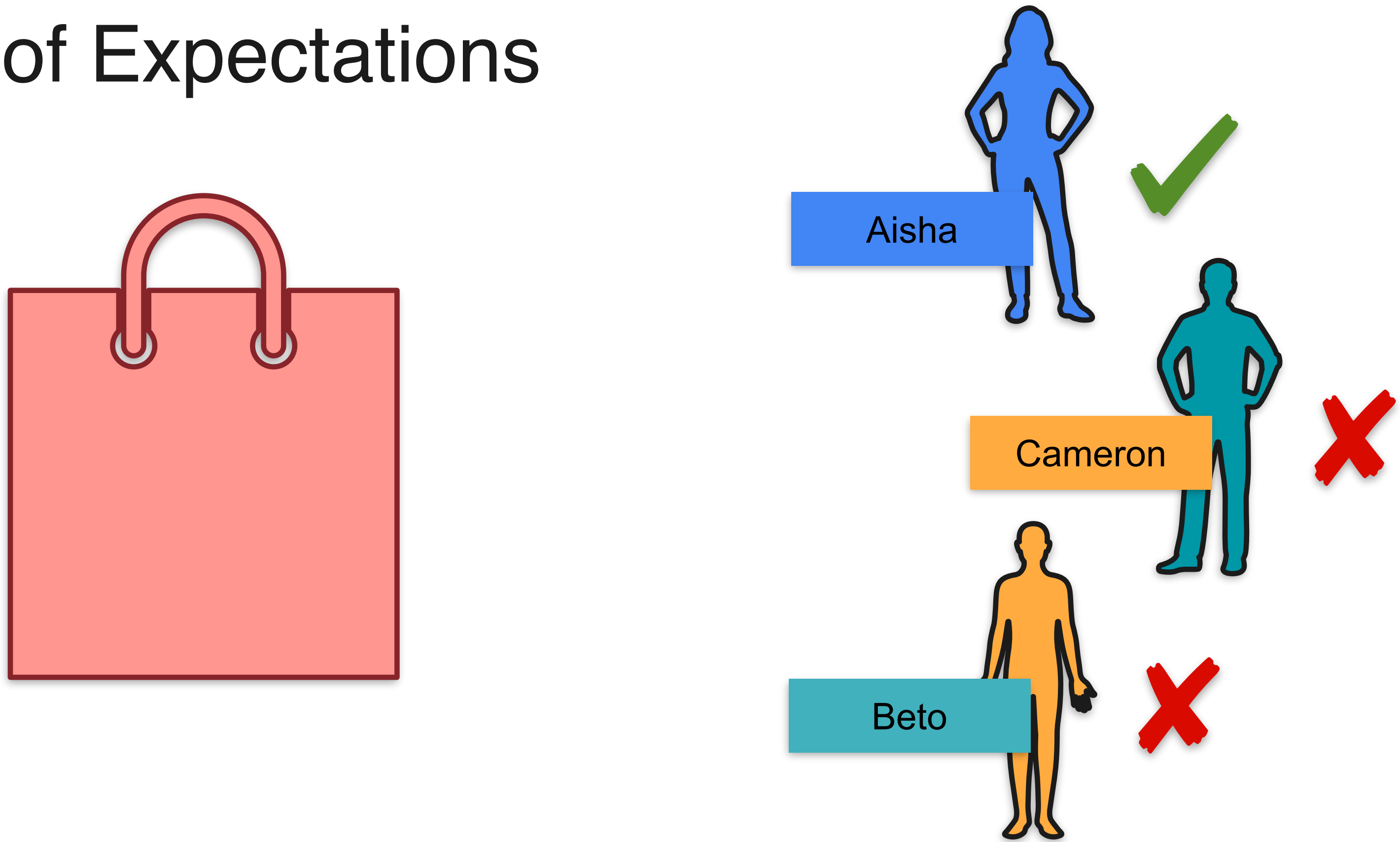
Sum of Expectations



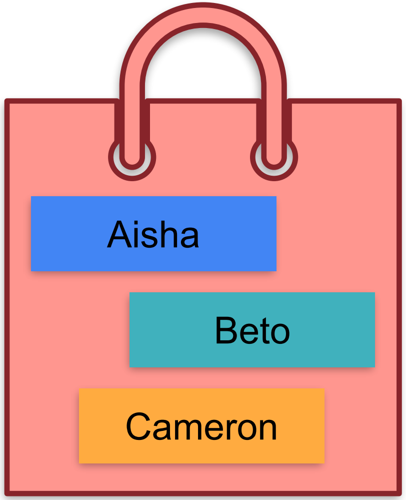
Sum of Expectations

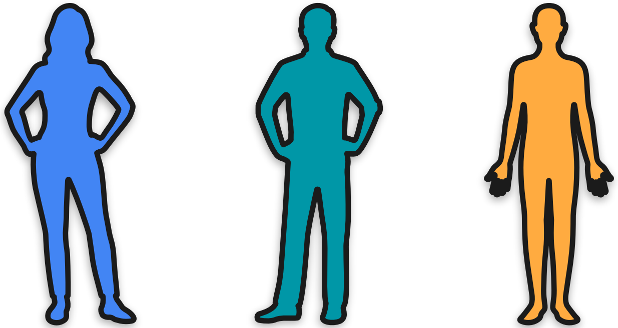


Sum of Expectations

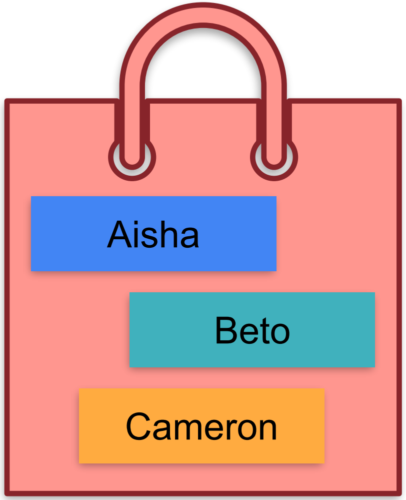


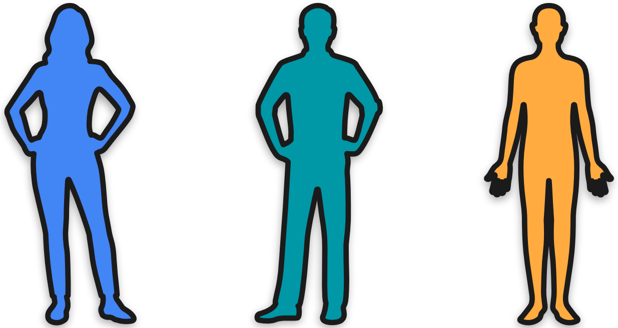
Sum of Expectations





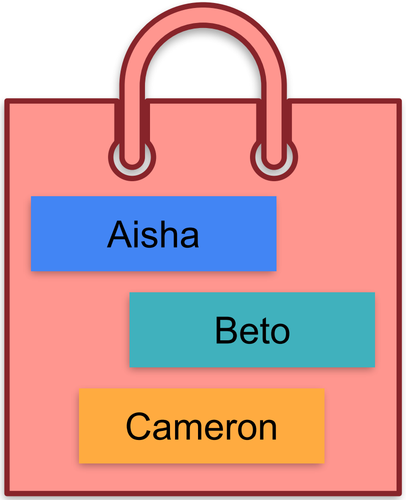
Sum of Expectations



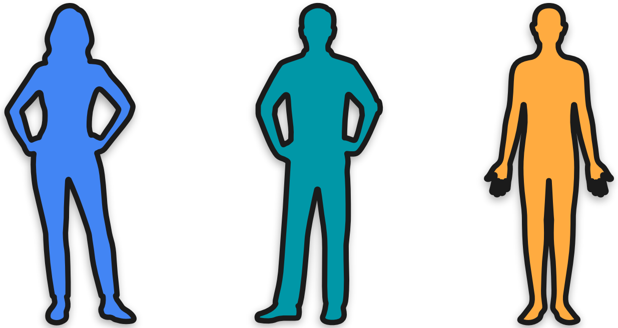


Aisha	Beto	Cameron
Aisha	Cameron	Beto
Beto	Aisha	Cameron
Beto	Cameron	Aisha
Cameron	Aisha	Beto
Cameron	Beto	Aisha

Sum of Expectations

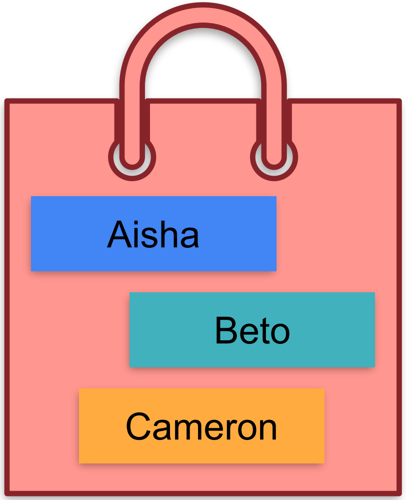


Correct

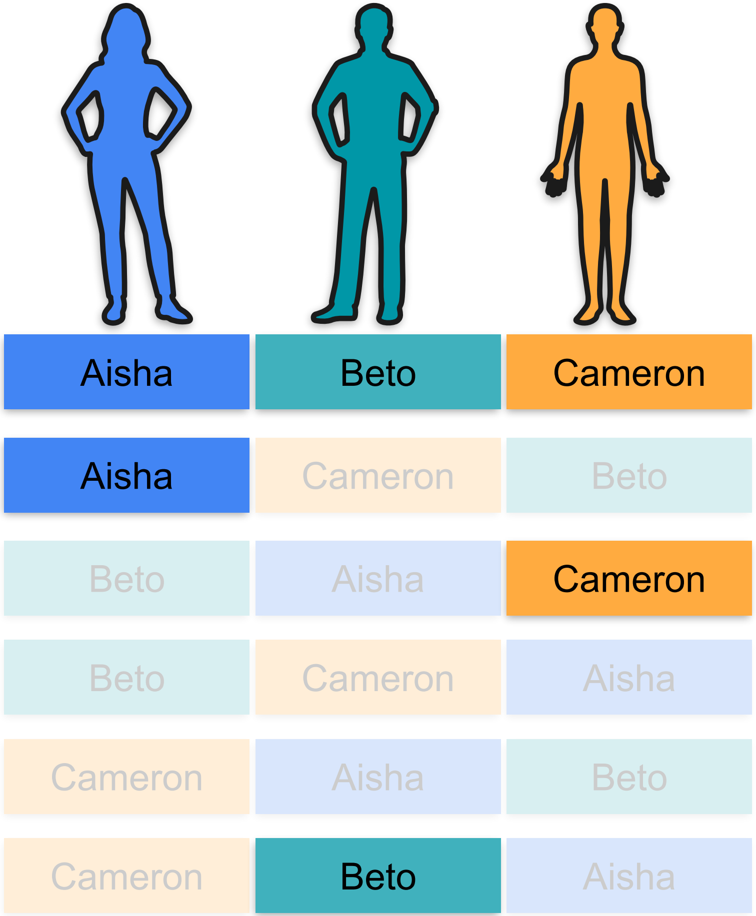


Aisha	Beto	Cameron
Aisha	Cameron	Beto
Beto	Aisha	Cameron
Beto	Cameron	Aisha
Cameron	Aisha	Beto
Cameron	Beto	Aisha

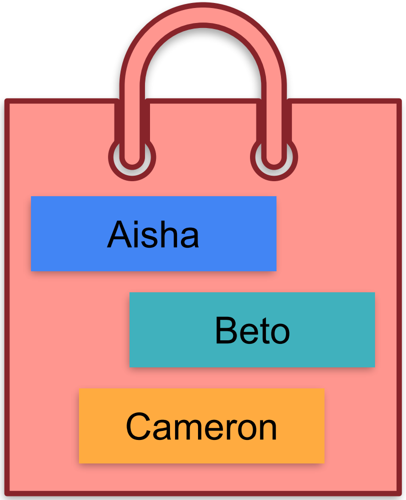
Sum of Expectations

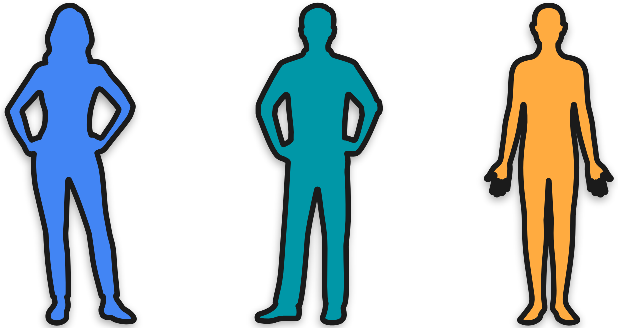


Correct



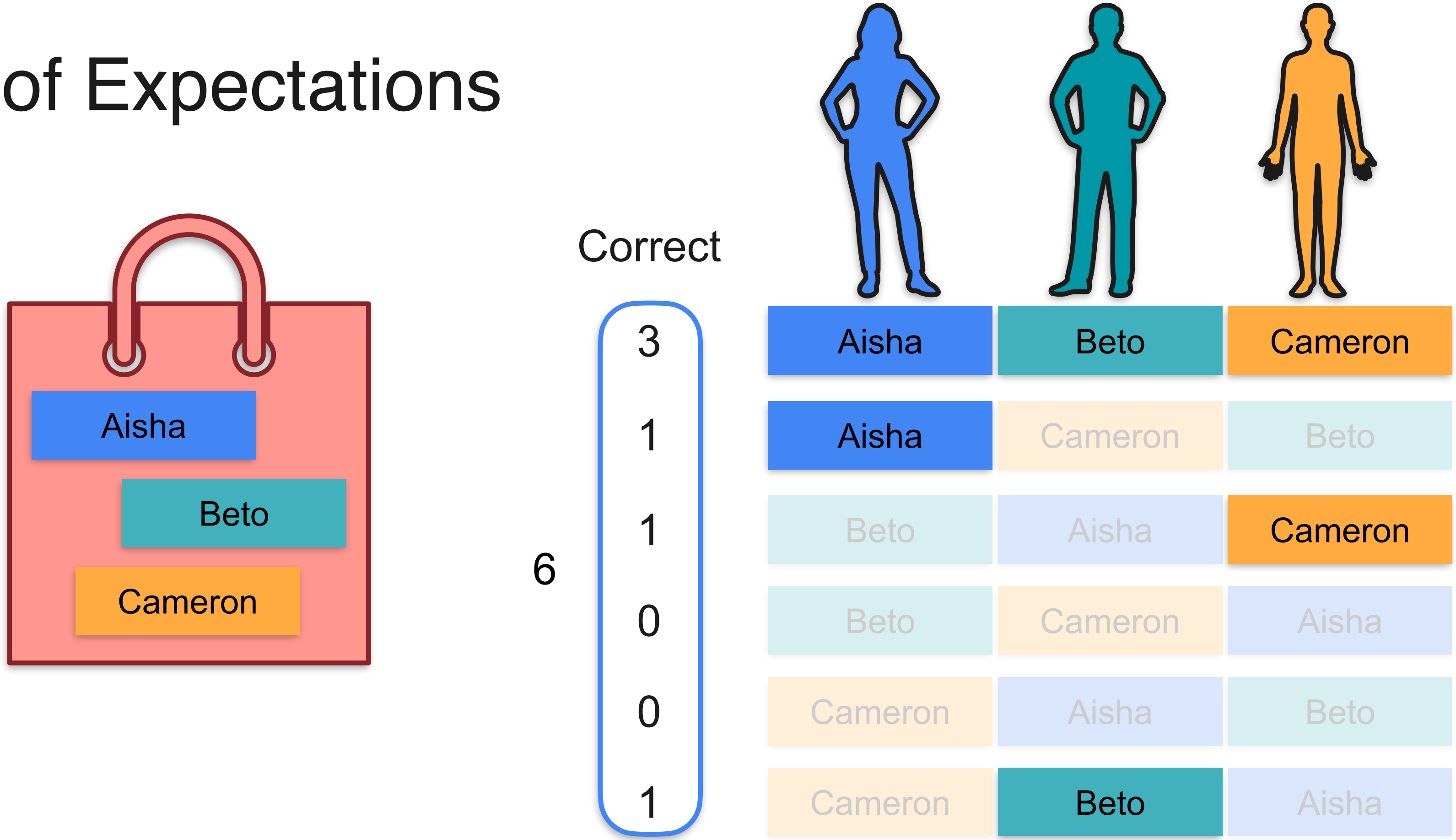
Sum of Expectations





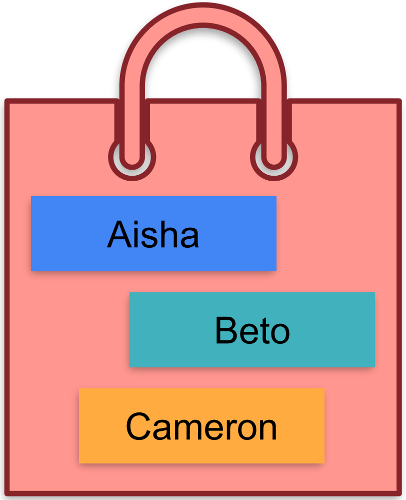
Correct			
3	Aisha	Beto	Cameron
1	Aisha	Cameron	Beto
1	Beto	Aisha	Cameron
0	Beto	Cameron	Aisha
0	Cameron	Aisha	Beto
1	Cameron	Beto	Aisha

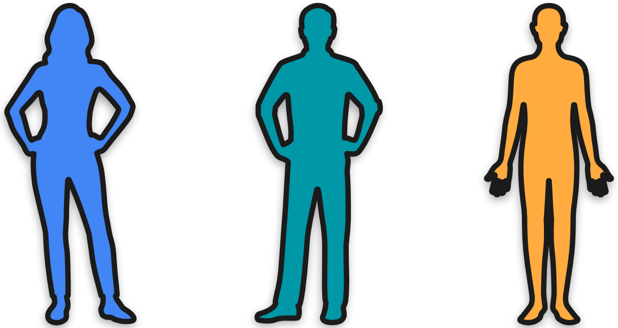
Sum of Expectations



Sum of Expectations

Average
1

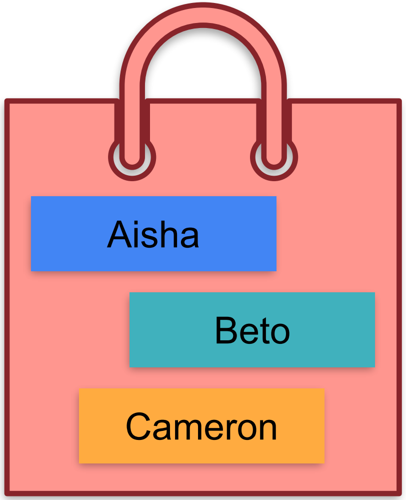


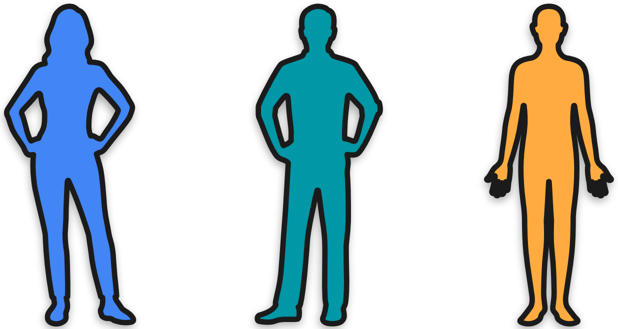


6

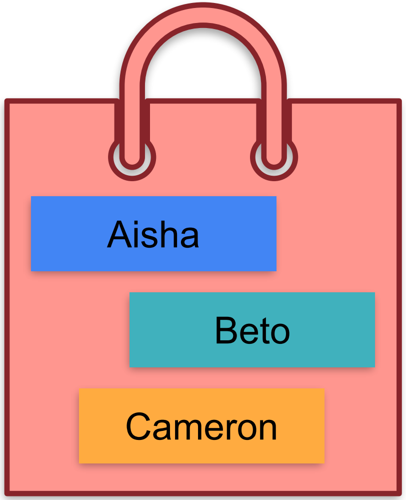
Correct			
3	Aisha	Beto	Cameron
1	Aisha	Cameron	Beto
1	Beto	Aisha	Cameron
0	Beto	Cameron	Aisha
0	Cameron	Aisha	Beto
1	Cameron	Beto	Aisha

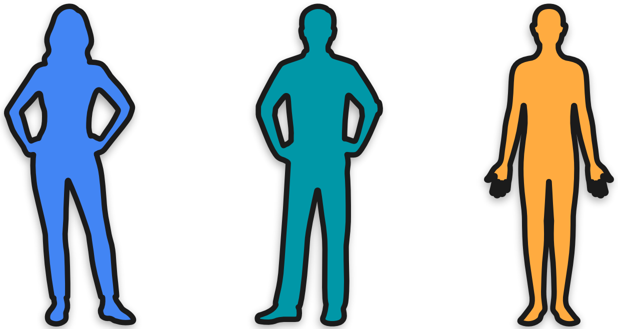
Sum of Expectations





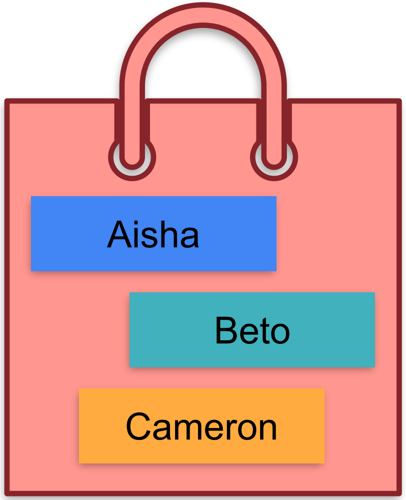
Sum of Expectations

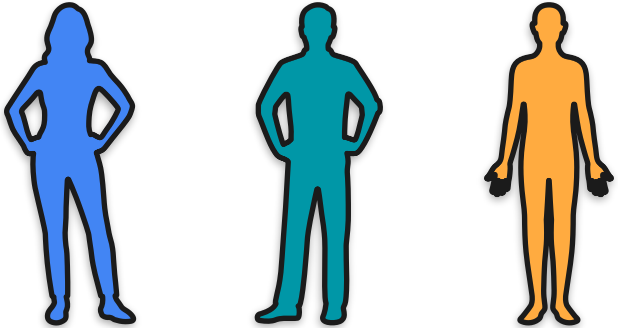




$\mathbb{E}[\text{Matches}]$

Sum of Expectations





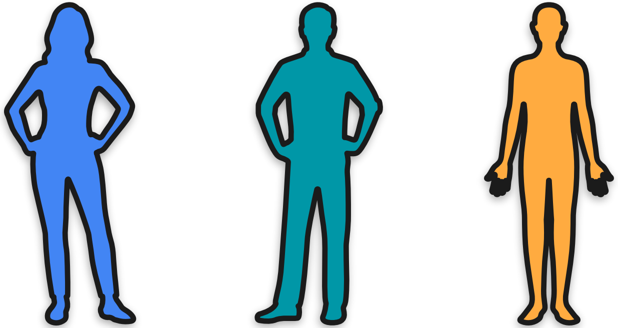
$$\mathbb{E}[\text{Matches}] \qquad = \mathbb{E}[A]$$

Sum of Expectations

Aisha

Beto

Cameron



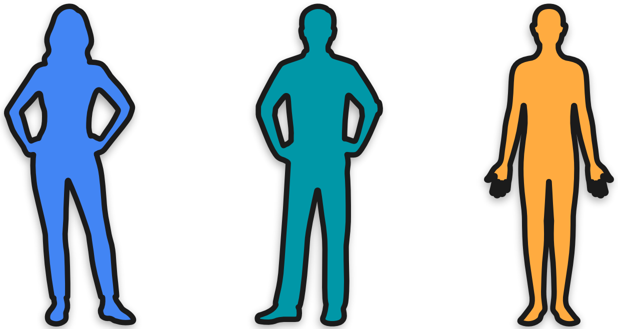
$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A]$$

Sum of Expectations

1/3 Aisha

1/3 Beto

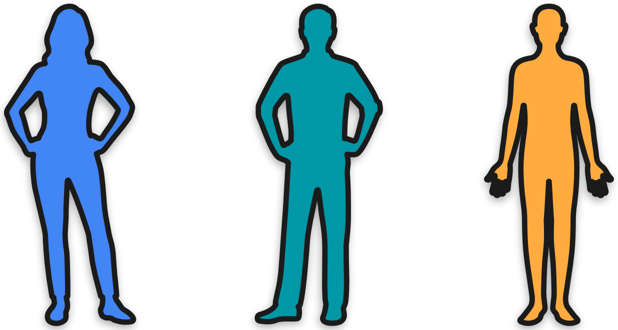
1/3 Cameron



$$\mathbb{E}[\text{Matches}] \qquad = \mathbb{E}[A]$$

Sum of Expectations

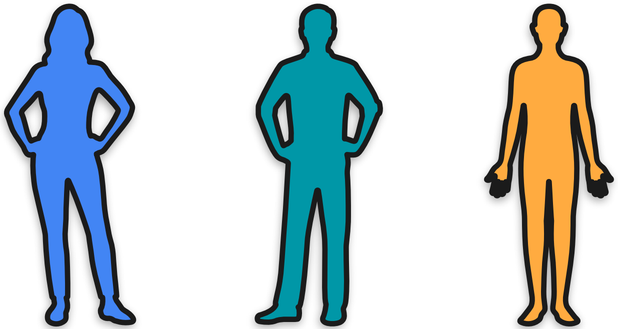
1/3	Aisha
1/3	Beto
1/3	Cameron

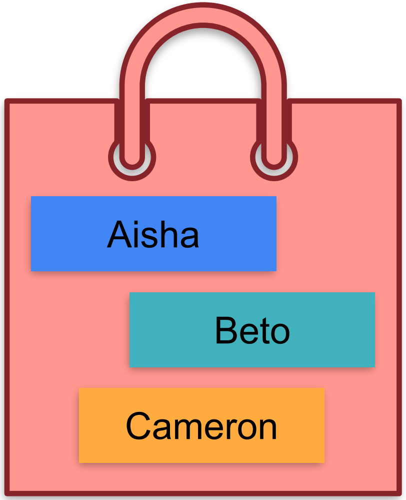


$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A]$$

$$= 1/3$$

Sum of Expectations

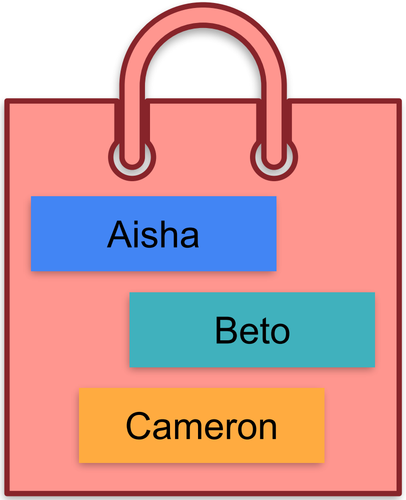


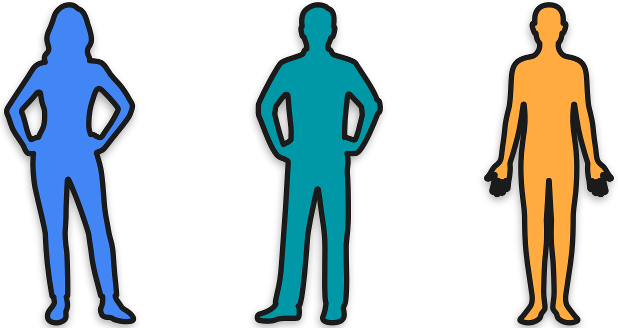


$$\mathbb{E}[\text{Matches}] \quad = \mathbb{E}[A]$$

$$= 1/3$$

Sum of Expectations

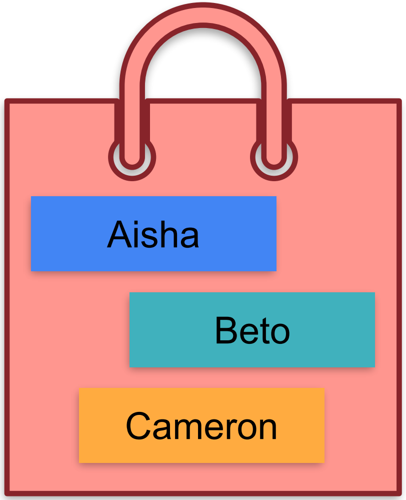


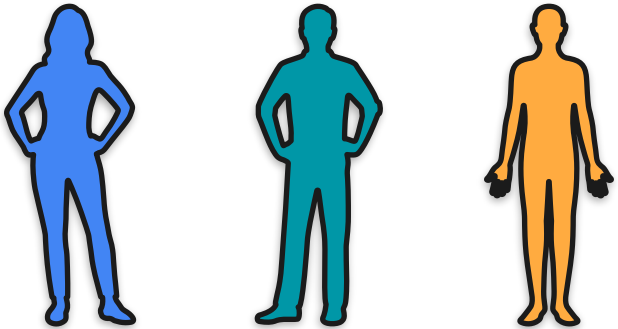


$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A] + \mathbb{E}[B]$$

$$= 1/3 + 1/3$$

Sum of Expectations

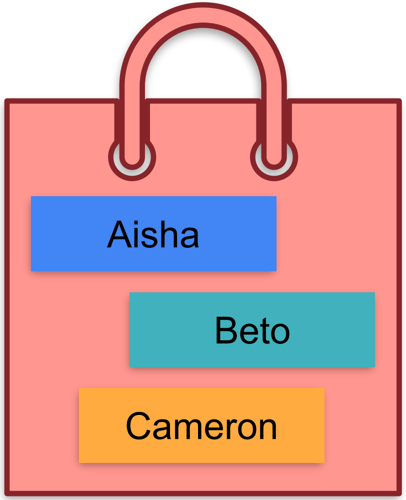


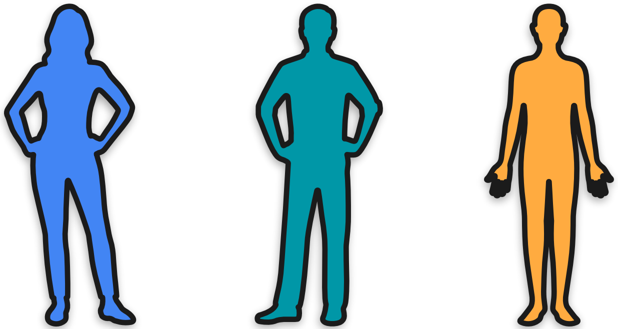


$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A] + \mathbb{E}[B] + \mathbb{E}[C]$$

$$= 1/3 + 1/3 + 1/3$$

Sum of Expectations



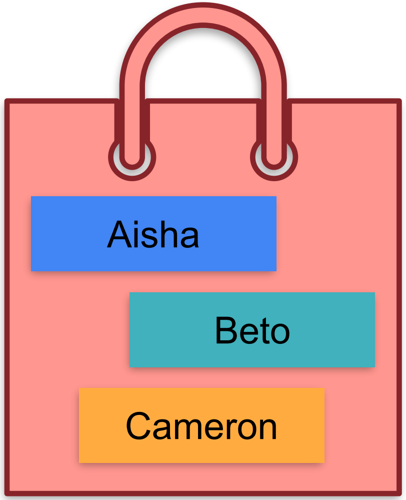


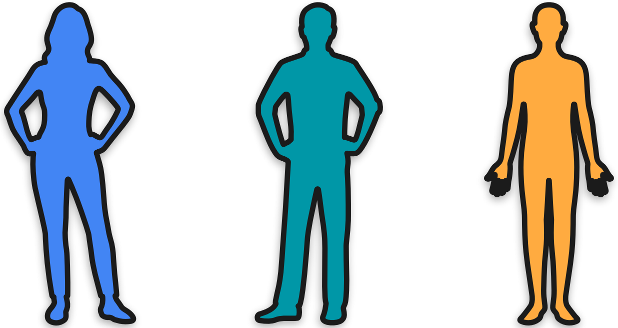
$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A] + \mathbb{E}[B] + \mathbb{E}[C]$$

$$= 1/3 + 1/3 + 1/3$$

$$= 1$$

Sum of Expectations





$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A] + \mathbb{E}[B] + \mathbb{E}[C]$$

$$= 1/3 + 1/3 + 1/3$$

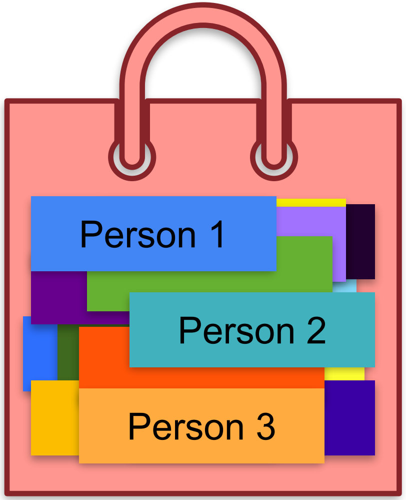
$$= 1$$

Average
1

Sum of Expectations

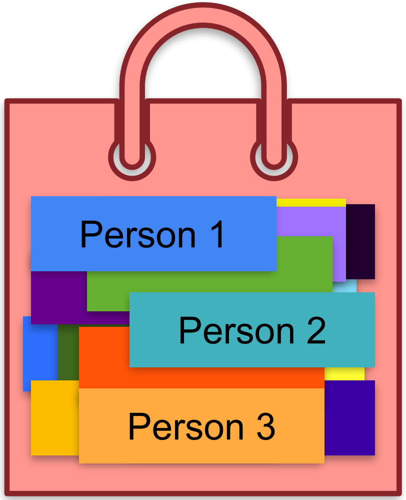
8 billion people

Sum of Expectations



8 billion people

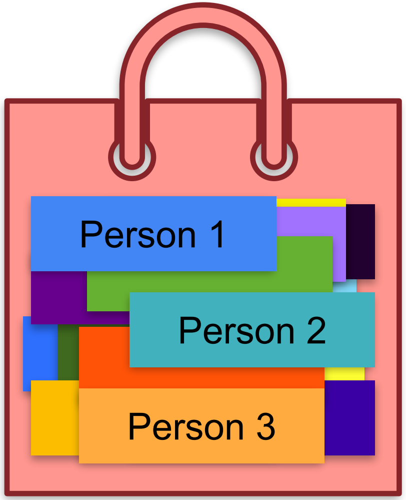
Sum of Expectations



8 billion people

Sum of Expectations

Expected number = ?



8 billion people

Sum of Expectations

Sum of Expectations

$$\mathbb{E}\left[\text{Matches}\right] = \mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right]$$

Sum of Expectations

n people (n = 8 billion)

$$\mathbb{E}\left[\text{Matches}\right] = \mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right]$$

Sum of Expectations

$$\mathbb{E}\,[\text{Matches}] = \overbrace{\mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right]}^{\text{n people (n = 8 billion)}} = \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}$$

Sum of Expectations

$$\mathbb{E}\,[\text{Matches}] = \overbrace{\mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right]}^{\text{n people (n = 8 billion)}}$$

$$= \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}$$

$$= n \cdot \frac{1}{n}$$

Sum of Expectations

n people (n = 8 billion)

$$\mathbb{E}\,[\text{Matches}] = \mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right] = \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}$$

$$= n \cdot \frac{1}{n} = 1$$

Sum of Expectations

n people (n = 8 billion)

$$\mathbb{E}\,[\text{Matches}] = \mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right] = \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}$$

$$= n \cdot \frac{1}{n} = 1$$

In general:

$$\mathbb{E}\left[X_1 + X_2 + \ldots + X_n\right] = \mathbb{E}\left[X_1\right] + \mathbb{E}\left[X_2\right] + \ldots + \mathbb{E}\left[X_n\right]$$

Describing Distributions

Variance

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 1 dollar

You lose 1 dollar

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 1 dollar

You lose 1 dollar

Game cost:

$0

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 100 dollars

You lose 100 dollars

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 100 dollars

You lose 100 dollars

Game cost:

$0

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

How do you tell these two games apart?

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 100 dollars

You lose 100 dollars

Variance Motivation: Fair Price To Play the Game

How do you tell these two games apart?

Game 1

H You win 1 dolla

T You lose 1 do

Ga

win 100 dollars

T You lose 100 dollars

Variance!

Variance Motivation: Measuring Spread

Variance Motivation: Measuring Spread

Variance Motivation: Measuring Spread

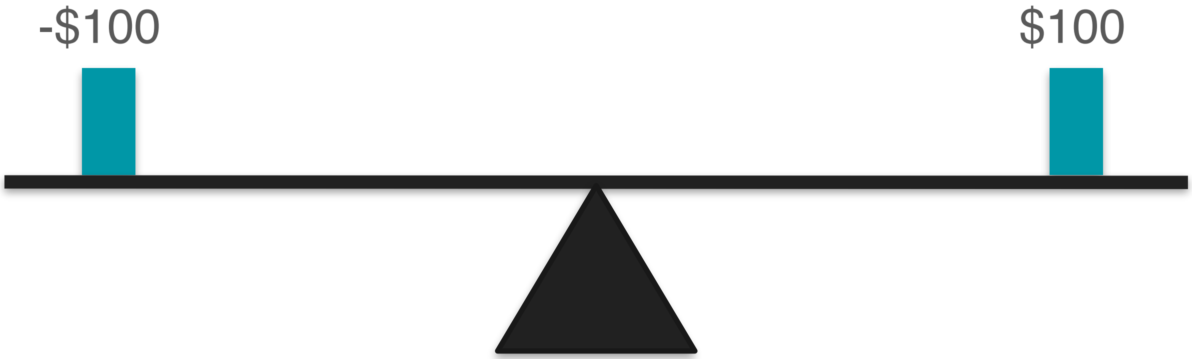
$$\mathbb{E}[X_1] = 0$$

Variance Motivation: Measuring Spread

X_1 = expected amount of money gained in game 1

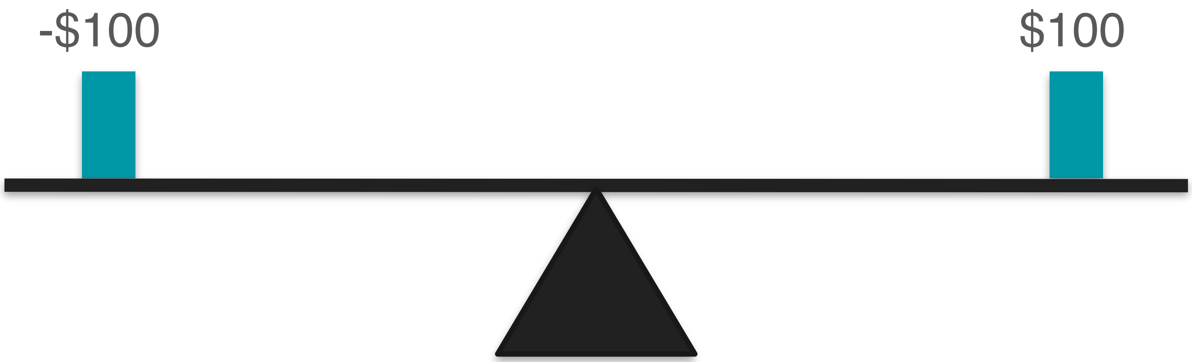
$$\mathbb{E}[X_1] = 0$$

Variance Motivation: Measuring Spread



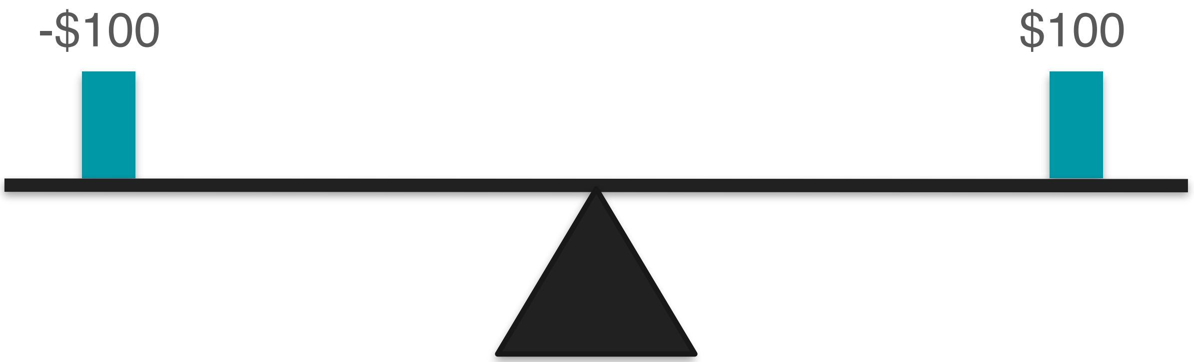
Variance Motivation: Measuring Spread

X_2 = expected amount of money gained in game 2



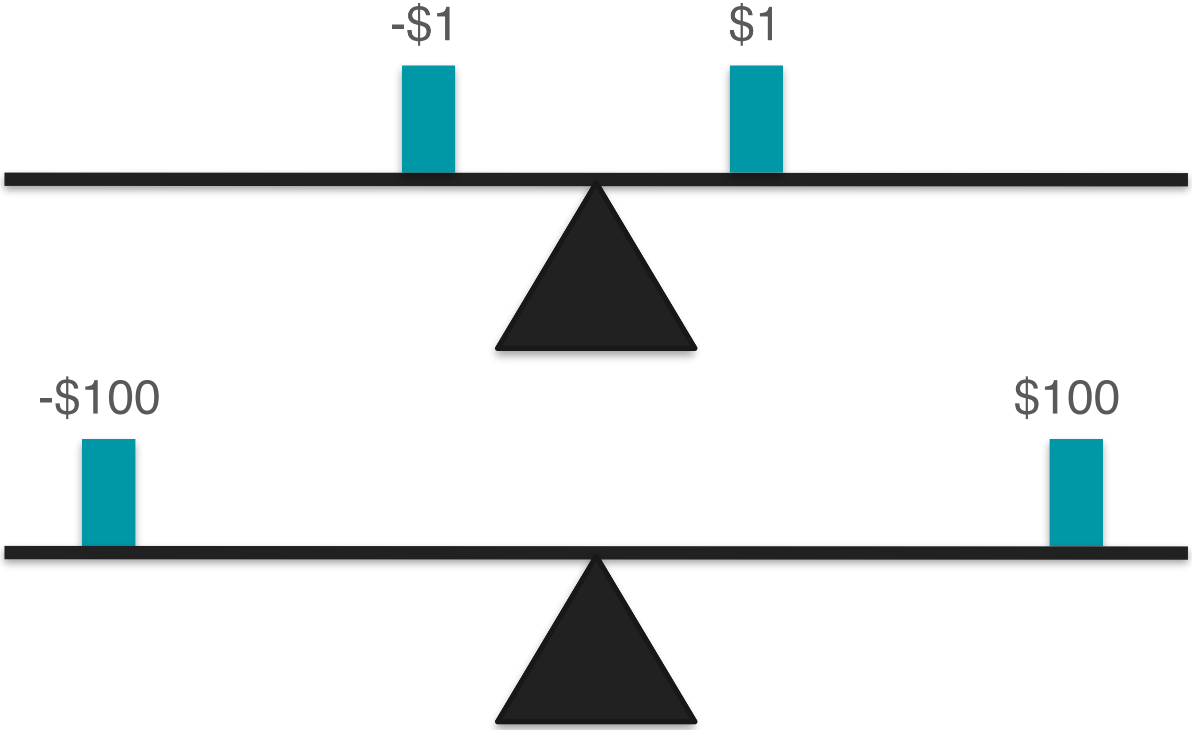
Variance Motivation: Measuring Spread

X_2 = expected amount of money gained in game 2



$\mathbb{E}[X_2] = 0$

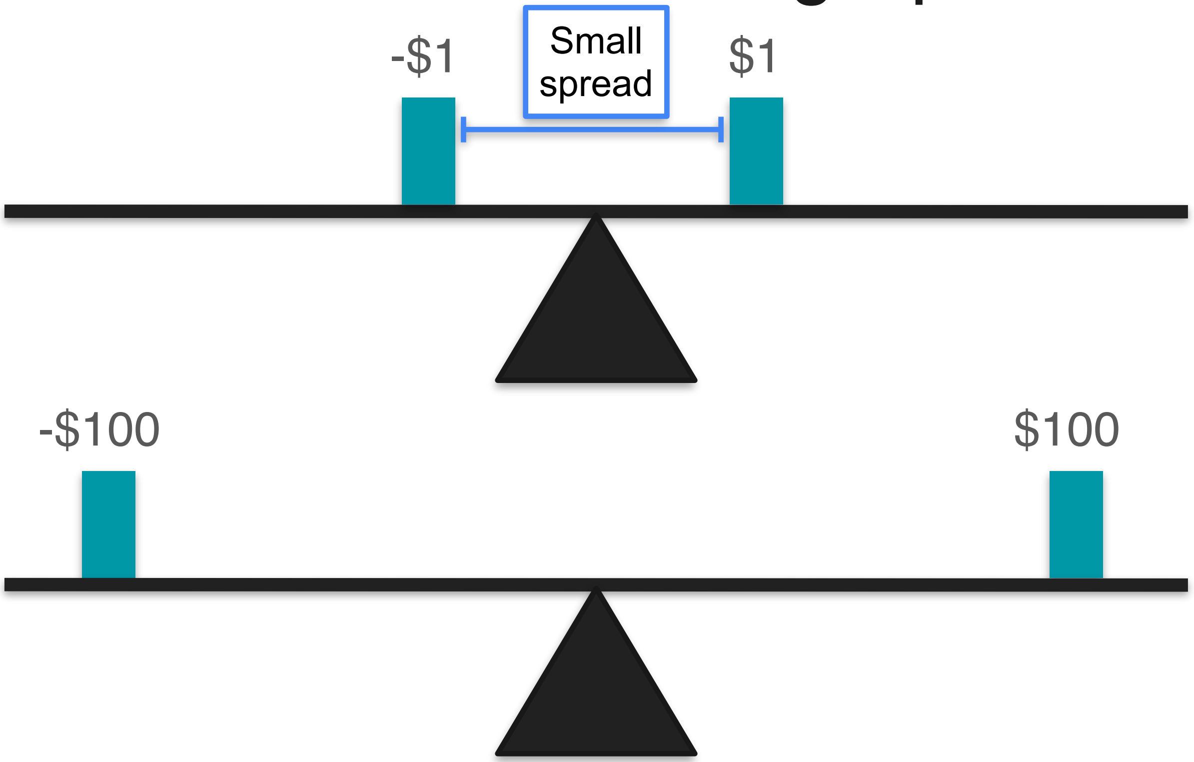
Variance Motivation: Measuring Spread



$\mathbb{E}[X_1] = 0$

$\mathbb{E}[X_2] = 0$

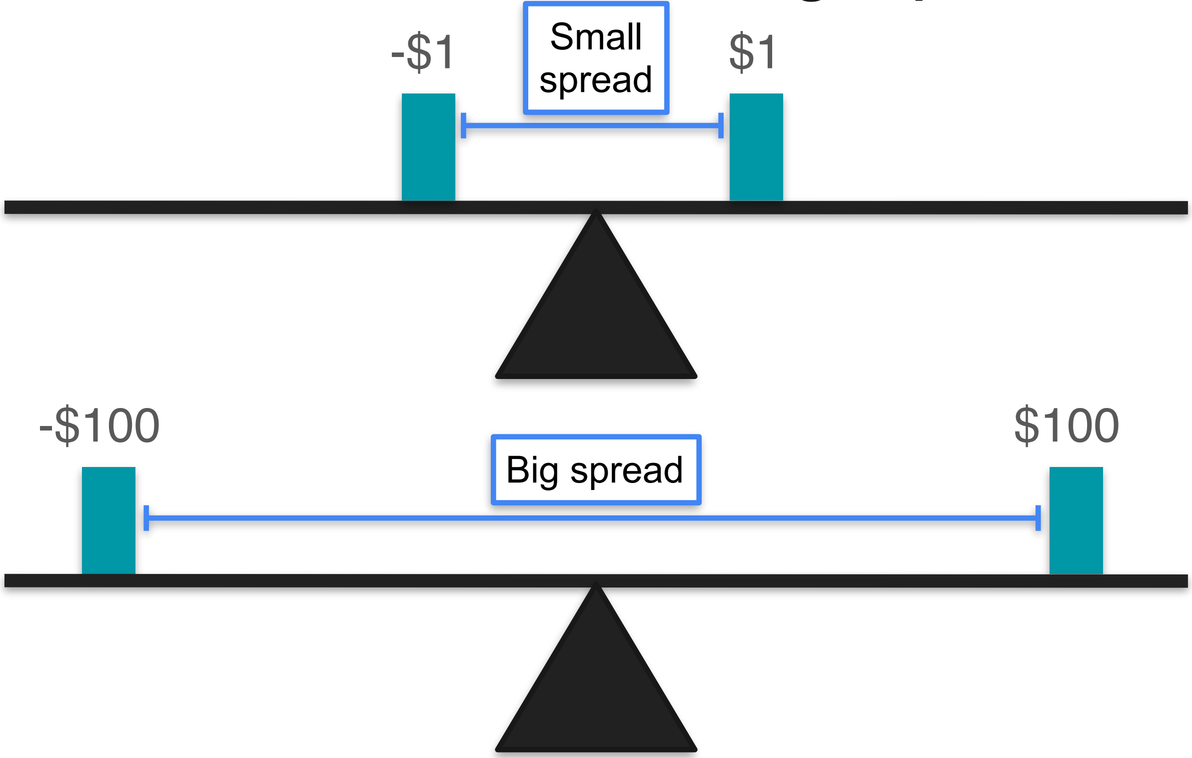
Variance Motivation: Measuring Spread



$\mathbb{E}[X_1] = 0$

$\mathbb{E}[X_2] = 0$

Variance Motivation: Measuring Spread



$$\mathbb{E}[X_1] = 0$$

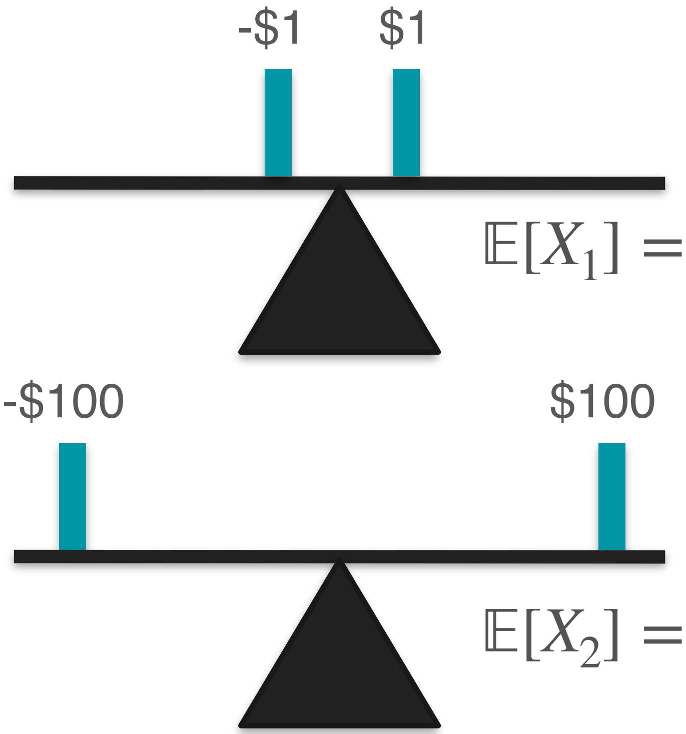
$$\mathbb{E}[X_2] = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

Variance Motivation: Measuring Spread



$\mathbb{E}[X_1] = 0$

$\mathbb{E}[X_2] = 0$

$$\mathbb{E}[X_1] = \frac{(1) \quad + (-1)}{2} \qquad = 0$$

Variance Motivation: Measuring Spread

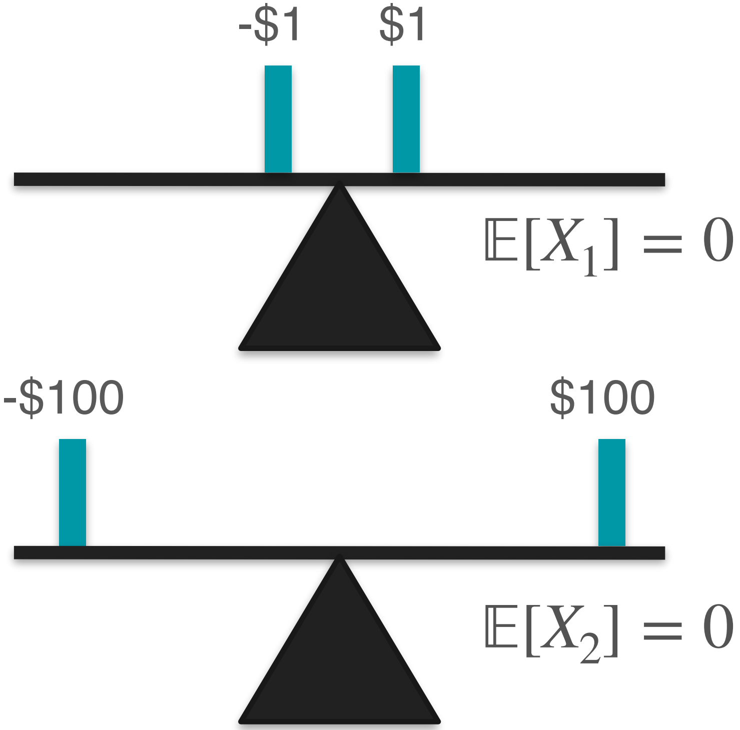
$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_1] = \frac{(1) + (-1)}{2} = 0$$

$$\mathbb{E}[X_2] = \frac{(100) + (-100)}{2} = 0$$

Variance Motivation: Measuring Spread



$$\mathbb{E}[X_1] = \frac{(1) \quad + (-1)}{2} \quad = 0$$

Turn into positive?

$$\mathbb{E}[X_2] = \frac{(100) \quad + (-100)}{2} = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1] = \frac{(1)^2 + (-1)^2}{2} = 0$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2] = \frac{(100) \ + (-100)}{2} = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 0$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = \frac{(100)^2 + (-100)^2}{2} = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 0$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = \frac{(100)^2 + (-100)^2}{2} = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 1$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = \frac{(100)^2 + (-100)^2}{2} = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 1$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = \frac{(100)^2 + (-100)^2}{2} = 10{,}000$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

Key for telling game 1 and game 2 apart!

$$\boxed{\mathbb{E}[X_1^2]} = \frac{(1)^2 + (-1)^2}{2} = 1$$

$$\boxed{\mathbb{E}[X_2^2]} = \frac{(100)^2 + (-100)^2}{2} = 10{,}000$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

Key for telling game 1 and game 2 apart!

$$\boxed{\mathbb{E}[X_1^2]} = \frac{(1)^2 + (-1)^2}{2} = 1$$

$$\boxed{\mathbb{E}[X_2^2]} = \frac{(100)^2 + (-100)^2}{2} = 10{,}000$$

Measure of spread

Variance

$$\mathbb{E}[X^2]$$

Variance

$$\mathbb{E}[X^2]$$

Almost…

Variance Motivation: Centering With Mean

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 6 dollars

You win 4 dollars

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 6 dollars

You win 4 dollars

How risky are these two games in comparison?

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 6 dollars

You win 4 dollars

How risky are these two games in comparison?

Hint: Think of the spread

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 6 dollars

You win 4 dollars

They are equally risky

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

$$\mathbb{E}[X_1^2] = \frac{(-1)^2 + (1)^2}{2} = 1$$

Game 2

You win 6 dollars

You win 4 dollars

$$\mathbb{E}[X_2^2] = \frac{(4)^2 + (6)^2}{2} = 26$$

Variance Motivation: Centering With Mean

Game 1

H You win 1 dollar

T You lose 1 dollar

Game 2

H You win 6 dollars

T You win 4 dollars

$$\mathbb{E}[X_1^2] = \frac{(-1)^2 + (1)^2}{2} = 1$$

$\uparrow$

Same risk?

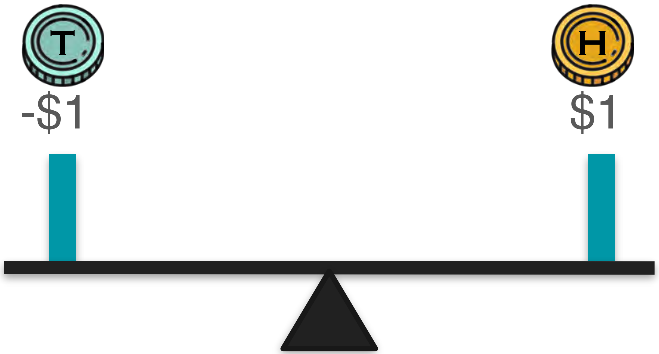
$\downarrow$

$$\mathbb{E}[X_2^2] = \frac{(4)^2 + (6)^2}{2} = 26$$

Variance Motivation: Centering With Mean

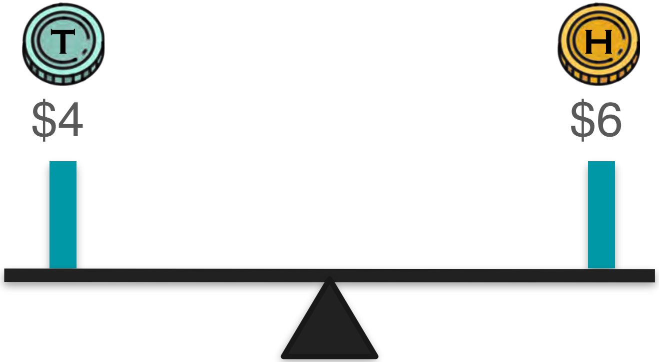
$$\mathbb{E}[X_1^2] = \frac{(-1)^2 + (1)^2}{2} = 1$$

$$\mathbb{E}[X_2^2] = \frac{(4)^2 + (6)^2}{2} = 26$$



$$\mathbb{E}[X_1] = 0$$

Game 1

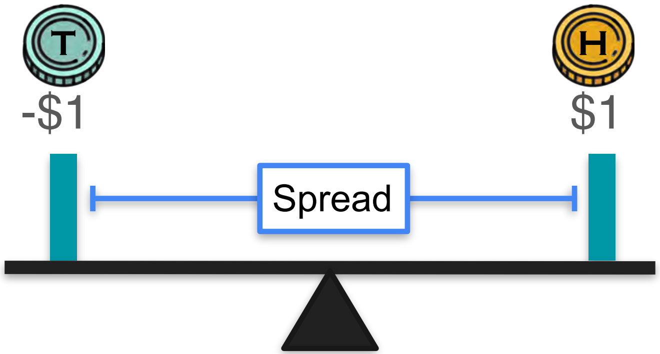


$$\mathbb{E}[X_2] = 5$$

Game 2

Variance Motivation: Centering With Mean

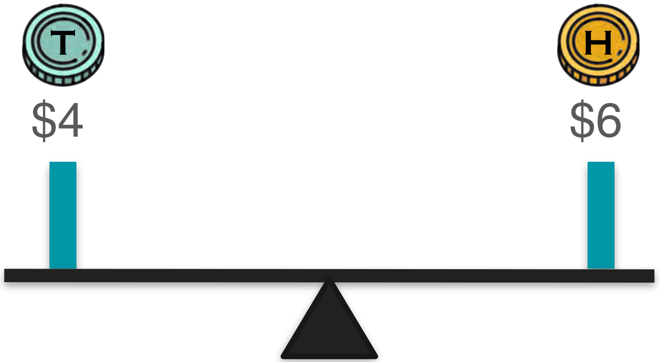
$$\mathbb{E}[X_1^2] = \frac{(-1)^2 + (1)^2}{2} = 1$$



$$\mathbb{E}[X_1] = 0$$

Game 1

$$\mathbb{E}[X_2^2] = \frac{(4)^2 + (6)^2}{2} = 26$$



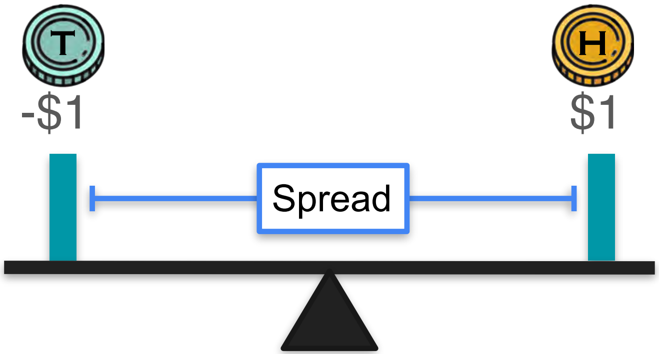
$$\mathbb{E}[X_2] = 5$$

Game 2

Variance Motivation: Centering With Mean

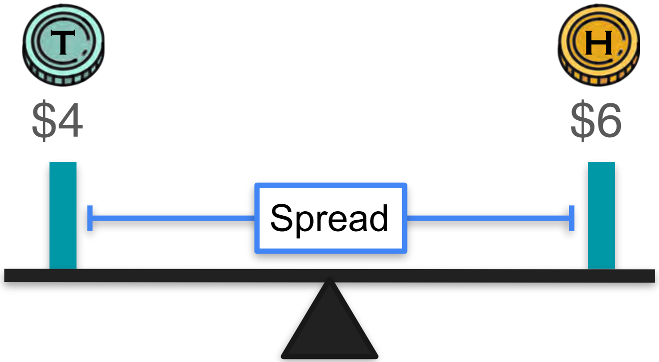
$$\mathbb{E}[X_1^2] = \frac{(-1)^2 + (1)^2}{2} = 1$$

$$\mathbb{E}[X_2^2] = \frac{(4)^2 + (6)^2}{2} = 26$$



$$\mathbb{E}[X_1] = 0$$

Game 1

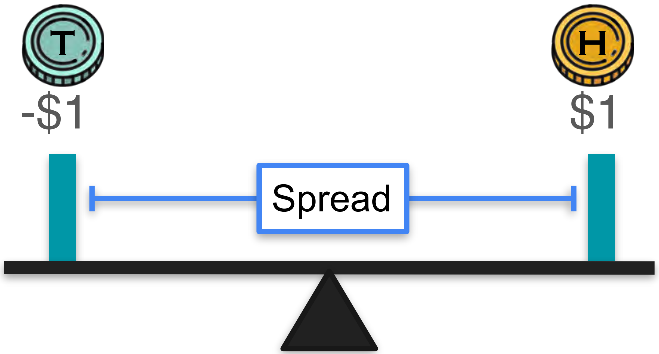


$$\mathbb{E}[X_2] = 5$$

Game 2

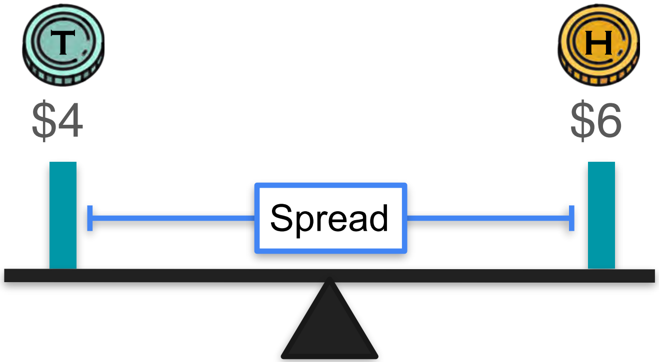
Variance Motivation: Centering With Mean

$$\mathbb{E}[X_1^2] = \frac{(-1)^2 + (1)^2}{2} = 1 \longleftarrow \text{Same spread?} \longrightarrow \mathbb{E}[X_2^2] = \frac{(4)^2 + (6)^2}{2} = 26$$



$\mathbb{E}[X_1] = 0$

Game 1



$\mathbb{E}[X_2] = 5$

Game 2

Variance: Spread and Shift

$\mathbb{E}[X_1^2] = 1$

$\mathbb{E}[X_1] = 0$

Game 1

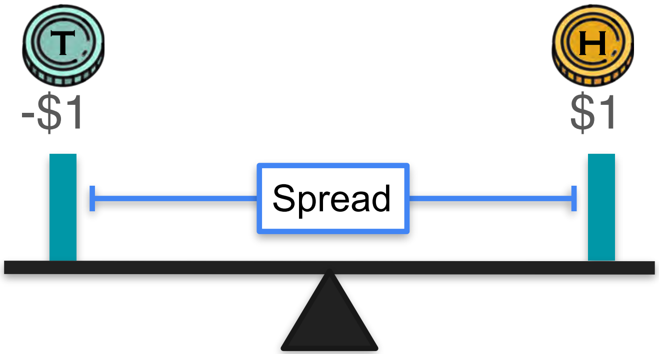
$\mathbb{E}[X_2] = 5$

Game 2

Variance: Spread and Shift

$$\mathbb{E}[X_1^2] = 1$$

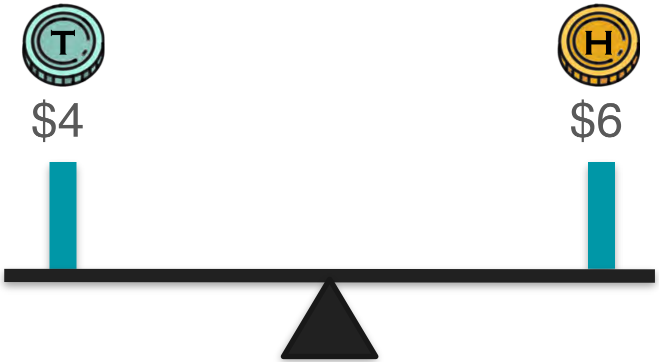
1 1



$$\mathbb{E}[X_1] = 0$$

Game 1

1 1



$$\mathbb{E}[X_2] = 5$$

Game 2

Variance: Spread and Shift

$$\mathbb{E}[X_1^2] = 1$$

$\mathbb{E}[X_1] = 0$

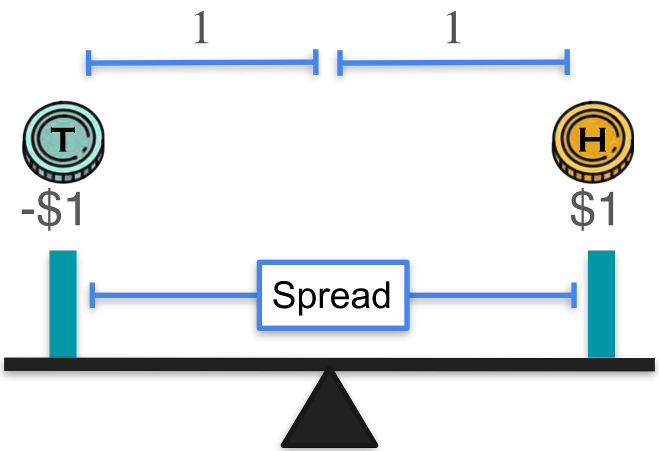
Game 1

$\mathbb{E}[X_2] = 5$

Game 2

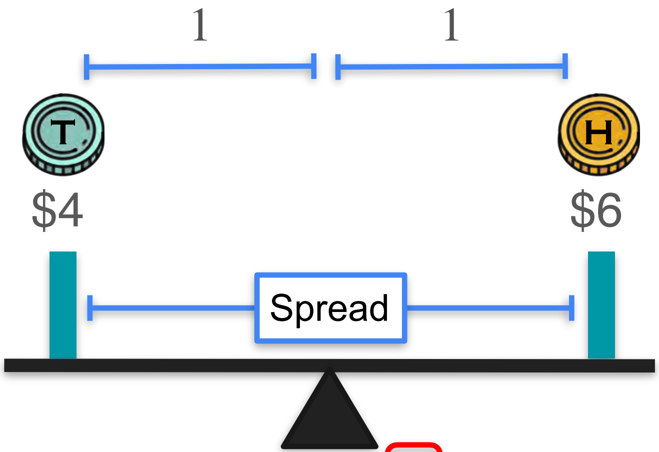
Variance: Spread and Shift

$$\mathbb{E}[X_1^2] = 1$$



$$\mathbb{E}[X_1] = 0$$

Game 1

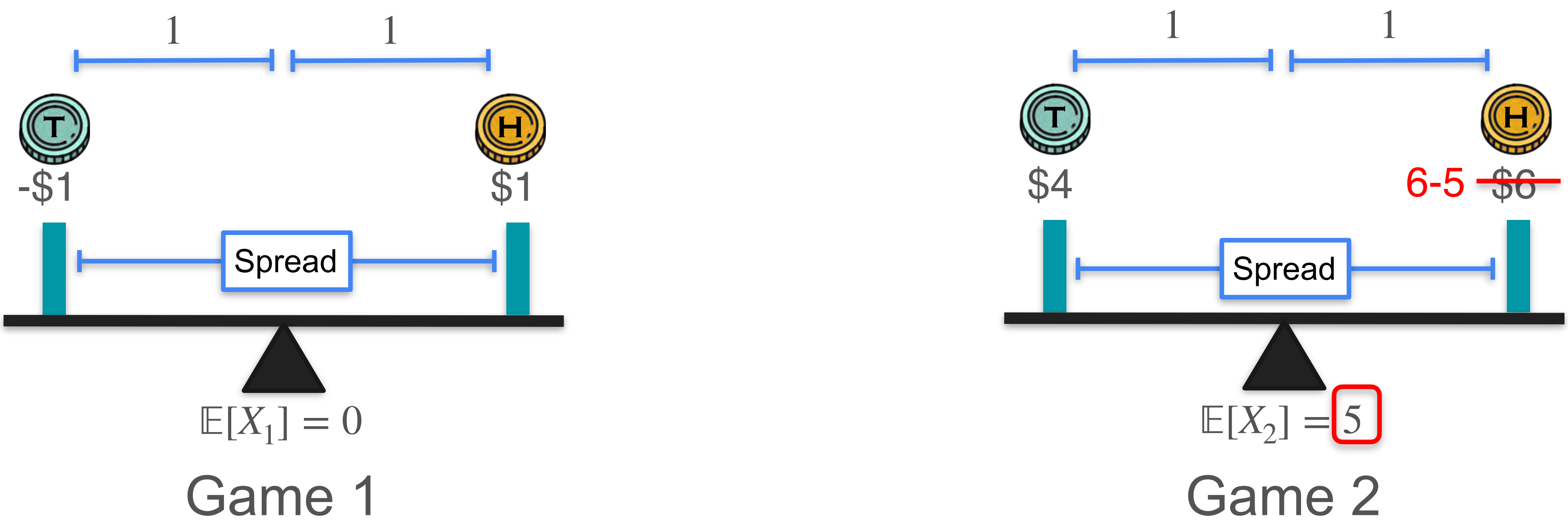


$$\mathbb{E}[X_2] = \boxed{5}$$

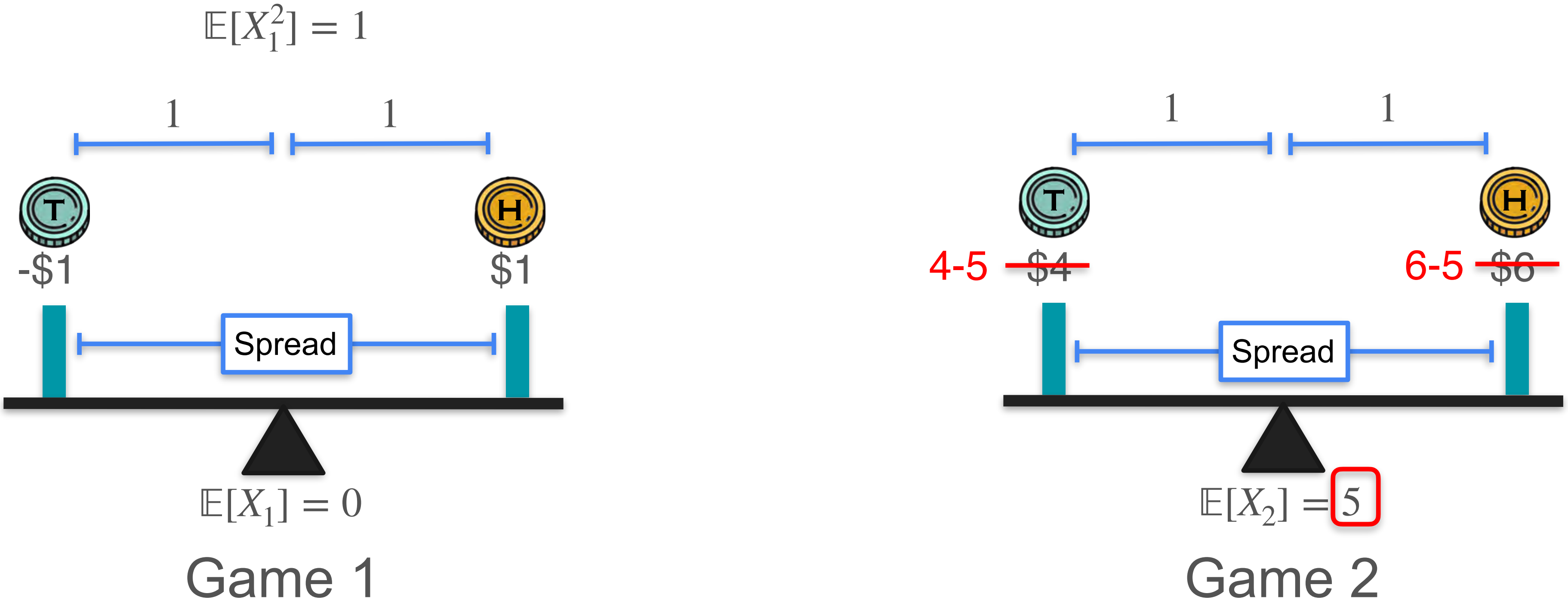
Game 2

Variance: Spread and Shift

$$\mathbb{E}[X_1^2] = 1$$

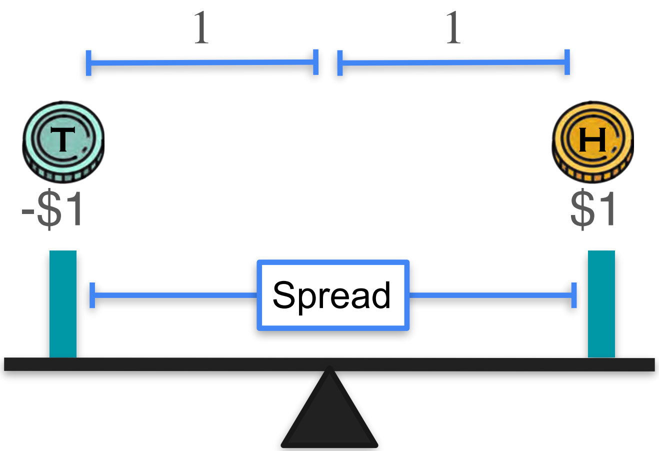


Variance: Spread and Shift



Variance: Spread and Shift

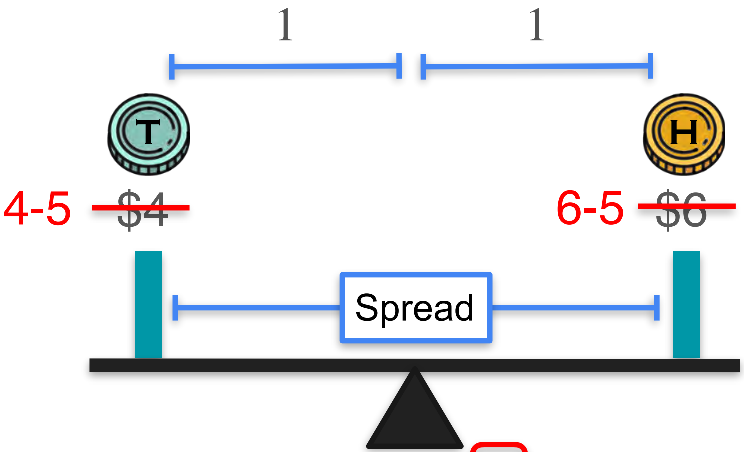
$\mathbb{E}[X_1^2] = 1$



$\mathbb{E}[X_1] = 0$

Game 1

$\mathbb{E}[(X_2 - 5)^2]$



$\mathbb{E}[X_2] = \boxed{5}$

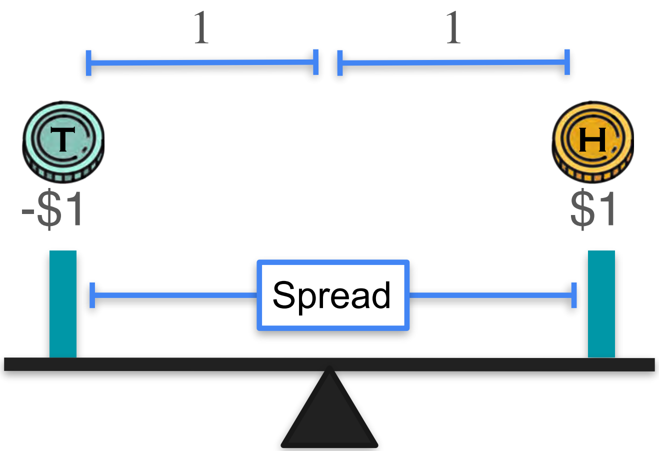
Game 2

Variance: Spread and Shift

$\mathbb{E}[X_1^2] = 1$

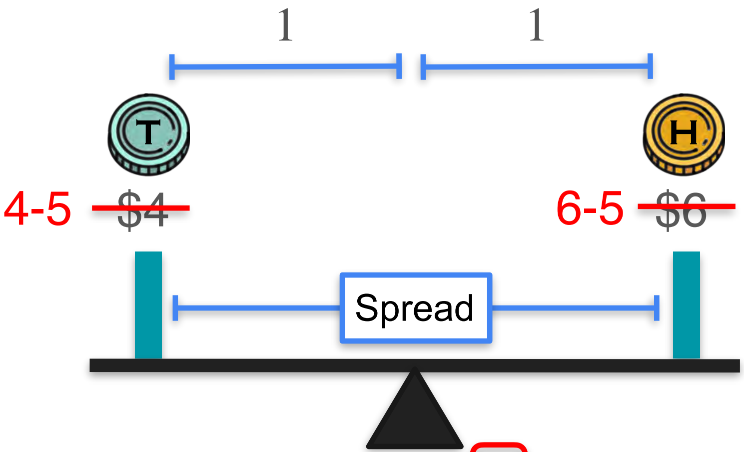
Mean: μ

$\mathbb{E}[(X_2 - 5)^2]$



$\mathbb{E}[X_1] = 0$

Game 1



$\mathbb{E}[X_2] = \boxed{5}$

Game 2

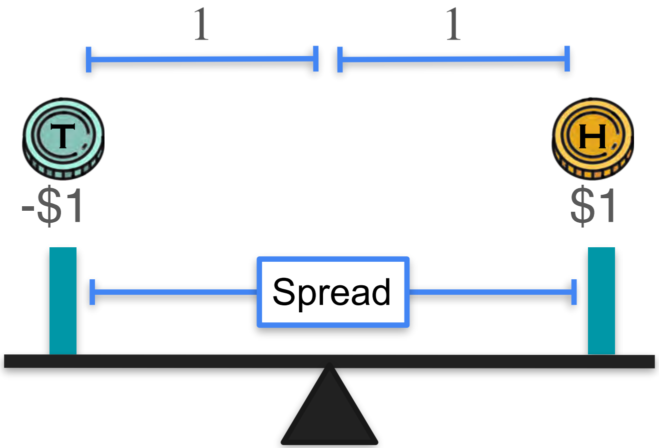
Variance: Spread and Shift

$\mathbb{E}[X_1^2] = 1$

Mean: μ

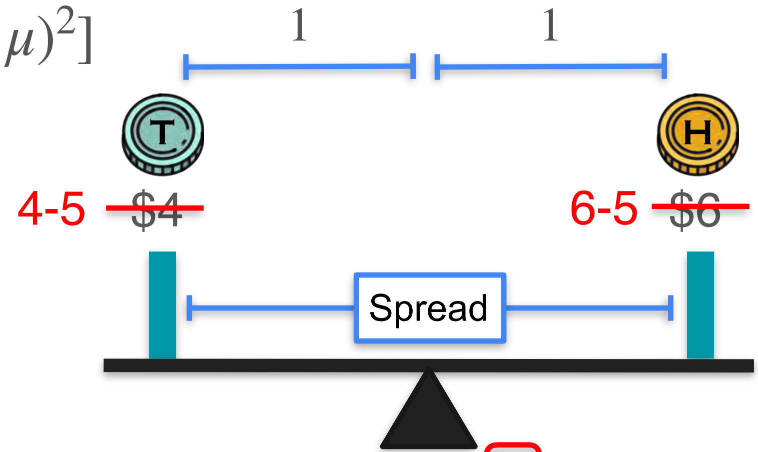
Variance: $\mathbb{E}[(X-\mu)^2]$

$\mathbb{E}[(X_2 - 5)^2]$



$\mathbb{E}[X_1] = 0$

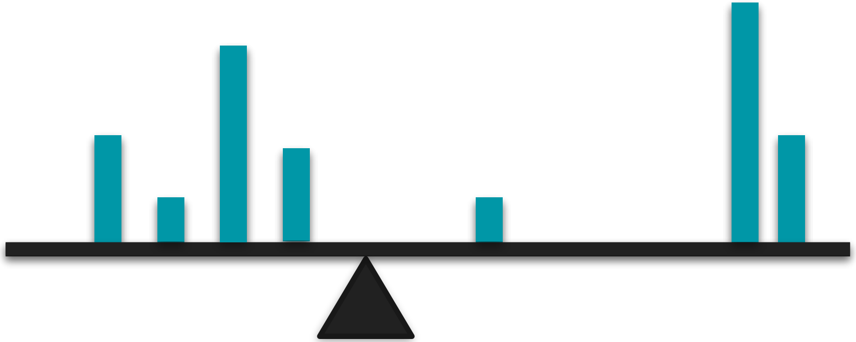
Game 1



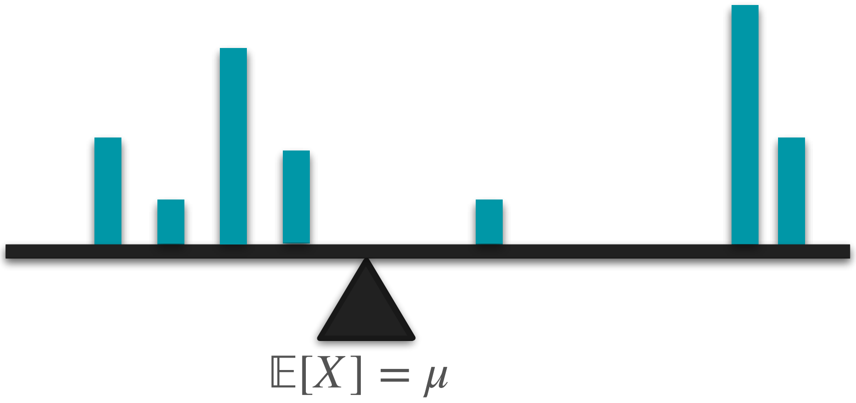
$\mathbb{E}[X_2] = \boxed{5}$

Game 2

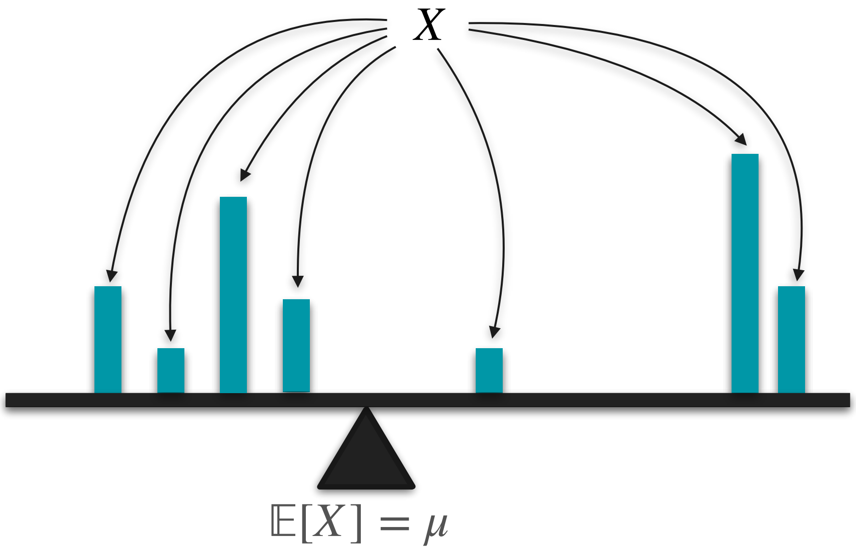
Variance Formula



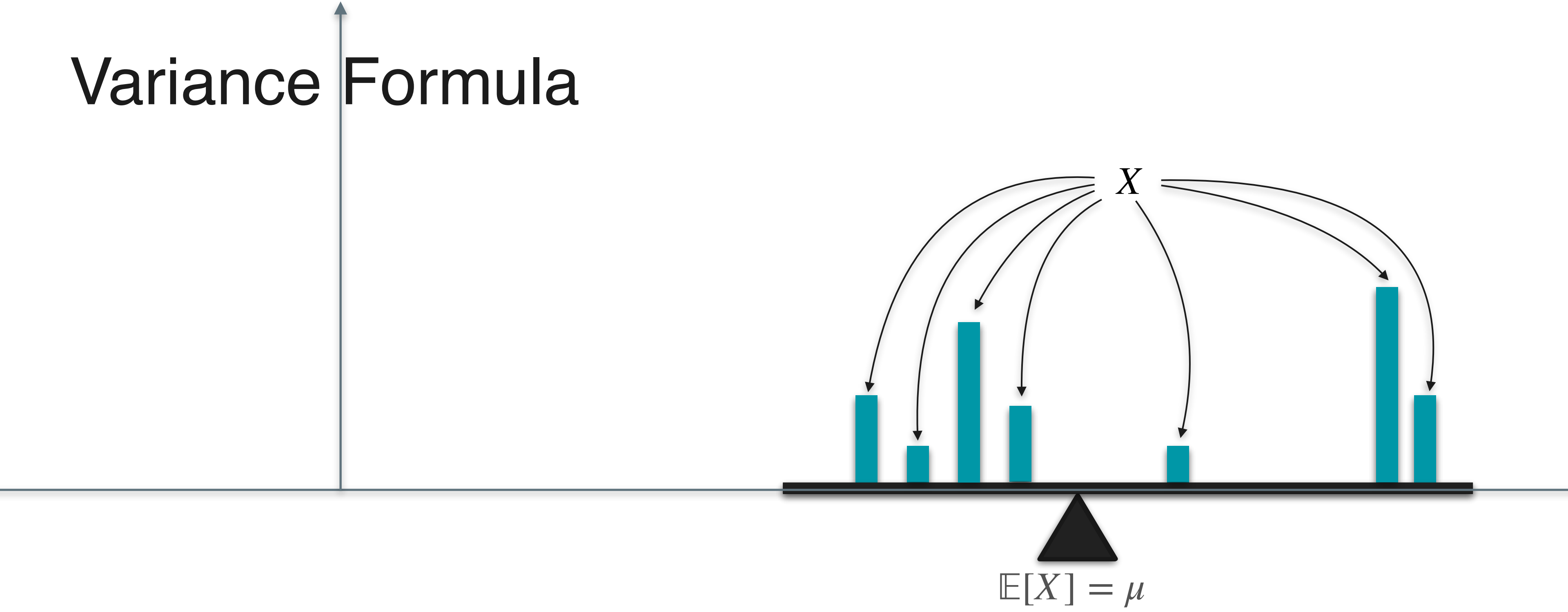
Variance Formula



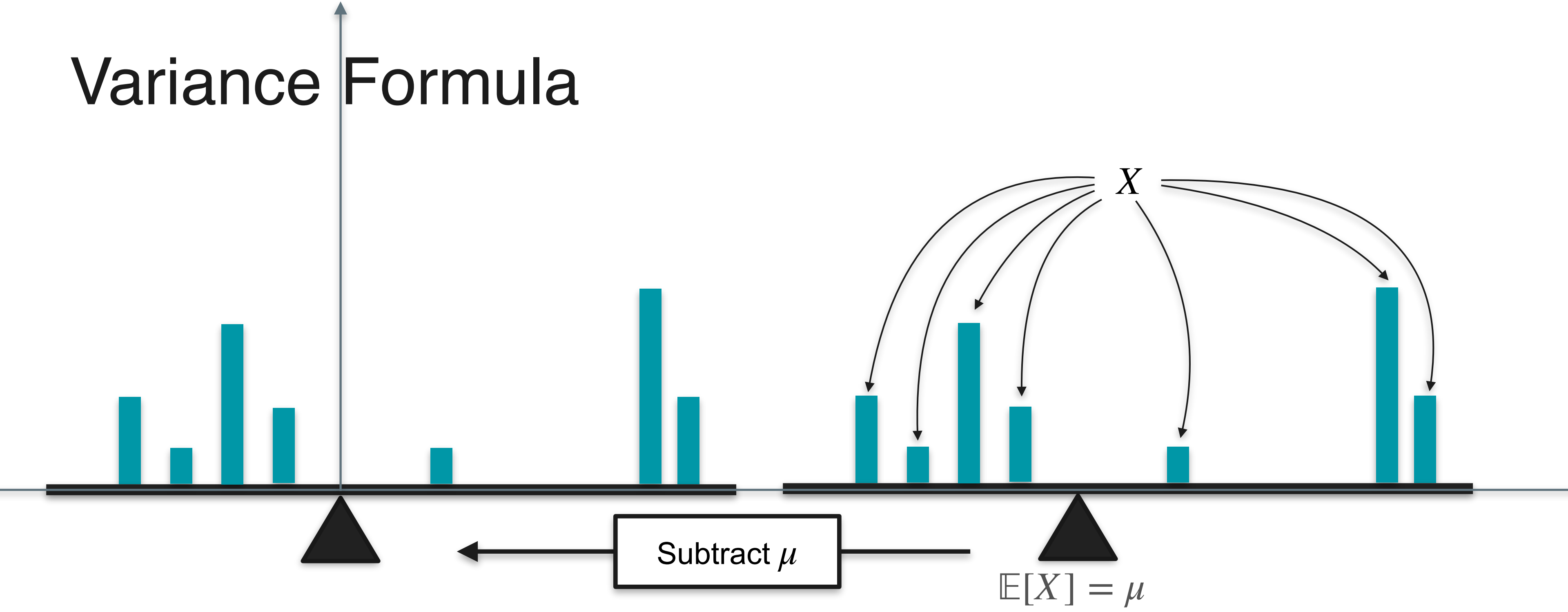
Variance Formula



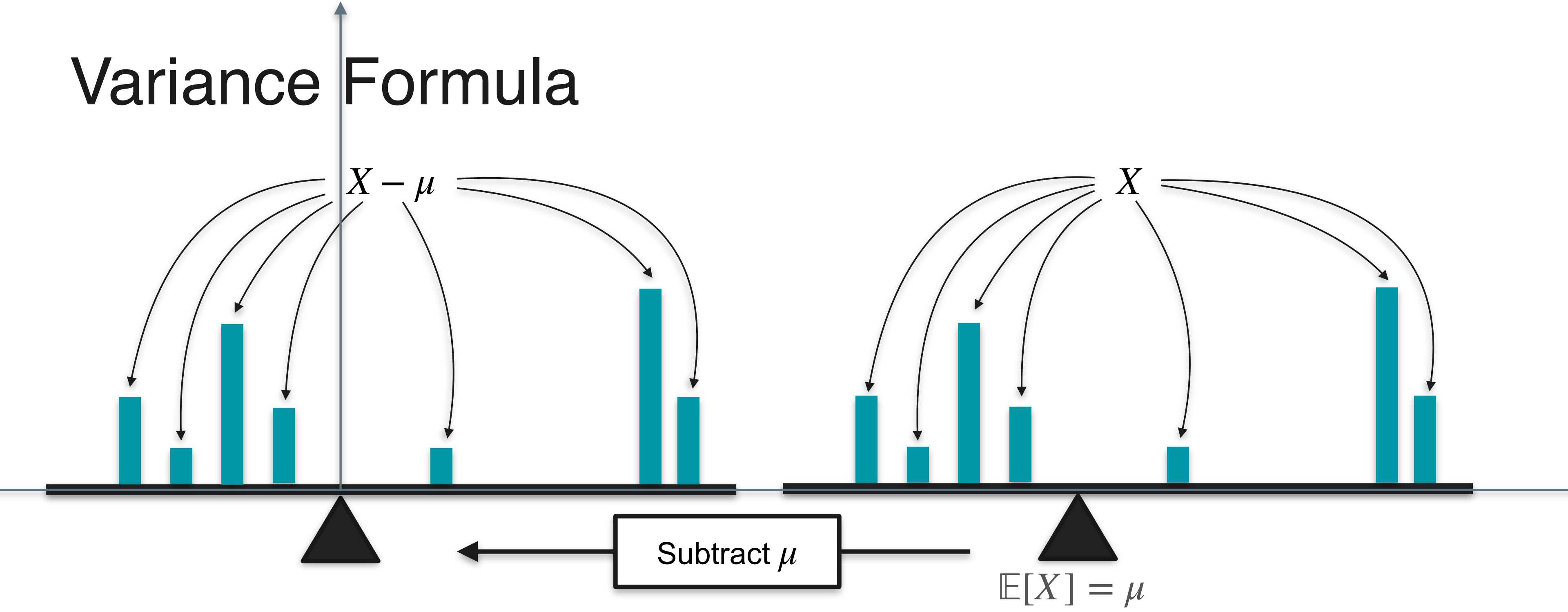
Variance Formula



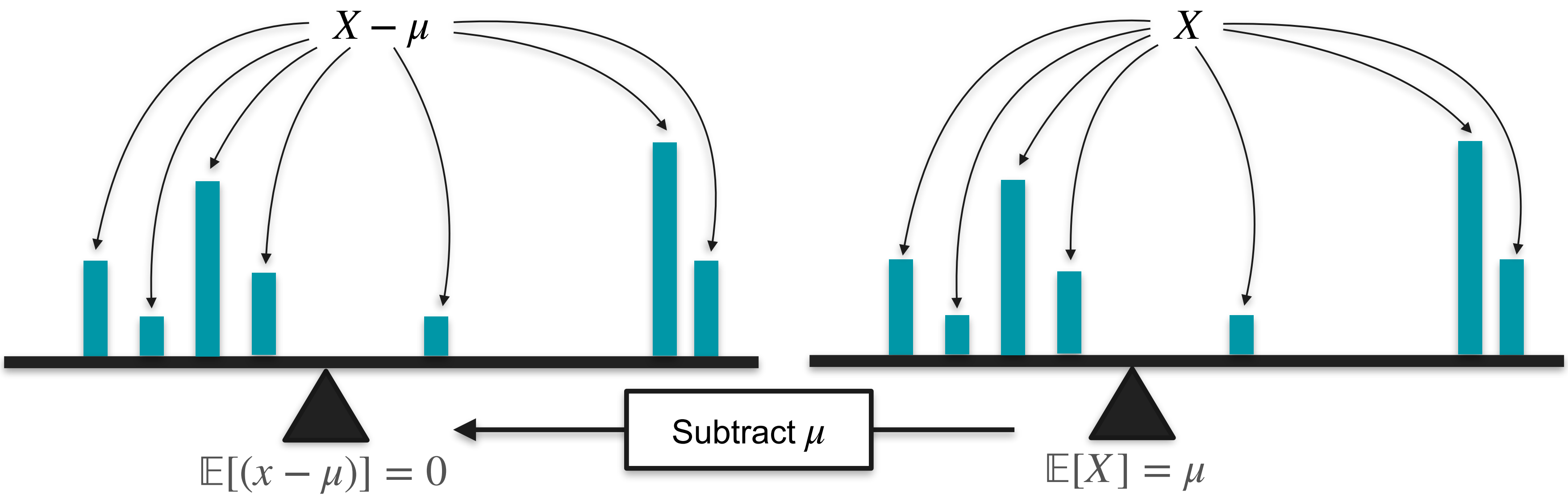
Variance Formula



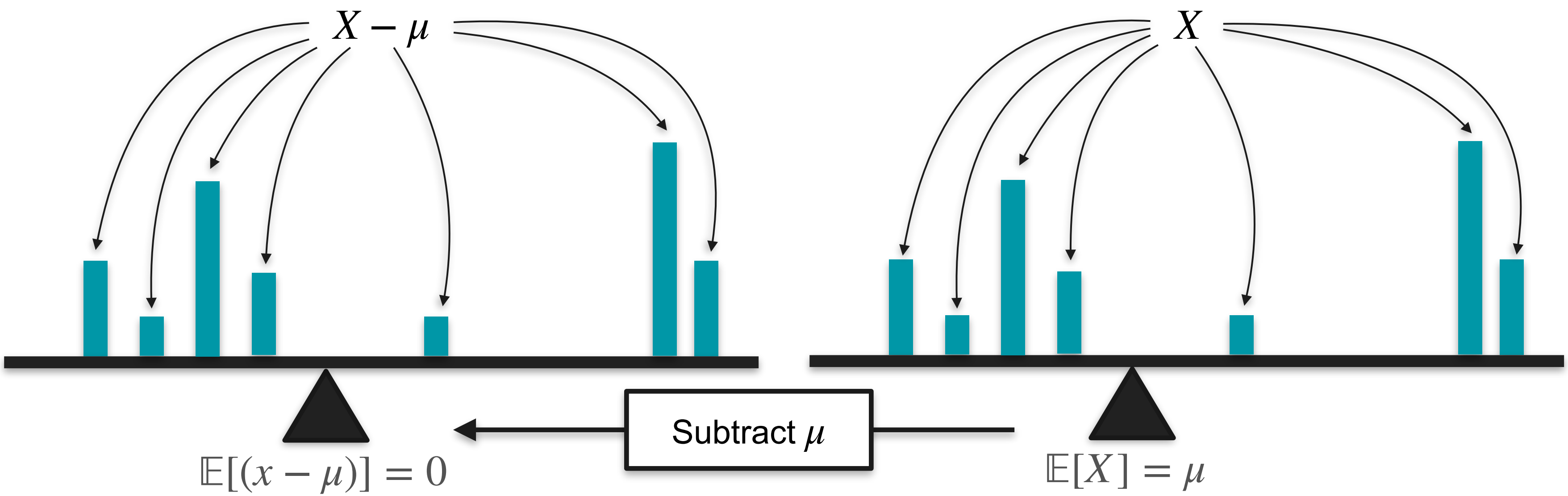
Variance Formula



Variance Formula



Variance Formula



$$Var(X) = \mathbb{E}[(X-\mu)^2]$$

Variance Formula

$$Var(X) = \mathbb{E}[(X - \mu)^2]$$

Variance Formula

$$Var(X) = \mathbb{E}[(X - \mu)^2]$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

Variance Formula

$$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$$\mathbb{E}[\text{constant} \cdot X] = \text{constant} \cdot \mathbb{E}[X]$$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$\mathbb{E}[\text{constant}] = \text{constant}$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2$$

$$\mu = \mathbb{E}[X]$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X] \cdot \mathbb{E}[X] + \mathbb{E}[X]^2$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X] \cdot \mathbb{E}[X] + \mathbb{E}[X]^2$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X]^2 + \mathbb{E}[X]^2$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X] \cdot \mathbb{E}[X] + \mathbb{E}[X]^2$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X]^2 + \mathbb{E}[X]^2$$

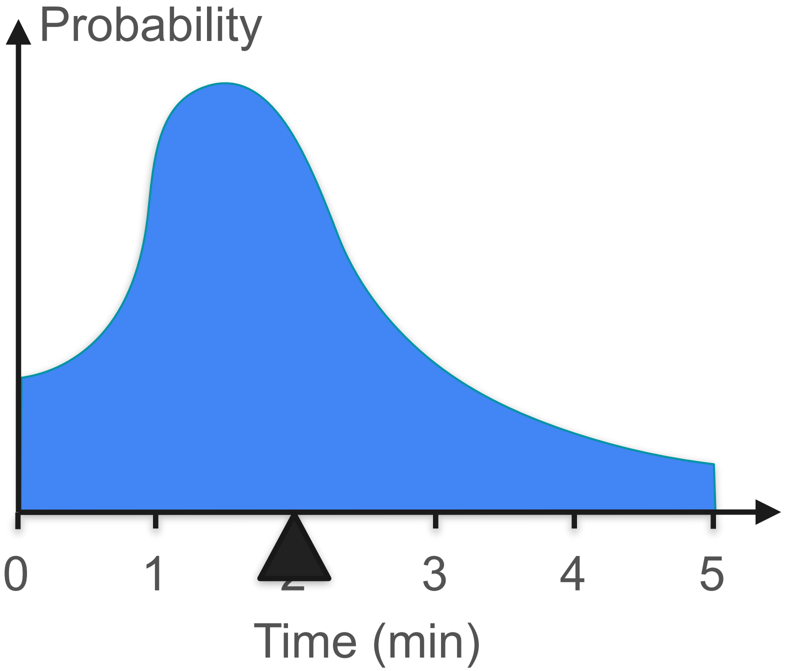
$$= \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

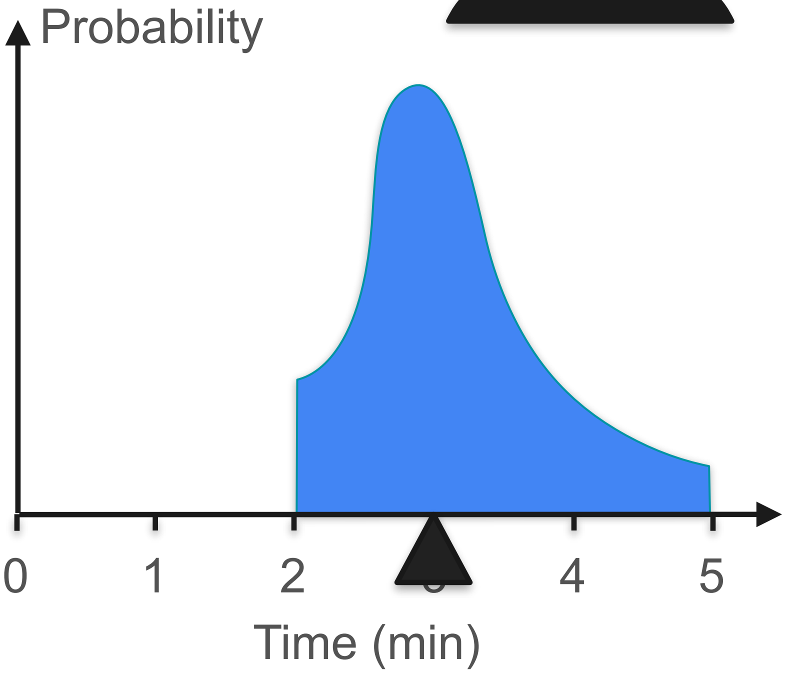
Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2]$$

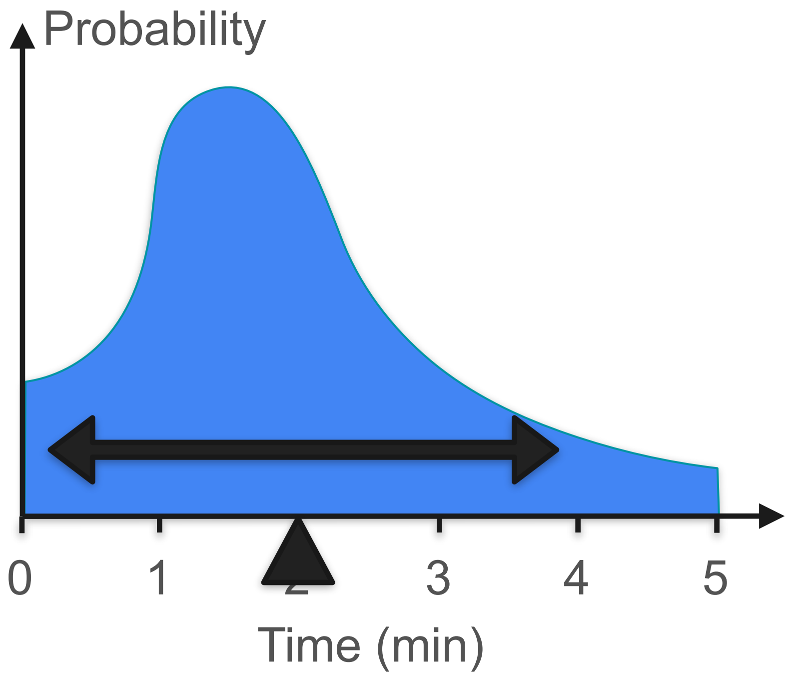
$$= \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

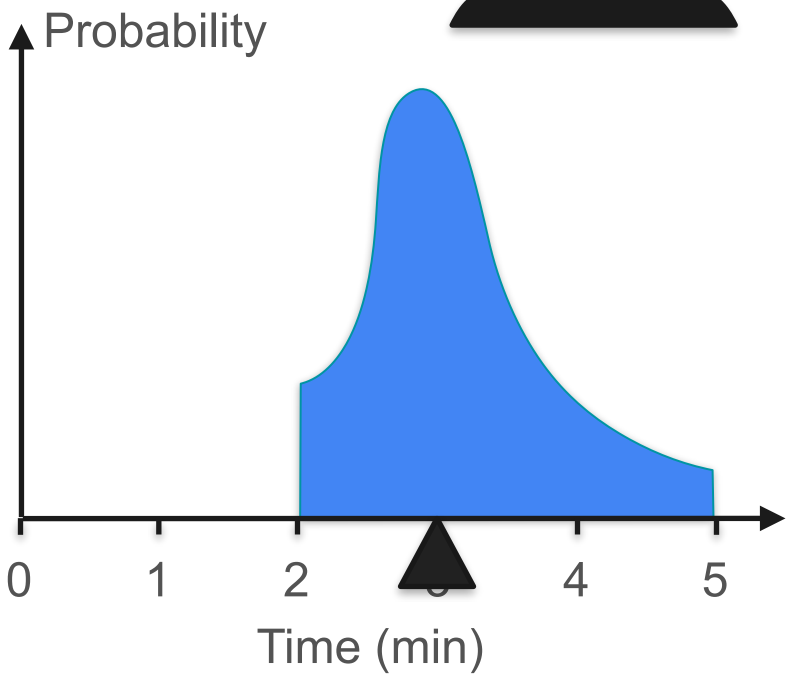
Variance for Continuous Distributions



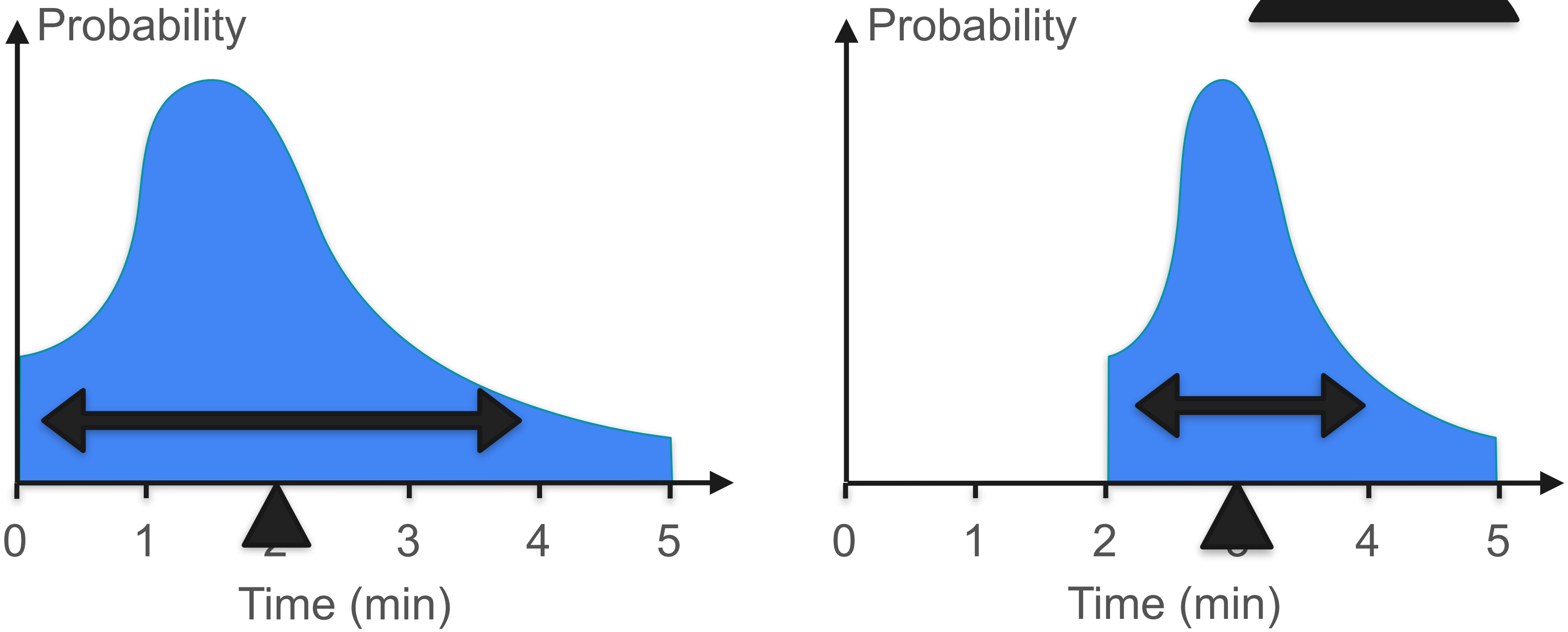


Variance for Continuous Distributions



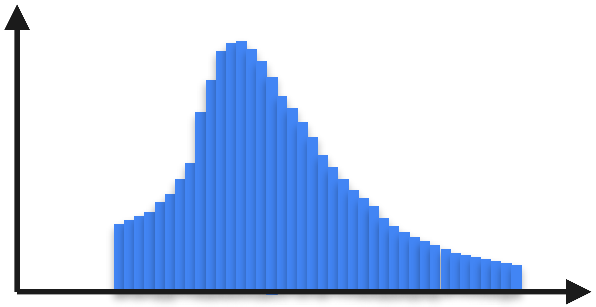


Variance for Continuous Distributions



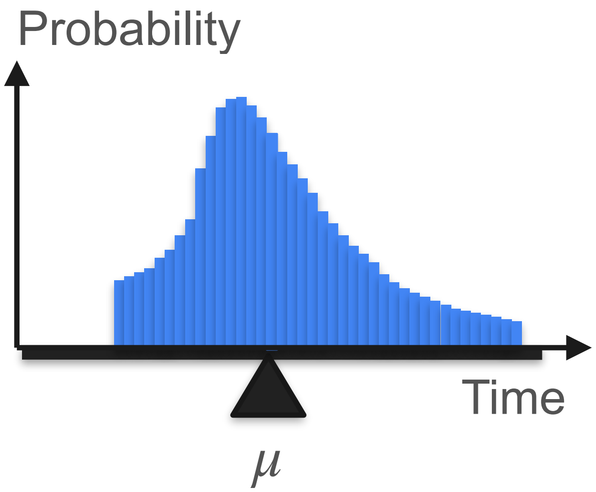
Variance

$$Var(X) = \mathbb{E}[(X - \mu)^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$



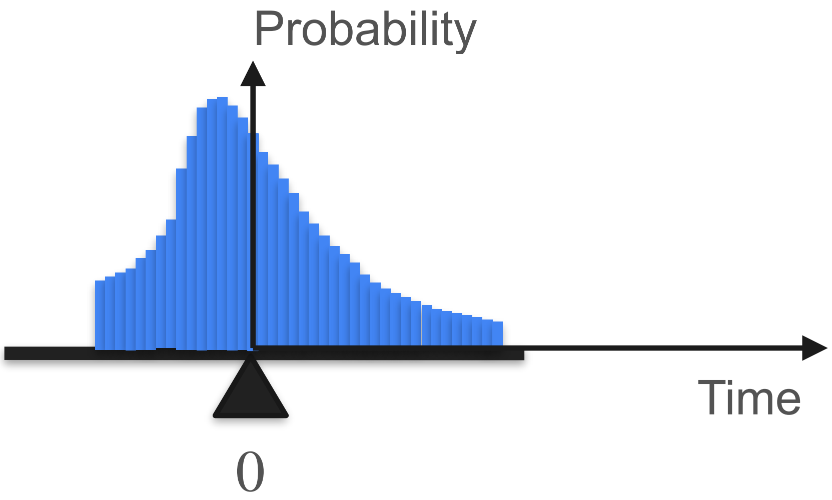
Variance

$$Var(X) = \mathbb{E}[(X - \mu)^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$



Variance

$$Var(X) = \mathbb{E}[(X-\mu)^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

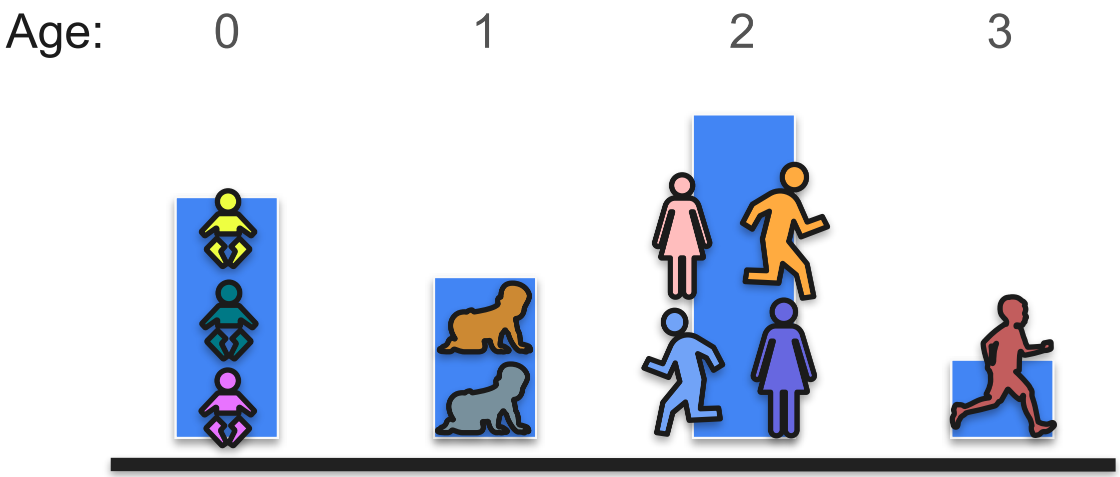


DeepLearning.AI

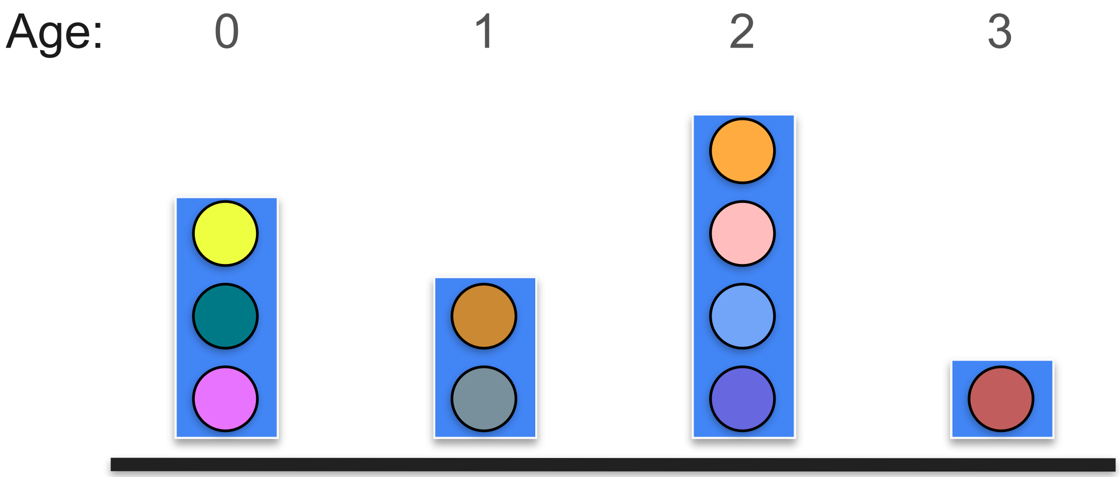
Describing Distributions

Measures of Central Tendency

Mean: Example

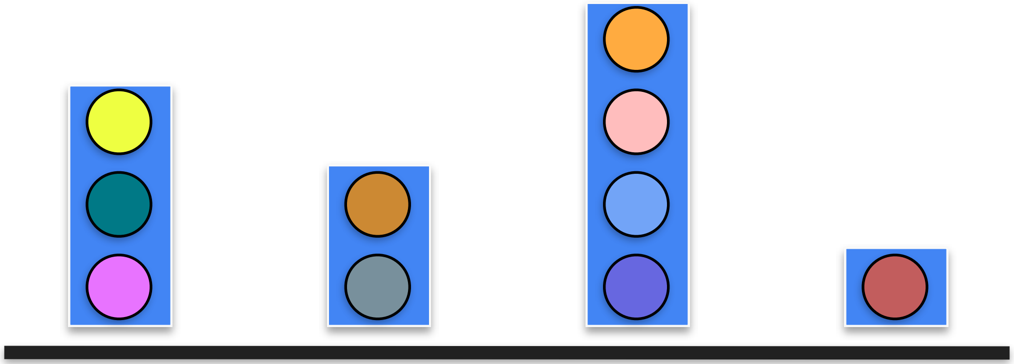


Mean: Example



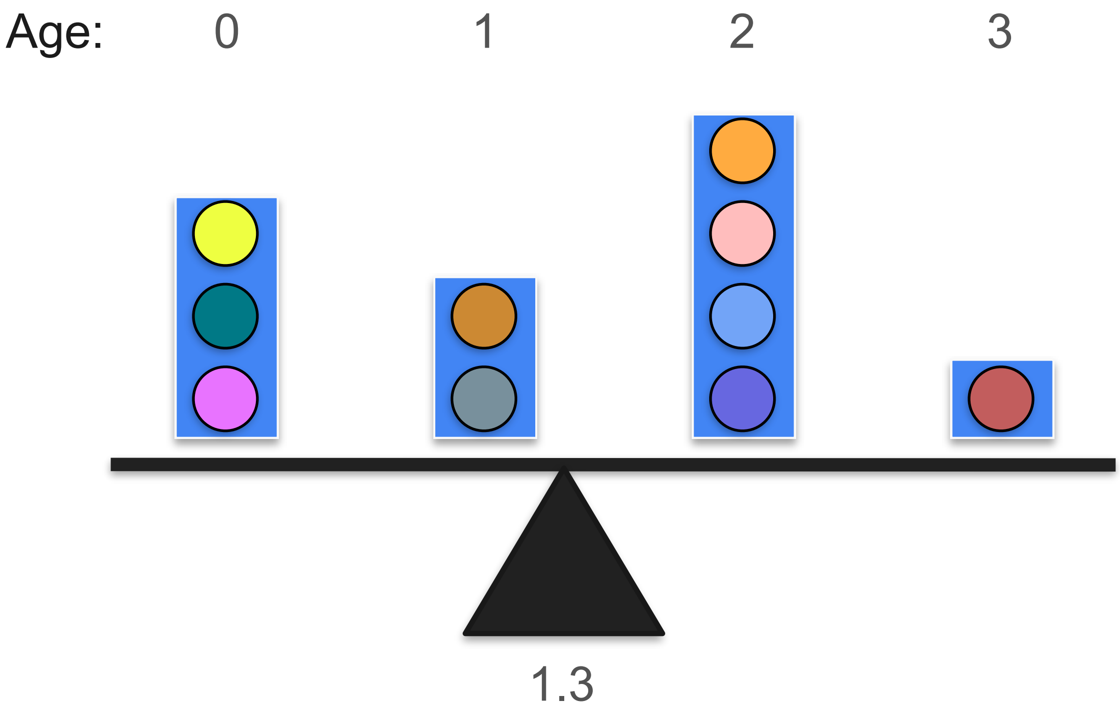
Mean: Example

Age: 0 1 2 3

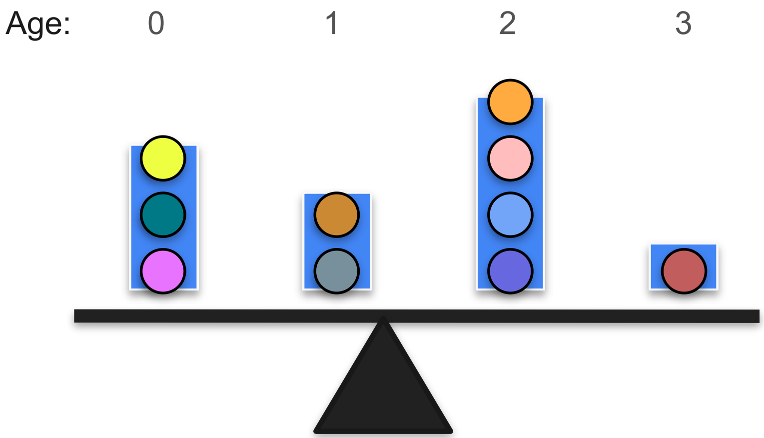


1.3

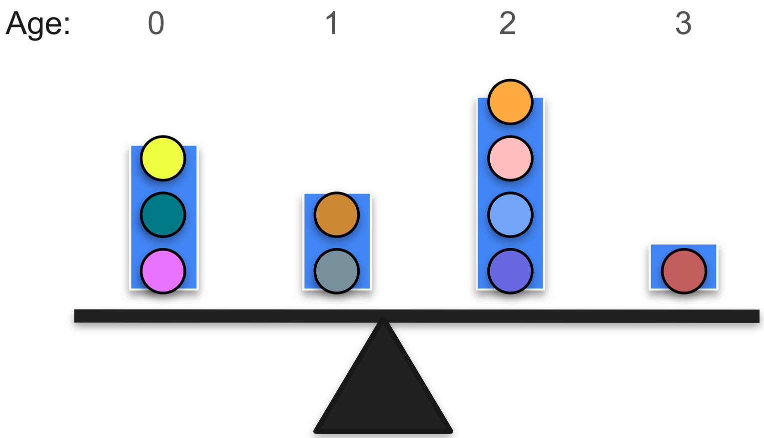
Mean: Example



Mean: Example

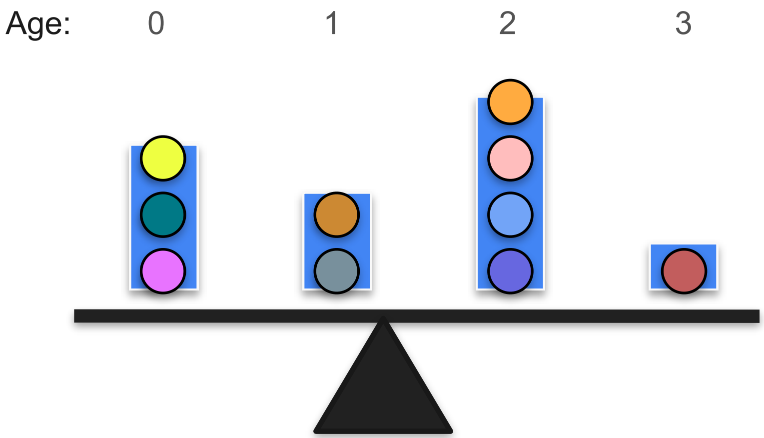


Mean: Example



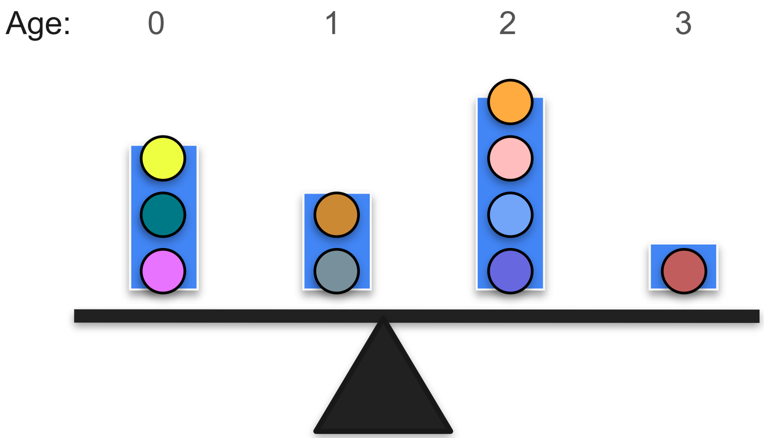
$$0 + 0 + 0$$

Mean: Example



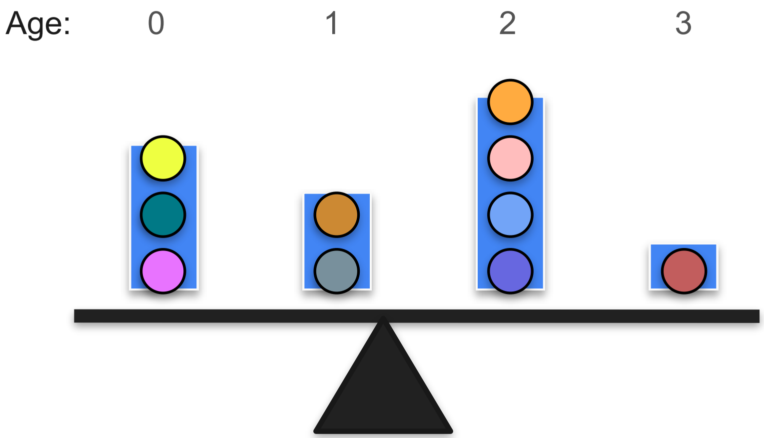
$$0 + 0 + 0 \ \ + 1 + 1$$

Mean: Example



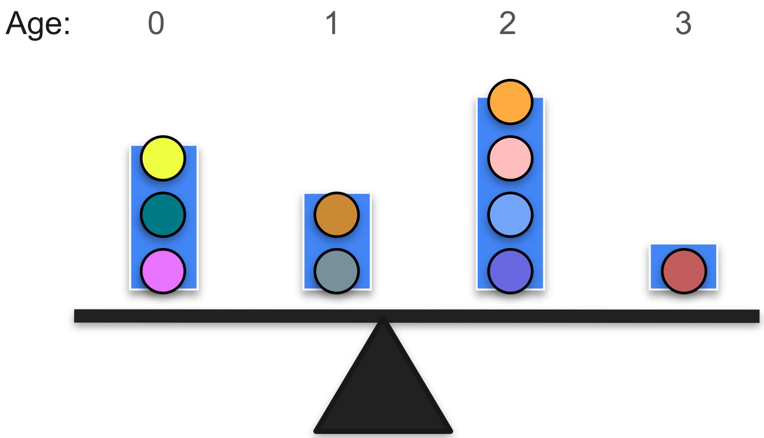
$$0+0+0 \;\; +1+1 \;\; +2+2+2+2$$

Mean: Example



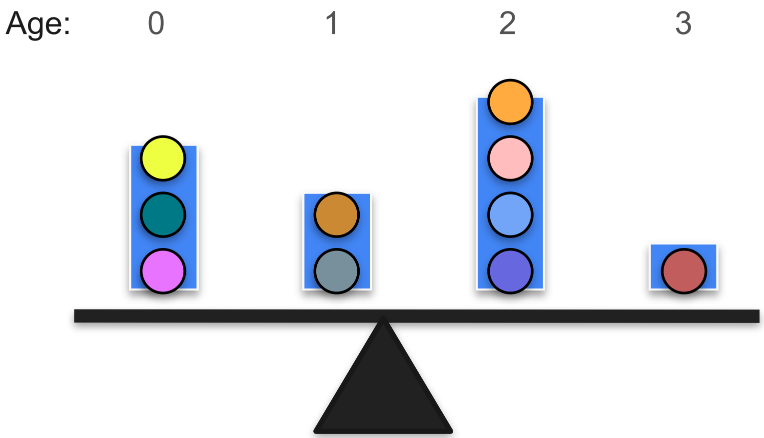
$$0 + 0 + 0 \ +1 + 1 \ +2 + 2 + 2 + 2 \ +3$$

Mean: Example



$$\frac{0+0+0\ +1+1\ +2+2+2+2\ +3}{10}$$

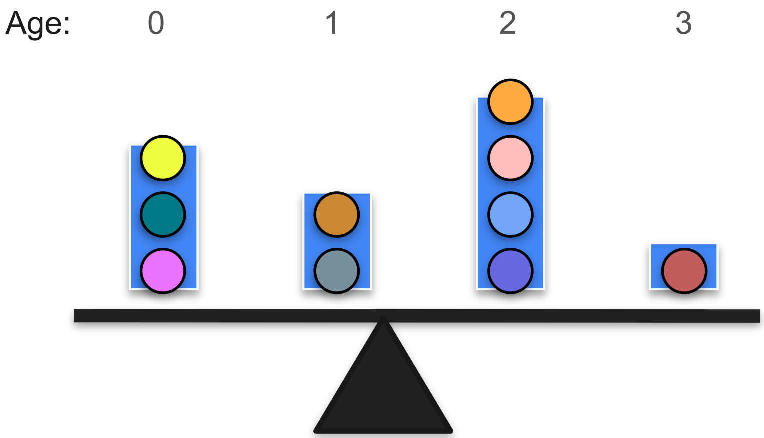
Mean: Example



$$\frac{0+0+0\ +1+1\ +2+2+2+2\ +3}{10}$$

$$=\frac{13}{10}$$

Mean: Example

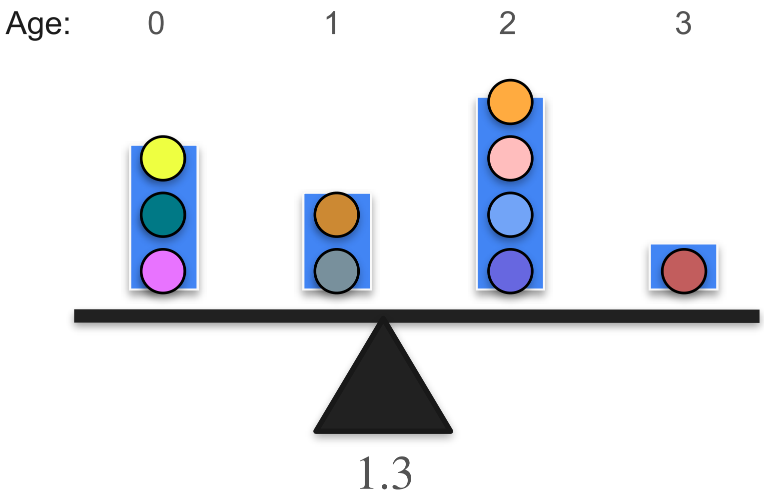


$$\frac{0+0+0\ +1+1\ +2+2+2+2\ +3}{10}$$

$$=\frac{13}{10}$$

$$=1.3$$

Mean: Example

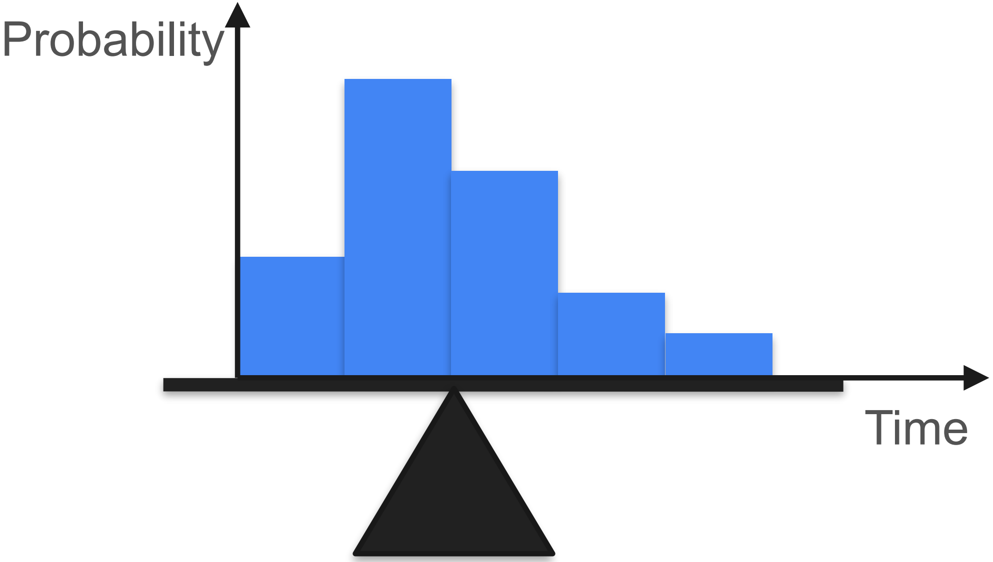


$$\frac{0+0+0\ +1+1\ +2+2+2+2\ +3}{10}$$

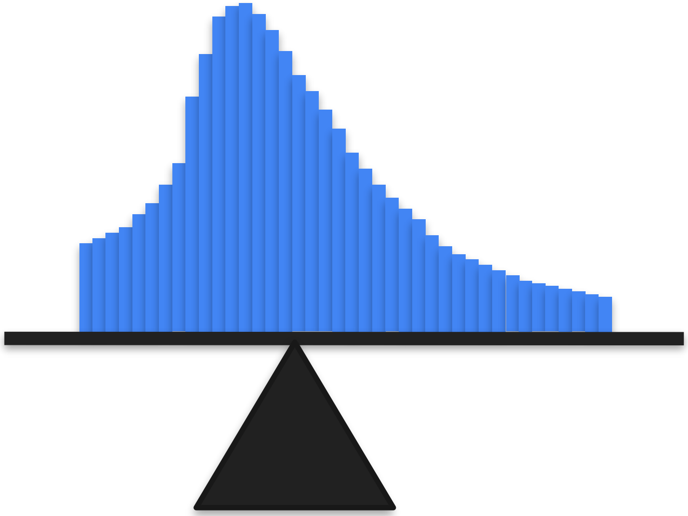
$$=\frac{13}{10}$$

$$=1.3$$

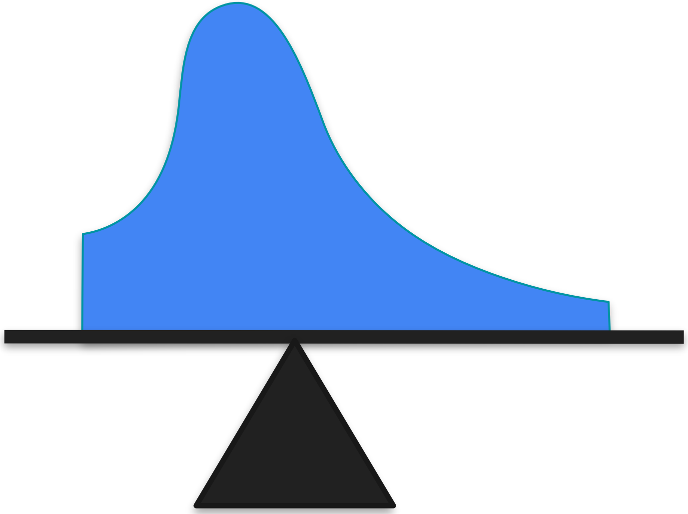
Mean



Mean



Mean



Median: Motivation

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Michael Jordan

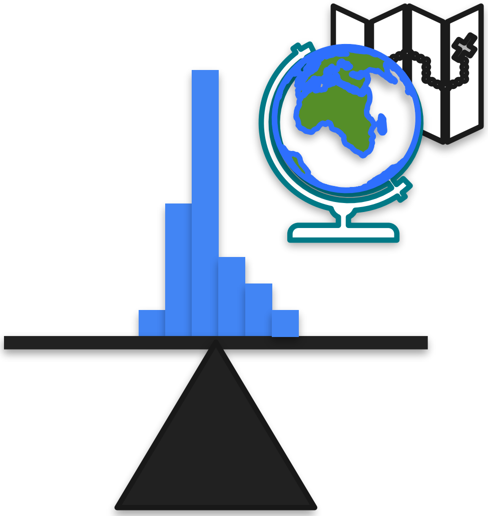
Outliers

Outliers

Outliers

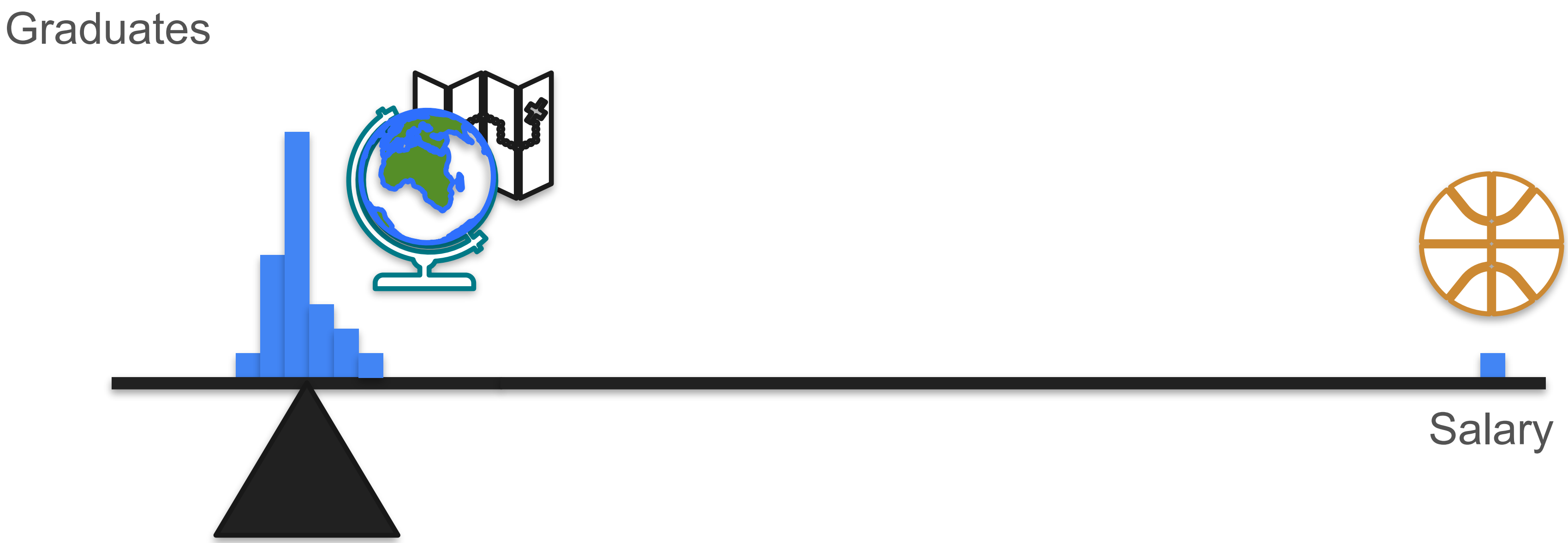
Outliers

Graduates

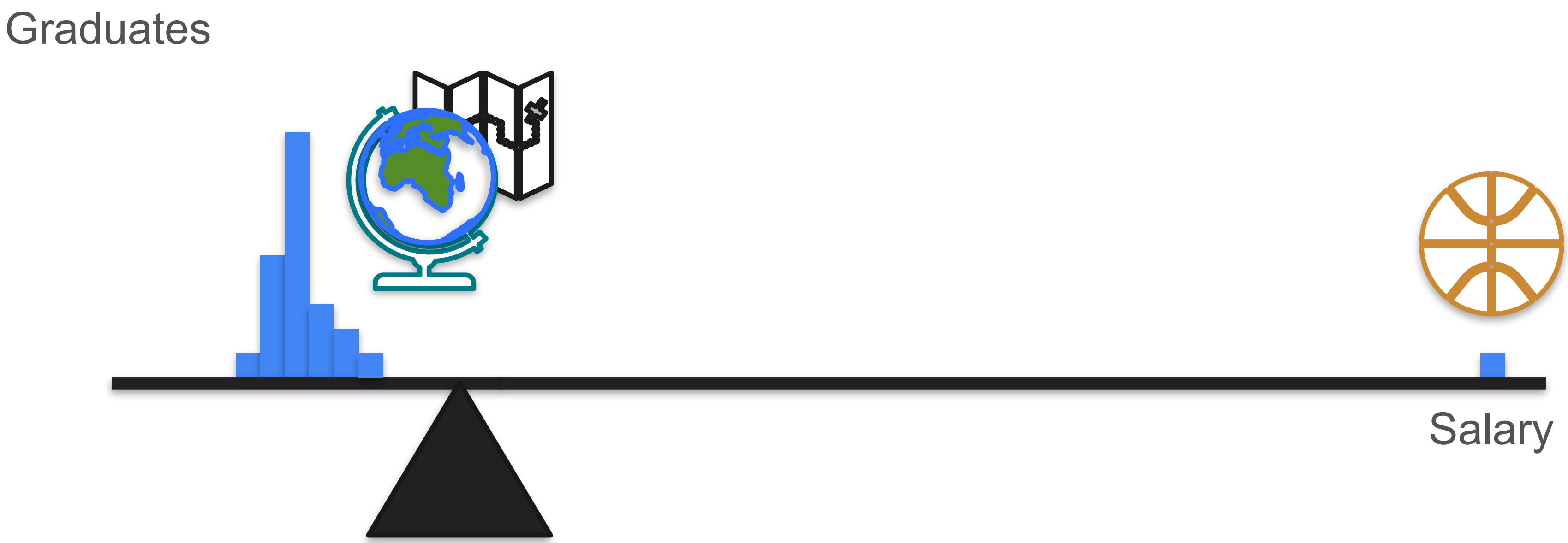


Salary

Outliers

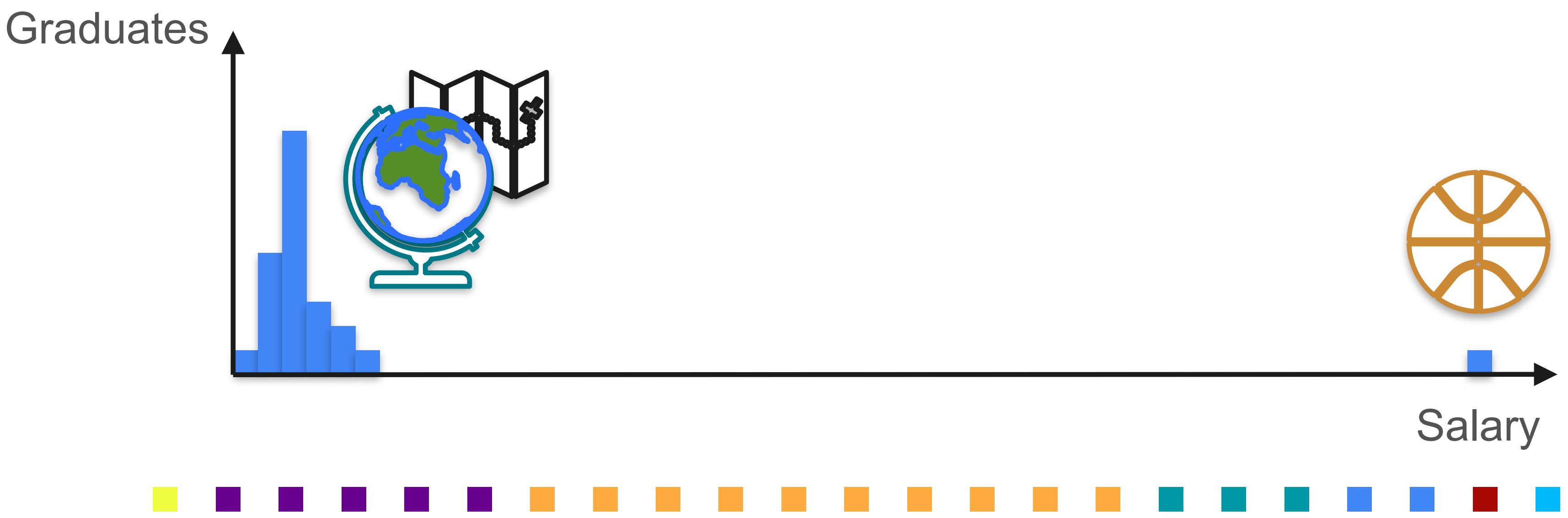


Outliers

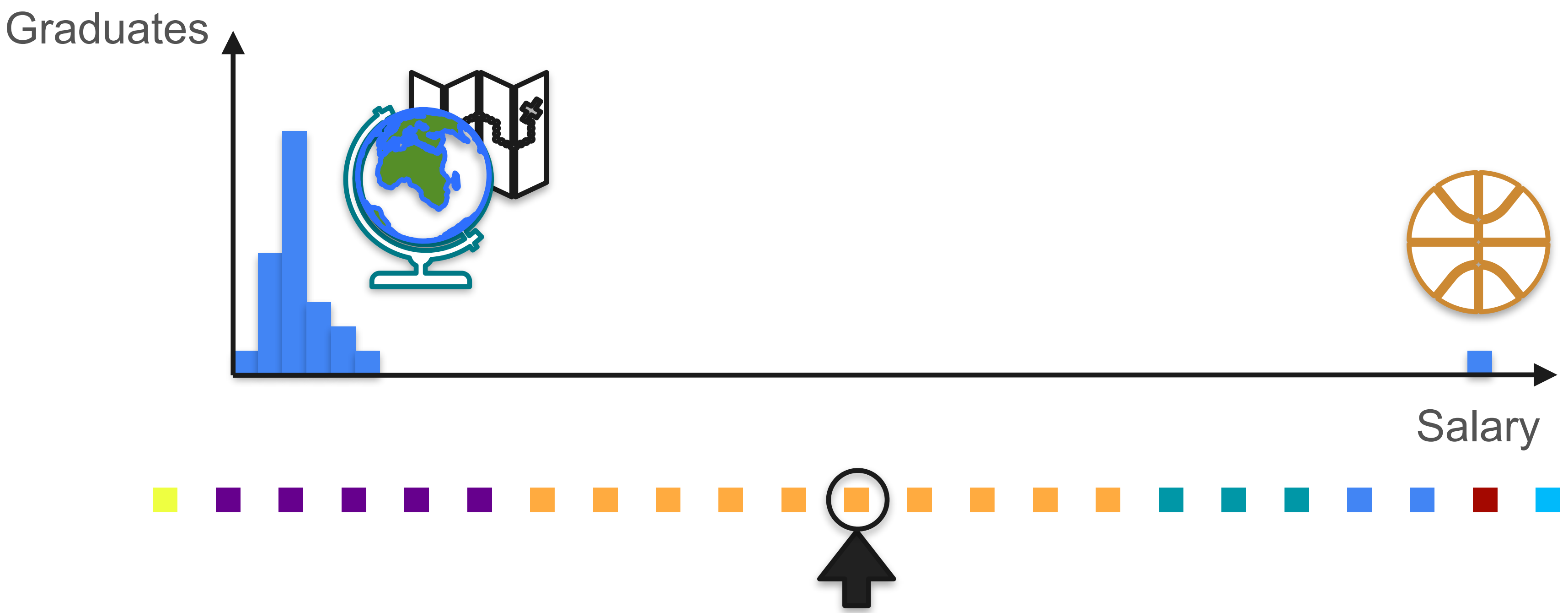


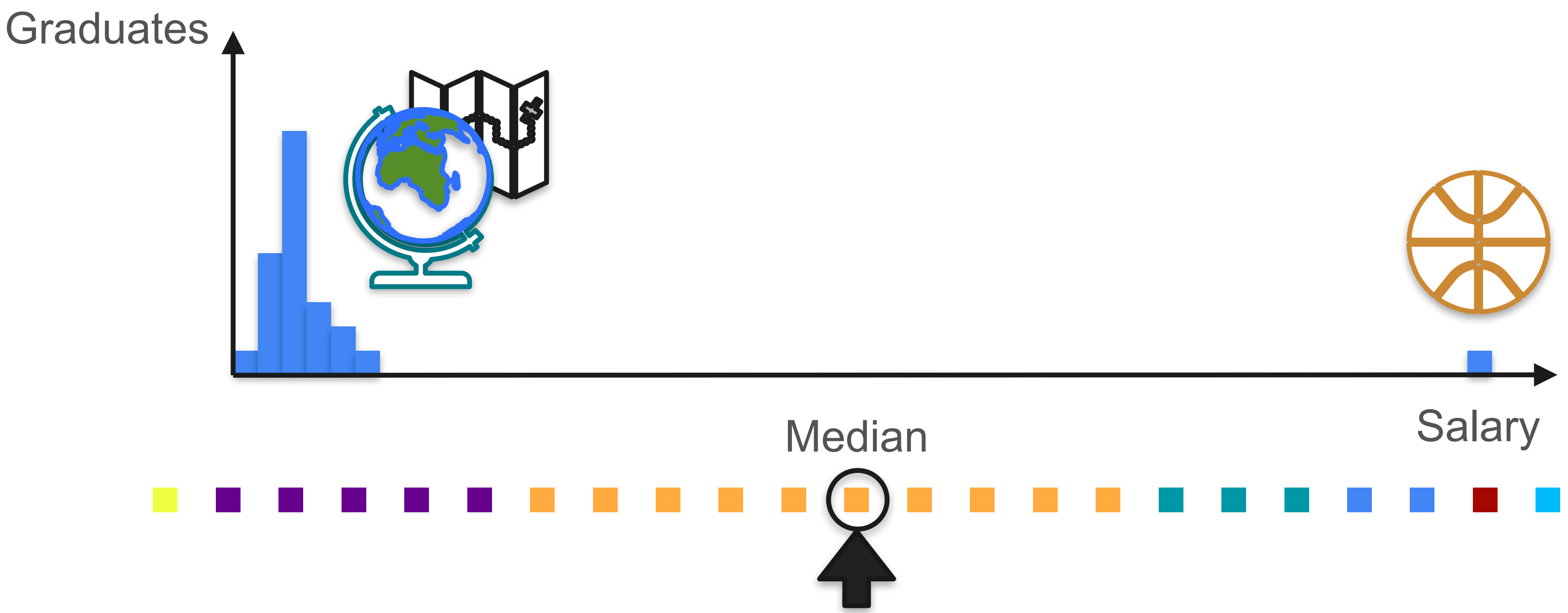
Outliers

Median



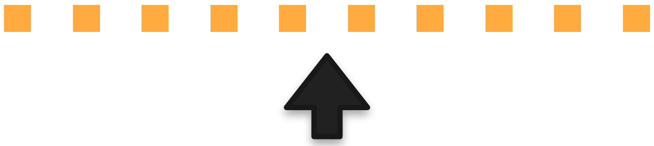
Median



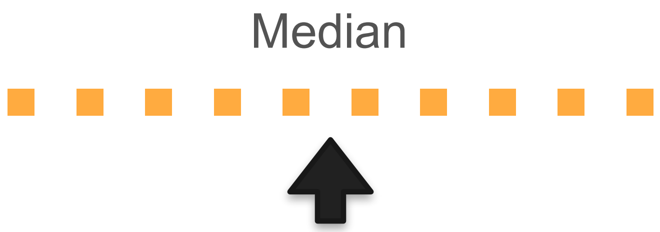

Graduates
Salary
Median

Median

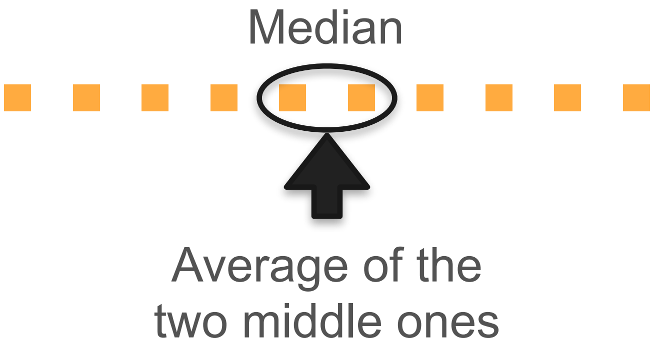
Median



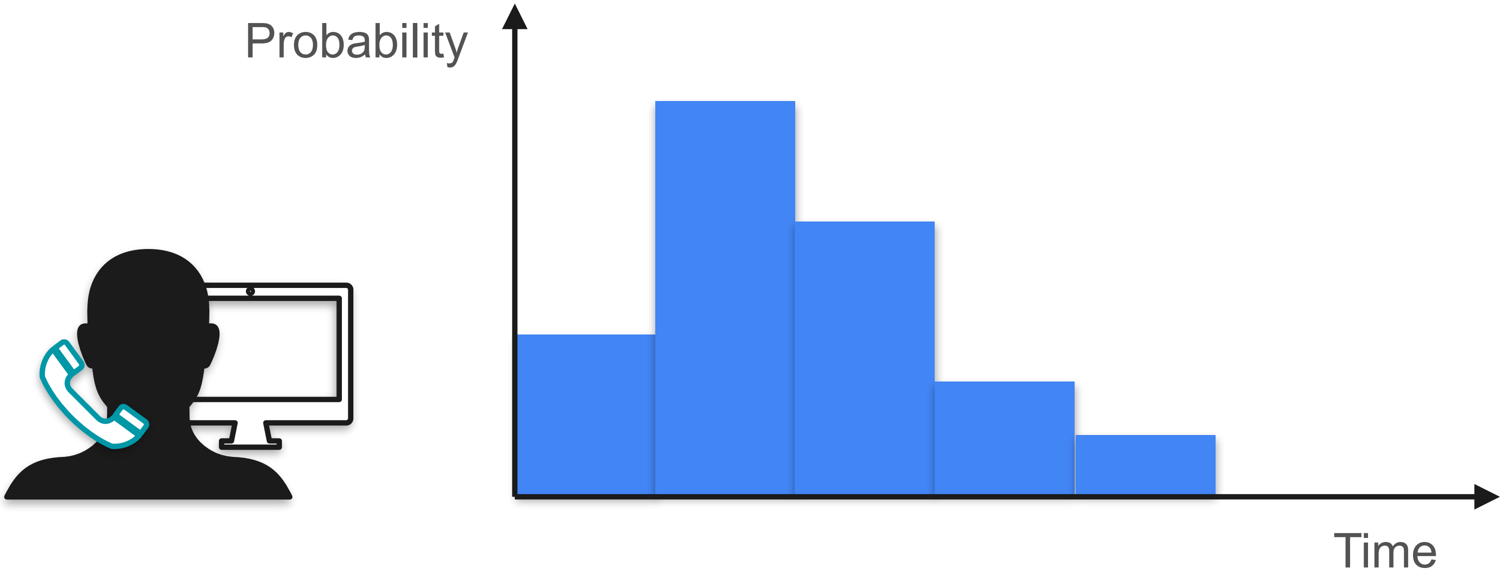
Median



Median



Mode



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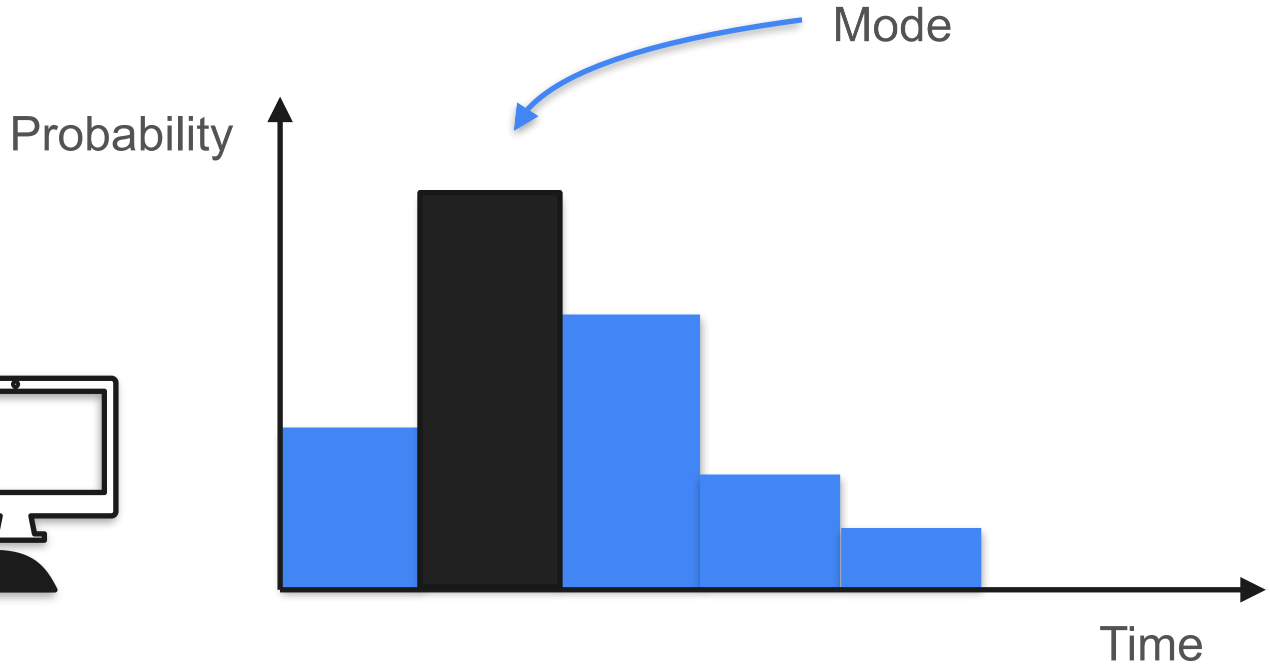
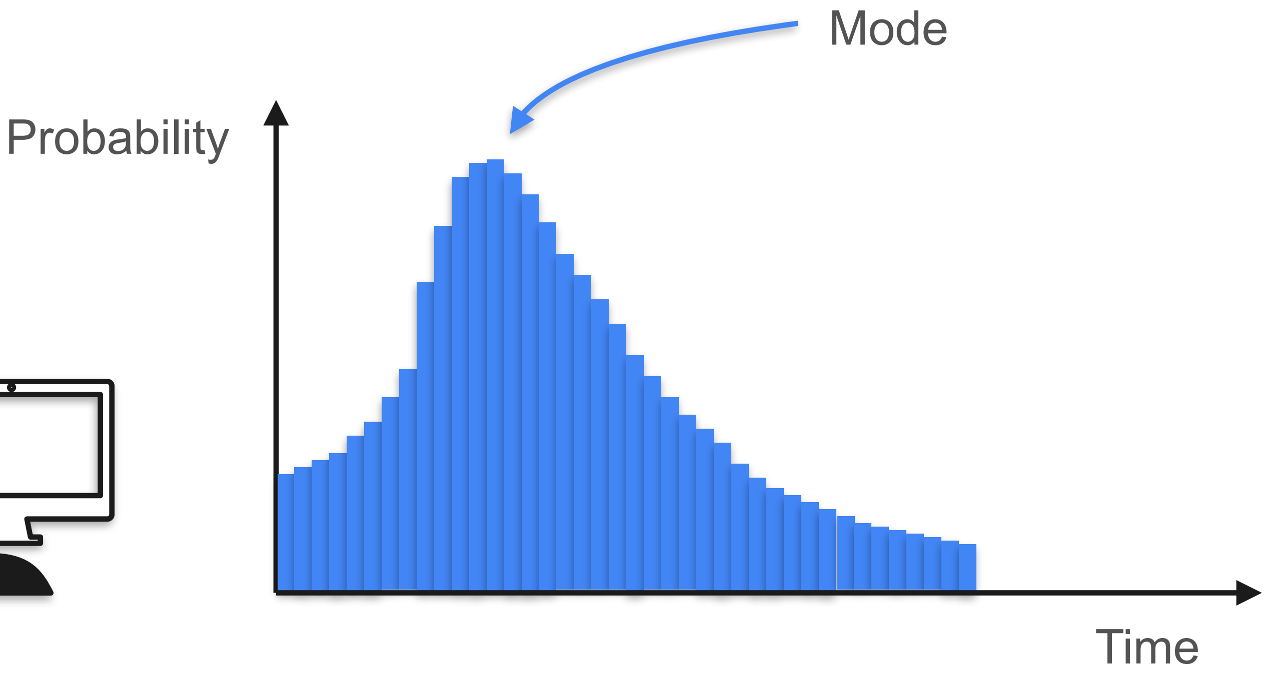


Mode



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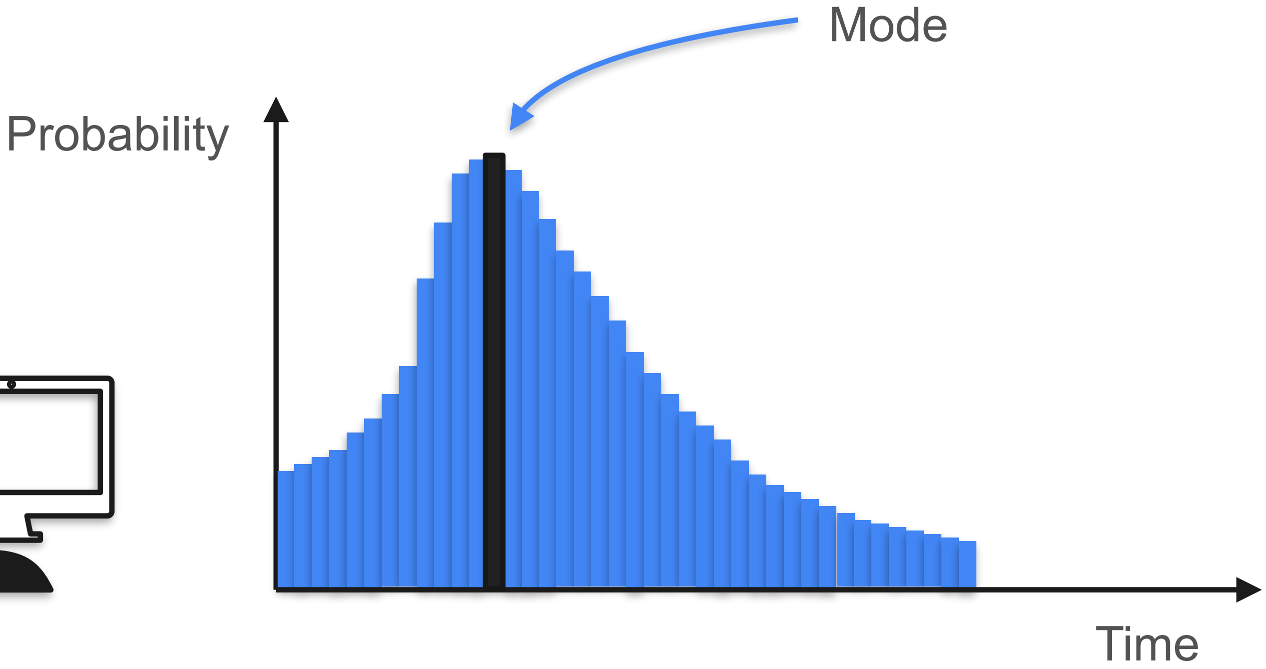
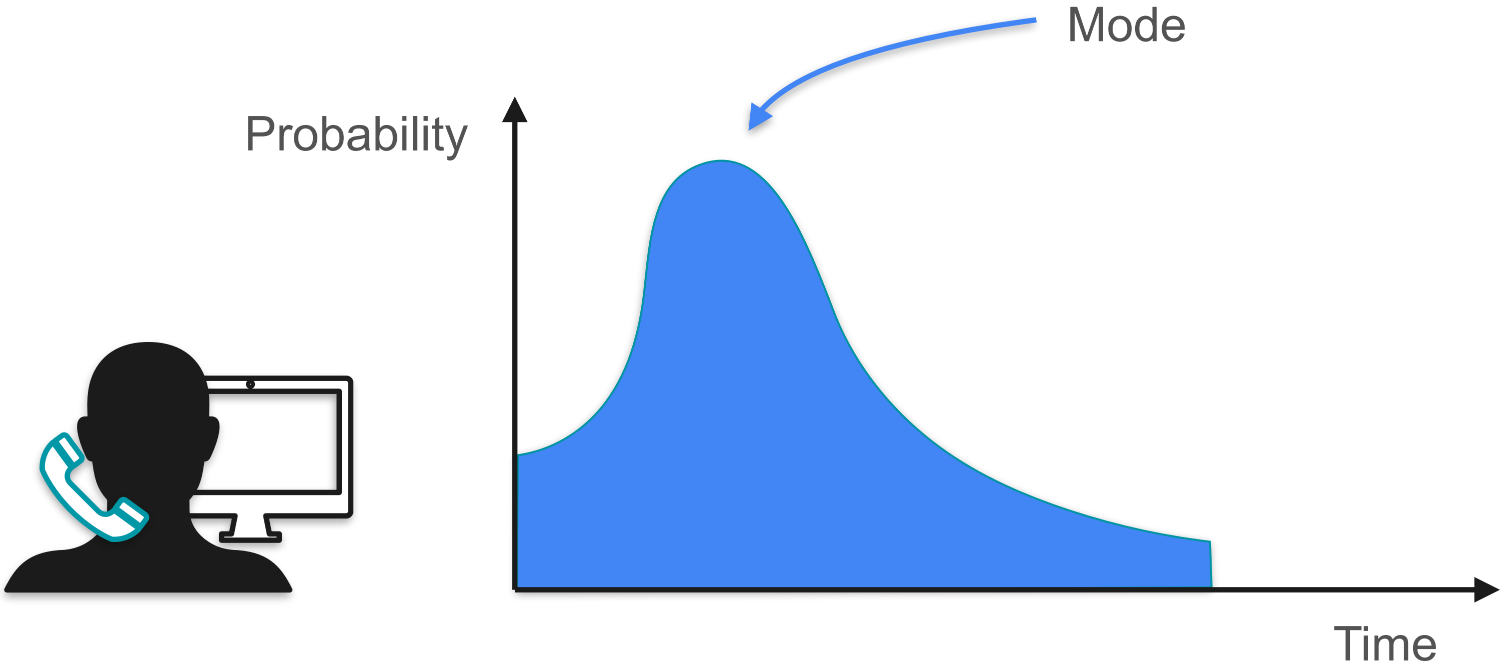
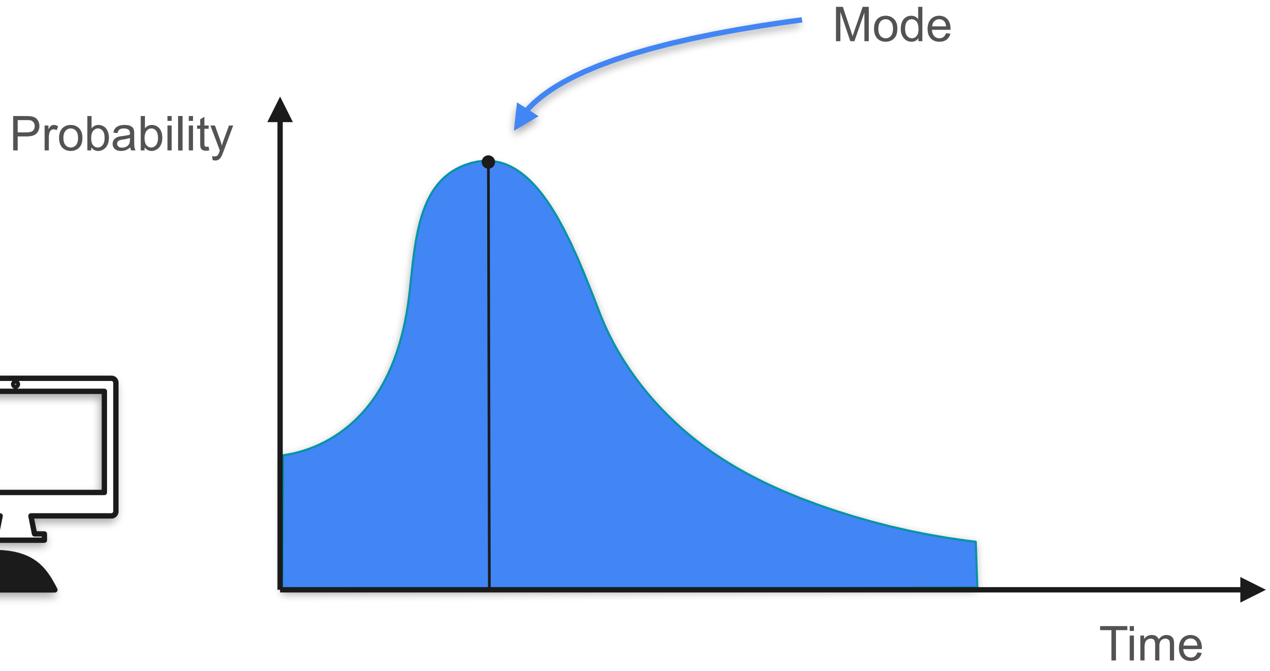


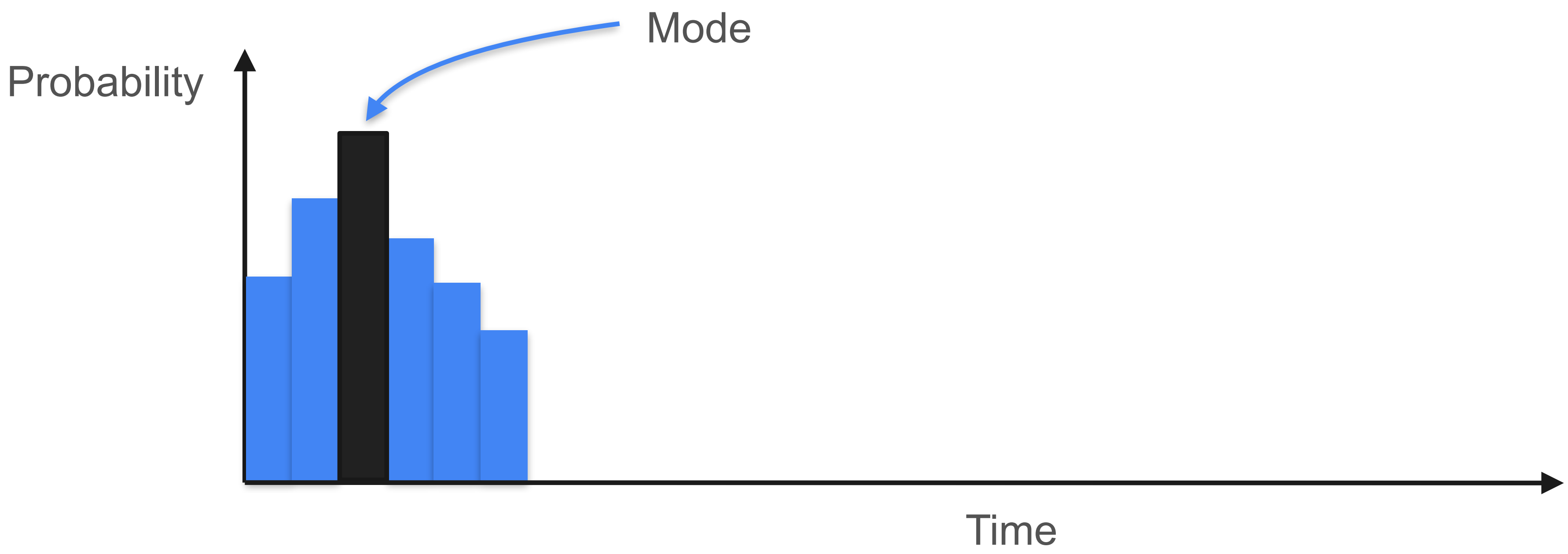
Mode



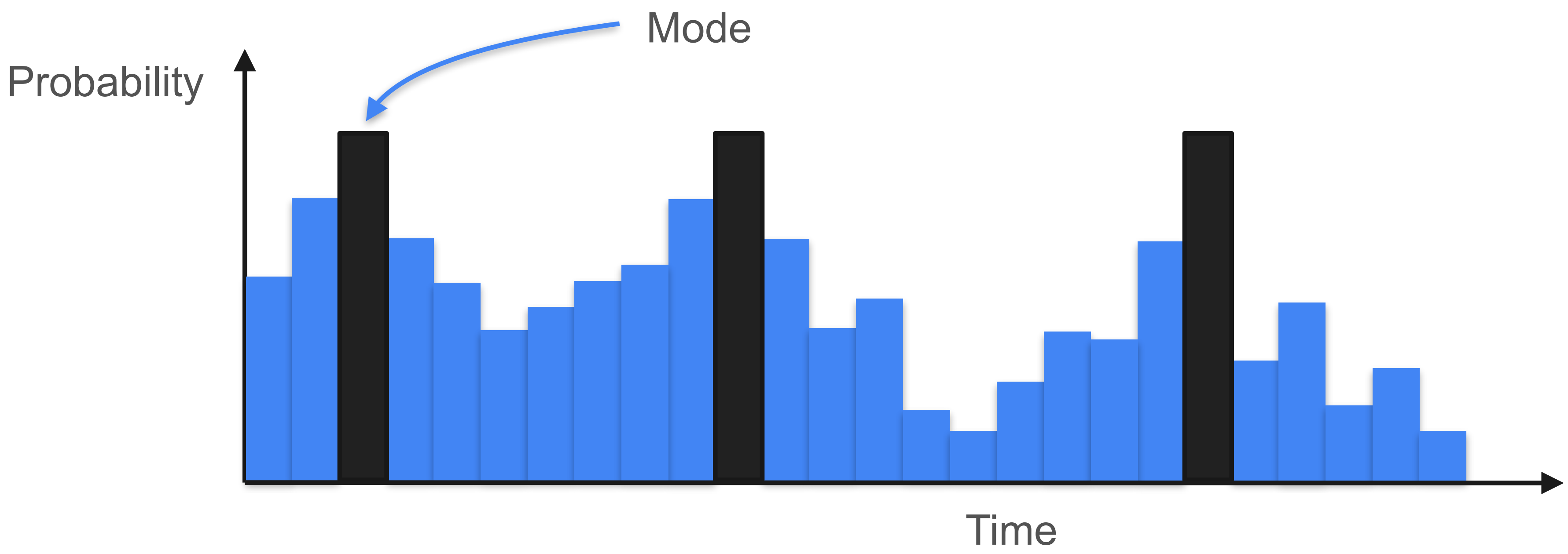
Mode



Mode: Multimodal Distribution



Mode: Multimodal Distribution



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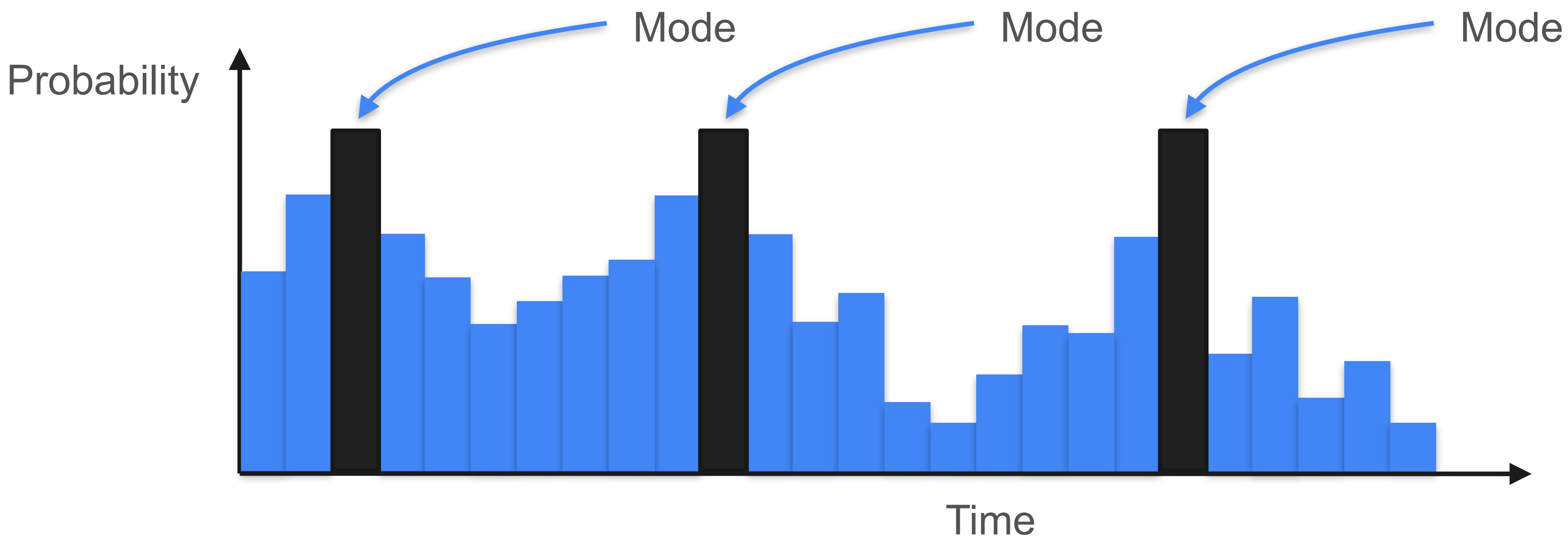
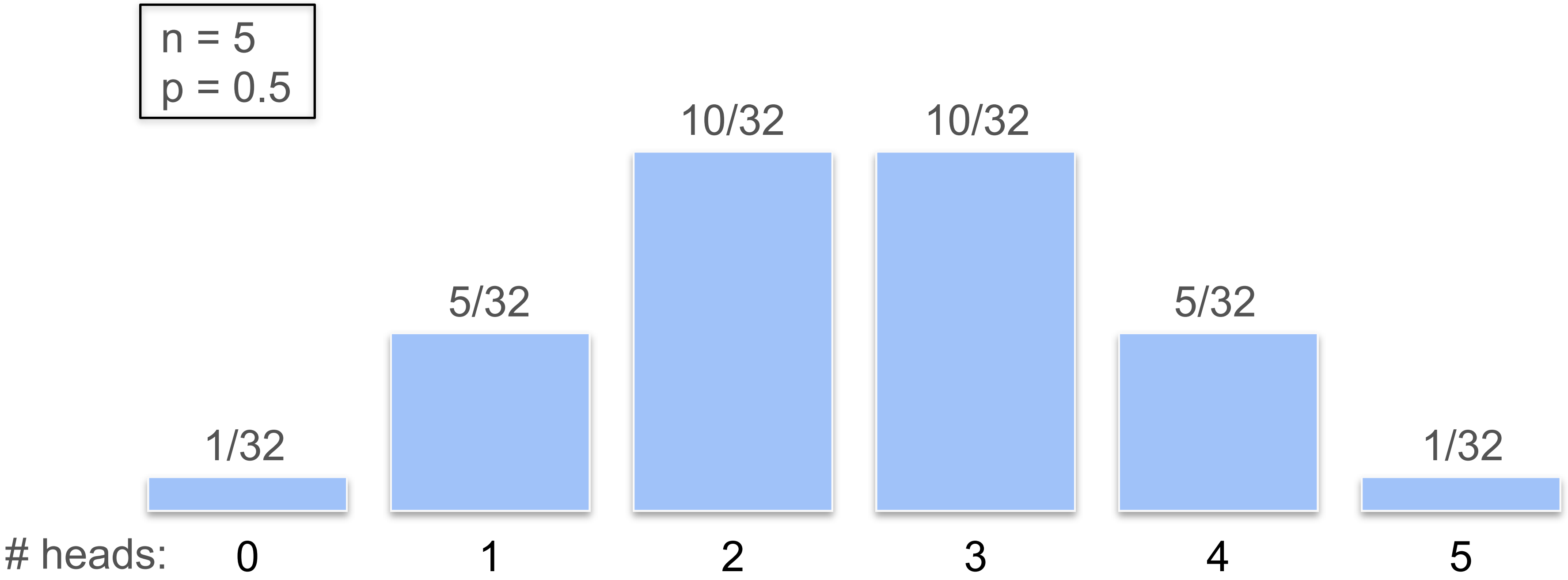
Mode: Multimodal Distribution



Mean, Median and Mode in Binomial Distribution



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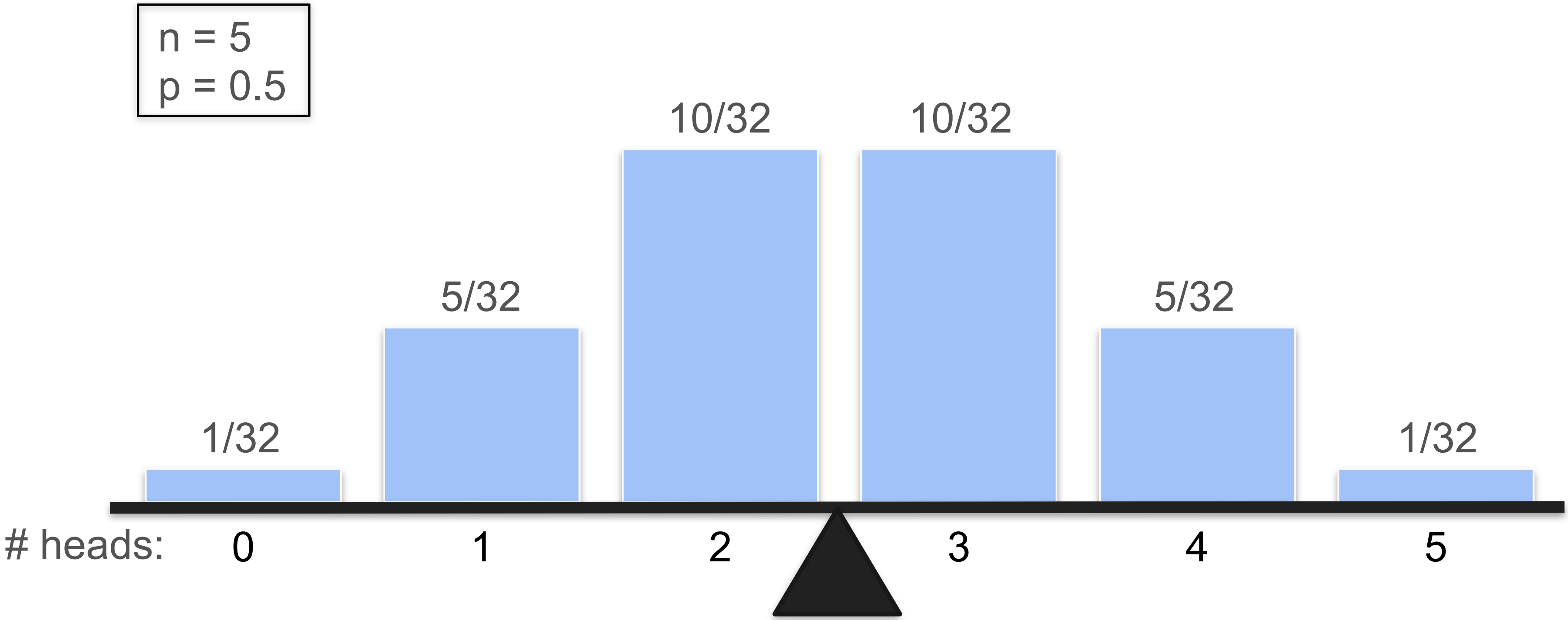

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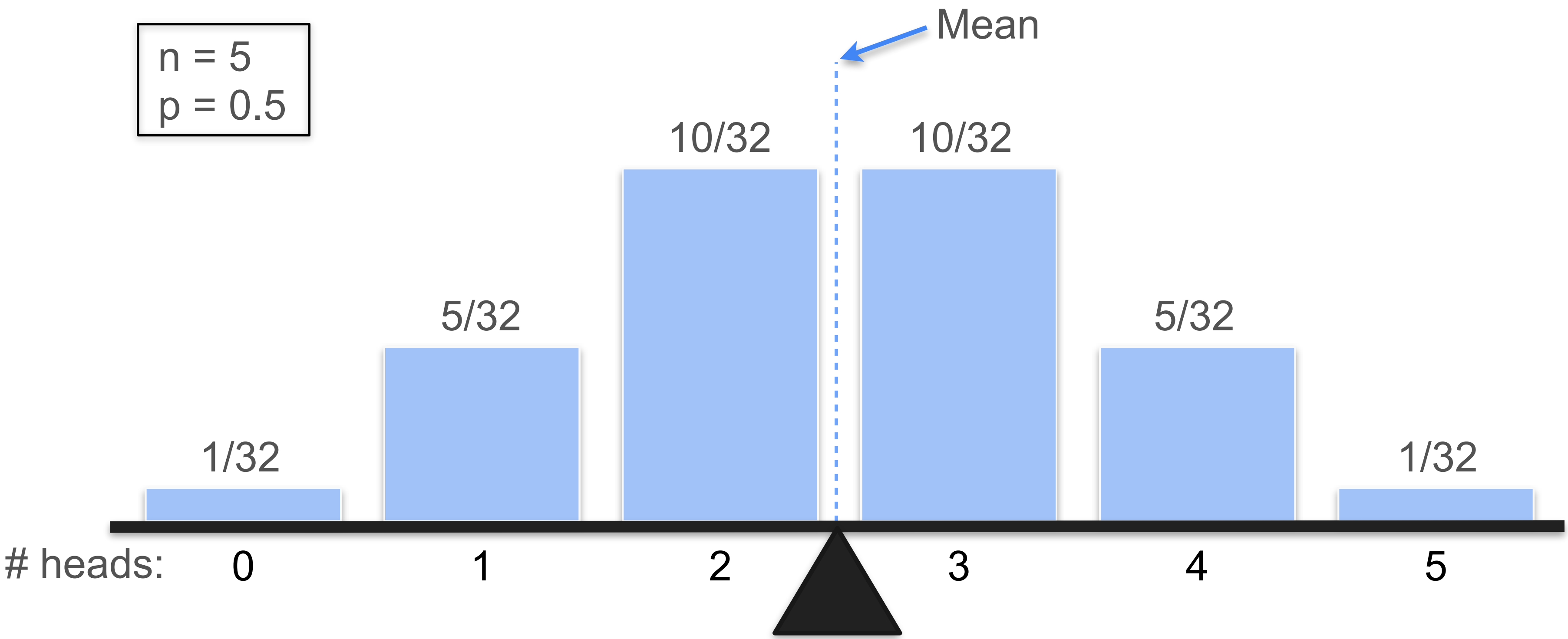

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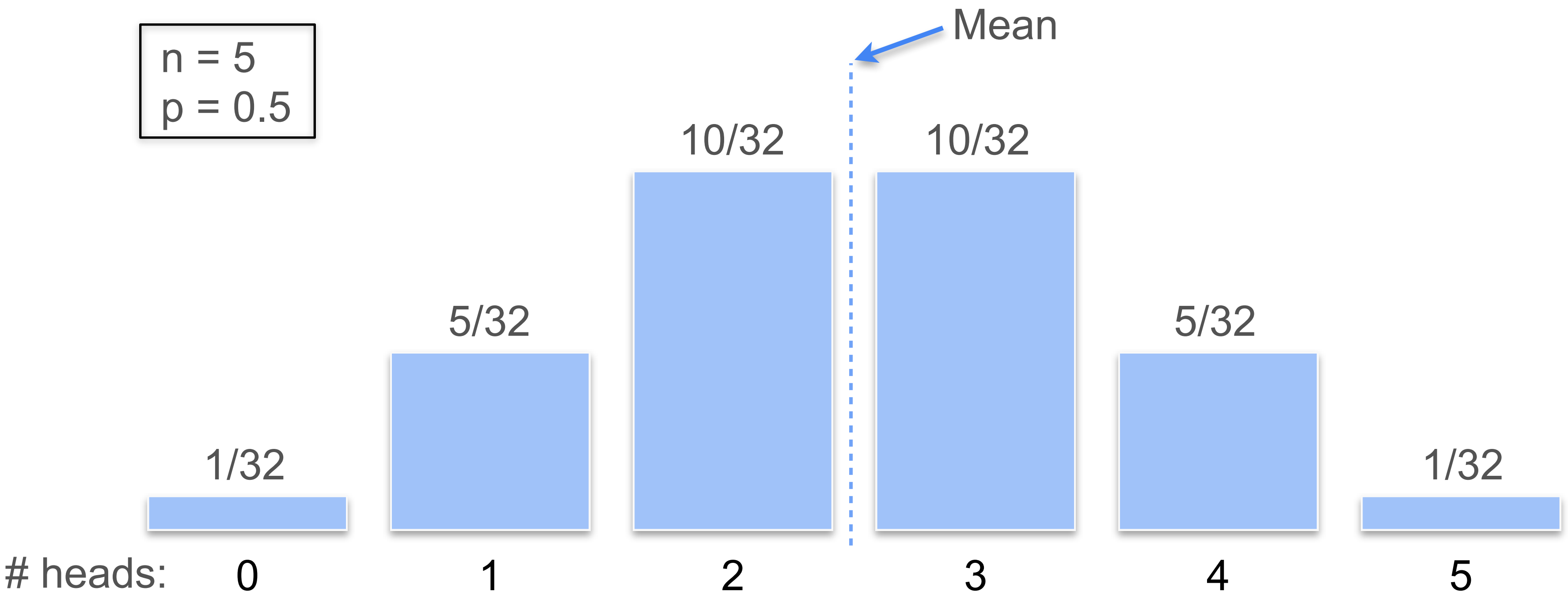

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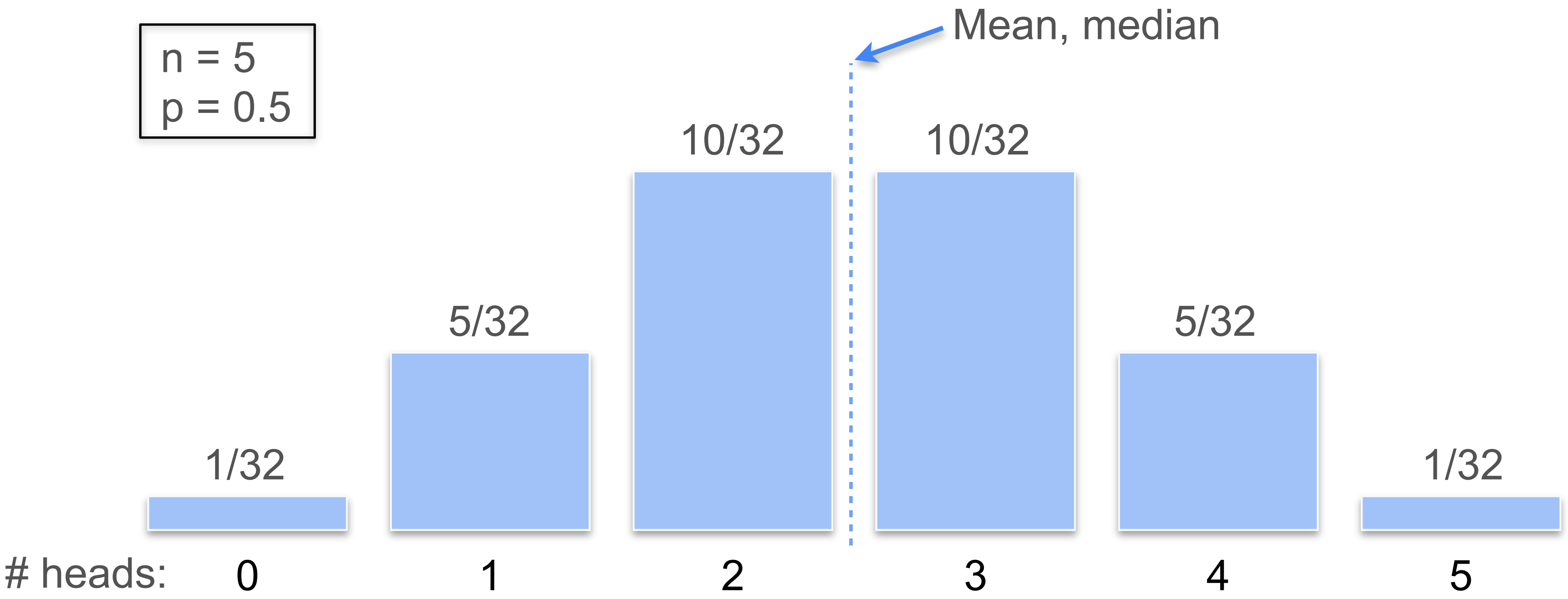
Mean, Median and Mode in Binomial Distribution



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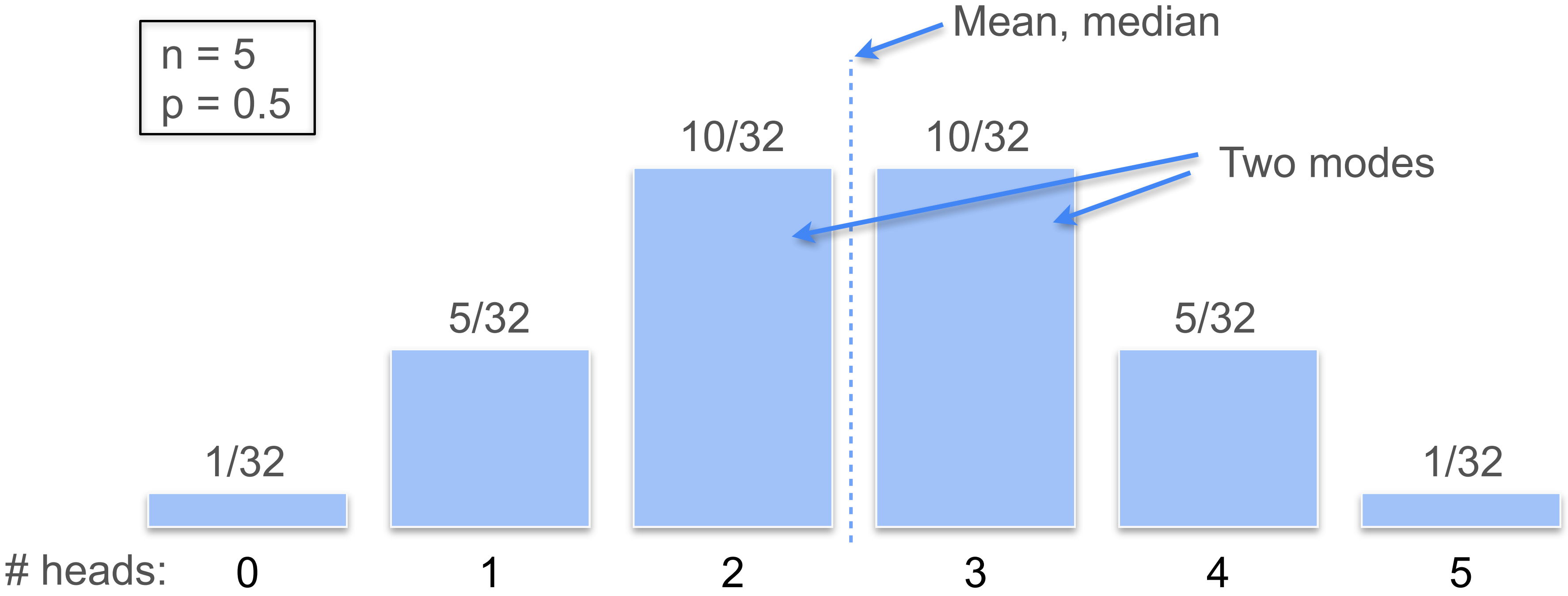
Mean, Median and Mode in Binomial Distribution



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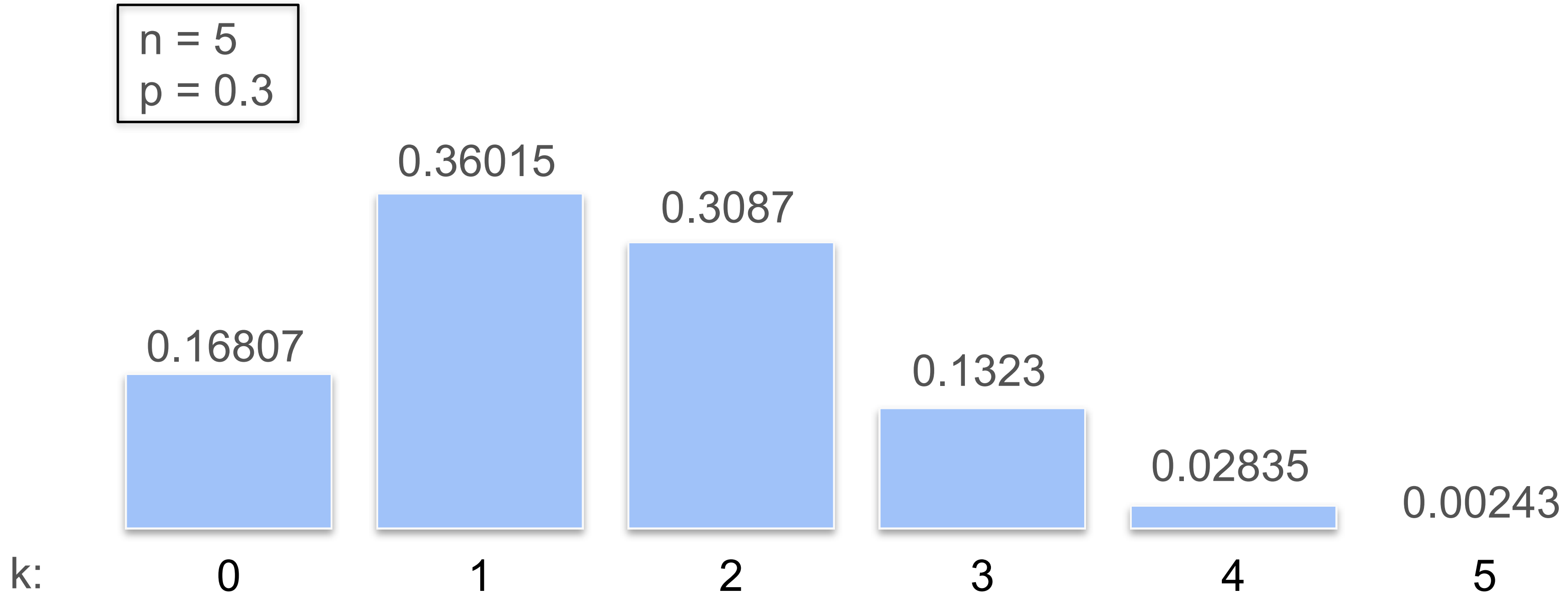

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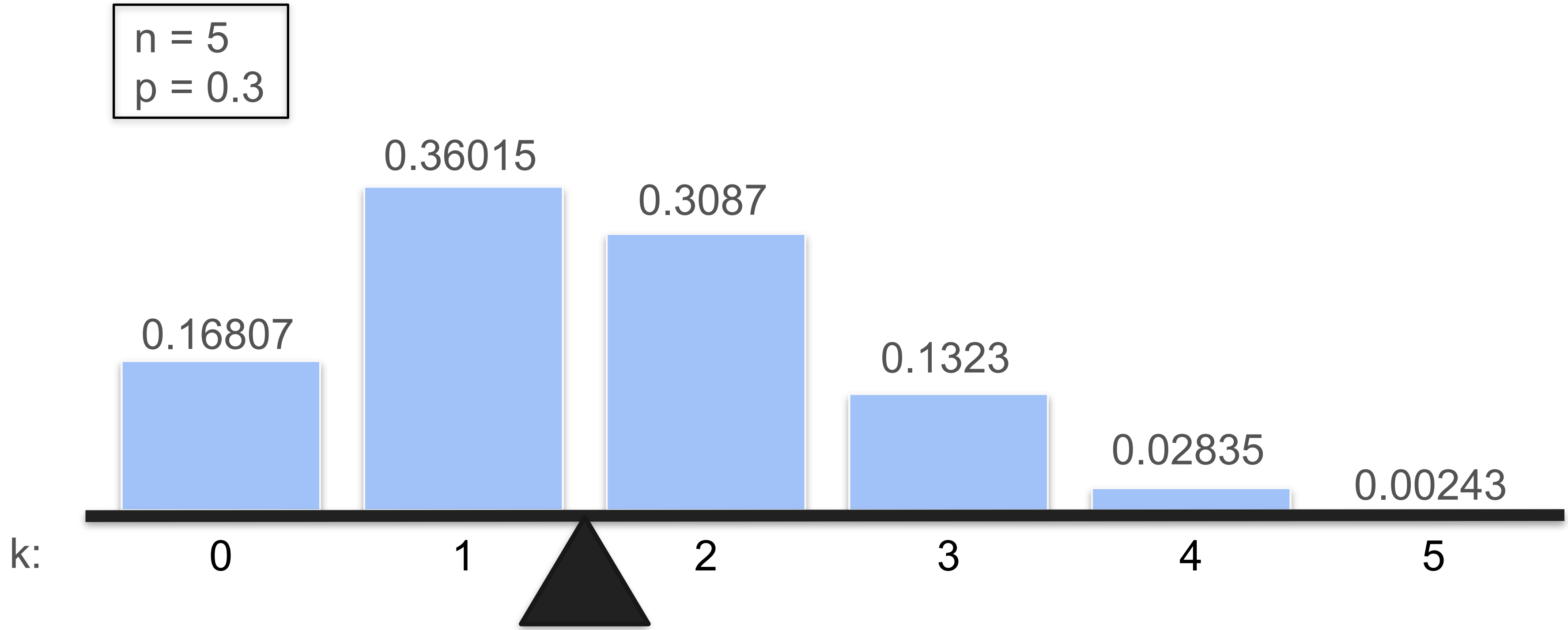

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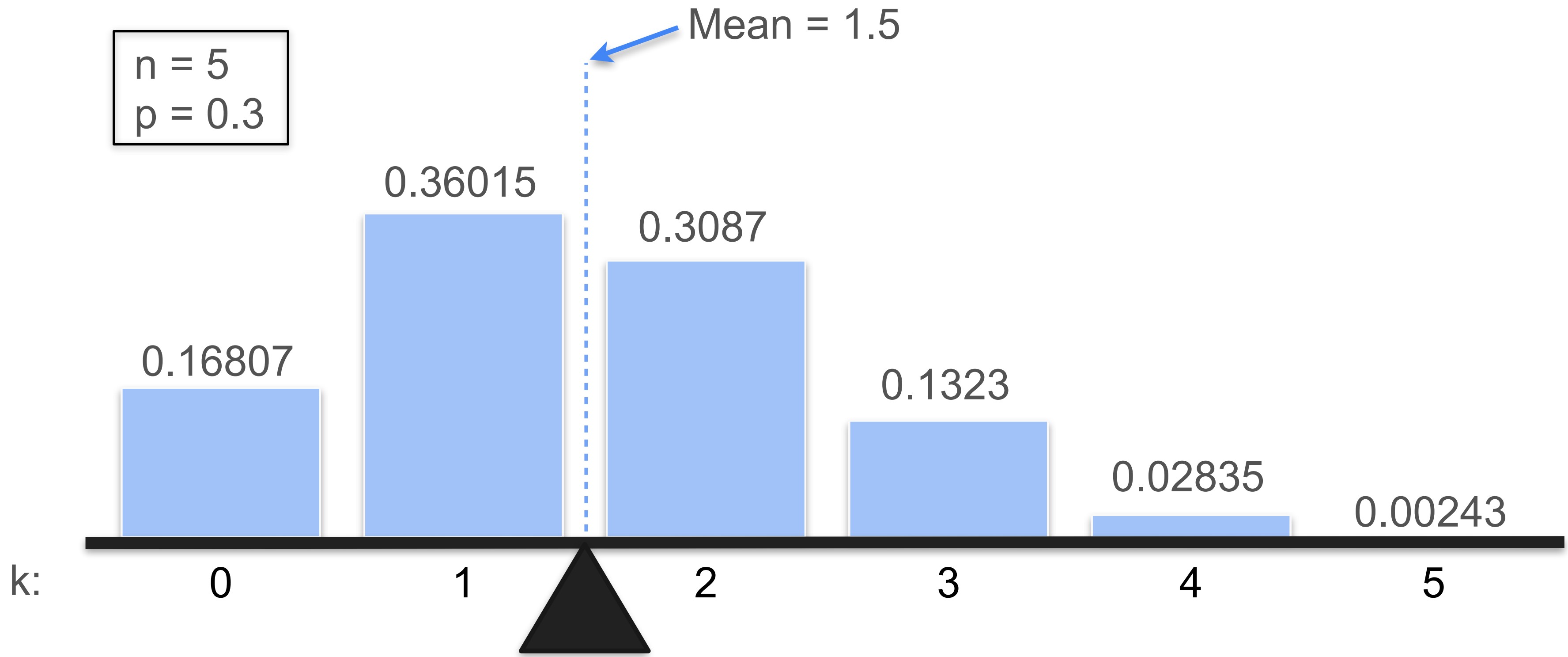

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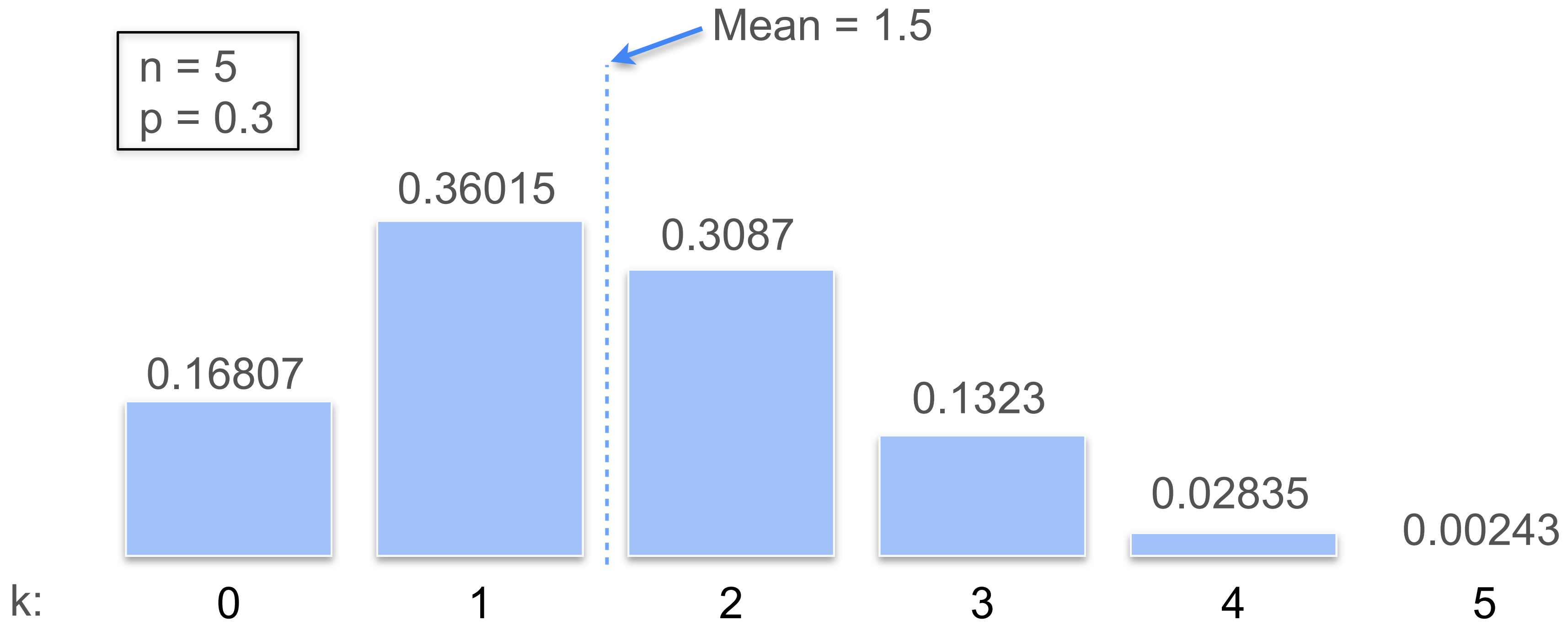

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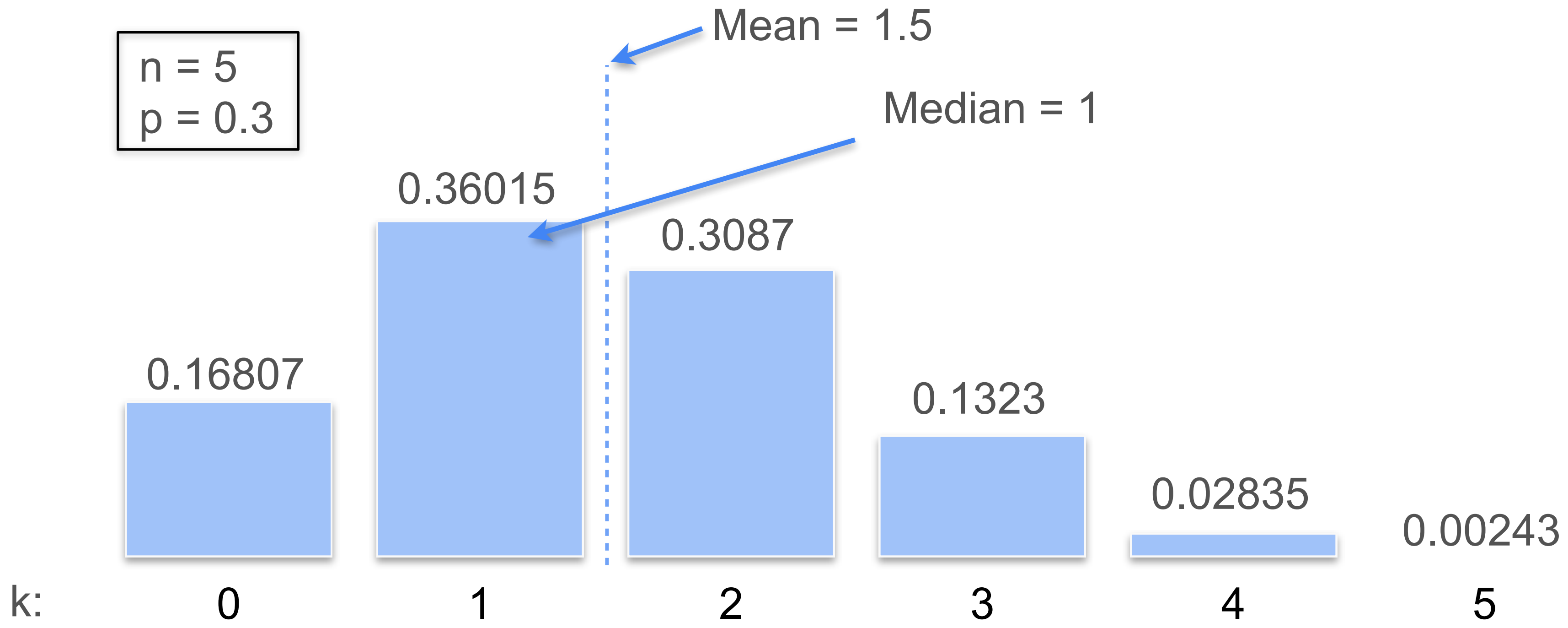

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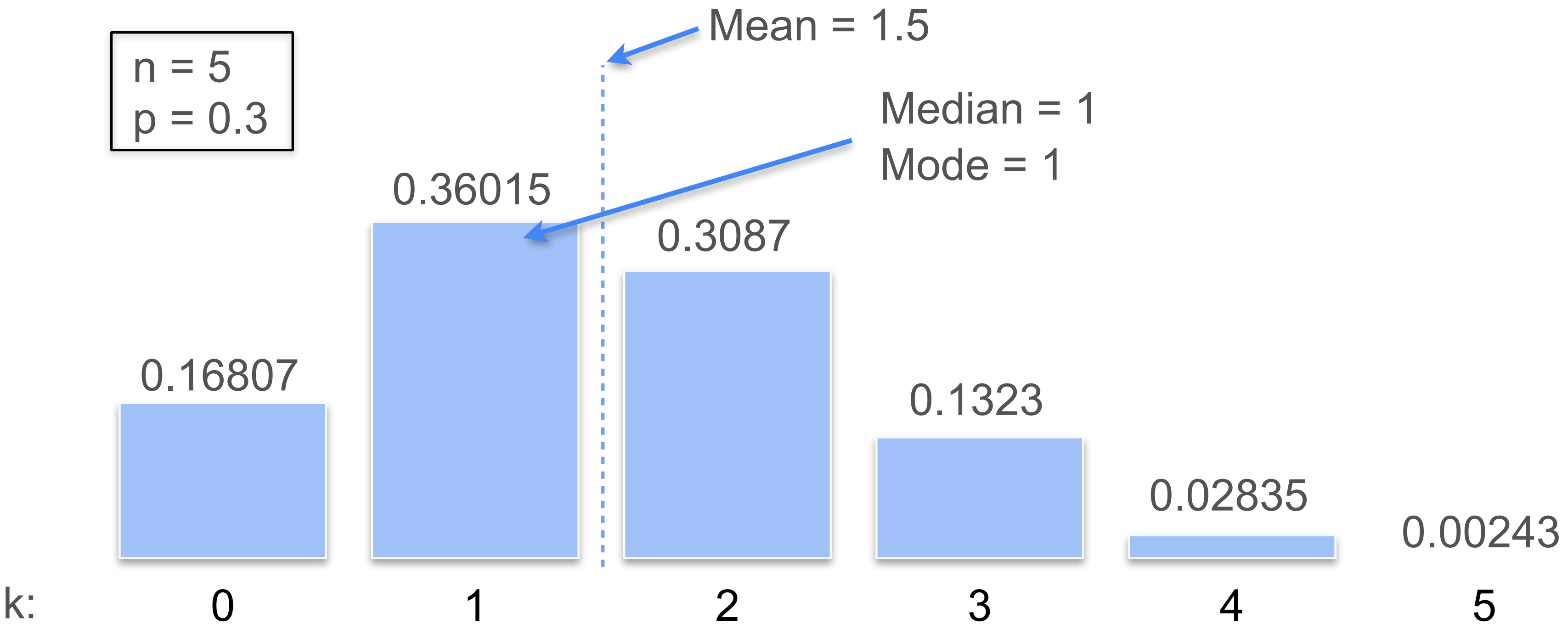
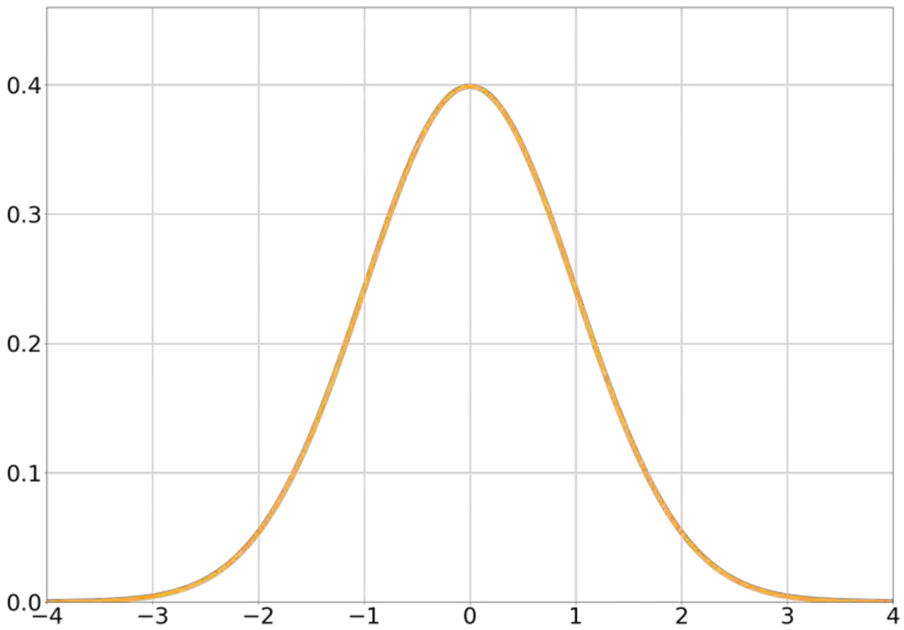
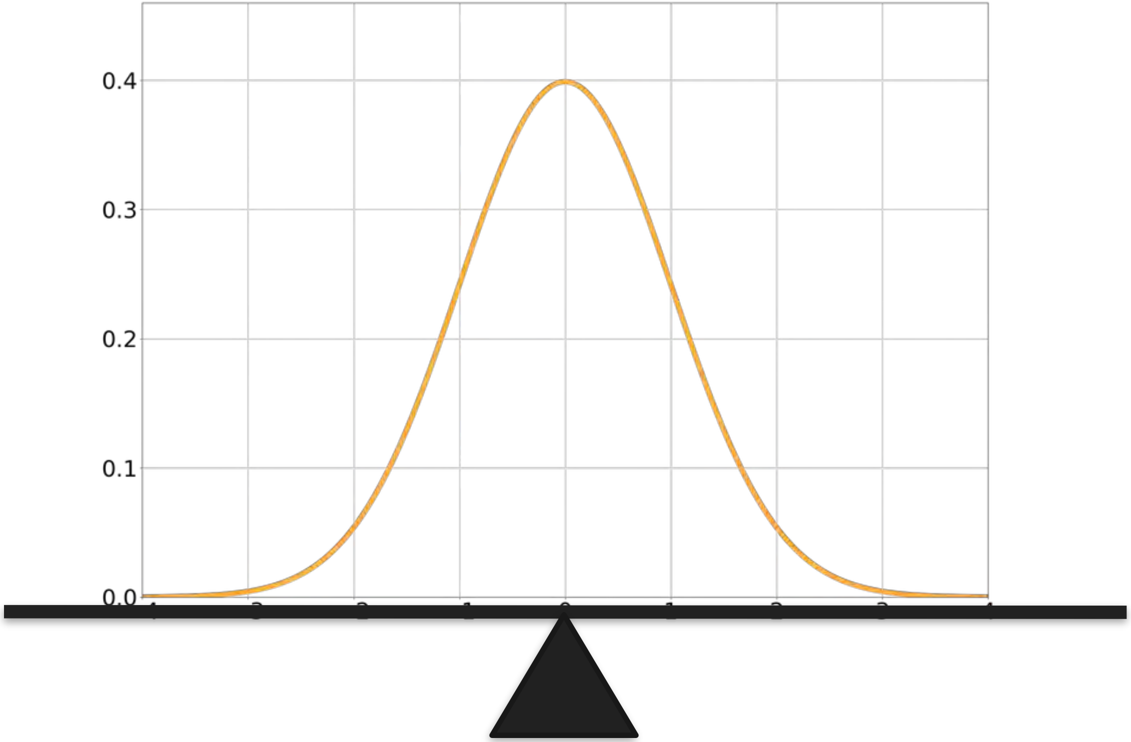


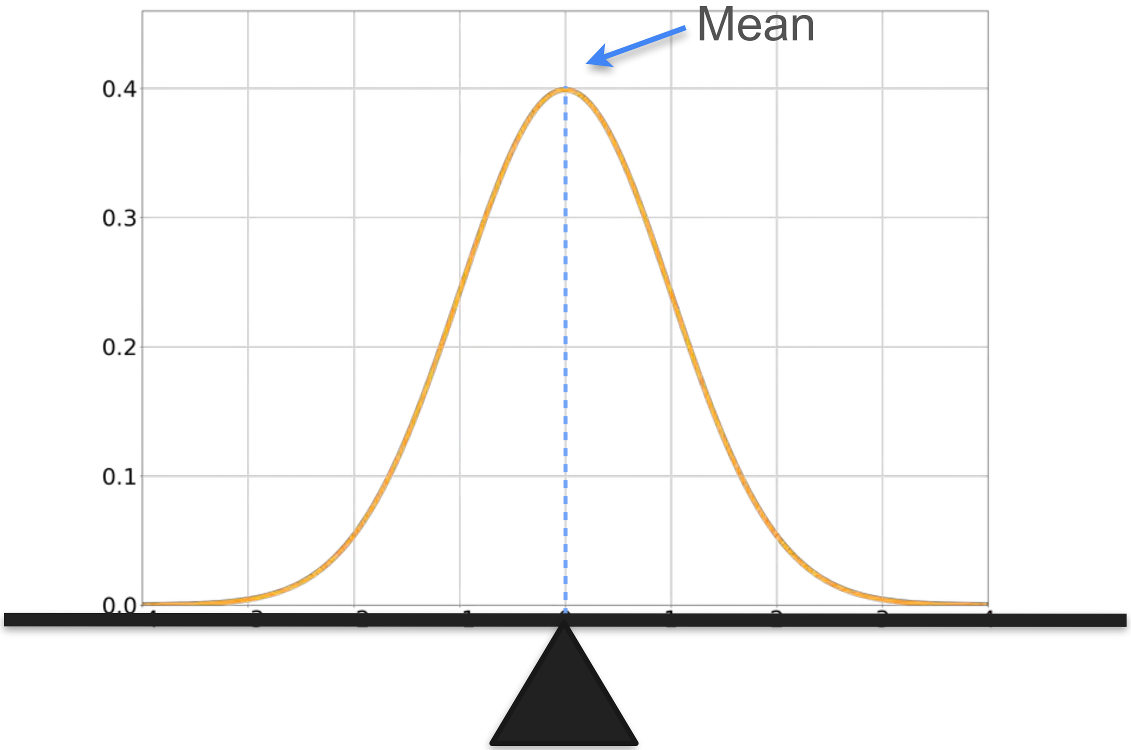
Mean, Median and Mode in Normal Distribution



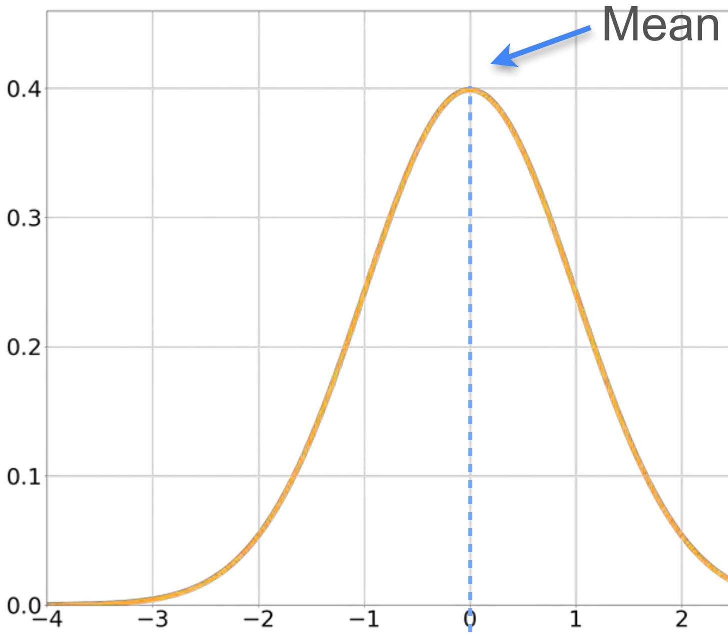
Mean, Median and Mode in Normal Distribution



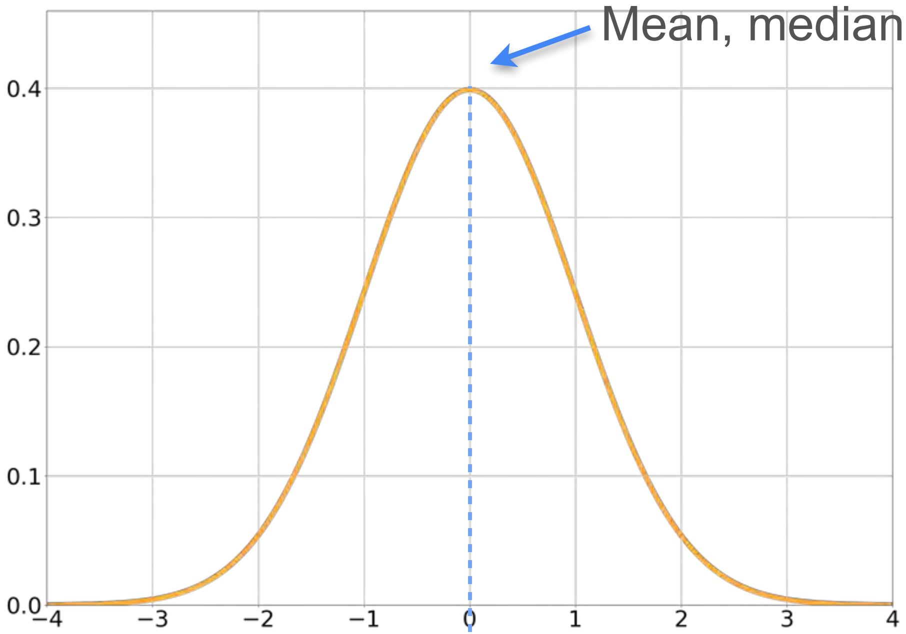
Mean, Median and Mode in Normal Distribution



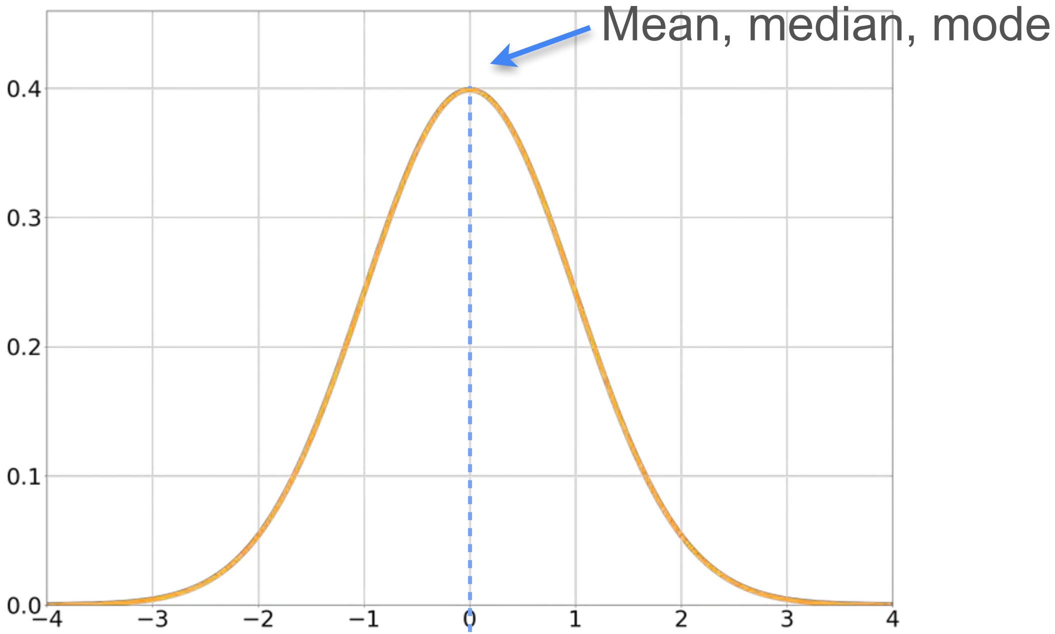
Mean, Median and Mode in Normal Distribution



Mean, Median and Mode in Normal Distribution



Mean, Median and Mode in Normal Distribution



Describing Distributions

Expected Value

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Game cost:

$6

Do you play the game?

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Game cost:

$4

Do you play the game?

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Game cost:

$4

Do you play the game?

What is the maximum amount of money you would pay to play this game?

Expected Value: Motivation Example 1

You play a game with a friend

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You win nothing

Expected Value: Motivation Example 1

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Long term:

Expected Value: Motivation Example 1

You play a game with a friend

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Long term: $0.5 \cdot \$10$

Expected Value: Motivation Example 1

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Long term: $0.5 \cdot \$10 \ +0.5 \cdot \$0$

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Long term: $0.5 \cdot \$10 \ +0.5 \cdot \$0 = \$5$

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You play a game with a friend

You win 10 dollars

You win nothing

Long term: $0.5 \cdot \$10 \ +0.5 \cdot \$0 = \$5$ You expect to win $\$5$ on average

$\mathbb{E}[X] = 5$

Expected Value: Motivation Example 1

You play a game with a friend

You win 10 dollars

You win nothing

Game cost:

$5

Long term: $0.5 \cdot \$10 \ \ + 0.5 \cdot \$0 = \$5$

You expect to win $\$5$ on average

$\mathbb{E}[X] = 5$

Expected Value: Motivation Example 1

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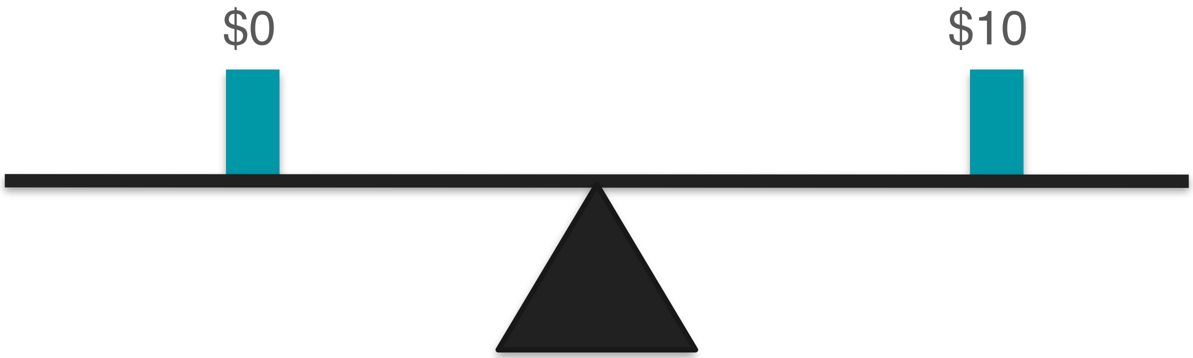
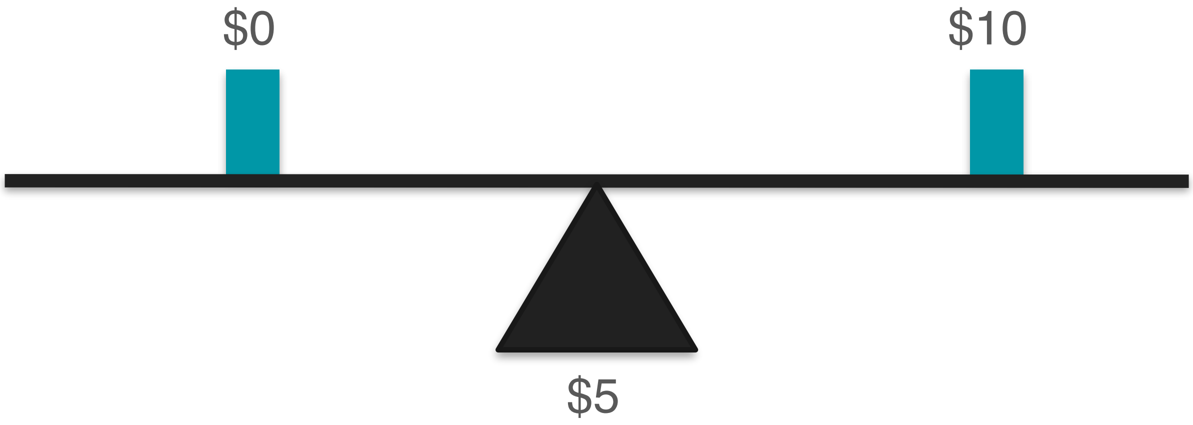
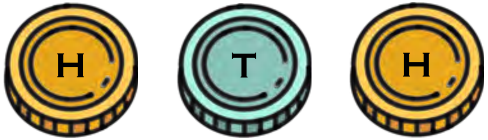


Expected Value: Motivation Example 1



Expected Value: Motivation Example 2

Play another game

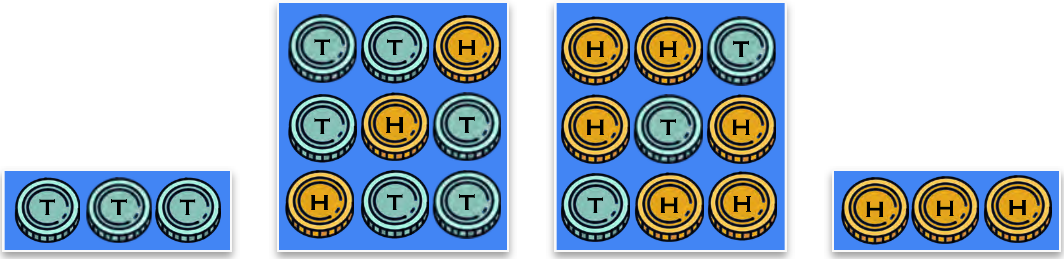


Flip three coins. For each heads you win $1

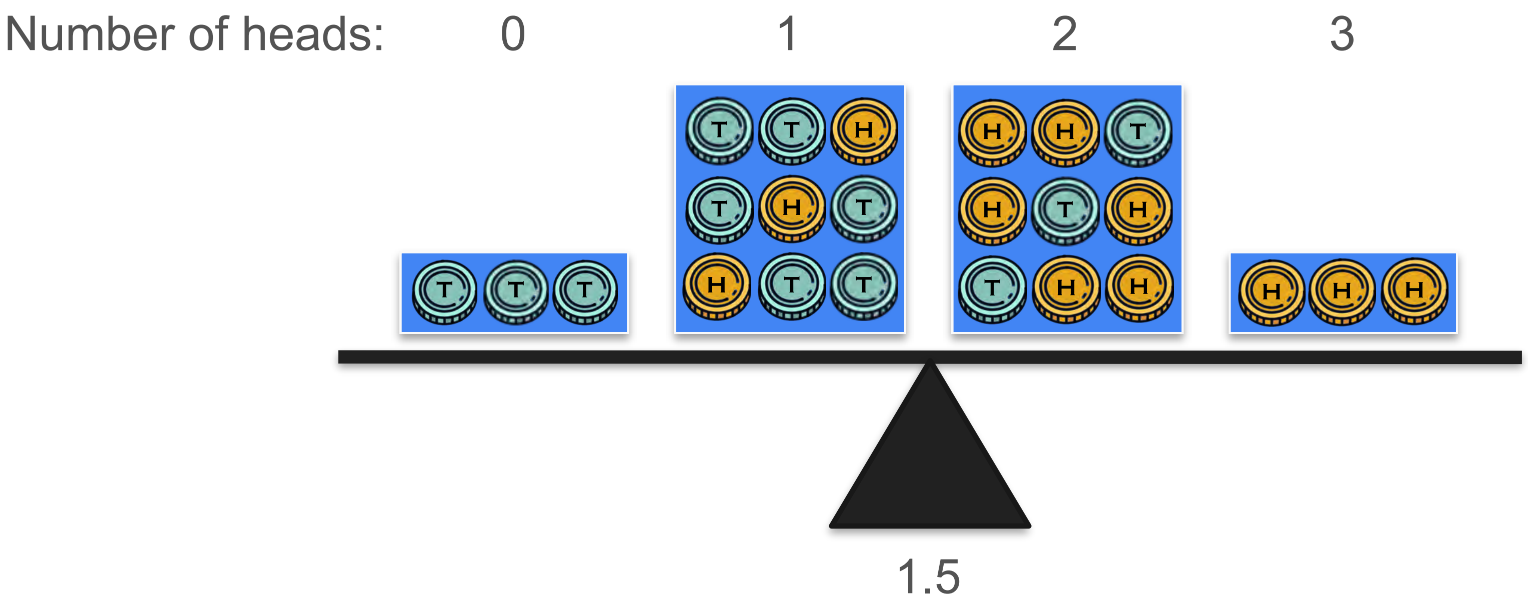
What is the maximum amount of money you would pay to play this game?

Expected Value: Motivation Example 2

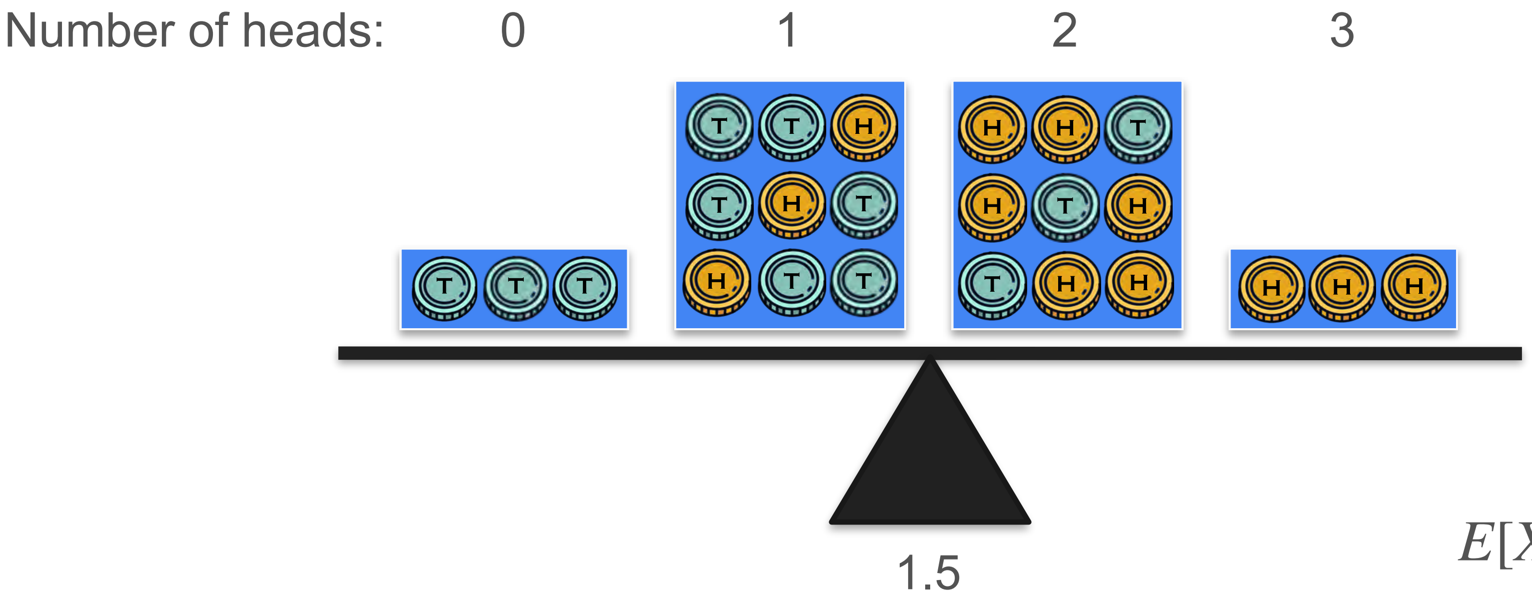
Number of heads: 0 1 2 3



Expected Value: Motivation Example 2



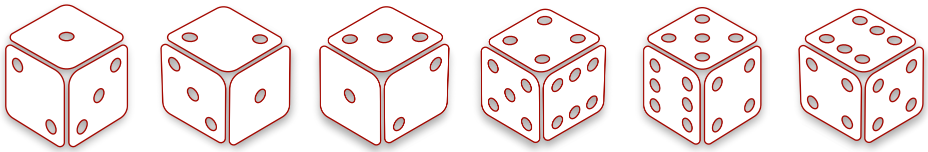
Expected Value: Motivation Example 2



$$E[X] = 1.5$$

Expected Value: Motivation Example 3

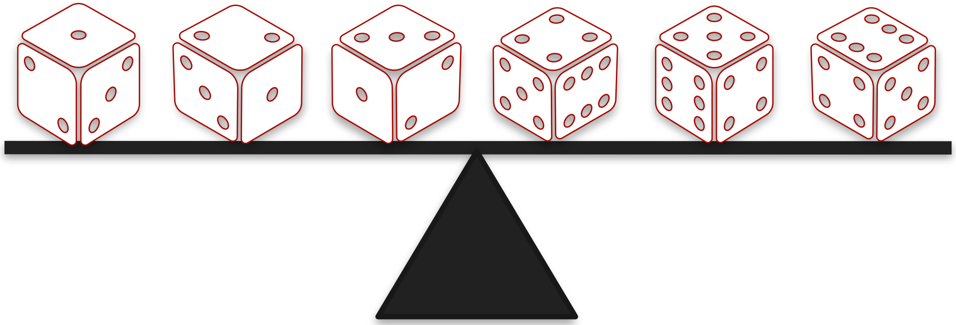
Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6



$$\frac{1+2+3+4+5+6}{6} = 3.5$$

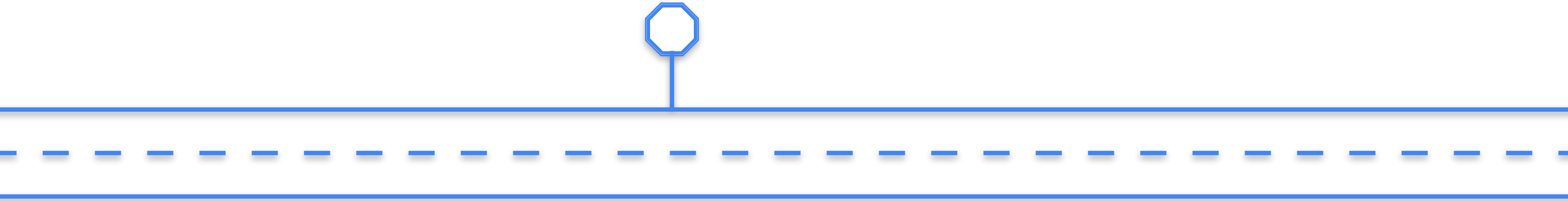
Expected Value: Motivation Example 3

Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6

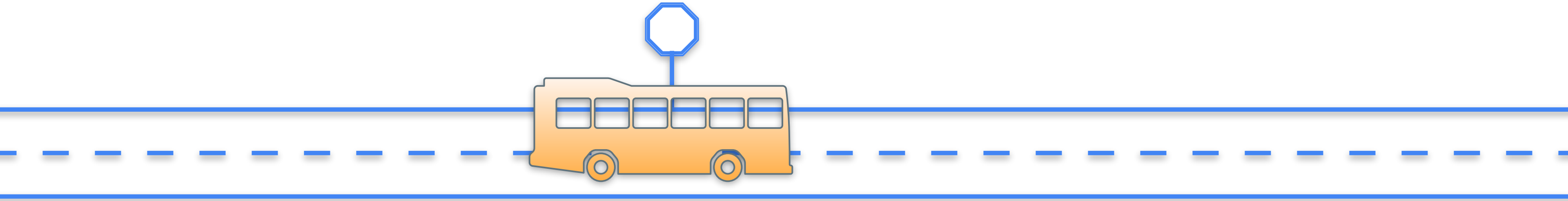


3.5

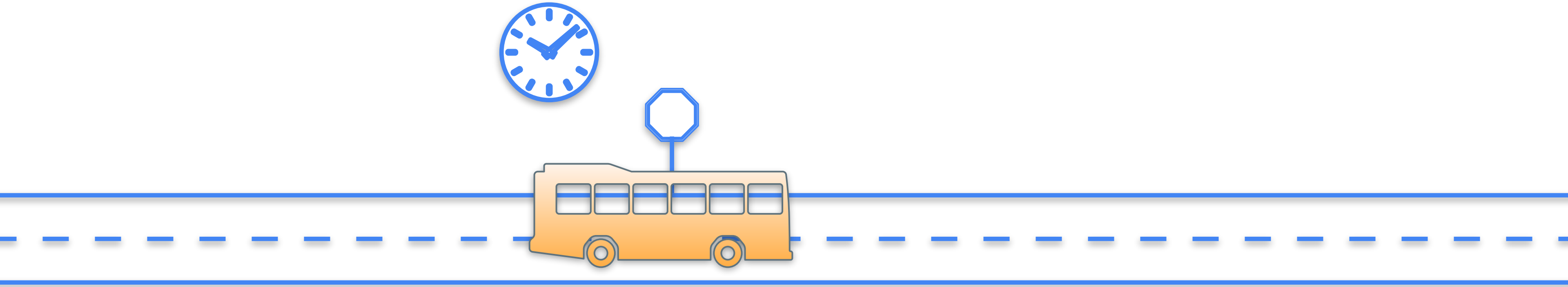
Expected Value



Expected Value

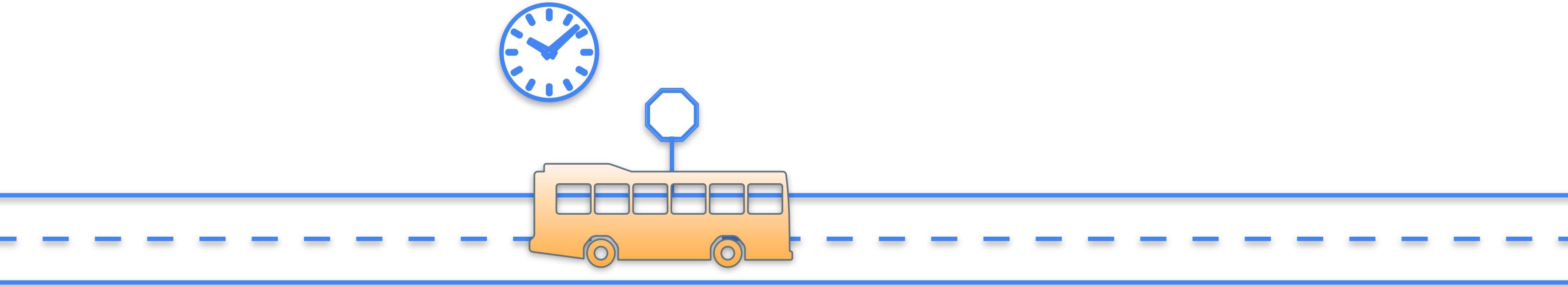


Expected Value



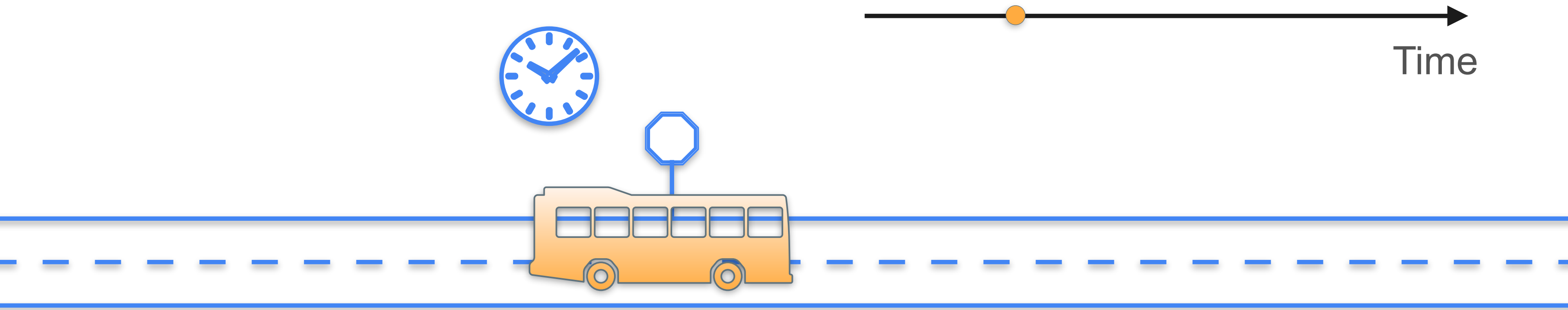
Expected Value

Waiting Time
15 min



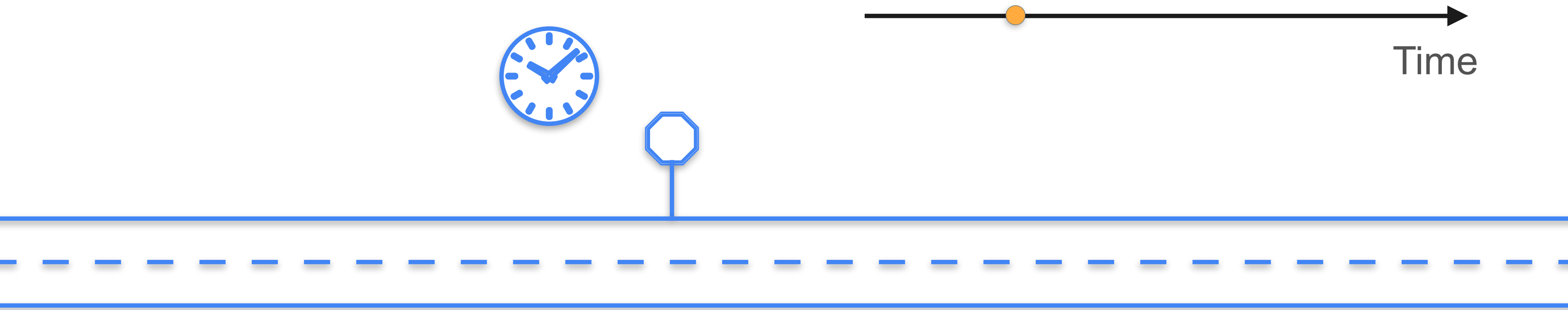
Expected Value

Waiting Time
15 min



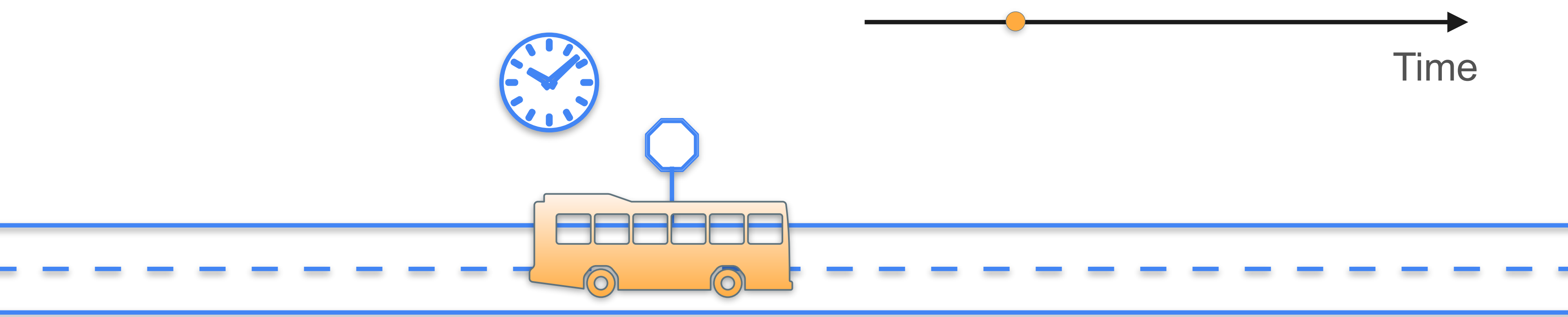
Expected Value

Waiting Time
15 min



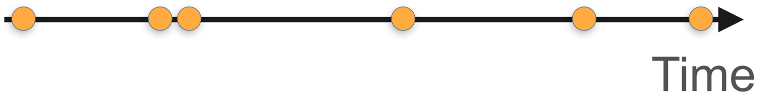
Expected Value

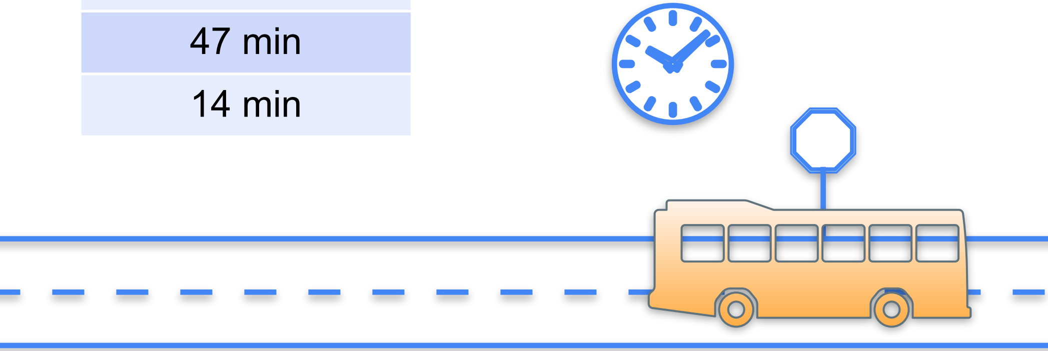
Waiting Time
15 min



Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min

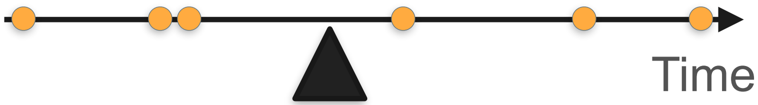


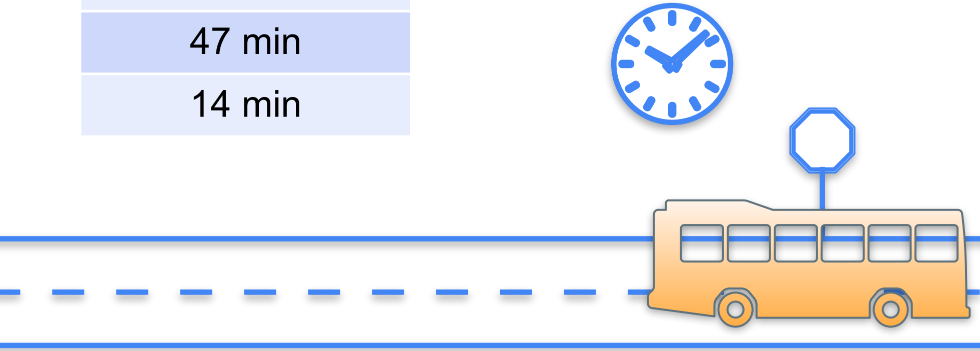


Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min

Average = 27.833

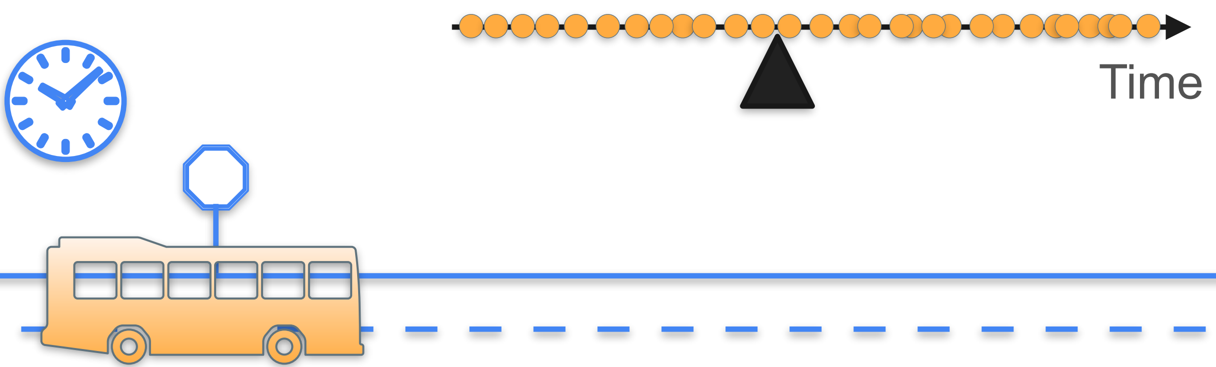




Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min
37 min
8 min
29 min
55 min
…

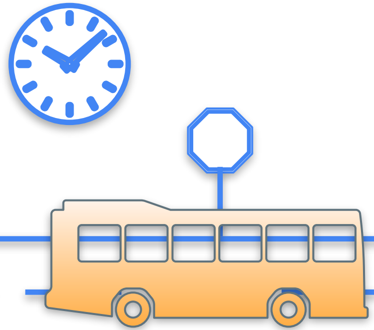
Average = 27.833

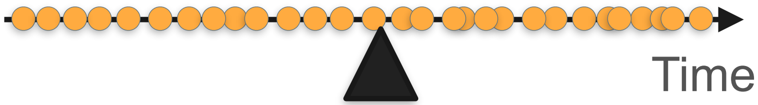


Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min
37 min
8 min
29 min
55 min
…

Average = 30

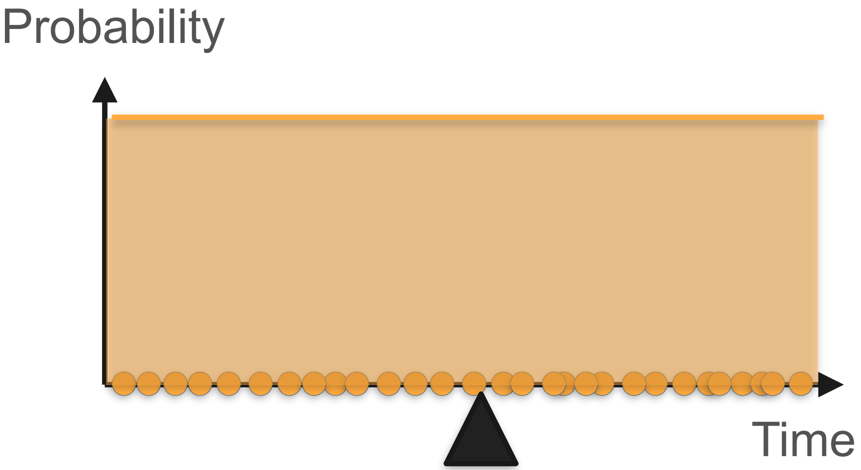


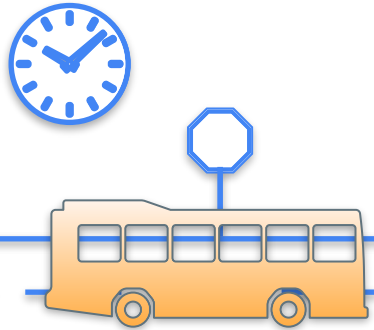


Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min
37 min
8 min
29 min
55 min
...

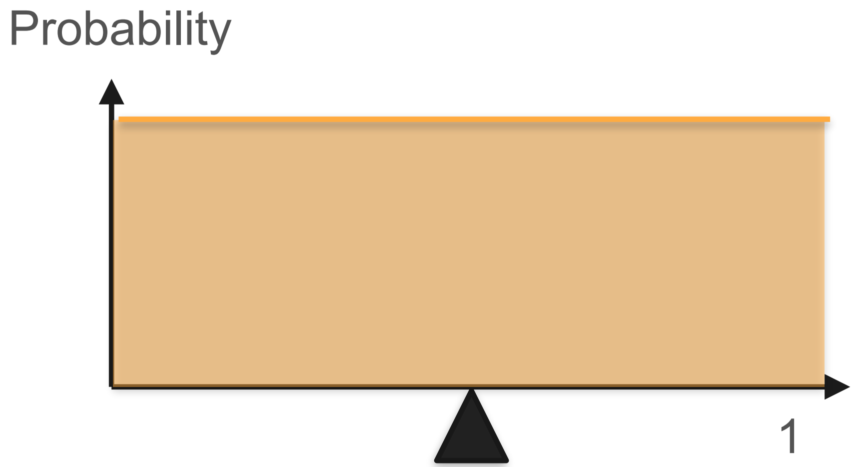
Average = 30





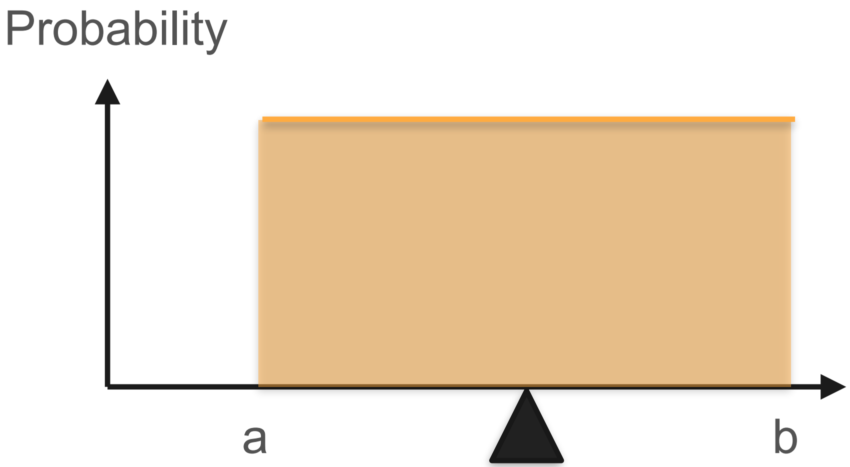
Expected Value: Uniform Distribution

Average = 0.5

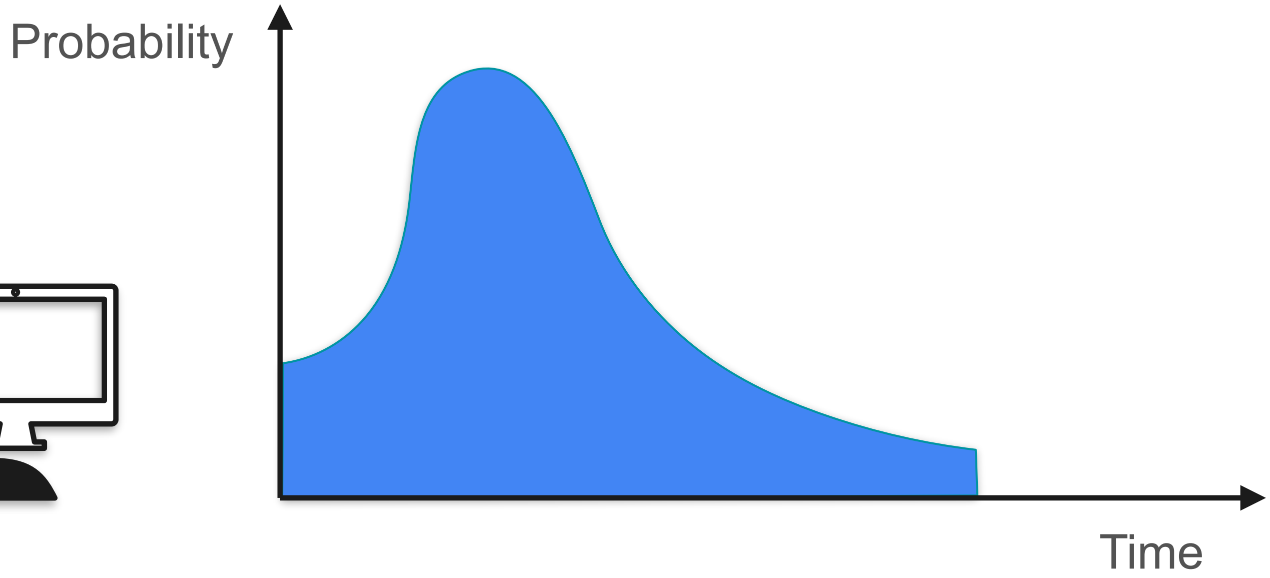


Expected Value: Uniform Distribution

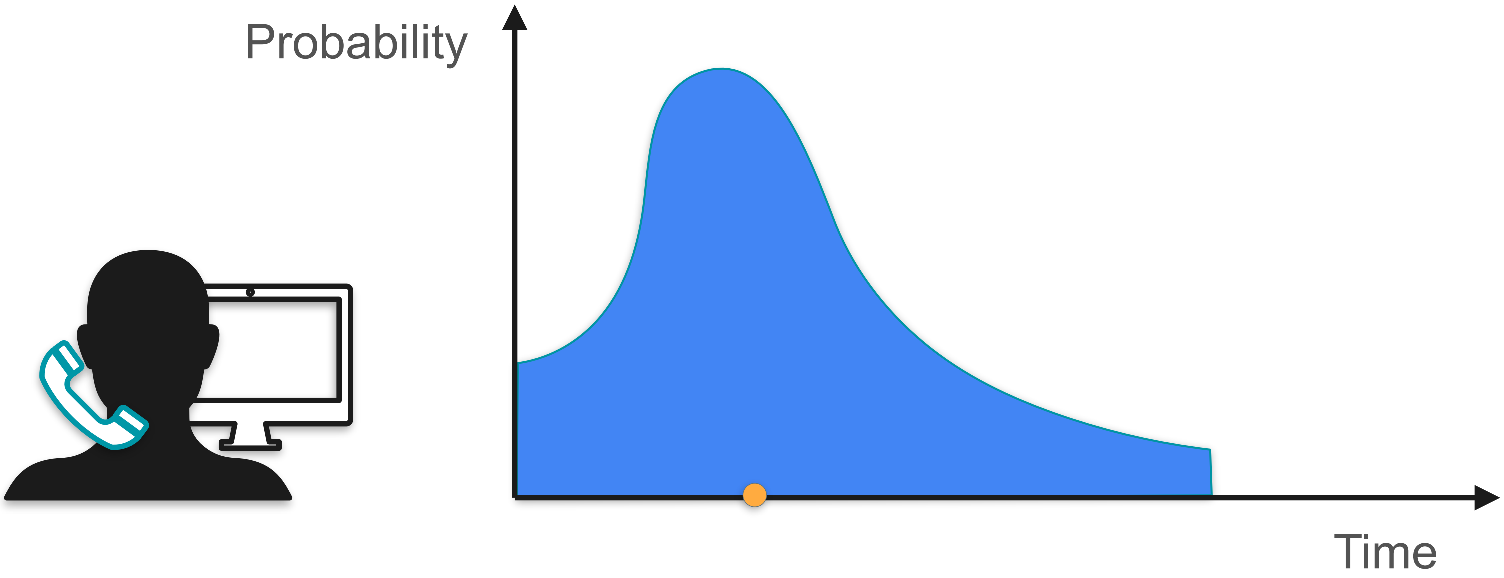
$$\text{Average} = \frac{a+b}{2}$$



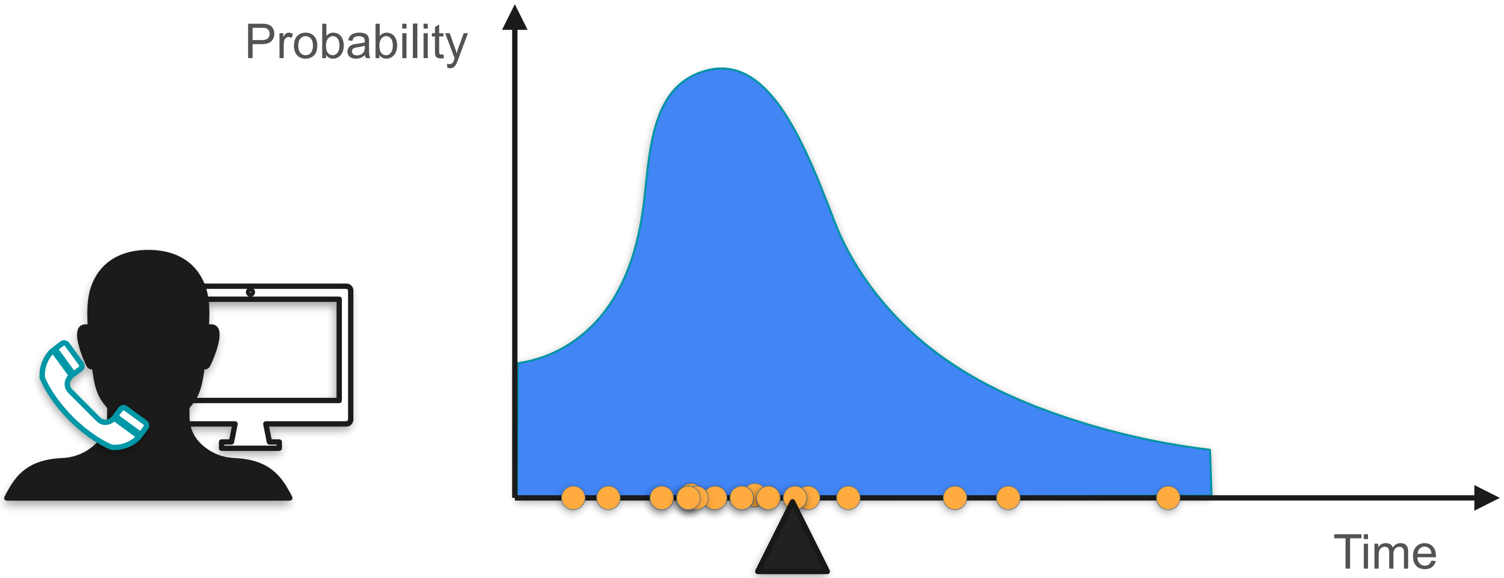
Expected Value



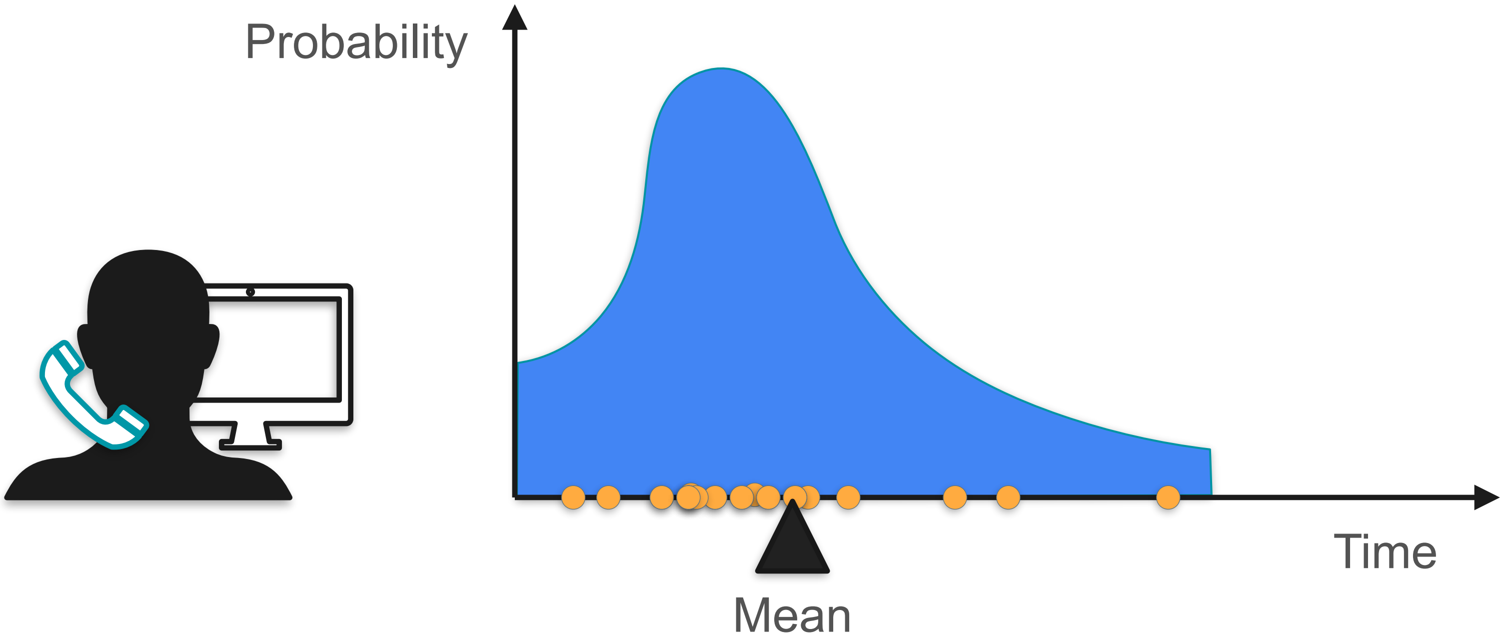
Expected Value



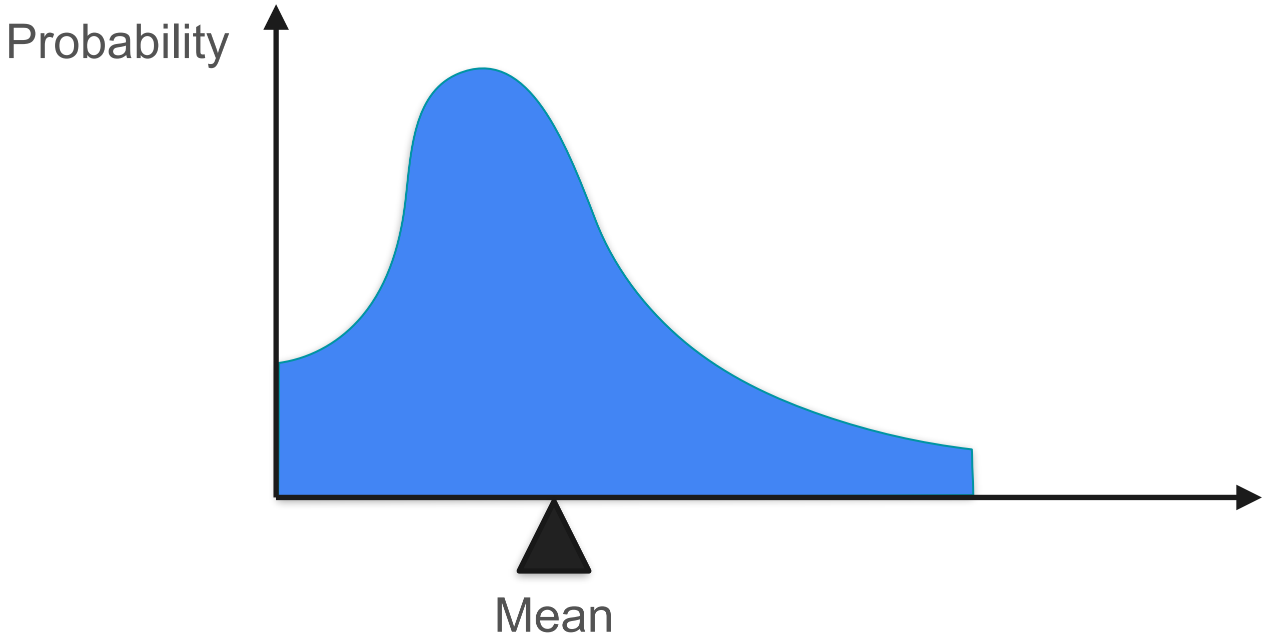
Expected Value



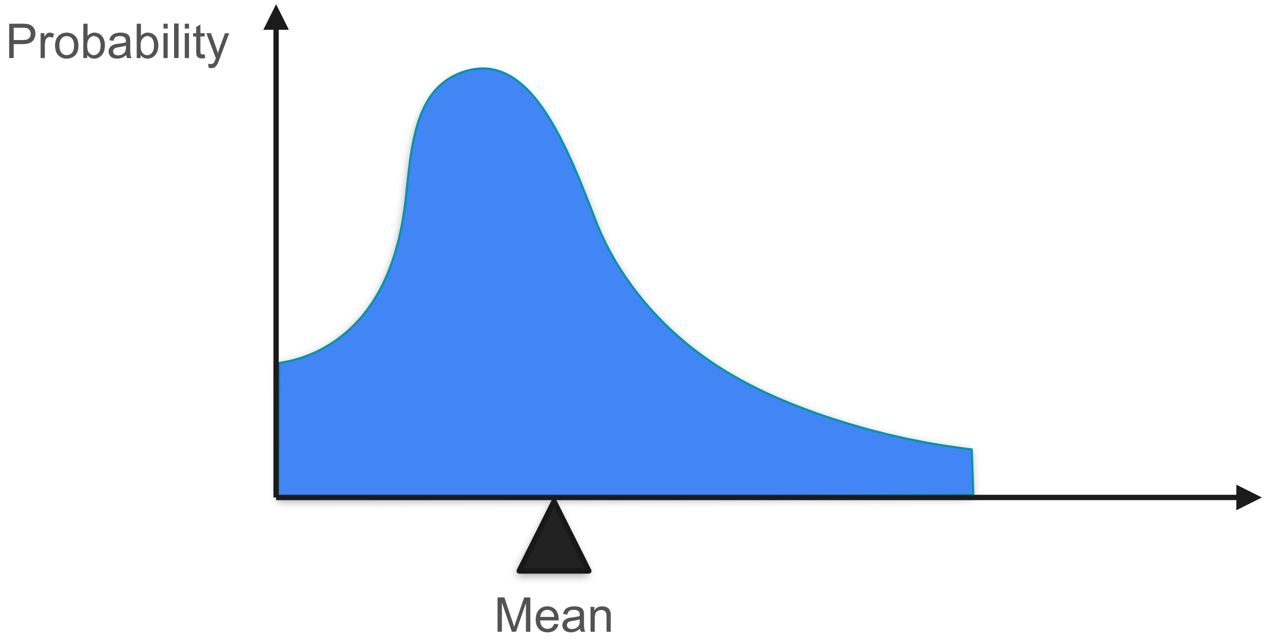
Expected Value



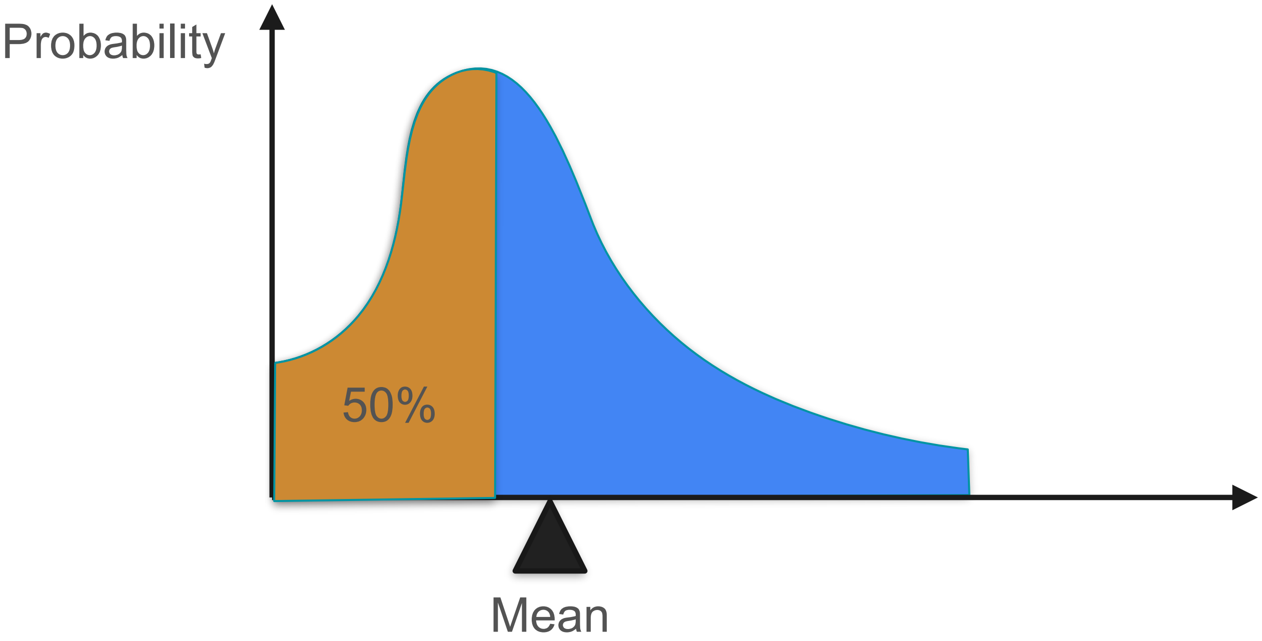
Expected Value: General Case



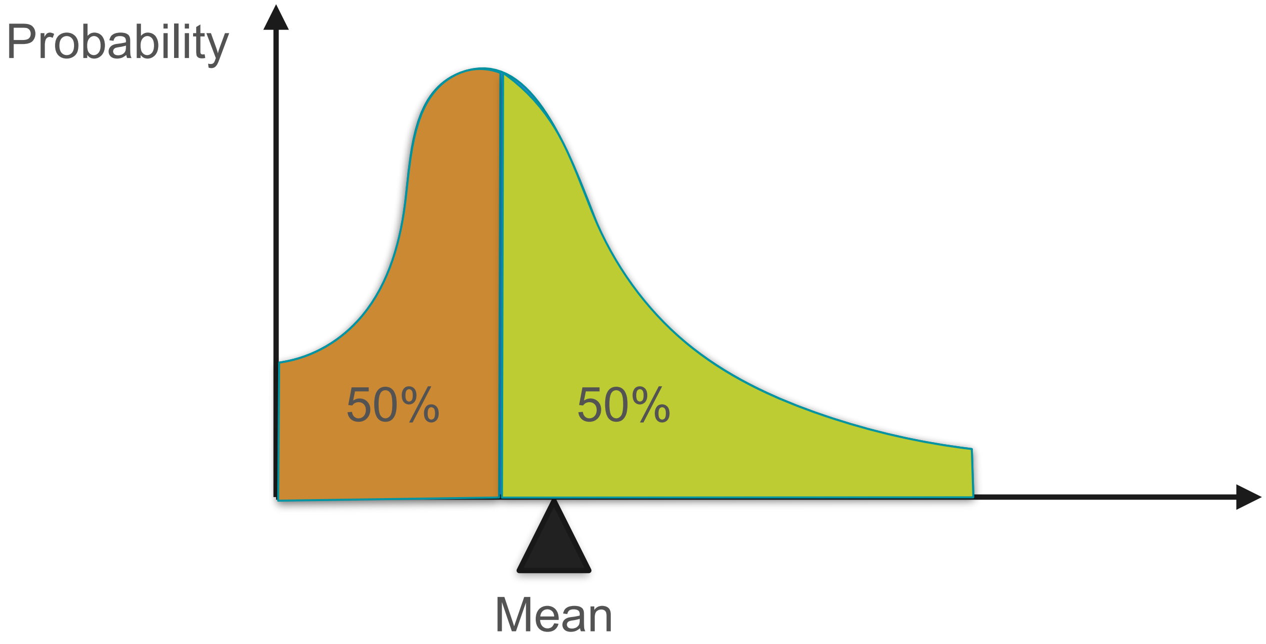
Expected Value: Common Misconception



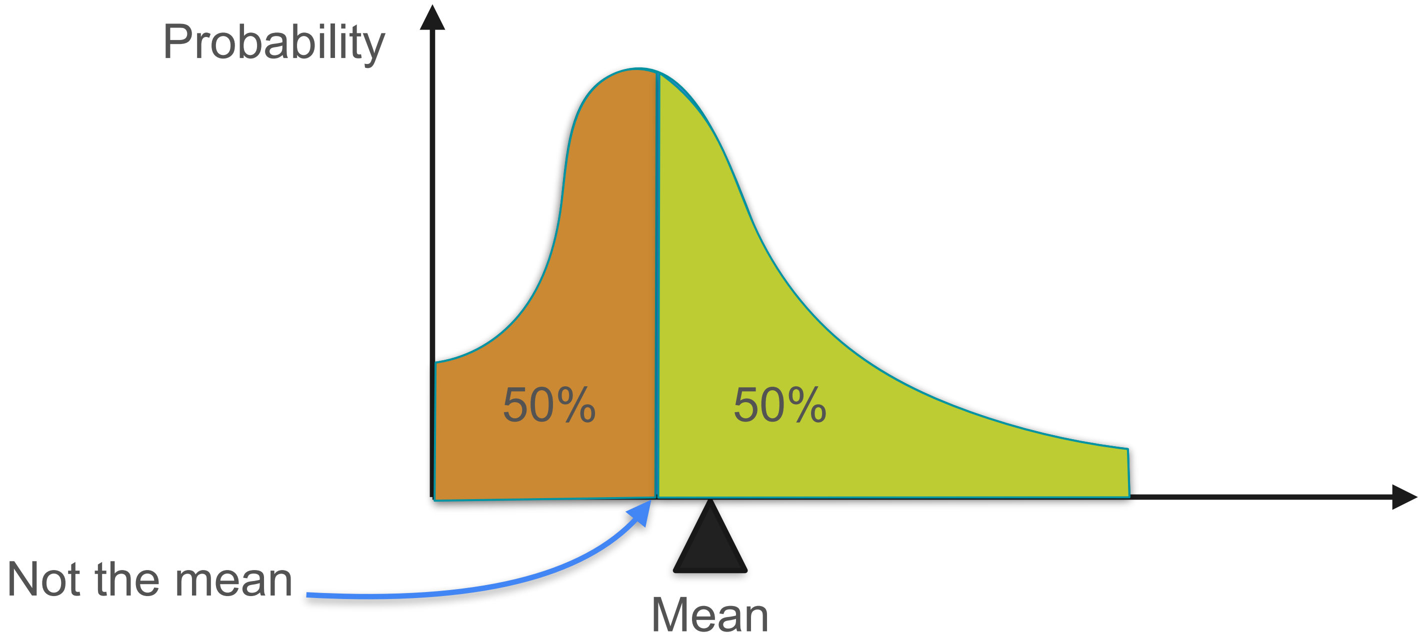
Expected Value: Common Misconception



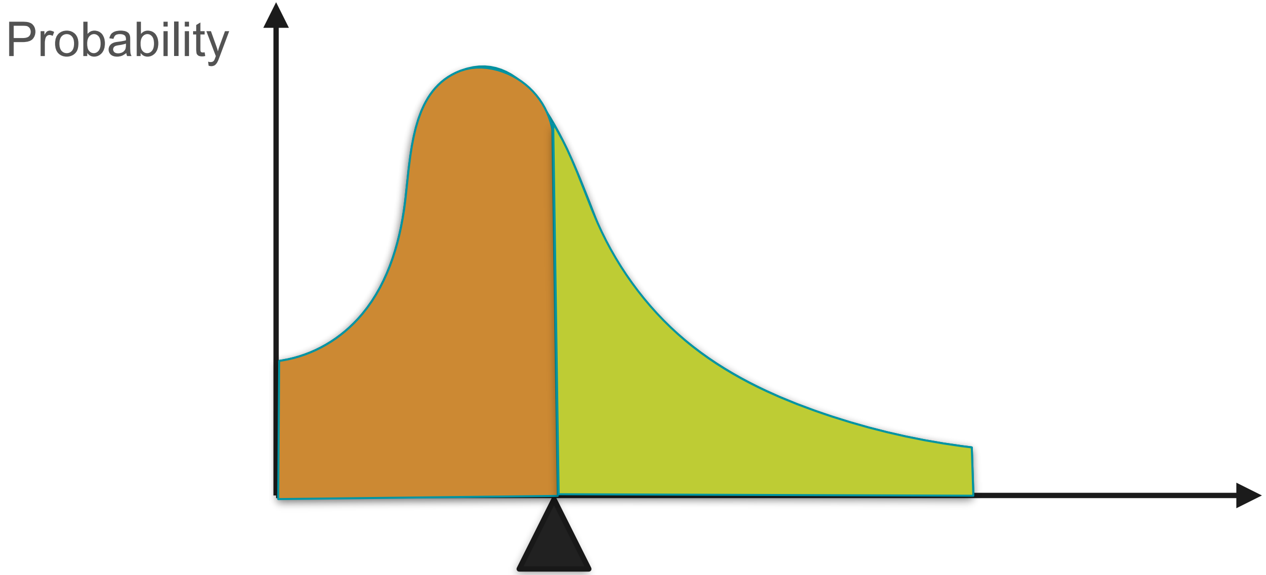
Expected Value: Common Misconception



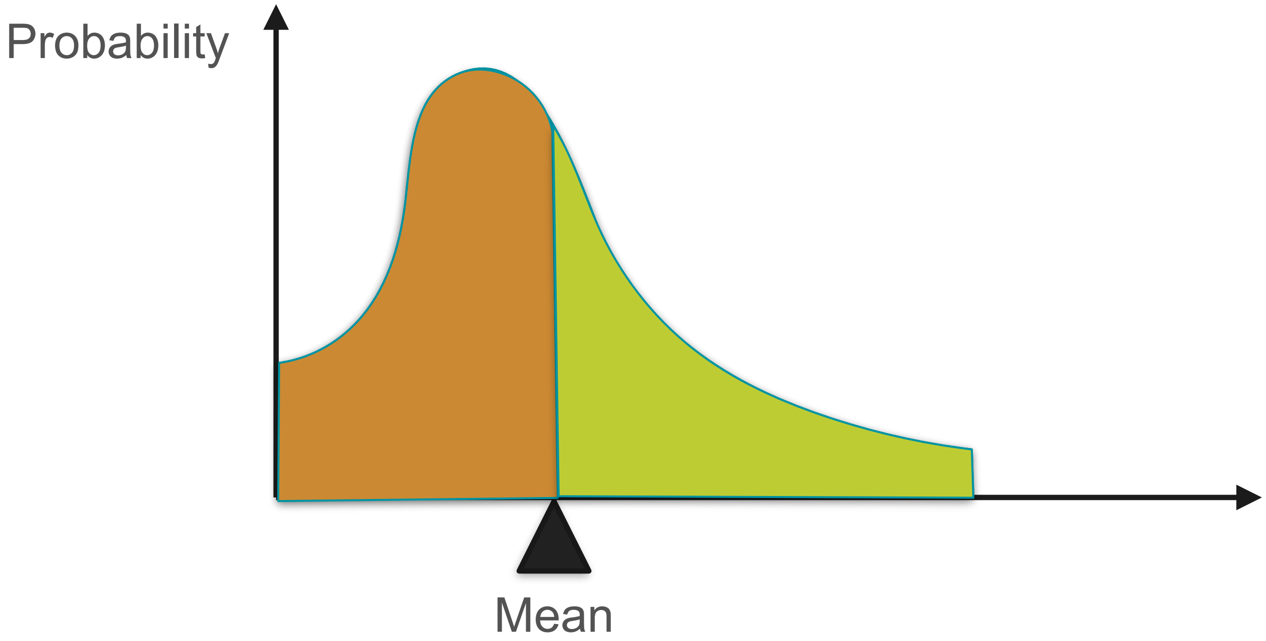
Expected Value: Common Misconception



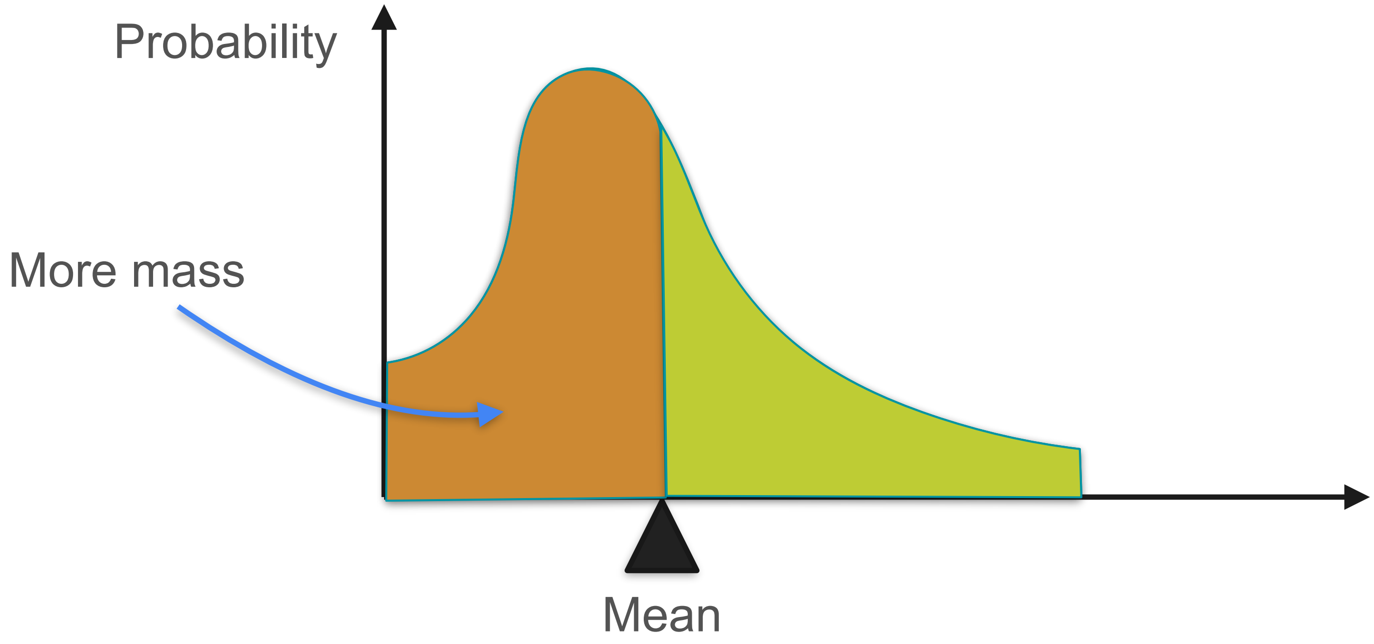
Expected Value: Common Misconception



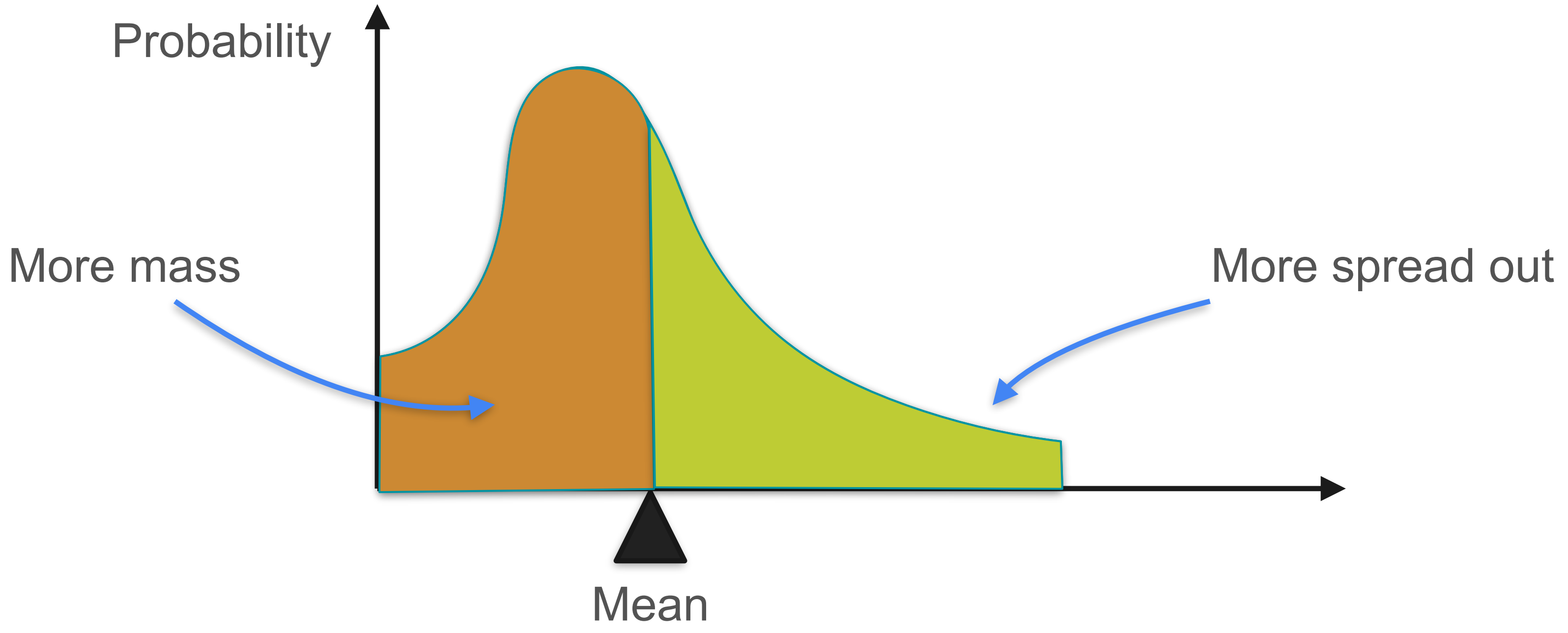
Expected Value: Common Misconception



Expected Value: Common Misconception



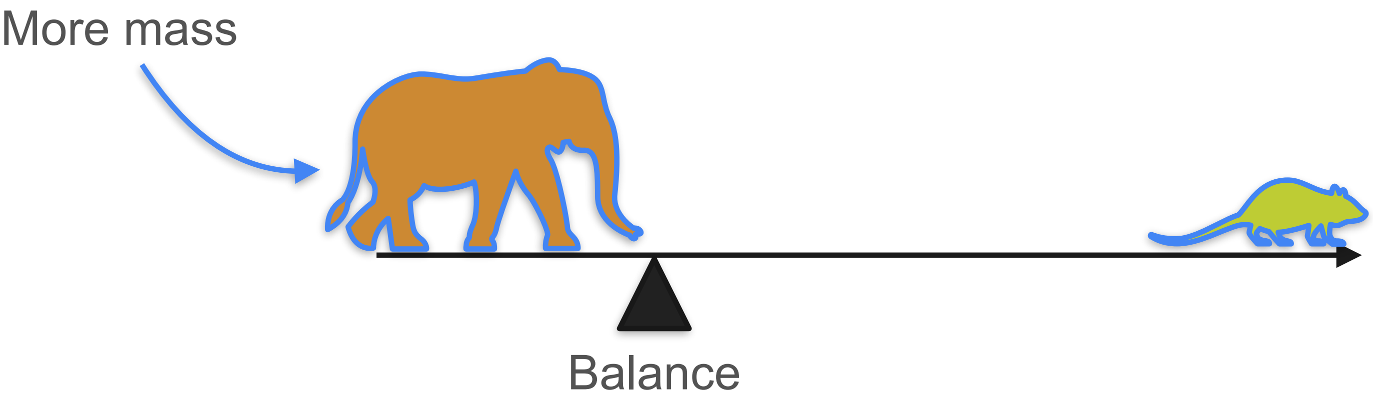
Expected Value: Common Misconception



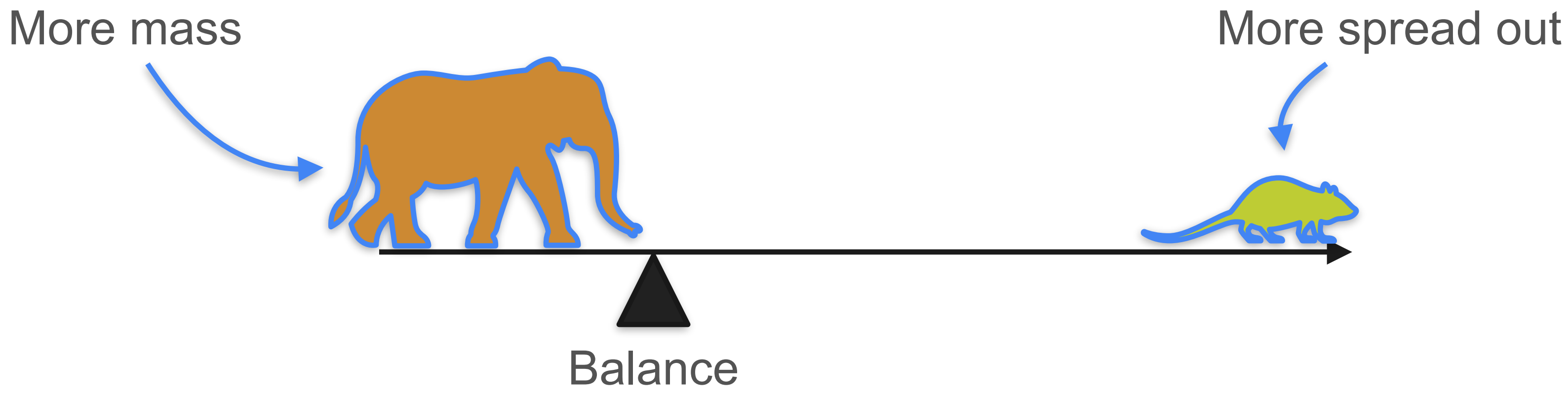
Expected Value: Common Misconception

Expected Value: Common Misconception

Expected Value: Common Misconception



Expected Value: Common Misconception

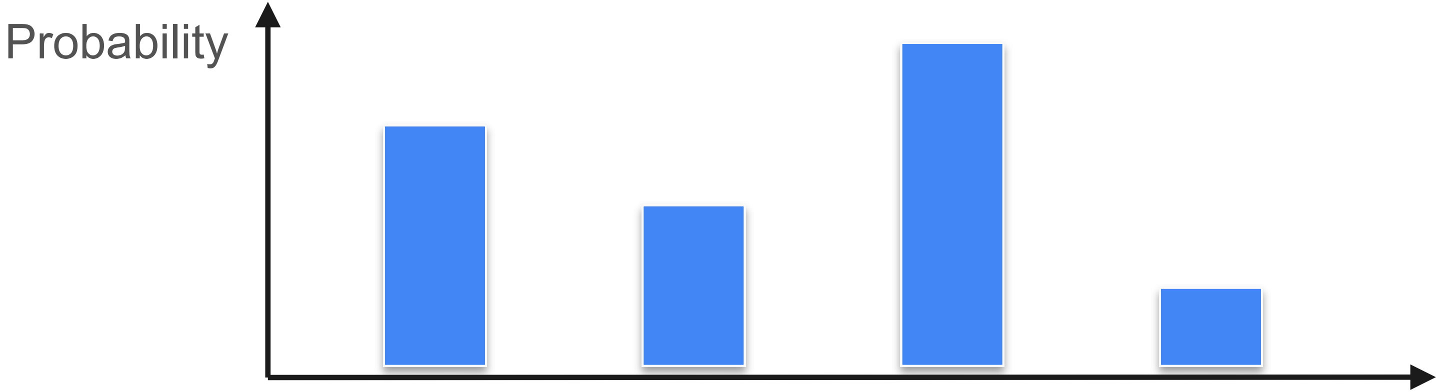


DeepLearning.AI

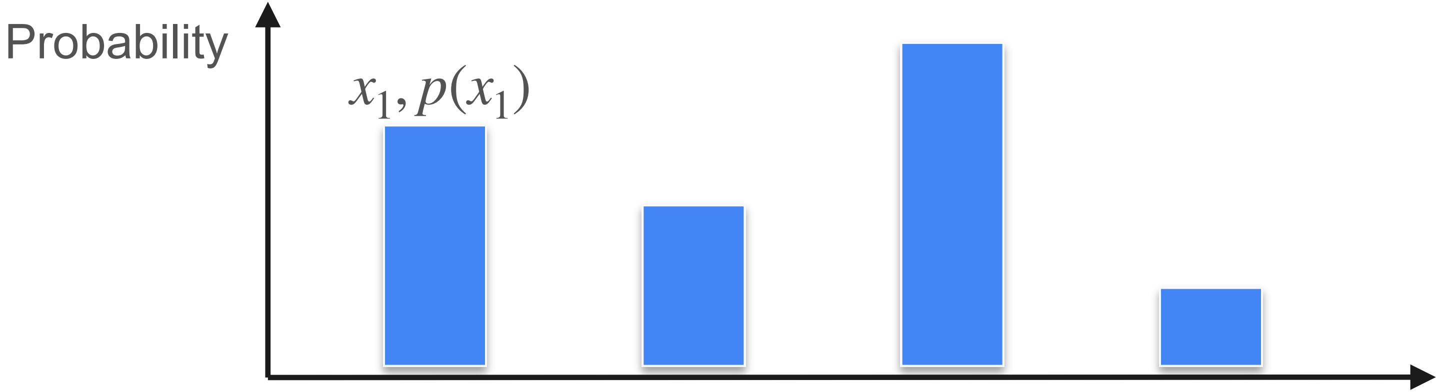
Describing Distributions

Expected value of a function

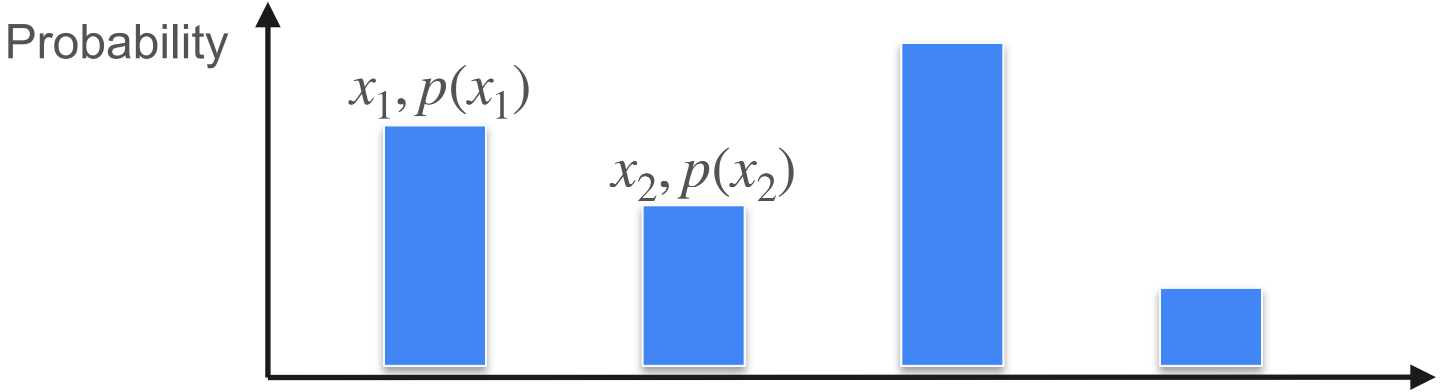
Expected Value of a Function



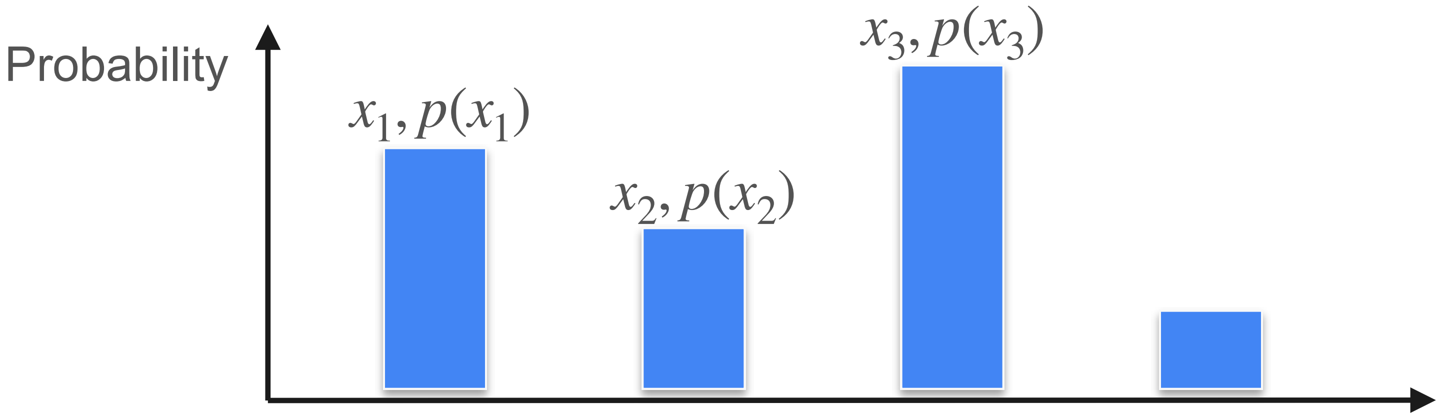
Expected Value of a Function



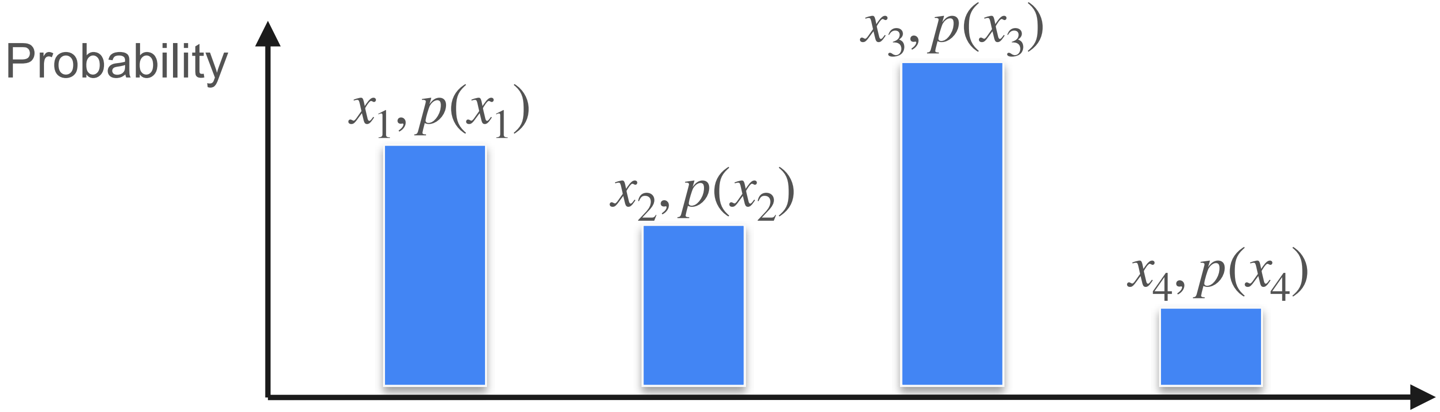
Expected Value of a Function



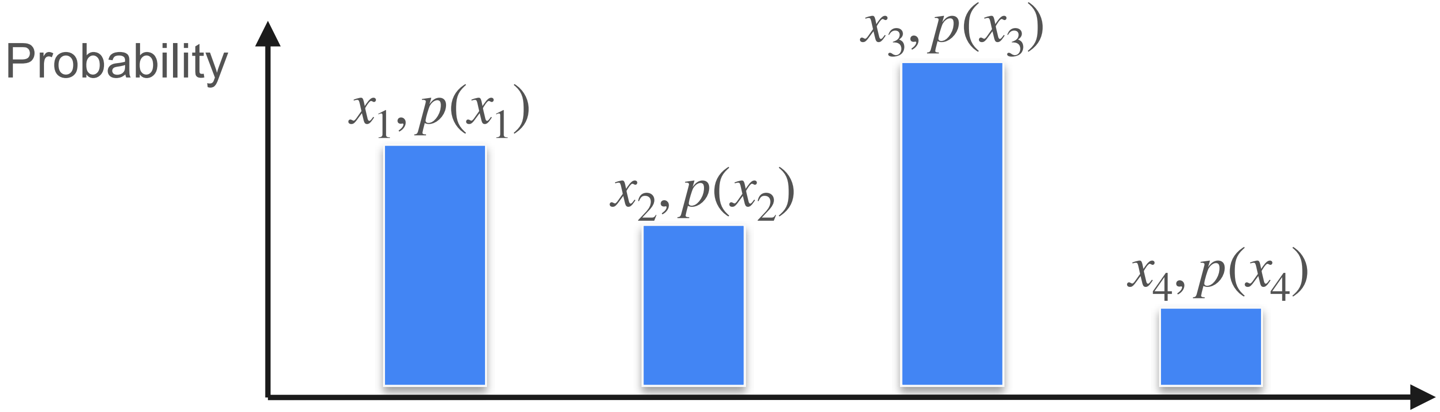
Expected Value of a Function



Expected Value of a Function

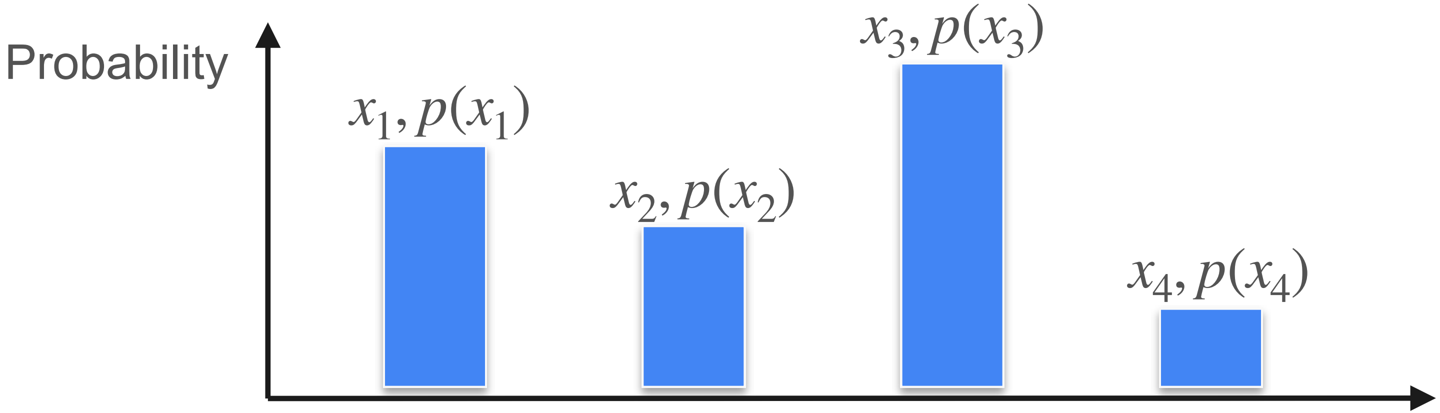


Expected Value of a Function



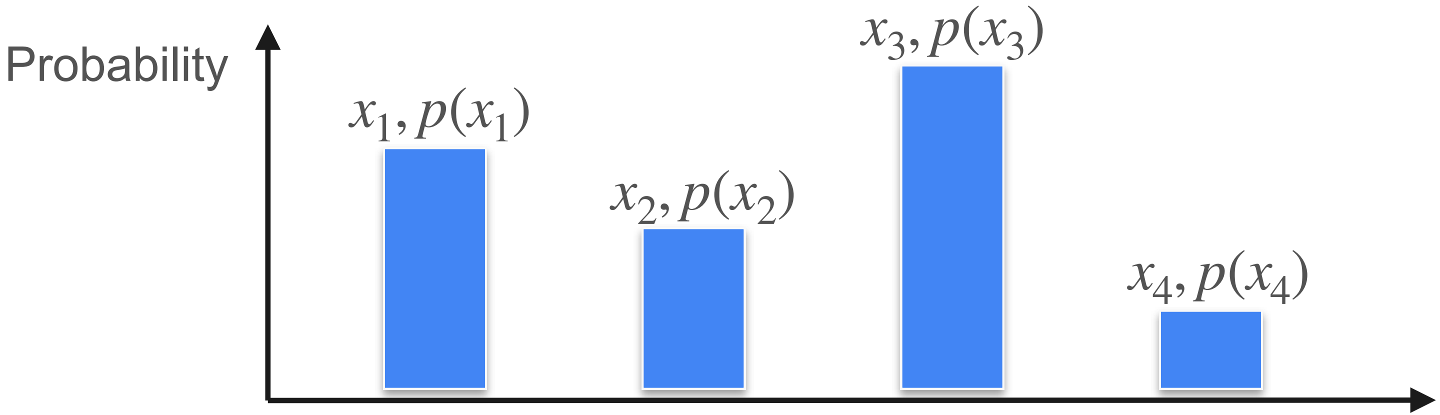
$$\mathbb{E}[X] =$$

Expected Value of a Function



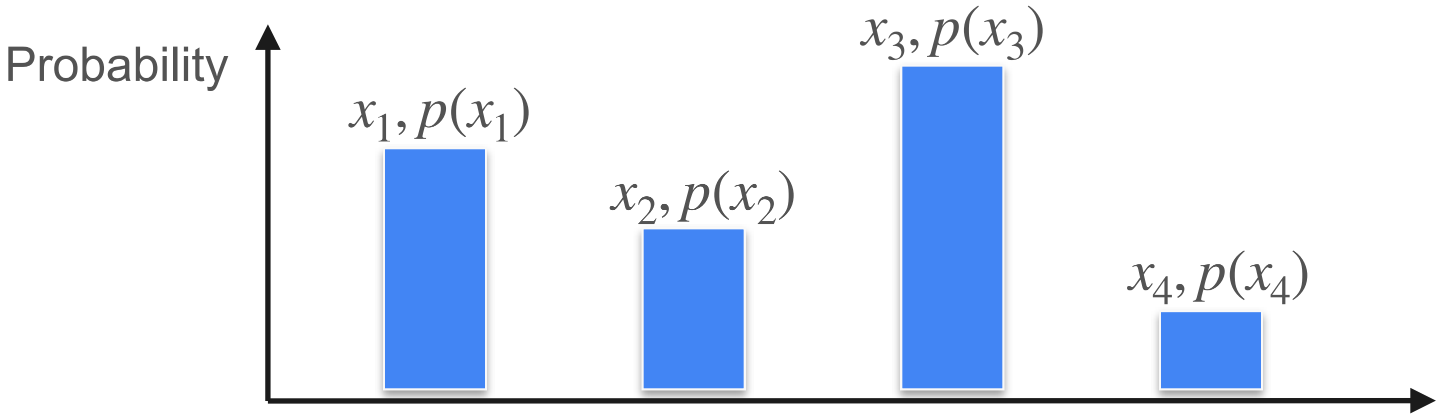
$$\mathbb{E}[X] = x_1 p(x_1)$$

Expected Value of a Function



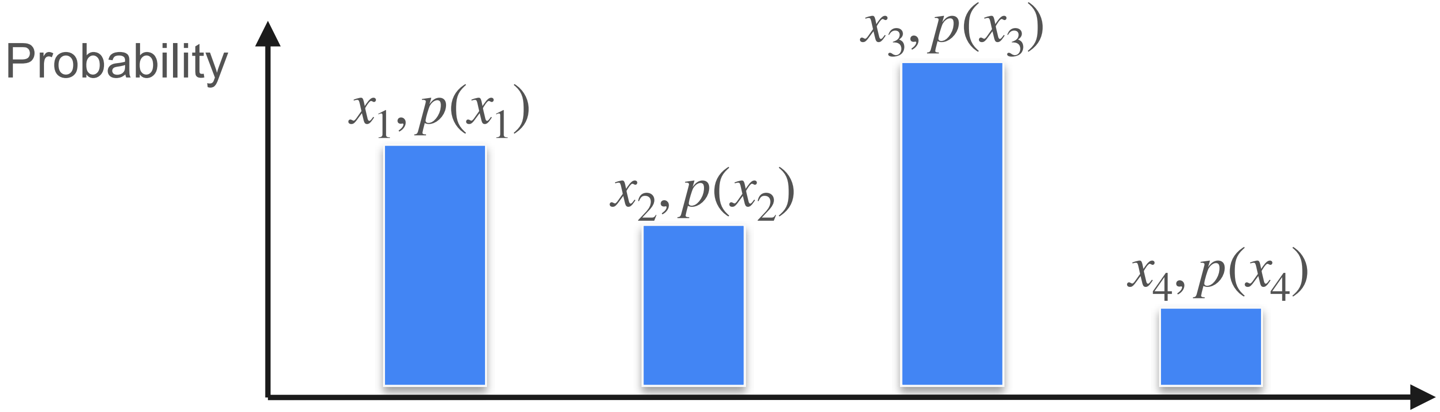
$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2)$$

Expected Value of a Function



$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

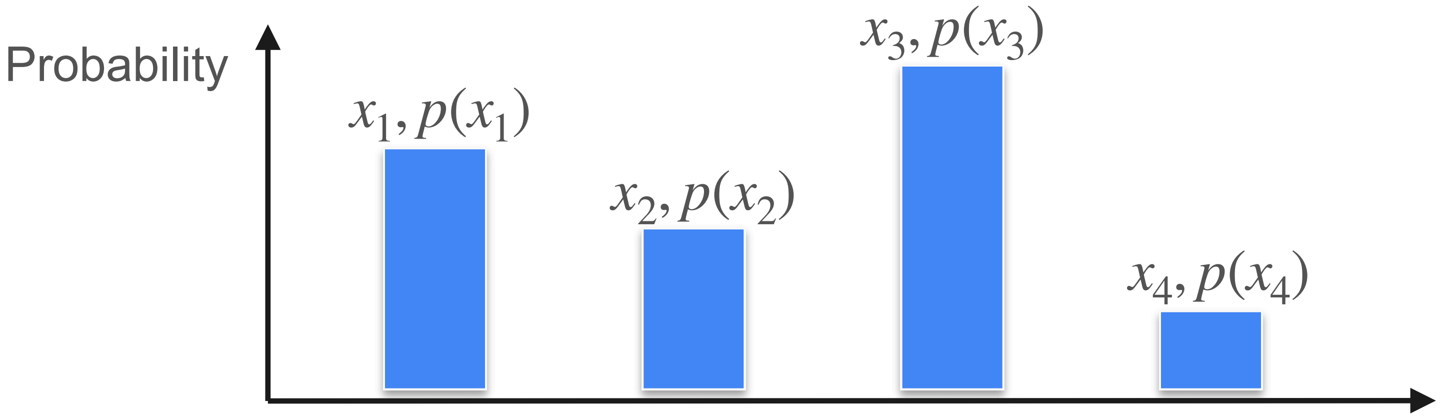
Expected Value of a Function



$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

$$E[f(X)] =$$

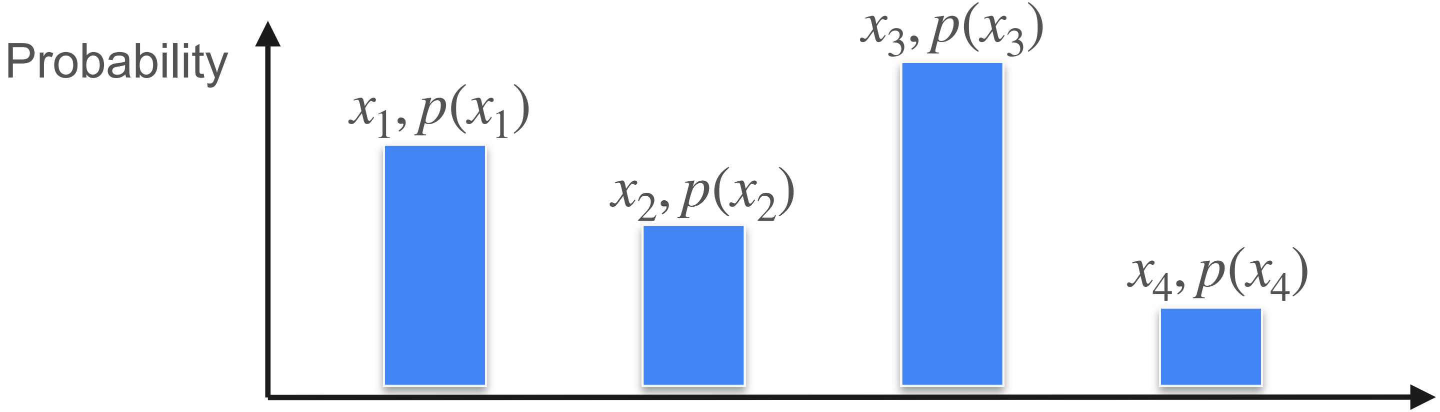
Expected Value of a Function



$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

$$E[f(X)] = f(x_1) p(x_1)$$

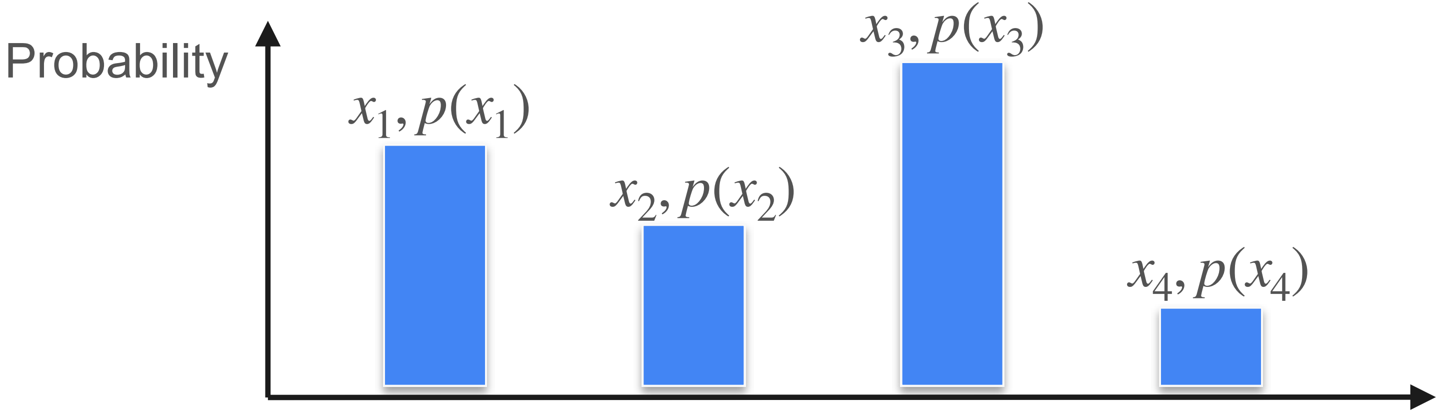
Expected Value of a Function



$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

$$E[f(X)] = f(x_1) p(x_1) + f(x_2) p(x_2)$$

Expected Value of a Function

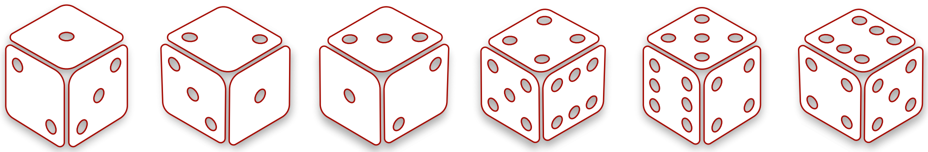


$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

$$E[f(X)] = f(x_1)p(x_1) + f(x_2)p(x_2) + f(x_3)p(x_3) + f(x_4)p(x_4)$$

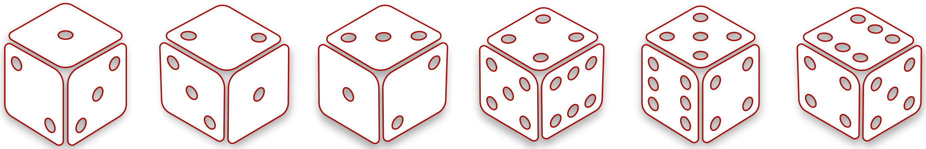
Expected Value of a Function

Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6



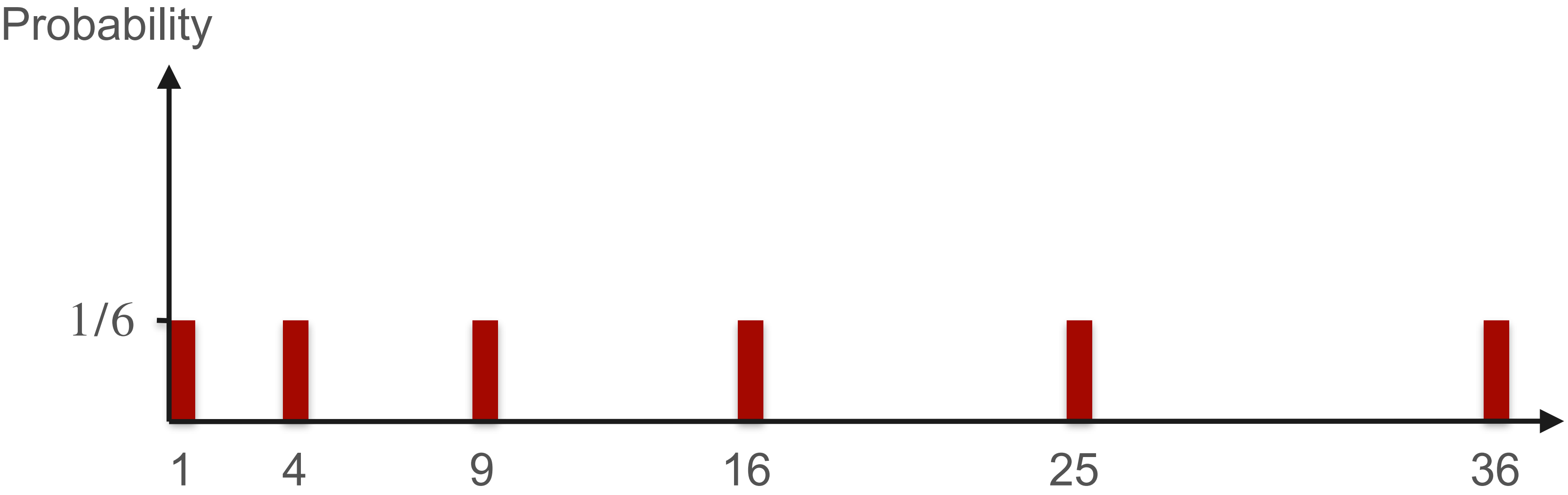
Expected Value of a Function

Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6

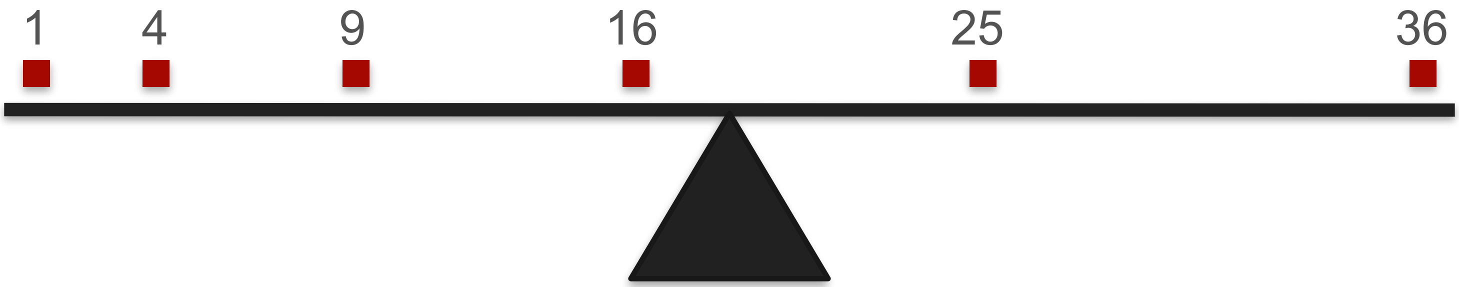


Square:	1	4	9	16	25	36

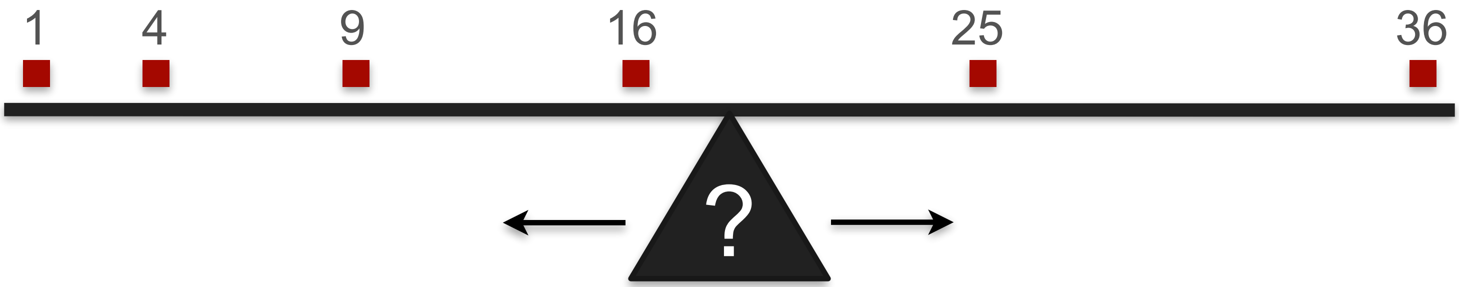
Expected Value of a Function



Expected Value of a Function

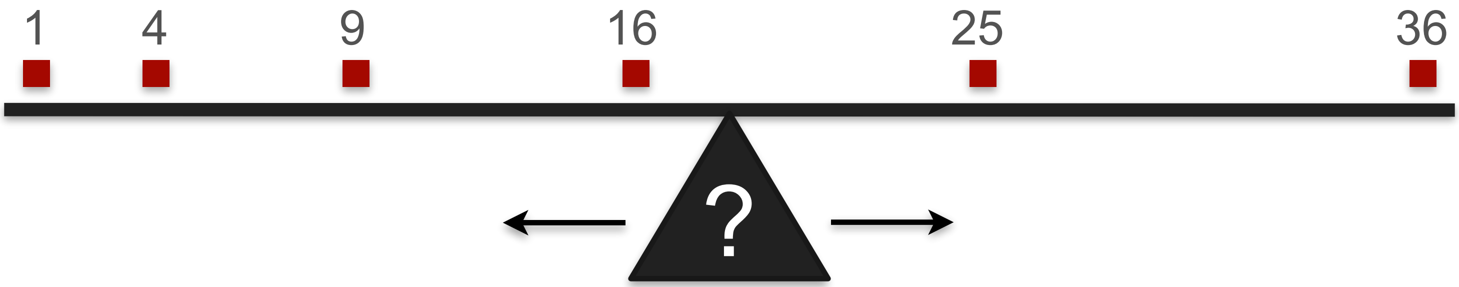


Expected Value of a Function



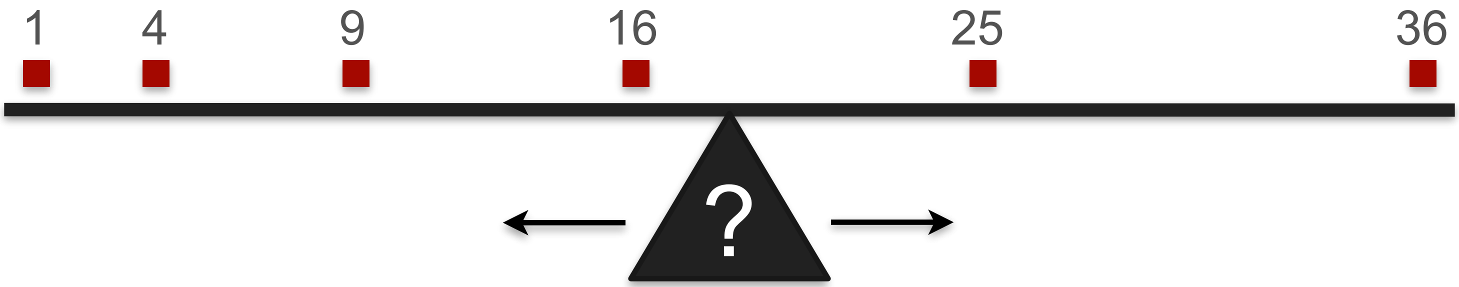
Expected Value of a Function

$$\frac{1 + 4 + 9 + 16 + 25 + 36}{6}$$



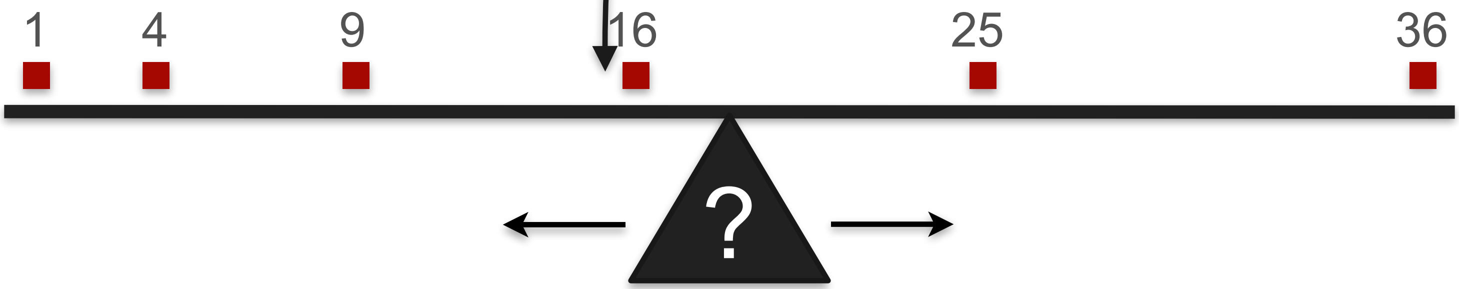
Expected Value of a Function

$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6}$$



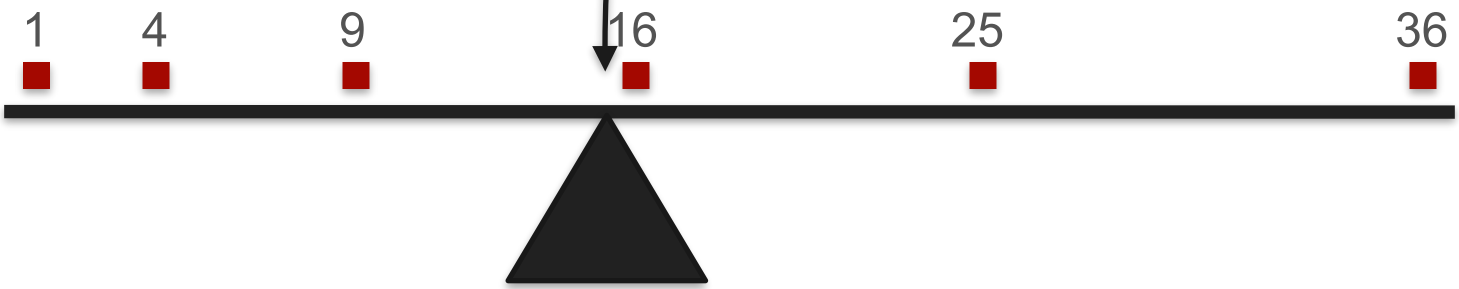
Expected Value of a Function

$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6}$$



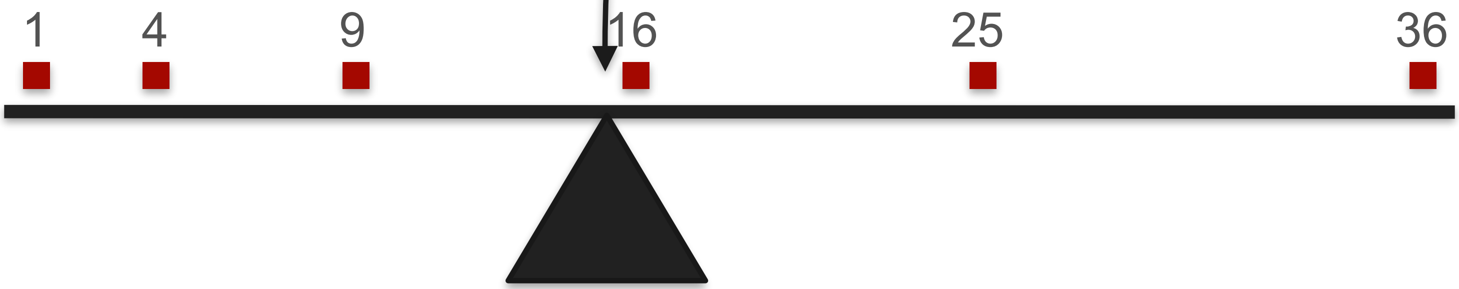
Expected Value of a Function

$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6}$$



Expected Value of a Function

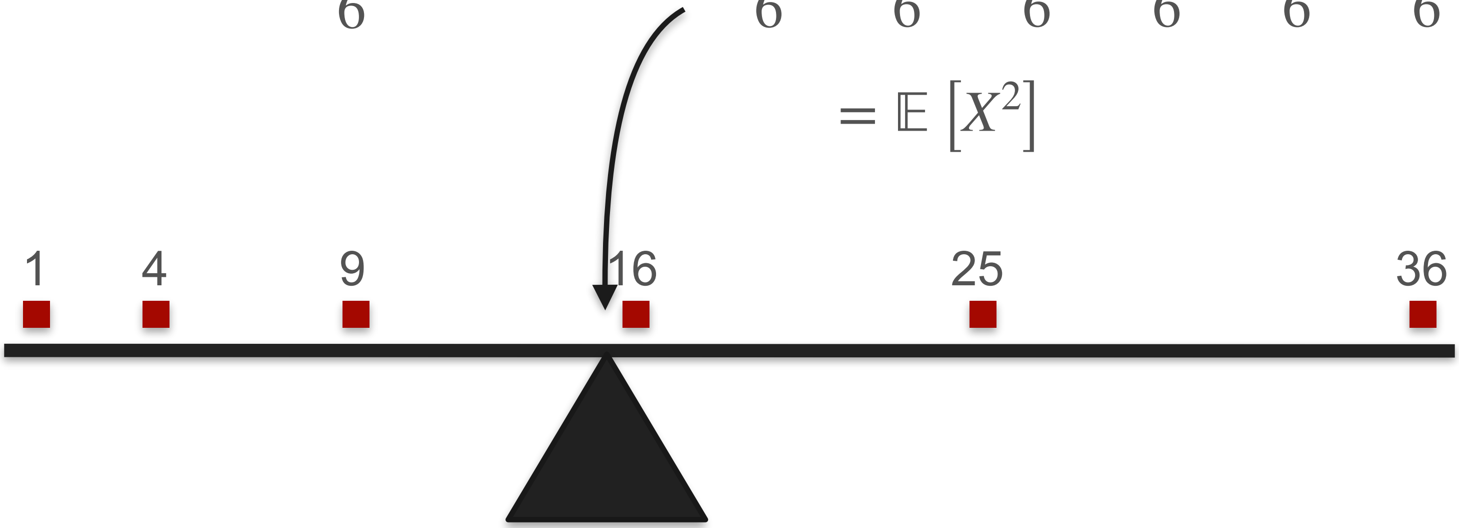
$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6} = \frac{1^2}{6} + \frac{2^2}{6} + \frac{3^2}{6} + \frac{4^2}{6} + \frac{5^2}{6} + \frac{6^2}{6}$$



Expected Value of a Function

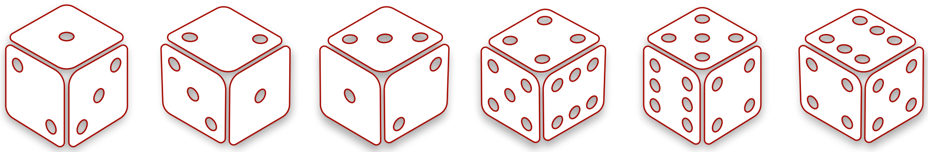
$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6} = \frac{1^2}{6} + \frac{2^2}{6} + \frac{3^2}{6} + \frac{4^2}{6} + \frac{5^2}{6} + \frac{6^2}{6}$$

$$= \mathbb{E}\left[X^2\right]$$



Expectation of Linear Function

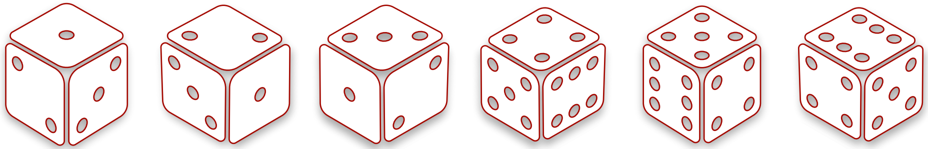
Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6



Wins

Expectation of Linear Function

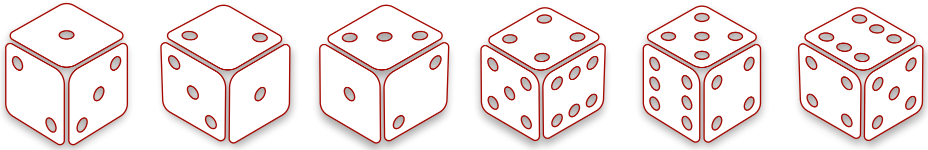
Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6



Double:	2	4	6	8	10	12
Wins	2 - 5	4 - 5	6 - 5	8 - 5	10 - 5	12 - 5

Expectation of Linear Function

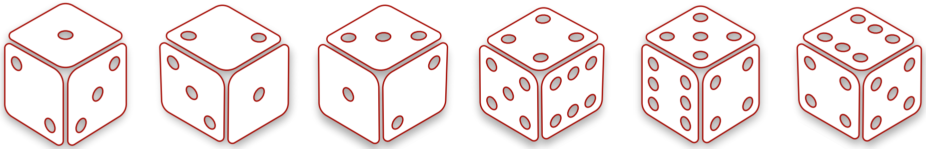
Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6



Wins

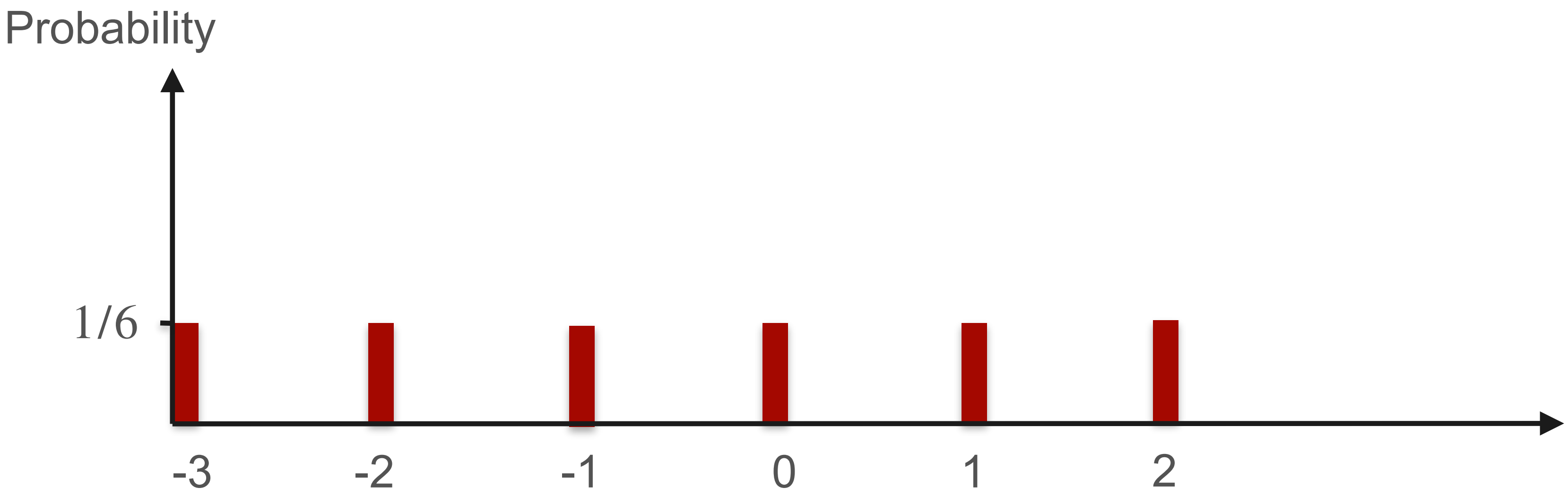
Expectation of Linear Function

Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6



Double:	2	4	6	8	10	12
Wins	-3	-2	-1	0	1	2

Expected Value of a Function



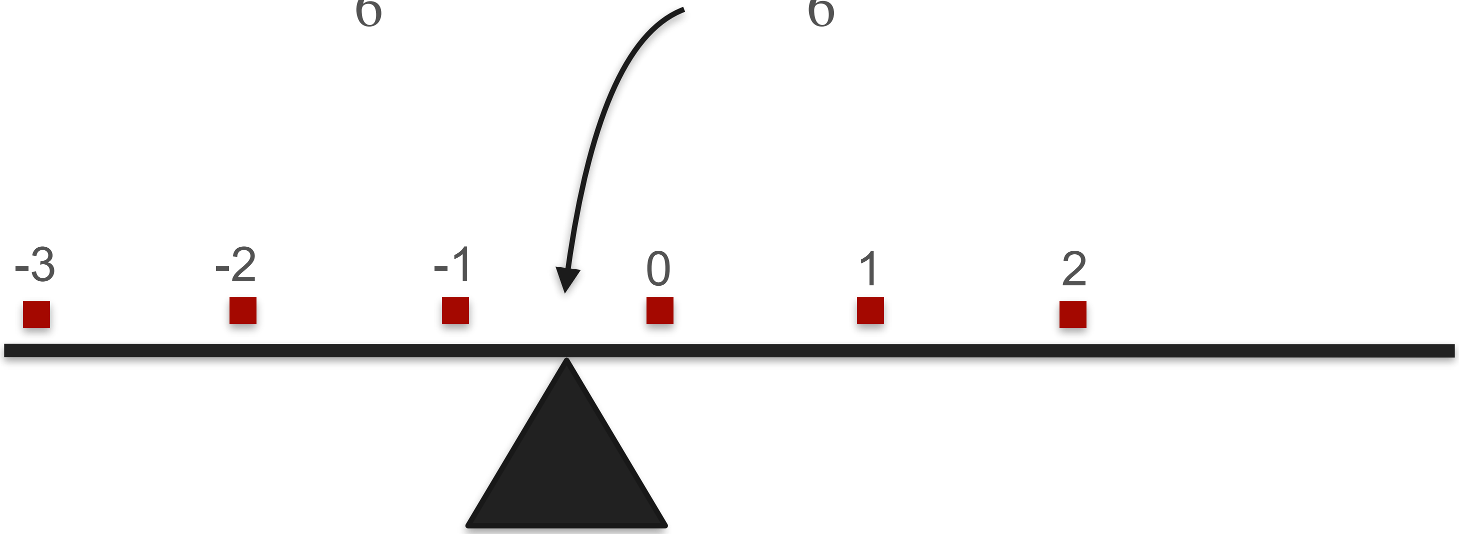
Expected Value of a Function

Expected Value of a Function

$$\frac{-3 + -2 + -1 + 0 + 1 + 2}{6}$$

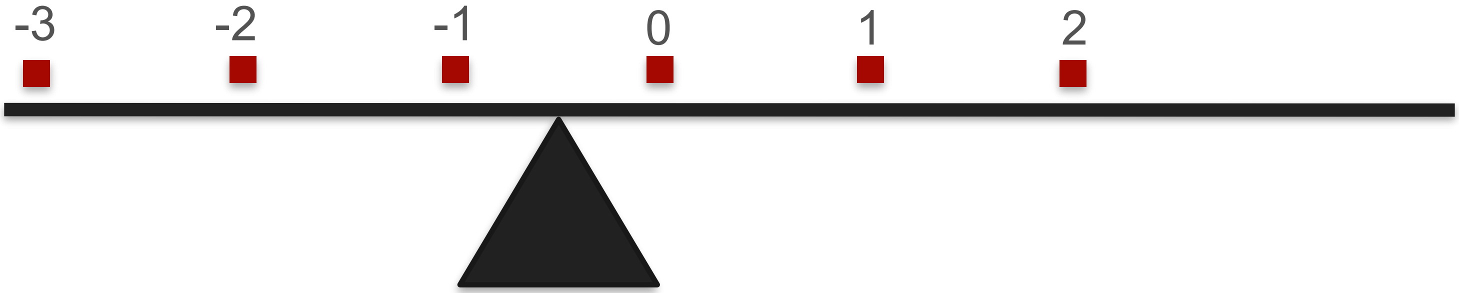
Expected Value of a Function

$$\frac{-3+-2+-1+0+1+2}{6} = \frac{-3}{6} = -0.5$$



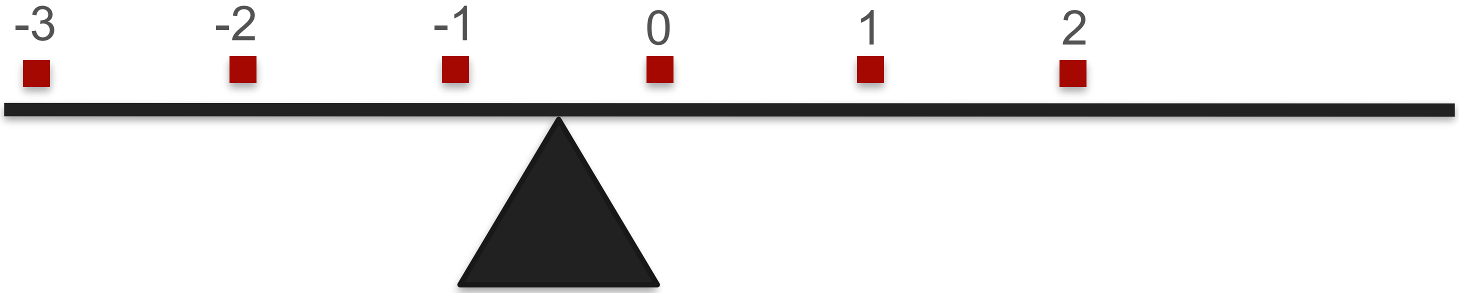
Expected Value of a Function

$$\frac{(2\cdot 1-5)+(2\cdot 2-5)+(2\cdot 3-5)+(2\cdot 4-5)+(2\cdot 5-5)+(2\cdot 6-5)}{6} = \frac{-3}{6} = -0.5$$



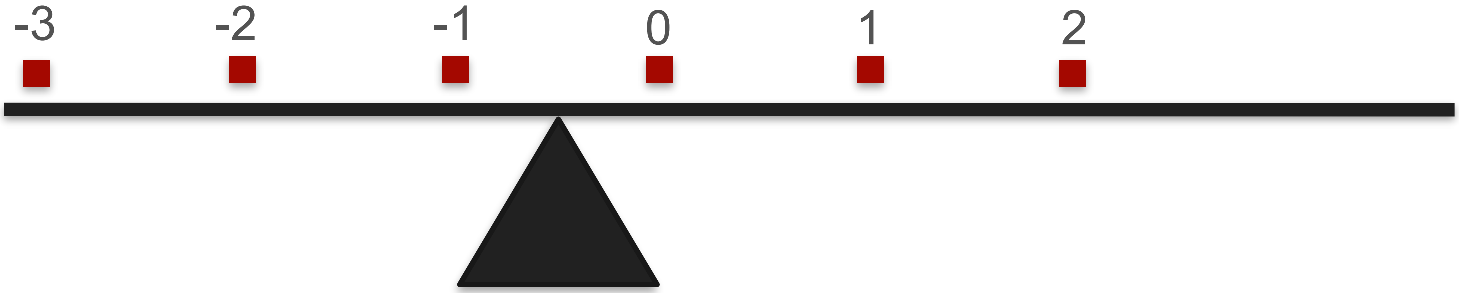
Expected Value of a Function

$$\frac{(2\cdot 1)+(2\cdot 2)+(2\cdot 3)+(2\cdot 4)+(2\cdot 5)+(2\cdot 6)+6\cdot(-5)}{6} = \frac{-3}{6} = -0.5$$



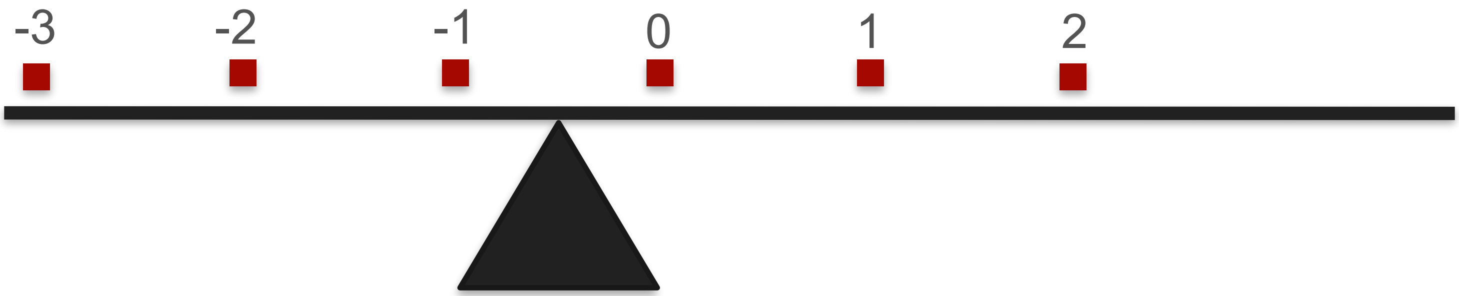
Expected Value of a Function

$$\frac{(2\cdot 1)+(2\cdot 2)+(2\cdot 3)+(2\cdot 4)+(2\cdot 5)+(2\cdot 6)}{6}+(-5) \quad = \frac{-3}{6} = -0.5$$



Expected Value of a Function

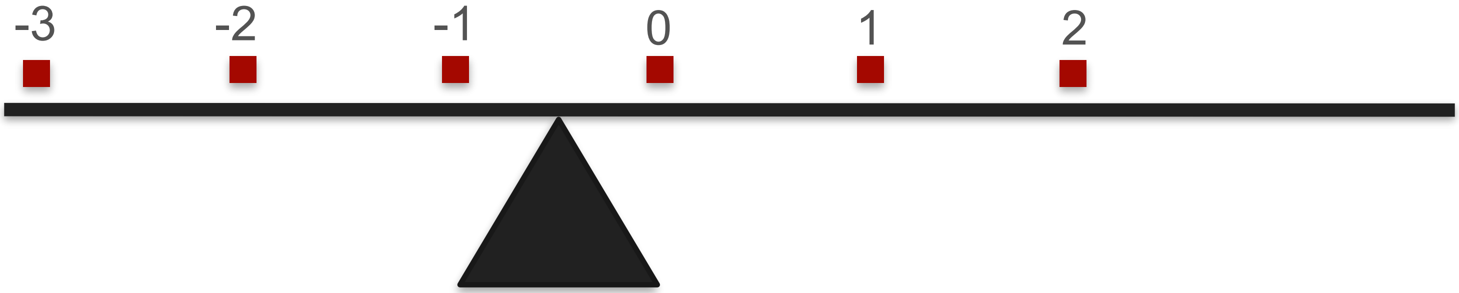
$$\frac{2 \cdot (1+2+3+4+5+6)}{6} + (-5) = \frac{-3}{6} = -0.5$$



Expected Value of a Function

$$\frac{2\cdot(1+2+3+4+5+6)}{6}+(-5) = \frac{-3}{6} = -0.5$$

$\mathbb{E}[X]$

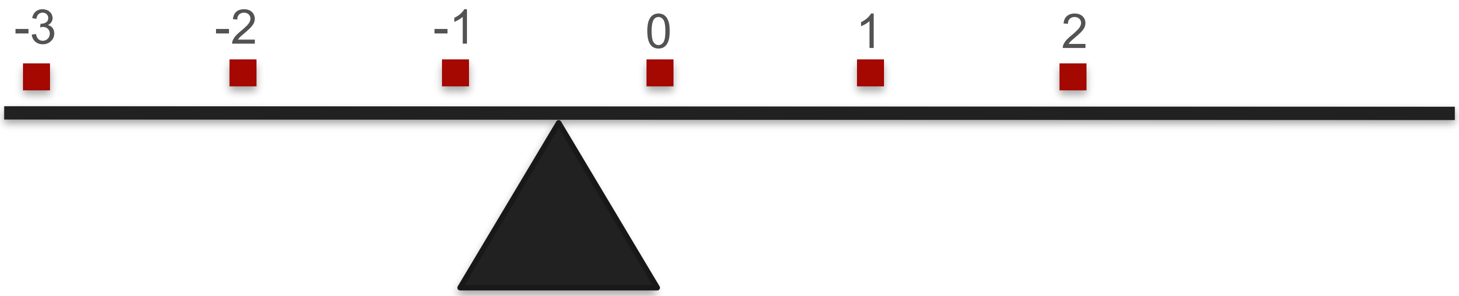


Expected Value of a Function

$$\mathbb{E}[2 \cdot X + (-5)] = \frac{2 \cdot \boxed{(1+2+3+4+5+6)}}{\boxed{6}} + (-5) = \frac{-3}{6} = -0.5$$

$\mathbb{E}[X]$

$$= 2 \cdot \mathbb{E}[X] + (-5)$$



Expected Value of a Function

$$\mathbb{E}[2 \cdot X + (-5)] = \frac{2 \cdot (1 + 2 + 3 + 4 + 5 + 6)}{6} + (-5) \quad = \frac{-3}{6} = -0.5$$

$$\mathbb{E}[X]$$

$$= 2 \cdot \mathbb{E}[X] + (-5)$$

In general:
$$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$$

Expected Value of a Function

$$\mathbb{E}[2 \cdot X + (-5)] = \frac{2 \cdot (1+2+3+4+5+6)}{6} + (-5) \quad = \frac{-3}{6} = -0.5$$

$\mathbb{E}[X]$

$$= 2 \cdot \mathbb{E}[X] + (-5)$$

In general:
$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$

$\mathbb{E}[aX] = a\mathbb{E}[X]$

Expected Value of a Function

$$\mathbb{E}[2 \cdot X + (-5)] = \frac{2 \cdot (1+2+3+4+5+6)}{6} + (-5) = \frac{-3}{6} = -0.5$$

$$\mathbb{E}[X]$$

$$= 2 \cdot \mathbb{E}[X] + (-5)$$

In general:
$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$

$$\mathbb{E}[aX] = a\mathbb{E}[X]$$

$$\mathbb{E}[b] = b$$

DeepLearning.AI

Describing Distributions

Sum of expectations

Sum of Expectation: Expected Winnings

You play a game:

Sum of Expectation: Expected Winnings

You play a game:

Flip a coin. If heads you win $ 1, else you win nothing.

Sum of Expectation: Expected Winnings

You play a game:

Flip a coin. If heads you win $ 1, else you win nothing.

Win $1

Win nothing

Sum of Expectation: Expected Winnings

You play a game:

Flip a coin. If heads you win $ 1, else you win nothing.

Then throw a dice. You win the amount you roll.

Win $1

Win nothing

Sum of Expectation: Expected Winnings

You play a game:

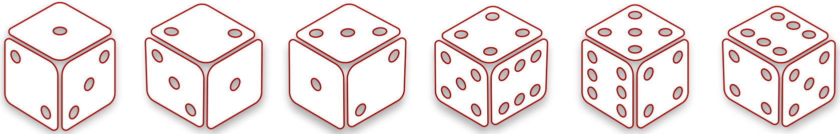
Flip a coin. If heads you win $ 1, else you win nothing.

Then throw a dice. You win the amount you roll.

Win $1

Win nothing

Win	$1	$2	$3	$4	$5	$6



Sum of Expectation: Expected Winnings

You play a game:

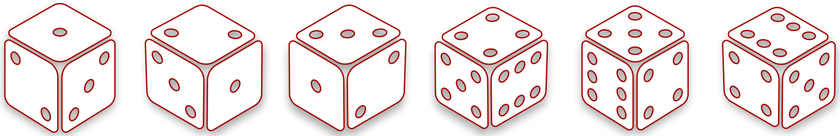
Flip a coin. If heads you win $ 1, else you win nothing.

Then throw a dice. You win the amount you roll.

Win $1

Win nothing

Win	$1	$2	$3	$4	$5	$6



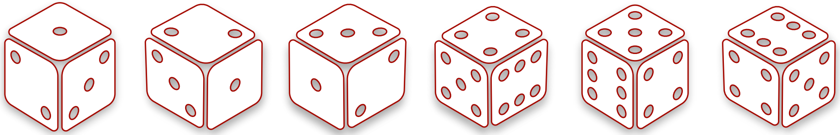
What are your expected winnings for the game?

Sum of Expectations: Expected Winnings

Win $1

Win nothing

Win $1 $2 $3 $4 $5 $6



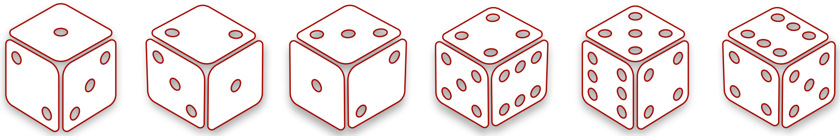
Sum of Expectations: Expected Winnings

Win $1

Win nothing

$$\mathbb{E}\left[X_{coin}\right] = \$0.5$$

Win $1 $2 $3 $4 $5 $6

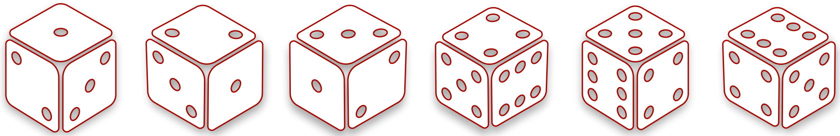


Sum of Expectations: Expected Winnings

Win $1

Win nothing

Win $1 $2 $3 $4 $5 $6



$$\mathbb{E}\left[X_{coin}\right] = \$0.5$$

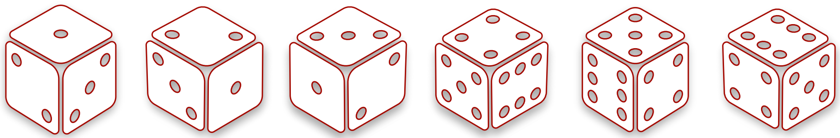
$$\mathbb{E}\left[X_{dice}\right] = \$3.5$$

Sum of Expectations: Expected Winnings

Win \$1

Win nothing

Win \$1 \$2 \$3 \$4 \$5 \$6



$$\mathbb{E}\left[X_{coin}\right] = \$0.5 \qquad \mathbb{E}\left[X_{dice}\right] = \$3.5$$

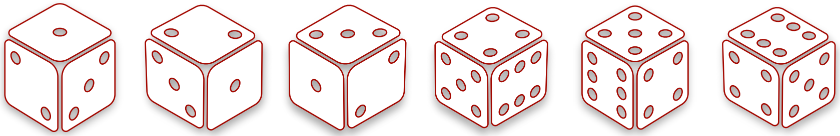
$$\mathbb{E}[X] = \mathbb{E}\left[X_{coin}\right] + \mathbb{E}\left[X_{dice}\right] = \$0.5 + \$3.5 = \$4$$

Sum of Expectations: Expected Winnings

Win \$1

Win nothing

Win \$1 \$2 \$3 \$4 \$5 \$6



$$\mathbb{E}\left[X_{coin}\right] = \$0.5 \qquad \mathbb{E}\left[X_{dice}\right] = \$3.5$$

$$\mathbb{E}[X] = \mathbb{E}\left[X_{coin}\right] + \mathbb{E}\left[X_{dice}\right] = \$0.5 + \$3.5 = \$4$$

In general: $\mathbb{E}\left[X_1 + X_2\right] = \mathbb{E}\left[X_1\right] + \mathbb{E}\left[X_2\right]$

Sum of Expectations: Bag With Names

8 billion people

Sum of Expectations: Bag With Names

8 billion people

Sum of Expectations: Bag With Names

8 billion people

Sum of Expectations: Bag With Names

Expected number of correct assignments?

8 billion people

Sum of Expectations: Bag With Names

1

Expected number of correct assignments?

8 billion people

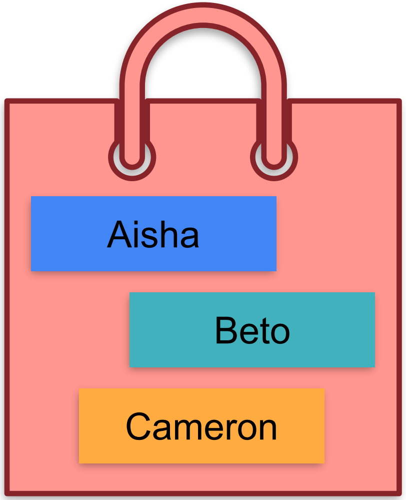
Sum of Expectations: Bag With Names

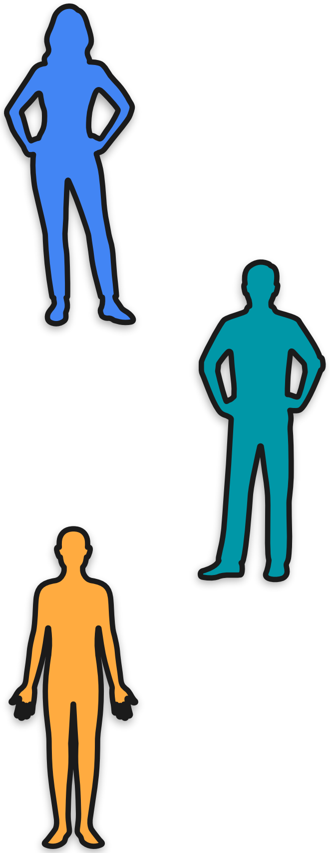
1

Expected number of correct assignments?

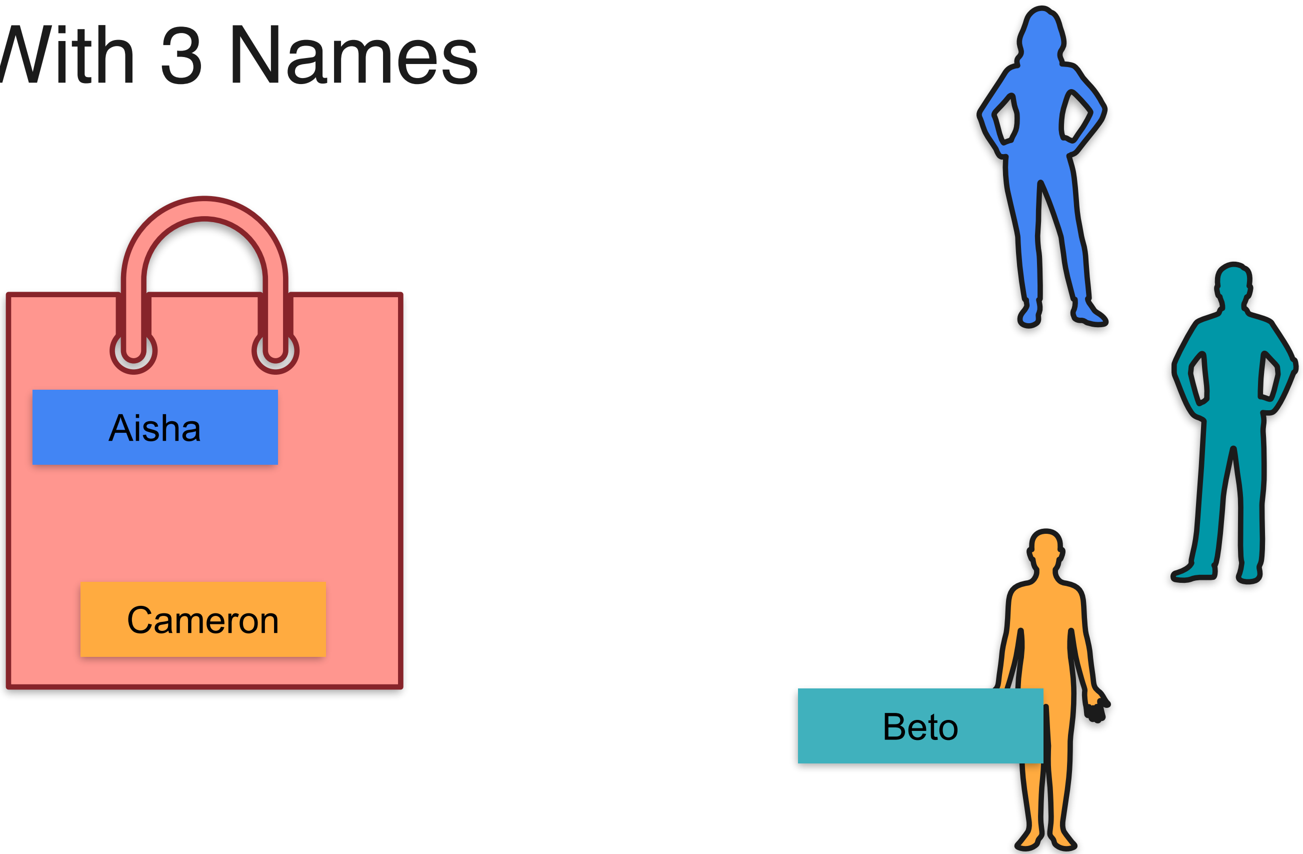
8 billion people

Bag With 3 Names

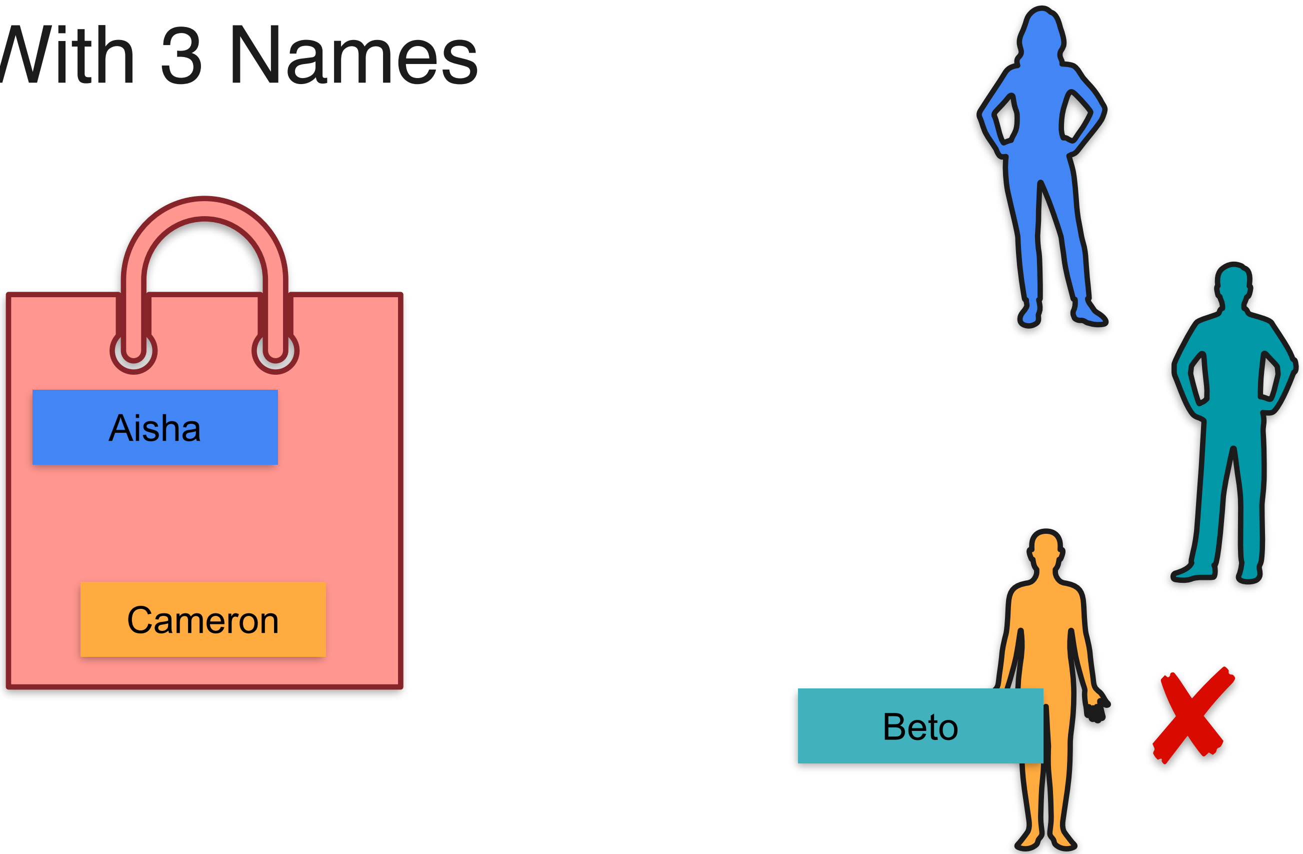




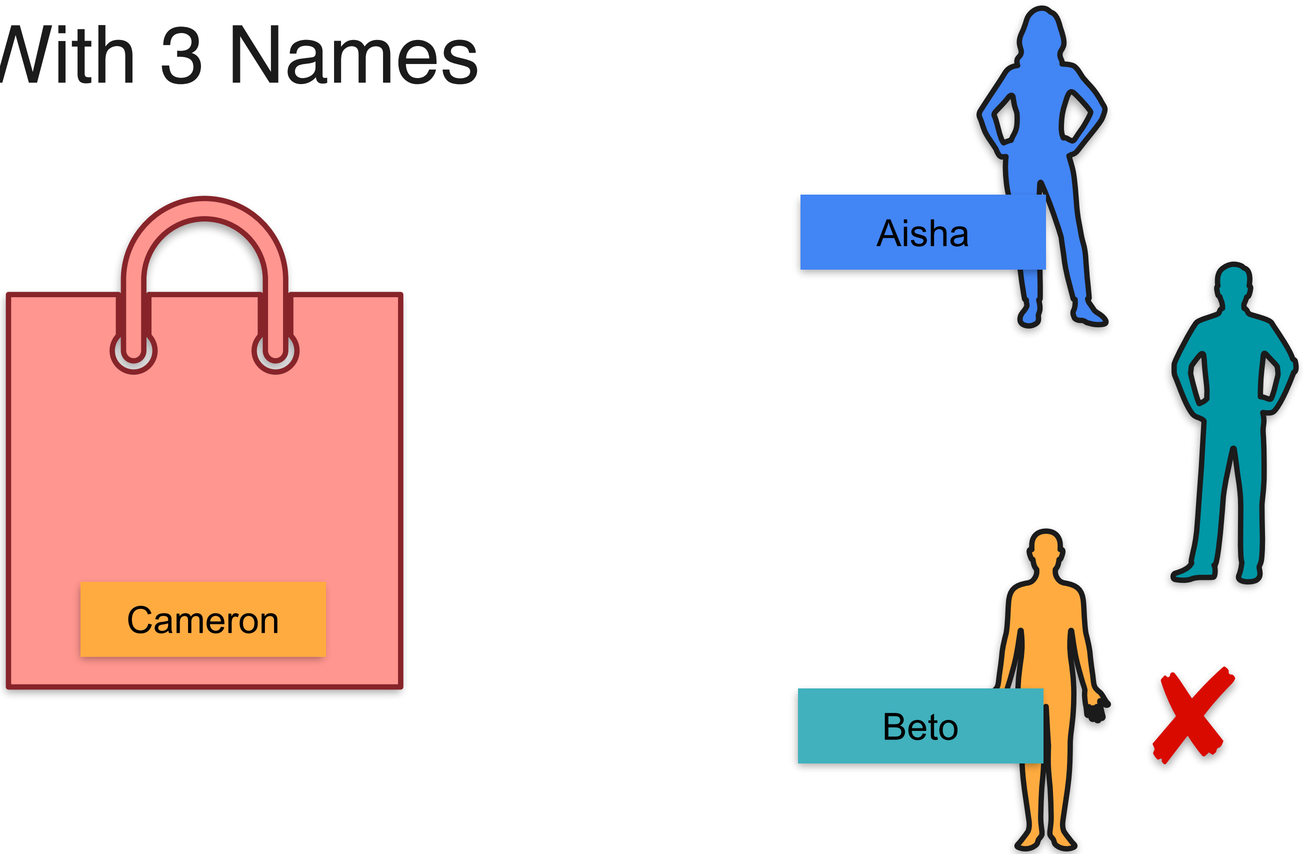
Bag With 3 Names



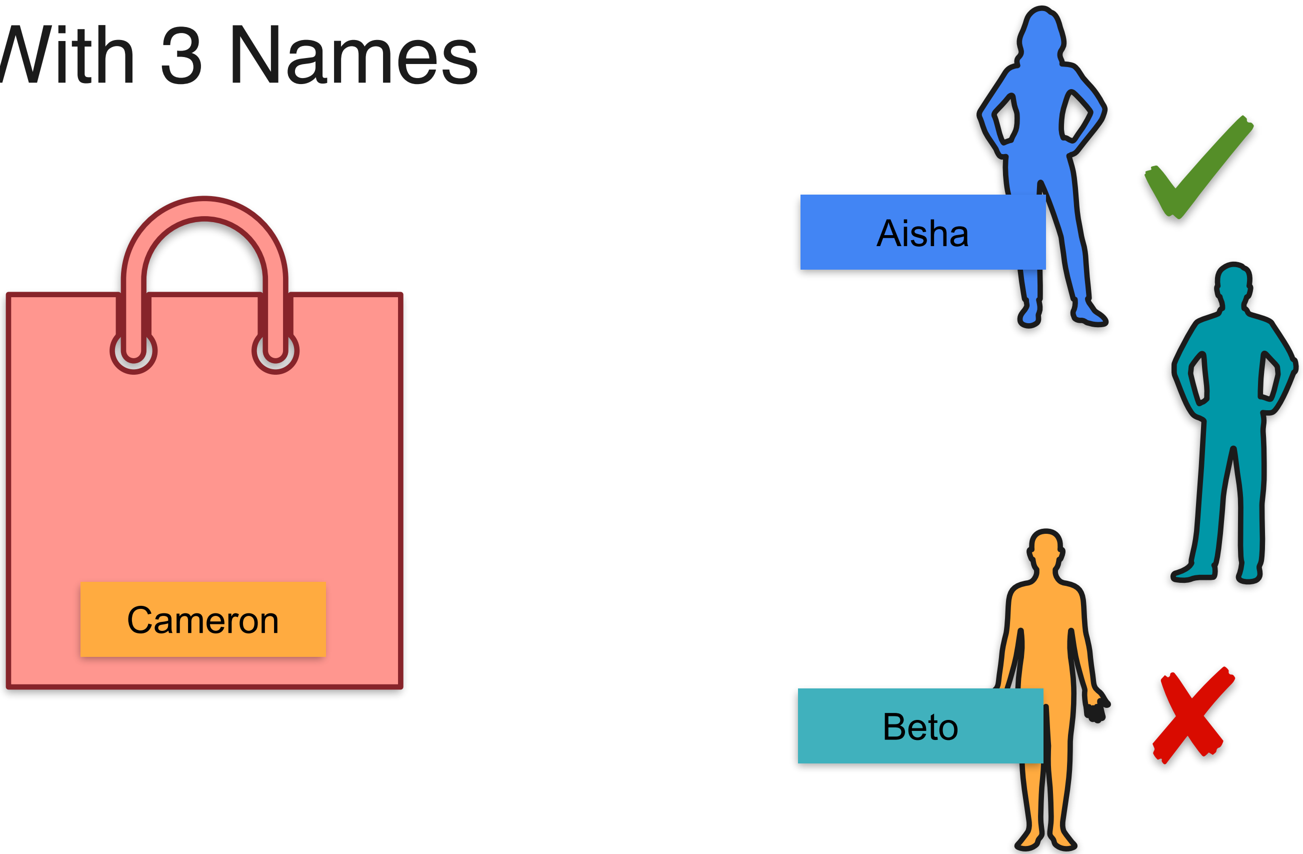
Bag With 3 Names



Bag With 3 Names



Bag With 3 Names



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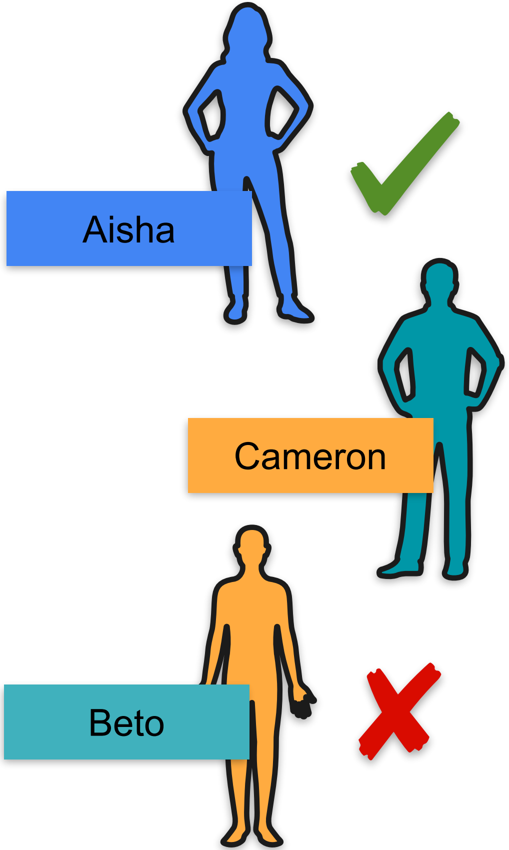

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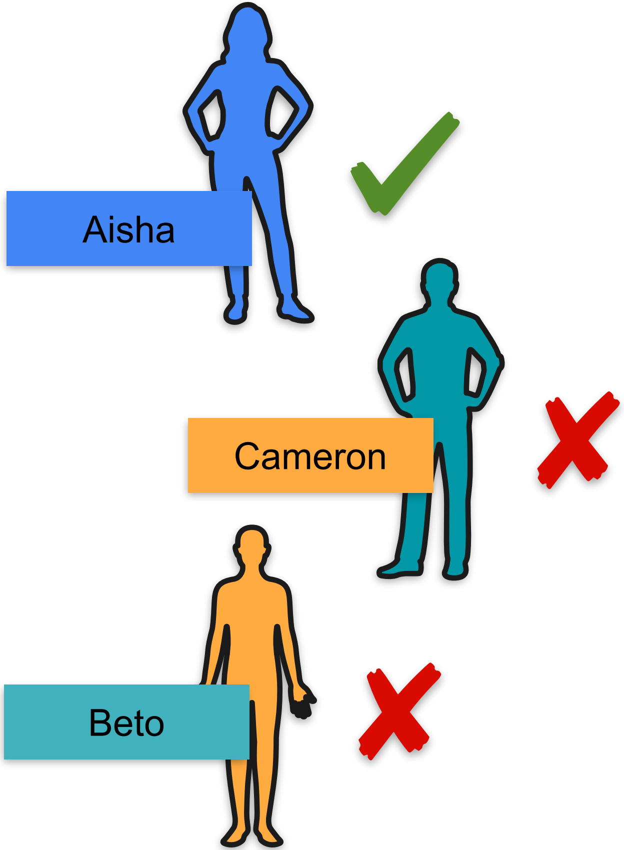

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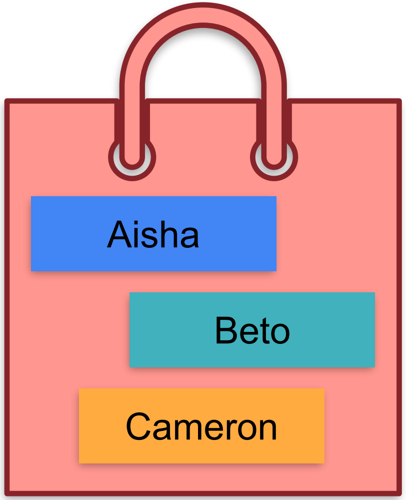
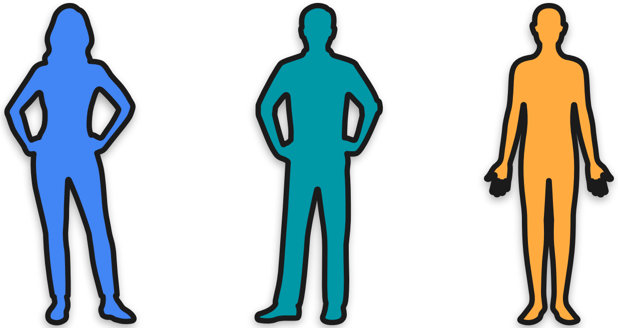
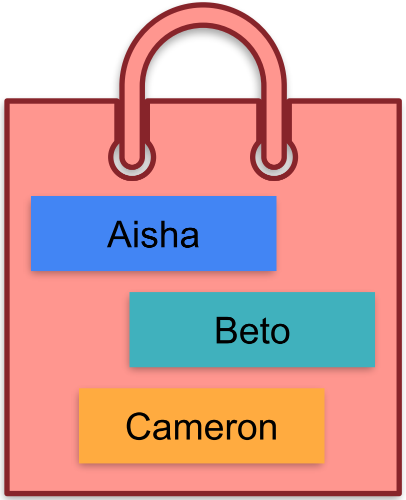
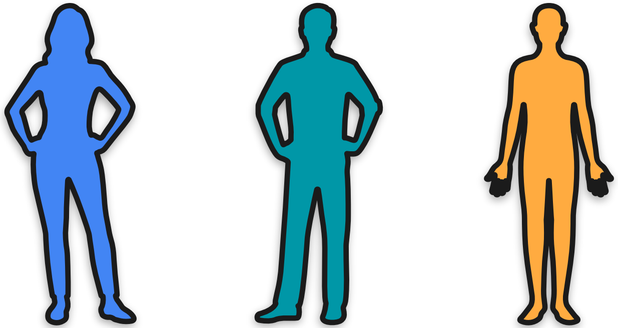




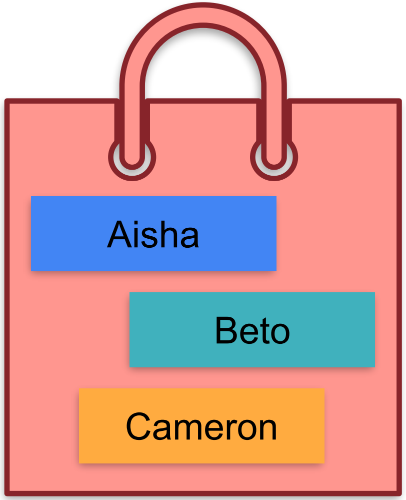
Bag With 3 Names



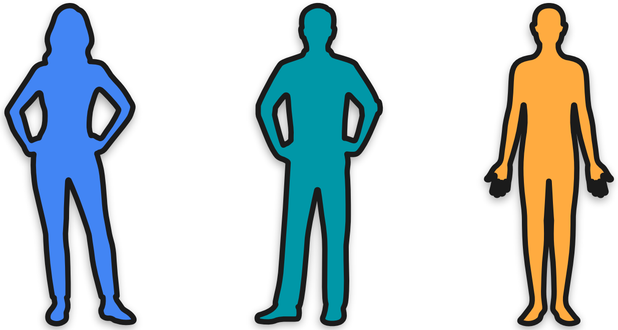


Aisha	Beto	Cameron
Aisha	Cameron	Beto
Beto	Aisha	Cameron
Beto	Cameron	Aisha
Cameron	Aisha	Beto
Cameron	Beto	Aisha

Bag With 3 Names

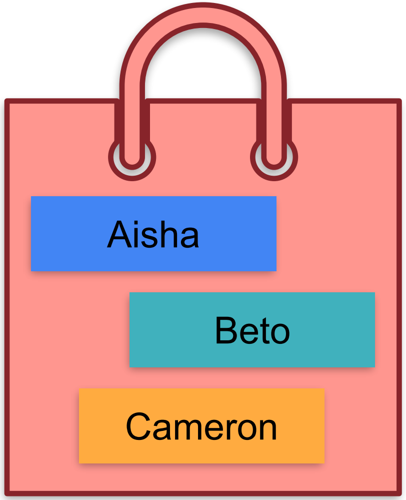


Correct

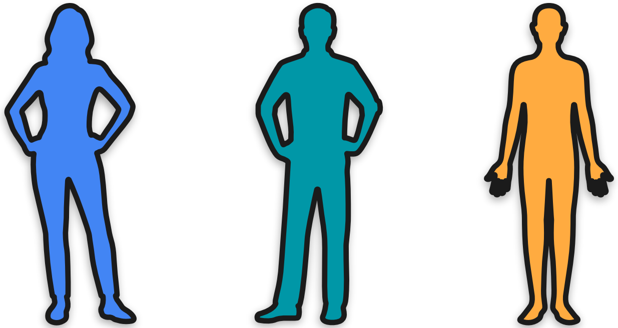


Aisha	Beto	Cameron
Aisha	Cameron	Beto
Beto	Aisha	Cameron
Beto	Cameron	Aisha
Cameron	Aisha	Beto
Cameron	Beto	Aisha

Bag With 3 Names

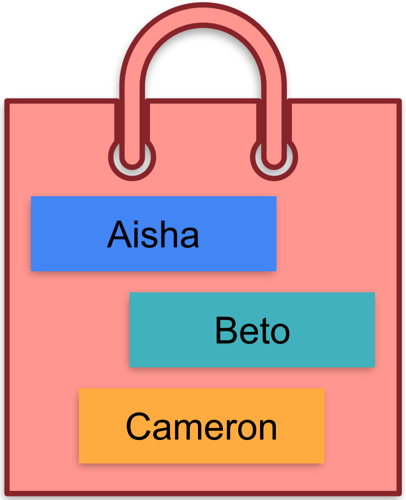


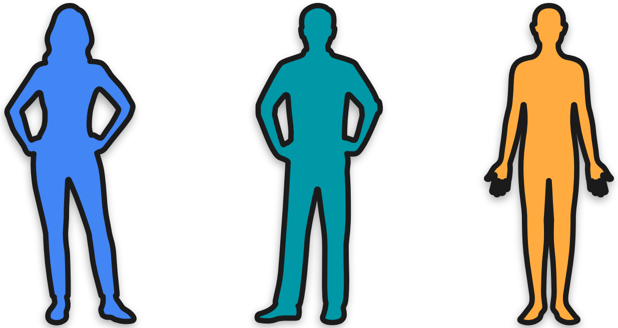
Correct



Aisha	Beto	Cameron
Aisha	Cameron	Beto
Beto	Aisha	Cameron
Beto	Cameron	Aisha
Cameron	Aisha	Beto
Cameron	Beto	Aisha

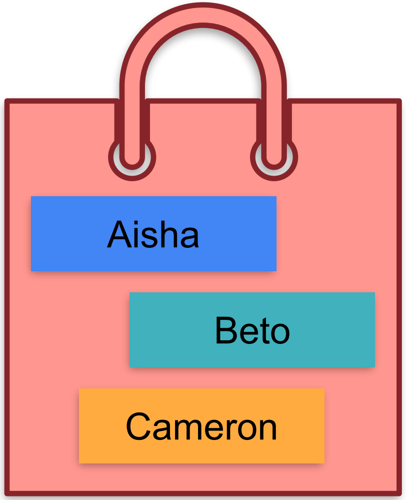
Bag With 3 Names

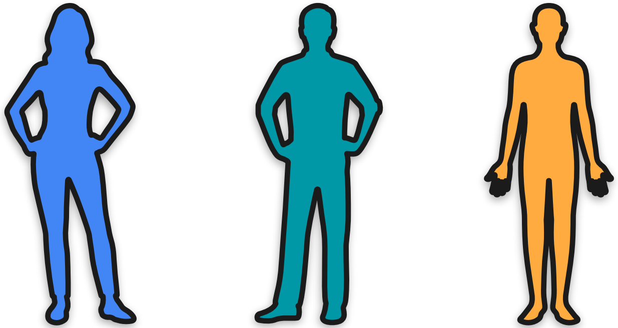




Correct			
3	Aisha	Beto	Cameron
1	Aisha	Cameron	Beto
1	Beto	Aisha	Cameron
0	Beto	Cameron	Aisha
0	Cameron	Aisha	Beto
1	Cameron	Beto	Aisha

Bag With 3 Names



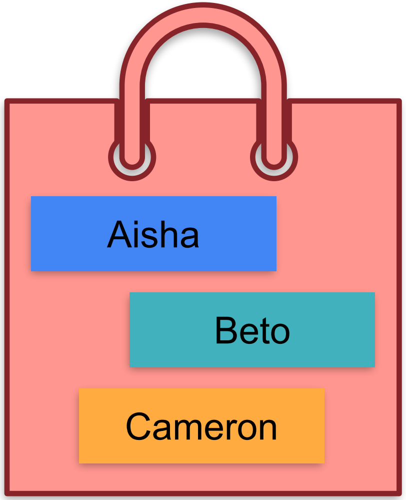


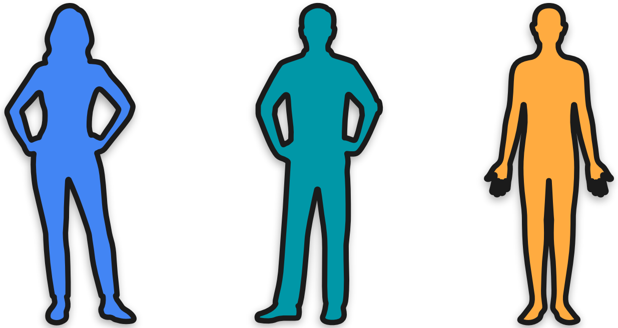
6

Correct			
3	Aisha	Beto	Cameron
1	Aisha	Cameron	Beto
1	Beto	Aisha	Cameron
0	Beto	Cameron	Aisha
0	Cameron	Aisha	Beto
1	Cameron	Beto	Aisha

Bag With 3 Names

Average
1

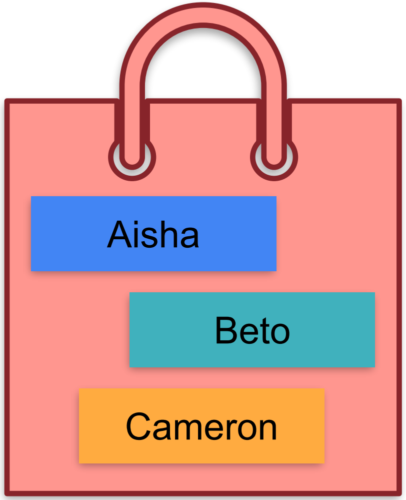


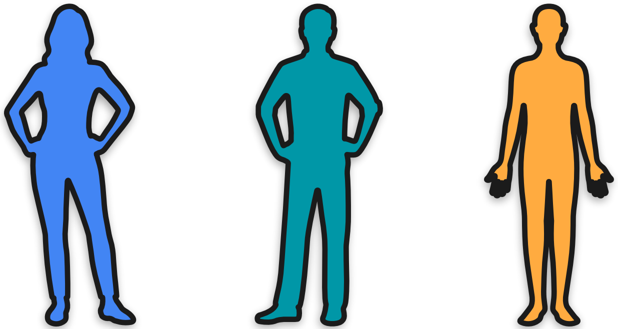


Correct			
3	Aisha	Beto	Cameron
1	Aisha	Cameron	Beto
1	Beto	Aisha	Cameron
0	Beto	Cameron	Aisha
0	Cameron	Aisha	Beto
1	Cameron	Beto	Aisha

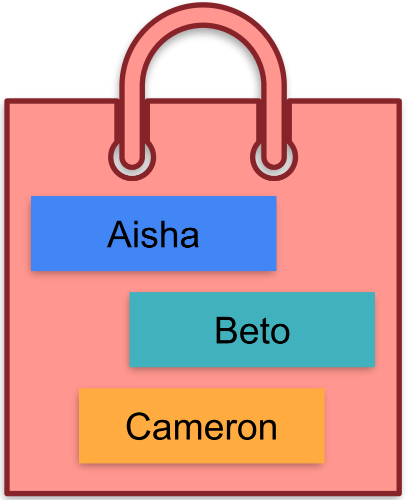
6

Adding Expectations

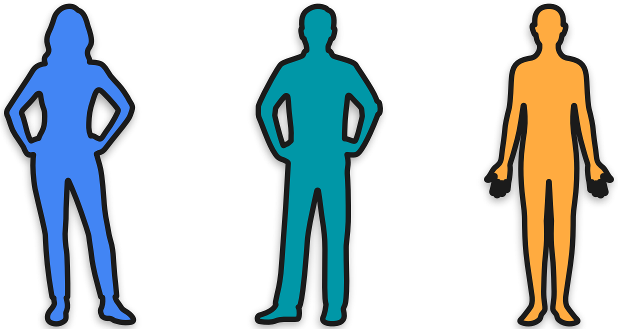




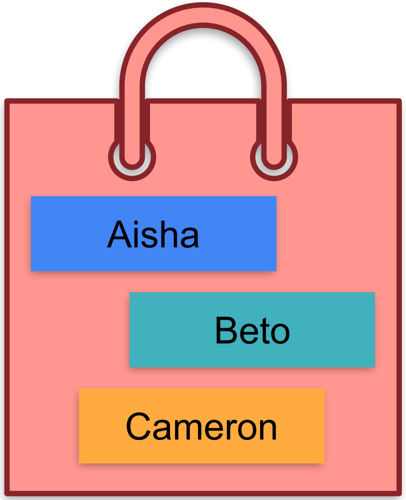
Adding Expectations

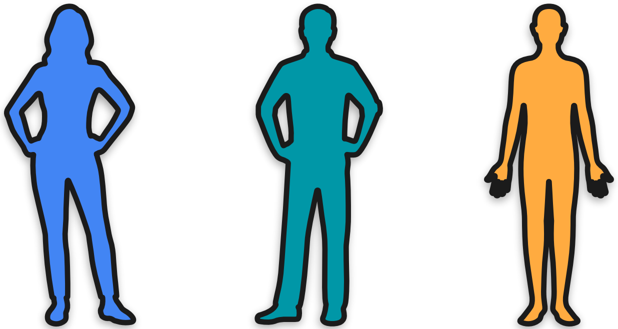


$\mathbb{E}[\text{Matches}]$



Adding Expectations





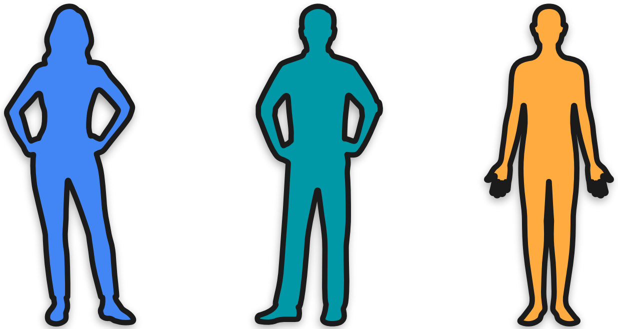
$$\mathbb{E}[\text{Matches}] \qquad = \mathbb{E}[A]$$

Adding Expectations

Aisha

Beto

Cameron



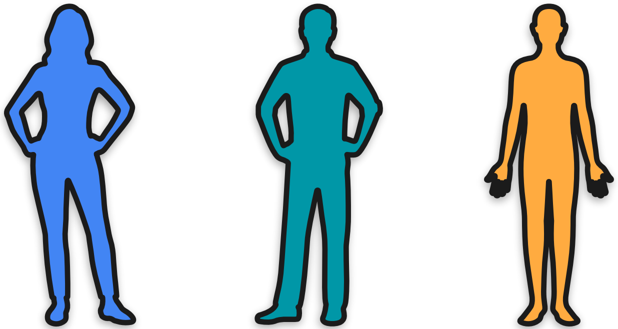
$$\mathbb{E}[\text{Matches}] \quad = \mathbb{E}[A]$$

Adding Expectations

1/3 Aisha

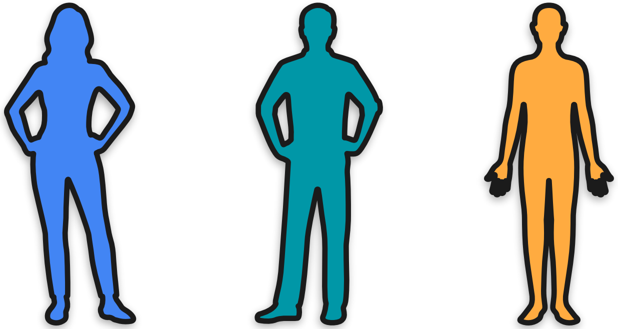
1/3 Beto

1/3 Cameron



$$\mathbb{E}[\text{Matches}] \quad = \mathbb{E}[A]$$

Adding Expectations

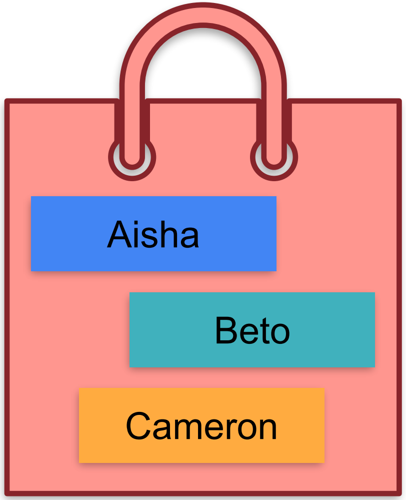


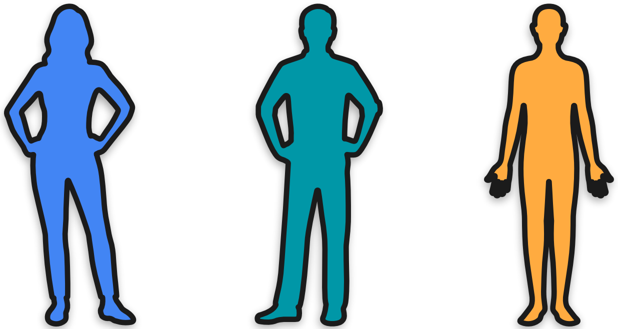
1/3	Aisha
1/3	Beto
1/3	Cameron

$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A]$$

$$= 1/3$$

Adding Expectations

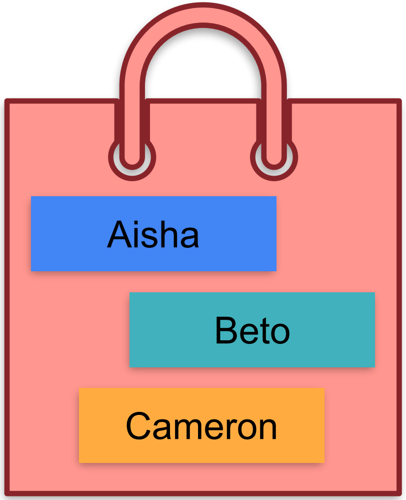


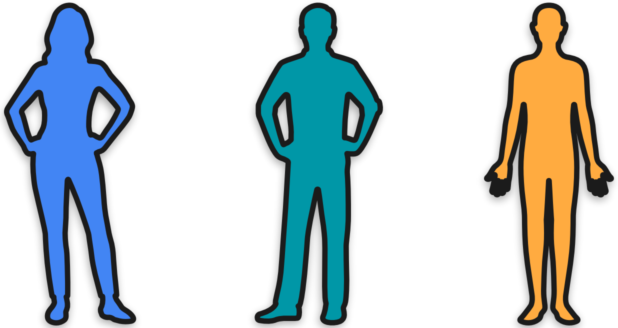


$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A]$$

$$= 1/3$$

Adding Expectations

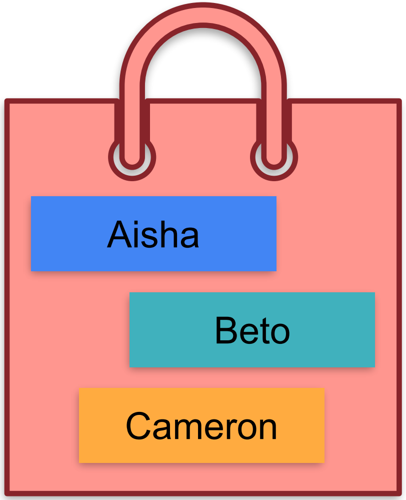


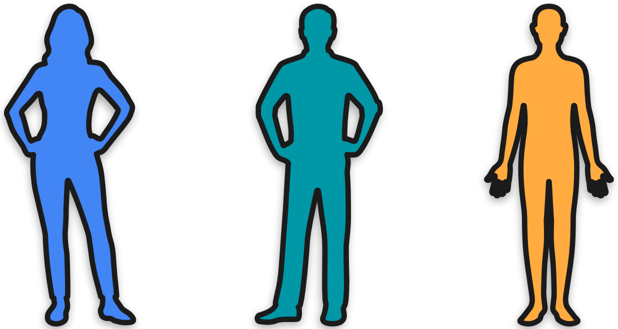


$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A] + \mathbb{E}[B]$$

$$= 1/3 + 1/3$$

Adding Expectations

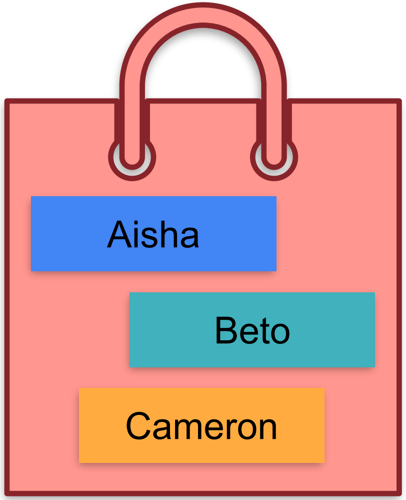


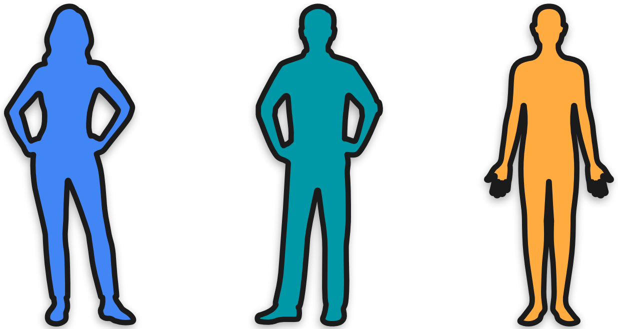


$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A] + \mathbb{E}[B] + \mathbb{E}[C]$$

$$= 1/3 + 1/3 + 1/3$$

Adding Expectations



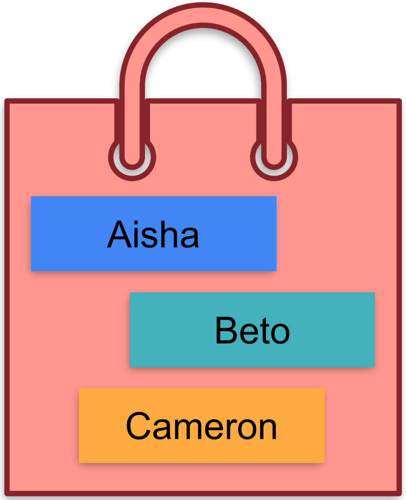


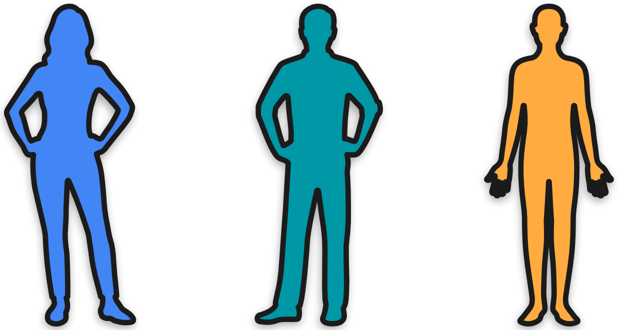
$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A] + \mathbb{E}[B] + \mathbb{E}[C]$$

$$= 1/3 + 1/3 + 1/3$$

$$= 1$$

Adding Expectations





$$\mathbb{E}[\text{Matches}] = \mathbb{E}[A] + \mathbb{E}[B] + \mathbb{E}[C]$$

$$= 1/3 + 1/3 + 1/3$$

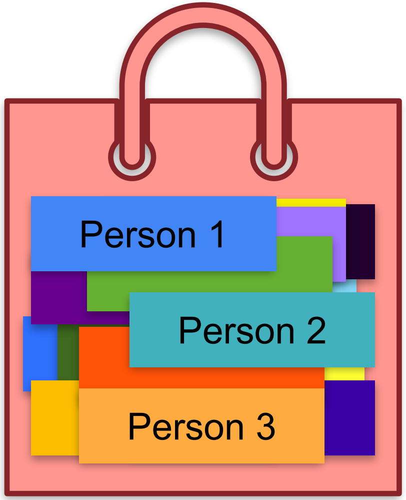
$$= 1$$

Average
1

Adding Expectations

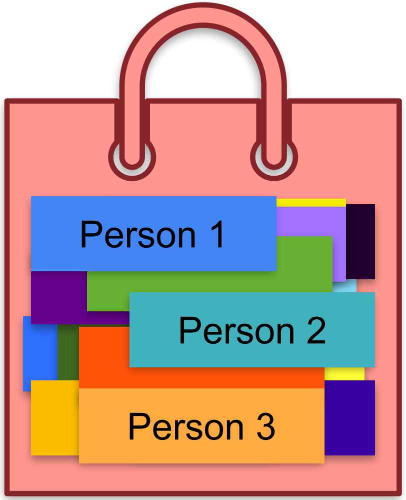
8 billion people

Adding Expectations



8 billion people

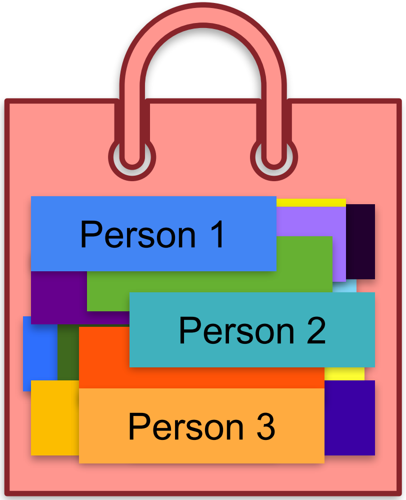
Adding Expectations



8 billion people

Adding Expectations

Expected number = ?



8 billion people

Adding Expectations

Adding Expectations

$$\mathbb{E}\left[\text{Matches}\right] = \mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \dots + \mathbb{E}\left[X_{\text{person}_n}\right]$$

Adding Expectations

$$\mathbb{E}\left[\text{Matches}\right] = \overbrace{\mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right]}^{\text{n people (n = 8 billion)}}$$

Adding Expectations

$$\mathbb{E}\,[\text{Matches}] = \overbrace{\mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right]}^{\text{n people (n = 8 billion)}} = \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}$$

Adding Expectations

n people (n = 8 billion)

$$\mathbb{E}[\text{Matches}] = \mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right]$$

$$= \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}$$

$$= n \cdot \frac{1}{n}$$

Adding Expectations

$$\mathbb{E}[\text{Matches}] = \overbrace{\mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right]}^{\text{n people (n = 8 billion)}}$$

$$= \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}$$

$$= n \cdot \frac{1}{n} = 1$$

Adding Expectations

n people (n = 8 billion)

$$\mathbb{E}\,[\text{Matches}] = \mathbb{E}\left[X_{\text{person}_1}\right] + \mathbb{E}\left[X_{\text{person}_2}\right] + \ldots + \mathbb{E}\left[X_{\text{person}_n}\right] = \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}$$

$$= n \cdot \frac{1}{n} = 1$$

In general:

$$\mathbb{E}\left[X_1 + X_2 + \ldots + X_n\right] = \mathbb{E}\left[X_1\right] + \mathbb{E}\left[X_2\right] + \ldots + \mathbb{E}\left[X_n\right]$$

Describing Distributions

Variance

DeepLearning.AI

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 1 dollar

You lose 1 dollar

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 1 dollar

You lose 1 dollar

$0

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 1 dollar

You lose 1 dollar

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

Game cost:

You win 1 dollar

You lose 1 dollar

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 1 dollar

You lose 1 dollar

Game cost:

$0

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 100 dollars

You lose 100 dollars

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 100 dollars

You lose 100 dollars

$0

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 100 dollars

You lose 100 dollars

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

Game cost:

You win 100 dollars

You lose 100 dollars

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

You play a game with a friend

You win 100 dollars

You lose 100 dollars

Game cost:

$0

What is the fair amount of money to pay to play this game?

Variance Motivation: Fair Price To Play the Game

How do you tell these two games apart?

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 100 dollars

You lose 100 dollars

Variance Motivation: Fair Price To Play the Game

How do you tell these two games apart?

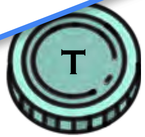
Game 1

You win 1 dolla

You lose 1 d

Ga

win 100 dollars



You lose 100 dollars

Variance!

Variance Motivation: Measuring Spread

Variance Motivation: Measuring Spread

Variance Motivation: Measuring Spread

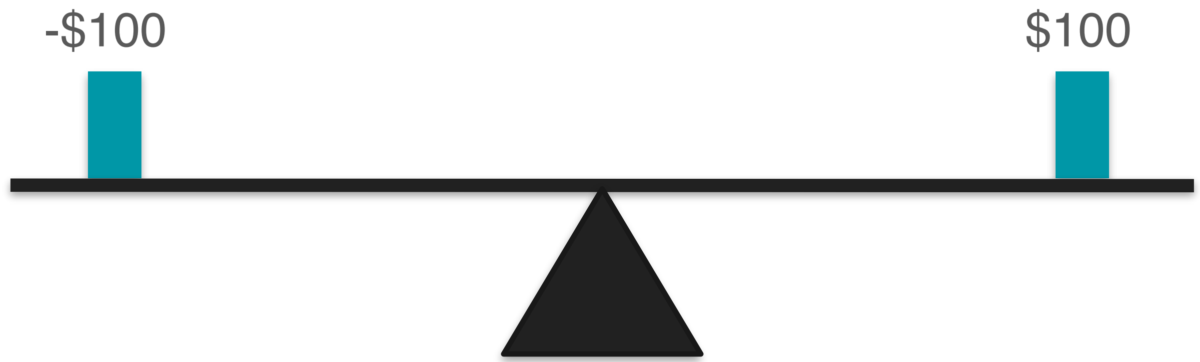
$$\mathbb{E}[X_1] = 0$$

Variance Motivation: Measuring Spread

X_1 = expected amount of money gained in game 1

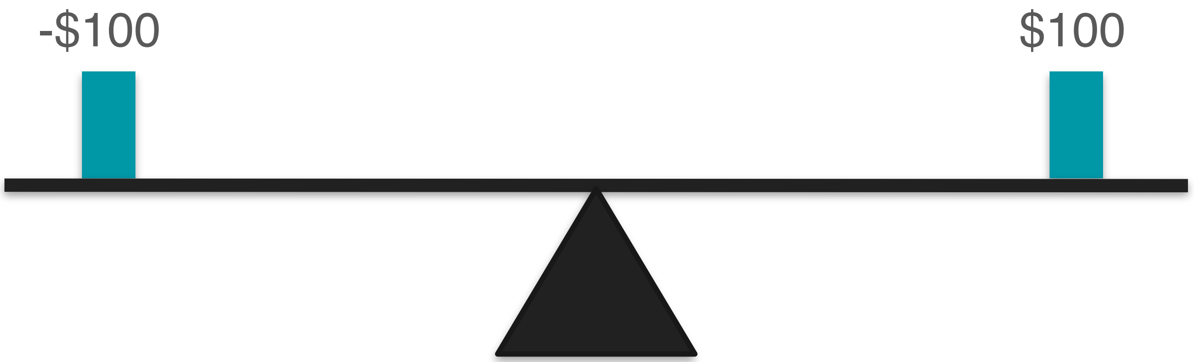
$$\mathbb{E}[X_1] = 0$$

Variance Motivation: Measuring Spread



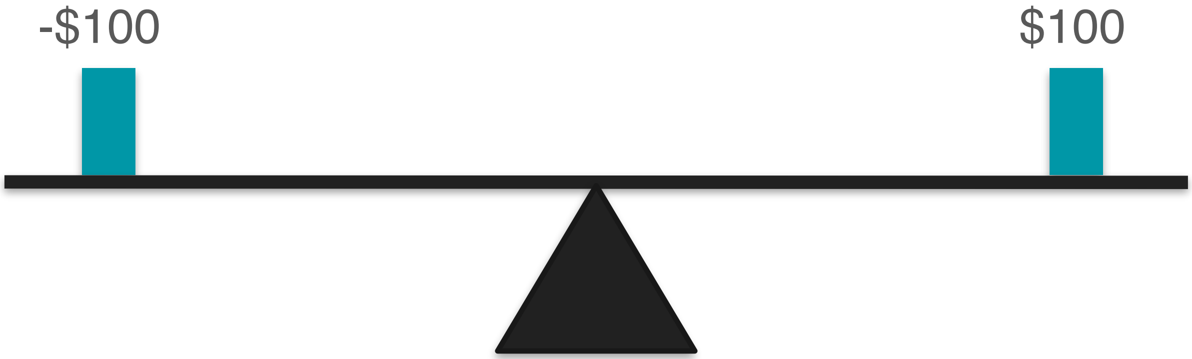
Variance Motivation: Measuring Spread

X_2 = expected amount of money gained in game 2



Variance Motivation: Measuring Spread

X_2 = expected amount of money gained in game 2



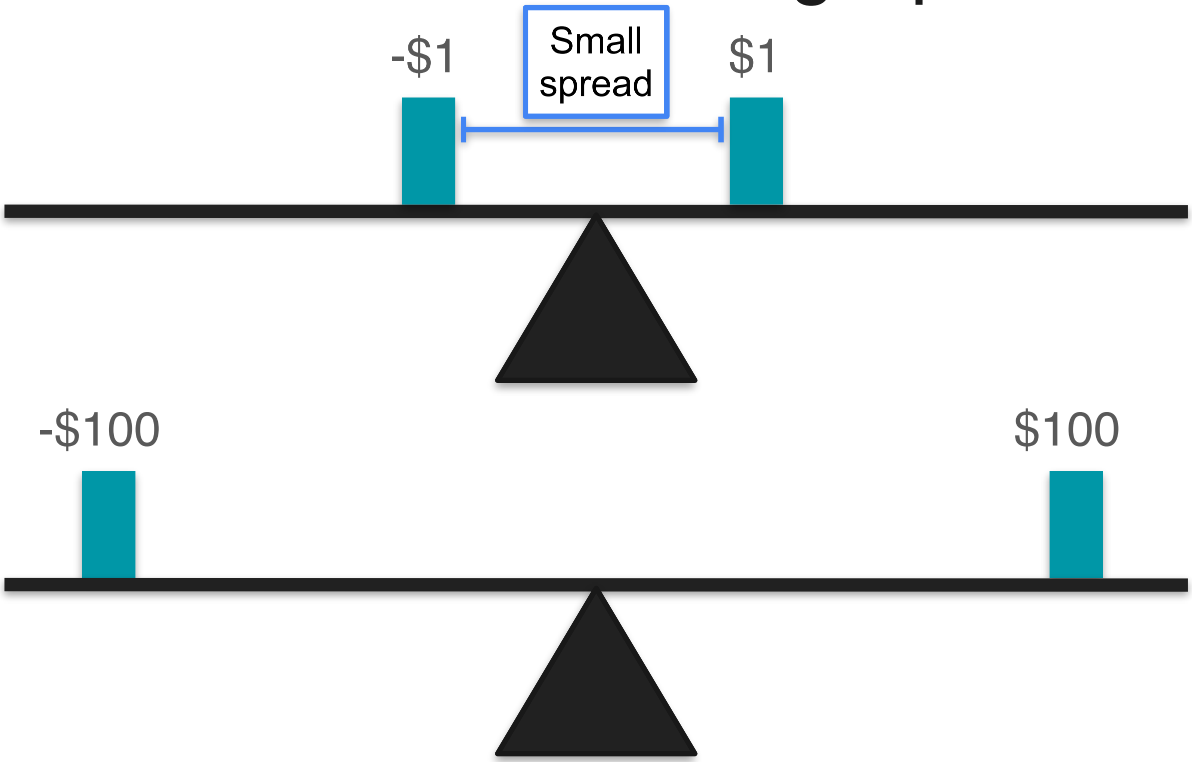
$$\mathbb{E}[X_2] = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

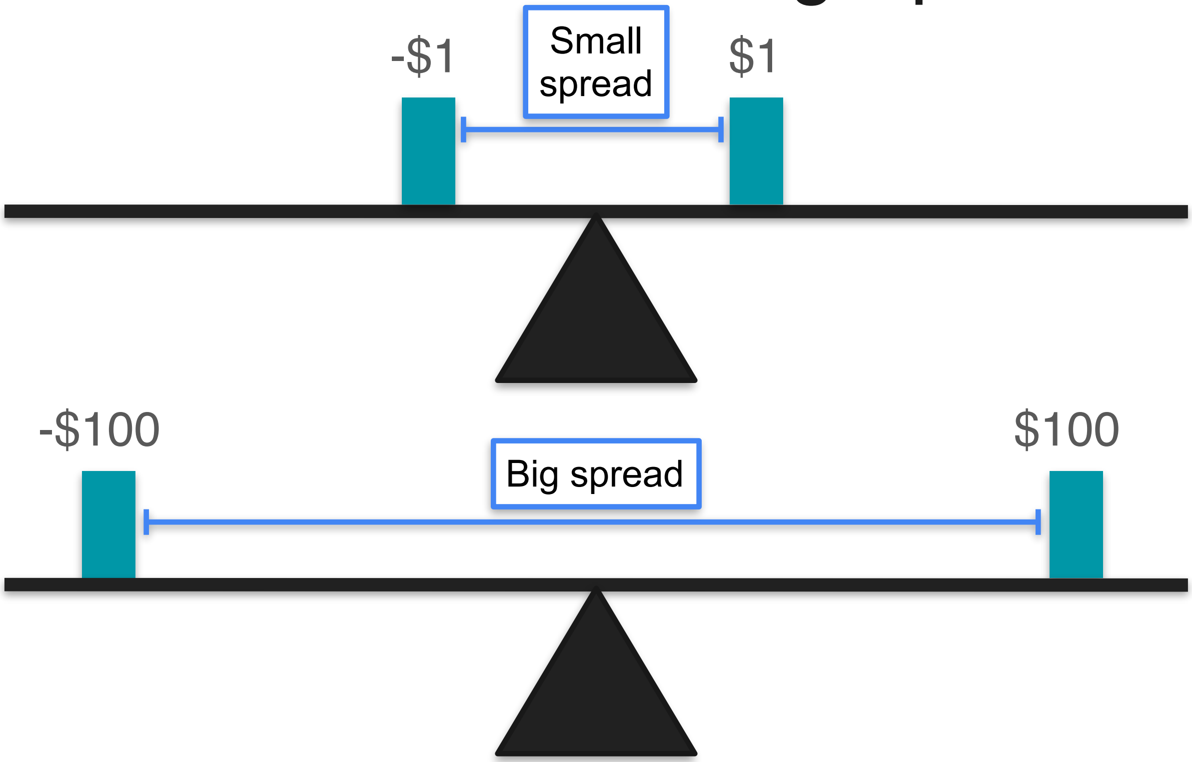
Variance Motivation: Measuring Spread



$\mathbb{E}[X_1] = 0$

$\mathbb{E}[X_2] = 0$

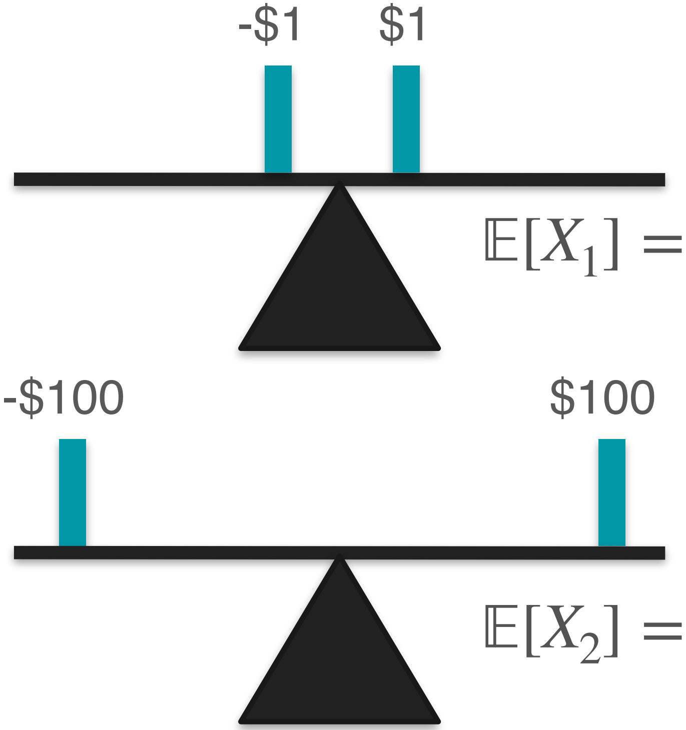
Variance Motivation: Measuring Spread



$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

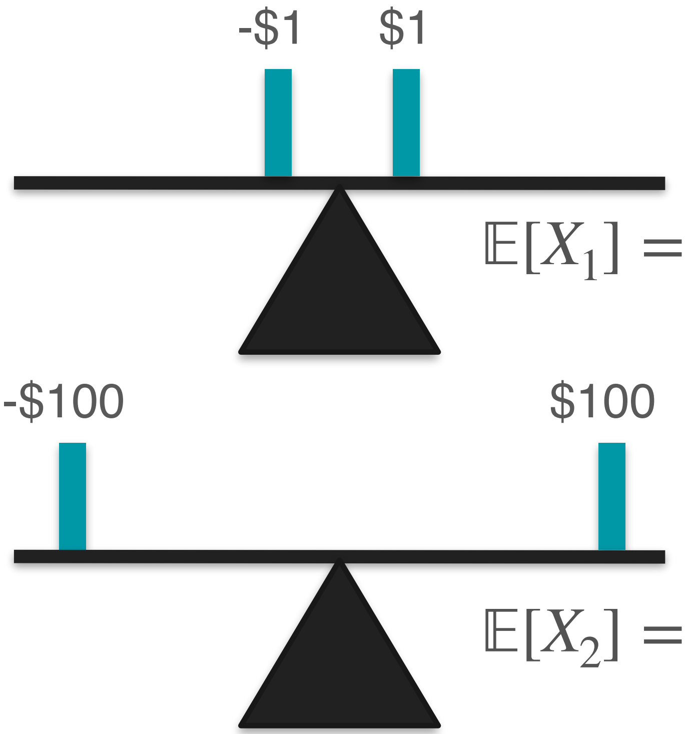
Variance Motivation: Measuring Spread



$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

Variance Motivation: Measuring Spread



$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 0$$

Variance Motivation: Measuring Spread

$\mathbb{E}[X_1] = 0$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 0$$

$\mathbb{E}[X_2] = 0$

$$\mathbb{E}[X_2^2] = \frac{(100) + (-100)}{2} = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 0$$

Turn into positive?

$$\mathbb{E}[X_2^2] = \frac{(100) \quad + (-100)}{2} = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 0$$

$$\mathbb{E}[X_2^2] = \frac{(100) \;\; + (-100)}{2} = 0$$

Variance Motivation: Measuring Spread

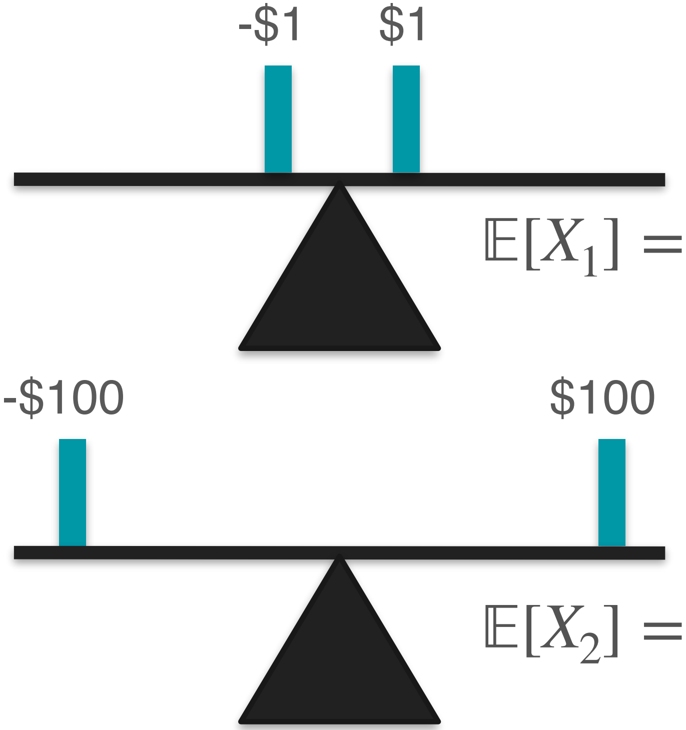
$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 0$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = \frac{(100)^2 + (-100)^2}{2} = 0$$

Variance Motivation: Measuring Spread



$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 0$$

$$\mathbb{E}[X_2^2] = \frac{(100)^2 + (-100)^2}{2} = 0$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 1$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = \frac{(100)^2 + (-100)^2}{2} = 0$$

Variance Motivation: Measuring Spread

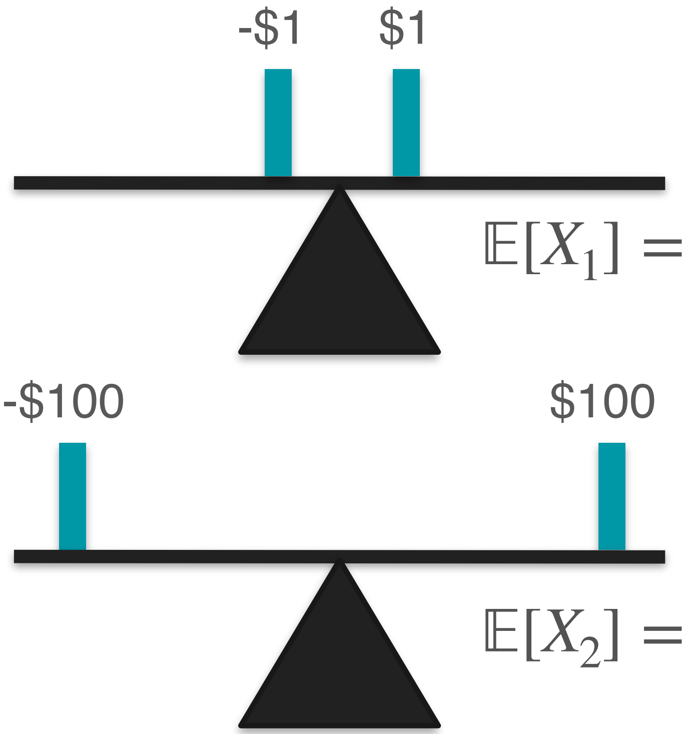
$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = \frac{(1)^2 + (-1)^2}{2} = 1$$

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = \frac{(100)^2 + (-100)^2}{2} = 10{,}000$$

Variance Motivation: Measuring Spread



$\mathbb{E}[X_1] = 0$

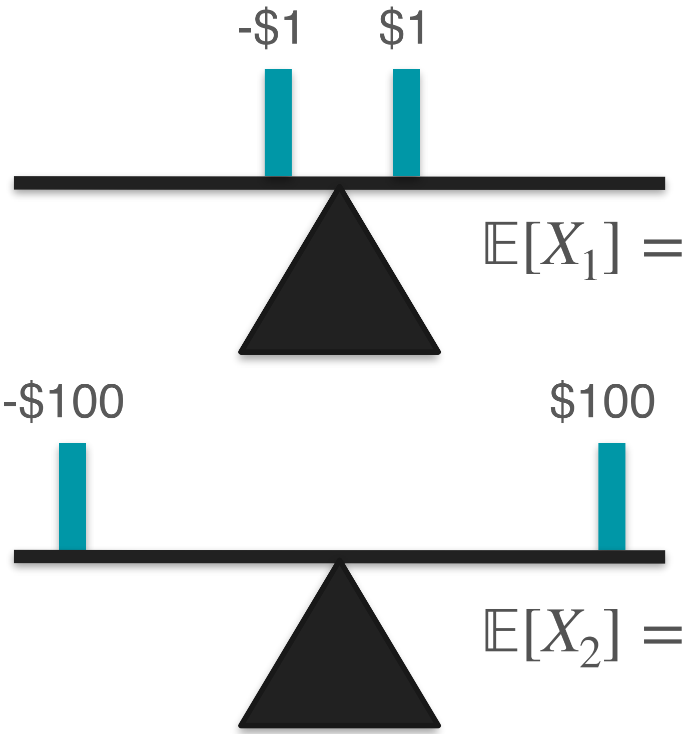
$\mathbb{E}[X_2] = 0$

Key for telling game 1 and game 2 apart!

$$\boxed{\mathbb{E}[X_1^2]} = \frac{(1)^2 + (-1)^2}{2} = 1$$

$$\boxed{\mathbb{E}[X_2^2]} = \frac{(100)^2 + (-100)^2}{2} = 10{,}000$$

Variance Motivation: Measuring Spread



$\mathbb{E}[X_1] = 0$

$\mathbb{E}[X_2] = 0$

Key for telling game 1 and game 2 apart!

$$\boxed{\mathbb{E}[X_1^2]} = \frac{(1)^2 + (-1)^2}{2} = 1$$

$$\boxed{\mathbb{E}[X_2^2]} = \frac{(100)^2 + (-100)^2}{2} = 10{,}000$$

Measure of spread

Variance Motivation: Measuring Spread

$$\mathbb{E}[X^2]$$

Variance Motivation: Measuring Spread

$$\mathbb{E}[X^2]$$

Almost…

Variance Motivation: Centering With Mean

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 6 dollars

You win 4 dollars

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 6 dollars

You win 4 dollars

How risky are these two games in comparison?

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 6 dollars

You win 4 dollars

How risky are these two games in comparison?

Hint: Think of the spread

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 6 dollars

You win 4 dollars

They are equally risky

Variance Motivation: Centering With Mean

Game 1

H You win 1 dollar

T You lose 1 dollar

$$\mathbb{E}[X_1]^2 = \frac{(-1)^2 + (1)^2}{2} = 1$$

Game 2

H You win 6 dollars

T You win 4 dollars

$$\mathbb{E}[X_2]^2 = \frac{(4)^2 + (6)^2}{2} = 26$$

Variance Motivation: Centering With Mean

Game 1

You win 1 dollar

You lose 1 dollar

Game 2

You win 6 dollars

You win 4 dollars

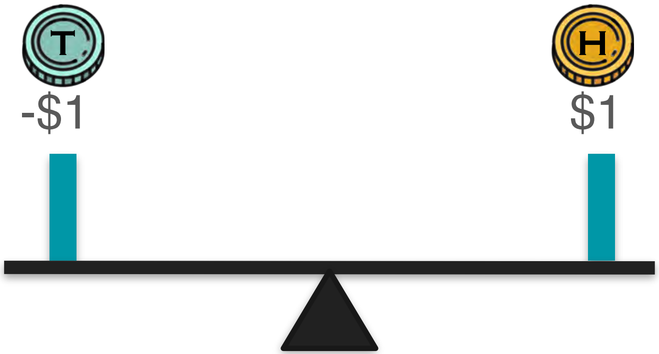
$$\mathbb{E}[X_1]^2 = \frac{(-1)^2 + (1)^2}{2} = 1$$

Same risk?

$$\mathbb{E}[X_2]^2 = \frac{(4)^2 + (6)^2}{2} = 26$$

Variance Motivation: Centering With Mean

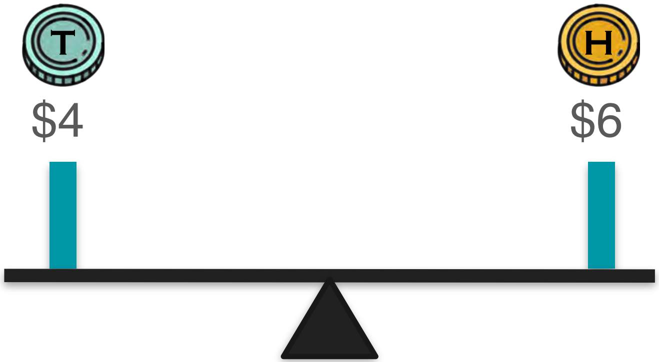
$$\mathbb{E}[X_1]^2 = \frac{(-1)^2 + (1)^2}{2} = 1$$



$$\mathbb{E}[X_1] = 0$$

Game 1

$$\mathbb{E}[X_2]^2 = \frac{(4)^2 + (6)^2}{2} = 26$$



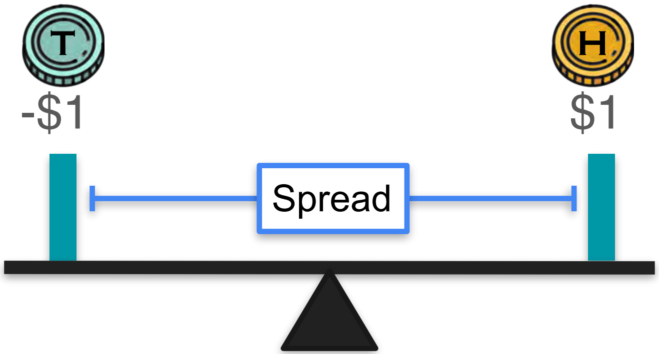
$$\mathbb{E}[X_2] = 5$$

Game 2

Variance Motivation: Centering With Mean

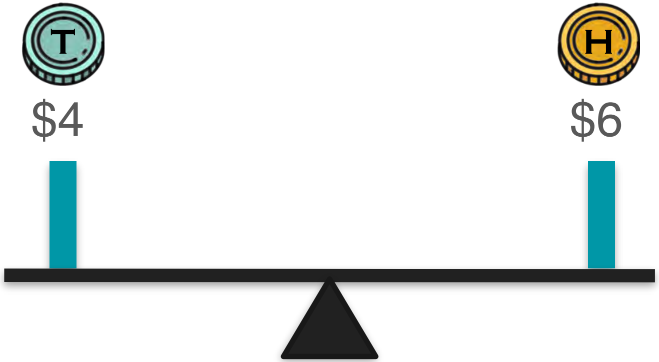
$$\mathbb{E}[X_1]^2 = \frac{(-1)^2 + (1)^2}{2} = 1$$

$$\mathbb{E}[X_2]^2 = \frac{(4)^2 + (6)^2}{2} = 26$$



$\mathbb{E}[X_1] = 0$

Game 1

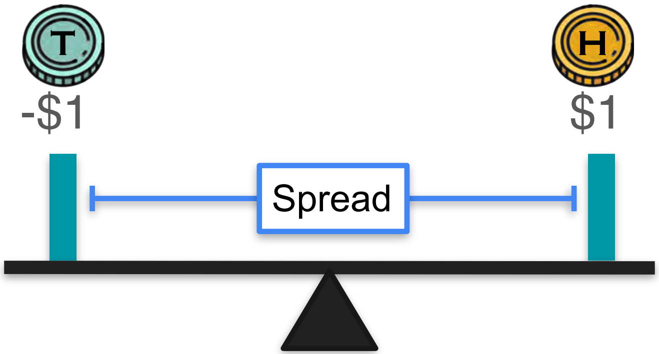


$\mathbb{E}[X_2] = 5$

Game 2

Variance Motivation: Centering With Mean

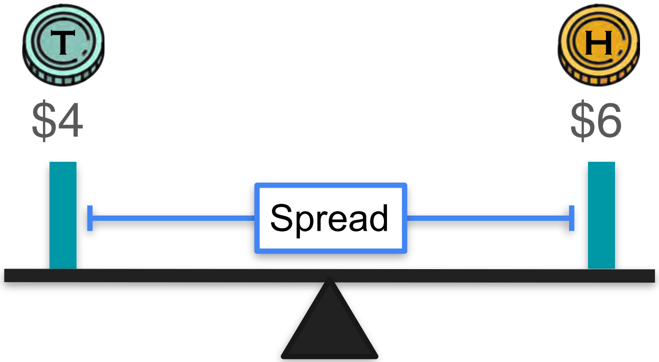
$$\mathbb{E}[X_1]^2 = \frac{(-1)^2 + (1)^2}{2} = 1$$



$$\mathbb{E}[X_1] = 0$$

Game 1

$$\mathbb{E}[X_2]^2 = \frac{(4)^2 + (6)^2}{2} = 26$$

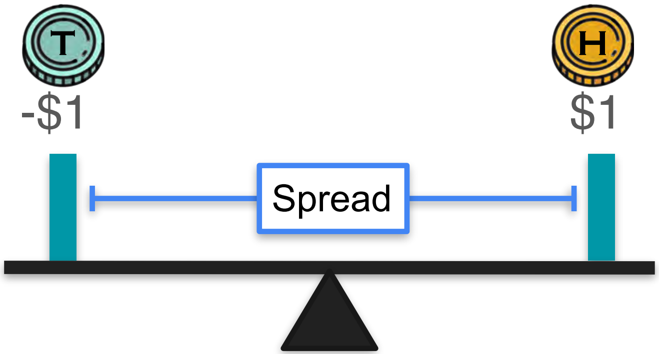


$$\mathbb{E}[X_2] = 5$$

Game 2

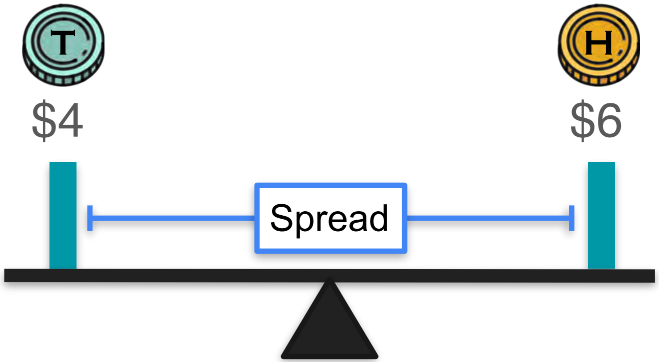
Variance Motivation: Centering With Mean

$$\mathbb{E}[X_1]^2 = \frac{(-1)^2 + (1)^2}{2} = 1 \longleftarrow \text{Same spread?} \longrightarrow \mathbb{E}[X_2]^2 = \frac{(4)^2 + (6)^2}{2} = 26$$



$$\mathbb{E}[X_1] = 0$$

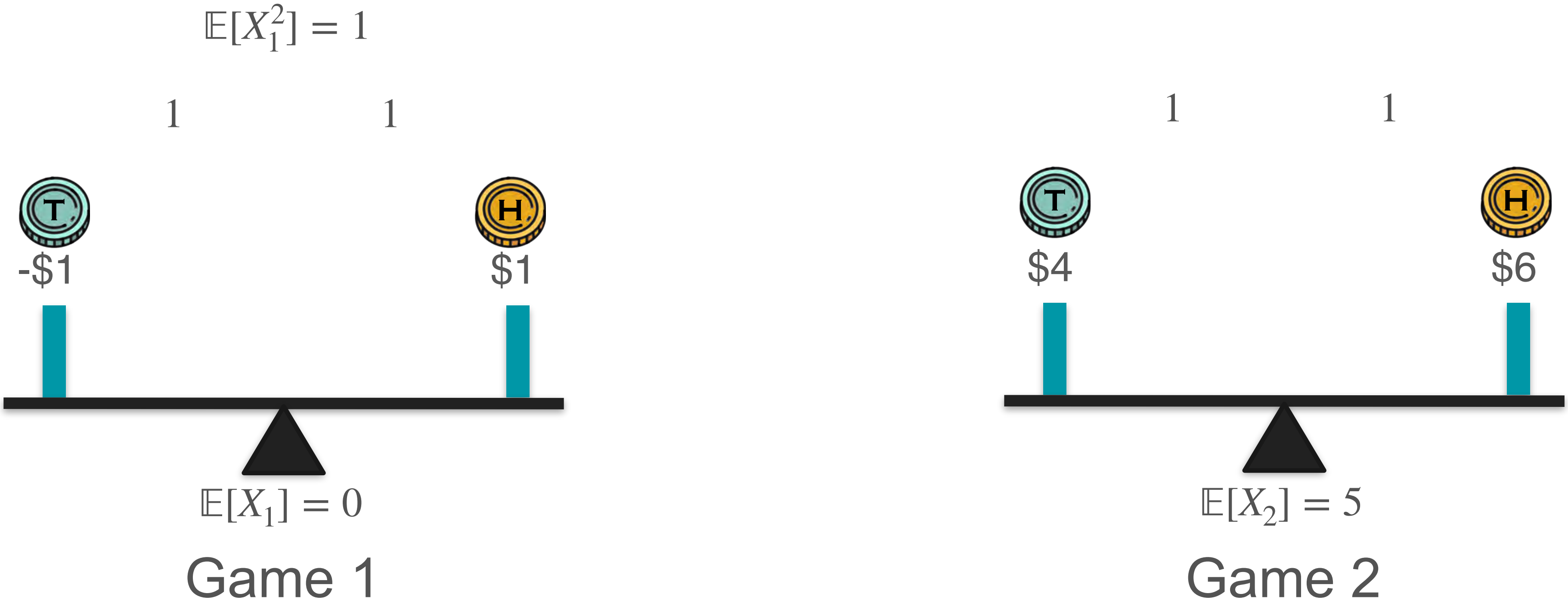
Game 1



$$\mathbb{E}[X_2] = 5$$

Game 2

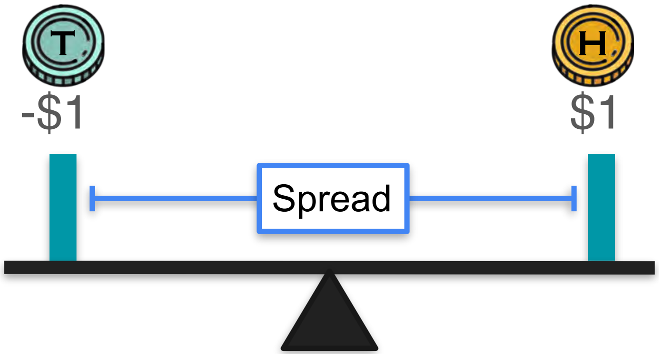
Variance: Spread and Shift



Variance: Spread and Shift

$\mathbb{E}[X_1^2] = 1$

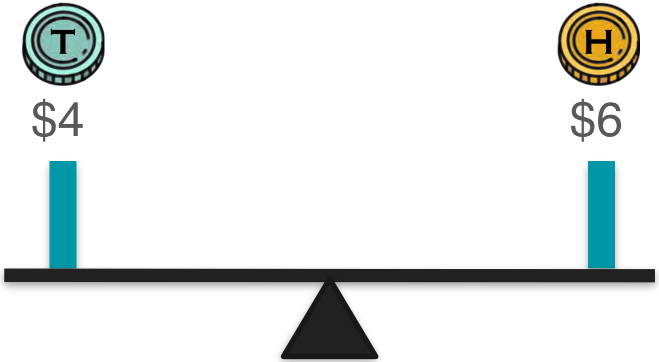
1 1



$\mathbb{E}[X_1] = 0$

Game 1

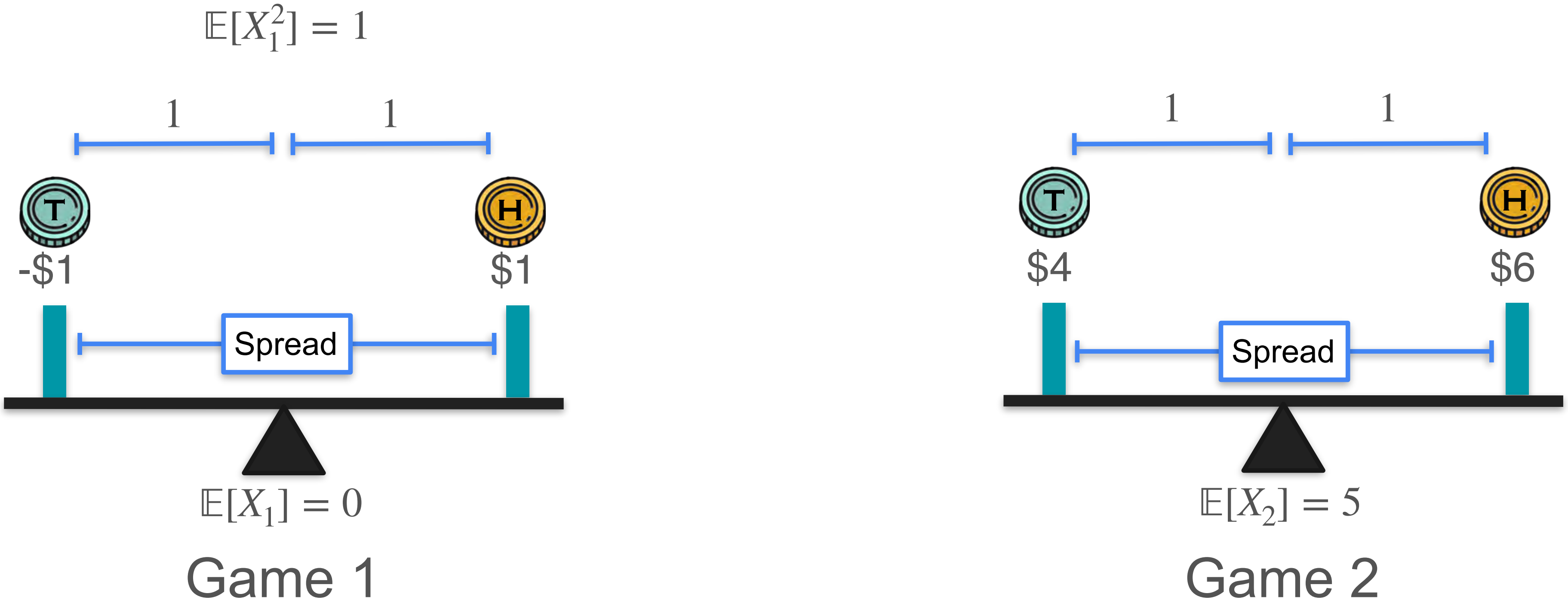
1 1



$\mathbb{E}[X_2] = 5$

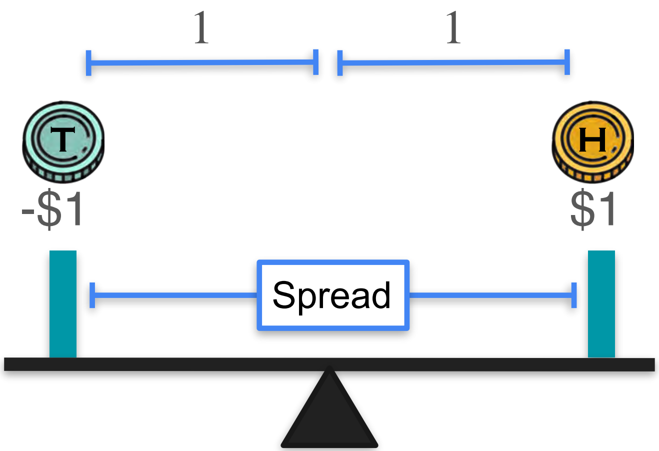
Game 2

Variance: Spread and Shift



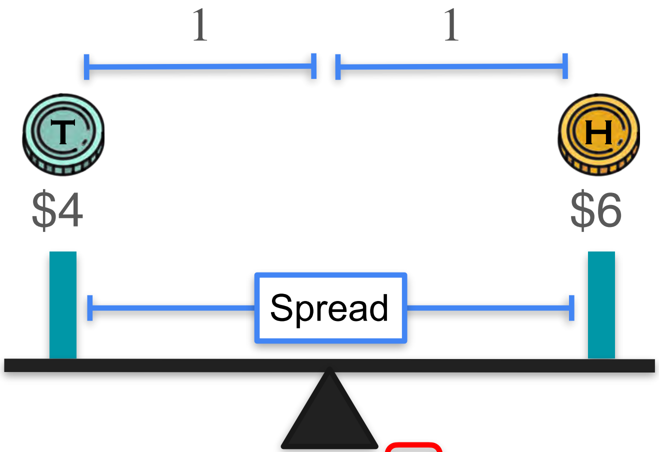
Variance: Spread and Shift

$$\mathbb{E}[X_1^2] = 1$$



$$\mathbb{E}[X_1] = 0$$

Game 1

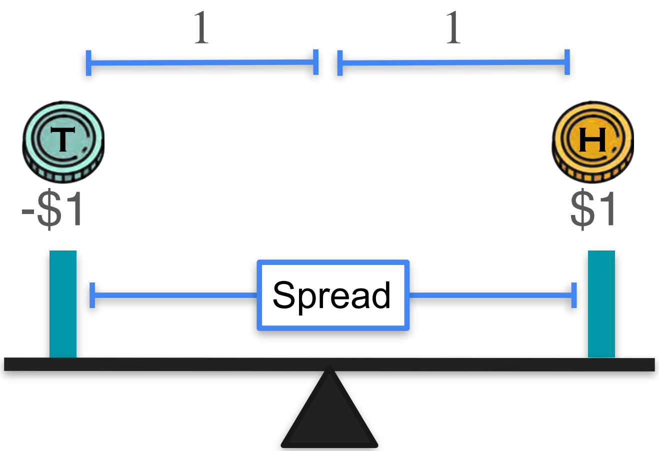


$$\mathbb{E}[X_2] = \boxed{5}$$

Game 2

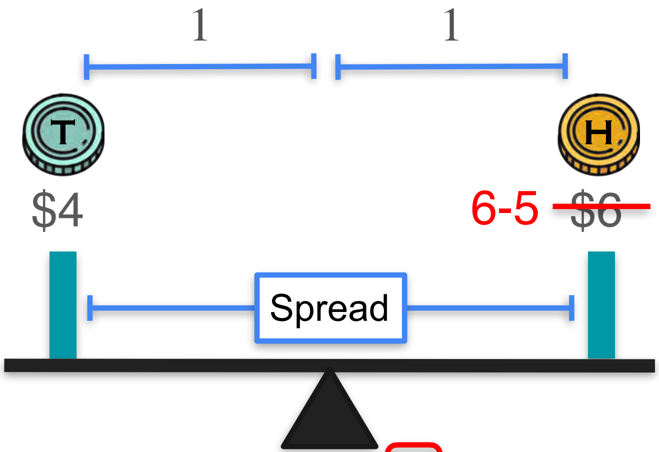
Variance: Spread and Shift

$$\mathbb{E}[X_1^2] = 1$$



$$\mathbb{E}[X_1] = 0$$

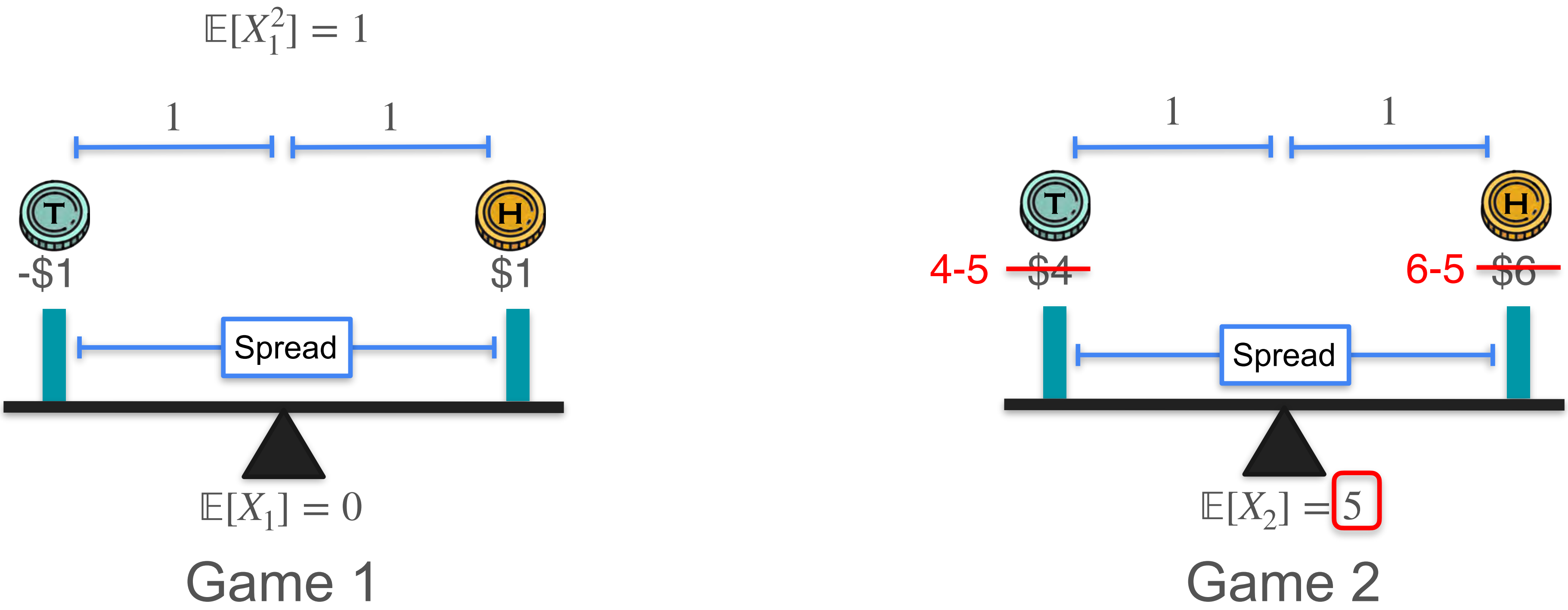
Game 1



$$\mathbb{E}[X_2] = \boxed{5}$$

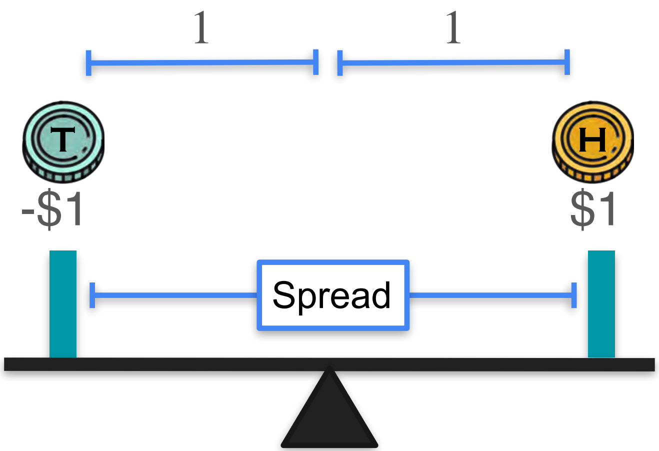
Game 2

Variance: Spread and Shift



Variance: Spread and Shift

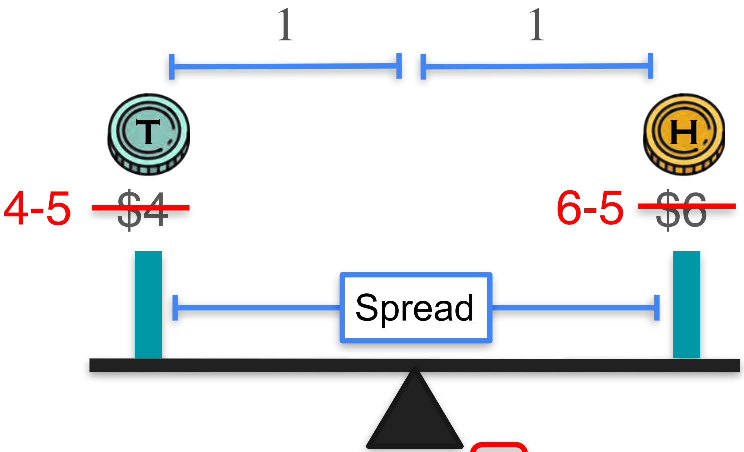
$\mathbb{E}[X_1^2] = 1$



$\mathbb{E}[X_1] = 0$

Game 1

$\mathbb{E}[(X_2 - 5)^2]$



$\mathbb{E}[X_2] = \boxed{5}$

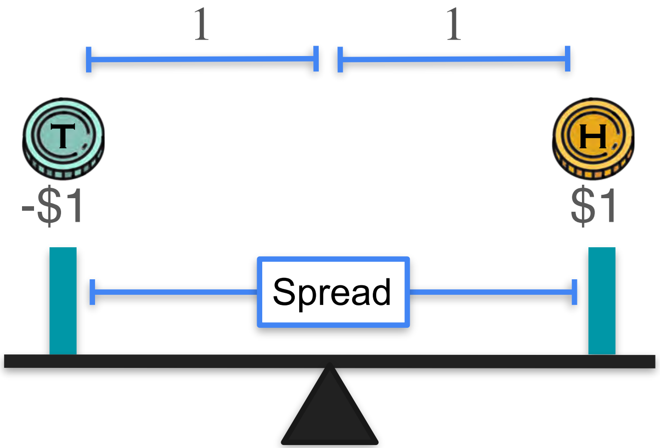
Game 2

Variance: Spread and Shift

$\mathbb{E}[X_1^2] = 1$

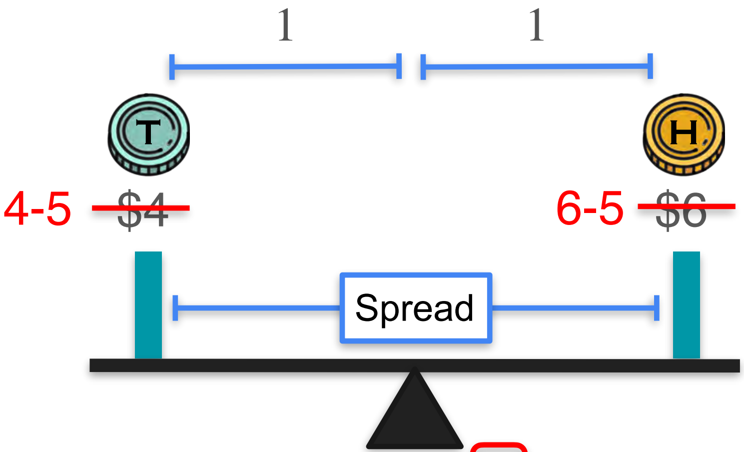
Mean: μ

$\mathbb{E}[(X_2 - 5)^2]$



$\mathbb{E}[X_1] = 0$

Game 1



$\mathbb{E}[X_2] = \boxed{5}$

Game 2

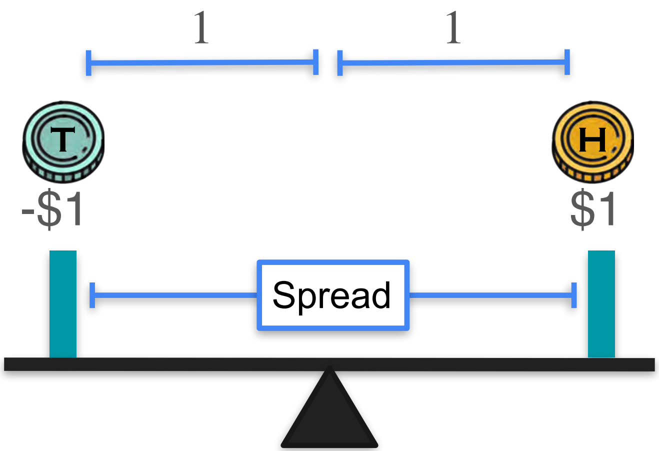
Variance: Spread and Shift

$\mathbb{E}[X_1^2] = 1$

Mean: μ

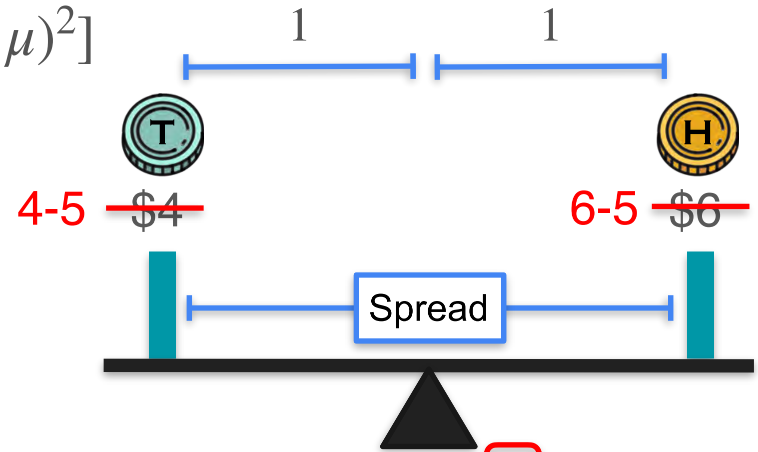
$\mathbb{E}[(X_2 - 5)^2]$

Variance: $\mathbb{E}[(X - \mu)^2]$



$\mathbb{E}[X_1] = 0$

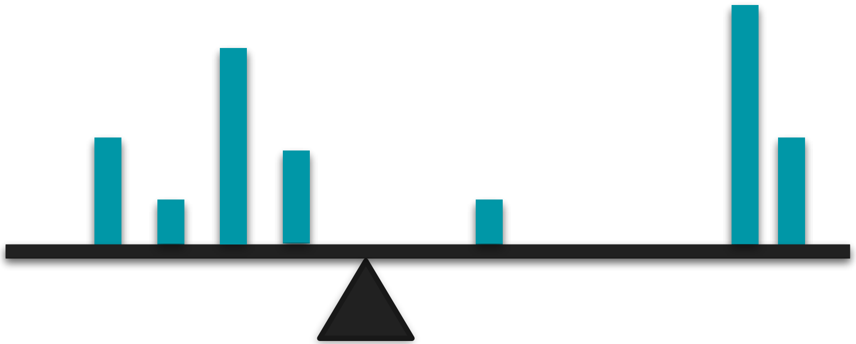
Game 1



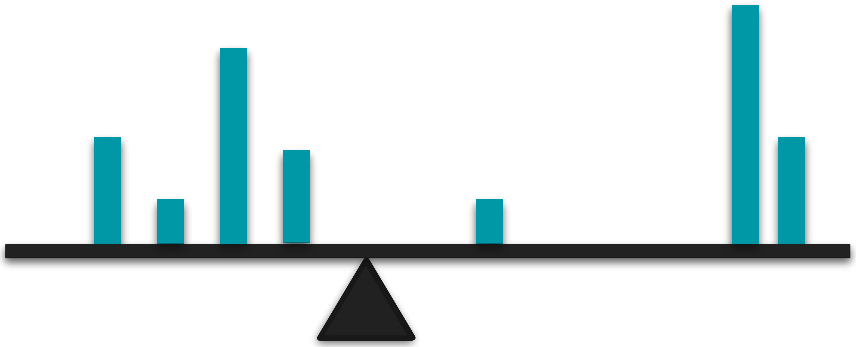
$\mathbb{E}[X_2] = \boxed{5}$

Game 2

Variance Formula

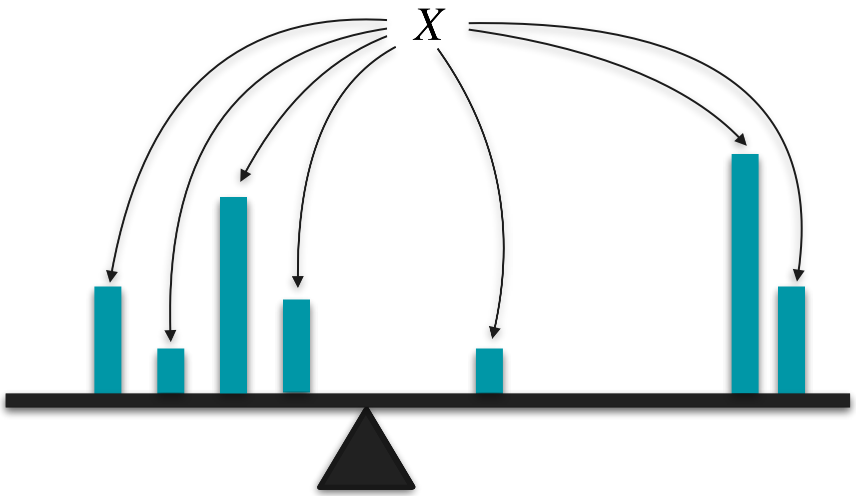


Variance Formula



$$\mathbb{E}[X] = \mu$$

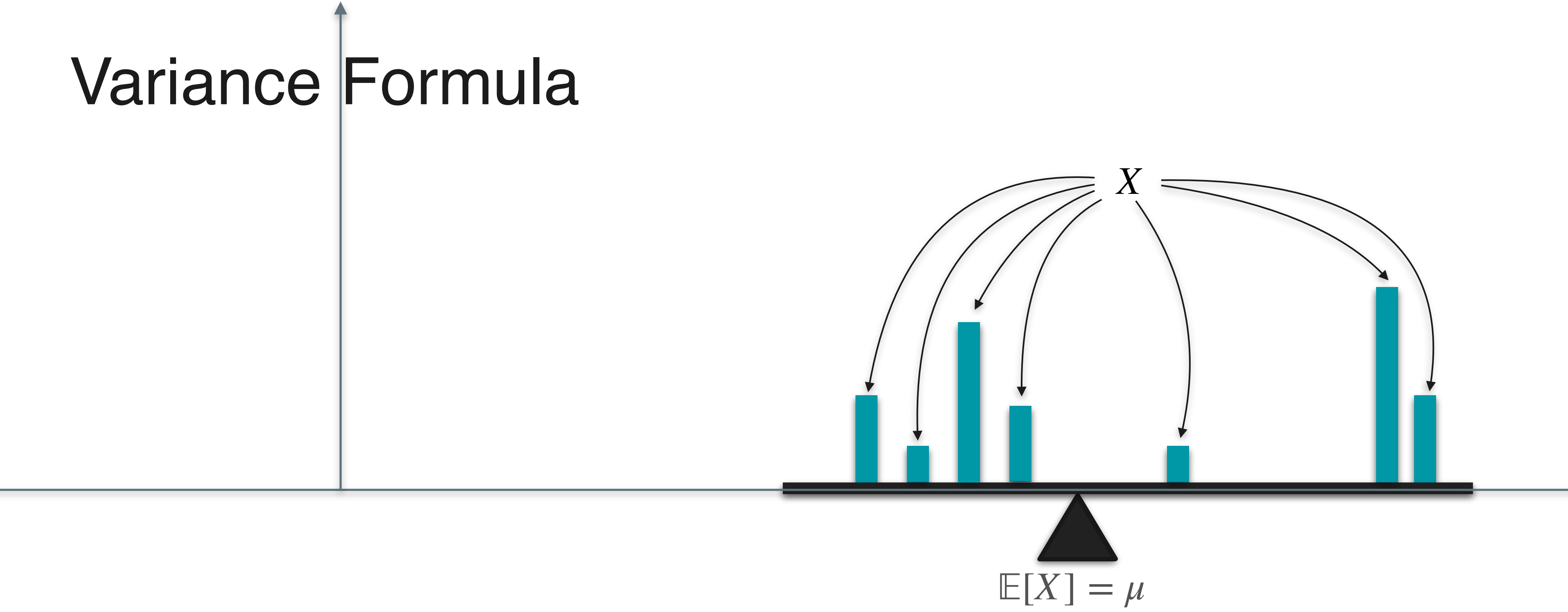
Variance Formula



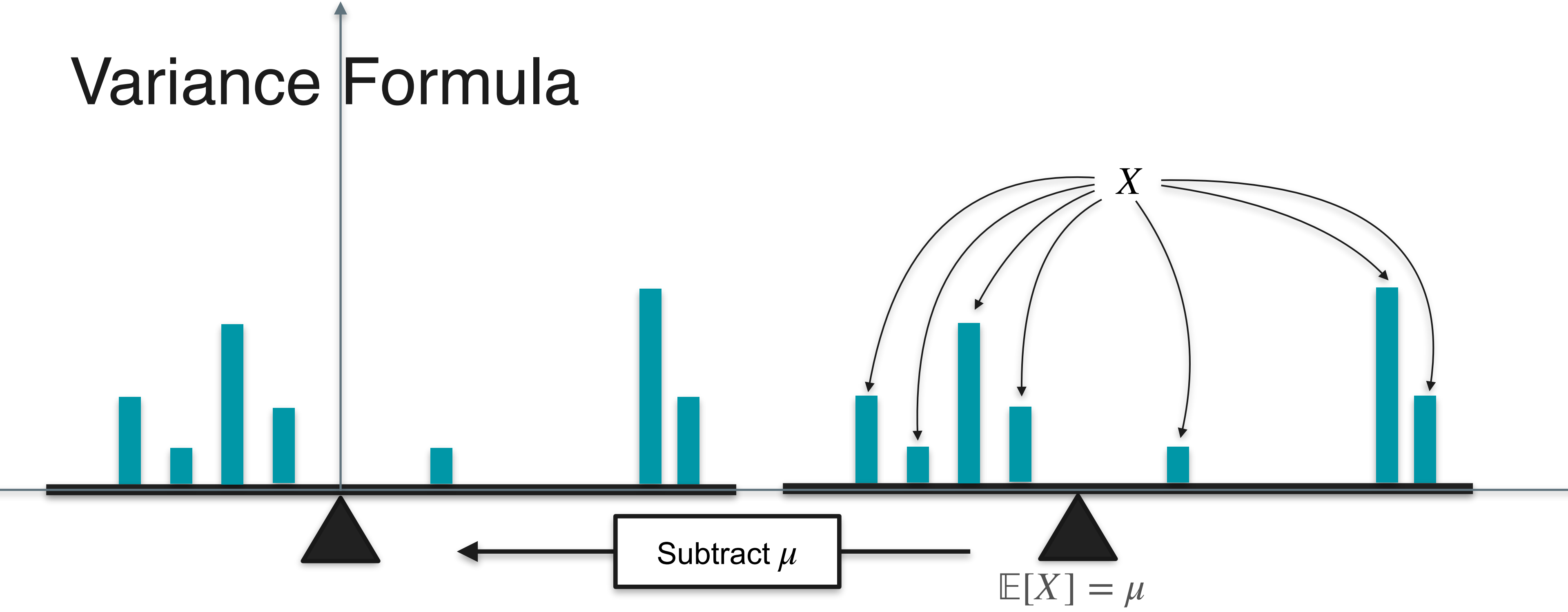
$\mathbb{E}[X] = \mu$

DeepLearning.AI

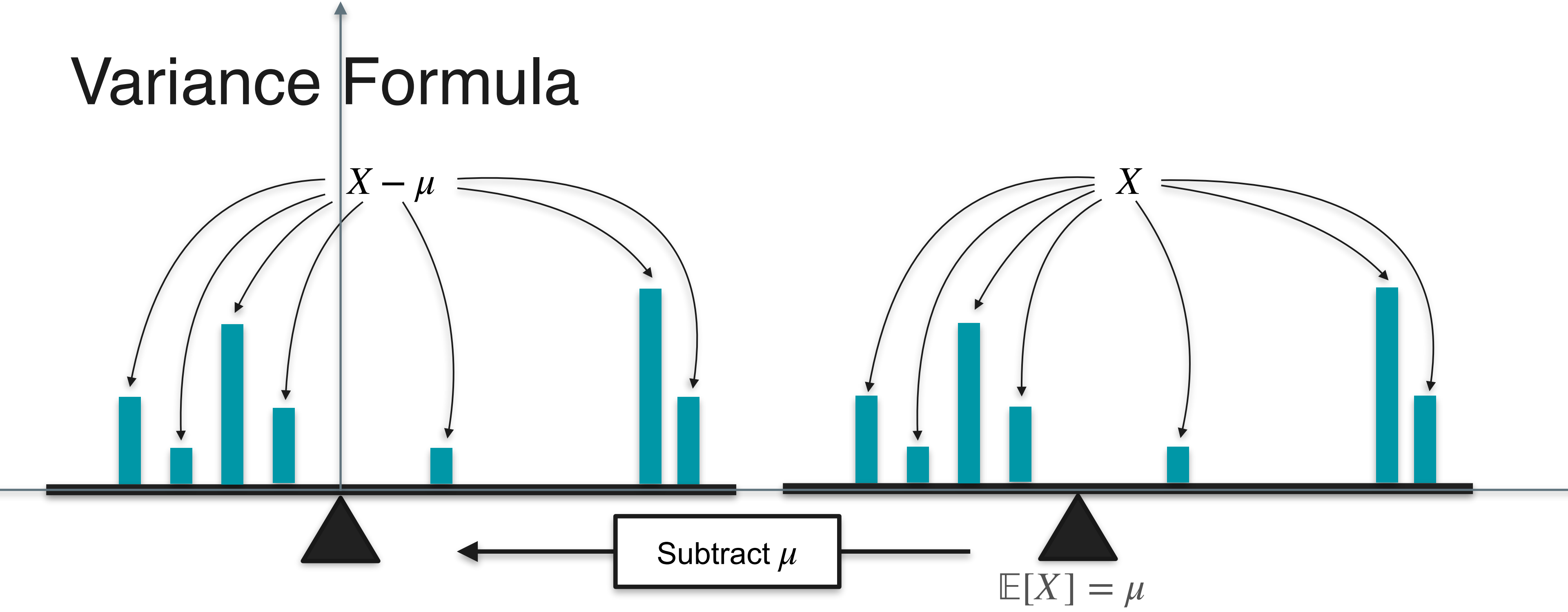
Variance Formula



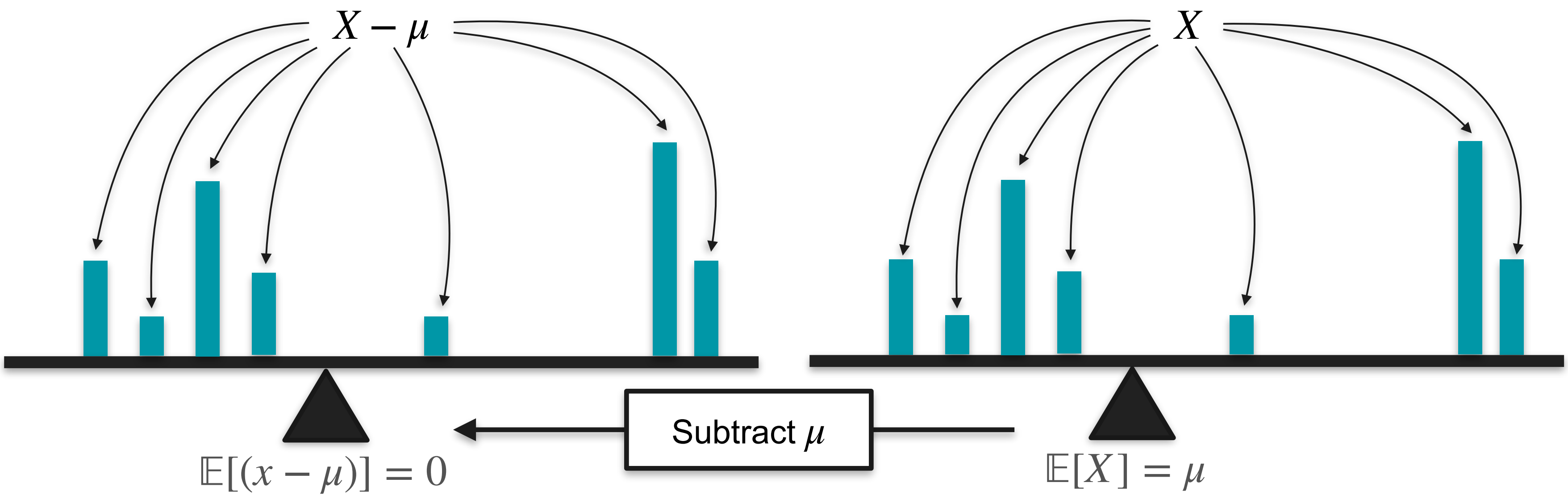
Variance Formula



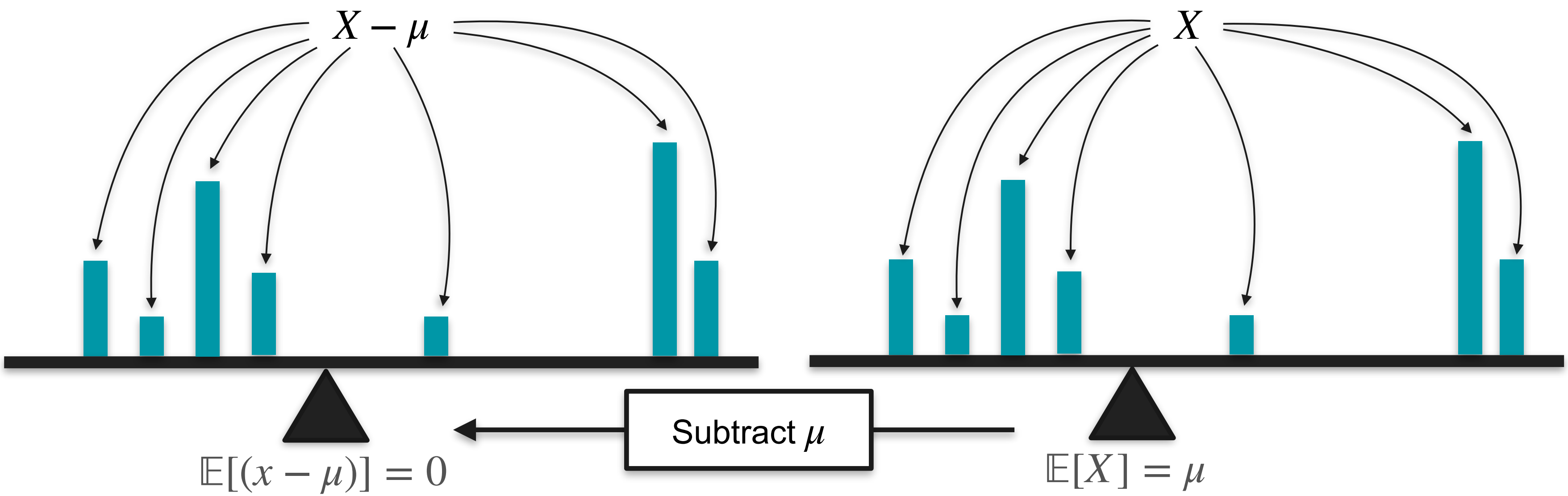
Variance Formula



Variance Formula



Variance Formula



$$Var(X) = \mathbb{E}[(X-\mu)^2]$$

Variance Formula

$$Var(X) = \mathbb{E}[(X - \mu)^2]$$

Variance Formula

$$Var(X) = \mathbb{E}[(X - \mu)^2]$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

Variance Formula

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$$\mathbb{E}[\text{constant} \cdot X] = \text{constant} \cdot \mathbb{E}[X]$$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$\mathbb{E}[\text{constant}] = \text{constant}$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2 \qquad \mu = \mathbb{E}[X]$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X] \cdot \mathbb{E}[X] + \mathbb{E}[X]^2$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X] \cdot \mathbb{E}[X] + \mathbb{E}[X]^2$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X]^2 + \mathbb{E}[X]^2$$

Variance Formula

$$Var(X) = \mathbb{E}[(X-\mu)^2] \quad = \mathbb{E}[X^2 - 2\mu X + \mu^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[2\mu X] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[X^2] - 2\mu\mathbb{E}[X] + \mu^2$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X] \cdot \mathbb{E}[X] + \mathbb{E}[X]^2$$

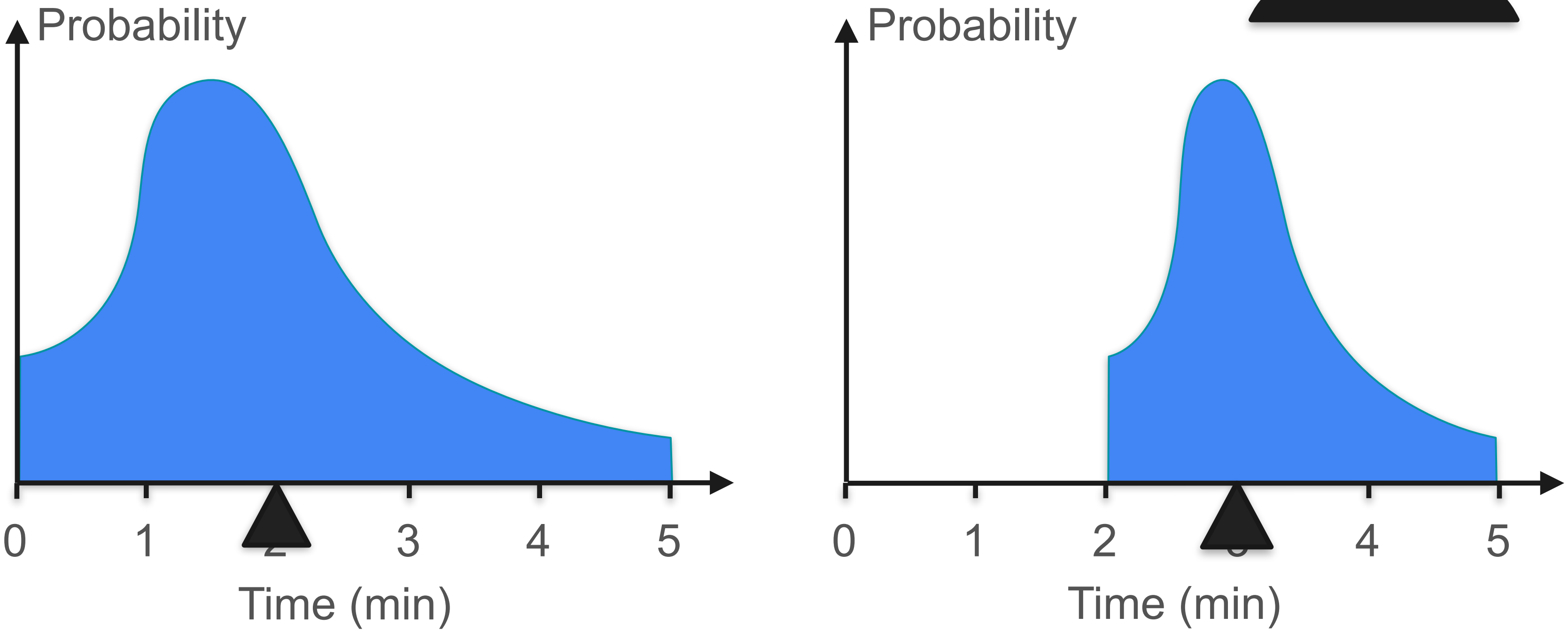
$$= \mathbb{E}[X^2] - 2\mathbb{E}[X]^2 + \mathbb{E}[X]^2$$

$$= \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

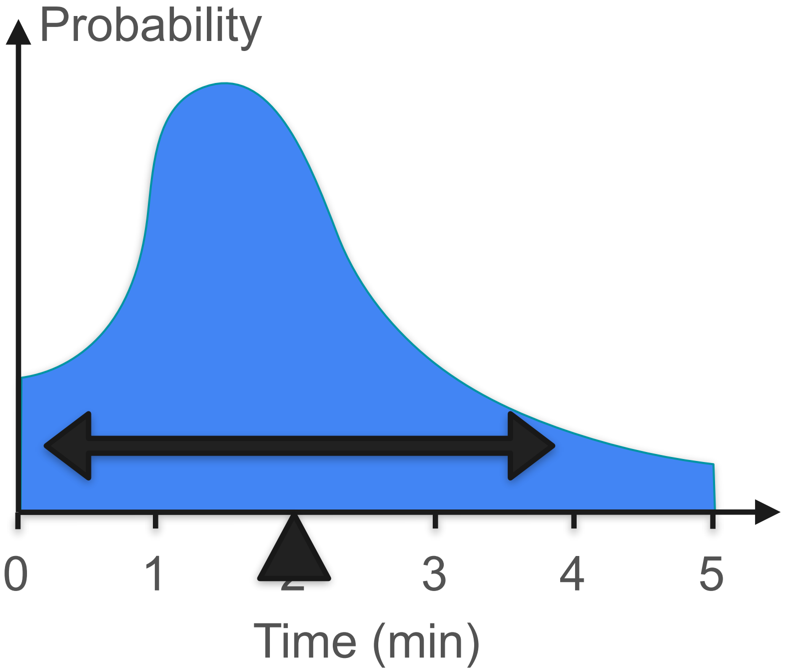
Variance Formula

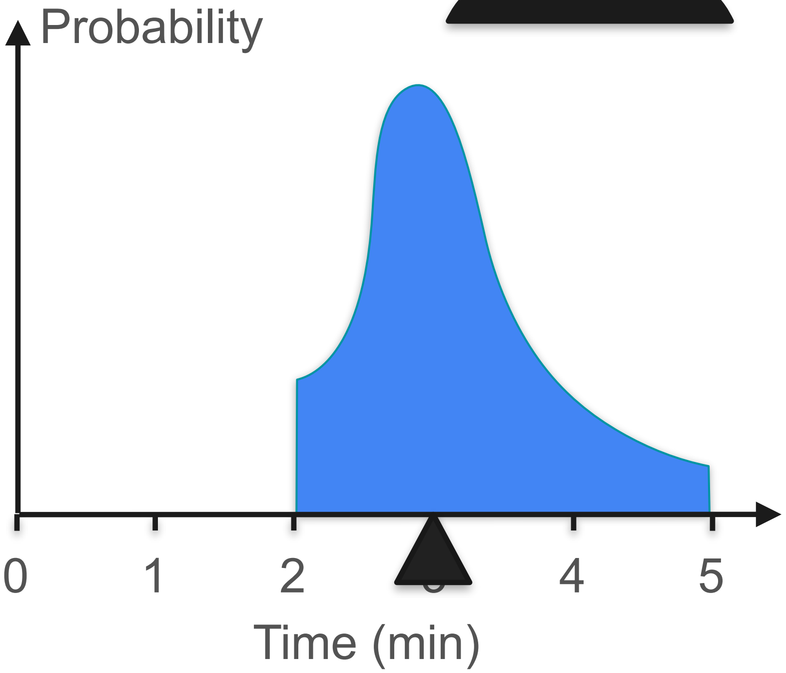
$$Var(X) = \mathbb{E}[(X - \mu)^2]$$
$$= \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

Variance for Continuous Distributions

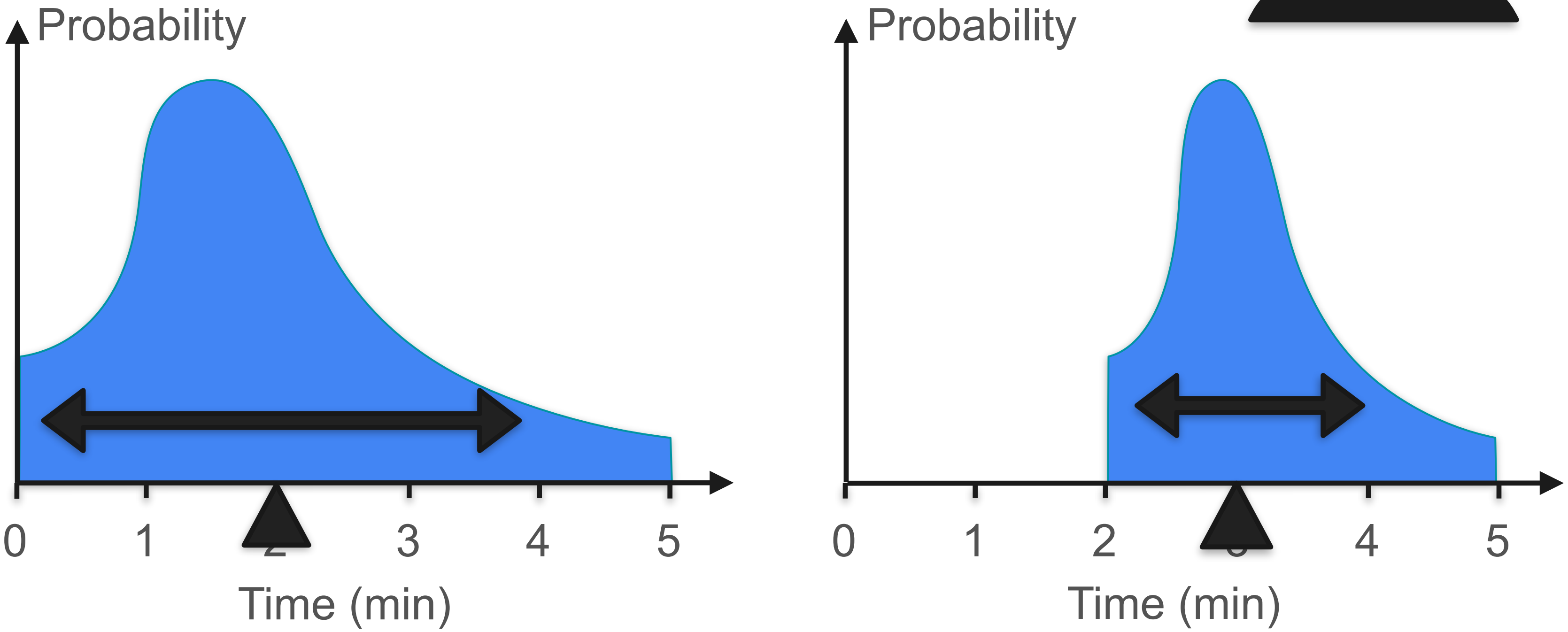


Variance for Continuous Distributions



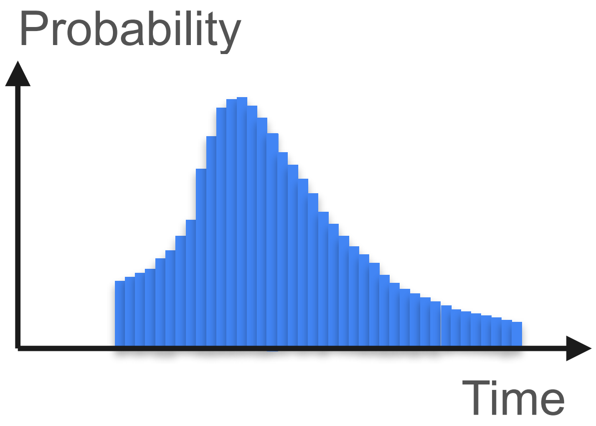


Variance for Continuous Distributions



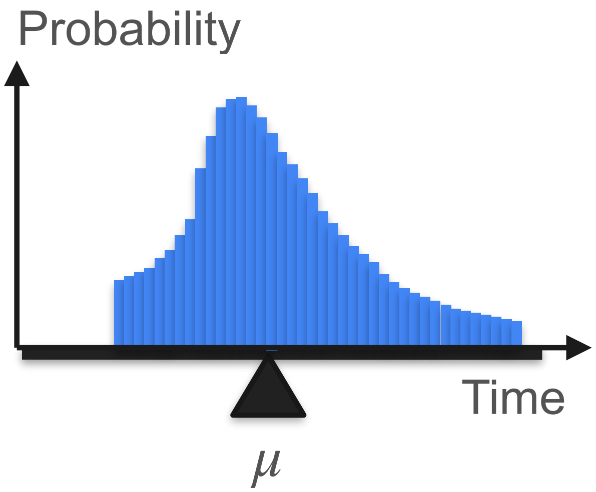
Variance

$$Var(X) = \mathbb{E}[(X - \mu)^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$



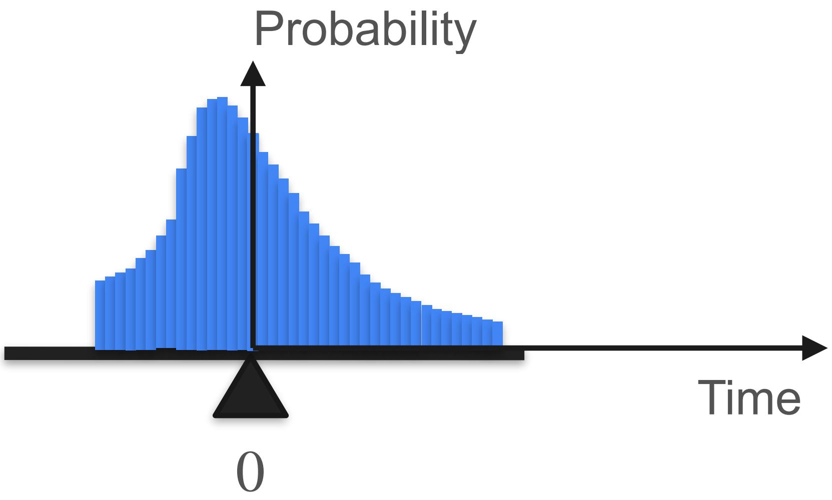
Variance

$$Var(X) = \mathbb{E}[(X-\mu)^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$



Variance

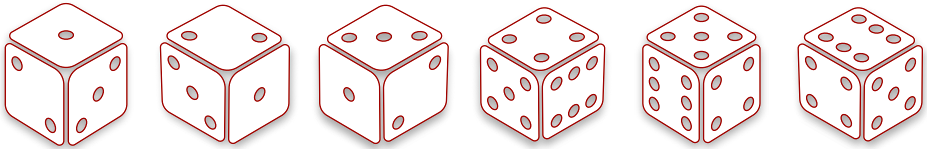
$$Var(X) = \mathbb{E}[(X-\mu)^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$



Properties of the Variance

Properties of the Variance

Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6



Double:	2	4	6	8	10	12
Wins	-3	-2	-1	0	1	2

Properties of the Variance

$$= 2.92$$

Properties of the Variance

$$= 2.92$$

Properties of the Variance

$$= 2.92$$

Properties of the Variance

$$= 2.92$$

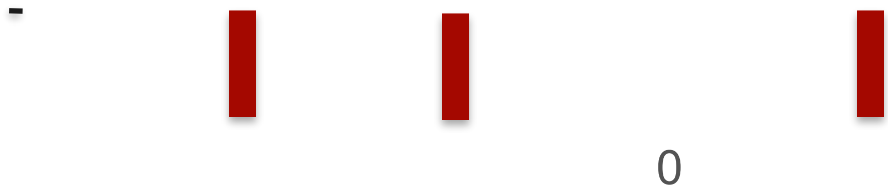
0

Properties of the Variance

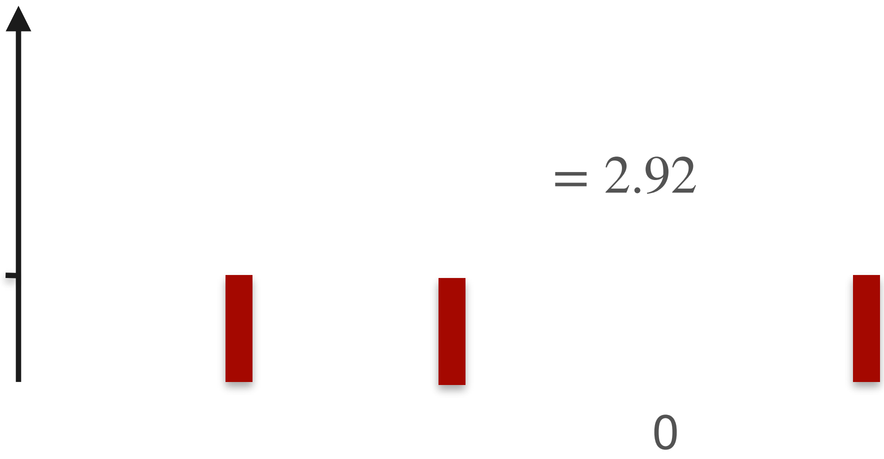
$$= 2.92$$

Properties of the Variance

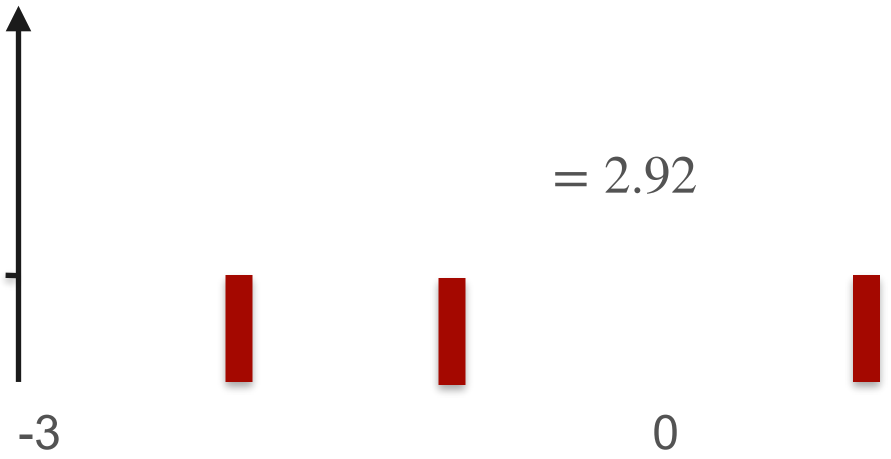
$$= 2.92$$



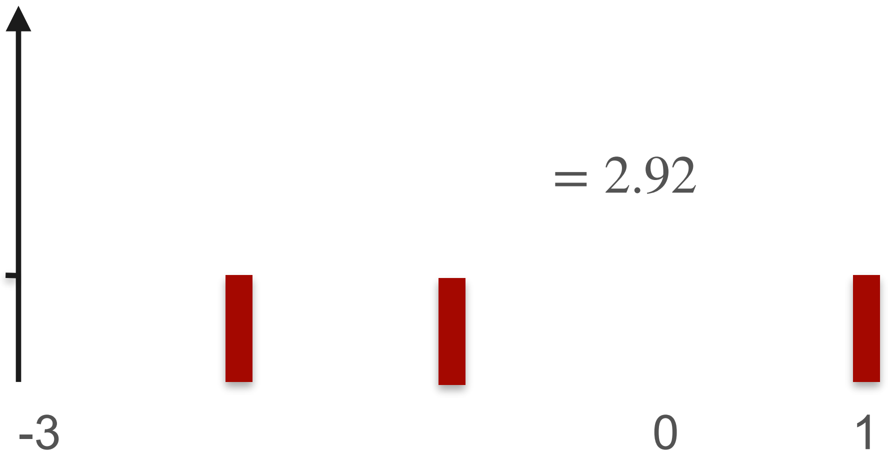
Properties of the Variance



Properties of the Variance

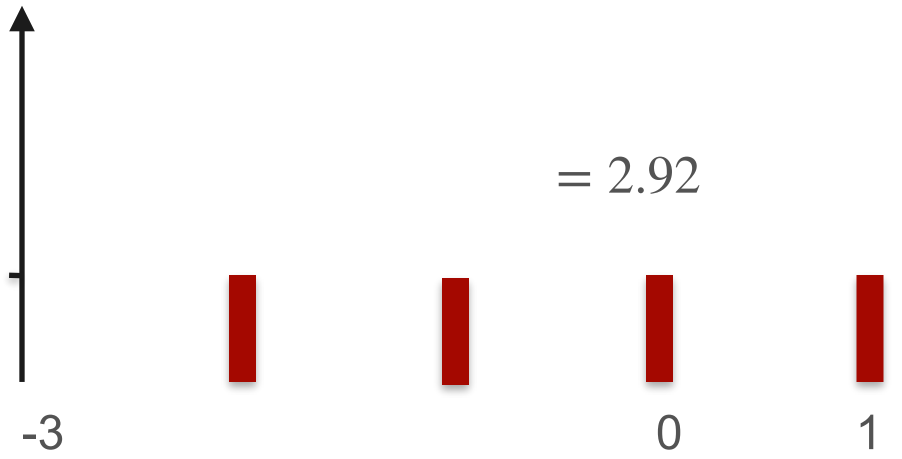


Properties of the Variance

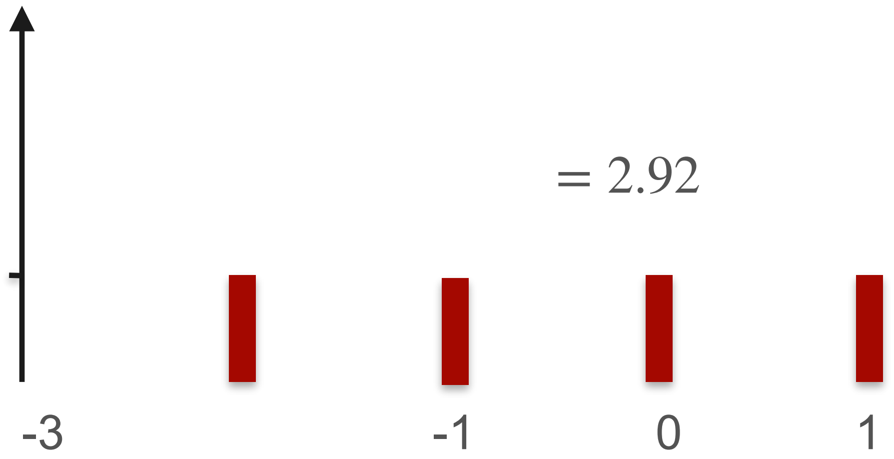


$= 2.92$

Properties of the Variance

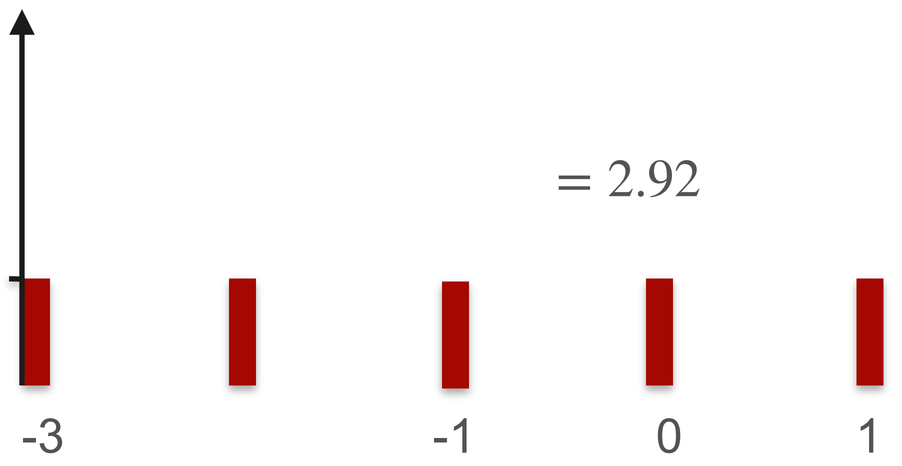


Properties of the Variance

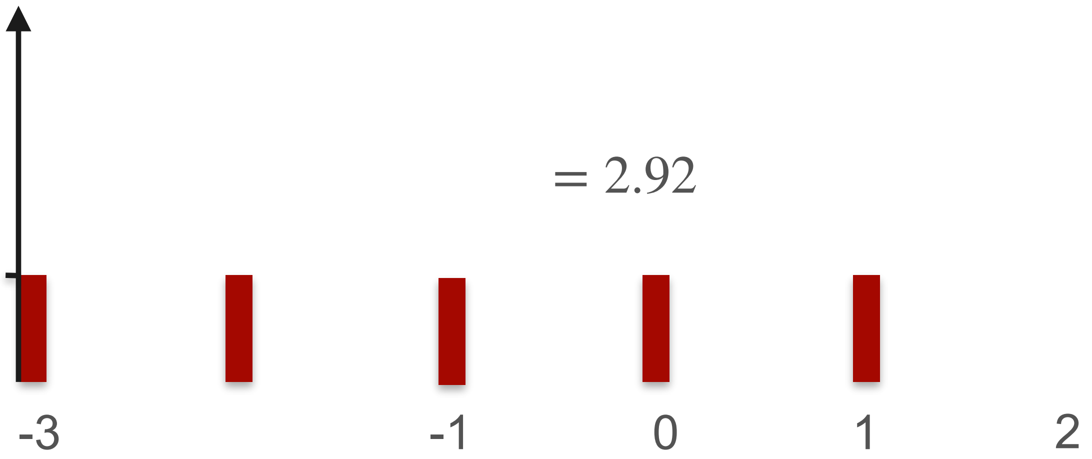


$= 2.92$

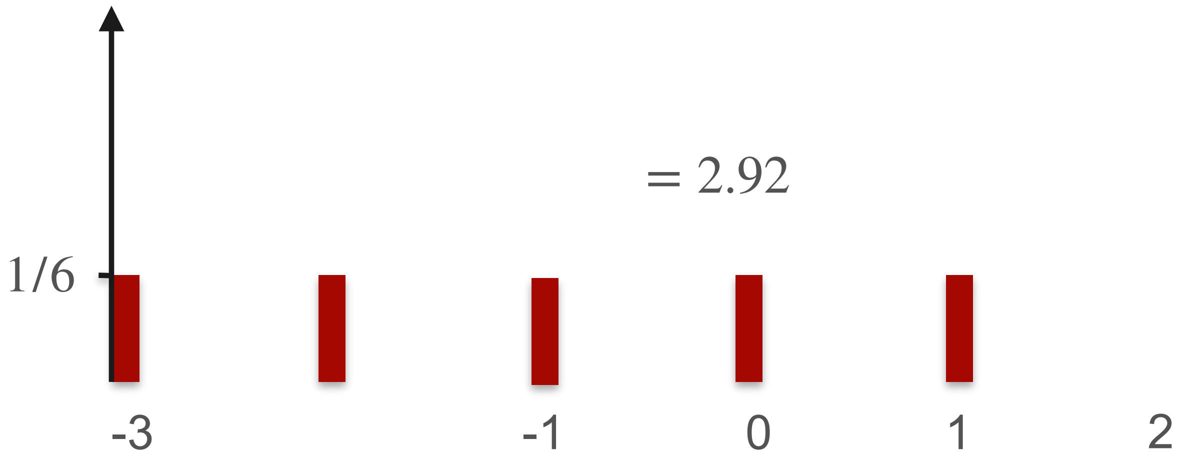
Properties of the Variance



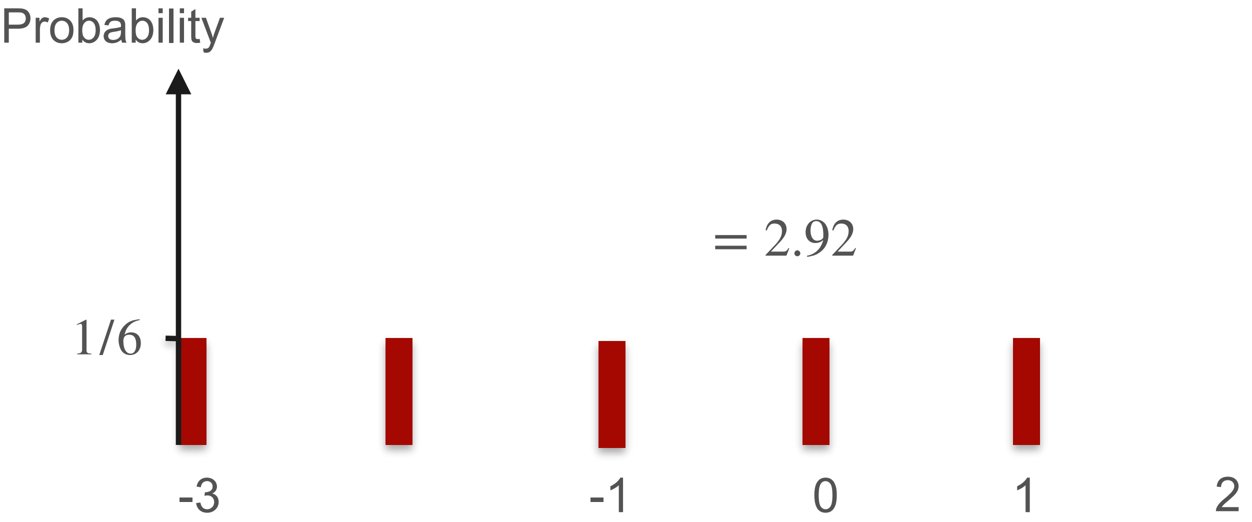
Properties of the Variance



Properties of the Variance

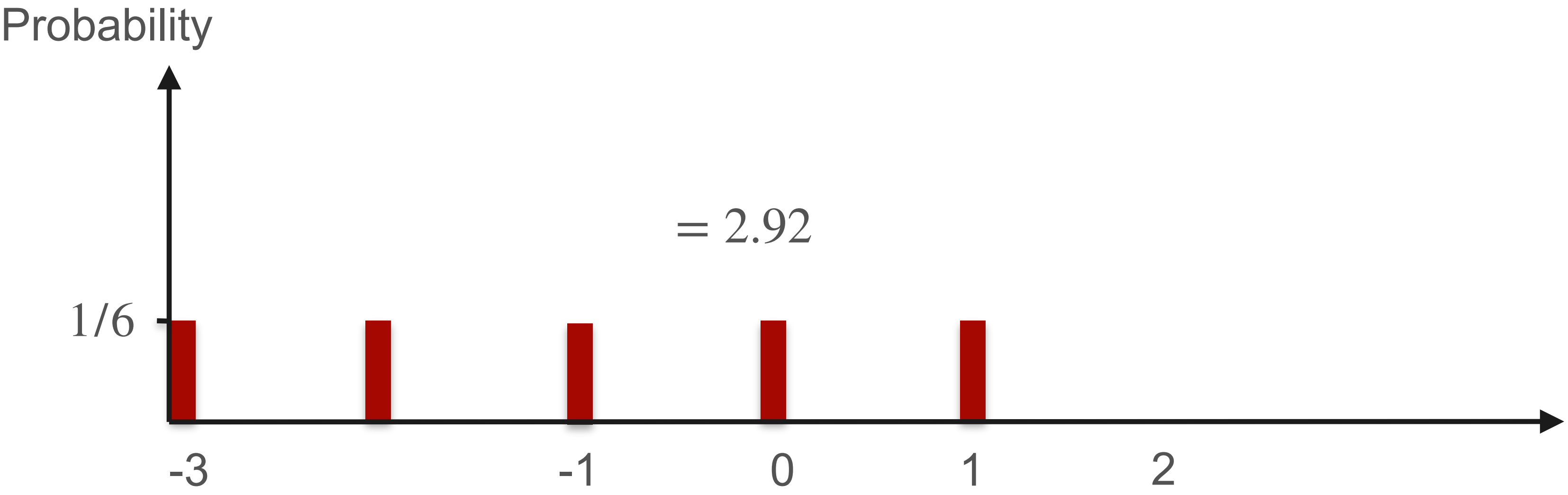


Properties of the Variance

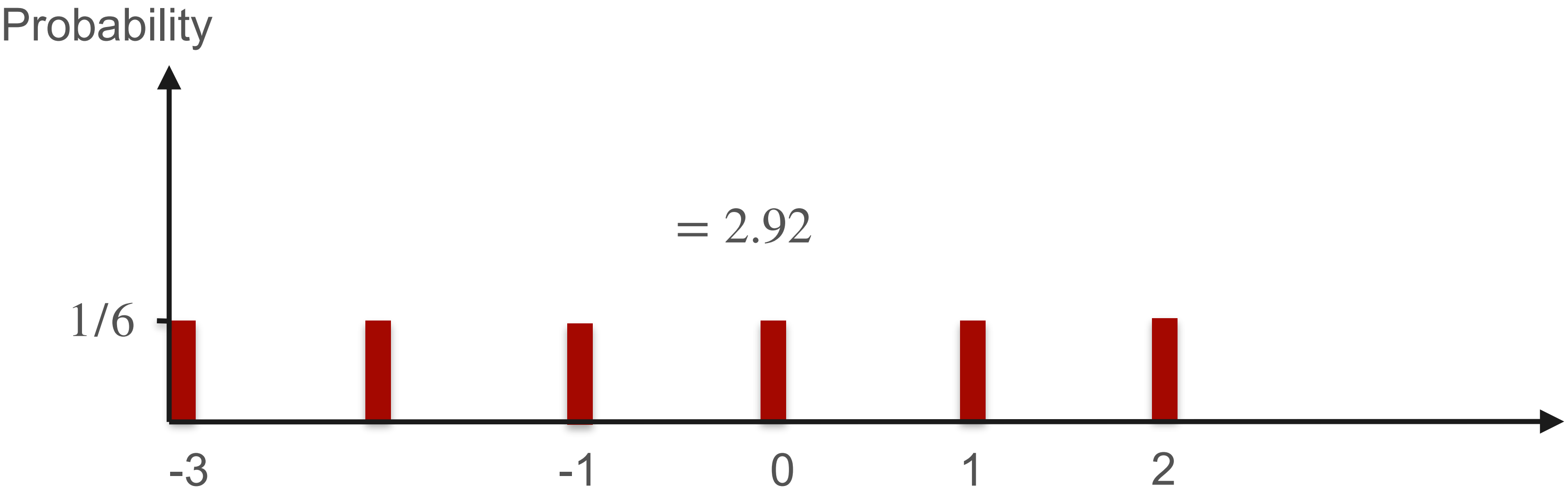


$= 2.92$

Properties of the Variance

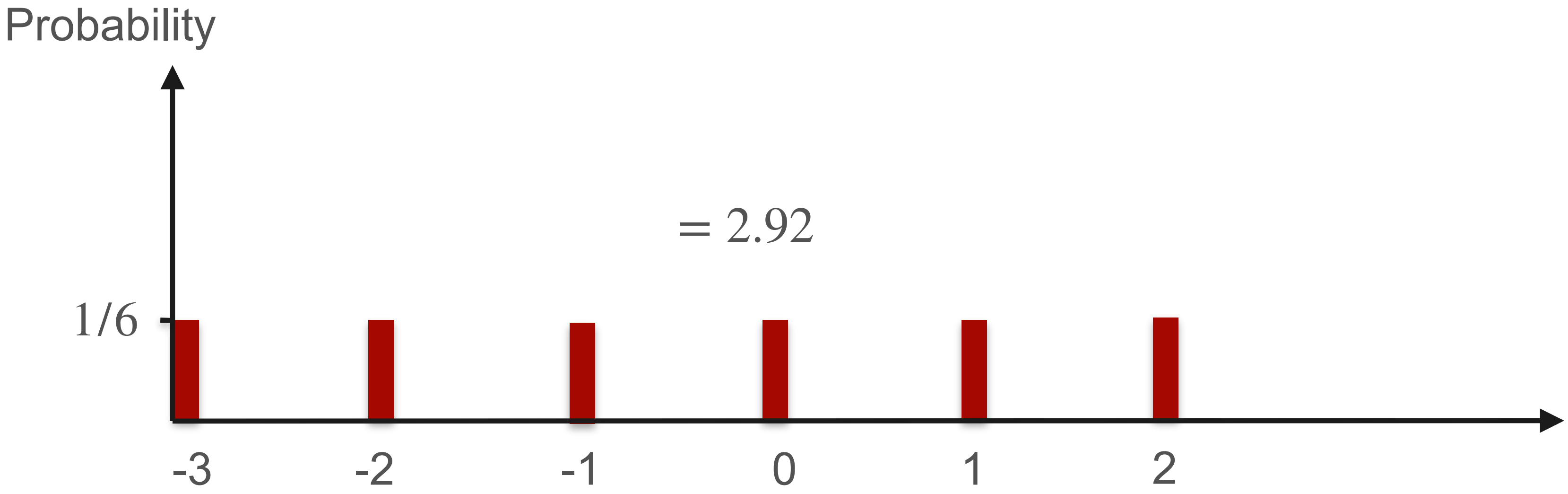


Properties of the Variance



$= 2.92$

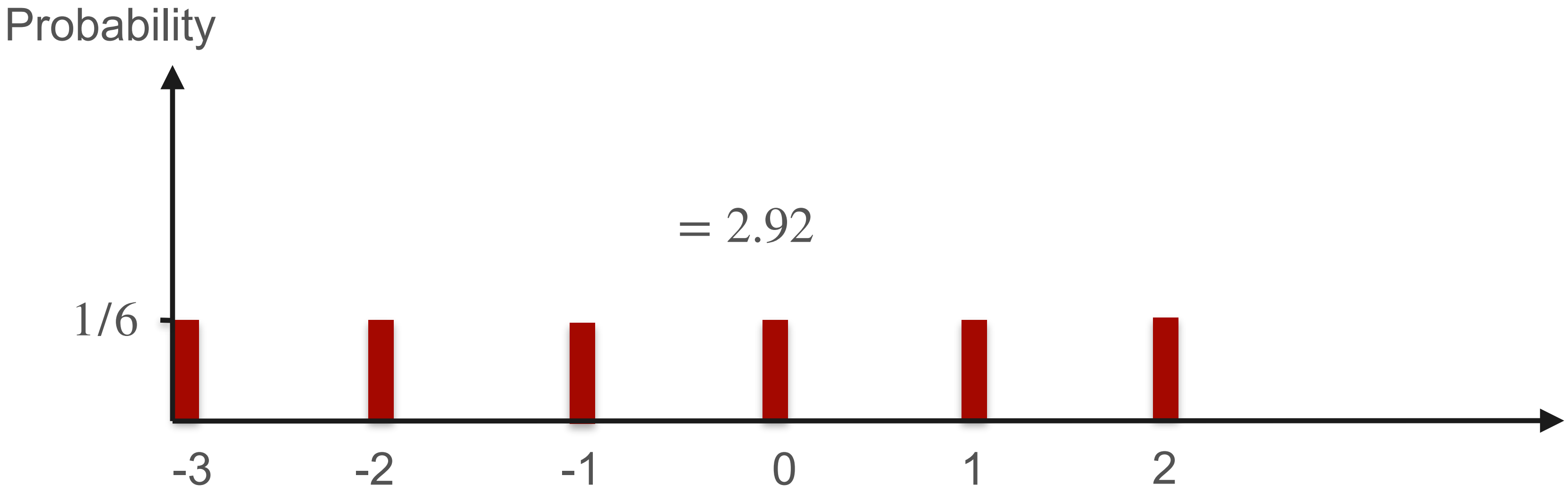
Properties of the Variance



Properties of the Variance

$$Var(2X - 5) = \mathbb{E}\left[\left(2X - 5 - \mathbb{E}[2X - 5]\right)^2\right]$$

$$= 2.92$$

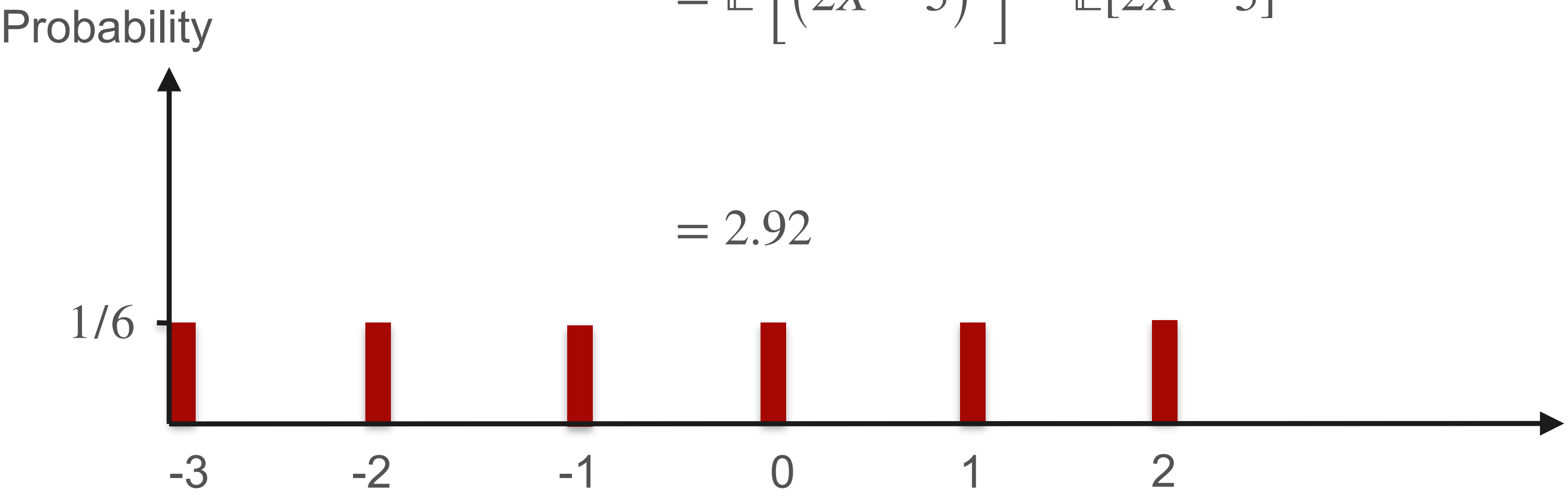


Properties of the Variance

$$Var(2X-5) = \mathbb{E}\left[\left(2X-5-\mathbb{E}[2X-5]\right)^2\right]$$

$$= \mathbb{E}\left[\left(2X-5\right)^2\right] - \mathbb{E}[2X-5]^2$$

$$= 2.92$$

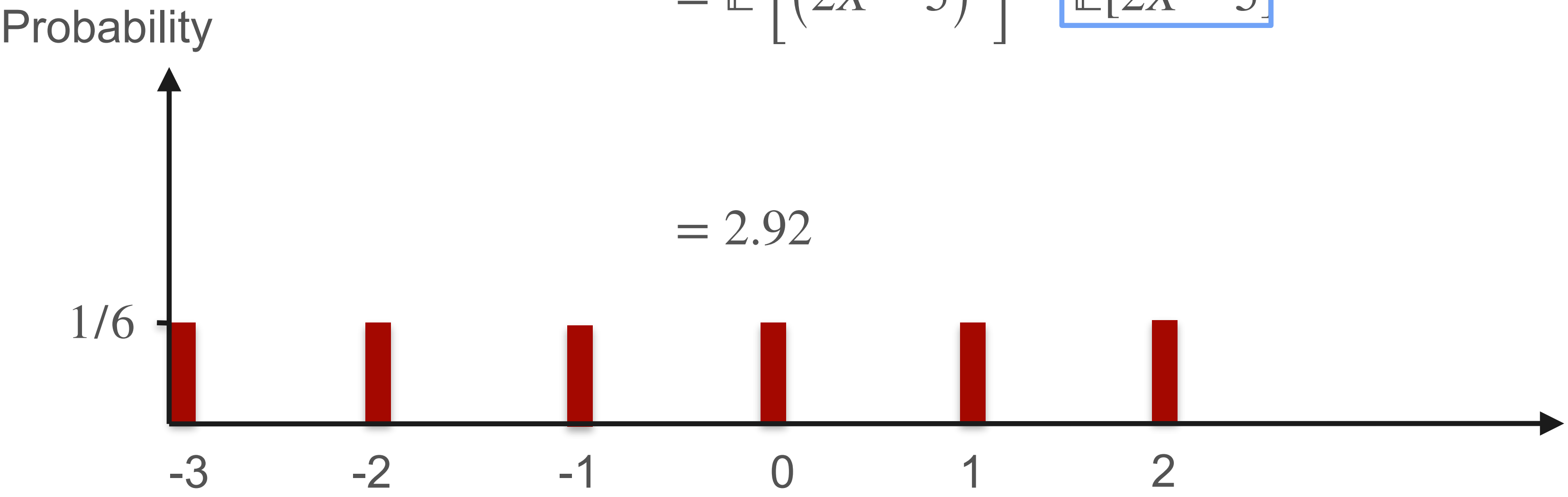


Properties of the Variance

$$Var(2X-5) = \mathbb{E}\left[\left(2X-5-\mathbb{E}[2X-5]\right)^2\right]$$

$$= \mathbb{E}\left[\left(2X-5\right)^2\right] - \boxed{\mathbb{E}[2X-5]}^2$$

$$= 2.92$$

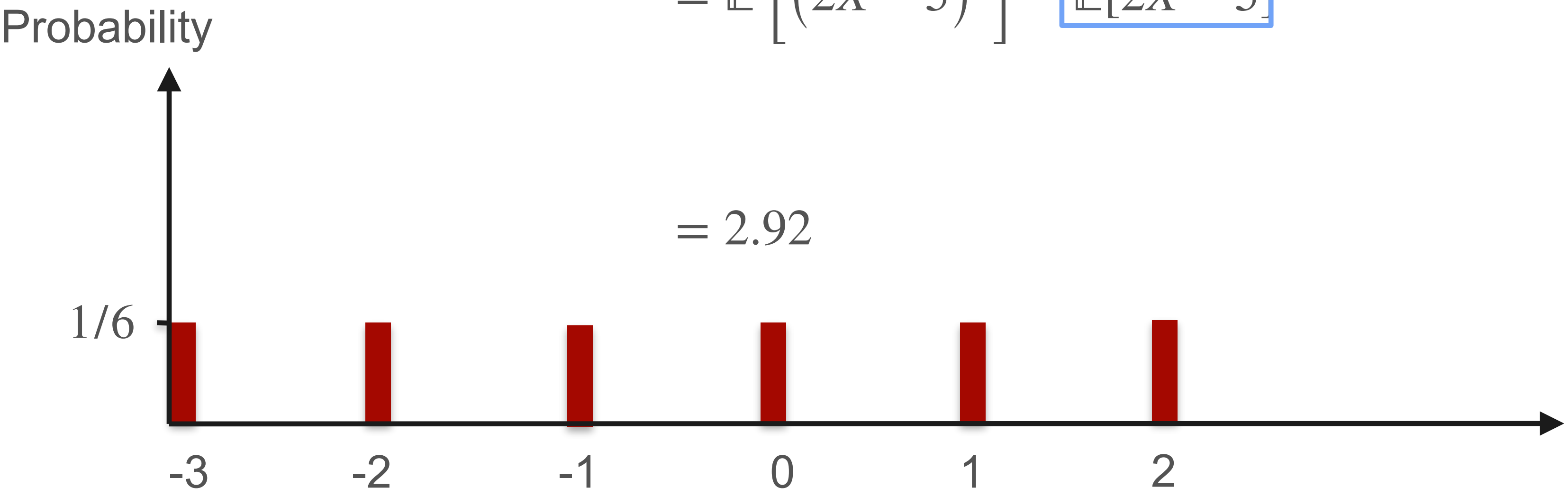


Properties of the Variance

$$Var(2X-5) = \mathbb{E}\left[\left(2X-5-\mathbb{E}[2X-5]\right)^2\right]$$

$$= \mathbb{E}\left[(2X-5)^2\right] - \overset{-0.5}{\boxed{\mathbb{E}[2X-5]}}^2$$

$$= 2.92$$



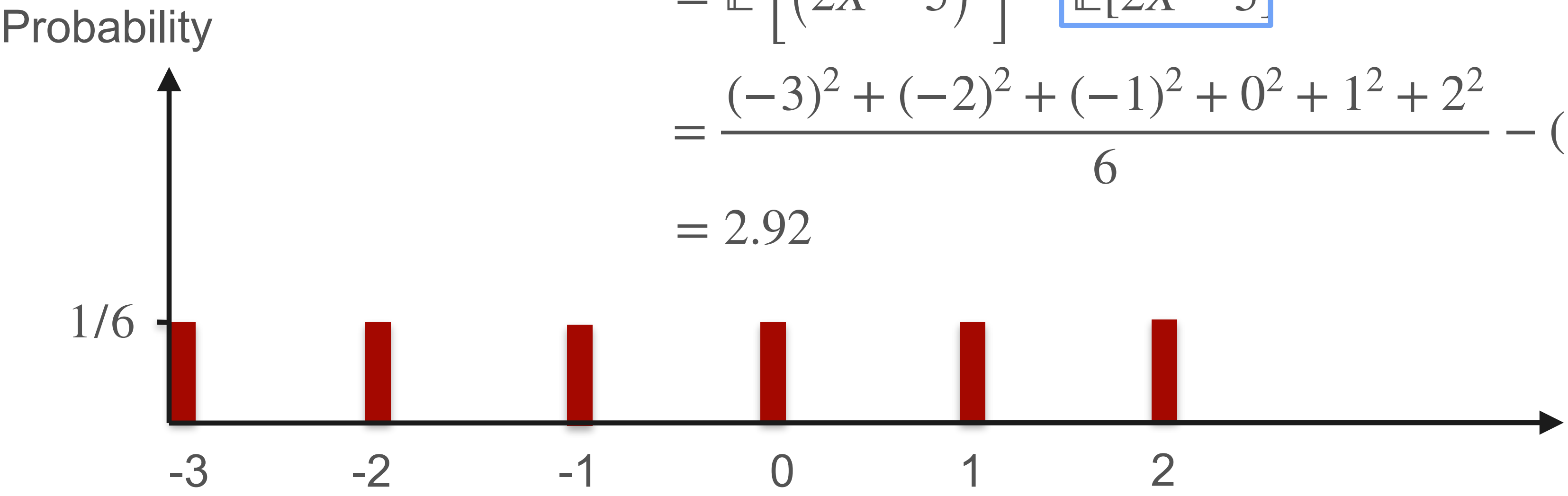
Properties of the Variance

$$Var(2X-5) = \mathbb{E}\left[\left(2X-5-\mathbb{E}[2X-5]\right)^2\right]$$

$$= \mathbb{E}\left[(2X-5)^2\right] - \overset{-0.5}{\boxed{\mathbb{E}[2X-5]}}^2$$

$$= \frac{(-3)^2+(-2)^2+(-1)^2+0^2+1^2+2^2}{6} - (-0.5)^2$$

$$= 2.92$$



Properties of the Variance

Properties of the Variance

$$\mathbb{E}\left[(2X-5)^2\right]$$

Properties of the Variance

$$\mathbb{E}\left[\left(2X - 5\right)^2\right] = \mathbb{E}\left[2^2X^2 + (-5)^2 - 2(-5)X\right]$$

Properties of the Variance

$$\mathbb{E}\left[(2X-5)^2\right] = \mathbb{E}\left[2^2X^2 + (-5)^2 - 2(-5)X\right]$$

$$= 2^2\mathbb{E}\left[X^2\right] + (-5)^2 - 2(-5)\mathbb{E}[X]$$

Properties of the Variance

$$\mathbb{E}\left[(2X-5)^2\right] = \mathbb{E}\left[2^2X^2 + (-5)^2 - 2(-5)X\right]$$

$$= 2^2\mathbb{E}\left[X^2\right] + (-5)^2 - 2(-5)\mathbb{E}[X]$$

$$Var(2X-5) = \mathbb{E}\left[(2X-5)^2\right] - \mathbb{E}[2X-5]^2$$

Properties of the Variance

$$\mathbb{E}\left[(2X-5)^2\right] = \mathbb{E}\left[2^2X^2 + (-5)^2 - 2(-5)X\right]$$

$$= 2^2\mathbb{E}\left[X^2\right] + (-5)^2 - 2(-5)\mathbb{E}[X]$$

$$Var(2X-5) = \mathbb{E}\left[(2X-5)^2\right] - \boxed{\mathbb{E}[2X-5]}^2$$

$$2\mathbb{E}[X]-5$$

Properties of the Variance

$$\mathbb{E}\left[(2X-5)^2\right] = \mathbb{E}\left[2^2X^2 + (-5)^2 - 2(-5)X\right]$$

$$= 2^2\mathbb{E}\left[X^2\right] + (-5)^2 - 2(-5)\mathbb{E}[X]$$

$2\mathbb{E}[X] - 5$

$$Var(2X-5) = \mathbb{E}\left[(2X-5)^2\right] - \boxed{\mathbb{E}[2X-5]}^2$$

$$= 2^2\mathbb{E}\left[X^2\right] + (-5)^2 - 2(-5)\mathbb{E}[X] - (2\mathbb{E}[X]-5)^2$$

Properties of the Variance

$$\mathbb{E}\left[(2X-5)^2\right] = \mathbb{E}\left[2^2X^2 + (-5)^2 - 2(-5)X\right]$$

$$= 2^2\mathbb{E}\left[X^2\right] + (-5)^2 - 2(-5)\mathbb{E}[X]$$

$$Var(2X-5) = \mathbb{E}\left[(2X-5)^2\right] - \boxed{\mathbb{E}[2X-5]}^2$$

$2\mathbb{E}[X]-5$

$$= 2^2\mathbb{E}\left[X^2\right] + (-5)^2 - 2(-5)\mathbb{E}[X] - (2\mathbb{E}[X]-5)^2$$

$$= 2^2\mathbb{E}\left[X^2\right] + (-5)^2 - 2(-5)\mathbb{E}[X] - \left(2^2\mathbb{E}[X]^2 + (-5)^2 - 2(-5)\mathbb{E}[X]\right)$$

Properties of the Variance

$$Var(2X-5) = \boxed{2^2\mathbb{E}\left[X^2\right]} + (-5)^2 - 2(-5)\mathbb{E}[X] - \left(\boxed{2^2\mathbb{E}[X]^2} + (-5)^2 - 2(-5)\mathbb{E}[X]\right)$$

Properties of the Variance

$$Var(2X-5) = \boxed{2^2\mathbb{E}\left[X^2\right]} + \cancel{(-5)^2} - 2(-5)\mathbb{E}[X] - \left(\boxed{2^2\mathbb{E}[X]^2} + \cancel{(-5)^2} - 2(-5)\mathbb{E}[X]\right)$$

Properties of the Variance

$$Var(2X-5) = \boxed{2^2\mathbb{E}\left[X^2\right]} + \cancel{(-5)^2} - \cancel{2(-5)\mathbb{E}[X]} - \left(\boxed{2^2\mathbb{E}[X]^2} + \cancel{(-5)^2} - \cancel{2(-5)\mathbb{E}[X]}\right)$$

Properties of the Variance

$$Var(2X-5) = \boxed{2^2\mathbb{E}\left[X^2\right]} + \cancel{(-5)^2} - \cancel{2(-5)\mathbb{E}[X]} - \left(\boxed{2^2\mathbb{E}[X]^2} + \cancel{(-5)^2} - \cancel{2(-5)\mathbb{E}[X]}\right)$$

$$= 2^2\left(\mathbb{E}[X^2] - \mathbb{E}[X]^2\right)$$

Properties of the Variance

$$Var(2X-5) = \boxed{2^2\mathbb{E}\left[X^2\right]} + \cancel{(-5)^2} - \cancel{2(-5)\mathbb{E}[X]} - \left(\boxed{2^2\mathbb{E}[X]^2} + \cancel{(-5)^2} - \cancel{2(-5)\mathbb{E}[X]}\right)$$

$$= 2^2 \underbrace{\left(\mathbb{E}[X^2] - \mathbb{E}[X]^2\right)}_{Var(X)}$$

Properties of the Variance

$$Var(2X-5) = \boxed{2^2\mathbb{E}\left[X^2\right]} + \cancel{(-5)^2} - \cancel{2(-5)\mathbb{E}[X]} - \left(\boxed{2^2\mathbb{E}[X]^2} + \cancel{(-5)^2} - \cancel{2(-5)\mathbb{E}[X]}\right)$$

$$= 2^2 \underbrace{\left(\mathbb{E}[X^2] - \mathbb{E}[X]^2\right)}_{Var(X)} = 2^2 Var(X)$$

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In general: $Var(aX+b) = a^2 Var(X)$

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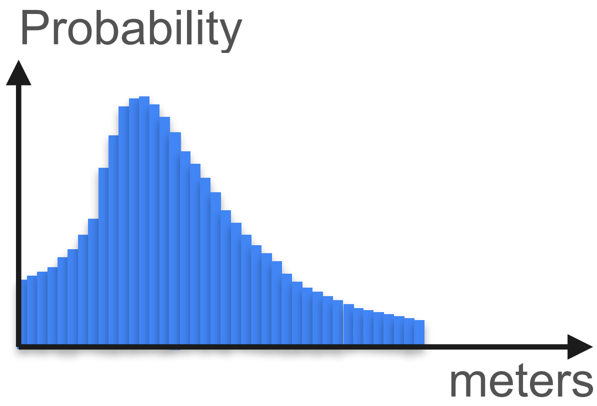
Describing Distributions

Standard deviation

Standard Deviation

$$Var(X) = \mathbb{E}[(X-\mu)^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$



Standard Deviation

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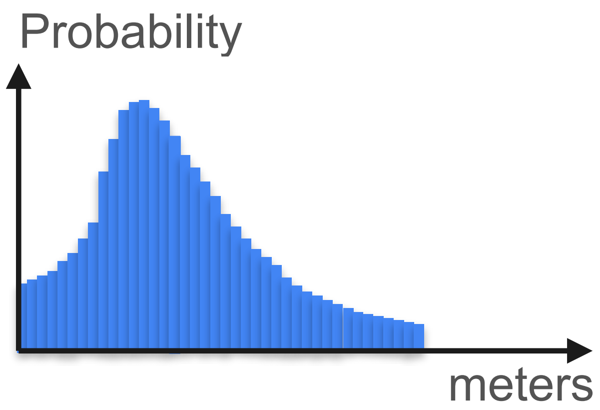
Say X is measured in meters.



Standard Deviation

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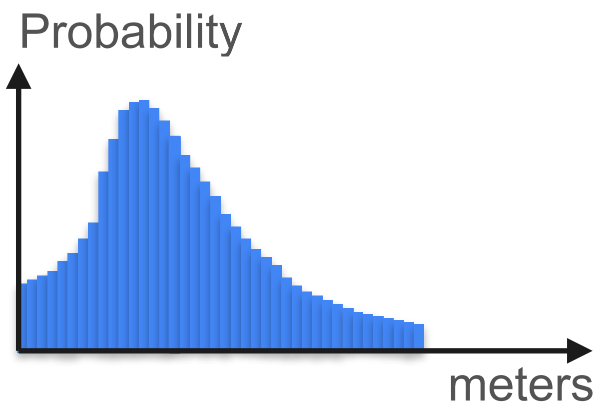
Say X is measured in meters.

Then $\mathbb{E}[X]$ is measured in meters.



Standard Deviation

$$Var(X) = \mathbb{E}[(X-\mu)^2] = \overset{m^2}{\boxed{\mathbb{E}[X^2]}} - \overset{m^2}{\boxed{\mathbb{E}[X]^2}}$$

Say X is measured in meters.

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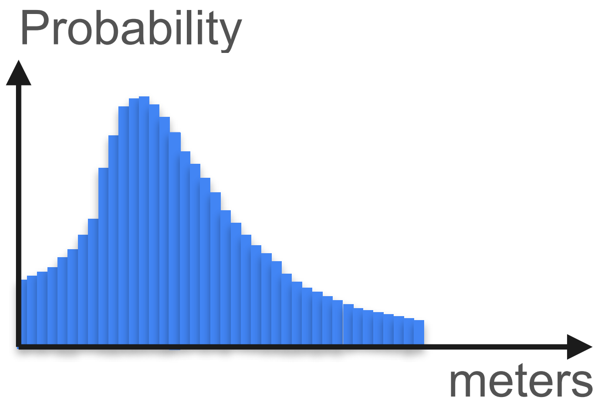

Standard Deviation

$$\underbrace{\boxed{Var(X)}}^{m^2} = \mathbb{E}[(X-\mu)^2] = \overbrace{\boxed{\mathbb{E}[X^2]}}^{m^2} - \overbrace{\boxed{\mathbb{E}[X]^2}}^{m^2}$$

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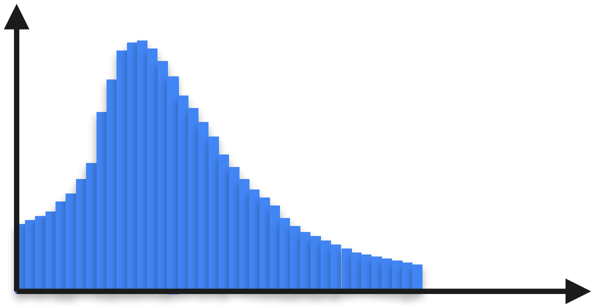


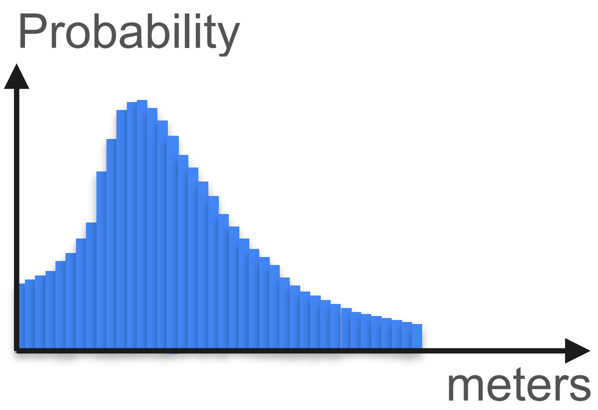
Standard Deviation

$$\overset{m^2}{\boxed{Var(X)}} = \mathbb{E}[(X-\mu)^2] = \overset{m^2}{\boxed{\mathbb{E}[X^2]}} - \overset{m^2}{\boxed{\mathbb{E}[X]^2}}$$

Say X is measured in meters.

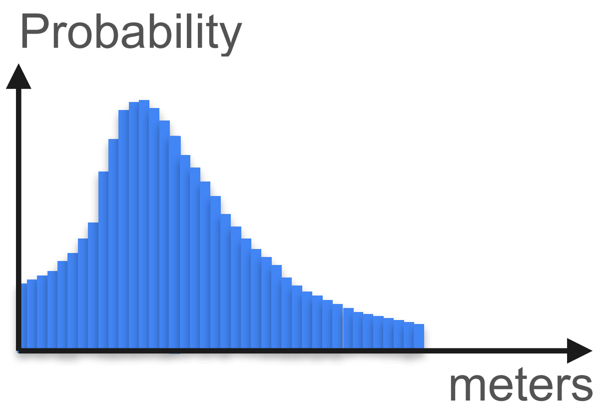
Then $\mathbb{E}[X]$ is measured in meters.

Then $Var(X)$ is measured in meters^2.



Standard Deviation

$$\overset{m^2}{\boxed{Var(X)}} = \mathbb{E}[(X-\mu)^2] = \overset{m^2}{\boxed{\mathbb{E}[X^2]}} - \overset{m^2}{\boxed{\mathbb{E}[X]^2}}$$



Say X is measured in meters.

Then $\mathbb{E}[X]$ is measured in meters.

Then $Var(X)$ is measured in meters^2.

Then $\sqrt{Var(X)}$ is measured in meters.

Standard Deviation

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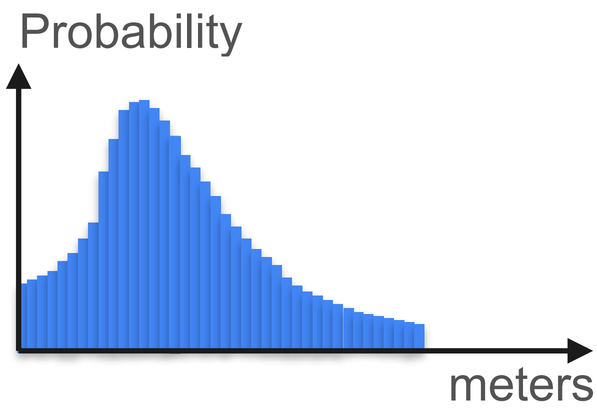

Say X is measured in meters.

Then $\mathbb{E}[X]$ is measured in meters.

Then $Var(X)$ is measured in meters^2.

Then $\sqrt{Var(X)}$ is measured in meters.

Let's call $std(X) = \sqrt{Var(X)}$, the *standard deviation* of X

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Normal Distribution: 68-95-99.7 Rule

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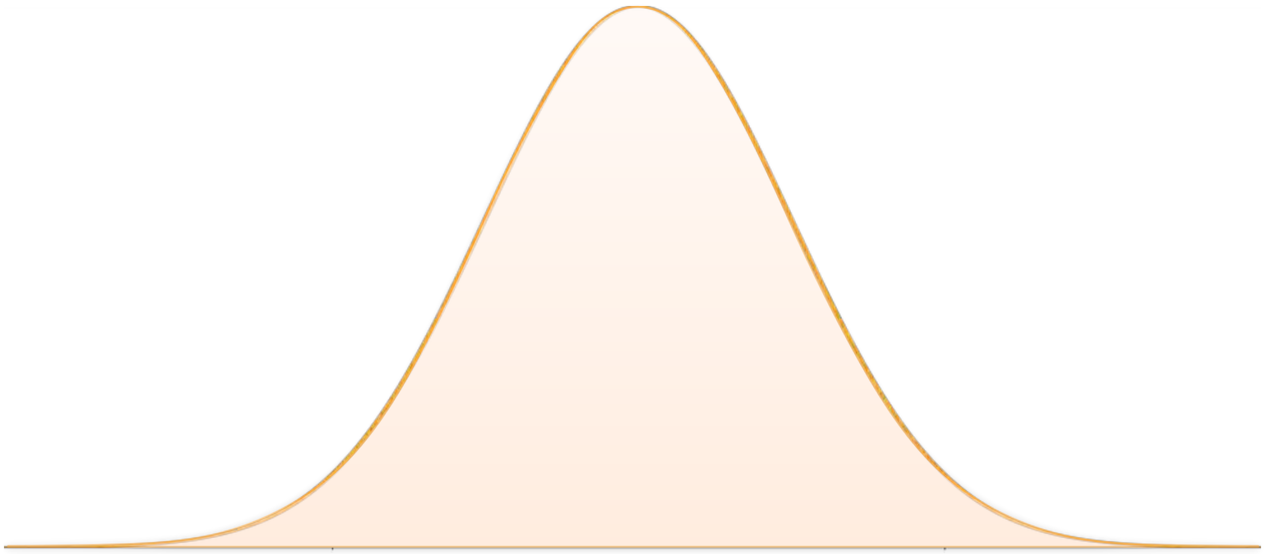

Normal Distribution: 68-95-99.7 Rule

Parameters:

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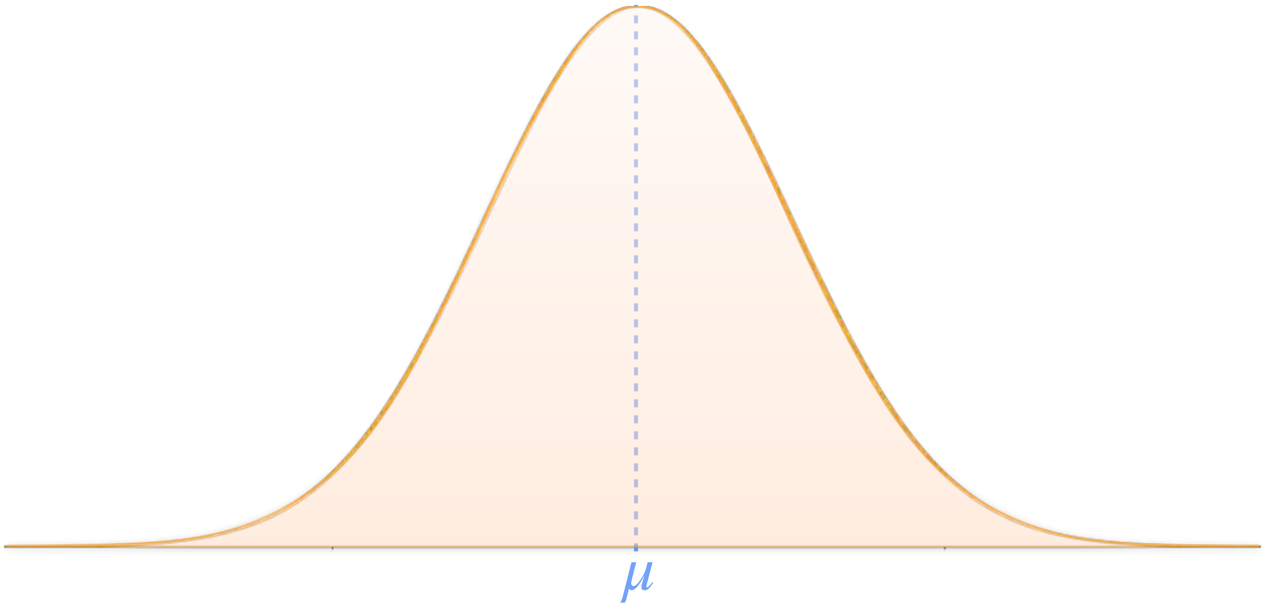
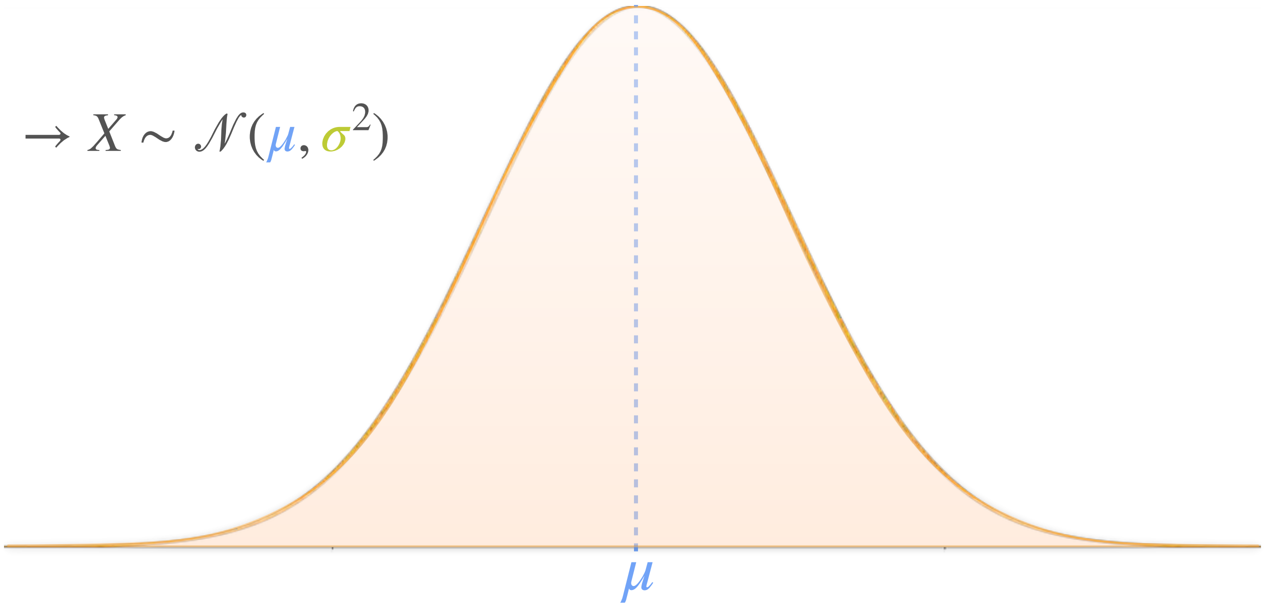


Normal Distribution: 68-95-99.7 Rule

Parameters:

- μ: center of the bell
- σ: spread of the bell

$\rightarrow X \sim \mathcal{N}(\mu, \sigma^2)$



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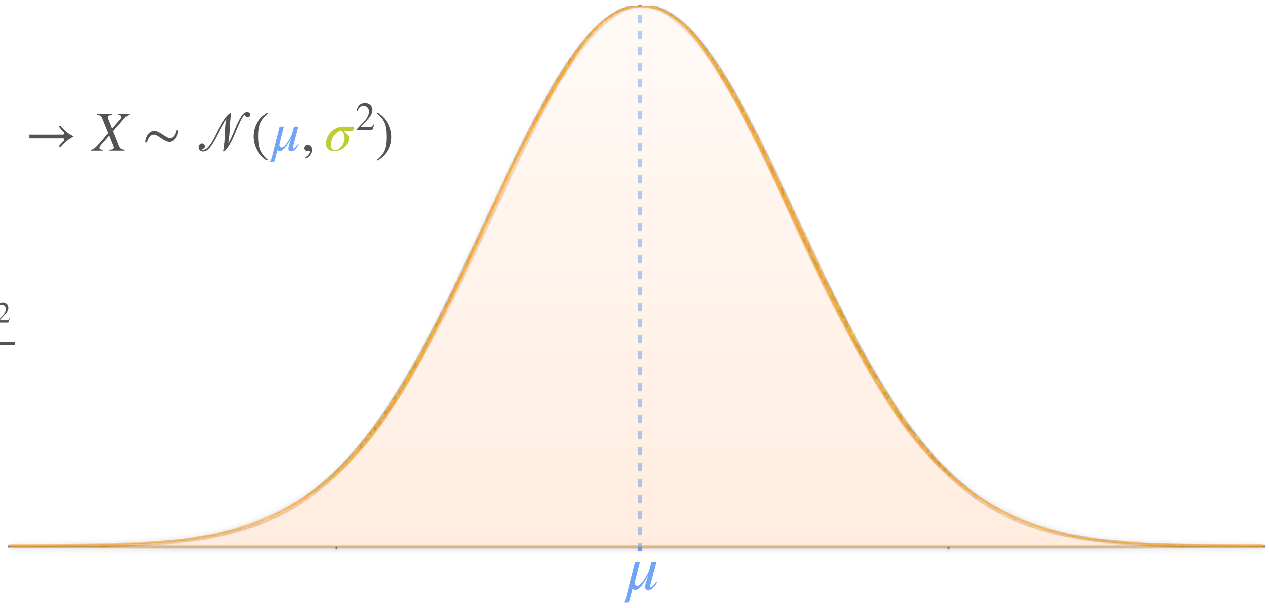
$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$



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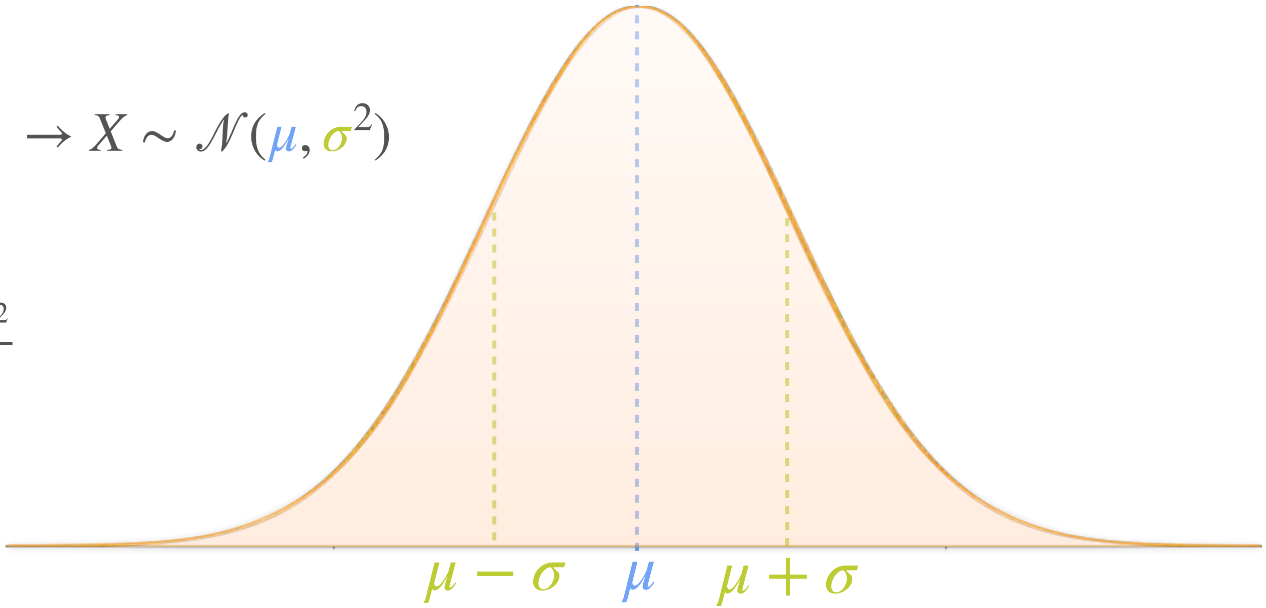

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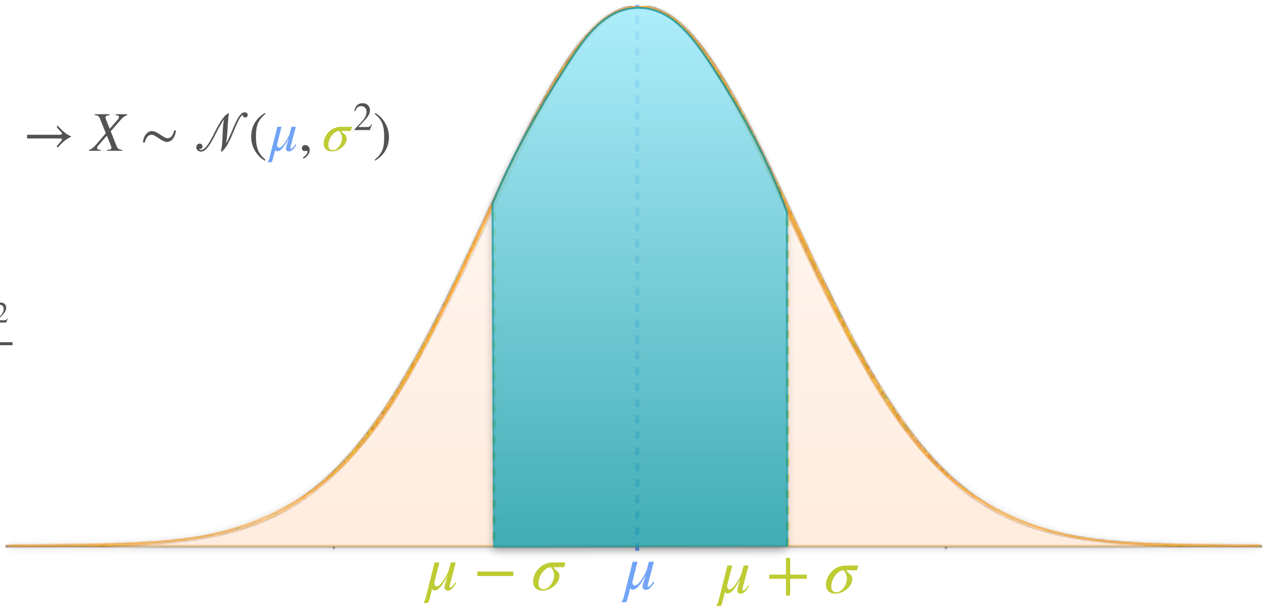

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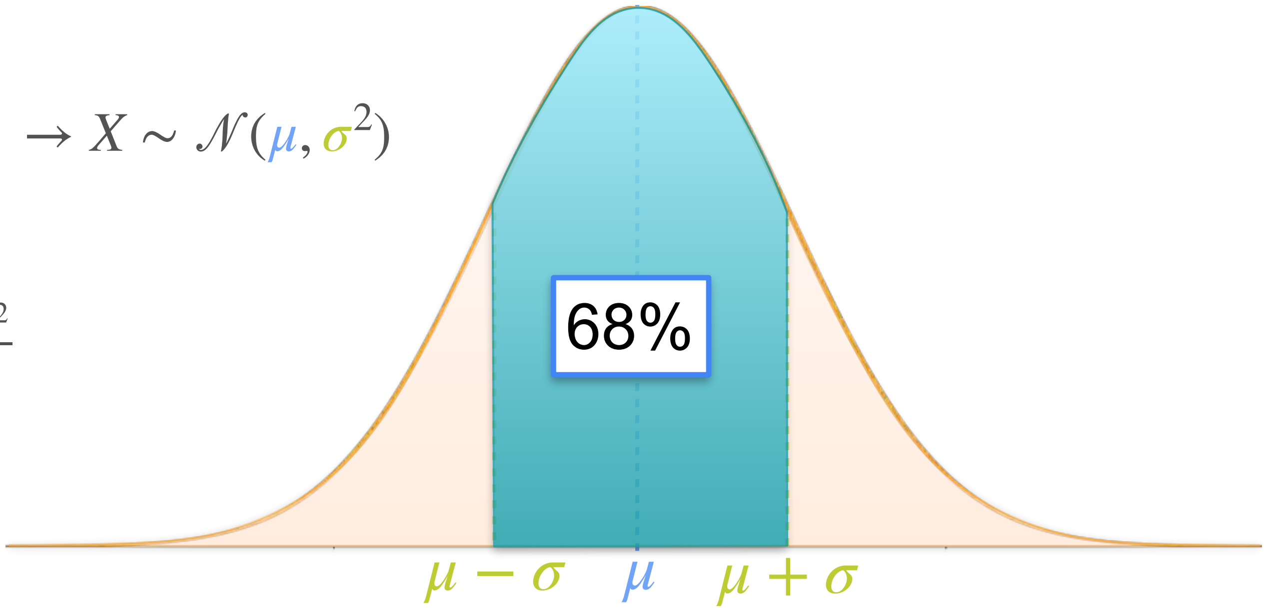

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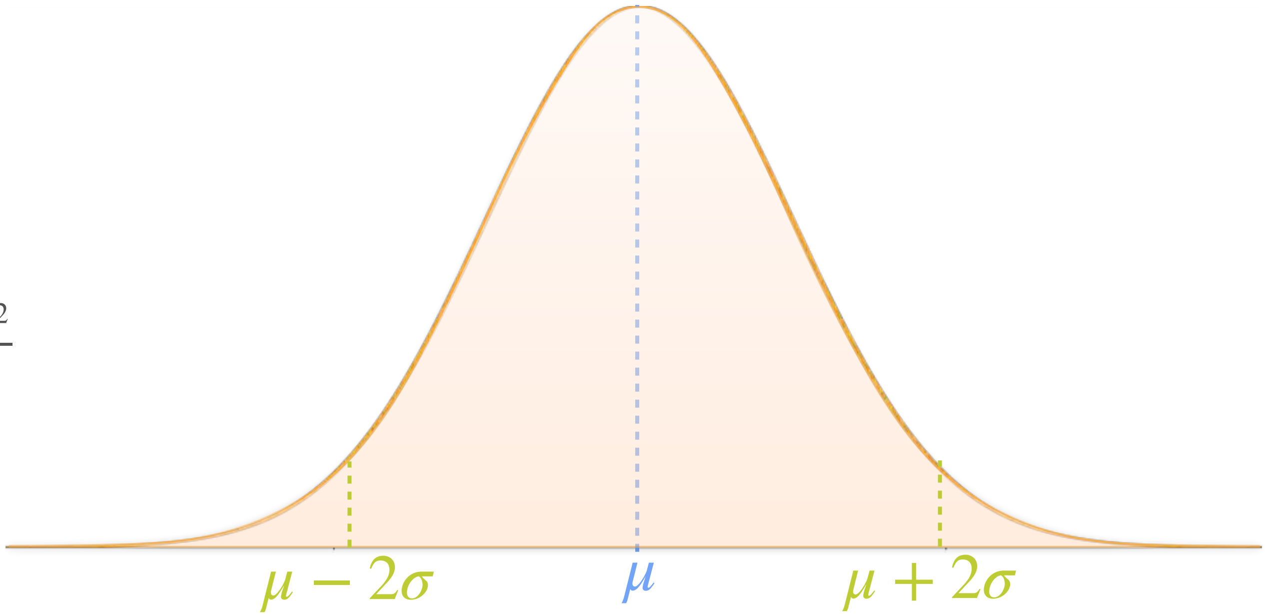

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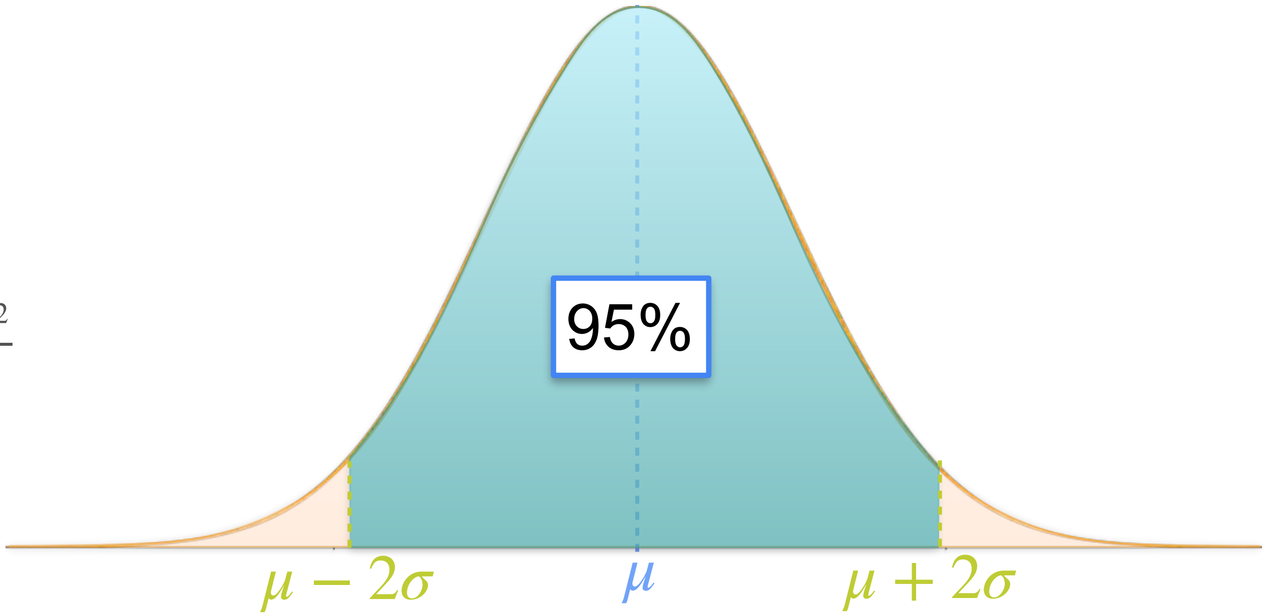

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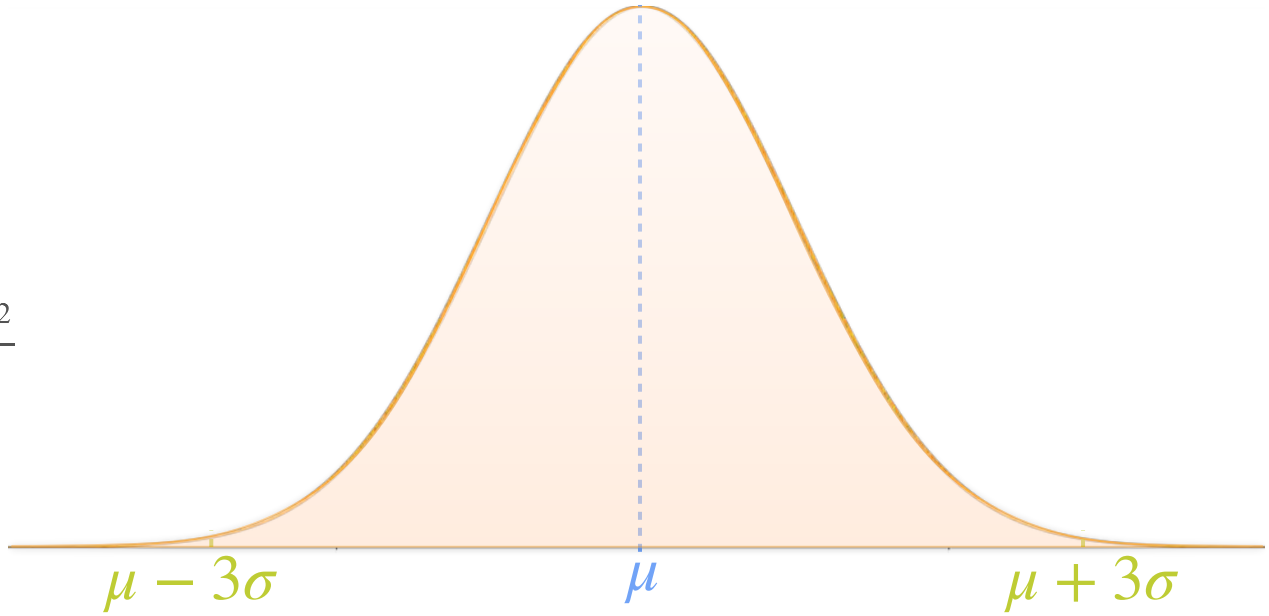

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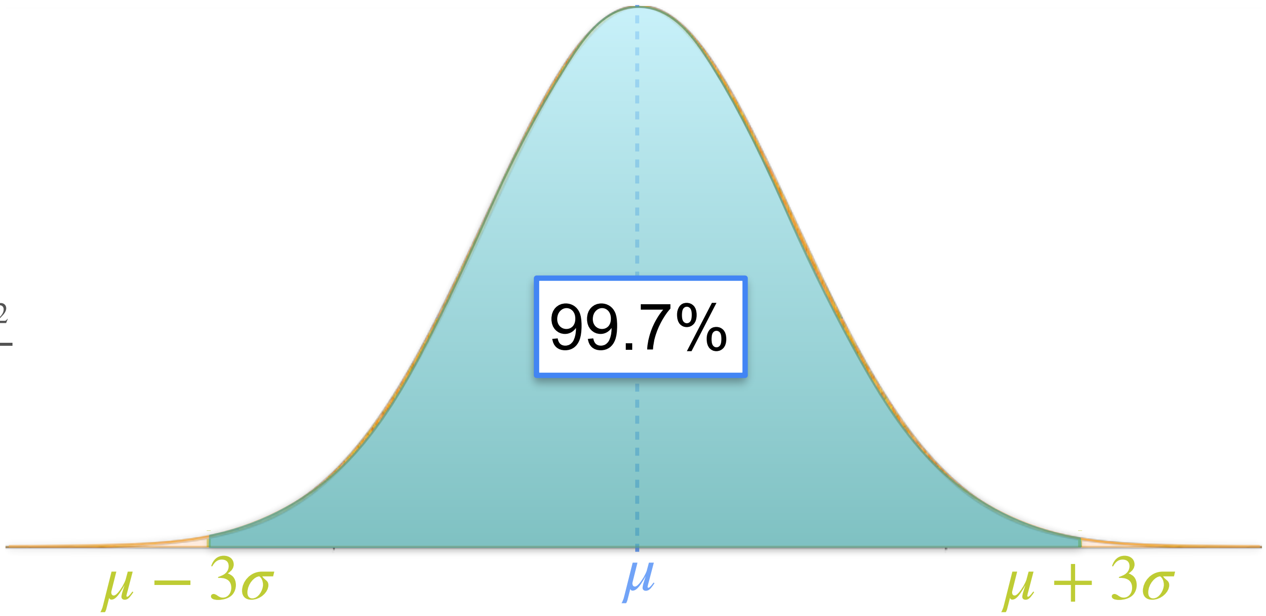

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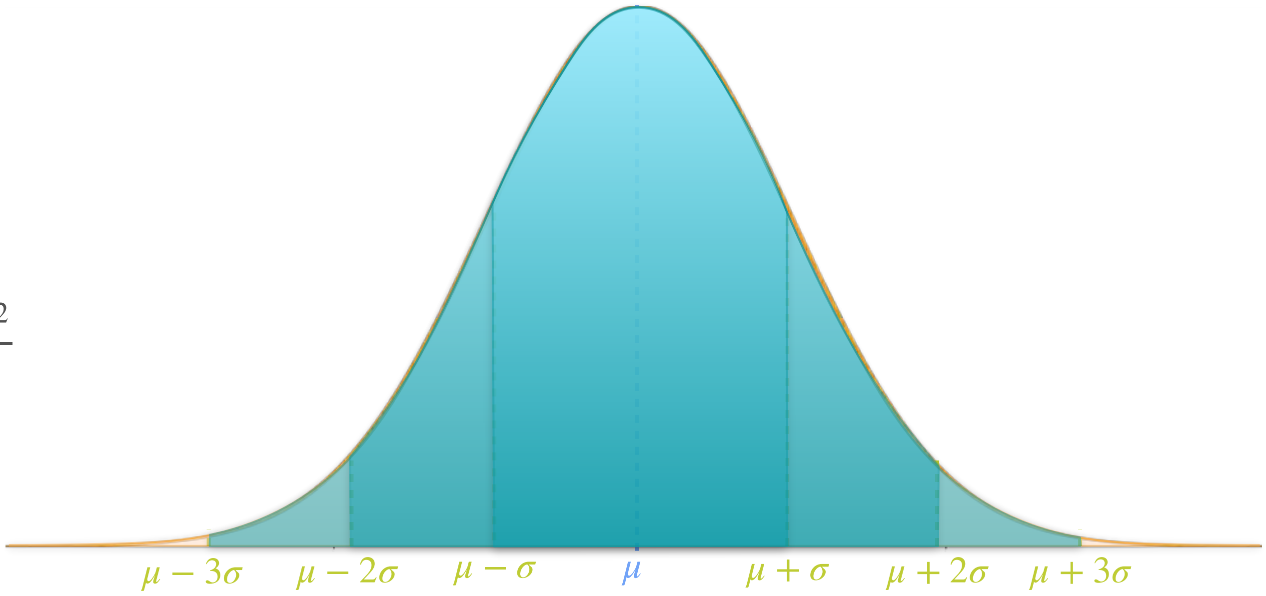

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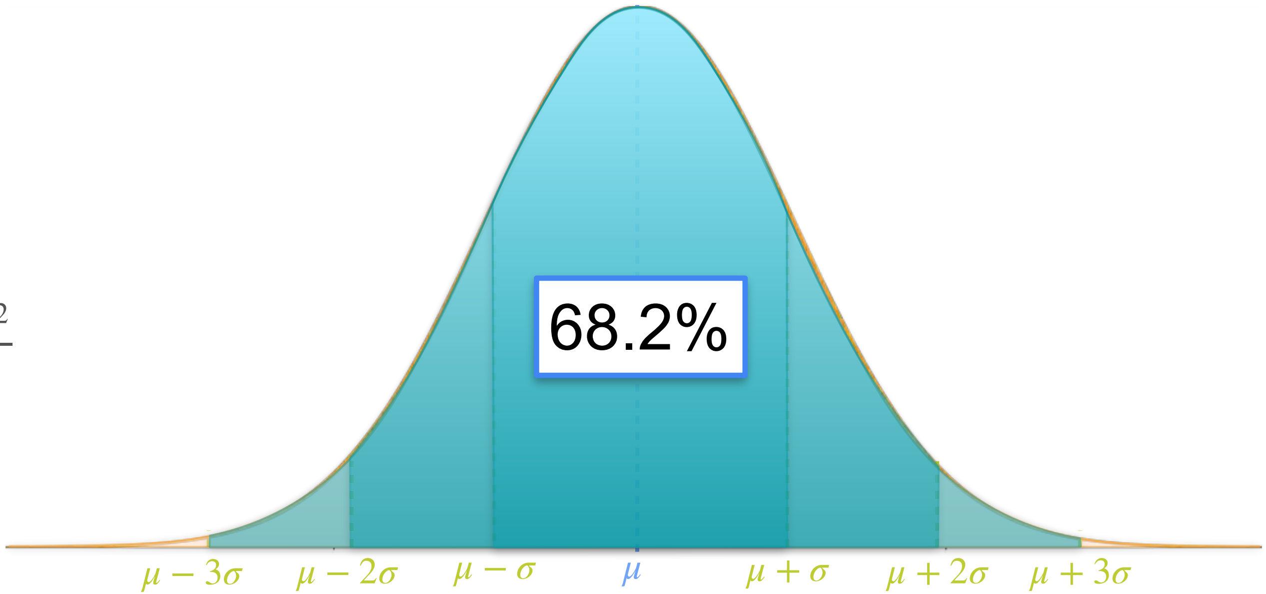

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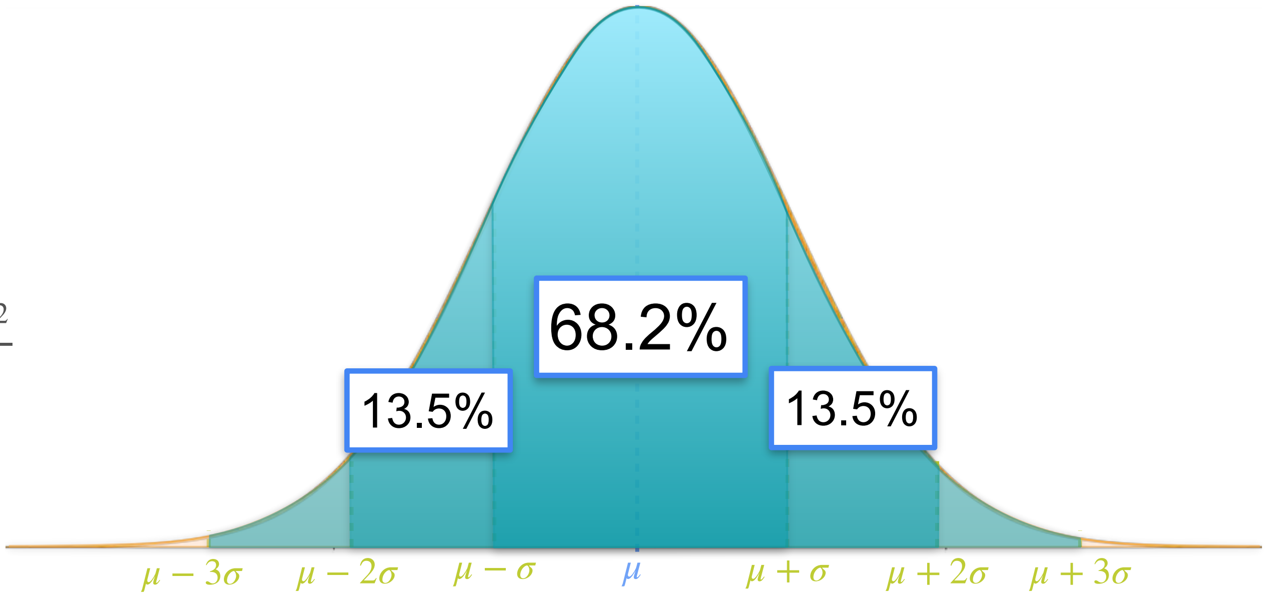

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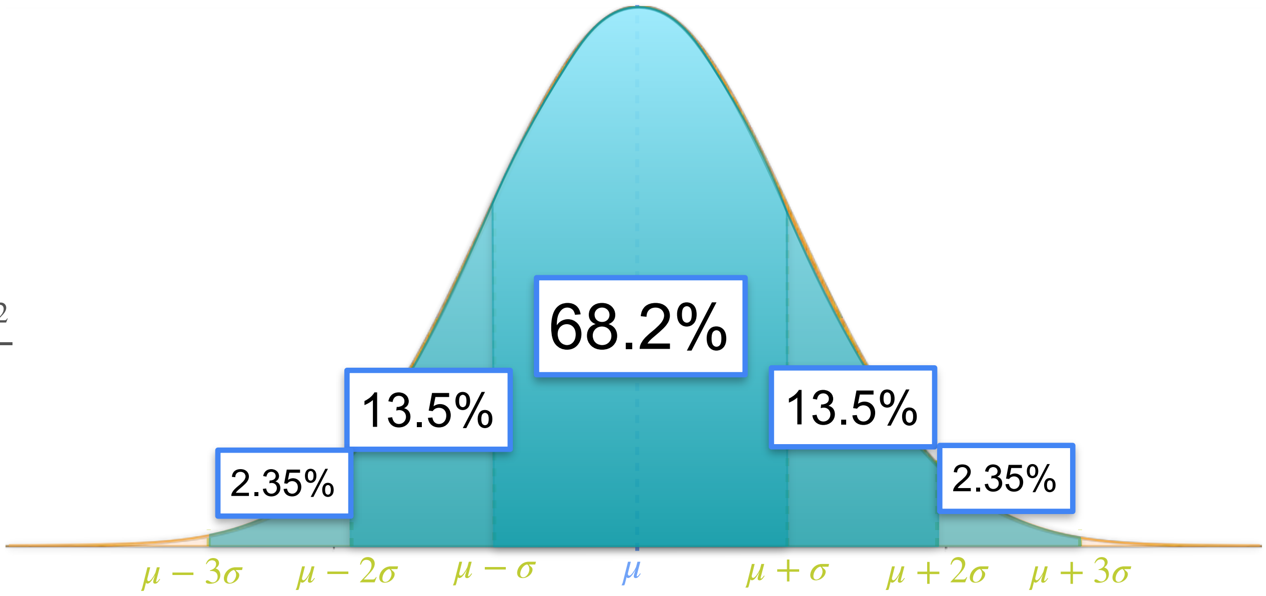

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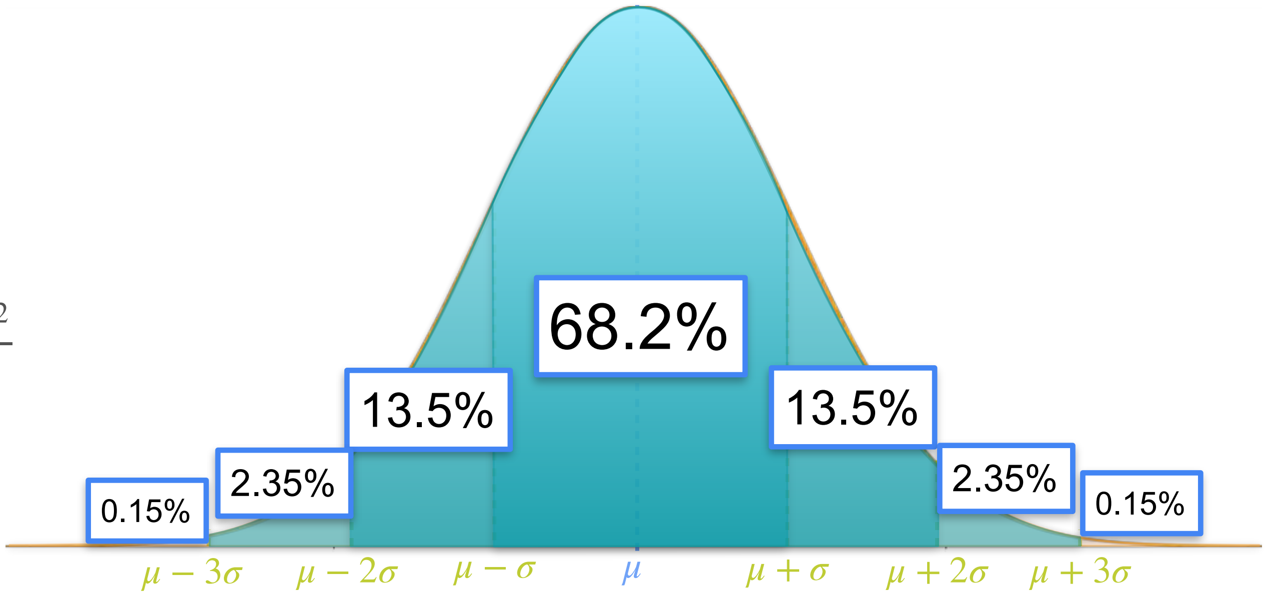


Sum of Gaussians: an Example

Sum of Gaussians: an Example

Total response time of a computer system

Sum of Gaussians: an Example

Total response time of a computer system

Processing time

Sum of Gaussians: an Example

Total response time of a computer system

Processing time

Network Latency

Sum of Gaussians: an Example

R : Total response time of a computer system

T : Processing time

L : Network Latency

Sum of Gaussians: an Example

R : Total response time of a computer system

T : Processing time

L : Network Latency

$$R = T + L$$

Sum of Gaussians: an Example

R : Total response time of a computer system

T : Processing time

L : Network Latency

$T \sim \mathcal{N}(10, 2^2)$

$$R = T + L$$

Sum of Gaussians: an Example

R : Total response time of a computer system

T : Processing time

L : Network Latency

$T \sim \mathcal{N}(10, 2^2)$

$L \sim \mathcal{N}(5, 1^2)$

$$R = T + L$$

Sum of Gaussians: an Example

R : Total response time of a computer system

Independent

T : Processing time $\longleftrightarrow$ L : Network Latency

$T \sim \mathcal{N}(10, 2^2)$

$L \sim \mathcal{N}(5, 1^2)$

$$R = T + L$$

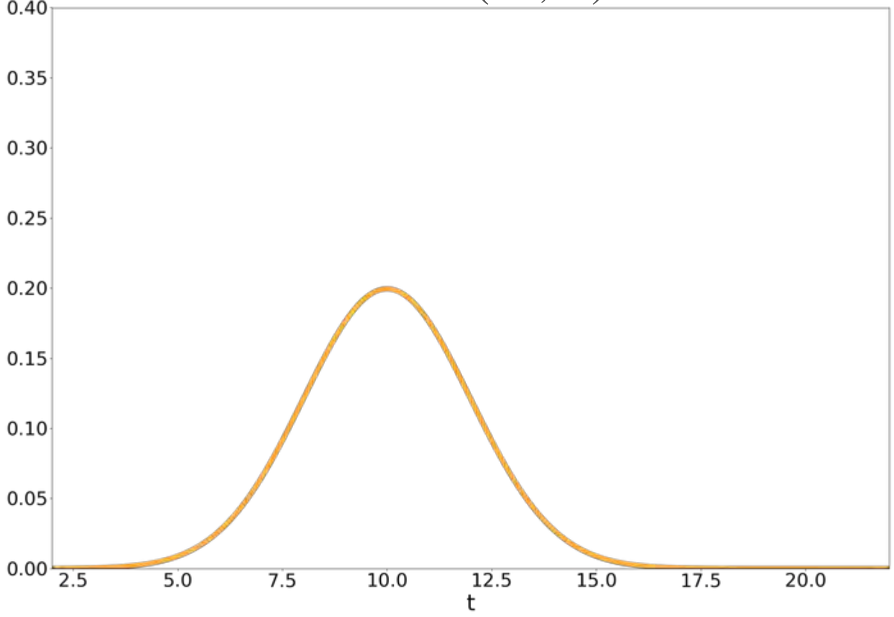
Sum Of Gaussians

$T \sim \mathcal{N}(10, 2^2)$

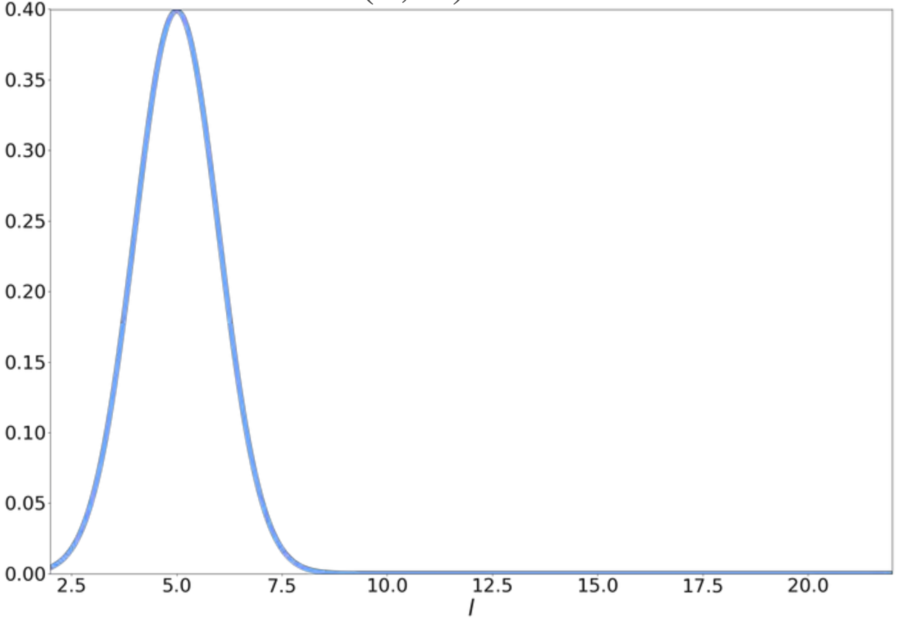
$L \sim \mathcal{N}(5, 1^2)$

Sum Of Gaussians

$T \sim \mathcal{N}(10, 2^2)$



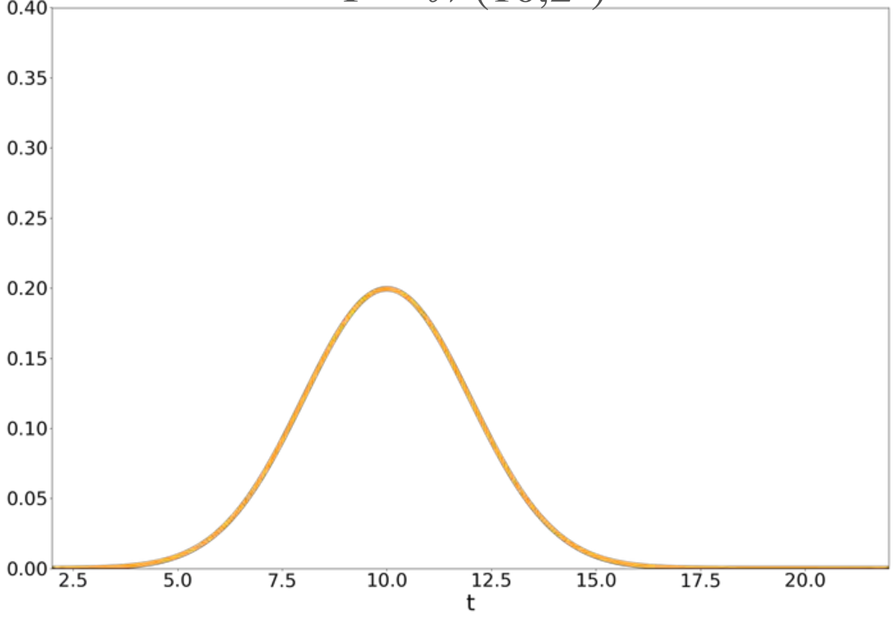
$L \sim \mathcal{N}(5, 1^2)$



Sum Of Gaussians

Sample each variable 10000 times

$T \sim \mathcal{N}(10, 2^2)$



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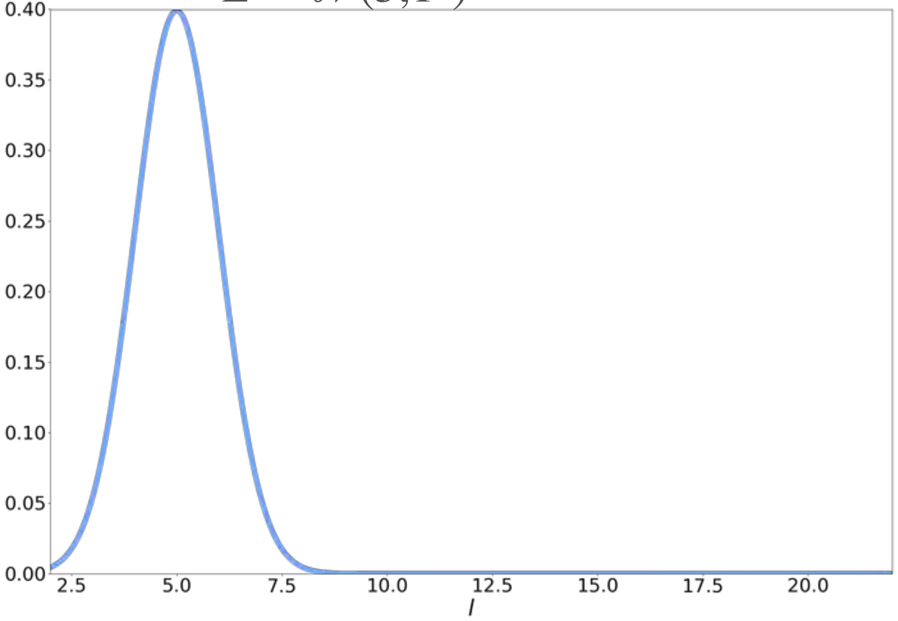
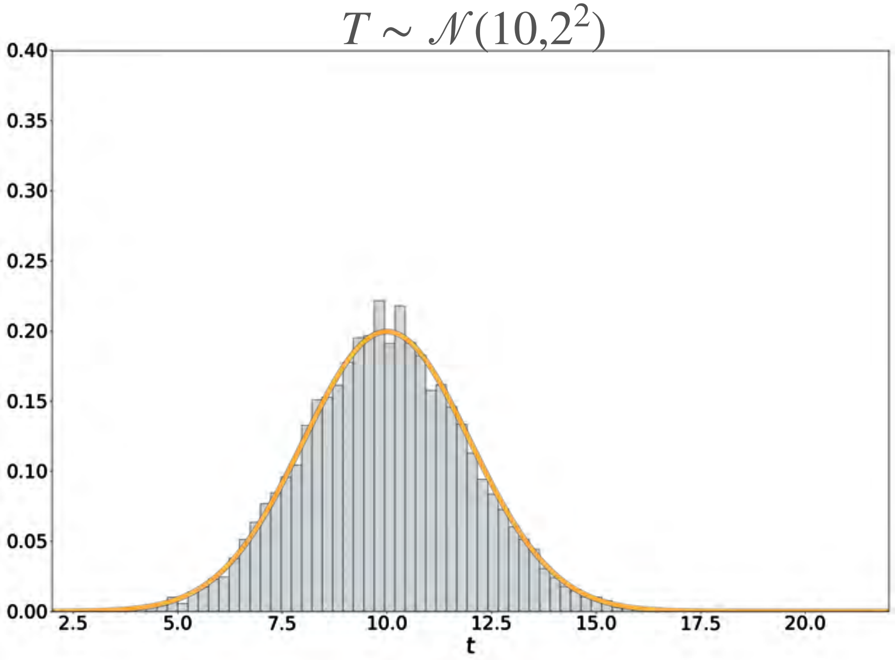
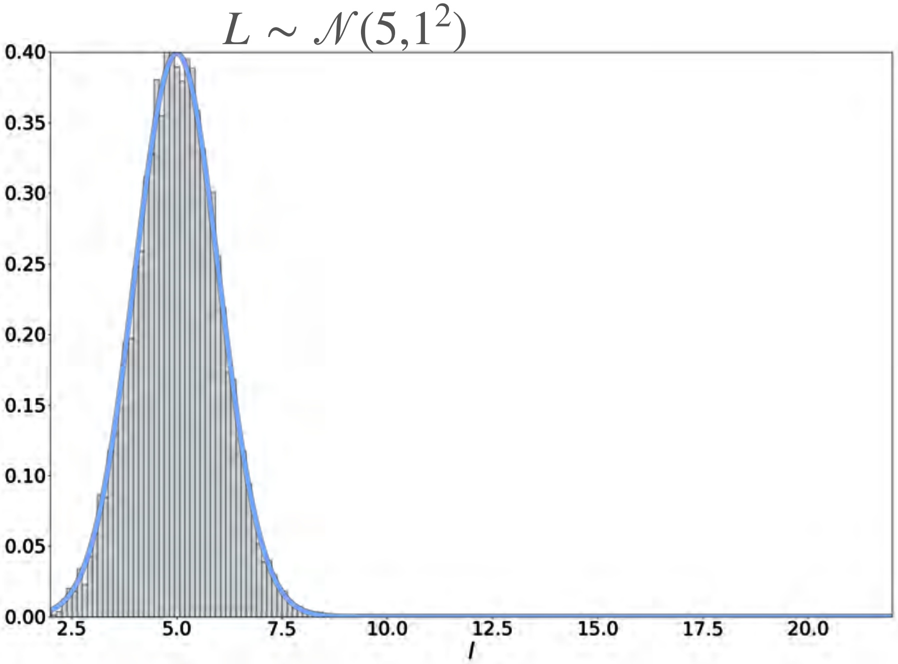


Sum Of Gaussians

Sample each variable 10000 times





Sum Of Gaussians

$$R = T + L$$

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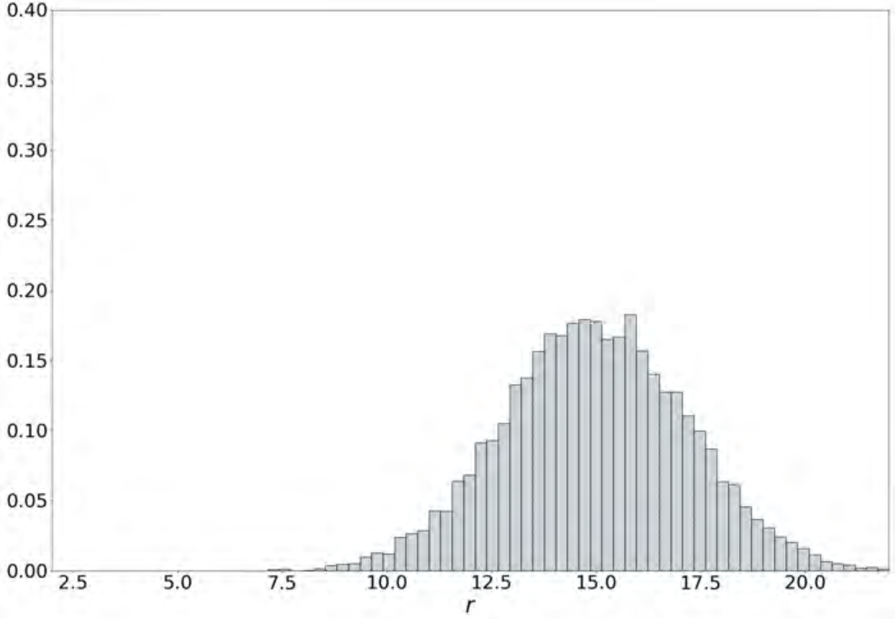

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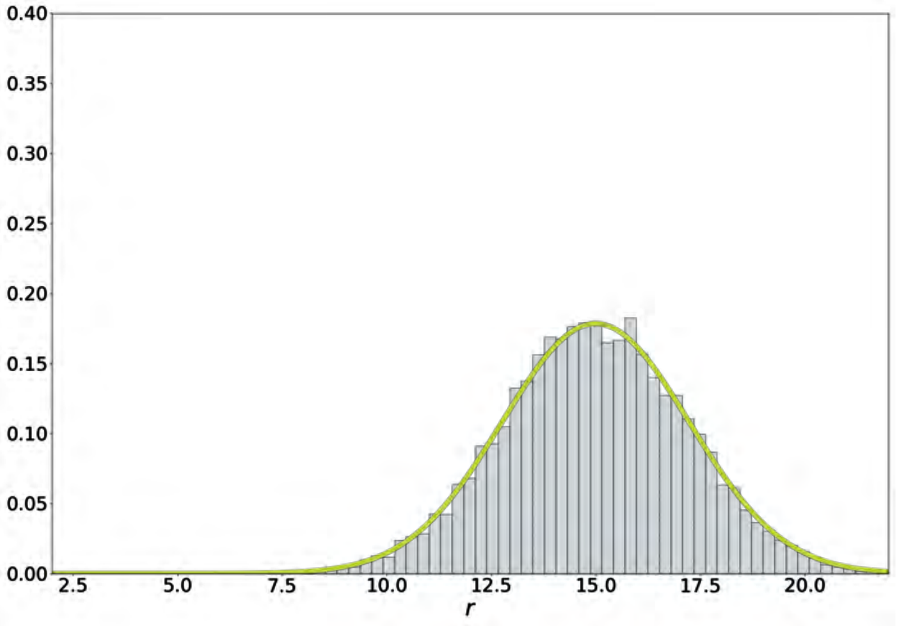
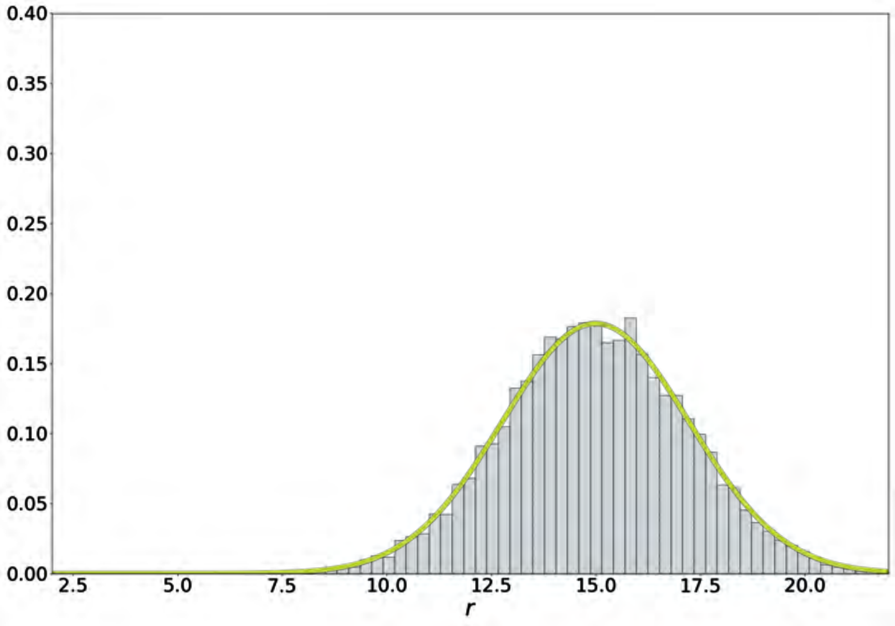


R is still Gaussian!

Sum Of Gaussians

$$R = T + L$$

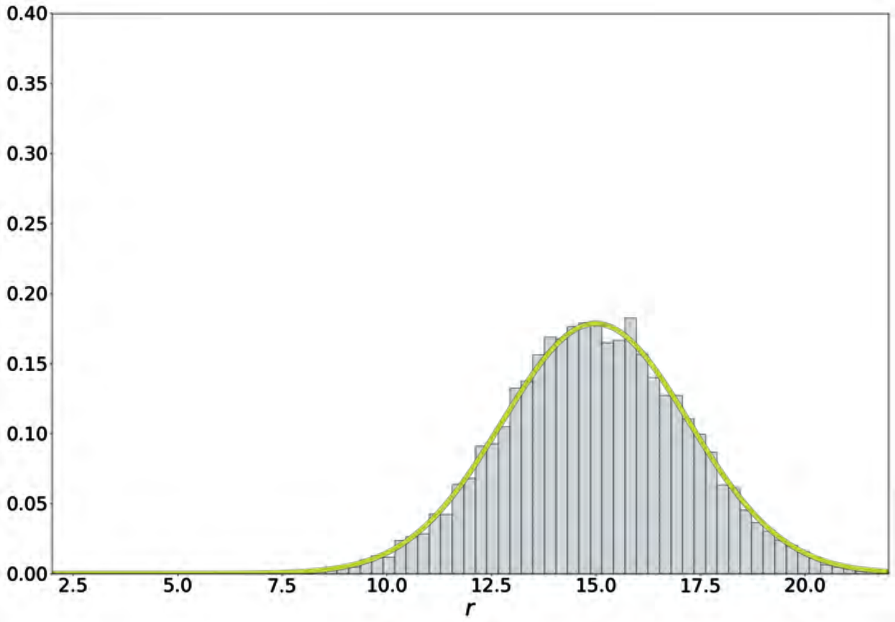


R is still Gaussian!

$$\mu_R = \mathbb{E}[R]$$

Sum Of Gaussians

$$R = T + L$$



R is still Gaussian!

$$\mu_R = \mathbb{E}[R] \quad = \mathbb{E}[T + L]$$

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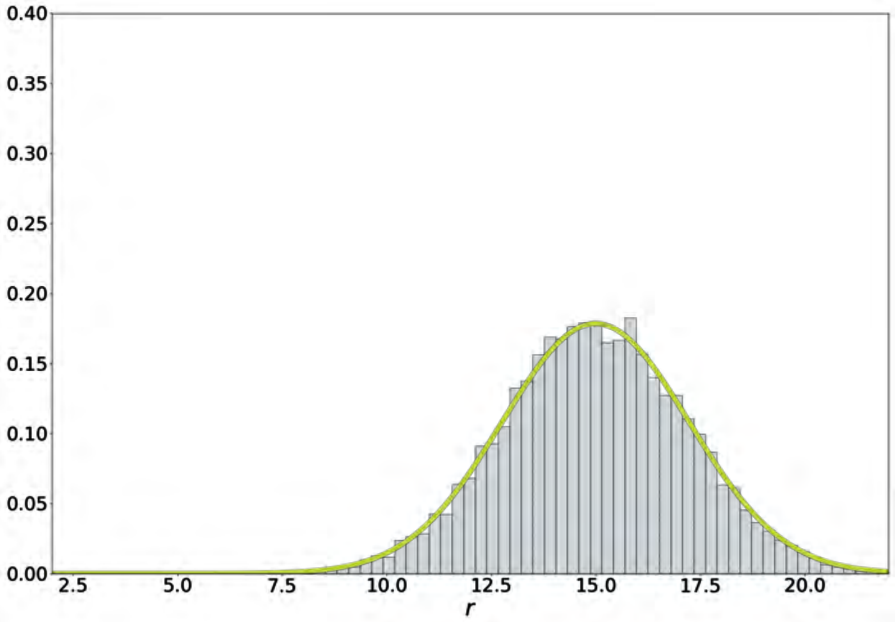
Sum Of Gaussians

$$R = T + L$$



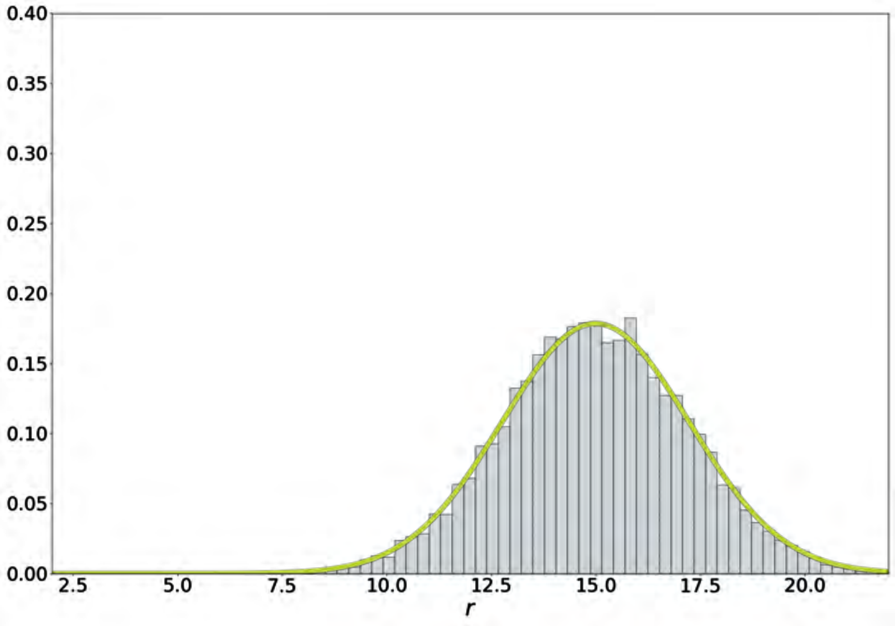
R is still Gaussian!

Expectation is linear

$$\mu_R = \mathbb{E}[R] \;\; = \mathbb{E}[T + L] \; = \mathbb{E}[T] + \mathbb{E}[L]$$

Sum Of Gaussians

$$R = T + L$$



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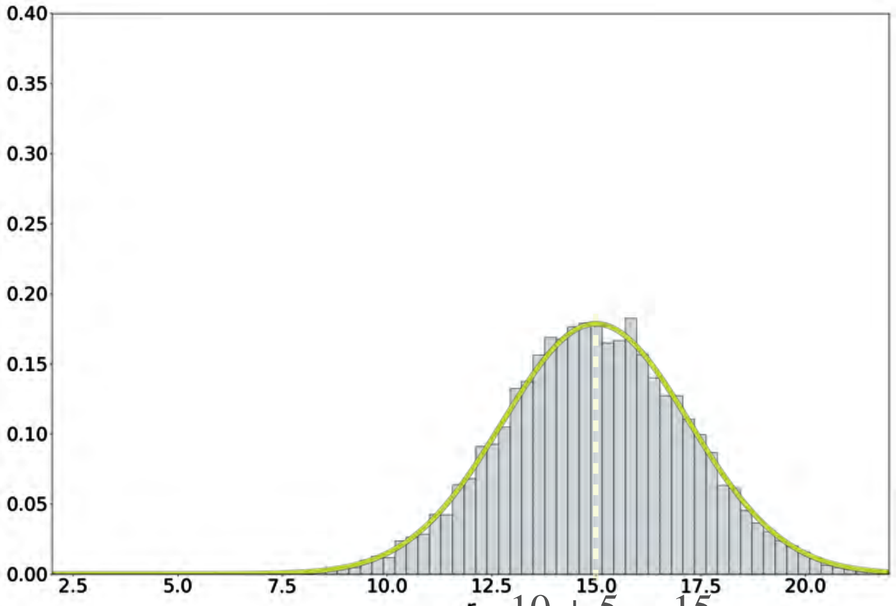
Expectation is linear

$$\mu_R = \mathbb{E}[R] \;= \mathbb{E}[T+L] = \mathbb{E}[T] + \mathbb{E}[L]$$

$$= \mu_T + \mu_L$$

Sum Of Gaussians

$$R = T + L$$



$$10 + 5 = 15$$

R is still Gaussian!

Expectation is linear

$$\mu_R = \mathbb{E}[R] \;\; = \mathbb{E}[T + L] \; = \mathbb{E}[T] + \mathbb{E}[L]$$

$$= \mu_T + \mu_L \; = 10 + 5 \; = 15$$

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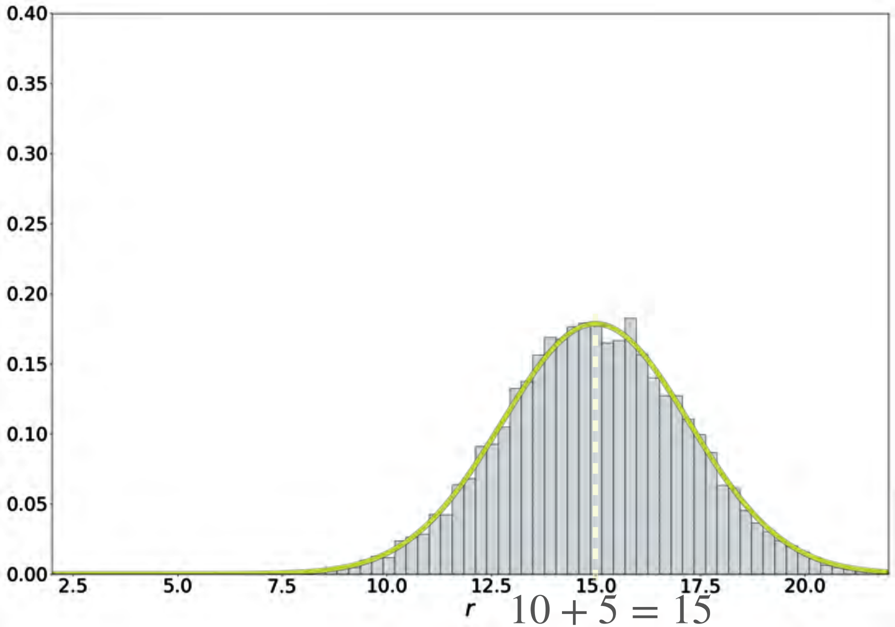
Sum Of Gaussians

$$R = T + L$$



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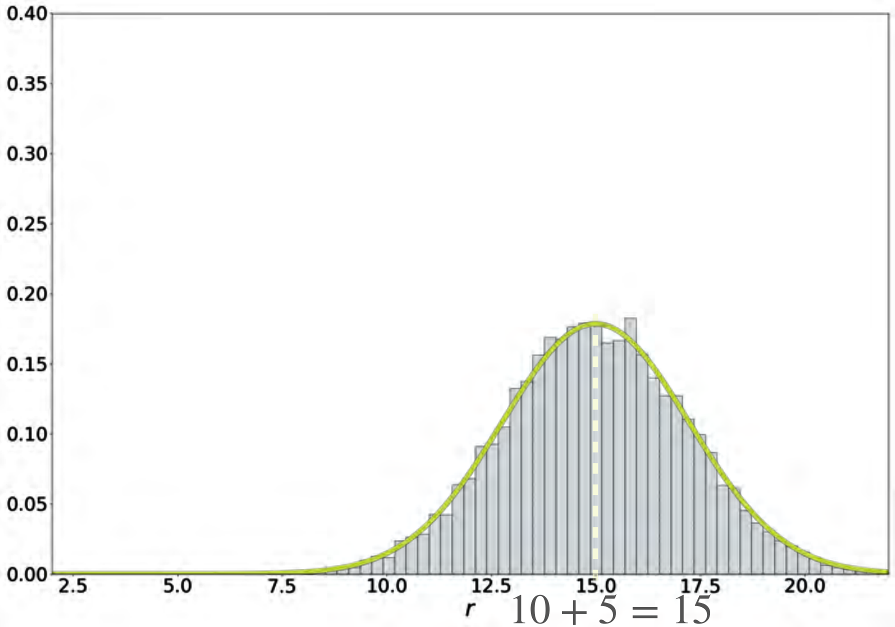
$$= \mu_T + \mu_L = 10 + 5 = 15$$

$$\sigma_R^2 = Var(R)$$

Sum Of Gaussians

$$R = T + L$$



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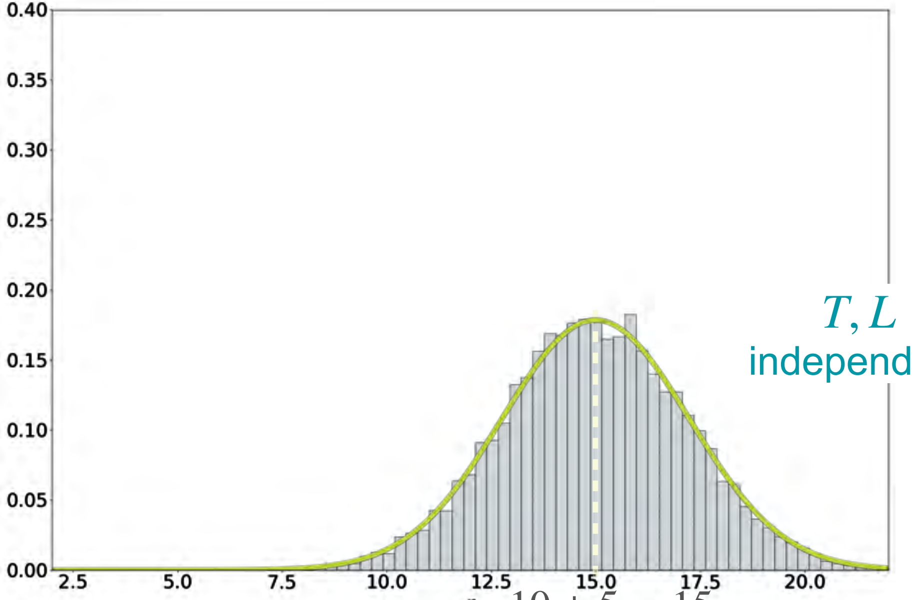
$$= \mu_T + \mu_L \; = 10 + 5 \; = 15$$

$$\sigma_R^2 = Var(R) \qquad Var(T + L)$$

Sum Of Gaussians

$$R = T + L$$



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$$= \mu_T + \mu_L = 10 + 5 = 15$$

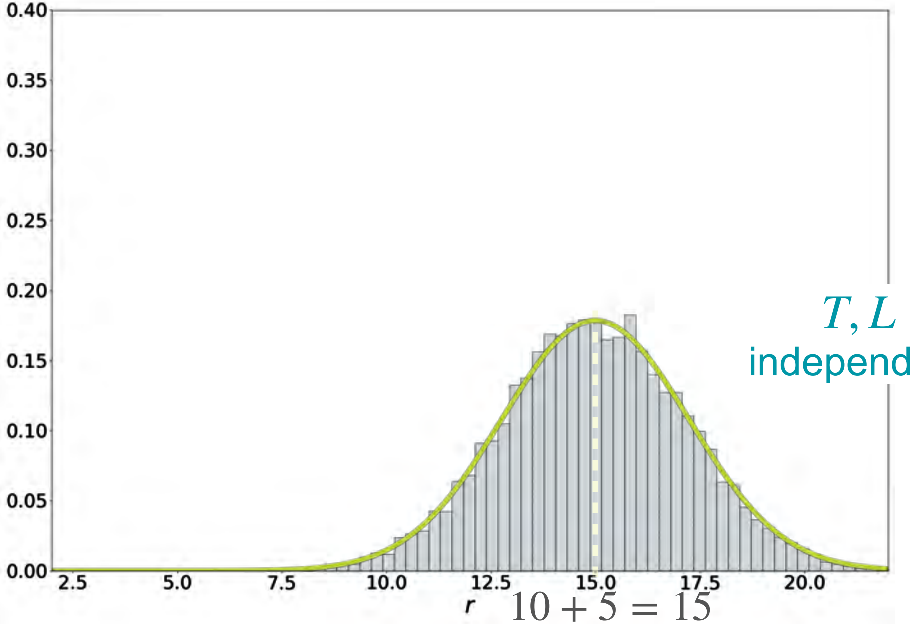
T, L independent

$$\sigma_R^2 = Var(R) \quad Var(T + L)$$

$$= Var(T) + Var(L)$$

Sum Of Gaussians

$$R = T + L$$



$$10 + 5 = 15$$

R is still Gaussian!

Expectation is linear

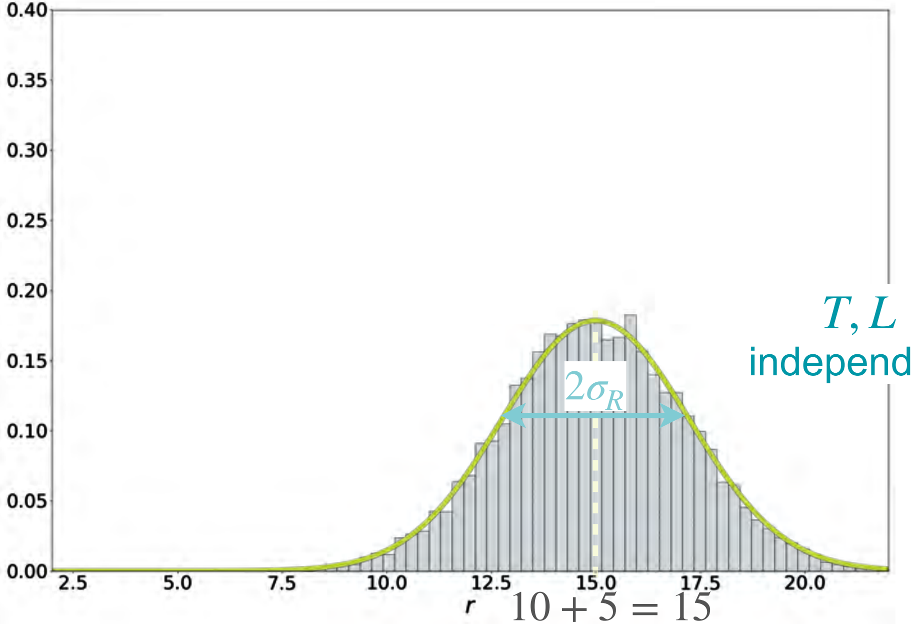
$$\mu_R = \mathbb{E}[R] = \mathbb{E}[T + L] = \mathbb{E}[T] + \mathbb{E}[L]$$
$$= \mu_T + \mu_L = 10 + 5 = 15$$

T, L independent

$$\sigma_R^2 = Var(R) \quad Var(T + L)$$
$$= Var(T) + Var(L) = \sigma_T^2 + \sigma_L^2$$

Sum Of Gaussians

$$R = T + L$$



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Expectation is linear

$$\mu_R = \mathbb{E}[R] \quad = \mathbb{E}[T + L] \quad = \mathbb{E}[T] + \mathbb{E}[L]$$

$$= \mu_T + \mu_L \quad = 10 + 5 \quad = 15$$

T, L independent

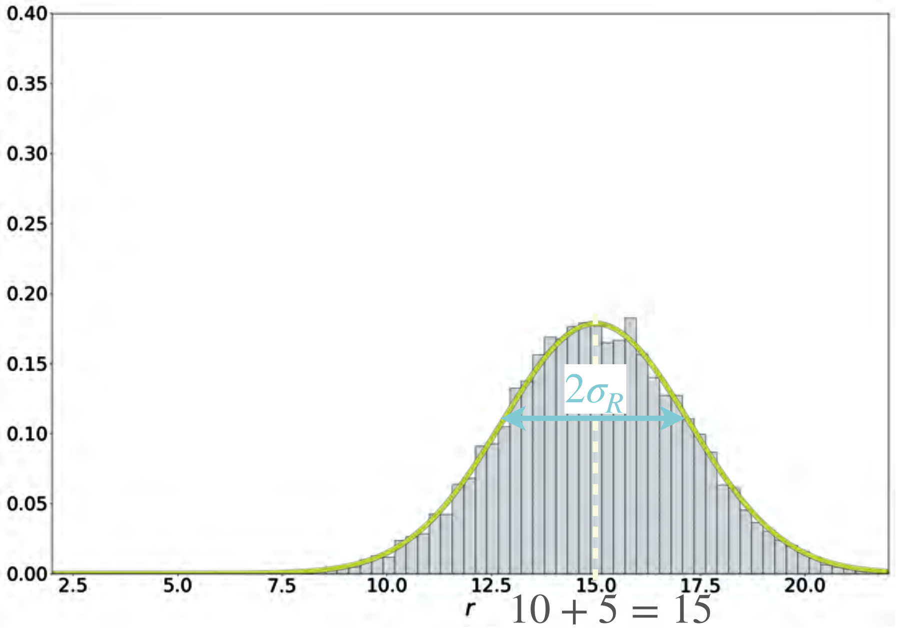
$$\sigma_R^2 = Var(R) \quad Var(T + L)$$

$$= Var(T) + Var(L) \quad = \sigma_T^2 + \sigma_L^2$$

$$= 4 + 1 \quad = 5$$

Sum Of Gaussians

$R = T + L$

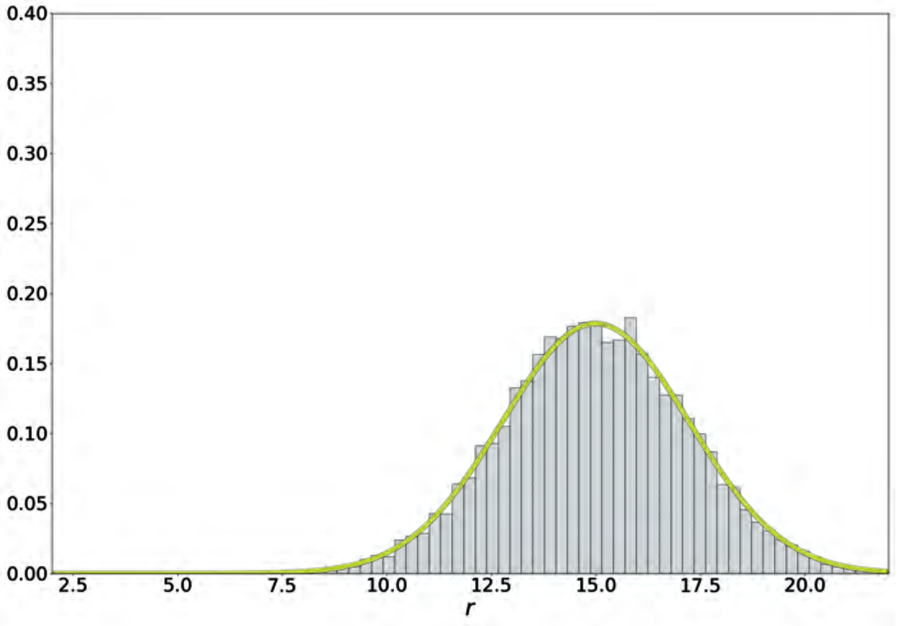


R is still Gaussian!

$$R = (T + L) \sim \mathcal{N}\left(10 + 5, \quad 4 + 1 \quad\right)$$

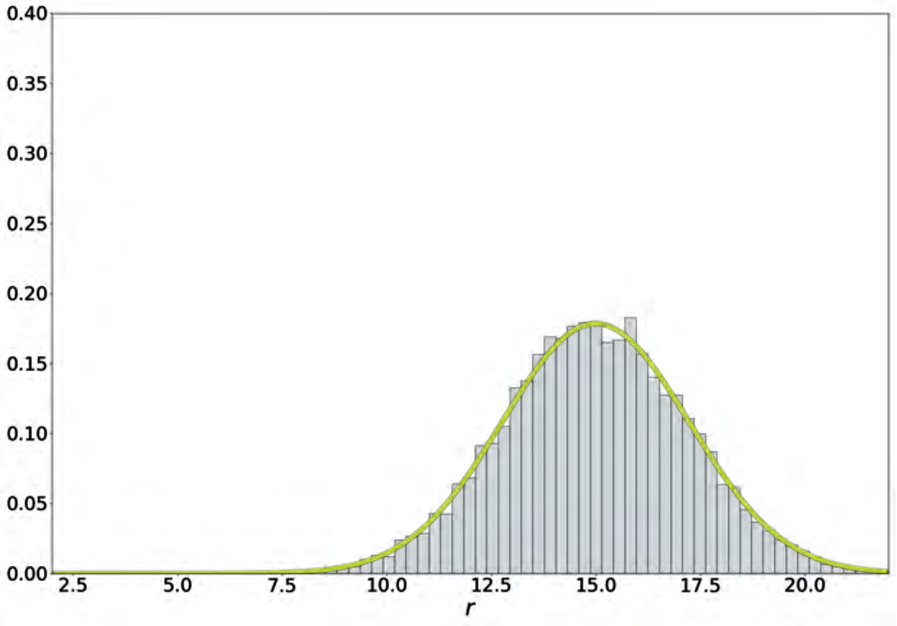
Sum Of Gaussians

$$R = T + L$$



Sum Of Gaussians

$$R = T + L$$

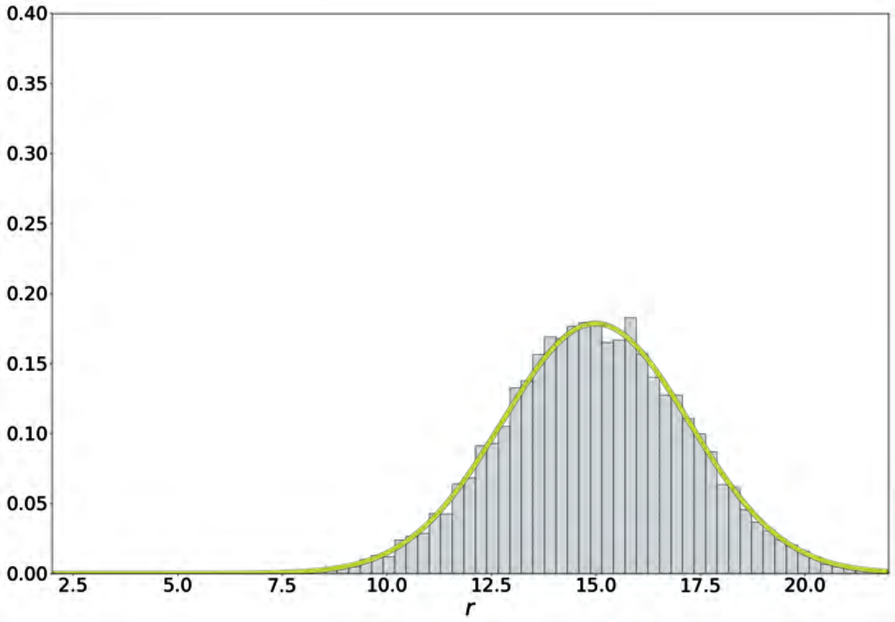


In general: $W = aX + bY$

Independent $\begin{cases} X \sim \mathcal{N}(\mu_X, \sigma_X^2) \\ Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2) \end{cases}$

Sum Of Gaussians

$$R = T + L$$



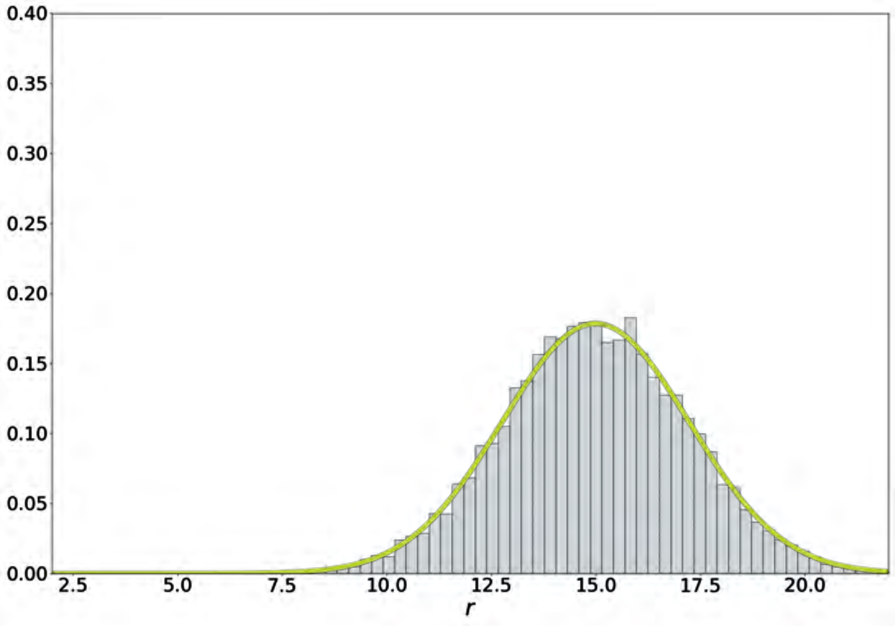
In general: $W = aX + bY$

Independent $\begin{cases} X \sim \mathcal{N}(\mu_X, \sigma_X^2) \\ Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2) \end{cases}$

$$\rightarrow W \sim \mathcal{N}\left(\qquad , \qquad \right)$$

Sum Of Gaussians

$$R = T + L$$



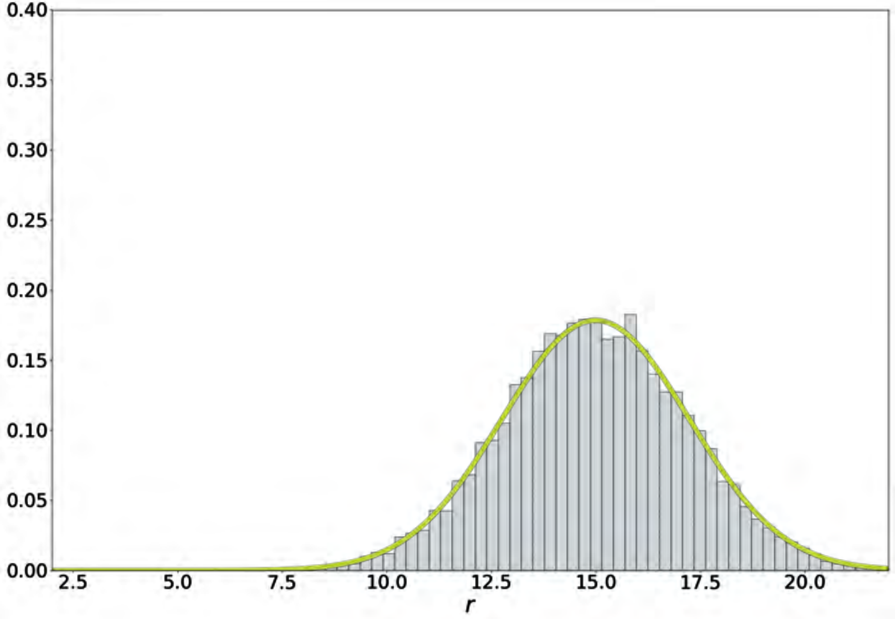
In general: $W = aX + bY$

Independent $\begin{cases} X \sim \mathcal{N}(\mu_X, \sigma_X^2) \\ Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2) \end{cases}$

$$\rightarrow W \sim \mathcal{N}\left(a\mu_x + b\mu_y, \qquad\qquad\qquad \right)$$

Sum Of Gaussians

$$R = T + L$$



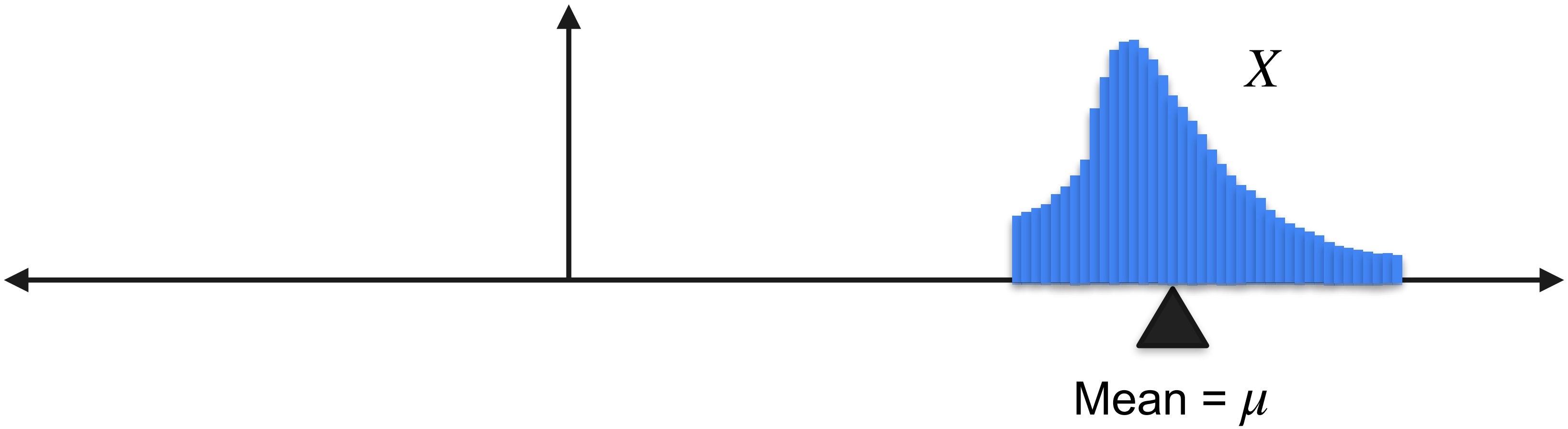
In general: $W = aX + bY$

Independent $\begin{cases} X \sim \mathcal{N}(\mu_X, \sigma_X^2) \\ Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2) \end{cases}$

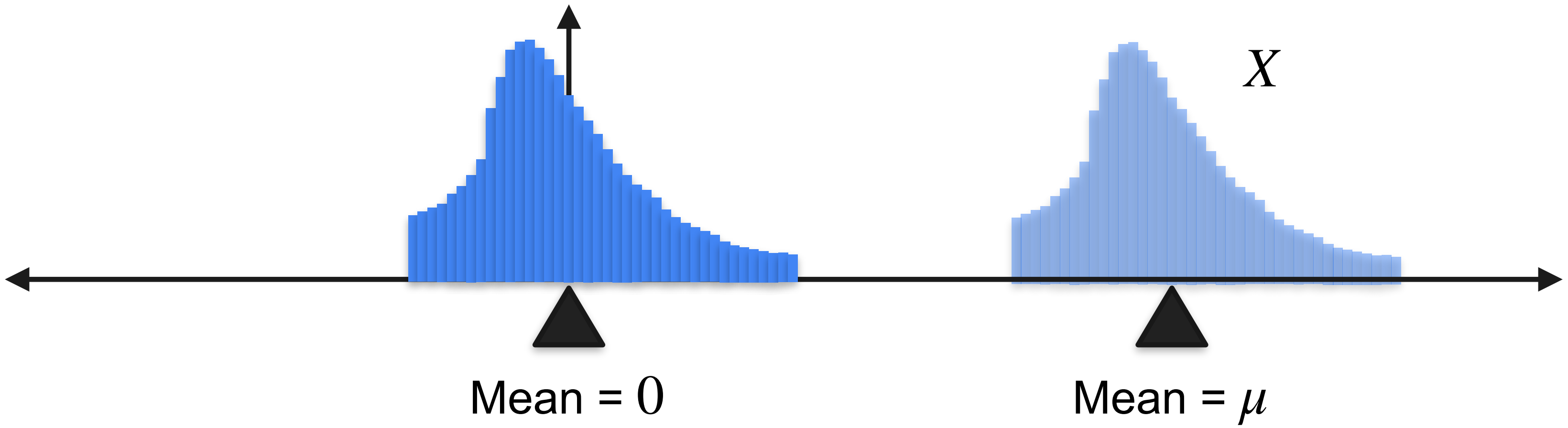
$$\rightarrow W \sim \mathcal{N}\left(a\mu_x + b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2\right)$$

Everything Is Nicer When the Mean Is 0

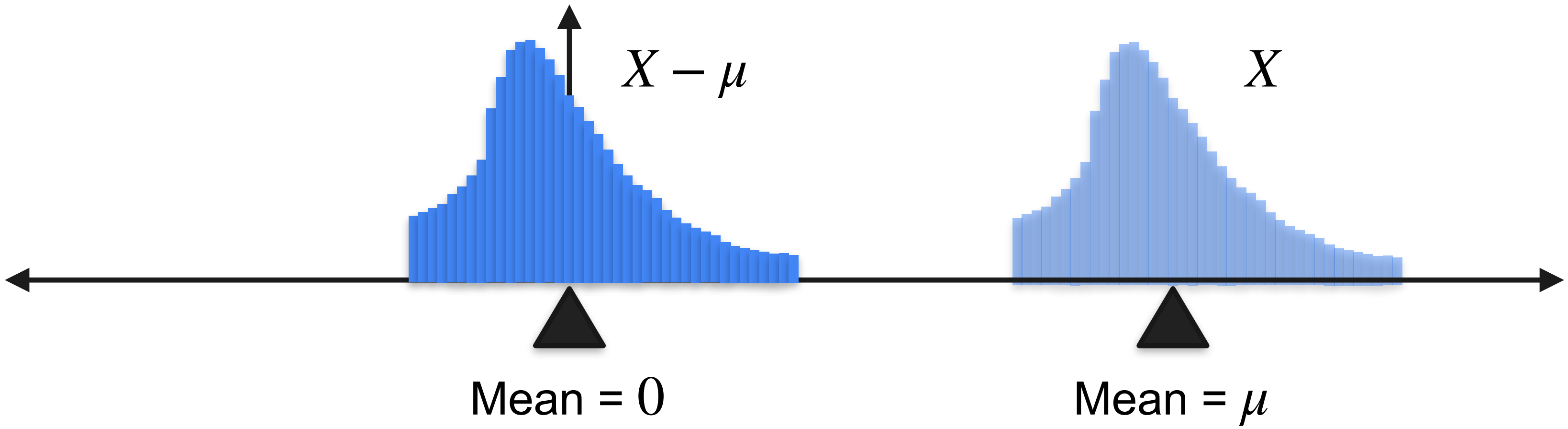
Everything Is Nicer When the Mean Is 0



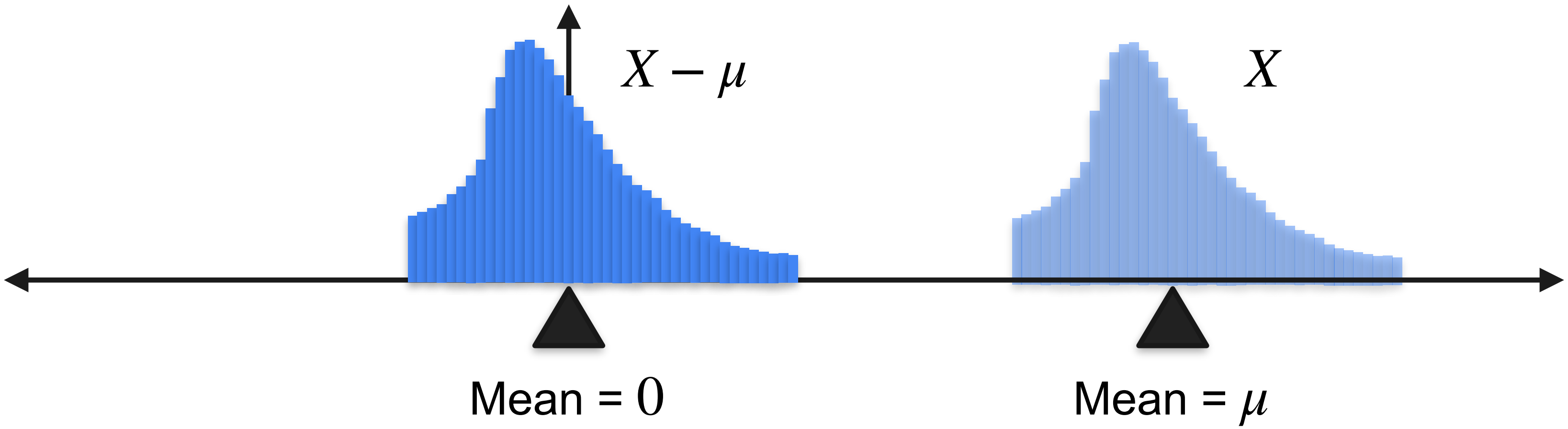
Everything Is Nicer When the Mean Is 0



Everything Is Nicer When the Mean Is 0



Everything Is Nicer When the Mean Is 0



$$X \to X - \mu$$

Everything Is Nicer When the Mean Is 0

Why?

Everything Is Nicer When the Mean Is 0

Why?

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

Everything Is Nicer When the Mean Is 0

Why?

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

$$\mathbb{E}[X - \mu]$$

Everything Is Nicer When the Mean Is 0

Why?

$$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

$$\mathbb{E}[X-\mu] \quad = \mathbb{E}[X] - \mathbb{E}[\mu]$$

Everything Is Nicer When the Mean Is 0

Why?

$$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

$$\begin{aligned}\mathbb{E}[X-\mu] &= \mathbb{E}[X] - \mathbb{E}[\mu] \\ &= \mathbb{E}[X] - \mu\end{aligned}$$

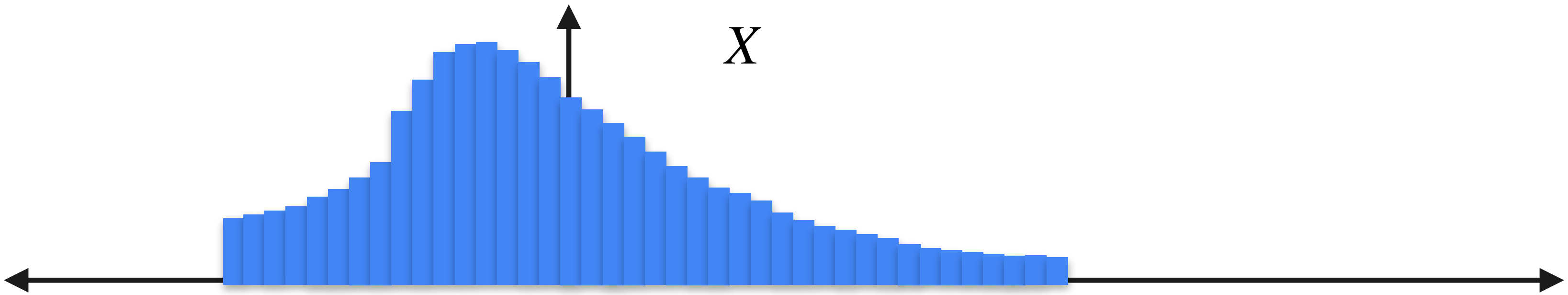
Everything Is Nicer When the Mean Is 0

Why?

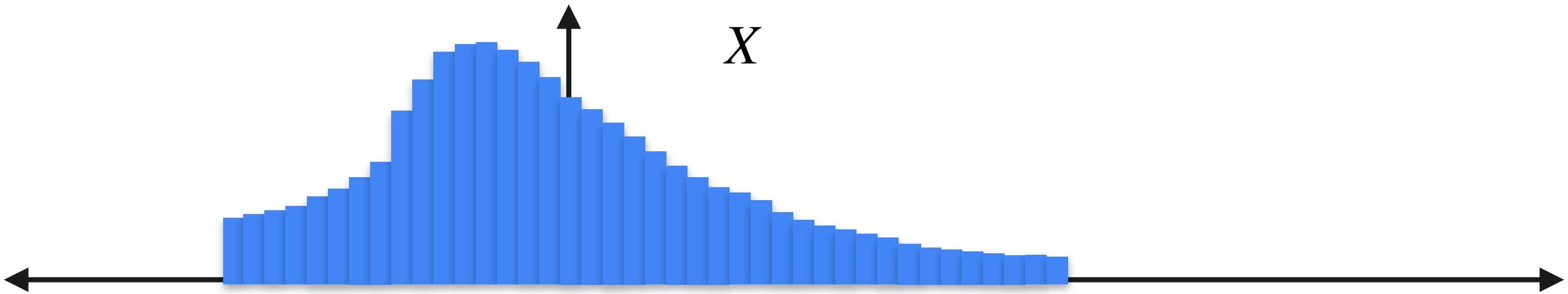
$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

$$\begin{aligned}\mathbb{E}[X - \mu] &= \mathbb{E}[X] - \mathbb{E}[\mu] \\ &= \mathbb{E}[X] - \mu \\ &= 0\end{aligned}$$

Everything Is Nicer When the Standard Deviation Is 1



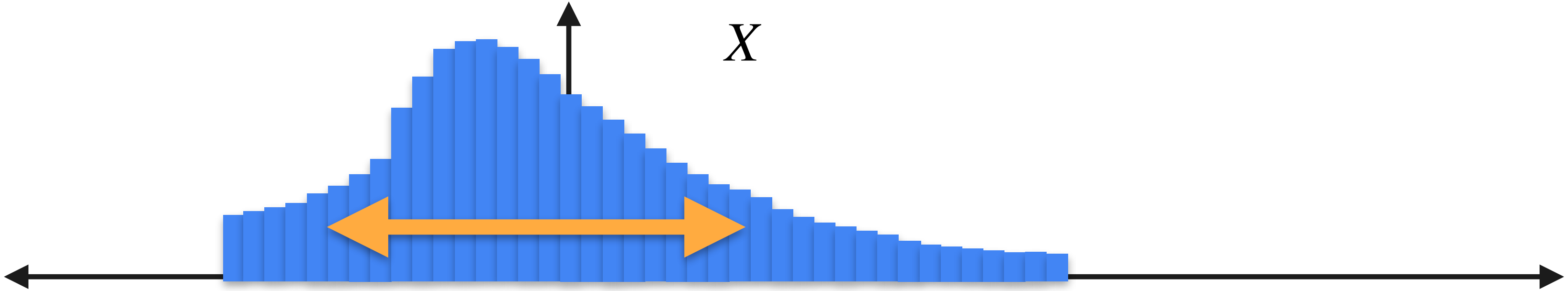
Everything Is Nicer When the Standard Deviation Is 1



Mean = 0

Standard deviation = σ

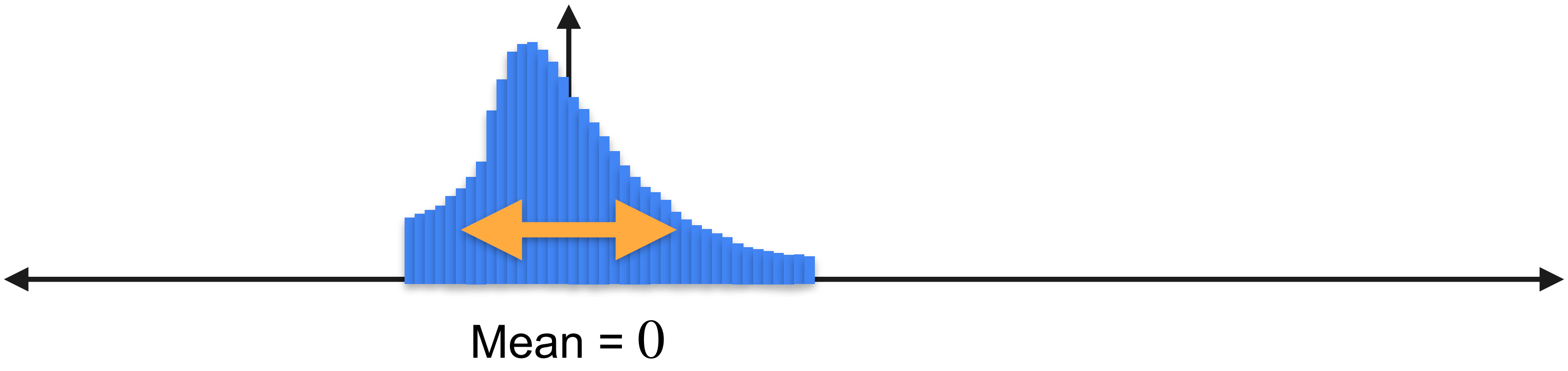
Everything Is Nicer When the Standard Deviation Is 1



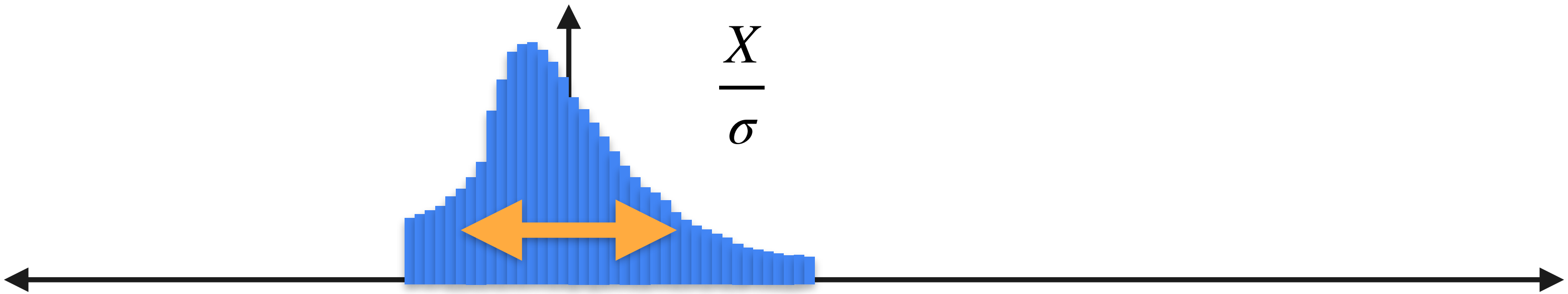
Mean = 0

Standard deviation = σ

Everything Is Nicer When the Standard Deviation Is 1



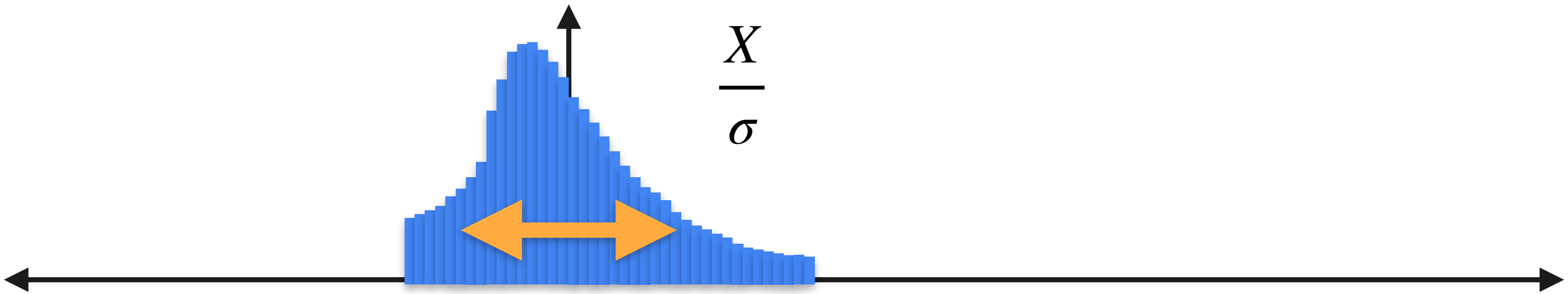
Everything Is Nicer When the Standard Deviation Is 1



Mean = 0

Standard deviation = 1

Everything Is Nicer When the Standard Deviation Is 1



Mean = 0

Standard deviation = 1

$$X \to \frac{X}{\sigma}$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$Var\left(\frac{X}{\sigma}\right)$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$Var(cX)$$

$$Var\left(\frac{X}{\sigma}\right)$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$Var(cX) = \mathbb{E}[(cX)^2] - \mathbb{E}[cX]^2$$

$$Var\left(\frac{X}{\sigma}\right)$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$Var(cX) = \mathbb{E}[(cX)^2] - \mathbb{E}[cX]^2$$

$$= \mathbb{E}[c^2X^2] - (c\mathbb{E}[X])^2$$

$$Var\left(\frac{X}{\sigma}\right)$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$Var\left(\frac{X}{\sigma}\right)$$

$$\begin{aligned} Var(cX) &= \mathbb{E}[(cX)^2] - \mathbb{E}[cX]^2 \\ &= \mathbb{E}[c^2X^2] - (c\mathbb{E}[X])^2 \\ &= c^2\mathbb{E}[X^2] - c^2\mathbb{E}[X]^2 \end{aligned}$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$Var\left(\frac{X}{\sigma}\right)$$

$$\begin{aligned} Var(cX) &= \mathbb{E}[(cX)^2] - \mathbb{E}[cX]^2 \\ &= \mathbb{E}[c^2X^2] - (c\mathbb{E}[X])^2 \\ &= c^2\mathbb{E}[X^2] - c^2\mathbb{E}[X]^2 \\ &= c^2(\mathbb{E}[X^2] - \mathbb{E}[X]^2) \end{aligned}$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$
\begin{aligned}
Var(cX) &= \mathbb{E}[(cX)^2] - \mathbb{E}[cX]^2 \\
&= \mathbb{E}[c^2X^2] - (c\mathbb{E}[X])^2 \\
&= c^2\mathbb{E}[X^2] - c^2\mathbb{E}[X]^2 \\
&= c^2(\mathbb{E}[X^2] - \mathbb{E}[X]^2) \\
&= c^2Var(X)
\end{aligned}
$$

$$Var\left(\frac{X}{\sigma}\right)$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$\begin{aligned}Var(cX) &= \mathbb{E}[(cX)^2] - \mathbb{E}[cX]^2\\ &= \mathbb{E}[c^2X^2] - (c\mathbb{E}[X])^2\\ &= c^2\mathbb{E}[X^2] - c^2\mathbb{E}[X]^2\\ &= c^2(\mathbb{E}[X^2] - \mathbb{E}[X]^2)\\ &= c^2 Var(X)\end{aligned}$$

$$Var\left(\frac{X}{\sigma}\right) = \frac{1}{\sigma^2} Var(X)$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$
\begin{aligned}
Var(cX) &= \mathbb{E}[(cX)^2] - \mathbb{E}[cX]^2 \\
&= \mathbb{E}[c^2X^2] - (c\mathbb{E}[X])^2 \\
&= c^2\mathbb{E}[X^2] - c^2\mathbb{E}[X]^2 \\
&= c^2(\mathbb{E}[X^2] - \mathbb{E}[X]^2) \\
&= c^2Var(X)
\end{aligned}
$$

$$Var\left(\frac{X}{\sigma}\right) = \frac{1}{\sigma^2}Var(X)$$

$$std\left(\frac{X}{\sigma}\right) = \frac{1}{\sigma}std(X)$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$
\begin{aligned}
Var(cX) &= \mathbb{E}[(cX)^2] - \mathbb{E}[cX]^2 \\
&= \mathbb{E}[c^2X^2] - (c\mathbb{E}[X])^2 \\
&= c^2\mathbb{E}[X^2] - c^2\mathbb{E}[X]^2 \\
&= c^2(\mathbb{E}[X^2] - \mathbb{E}[X]^2) \\
&= c^2Var(X)
\end{aligned}
$$

$$Var\left(\frac{X}{\sigma}\right) = \frac{1}{\sigma^2}Var\,(X)$$

$$
\begin{aligned}
std\left(\frac{X}{\sigma}\right) &= \frac{1}{\sigma}std(X) \\
&= \frac{\sigma}{\sigma}
\end{aligned}
$$

Everything Is Nicer When the Standard Deviation Is 1

Why?

$$\begin{aligned} Var(cX) &= \mathbb{E}[(cX)^2] - \mathbb{E}[cX]^2 \\ &= \mathbb{E}[c^2X^2] - (c\mathbb{E}[X])^2 \\ &= c^2\mathbb{E}[X^2] - c^2\mathbb{E}[X]^2 \\ &= c^2(\mathbb{E}[X^2] - \mathbb{E}[X]^2) \\ &= c^2 Var(X) \end{aligned}$$

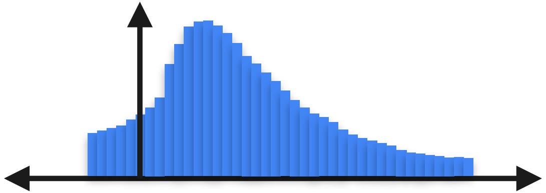
$$Var\left(\frac{X}{\sigma}\right) = \frac{1}{\sigma^2} Var(X)$$

$$\begin{aligned} std\left(\frac{X}{\sigma}\right) &= \frac{1}{\sigma} std(X) \\ &= \frac{\sigma}{\sigma} \\ &= 1 \end{aligned}$$

Standardize a Distribution

Standardize a Distribution

$$X$$

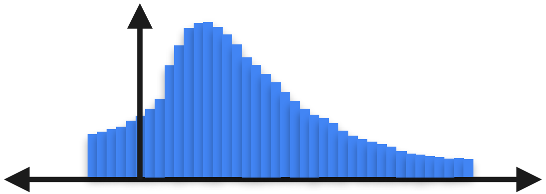


Mean = μ

std = σ

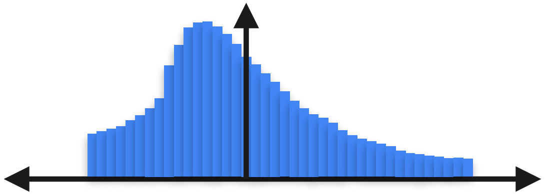
Standardize a Distribution

$$X$$



Mean = μ
std = σ

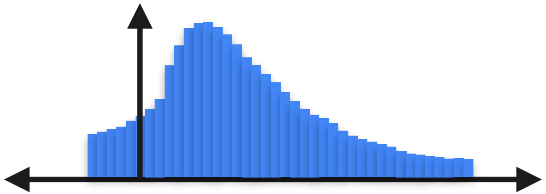
$$X - \mu$$



Mean = 0
std = σ

Standardize a Distribution

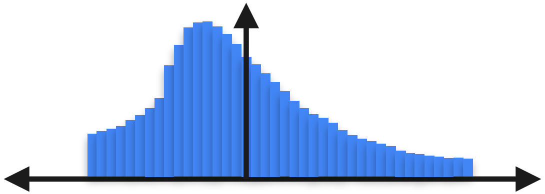
X



Mean = μ
std = σ

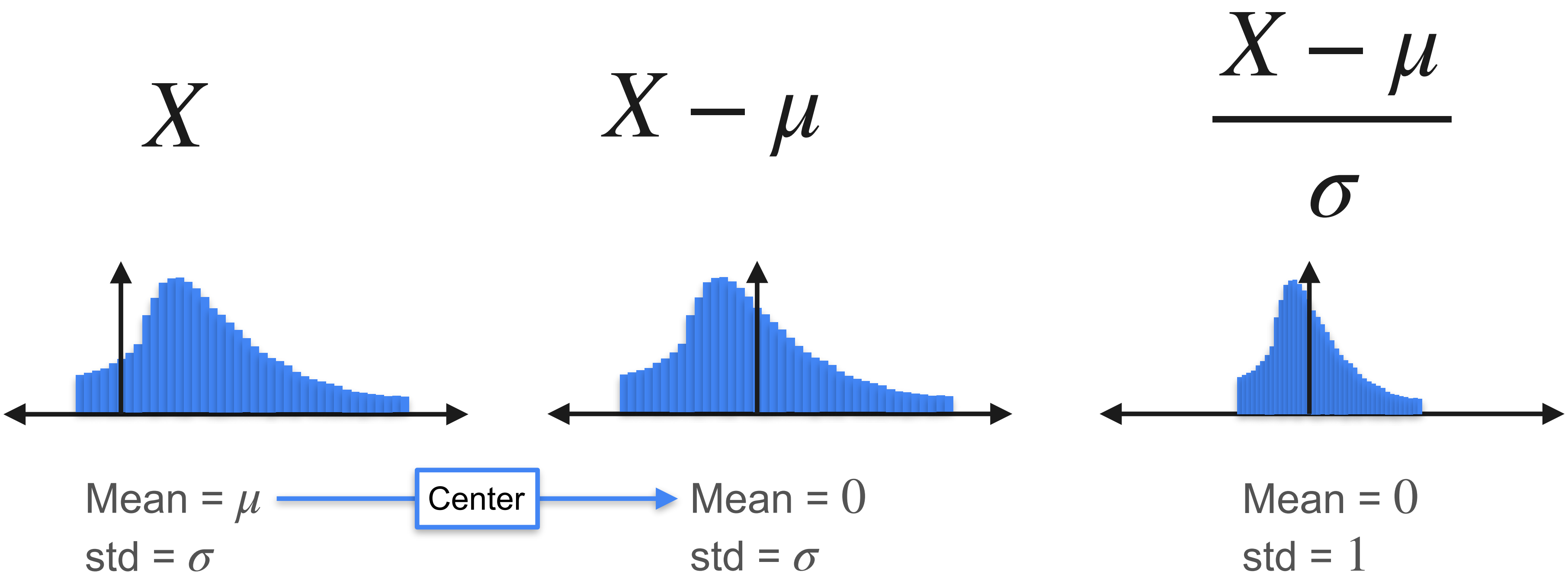
Center

$X - \mu$



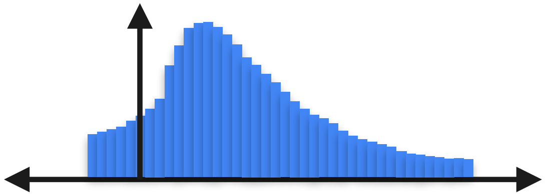
Mean = 0
std = σ

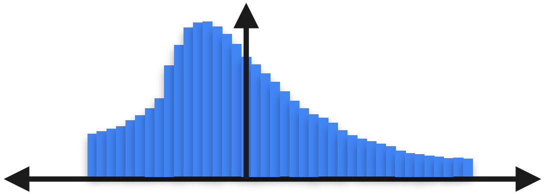
Standardize a Distribution

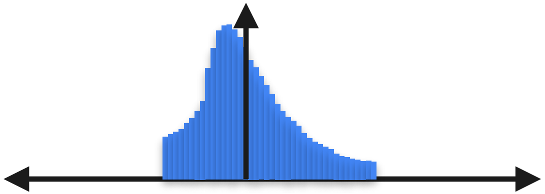


Standardize a Distribution

$$X \qquad\qquad X - \mu \qquad\qquad \frac{X - \mu}{\sigma}$$



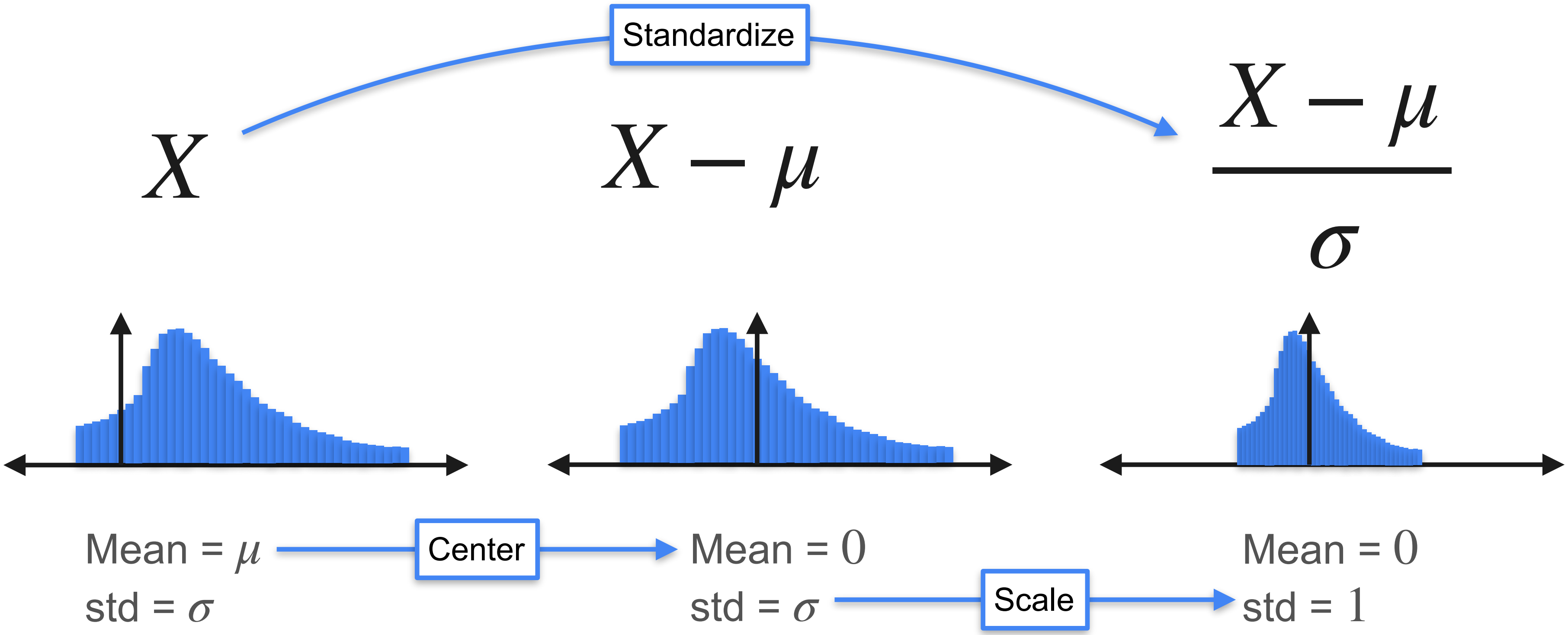




Mean = μ → **Center** → Mean = 0 Mean = 0

std = σ std = σ → **Scale** → std = 1

Standardize a Distribution



DeepLearning.AI

Describing Distributions

Skewness and Kurtosis

Moments of a Distribution

Random variable X

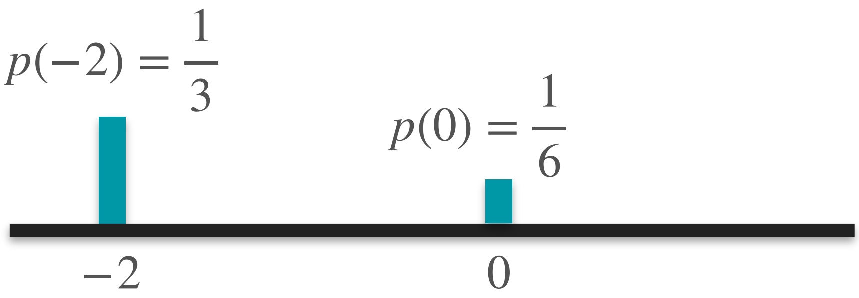
Moments of a Distribution

Random variable X

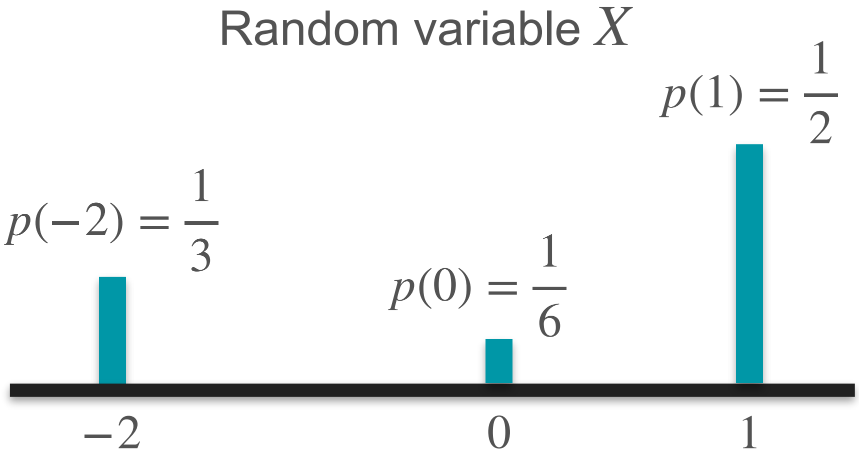
$$p(-2) = \frac{1}{3}$$

Moments of a Distribution

Random variable X

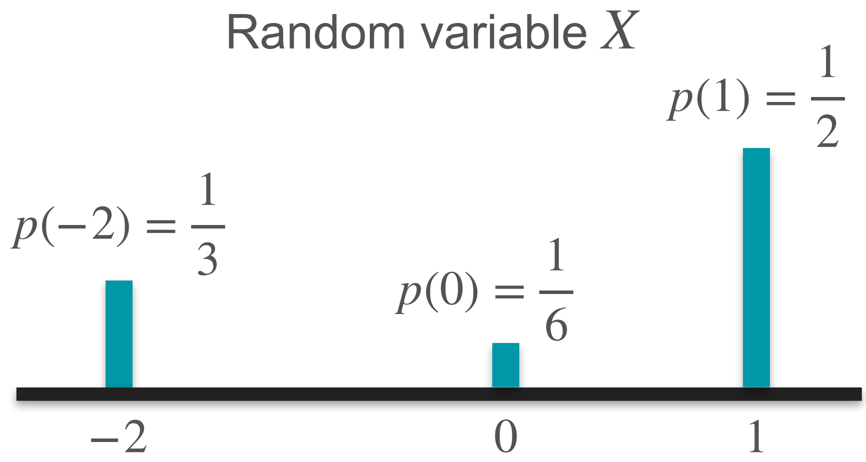


Moments of a Distribution



Moments of a Distribution

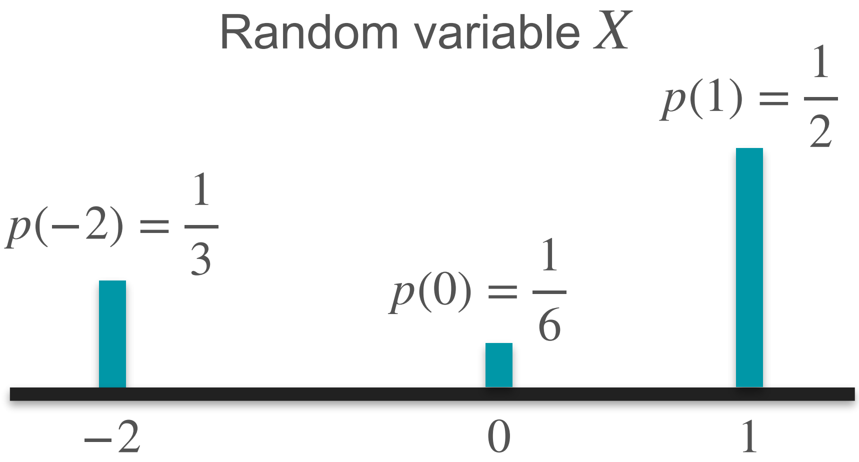
$$\mathbb{E}[X] = \frac{1}{3}(-2) + \frac{1}{6}(0) + \frac{1}{2}(1)$$



Moments of a Distribution

$$\mathbb{E}[X] = \frac{1}{3}(-2) + \frac{1}{6}(0) + \frac{1}{2}(1)$$

$$\mathbb{E}[X^2] = \frac{1}{3}(-2)^2 + \frac{1}{6}(0)^2 + \frac{1}{2}(1)^2$$



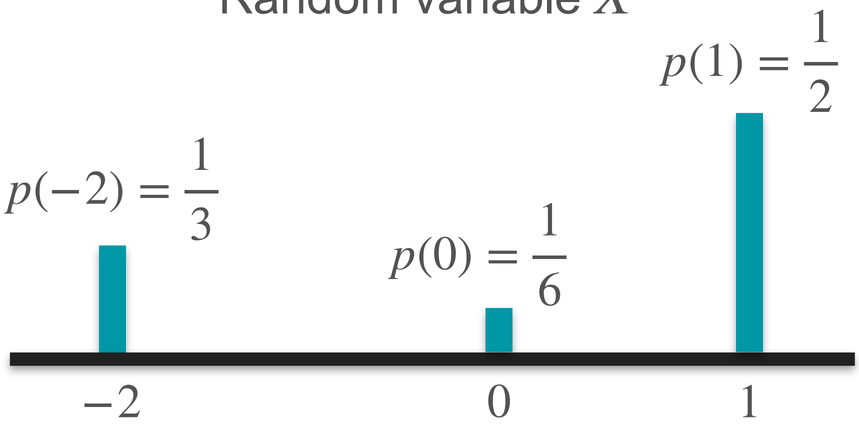
Moments of a Distribution

$$\mathbb{E}[X] = \frac{1}{3}(-2) + \frac{1}{6}(0) + \frac{1}{2}(1)$$

$$\mathbb{E}[X^2] = \frac{1}{3}(-2)^2 + \frac{1}{6}(0)^2 + \frac{1}{2}(1)^2$$

$$\mathbb{E}[X^3] = \frac{1}{3}(-2)^3 + \frac{1}{6}(0)^3 + \frac{1}{2}(1)^3$$

Random variable X



Moments of a Distribution

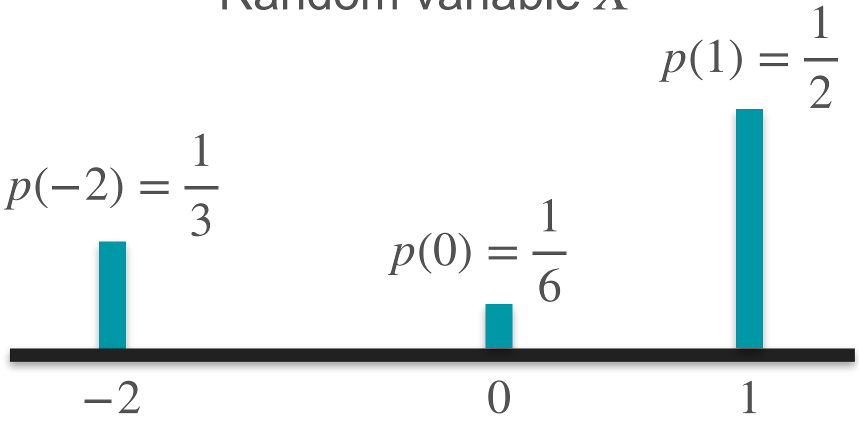
$$\mathbb{E}[X] = \frac{1}{3}(-2) + \frac{1}{6}(0) + \frac{1}{2}(1)$$

$$\mathbb{E}[X^{2}] = \frac{1}{3}(-2)^{2} + \frac{1}{6}(0)^{2} + \frac{1}{2}(1)^{2}$$

$$\mathbb{E}[X^{3}] = \frac{1}{3}(-2)^{3} + \frac{1}{6}(0)^{3} + \frac{1}{2}(1)^{3}$$

$$\cdots$$

Random variable X



Moments of a Distribution

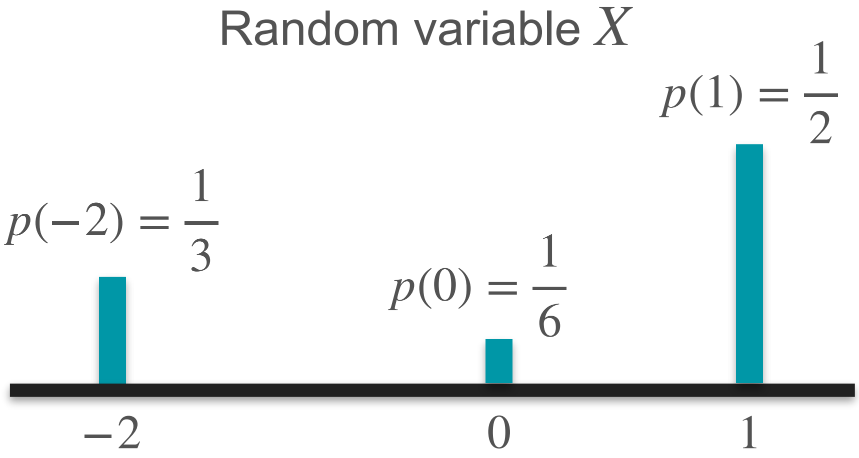
$$\mathbb{E}[X] = \frac{1}{3}(-2) + \frac{1}{6}(0) + \frac{1}{2}(1)$$

$$\mathbb{E}[X^{2}] = \frac{1}{3}(-2)^{2} + \frac{1}{6}(0)^{2} + \frac{1}{2}(1)^{2}$$

$$\mathbb{E}[X^{3}] = \frac{1}{3}(-2)^{3} + \frac{1}{6}(0)^{3} + \frac{1}{2}(1)^{3}$$

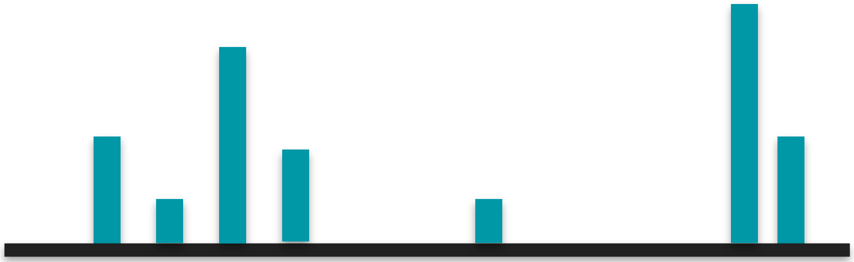
$$\cdots$$

$$\mathbb{E}[X^{k}] = \frac{1}{3}(-2)^{k} + \frac{1}{6}(0)^{k} + \frac{1}{2}(1)^{k}$$



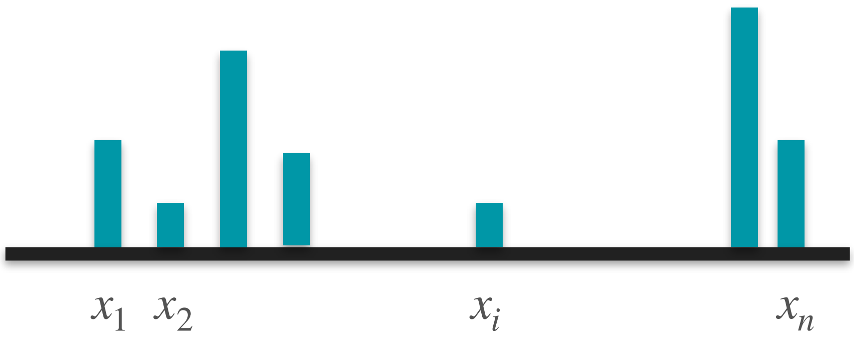
Moments of a Distribution

Random variable X

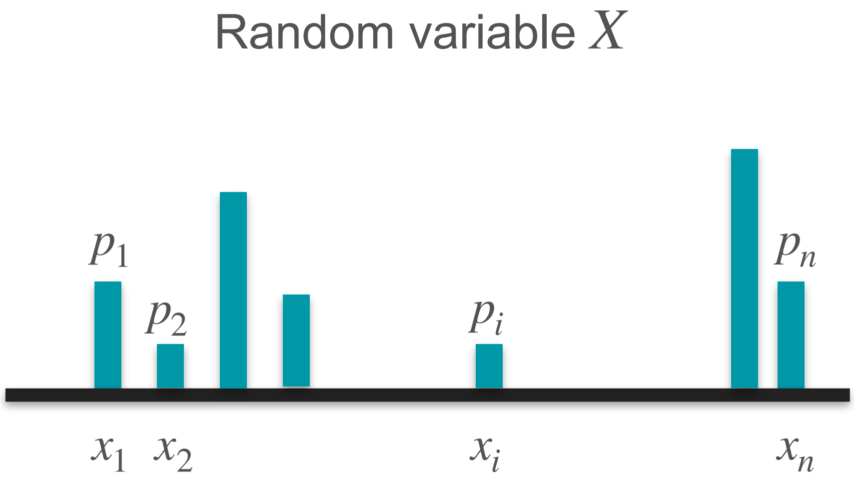


Moments of a Distribution

Random variable X



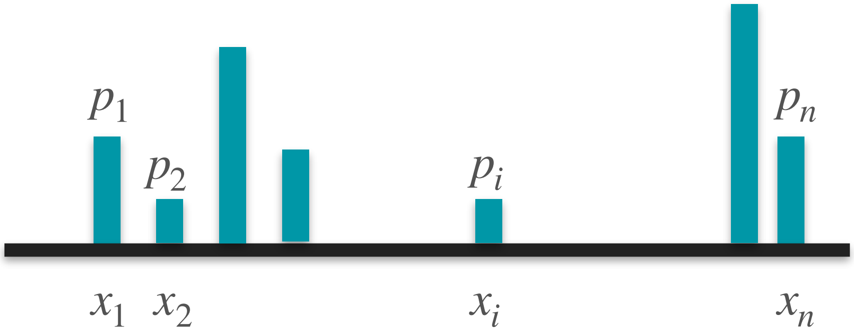
Moments of a Distribution



Moments of a Distribution

$$\mathbb{E}[X] = p_1 x_1 + p_2 x_2 + \cdots + p_n x_n$$

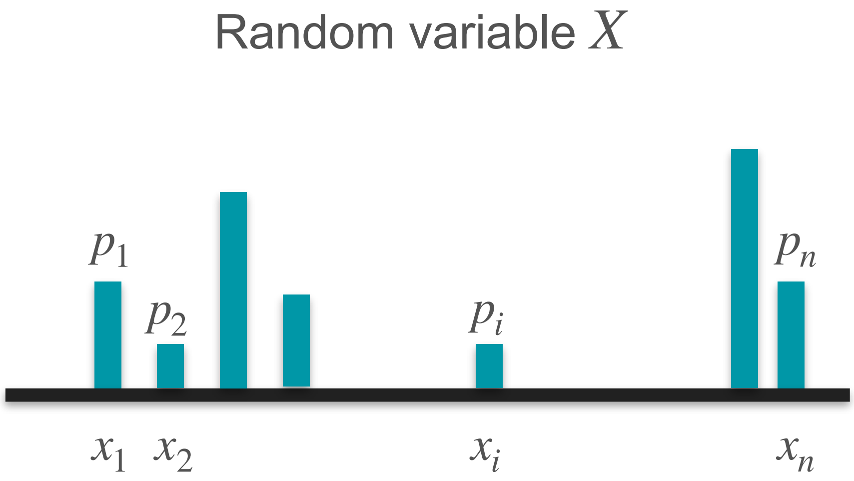
Random variable X



Moments of a Distribution

$$\mathbb{E}[X] = p_1x_1 + p_2x_2 + \cdots + p_nx_n$$

$$\mathbb{E}[X^2] = p_1x_1^2 + p_2x_2^2 + \cdots + p_nx_n^2$$



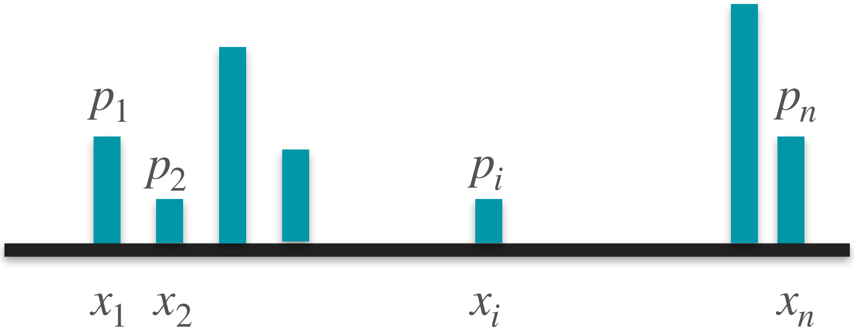
Moments of a Distribution

$$\mathbb{E}[X] = p_1x_1 + p_2x_2 + \cdots + p_nx_n$$

$$\mathbb{E}[X^2] = p_1x_1^2 + p_2x_2^2 + \cdots + p_nx_n^2$$

$$\mathbb{E}[X^3] = p_1x_1^3 + p_2x_2^3 + \cdots + p_nx_n^3$$

Random variable X



Moments of a Distribution

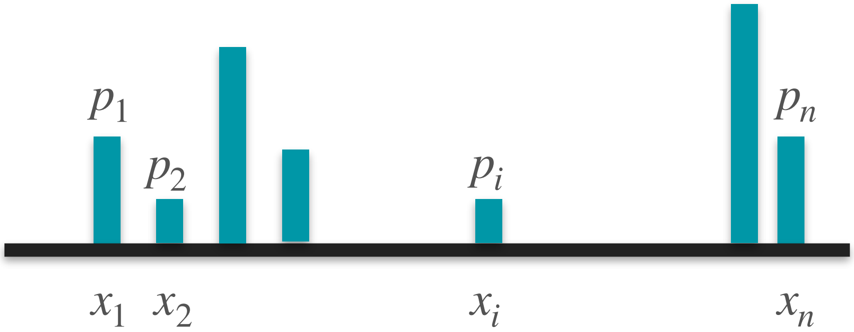
$$\mathbb{E}[X] = p_1x_1 + p_2x_2 + \cdots + p_nx_n$$

$$\mathbb{E}[X^2] = p_1x_1^2 + p_2x_2^2 + \cdots + p_nx_n^2$$

$$\mathbb{E}[X^3] = p_1x_1^3 + p_2x_2^3 + \cdots + p_nx_n^3$$

$$\mathbb{E}[X^4] = p_1x_1^4 + p_2x_2^4 + \cdots + p_nx_n^4$$

Random variable X



Moments of a Distribution

$$\mathbb{E}[X] = p_1x_1 + p_2x_2 + \cdots + p_nx_n$$

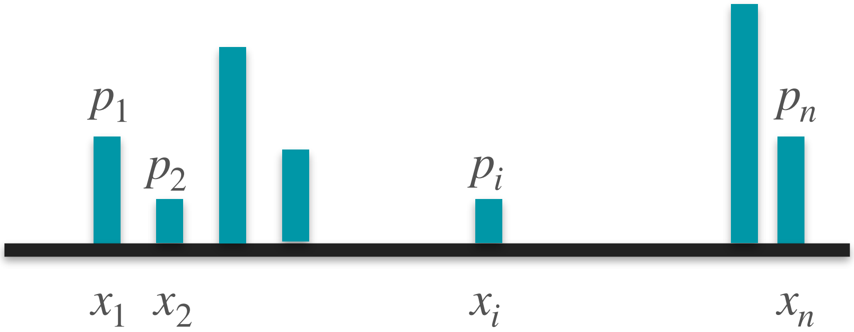
$$\mathbb{E}[X^2] = p_1x_1^2 + p_2x_2^2 + \cdots + p_nx_n^2$$

$$\mathbb{E}[X^3] = p_1x_1^3 + p_2x_2^3 + \cdots + p_nx_n^3$$

$$\mathbb{E}[X^4] = p_1x_1^4 + p_2x_2^4 + \cdots + p_nx_n^4$$

$$\cdots$$

Random variable X



Moments of a Distribution

$$\mathbb{E}[X] = p_1x_1 + p_2x_2 + \cdots + p_nx_n$$

$$\mathbb{E}[X^2] = p_1x_1^2 + p_2x_2^2 + \cdots + p_nx_n^2$$

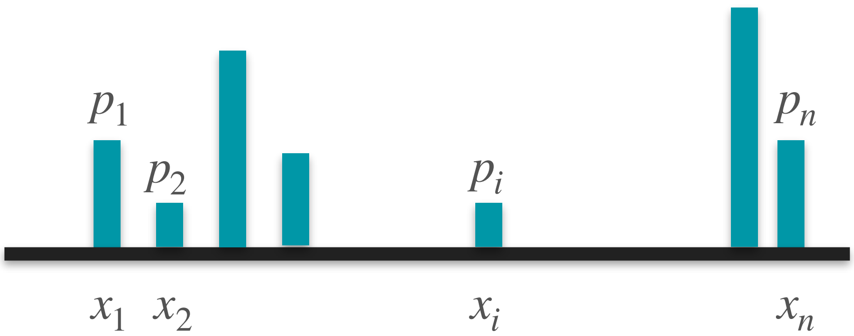
$$\mathbb{E}[X^3] = p_1x_1^3 + p_2x_2^3 + \cdots + p_nx_n^3$$

$$\mathbb{E}[X^4] = p_1x_1^4 + p_2x_2^4 + \cdots + p_nx_n^4$$

$$\cdots$$

$$\mathbb{E}[X^k] = p_1x_1^k + p_2x_2^k + \cdots + p_nx_n^k$$

Random variable X



Video 7b:

- Skewness

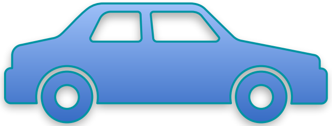
Lottery vs Insurance

Lottery vs Insurance

Lottery

Lottery vs Insurance

Lottery

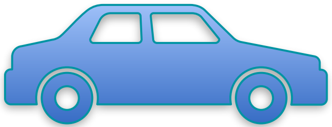


Car insurance

Lottery vs Insurance

Ticket: $1
Jackpot: $100

Lottery

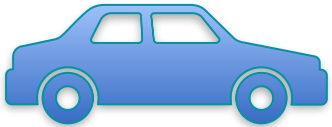


Car insurance

Lottery vs Insurance

Ticket: $1
Jackpot: $100

Lottery



Car insurance

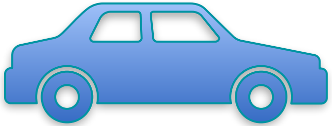
You **win** $99 with 1% probability

You **lose** $1 with 99% probability

Lottery vs Insurance

Ticket: $1
Jackpot: $100

Lottery



Cost: $1
Crash Reparation: $100

Car insurance

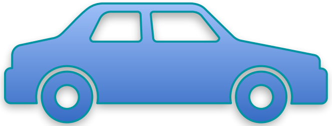
You **win** $99 with 1% probability

You **lose** $1 with 99% probability

Lottery vs Insurance

Ticket: $1
Jackpot: $100

Lottery



Cost: $1
Crash Reparation: $100

Car insurance

You **win** $99 with 1% probability

You **lose** $1 with 99% probability

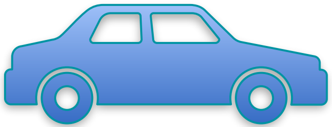
You **win** $1 with 99% probability

You **lose** $99 with 1% probability

Lottery vs Insurance

Ticket: $1
Jackpot: $100

Lottery



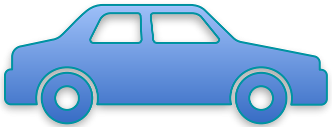
Cost: $1
Crash Reparation: $100

Car insurance

Lottery vs Insurance

Ticket: $1
Jackpot: $100

Lottery



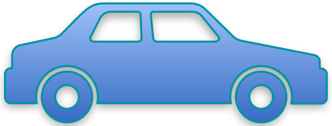
Cost: $1
Crash Reparation: $100

Car insurance

Lottery vs Insurance

Ticket: $1
Jackpot: $100

Lottery



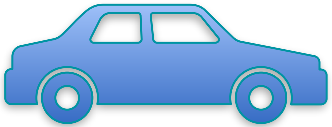
Cost: $1
Crash Reparation: $100

Car insurance

Lottery vs Insurance

Ticket: $1
Jackpot: $100

Lottery



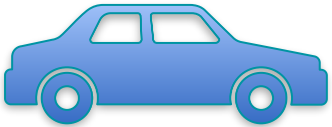
Cost: $1
Crash Reparation: $100

Car insurance

Lottery vs Insurance

Ticket: $1
Jackpot: $100

Lottery

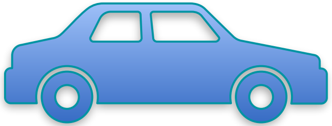


Cost: $1
Crash Reparation: $100

Car insurance

Ticket: $1
Jackpot: $100

Lottery

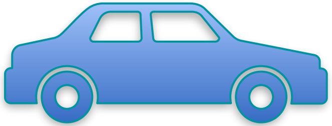


Cost: $1
Crash Reparation: $100

Car insurance

Ticket: $1
Jackpot: $100

Lottery

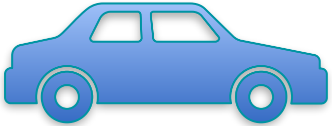


Cost: $1
Crash Reparation: $100

Car insurance

Ticket: $1
Jackpot: $100

Lottery



Cost: $1
Crash Reparation: $100

Car insurance

$$\mathbb{E}[X_1] = -1 \cdot 0.99 + 99 \cdot 0.01 = 0$$

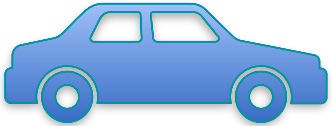
Ticket: $1
Jackpot: $100

Lottery

$$\mathbb{E}[X_1] = -1 \cdot 0.99 + 99 \cdot 0.01 = 0$$

Lose 1
With probability 0.99

Win 99
With probability 0.01



Cost: $1
Crash Reparation: $100

Car insurance

Lottery

Ticket: $1
Jackpot: $100

$$\mathbb{E}[X_1] = -1 \cdot 0.99 + 99 \cdot 0.01 = 0$$

Lose 1
With probability 0.99

Win 99
With probability 0.01

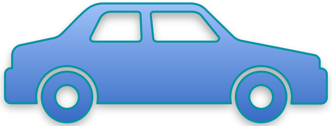
Car insurance

Cost: $1
Crash Reparation: $100

$$\mathbb{E}[X_2] = -99 \cdot 0.01 + 1 \cdot 0.99 = 0$$

Ticket: $1
Jackpot: $100

Lottery



Cost: $1
Crash Reparation: $100

Car insurance

$$\mathbb{E}[X_1] = -1 \cdot 0.99 + 99 \cdot 0.01 = 0$$

Lose 1
With probability 0.99

Win 99
With probability 0.01

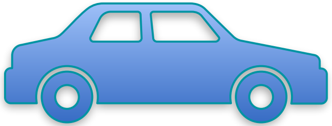
$$\mathbb{E}[X_2] = -99 \cdot 0.01 + 1 \cdot 0.99 = 0$$

Lose 99
With probability 0.01

Win 1
With probability 0.99

Ticket: $1
Jackpot: $100

Lottery



Cost: $1
Crash Reparation: $100

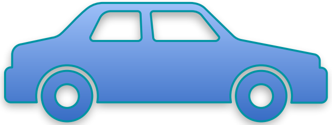
Car insurance

$$\mathbb{E}[X_1] = -1 \cdot 0.99 + 99 \cdot 0.01 = 0$$

$$\mathbb{E}[X_2] = -99 \cdot 0.01 + 1 \cdot 0.99 = 0$$

Ticket: $1
Jackpot: $100

Lottery

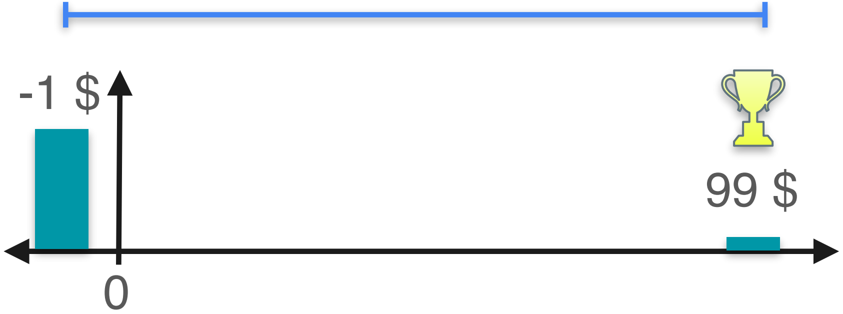


Cost: $1
Crash Reparation: $100

Car insurance

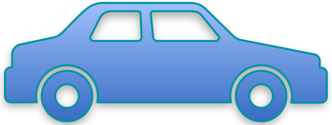
$$\mathbb{E}[X_1] = -1 \cdot 0.99 + 99 \cdot 0.01 = 0$$

$$\mathbb{E}[X_2] = -99 \cdot 0.01 + 1 \cdot 0.99 = 0$$



Ticket: $1
Jackpot: $100

Lottery

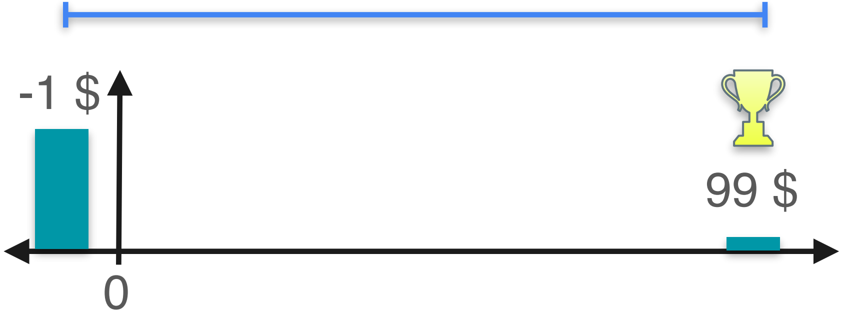


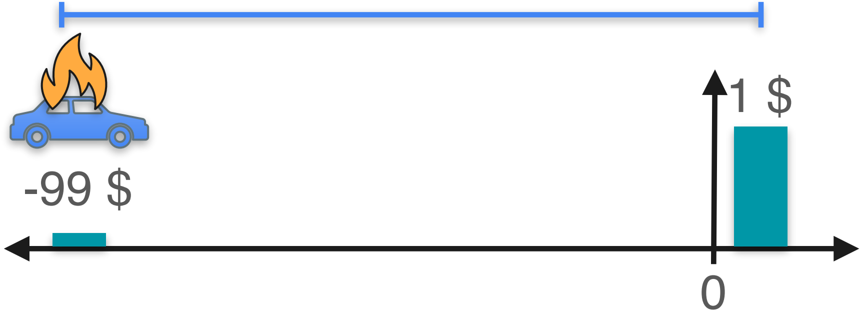
Cost: $1
Crash Reparation: $100

Car insurance

$$\mathbb{E}[X_1] = -1 \cdot 0.99 + 99 \cdot 0.01 = 0$$

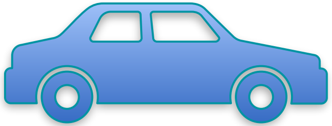
$$\mathbb{E}[X_2] = -99 \cdot 0.01 + 1 \cdot 0.99 = 0$$





Ticket: \$1
Jackpot: \$100

Lottery



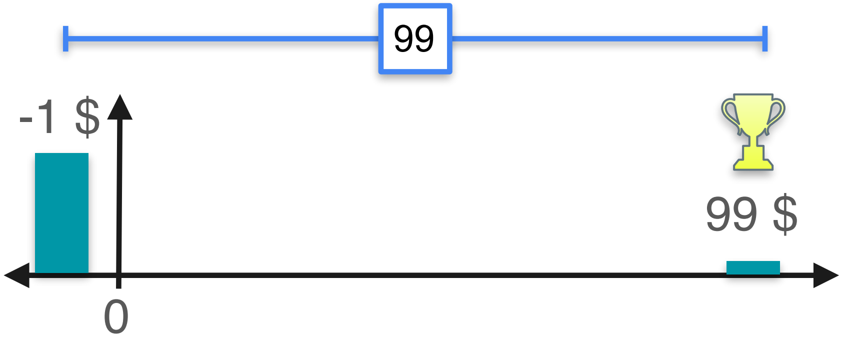
Cost: \$1
Crash Reparation: \$100

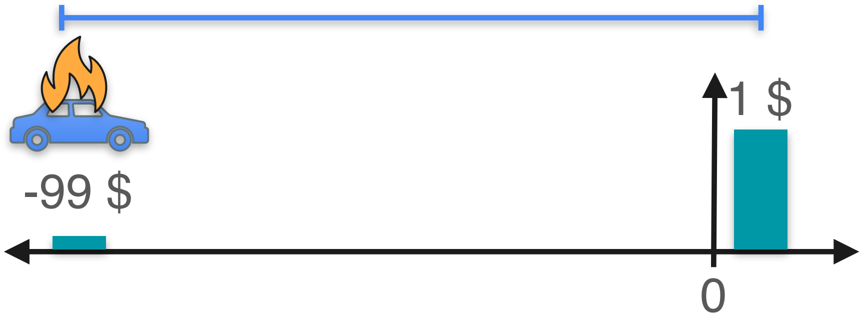
Car insurance

$$\mathbb{E}[X_1] = -1 \cdot 0.99 + 99 \cdot 0.01 = 0$$

$$\mathbb{E}[X_2] = -99 \cdot 0.01 + 1 \cdot 0.99 = 0$$

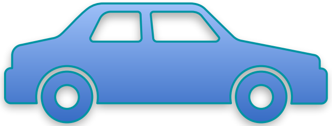
$$Var(X_1) = (-1)^2 \cdot 0.99 + (99)^2 \cdot 0.01 = 99$$





Ticket: \$1
Jackpot: \$100

Lottery



Cost: \$1
Crash Reparation: \$100

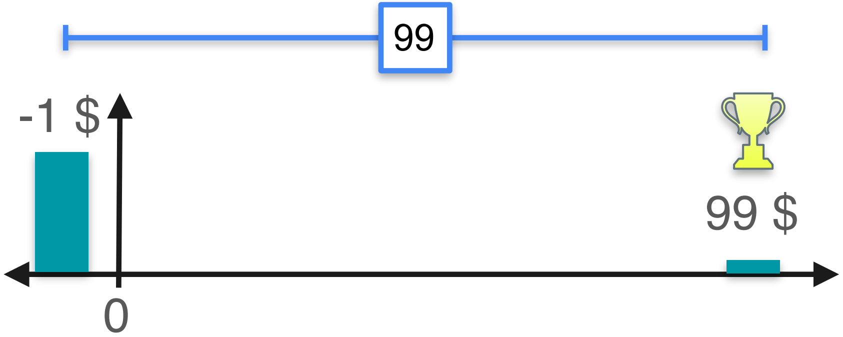
Car insurance

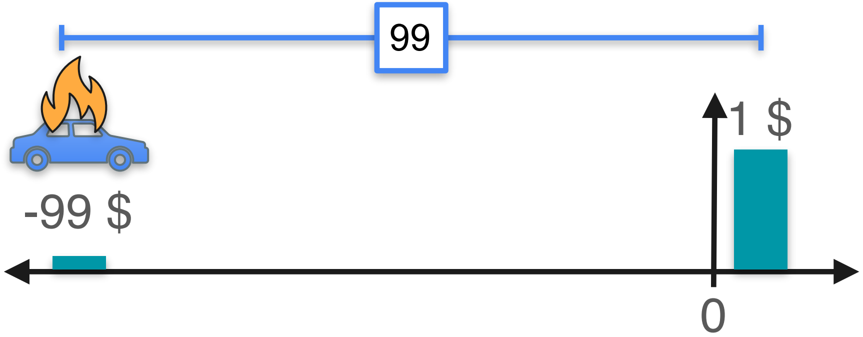
$$\mathbb{E}[X_1] = -1 \cdot 0.99 + 99 \cdot 0.01 = 0$$

$$\mathbb{E}[X_2] = -99 \cdot 0.01 + 1 \cdot 0.99 = 0$$

$$Var(X_1) = (-1)^2 \cdot 0.99 + (99)^2 \cdot 0.01 = 99$$

$$Var(X_2) = (-99)^2 \cdot 0.01 + (1)^2 \cdot 0.99 = 99$$



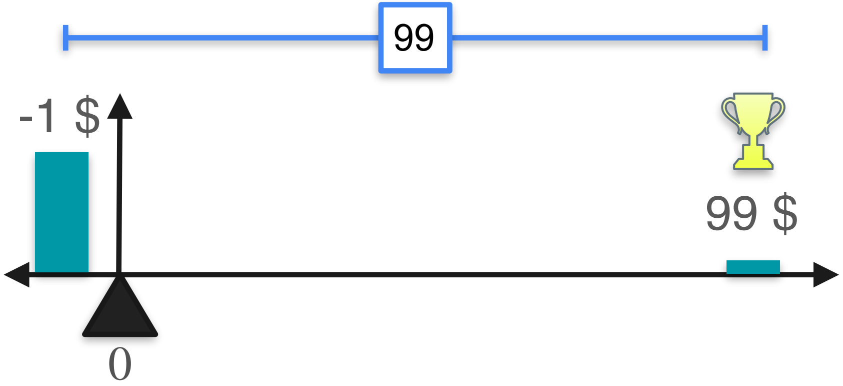


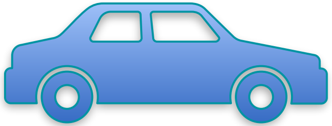
Ticket: $1
Jackpot: $100

Lottery

$$\mathbb{E}[X_1] = 0$$

$$Var(X_1) = 99$$



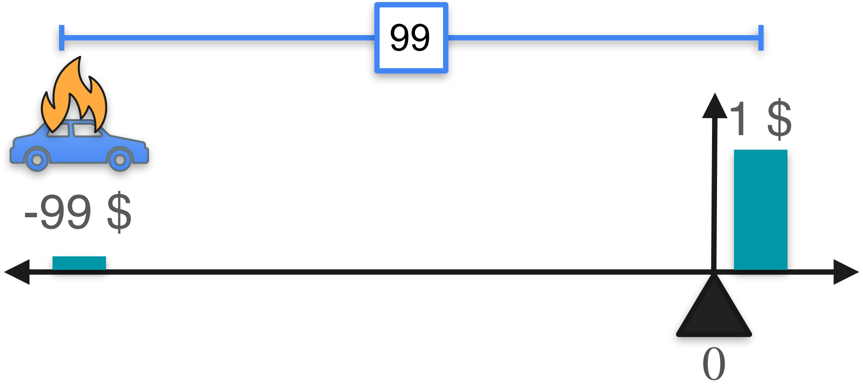


Cost: $1
Crash Reparation: $100

Car insurance

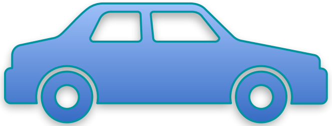
$$\mathbb{E}[X_2] = 0$$

$$Var(X_2) = 99$$



Ticket: $1
Jackpot: $100

Lottery



Cost: $1
Crash Reparation: $100

Car insurance

$$\mathbb{E}[X_1] = 0$$

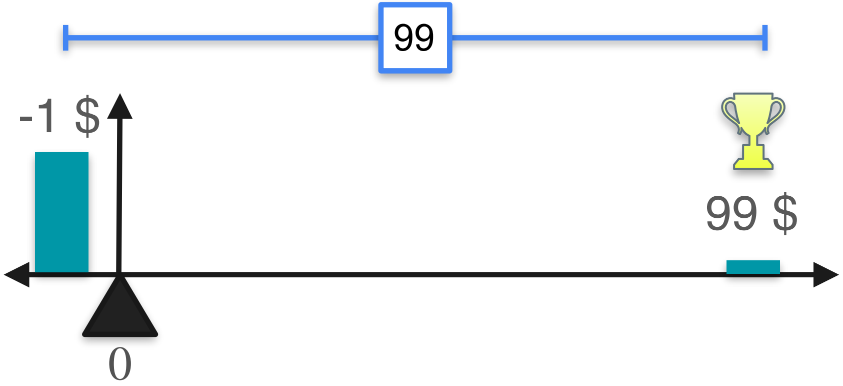
$$Var(X_1) = 99$$

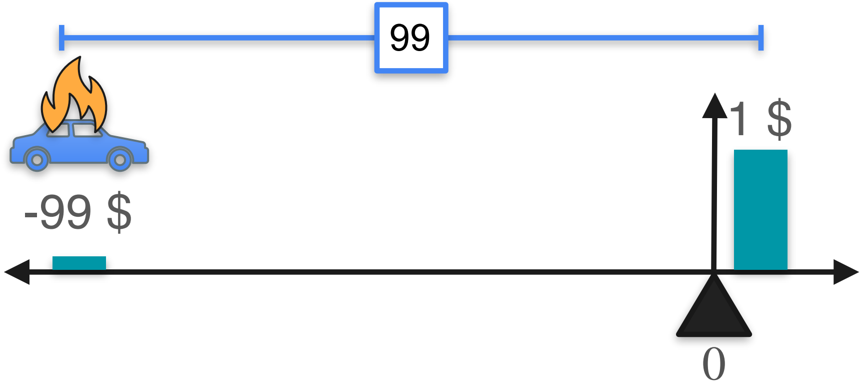
Same expectation
Same variance

How to tell them apart?

$$\mathbb{E}[X_2] = 0$$

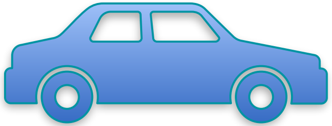
$$Var(X_2) = 99$$





Ticket: $1
Jackpot: $99

Lottery



Cost: $1
Crash Reparation: $99

Car insurance

$$\mathbb{E}[X_1] = 0$$

$$Var(X_1) = 99$$

Same expectation
Same variance

How to tell them apart?

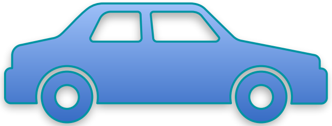
$$\mathbb{E}[X_2] = 0$$

$$Var(X_2) = 99$$

Ticket: $1

Jackpot: $99

Lottery



Cost: $1

Crash Reparation: $99

Car insurance

$$\mathbb{E}[X_1] = 0$$

$$Var(X_1) = 99$$

$$\mathbb{E}[X_1^2] - \mathbb{E}[X_1]^2 = 99$$

Same expectation
Same variance

How to tell them apart?

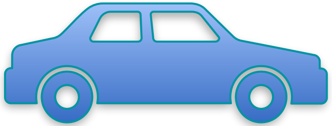
$$\mathbb{E}[X_2] = 0$$

$$Var(X_2) = 99$$

$$\mathbb{E}[X_2^2] - \mathbb{E}[X_2]^2 = 99$$

Ticket: $1
Jackpot: $99

Lottery



Cost: $1
Crash Reparation: $99

Car insurance

$$\mathbb{E}[X_1] = 0$$

$$Var(X_1) = 99$$

$$\mathbb{E}[X_1^2] - \cancel{\mathbb{E}[X_1]^2} = 99$$

Same expectation
Same variance

How to tell them apart?

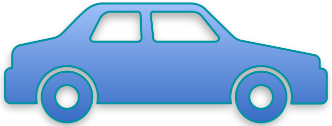
$$\mathbb{E}[X_2] = 0$$

$$Var(X_2) = 99$$

$$\mathbb{E}[X_2^2] - \cancel{\mathbb{E}[X_2]^2} = 99$$

Ticket: $1
Jackpot: $99

Lottery



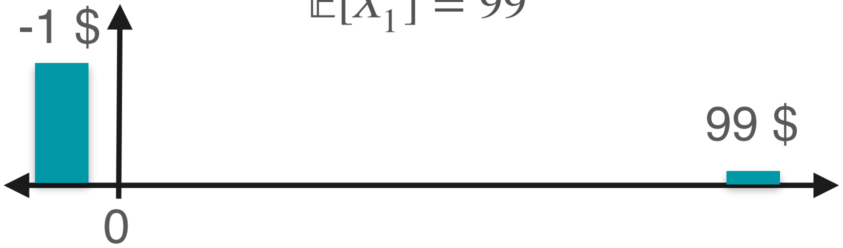
Cost: $1
Crash Reparation: $99

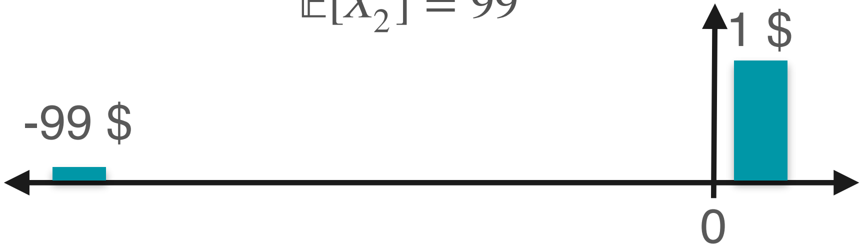
Car insurance

Same expectation
Same variance

How to tell them apart?

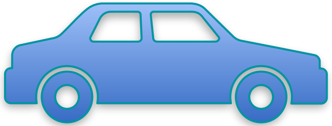
Lottery	Car insurance
$\mathbb{E}[X_1] = 0$	$\mathbb{E}[X_2] = 0$
$Var(X_1) = 99$	$Var(X_2) = 99$
$\mathbb{E}[X_1^2] - \cancel{\mathbb{E}[X_1]^2} = 99$	$\mathbb{E}[X_2^2] - \cancel{\mathbb{E}[X_2]^2} = 99$
$\mathbb{E}[X_1^2] = 99$	$\mathbb{E}[X_2^2] = 99$





Ticket: $1
Jackpot: $99

Lottery



Cost: $1
Crash Reparation: $99

Car insurance

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = 99$$

Same expectation
Same variance

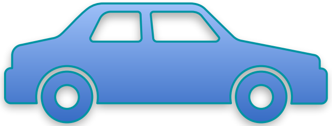
How to tell them apart?

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = 99$$

Ticket: \$1
Jackpot: \$99

Lottery



Cost: \$1
Crash Reparation: \$99

Car insurance

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = 99$$

$$\mathbb{E}[X_1^3] = (-1)^3 \cdot 0.99 + (99)^3 \cdot 0.01$$
$$= 9702$$

Same expectation
Same variance

How to tell them apart?

$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = 99$$

Ticket: \$1
Jackpot: \$99

Lottery

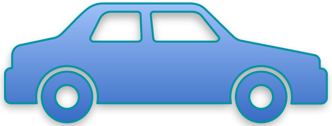
$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = 99$$

$$\mathbb{E}[X_1^3] = (-1)^3 \cdot 0.99 + (99)^3 \cdot 0.01 = 9702$$

Same expectation
Same variance

How to tell them apart?



Cost: \$1
Crash Reparation: \$99

Car insurance

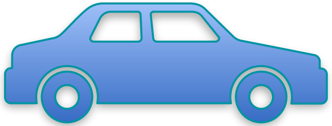
$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[X_2^2] = 99$$

$$\mathbb{E}[X_2^3] = (1)^3 \cdot 0.99 + (-99)^3 \cdot 0.01 = -9702$$

Ticket: \$1
Jackpot: \$99

Lottery



Cost: \$1
Crash Reparation: \$99

Car insurance

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = 99$$

$$\mathbb{E}[X_1^3] = (-1)^3 \cdot 0.99 + (99)^3 \cdot 0.01$$
$$= 9702$$

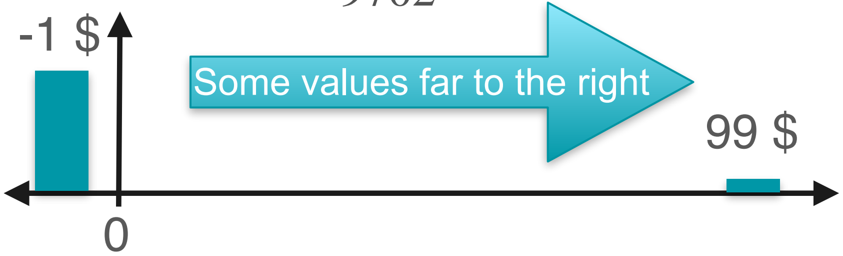
Same expectation
Same variance

How to tell them apart?

$$\mathbb{E}[X_2] = 0$$

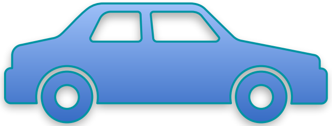
$$\mathbb{E}[X_2^2] = 99$$

$$\mathbb{E}[X_2^3] = (1)^3 \cdot 0.99 + (-99)^3 \cdot 0.01$$
$$= -9702$$



Ticket: \$1
Jackpot: \$99

Lottery



Cost: \$1
Crash Reparation: \$99

Car insurance

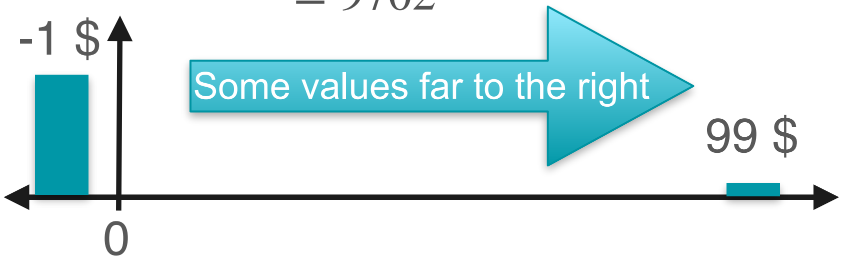
Same expectation
Same variance

How to tell them apart?

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_1^2] = 99$$

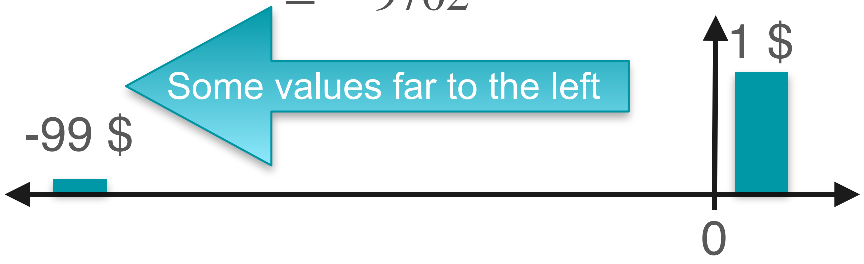
$$\mathbb{E}[X_1^3] = (-1)^3 \cdot 0.99 + (99)^3 \cdot 0.01$$
$$= 9702$$



$$\mathbb{E}[X_2] = 0$$

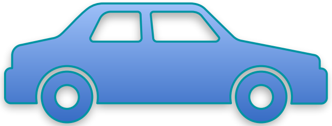
$$\mathbb{E}[X_2^2] = 99$$

$$\mathbb{E}[X_2^3] = (1)^3 \cdot 0.99 + (-99)^3 \cdot 0.01$$
$$= -9702$$



Ticket: \$1
Jackpot: \$99

Lottery



Cost: \$1
Crash Reparation: \$99

Car insurance

Same expectation
Same variance

How to tell them apart?

$\mathbb{E}[X_1] = 0$

$\mathbb{E}[X_1^2] = 99$

$\mathbb{E}[X_1^3]$ = Large positive value

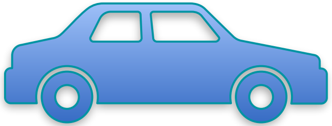
$\mathbb{E}[X_2] = 0$

$\mathbb{E}[X_1^2] = 99$

$\mathbb{E}[X_1^3]$ = Large negative value

Ticket: \$1
Jackpot: \$99

Lottery



Cost: \$1
Crash Reparation: \$99

Car insurance

$\mathbb{E}[X_1] = 0$

$\mathbb{E}[X_1^2] = 99$

$\mathbb{E}[X_1^3]$ = Large positive value

Same expectation
Same variance

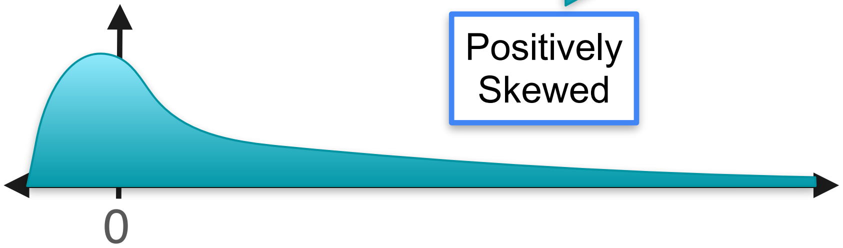
How to tell them apart?

$\mathbb{E}[X_2] = 0$

$\mathbb{E}[X_1^2] = 99$

$\mathbb{E}[X_1^3]$ = Large negative value

Positively Skewed



Ticket: \$1
Jackpot: \$99

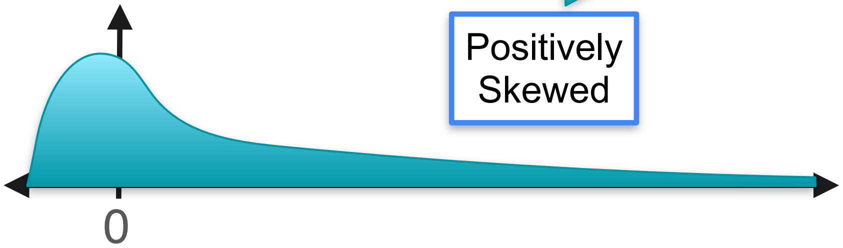
Lottery

$\mathbb{E}[X_1] = 0$

$\mathbb{E}[X_1^2] = 99$

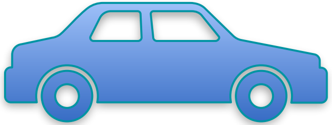
$\mathbb{E}[X_1^3]$ = Large positive value

Positively Skewed



Same expectation
Same variance

How to tell them apart?



Cost: \$1
Crash Reparation: \$99

Car insurance

$\mathbb{E}[X_2] = 0$

$\mathbb{E}[X_1^2] = 99$

$\mathbb{E}[X_1^3]$ = Large negative value

Negatively Skewed

Skewness

$$\mathbb{E}[X^3]$$

Skewness

$$\mathbb{E}[X^3]$$

Almost…

Skewness

$$\mathbb{E}[X^3]$$

Almost…

Need to standardize…

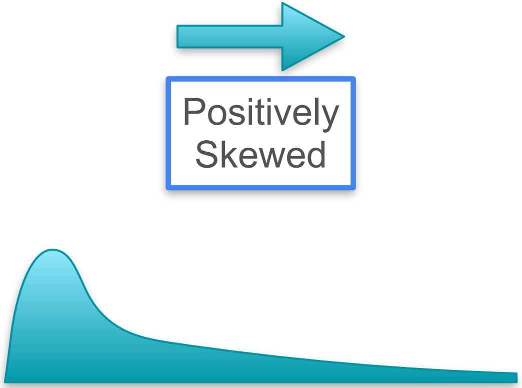
Skewness

Skewness

$$\text{Skewness} = \mathbb{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right]$$

Skewness

Skewness



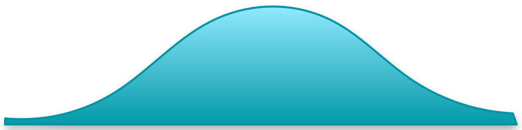
$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] > 0$$

Skewness

Positively Skewed

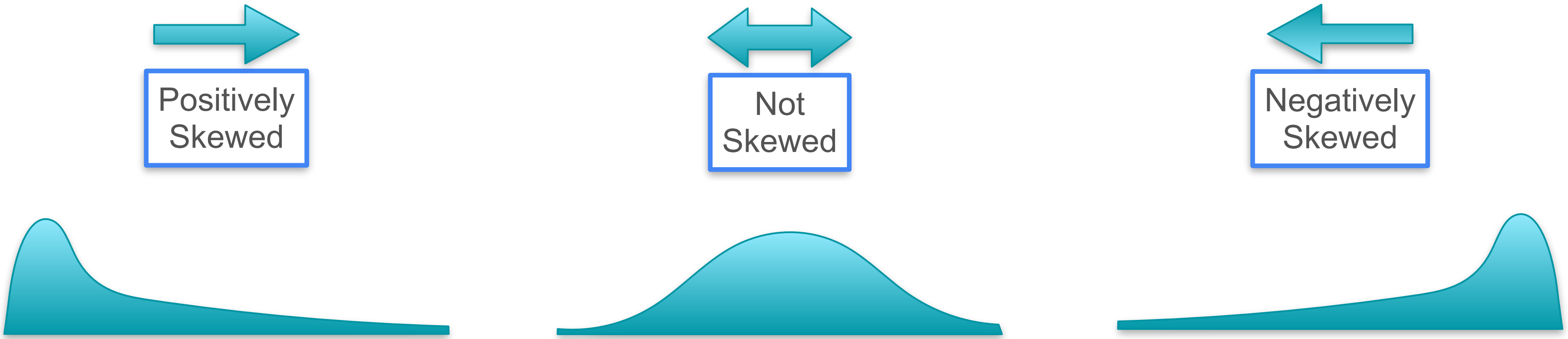
$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] > 0$$

Not Skewed



$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0$$

Skewness



$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] > 0 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] < 0$$

Video 7c:

- Kurtosis

Kurtosis: Example

Game 1

Game 2

Kurtosis: Example

Game 1

Game 2

probability $\frac{1}{2}$: You win 1 dollar

Kurtosis: Example

Game 1

Game 2

probability $\frac{1}{2}$: You win 1 dollar

probability $\frac{1}{2}$: You lose 1 dollar

Kurtosis: Example

Game 1

probability $\frac{1}{2}$**:** You win 1 dollar

probability $\frac{1}{2}$**:** You lose 1 dollar

Game 2

probability $\frac{100}{202}$**:** You win 10 cents

Kurtosis: Example

Game 1

probability $\frac{1}{2}$**:** You win 1 dollar

probability $\frac{1}{2}$**:** You lose 1 dollar

Game 2

probability $\frac{100}{202}$**:** You win 10 cents

probability $\frac{100}{202}$**:** You lose 10 cents

Kurtosis: Example

Game 1

probability $\frac{1}{2}$**:** You win 1 dollar

probability $\frac{1}{2}$**:** You lose 1 dollar

Game 2

probability $\frac{100}{202}$**:** You win 10 cents

probability $\frac{100}{202}$**:** You lose 10 cents

probability $\frac{1}{202}$**:** You win 10 dollars

Kurtosis: Example

Game 1

probability $\frac{1}{2}$: You win 1 dollar

probability $\frac{1}{2}$: You lose 1 dollar

Game 2

probability $\frac{100}{202}$: You win 10 cents

probability $\frac{100}{202}$: You lose 10 cents

probability $\frac{1}{202}$: You win 10 dollars

probability $\frac{1}{202}$: You lose 10 dollars

Kurtosis: Example

Game 1

Which one is riskier?

probability $\frac{1}{2}$: You win 1 dollar

probability $\frac{1}{2}$: You lose 1 dollar

Game 2

probability $\frac{100}{202}$: You win 10 cents

probability $\frac{100}{202}$: You lose 10 cents

probability $\frac{1}{202}$: You win 10 dollars

probability $\frac{1}{202}$: You lose 10 dollars

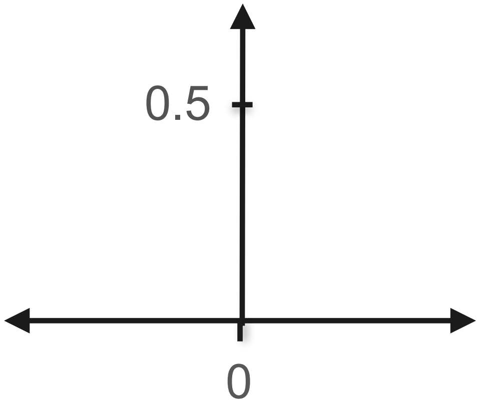
Kurtosis: Example

Game 1

Game 2

Kurtosis: Example

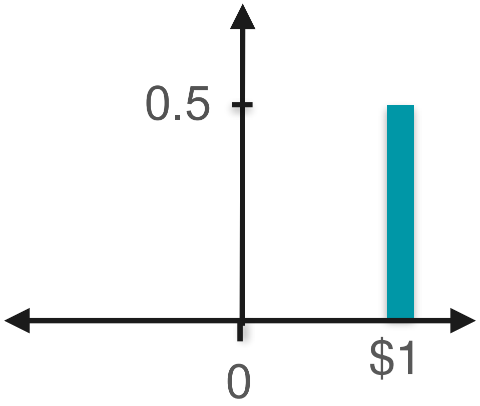
Game 1



Game 2

Kurtosis: Example

Game 1



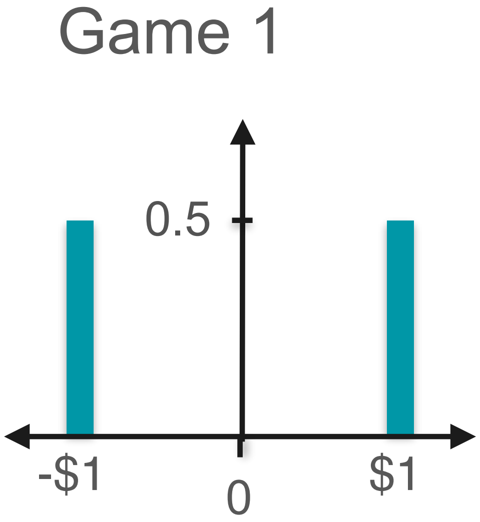
Game 2

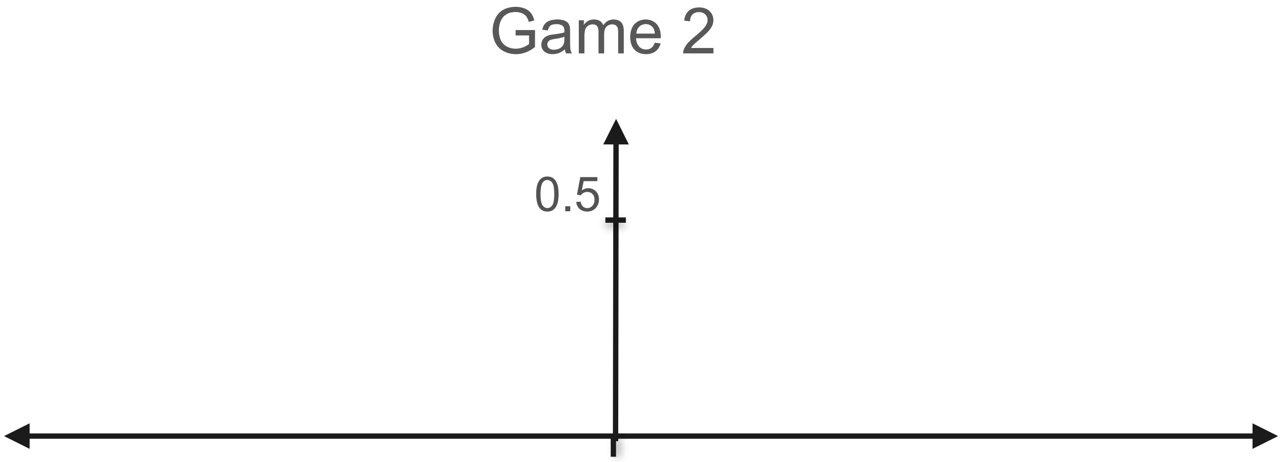
Kurtosis: Example

Game 1

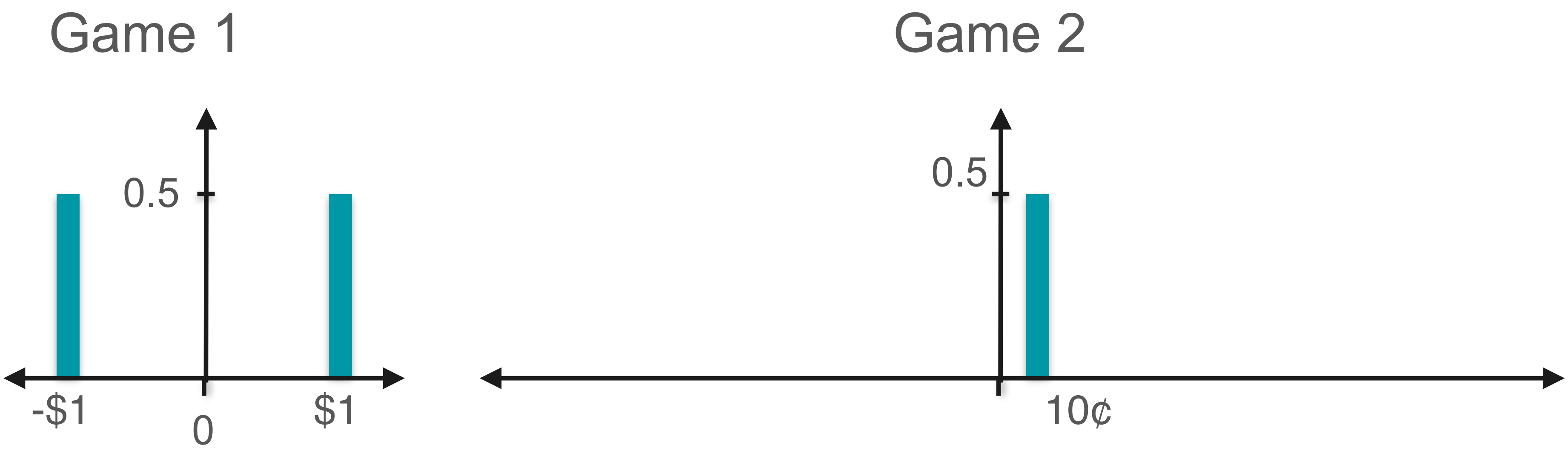
Game 2

Kurtosis: Example

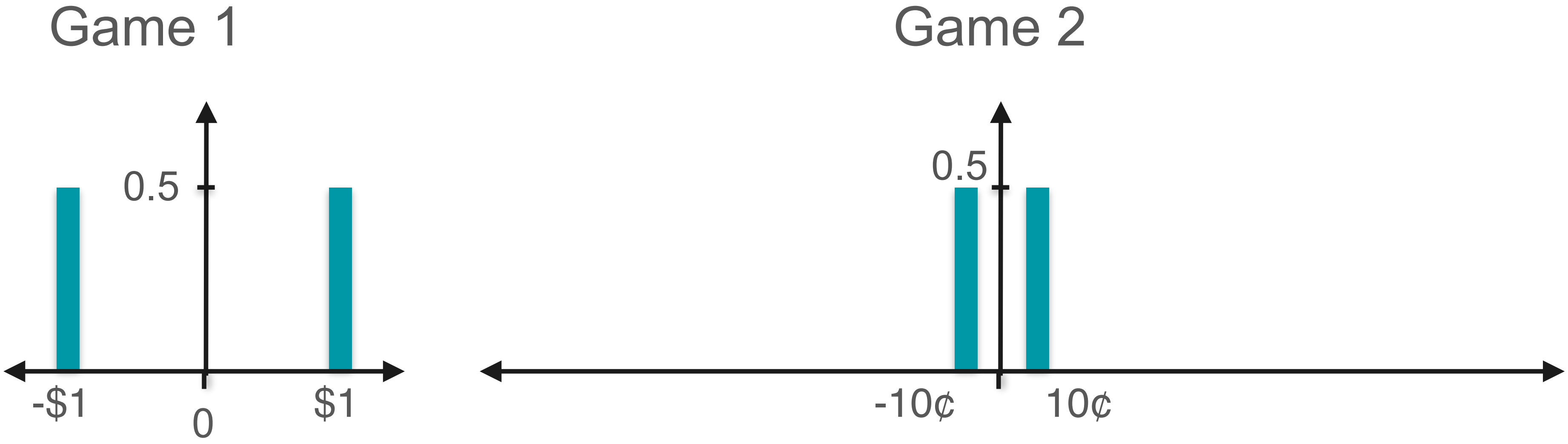




Kurtosis: Example



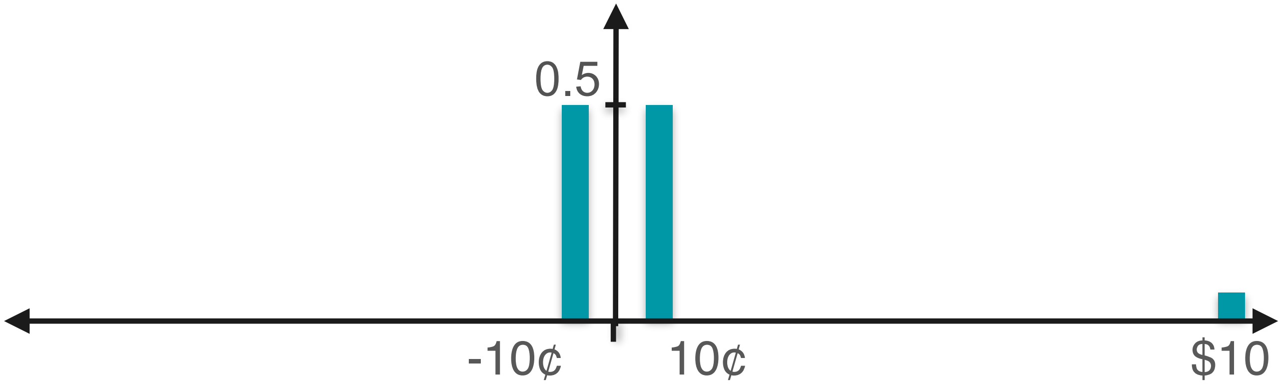
Kurtosis: Example



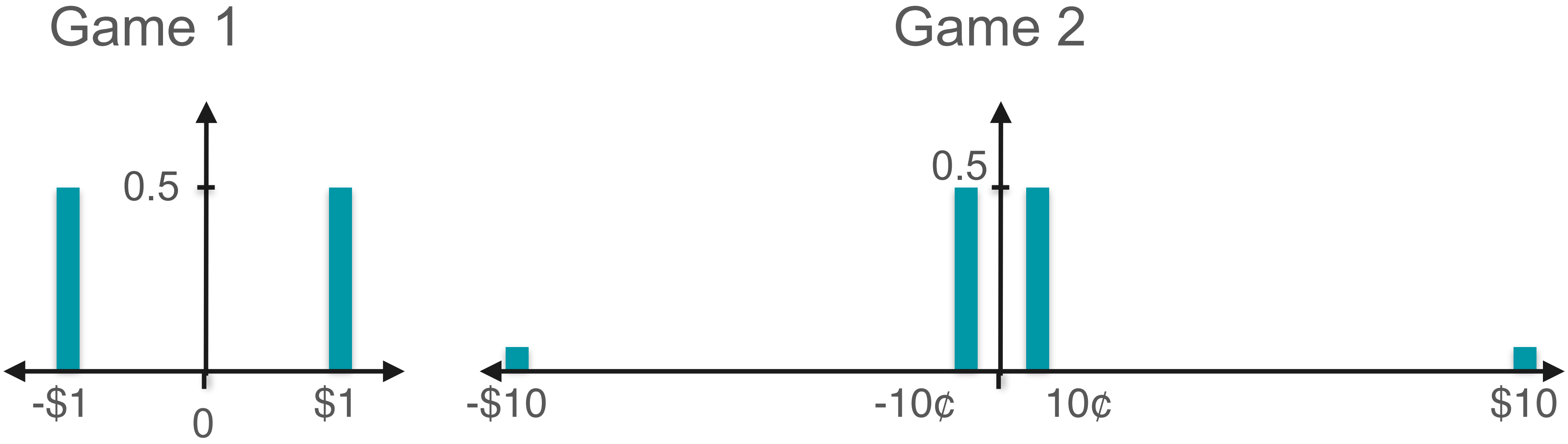
Kurtosis: Example

Game 1

Game 2



Kurtosis: Example

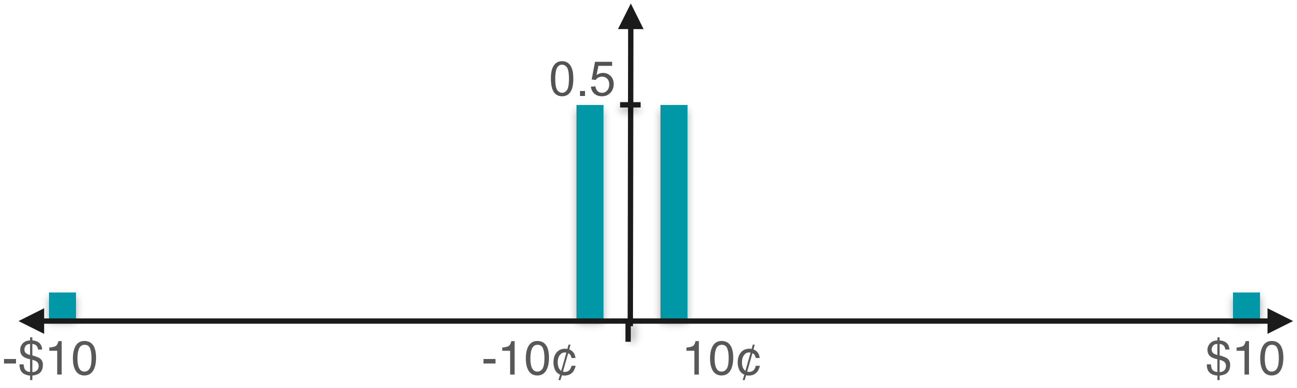


Kurtosis: Example

Expected value?

Game 1

Game 2



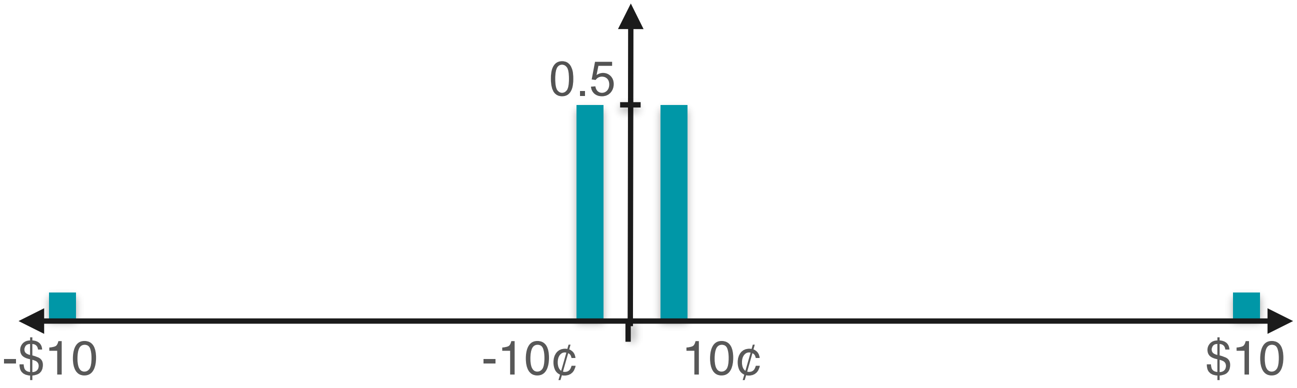
Kurtosis: Example

Game 1

Expected value?

Standard deviation?

Game 2



Kurtosis: Example

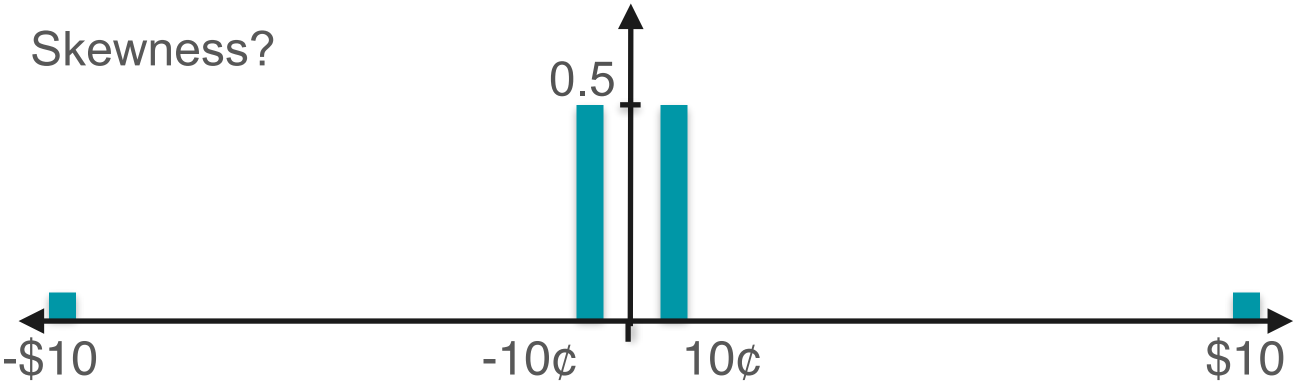
Game 1

Expected value?

Standard deviation?

Skewness?

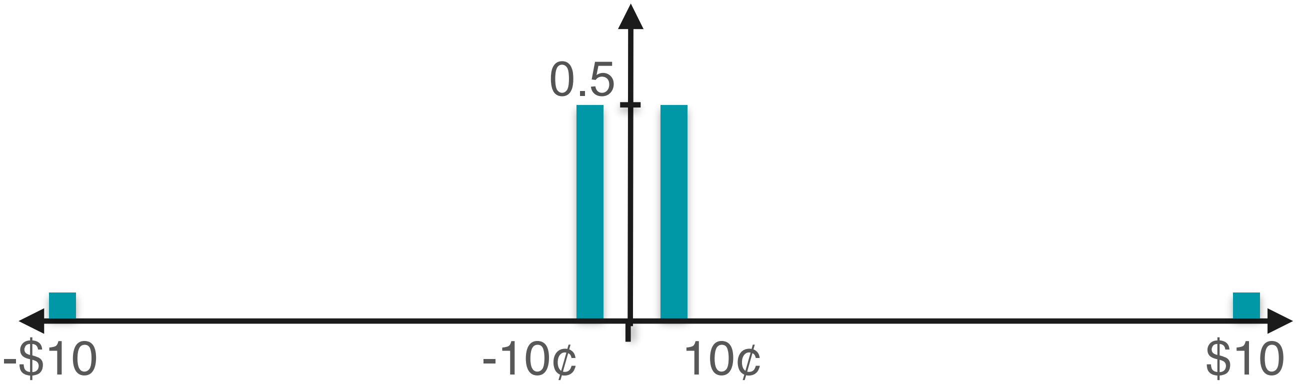
Game 2



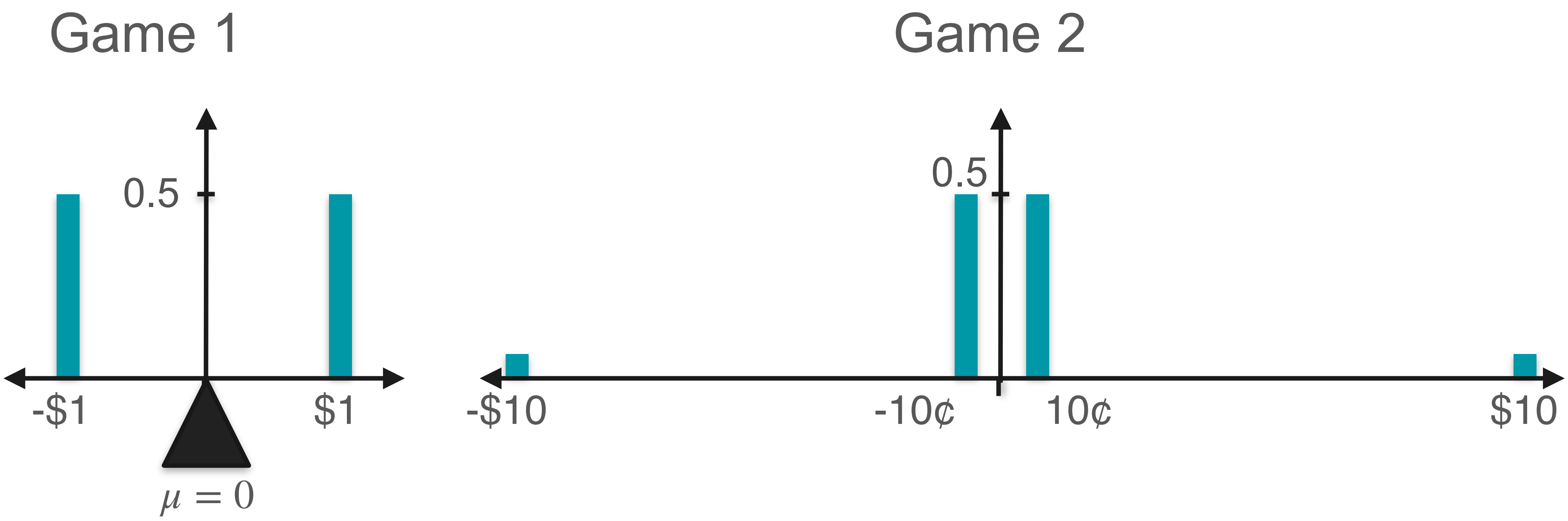
Kurtosis: Example Expected Value

Game 1

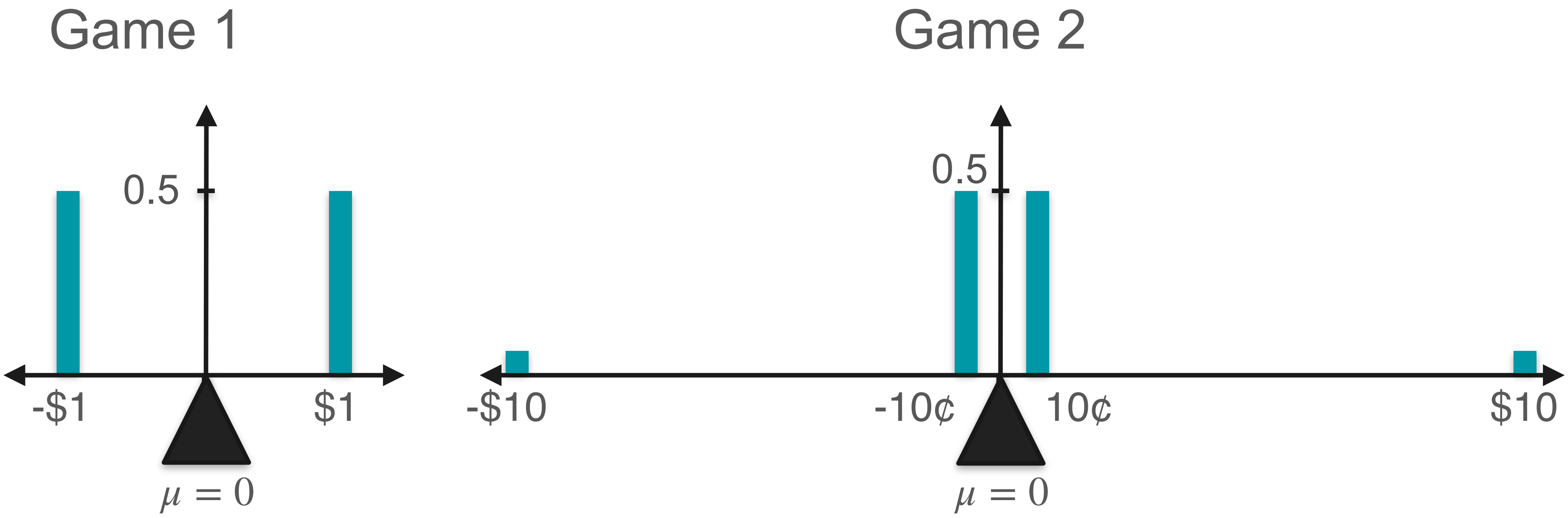
Game 2



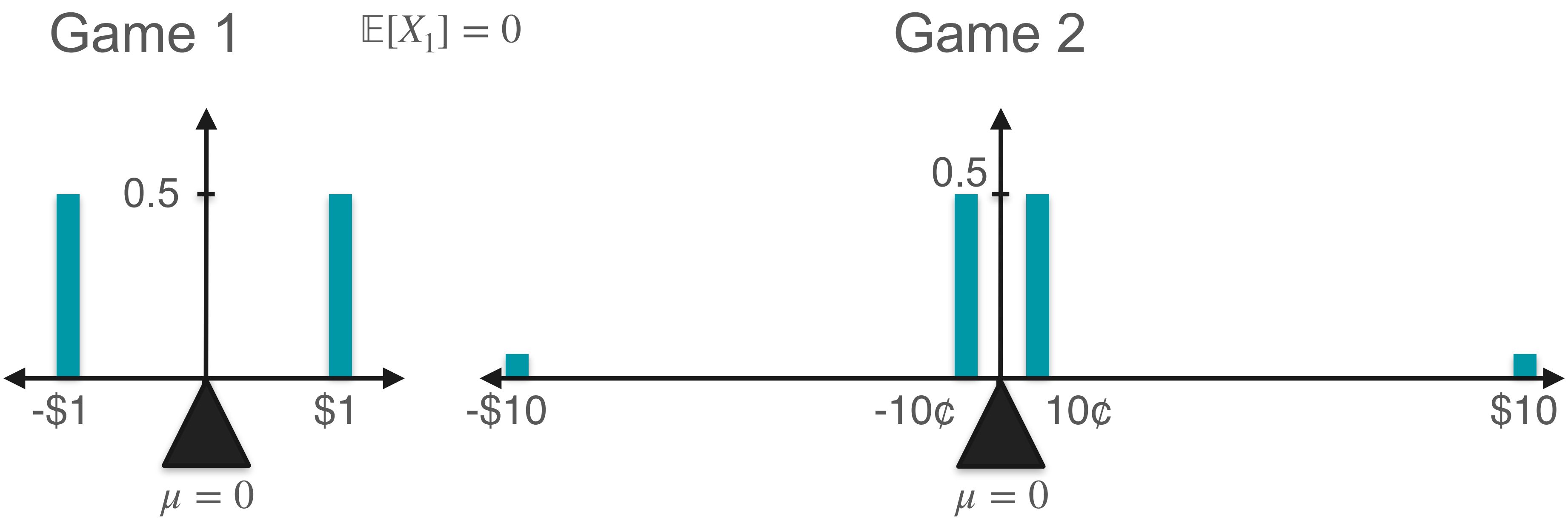
Kurtosis: Example Expected Value



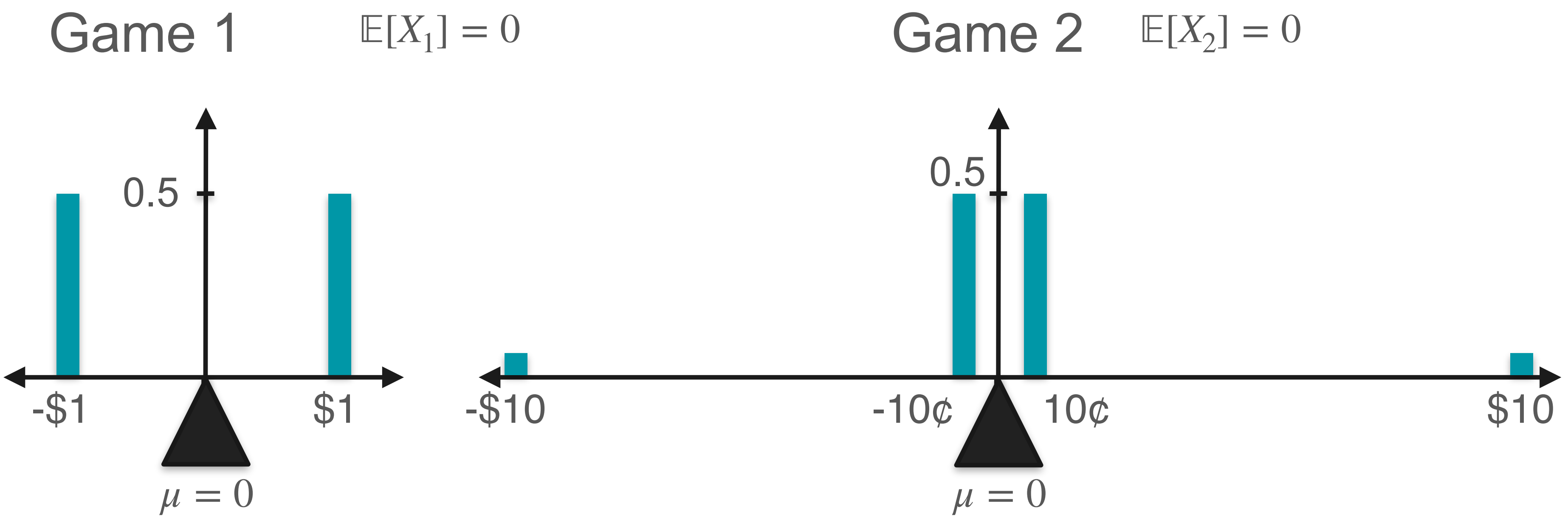
Kurtosis: Example Expected Value



Kurtosis: Example Expected Value

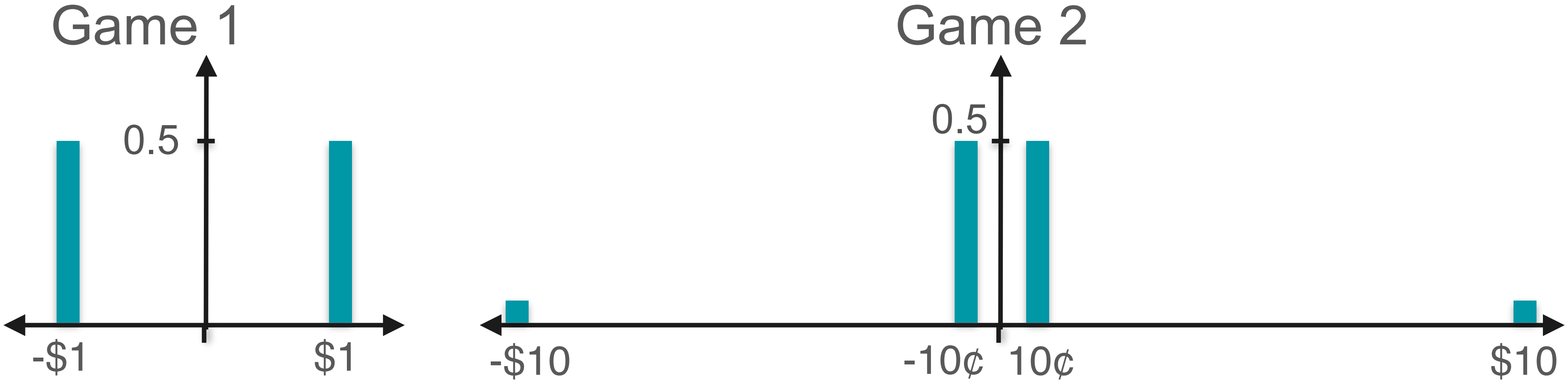


Kurtosis: Example Expected Value

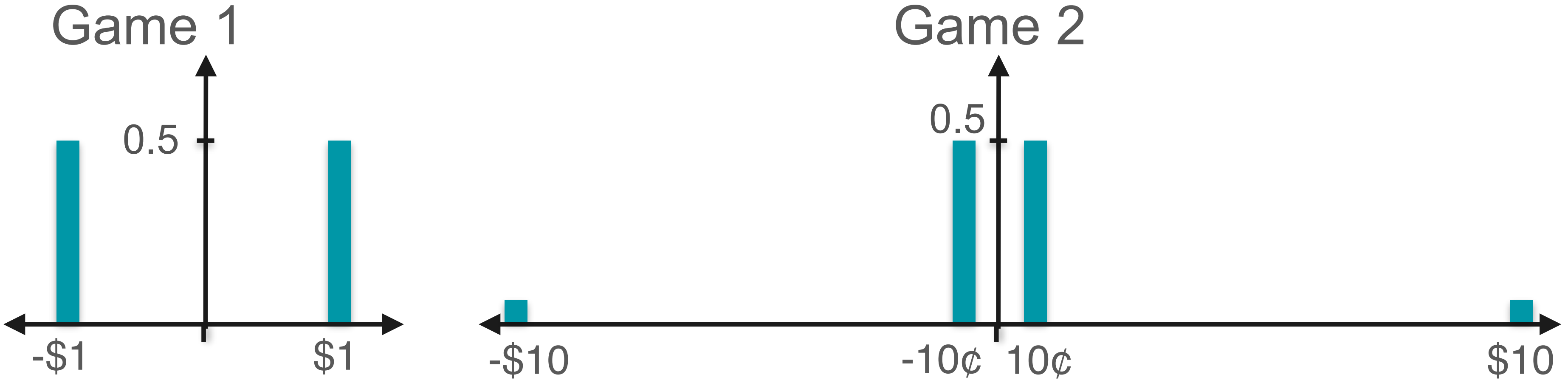


DeepLearning.AI

Kurtosis: Example Variance

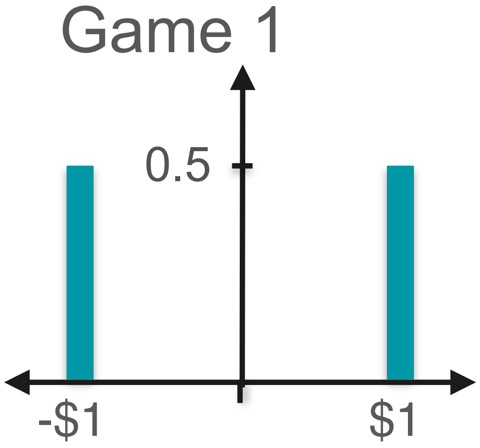


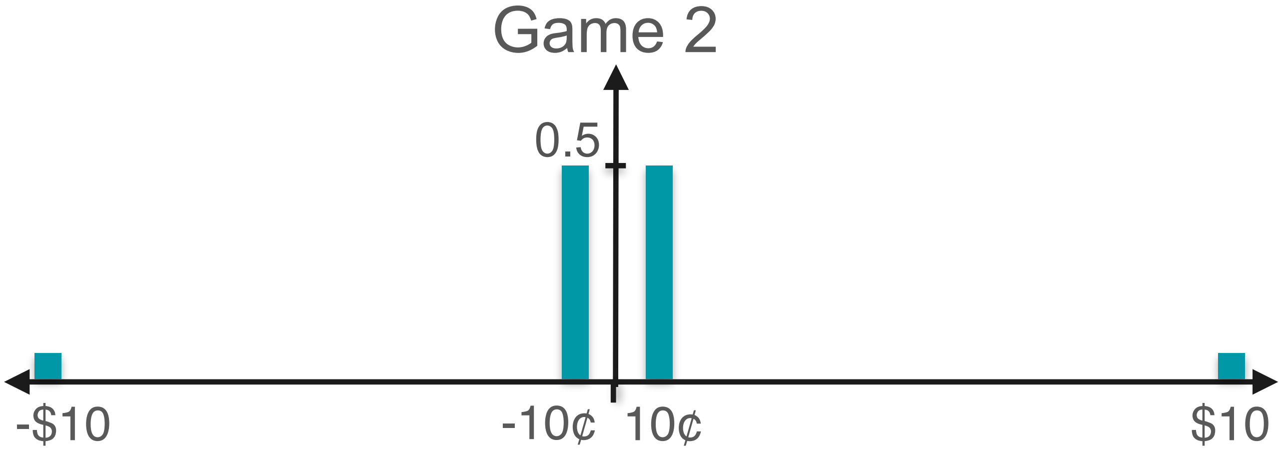
Kurtosis: Example Variance



$\mathbb{E}[X_1^2]$

Kurtosis: Example Variance

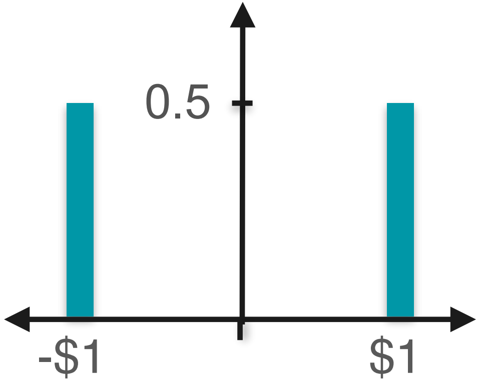




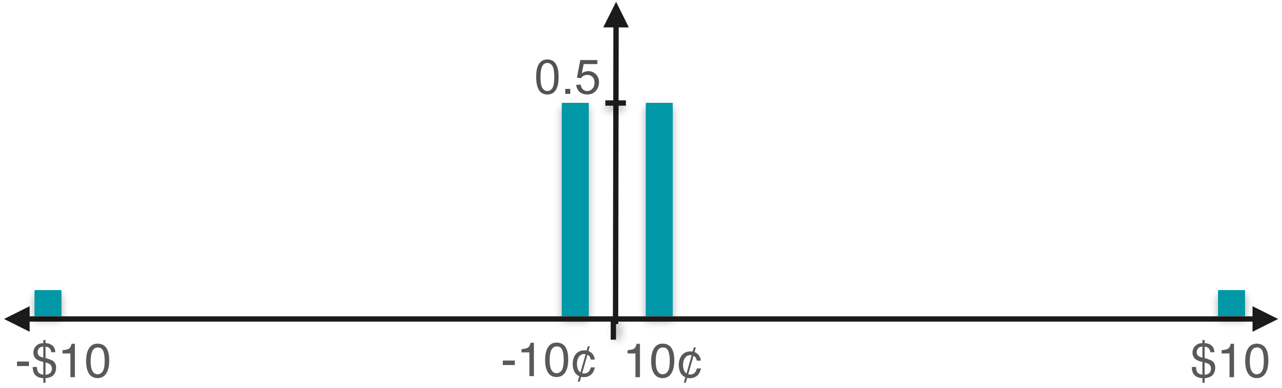
$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$

Kurtosis: Example Variance

Game 1



Game 2

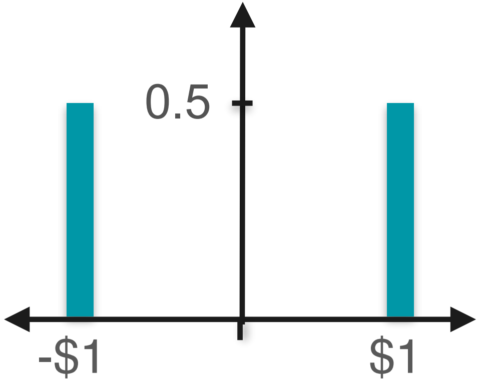


$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$

$$= 1$$

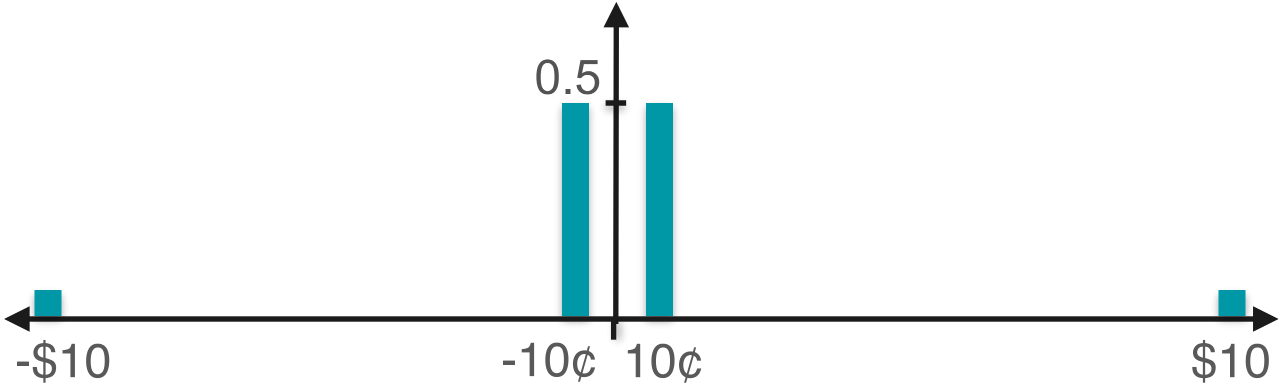
Kurtosis: Example Variance

Game 1



$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$
$$= 1$$

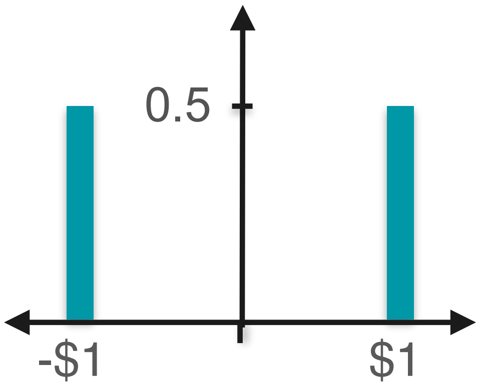
Game 2



$$\mathbb{E}[X_2^2]$$

Kurtosis: Example Variance

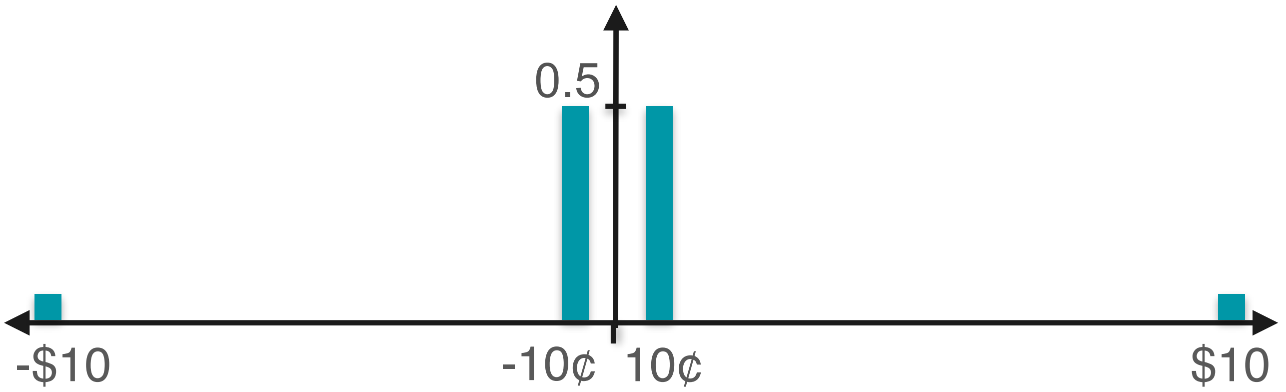
Game 1



$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$

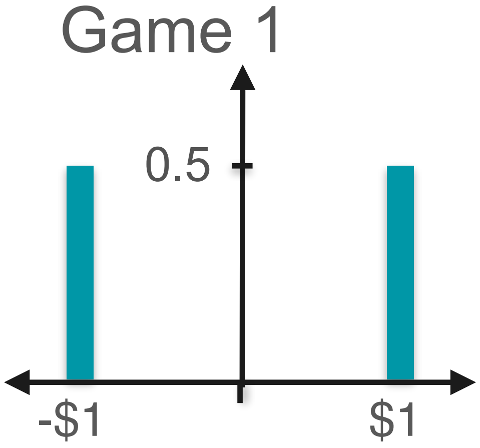
$$= 1$$

Game 2

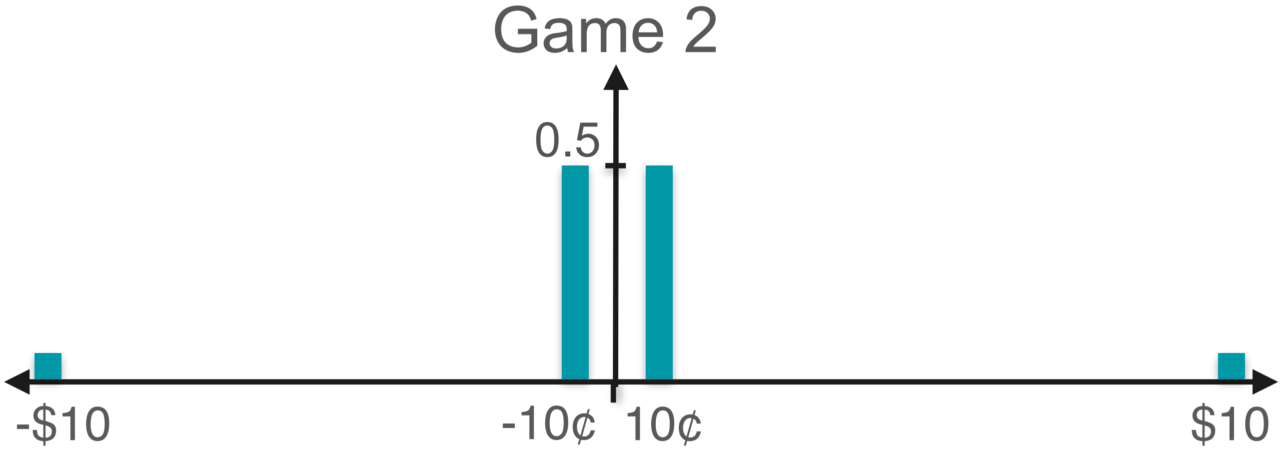


$$\mathbb{E}[X_2^2] = \frac{100}{202}(-0.1)^2 + \frac{100}{202}(0.1)^2 + \frac{1}{202}(-10)^2 + \frac{1}{202}(10)^2$$

Kurtosis: Example Variance

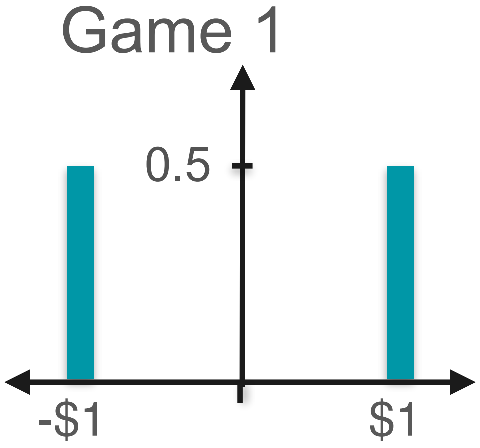


$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$
$$= 1$$

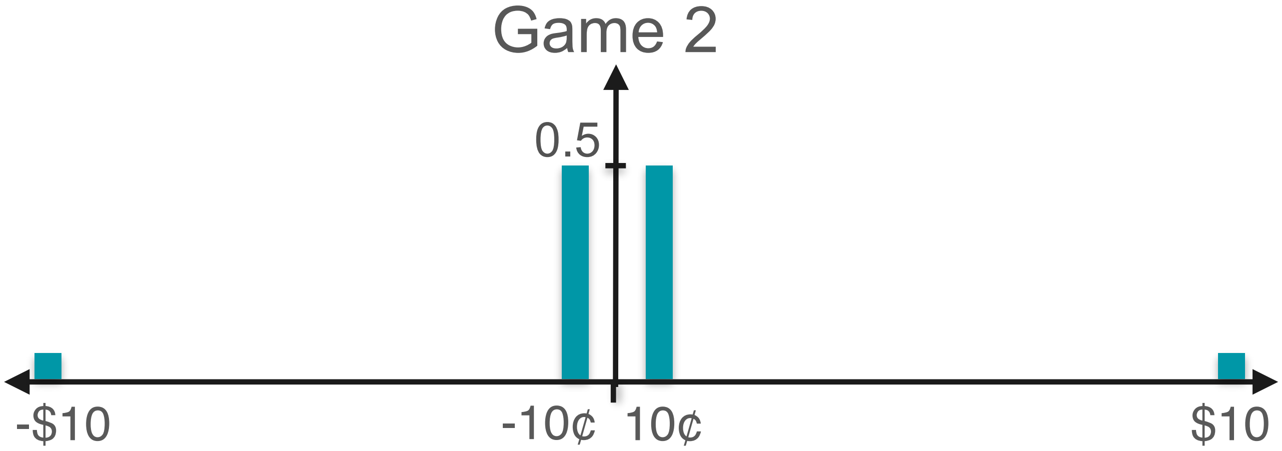


$$\mathbb{E}[X_2^2] = \frac{100}{202}(-0.1)^2 + \frac{100}{202}(0.1)^2 + \frac{1}{202}(-10)^2 + \frac{1}{202}(10)^2$$
$$= \frac{100}{202}\frac{1}{100} + \frac{100}{202}\frac{1}{100} + \frac{1}{202}\cdot 100 + \frac{1}{202}\cdot 100$$

Kurtosis: Example Variance



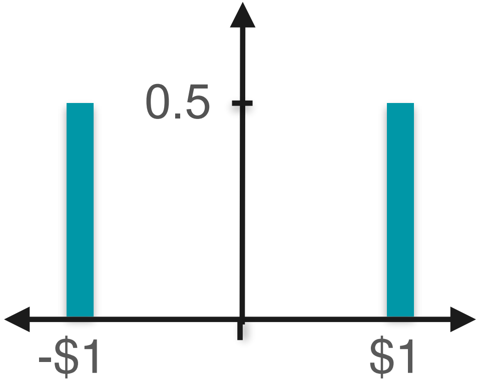
$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$
$$= 1$$



$$\mathbb{E}[X_2^2] = \frac{100}{202}(-0.1)^2 + \frac{100}{202}(0.1)^2 + \frac{1}{202}(-10)^2 + \frac{1}{202}(10)^2$$
$$= \frac{100}{202}\frac{1}{100} + \frac{100}{202}\frac{1}{100} + \frac{1}{202}\cdot 100 + \frac{1}{202}\cdot 100$$
$$= \frac{1}{202} + \frac{1}{202} + \frac{100}{202} + \frac{100}{202}$$

Kurtosis: Example Variance

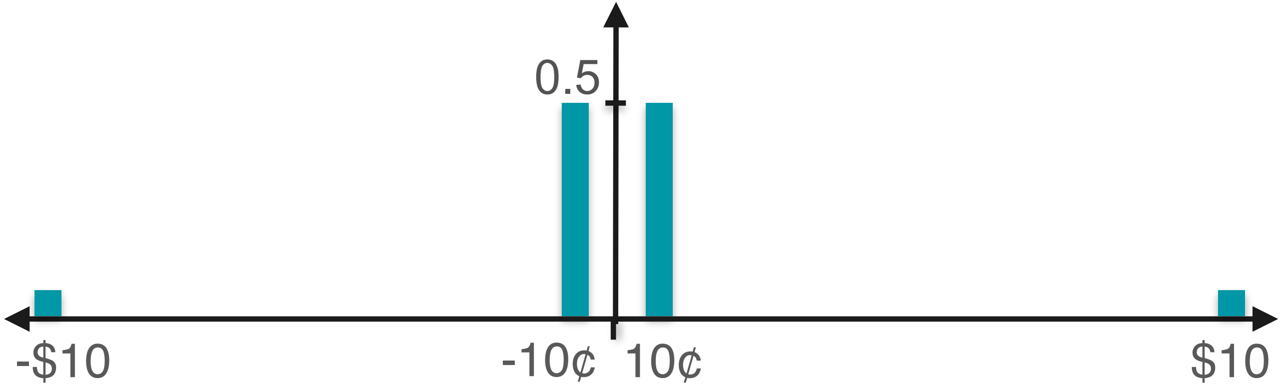
Game 1



$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$

$$= 1$$

Game 2



$$\mathbb{E}[X_2^2] = \frac{100}{202}(-0.1)^2 + \frac{100}{202}(0.1)^2 + \frac{1}{202}(-10)^2 + \frac{1}{202}(10)^2$$

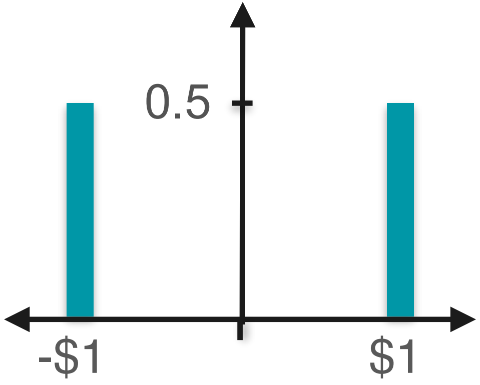
$$= \frac{100}{202}\frac{1}{100} + \frac{100}{202}\frac{1}{100} + \frac{1}{202}\cdot 100 + \frac{1}{202}\cdot 100$$

$$= \frac{1}{202} + \frac{1}{202} + \frac{100}{202} + \frac{100}{202} = 1$$

Kurtosis: Example Variance

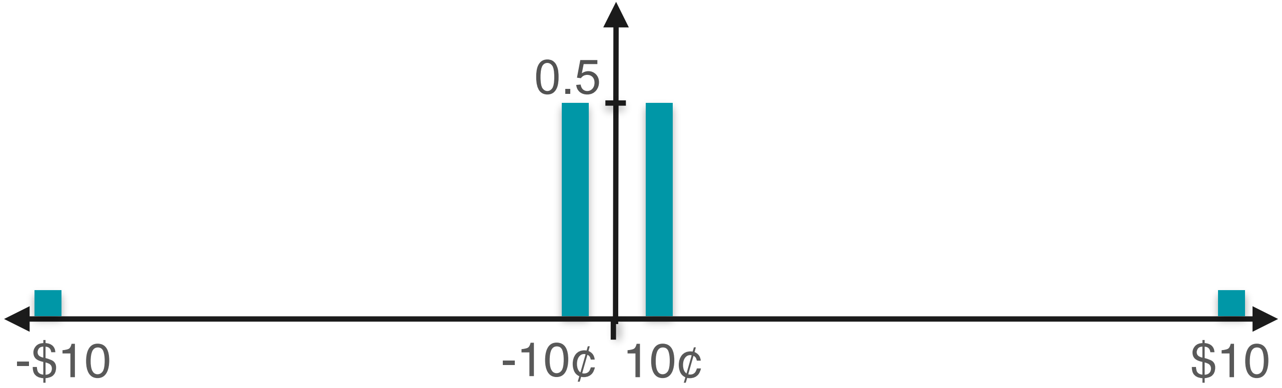
Game 1

$Var(X_1) = 1$



$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$
$$= 1$$

Game 2

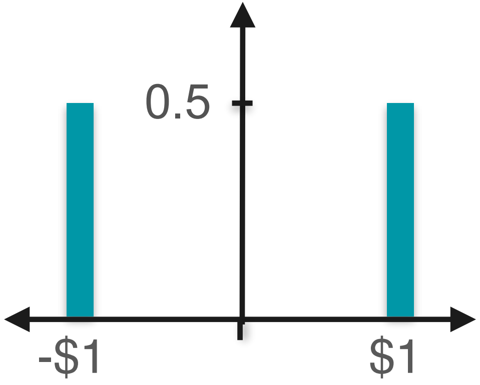


$$\mathbb{E}[X_2^2] = \frac{100}{202}(-0.1)^2 + \frac{100}{202}(0.1)^2 + \frac{1}{202}(-10)^2 + \frac{1}{202}(10)^2$$
$$= \frac{100}{202}\frac{1}{100} + \frac{100}{202}\frac{1}{100} + \frac{1}{202} \cdot 100 + \frac{1}{202} \cdot 100$$
$$= \frac{1}{202} + \frac{1}{202} + \frac{100}{202} + \frac{100}{202} = 1$$

Kurtosis: Example Variance

Game 1

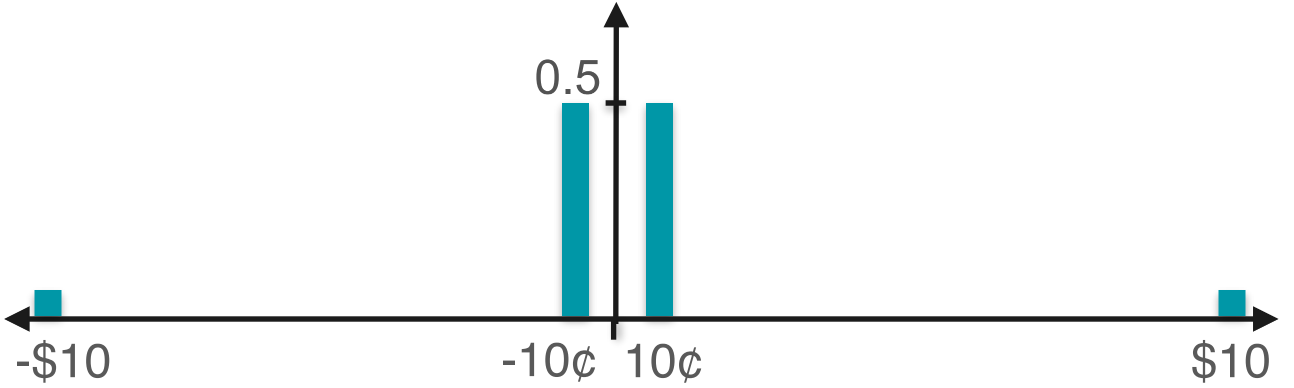
$Var(X_1) = 1$



$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$
$$= 1$$

Game 2

$Var(X_2) = 1$



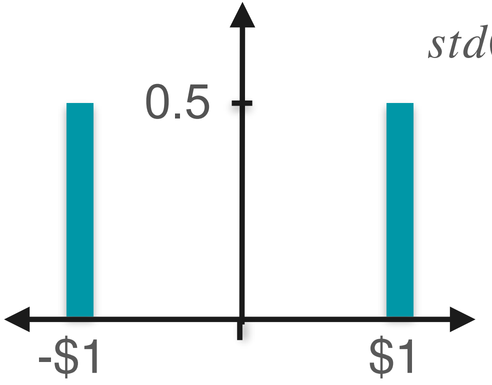
$$\mathbb{E}[X_2^2] = \frac{100}{202}(-0.1)^2 + \frac{100}{202}(0.1)^2 + \frac{1}{202}(-10)^2 + \frac{1}{202}(10)^2$$
$$= \frac{100}{202}\frac{1}{100} + \frac{100}{202}\frac{1}{100} + \frac{1}{202}\cdot 100 + \frac{1}{202}\cdot 100$$
$$= \frac{1}{202} + \frac{1}{202} + \frac{100}{202} + \frac{100}{202} = 1$$

Kurtosis: Example Variance

Game 1

$Var(X_1) = 1$

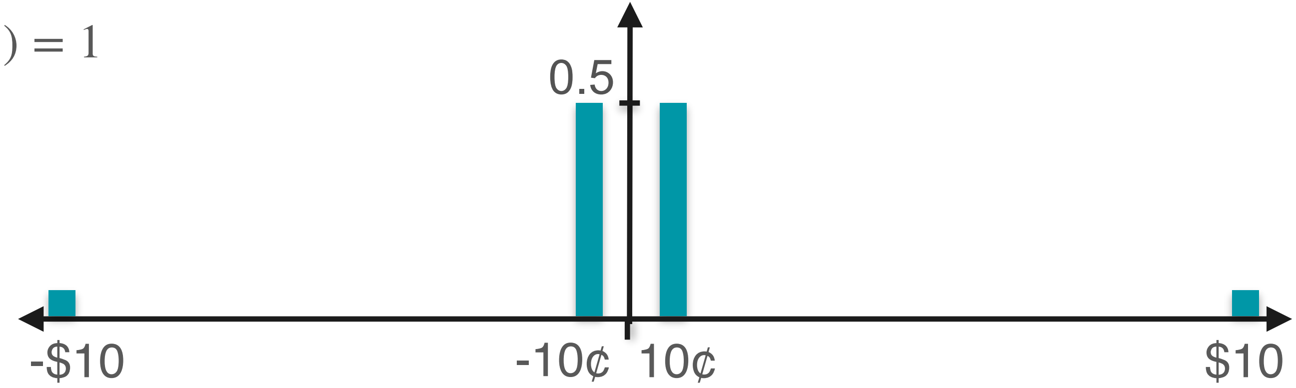
$std(X_1) = 1$



$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1$$

Game 2

$Var(X_2) = 1$



$$\mathbb{E}[X_2^2] = \frac{100}{202}(-0.1)^2 + \frac{100}{202}(0.1)^2 + \frac{1}{202}(-10)^2 + \frac{1}{202}(10)^2$$

$$= \frac{100}{202}\frac{1}{100} + \frac{100}{202}\frac{1}{100} + \frac{1}{202}\cdot 100 + \frac{1}{202}\cdot 100$$

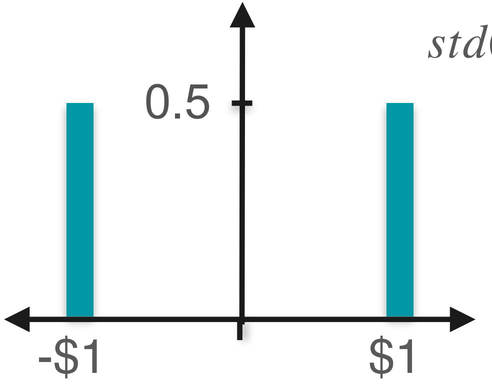
$$= \frac{1}{202} + \frac{1}{202} + \frac{100}{202} + \frac{100}{202} = 1$$

Kurtosis: Example Variance

Game 1

$Var(X_1) = 1$

$std(X_1) = 1$

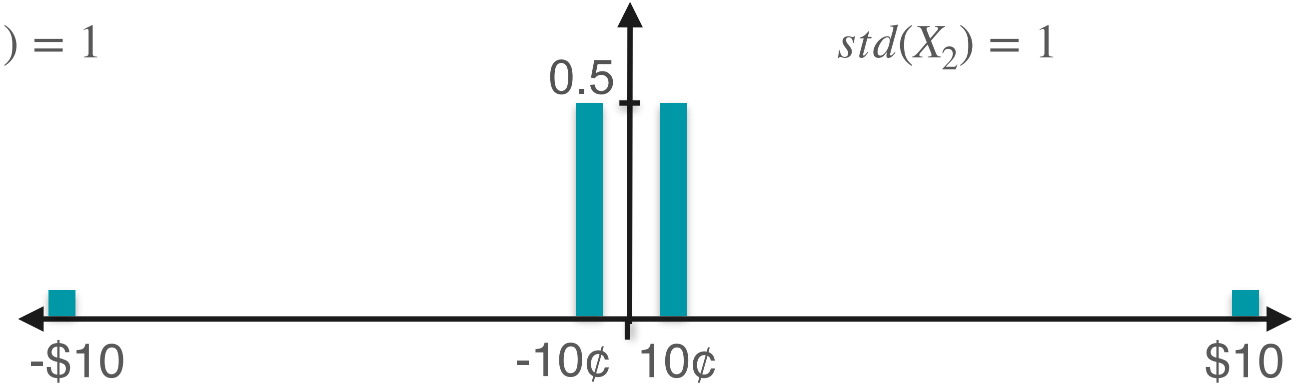


$$\mathbb{E}[X_1^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$
$$= 1$$

Game 2

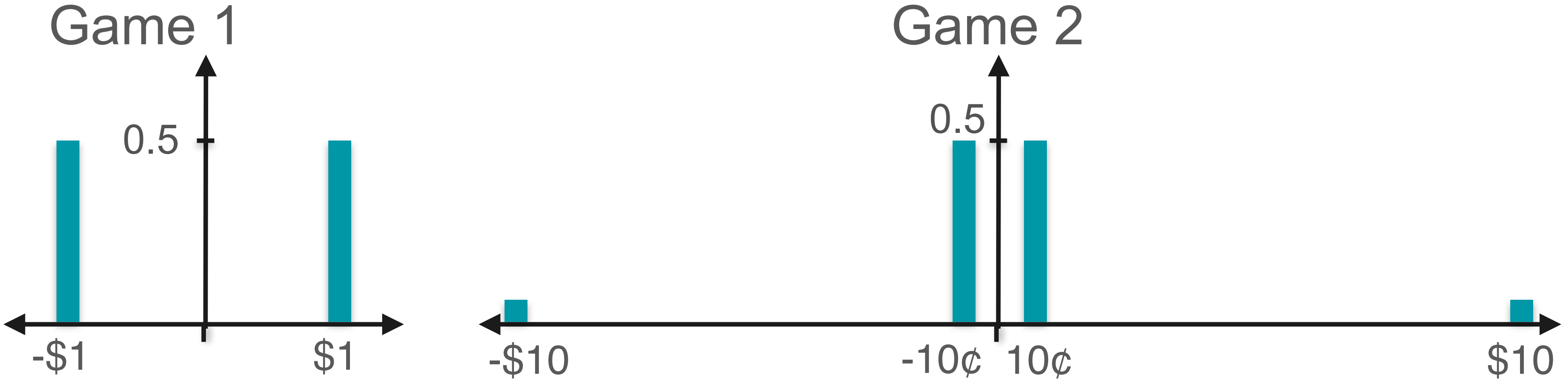
$Var(X_2) = 1$

$std(X_2) = 1$

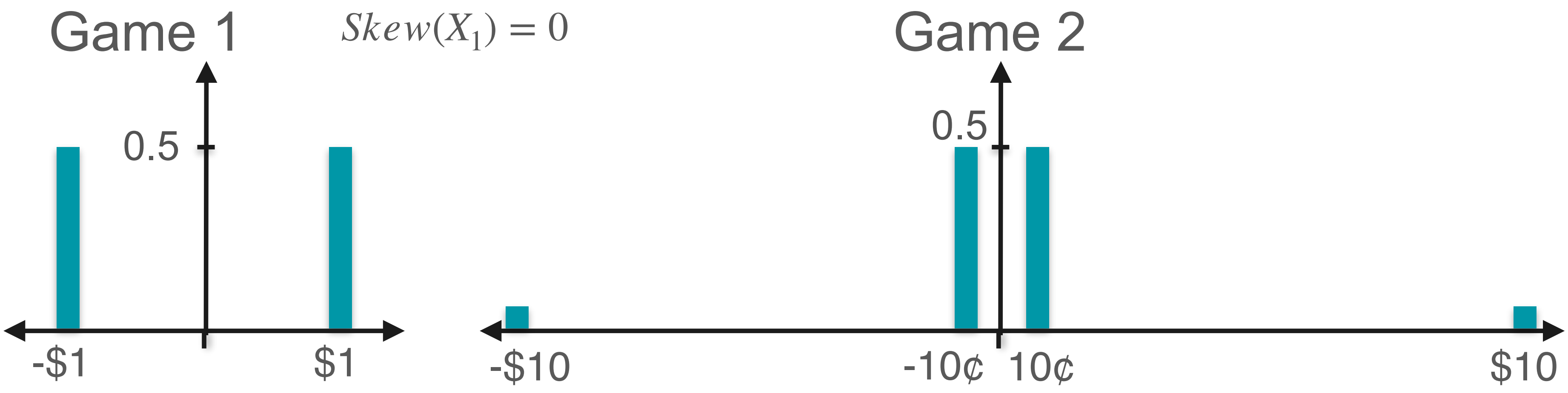


$$\mathbb{E}[X_2^2] = \frac{100}{202}(-0.1)^2 + \frac{100}{202}(0.1)^2 + \frac{1}{202}(-10)^2 + \frac{1}{202}(10)^2$$
$$= \frac{100}{202}\frac{1}{100} + \frac{100}{202}\frac{1}{100} + \frac{1}{202}\cdot 100 + \frac{1}{202}\cdot 100$$
$$= \frac{1}{202} + \frac{1}{202} + \frac{100}{202} + \frac{100}{202} = 1$$

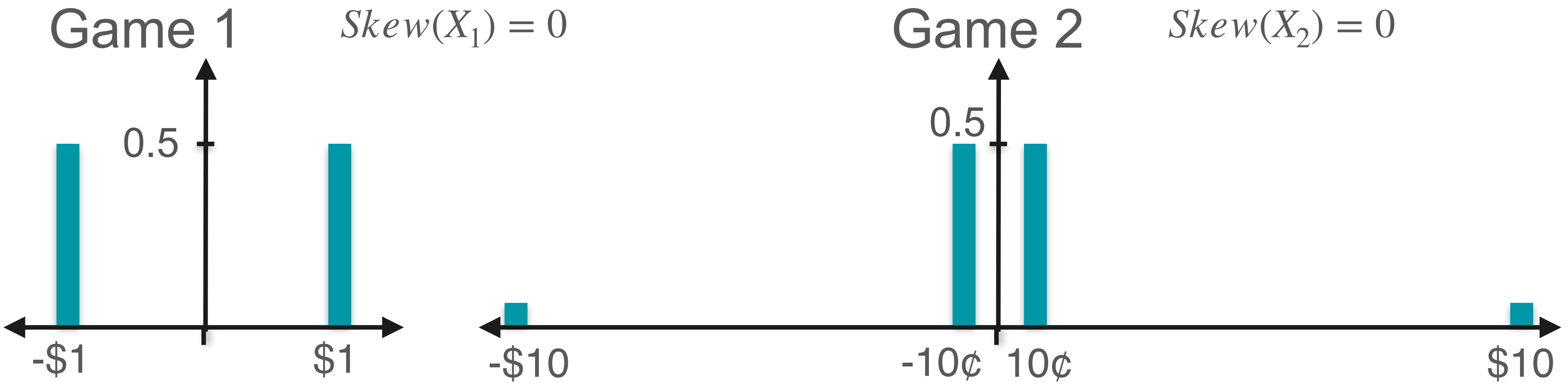
Kurtosis: Example Skewness



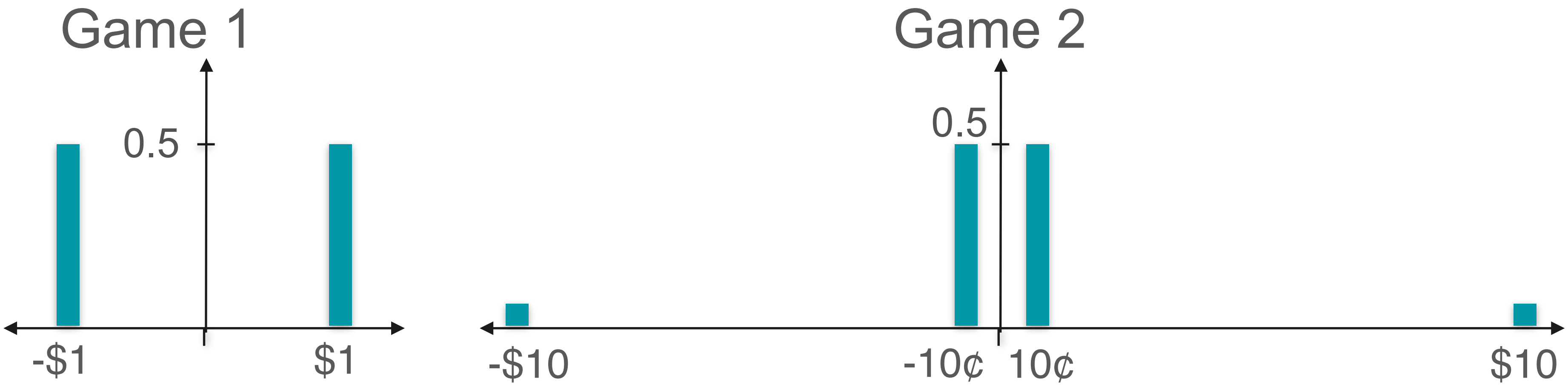
Kurtosis: Example Skewness



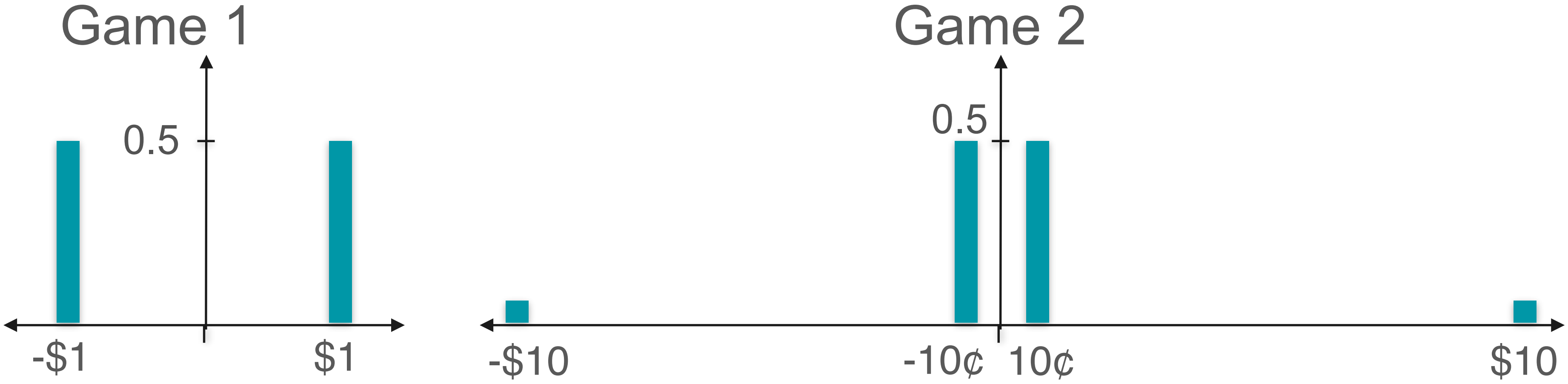
Kurtosis: Example Skewness



Kurtosis

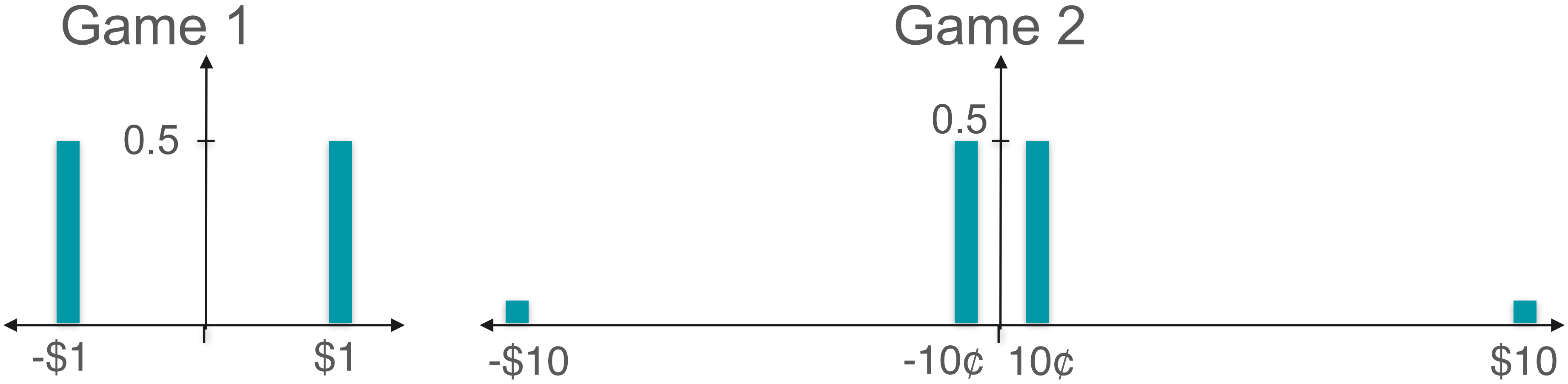


Kurtosis



$$Skew(X_1) = 0$$

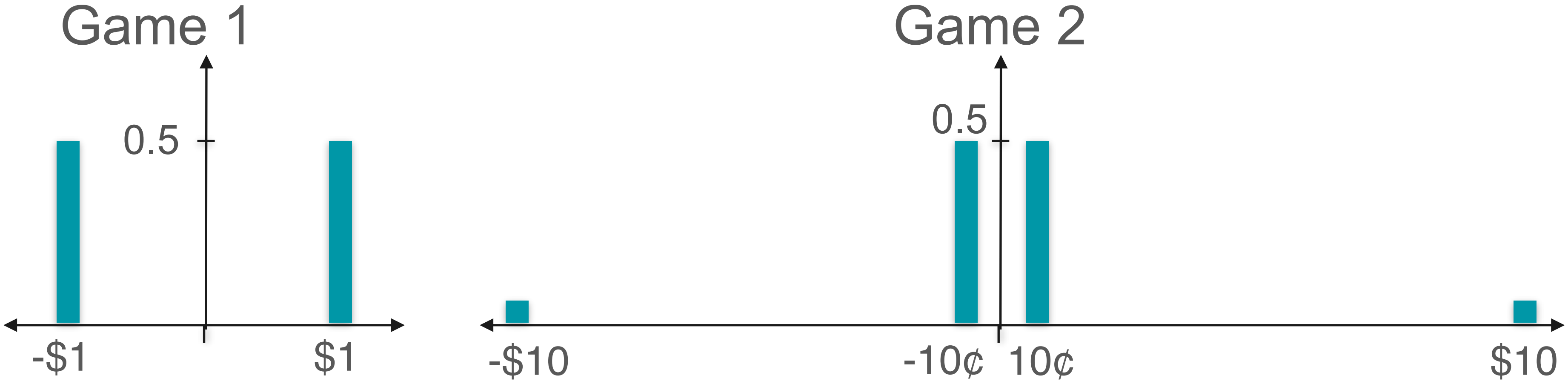
Kurtosis



$Skew(X_1) = 0$

$Skew(X_2) = 0$

Kurtosis

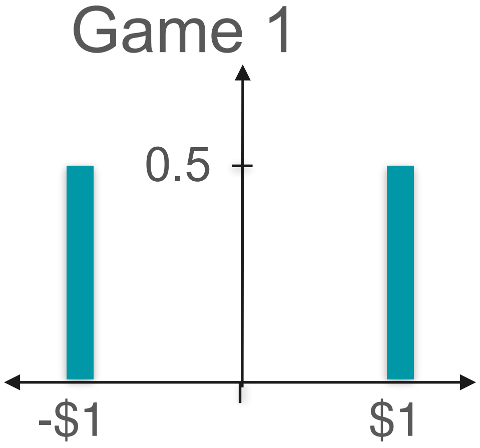


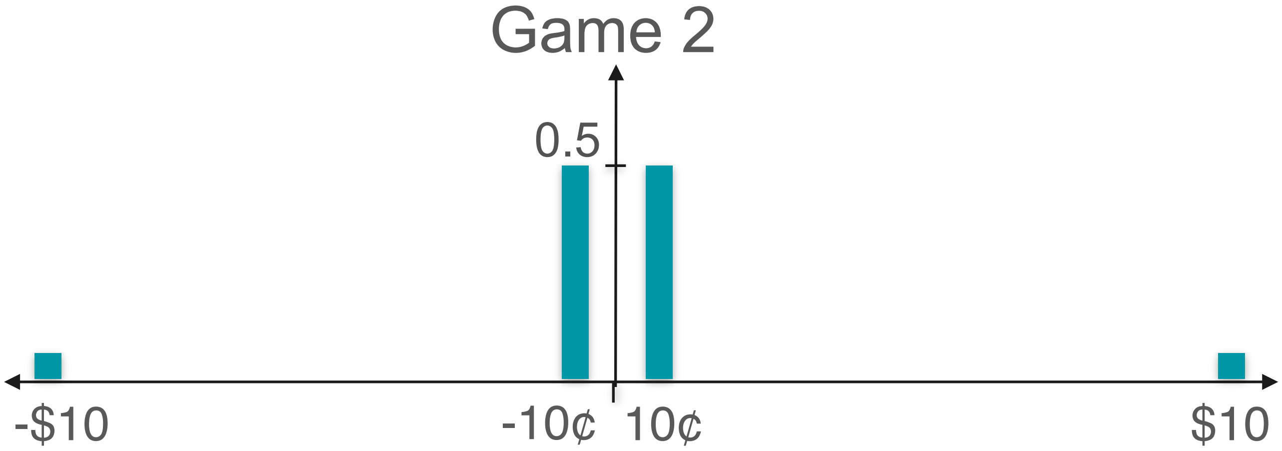
$Var(X_1) = 1$

$Skew(X_1) = 0$

$Skew(X_2) = 0$

Kurtosis





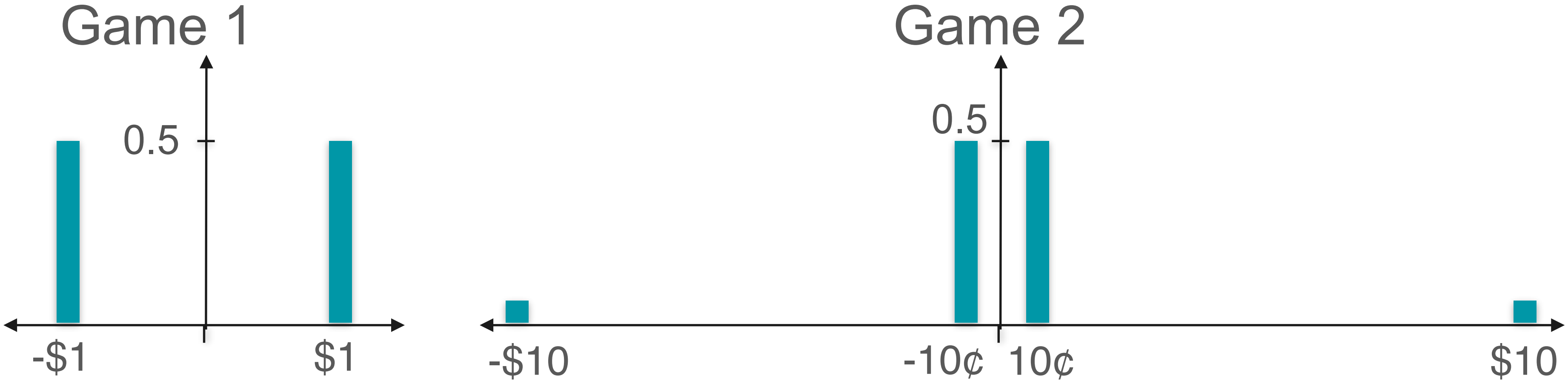
$Var(X_1) = 1$

$Skew(X_1) = 0$

$Var(X_2) = 1$

$Skew(X_2) = 0$

Kurtosis



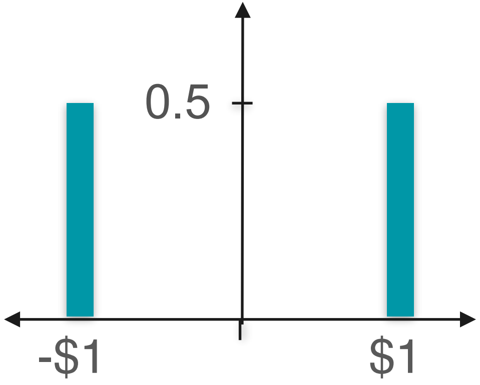
$$E[X_1] = 0$$

$$Var(X_1) = 1 \qquad Var(X_2) = 1$$

$$Skew(X_1) = 0 \qquad Skew(X_2) = 0$$

Kurtosis

Game 1

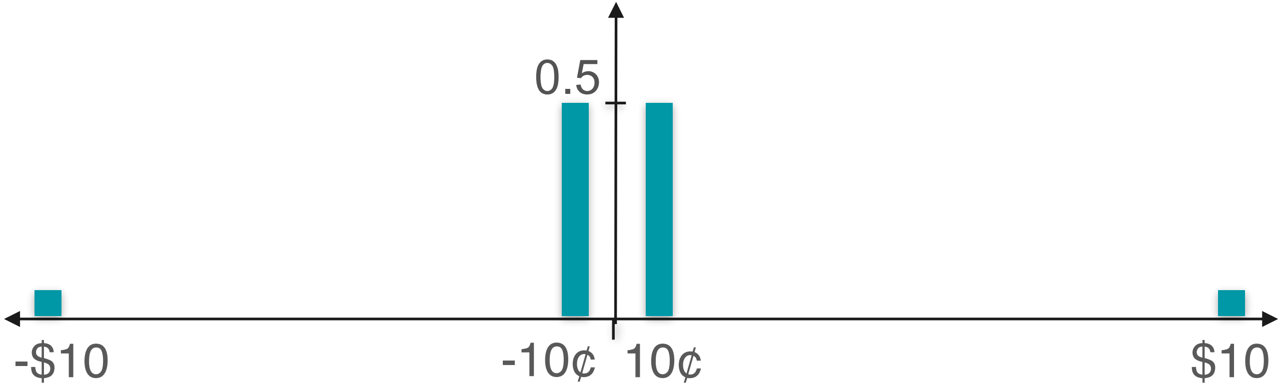


$$E[X_1] = 0$$

$$Var(X_1) = 1$$

$$Skew(X_1) = 0$$

Game 2



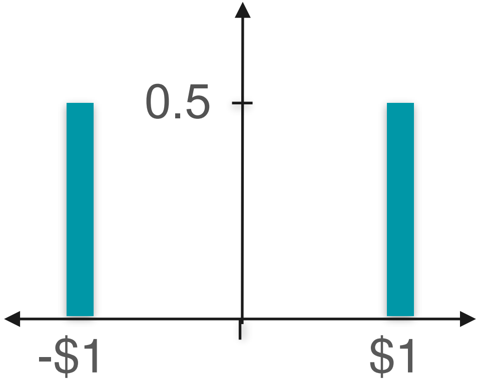
$$E[X_2] = 0$$

$$Var(X_2) = 1$$

$$Skew(X_2) = 0$$

Kurtosis

Game 1

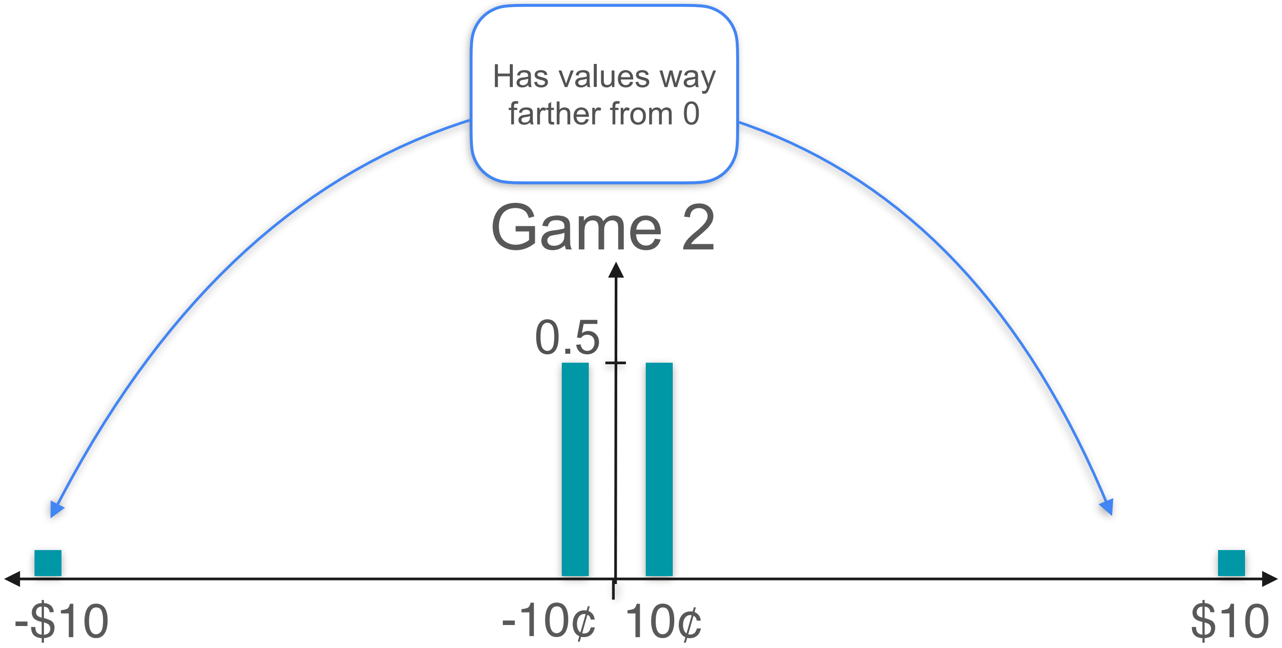


$$E[X_1] = 0$$

$$Var(X_1) = 1$$

$$Skew(X_1) = 0$$

Game 2



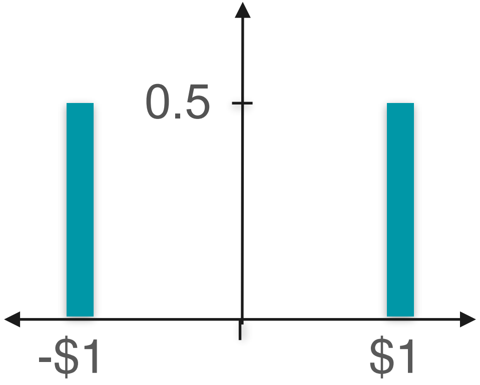
$$E[X_2] = 0$$

$$Var(X_2) = 1$$

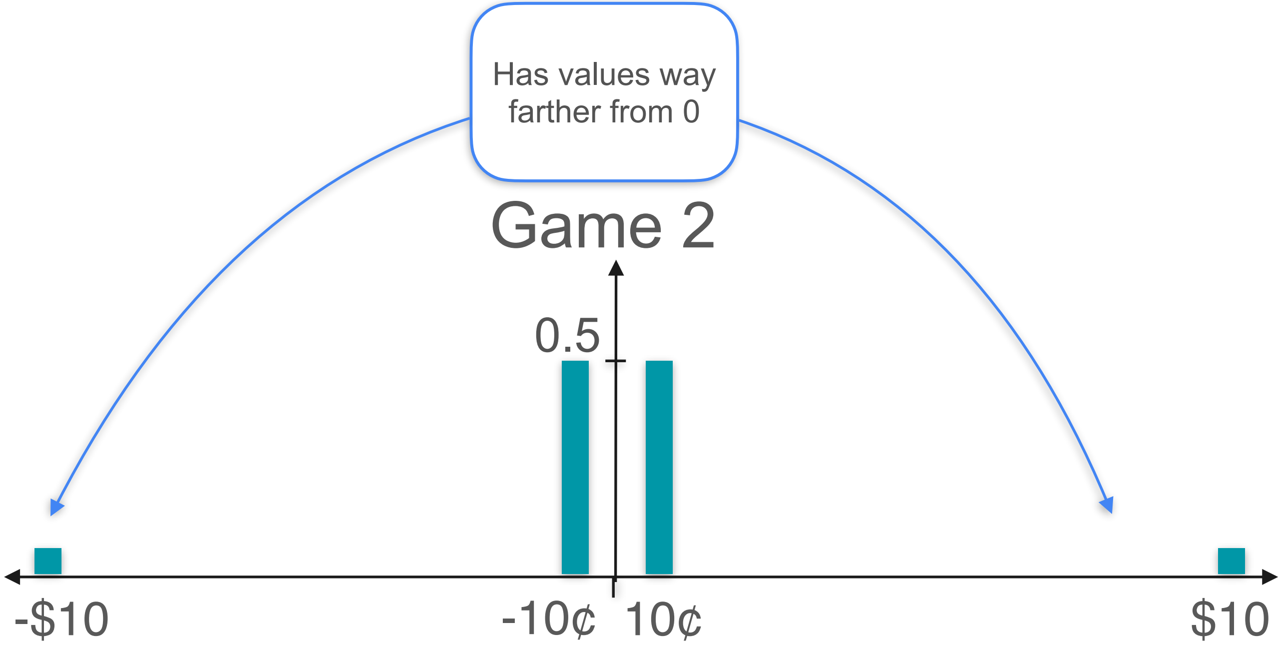
$$Skew(X_2) = 0$$

Kurtosis

Game 1



Game 2



$E[X_1] = 0$ $\quad$ $E[X_1] = 0$

$Var(X_1) = 1$

$Skew(X_1) = 0$

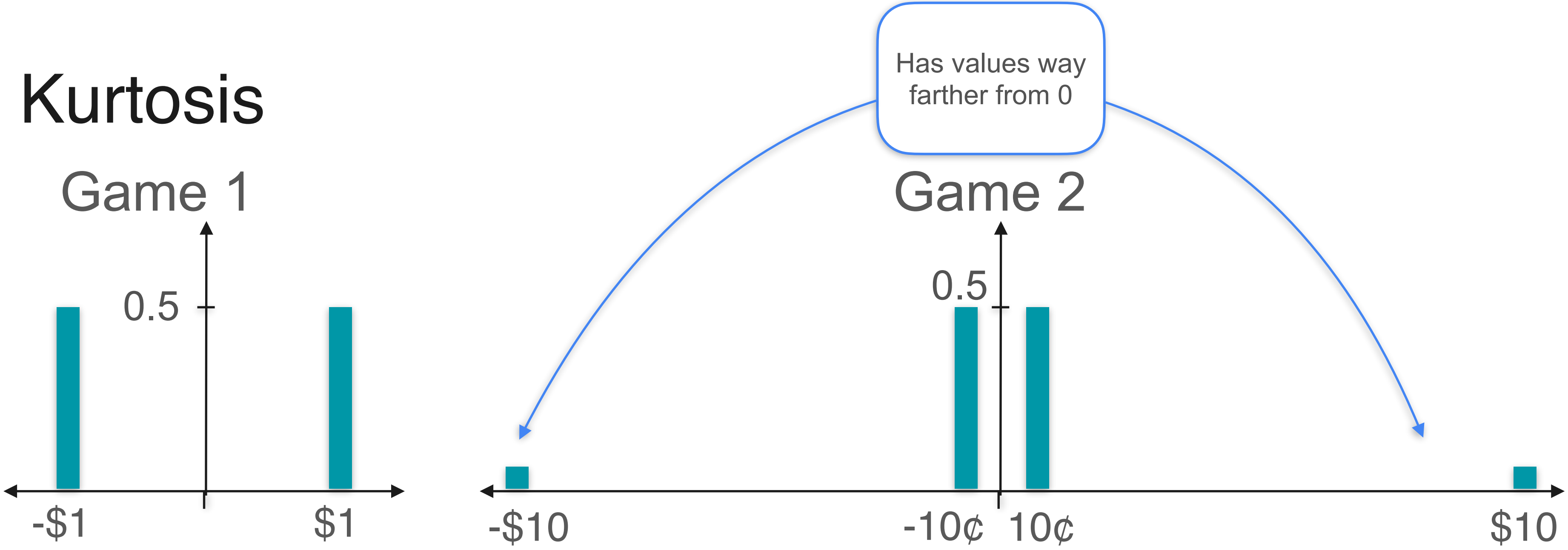
$E[X_2] = 0$

$Var(X_2) = 1$

$Skew(X_2) = 0$

DeepLearning.AI

Kurtosis



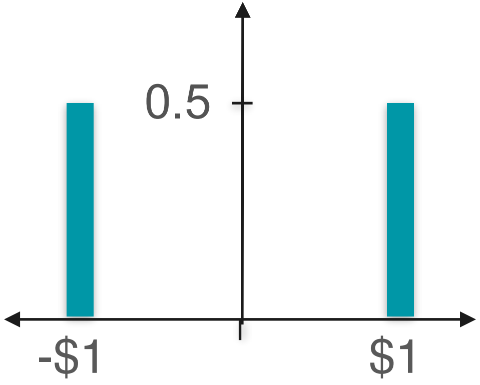
$E[X_1] = 0$ $\qquad E[X_1] = 0$ $\qquad E[X_2] = 0$

$Var(X_1) = 1$ $\qquad E[X_1^2] = 1$ $\qquad Var(X_2) = 1$

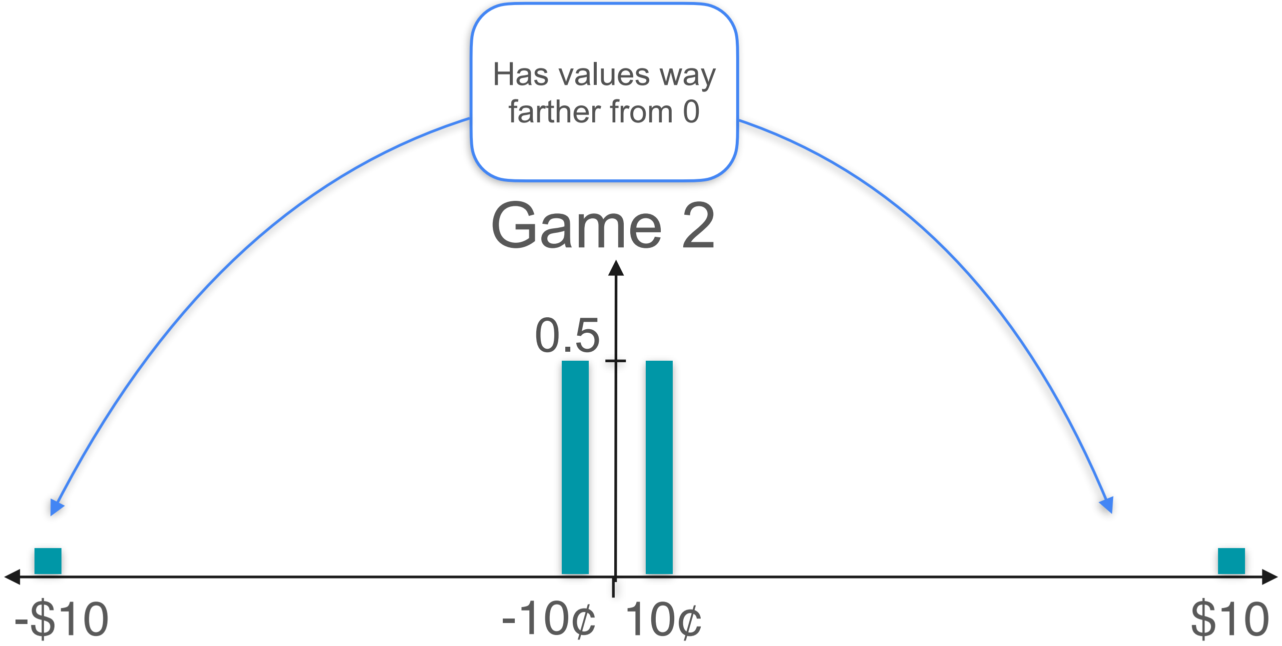
$Skew(X_1) = 0$ $\qquad Skew(X_2) = 0$

Kurtosis

Game 1



Game 2



$E[X_1] = 0$ $\quad E[X_1] = 0$

$Var(X_1) = 1$ $\quad E[X_1^2] = 1$

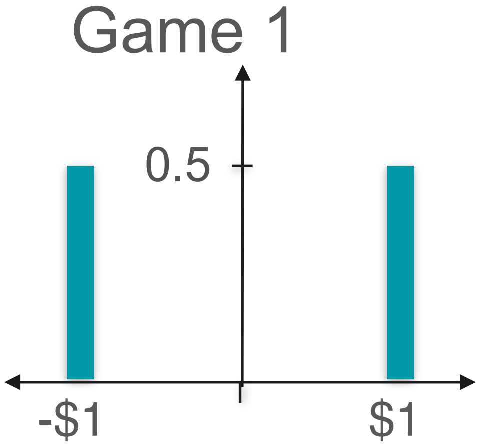
$Skew(X_1) = 0$ $\quad E[X_1^3] = 0$

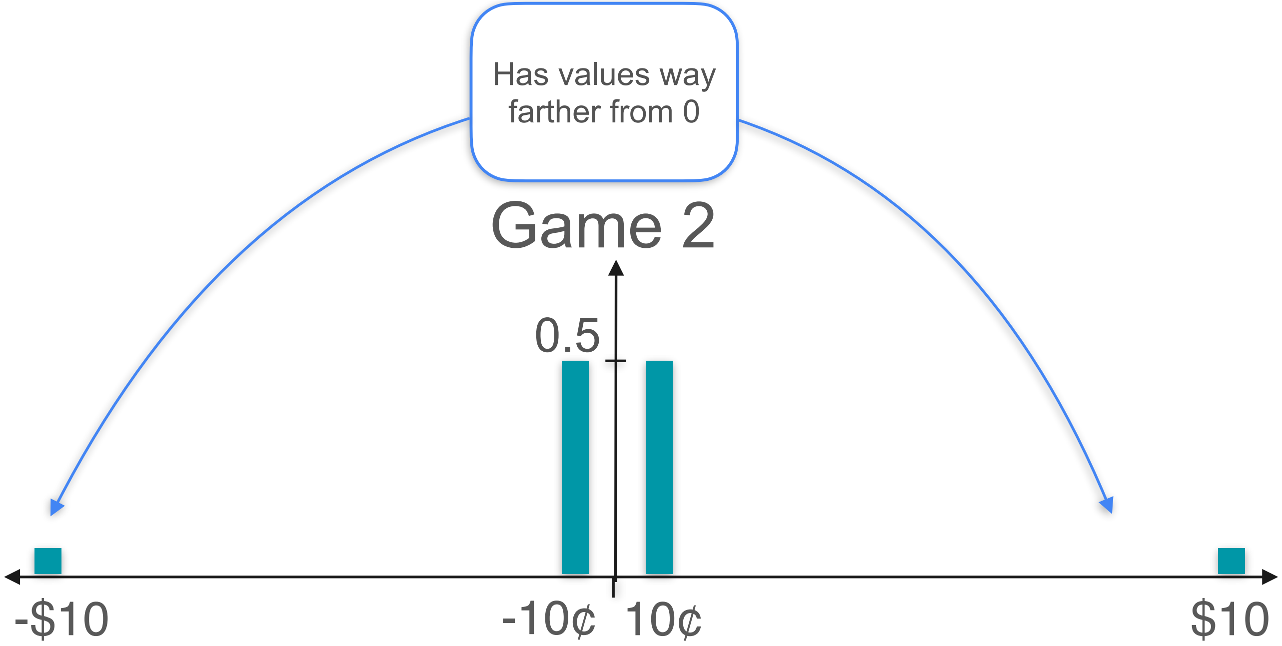
$E[X_2] = 0$

$Var(X_2) = 1$

$Skew(X_2) = 0$

Kurtosis





$E[X_1] = 0$ $E[X_1] = 0$

$Var(X_1) = 1$ $E[X_1^2] = 1$

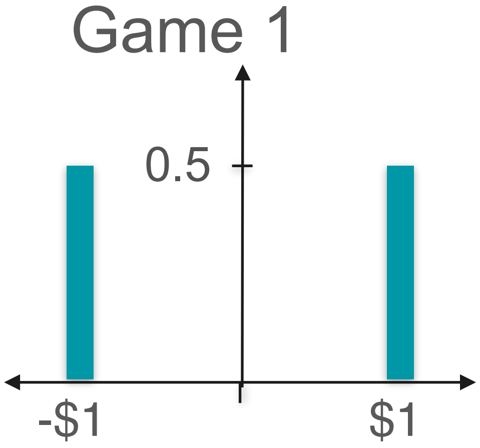
$Skew(X_1) = 0$ $E[X_1^3] = 0$

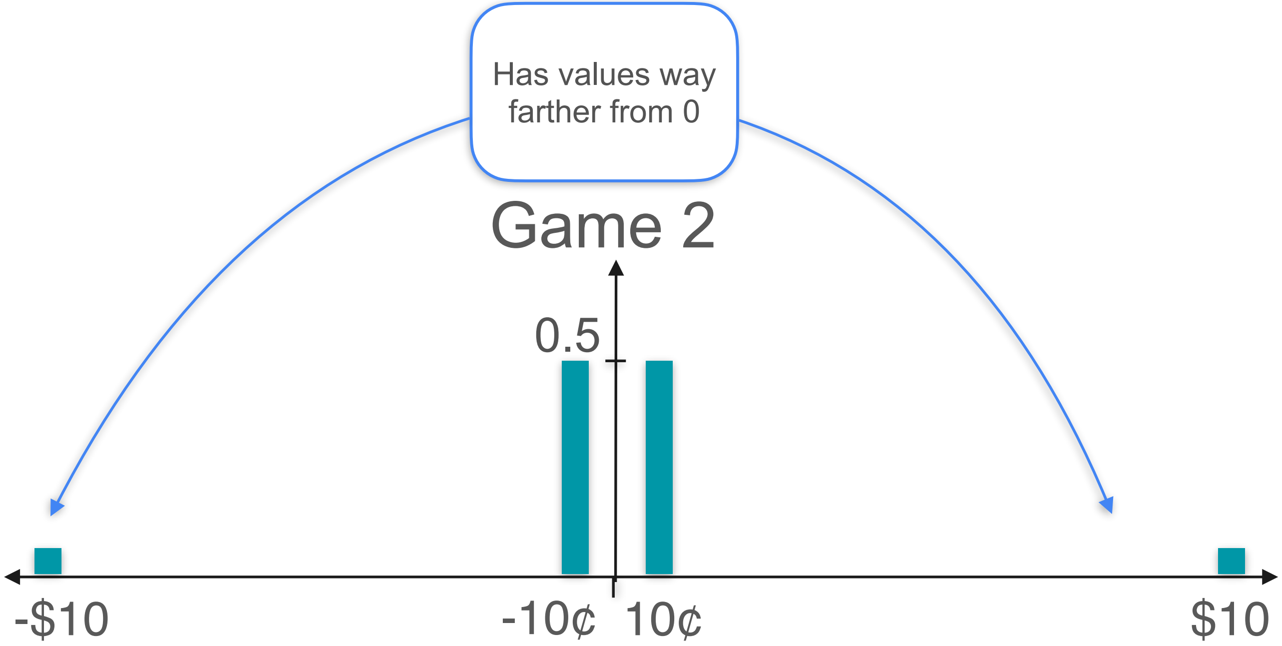
$E[X_2] = 0$ $E[X_2] = 0$

$Var(X_2) = 1$

$Skew(X_2) = 0$

Kurtosis





$$E[X_1] = 0 \qquad E[X_1] = 0$$

$$Var(X_1) = 1 \qquad E[X_1^2] = 1$$

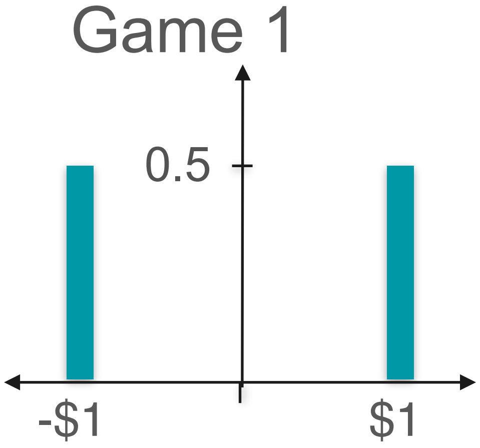
$$Skew(X_1) = 0 \qquad E[X_1^3] = 0$$

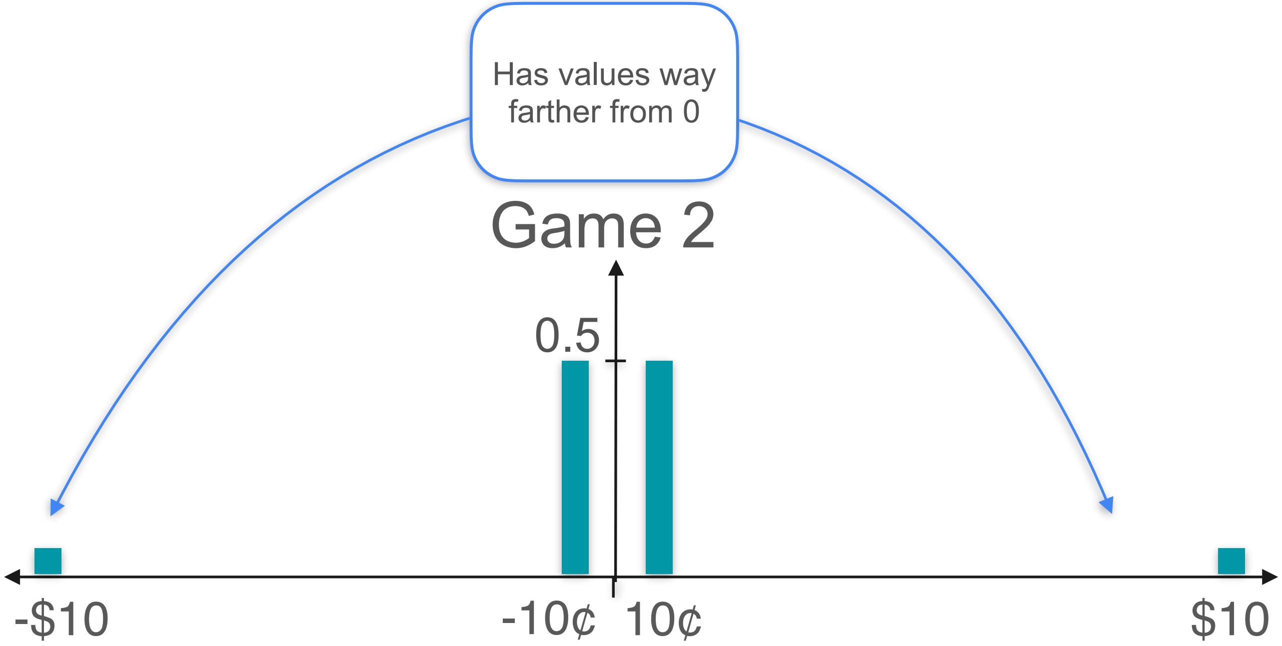
$$E[X_2] = 0 \qquad E[X_2] = 0$$

$$Var(X_2) = 1 \qquad E[X_2^2] = 1$$

$$Skew(X_2) = 0$$

Kurtosis



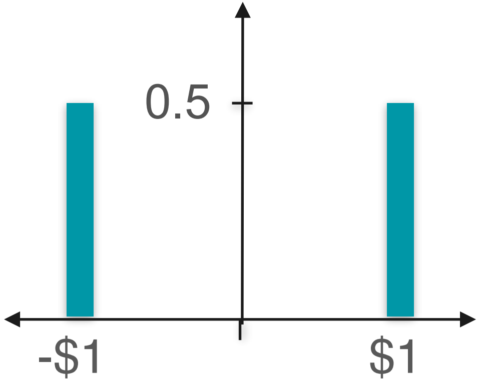


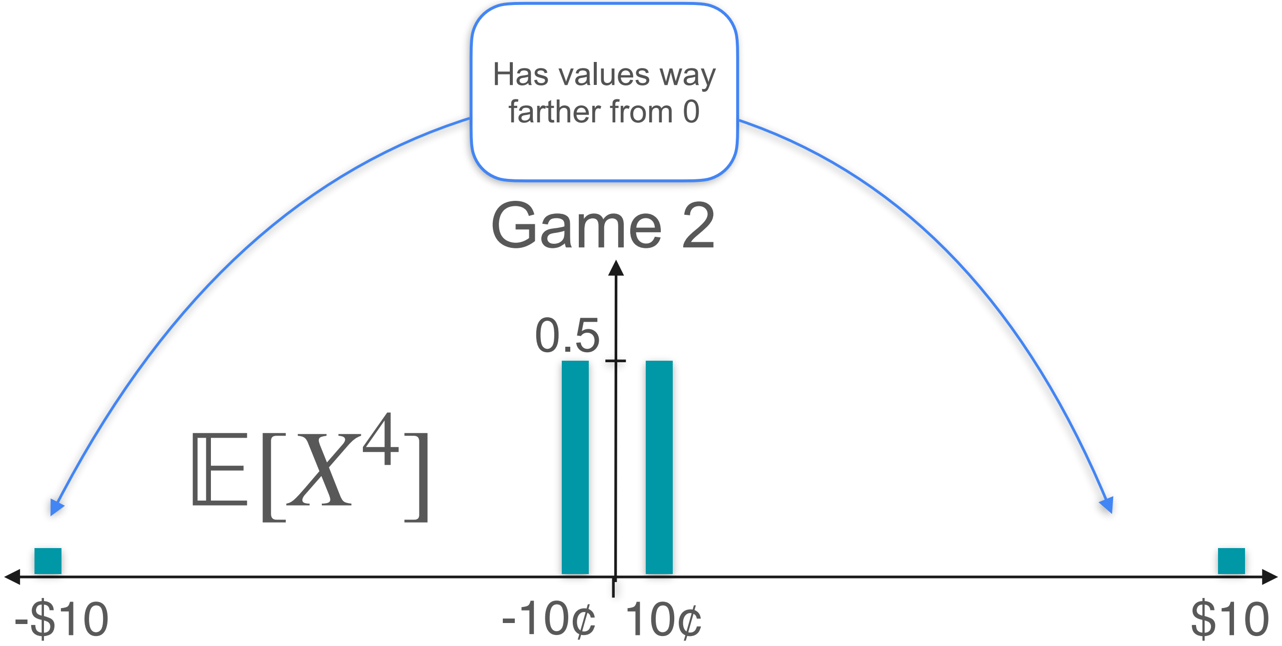
$E[X_1] = 0$	$E[X_1] = 0$
$Var(X_1) = 1$	$E[X_1^2] = 1$
$Skew(X_1) = 0$	$E[X_1^3] = 0$

$E[X_2] = 0$	$E[X_2] = 0$
$Var(X_2) = 1$	$E[X_2^2] = 1$
$Skew(X_2) = 0$	$E[X_2^3] = 0$

Kurtosis

Game 1





$E[X_1] = 0 \quad E[X_1] = 0$

$Var(X_1) = 1 \quad E[X_1^2] = 1$

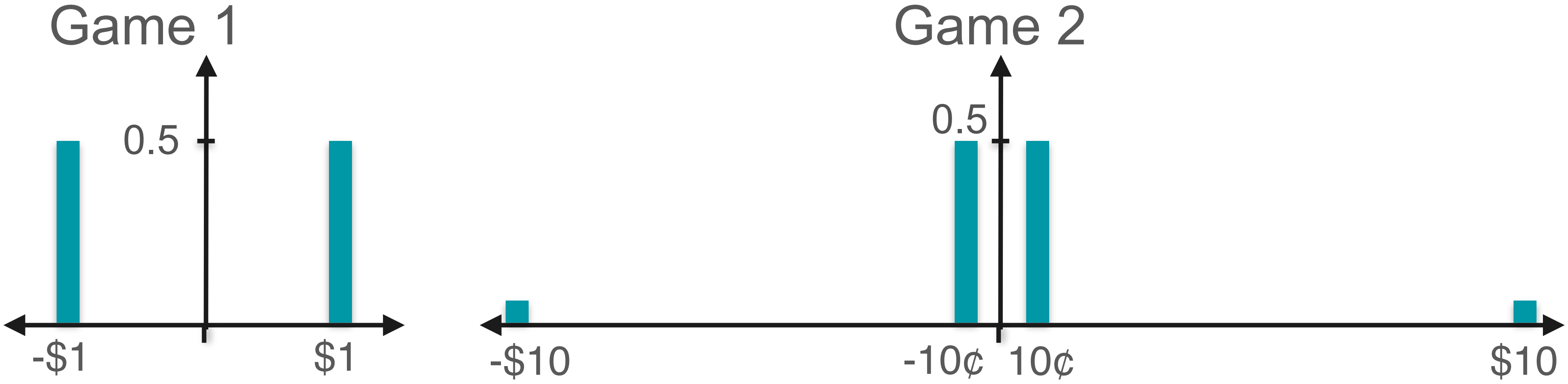
$Skew(X_1) = 0 \quad E[X_1^3] = 0$

$E[X_2] = 0 \quad E[X_2] = 0$

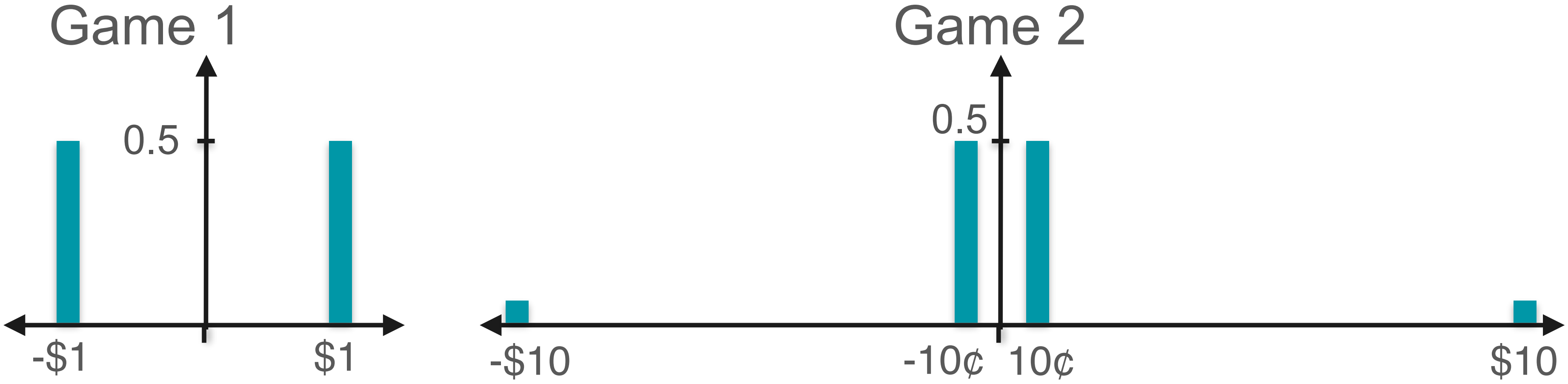
$Var(X_2) = 1 \quad E[X_2^2] = 1$

$Skew(X_2) = 0 \quad E[X_2^3] = 0$

Kurtosis

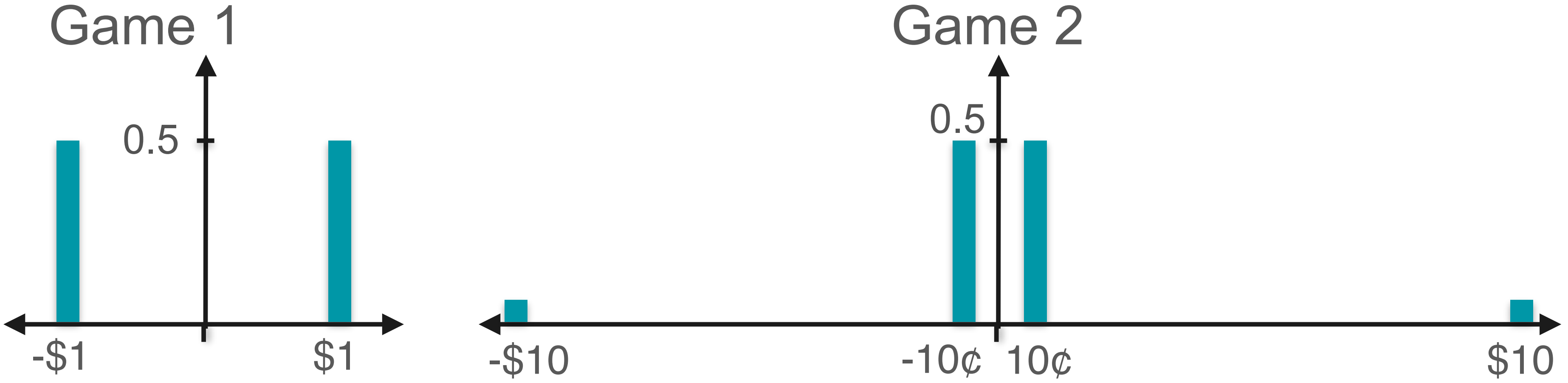


Kurtosis



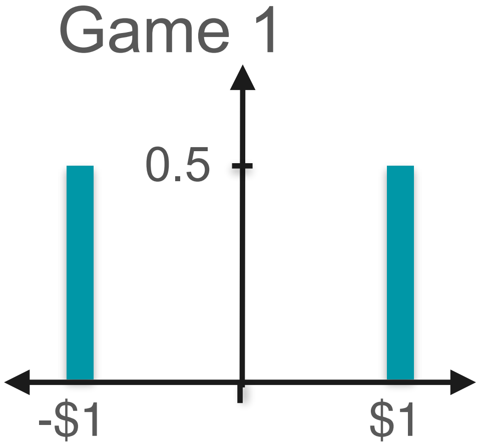
$\mathbb{E}[X_1^4]$

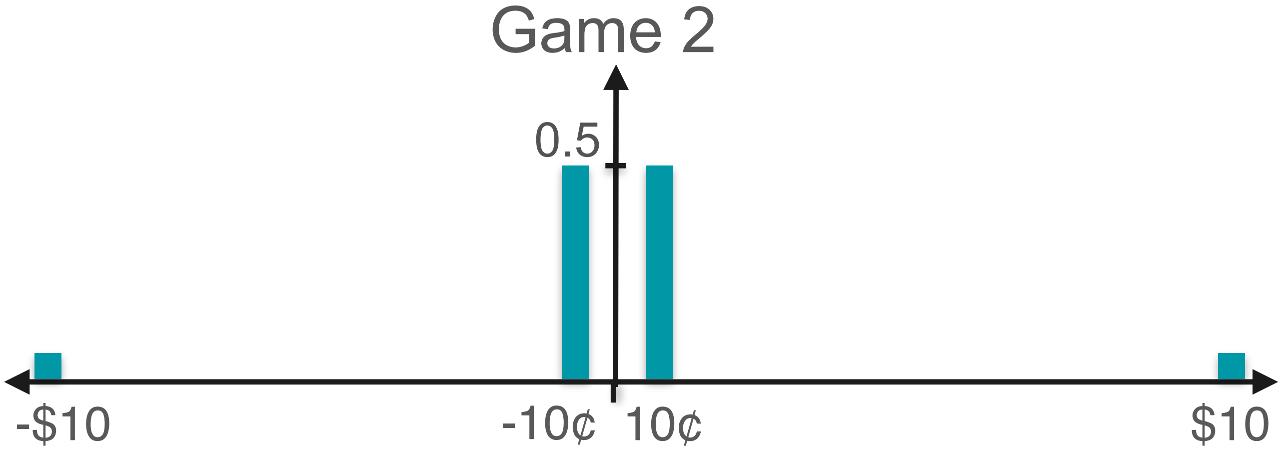
Kurtosis



$$\mathbb{E}[X_1^4] = \frac{1}{2}(-1)^4 + \frac{1}{2}(1)^4$$

Kurtosis

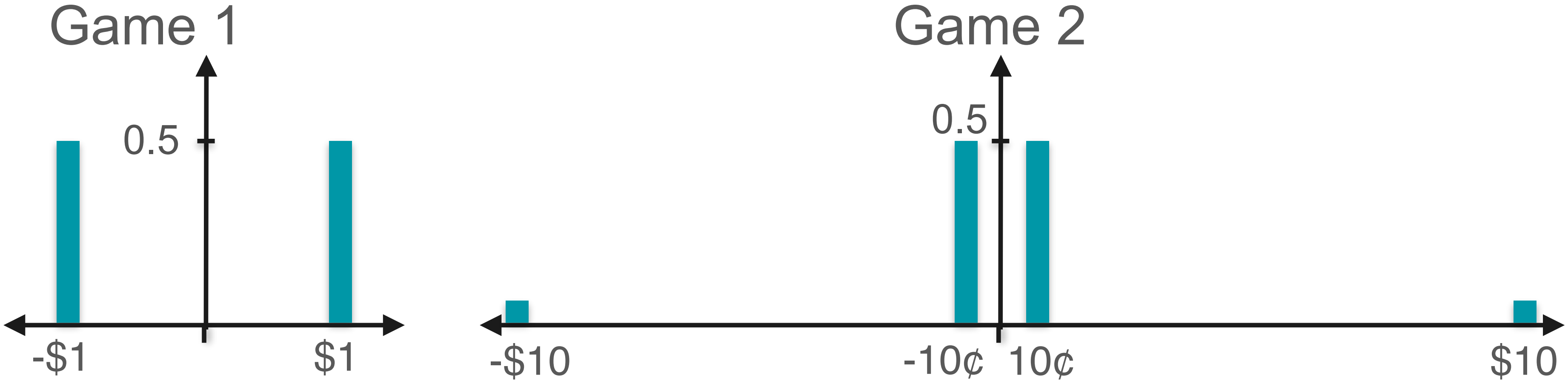




$$\mathbb{E}[X_1^4] = \frac{1}{2}(-1)^4 + \frac{1}{2}(1)^4$$

$$= 1$$

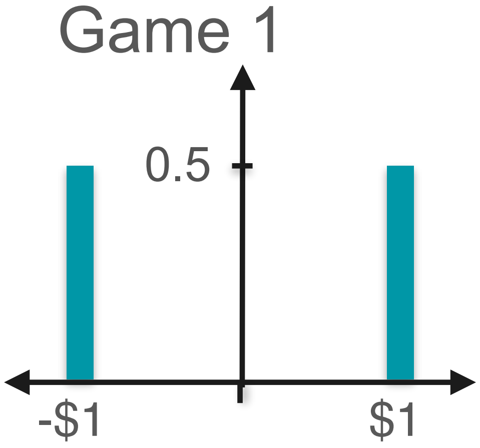
Kurtosis



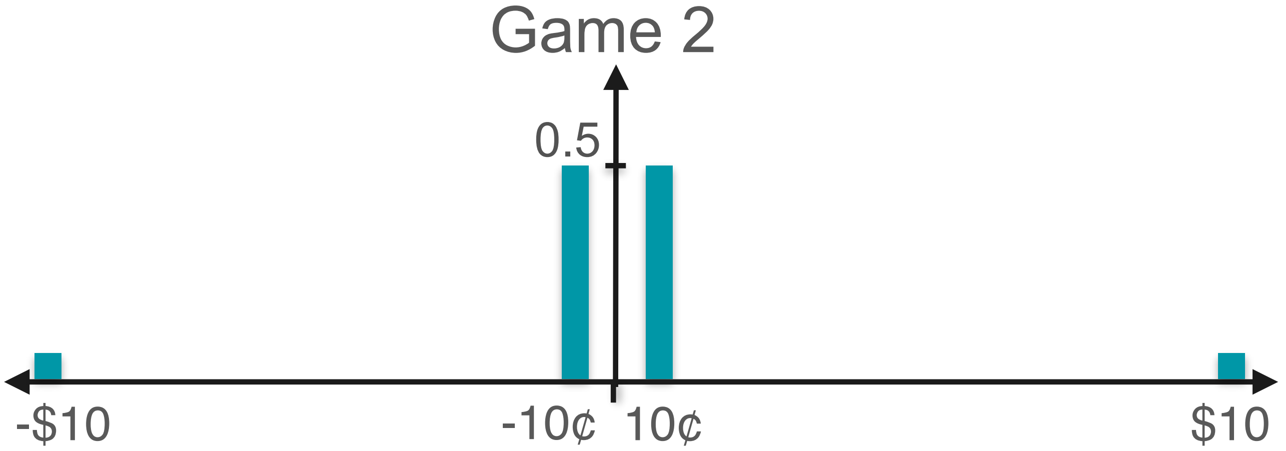
$$\mathbb{E}[X_1^4] = \frac{1}{2}(-1)^4 + \frac{1}{2}(1)^4$$
$$= 1$$

$$\mathbb{E}[X_2^4]$$

Kurtosis



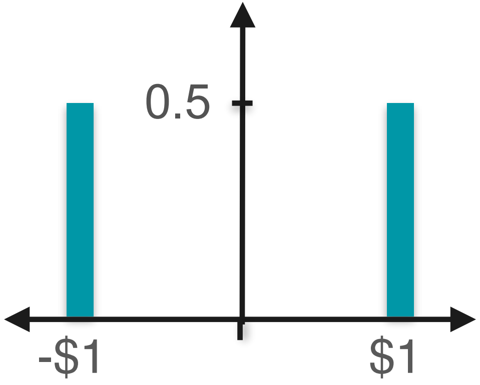
$$\mathbb{E}[X_1^4] = \frac{1}{2}(-1)^4 + \frac{1}{2}(1)^4$$
$$= 1$$



$$\mathbb{E}[X_2^4] = \frac{100}{202}(-0.1)^4 + \frac{100}{202}(0.1)^4 + \frac{1}{202}(-10)^4 + \frac{1}{202}(10)^4$$

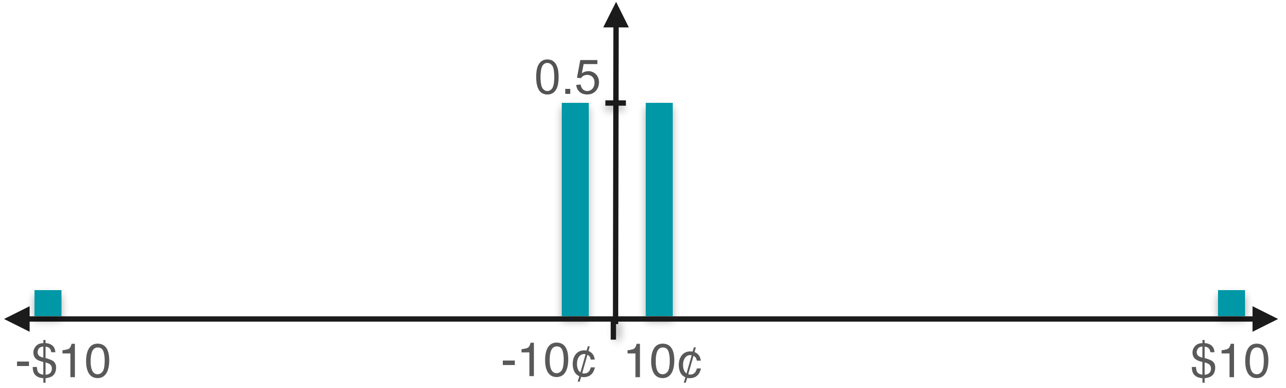
Kurtosis

Game 1



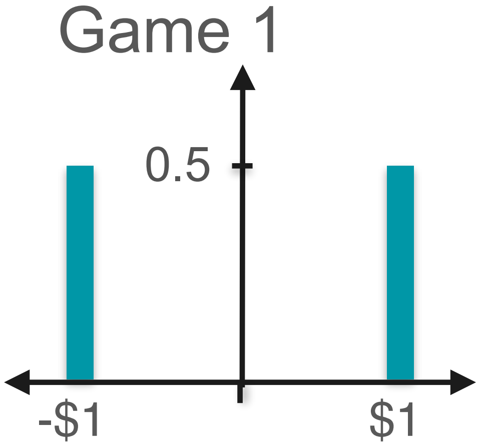
$$\mathbb{E}[X_1^4] = \frac{1}{2}(-1)^4 + \frac{1}{2}(1)^4$$
$$= 1$$

Game 2



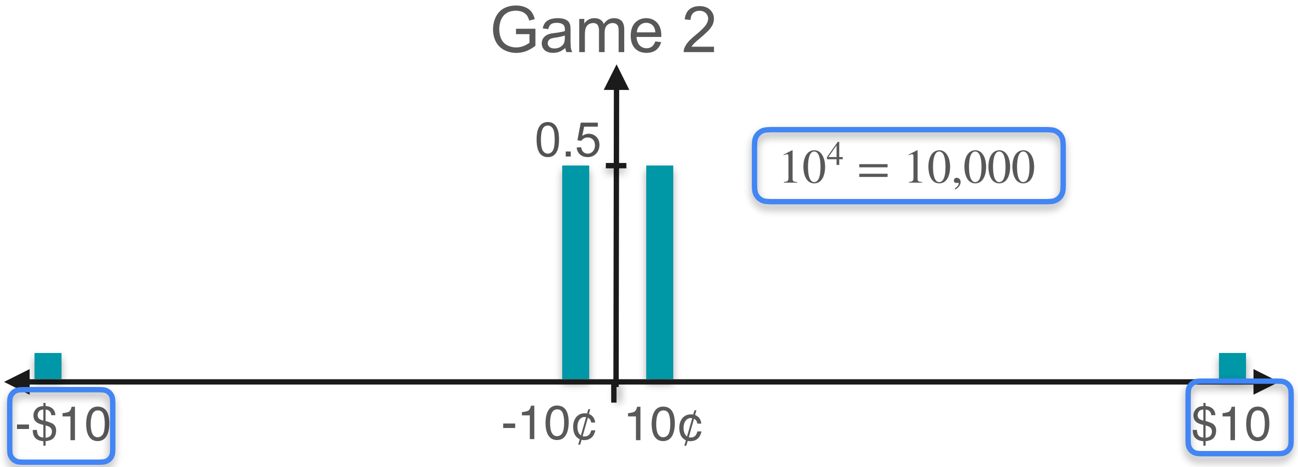
$$\mathbb{E}[X_2^4] = \frac{100}{202}(-0.1)^4 + \frac{100}{202}(0.1)^4 + \frac{1}{202}(-10)^4 + \frac{1}{202}(10)^4$$
$$= 99.01$$

Kurtosis



$$\mathbb{E}[X_1^4] = \frac{1}{2}(-1)^4 + \frac{1}{2}(1)^4$$

$$= 1$$



$$\mathbb{E}[X_2^4] = \frac{100}{202}(-0.1)^4 + \frac{100}{202}(0.1)^4 + \frac{1}{202}\boxed{(-10)^4} + \frac{1}{202}\boxed{(10)^4}$$

$$= 99.01$$

Kurtosis

$$\mathbb{E}[X^4]$$

Kurtosis

$$\mathbb{E}[X^4]$$

Almost…

Kurtosis

$$\mathbb{E}[X^4]$$

Almost…

Need to standardize…

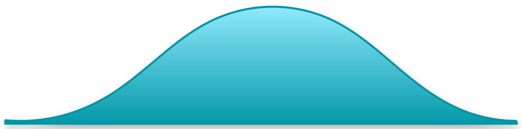
Kurtosis

Kurtosis

$$\text{Kurtosis} = E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right]$$

Kurtosis: High and Low

Kurtosis: High and Low

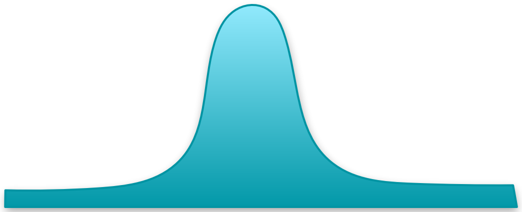


$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{small}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{large}$$

Kurtosis: High and Low

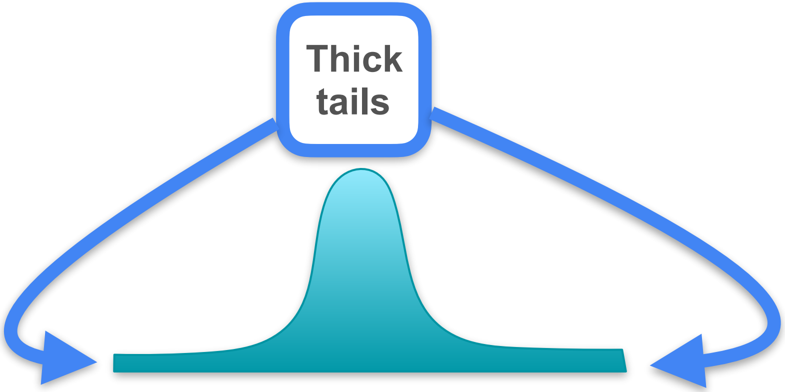
$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{small}$$



$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{large}$$

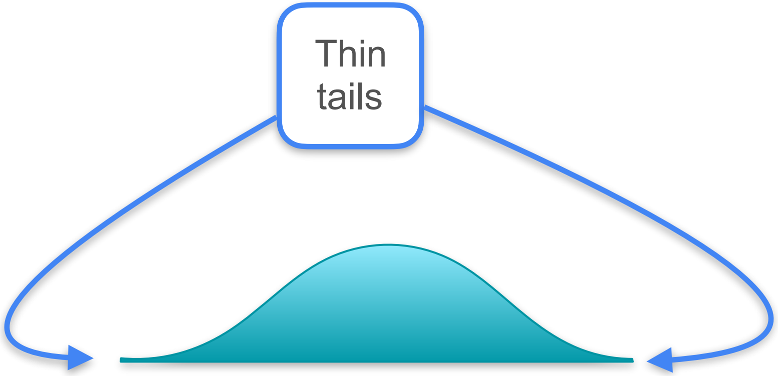
Kurtosis: High and Low

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{small}$$

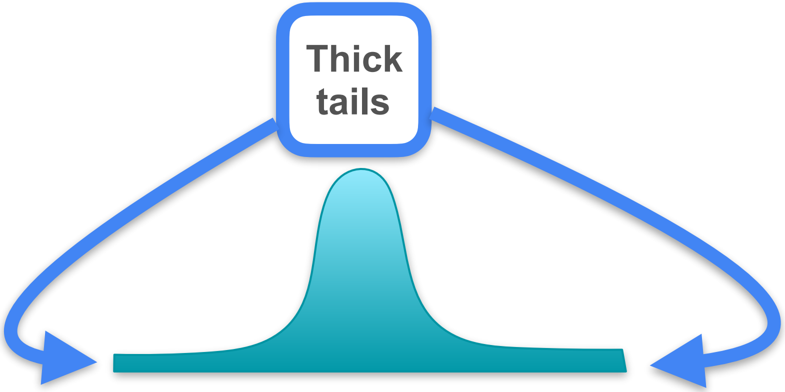


$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{large}$$

Kurtosis: High and Low

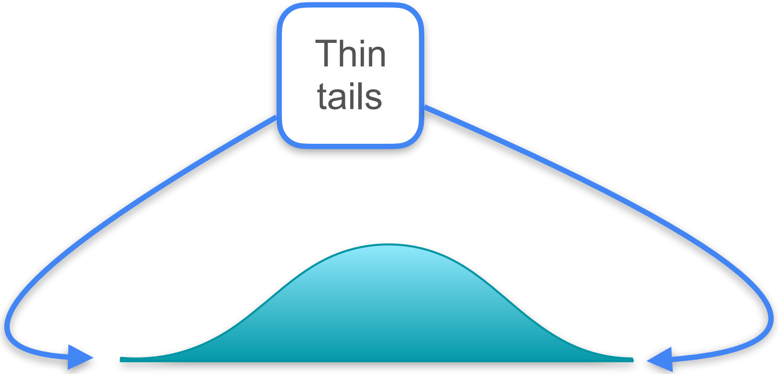


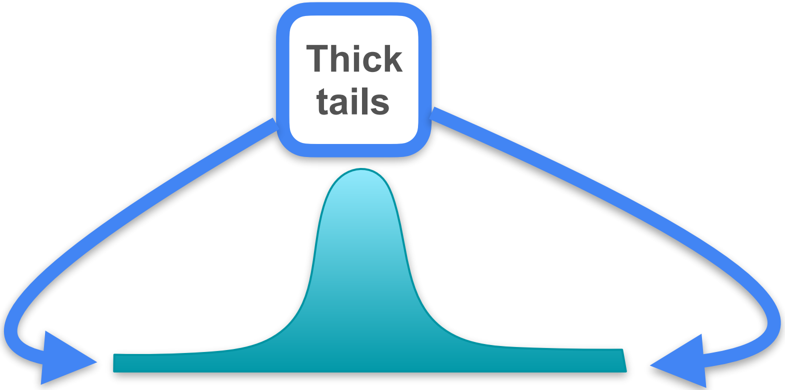
$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{small}$$



$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{large}$$

Kurtosis: High and Low





$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{small}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \text{large}$$

Even if they have the same variance!

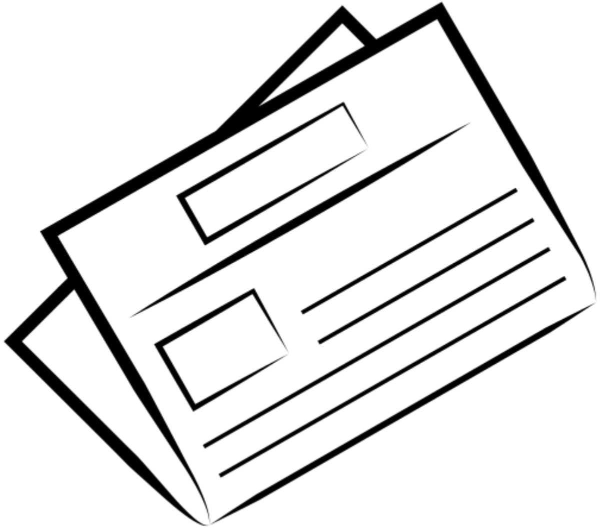
DeepLearning.AI

Describing Distributions

Quantiles and box-plots

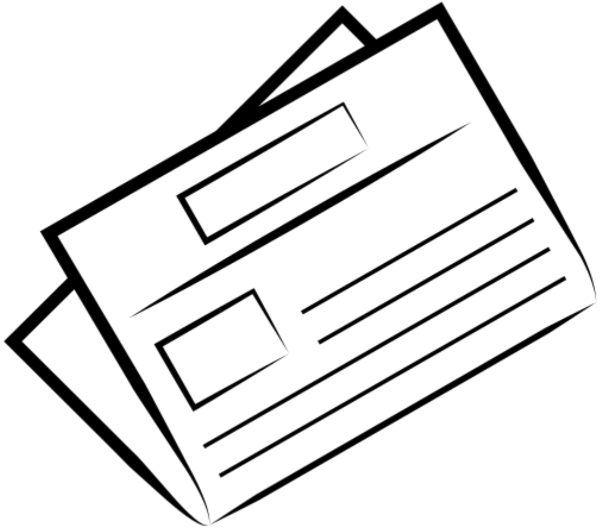
Quantiles: Example

Quantiles: Example



Newspaper advertisement

Quantiles: Example



Newspaper advertisement

Newspaper Advertisement (X)			
18.3	18.4	23.2	51.2
35.2	29.7	75	8.7
65.9	14.2	54.7	25.9

Quantiles: Example

Newspaper Advertisement (X)			
18.3	18.4	23.2	51.2
35.2	29.7	75	8.7
65.9	14.2	54.7	25.9

Quantiles: Example

What is the median here?

Newspaper Advertisement (X)			
18.3	18.4	23.2	51.2
35.2	29.7	75	8.7
65.9	14.2	54.7	25.9

Quantiles: Example

What is the median here?

The point that splits your data in half

Newspaper Advertisement (X)			
18.3	18.4	23.2	51.2
35.2	29.7	75	8.7
65.9	14.2	54.7	25.9

Quantiles: Example

What is the median here?

The point that splits your data in half

Newspaper Advertisement (X)			
8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Quantiles: Example

What is the median here?

The point that splits your data in half

Newspaper Advertisement (X)			
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8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Quantiles: Example

What is the median here?

The point that splits your data in half

$$\text{Median} = \frac{25.9 + 29.7}{2} = 27.8$$

50% quantile

Second quartile

Newspaper Advertisement (X)			
8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Quantiles: Example

Newspaper Advertisement (X)			
8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Quantiles: Example

What about the point that leaves 1/4 of your data to the left and 3/4 to the right?

Newspaper Advertisement (X)			
8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Quantiles: Example

What about the point that leaves 1/4 of your data to the left and 3/4 to the right?

$$\frac{18.3 + 18.4}{2} = 18.35$$

Newspaper Advertisement (X)			
8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Quantiles: Example

What about the point that leaves 1/4 of your data to the left and 3/4 to the right?

$$\frac{18.3 + 18.4}{2} = 18.35$$

25% quantile

First quartile

Newspaper Advertisement (X)			
8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Quantiles: Example

What about the point that leaves 1/4 of your data to the left and 3/4 to the right?

$$q_{0.25} = Q1 = \frac{18.3 + 18.4}{2} = 18.35$$

25% quantile

First quartile

Newspaper Advertisement (X)			
8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Quantiles

Quantiles

In general:

The **k% quantile (**$q_{k/100}$**)** is the value that leaves k% of your data to the left and (100-k)% of your data to the right

Quantiles

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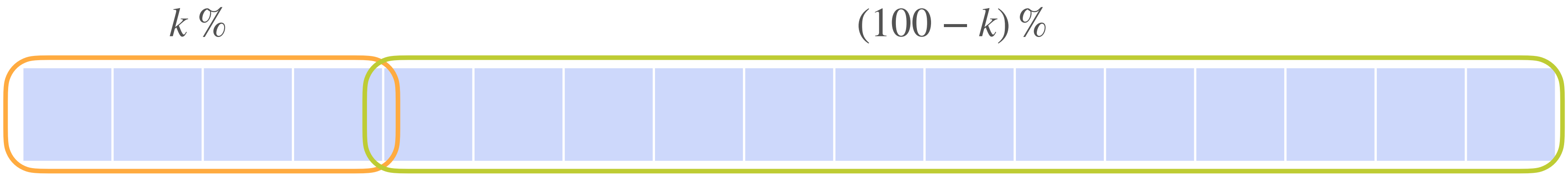
The **k% quantile ($q_{k/100}$)** is the value that leaves k% of your data to the left and (100-k)% of your data to the right



Quantiles

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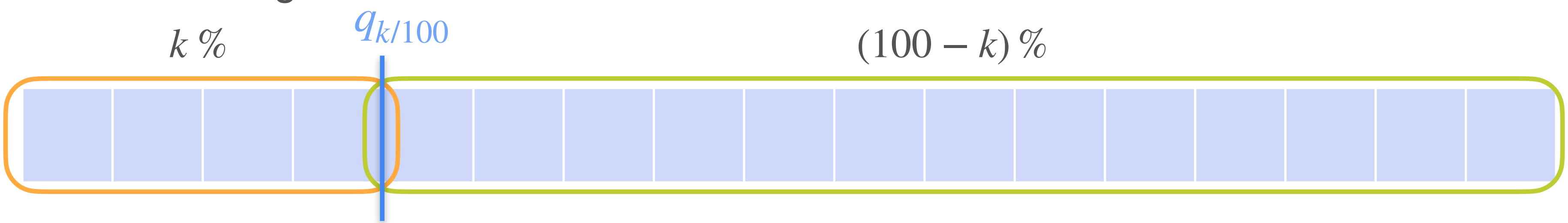


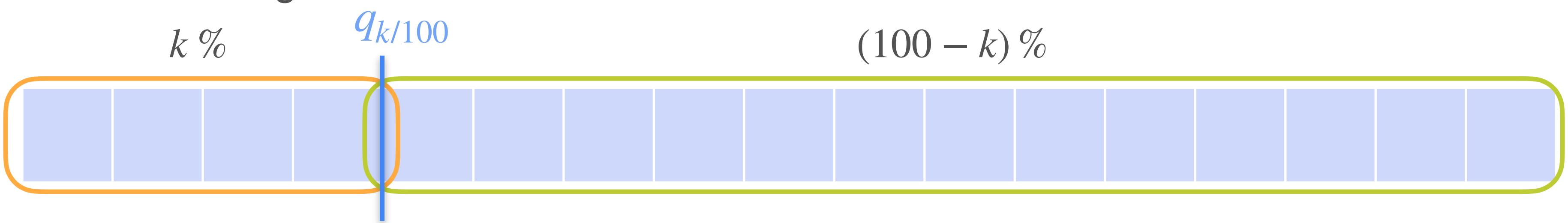
Quantiles

In general:

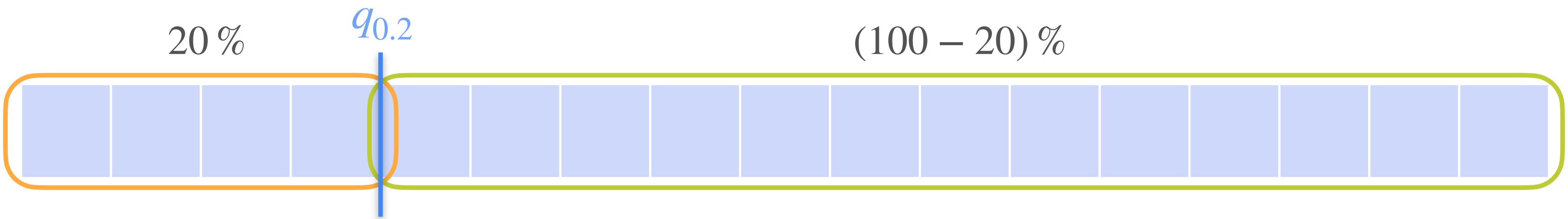
The **k% quantile ($q_{k/100}$)** is the value that leaves k% of your data to the left and (100-k)% of your data to the right

Some common quantiles:

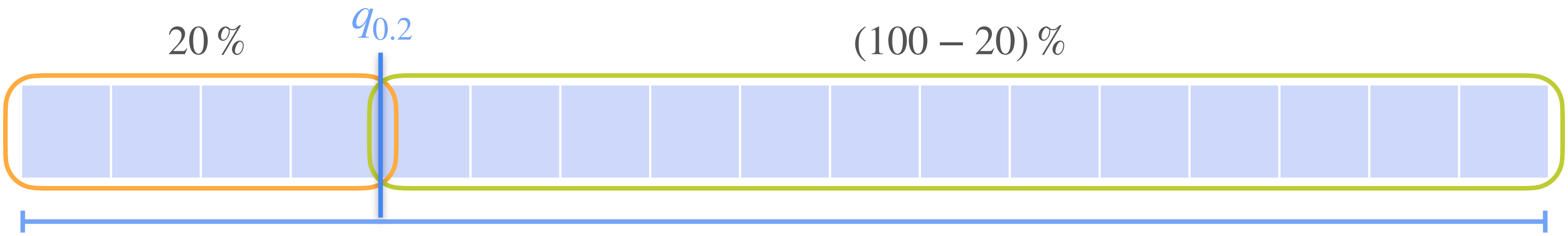
- 25% quantile (first quartile - Q1)
- 50% quantile (median - Q2)
- 75% quantile (third quartile - Q3)



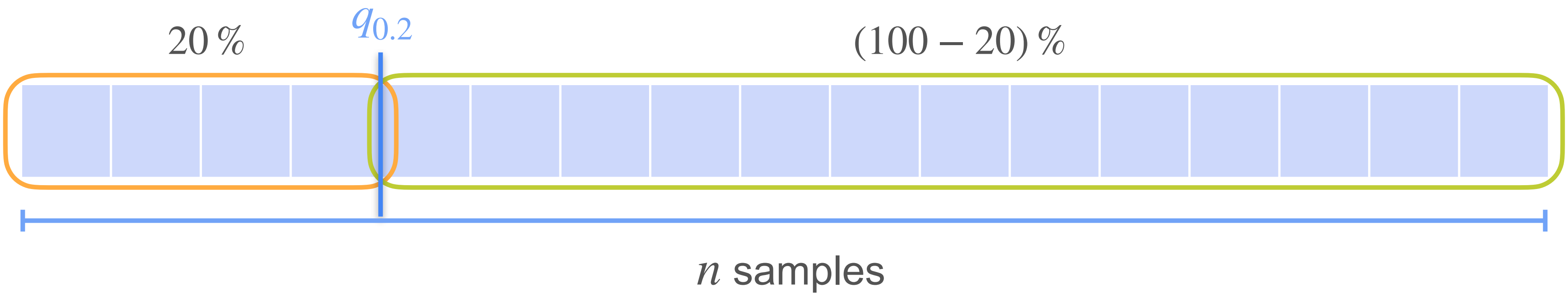
Quantiles



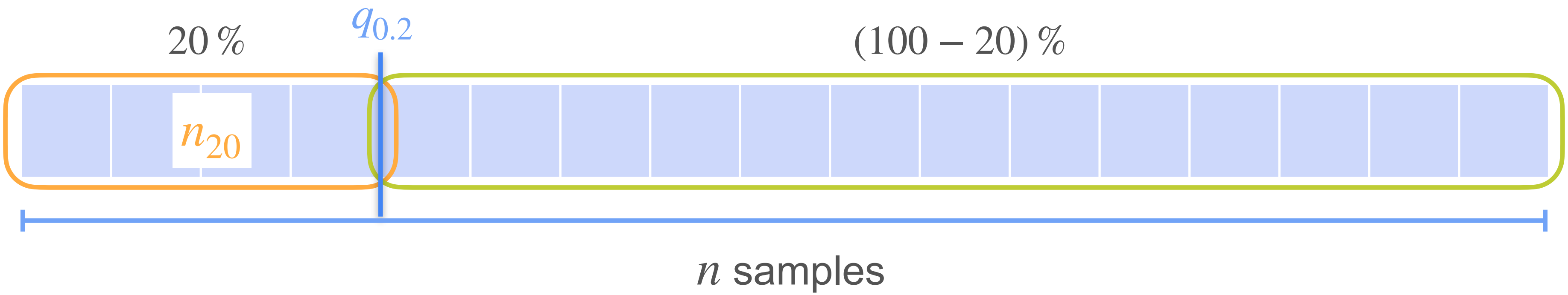
Quantiles



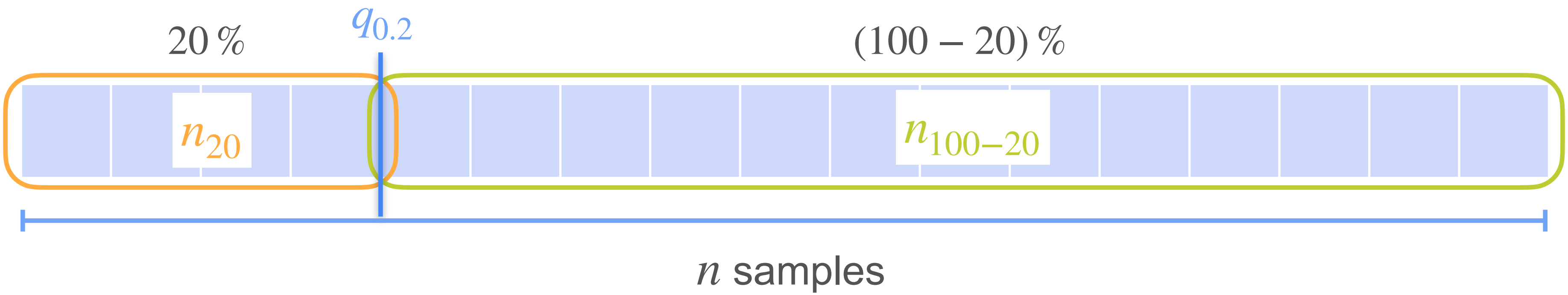
Quantiles



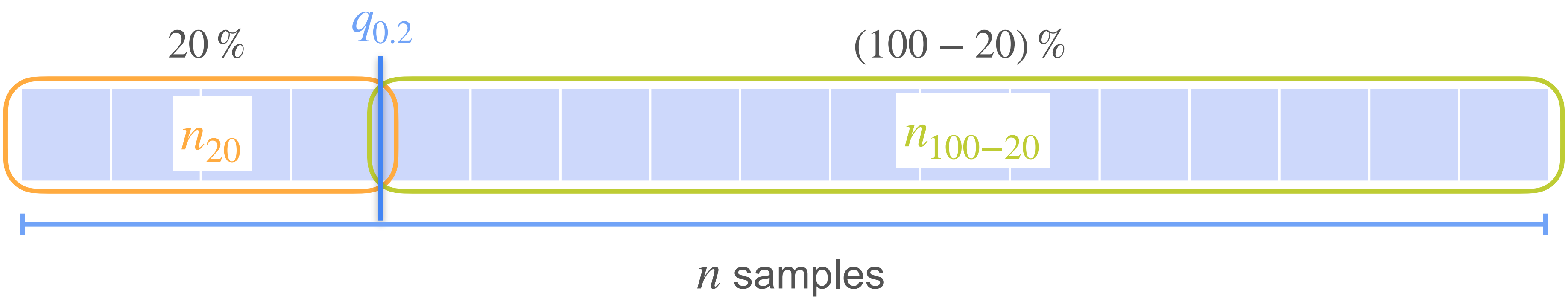
Quantiles



Quantiles

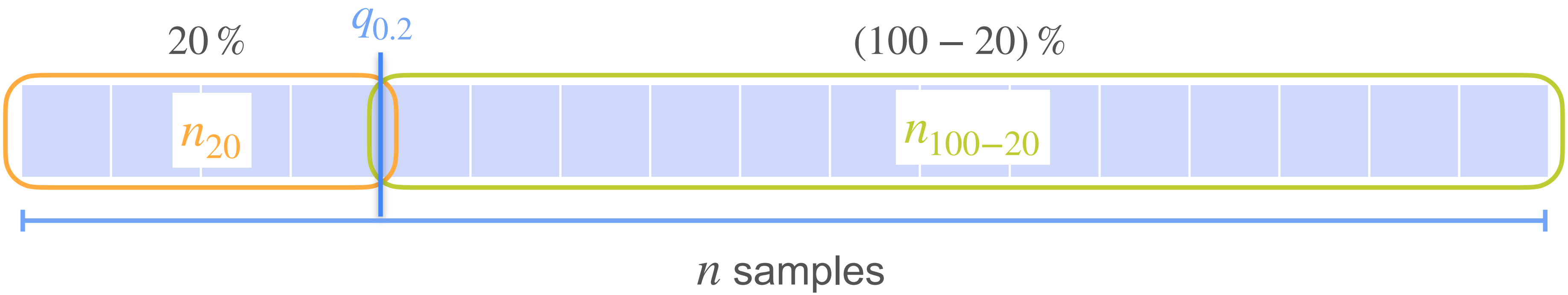


Quantiles



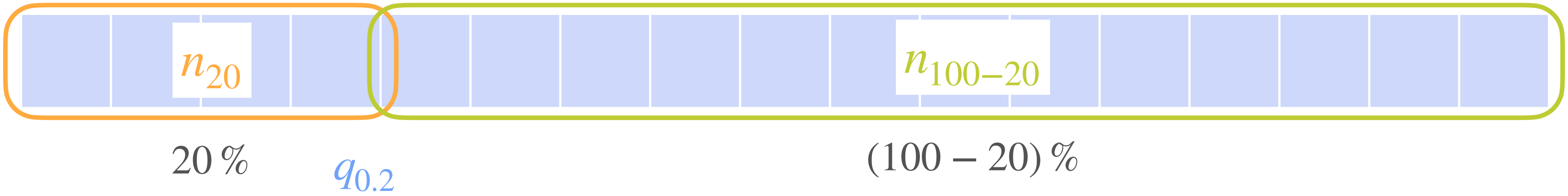
$$\frac{20}{100} = \frac{n_{20}}{n}$$

Quantiles

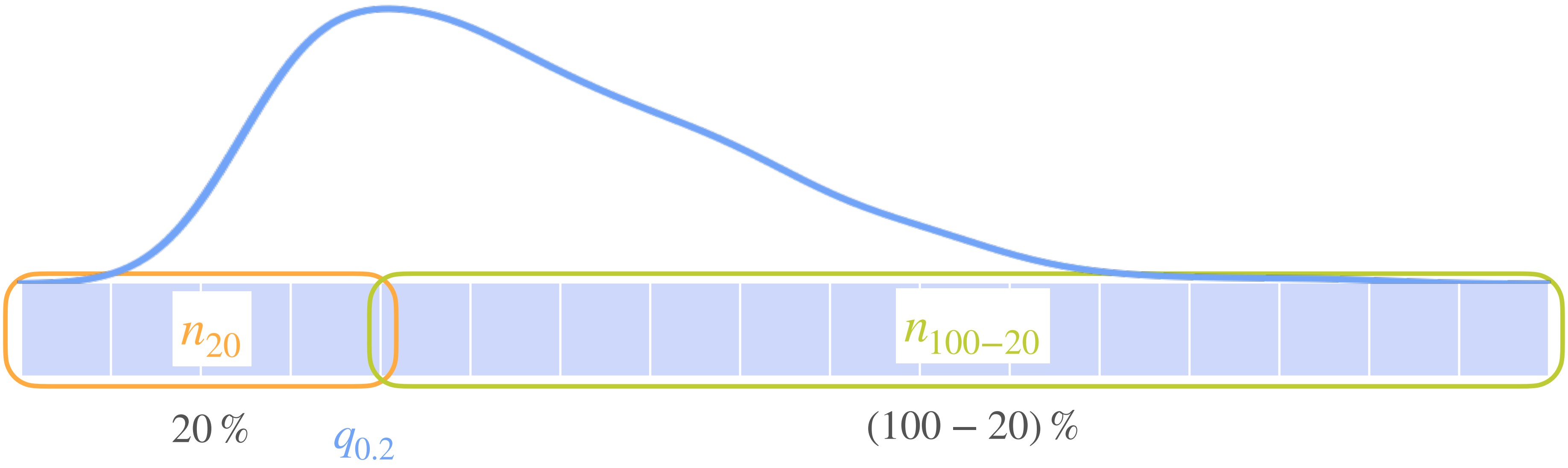


$$\frac{20}{100} = \frac{n_{20}}{n} \approx \mathbf{P}(X \leq q_{0.2})$$

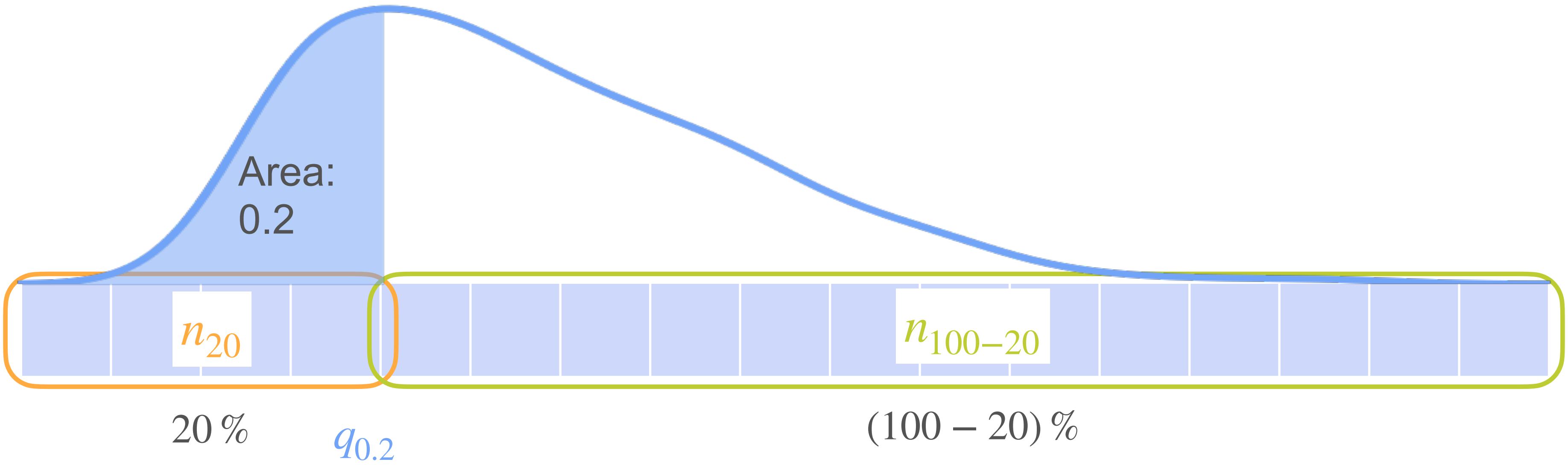
Quantiles



Quantiles



Quantiles



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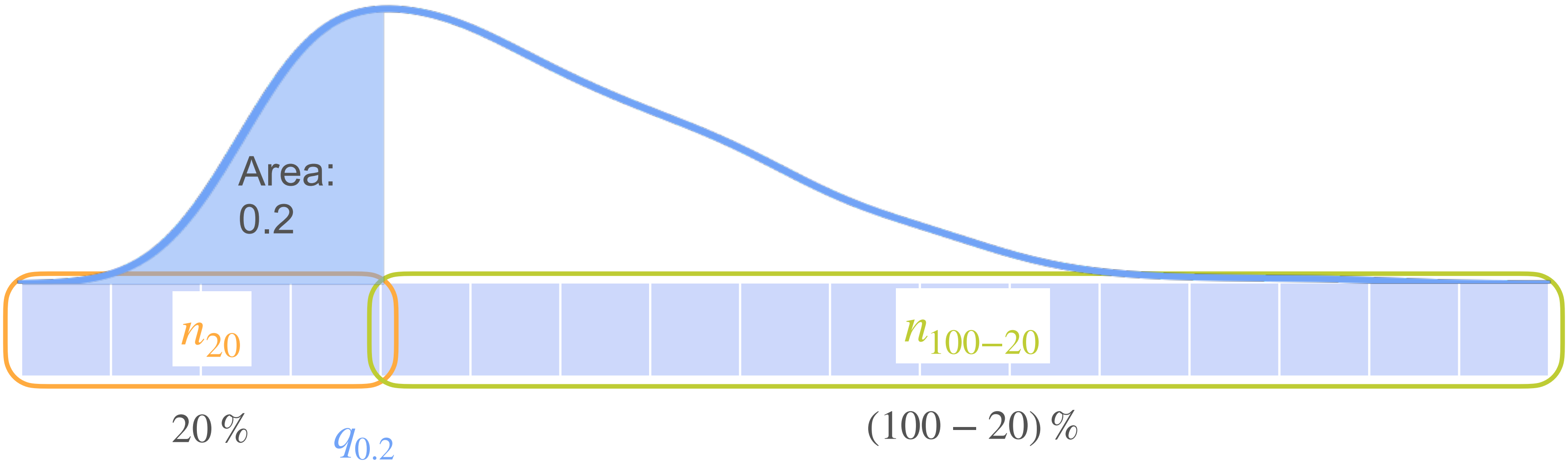
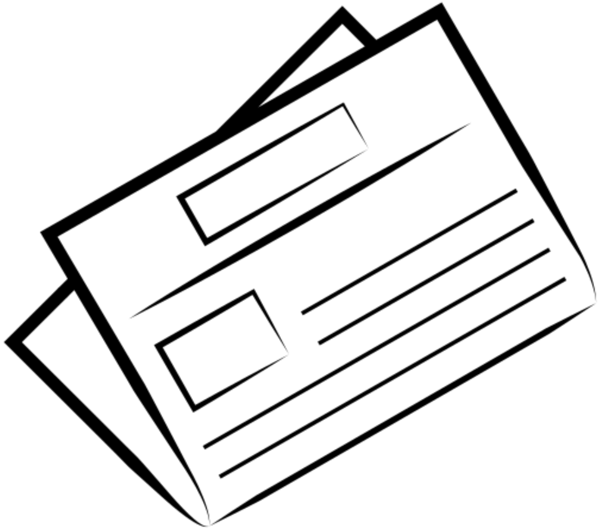


k% quantile ($q_{k/100}$) is the value such that $\mathbf{P}\left(X \leq q_{k/100}\right) = \dfrac{k}{100}$

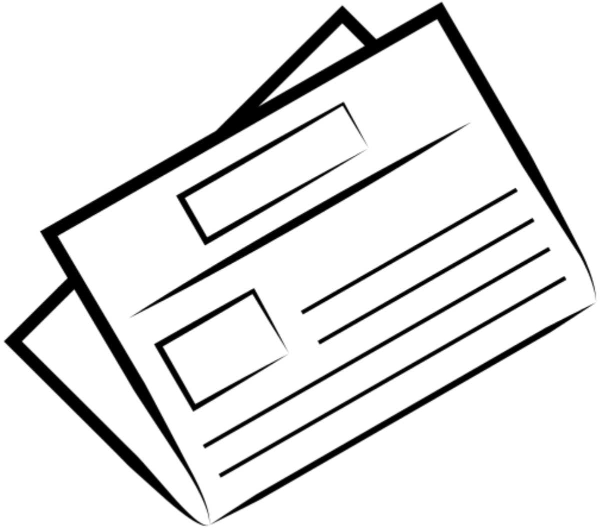
Box-Plots

Box-Plots



Newspaper advertisement

Box-Plots



Newspaper advertisement

Newspaper Advertisement (X)			
8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Box-Plots

Newspaper Advertisement (X)			
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23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Box-Plots

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Box-Plots

$$Q1 = q_{0.25} = \frac{18.3 + 18.4}{2} = 18.35$$

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Interquartile range (IQR)

$$IQR = Q3 - Q1$$

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Interquartile range (IQR)

$$IQR = Q3 - Q1 = 52.95 - 18.35 = 34.6$$

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$$IQR = Q3 - Q1 = 52.95 - 18.35 = 34.6$$

$$\min = 8.7$$

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$$\min = 8.7 \qquad \max = 75$$

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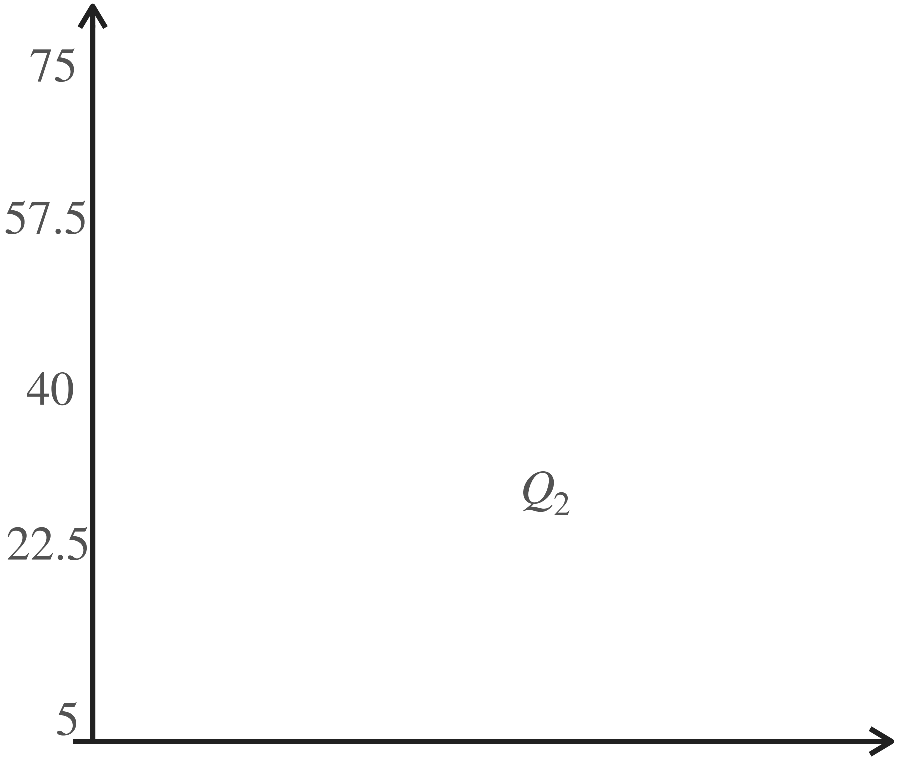

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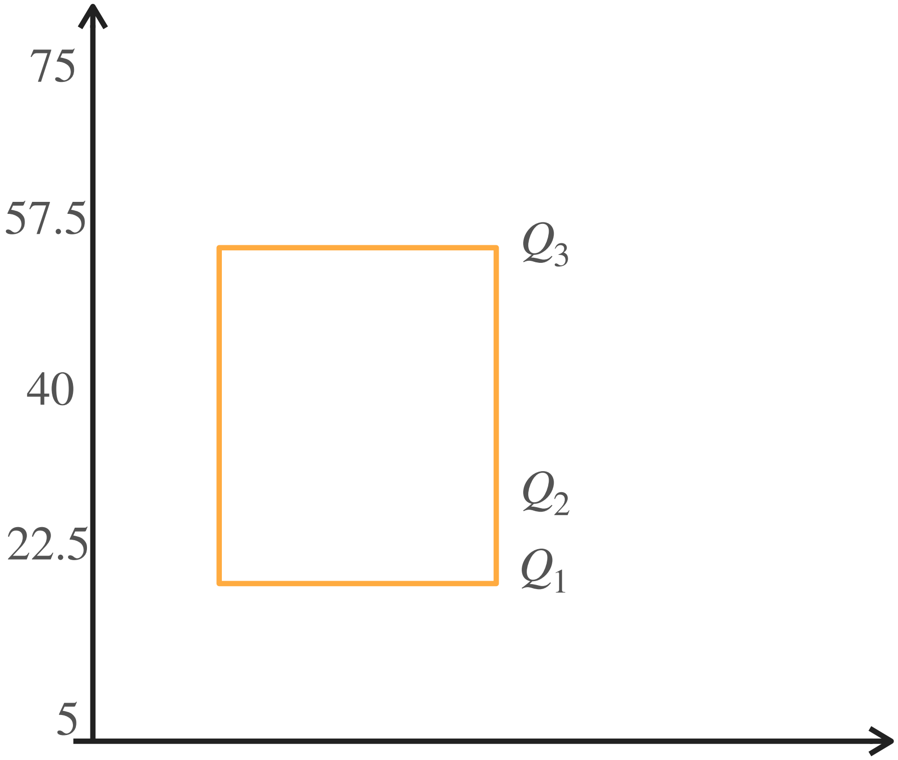

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Box-Plots

$$Q1 = q_{0.25} = \frac{18.3 + 18.4}{2} = 18.35$$

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$$IQR = Q3 - Q1 = 52.95 - 18.35 = 34.6$$

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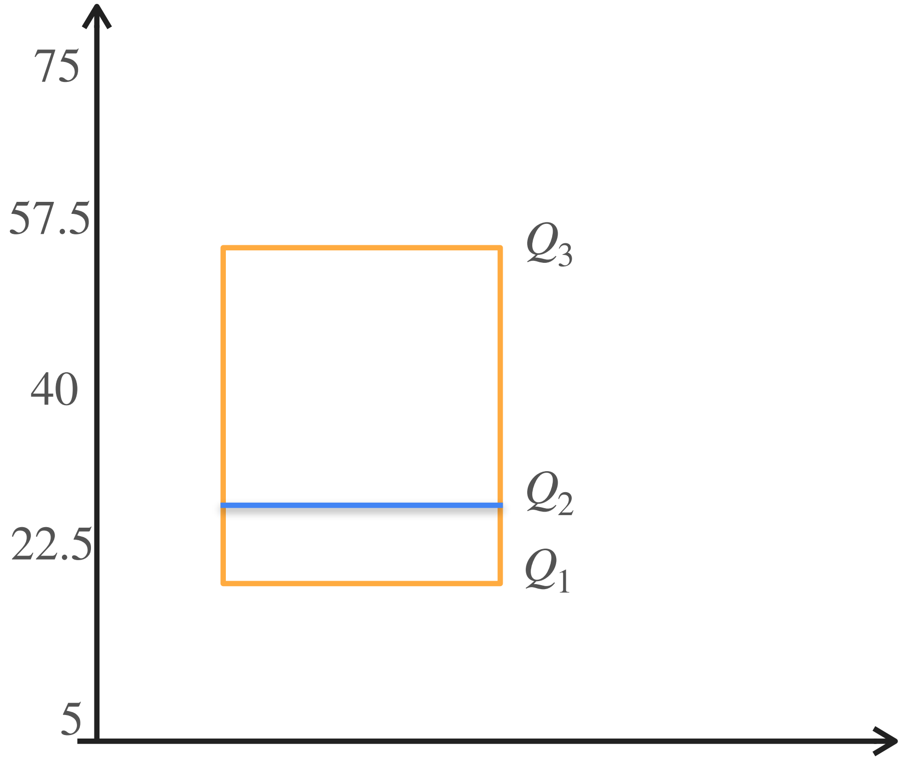

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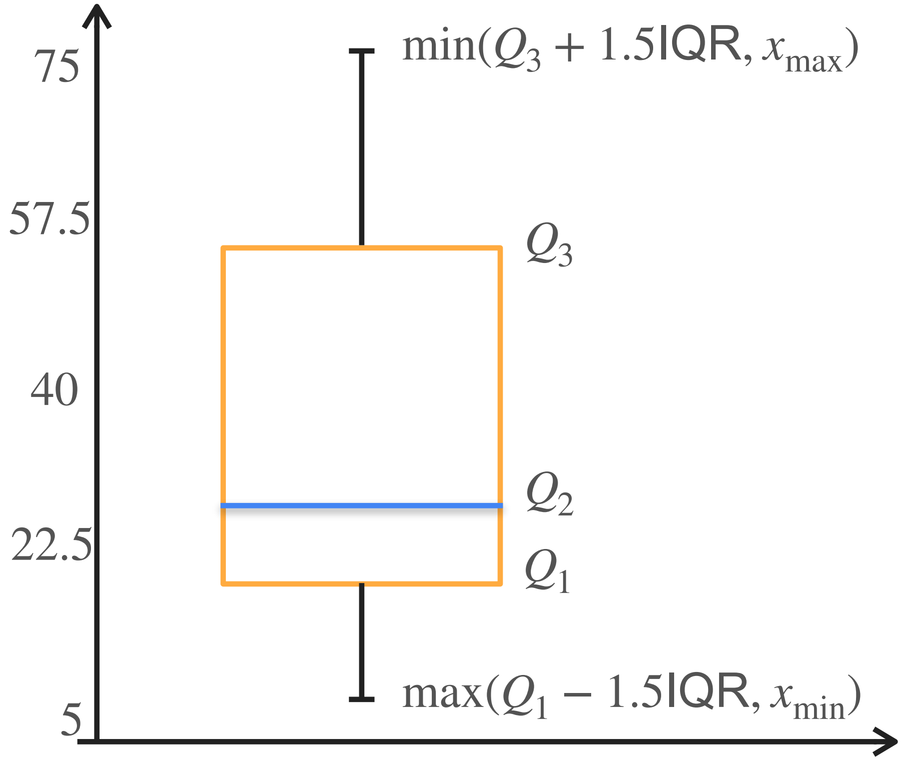
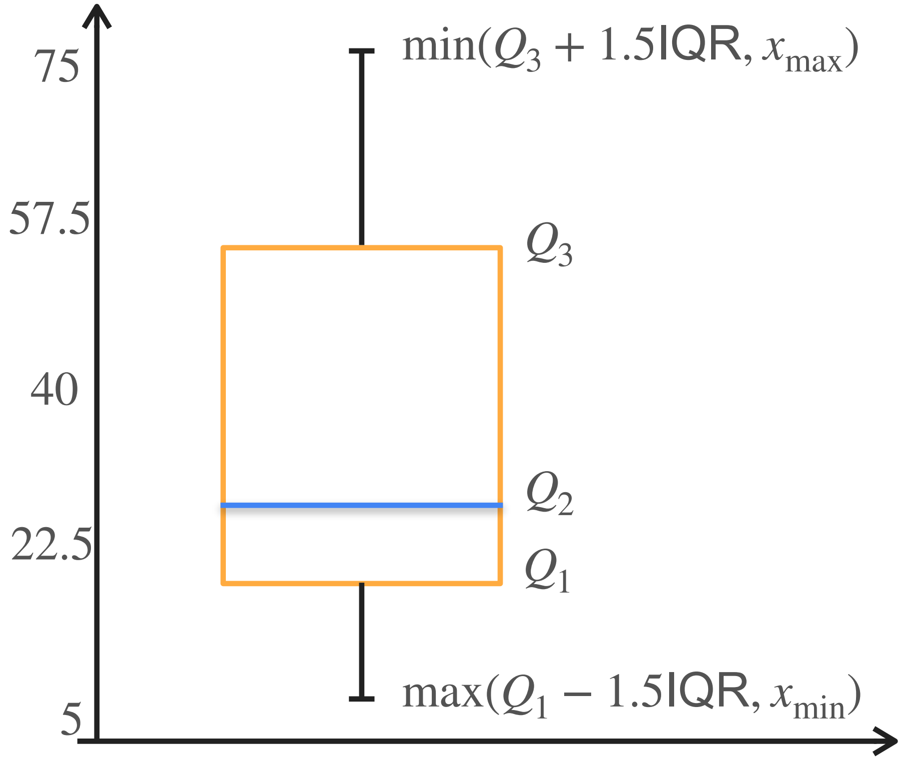


Box-Plots

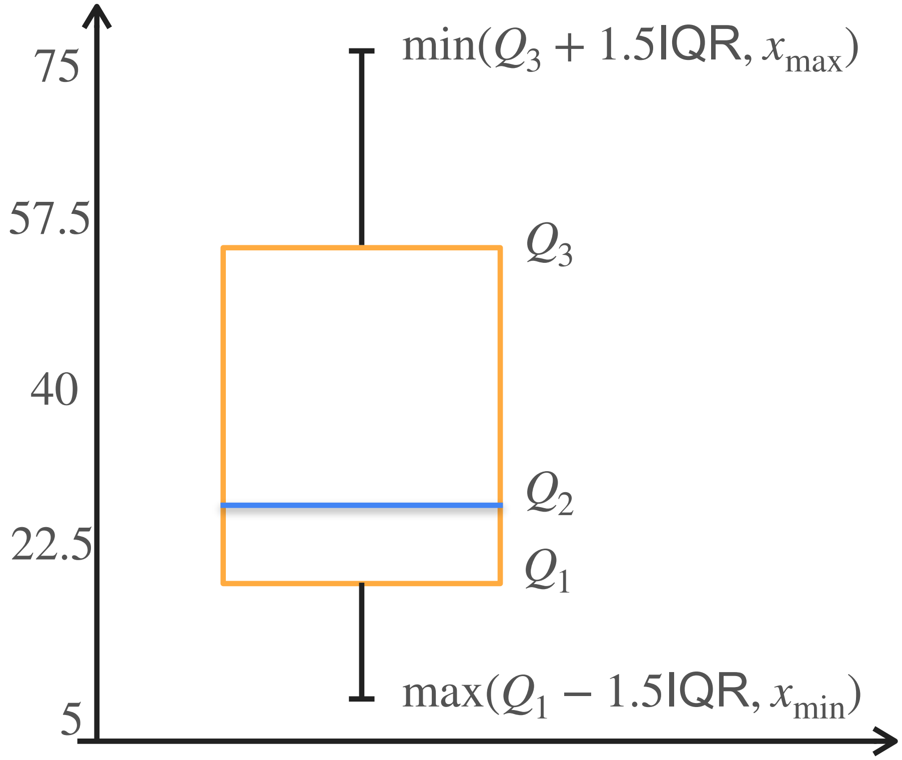


75
57.5
40
22.5
5
$\min(Q_3 + 1.5\text{IQR}, x_{\max})$
Q_3
Q_2
Q_1
$\max(Q_1 - 1.5\text{IQR}, x_{\min})$

Box-Plots

What can you tell from this plot?

- Data is skewed
- No outliers (whiskers were cut at max and min value)
- Analyze dispersion



Box-Plots

Box-Plots

Let's see how the box-plot looks for the whole dataset

Box-Plots

Let’s see how the box-plot looks for the whole dataset

$Q_1 = 12.75$ $Q_2 = 25.75$ $Q_3 = 45.1$

IQR $= 32.35$ $x_{\text{min}} = 0.3$ $x_{\text{max}} = 114$

Box-Plots

Let's see how the box-plot looks for the whole dataset

$Q_1 = 12.75$ $Q_2 = 25.75$ $Q_3 = 45.1$

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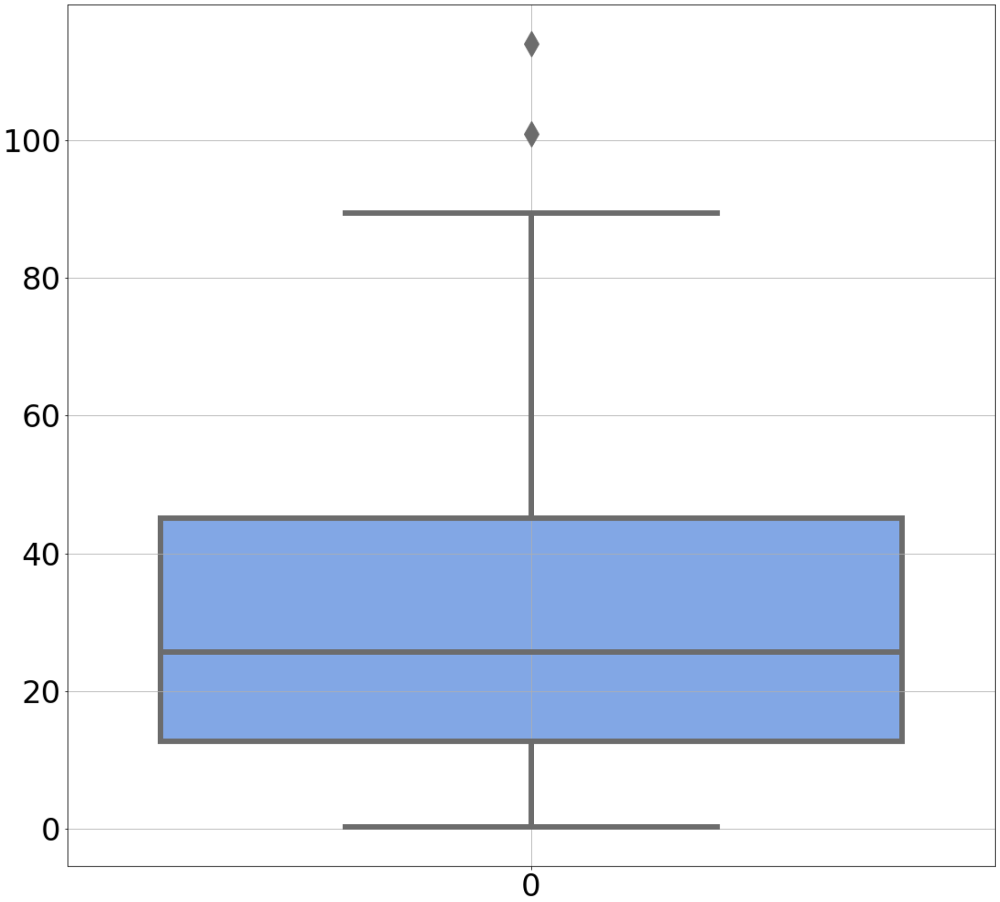

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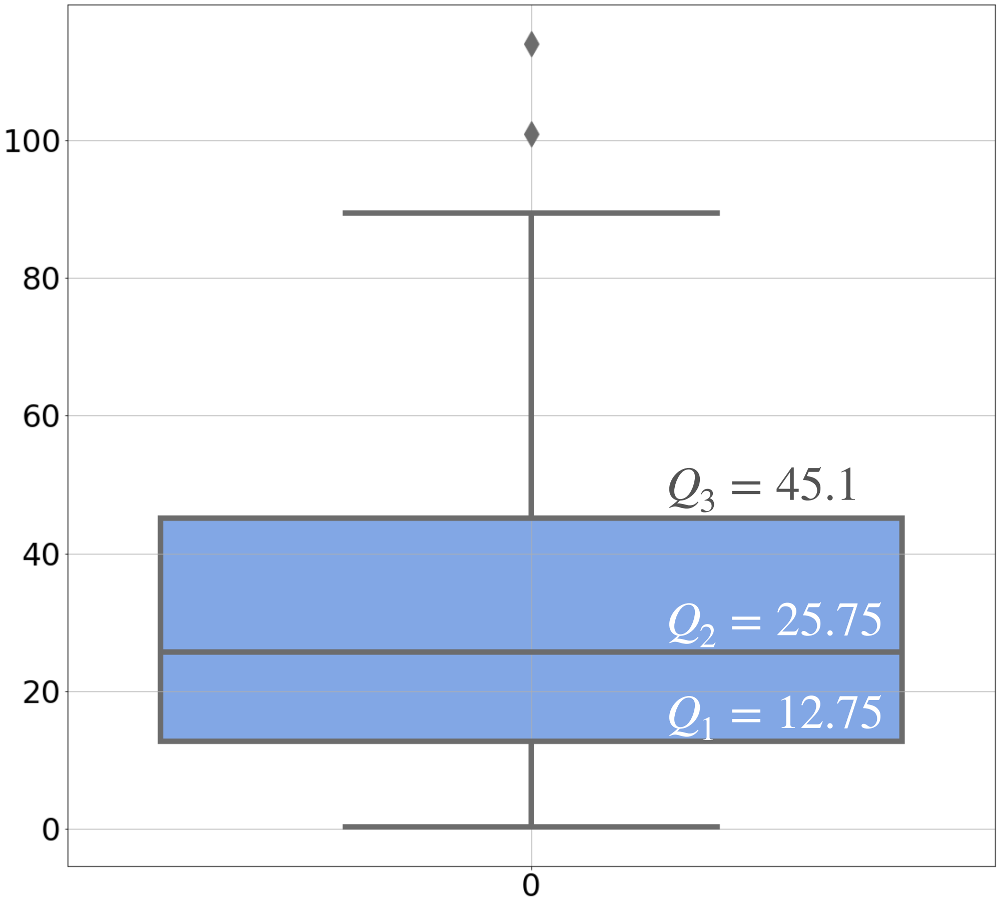

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$Q_1 = 12.75$ $Q_2 = 25.75$ $Q_3 = 45.1$

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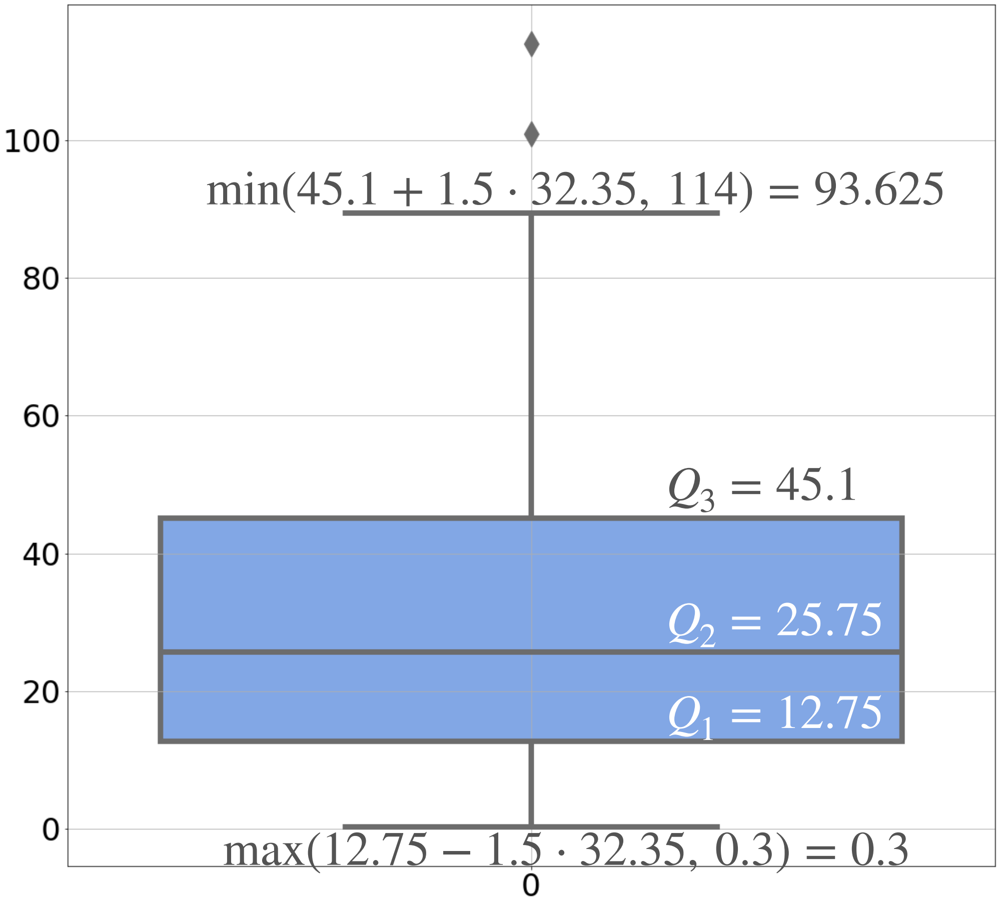


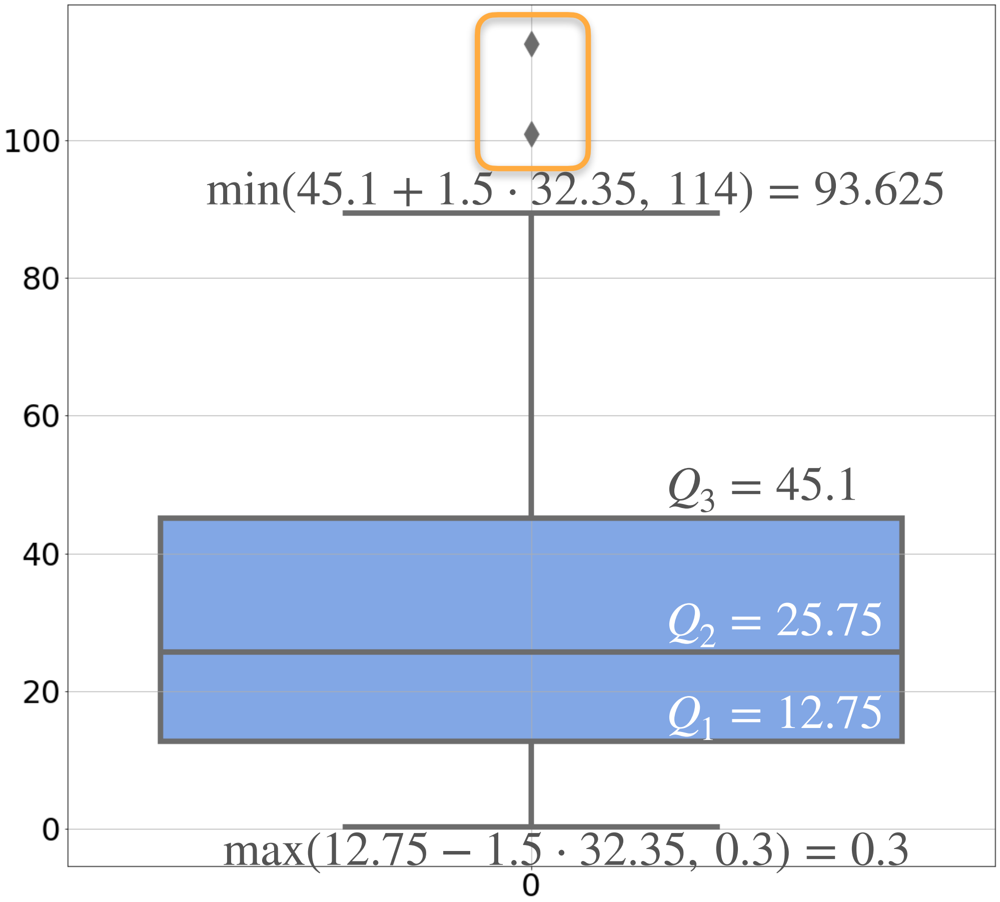
Box-Plots

Let's see how the box-plot looks for the whole dataset

$Q_1 = 12.75$ $Q_2 = 25.75$ $Q_3 = 45.1$

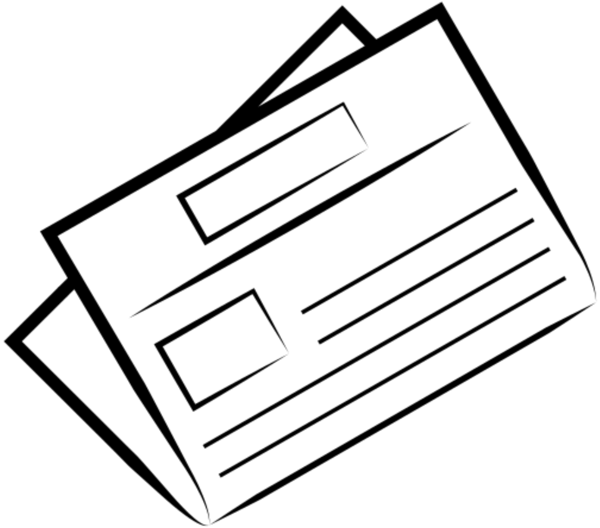
IQR $= 32.35$ $x_{\min} = 0.3$ $x_{\max} = 114$

Now you can see two outliers



Density Estimation

Density Estimation

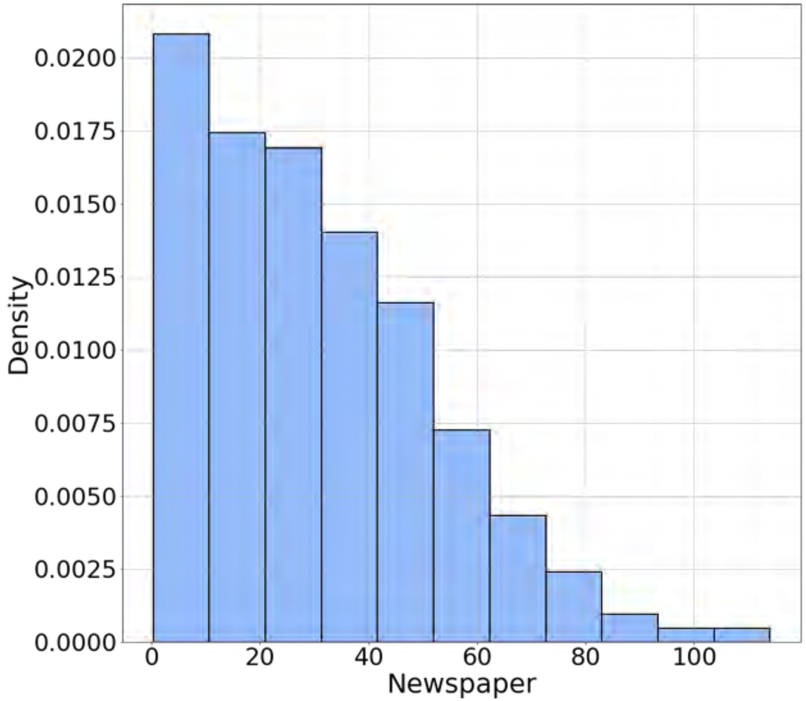


Newspaper advertisement

Newspaper Advertisement (X)			
8.7	14.2	18.3	18.4
23.2	25.9	29.7	35.2
51.2	54.7	65.9	75

Histograms

Histograms

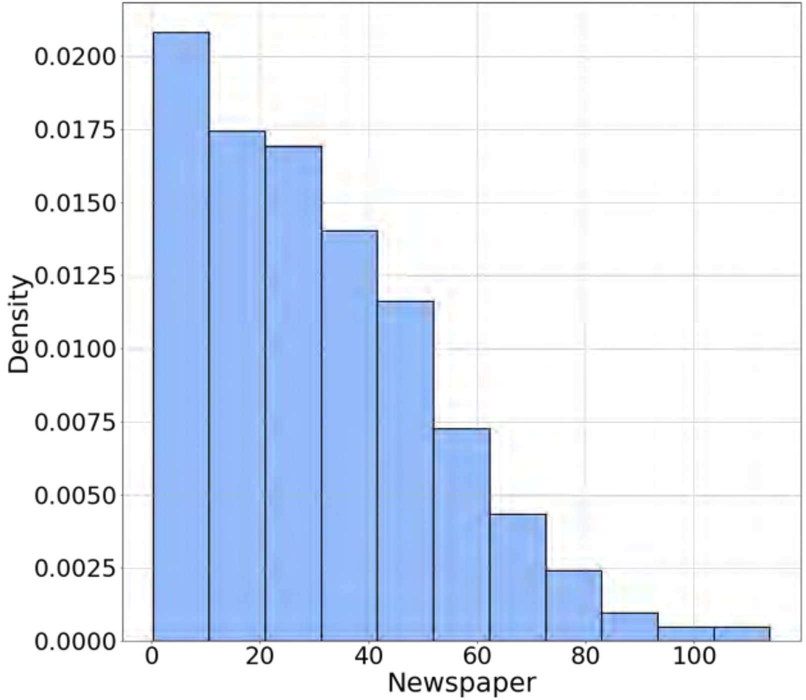


0.0200
0.0175
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
Density
0
20
40
60
80
100
Newspaper

Histograms

It represents a density function

- It is positive
- Area under the curve is 1



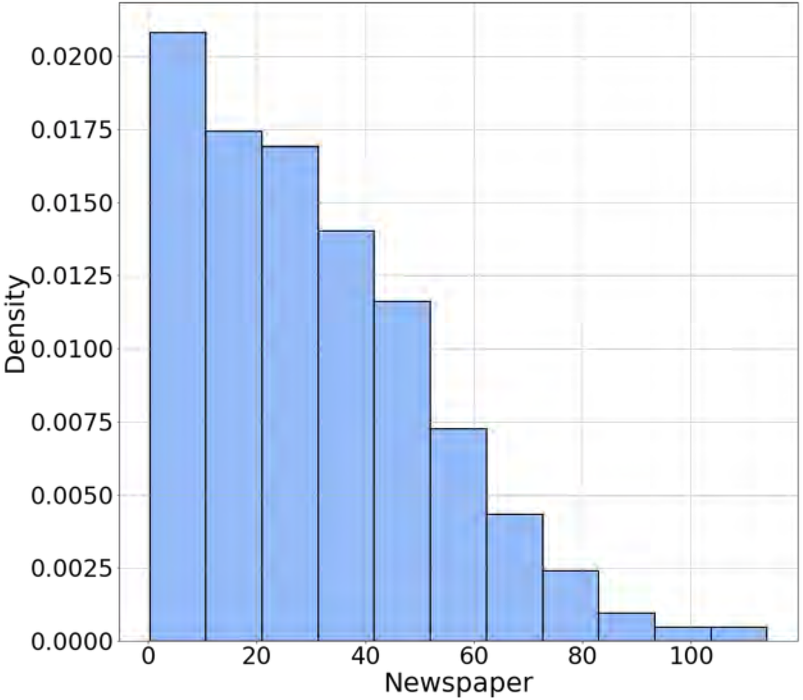
Histograms

It represents a density function

- It is positive
- Area under the curve is 1

But…

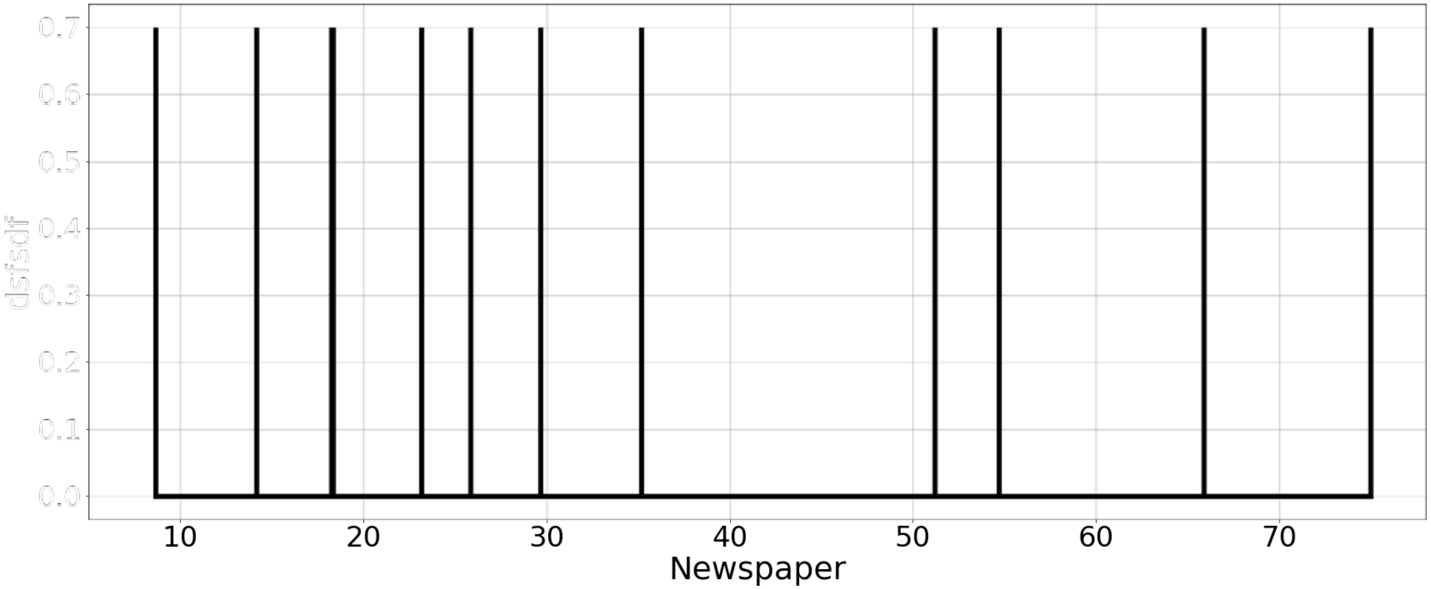
- PDFs are usually smooth function
- The discontinuities come from the method and not the data



Kernel Density Estimation

Kernel Density Estimation

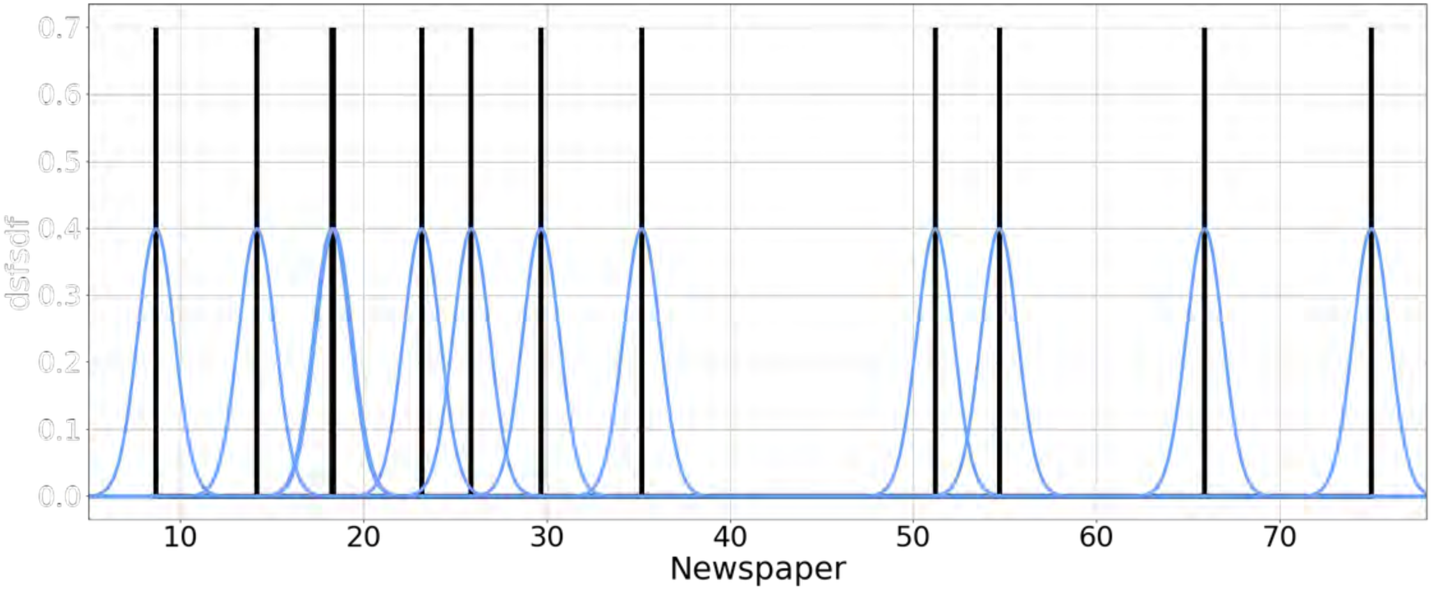
First: draw your observations along the x axis



Kernel Density Estimation

Kernel Density Estimation

Second: draw a gaussian centered at each observation



Kernel Density Estimation

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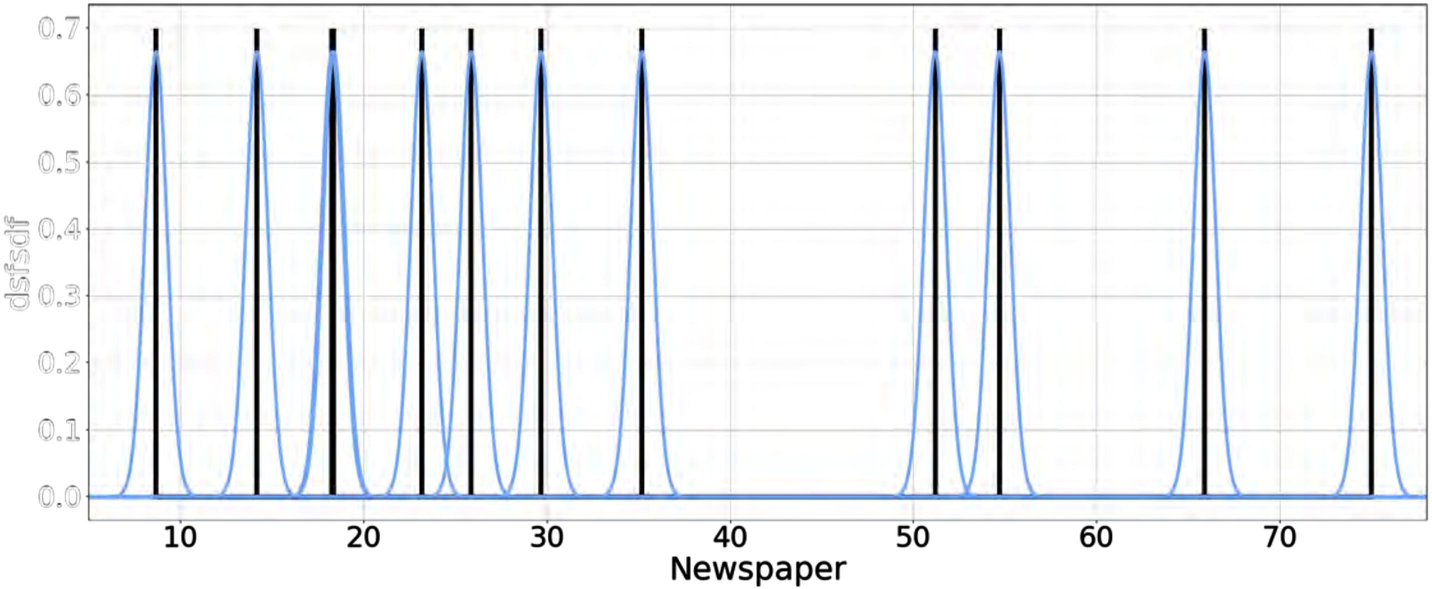

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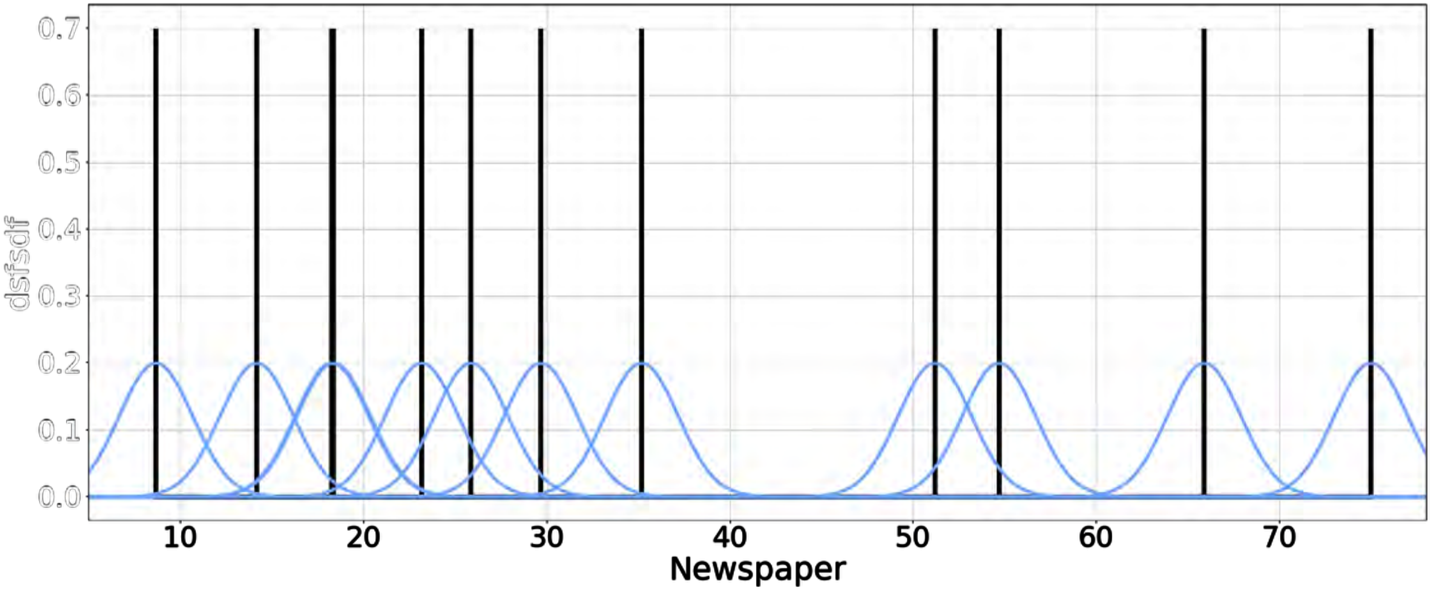

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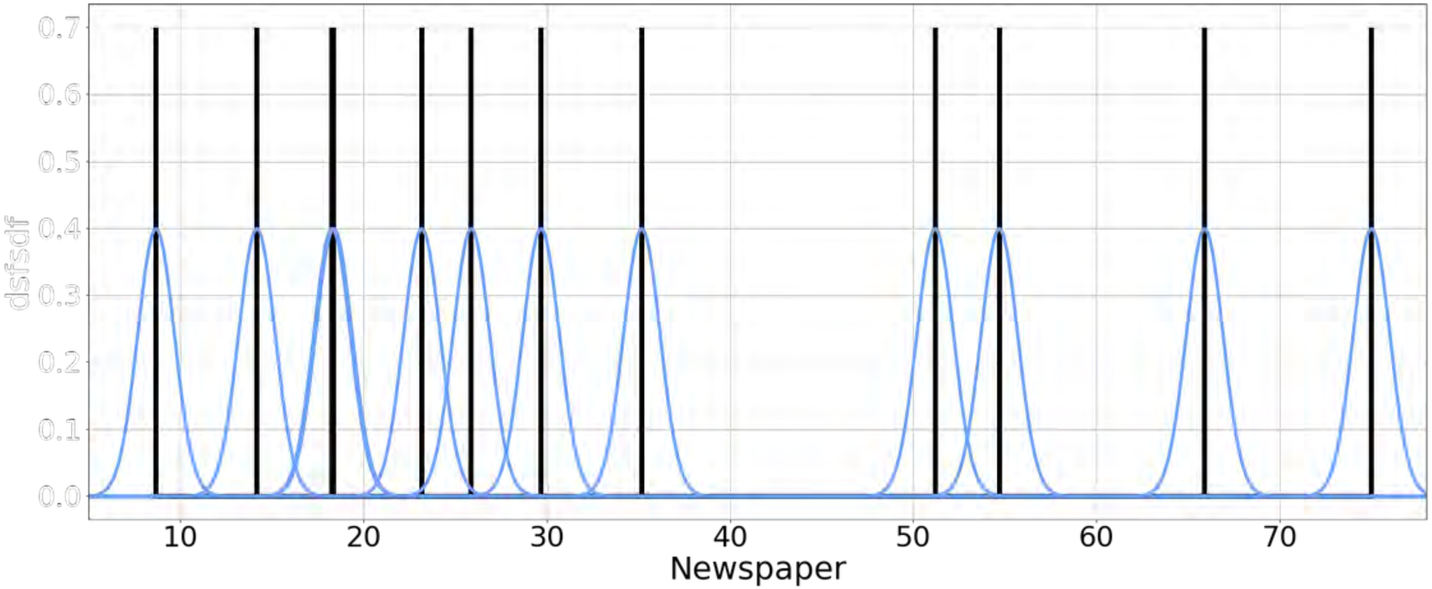
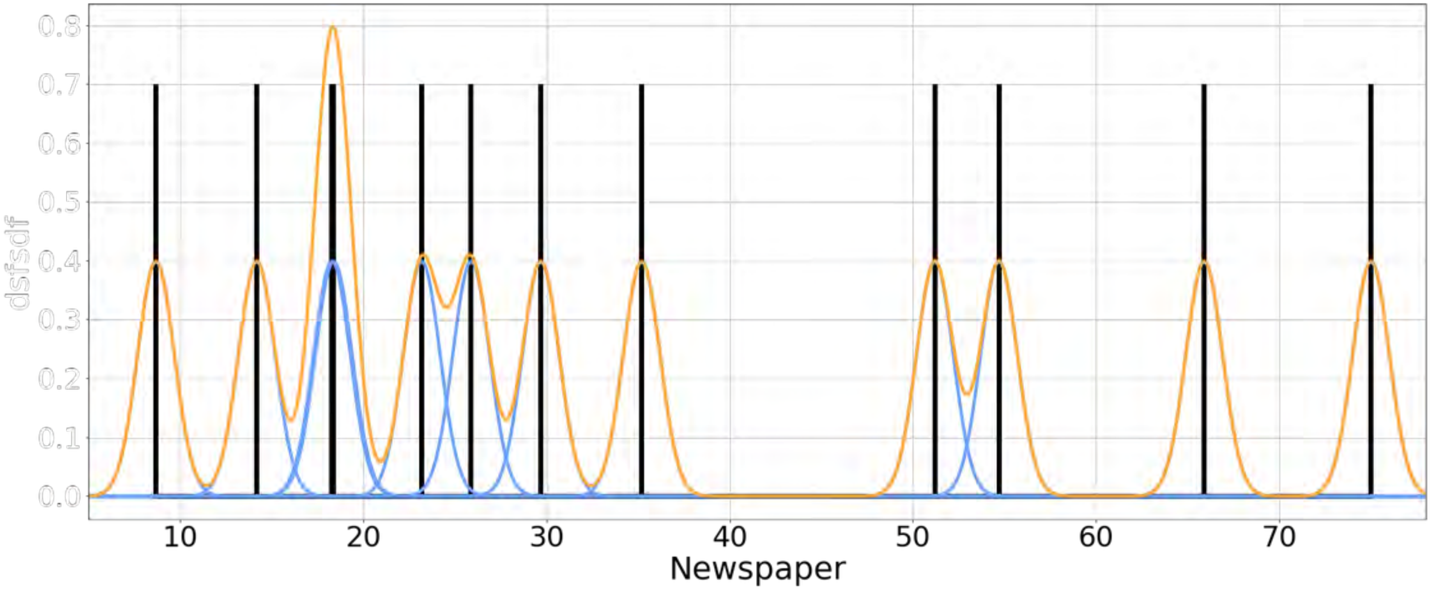
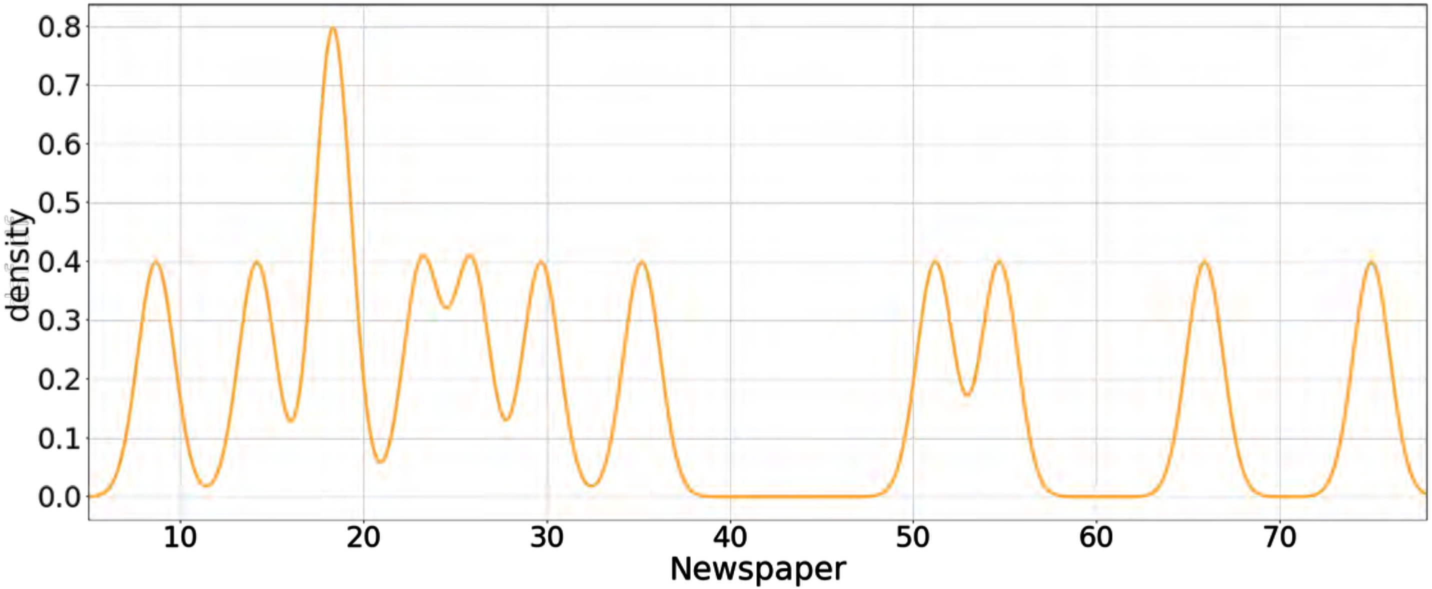


Kernel Density Estimation



Kernel Density Estimation

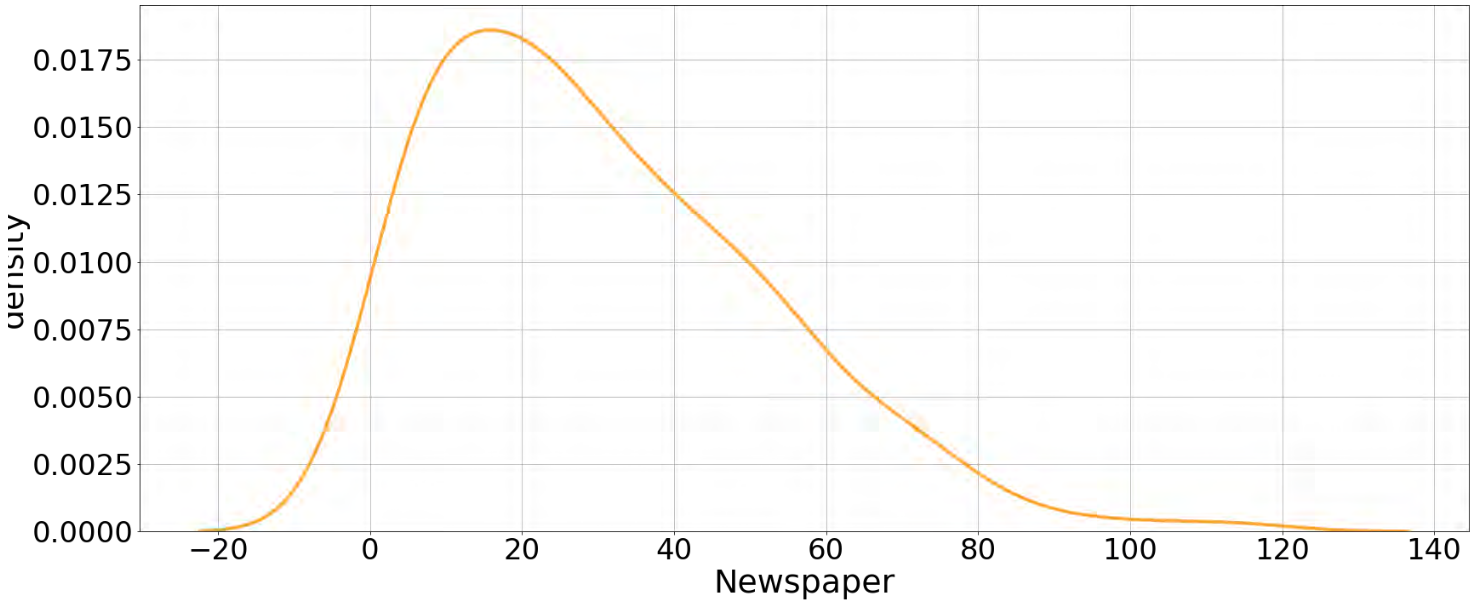
Third: multiply everything by 1/n and sum the curves



Kernel Density Estimation

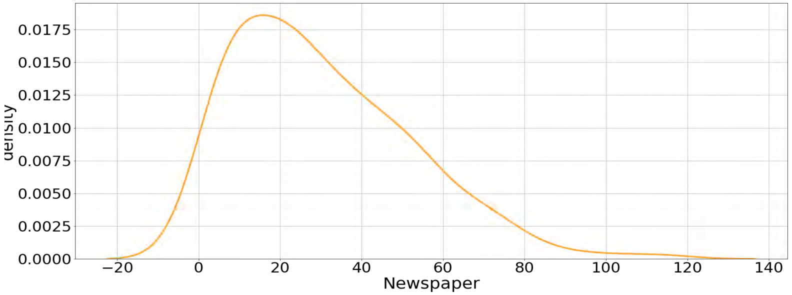
Kernel Density Estimation

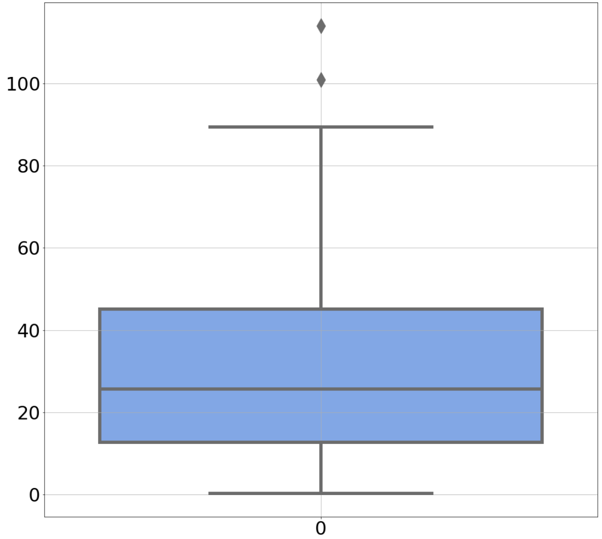
What if you used all the dataset?



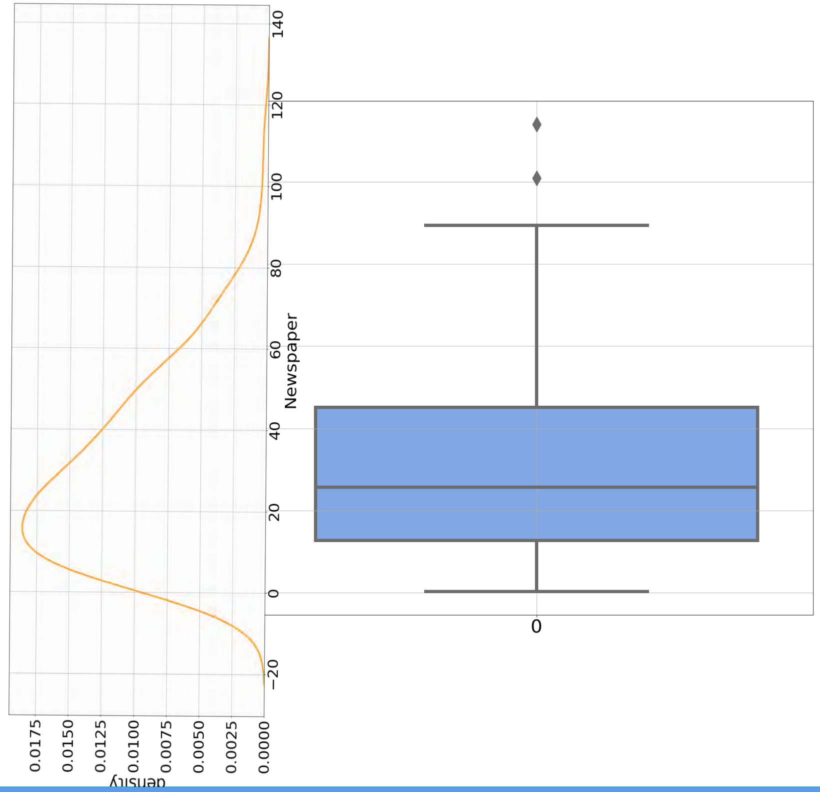
Violin Plots

Violin Plots





Violin Plots



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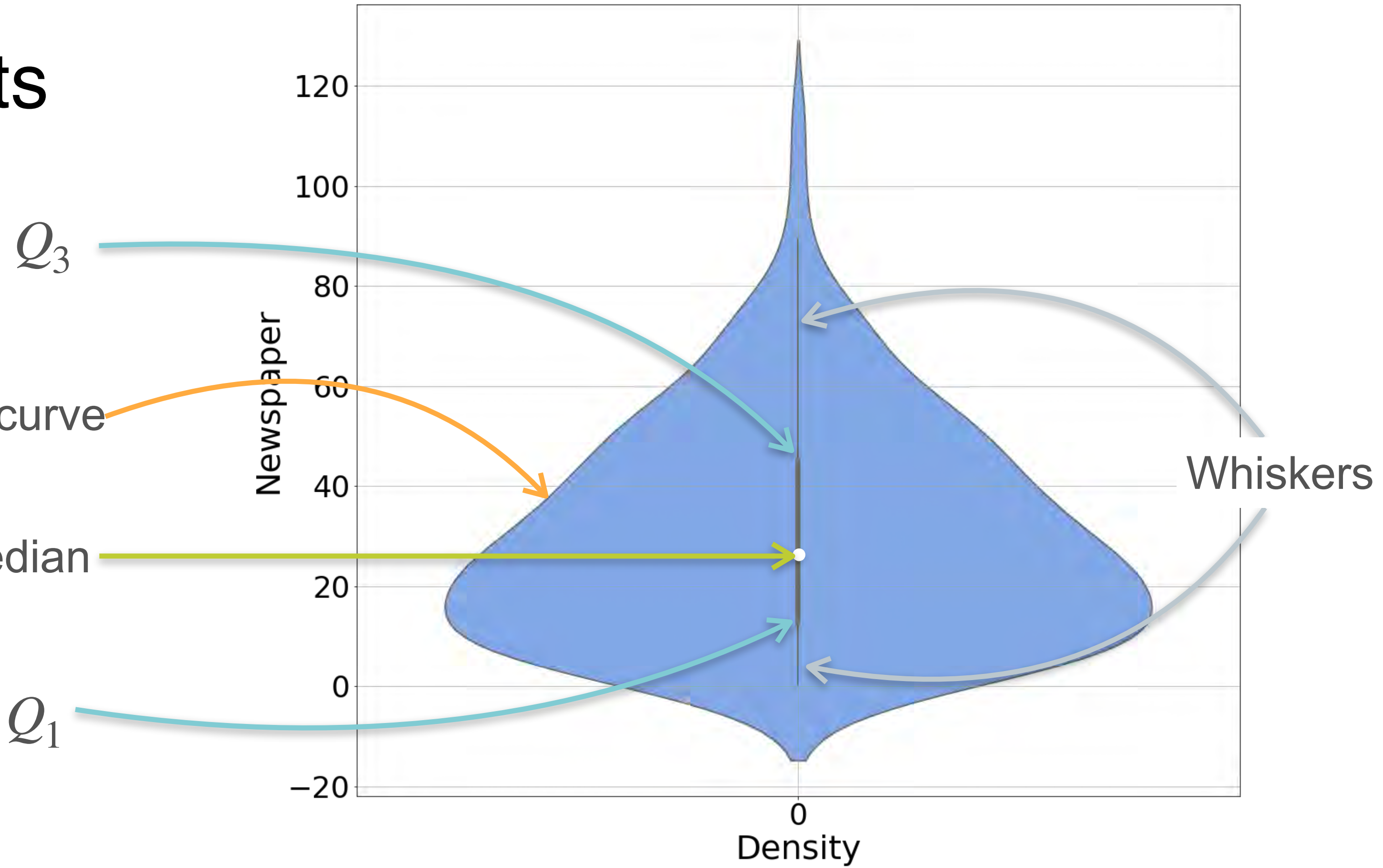
Violin Plots

Violin Plots



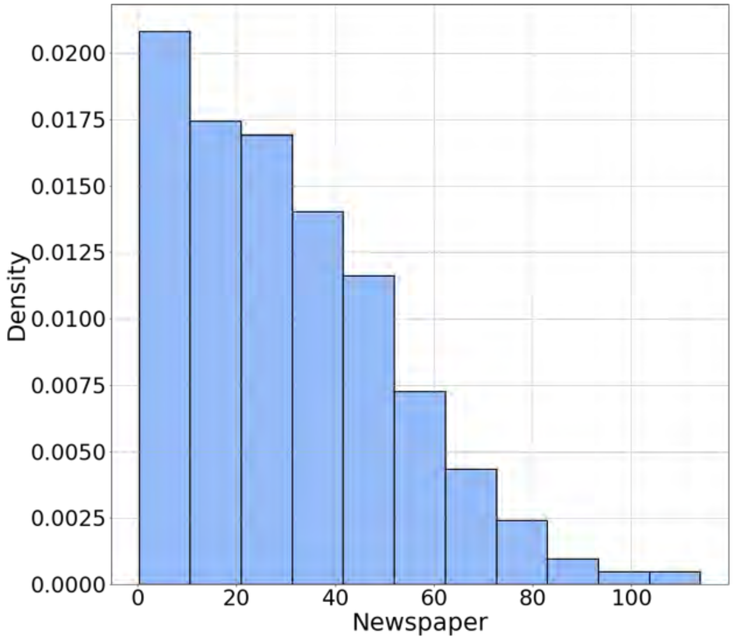
Assessing Normality of Data

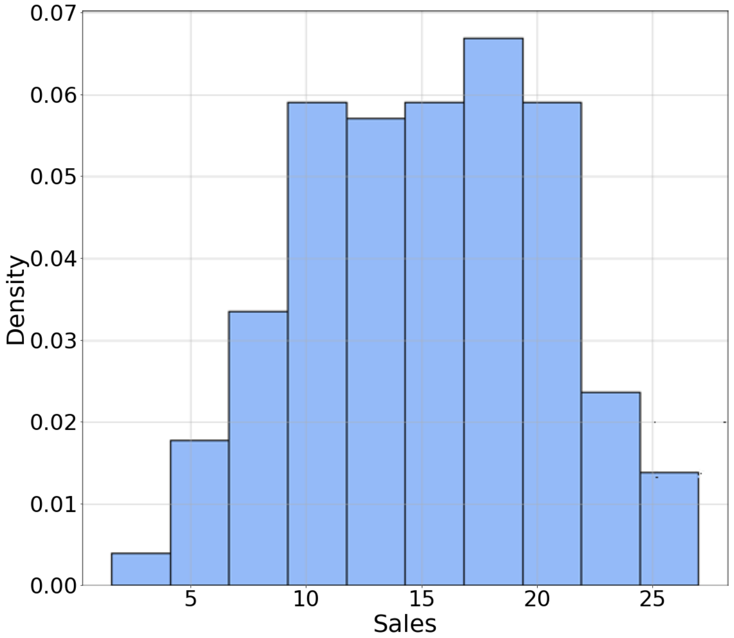
Some models assume normally distributed data

- Linear regression
- Logistic regression
- Gaussian Naive Bayes
- Others

Some tests used in Data Science also assume normality.

Assessing Normality of Data





QQ Plots

Newspaper

QQ Plots

Newspaper

Quantile-Quantile plots (QQ Plots) compare quantiles

QQ Plots

Newspaper

Quantile-Quantile plots (QQ Plots) compare quantiles

- Standardize your data:

$$\left(\frac{x-\mu}{\sigma}\right)$$

QQ Plots

Newspaper

Quantile-Quantile plots (QQ Plots) compare quantiles

- Standardize your data:

$$\left(\frac{x - \mu}{\sigma}\right)$$

- Compute quantiles

QQ Plots

Newspaper

Quantile-Quantile plots (QQ Plots) compare quantiles

- Standardize your data:
$$\left(\frac{x-\mu}{\sigma}\right)$$
- Compute quantiles
- Compare to gaussian quantiles

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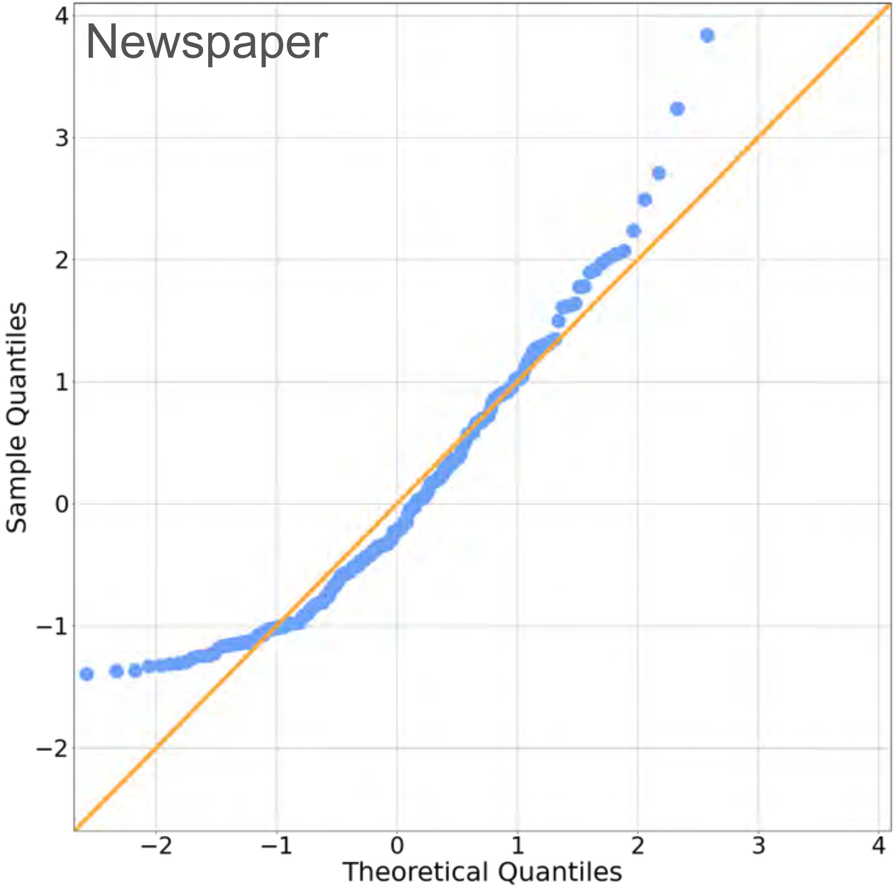

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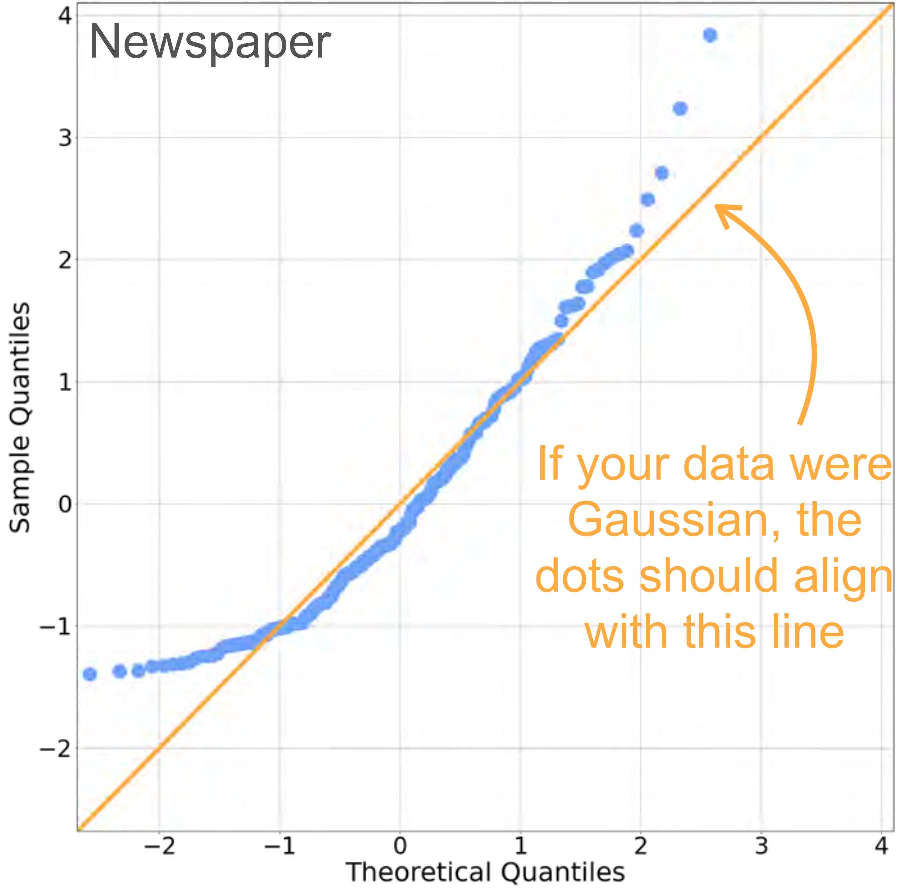

QQ Plots

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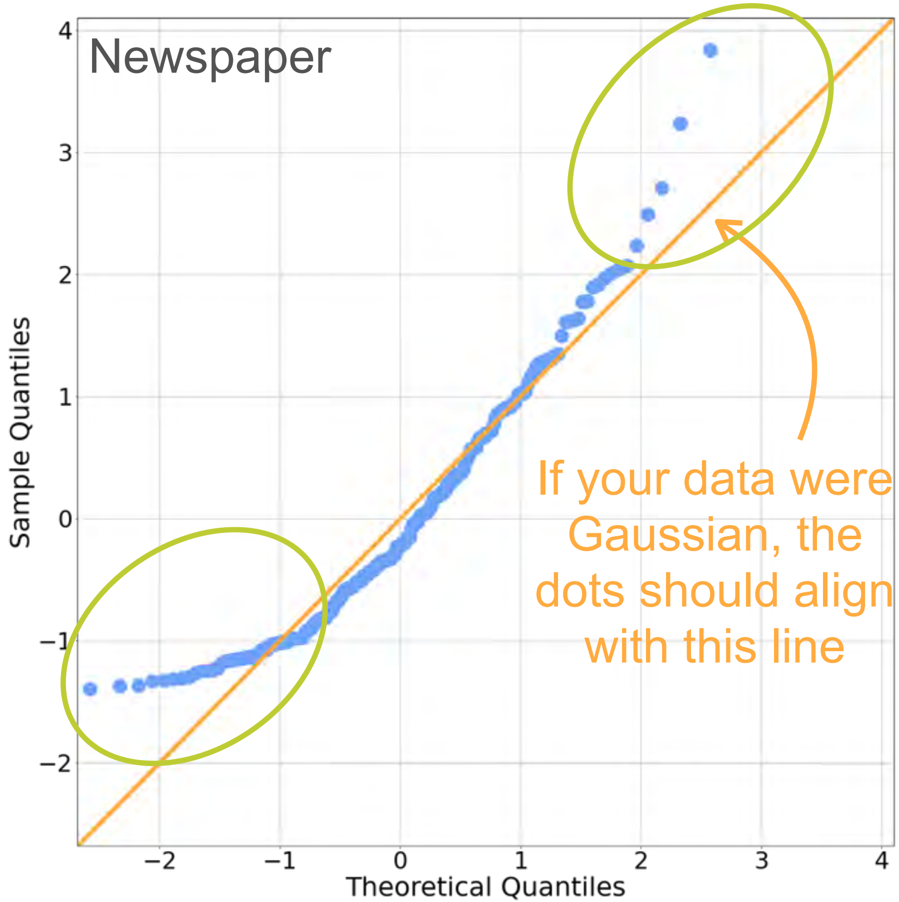
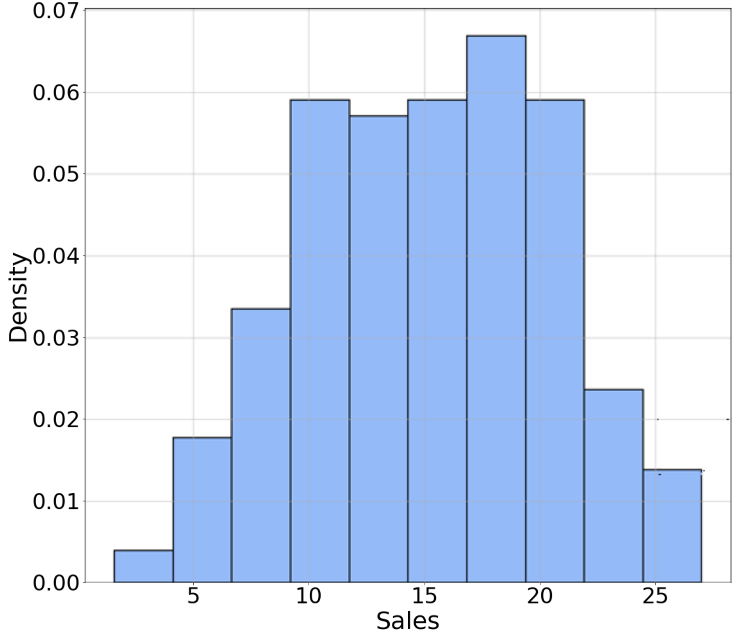
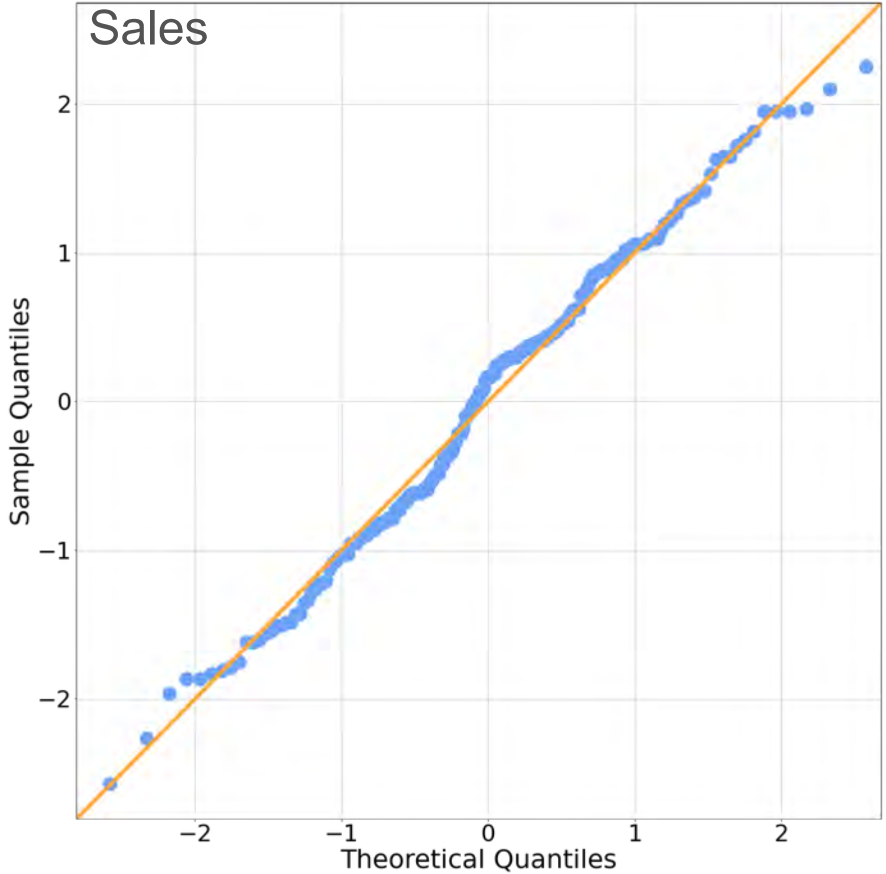


QQ Plots



0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
Density
5
10
15
20
25
Sales



Sales
2
1
0
−1
−2
Sample Quantiles
−2
−1
0
1
2
Theoretical Quantiles

W2 Lesson 2

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Probability Distributions with Multiple Variables

Joint Distribution (Discrete)

Joint Distributions (Discrete): Example 1

Joint Distributions (Discrete): Example 1

Age (Year)	Count
7	3
8	2
9	4
10	1

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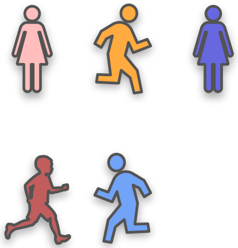

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Age (Year)	Count
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8	2
9	4
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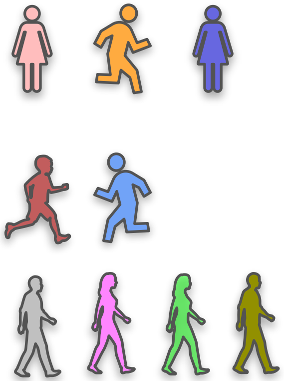

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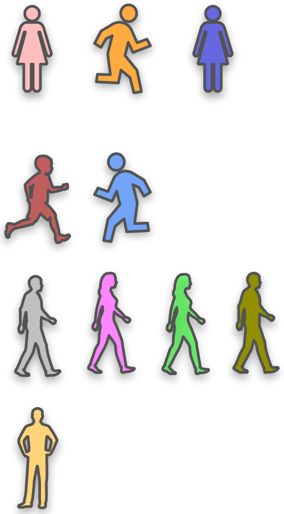
Age (Year)	Count
7	3
8	2
9	4
10	1



Joint Distributions (Discrete): Example 1

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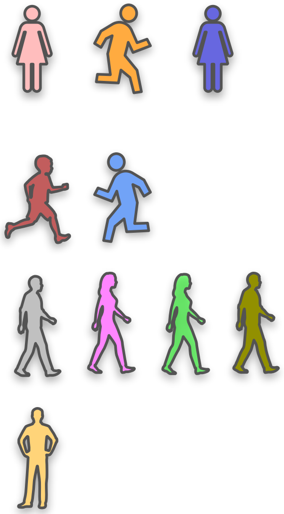
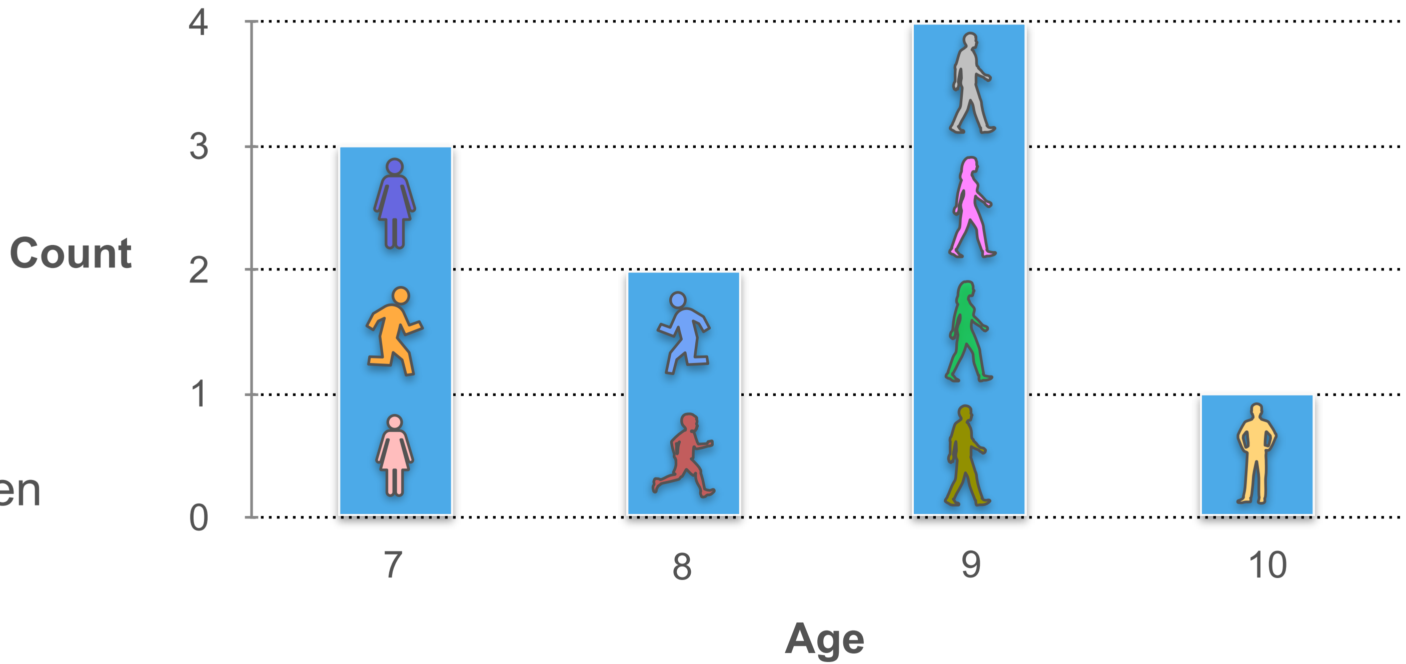


10

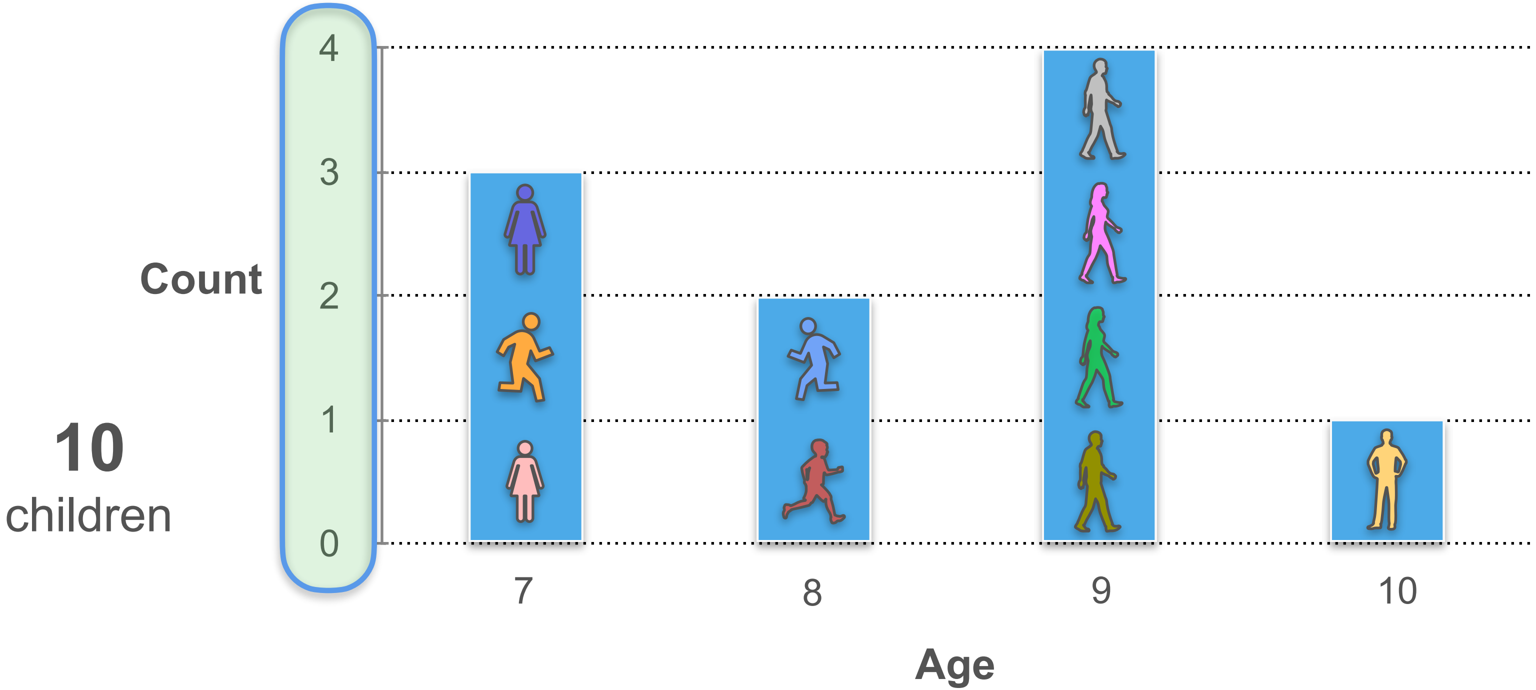
children

Joint Distributions (Discrete): Example 1



10
children

Joint Distributions (Discrete): Example 1



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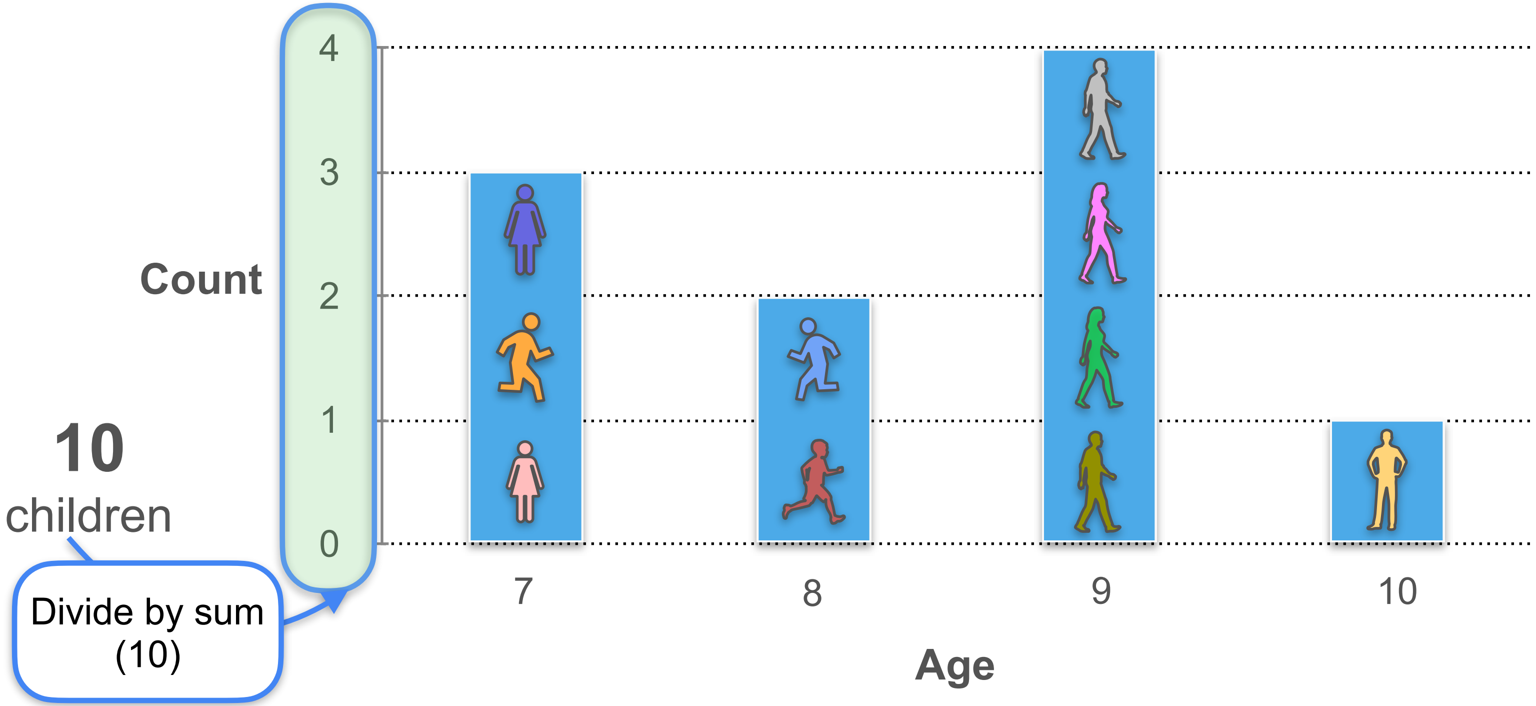

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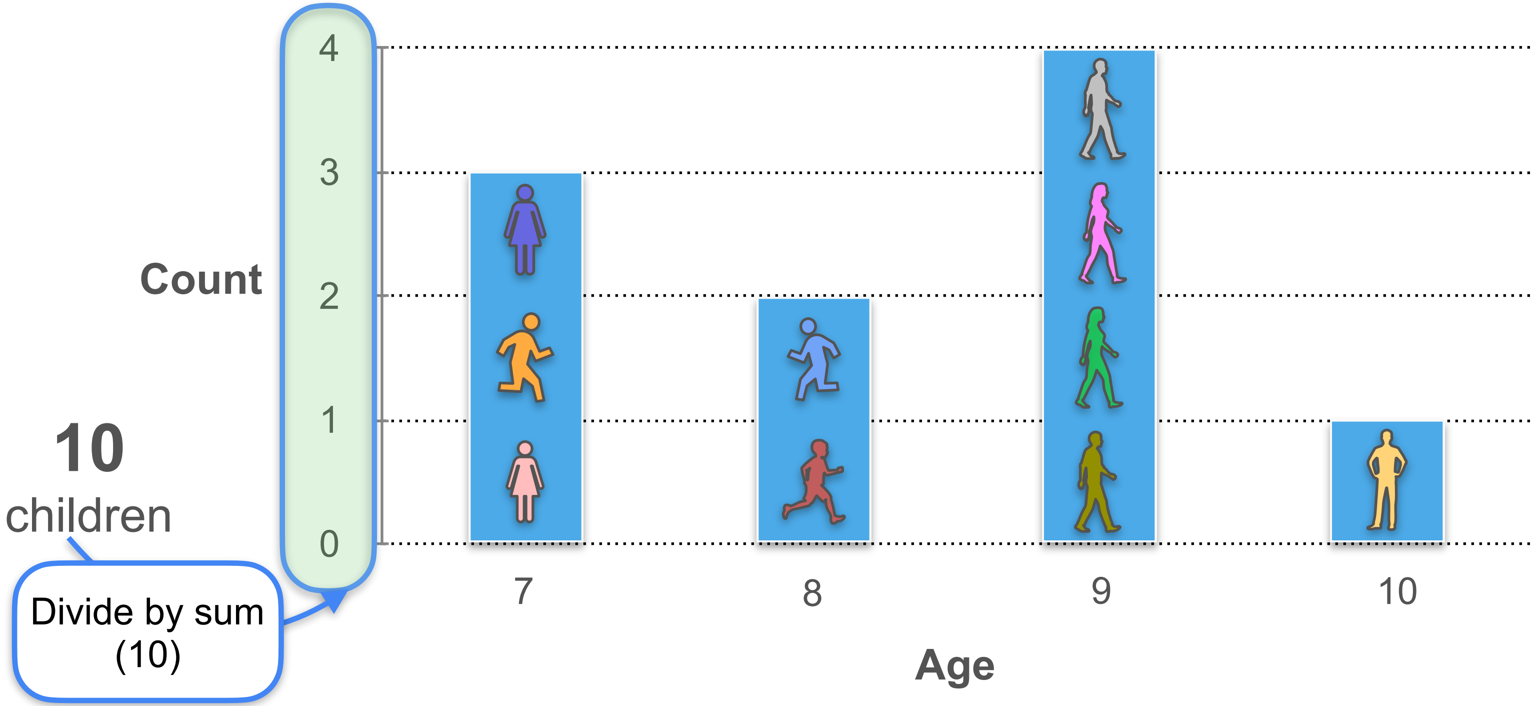
Joint Distributions (Discrete): Example 1



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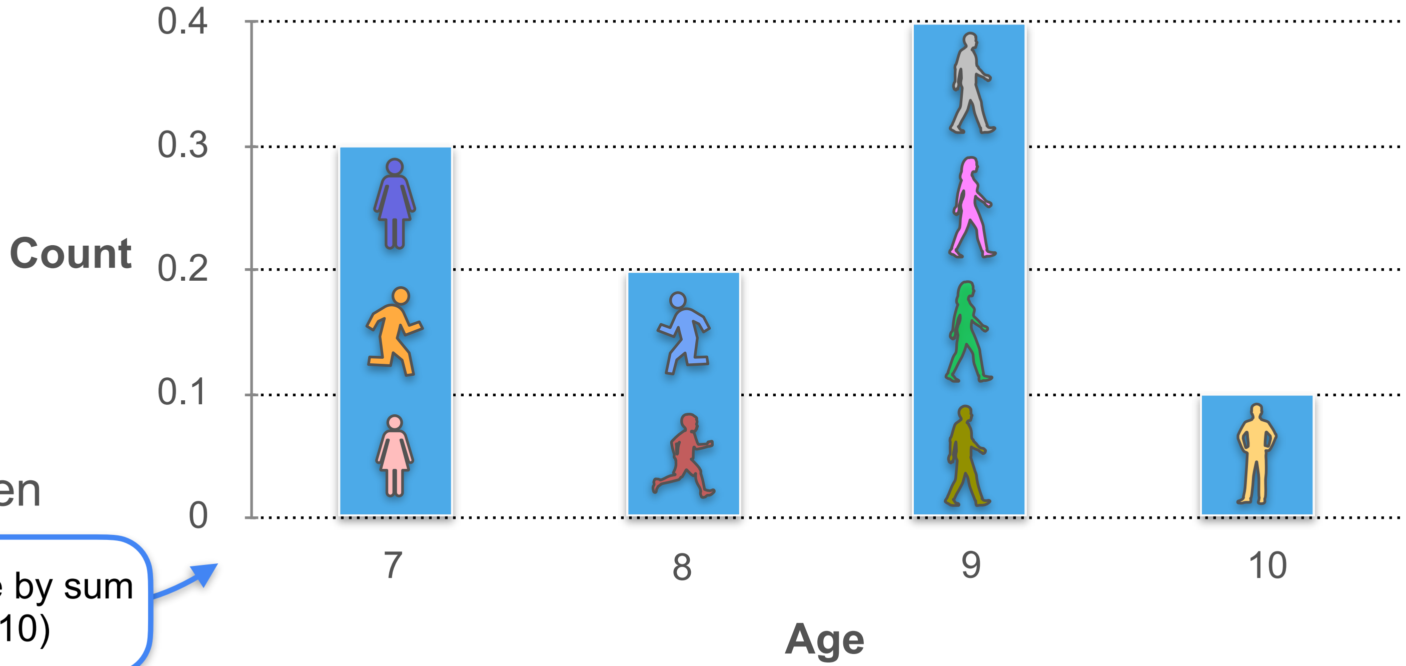
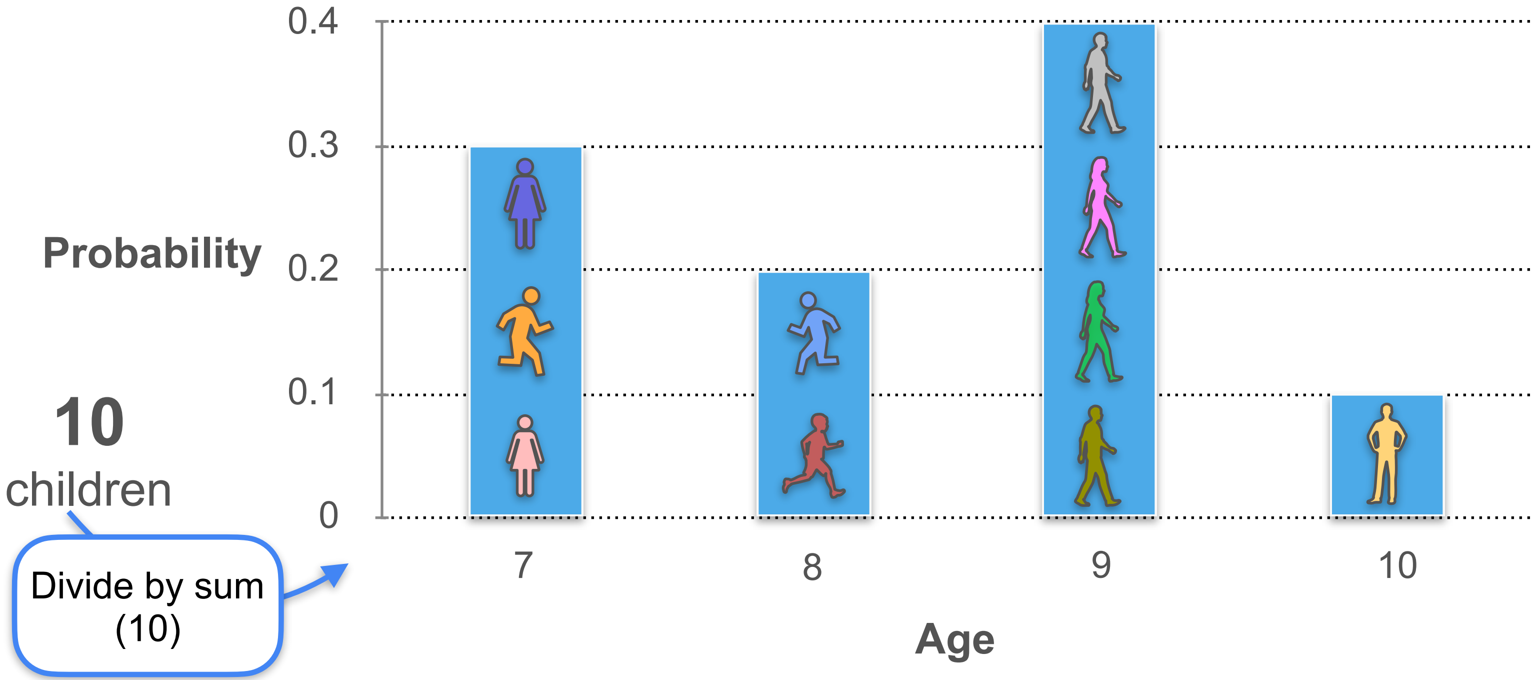


10
children

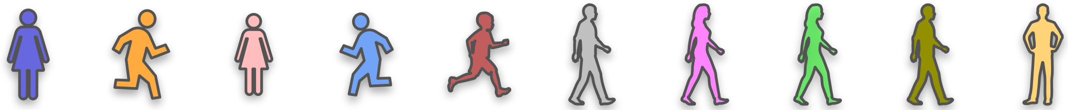
Divide by sum (10)

Joint Distributions (Discrete): Example 1



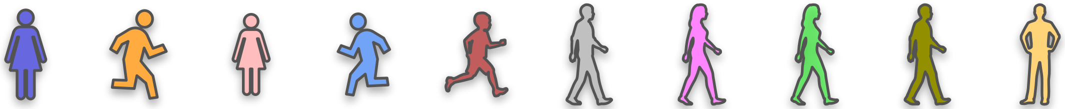
Joint Distributions (Discrete): Example 1

Age (years): 7 7 7 8 8 9 9 9 9 10



Joint Distributions (Discrete): Example 1

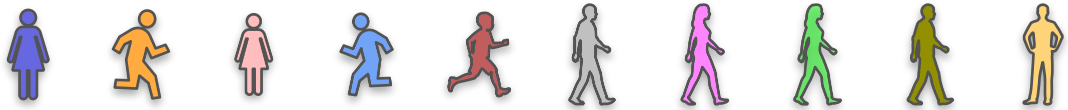
Age (years): 7 7 7 8 8 9 9 9 9 10



Height (in):

Joint Distributions (Discrete): Example 1

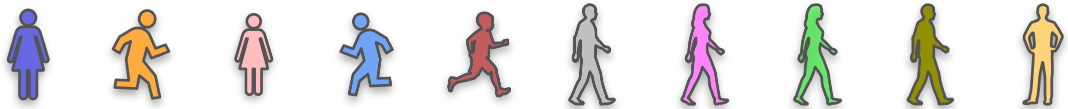
Age (years):	7	7	7	8	8	9	9	9	9	10



Height (in):	45	46	46	47	47	49	49	49	49	50

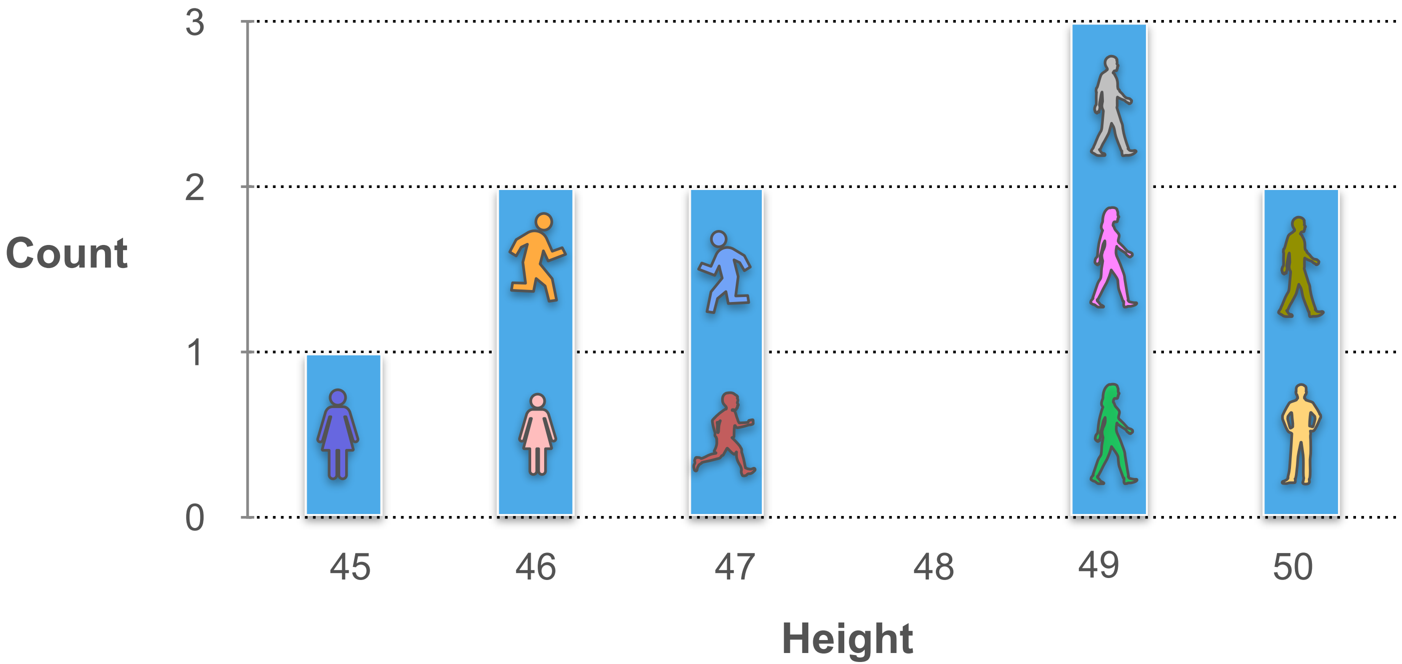
Joint Distributions (Discrete): Example 1

Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	49	50



Height (in)	Count
45	1
46	2
47	2
48	0
49	4
50	1

Joint Distributions (Discrete): Example 1



Joint Distributions (Discrete): Example 1

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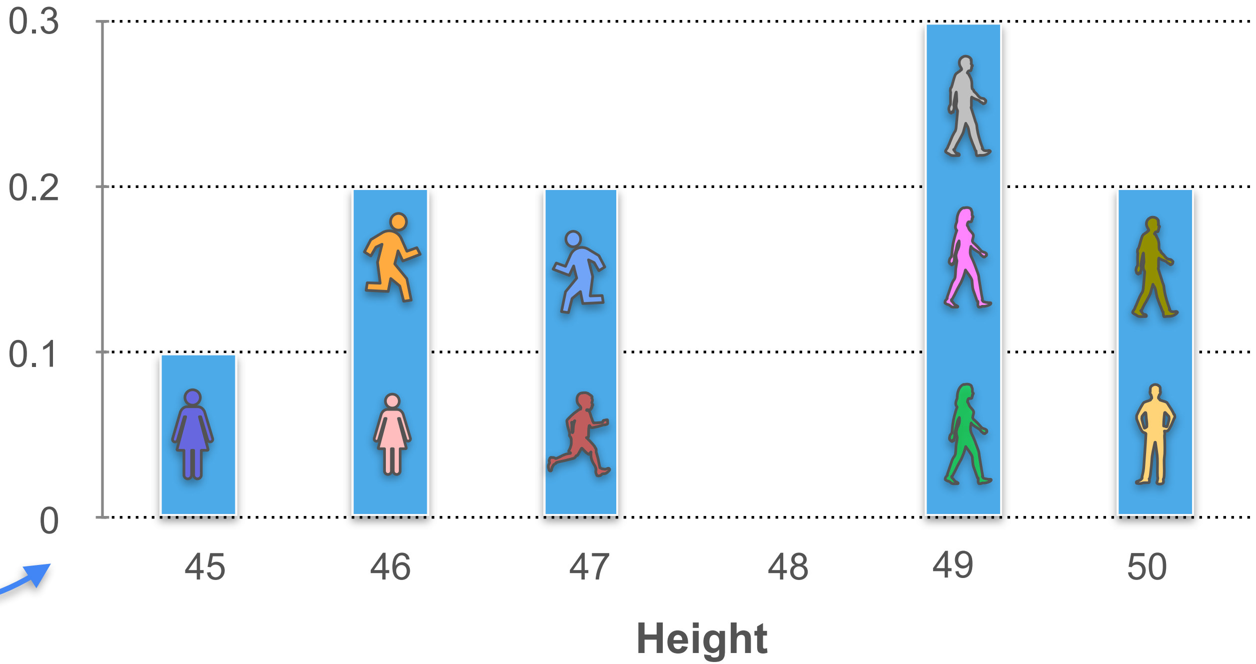
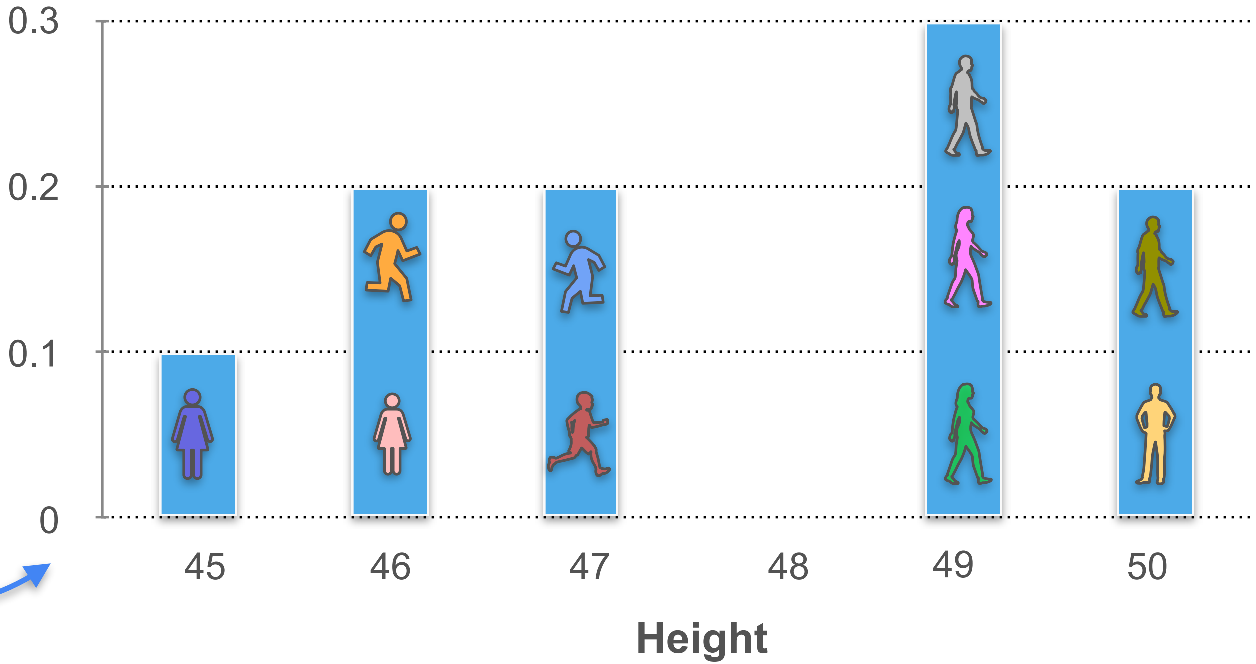


10
children

Divide by sum (10)

Joint Distributions (Discrete): Example 1



10
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Divide by sum (10)

Joint Distributions (Discrete): Example 1

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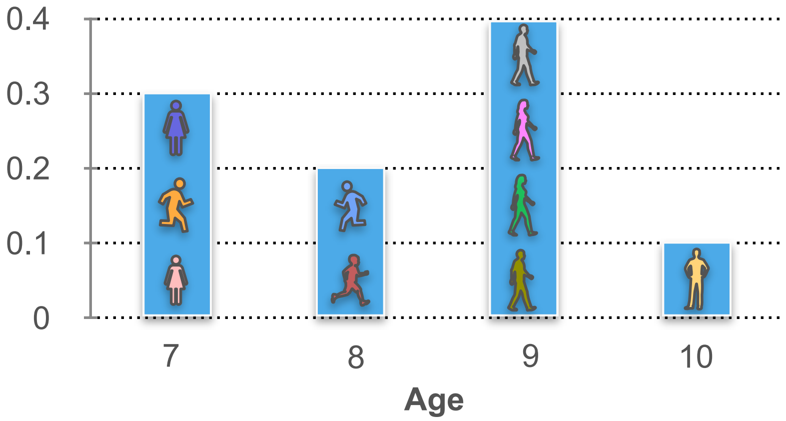
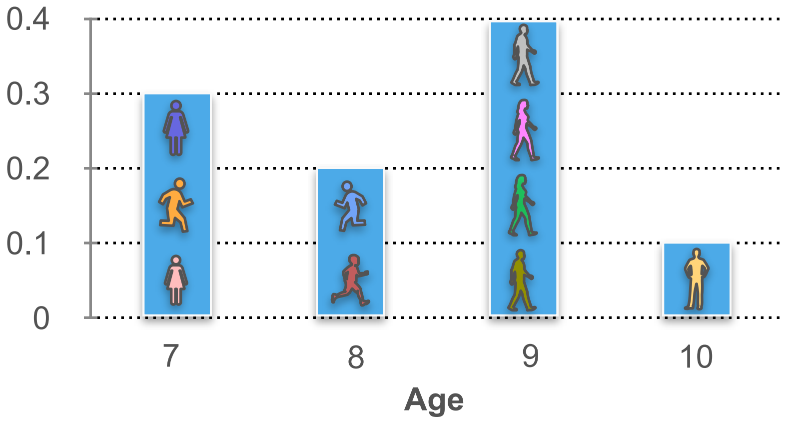
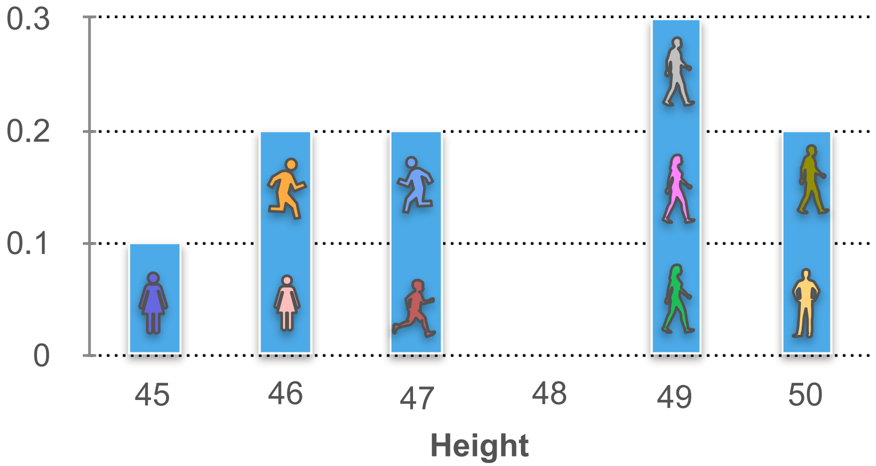
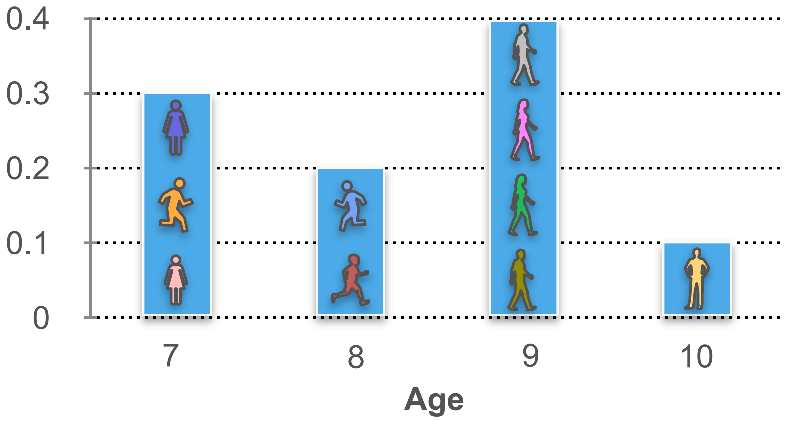


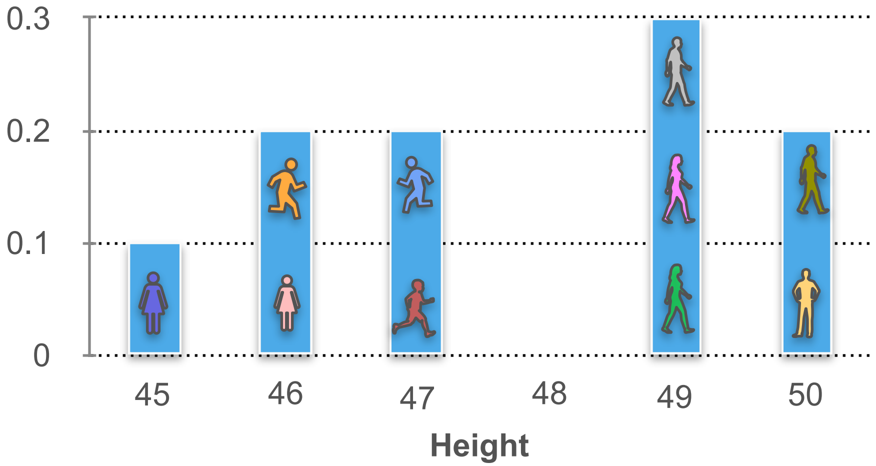
Joint Distributions (Discrete): Example 1





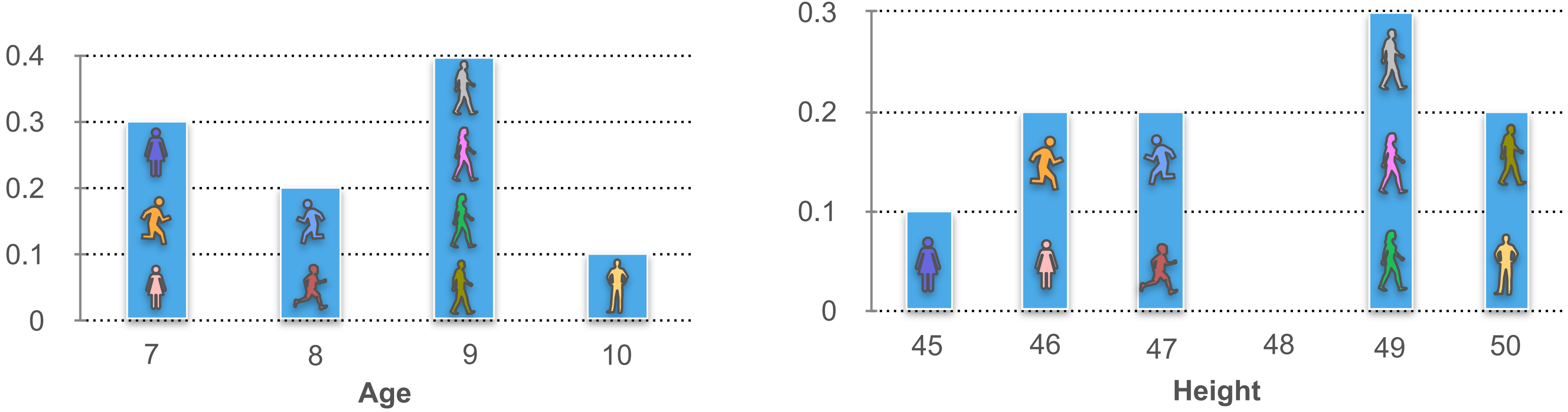
Joint Distributions (Discrete): Example 1





What is the probability that a child is **9 years** old and **49 inches** tall?

Joint Distributions (Discrete): Example 1



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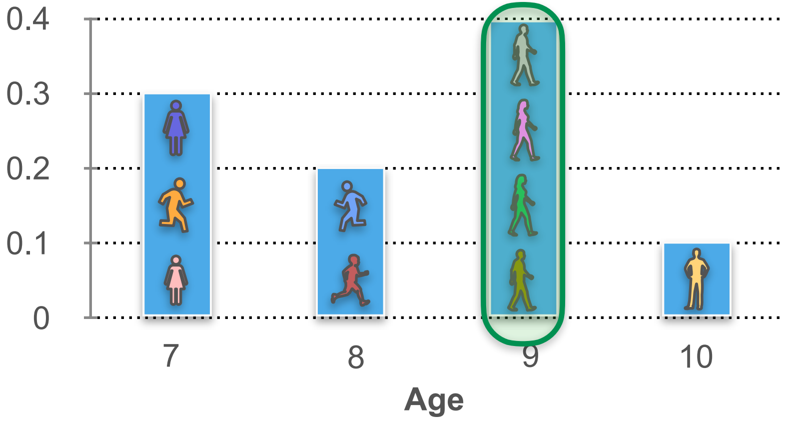

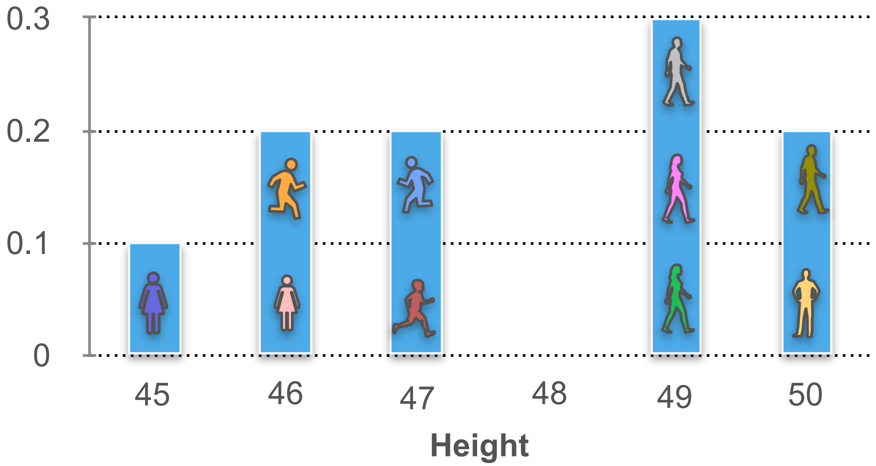

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Joint Distributions (Discrete): Example 1

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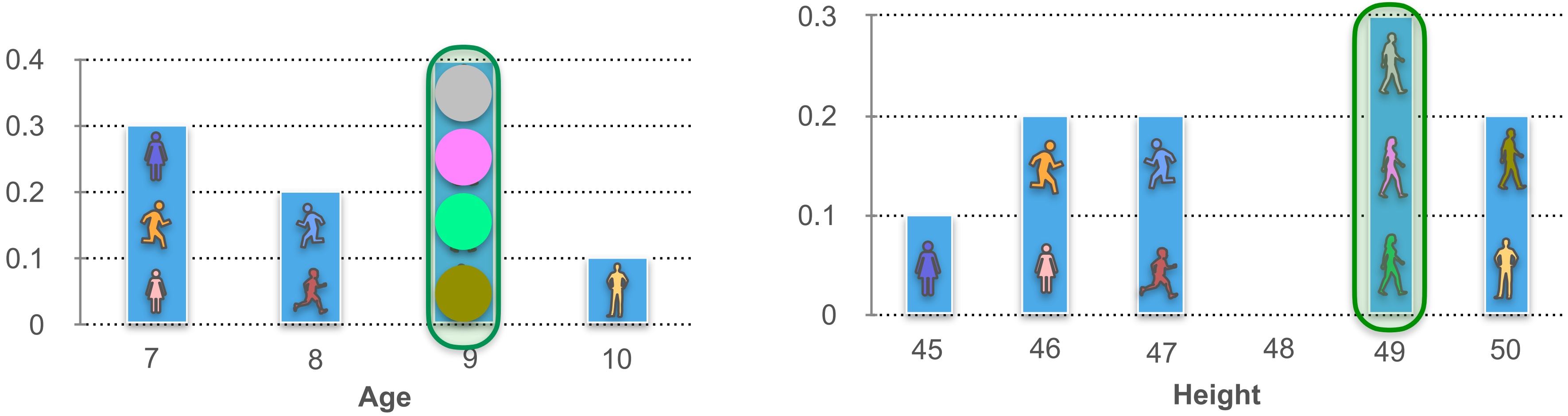

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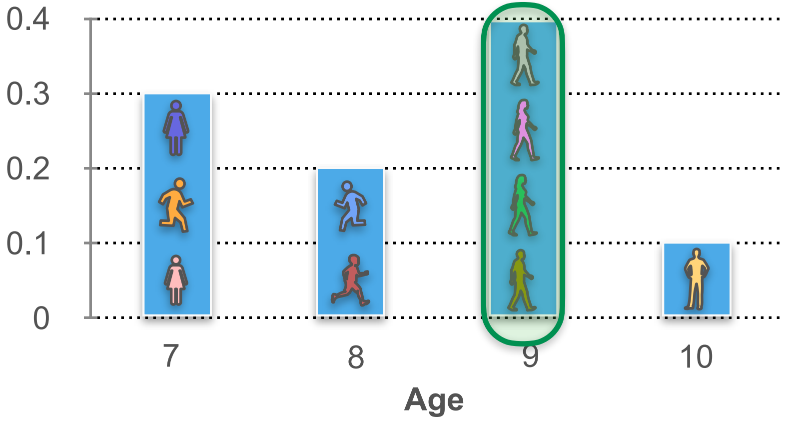

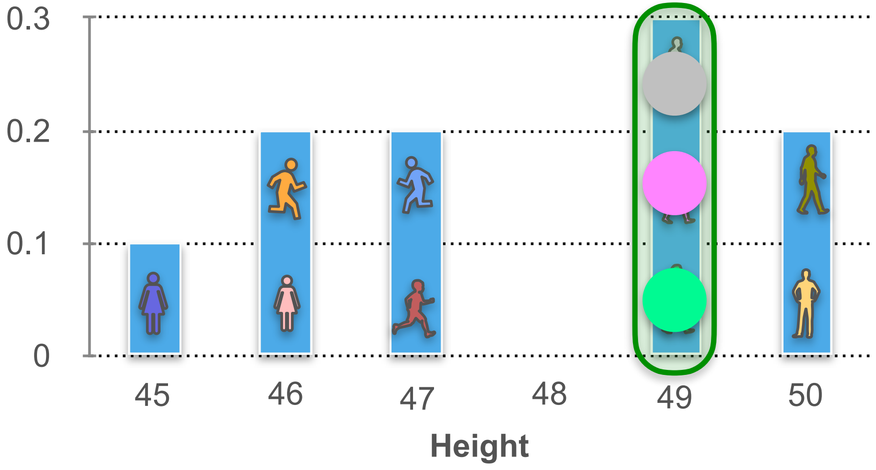

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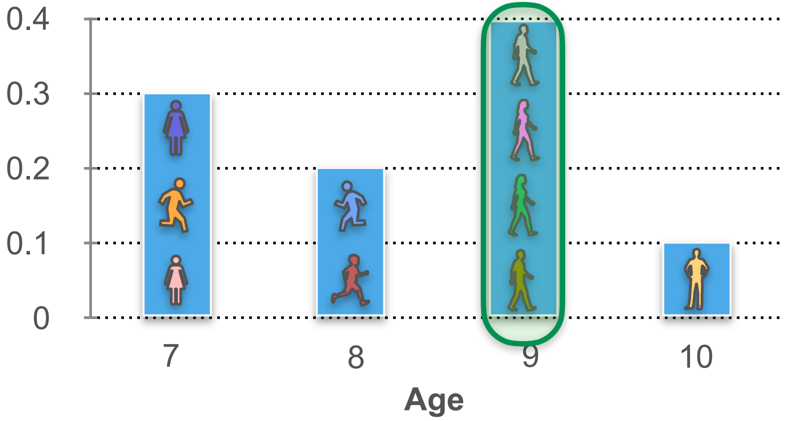

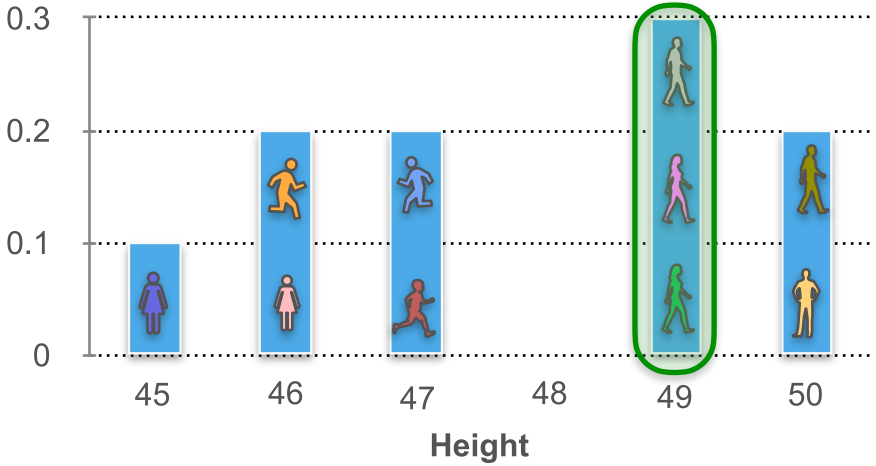

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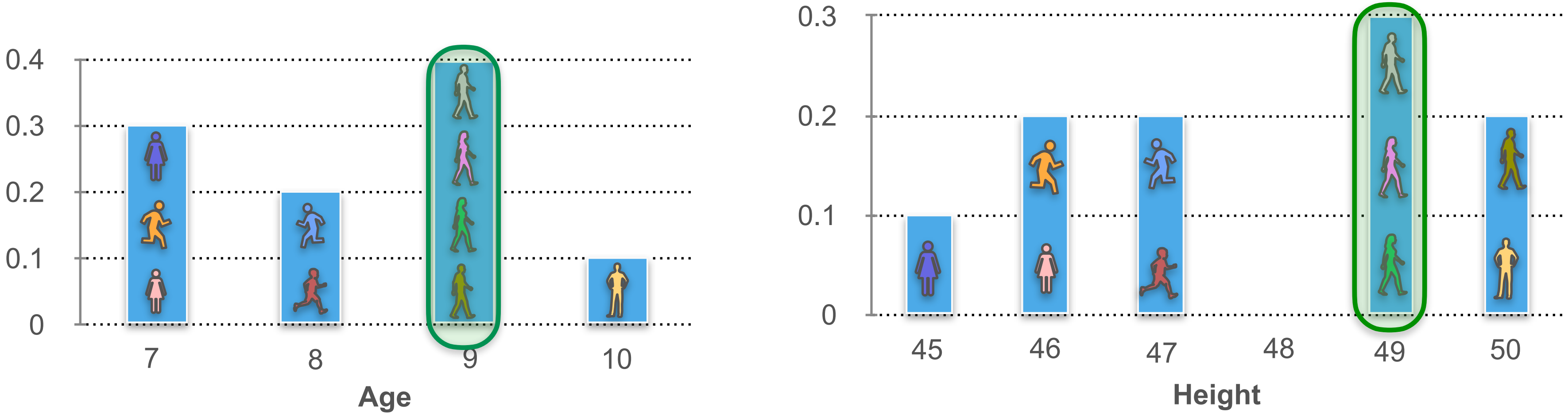
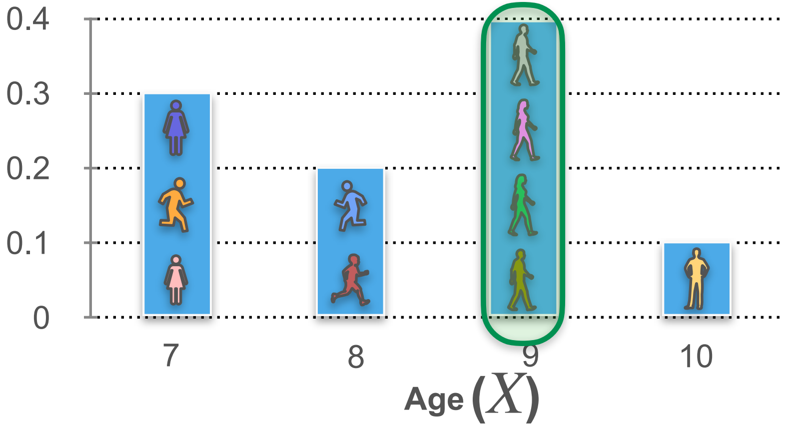
Joint Distributions (Discrete): Example 1

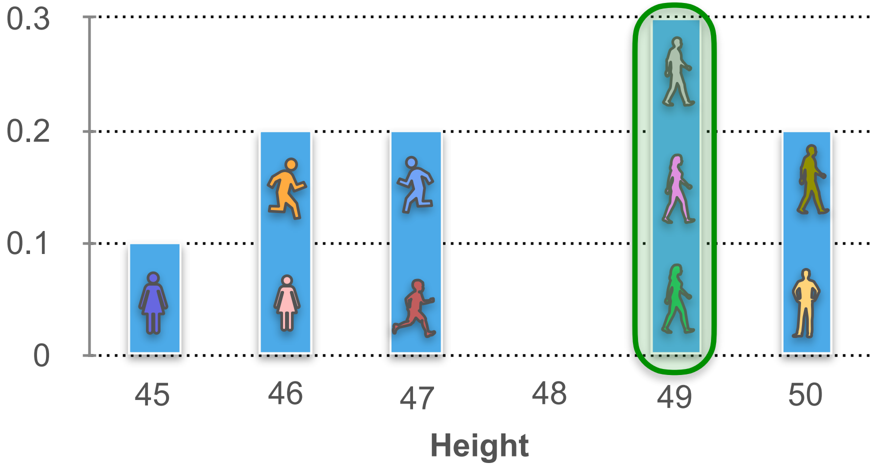


What is the probability that a child is **9 years** old and **49 inches** tall?

$$\frac{\bullet\bullet\bullet}{10} = \frac{3}{10}$$

Joint Distributions (Discrete): Example 1





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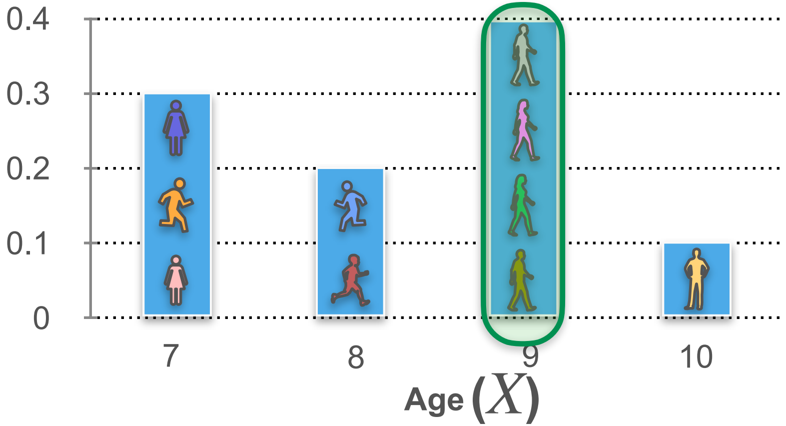

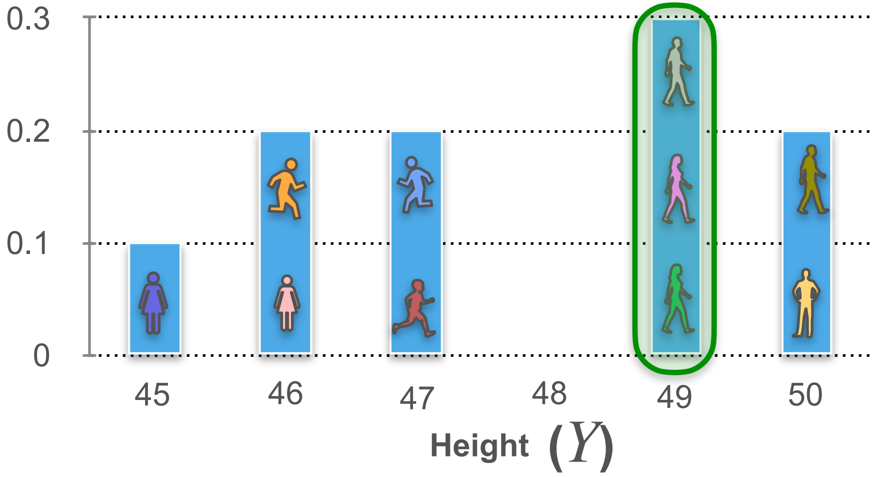

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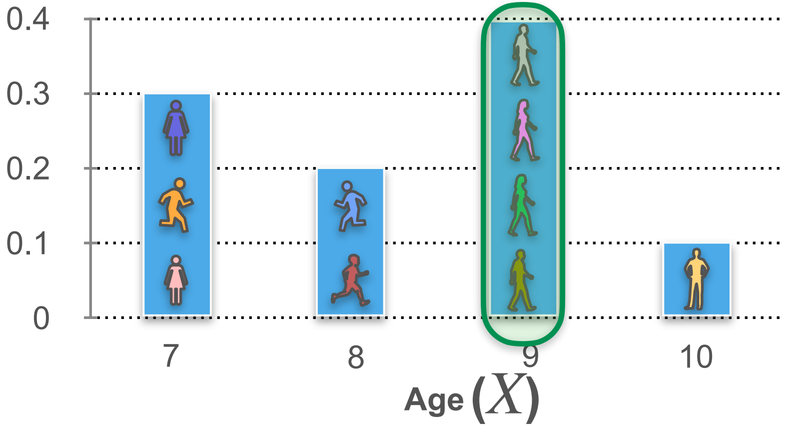
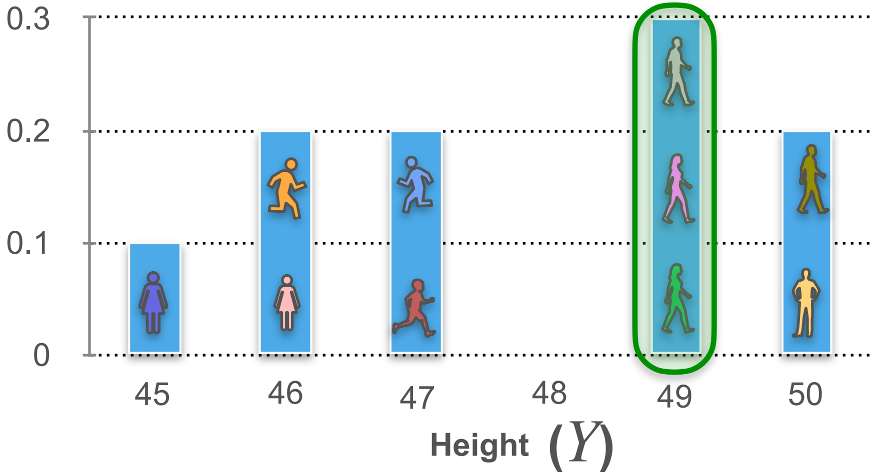
$$\frac{\bullet\bullet\bullet}{10} = \frac{3}{10}$$

Joint Distributions (Discrete): Example 1





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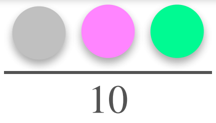
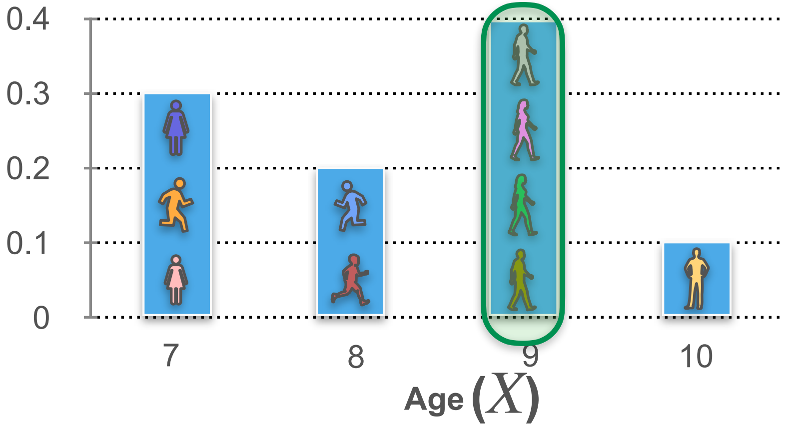
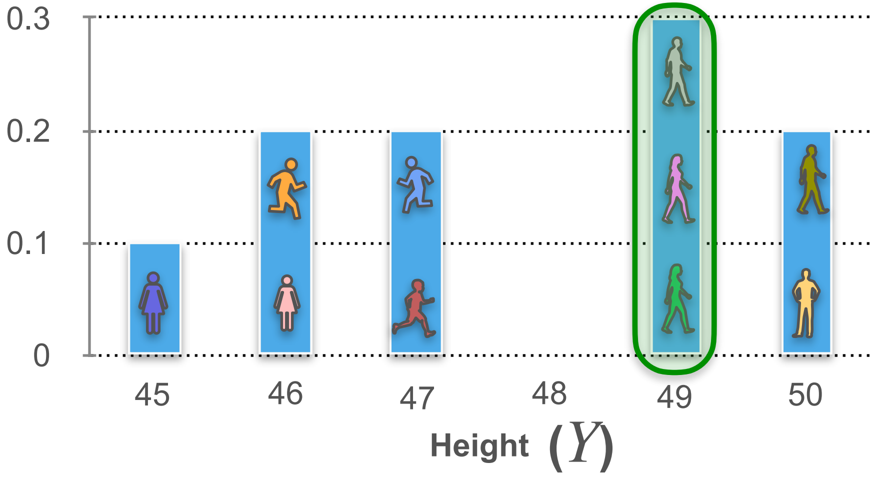


$$= \frac{3}{10}$$

Joint Distributions (Discrete): Example 1



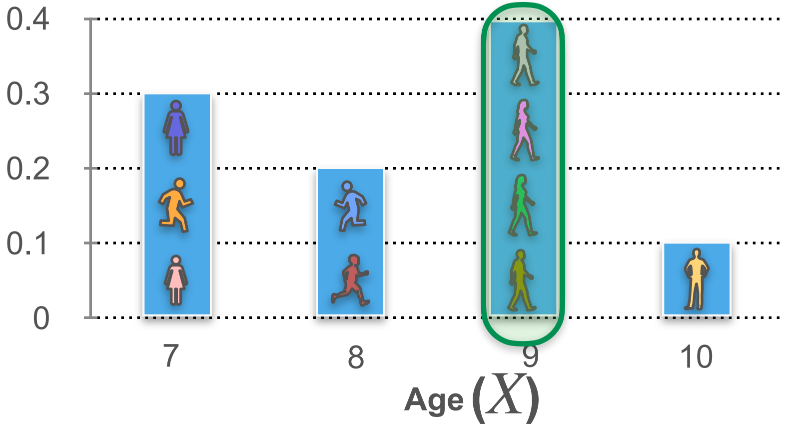


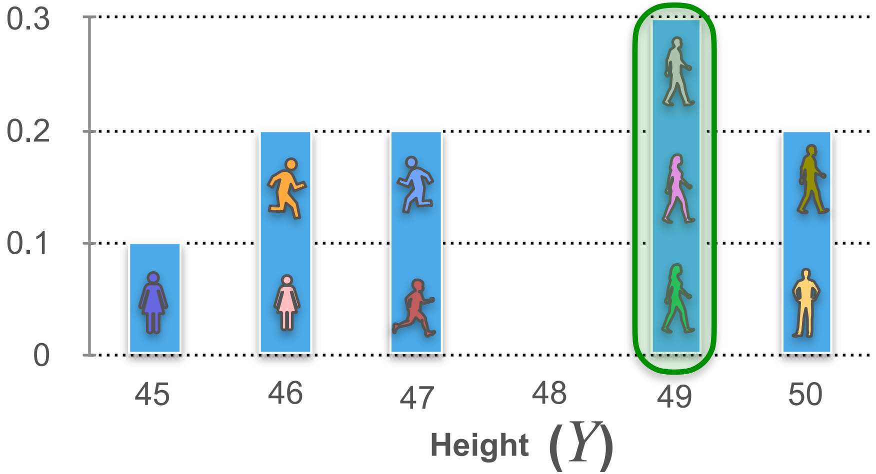
What is the probability that a child is **9 years** old and **49 inches** tall?

$p_{XY}(9, 49)$

$$\frac{\bullet\ \bullet\ \bullet}{10} = \frac{3}{10}$$

Joint Distributions (Discrete): Example 1





What is the probability that a child is **9 years** old and **49 inches** tall?

$$p_{XY}(9, 49) = \mathbf{P}(X = 9, Y = 49) = \frac{\bullet\ \bullet\ \bullet}{10} = \frac{3}{10}$$

Joint Distributions (Discrete): Example 1

What is the probability that a child is **9 years** old and **49 inches** tall?

$$p_{XY}(9, 49) = \mathbf{P}(X = 9, Y = 49) = \frac{\quad}{10} = \frac{3}{10}$$

Joint Distributions (Discrete): Example 1

What is the probability that a child is **9 years** old and **49 inches** tall?

$$p_{XY}(9, 49) = \mathbf{P}(X = 9, Y = 49) = \frac{\quad}{10} = \frac{3}{10}$$

$$p_{XY}(x, y)$$

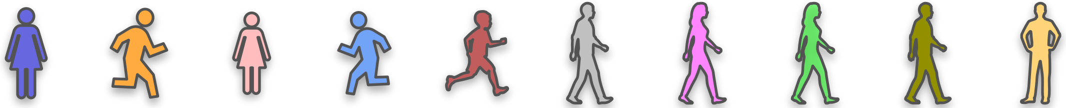
Joint Distributions (Discrete): Example 1

What is the probability that a child is **9 years** old and **49 inches** tall?

$$p_{XY}(9, 49) = \mathbf{P}(X = 9, Y = 49) = \frac{\quad}{10} = \frac{3}{10}$$

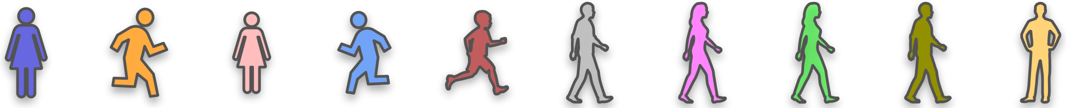
$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

Joint Distributions: Example 1



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Joint Distributions: Example 1



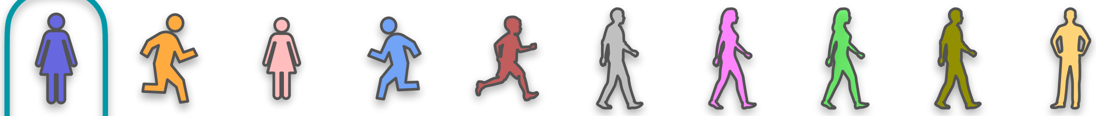
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	46	47	48	49	50
7						
8						
9						
10						

Joint Distributions: Example 1



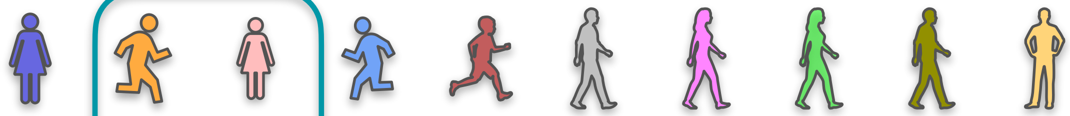
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1					
8						
9						
10						

Joint Distributions: Example 1



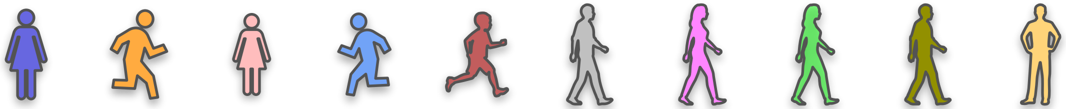
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1	2				
8						
9						
10						

Joint Distributions: Example 1

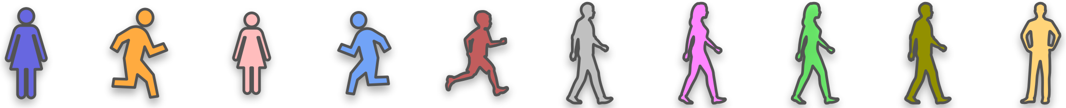


Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age		**45**	**46**	**47**	**48**	**49**	**50**
	7	1	2	0	0	0	0
	8						
	9						
	10						

Joint Distributions: Example 1



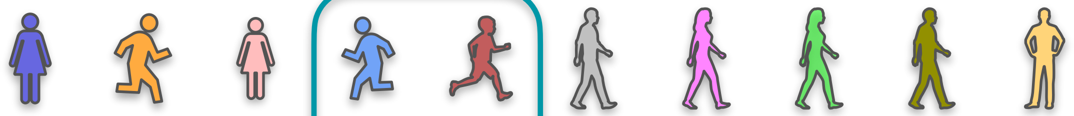
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1	2	0	0	0	0
8	0	0				
9						
10						

Joint Distributions: Example 1

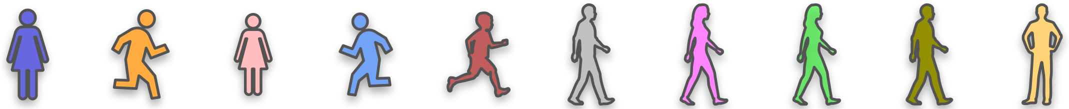


Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age		**45**	**46**	**47**	**48**	**49**	**50**
	7	1	2	0	0	0	0
	8	0	0	2			
	9						
	10						

Joint Distributions: Example 1



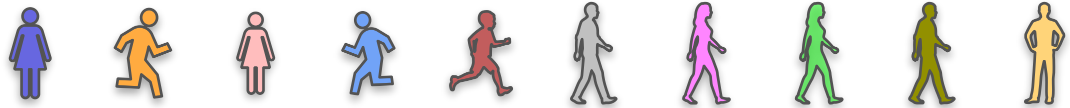
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9						
10						

Joint Distributions: Example 1



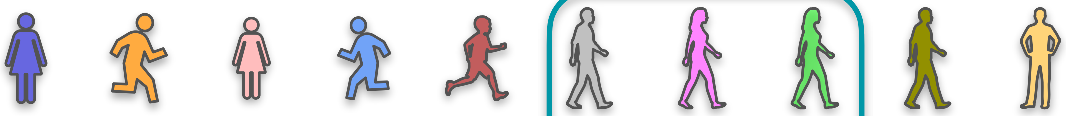
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0		
10						

Joint Distributions: Example 1



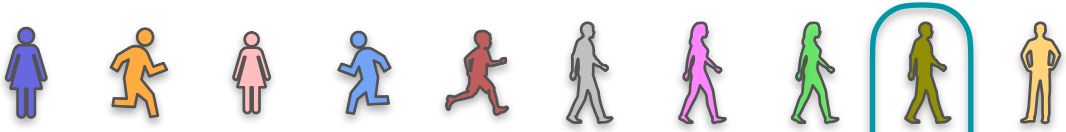
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	
10						

Joint Distributions: Example 1



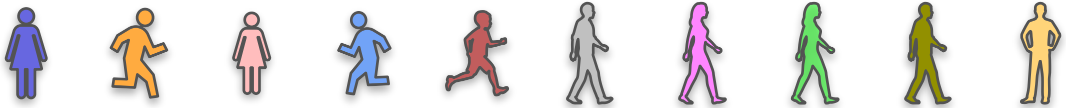
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	1
10						

Joint Distributions: Example 1



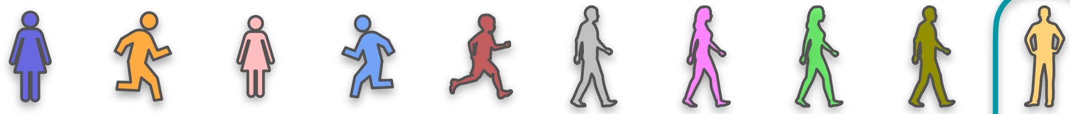
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	1
10	0	0	0	0	0	

Joint Distributions: Example 1



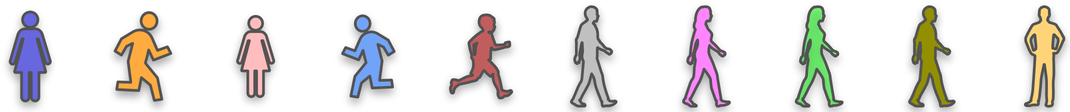
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	1
10	0	0	0	0	0	1

Joint Distributions: Example 1



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height

Age

	45	**46**	**47**	**48**	**49**	**50**
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	1
10	0	0	0	0	0	1

Joint Distributions: Example 1

Height

Age

	45	46	47	48	49	50
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	1
10	0	0	0	0	0	1

Counts

Joint Distributions: Example 1

Divide by sum (10)

Counts

Age \ Height	45	46	47	48	49	50
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	1
10	0	0	0	0	0	1

Age \ Height	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Joint Distributions: Example 1

Divide by sum (10)

Counts

Age \ Height	45	46	47	48	49	50
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	1
10	0	0	0	0	0	1

Age \ Height	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Joint Distributions: Example 1

Divide by sum (10)

Counts (rows: Age; columns: Height)

Age \ Height	45	46	47	48	49	50
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	1
10	0	0	0	0	0	1

Probabilities (rows: Age; columns: Height)

Age \ Height	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Joint Distributions: Example 1

Height

Age

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

All probabilities for all possible combinations of X and Y

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

Joint Distribution

All probabilities for all possible combinations of X and Y

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

$$p_{XY}(8, 48) = \mathbf{P}(X = 8, Y = 48)$$

Height (Y)

Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

$$p_{XY}(8, 48) = \mathbf{P}(X = 8, Y = 48)$$

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0		0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

$$p_{XY}(8, 48) = \mathbf{P}(X = 8, Y = 48)$$

$$p_{XY}(8, 48) = 0$$

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0		0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

$$p_{XY}(8, 48) = \mathbf{P}(X = 8, Y = 48)$$

$$p_{XY}(8, 48) = 0$$

$$p_{XY}(7, 46) =$$

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

$$p_{XY}(8, 48) = \mathbf{P}(X = 8, Y = 48)$$

$$p_{XY}(8, 48) = 0$$

$$p_{XY}(7, 46) =$$

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/1	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

$$p_{XY}(8, 48) = \mathbf{P}(X = 8, Y = 48)$$

$$p_{XY}(8, 48) = 0$$

$$p_{XY}(7, 46) =$$

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/1	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

$$p_{XY}(x,\ y) = \mathbf{P}(X = x, Y = y)$$

$$p_{XY}(8,\ 48) = \mathbf{P}(X = 8, Y = 48)$$

$$p_{XY}(8,\ 48) = 0$$

$$p_{XY}(7,\ 46) = \frac{2}{10}$$

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/1	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

Joint Distributions (Discrete): Example 2

Joint Distributions (Discrete): Example 2

X

Joint Distributions (Discrete): Example 2

X

the number rolled on the 1st dice

Joint Distributions (Discrete): Example 2

X

Y

the number rolled on the 1st dice

Joint Distributions (Discrete): Example 2

X

the number rolled on the 1st dice

Y

the number rolled on the 2nd dice

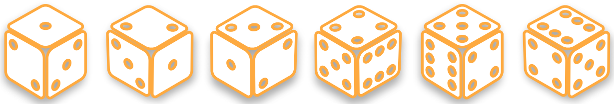
Joint Distributions (Discrete): Example 2

X

the number rolled on the 1st dice

Y

the number rolled on the 2nd dice



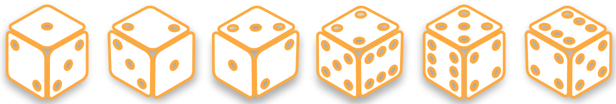
Joint Distributions (Discrete): Example 2

X

the number rolled on the 1st dice

Y

the number rolled on the 2nd dice

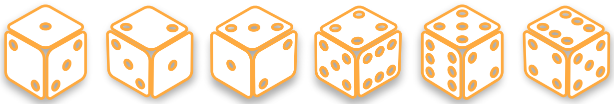


$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

Joint Distributions (Discrete): Example 2

X

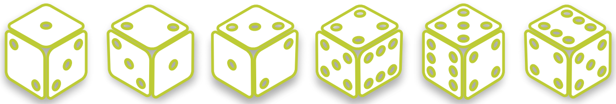
the number rolled on the 1st dice



$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

Y

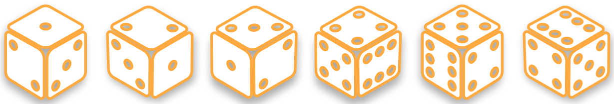
the number rolled on the 2nd dice



Joint Distributions (Discrete): Example 2

X

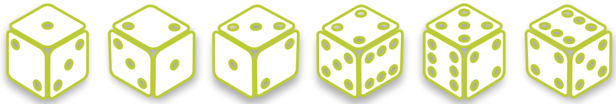
the number rolled on the 1st dice



$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

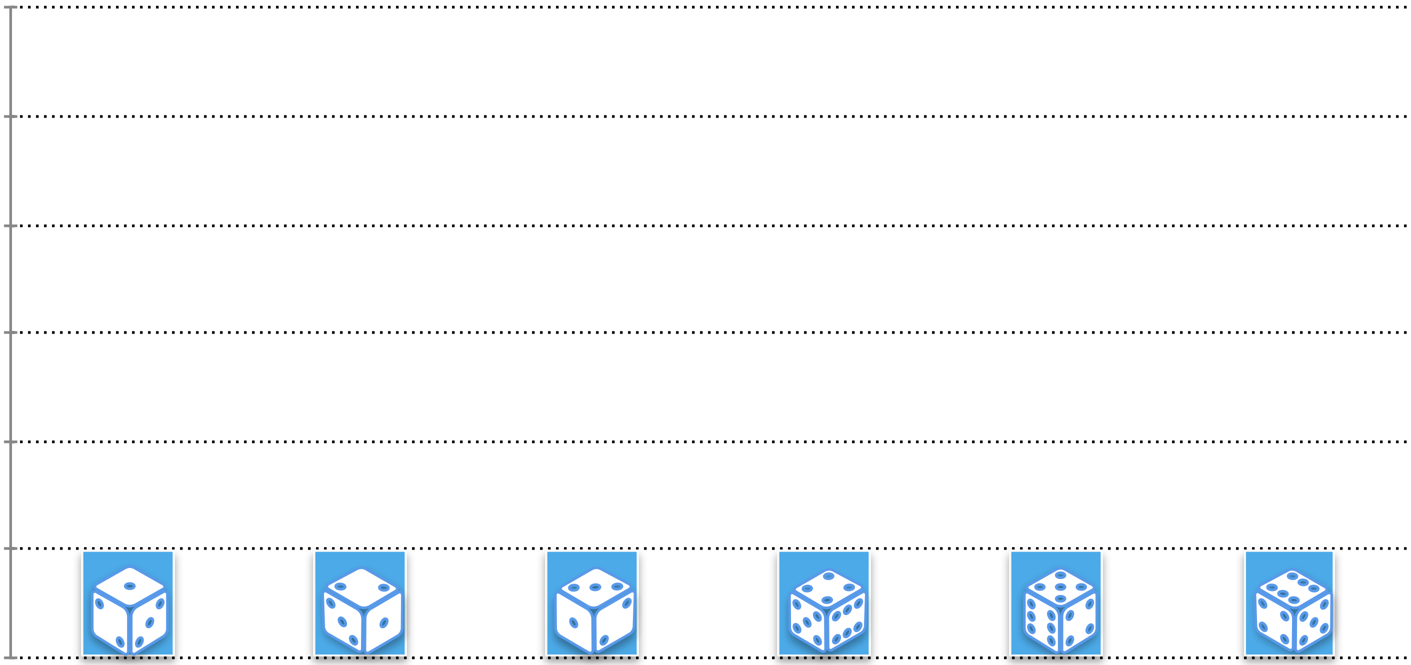
Y

the number rolled on the 2nd dice

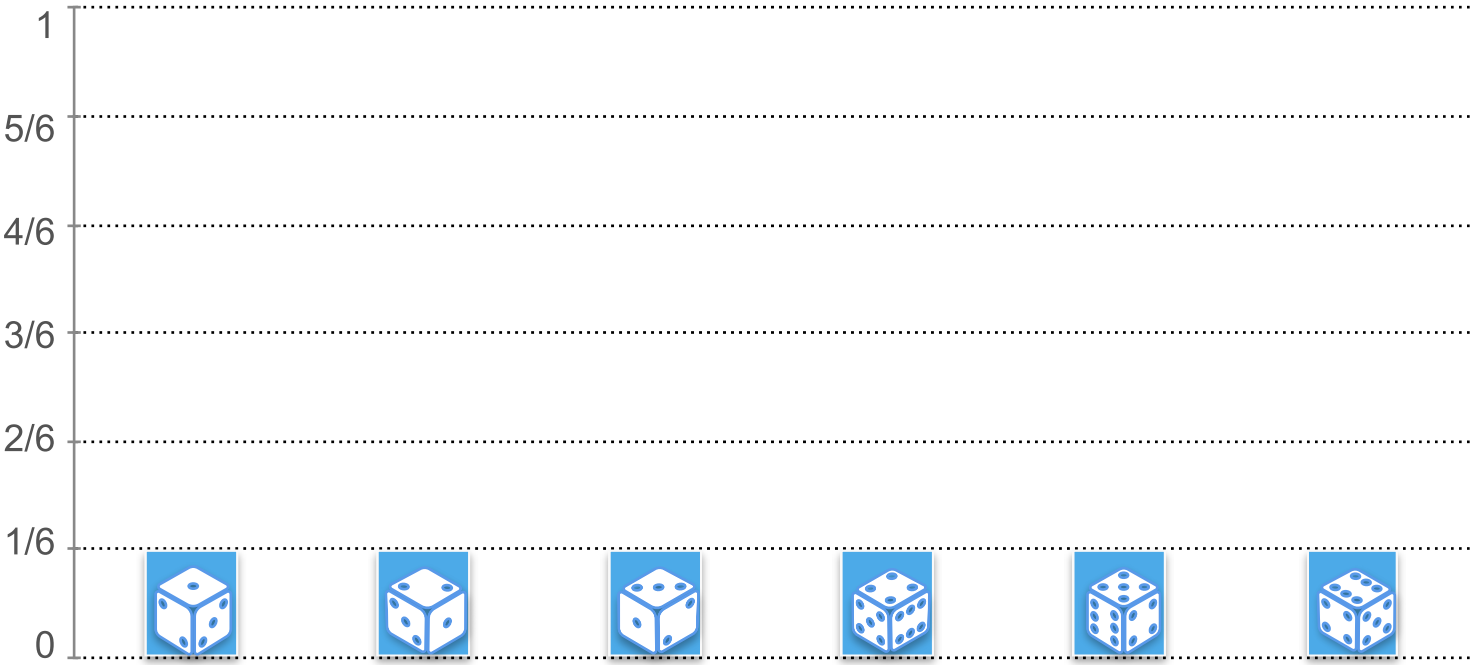


$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

Joint Distributions: Example 2



Joint Distributions: Example 2



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Joint Distributions: Example 2

Joint Distributions: Example 2

X

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Joint Distributions: Example 2

X: the number rolled on the 1st dice

Y

Joint Distributions: Example 2

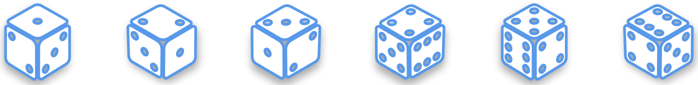
X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

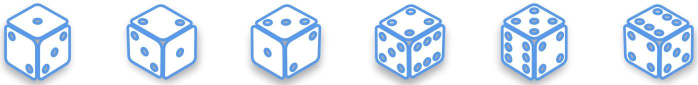


$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice



$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

Y

X	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

Y

X	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$p_{XY}($  $)$

Y

X	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$p_{XY}($  ,  $)$

Y

X	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$p_{XY}($ ,  $) = \mathbf{P}($  $) \cdot \mathbf{P}($  $)$

X \ Y	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$p_{XY}($  $,$  $) = \mathbf{P}($  $) \cdot \mathbf{P}($  $) \quad = \frac{1}{6} \cdot \frac{1}{6}$

Y

X	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$p_{XY}($  $,$  $) = \mathbf{P}($  $) \cdot \mathbf{P}($  $) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

Y

X	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$p_{XY}($  $,$  $) = \mathbf{P}($  $) \cdot \mathbf{P}($  $) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

Y

X	1	2	3	4	5	6
1						
2					1/36	
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$p_{XY}($  ,  $) = \mathbf{P}($  $) \cdot \mathbf{P}($  $) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

X \ Y	1	2	3	4	5	6
1						
2					1/36	
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$p_{XY}($  ,  $) = \mathbf{P}($  $) \cdot \mathbf{P}($  $) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y) = \frac{1}{36}$$

Y

X	1	2	3	4	5	6
1						
2					1/36	
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$p_{XY}($  ,  $) = \mathbf{P}($  $) \cdot \mathbf{P}($  $) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y) = \frac{1}{36}$$

X \ Y	1	2	3	4	5	6
1						
2					1/36	
3						
4						
5						
6						

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$$p_{XY}(\ ,\ ) = \mathbf{P}(\ ) \cdot \mathbf{P}(\ ) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y) = \frac{1}{36}$$

X \ Y	1	2	3	4	5	6
1	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	1/36

Joint Distributions: Example 2

Joint Distributions: Example 2

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y) = \mathbf{P}(x) \cdot \mathbf{P}(y)$$

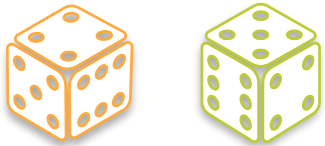
Joint Distributions: Example 2

Thus for independent discrete random variables:

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y) = \mathbf{P}(x) \cdot \mathbf{P}(y)$$

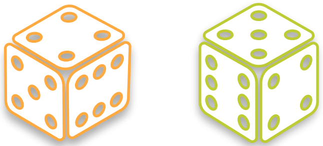
Joint Distributions (Discrete): Example 2

Joint Distributions (Discrete): Example 2



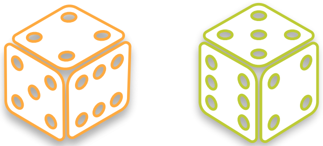
Joint Distributions (Discrete): Example 2

X



Joint Distributions (Discrete): Example 2

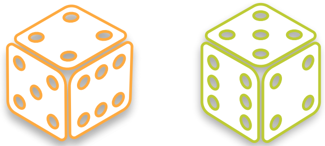
X



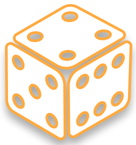
the number rolled on the 1st dice

Joint Distributions (Discrete): Example 2

X

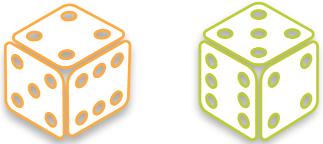


the number rolled on the 1st dice

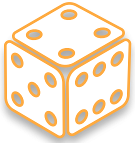


Joint Distributions (Discrete): Example 2

X



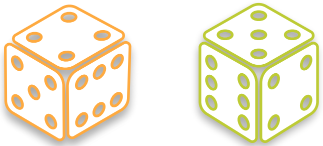
the number rolled on the 1st dice



$$X = 4$$

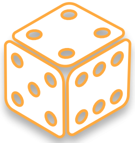
Joint Distributions (Discrete): Example 2

X



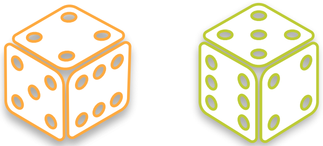
Y

the number rolled on the 1st dice



$$X = 4$$

Joint Distributions (Discrete): Example 2

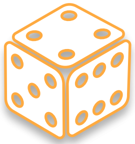


X

the number rolled on the 1st dice

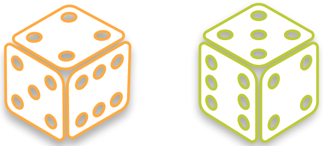
Y

sum of the two dice



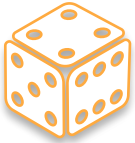
$X = 4$

Joint Distributions (Discrete): Example 2



X

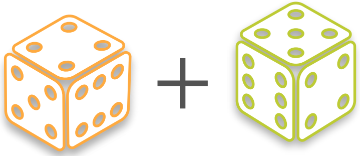
the number rolled on the 1st dice



$$X = 4$$

Y

sum of the two dice



$$Y = 4 + 5$$

Joint Distributions: Example 2

Joint Distributions: Example 2

X

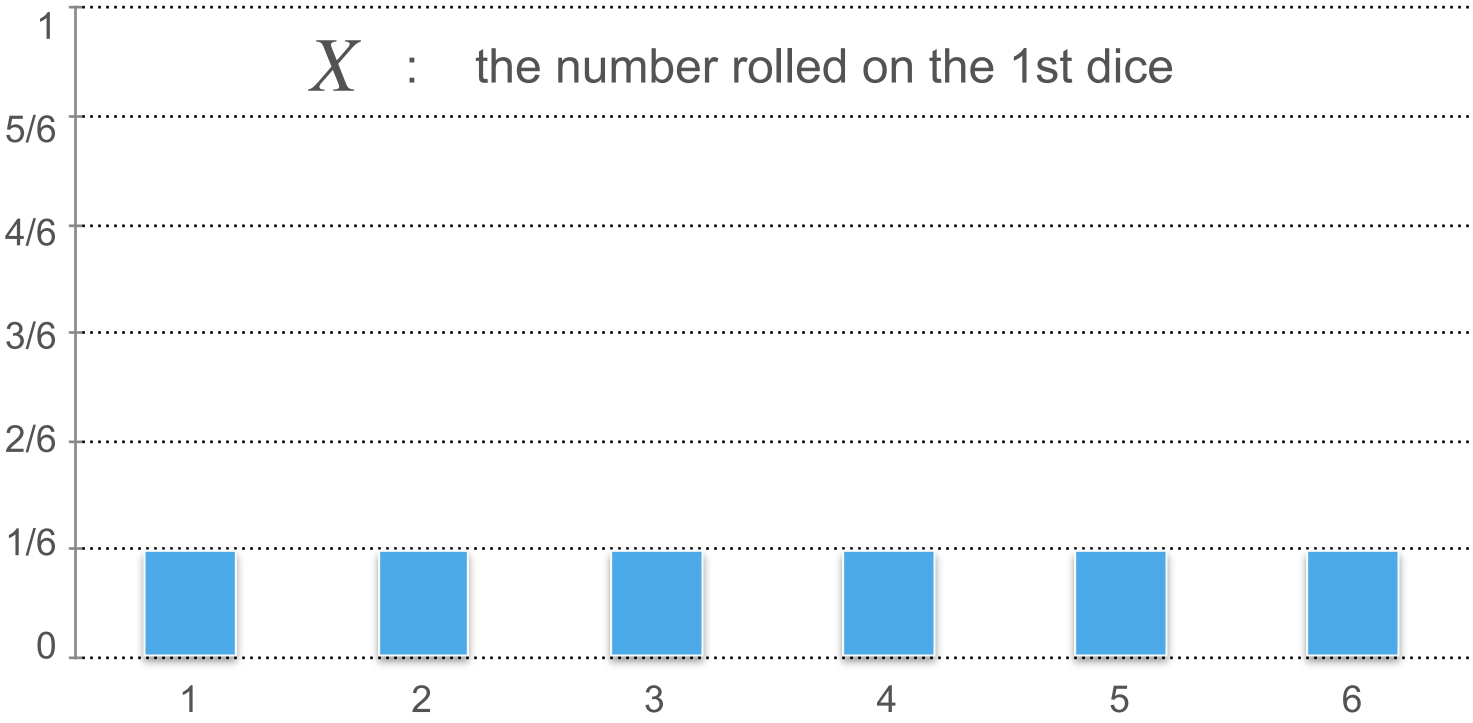
Joint Distributions: Example 2

X the number rolled on the 1st dice

Joint Distributions: Example 2

X : the number rolled on the 1st dice

Joint Distributions: Example 2



Joint Distributions - Example 3

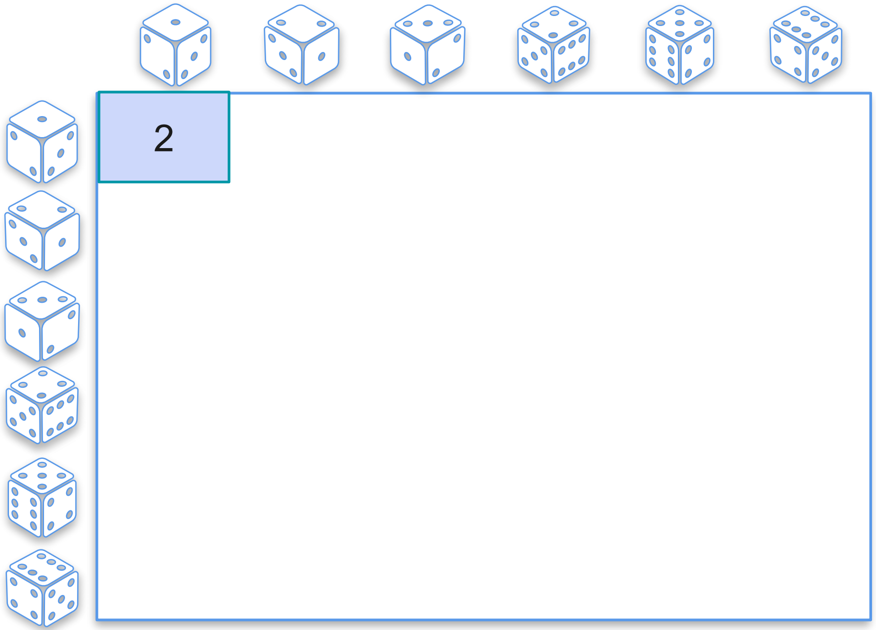
Y: Sum of both dice

Joint Distributions - Example 3

Y: Sum of both dice

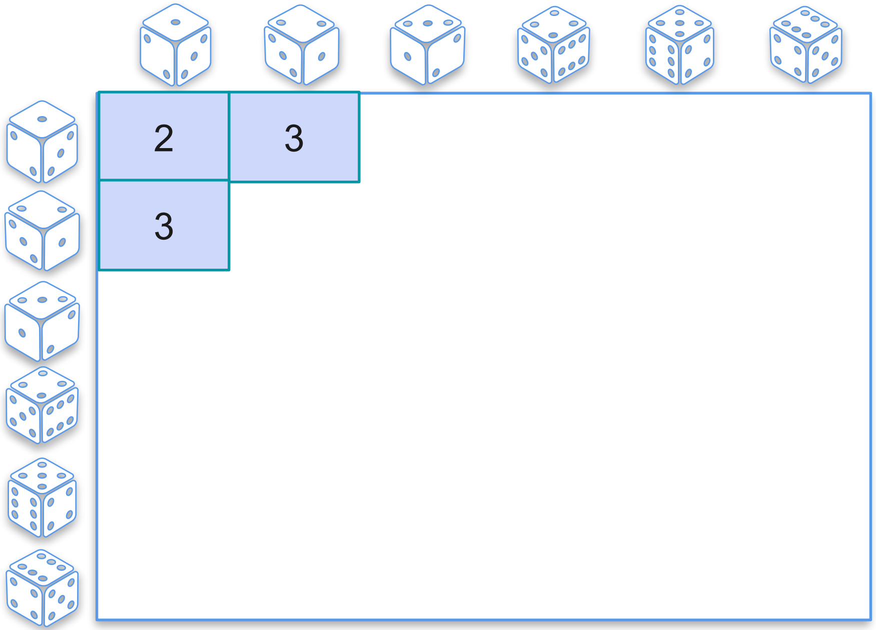
Joint Distributions - Example 3

Y: Sum of both dice



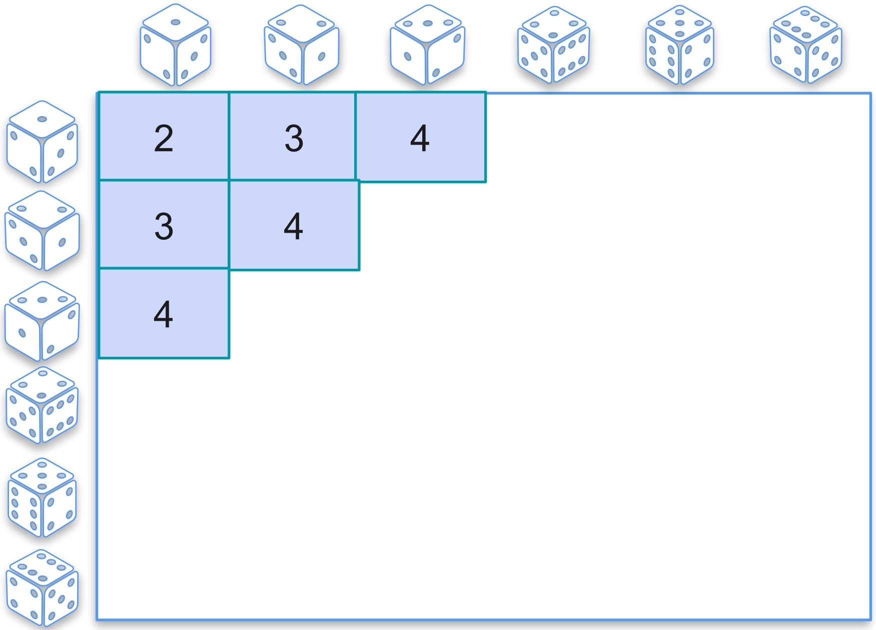
Joint Distributions - Example 3

Y: Sum of both dice



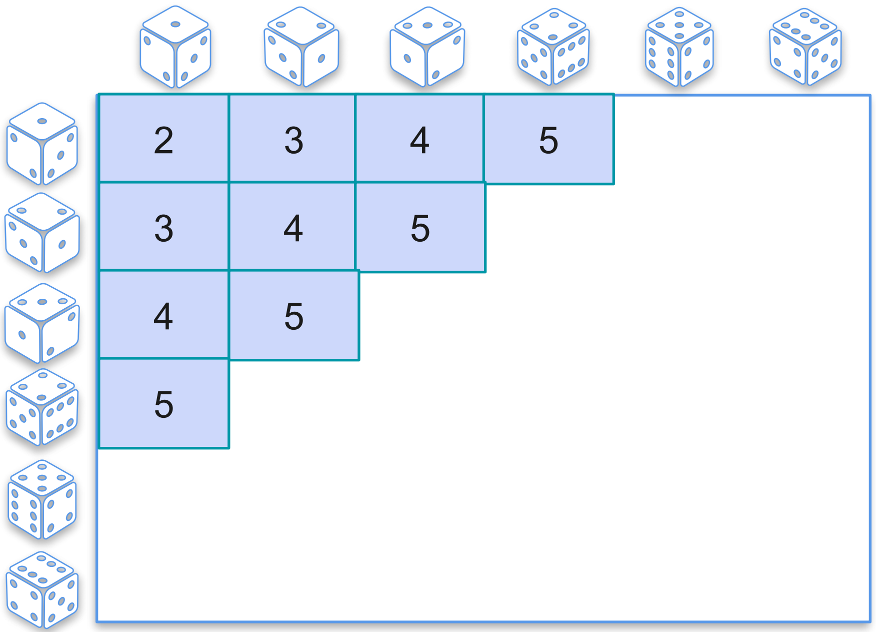
Joint Distributions - Example 3

Y: Sum of both dice



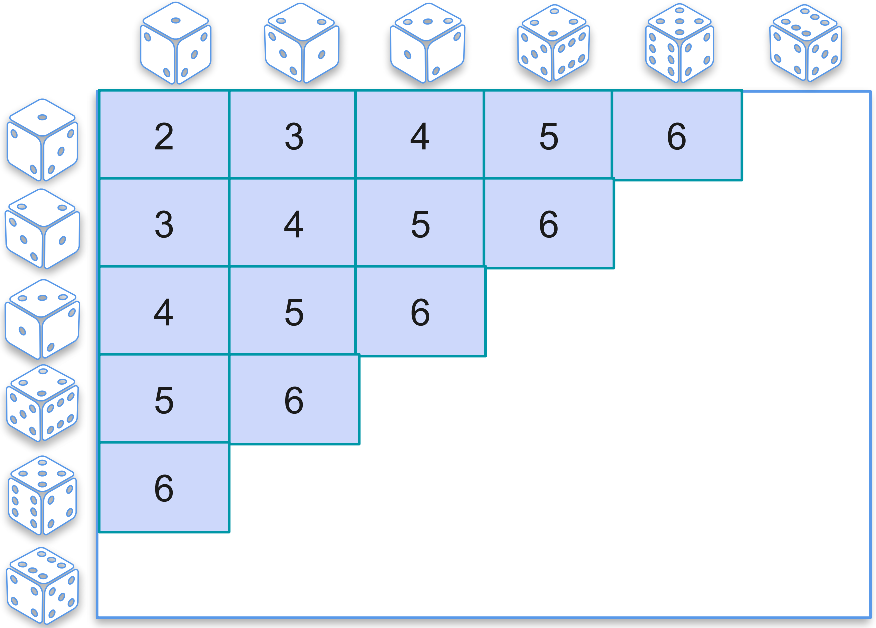
Joint Distributions - Example 3

Y: Sum of both dice



Joint Distributions - Example 3

Y: Sum of both dice



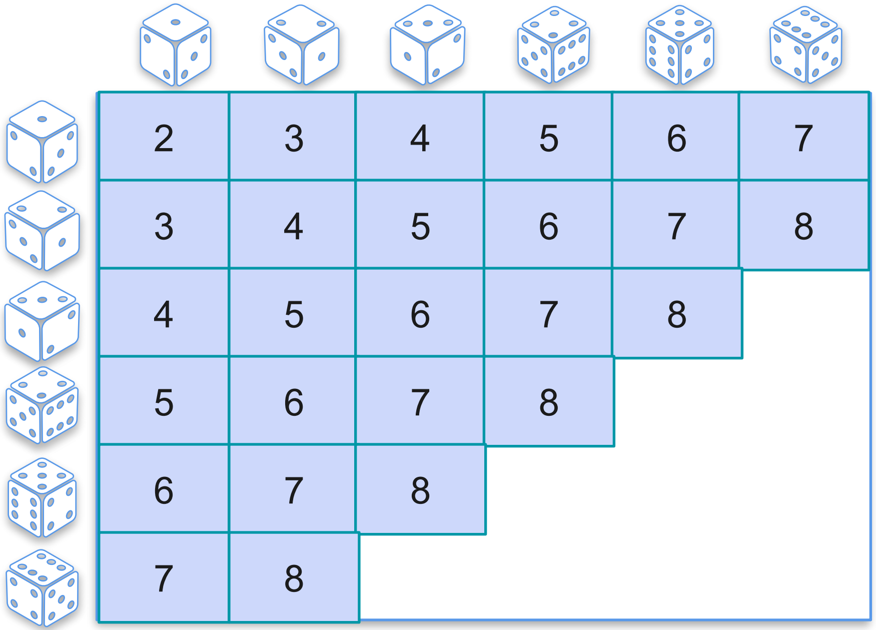
Joint Distributions - Example 3

Y: Sum of both dice

	2	3	4	5	6	7
	3	4	5	6	7	
	4	5	6	7		
	5	6	7			
	6	7				
	7					

Joint Distributions - Example 3

Y: Sum of both dice



Joint Distributions - Example 3

Y: Sum of both dice

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	
6	7	8	9		
7	8	9			

Joint Distributions - Example 3

Y: Sum of both dice

	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	
	7	8	9	10		

Joint Distributions - Example 3

Y: Sum of both dice

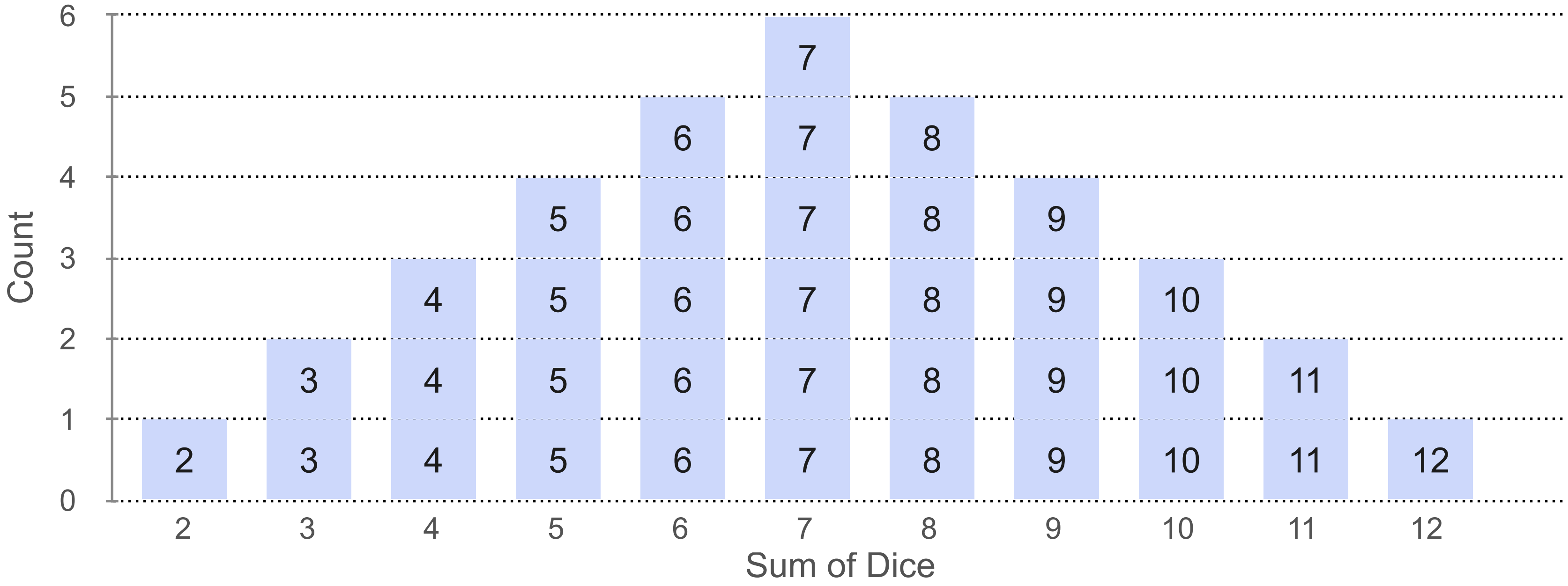
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	

Joint Distributions - Example 3

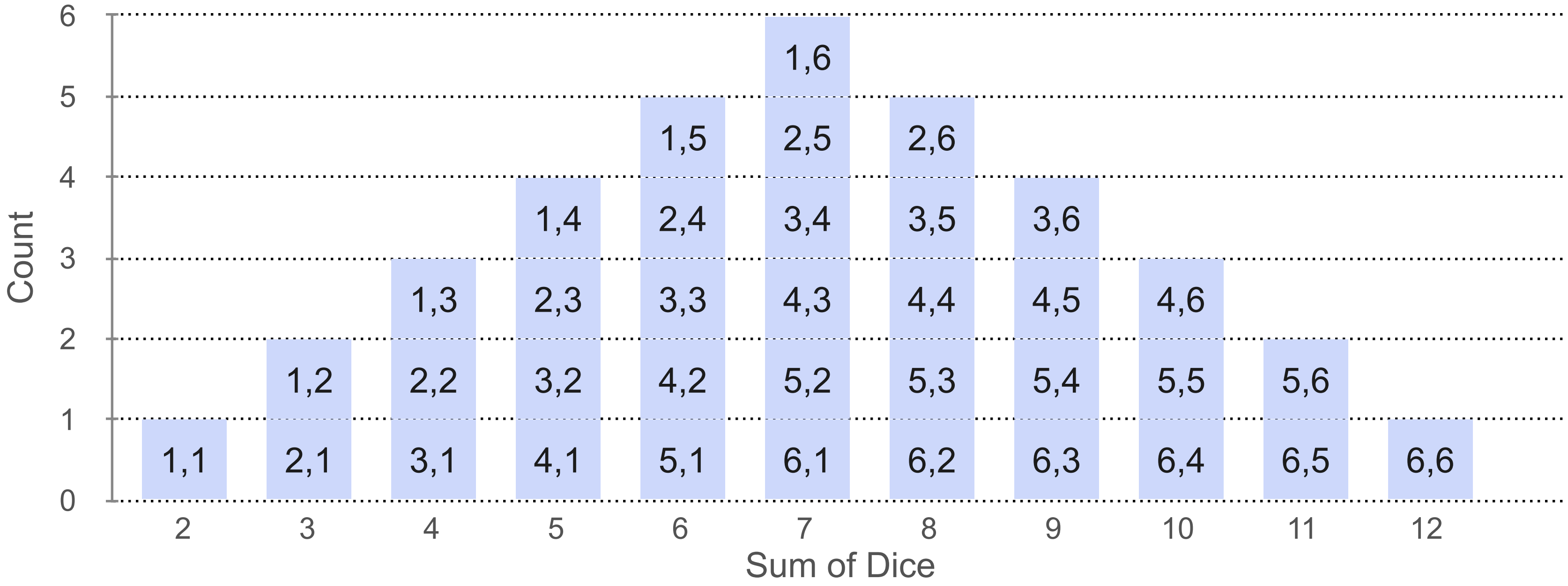
Y: Sum of both dice

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

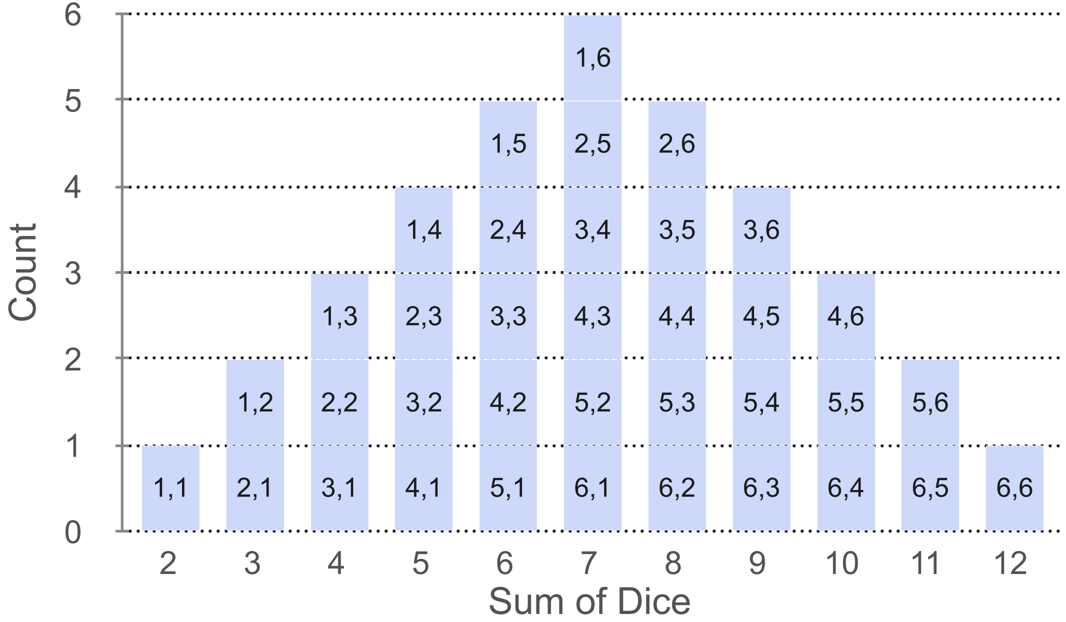
Joint Distributions: Example 3



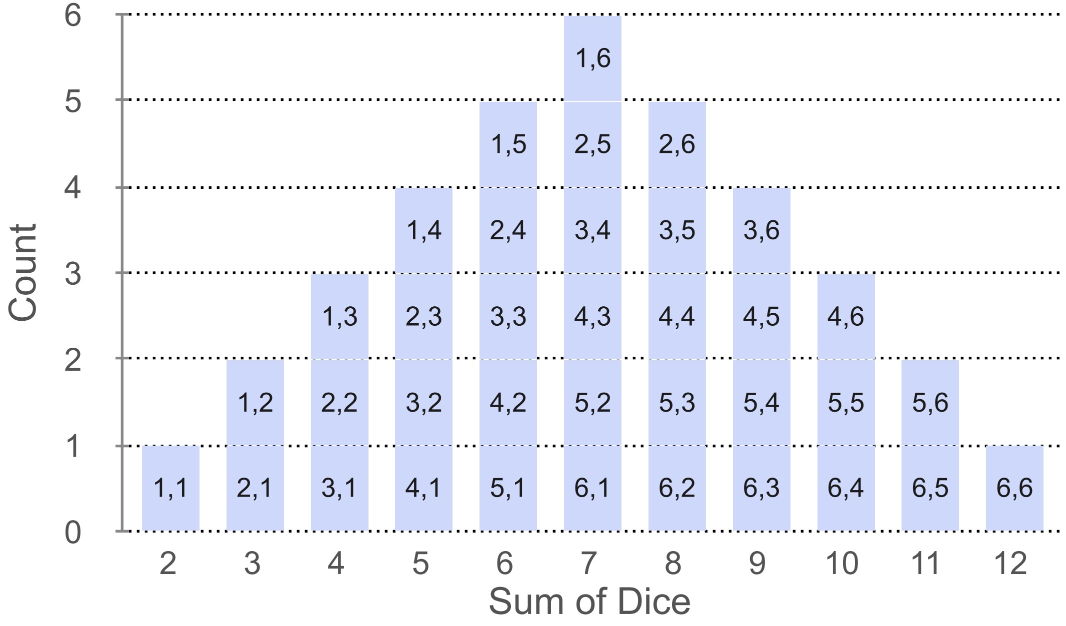
Joint Distributions: Example 3

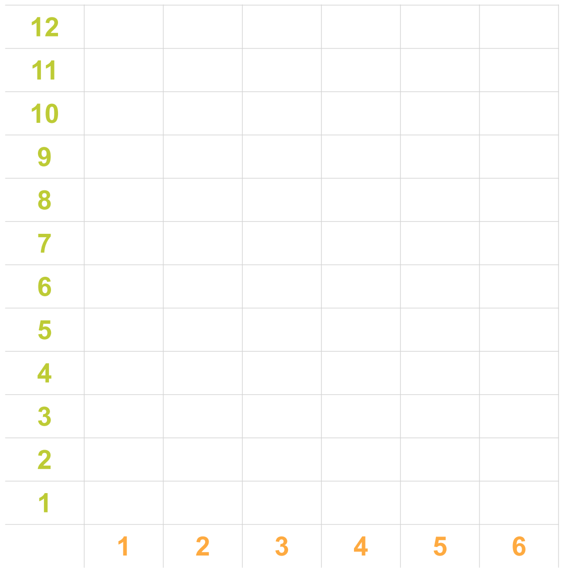


Joint Distributions: Example 3

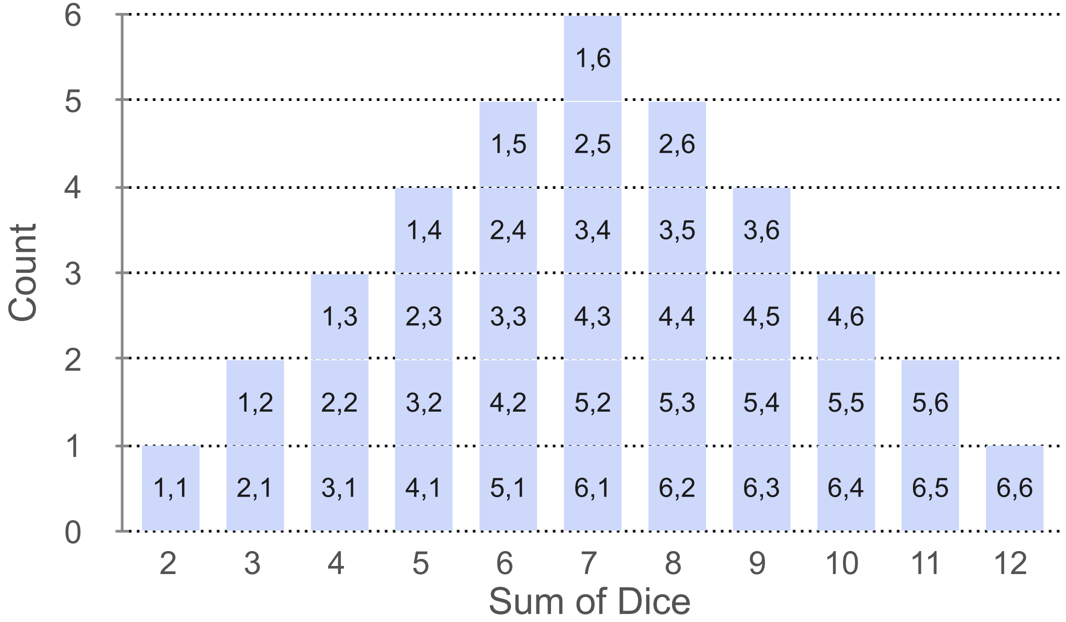


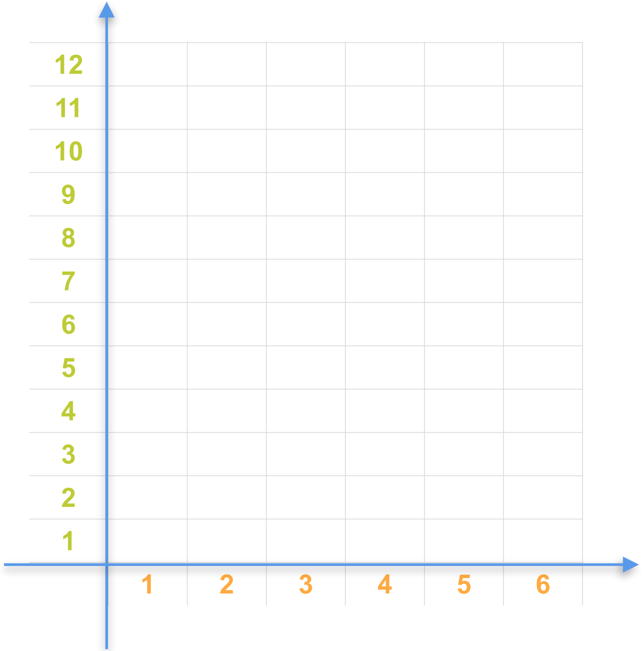
Joint Distributions: Example 3



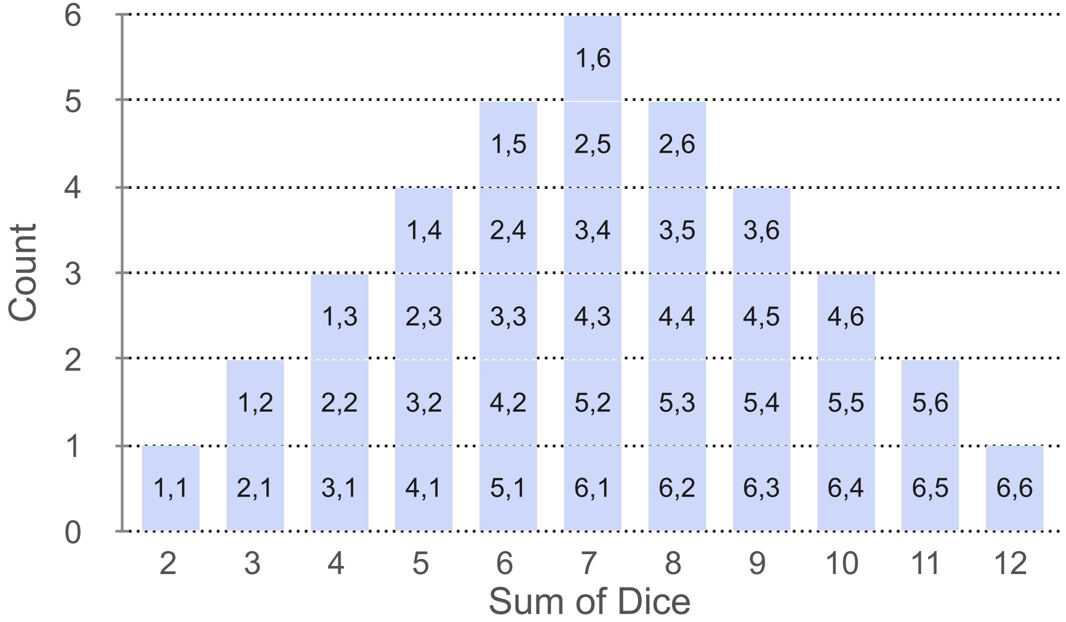


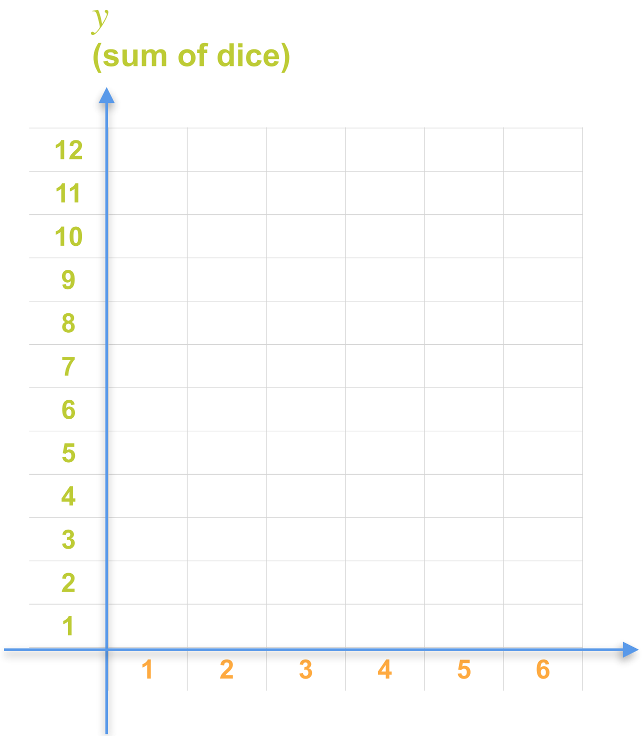
Joint Distributions: Example 3



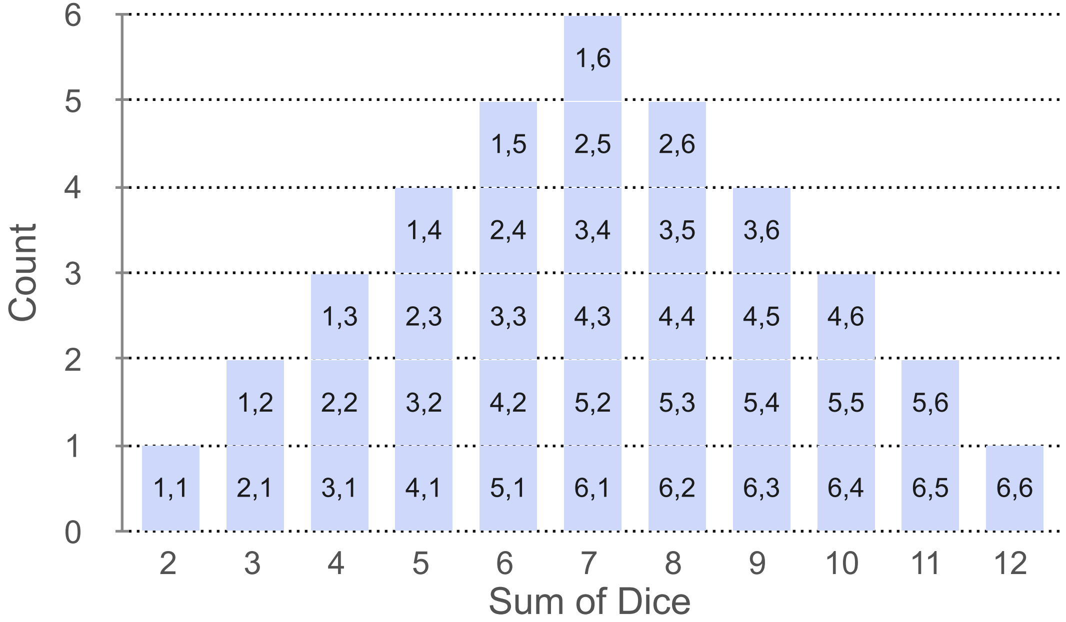


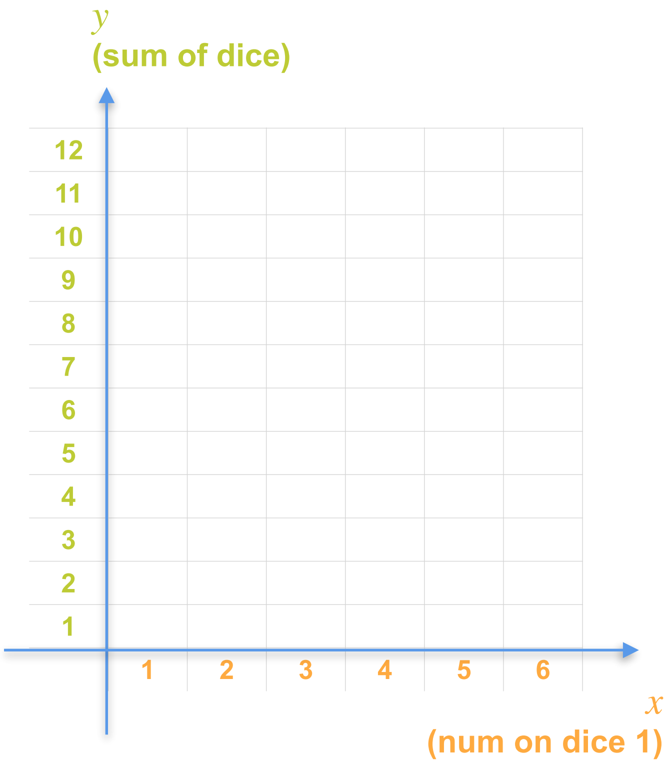
Joint Distributions: Example 3



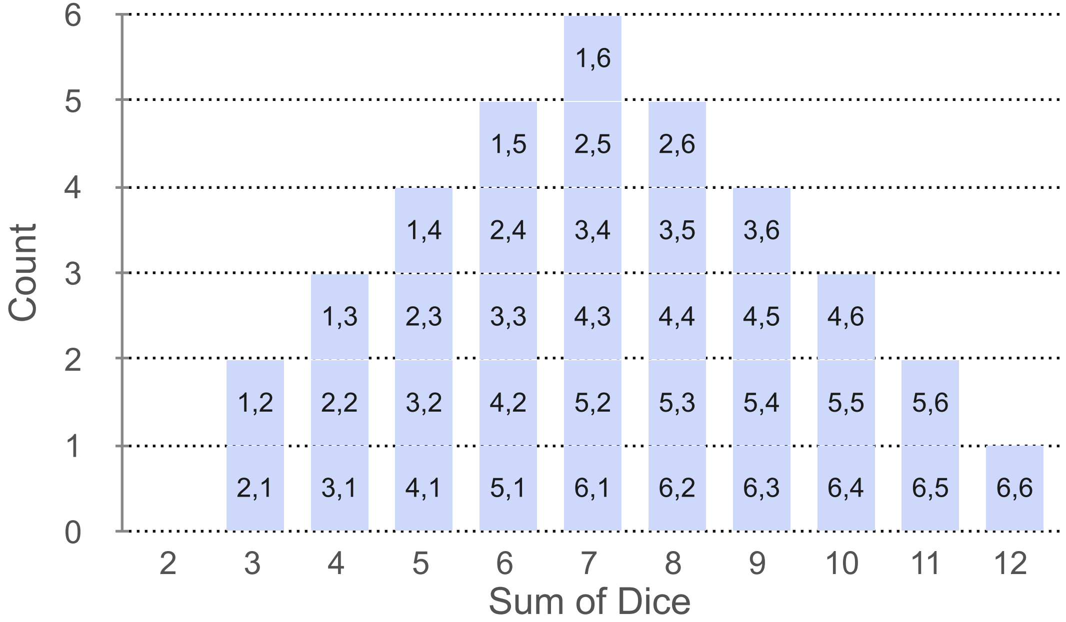


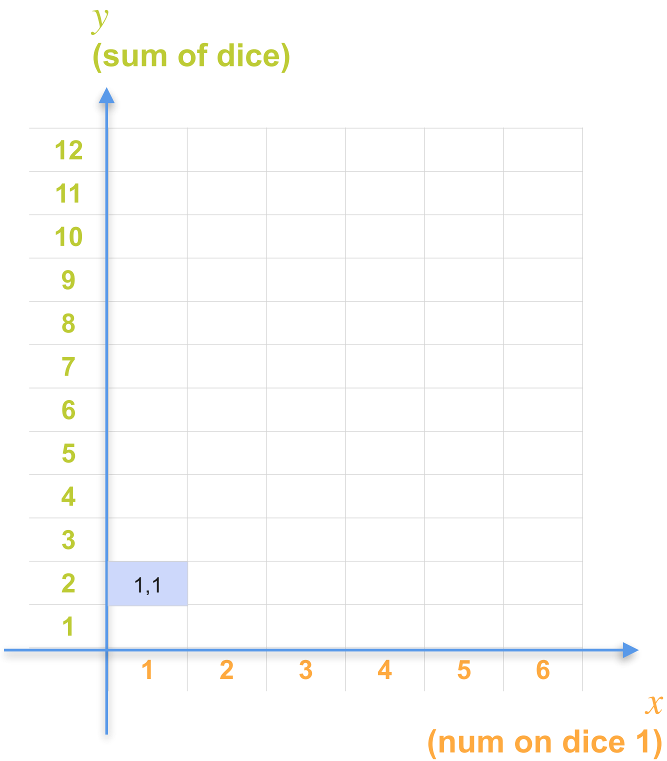
Joint Distributions: Example 3



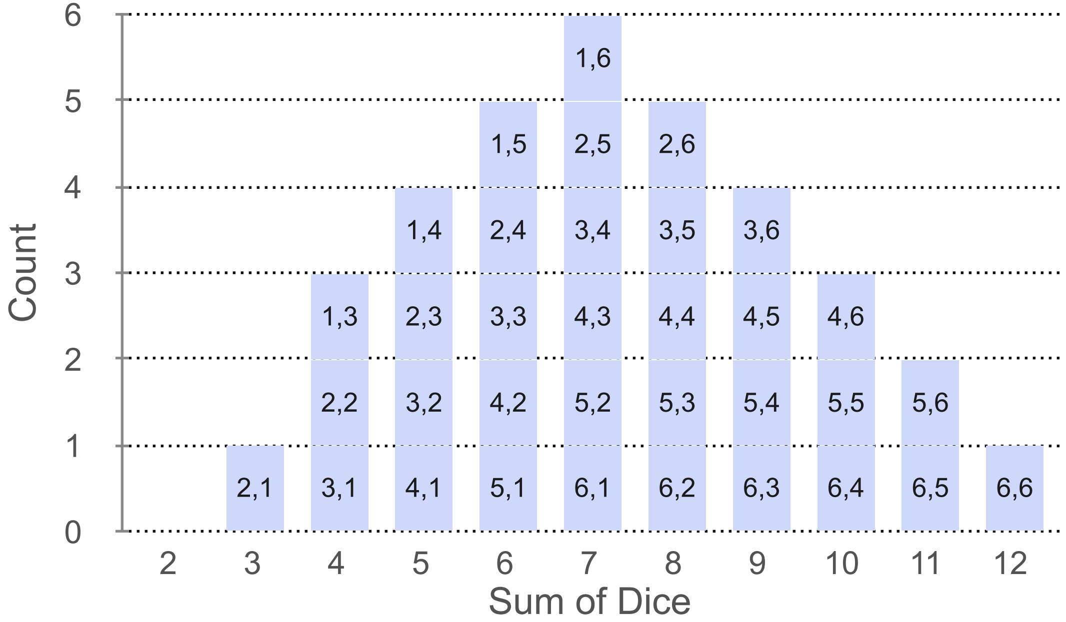


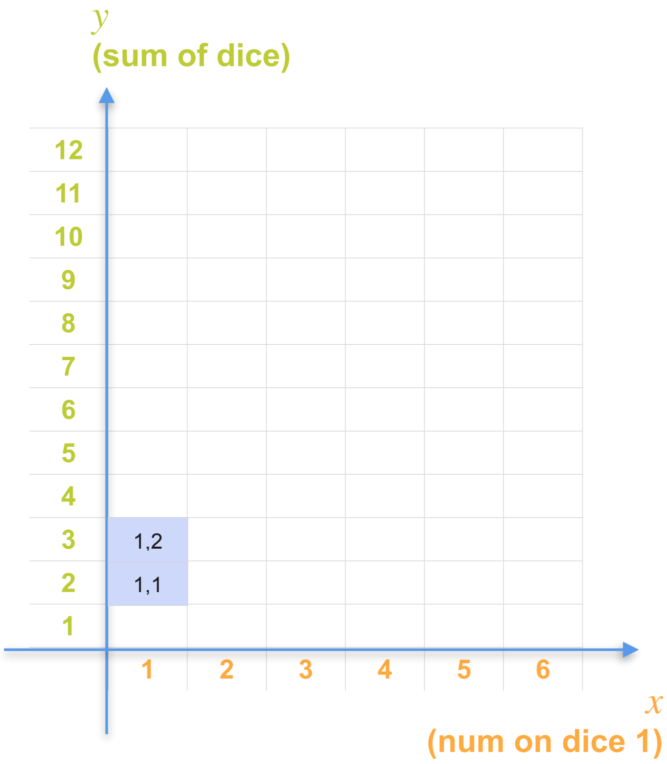
Joint Distributions: Example 3



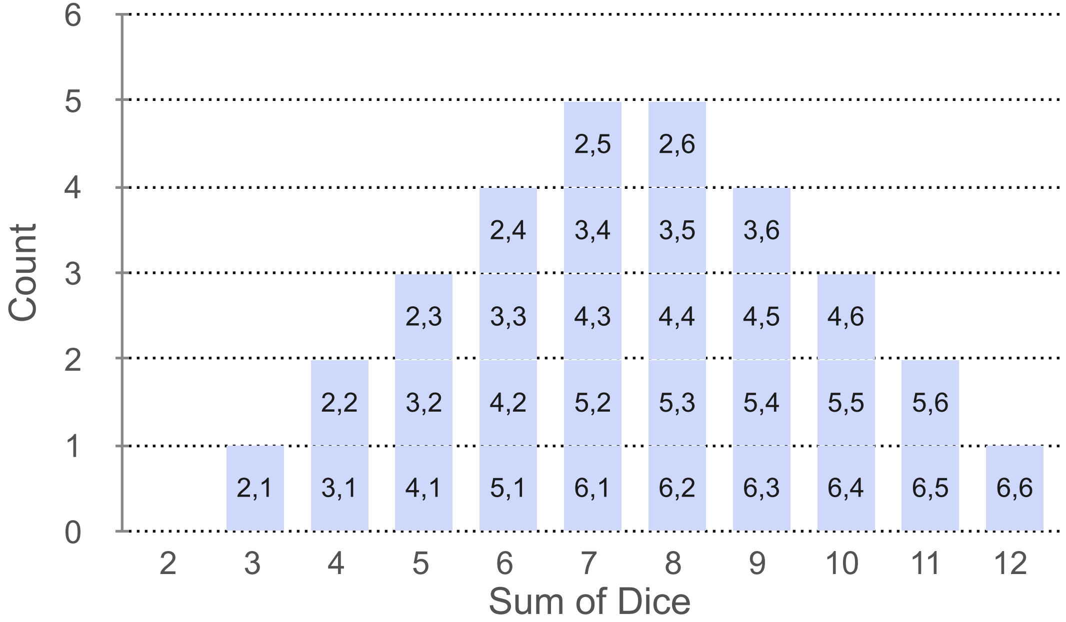


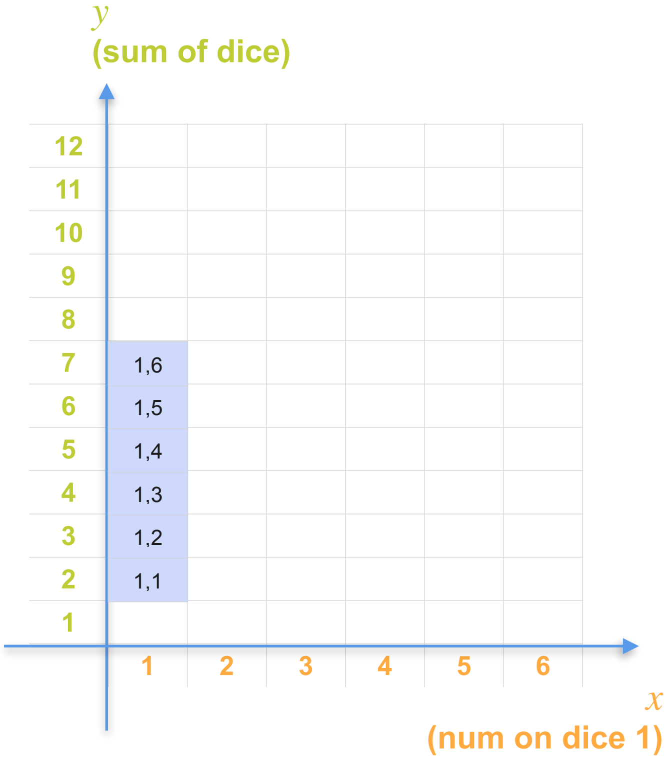
Joint Distributions: Example 3



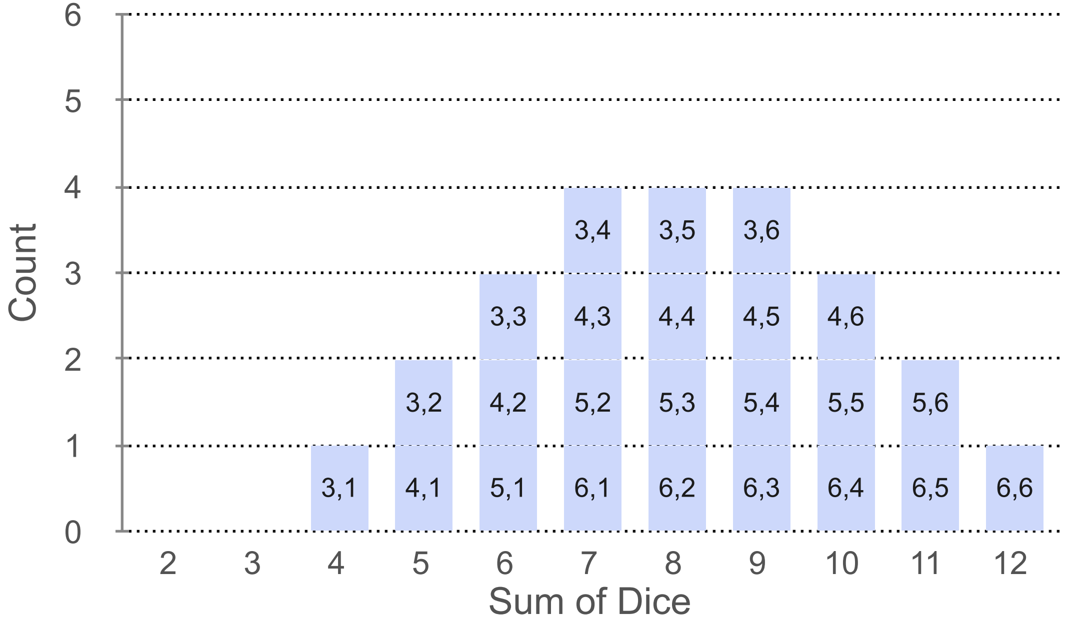


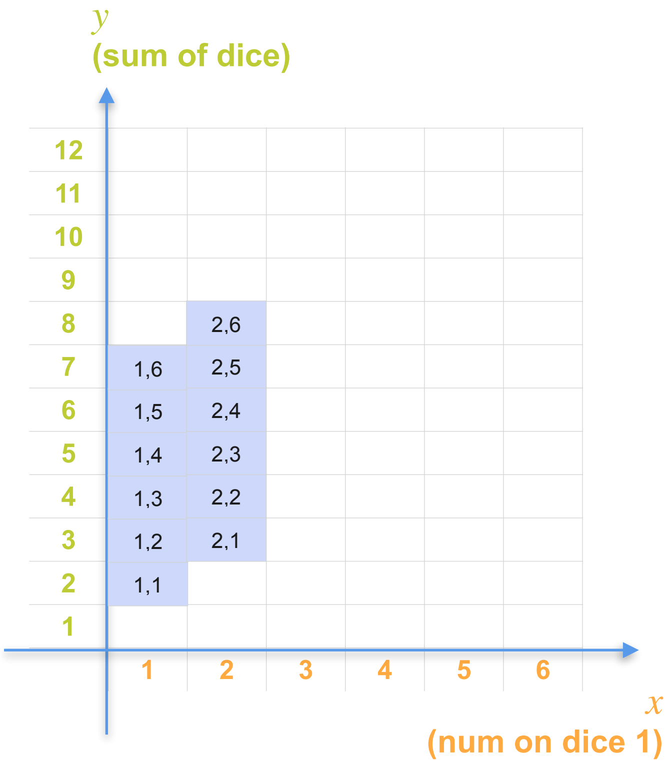
Joint Distributions: Example 3



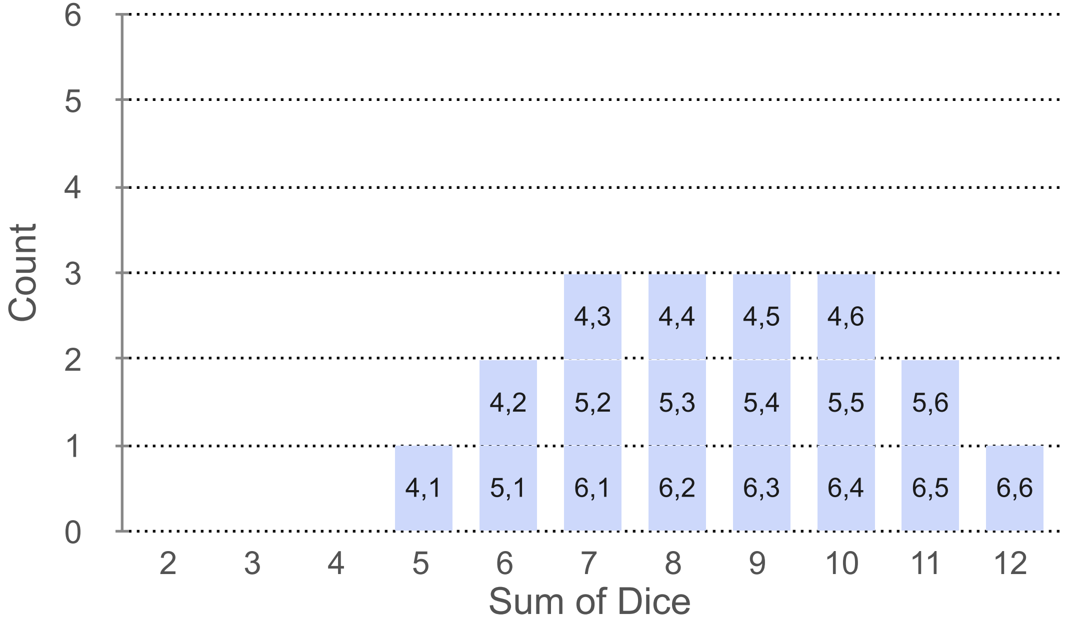


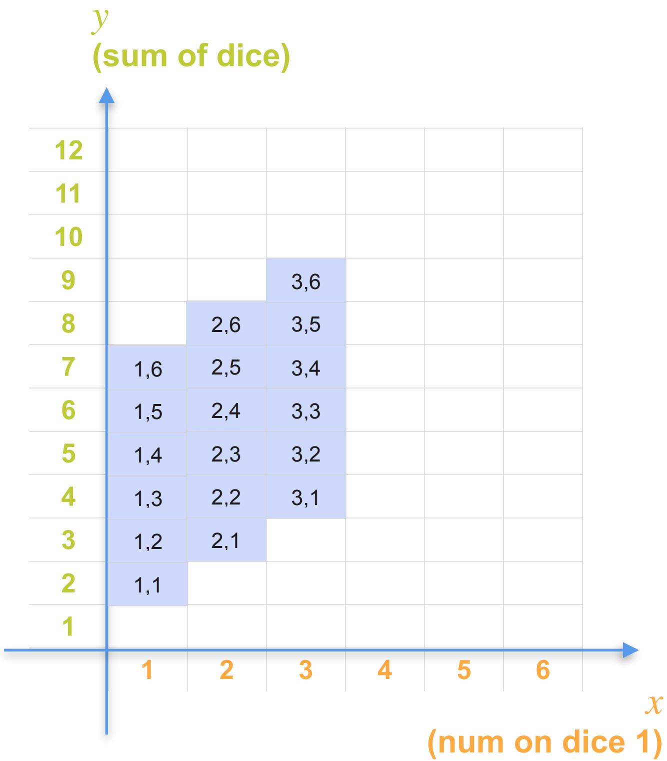
Joint Distributions: Example 3



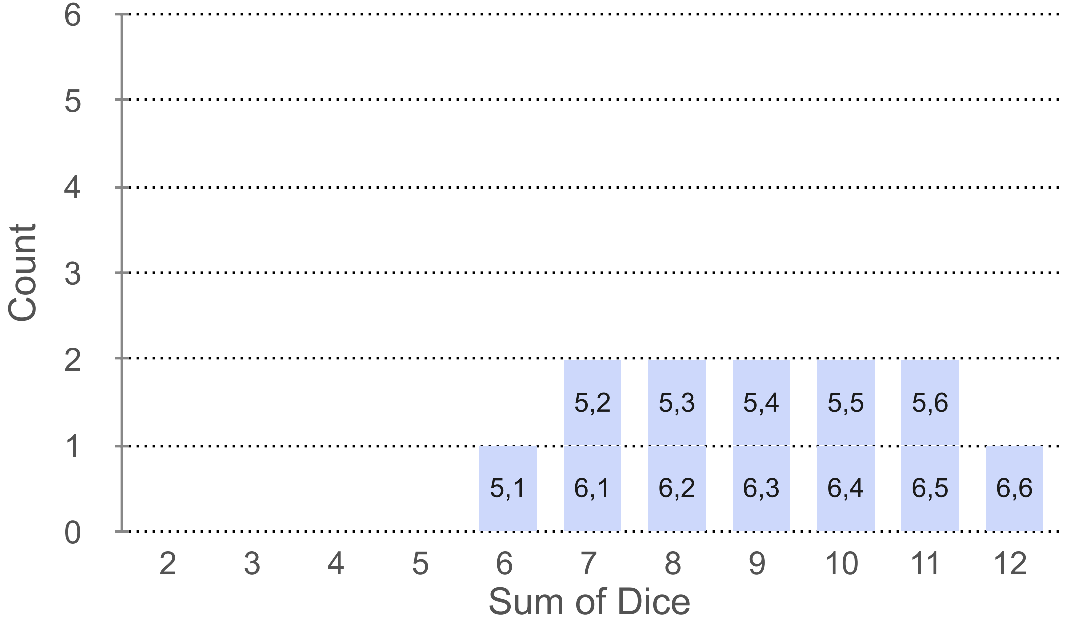


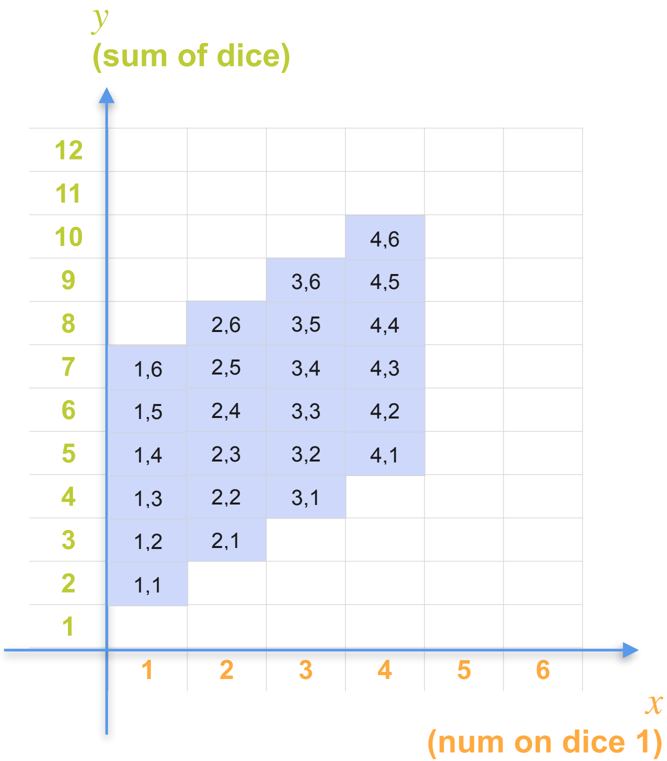
Joint Distributions: Example 3



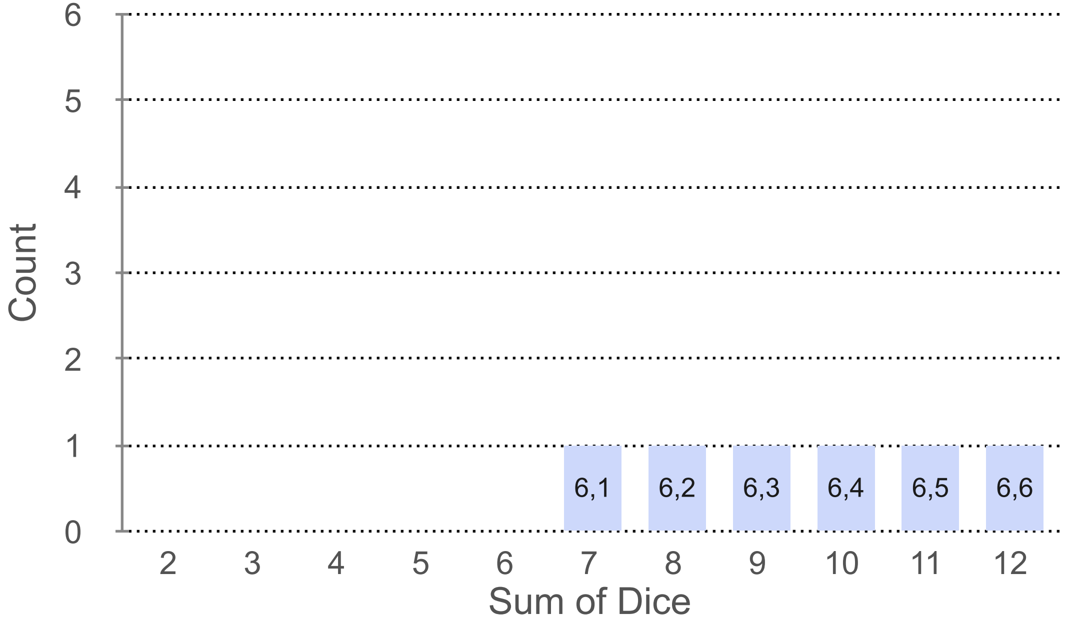


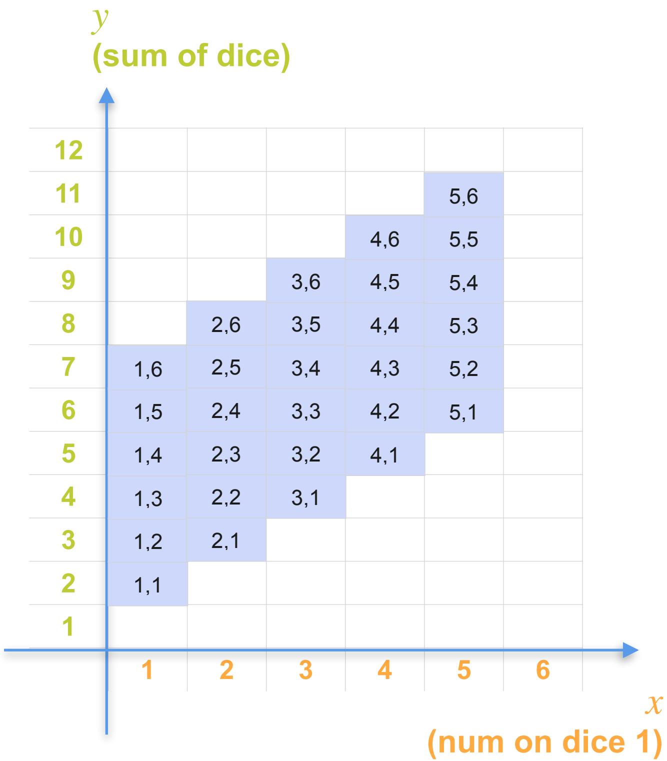
Joint Distributions: Example 3



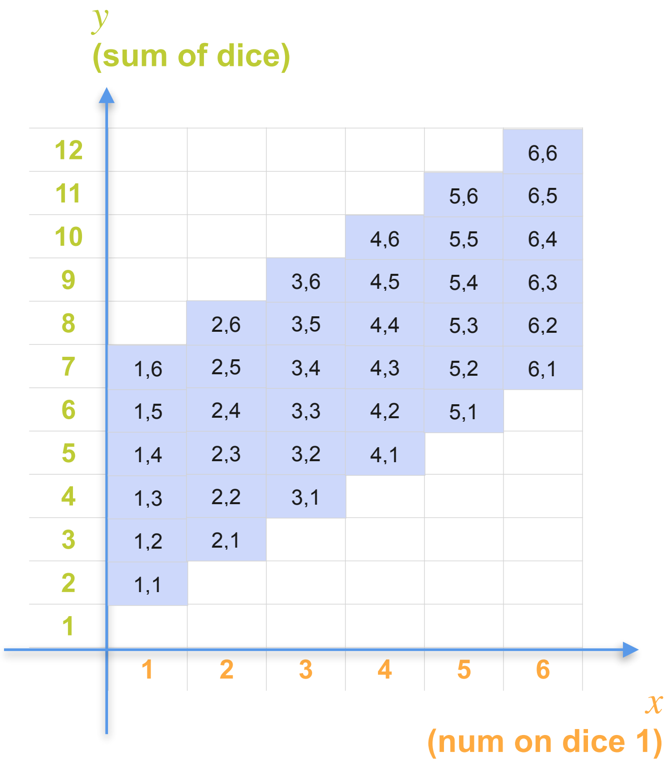


Joint Distributions: Example 3

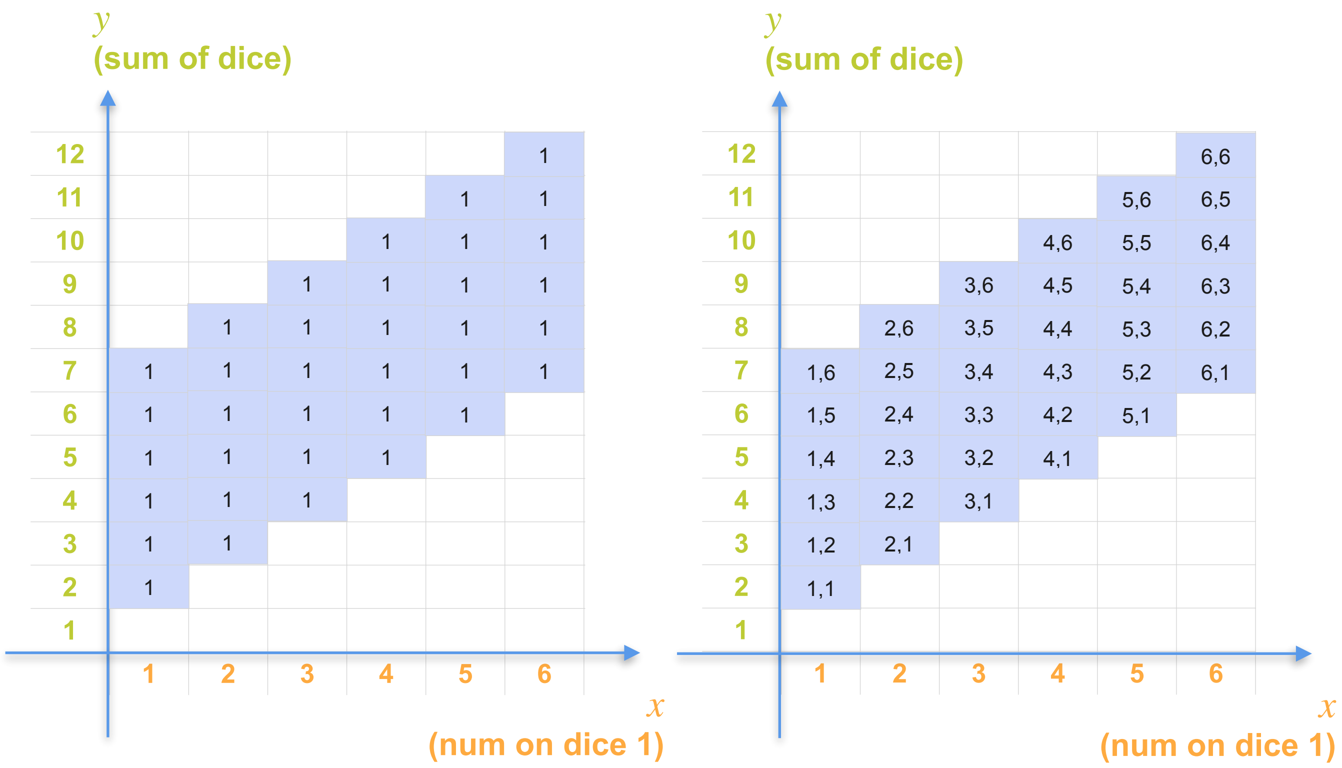




Joint Distributions: Example 3



Joint Distributions: Example 3



Joint Distributions: Example 3

36 possible outcomes

Divide by sum (36)

y (sum of dice)

y \ x	1	2	3	4	5	6
12						1
11					1	1
10				1	1	1
9			1	1	1	1
8		1	1	1	1	1
7	1	1	1	1	1	1
6	1	1	1	1	1	
5	1	1	1	1		
4	1	1	1			
3	1	1				
2	1					
1						

x (num on dice 1)

y (sum of dice)

y \ x	1	2	3	4	5	6
12						6,6
11					5,6	6,5
10				4,6	5,5	6,4
9			3,6	4,5	5,4	6,3
8		2,6	3,5	4,4	5,3	6,2
7	1,6	2,5	3,4	4,3	5,2	6,1
6	1,5	2,4	3,3	4,2	5,1	
5	1,4	2,3	3,2	4,1		
4	1,3	2,2	3,1			
3	1,2	2,1				
2	1,1					
1						

x (num on dice 1)

Joint Distributions: Example 3

36 possible outcomes

Divide by sum (36)

y (sum of dice) \ x (num on dice 1)	1	2	3	4	5	6
12						1/36
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1						

y (sum of dice) \ x (num on dice 1)	1	2	3	4	5	6
12						6,6
11					5,6	6,5
10				4,6	5,5	6,4
9			3,6	4,5	5,4	6,3
8		2,6	3,5	4,4	5,3	6,2
7	1,6	2,5	3,4	4,3	5,2	6,1
6	1,5	2,4	3,3	4,2	5,1	
5	1,4	2,3	3,2	4,1		
4	1,3	2,2	3,1			
3	1,2	2,1				
2	1,1					
1						

Joint Distributions: Example 3

Joint Distribution for X and Y

36 possible outcomes

Divide by sum (36)

y (sum of dice)

	1	2	3	4	5	6
12						1/36
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1						

x (num on dice 1)

y (sum of dice)

	1	2	3	4	5	6
12						6,6
11					5,6	6,5
10				4,6	5,5	6,4
9			3,6	4,5	5,4	6,3
8		2,6	3,5	4,4	5,3	6,2
7	1,6	2,5	3,4	4,3	5,2	6,1
6	1,5	2,4	3,3	4,2	5,1	
5	1,4	2,3	3,2	4,1		
4	1,3	2,2	3,1			
3	1,2	2,1				
2	1,1					
1						

x (num on dice 1)

Joint Distributions: Example 3

Joint Distribution for X and Y

y (sum of dice) \ x (num on dice 1)	1	2	3	4	5	6
12						1/36
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1						

Joint Distributions: Example 3

Joint Distribution for X and Y

y
(sum of dice)

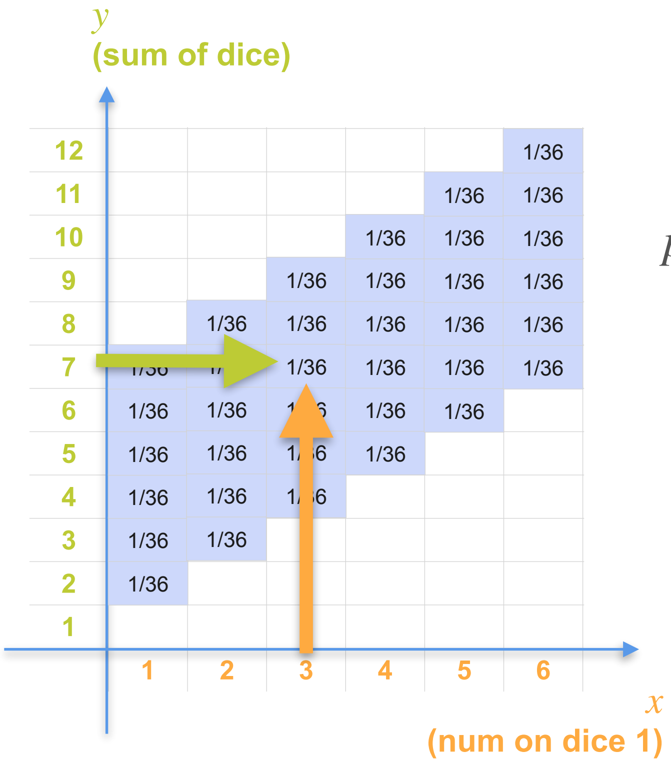
12						1/36
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1						
	1	2	3	4	5	6

x
(num on dice 1)

$$p_{XY}(3, 7) = \mathbf{P}(X = 3, Y = 7)$$

Joint Distributions: Example 3

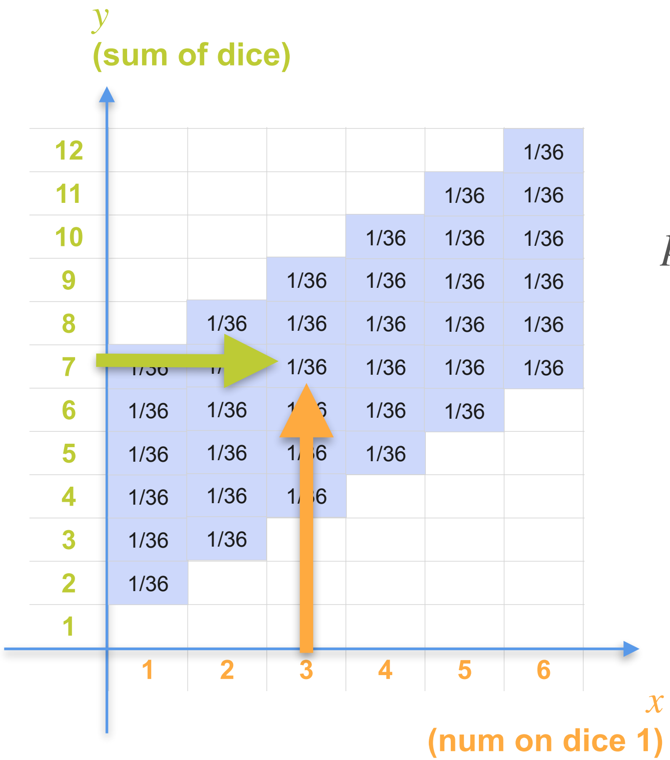
Joint Distribution for X and Y



$$p_{XY}(3, 7) = \mathbf{P}(X = 3, Y = 7)$$

Joint Distributions: Example 3

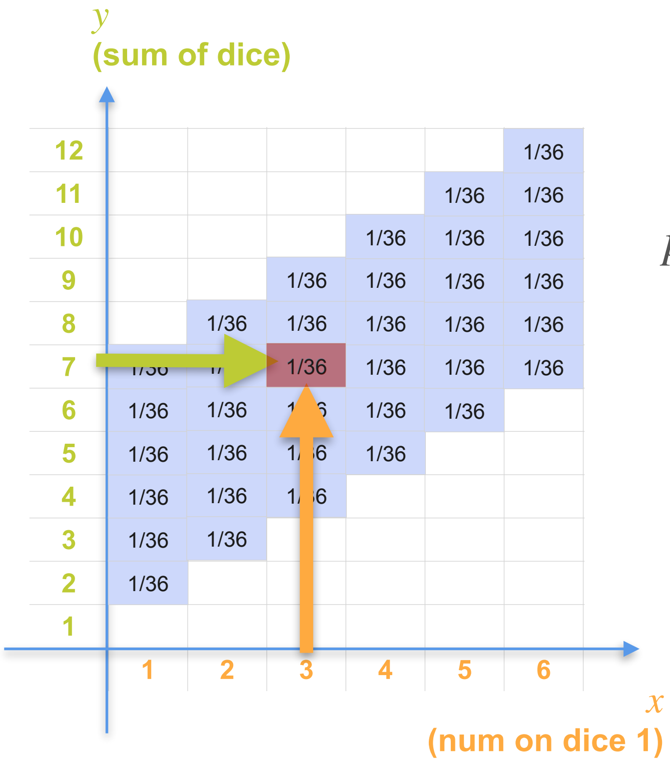
Joint Distribution for X **and** Y



$$p_{XY}(3, 7) = \mathbf{P}(X = 3, Y = 7) = \frac{1}{36}$$

Joint Distributions: Example 3

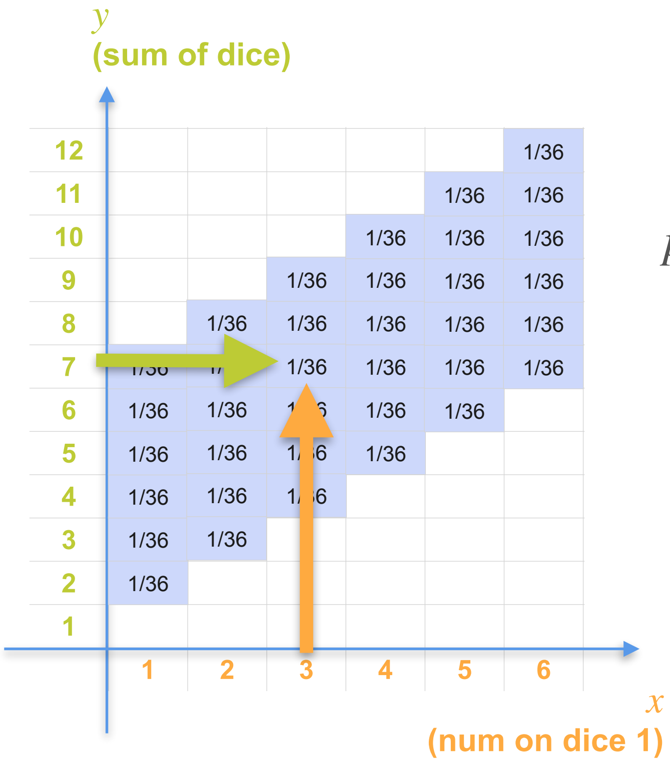
Joint Distribution for
X **and** Y



$$p_{XY}(3, 7) = \mathbf{P}(X = 3, Y = 7) = \frac{1}{36}$$

Joint Distributions: Example 3

Joint Distribution for X and Y



$$p_{XY}(3, 7) = \mathbf{P}(X = 3, Y = 7) = \frac{1}{36}$$

Joint Distributions: Example 3

Joint Distribution for X and Y

y (sum of dice) / x (num on dice 1)

y \ x	1	2	3	4	5	6
12						1/36
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1						

$$p_{XY}(3,7) = \mathbf{P}(X=3, Y=7) = \frac{1}{36}$$

$$p_{XY}(1,1) = \mathbf{P}(X=1, Y=1)$$

Joint Distributions: Example 3

Joint Distribution for
X **and** Y

y (sum of dice) / x (num on dice 1)

y \ x	1	2	3	4	5	6
12						1/36
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1	0					

$$p_{XY}(3,7) = \mathbf{P}(X=3, Y=7) = \frac{1}{36}$$

$$p_{XY}(1,1) = \mathbf{P}(X=1, Y=1)$$

Joint Distributions: Example 3

Joint Distribution for X and Y

y (sum of dice) \ x (num on dice 1)	1	2	3	4	5	6
12						1/36
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1	0					

$$p_{XY}(3,7) = \mathbf{P}(X=3, Y=7) = \frac{1}{36}$$

$$p_{XY}(1,1) = \mathbf{P}(X=1, Y=1) = 0$$

DeepLearning.AI

Probability Distributions with Multiple Variables

Joint Distribution (Continuous)

Joint Continuous Distributions

Joint Continuous Distributions

X : age of a child in year

Y : discrete values of height of child in inches

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X : the number rolled on the 1st dice

Y : sum of both dice

Joint Continuous Distributions

X : age of a child in year

Y : discrete values of height of child in inches

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are
Discrete Random Variables

X : the number rolled on the 1st dice

Y : sum of both dice

Joint Continuous Distributions

X : age of a child in year

Y : discrete values of height of child in inches

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are
Discrete Random Variables

X : the number rolled on the 1st dice

Y : sum of both dice

What about when X and Y are
Continuous Random Variables?

Joint Continuous Distributions

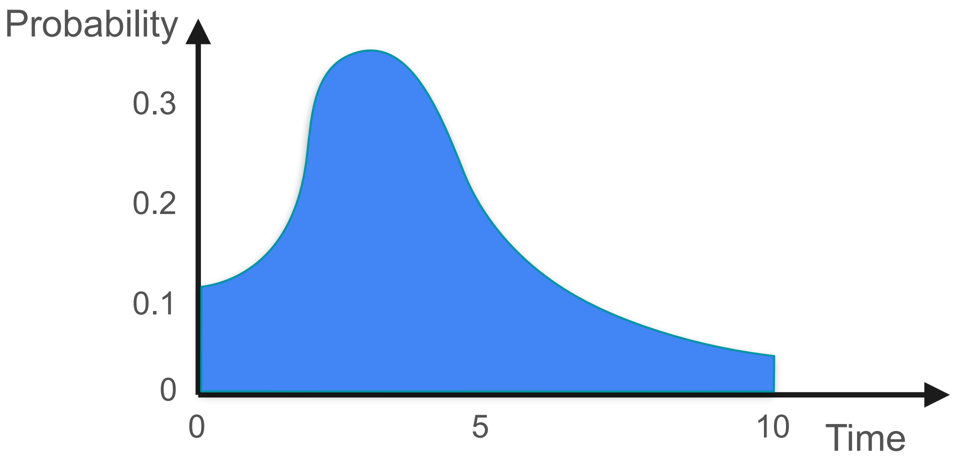
Joint Continuous Distributions

Joint Continuous Distributions

X variable: Waiting time

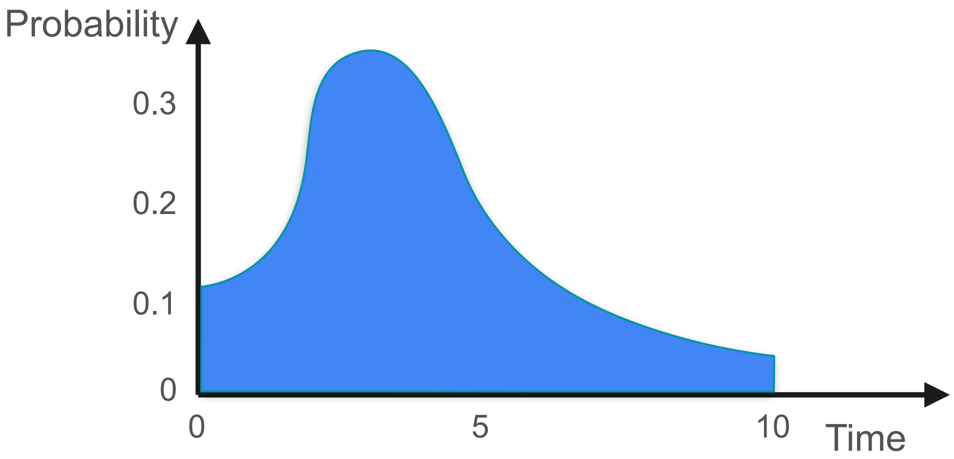
Joint Continuous Distributions

X variable: Waiting time



Joint Continuous Distributions

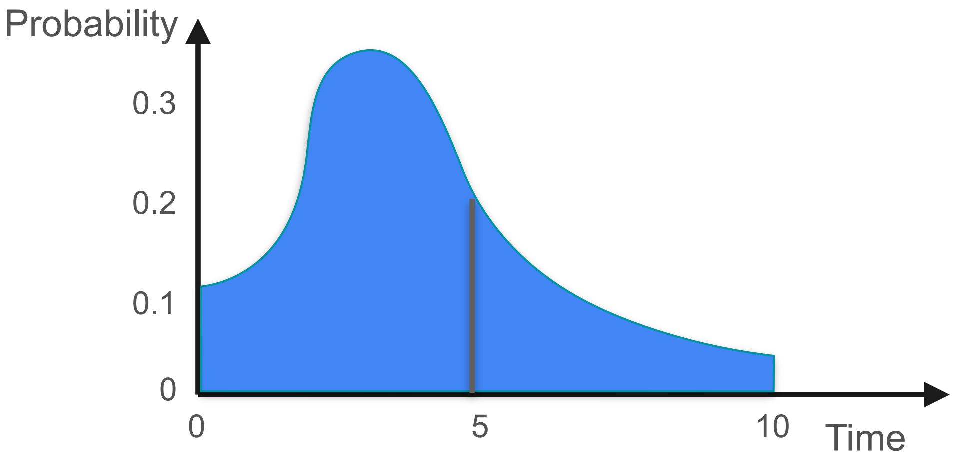
X variable: Waiting time



$\mathbf{P}(X$ between 0 and 5 mins$)$

Joint Continuous Distributions

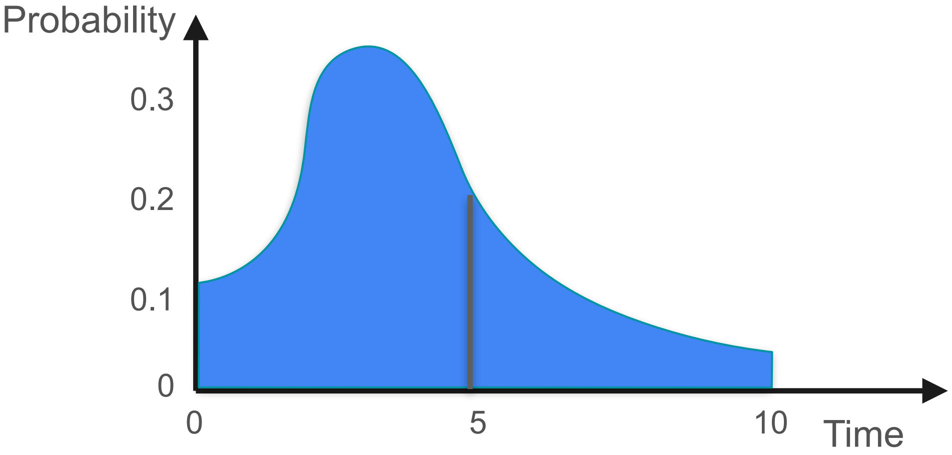
X variable: Waiting time



$\mathbf{P}(X$ between 0 and 5 mins$)$

Joint Continuous Distributions

X variable: Waiting time

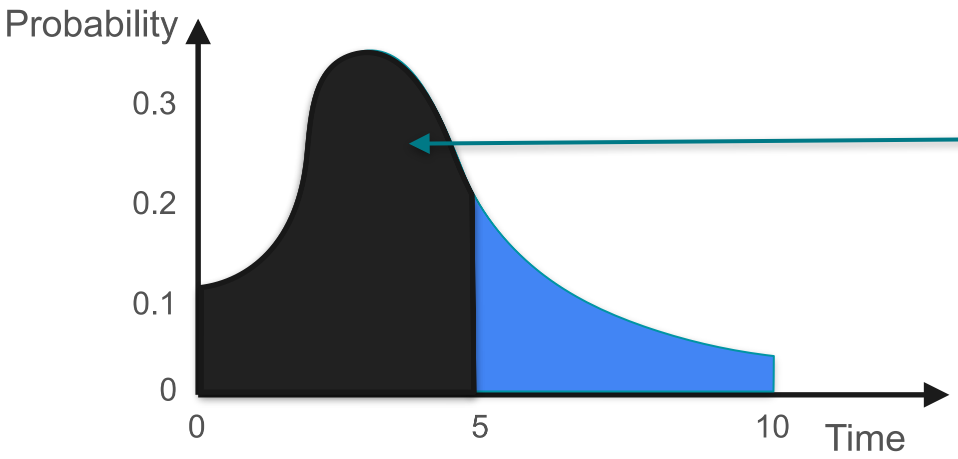


$\mathbf{P}(X$ between 0 and 5 mins)

Area occupied by this region

Joint Continuous Distributions

X variable: Waiting time



$\mathbf{P}(X$ between 0 and 5 mins)

Area occupied by this region

Joint Continuous Distributions

Joint Continuous Distributions

X Y

Joint Continuous Distributions

X

Waiting time
before a call is picked up
[0 - 10 minutes]

Y

Joint Continuous Distributions

X

Waiting time
before a call is picked up
[0 - 10 minutes]

Y

Customer
satisfaction rating
[0 - 10]

Joint Continuous Distributions

X

Waiting time
before a call is picked up
[0 - 10 minutes]

Y

Customer
satisfaction rating
[0 - 10]

Both variables are continuous

Joint Continuous Distributions

X

Waiting time
before a call is picked up
[0 - 10 minutes]

Y

Customer
satisfaction rating
[0 - 10]

Both variables are continuous

2.4 minutes

1.5 minutes

Joint Continuous Distributions

X

Waiting time
before a call is picked up
[0 - 10 minutes]

Y

Customer
satisfaction rating
[0 - 10]

Both variables are continuous

2.4 minutes

1.5 minutes

0.0

5.7

Joint Continuous Distributions: Dataset

Joint Continuous Distributions: Dataset

X variable: Waiting time (mins)
0 - 10 mins

Joint Continuous Distributions: Dataset

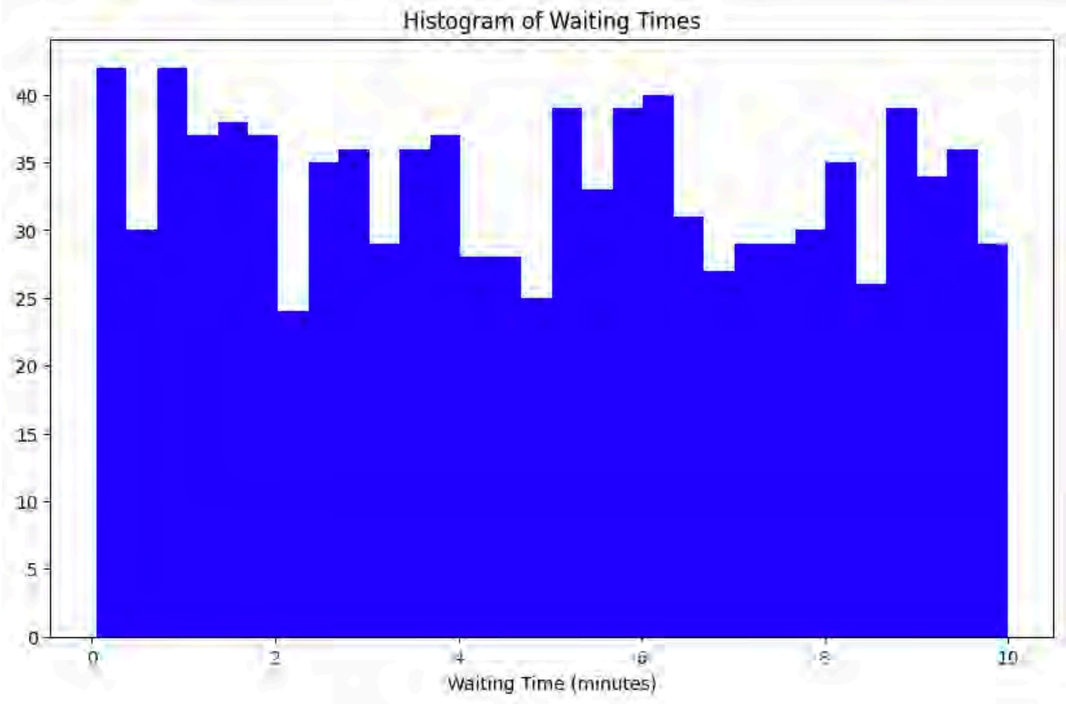
X variable: Waiting time (mins)
0 - 10 mins

1000 customers

Joint Continuous Distributions: Dataset

X variable: Waiting time (mins)
0 - 10 mins

1000 customers



Joint Continuous Distributions: Dataset

Joint Continuous Distributions: Dataset

Y variable: Satisfaction rating
0 - 10

Joint Continuous Distributions: Dataset

Y variable: Satisfaction rating
0 - 10

1000 customers

Joint Continuous Distributions: Dataset

Y variable: Satisfaction rating
0 - 10

1000 customers

Joint Continuous Distributions: Dataset

Joint Continuous Distributions: Dataset

X variable: Waiting time (mins)
0 - 10 mins

Joint Continuous Distributions: Dataset

X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

Joint Continuous Distributions: Dataset

X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

1000 customers

Joint Continuous Distributions: Dataset

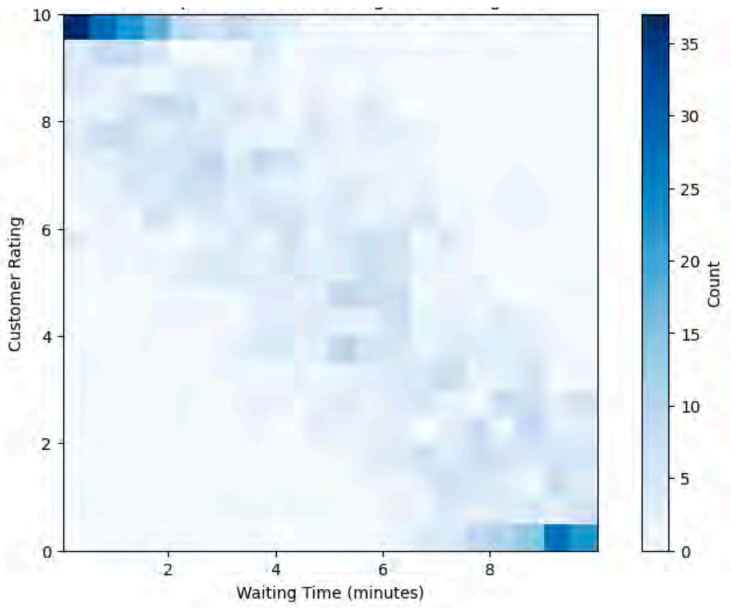
X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

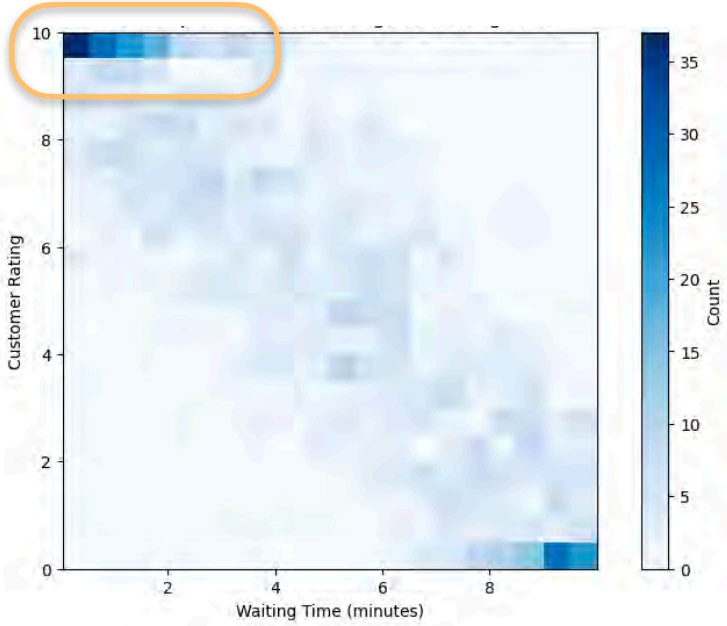
1000 customers

Joint Continuous Distributions: Dataset

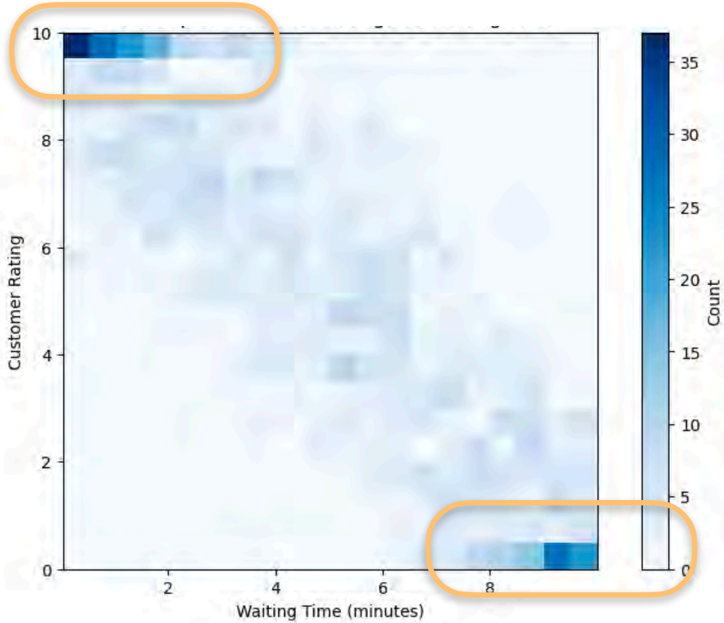
Joint Continuous Distributions: Dataset



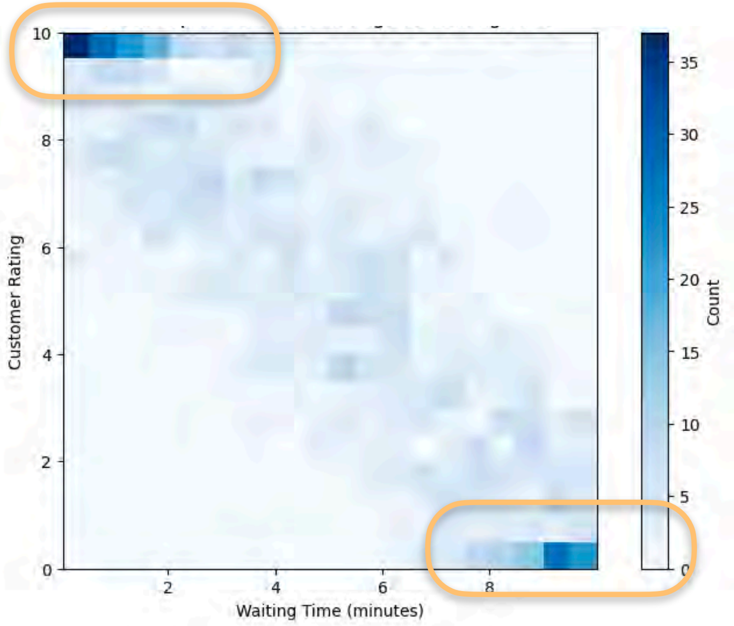
Joint Continuous Distributions: Dataset



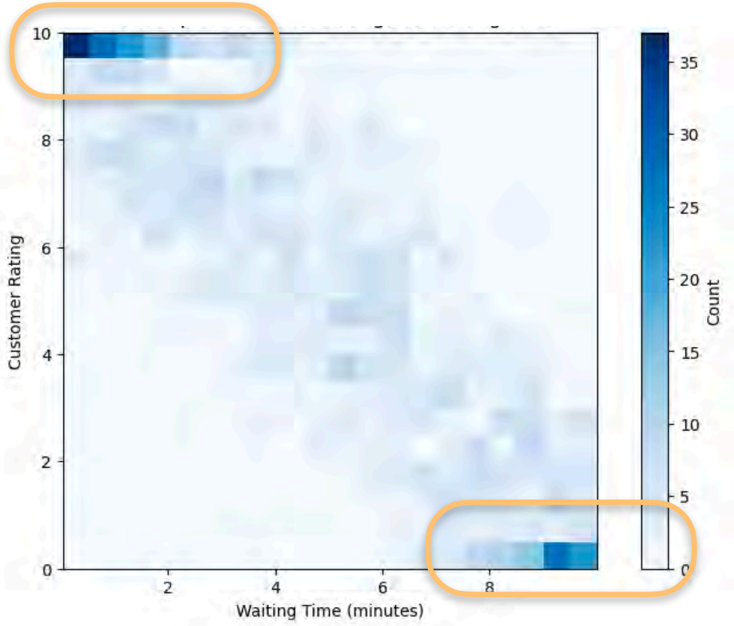
Joint Continuous Distributions: Dataset

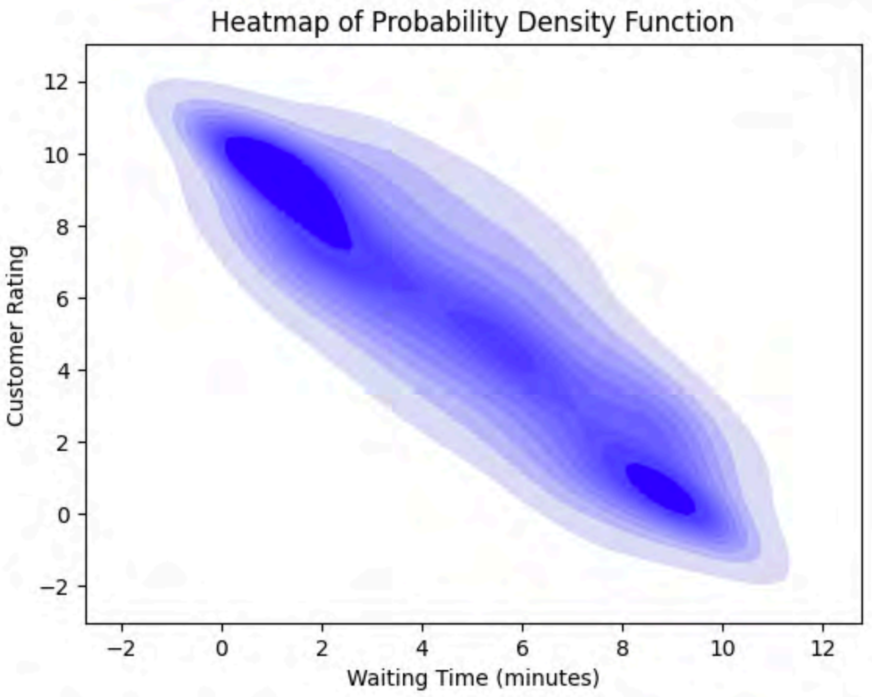


Joint Continuous Distributions: Dataset

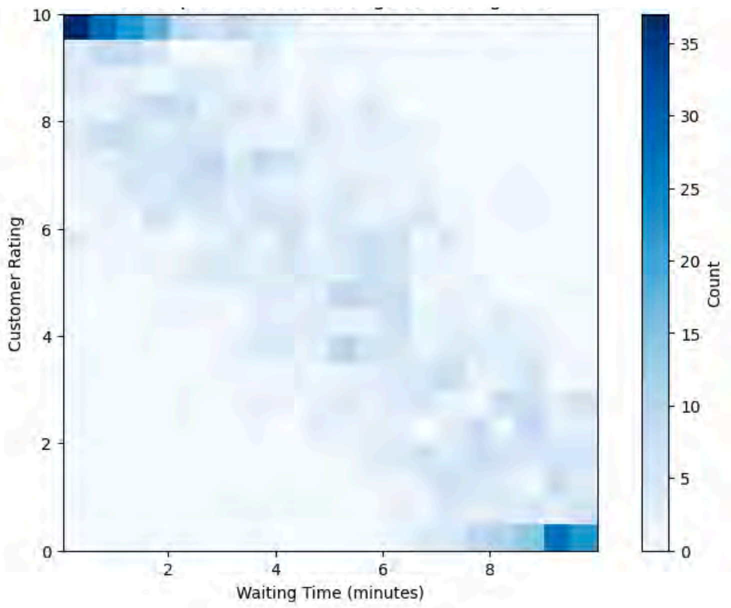


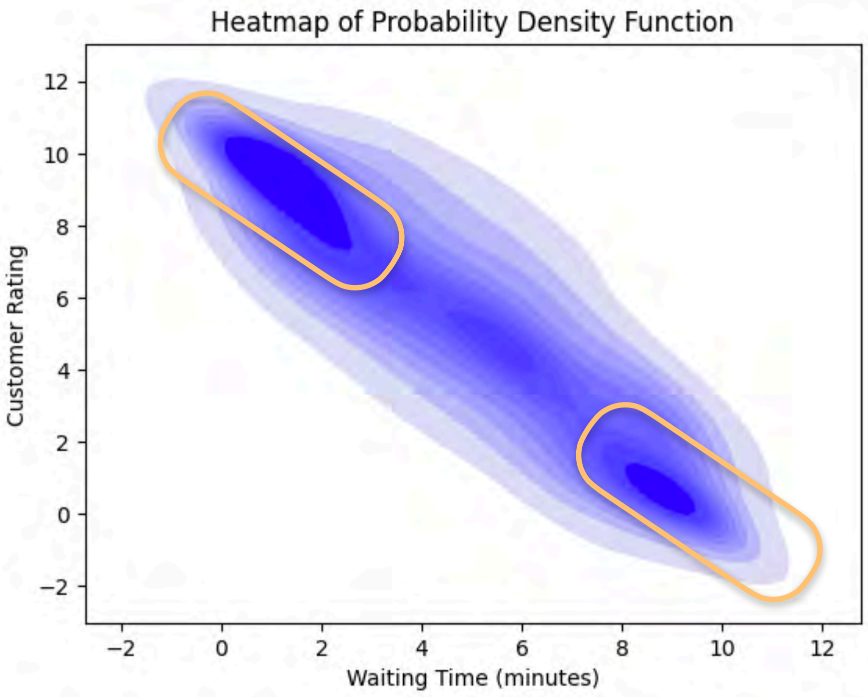
Joint Continuous Distributions: Dataset



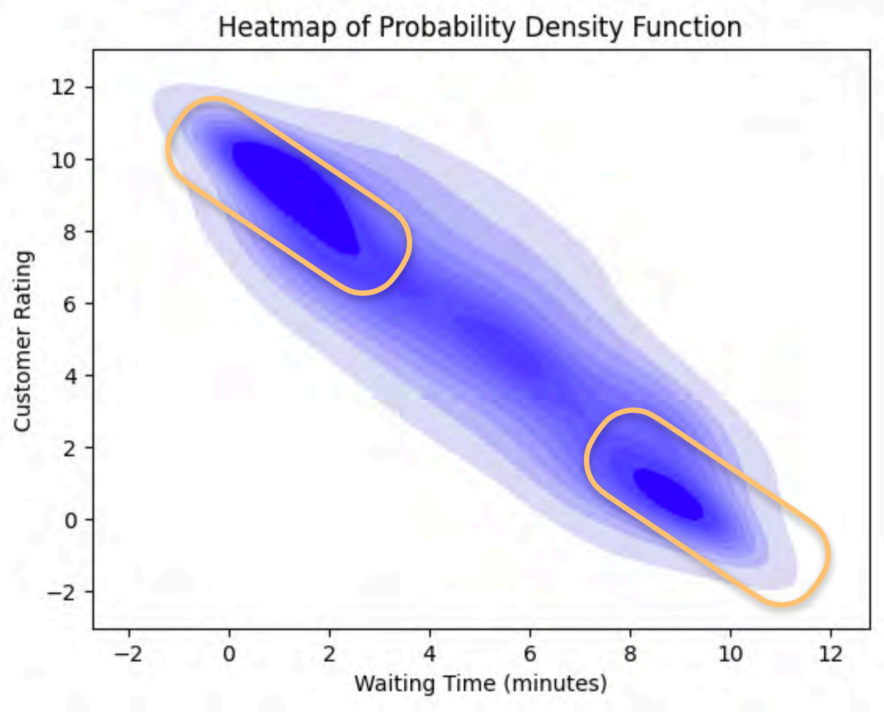


Joint Continuous Distributions: Dataset

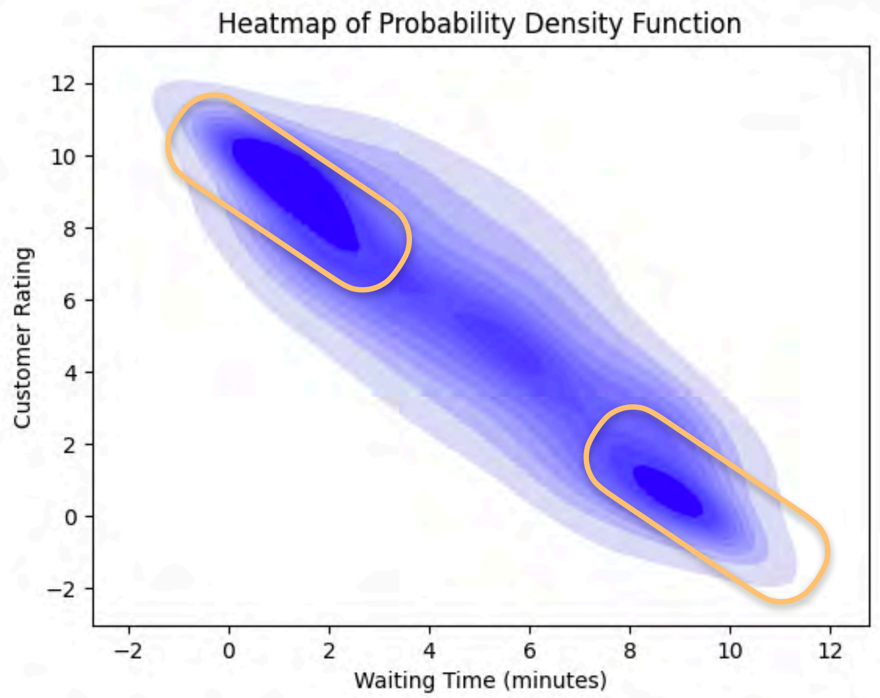




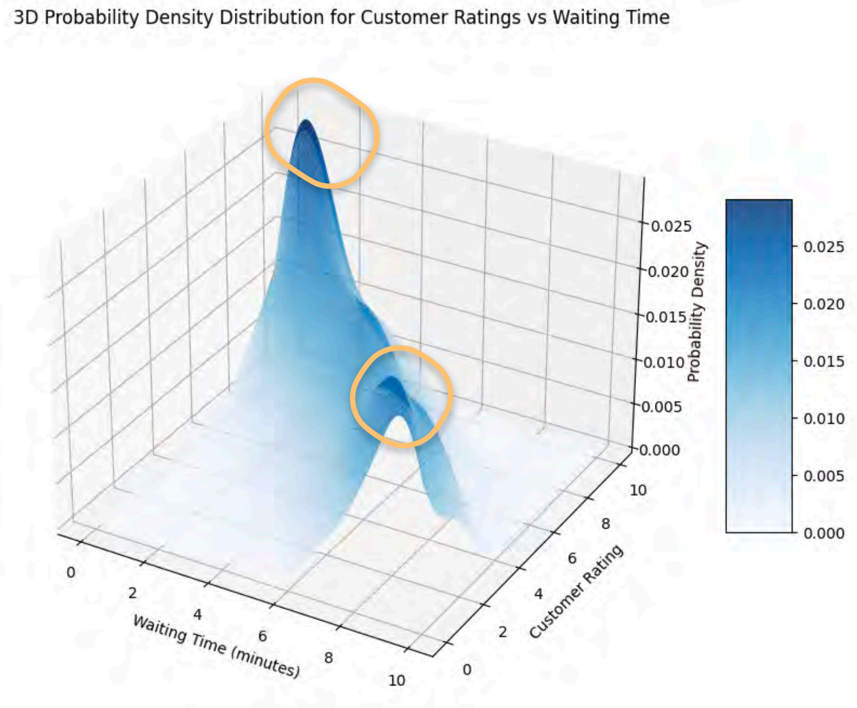
Joint Continuous Distributions: Dataset

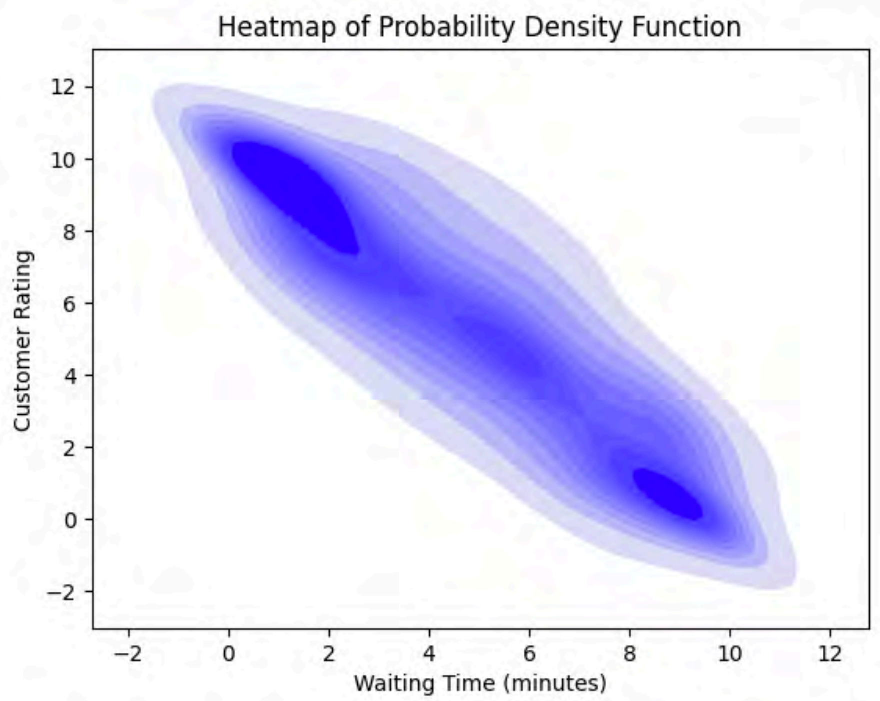


Joint Continuous Distributions: Dataset



Joint Continuous Distributions: Dataset





Joint Continuous Distributions: Dataset

X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

1000 customers

Expected Value

X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

1000 customers

Expected Value

X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

1000 customers

$\mathbb{E}[X] =$ 4.903 minutes

Expected Value

X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

1000 customers

$\mathbb{E}[X] =$ 4.903 minutes

$\mathbb{E}[Y] =$ 5.280

Expected Value

X variable: Waiting time (mins)
0 - 10 mins

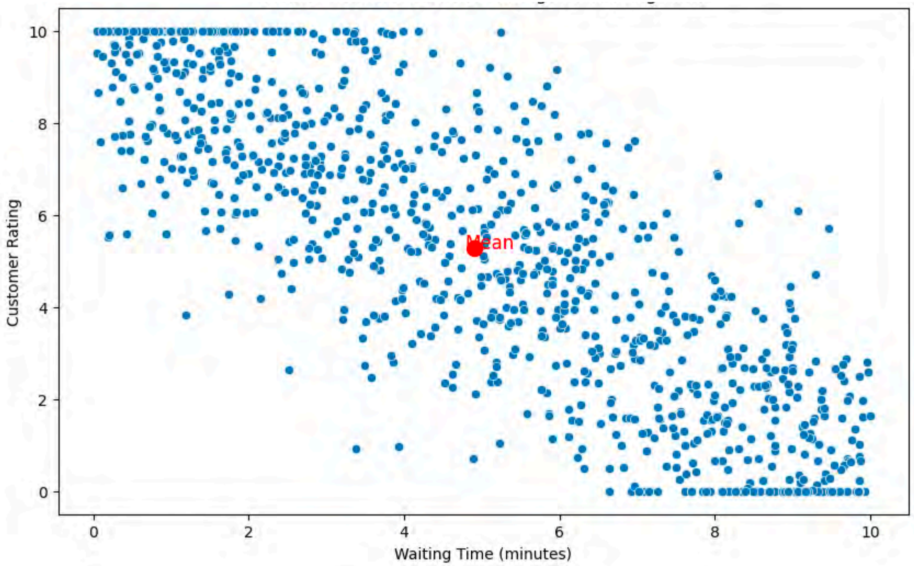
Y variable: Satisfaction rating
0 - 10

1000 customers

$\mathbb{E}[X] =$ 4.903 minutes

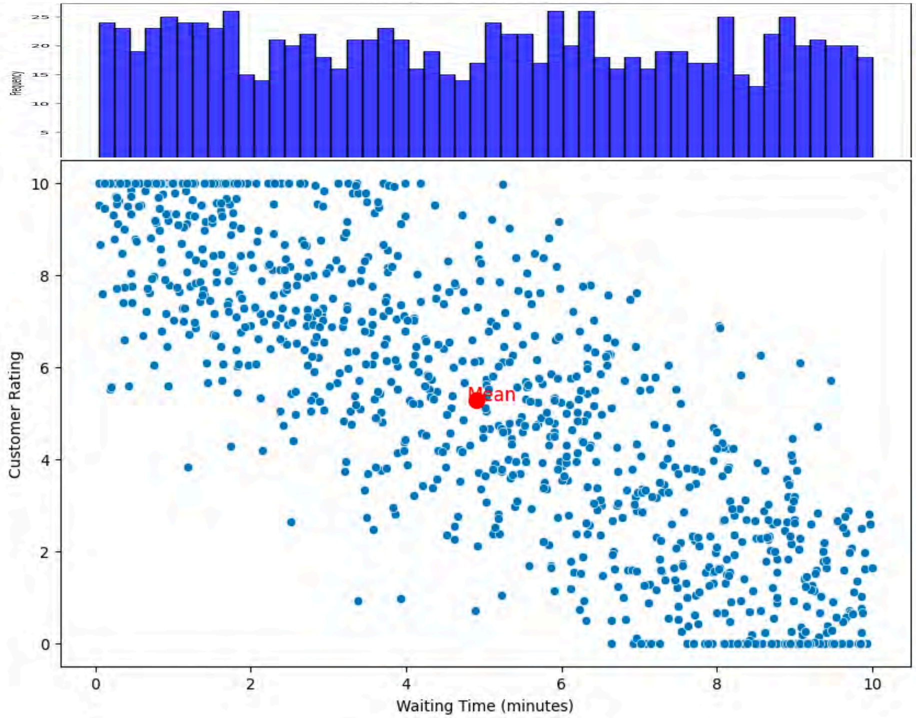
$\mathbb{E}[Y] =$ 5.280

Variances



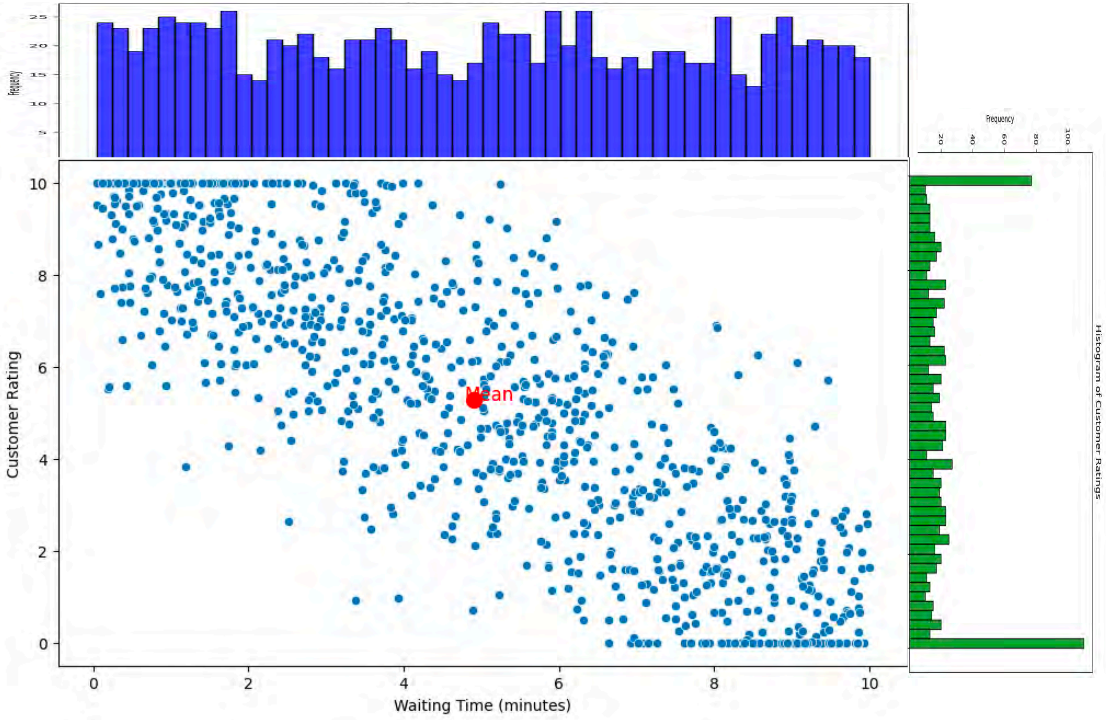
Mean
Customer Rating
Waiting Time (minutes)
0
2
4
6
8
10
0
2
4
6
8
10

Variances



Frequency
25
20
15
10
5
Customer Rating
10
8
6
4
2
0
Mean
0
2
4
6
8
10
Waiting Time (minutes)

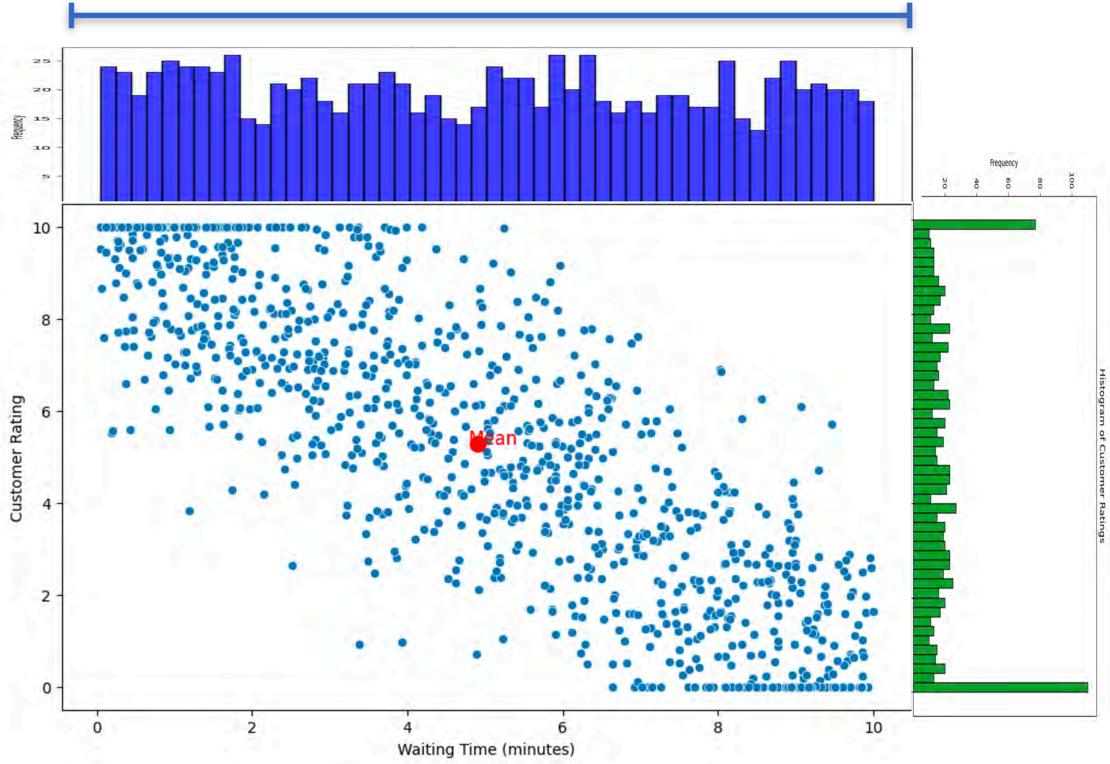
Variances



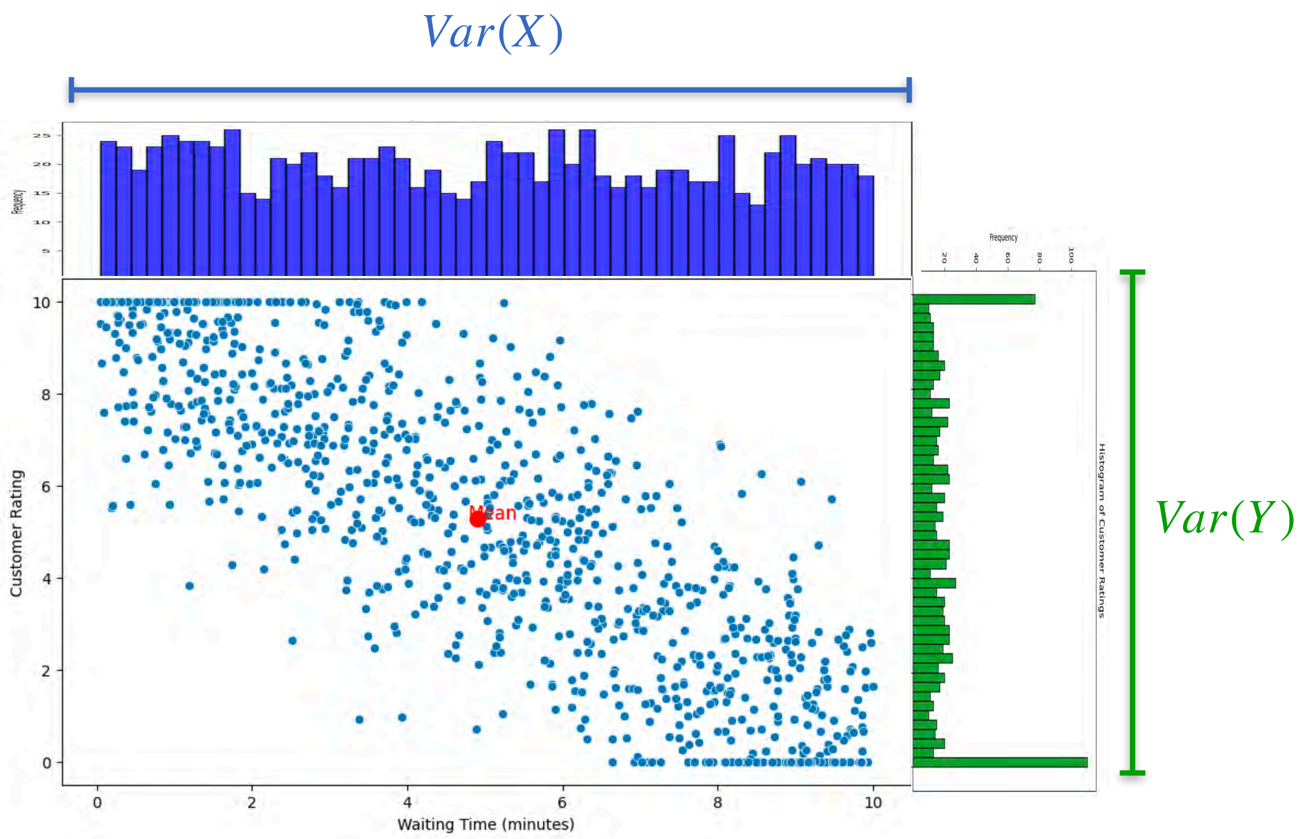
Frequency
Customer Rating
Waiting Time (minutes)
Mean
Histogram of Customer Ratings

Variances

$Var(X)$



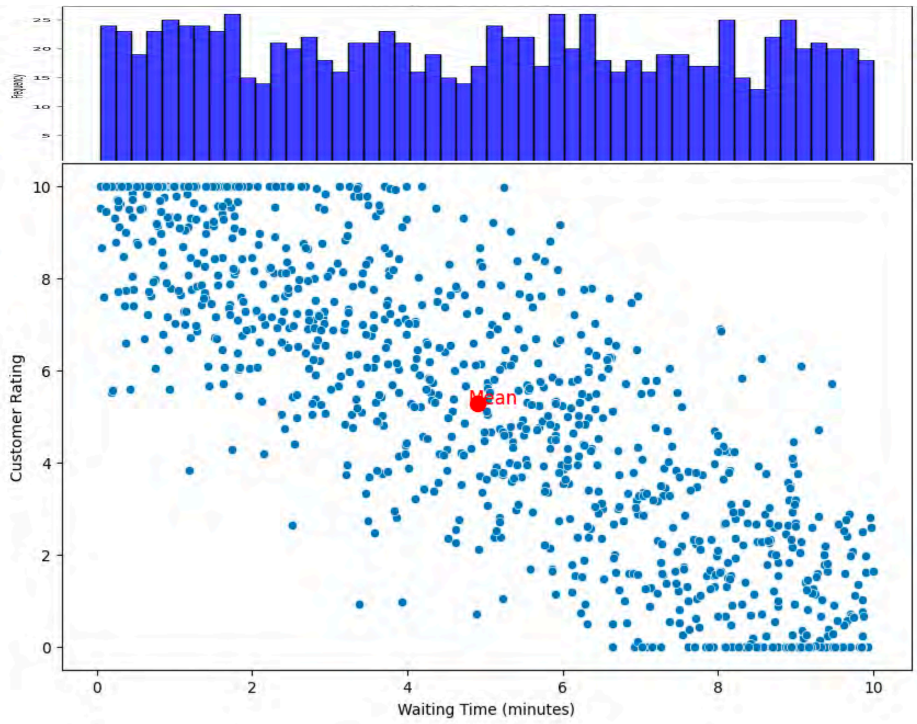
Variances



$Var(X)$
Frequency
Customer Rating
Waiting Time (minutes)
Mean
Frequency
Histogram of Customer Ratings
$Var(Y)$

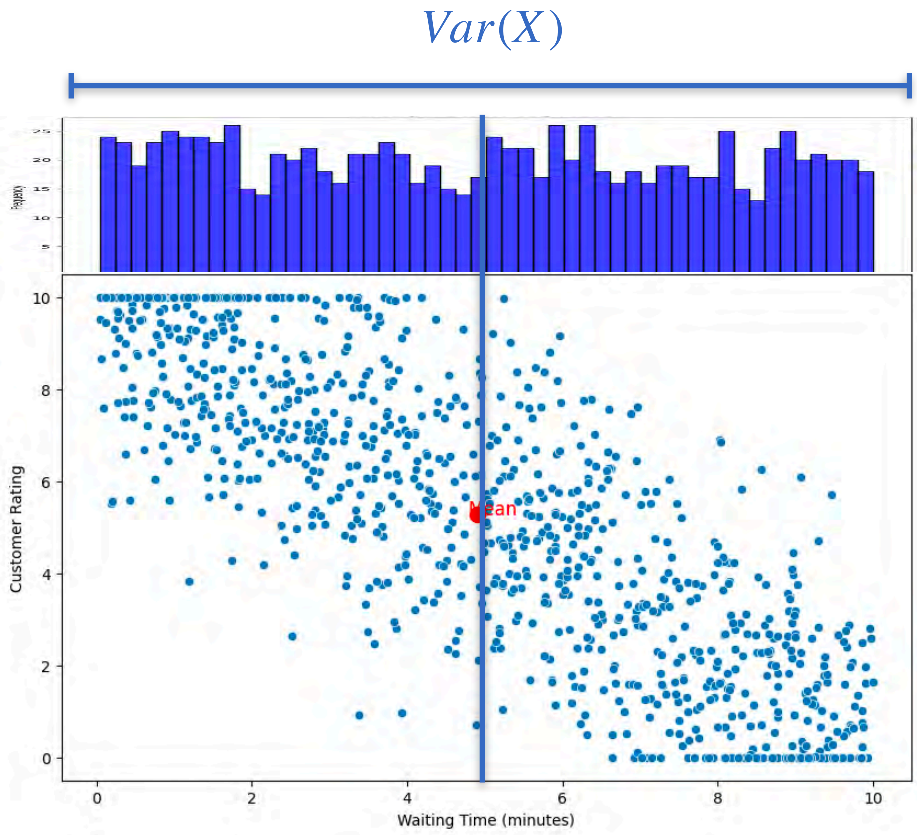
Variances

$Var(X)$



Variances

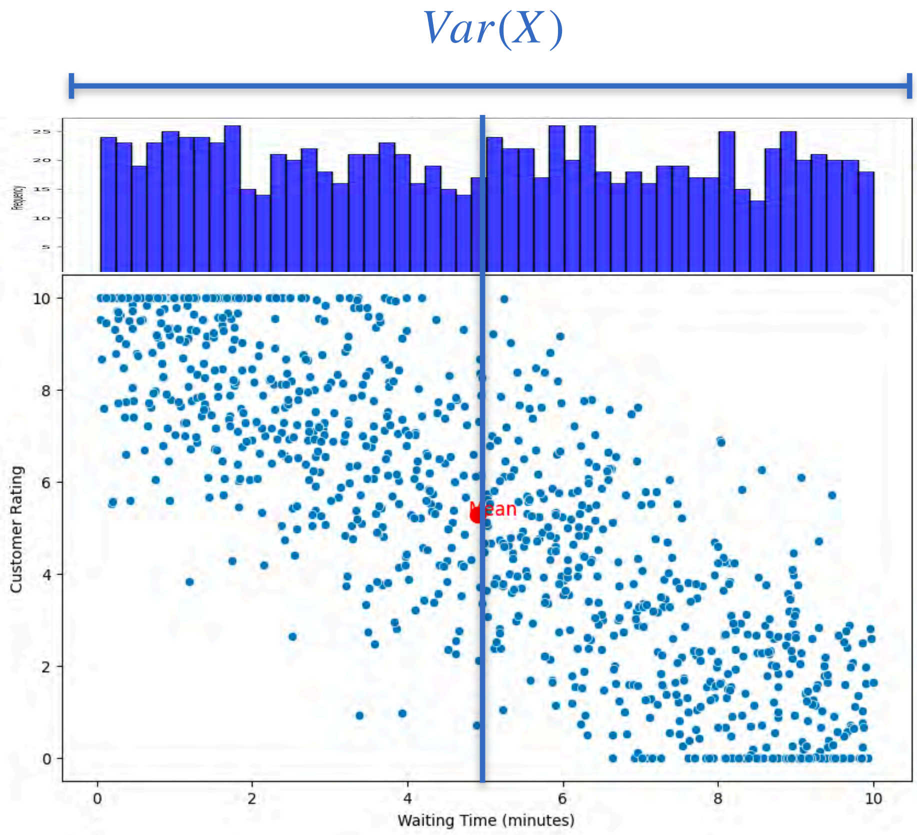
$\mathbb{E}[X] = 4.903$ minutes



Variances

$\mathbb{E}[X] = 4.903$ minutes

$\mathbb{E}[X^2] = 32.561$

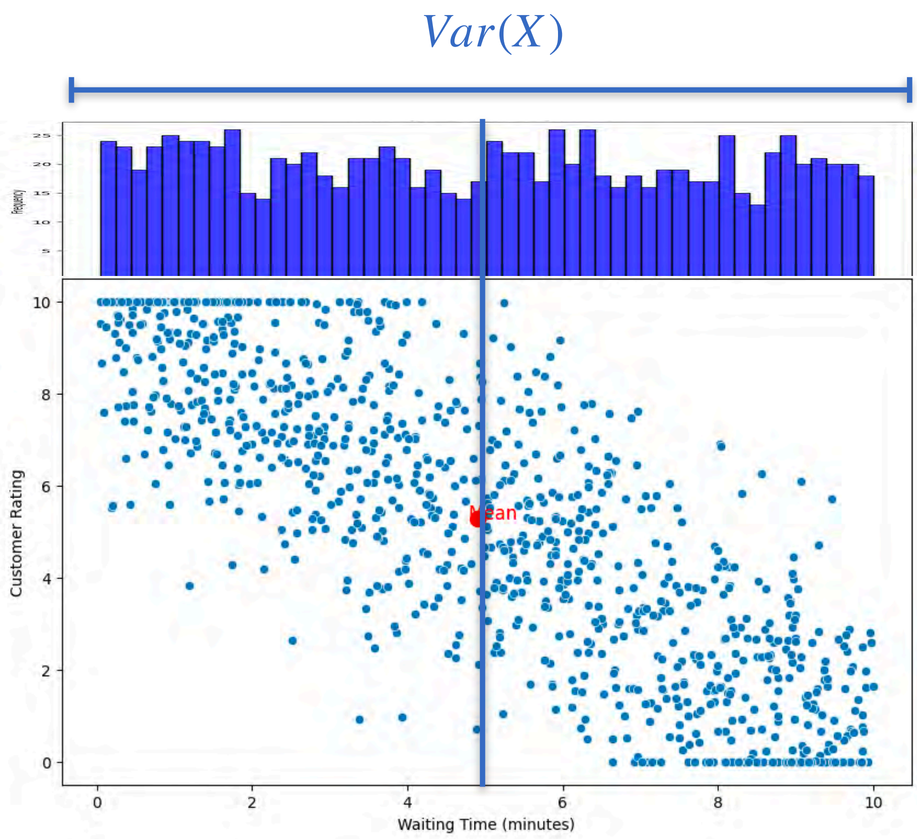


Variances

$\mathbb{E}[X] = 4.903$ minutes

$\mathbb{E}[X^2] = 32.561$

$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$



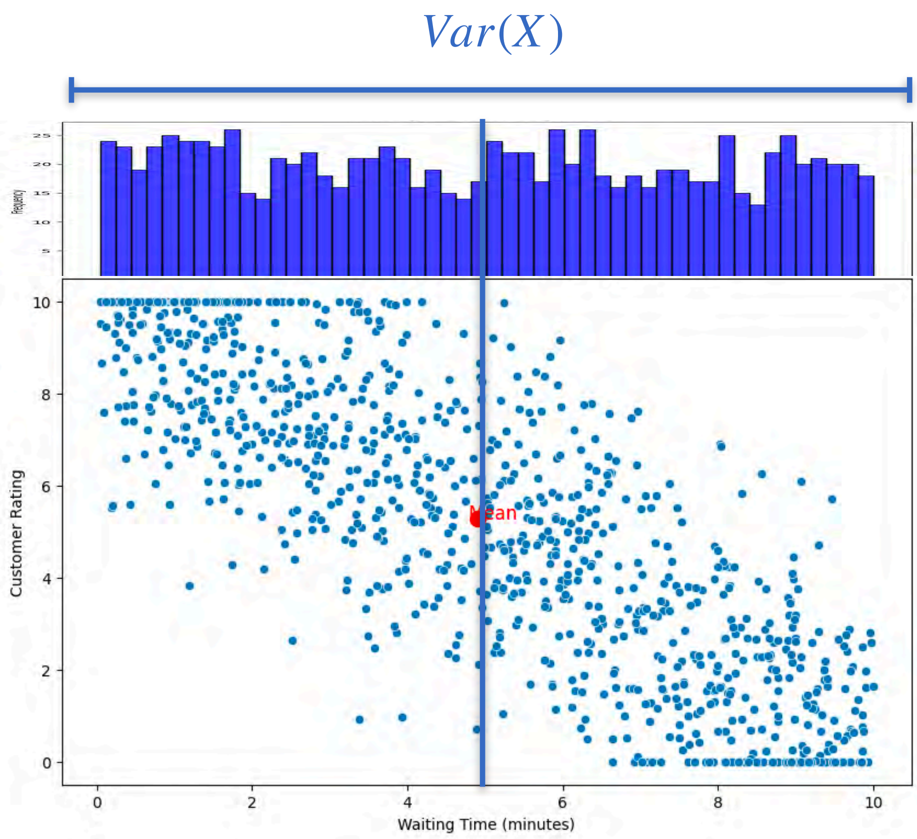
Variances

$\mathbb{E}[X] = 4.903$ minutes

$\mathbb{E}[X^2] = 32.561$

$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

$= 32.561 - 4.903^2$



Variances

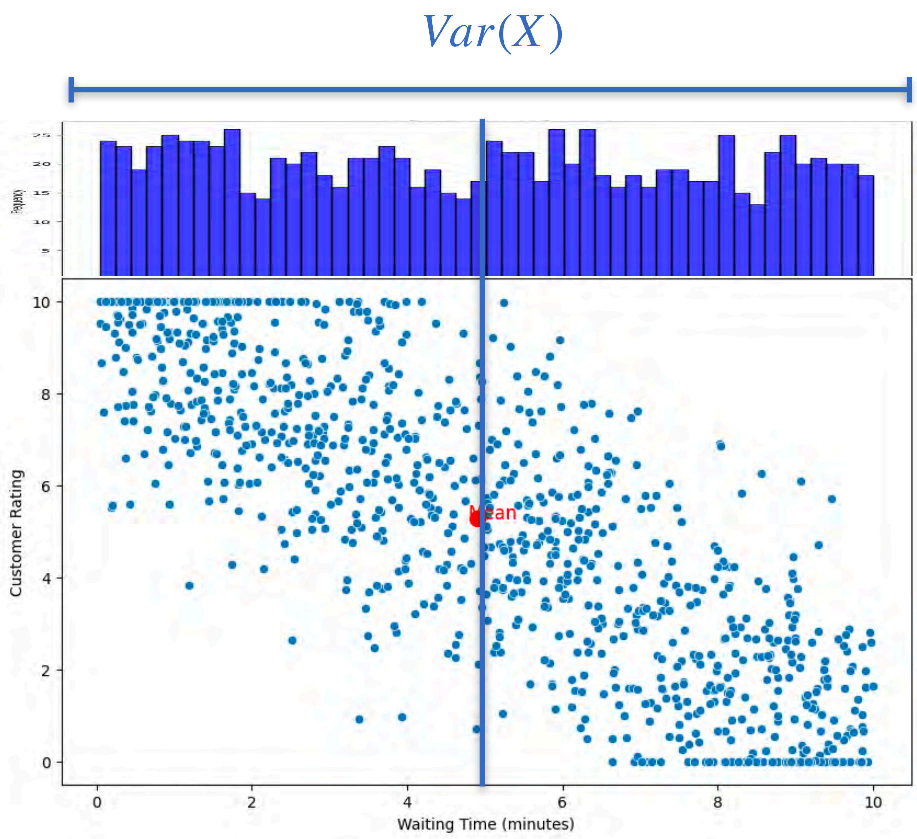
$\mathbb{E}[X] = 4.903$ minutes

$\mathbb{E}[X^2] = 32.561$

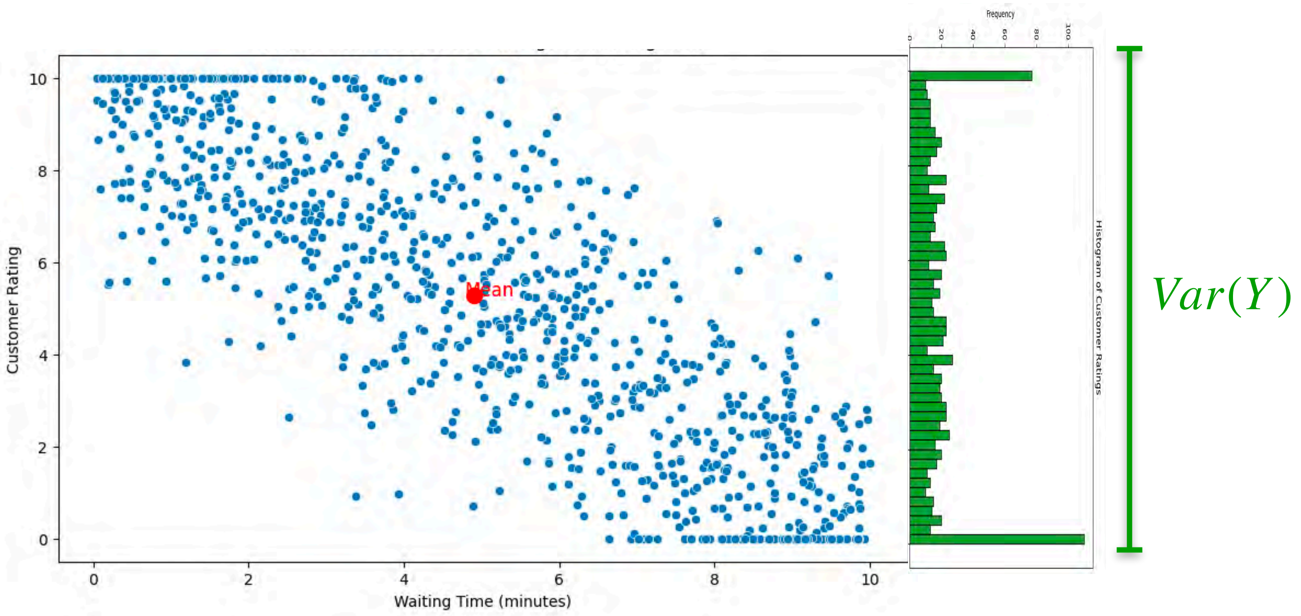
$$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$= 32.561 - 4.903^2$$

$$= 8.526$$

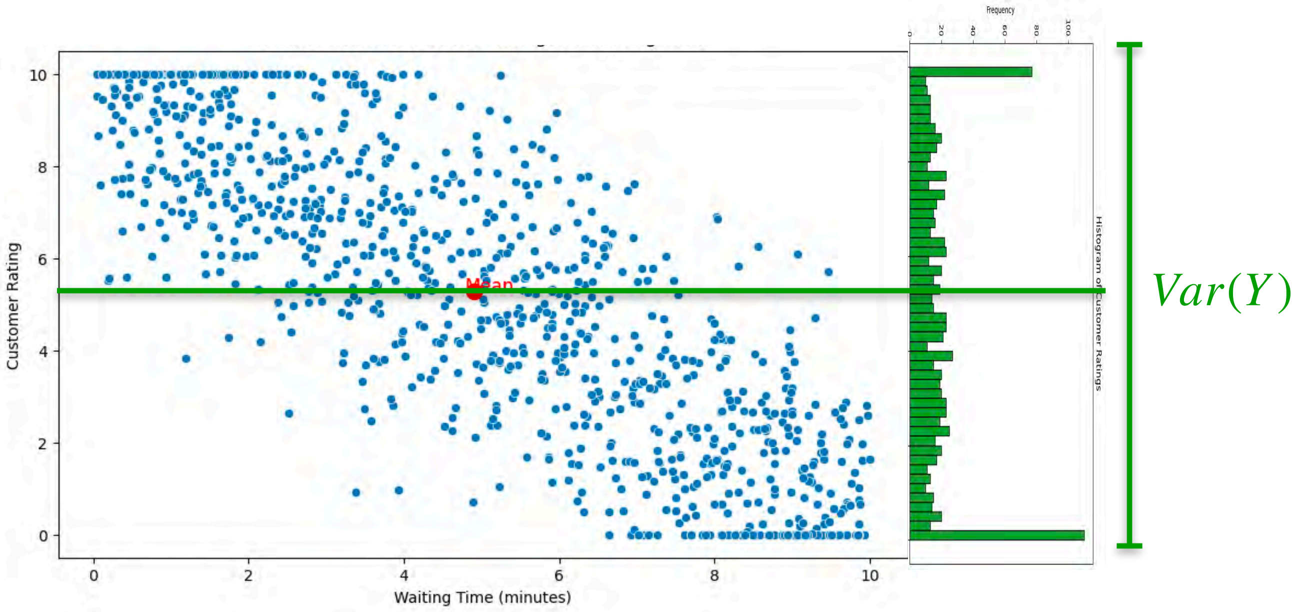


Variances


Mean
Customer Rating
Waiting Time (minutes)
Frequency
Histogram of Customer Ratings
$Var(Y)$

Variances

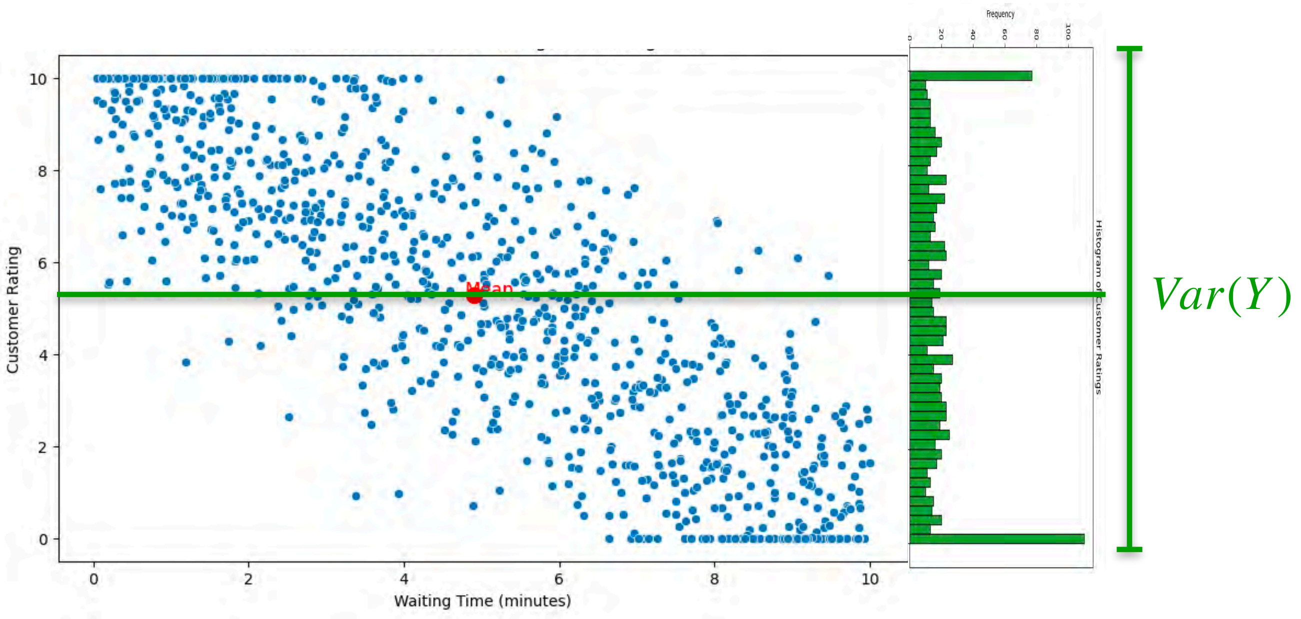
$\mathbb{E}[Y] = 5.280$



Variances

$\mathbb{E}[Y] = 5.280$

$\mathbb{E}[Y^2] = 38.037$

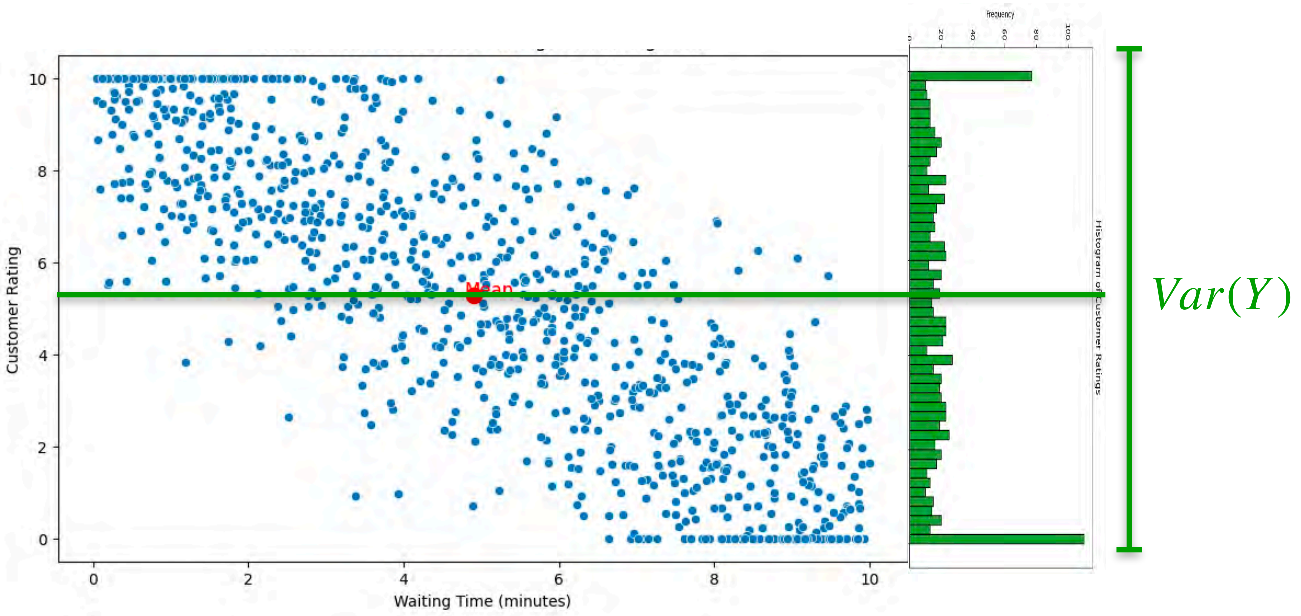


Variances

$$\mathbb{E}[Y] = 5.280$$

$$\mathbb{E}[Y^2] = 38.037$$

$$Var(Y) = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2$$



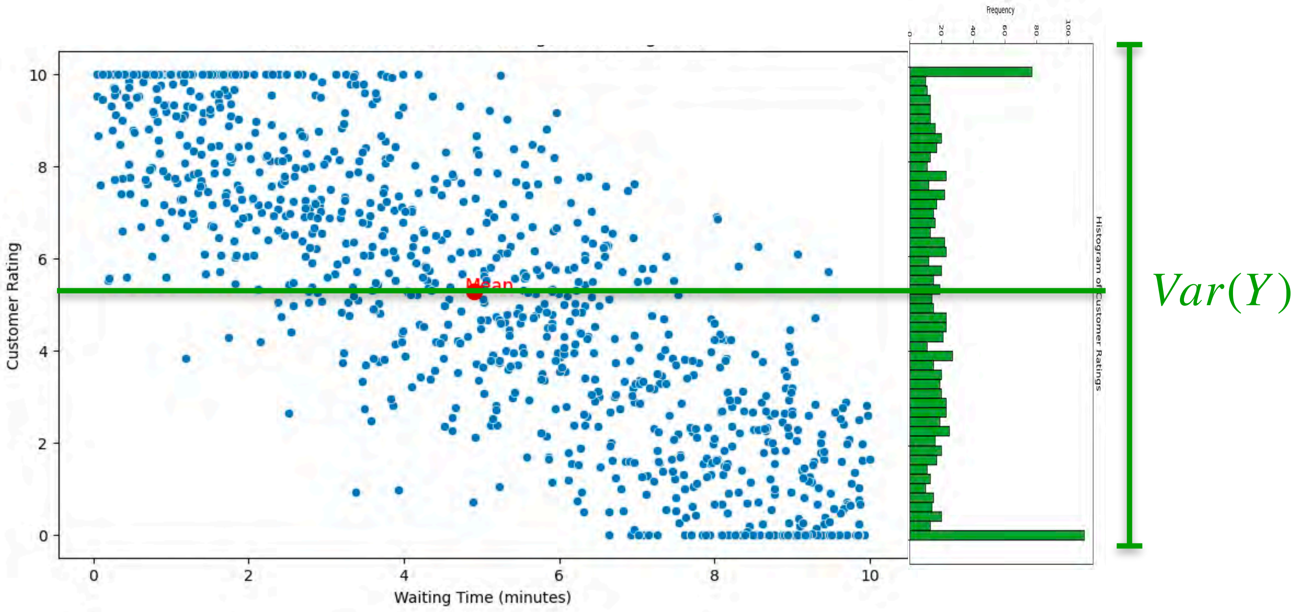
Variances

$\mathbb{E}[Y] = 5.280$

$\mathbb{E}[Y^2] = 38.037$

$$Var(Y) = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2$$

$$= 38.037 - 5.280^2$$



Variances

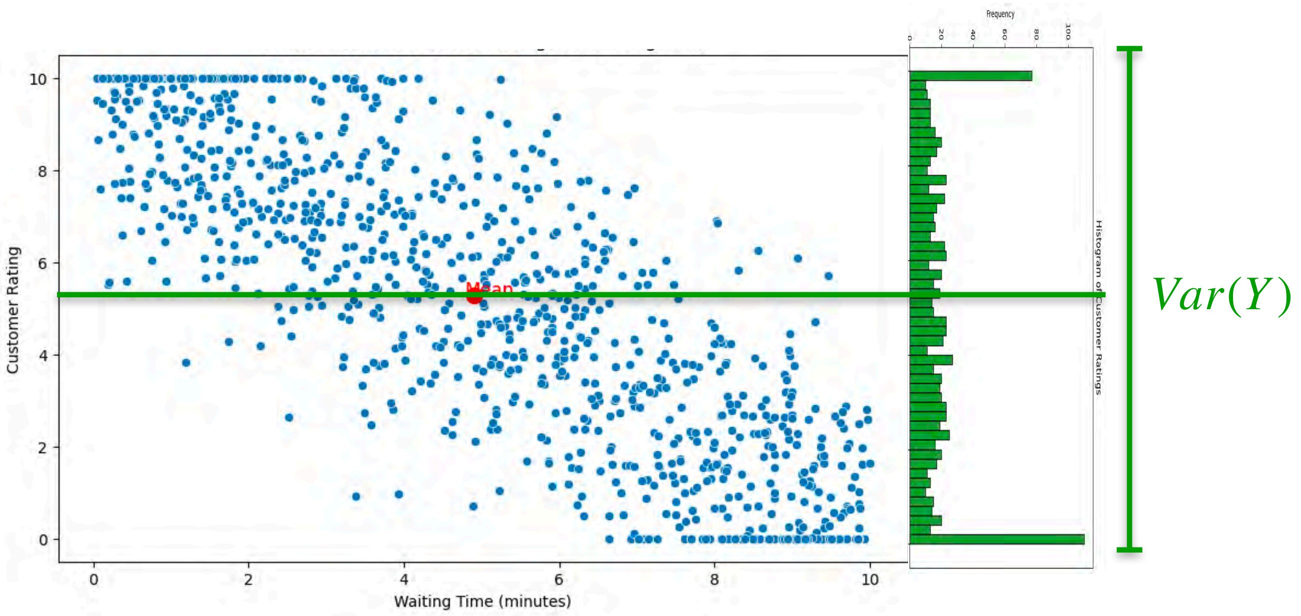
$$\mathbb{E}[Y] = 5.280$$

$$\mathbb{E}[Y^2] = 38.037$$

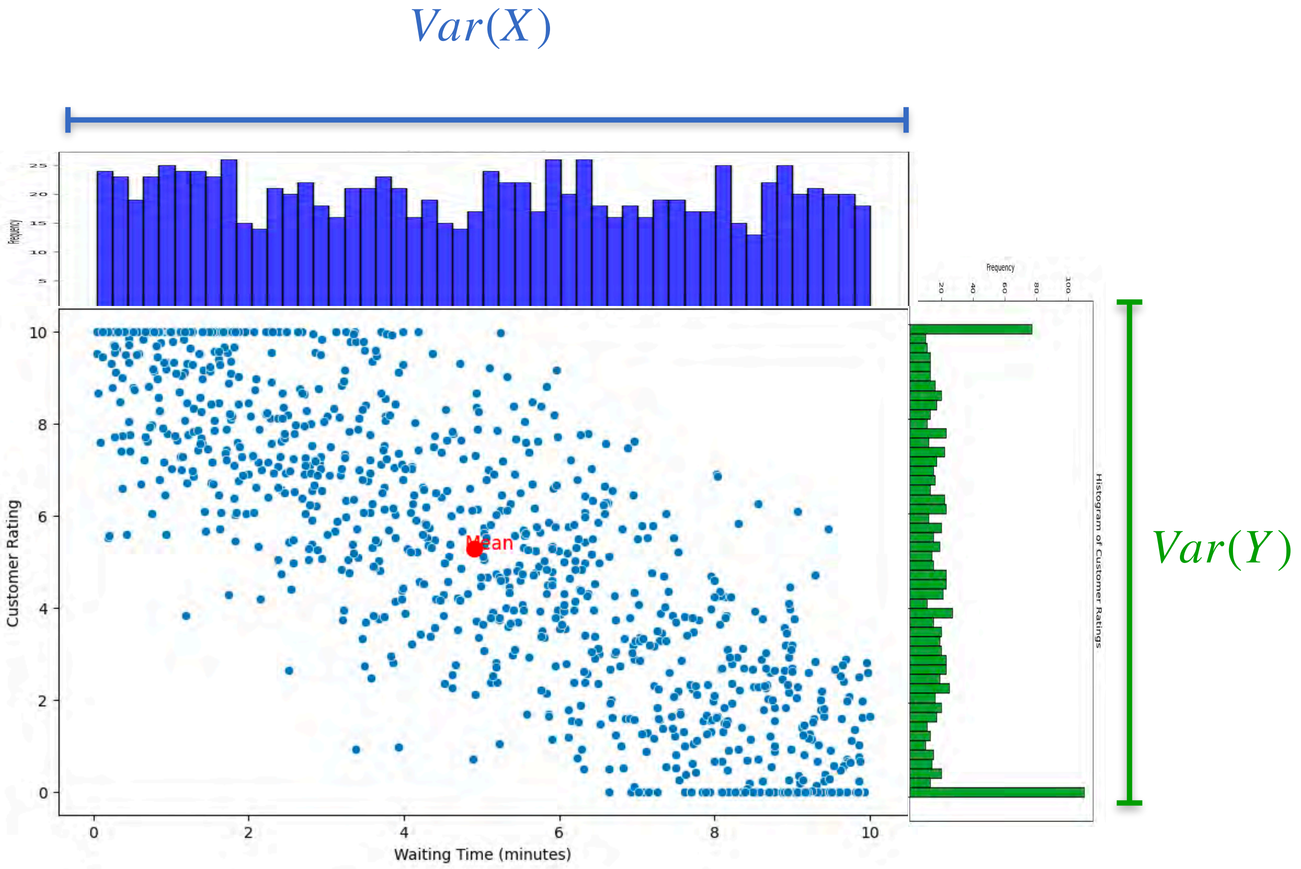
$$Var(Y) = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2$$

$$= 38.037 - 5.280^2$$

$$= 10.163$$

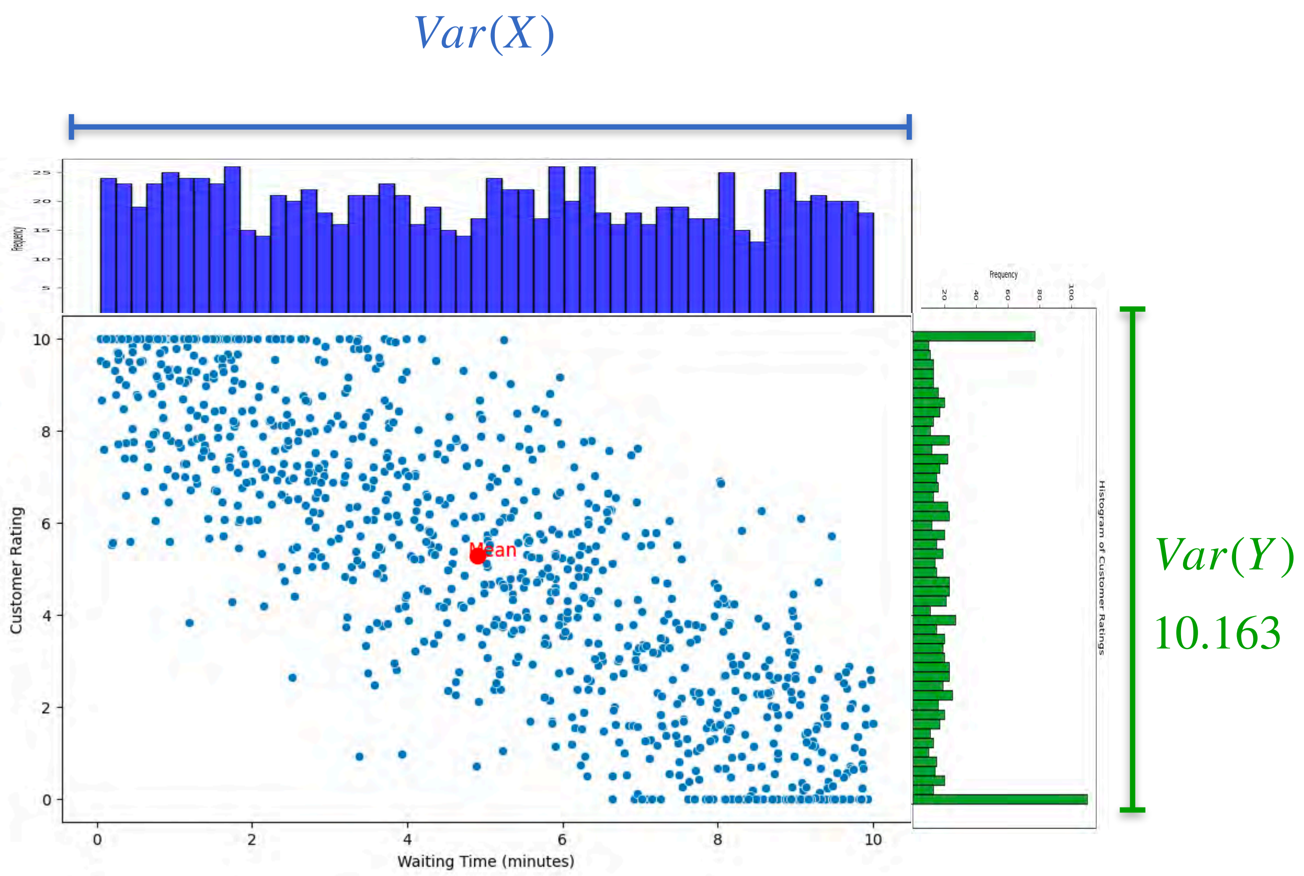


Variances

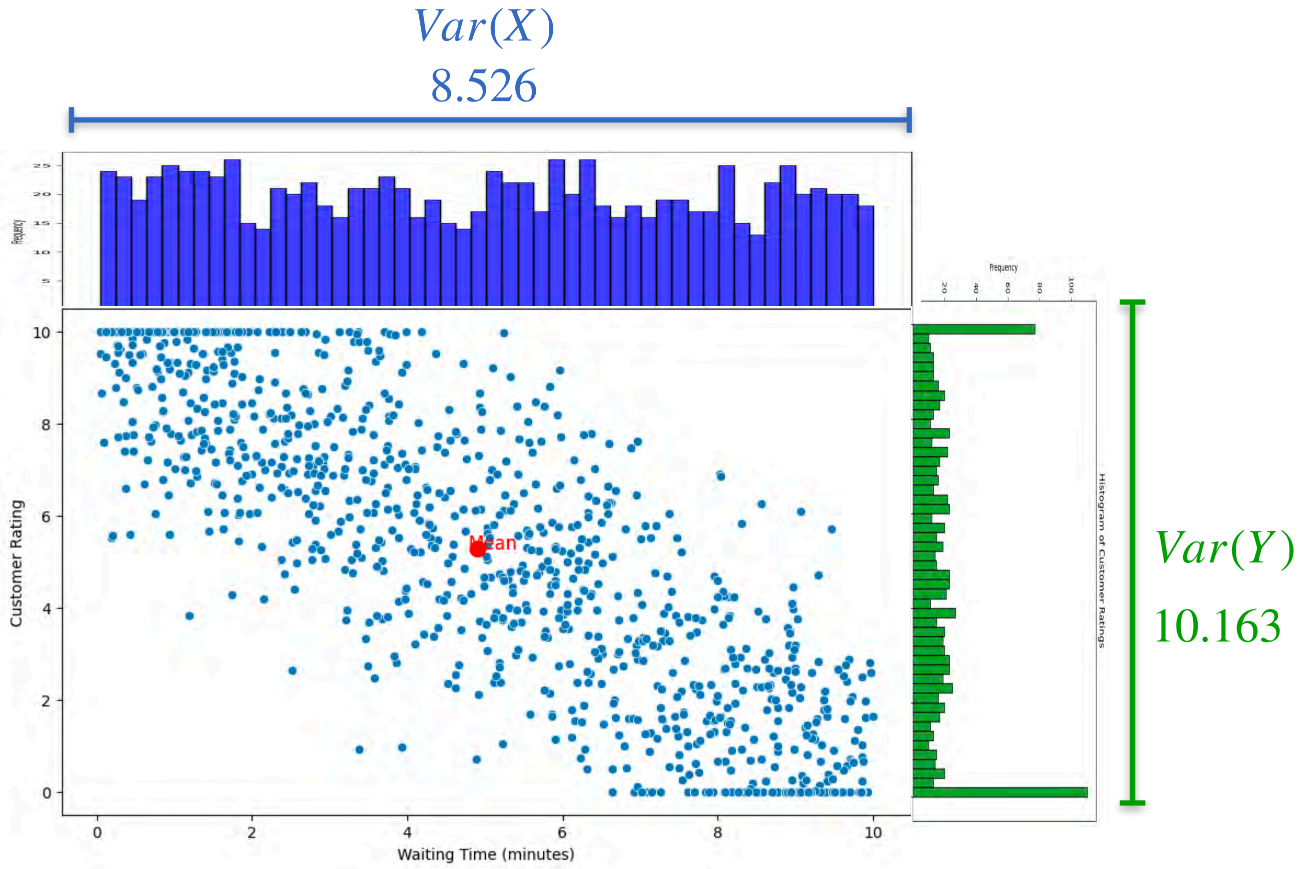


$Var(X)$
Frequency
Frequency
Histogram of Customer Ratings
Customer Rating
Mean
Waiting Time (minutes)
$Var(Y)$

Variances


$Var(X)$
Frequency
Customer Rating
Mean
Waiting Time (minutes)
Frequency
Histogram of Customer Ratings
$Var(Y)$
10.163

Variances



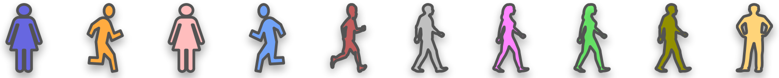
$Var(X)$
8.526
Frequency
Customer Rating
Mean
Waiting Time (minutes)
Histogram of Customer Ratings
$Var(Y)$
10.163

Probability Distributions with Multiple Variables

Marginal and Conditional Distribution

Marginal Distribution: Example 1

Marginal Distribution: Example 1

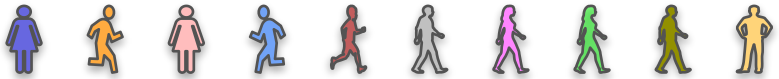


Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Marginal Distribution: Example 1

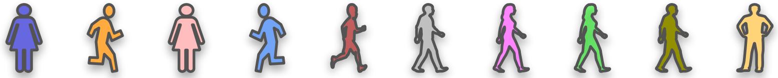
Height (Y)

Age (X)	**45**	**46**	**47**	**48**	**49**	**50**
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Marginal Distribution: Example 1



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (Y)

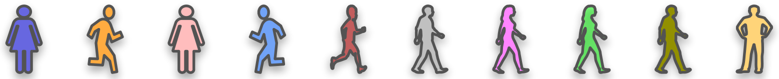
Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Marginal Distribution: Example 1

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



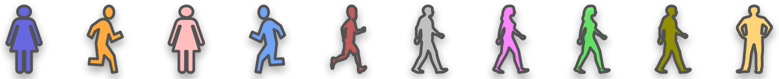
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Marginal Distribution

Marginal Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Marginal Distribution

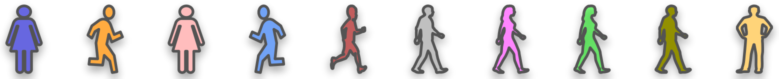
Distribution of one variable while ignoring the other

Marginal Distribution: Example 1

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



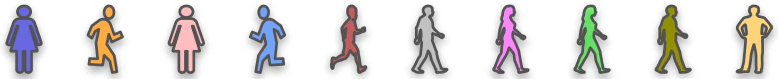
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Marginal Distribution: Example 1

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



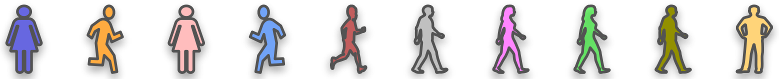
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

To find the marginal distribution of height:

Marginal Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

To find the marginal distribution of height:

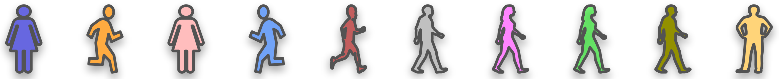
sum the joint probability distribution over all values of age

Marginal Distribution: Example 1

Height (Y)

Age (X)

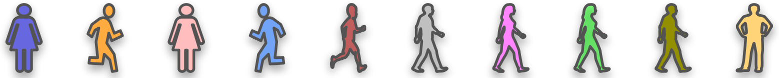
	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Marginal Distribution: Example 1

Age (X) \ Height (Y)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10
	1/10	2/10	2/10	0	3/10	2/10



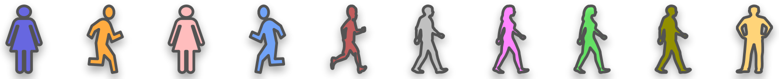
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j)$$

Marginal Distribution: Example 1

Height (Y)

Age (X)	**45**	**46**	**47**	**48**	**49**	**50**
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10
	1/10	2/10	2/10	0	3/10	2/10

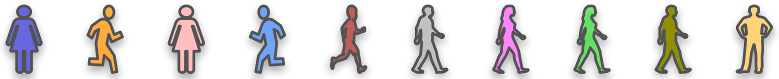


Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j)$$

Marginal Distribution: Example 1

Age (X) \ Height (Y)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10
	1/10	2/10	2/10	0	3/10	2/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

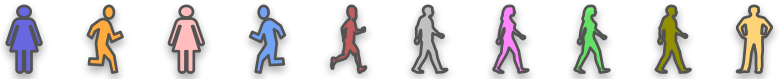
$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j)$$

$$p_Y(50) =$$

DeepLearning.AI

Marginal Distribution: Example 1

Age (X) \ Height (Y)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10
	1/10	2/10	2/10	0	3/10	2/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

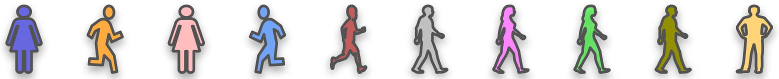
$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j)$$

Marginal Distribution: Example 1

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10
	1/10	2/10	2/10	0	3/10	2/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j)$$

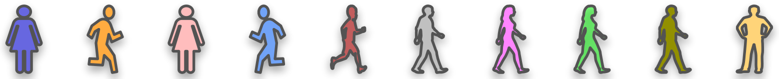
$$p_Y(50) = \sum_i p_{XY}(x_i, 50)$$

Marginal Distribution: Example 1

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10
	1/10	2/10	2/10	0	3/10	2/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

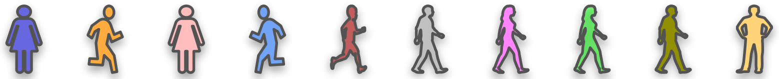
$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j)$$

$$p_Y(50) = \sum_i p_{XY}(x_i, 50)$$

$$p_Y(50) = \frac{2}{10}$$

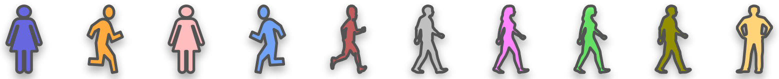
Marginal Distribution: Example 1

Age (X) \ Height (Y)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10
	1/10	2/10	2/10	0	3/10	2/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Marginal Distribution: Example 1



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (Y)

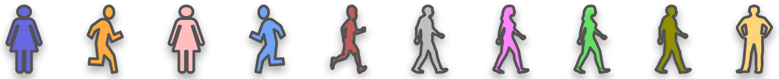
Age (X)	45	46	47	48	49	50	
7	1/10	2/10	0	0	0	0	3/10
8	0	0	2/10	0	0	0	2/10
9	0	0	0	0	3/10	1/10	4/10
10	0	0	0	0	0	1/10	1/10
	1/10	2/10	2/10	0	3/10	2/10	

$$p_X(x_i) = \sum_j p_{XY}(x_i, y_j)$$

Marginal Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50	
7	1/10	2/10	0	0	0	0	3/10
8	0	0	2/10	0	0	0	2/10
9	0	0	0	0	3/10	1/10	4/10
10	0	0	0	0	0	1/10	1/10
	1/10	2/10	2/10	0	3/10	2/10	



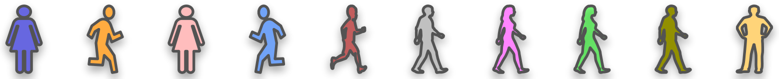
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

$$p_X(x_i) = \sum_j p_{XY}(x_i, y_j)$$

Marginal Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50	
7	1/10	2/10	0	0	0	0	3/10
8	0	0	2/10	0	0	0	2/10
9	0	0	0	0	3/10	1/10	4/10
10	0	0	0	0	0	1/10	1/10
	1/10	2/10	2/10	0	3/10	2/10	



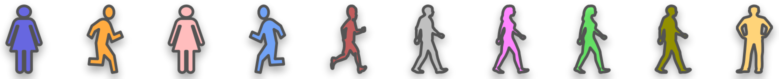
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

$$p_X(x_i) = \sum_j p_{XY}(x_i, y_j)$$

$$p_X(7) = \sum_j p_{XY}(7, y_j)$$

$$p_X(7) = \frac{3}{10}$$

Marginal Distribution: Example 1



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (Y)

Age (X)

	45	**46**	**47**	**48**	**49**	**50**	
7	1/10	2/10	0	0	0	0	3/10
8	0	0	2/10	0	0	0	2/10
9	0	0	0	0	3/10	1/10	4/10
10	0	0	0	0	0	1/10	1/10
	1/10	2/10	2/10	0	3/10	2/10	

Marginal Distribution: Example 1

Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (Y)

Age (X)

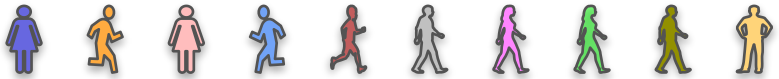
	45	46	47	48	49	50	
7	1/10	2/10	0	0	0	0	3/10
8	0	0	2/10	0	0	0	2/10
9	0	0	0	0	3/10	1/10	4/10
10	0	0	0	0	0	1/10	1/10
	1/10	2/10	2/10	0	3/10	2/10	

Age (X)

7	3/10
8	2/10
9	4/10
10	1/10

Marginal Distribution of Age

Marginal Distribution: Example 1



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (Y)

Age (X)

	45	46	47	48	49	50	
7	1/10	2/10	0	0	0	0	3/10
8	0	0	2/10	0	0	0	2/10
9	0	0	0	0	3/10	1/10	4/10
10	0	0	0	0	0	1/10	1/10
	1/10	2/10	2/10	0	3/10	2/10	

Age (X)

7	3/10
8	2/10
9	4/10
10	1/10

Marginal Distribution of Age

Height (Y)

45	46	47	48	49	50
1/10	2/10	2/10	0	3/10	2/10

Marginal Distribution of Height

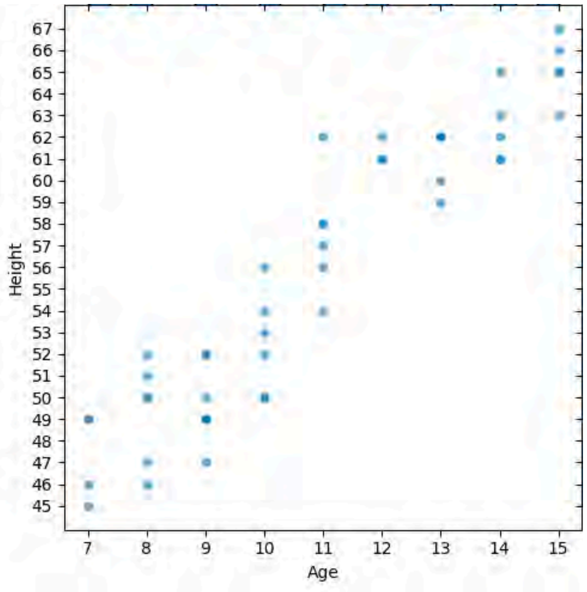
Marginal Distribution: Example 1

Marginal Distribution: Example 1

Age and Height Dataset
for 50 children

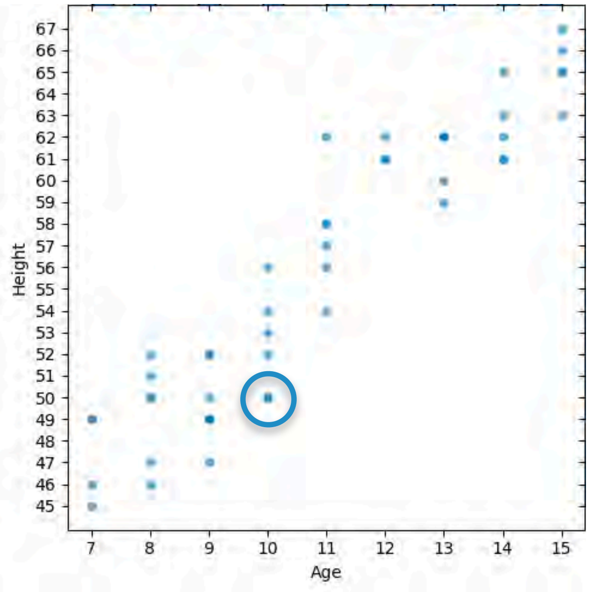
Marginal Distribution: Example 1

Age and Height Dataset
for 50 children



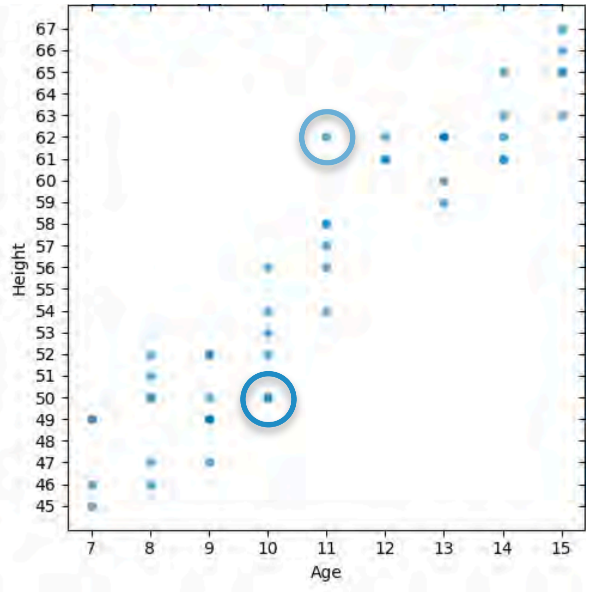
Marginal Distribution: Example 1

Age and Height Dataset
for 50 children



Marginal Distribution: Example 1

Age and Height Dataset
for 50 children



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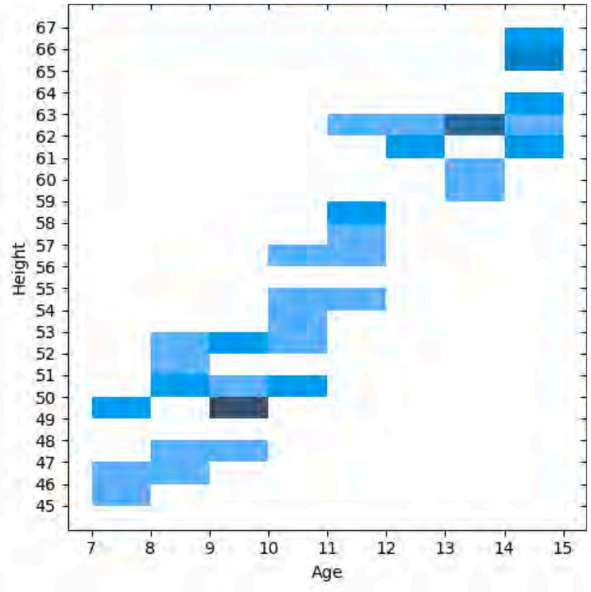
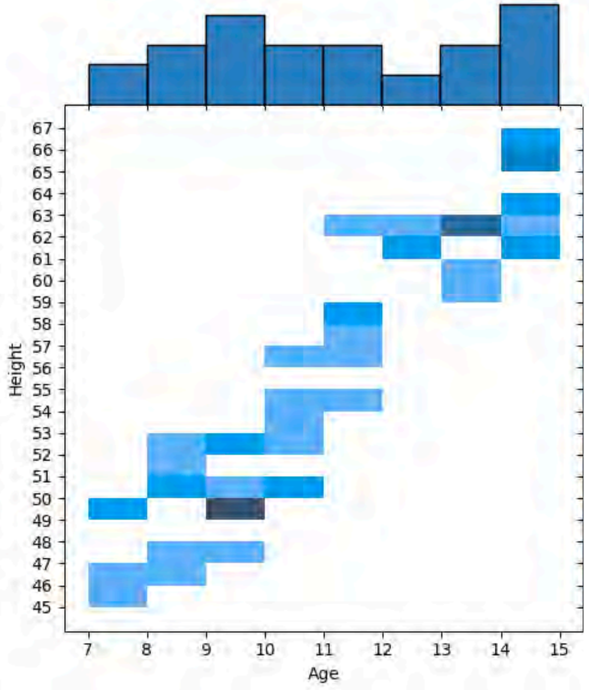
Marginal Distribution: Example 1

Age and Height Dataset
for 50 children



Marginal Distribution: Example 1

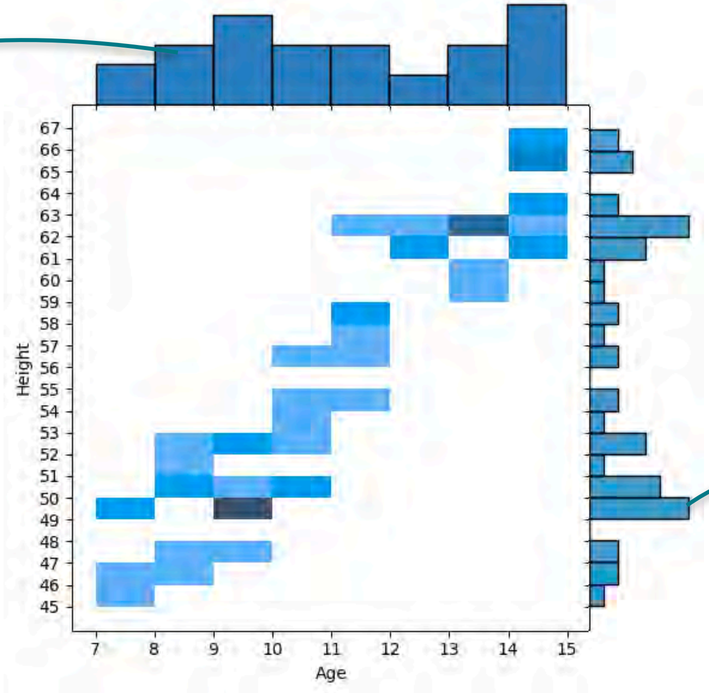
Age and Height Dataset
for 50 children



Marginal Distribution: Example 1

Marginal Distribution of Age

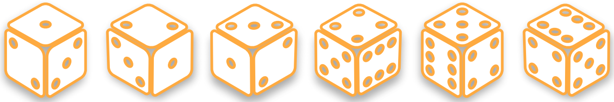
Age and Height Dataset
for 50 children



Marginal Distribution of Height

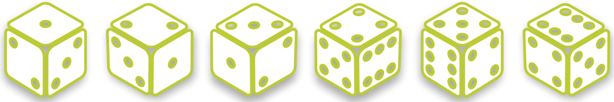
Marginal Distributions: Example 2

X : the number rolled on the 1st dice



$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

Y : the number rolled on the 2nd dice



$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

Marginal Distributions: Example 2

Y

X

	1	2	3	4	5	6
1	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	1/36

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

Marginal Distributions: Example 2

Y

X	1	2	3	4	5	6	
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

Marginal Distributions: Example 2

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X \ Y	1	2	3	4	5	6	
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6
	1/6	1/6	1/6	1/6	1/6	1/6	

Marginal Distributions: Example 2

	Y: 1	2	3	4	5	6	
X: 1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6
	1/6	1/6	1/6	1/6	1/6	1/6	

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

$$p_X(x_i) = \frac{1}{6}$$

Marginal Distributions: Example 2

X \ Y	1	2	3	4	5	6	
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6
	1/6	1/6	1/6	1/6	1/6	1/6	

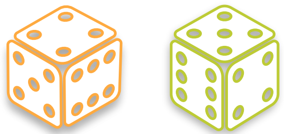
X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

$$p_X(x_i) = \frac{1}{6}$$

$$p_Y(y_j) = \frac{1}{6}$$

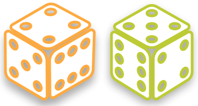
Marginal Distributions: Example 3



X : the number rolled on the 1st dice

Y : sum of the two dice

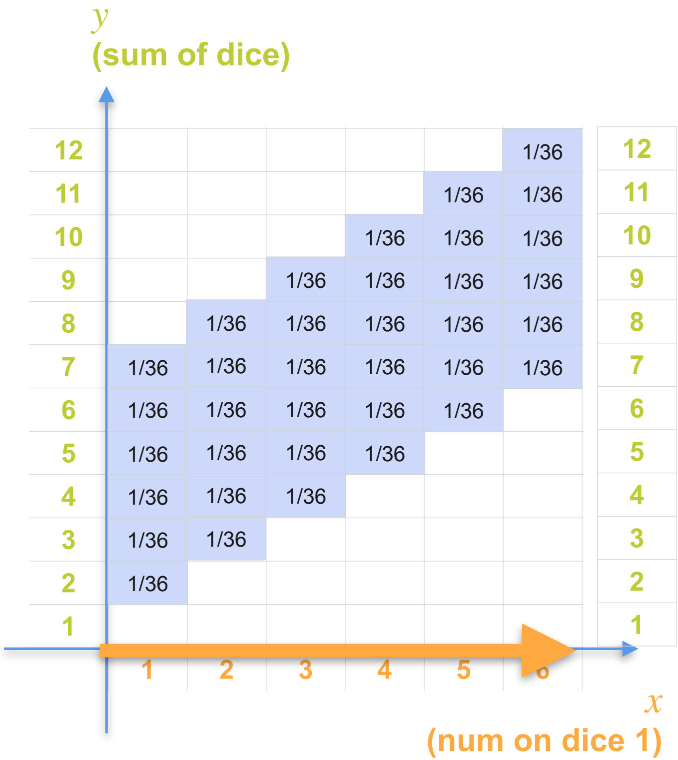
Marginal Distributions: Example 3

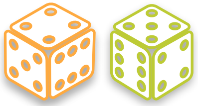


X : the number rolled on the 1st dice

Y : sum of the two dice

Marginal Distributions: Example 3

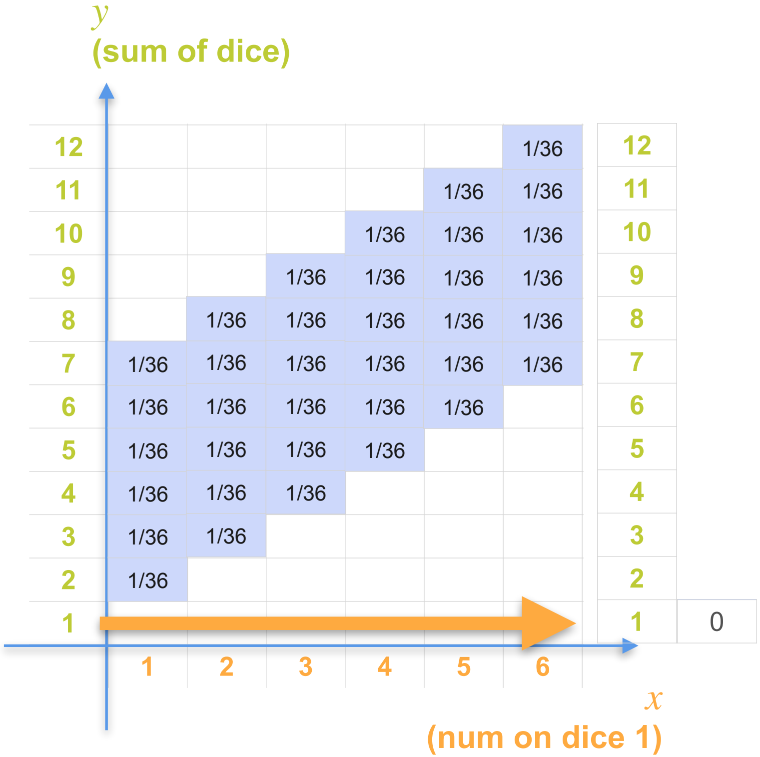


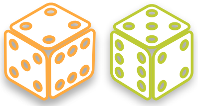


X : the number rolled on the 1st dice

Y : sum of the two dice

Marginal Distributions: Example 3

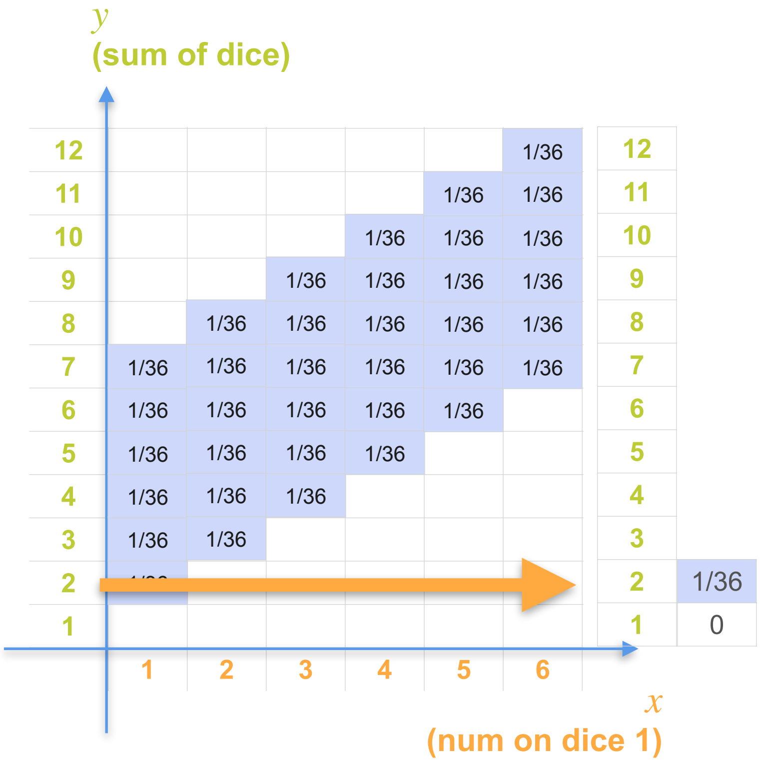


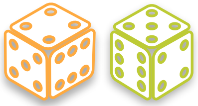


X : the number rolled on the 1st dice

Y : sum of the two dice

Marginal Distributions: Example 3





X : the number rolled on the 1st dice

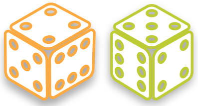
Y : sum of the two dice

Marginal Distributions: Example 3

y (sum of dice)

y							y	
12						1/36	12	
11					1/36	1/36	11	
10				1/36	1/36	1/36	10	
9			1/36	1/36	1/36	1/36	9	
8		1/36	1/36	1/36	1/36	1/36	8	
7	1/36	1/36	1/36	1/36	1/36	1/36	7	
6	1/36	1/36	1/36	1/36	1/36		6	
5	1/36	1/36	1/36	1/36			5	
4	1/36	1/36	1/36				4	
3	[illegible]	[illegible]					3	2/36
2	1/36						2	1/36
1							1	0
	1	2	3	4	5	6		

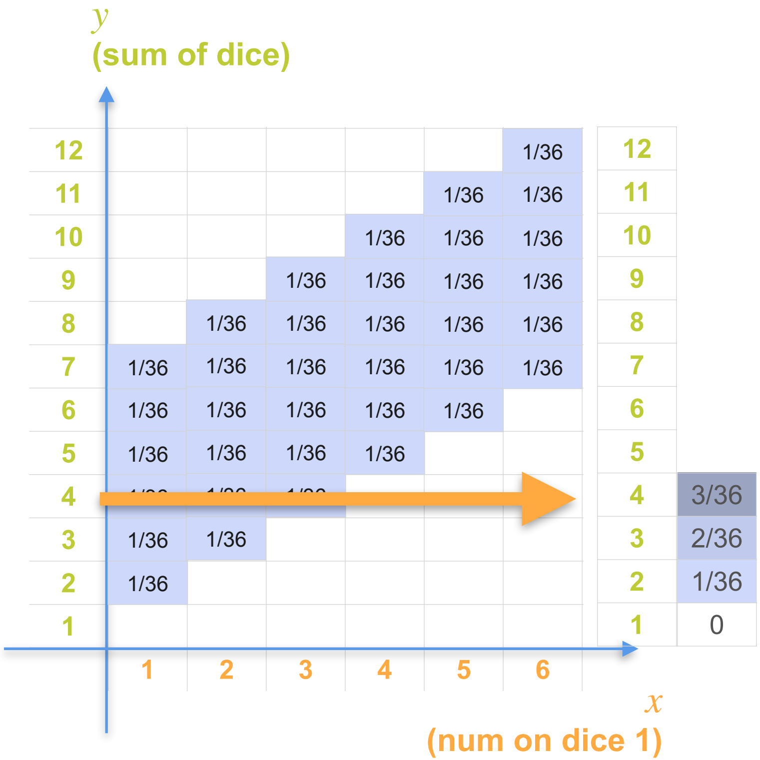
x (num on dice 1)

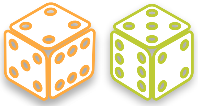


X : the number rolled on the 1st dice

Y : sum of the two dice

Marginal Distributions: Example 3

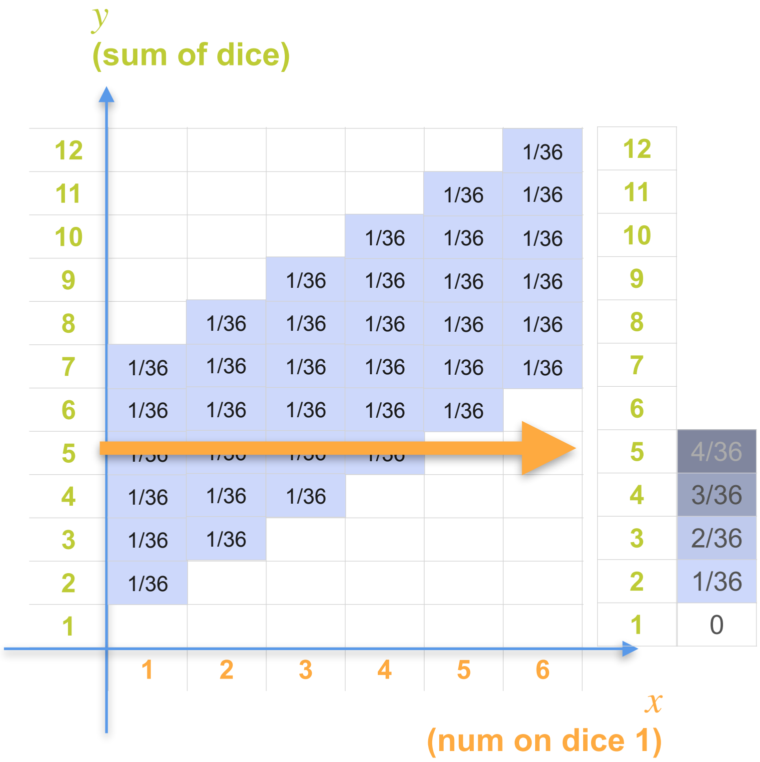


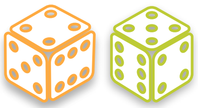


X : the number rolled on the 1st dice

Y : sum of the two dice

Marginal Distributions: Example 3

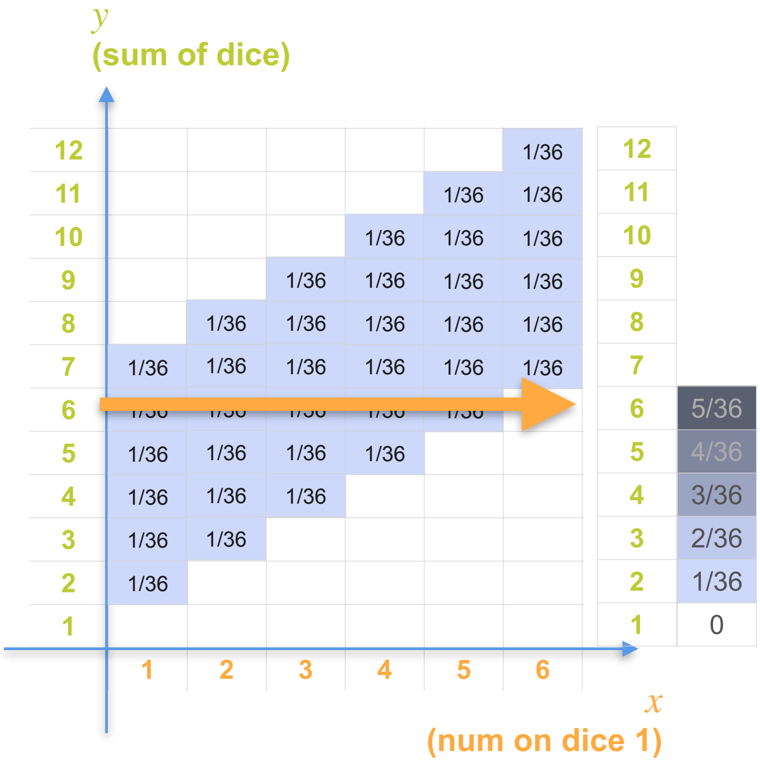


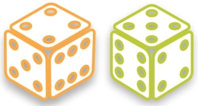


X : the number rolled on the 1st dice

Y : sum of the two dice

Marginal Distributions: Example 3

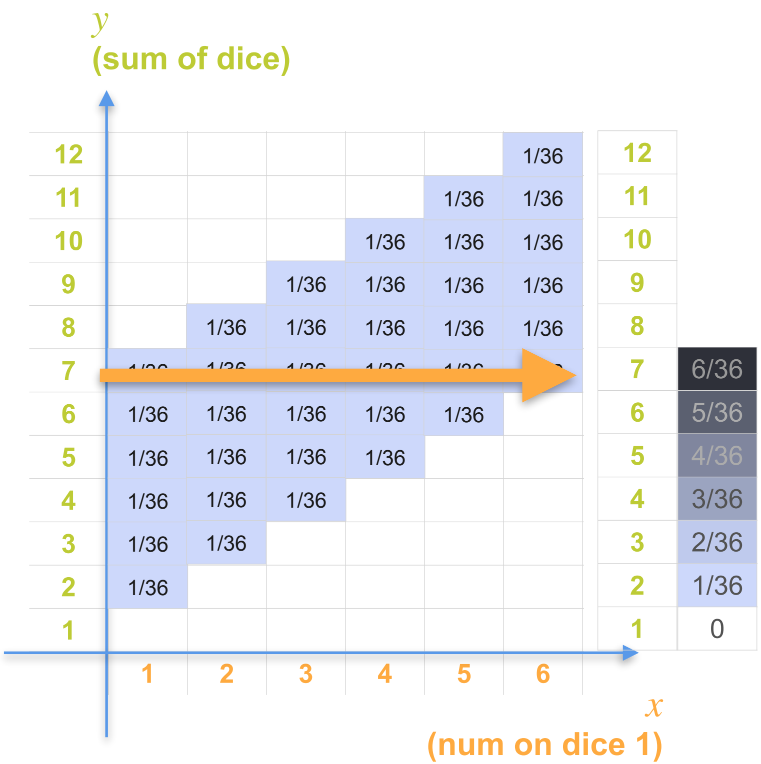


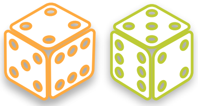


X : the number rolled on the 1st dice

Y : sum of the two dice

Marginal Distributions: Example 3





X : the number rolled on the 1st dice

Y : sum of the two dice

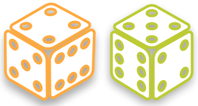
Marginal Distributions: Example 3

y (sum of dice)

y \ x	1	2	3	4	5	6
12						[illegible]
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1						

x (num on dice 1)

12	1/36
11	2/36
10	3/36
9	4/36
8	5/36
7	6/36
6	5/36
5	4/36
4	3/36
3	2/36
2	1/36
1	0



X : the number rolled on the 1st dice

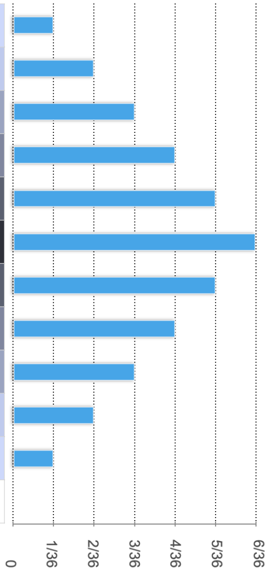
Y : sum of the two dice

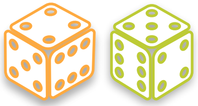
Marginal Distributions: Example 3

y (sum of dice)

y \ x	1	2	3	4	5	6	
12						[illegible]	1/36
11					1/36	1/36	2/36
10				1/36	1/36	1/36	3/36
9			1/36	1/36	1/36	1/36	4/36
8		1/36	1/36	1/36	1/36	1/36	5/36
7	1/36	1/36	1/36	1/36	1/36	1/36	6/36
6	1/36	1/36	1/36	1/36	1/36		5/36
5	1/36	1/36	1/36	1/36			4/36
4	1/36	1/36	1/36				3/36
3	1/36	1/36					2/36
2	1/36						1/36
1							0

x (num on dice 1)



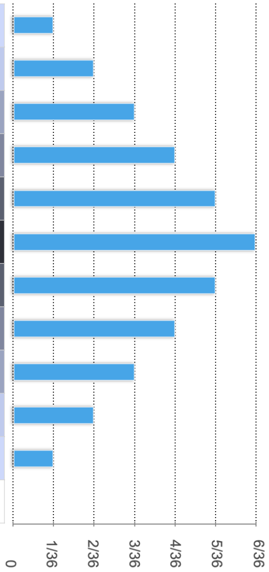


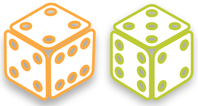
X : the number rolled on the 1st dice

Y : sum of the two dice

Marginal Distributions: Example 3

y (sum of dice) \ x (num on dice 1)	1	2	3	4	5	6		p_Y
12						[illegible]	12	1/36
11					1/36	1/36	11	2/36
10				1/36	1/36	1/36	10	3/36
9			1/36	1/36	1/36	1/36	9	4/36
8		1/36	1/36	1/36	1/36	1/36	8	5/36
7	1/36	1/36	1/36	1/36	1/36	1/36	7	6/36
6	1/36	1/36	1/36	1/36	1/36		6	5/36
5	1/36	1/36	1/36	1/36			5	4/36
4	1/36	1/36	1/36				4	3/36
3	1/36	1/36					3	2/36
2	1/36						2	1/36
1							1	0





X : the number rolled on the 1st dice

Y : sum of the two dice

$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j) = ?$$

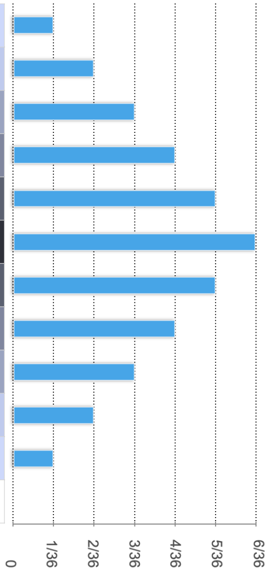
Marginal Distributions: Example 3

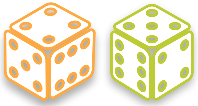
y (sum of dice)

y \ x	1	2	3	4	5	6
12						[illegible]
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1						

x (num on dice 1)

12	1/36
11	2/36
10	3/36
9	4/36
8	5/36
7	6/36
6	5/36
5	4/36
4	3/36
3	2/36
2	1/36
1	0





X : the number rolled on the 1st dice

Y : sum of the two dice

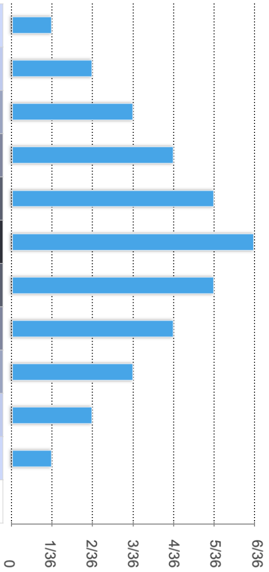
$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j) = ?$$

$$p_Y(10) = ?$$

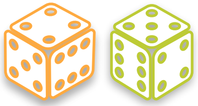
Marginal Distributions: Example 3

y (sum of dice)

y \ x (num on dice 1)	1	2	3	4	5	6	p_Y
12						[illegible]	1/36
11					1/36	1/36	2/36
10				1/36	1/36	1/36	3/36
9			1/36	1/36	1/36	1/36	4/36
8		1/36	1/36	1/36	1/36	1/36	5/36
7	1/36	1/36	1/36	1/36	1/36	1/36	6/36
6	1/36	1/36	1/36	1/36	1/36		5/36
5	1/36	1/36	1/36	1/36			4/36
4	1/36	1/36	1/36				3/36
3	1/36	1/36					2/36
2	1/36						1/36
1							0



x (num on dice 1)



X : the number rolled on the 1st dice

Y : sum of the two dice

$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j) = ?$$

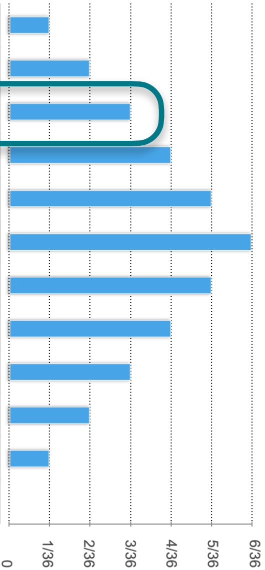
$$p_Y(10) = ?$$

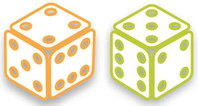
Marginal Distributions: Example 3

y (sum of dice)						
12						1/36
11					1/36	1/36
10				1/36	1/36	1/36
9			1/36	1/36	1/36	1/36
8		1/36	1/36	1/36	1/36	1/36
7	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	
5	1/36	1/36	1/36	1/36		
4	1/36	1/36	1/36			
3	1/36	1/36				
2	1/36					
1						
	1	2	3	4	5	6

x (num on dice 1)

12	1/36
11	2/36
10	3/36
9	4/36
8	5/36
7	6/36
6	5/36
5	4/36
4	3/36
3	2/36
2	1/36
1	0





X : the number rolled on the 1st dice

Y : sum of the two dice

$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j) = ?$$

$$p_Y(10) = ? \quad = \frac{3}{36}$$

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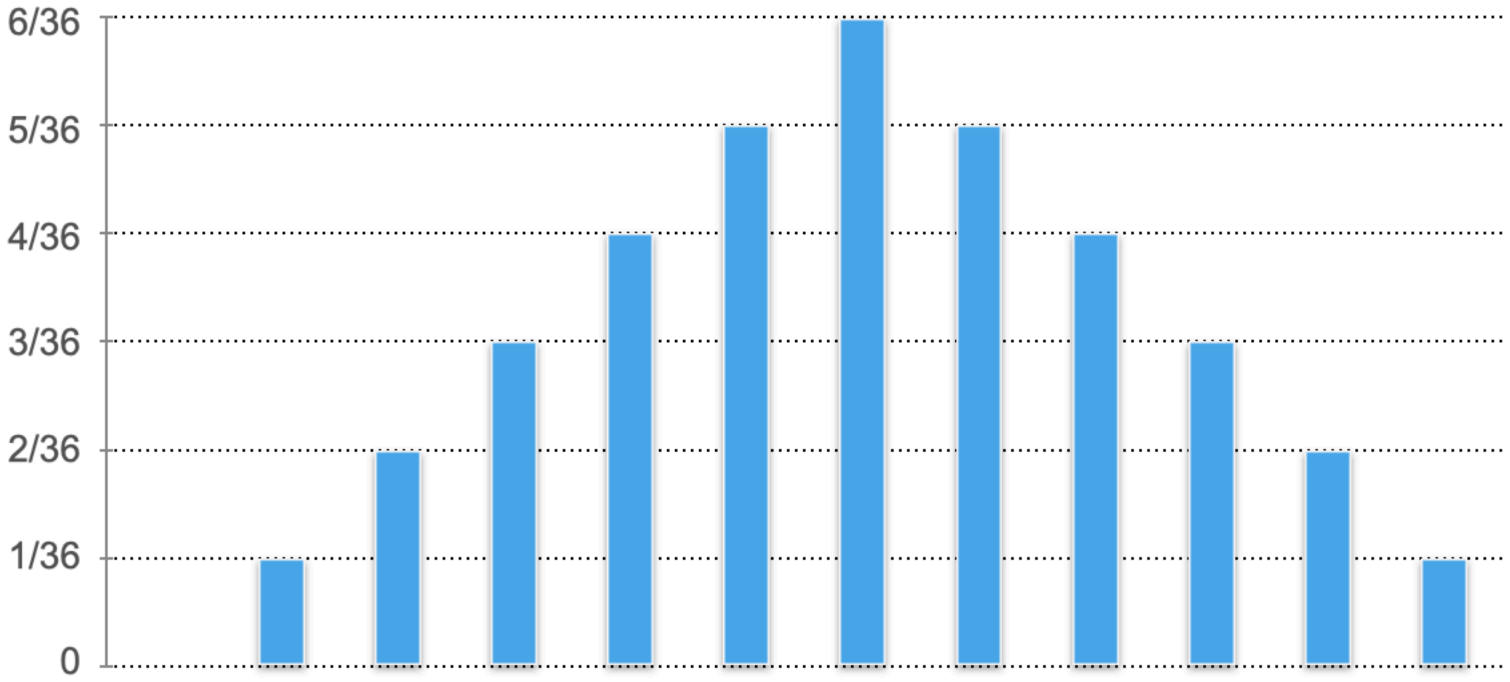
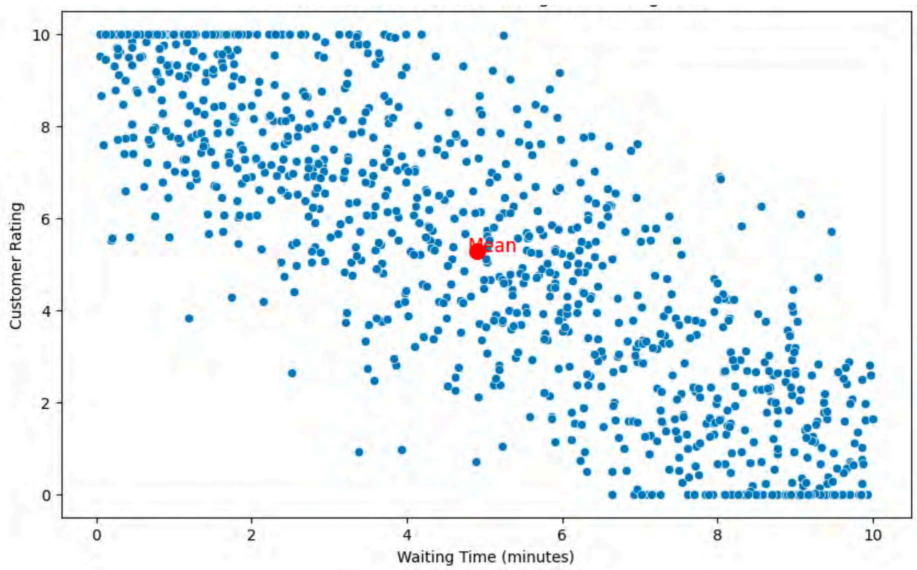
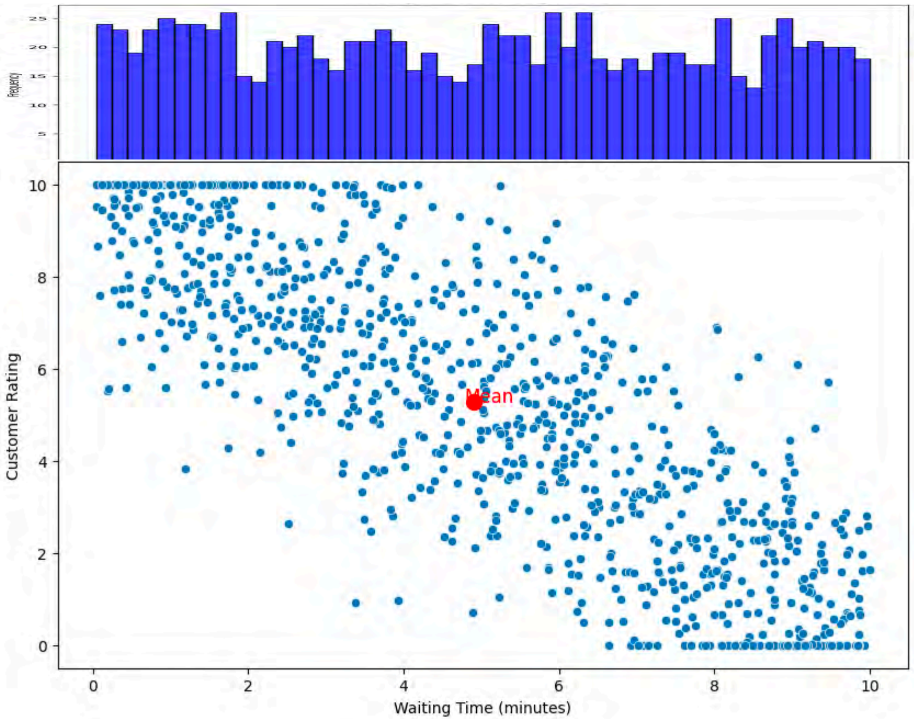
Marginal Distributions: Example 2



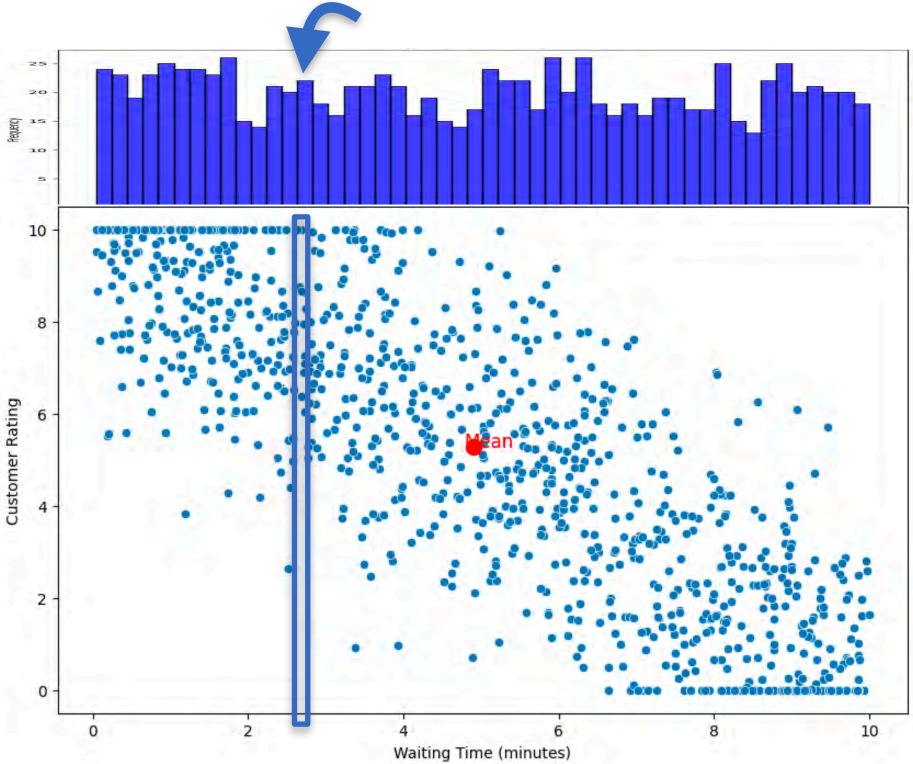
Marginal Distributions



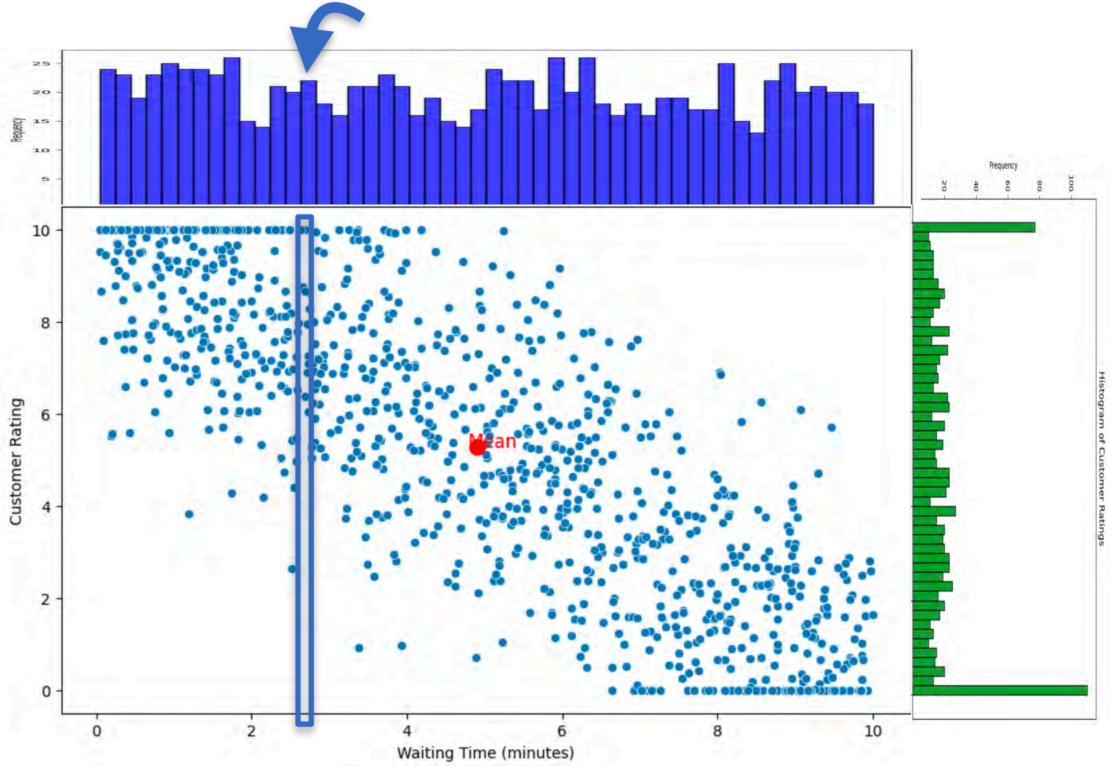
Marginal Distributions



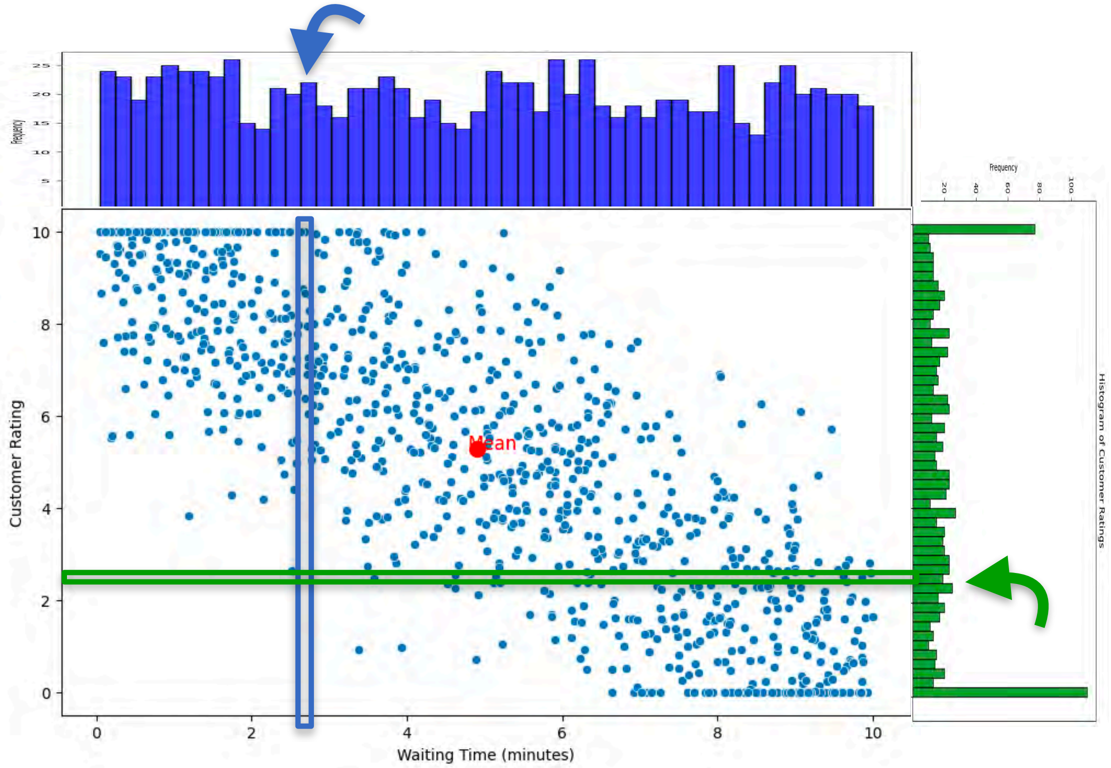
Marginal Distributions



Marginal Distributions



Marginal Distributions



Continuous Marginal Distribution

Continuous Marginal Distribution

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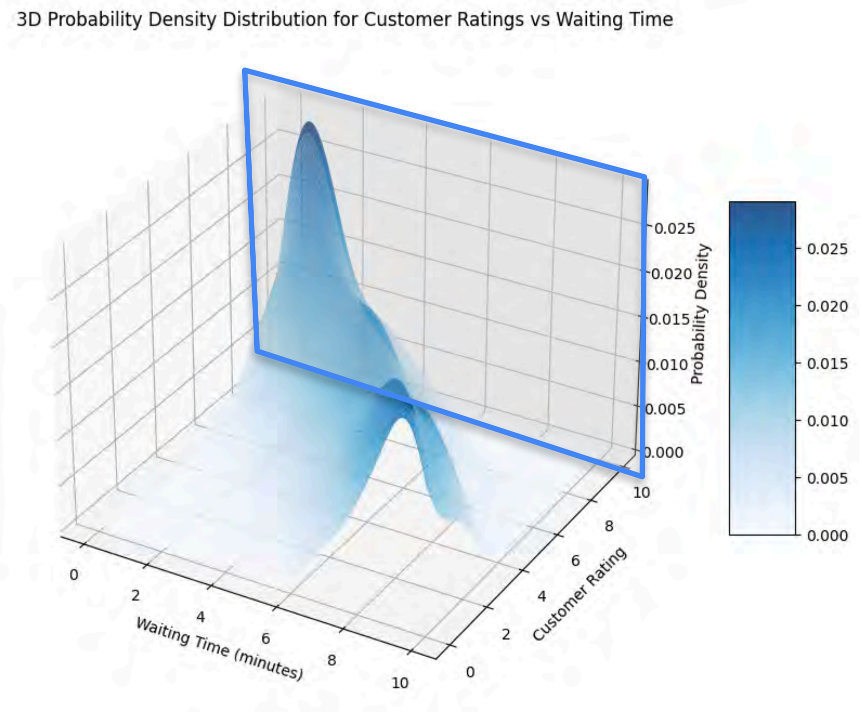
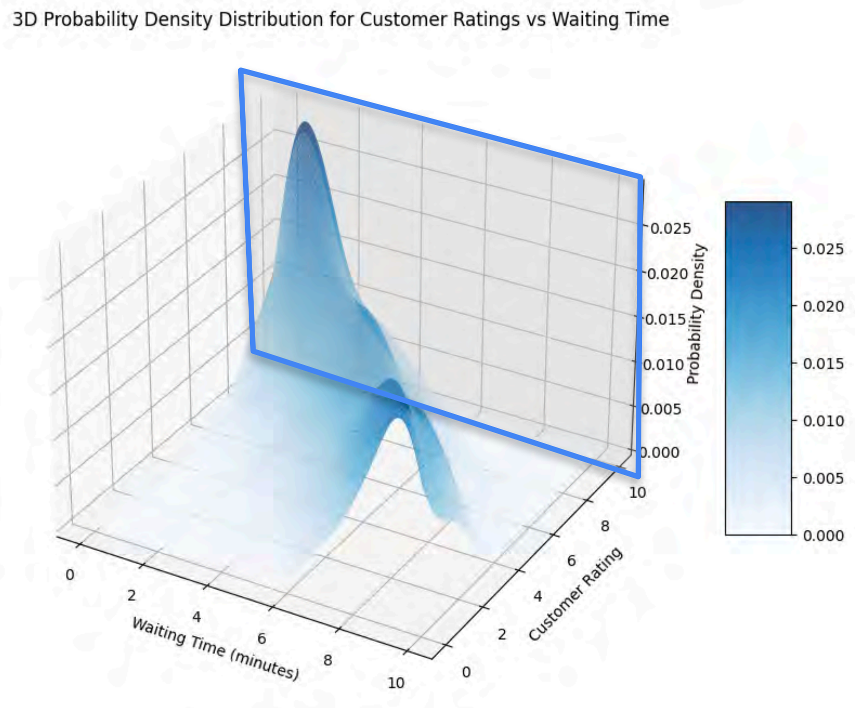
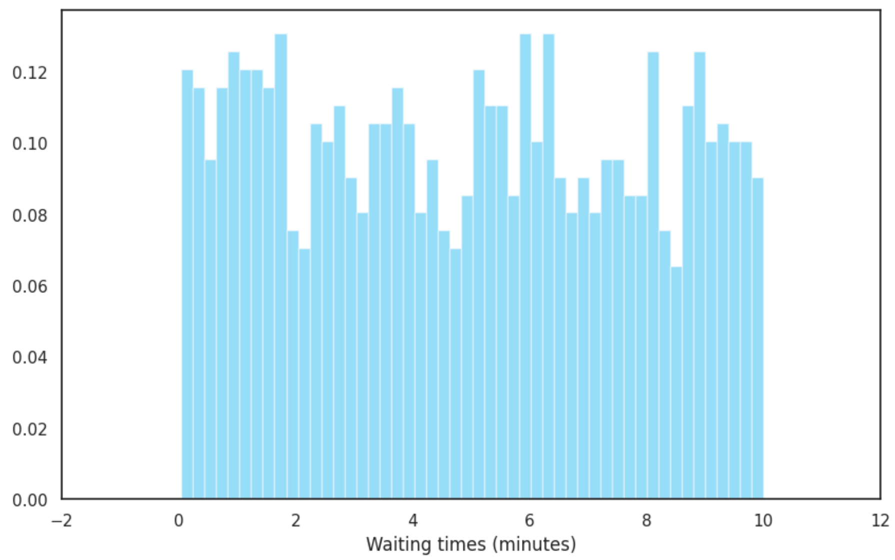
Continuous Marginal Distribution

Continuous Marginal Distribution

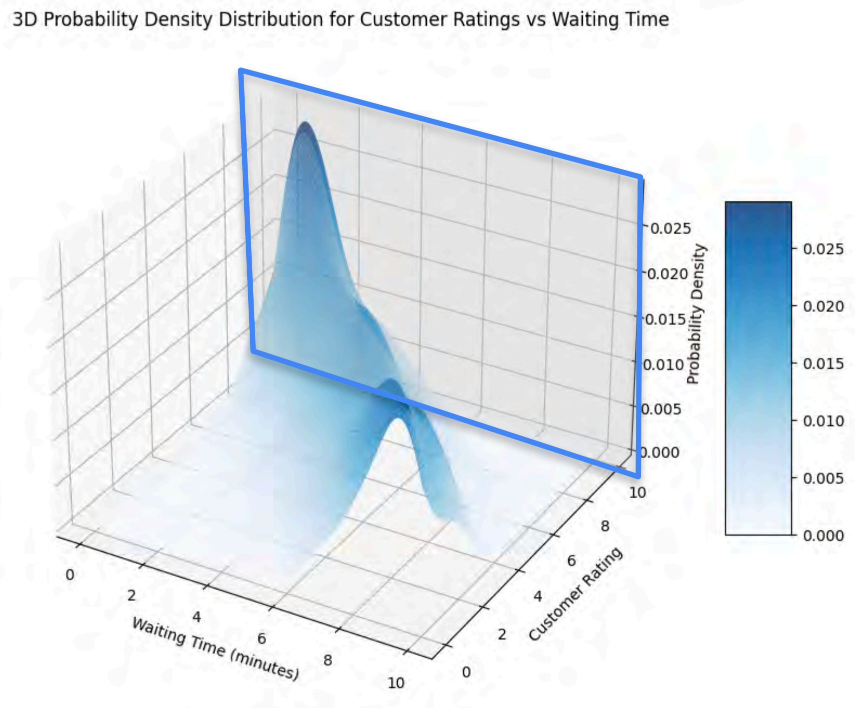


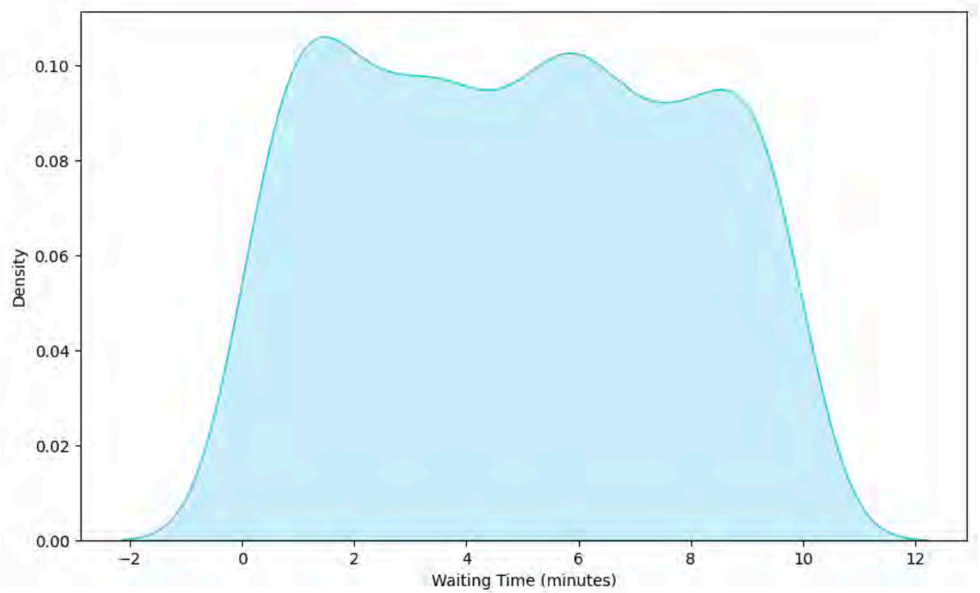
Continuous Marginal Distribution



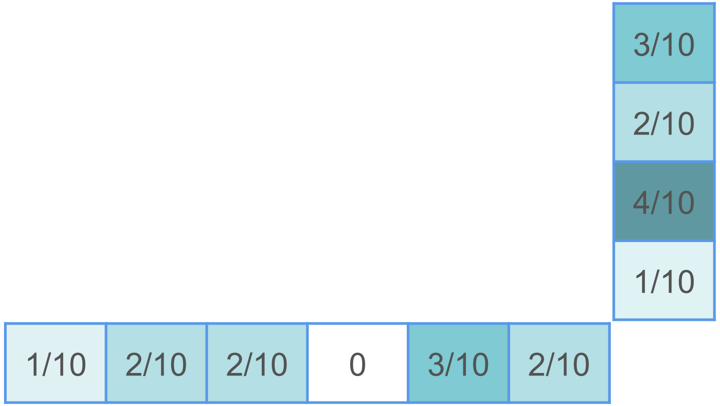


Continuous Marginal Distribution

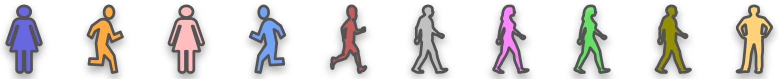




Conditional Distribution: Example 1



Conditional Distribution: Example 1

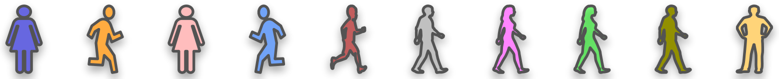


Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

3/10
2/10
4/10
1/10

1/10	2/10	2/10	0	3/10	2/10

Conditional Distribution: Example 1

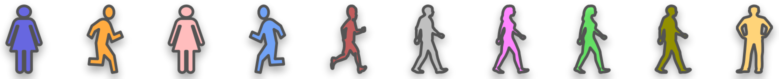


Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (Y)

Age (X)	45	46	47	48	49	50	
7	1/10	2/10	0	0	0	0	3/10
8	0	0	2/10	0	0	0	2/10
9	0	0	0	0	3/10	1/10	4/10
10	0	0	0	0	0	1/10	1/10
	1/10	2/10	2/10	0	3/10	2/10	

Conditional Distribution: Example 1



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (Y)

Age (X)

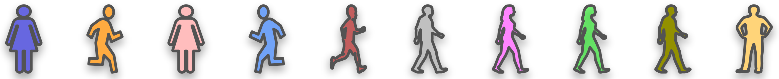
	45	46	47	48	49	50	
7	1/10	2/10	0	0	0	0	3/10
8	0	0	2/10	0	0	0	2/10
9	0	0	0	0	3/10	1/10	4/10
10	0	0	0	0	0	1/10	1/10
	1/10	2/10	2/10	0	3/10	2/10	

Age (X)

7	3/10
8	2/10
9	4/10
10	1/10

Marginal Distribution of Age

Conditional Distribution: Example 1



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (Y)

Age (X)	45	46	47	48	49	50	
7	1/10	2/10	0	0	0	0	3/10
8	0	0	2/10	0	0	0	2/10
9	0	0	0	0	3/10	1/10	4/10
10	0	0	0	0	0	1/10	1/10
	1/10	2/10	2/10	0	3/10	2/10	

Age (X)	
7	3/10
8	2/10
9	4/10
10	1/10

Marginal Distribution of Age

Height (Y)

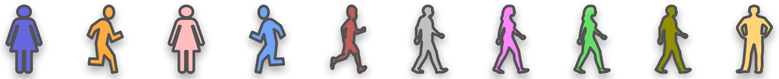
45	46	47	48	49	50
1/10	2/10	2/10	0	3/10	2/10

Marginal Distribution of Height

Conditional Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



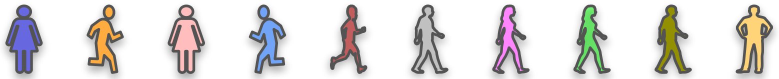
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Conditional Distribution: Example 1

Height (Y)

Age (X)

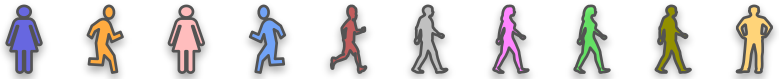
	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

If age = 9, what is the distribution across the height variable?

Conditional Distribution: Example 1



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

If age = 9, what is the distribution across the height variable?

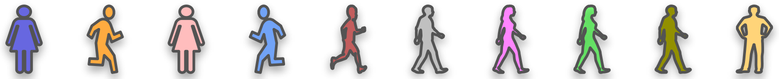
Conditional Distribution

Conditional Distribution: Example 1

Height (Y)

Age (X)

	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



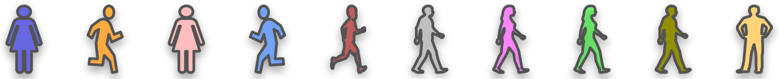
Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

If age = 9, what is the distribution across the height variable?

Conditional Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

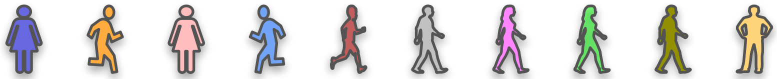
If age = 9, what is the distribution across the height variable?

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

Conditional Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10



Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

If age = 9, what is the distribution across the height variable?

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

Conditional Distribution: Example 1

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

Height (Y)

Age (X)

	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Conditional Distribution: Example 1

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

Height (Y)

Age (X)

	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Conditional Distribution: Example 1

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

Height (Y)

Age (X)		45	46	47	48	49	50
	9	0	0	0	0	3/10	1/10

Normalize

9	0	0	0	0	3/4	1/4

Conditional Distribution: Example 1

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

Height (Y)

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

Conditional Distribution: Example 1

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

Height (Y)

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3}{4}$$

Conditional Distribution: Example 1

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

$$\mathbf{P}(A, B) = \mathbf{P}(A) \cdot \mathbf{P}(B \,|\, A)$$

Height (Y)

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3}{4}$$

Conditional Distribution: Example 1

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

$$\mathbf{P}(A, B) = \mathbf{P}(A) \cdot \mathbf{P}(B \,|\, A)$$

$$\mathbf{P}(X = 9, Y = 49) = P(X = 9) \cdot P(Y = 49 \,|\, X = 9)$$

Height (Y)

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3}{4}$$

Conditional Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(Y = 49 \mid X = 9) = \frac{3}{4}$$

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \mid X = 9)$$

$$\mathbf{P}(A, B) = \mathbf{P}(A) \cdot \mathbf{P}(B \mid A)$$

$$\mathbf{P}(X = 9, Y = 49) = P(X = 9) \cdot P(Y = 49 \mid X = 9)$$

$$P(Y = 49 \mid X = 9) = \frac{\mathbf{P}(X = 9, Y = 49)}{P(X = 9)}$$

Row sum

Conditional Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3}{4}$$

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

$$\mathbf{P}(A, B) = \mathbf{P}(A) \cdot \mathbf{P}(B \,|\, A)$$

$$\mathbf{P}(X = 9, Y = 49) = P(X = 9) \cdot P(Y = 49 \,|\, X = 9)$$

$$P(Y = 49 \,|\, X = 9) = \frac{\mathbf{P}(X = 9, Y = 49)}{P(X = 9)}$$

Row sum

$$\mathbf{P}(Y = 49 \,|\, X = 9)$$

Conditional Distribution: Example 1

Height (Y)

Age (X)		45	46	47	48	49	50
	9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3}{4}$$

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

$$\mathbf{P}(A, B) = \mathbf{P}(A) \cdot \mathbf{P}(B \,|\, A)$$

$$\mathbf{P}(X = 9, Y = 49) = P(X = 9) \cdot P(Y = 49 \,|\, X = 9)$$

$$P(Y = 49 \,|\, X = 9) = \frac{\mathbf{P}(X = 9, Y = 49)}{P(X = 9)}$$

Row sum

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3/10}{4/10}$$

Conditional Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(Y = 49 \mid X = 9) = \frac{3}{4}$$

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \mid X = 9)$$

$$\mathbf{P}(A, B) = \mathbf{P}(A) \cdot \mathbf{P}(B \mid A)$$

$$\mathbf{P}(X = 9, Y = 49) = P(X = 9) \cdot P(Y = 49 \mid X = 9)$$

$$P(Y = 49 \mid X = 9) = \frac{\mathbf{P}(X = 9, Y = 49)}{P(X = 9)}$$

Row sum

$$\mathbf{P}(Y = 49 \mid X = 9) = \frac{3/10}{4/10} = \frac{3}{4}$$

Conditional Distribution: Example 1

Height (Y)

Age (X)		45	46	47	48	49	50
	9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3}{4}$$

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

$$\mathbf{P}(A, B) = \mathbf{P}(A) \cdot \mathbf{P}(B \,|\, A)$$

$$\mathbf{P}(X = 9, Y = 49) = P(X = 9) \cdot P(Y = 49 \,|\, X = 9)$$

$$P(Y = 49 \,|\, X = 9) = \frac{\mathbf{P}(X = 9, Y = 49)}{P(X = 9)}$$

Row sum

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3/10}{4/10} = \frac{3}{4}$$

Conditional Distribution: Example 1

Height (Y) — Sum $\mathbf{P}(X=9)$

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(Y=49\,|\,X=9)=\frac{3}{4}$$

$$p_{Y|X=9}(y)=\mathbf{P}(Y=y\,|\,X=9)$$

$$\mathbf{P}(A,B)=\mathbf{P}(A)\cdot\mathbf{P}(B\,|\,A)$$

$$\mathbf{P}(X=9,Y=49)=P(X=9)\cdot P(Y=49\,|\,X=9)$$

$$P(Y=49\,|\,X=9)=\frac{\mathbf{P}(X=9,Y=49)}{P(X=9)}$$

Row sum

$$\mathbf{P}(Y=49\,|\,X=9)=\frac{3/10}{4/10}=\frac{3}{4}$$

Conditional Distribution: Example 1

Height (Y) | Sum $\mathbf{P}(X = 9)$

(X)						
	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

$$\mathbf{P}(A, B) = \mathbf{P}(A) \cdot \mathbf{P}(B \,|\, A)$$

$$\mathbf{P}(X = 9, Y = 49) = P(X = 9) \cdot P(Y = 49 \,|\, X = 9)$$

$$P(Y = 49 \,|\, X = 9) = \frac{\mathbf{P}(X = 9, Y = 49)}{P(X = 9)}$$

Row sum

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3/10}{4/10} = \frac{3}{4}$$

Conditional Distribution: Example 1

	Height (Y)						Sum
(X)	45	46	47	48	49	50	$\mathbf{P}(X=9)$
9	0	0	0	0	3/10	1/10	

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

$$\mathbf{P}(A,B) = \mathbf{P}(A) \cdot \mathbf{P}(B \,|\, A)$$

$$\mathbf{P}(X=9, Y=49) = P(X=9) \cdot P(Y=49 \,|\, X=9)$$

$$P(Y=49 \,|\, X=9) = \frac{\mathbf{P}(X=9, Y=49)}{P(X=9)}$$

Row sum

$$p_{Y|X=x}(y) = \frac{p_{XY}(x,y)}{p_X(x)}$$

$$\mathbf{P}(Y=49 \,|\, X=9) = \frac{3/10}{4/10} = \frac{3}{4}$$

Discrete Conditional Distribution: Formula

$$p_{Y|X=x}(y) = \frac{p_{XY}(x, y)}{p_X(x)}$$

Discrete Conditional Distribution: Formula

Joint PDF of X and Y

$$p_{Y|X=x}(y) = \frac{p_{XY}(x, y)}{p_X(x)}$$

Discrete Conditional Distribution: Formula

Joint PDF of X and Y

Conditional PDF of Y

$$p_{Y|X=x}(y) = \frac{p_{XY}(x, y)}{p_X(x)}$$

Discrete Conditional Distribution: Formula

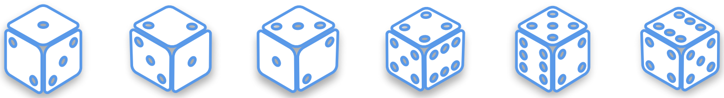
Joint PDF of X and Y

Conditional PDF of Y

$$p_{Y|X=x}(y) = \frac{p_{XY}(x, y)}{p_X(x)}$$

Marginal distribution of X

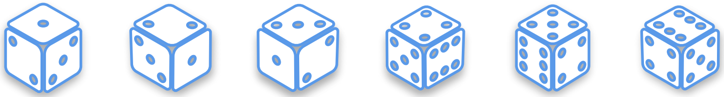
Conditional Distributions: Example 2



Die 1:	1/6	1/6	1/6	1/6	1/6	1/6
Die 2:	1/6	1/6	1/6	1/6	1/6	1/6

		Y					
		1	**2**	**3**	**4**	**5**	**6**
	1	1/36	1/36	1/36	1/36	1/36	1/36
	2	1/36	1/36	1/36	1/36	1/36	1/36
X	**3**	1/36	1/36	1/36	1/36	1/36	1/36
	4	1/36	1/36	1/36	1/36	1/36	1/36
	5	1/36	1/36	1/36	1/36	1/36	1/36
	6	1/36	1/36	1/36	1/36	1/36	1/36

Conditional Distributions: Example 2

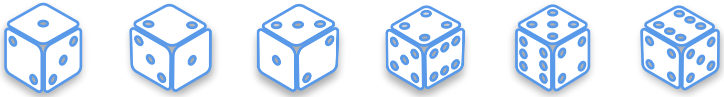


Die 1:	1/6	1/6	1/6	1/6	1/6	1/6
Die 2:	1/6	1/6	1/6	1/6	1/6	1/6

$$p_{Y|X=4}(y = 1)$$

X \ Y	1	2	3	4	5	6
1	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	1/36

Conditional Distributions: Example 2



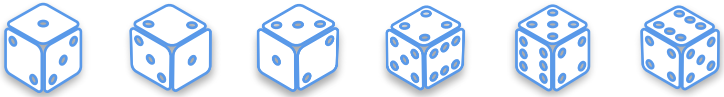
Die 1:	1/6	1/6	1/6	1/6	1/6	1/6
Die 2:	1/6	1/6	1/6	1/6	1/6	1/6

$$p_{Y|X=4}(y=1) = \frac{p_{XY}(x=4, y=1)}{p_X(x=4)}$$

Y

X	1	2	3	4	5	6
1	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	1/36

Conditional Distributions: Example 2



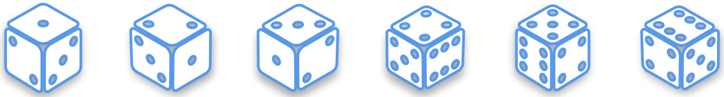
Die 1:	1/6	1/6	1/6	1/6	1/6	1/6
Die 2:	1/6	1/6	1/6	1/6	1/6	1/6

$$p_{Y|X=4}(y=1) = \frac{p_{XY}(x=4, y=1)}{p_X(x=4)}$$

Y (columns), X (rows)

X \ Y	1	2	3	4	5	6	Sum
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6

Conditional Distributions: Example 2



Die 1:	1/6	1/6	1/6	1/6	1/6	1/6
Die 2:	1/6	1/6	1/6	1/6	1/6	1/6

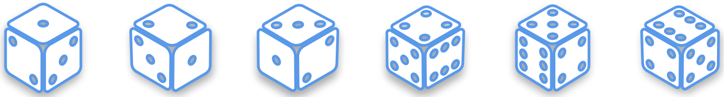
$$p_{Y|X=4}(y=1) = \frac{p_{XY}(x=4, y=1)}{p_X(x=4)}$$

$$= \frac{1/36}{1/6}$$

Y (columns), X (rows)

	1	**2**	**3**	**4**	**5**	**6**	**Sum**
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6

Conditional Distributions: Example 2



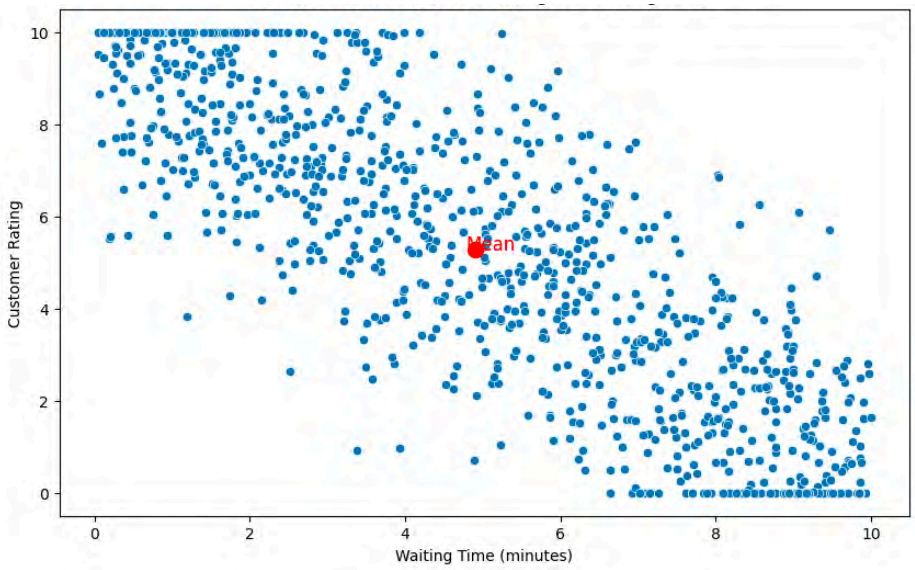
Die 1:	1/6	1/6	1/6	1/6	1/6	1/6
Die 2:	1/6	1/6	1/6	1/6	1/6	1/6

$$p_{Y|X=4}(y=1) = \frac{p_{XY}(x=4, y=1)}{p_X(x=4)} = \frac{1/36}{1/6} = \frac{1}{6}$$

Y (columns), X (rows)

	1	2	3	4	5	6	Sum
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6

Conditional Distributions: Example 4



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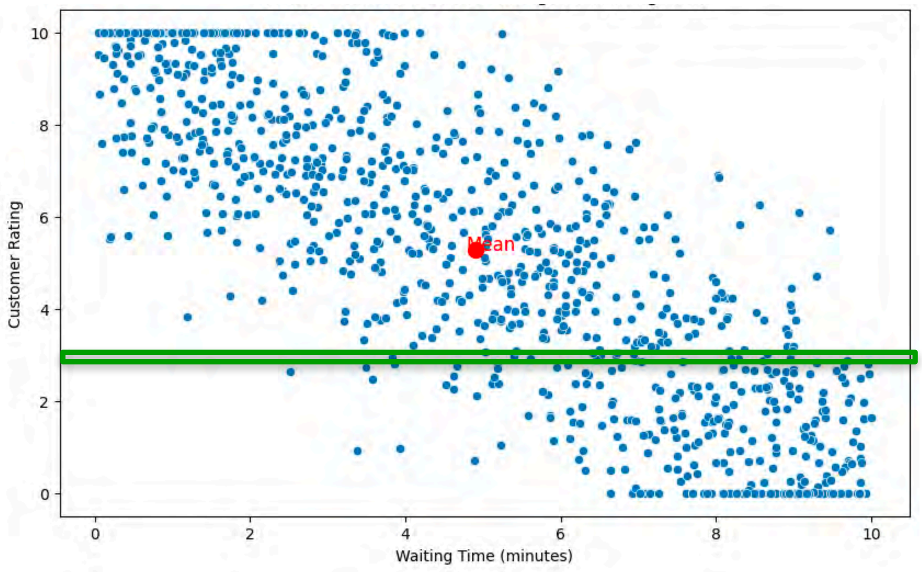
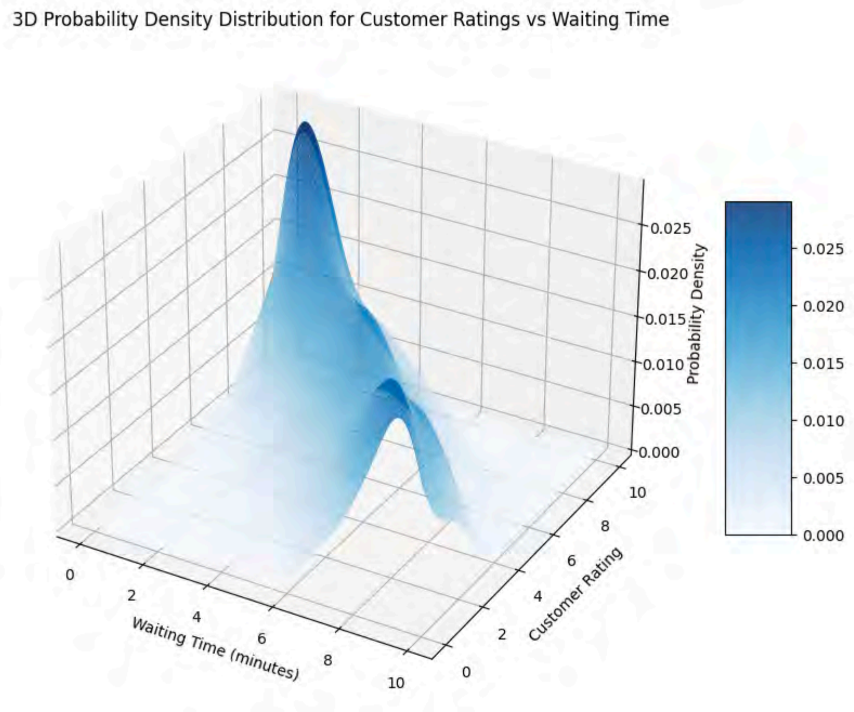
Conditional Distributions: Example 4



Continuous Conditional Distribution

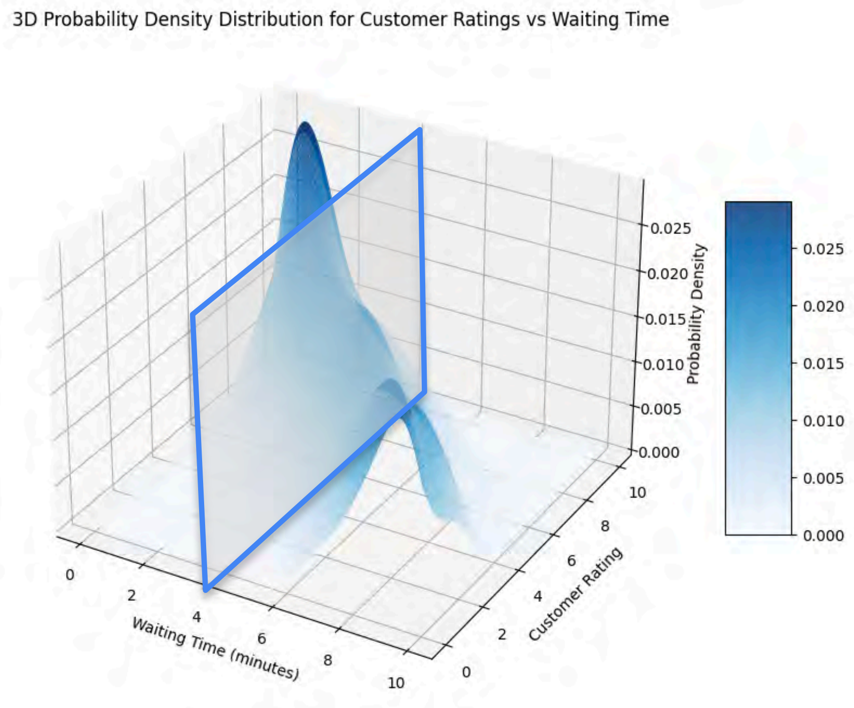
Continuous Conditional Distribution



Continuous Conditional Distribution

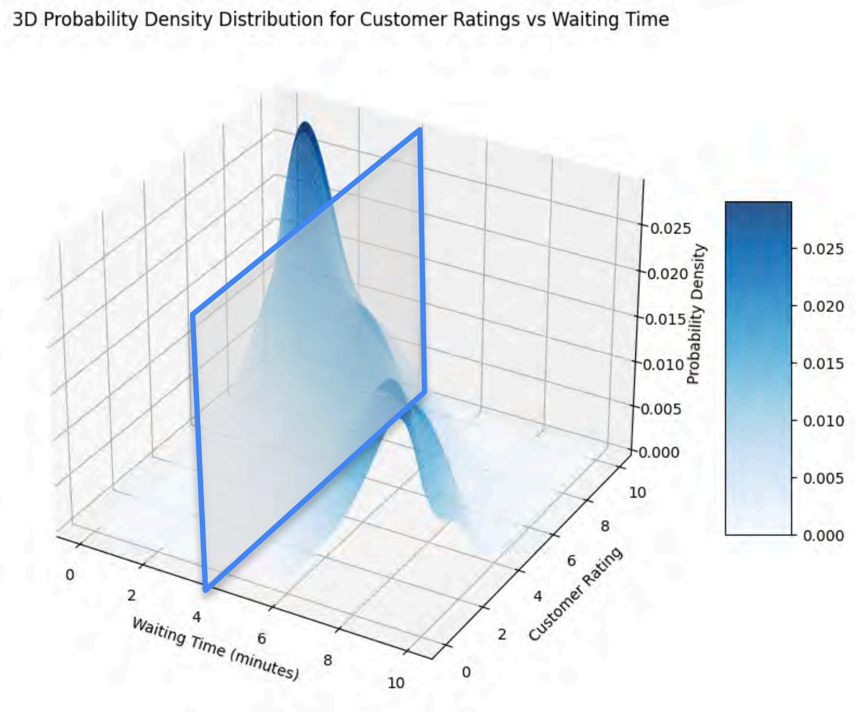
Probability distribution for rating given that waiting time was 4 minutes

Continuous Conditional Distribution

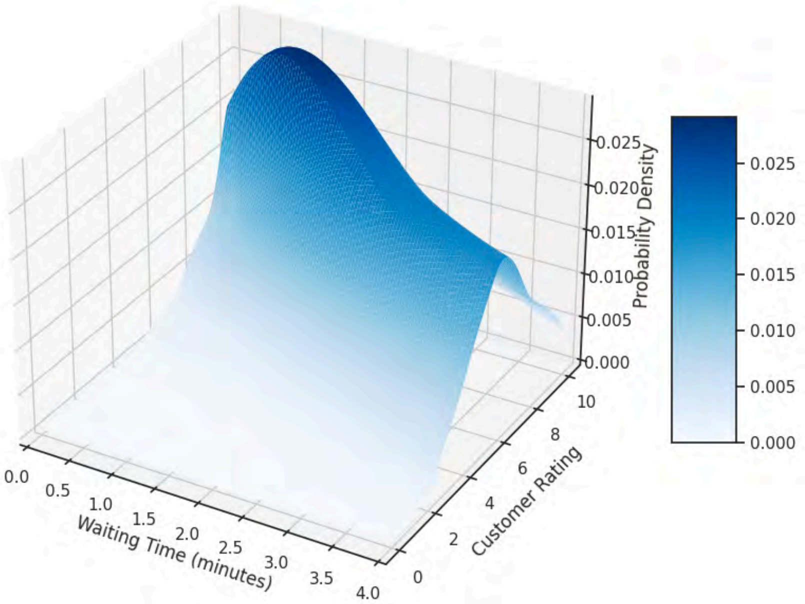


Probability distribution for rating given that waiting time was 4 minutes

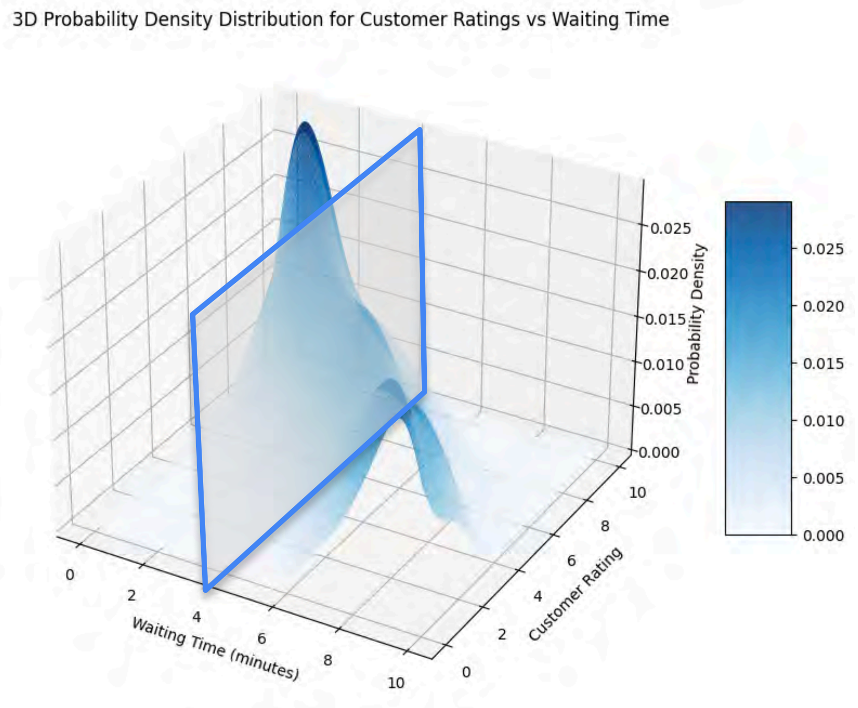
Continuous Conditional Distribution



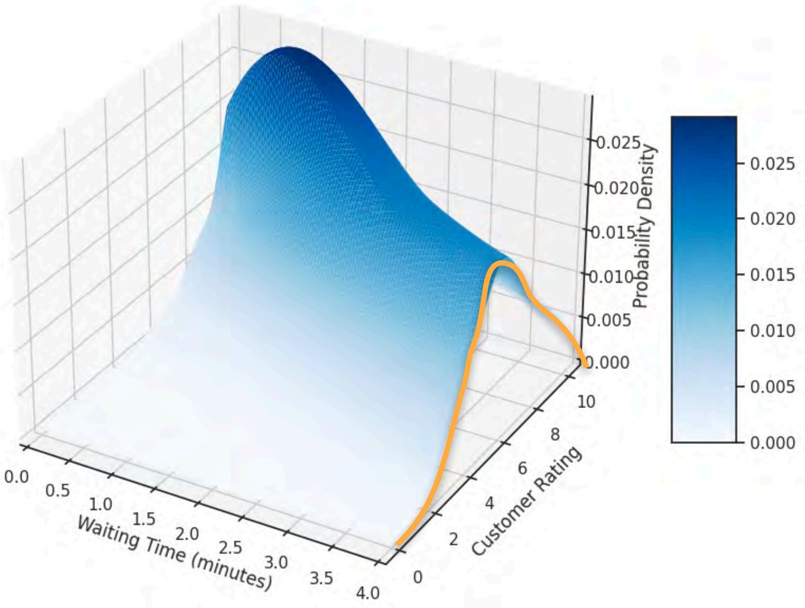
Probability distribution for rating given that waiting time was 4 minutes



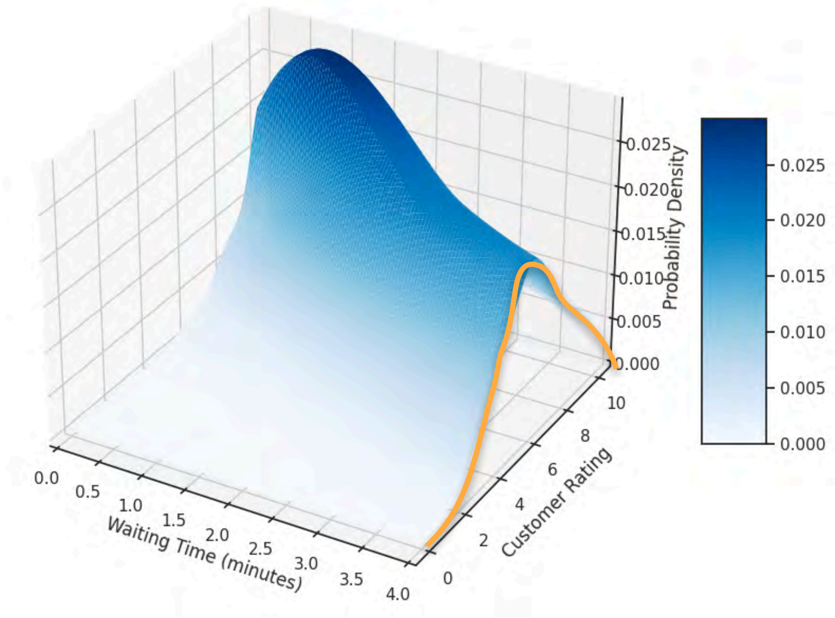
Continuous Conditional Distribution



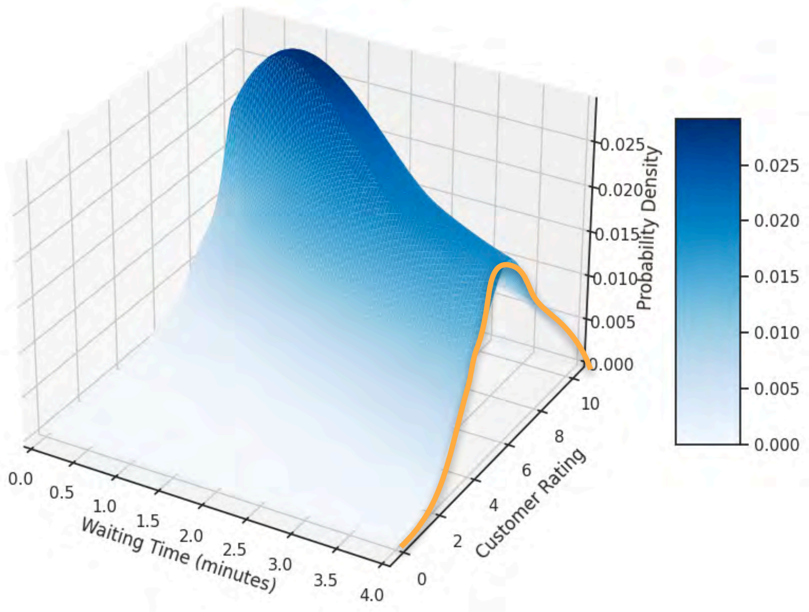
Probability distribution for rating given that waiting time was 4 minutes



Continuous Conditional Distribution

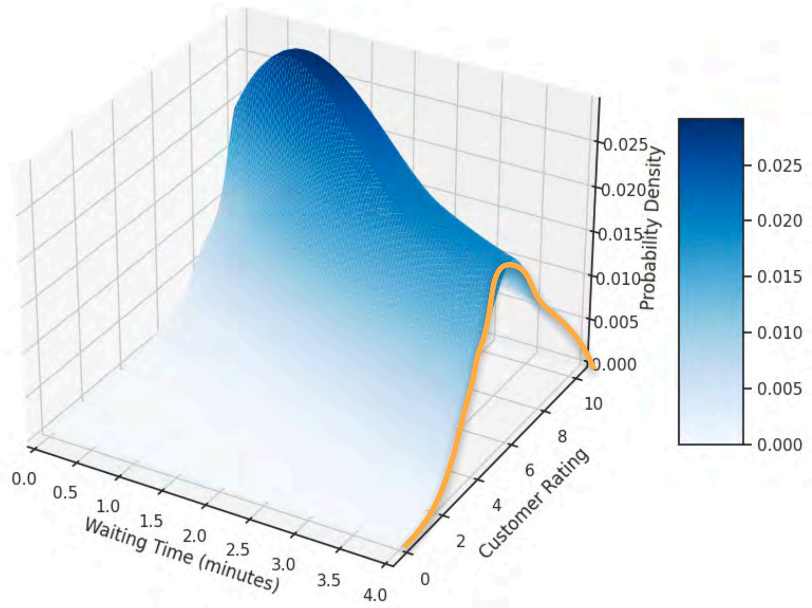


Continuous Conditional Distribution



Continuous Conditional Distribution

Conditional PDF of y given x = 4



Discrete Conditional Distribution

$$p_{Y|X=x}(y) = \frac{p_{XY}(x, y)}{p_X(x)}$$

Discrete Conditional Distribution

Joint PDF of X and Y

$$p_{Y|X=x}(y) = \frac{p_{XY}(x, y)}{p_X(x)}$$

Discrete Conditional Distribution

Joint PDF of X and Y

Conditional PDF of Y

$$p_{Y|X=x}(y) = \frac{p_{XY}(x, y)}{p_X(x)}$$

Discrete Conditional Distribution

Joint PDF of X and Y

Conditional PDF of Y

$$p_{Y|X=x}(y) = \frac{p_{XY}(x, y)}{p_X(x)}$$

Marginal distribution of X

Continuous Conditional Distribution: Formula

Joint PDF of X and Y

Conditional PDF of Y

$$f_{Y|X=x}(y) = \frac{f_{XY}(x, y)}{f_X(x)}$$

Marginal distribution of X

Probability Distributions with Multiple Variables

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Covariance

Introduction to Covariance

Introduction to Covariance

X : age of a child

Introduction to Covariance

X : age of a child

Y_1: height of the child (in)

Introduction to Covariance

X : age of a child

Y_1: height of the child (in)

Y_2 : grades in a test

Introduction to Covariance

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Y_2 : grades in a test

Y_3: number of naps per day

Introduction to Covariance

X : age of a child

Y_1: height of the child (in)

Y_2 : grades in a test

Y_3: number of naps per day

Age (X)	Height (Y_1)
6	50
7	52
8	55
9	57
10	60
11	62
12	64
13	65
14	67
15	68

Introduction to Covariance

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6	50
7	52
8	55
9	57
10	60
11	62
12	64
13	65
14	67
15	68

Y_2 : grades in a test

Age (X)	Grades (Y_2)
6	5
7	7
8	8
9	3
10	1
11	1
12	6
13	10
14	2
15	7

Y_3: number of naps per day

Introduction to Covariance

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14	2
15	7

Y_3: number of naps per day

Age (X)	Naps per day (Y_3)
6	8
7	7
8	6
9	5
10	4
11	3
12	2
13	1
14	1
15	0

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11	3
12	2
13	1
14	1
15	0

Introduction to Covariance

- **How is variable X related to each of the Y variables?**
- **How do you compare these relations?**

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14	1
15	0

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12	2
13	1
14	1
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Introduction to Covariance

X : age of a child

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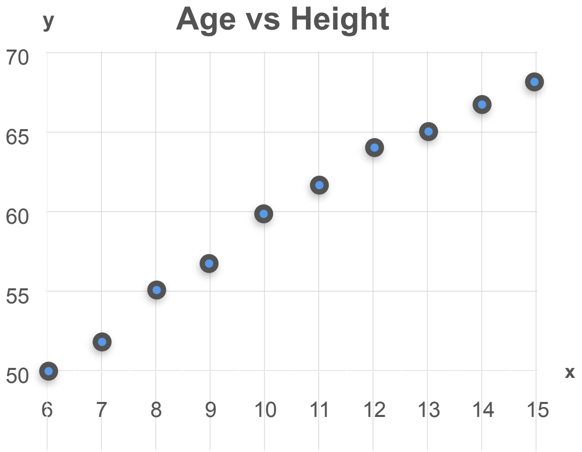

Y_2 : grades in a test

Age (X)	**Grades (Y_2)**
6	5
7	7
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10	1
11	1
12	6
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Introduction to Covariance

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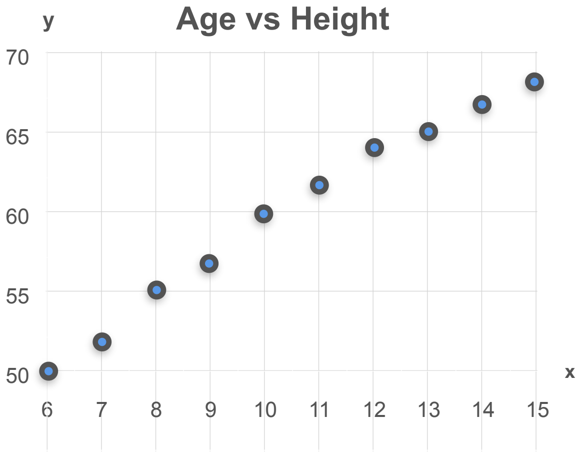
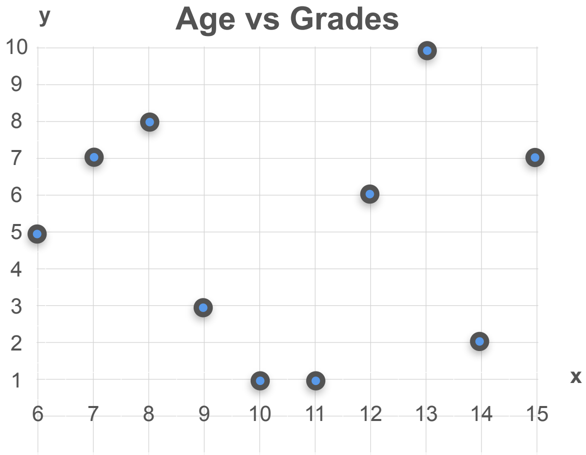


Y_2 : grades in a test



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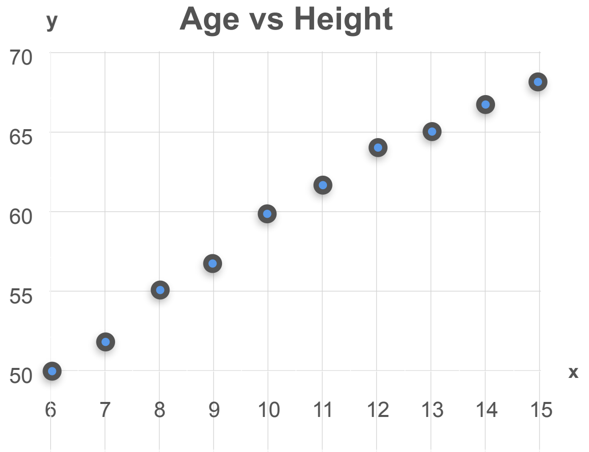

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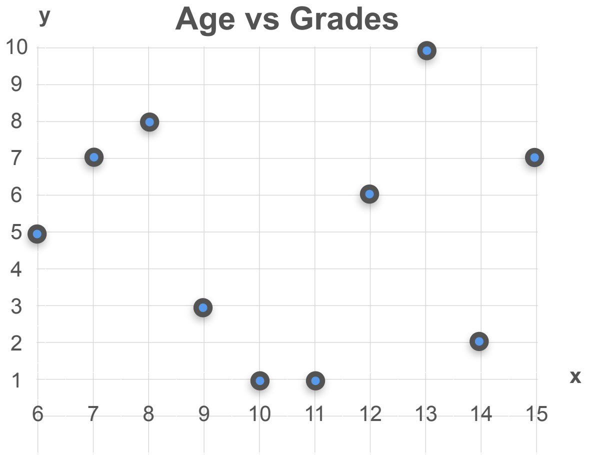

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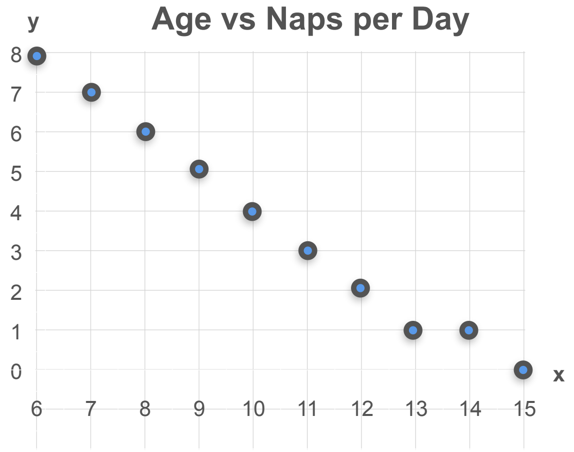
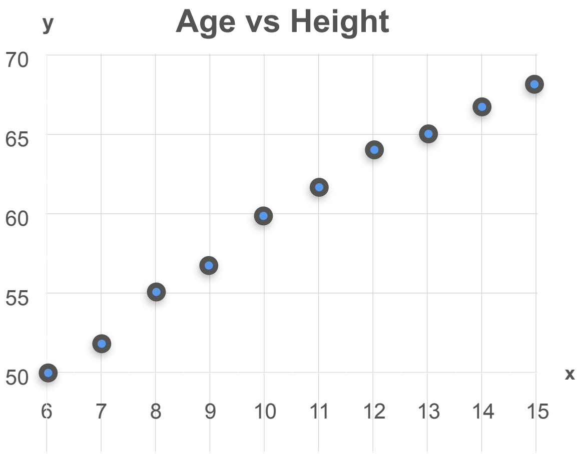
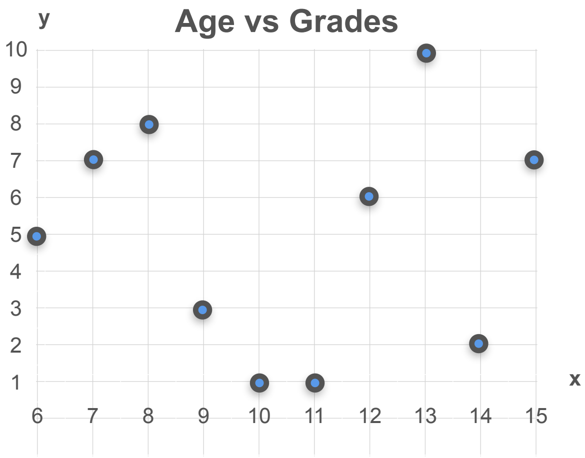
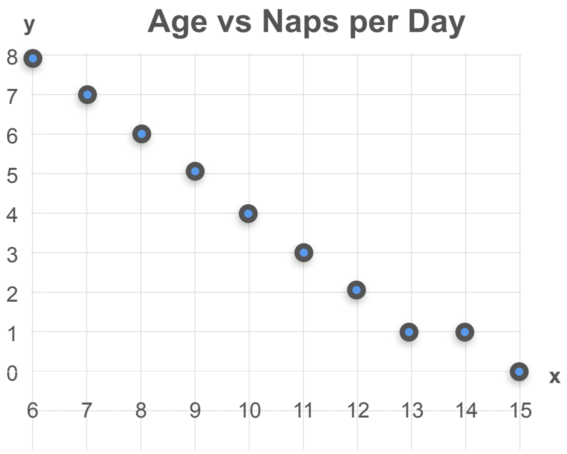


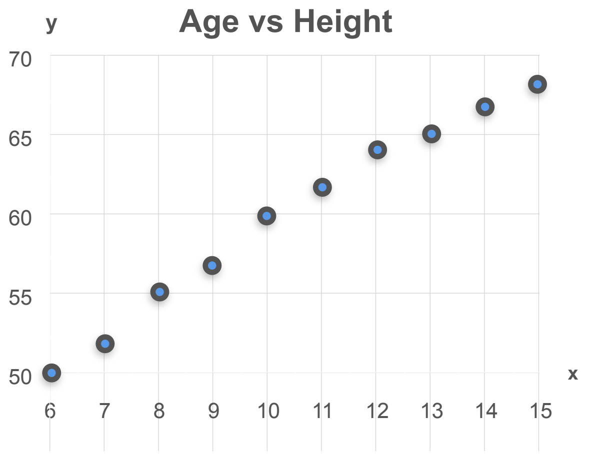
How To Compare These?

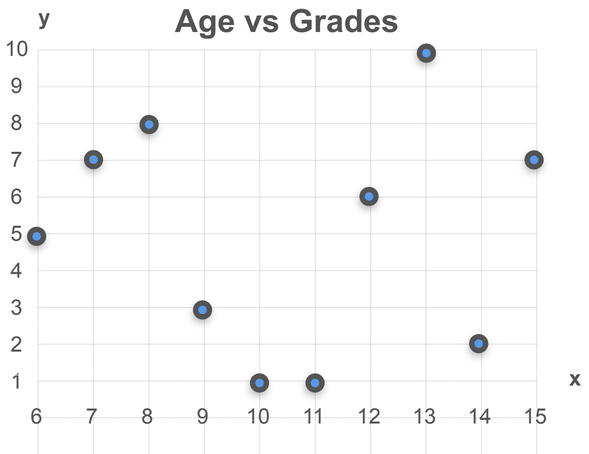


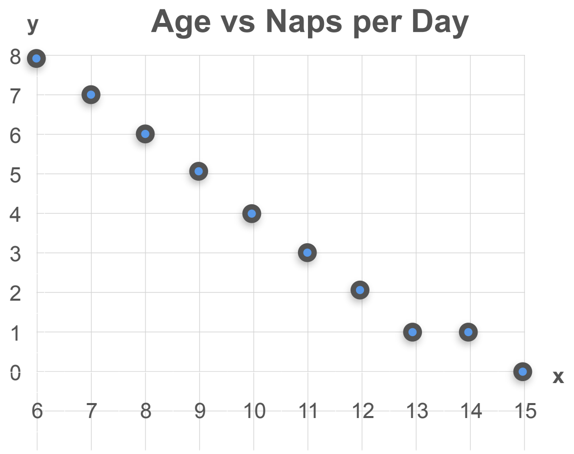




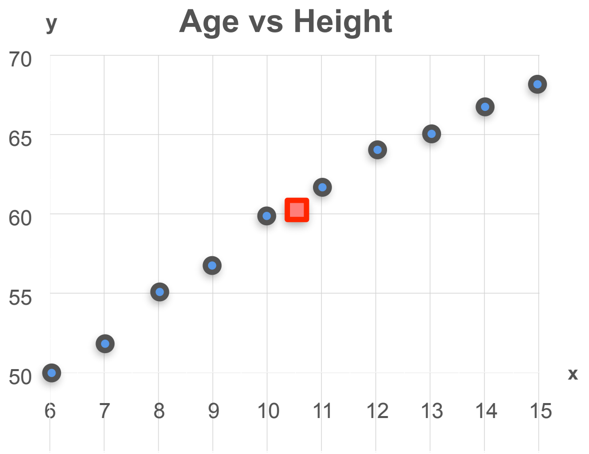
Mean?

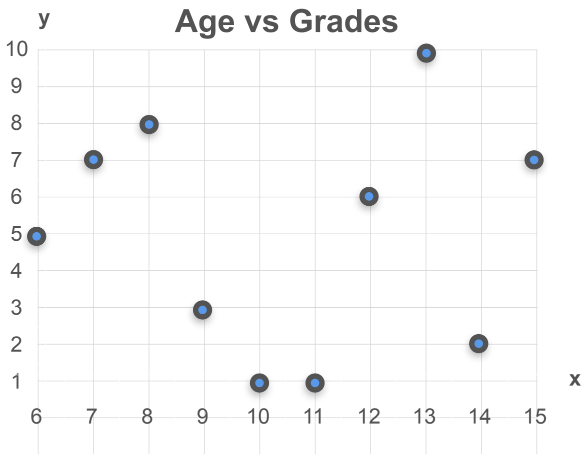

Age vs Height
y
70
65
60
55
50
6 7 8 9 10 11 12 13 14 15
x

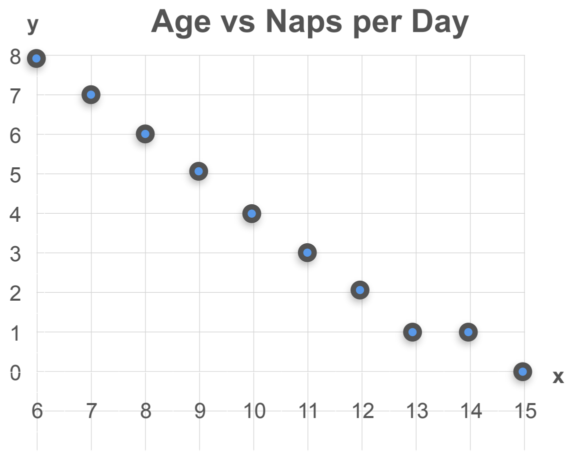

Age vs Grades
y
10
9
8
7
6
5
4
3
2
1
6 7 8 9 10 11 12 13 14 15
x


Age vs Naps per Day
y
8
7
6
5
4
3
2
1
0
6 7 8 9 10 11 12 13 14 15
x

Mean?

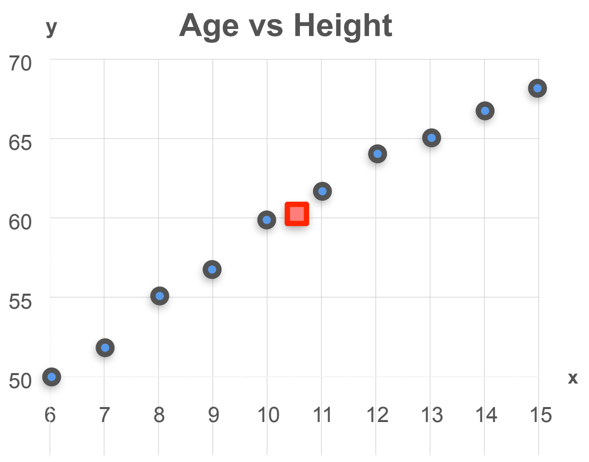


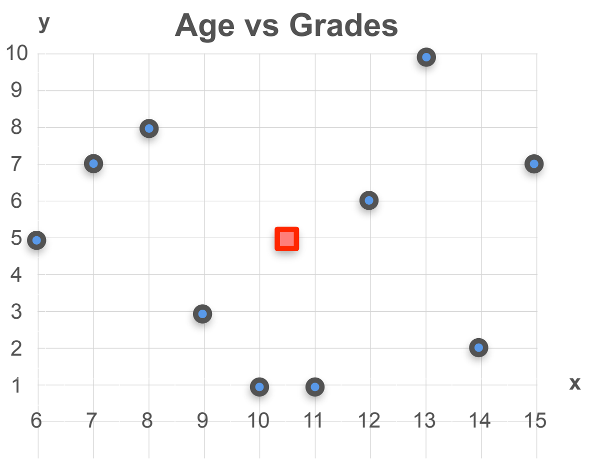


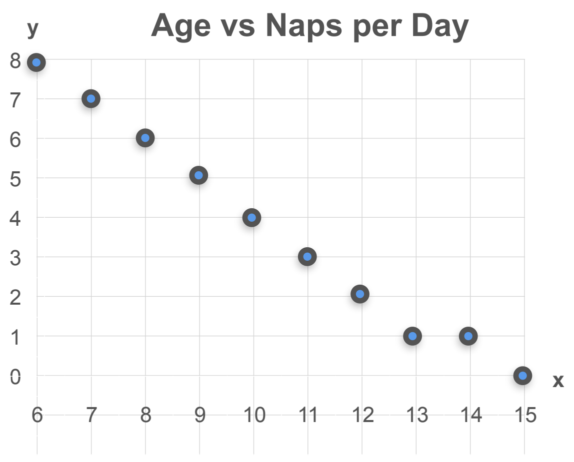


$\mu_x = 10.5 \qquad \mu_y = 60$

Mean?



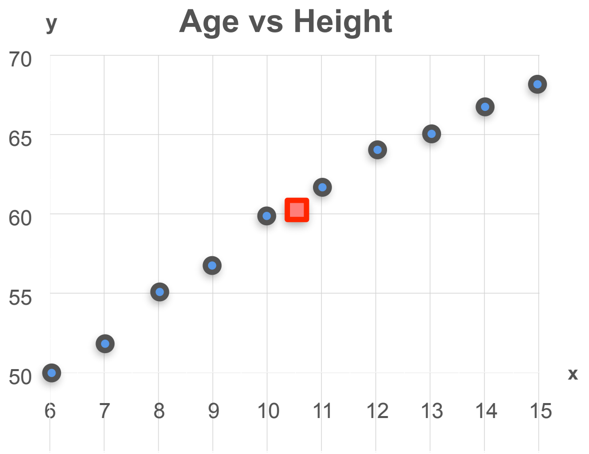




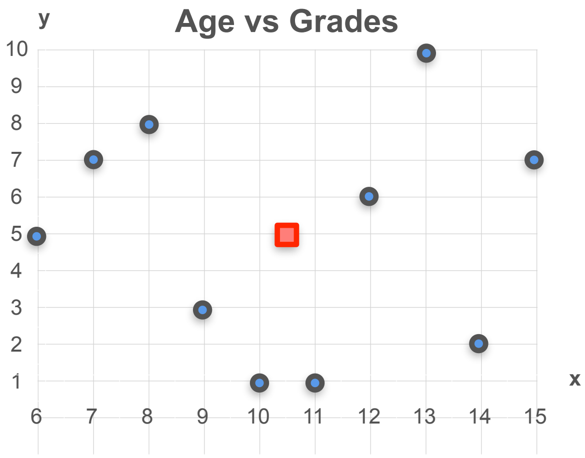
$\mu_x = 10.5 \quad \mu_y = 60$

$\mu_x = 10.5 \quad \mu_y = 5$

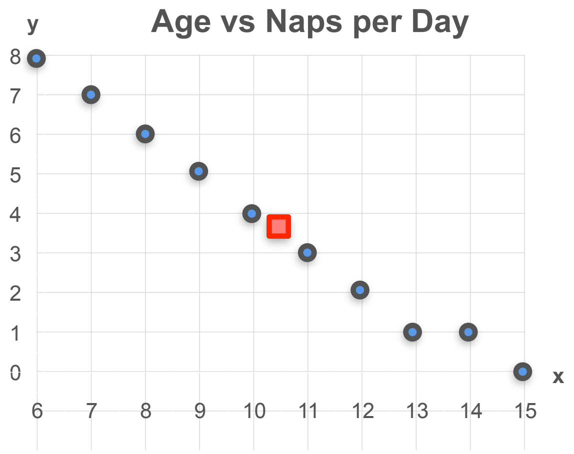
Mean?



$\mu_x = 10.5 \quad \mu_y = 60$

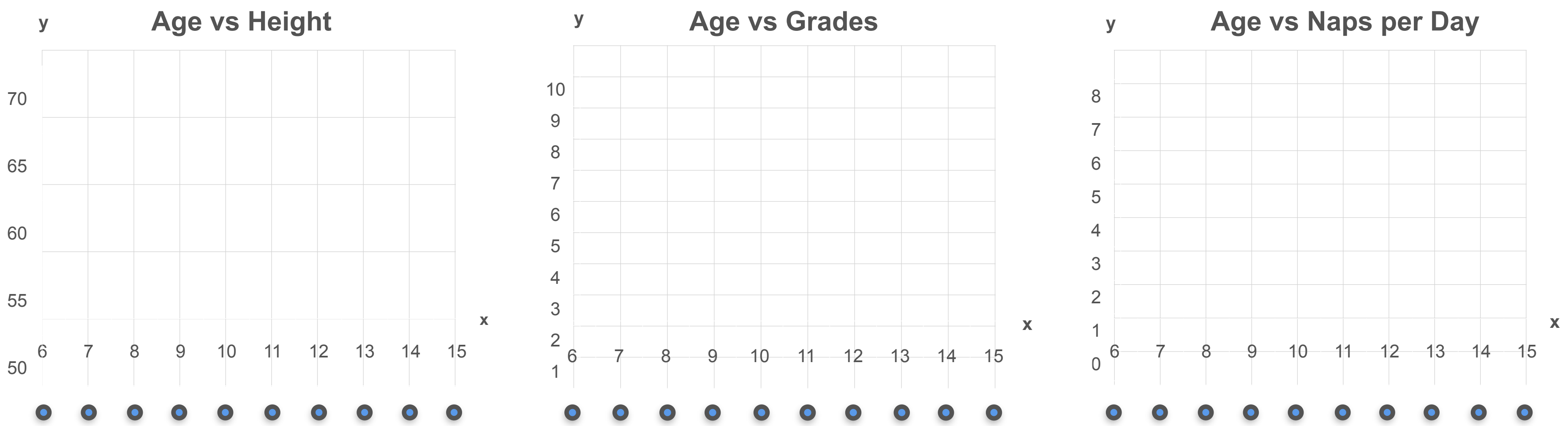


$\mu_x = 10.5 \quad \mu_y = 5$

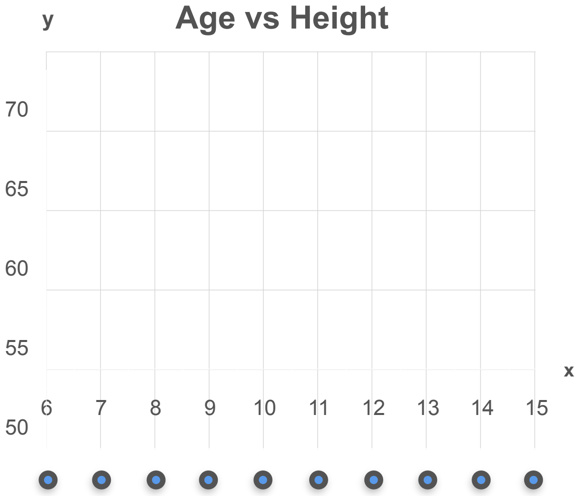


$\mu_x = 10.5 \quad \mu_y = 3.7$

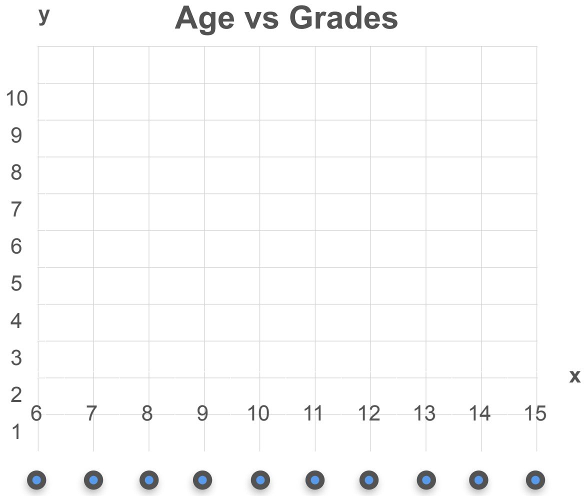
Horizontal (X) Variance



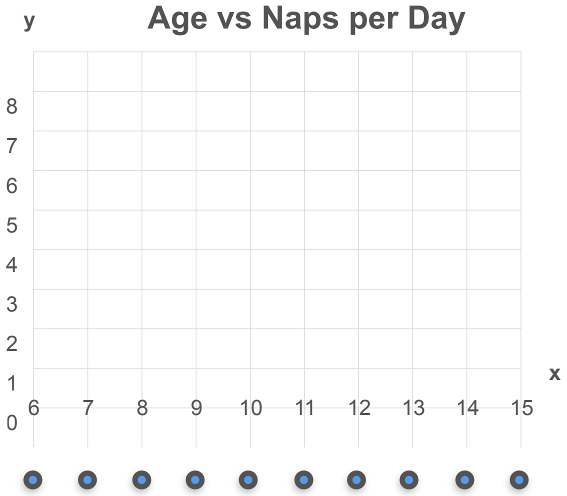
Horizontal (X) Variance



$$Var(X) = 9.17$$

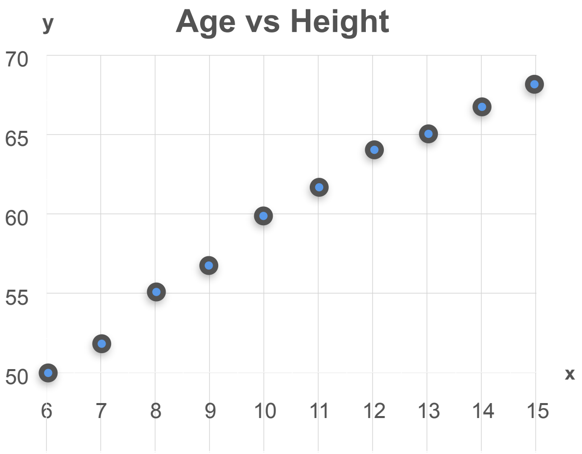


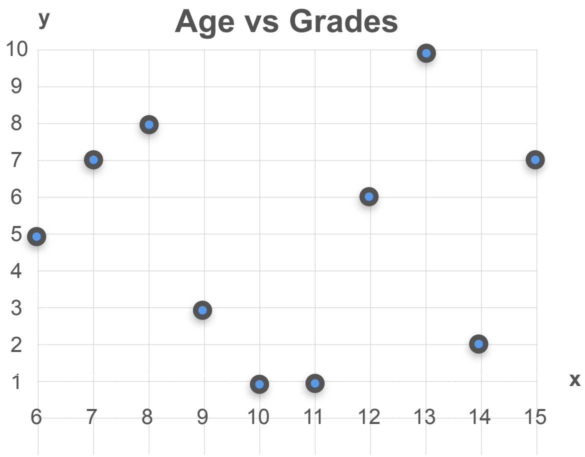
$$Var(X) = 9.17$$

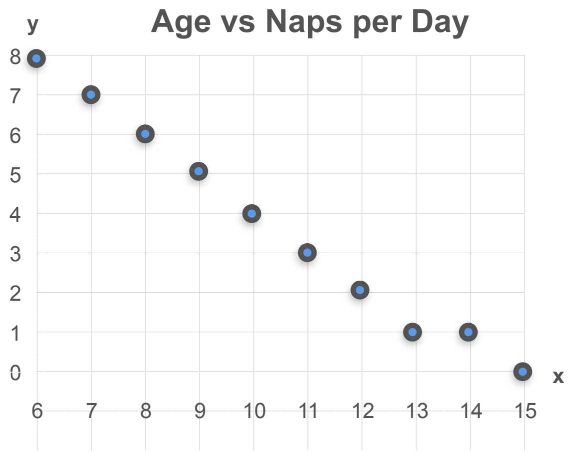


$$Var(X) = 9.17$$

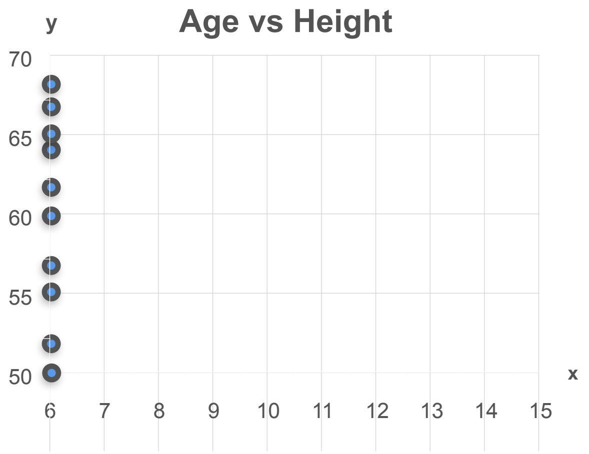
Anything Else?

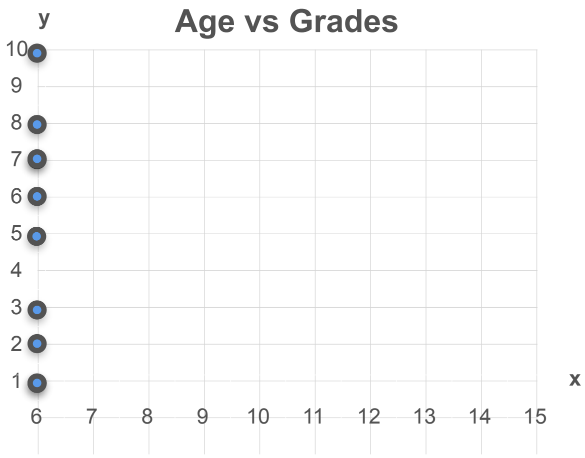


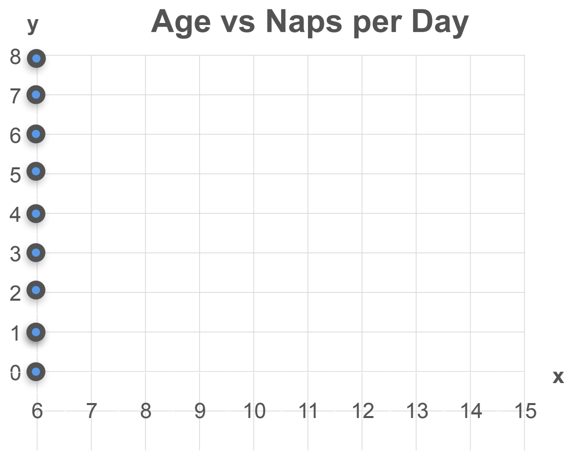




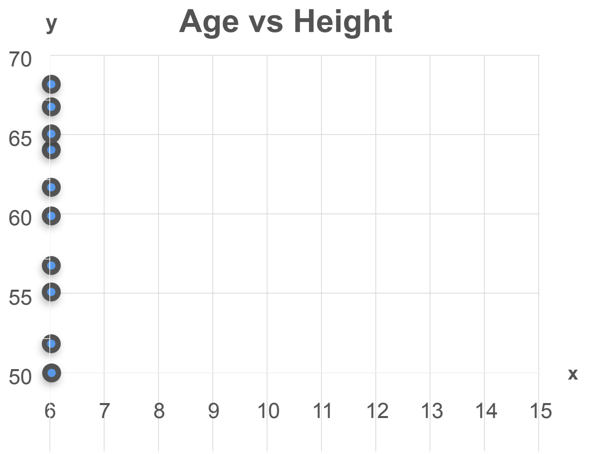
Vertical (Y) Variance

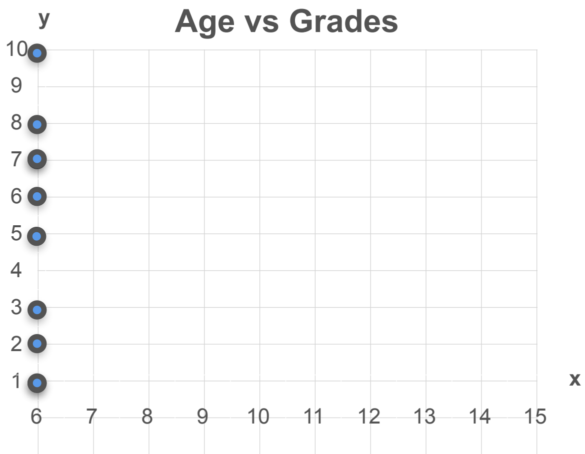


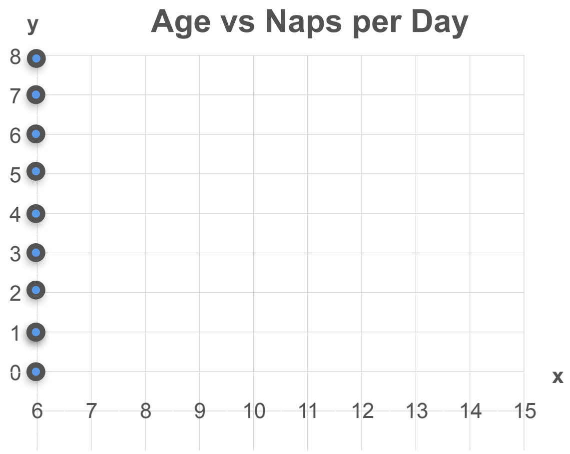




Vertical (Y) Variance





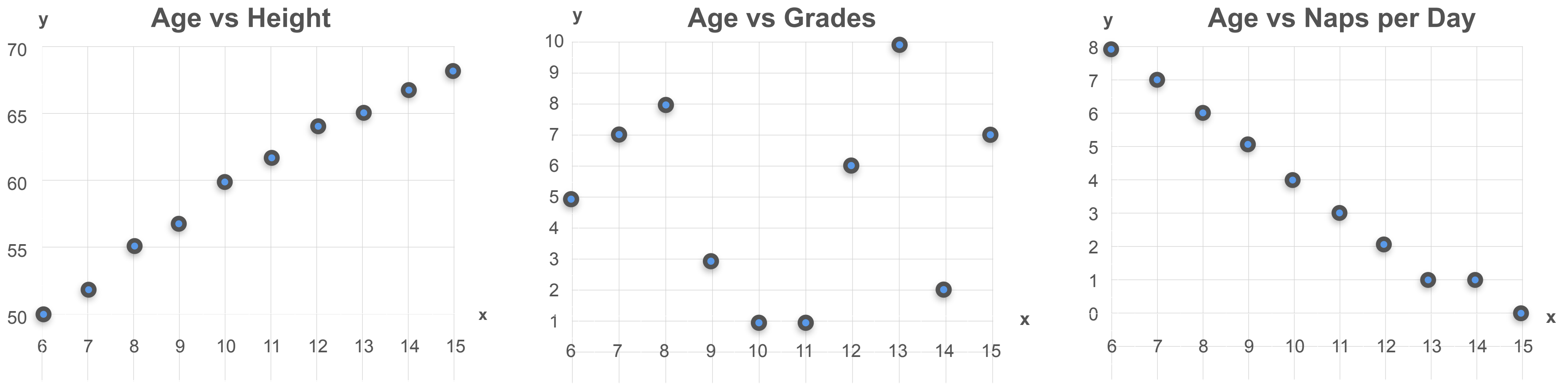


$$Var(Y) = 39.56$$

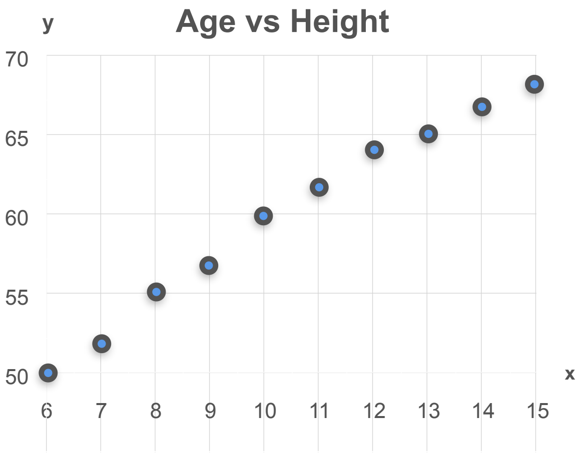
$$Var(Y) = 9.78$$

$$Var(Y) = 7.57$$

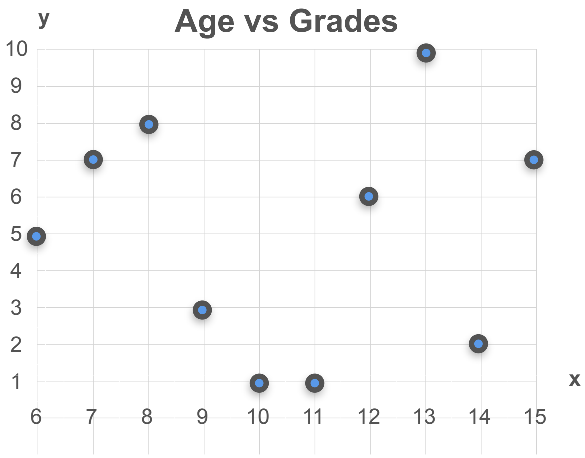
Still no Way To Compare Them



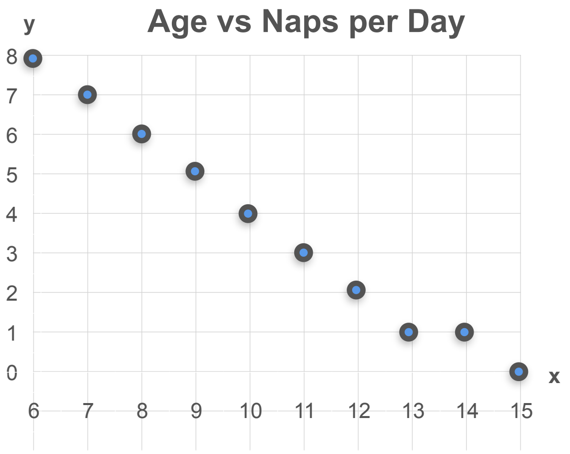
Still no Way To Compare Them



Covariance > 0

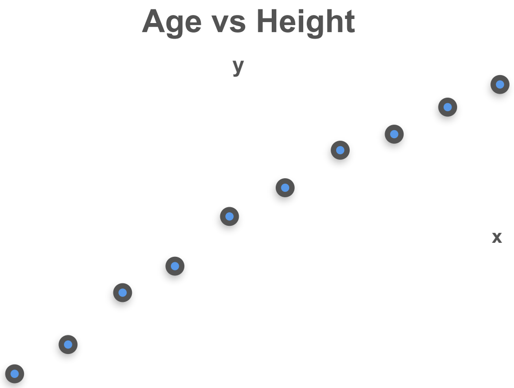


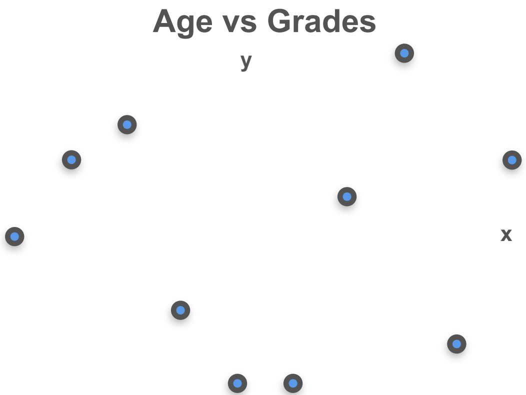
Covariance ≈ 0

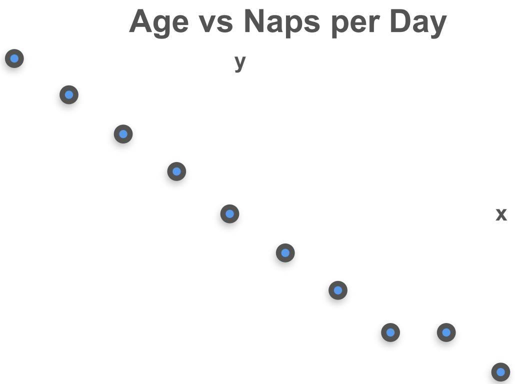


Covariance < 0

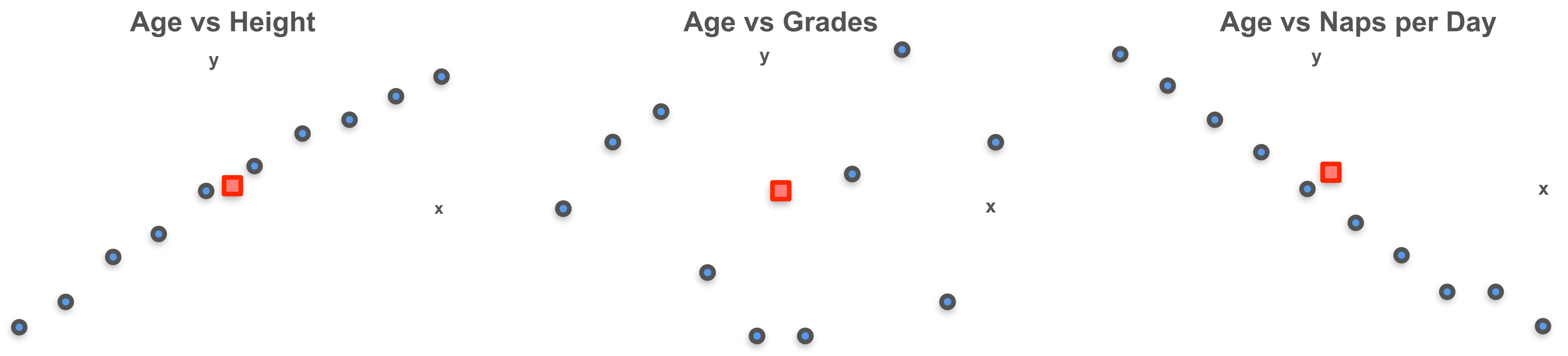
First Step: Center Them



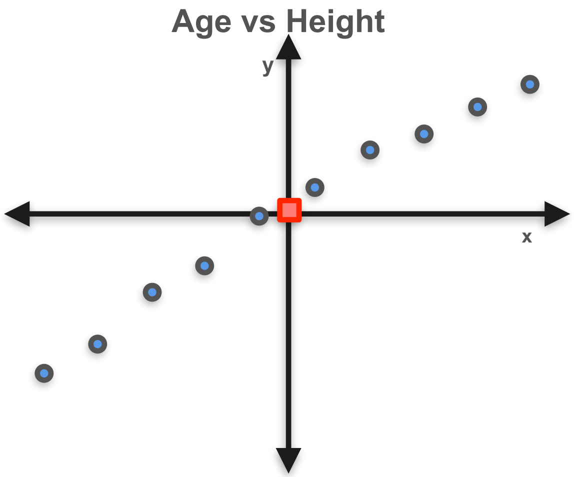


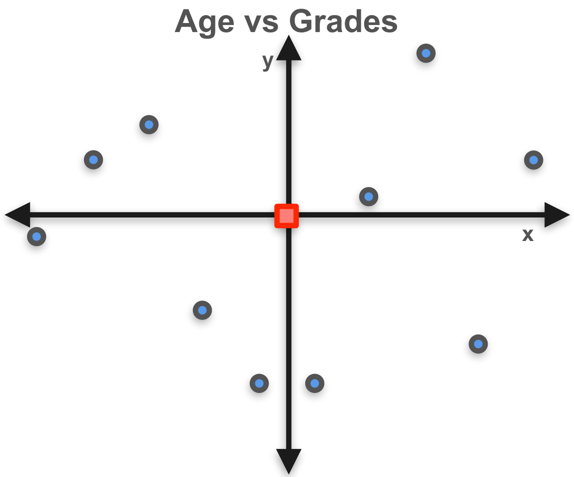


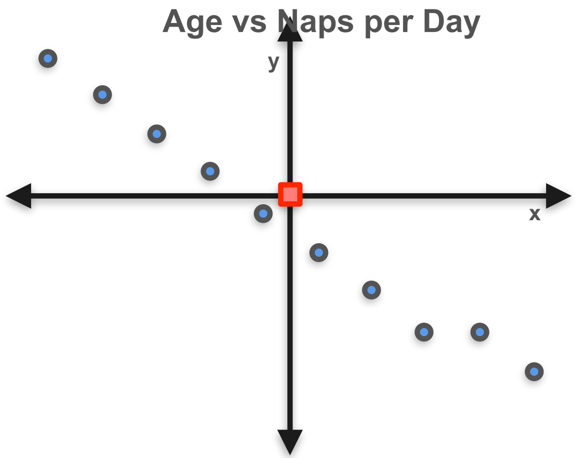
First Step: Center Them



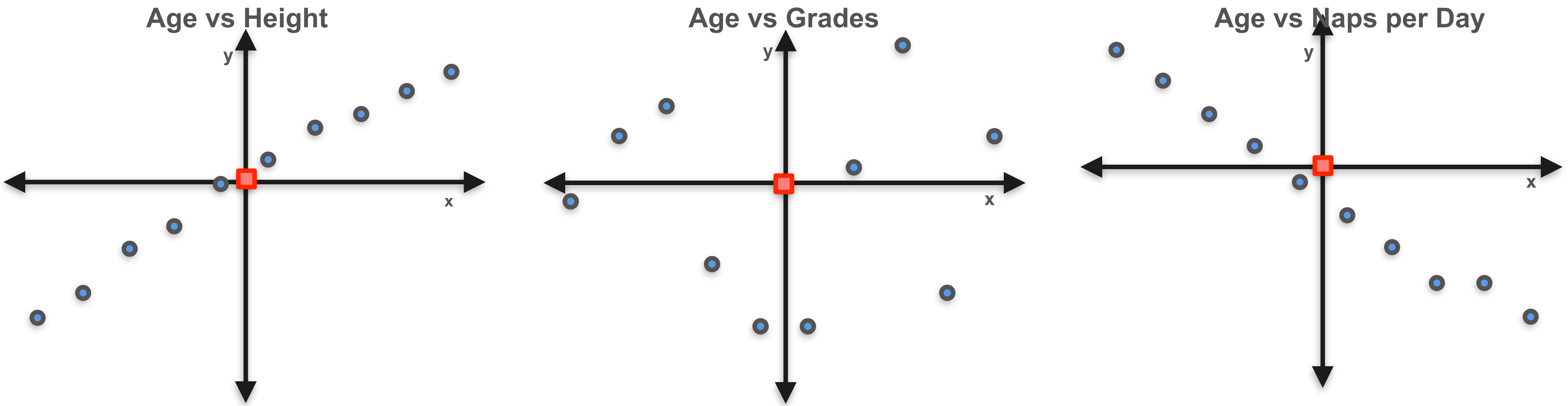
First Step: Center Them







First Step: Center Them

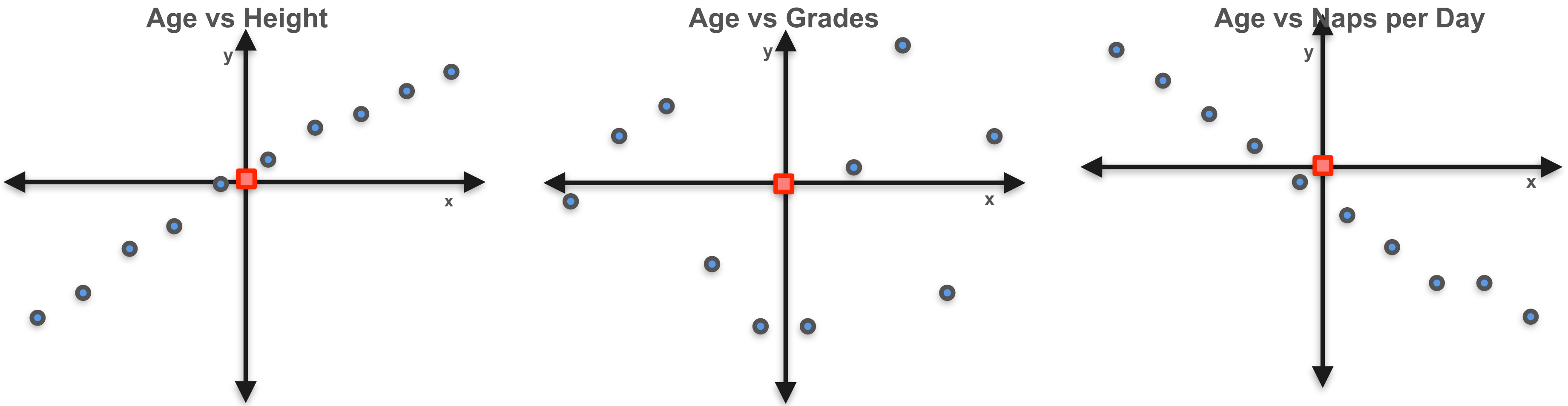


$\mu_x = 0$
$\mu_y = 0$
$Var(X) = 1$
$Var(Y) = 1$

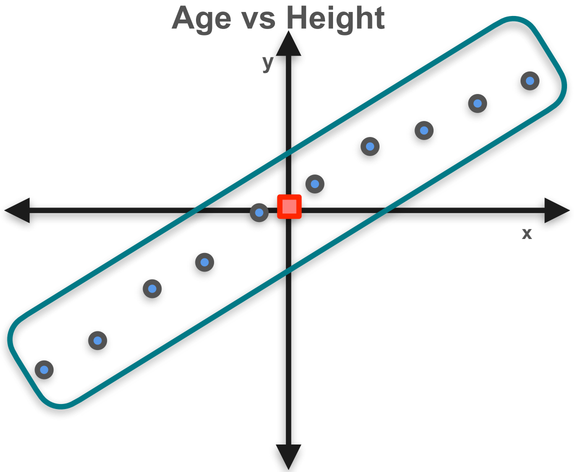
$\mu_x = 0$
$\mu_y = 0$
$Var(X) = 1$
$Var(Y) = 1$

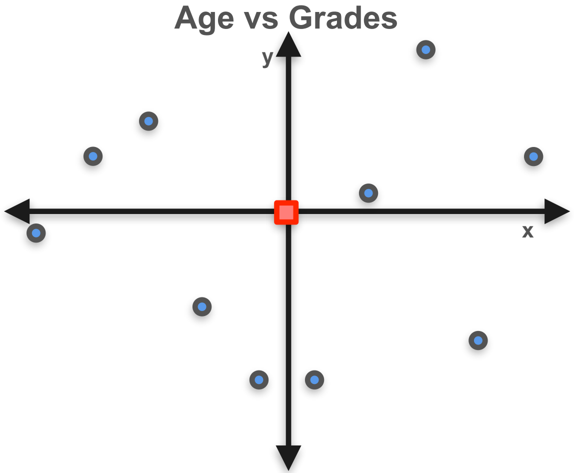
$\mu_x = 0$
$\mu_y = 0$
$Var(X) = 1$
$Var(Y) = 1$

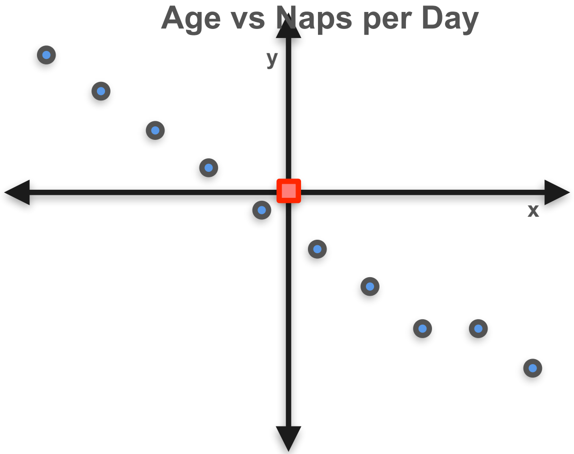
Second Step: Notice Trend



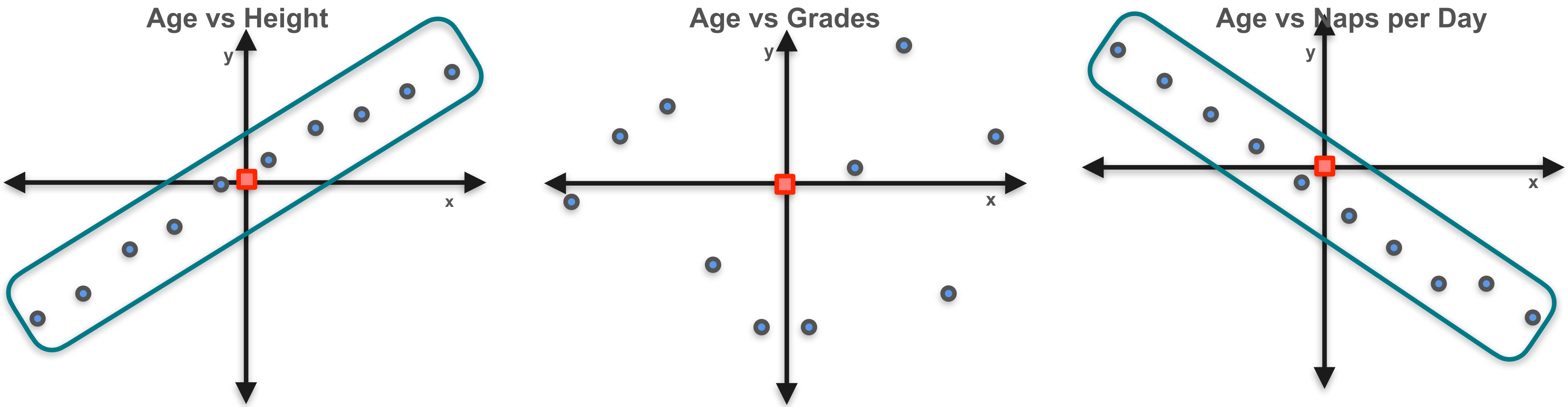
Second Step: Notice Trend



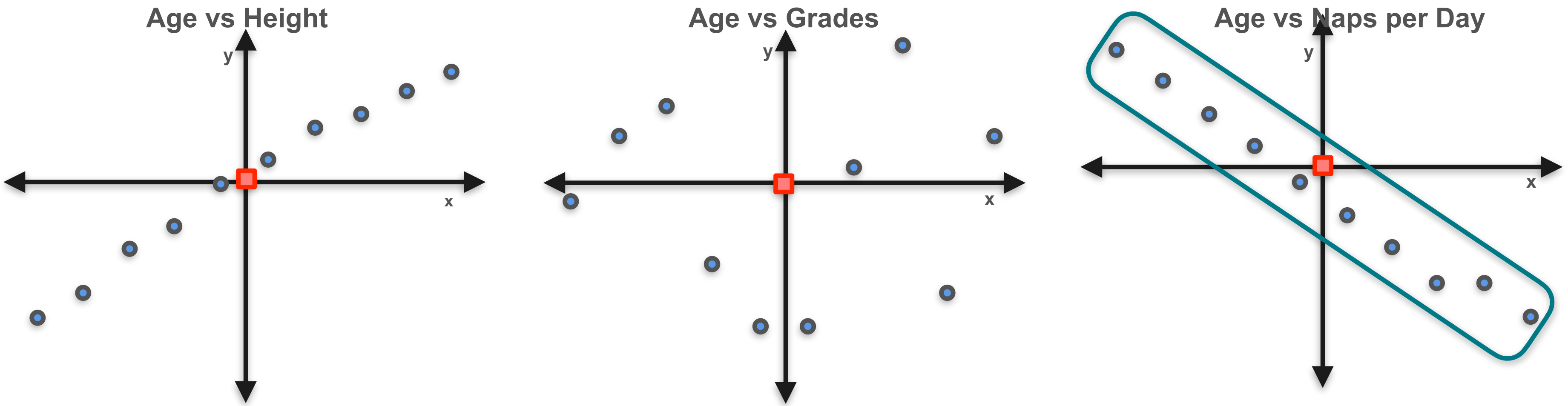




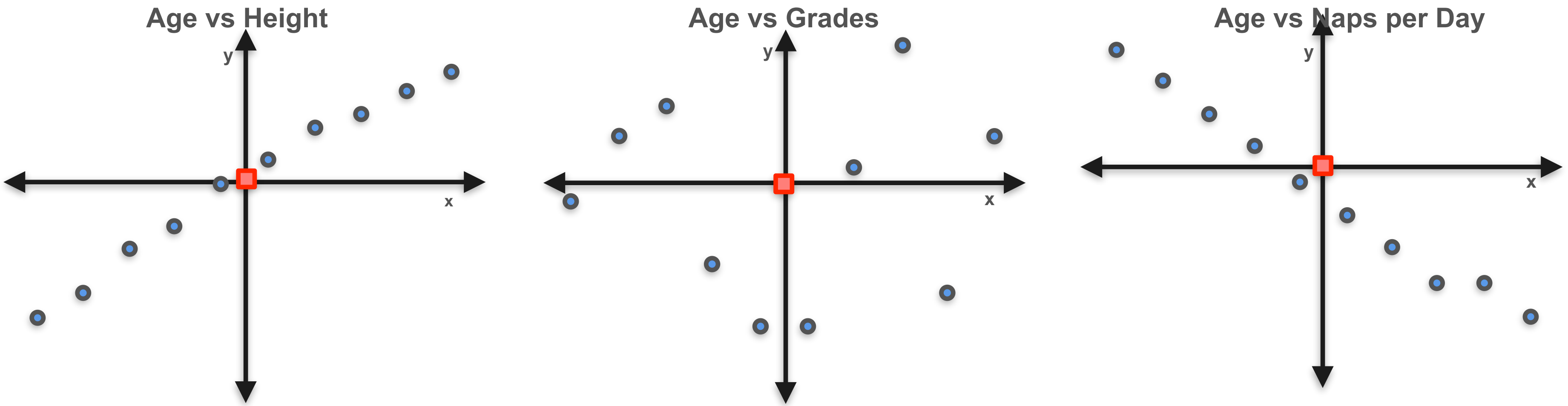
Second Step: Notice Trend



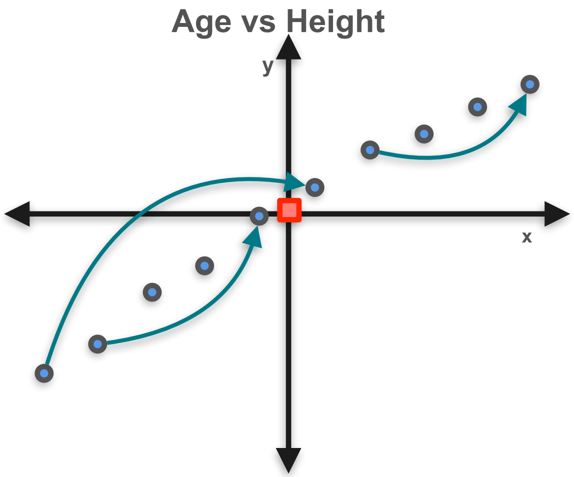
Second Step: Notice Trend

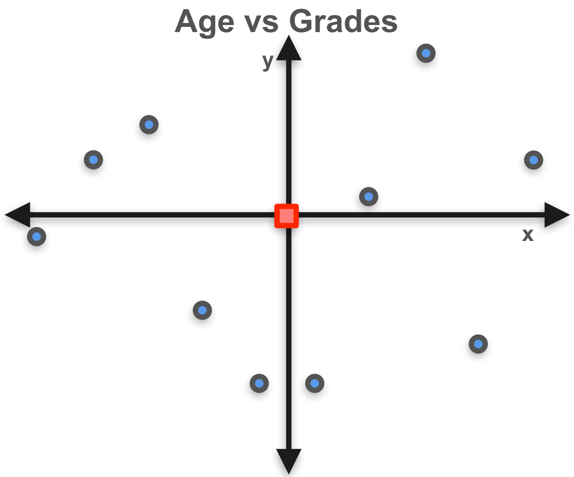


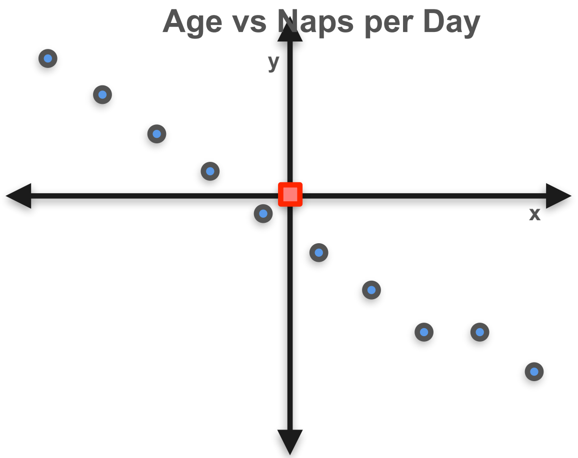
Second Step: Notice Trend



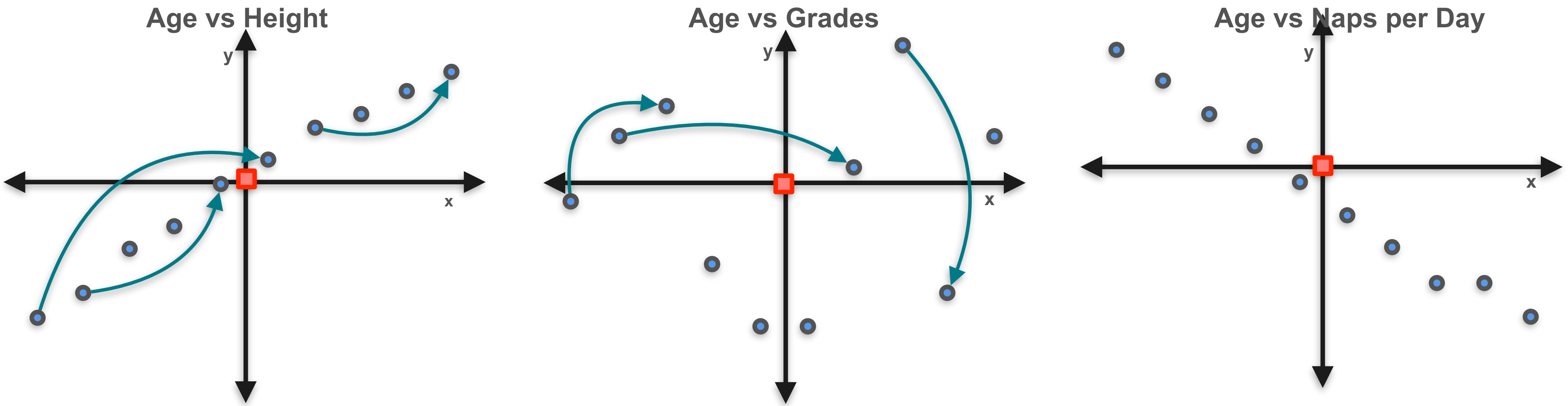
Second Step: Notice Trend



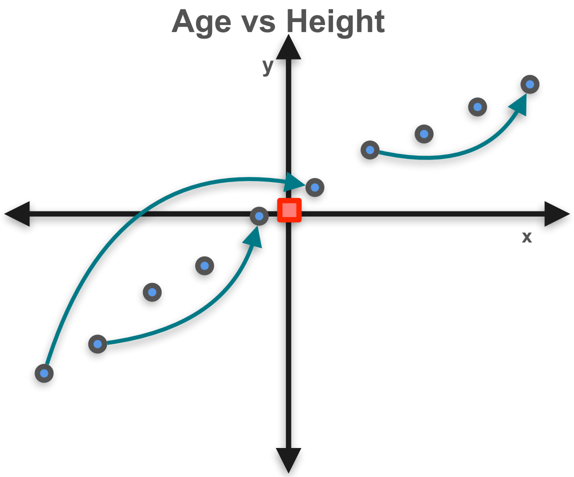


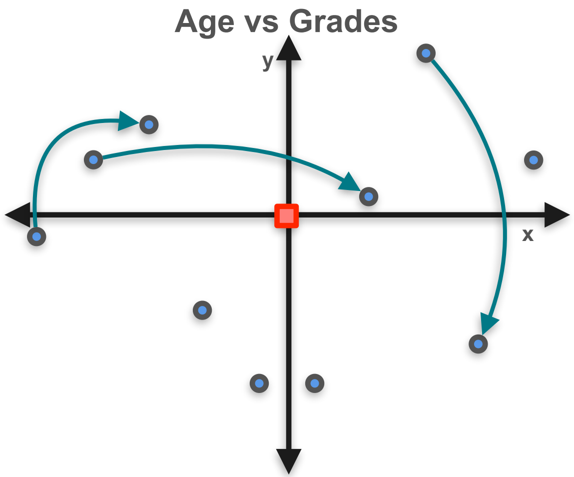


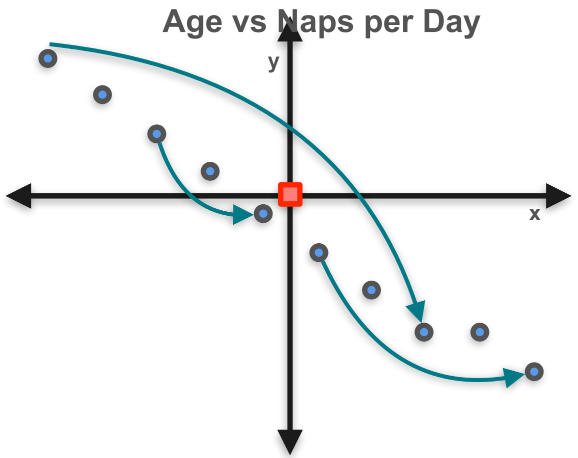
Second Step: Notice Trend



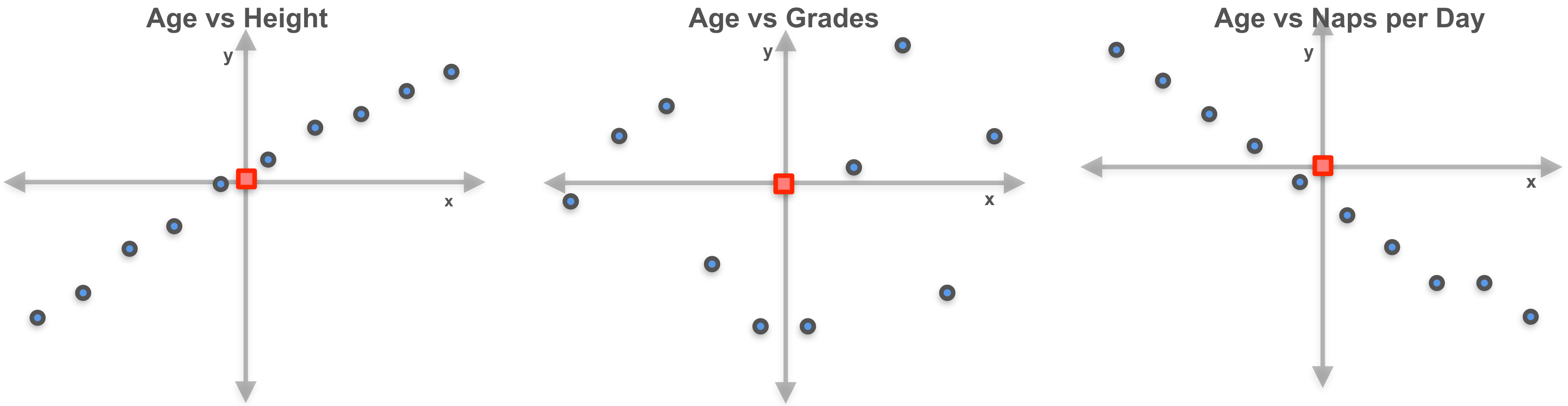
Second Step: Notice Trend



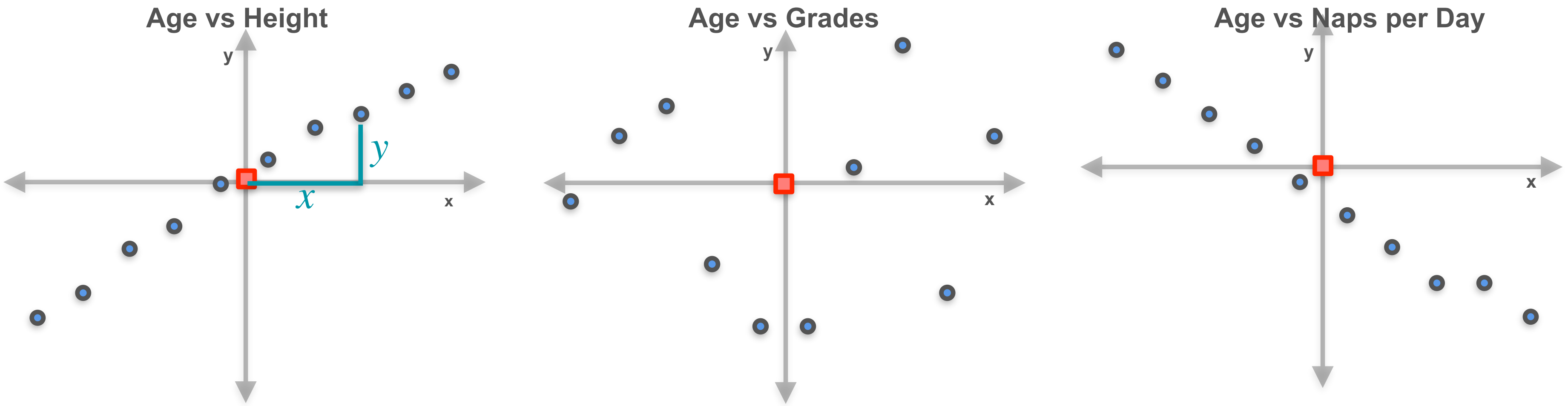




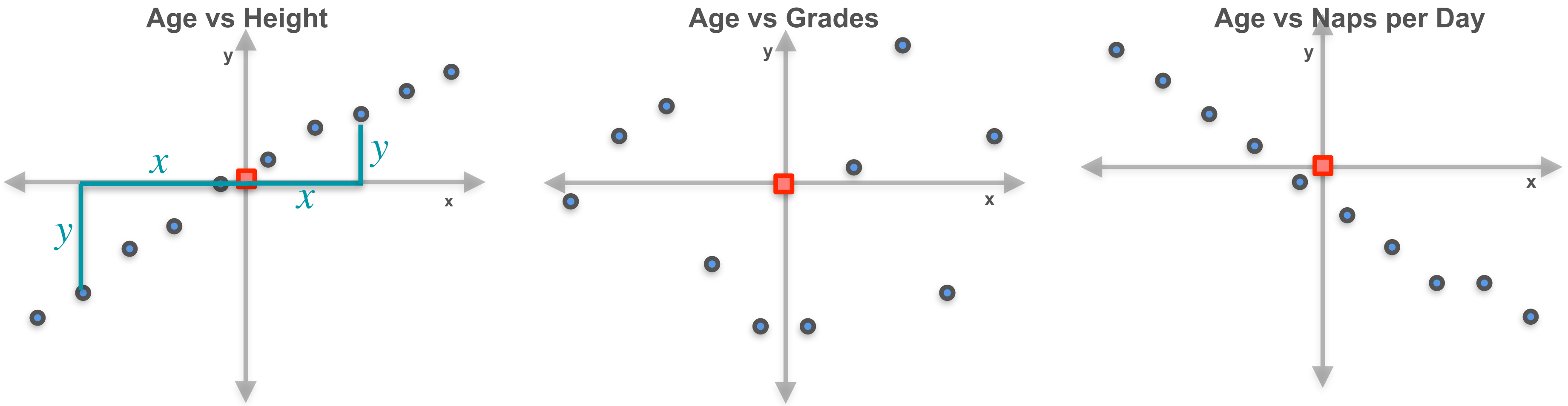
Positives and Negatives



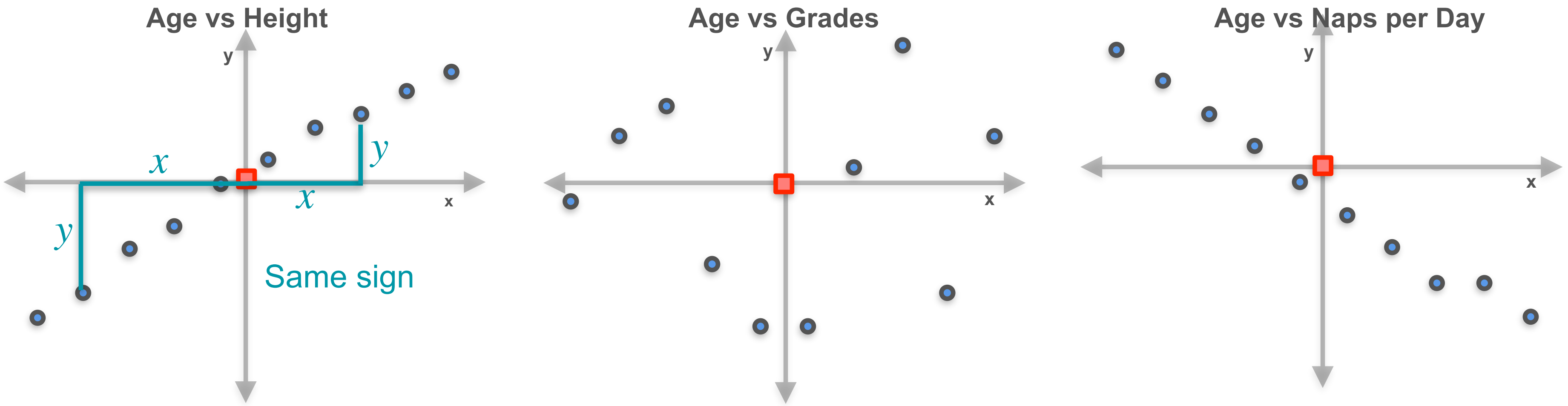
Positives and Negatives



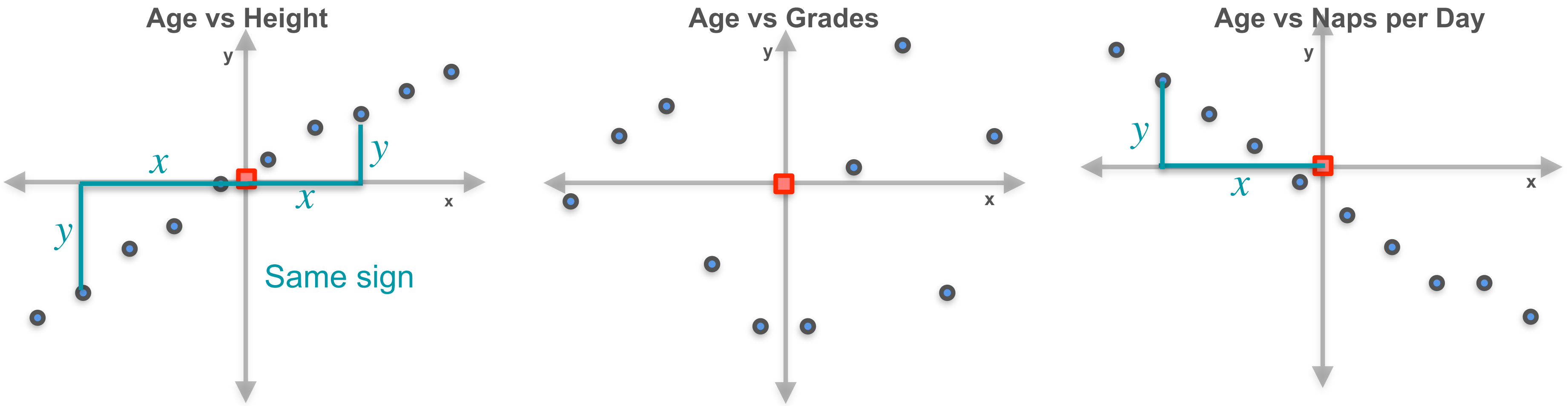
Positives and Negatives



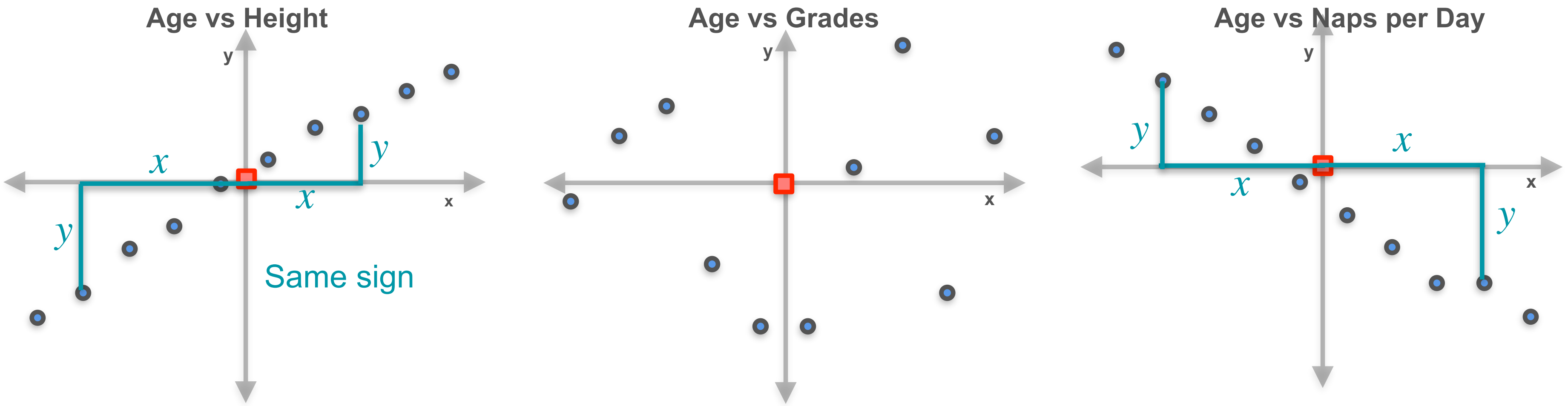
Positives and Negatives



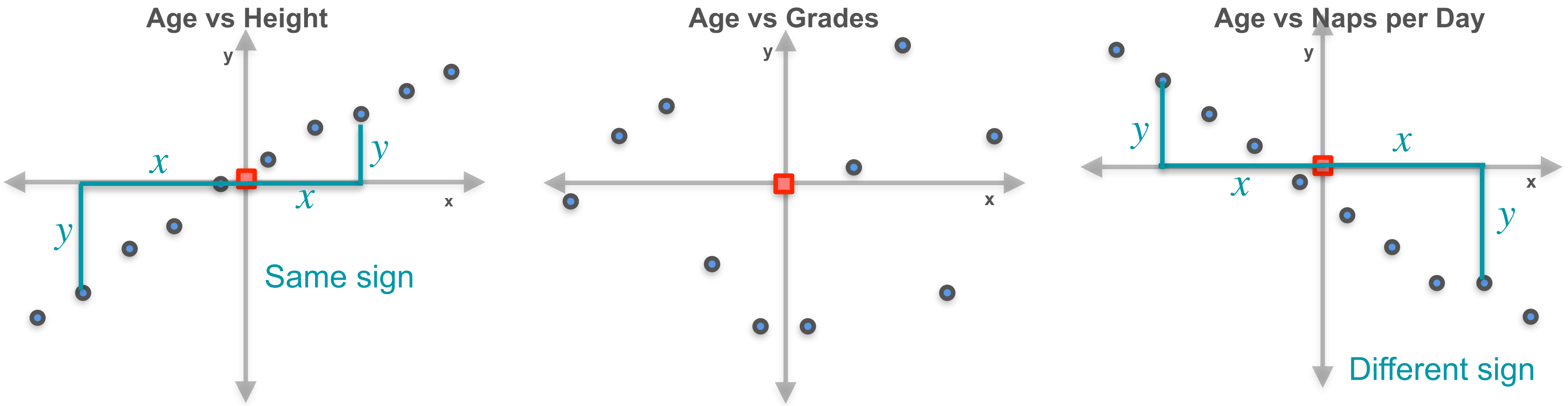
Positives and Negatives



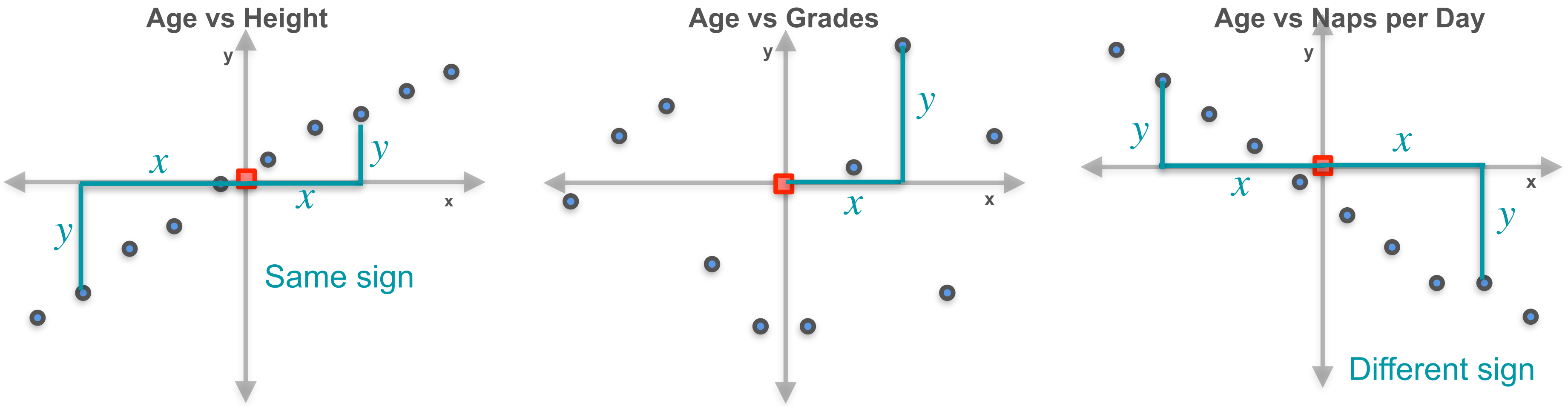
Positives and Negatives



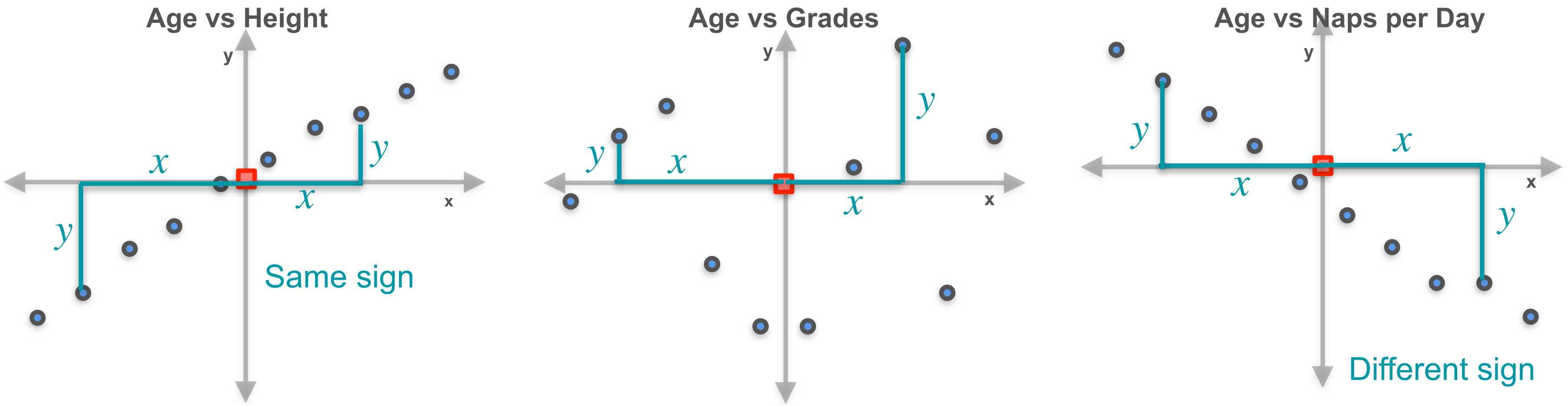
Positives and Negatives



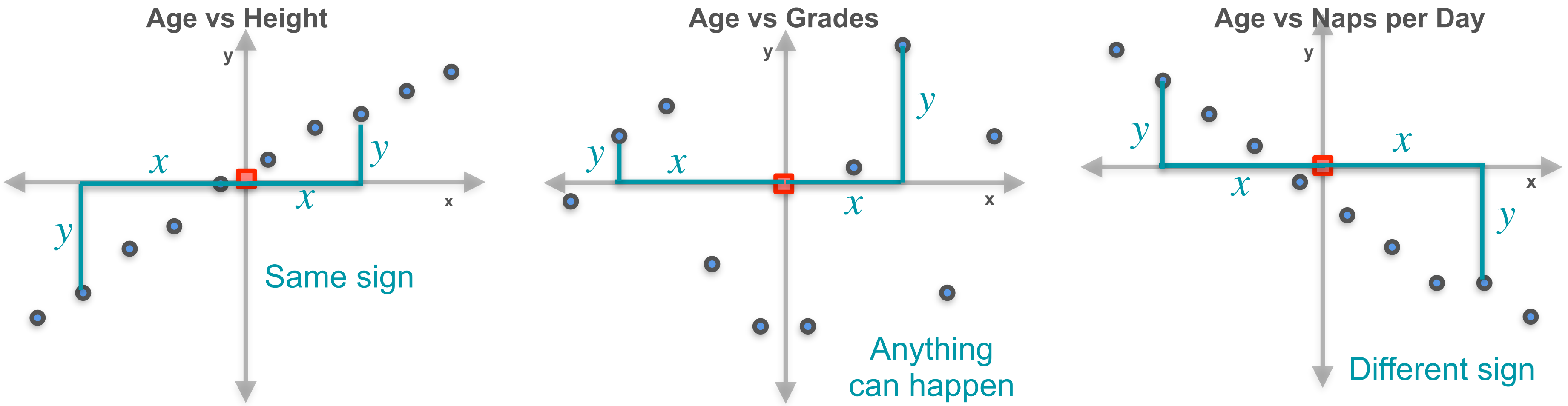
Positives and Negatives



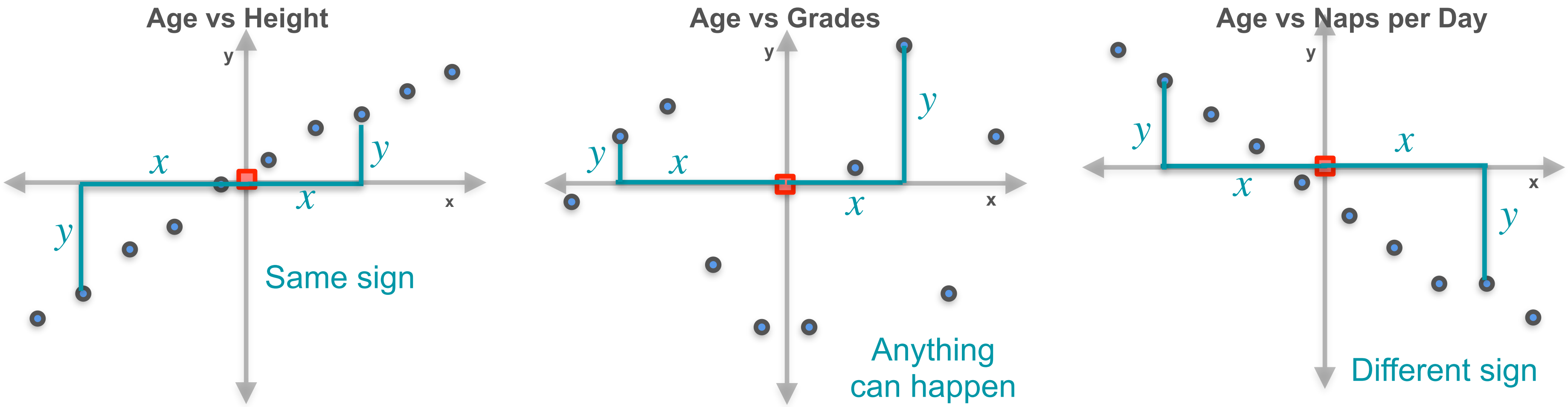
Positives and Negatives



Positives and Negatives



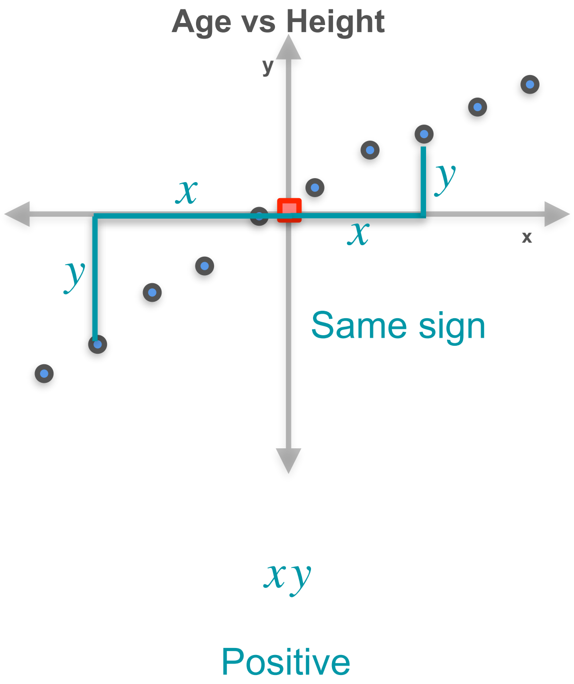
Positives and Negatives

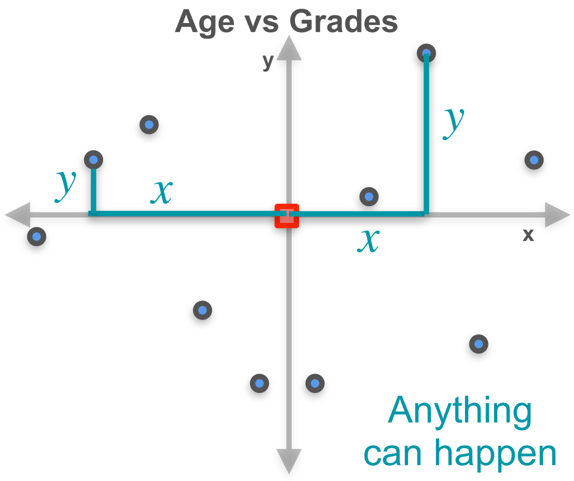


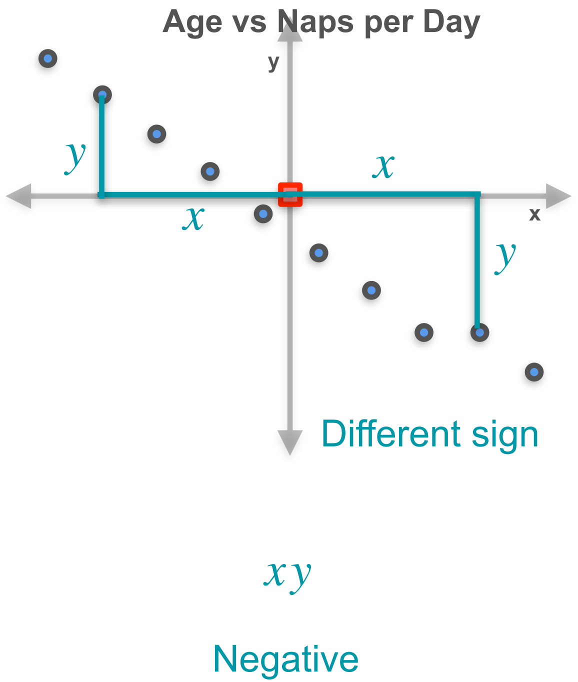
xy

Positive

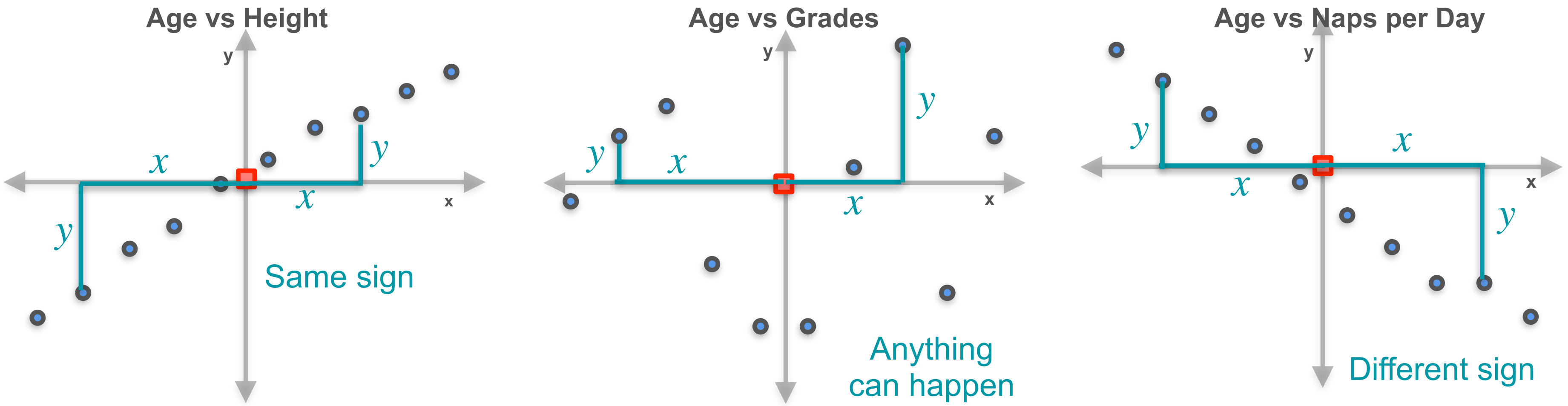
Positives and Negatives







Positives and Negatives



xy

Positive

xy

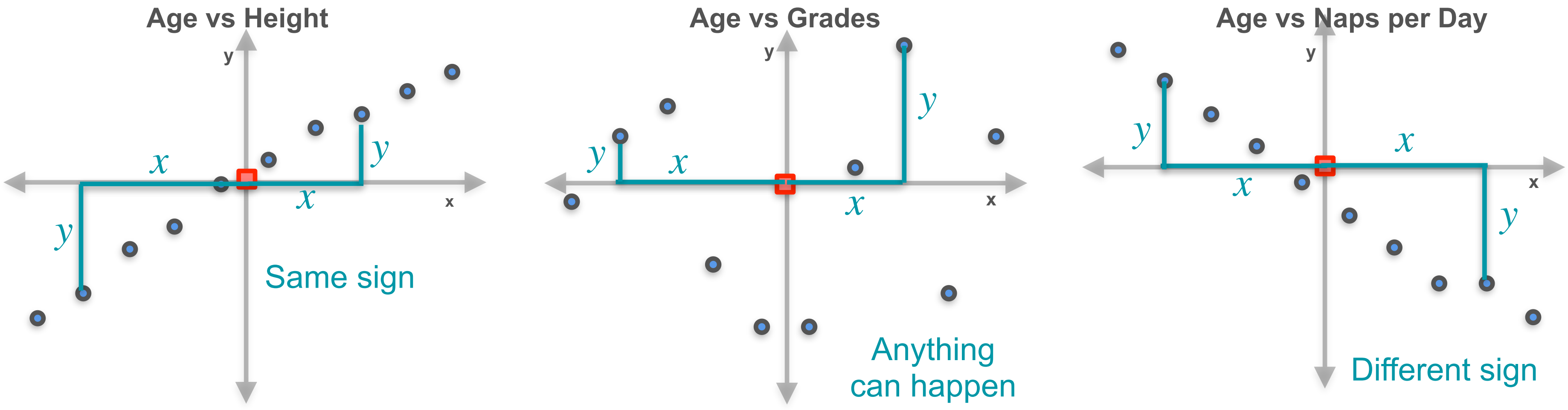
Both positive and negative

xy

Negative

DeepLearning.AI

Positives and Negatives



$\sum xy$

Positive

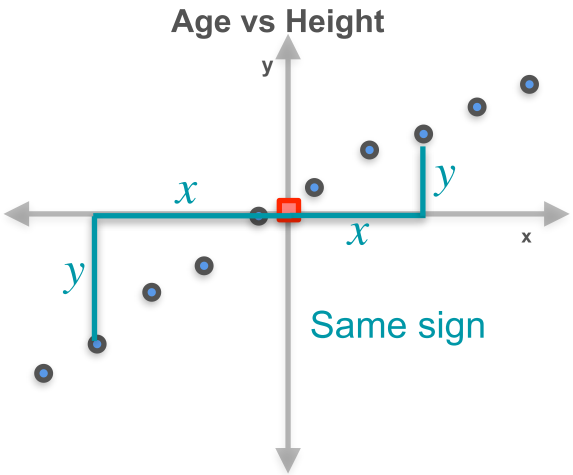
$\sum xy$

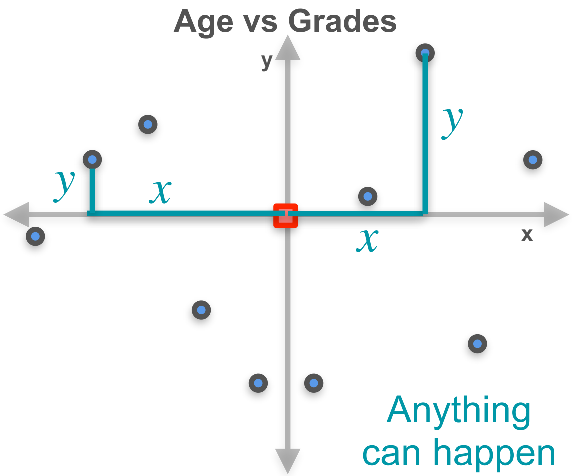
Both positive and negative

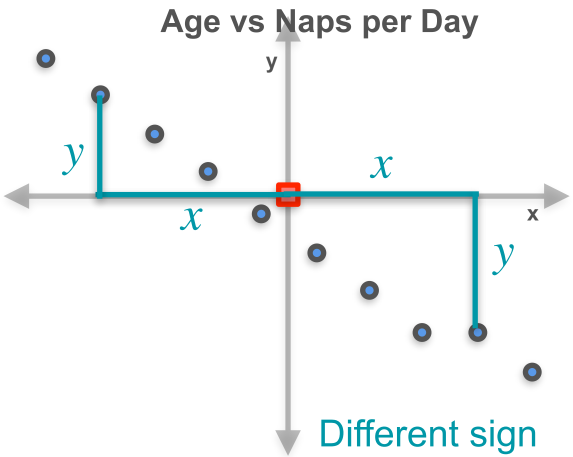
$\sum xy$

Negative

Positives and Negatives







$$\sum xy > 0$$

Positive

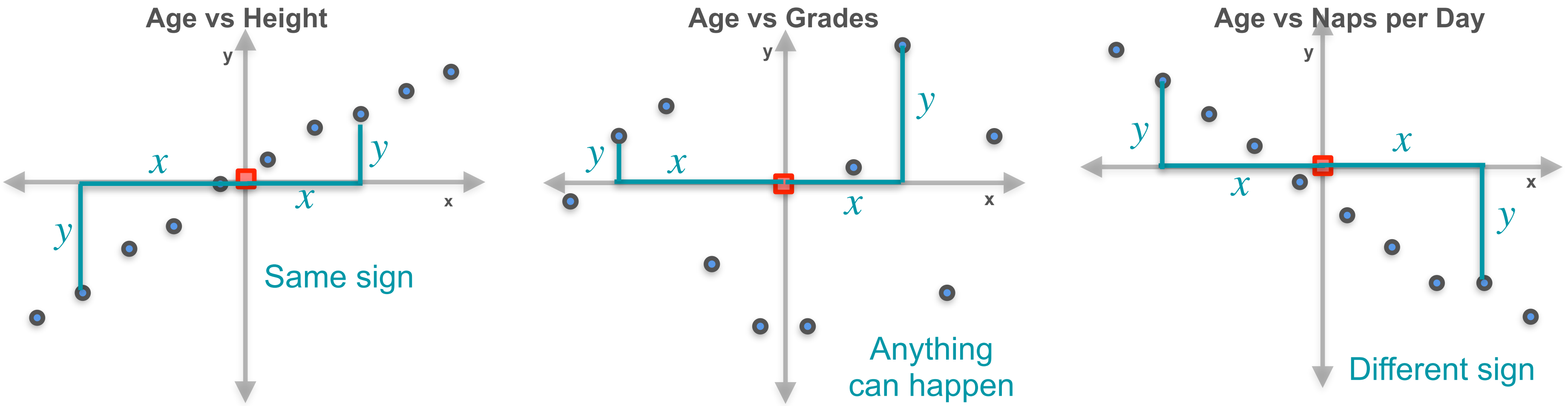
$$\sum xy$$

Both positive and negative

$$\sum xy$$

Negative

Positives and Negatives



$\sum xy > 0$

Positive

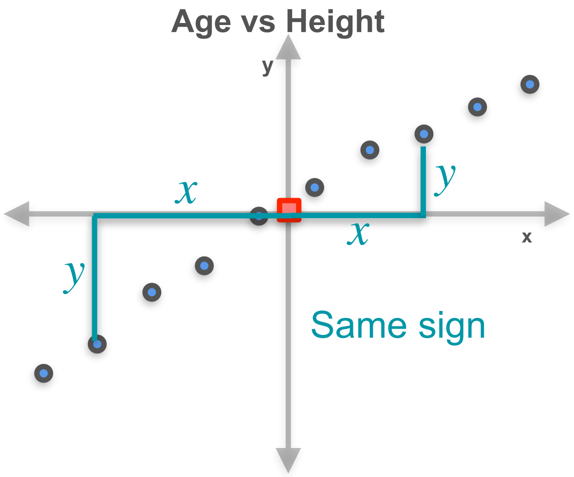
$\sum xy$

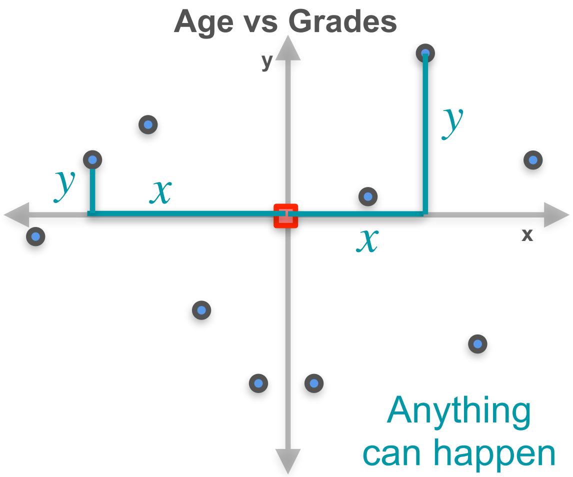
Both positive and negative

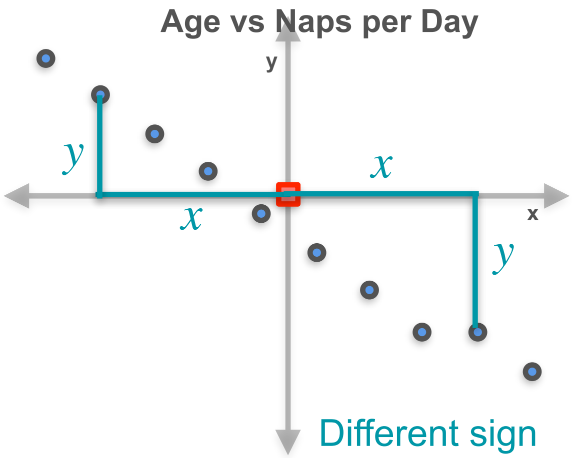
$\sum xy < 0$

Negative

Positives and Negatives







$$\sum xy > 0$$

Positive

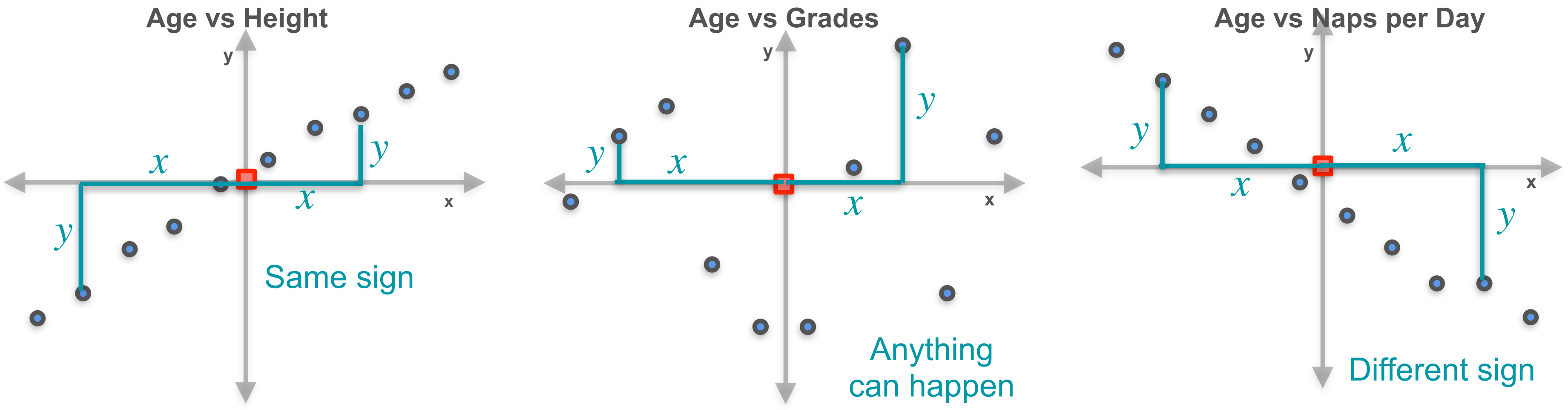
$$\sum xy \approx 0$$

Both positive
and negative

$$\sum xy < 0$$

Negative

Covariance



$$\sum xy > 0$$

Positive

$$\sum xy \approx 0$$

Both positive
and negative

$$\sum xy < 0$$

Negative

Covariance

Covariance

$$Cov(X, Y) = \sum xy$$

Covariance

$$Cov(X, Y) = \sum xy$$

Almost…

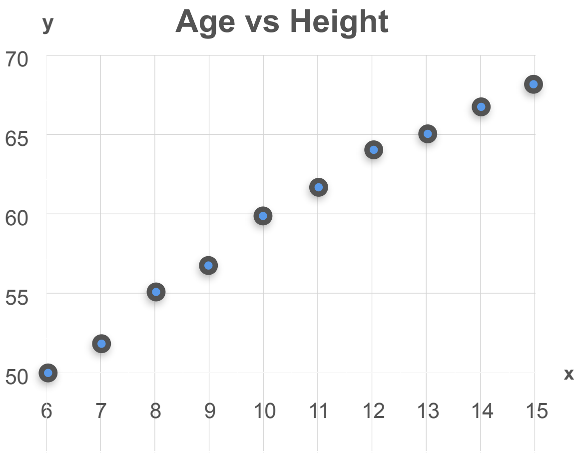
Covariance

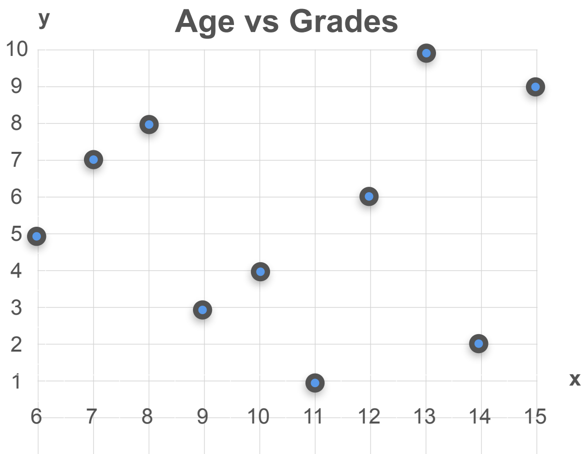
$$Cov(X, Y) = \sum xy$$

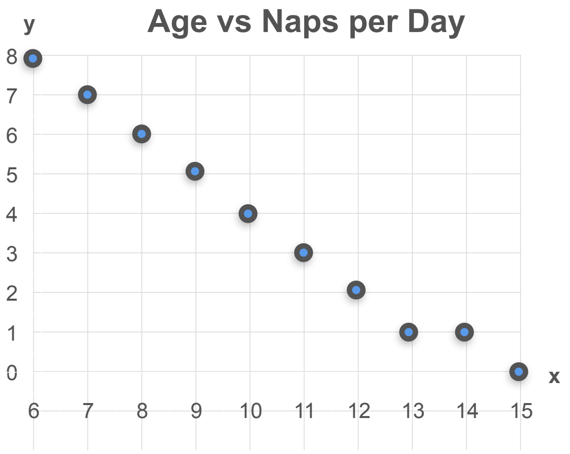
Almost…

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

Covariance







$$Cov(X, Y) > 0$$

$$Cov(X, Y) \approx 0$$

$$Cov(X, Y) < 0$$

Covariance Formula

Covariance Formula

Age vs Height

Covariance Formula

Age vs Height

Covariance > 0

Covariance Formula

Age vs Height

Covariance > 0

$$\mu_x = 10.5$$

Covariance Formula

Age vs Height

Covariance > 0

$\mu_x = 10.5 \quad \mu_y = 60$

Covariance Formula

Age vs Height

Covariance > 0

$\mu_x = 10.5 \quad \mu_y = 60$

Age (x)	Height (y)
6	50
7	52
8	55
9	57
10	60
11	62
12	64
13	65
14	67
15	68

Covariance Formula

Age vs Height

Covariance > 0

$\mu_x = 10.5 \quad \mu_y = 60$

Age (x)	Height (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	50	-4.5	-10	
7	52	-3.5	-8	
8	55	-2.5	-5	
9	57	-1.5	-3	
10	60	-0.5	0	
11	62	0.5	2	
12	64	1.5	4	
13	65	2.5	5	
14	67	3.5	7	
15	68	4.5	8	

Covariance Formula

Age vs Height

Covariance > 0

$\mu_x = 10.5 \quad \mu_y = 60$

Age (x)	Height (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	50	-4.5	-10	45
7	52	-3.5	-8	28
8	55	-2.5	-5	12.5
9	57	-1.5	-3	4.5
10	60	-0.5	0	0
11	62	0.5	2	1
12	64	1.5	4	6
13	65	2.5	5	12.5
14	67	3.5	7	24.5
15	68	4.5	8	36

Covariance Formula

Age vs Height

Covariance > 0

$\mu_x = 10.5 \quad \mu_y = 60$

Age (x)	Height (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	50	-4.5	-10	45
7	52	-3.5	-8	28
8	55	-2.5	-5	12.5
9	57	-1.5	-3	4.5
10	60	-0.5	0	0
11	62	0.5	2	1
12	64	1.5	4	6
13	65	2.5	5	12.5
14	67	3.5	7	24.5
15	68	4.5	8	36

$\sum = 170$

Covariance Formula

Age vs Height

Covariance > 0

$\mu_x = 10.5 \quad \mu_y = 60$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

Age (x)	Height (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	50	-4.5	-10	45
7	52	-3.5	-8	28
8	55	-2.5	-5	12.5
9	57	-1.5	-3	4.5
10	60	-0.5	0	0
11	62	0.5	2	1
12	64	1.5	4	6
13	65	2.5	5	12.5
14	67	3.5	7	24.5
15	68	4.5	8	36

$\sum = 170$

Covariance Formula

Age vs Height

Covariance > 0

$\mu_x = 10.5 \quad \mu_y = 60$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

$$Cov(X, Y) = \frac{170}{10}$$

Age (x)	Height (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	50	-4.5	-10	45
7	52	-3.5	-8	28
8	55	-2.5	-5	12.5
9	57	-1.5	-3	4.5
10	60	-0.5	0	0
11	62	0.5	2	1
12	64	1.5	4	6
13	65	2.5	5	12.5
14	67	3.5	7	24.5
15	68	4.5	8	36

$\sum = 170$

Covariance Formula

Age vs Height

Covariance > 0

$\mu_x = 10.5 \quad \mu_y = 60$

$$Cov(X,Y) = \frac{\sum(x_i - \mu_x)(y_i - \mu_y)}{n}$$

$$Cov(X,Y) = \frac{170}{10} = 17$$

Age (x)	Height (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	50	-4.5	-10	45
7	52	-3.5	-8	28
8	55	-2.5	-5	12.5
9	57	-1.5	-3	4.5
10	60	-0.5	0	0
11	62	0.5	2	1
12	64	1.5	4	6
13	65	2.5	5	12.5
14	67	3.5	7	24.5
15	68	4.5	8	36

$\sum = 170$

Covariance Formula

Age vs Height

Covariance > 0

$\mu_x = 10.5 \quad \mu_y = 60$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

$$Cov(X, Y) = \frac{170}{10} = 17 > 0$$

Age (x)	Height (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	50	-4.5	-10	45
7	52	-3.5	-8	28
8	55	-2.5	-5	12.5
9	57	-1.5	-3	4.5
10	60	-0.5	0	0
11	62	0.5	2	1
12	64	1.5	4	6
13	65	2.5	5	12.5
14	67	3.5	7	24.5
15	68	4.5	8	36

$\sum = 170$

Covariance Formula

Covariance Formula

Age vs Naps per Day

Covariance Formula

Age vs Naps per Day

Covariance < 0

Covariance Formula

Age vs Naps per Day

Covariance < 0

$$\mu_x = 10.5$$

Covariance Formula

Age vs Naps per Day

Covariance < 0

$\mu_x = 10.5 \quad \mu_y = 3.7$

Covariance Formula

Age vs Naps per Day

Covariance < 0

$\mu_x = 10.5 \quad \mu_y = 3.7$

Age (x)
6
7
8
9
10
11
12
13
14
15

Covariance Formula

Age vs Naps per Day

Covariance < 0

$\mu_x = 10.5 \qquad \mu_y = 3.7$

Age (x)	Naps (y)
6	8
7	7
8	6
9	5
10	4
11	3
12	2
13	1
14	1
15	0

Covariance Formula

Age vs Naps per Day

Covariance < 0

$\mu_x = 10.5 \quad \mu_y = 3.7$

Age (x)	Naps (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	8	-4.5	4.3	-19.35
7	7	-3.5	3.3	-11.55
8	6	-2.5	2.3	-5.75
9	5	-1.5	1.3	-1.95
10	4	-0.5	0.3	-0.15
11	3	0.5	-0.7	-0..35
12	2	1.5	-1.7	-2.55
13	1	2.5	-2.7	-6.75
14	1	3.5	-2.7	-9.45
15	0	4.5	-3.7	-16.65

Covariance Formula

Age vs Naps per Day

Covariance < 0

$\mu_x = 10.5 \quad \mu_y = 3.7$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

Age (x)	Naps (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	8	-4.5	4.3	-19.35
7	7	-3.5	3.3	-11.55
8	6	-2.5	2.3	-5.75
9	5	-1.5	1.3	-1.95
10	4	-0.5	0.3	-0.15
11	3	0.5	-0.7	-0..35
12	2	1.5	-1.7	-2.55
13	1	2.5	-2.7	-6.75
14	1	3.5	-2.7	-9.45
15	0	4.5	-3.7	-16.65

$\sum$ = -74.5

Covariance Formula

Age vs Naps per Day

Covariance < 0

$\mu_x = 10.5 \quad \mu_y = 3.7$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

$$Cov(X, Y) = \frac{-74.5}{10}$$

Age (x)	Naps (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	8	-4.5	4.3	-19.35
7	7	-3.5	3.3	-11.55
8	6	-2.5	2.3	-5.75
9	5	-1.5	1.3	-1.95
10	4	-0.5	0.3	-0.15
11	3	0.5	-0.7	-0..35
12	2	1.5	-1.7	-2.55
13	1	2.5	-2.7	-6.75
14	1	3.5	-2.7	-9.45
15	0	4.5	-3.7	-16.65

$\sum$ = -74.5

Covariance Formula

Age vs Naps per Day

Covariance < 0

$\mu_x = 10.5 \quad \mu_y = 3.7$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

$$Cov(X, Y) = \frac{-74.5}{10} = -7.45$$

Age (x)	Naps (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	8	-4.5	4.3	-19.35
7	7	-3.5	3.3	-11.55
8	6	-2.5	2.3	-5.75
9	5	-1.5	1.3	-1.95
10	4	-0.5	0.3	-0.15
11	3	0.5	-0.7	-0..35
12	2	1.5	-1.7	-2.55
13	1	2.5	-2.7	-6.75
14	1	3.5	-2.7	-9.45
15	0	4.5	-3.7	-16.65

$\sum$ = -74.5

Covariance Formula

Age vs Naps per Day

Covariance < 0

$\mu_x = 10.5 \quad \mu_y = 3.7$

$$Cov(X,Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

$$Cov(X,Y) = \frac{-74.5}{10} = -7.45 \; < 0$$

Age (x)	Naps (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	8	-4.5	4.3	-19.35
7	7	-3.5	3.3	-11.55
8	6	-2.5	2.3	-5.75
9	5	-1.5	1.3	-1.95
10	4	-0.5	0.3	-0.15
11	3	0.5	-0.7	-0..35
12	2	1.5	-1.7	-2.55
13	1	2.5	-2.7	-6.75
14	1	3.5	-2.7	-9.45
15	0	4.5	-3.7	-16.65

$\sum$ = -74.5

Covariance Formula

$$(x_i - \mu_x)$$

Covariance Formula

Age vs Grades

Covariance ≈ 0

$\mu_x = 10.5 \quad \mu_y = 5$

Age (x)	Grades (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	5	-4.5	0	0
7	7	-3.5	2	-7
8	8	-2.5	3	-7.5
9	3	-1.5	-2	3
10	1	-0.5	-4	2
11	1	0.5	-4	-2
12	6	1.5	1	1.5
13	10	2.5	5	12.5
14	2	3.5	-3	-10.5
15	7	4.5	2	9

Covariance Formula

Age vs Grades

Covariance ≈ 0

$\mu_x = 10.5 \quad \mu_y = 5$

Age (x)	Grades (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	5	-4.5	0	0
7	7	-3.5	2	-7
8	8	-2.5	3	-7.5
9	3	-1.5	-2	3
10	1	-0.5	-4	2
11	1	0.5	-4	-2
12	6	1.5	1	1.5
13	10	2.5	5	12.5
14	2	3.5	-3	-10.5
15	7	4.5	2	9

$\sum = 1$

DeepLearning.AI

Covariance Formula

Age vs Grades

Covariance ≈ 0

$\mu_x = 10.5 \quad \mu_y = 5$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

Age (x)	Grades (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	5	-4.5	0	0
7	7	-3.5	2	-7
8	8	-2.5	3	-7.5
9	3	-1.5	-2	3
10	1	-0.5	-4	2
11	1	0.5	-4	-2
12	6	1.5	1	1.5
13	10	2.5	5	12.5
14	2	3.5	-3	-10.5
15	7	4.5	2	9

$\sum = 1$

Covariance Formula

Age vs Grades

Covariance ≈ 0

$\mu_x = 10.5 \quad \mu_y = 5$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

$$Cov(X, Y) = \frac{1}{10}$$

Age (x)	Grades (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	5	-4.5	0	0
7	7	-3.5	2	-7
8	8	-2.5	3	-7.5
9	3	-1.5	-2	3
10	1	-0.5	-4	2
11	1	0.5	-4	-2
12	6	1.5	1	1.5
13	10	2.5	5	12.5
14	2	3.5	-3	-10.5
15	7	4.5	2	9

$\sum = 1$

Covariance Formula

Age vs Grades

Covariance ≈ 0

$\mu_x = 10.5 \quad \mu_y = 5$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

$$Cov(X, Y) = \frac{1}{10} = 0.1$$

Age (x)	Grades (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	5	-4.5	0	0
7	7	-3.5	2	-7
8	8	-2.5	3	-7.5
9	3	-1.5	-2	3
10	1	-0.5	-4	2
11	1	0.5	-4	-2
12	6	1.5	1	1.5
13	10	2.5	5	12.5
14	2	3.5	-3	-10.5
15	7	4.5	2	9

$\sum = 1$

Covariance Formula

Age vs Grades

Covariance ≈ 0

$\mu_x = 10.5 \quad \mu_y = 5$

$$Cov(X,Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

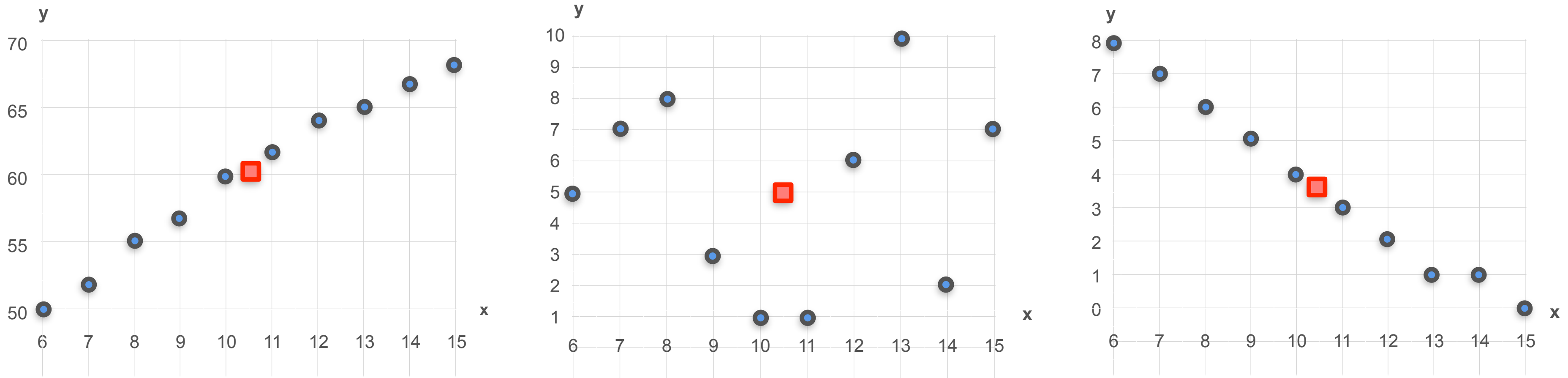
$$Cov(X,Y) = \frac{1}{10} = 0.1 \quad \approx 0$$

Age (x)	Grades (y)	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
6	5	-4.5	0	0
7	7	-3.5	2	-7
8	8	-2.5	3	-7.5
9	3	-1.5	-2	3
10	1	-0.5	-4	2
11	1	0.5	-4	-2
12	6	1.5	1	1.5
13	10	2.5	5	12.5
14	2	3.5	-3	-10.5
15	7	4.5	2	9

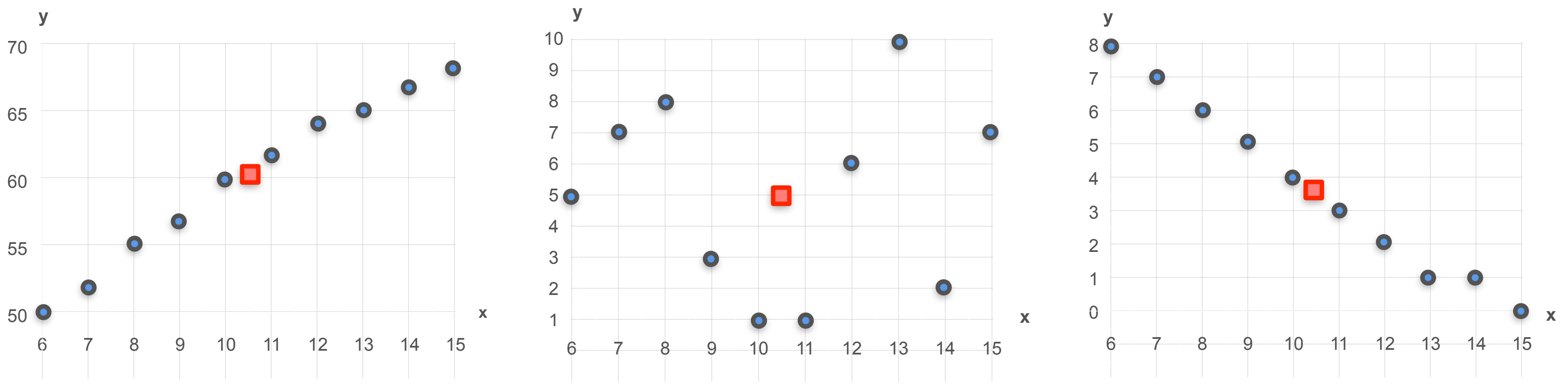
$\sum = 1$

Comparing Correlations

Comparing Correlations



Comparing Correlations

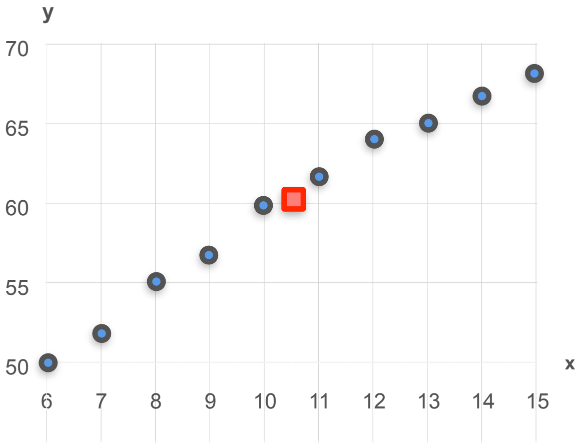


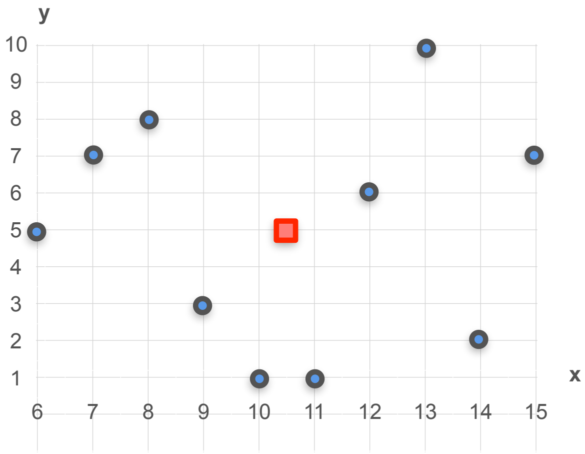
Age vs Height

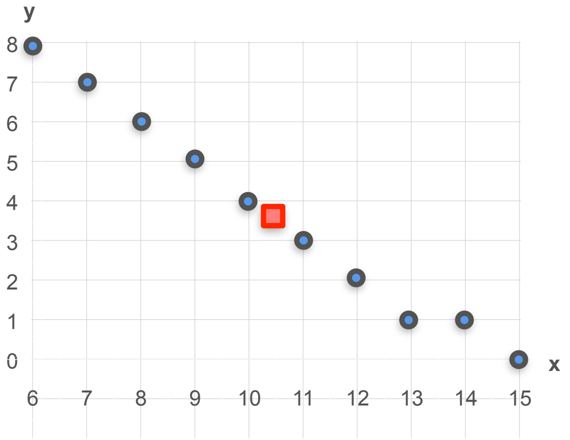
Covariance > 0

$$Cov(x, y) = 17$$

Comparing Correlations







Age vs Height

Covariance > 0

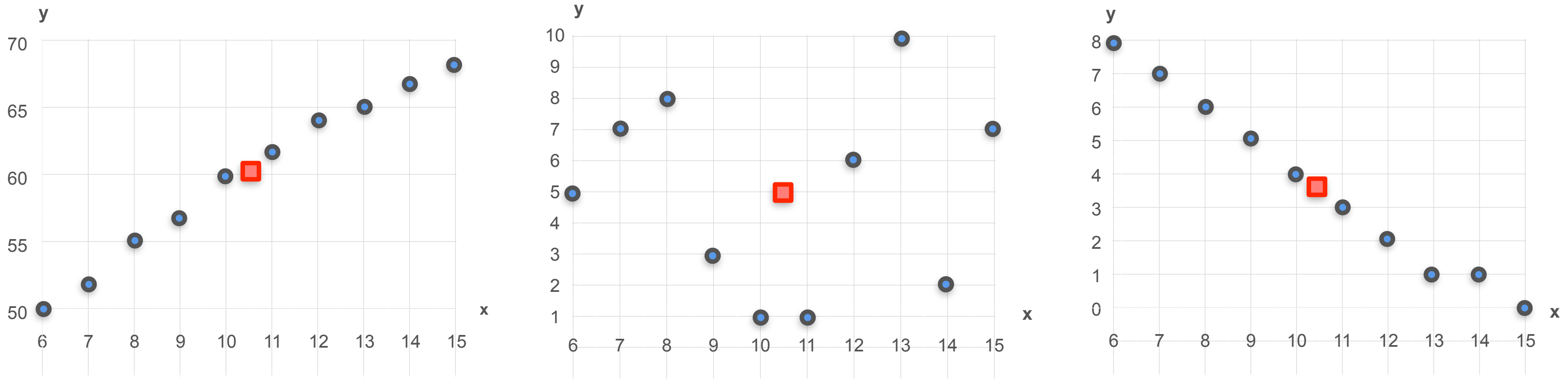
$$Cov(x, y) = 17$$

Age vs Grades

Covariance $\approx$ 0

$$Cov(x, y) = 0.1$$

Comparing Correlations



Age vs Height

Covariance > 0

$$Cov(x, y) = 17$$

Age vs Grades

Covariance ≈ 0

$$Cov(x, y) = 0.1$$

Age vs Naps per Day

Covariance < 0

$$Cov(x, y) = -7.45$$

Covariance of a Probability Distribution: Motivation

Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1 **GAME 2** **GAME 3**

Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win \$1 each

GAME 2

GAME 3

Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1	GAME 2	GAME 3
a: Both players win $1 each b: Both players lose $1 each		

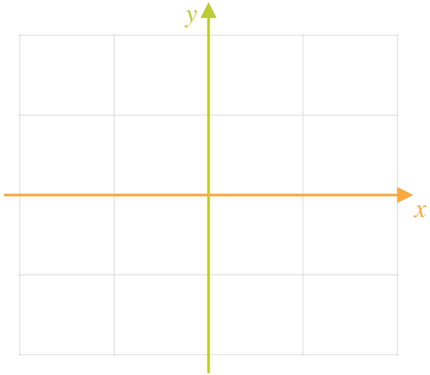
Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win \$1 each

b: Both players lose \$1 each



GAME 2

GAME 3

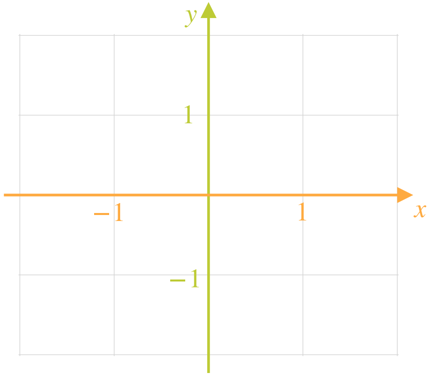
Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win \$1 each

b: Both players lose \$1 each



GAME 2

GAME 3

Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

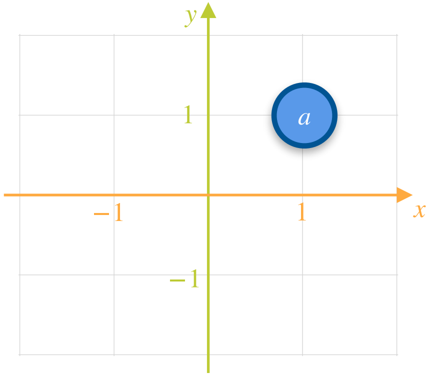
GAME 1

GAME 2

GAME 3

a: Both players win $1 each

b: Both players lose $1 each



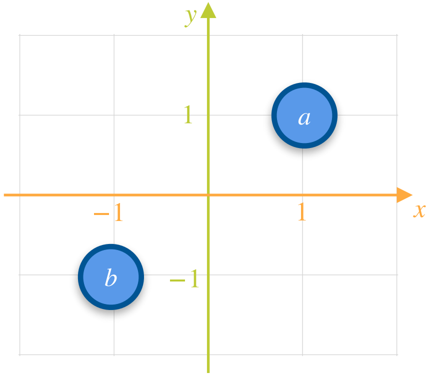
Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win \$1 each

b: Both players lose \$1 each



GAME 2

GAME 3

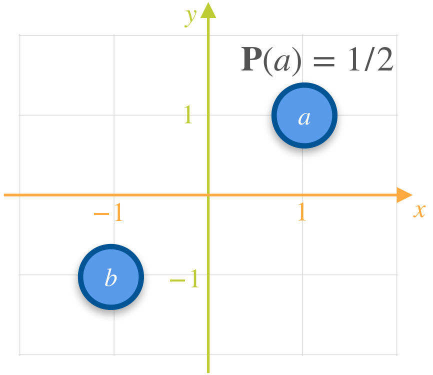
Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win \$1 each

b: Both players lose \$1 each



GAME 2

GAME 3

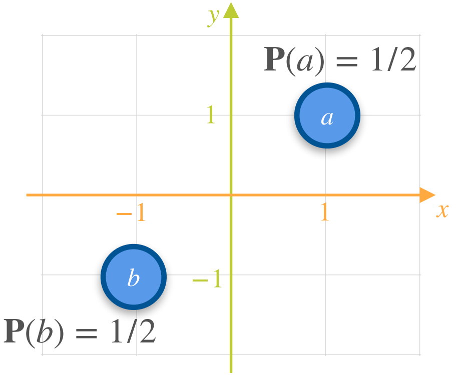
Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win \$1 each

b: Both players lose \$1 each



GAME 2

GAME 3

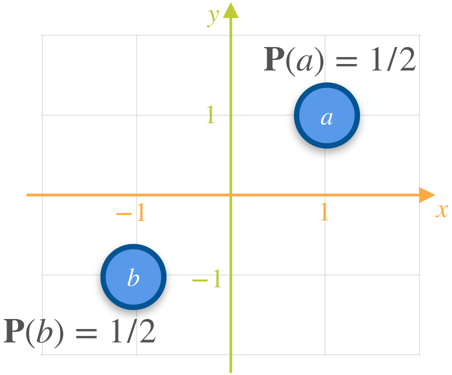
Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win \$1 each

b: Both players lose \$1 each



GAME 2

c: X wins \$1 and Y loses \$1

GAME 3

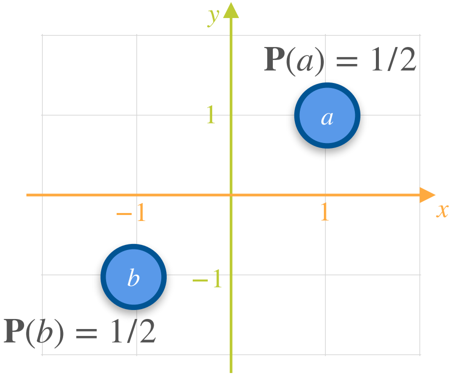
Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win \$1 each

b: Both players lose \$1 each



GAME 2

c: X wins \$1 and Y loses \$1

d: X loses \$1 and Y win \$1

GAME 3

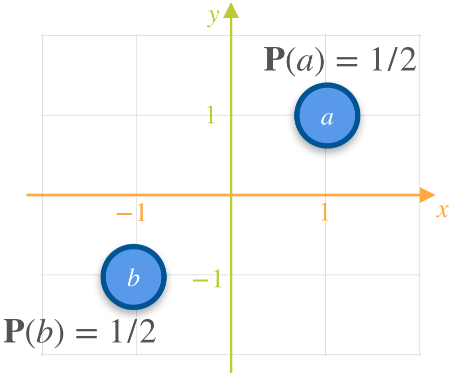
Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win \$1 each

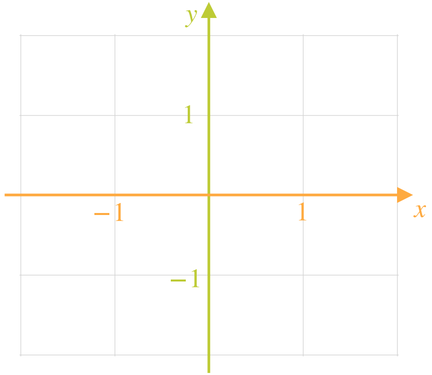
b: Both players lose \$1 each



GAME 2

c: X wins \$1 and Y loses \$1

d: X loses \$1 and Y win \$1



GAME 3

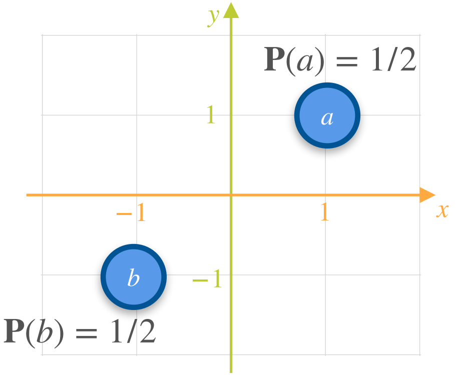
Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

GAME 1

a: Both players win $1 each

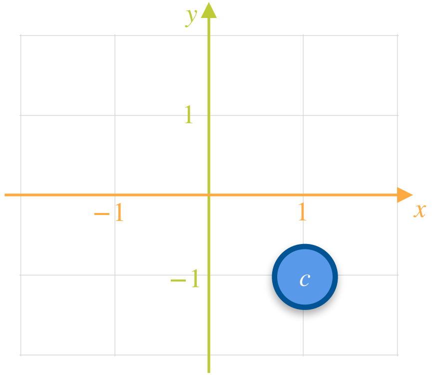
b: Both players lose $1 each



GAME 2

c: X wins $1 and Y loses $1

d: X loses $1 and Y win $1



GAME 3

DeepLearning.AI

Covariance of a Probability Distribution: Motivation

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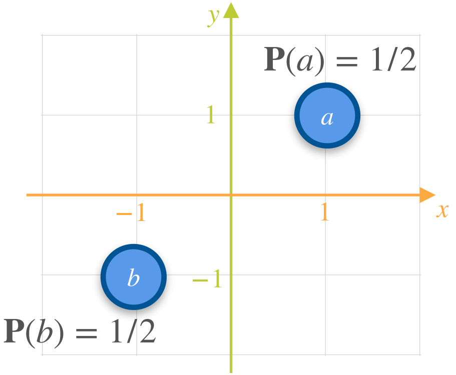

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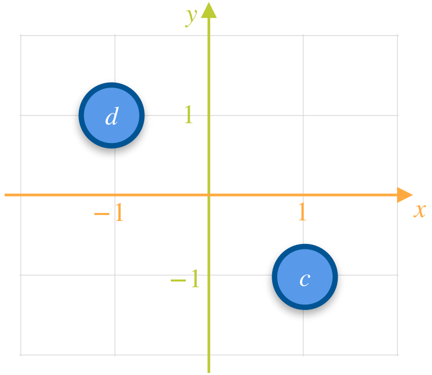

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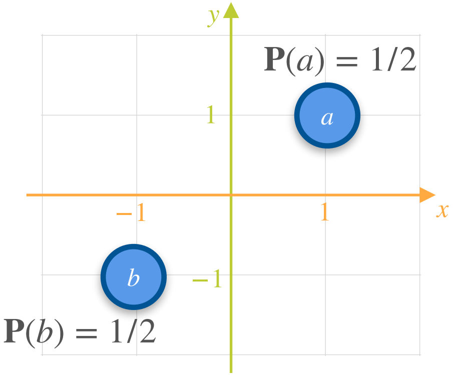

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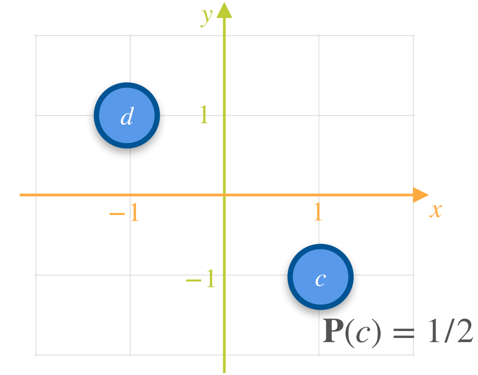

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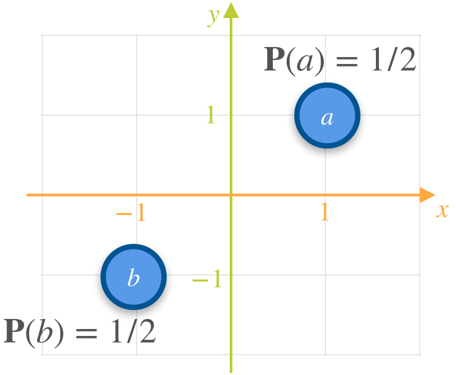

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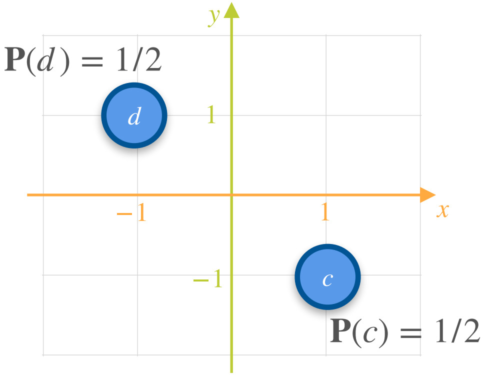

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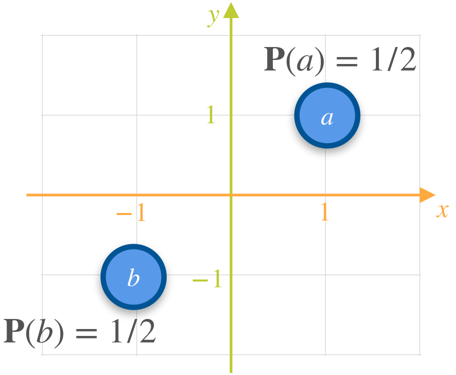

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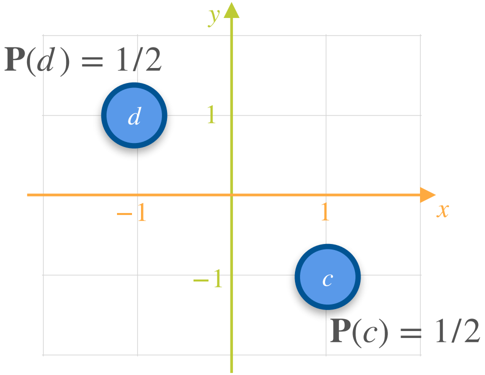

GAME 3

a, b, c or d

Covariance of a Probability Distribution: Motivation

X and Y are playing 3 games to either win or lose a dollar

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a: Both players win \$1 each

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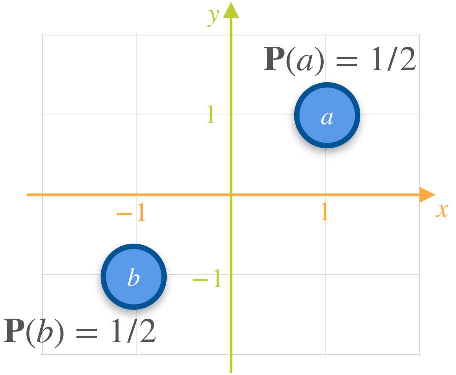

GAME 2

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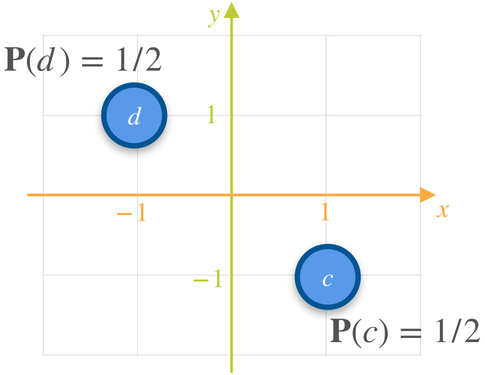

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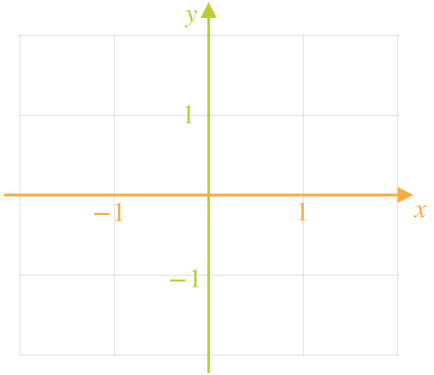

DeepLearning.AI

Covariance of a Probability Distribution: Motivation

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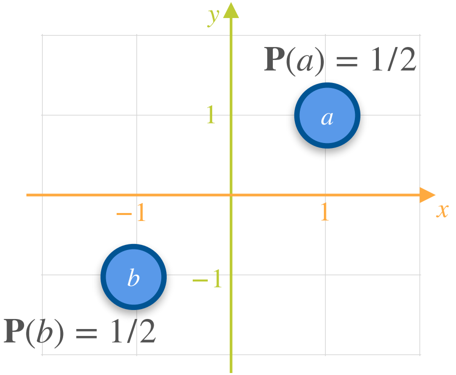

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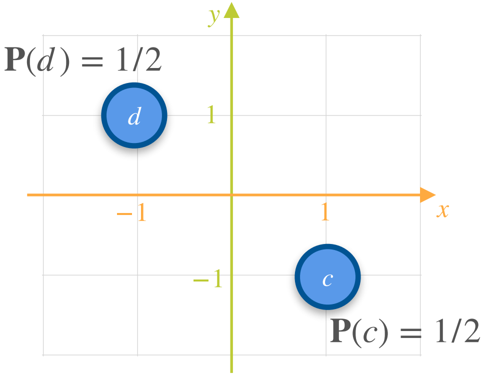
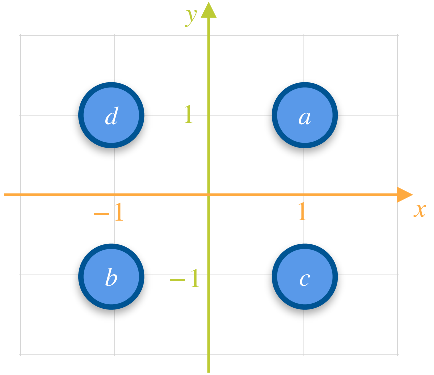


GAME 3

a, b, c or d



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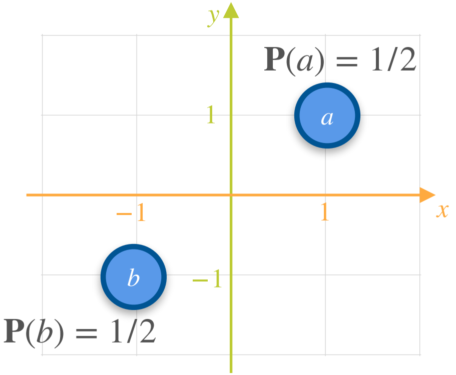
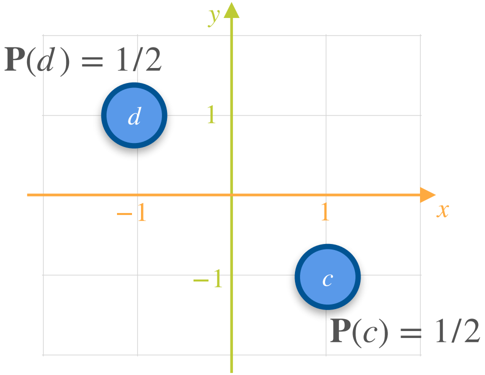


GAME 2

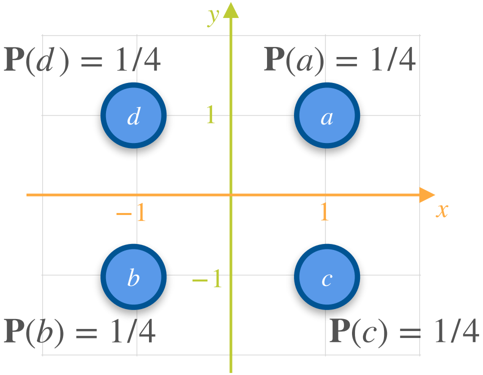
c: X wins \$1 and Y loses \$1

d: X loses \$1 and Y win \$1



GAME 3

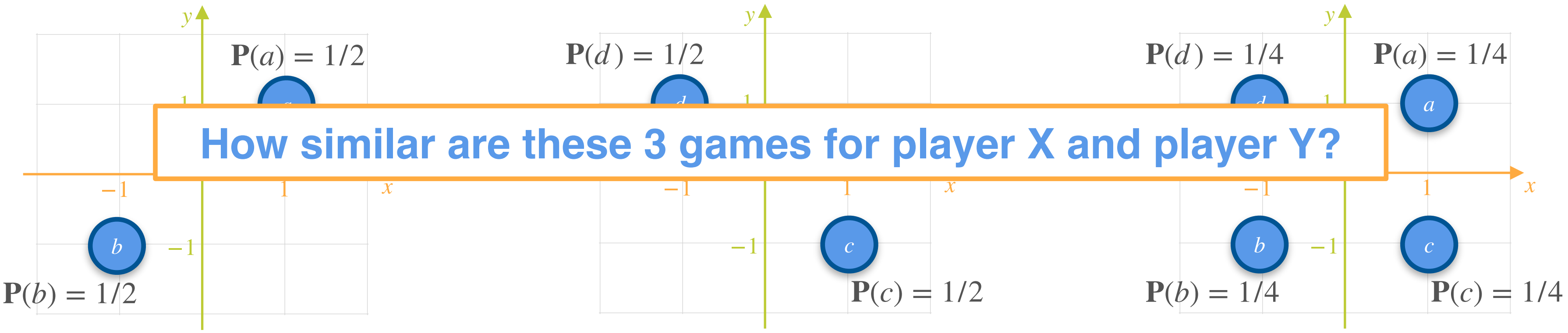
a, b, c or d



Covariance of a Probability Distribution: Motivation

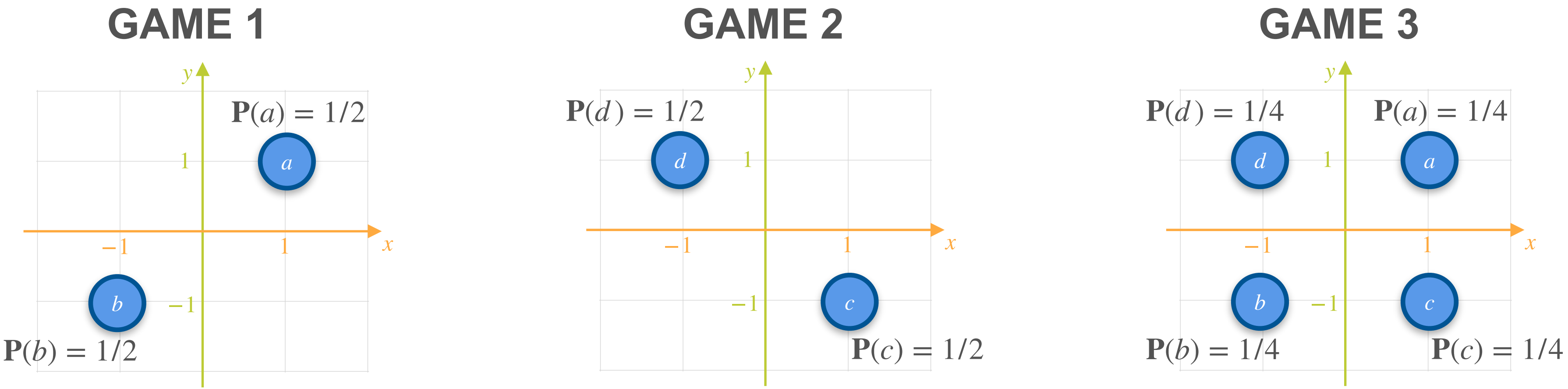
X and Y are playing 3 games to either win or lose a dollar

GAME 1	GAME 2	GAME 3
a: Both players win \$1 each	c: X wins \$1 and Y loses \$1	a, b, c or d
b: Both players lose \$1 each	d: X loses \$1 and Y win \$1	



How similar are these 3 games for player X and player Y?

Covariance of a Probability Distribution: Motivation



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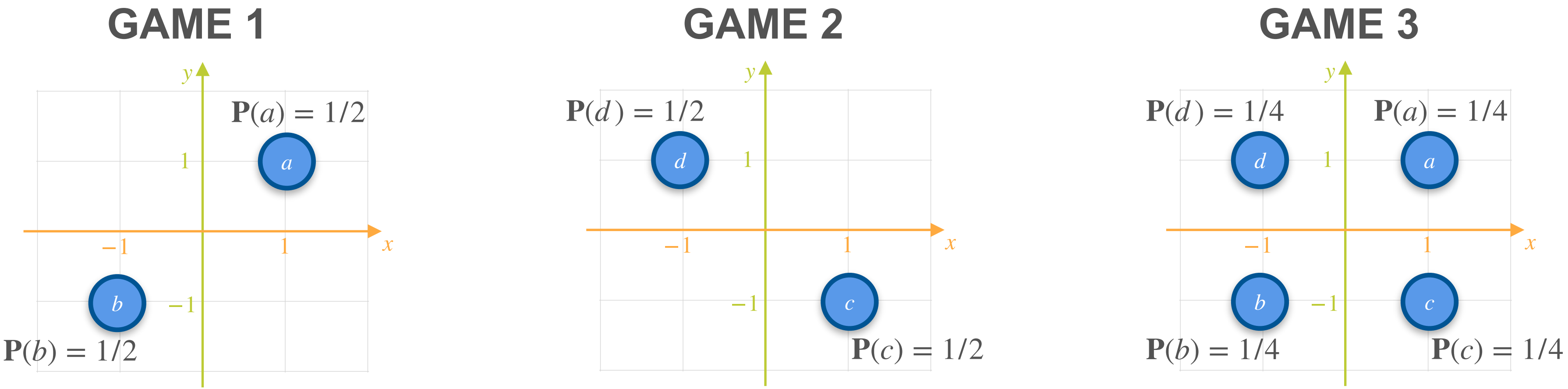
Covariance of a Probability Distribution: Motivation

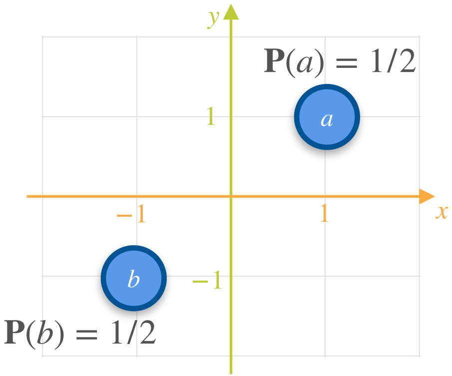


How similar are these 3 games for player X and player Y?

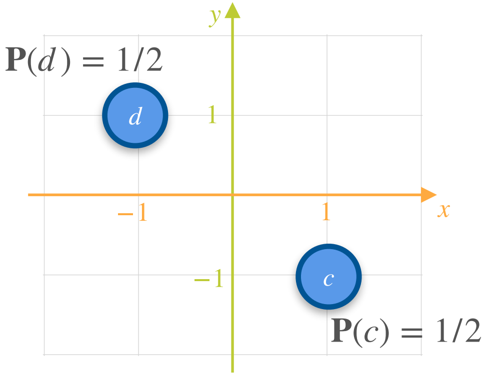
X : how much money in dollars player X wins

Covariance of a Probability Distribution: Motivation

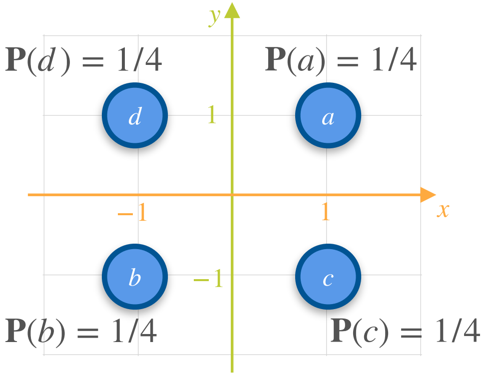
GAME 1



GAME 2



GAME 3

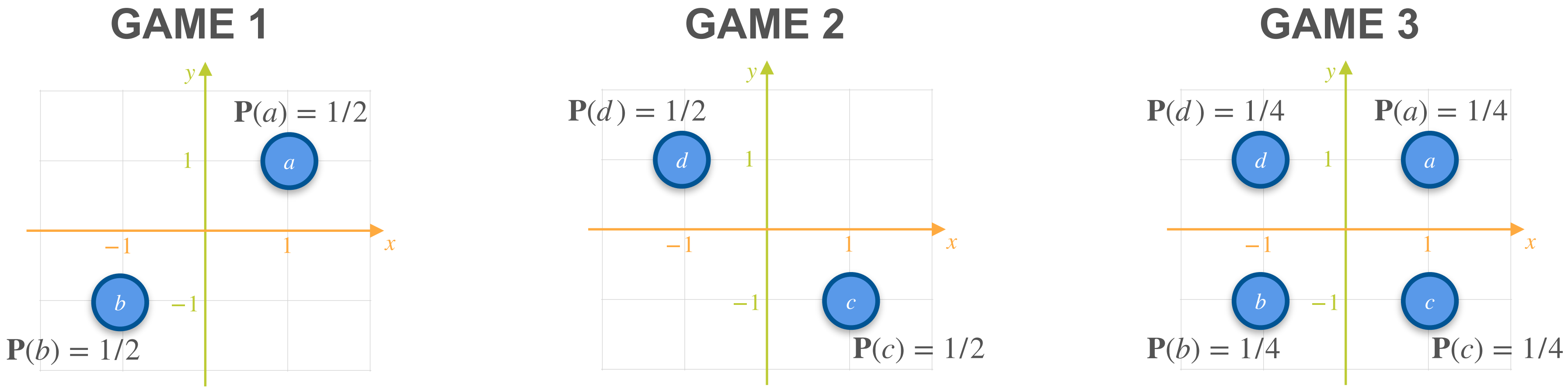


How similar are these 3 games for player X and player Y?

X : how much money in dollars player X wins

Y : how much money in dollars player Y wins

Covariance of a Probability Distribution: Motivation



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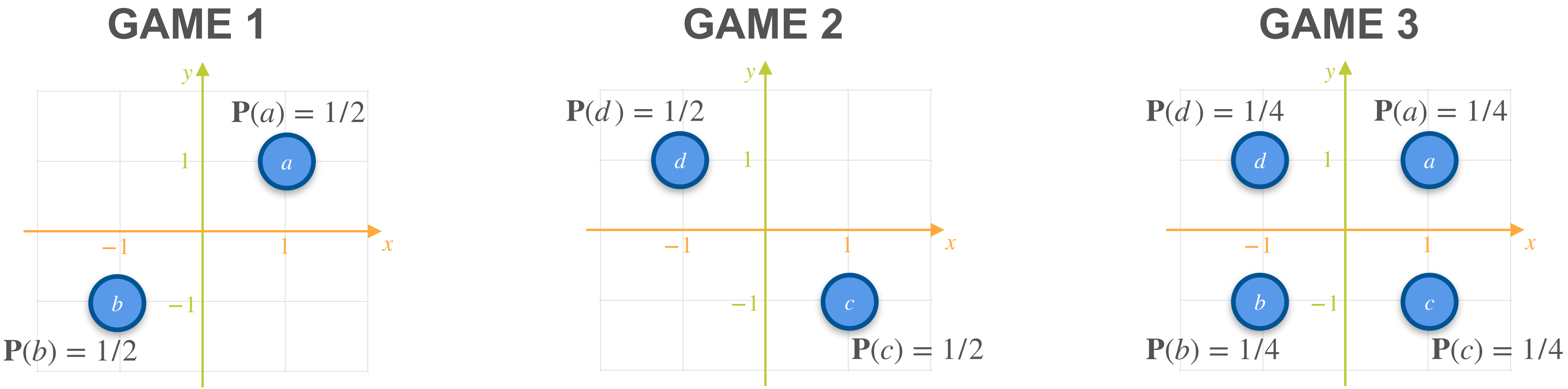
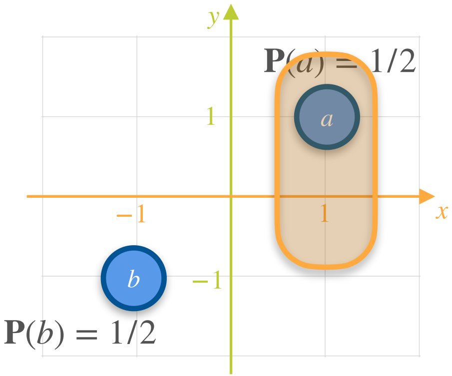


$$\mathbb{E}[X_1] =$$

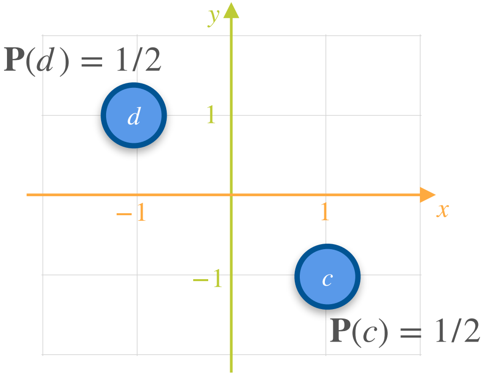
Covariance of a Probability Distribution: Motivation

GAME 1

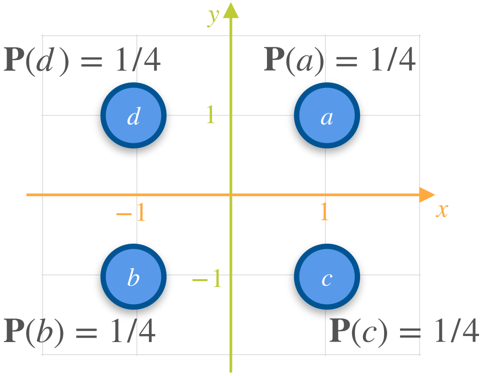


$$\mathbb{E}[X_1] = \frac{1}{2}(1)$$

GAME 2



GAME 3



Covariance of a Probability Distribution: Motivation

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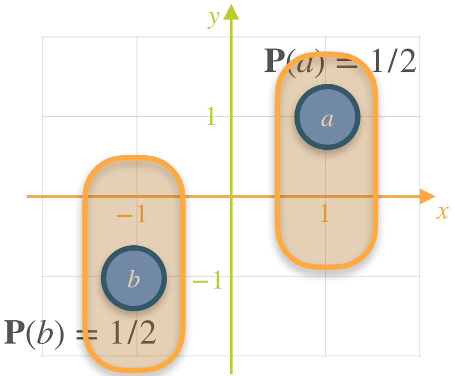
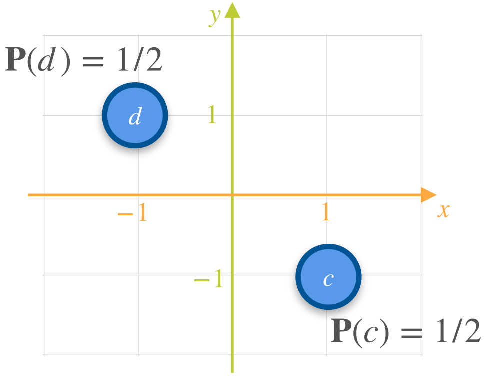
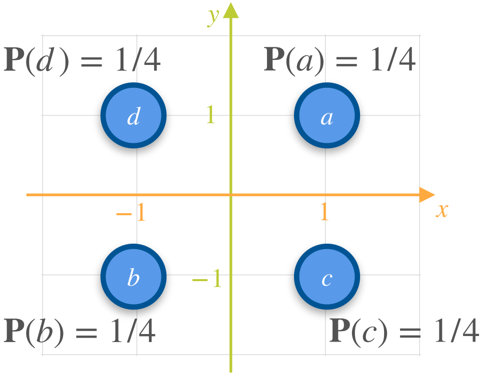


$$\mathbb{E}[X_1] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

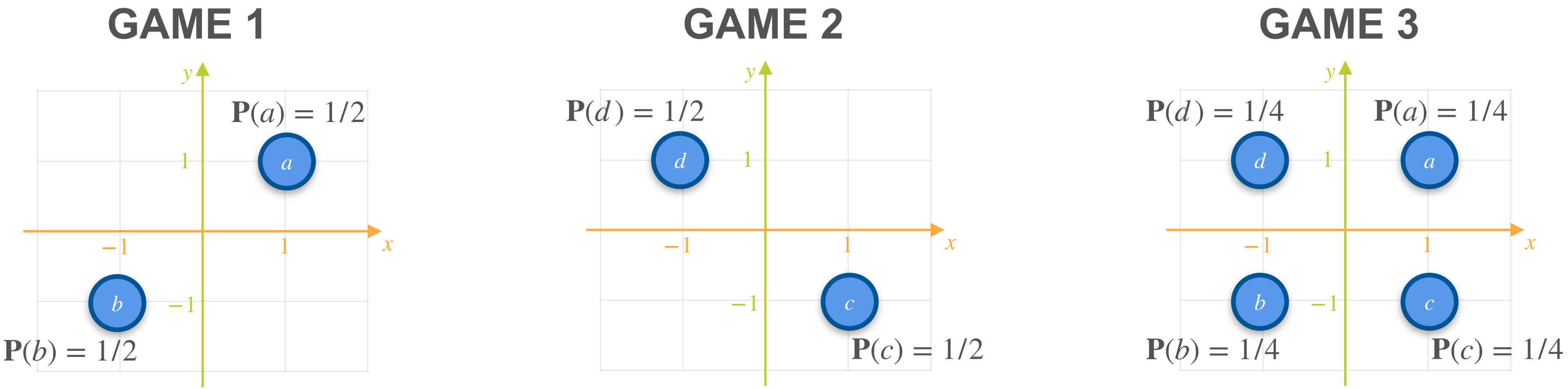
GAME 2



GAME 3



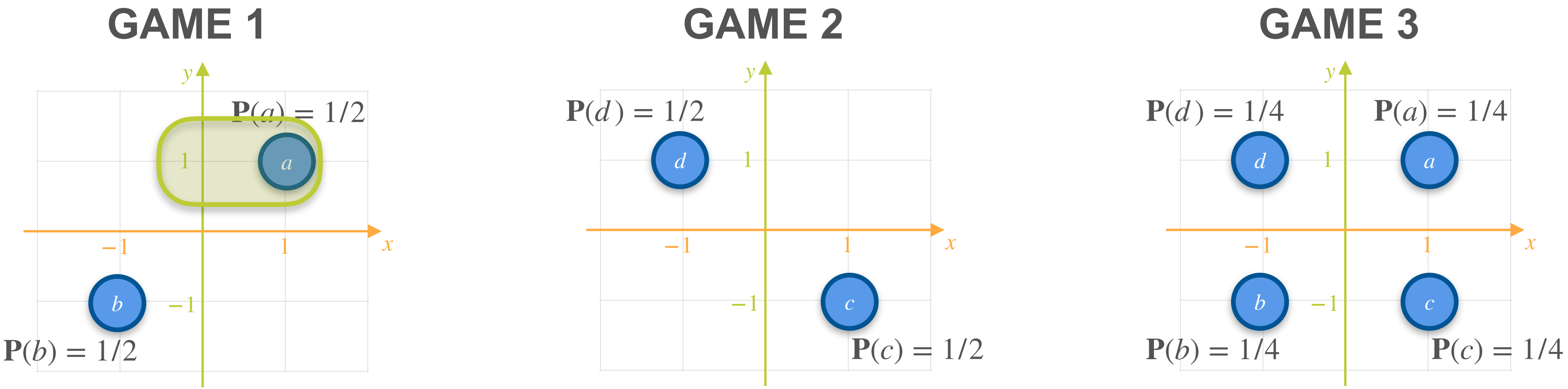
Covariance of a Probability Distribution: Motivation



$$\mathbb{E}[X_1] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

$$\mathbb{E}[Y_1] =$$

Covariance of a Probability Distribution: Motivation



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Covariance of a Probability Distribution: Motivation

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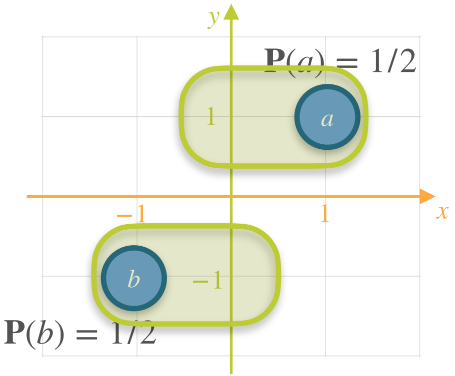
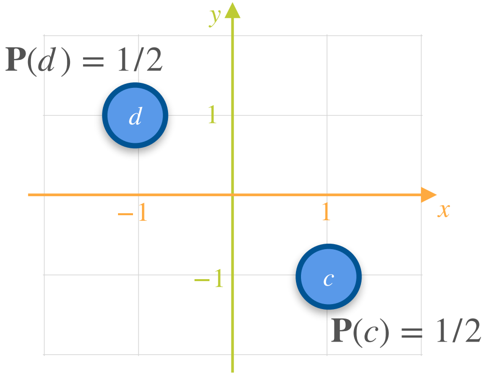
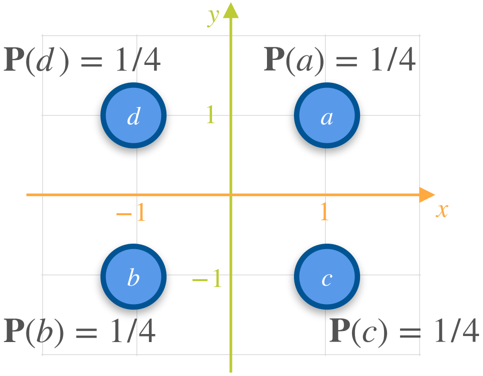


GAME 2



GAME 3



$$\mathbb{E}[X_1] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

$$\mathbb{E}[Y_1] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

Covariance of a Probability Distribution: Motivation

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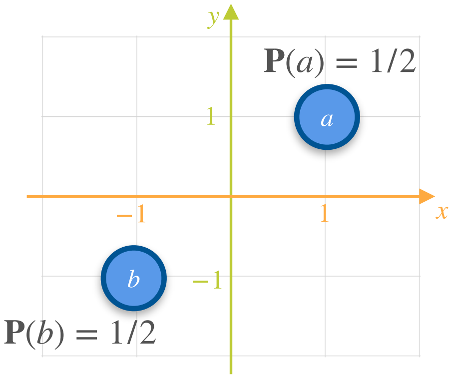
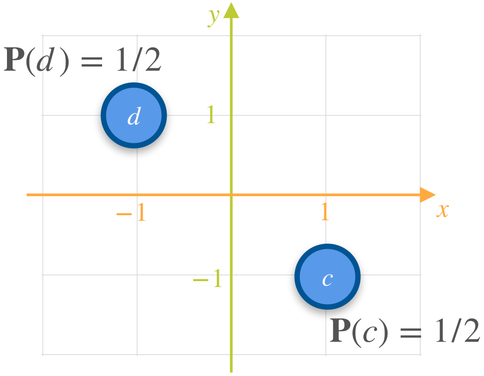


GAME 2



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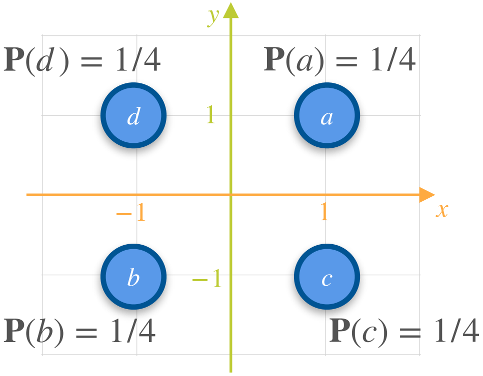

$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[Y_1] = 0$$

Covariance of a Probability Distribution: Motivation

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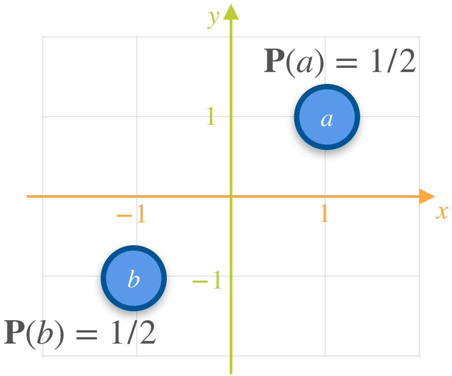
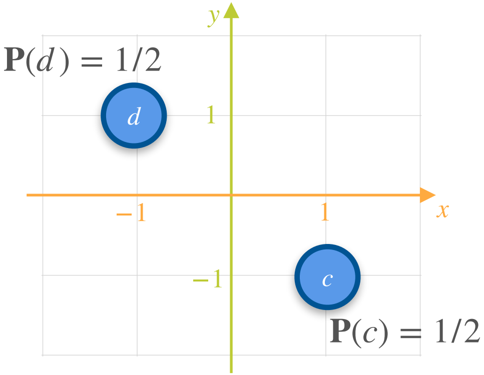
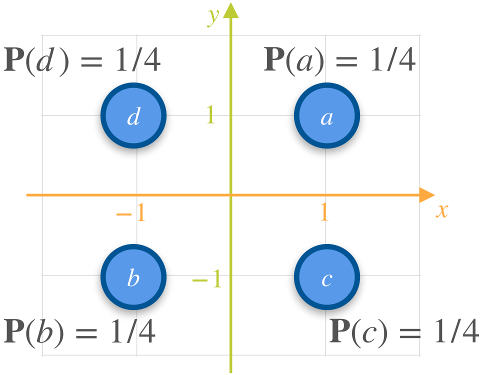


GAME 2



GAME 3

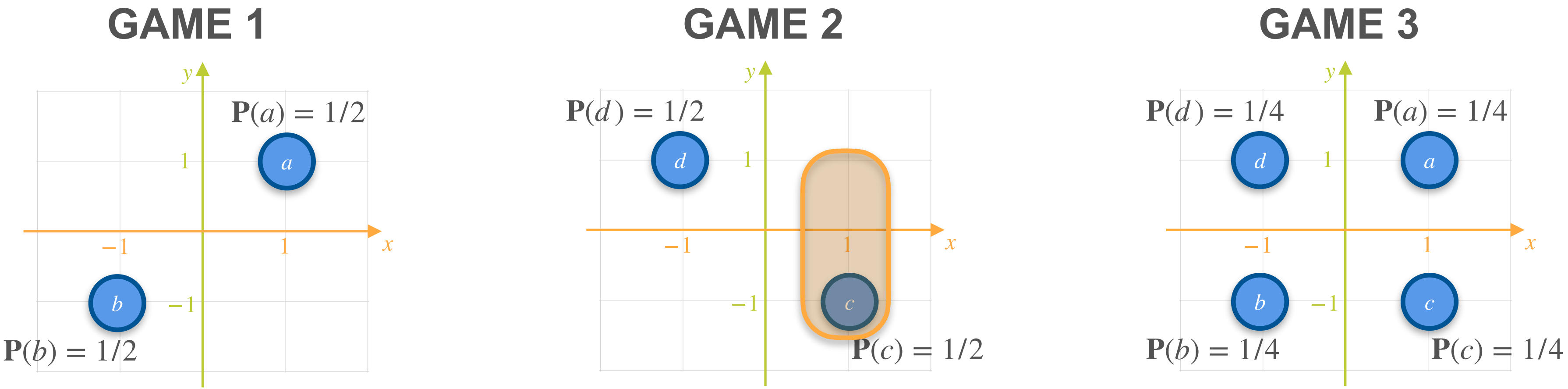


$\mathbb{E}[X_1] = 0$

$\mathbb{E}[X_2] =$

$\mathbb{E}[Y_1] = 0$

Covariance of a Probability Distribution: Motivation

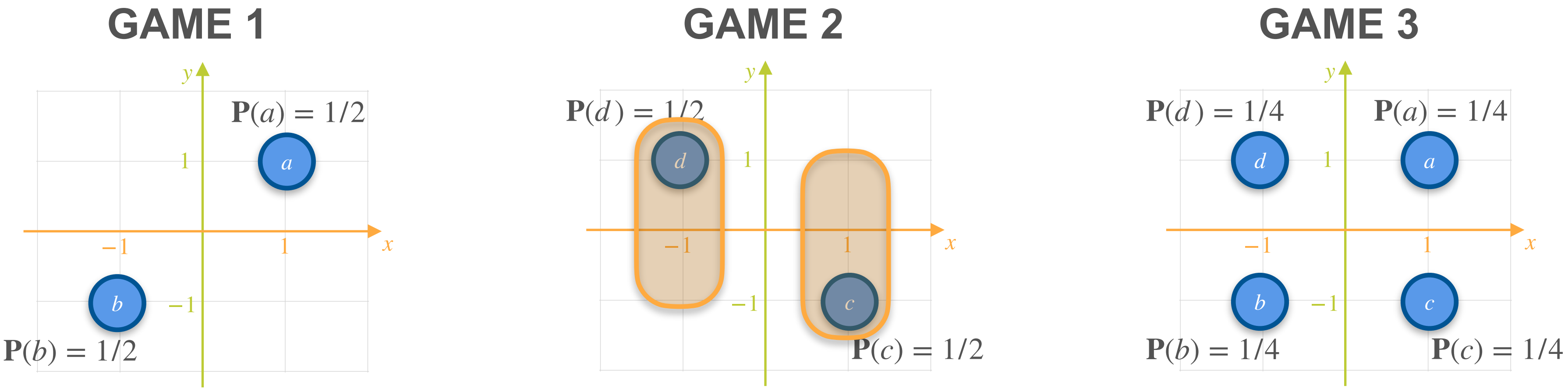


$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = \frac{1}{2}(1)$$

$$\mathbb{E}[Y_1] = 0$$

Covariance of a Probability Distribution: Motivation



$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[X_2] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

$$\mathbb{E}[Y_1] = 0$$

Covariance of a Probability Distribution: Motivation

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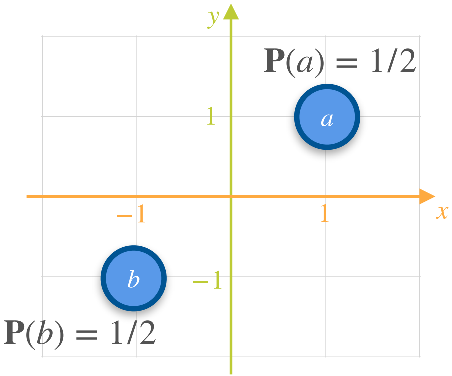
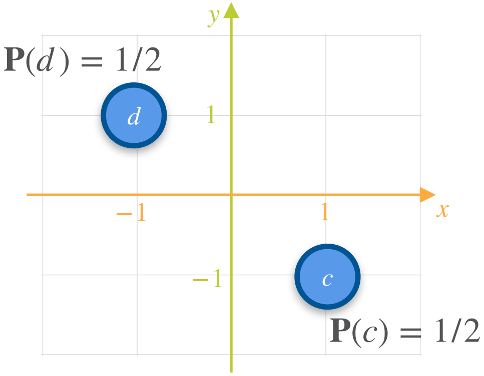


GAME 2



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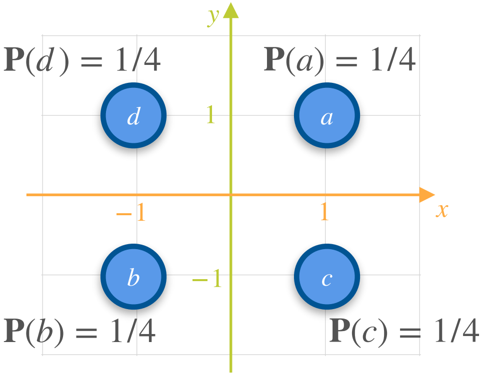

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$$\mathbb{E}[Y_2] =$$

Covariance of a Probability Distribution: Motivation

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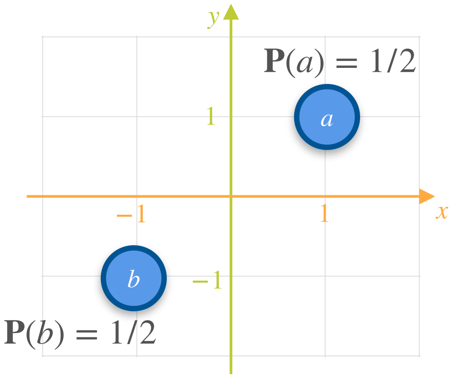

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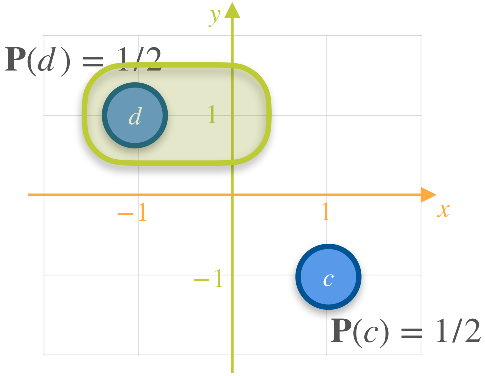

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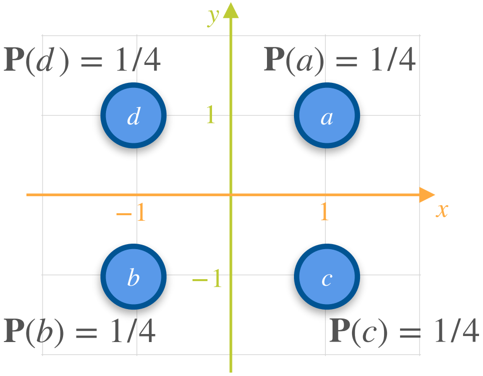

$$\mathbb{E}[X_1] = 0$$

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$$\mathbb{E}[X_2] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

$$\mathbb{E}[Y_2] = \frac{1}{2}(1)$$

DeepLearning.AI

Covariance of a Probability Distribution: Motivation

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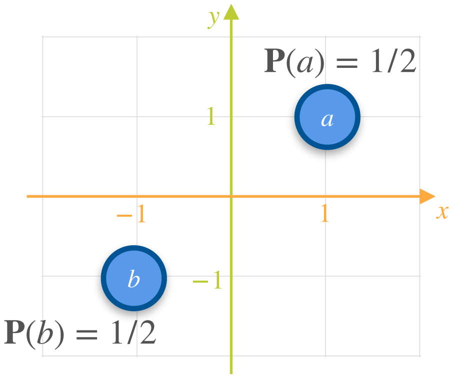
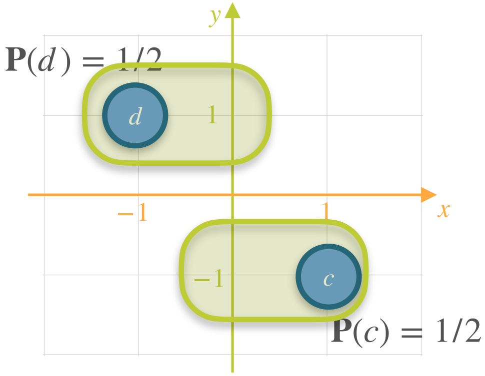
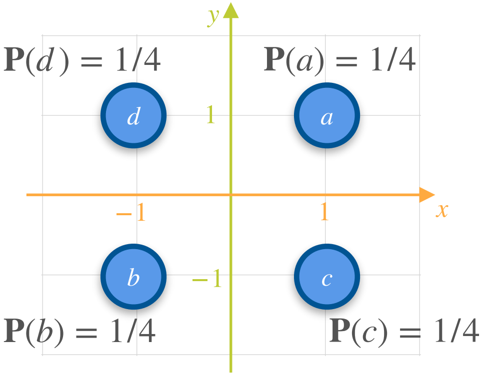


GAME 2



GAME 3



$$\mathbb{E}[X_1] = 0$$

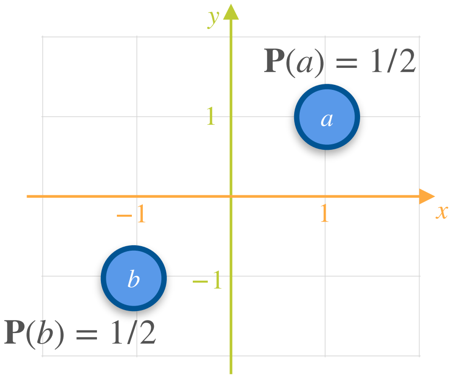
$$\mathbb{E}[Y_1] = 0$$

$$\mathbb{E}[X_2] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

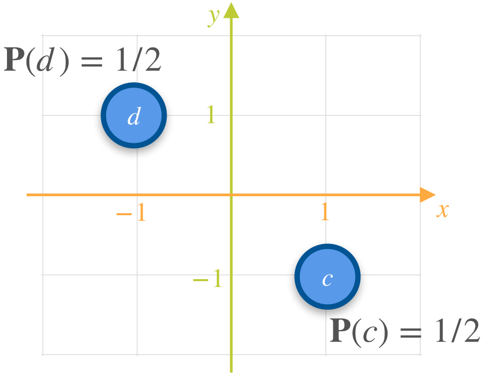
$$\mathbb{E}[Y_2] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

Covariance of a Probability Distribution: Motivation

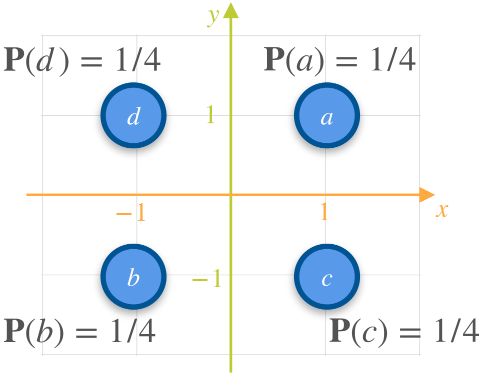
GAME 1



GAME 2



GAME 3



$$\mathbb{E}[X_1] = 0$$

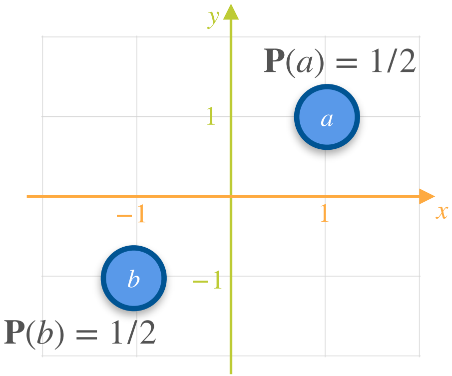
$$\mathbb{E}[Y_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

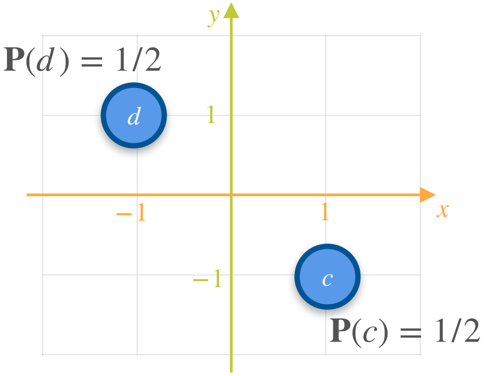
$$\mathbb{E}[Y_2] = 0$$

Covariance of a Probability Distribution: Motivation

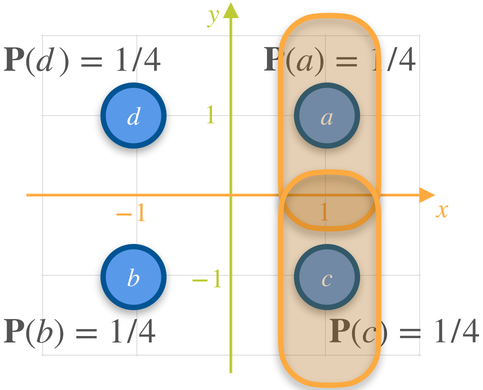
GAME 1



GAME 2



GAME 3



$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[Y_1] = 0$$

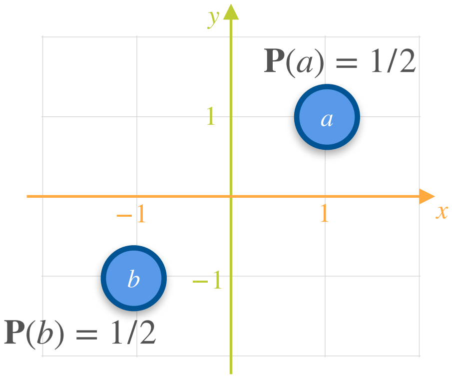
$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[Y_2] = 0$$

$$\mathbb{E}[X_3] = 2\left(\frac{1}{4}(1)\right)$$

Covariance of a Probability Distribution: Motivation

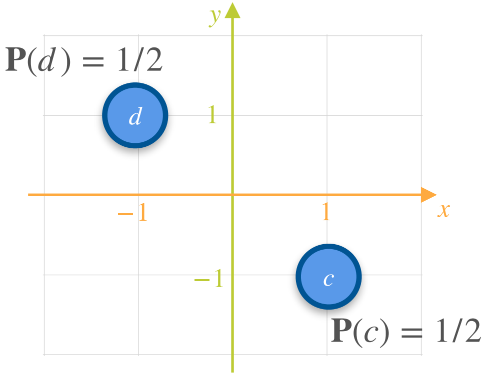
GAME 1



$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[Y_1] = 0$$

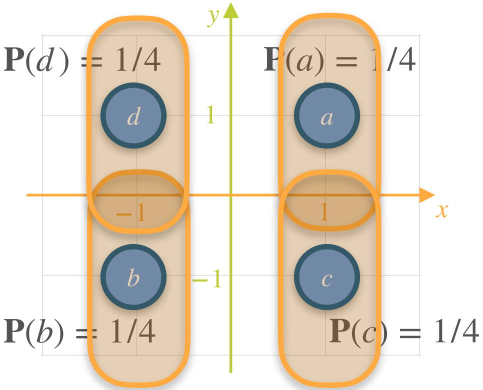
GAME 2



$$\mathbb{E}[X_2] = 0$$

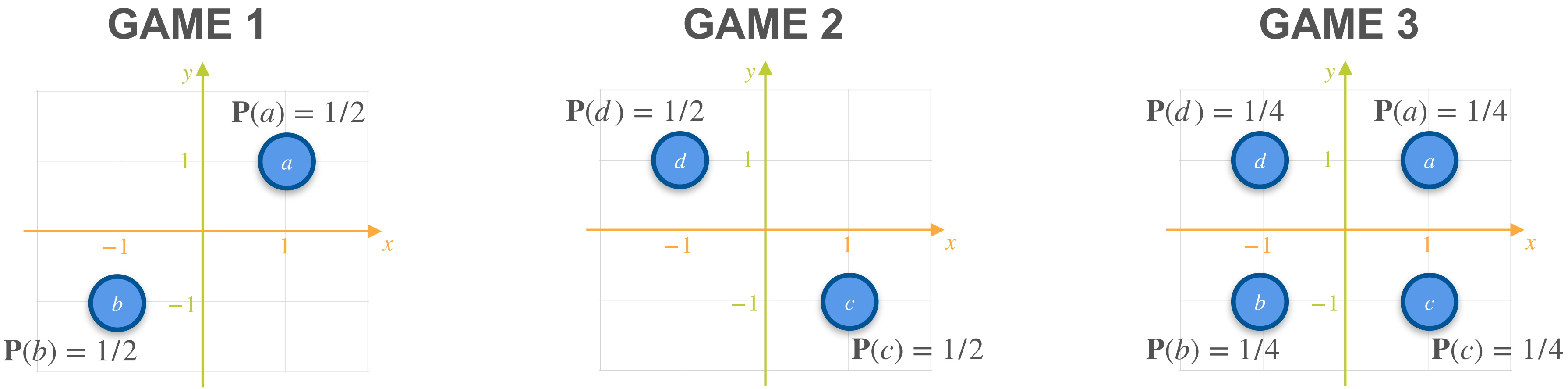
$$\mathbb{E}[Y_2] = 0$$

GAME 3



$$\mathbb{E}[X_3] = 2\left(\frac{1}{4}(1)\right) + 2\left(\frac{1}{4}(-1)\right) = 0$$

Covariance of a Probability Distribution: Motivation



$\mathbb{E}[X_1] = 0$

$\mathbb{E}[Y_1] = 0$

$\mathbb{E}[X_2] = 0$

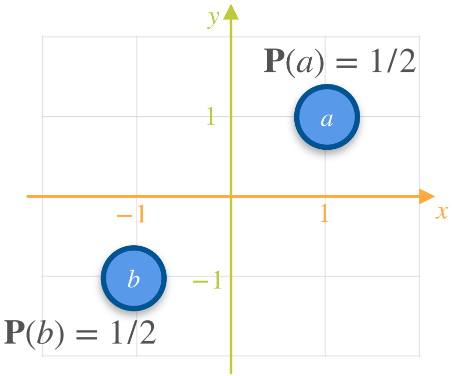
$\mathbb{E}[Y_2] = 0$

$$\mathbb{E}[X_3] = 2\left(\frac{1}{4}(1)\right) + 2\left(\frac{1}{4}(-1)\right) = 0$$

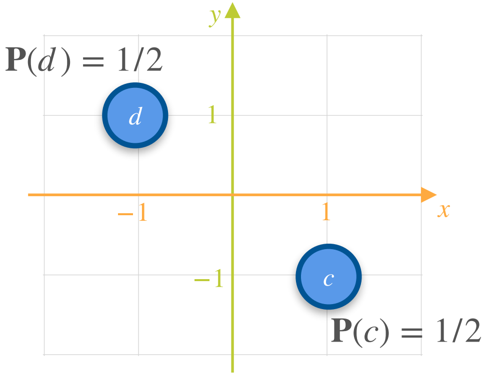
$\mathbb{E}[Y_3] =$

Covariance of a Probability Distribution: Motivation

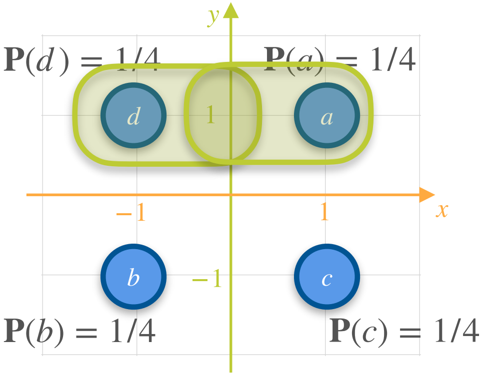
GAME 1



GAME 2



GAME 3



$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[Y_1] = 0$$

$$\mathbb{E}[X_2] = 0$$

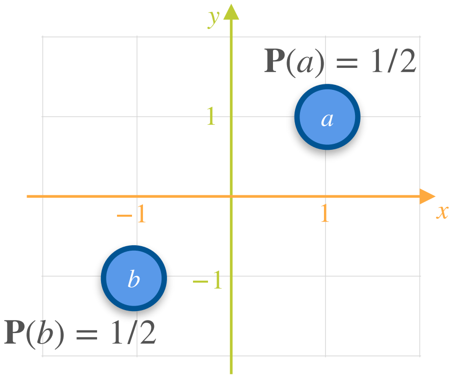
$$\mathbb{E}[Y_2] = 0$$

$$\mathbb{E}[X_3] = 2\left(\frac{1}{4}(1)\right) + 2\left(\frac{1}{4}(-1)\right) = 0$$

$$\mathbb{E}[Y_3] = 2\left(\frac{1}{4}(1)\right)$$

Covariance of a Probability Distribution: Motivation

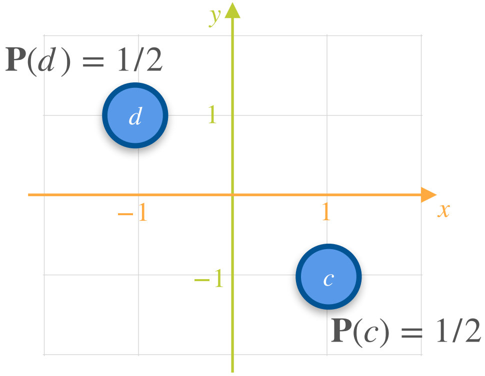
GAME 1



$$\mathbb{E}[X_1] = 0$$

$$\mathbb{E}[Y_1] = 0$$

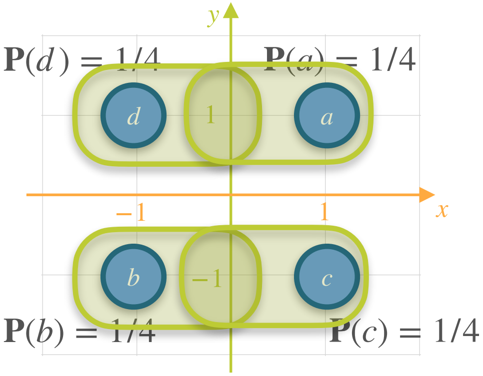
GAME 2



$$\mathbb{E}[X_2] = 0$$

$$\mathbb{E}[Y_2] = 0$$

GAME 3

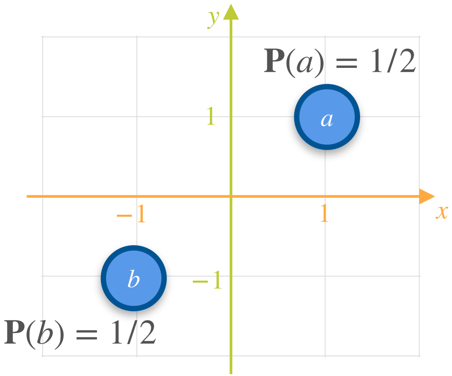


$$\mathbb{E}[X_3] = 2\left(\frac{1}{4}(1)\right) + 2\left(\frac{1}{4}(-1)\right) = 0$$

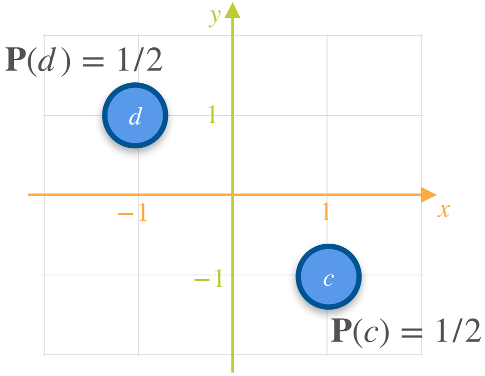
$$\mathbb{E}[Y_3] = 2\left(\frac{1}{4}(1)\right) + 2\left(\frac{1}{4}(-1)\right) = 0$$

Covariance of a Probability Distribution: Motivation

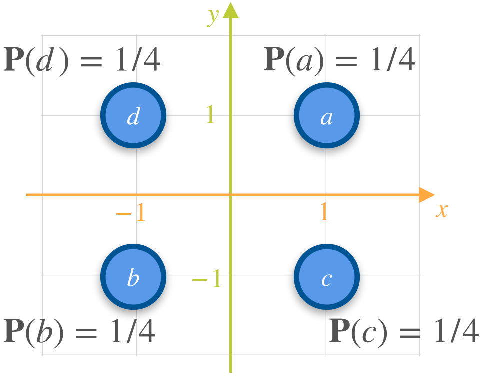
GAME 1



GAME 2



GAME 3



$\mathbb{E}[X_1] = 0$

$\mathbb{E}[Y_1] = 0$

$\mathbb{E}[X_2] = 0$

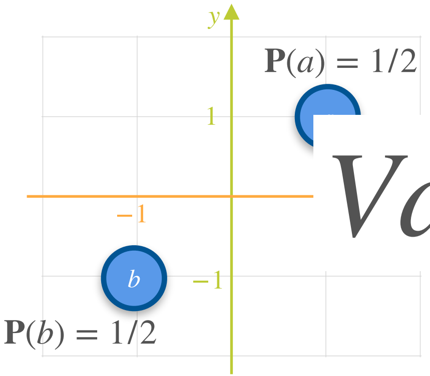
$\mathbb{E}[Y_2] = 0$

$\mathbb{E}[X_3] = 0$

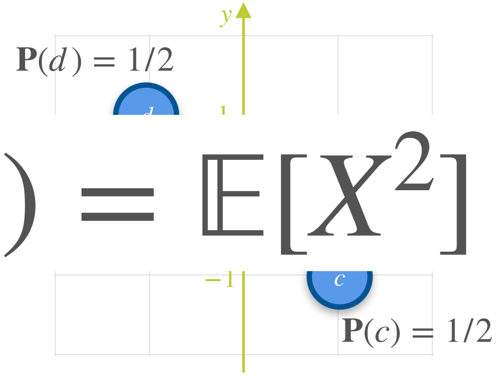
$\mathbb{E}[Y_3] = 0$

Covariance of a Probability Distribution: Motivation

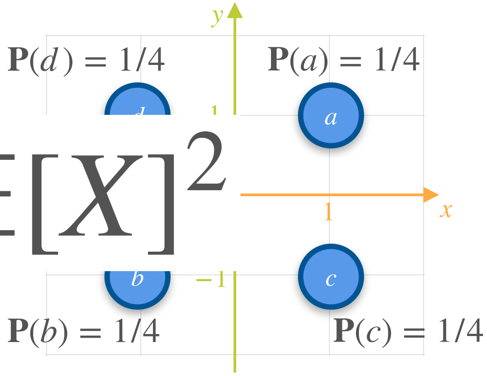
GAME 1



GAME 2



GAME 3



$$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$\mathbb{E}[X_1] = 0$

$\mathbb{E}[Y_1] = 0$

$\mathbb{E}[X_2] = 0$

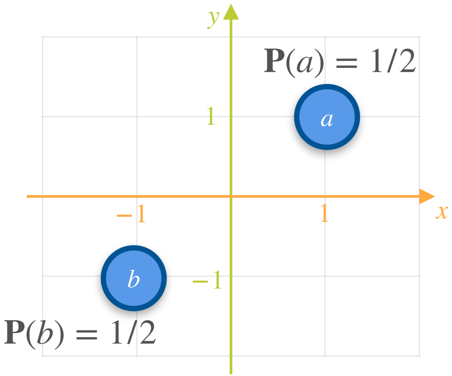
$\mathbb{E}[Y_2] = 0$

$\mathbb{E}[X_3] = 0$

$\mathbb{E}[Y_3] = 0$

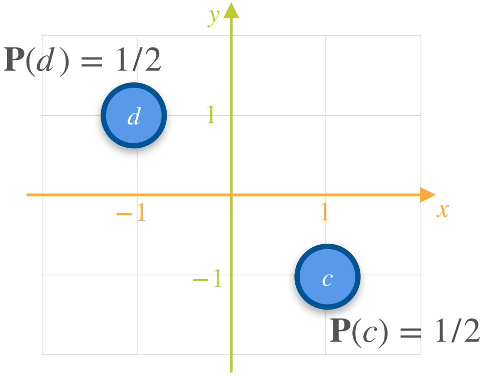
Covariance of a Probability Distribution: Motivation

GAME 1



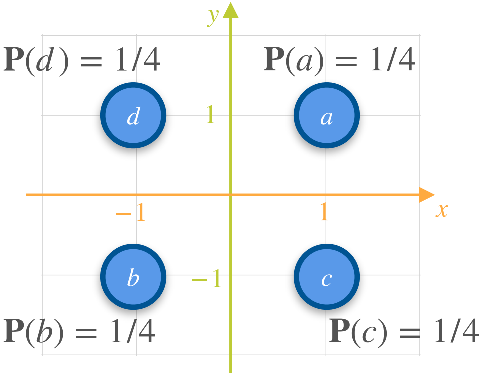
$\mathbb{E}[X_1] = 0 \quad \mathbb{E}[Y_1] = 0$

GAME 2



$\mathbb{E}[X_2] = 0 \quad \mathbb{E}[Y_2] = 0$

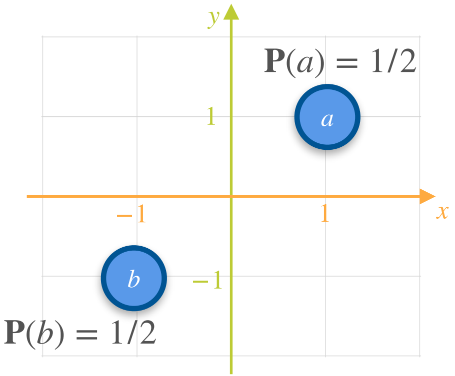
GAME 3



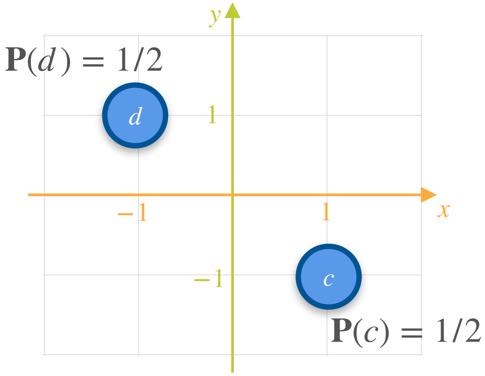
$\mathbb{E}[X_3] = 0 \quad \mathbb{E}[Y_3] = 0$

Covariance of a Probability Distribution: Motivation

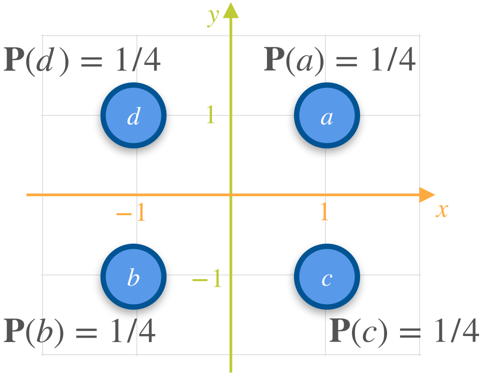
GAME 1



GAME 2



GAME 3



$\mathbb{E}[X_1] = 0 \quad \mathbb{E}[Y_1] = 0$

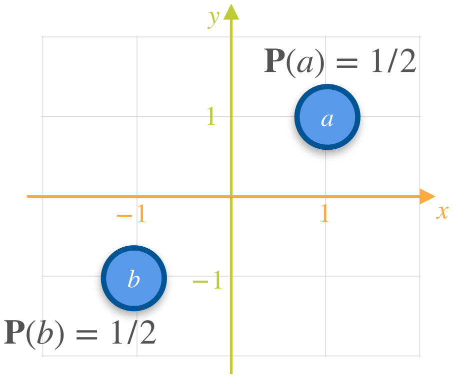
$\mathbb{E}[X_2] = 0 \quad \mathbb{E}[Y_2] = 0$

$\mathbb{E}[X_3] = 0 \quad \mathbb{E}[Y_3] = 0$

$$Var(X_1) =$$

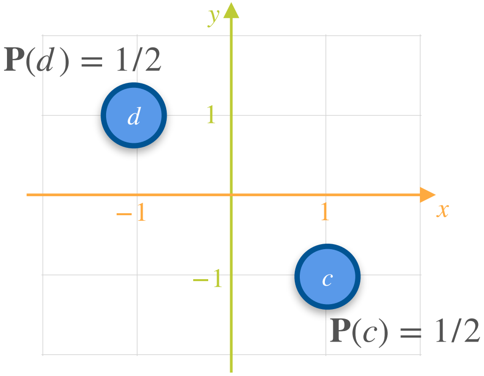
Covariance of a Probability Distribution: Motivation

GAME 1



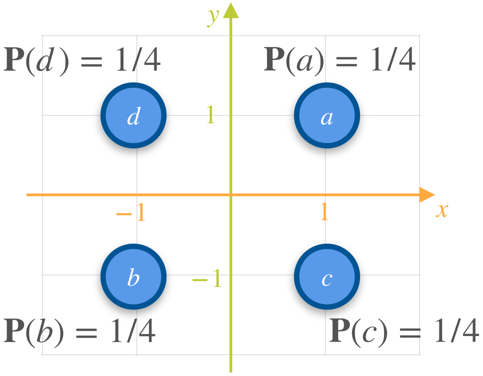
$\mathbb{E}[X_1] = 0 \quad \mathbb{E}[Y_1] = 0$

GAME 2



$\mathbb{E}[X_2] = 0 \quad \mathbb{E}[Y_2] = 0$

GAME 3

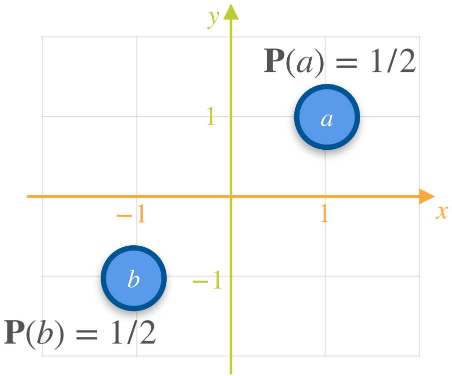


$\mathbb{E}[X_3] = 0 \quad \mathbb{E}[Y_3] = 0$

$$Var(X_1) = \mathbb{E}[X_1^2] - E[X_1]^2$$

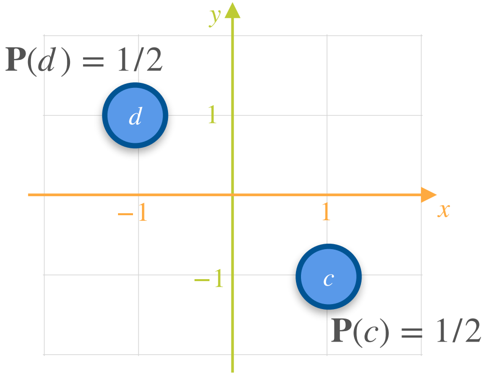
Covariance of a Probability Distribution: Motivation

GAME 1



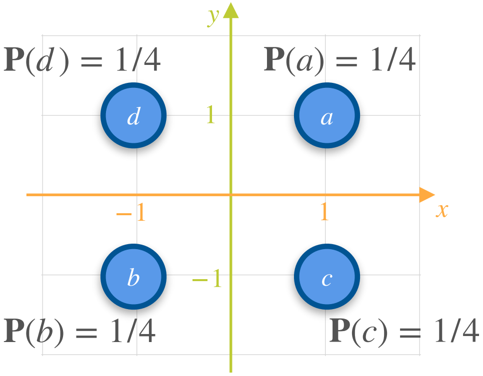
$\mathbb{E}[X_1] = 0 \quad \mathbb{E}[Y_1] = 0$

GAME 2



$\mathbb{E}[X_2] = 0 \quad \mathbb{E}[Y_2] = 0$

GAME 3

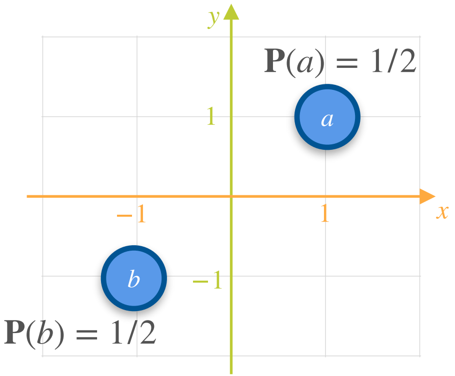


$\mathbb{E}[X_3] = 0 \quad \mathbb{E}[Y_3] = 0$

$$Var(X_1) = \mathbb{E}[X_1^2] - E[X_1]^2 = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 - 0^2 =$$

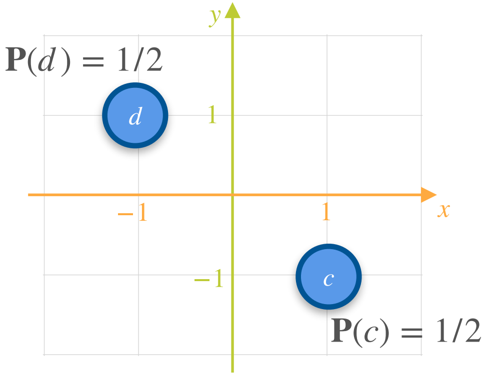
Covariance of a Probability Distribution: Motivation

GAME 1



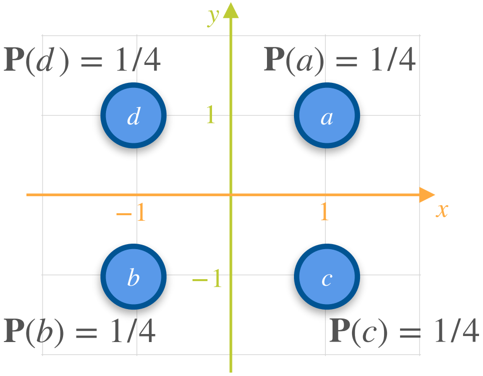
$\mathbb{E}[X_1] = 0 \quad \mathbb{E}[Y_1] = 0$

GAME 2



$\mathbb{E}[X_2] = 0 \quad \mathbb{E}[Y_2] = 0$

GAME 3

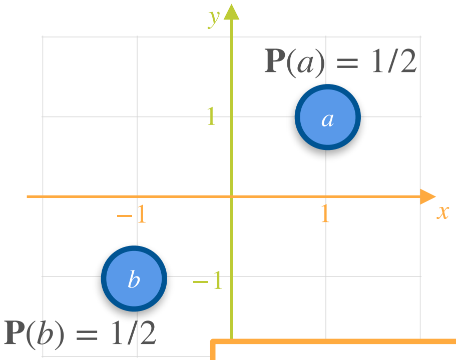


$\mathbb{E}[X_3] = 0 \quad \mathbb{E}[Y_3] = 0$

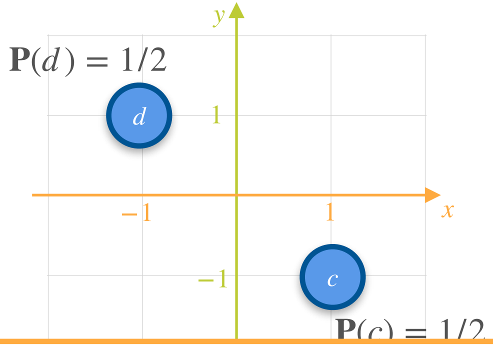
$$Var(X_1) = \mathbb{E}[X_1^2] - E[X_1]^2 = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 - 0^2 = 1$$

Covariance of a Probability Distribution: Motivation

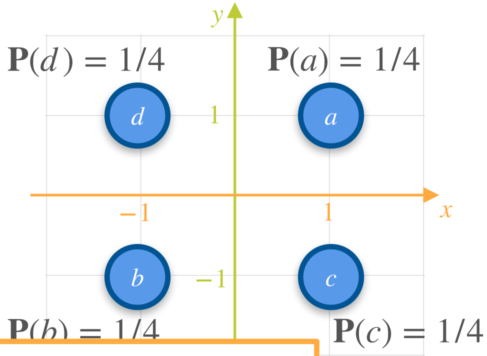
GAME 1



GAME 2



GAME 3



How similar are these 3 games for player X and player Y?

$\mathbb{E}[X_1] = 0$ $\mathbb{E}[Y_1] = 0$

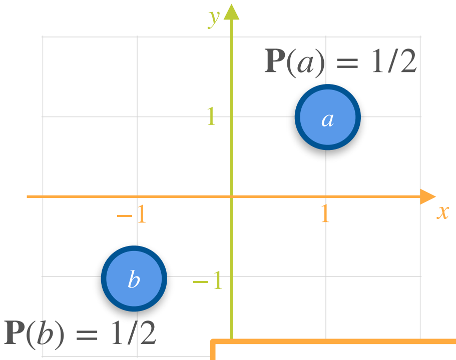
$\mathbb{E}[X_2] = 0$ $\mathbb{E}[Y_2] = 0$

$\mathbb{E}[X_3] = 0$ $\mathbb{E}[Y_3] = 0$

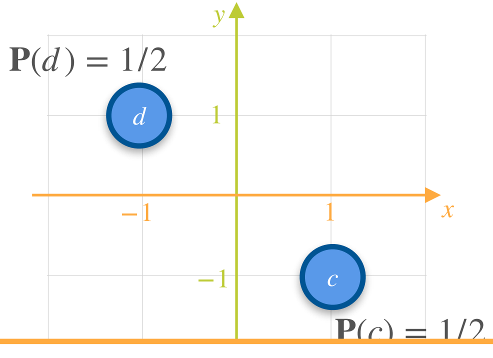
$$Var(X_1) = 1$$

Covariance of a Probability Distribution: Motivation

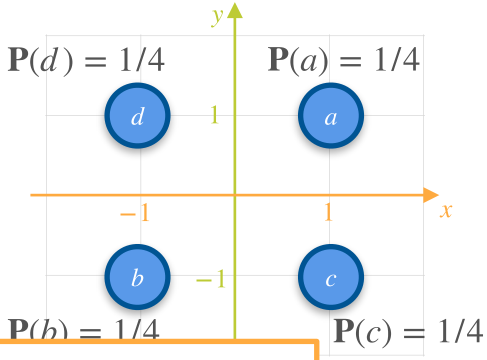
GAME 1



GAME 2



GAME 3



How similar are these 3 games for player X and player Y?

$\mathbb{E}[X_1] = 0$ $\mathbb{E}[Y_1] = 0$ $\mathbb{E}[X_2] = 0$ $\mathbb{E}[Y_2] = 0$ $\mathbb{E}[X_3] = 0$ $\mathbb{E}[Y_3] = 0$

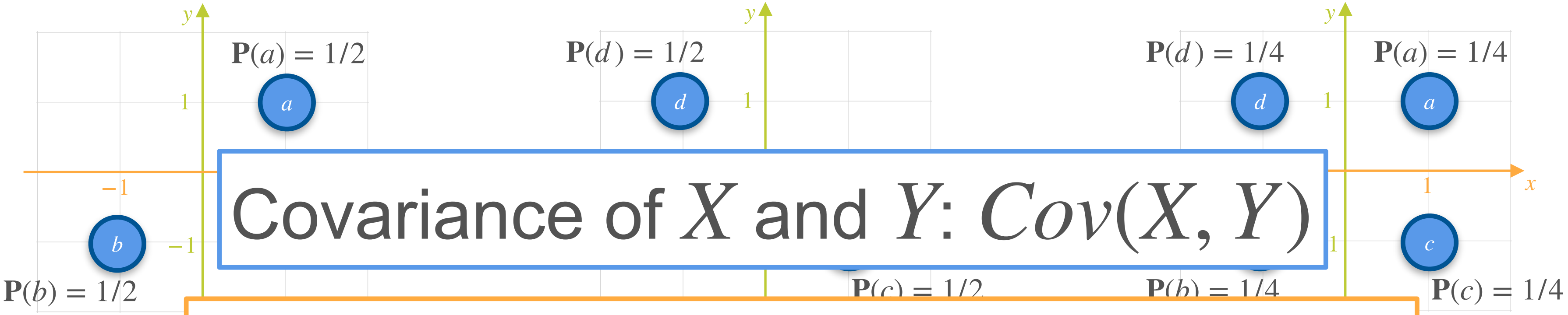
$Var(X_1) = 1$ $Var(X_2) = 1$ $Var(X_3) = 1$

$Var(Y_1) = 1$ $Var(Y_2) = 1$ $Var(Y_3) = 1$

DeepLearning.AI

Covariance of a Probability Distribution: Motivation

GAME 1 **GAME 2** **GAME 3**



Covariance of X and Y: $Cov(X, Y)$

How similar are these 3 games for player X and player Y?

$\mathbb{E}[X_1] = 0 \quad \mathbb{E}[Y_1] = 0$

$Var(X_1) = 1$

$Var(Y_1) = 1$

$\mathbb{E}[X_2] = 0 \quad \mathbb{E}[Y_2] = 0$

$Var(X_2) = 1$

$Var(Y_2) = 1$

$\mathbb{E}[X_3] = 0 \quad \mathbb{E}[Y_3] = 0$

$Var(X_3) = 1$

$Var(Y_3) = 1$

Covariance of a Probability Distribution: Motivation

Covariance of a Probability Distribution: Motivation

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

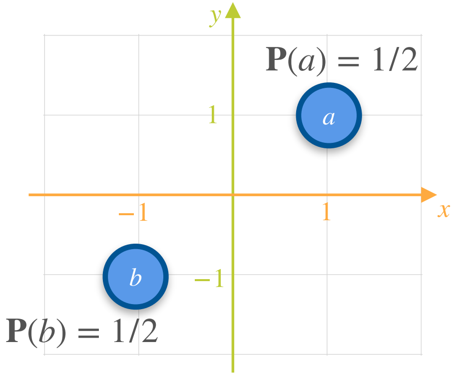
Covariance of a Probability Distribution: Motivation

$\boxed{\text{Covariance of } X \text{ and } Y\text{: } Cov(X, Y)}$

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

Covariance of a Probability Distribution: Motivation

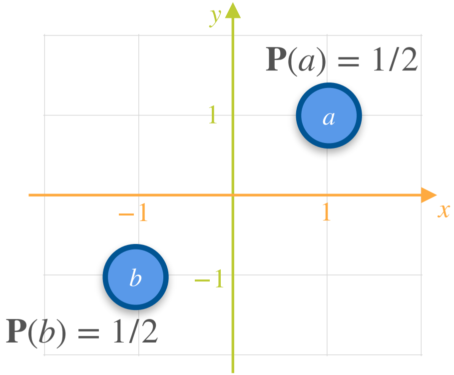
GAME 1



$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

Covariance of a Probability Distribution: Motivation

GAME 1

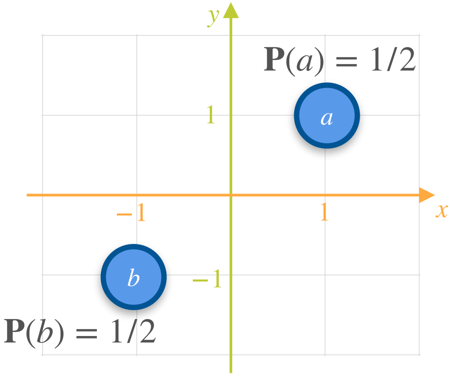


$$Cov(X,Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

x	y	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$

Covariance of a Probability Distribution: Motivation

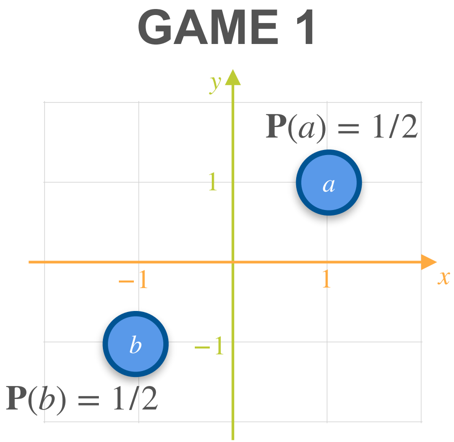
GAME 1



$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

x	y	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
1				
-1				

Covariance of a Probability Distribution: Motivation

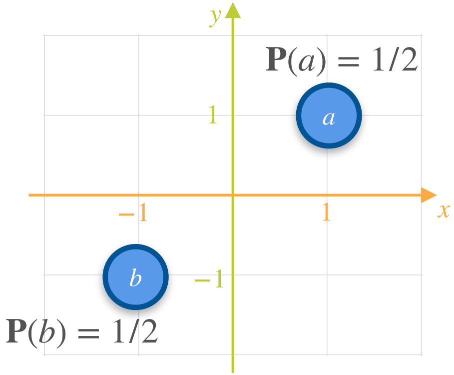


$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

x	y	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
1	1			
-1	-1			

Covariance of a Probability Distribution: Motivation

GAME 1

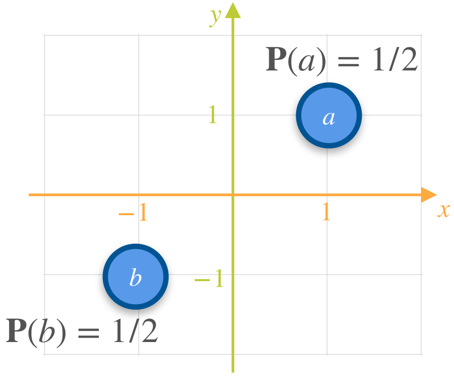


$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

x	y	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
1	1	1		
-1	-1	-1		

Covariance of a Probability Distribution: Motivation

GAME 1

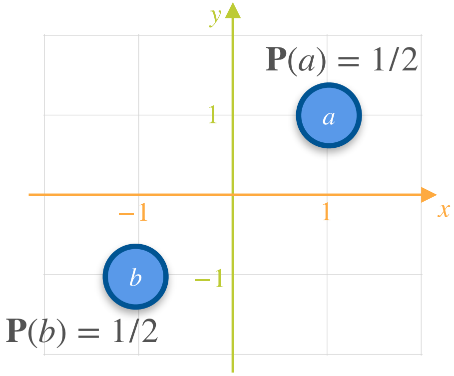


$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

x	y	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
1	1	1	1	
-1	-1	-1	-1	

Covariance of a Probability Distribution: Motivation

GAME 1

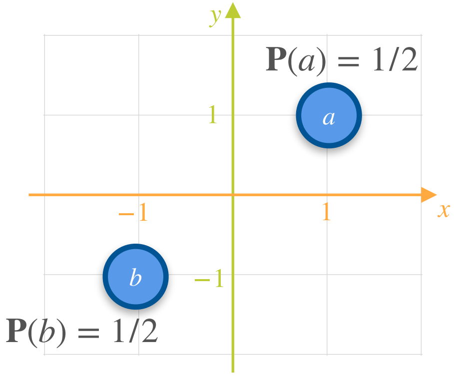


$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

x	y	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
1	1	1	1	1
-1	-1	-1	-1	1

Covariance of a Probability Distribution: Motivation

GAME 1



$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

x	y	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
1	1	1	1	1
-1	-1	-1	-1	1

$$\sum (x_i - \mu_x)(y_i - \mu_y) \quad =$$

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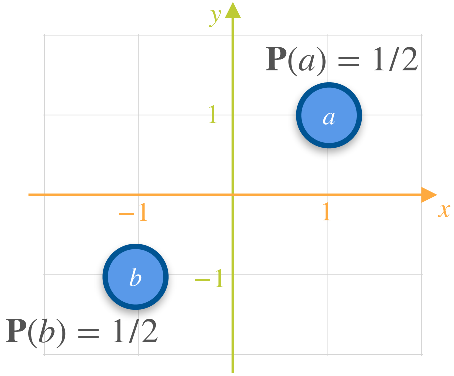

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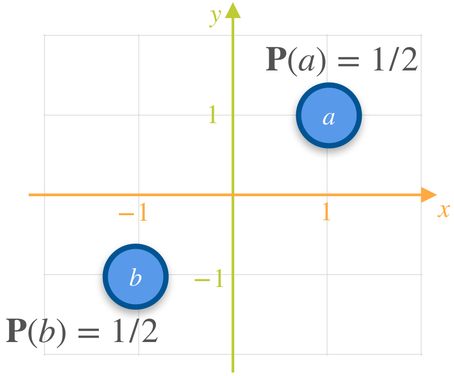

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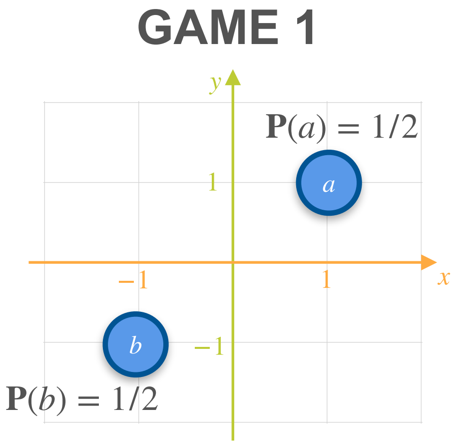

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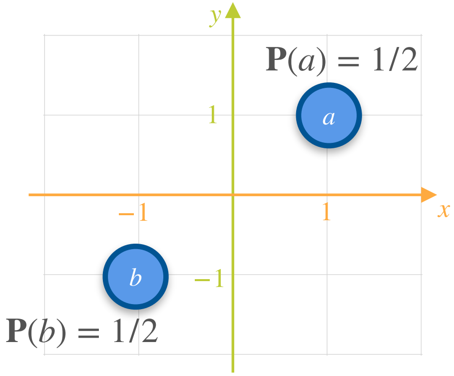

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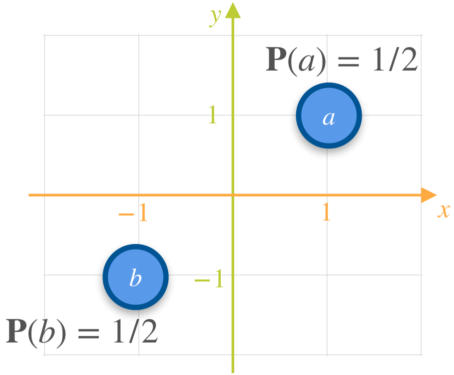

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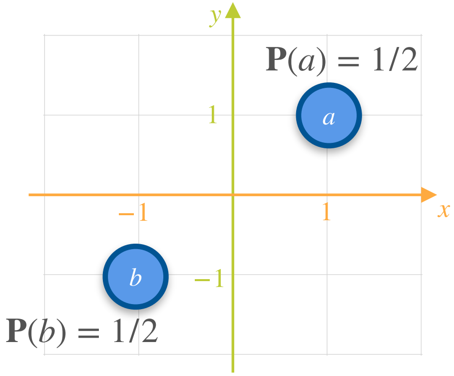

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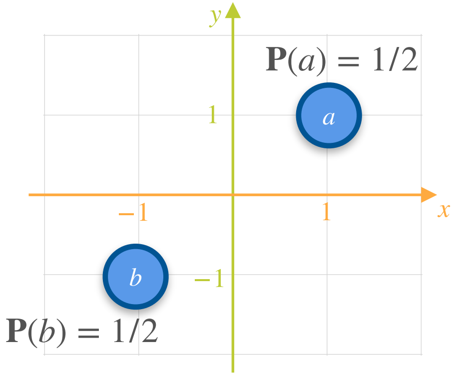

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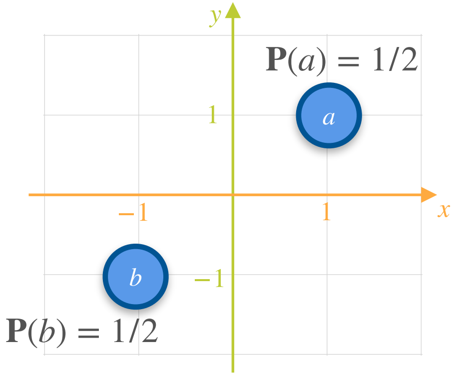

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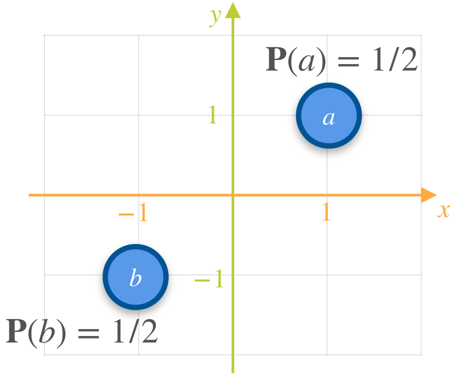

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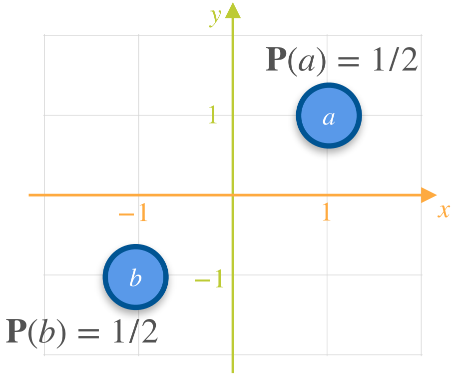

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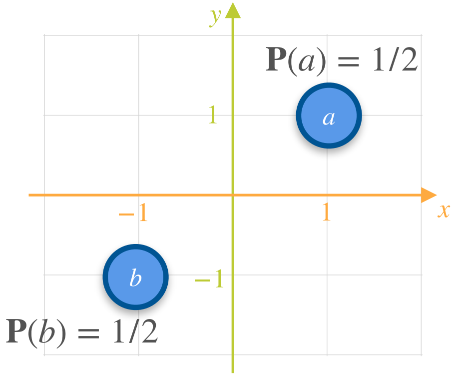

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1				
-1				

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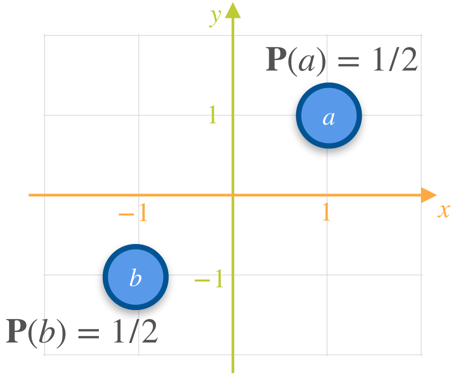

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-1	-1			

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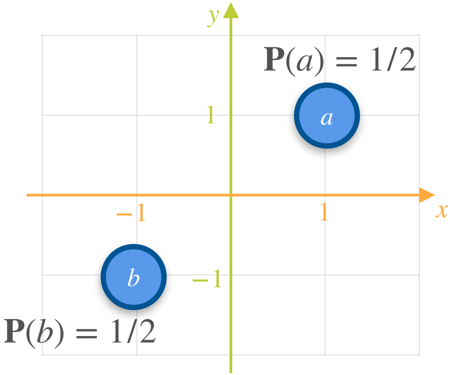

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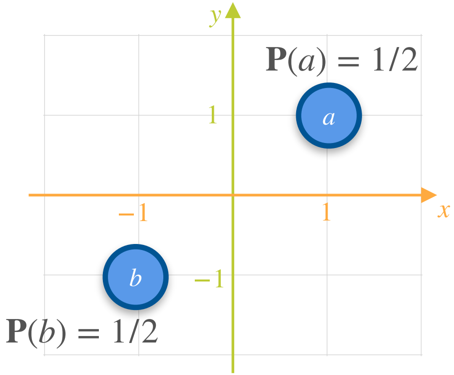

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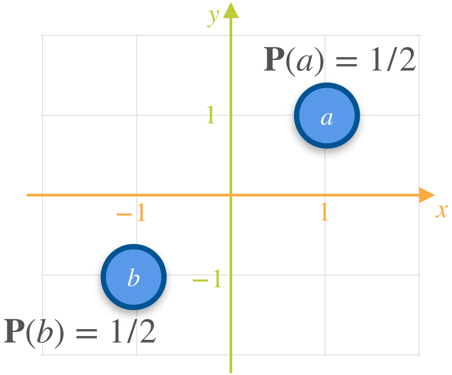

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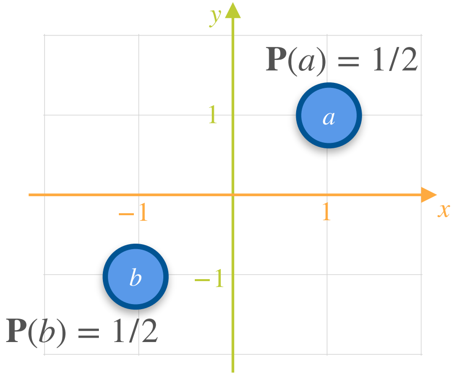

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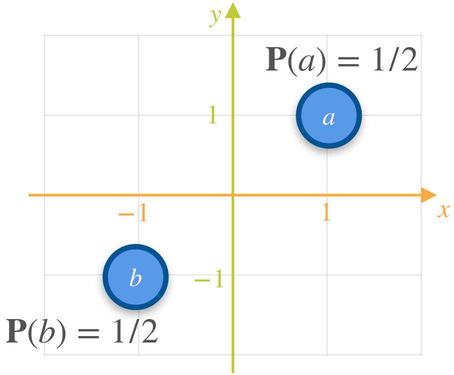

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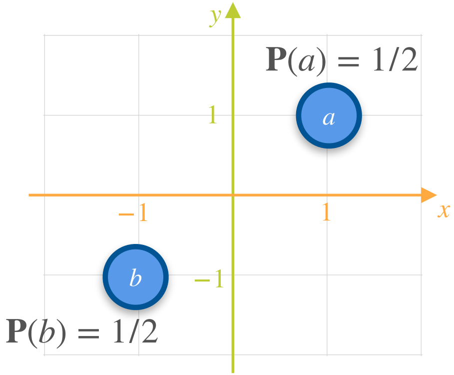

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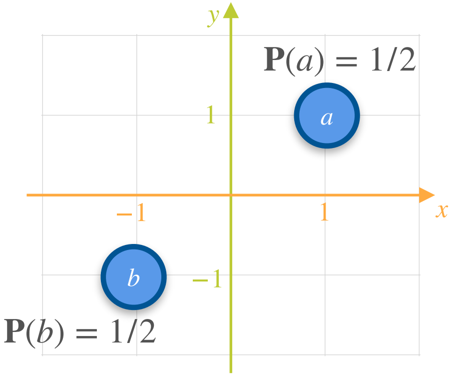

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DeepLearning.AI

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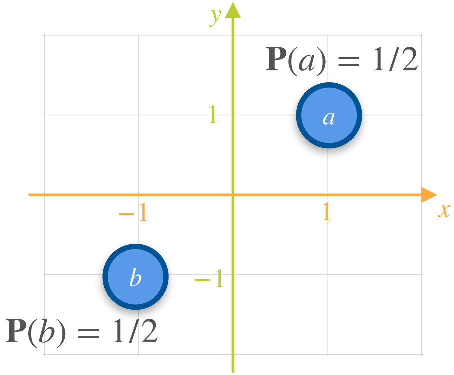

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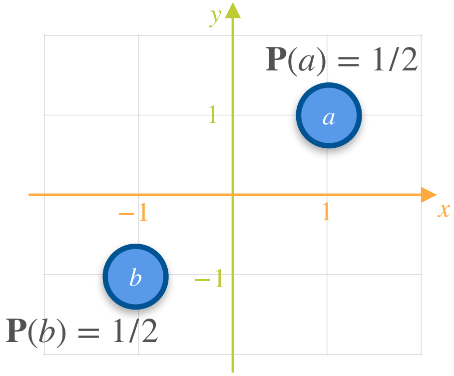

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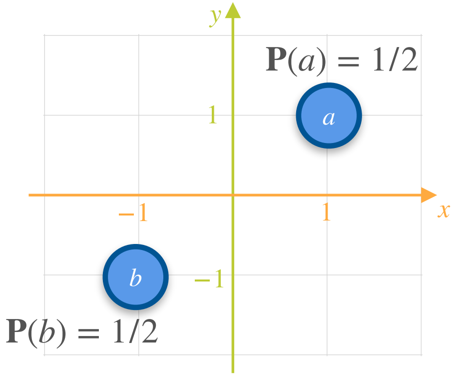

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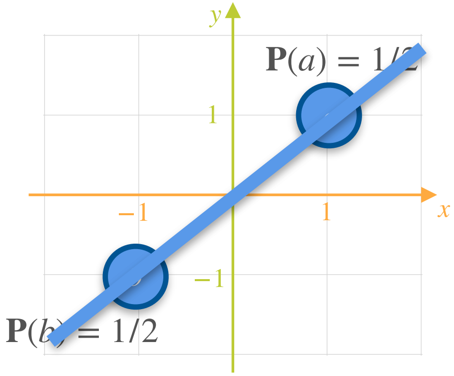

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Covariance of a Probability Distribution: Motivation

GAME 2

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Covariance of a Probability Distribution: Motivation

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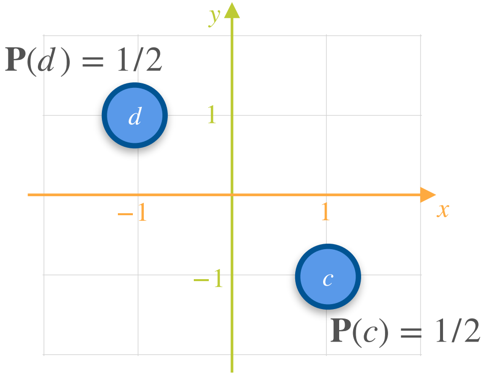

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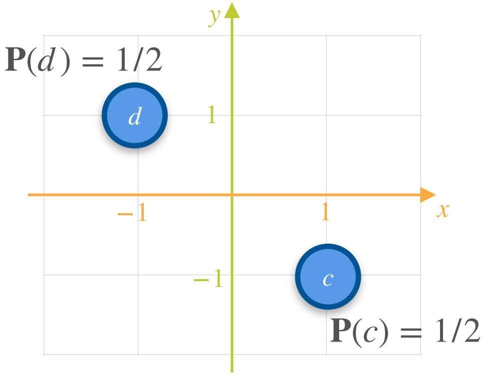

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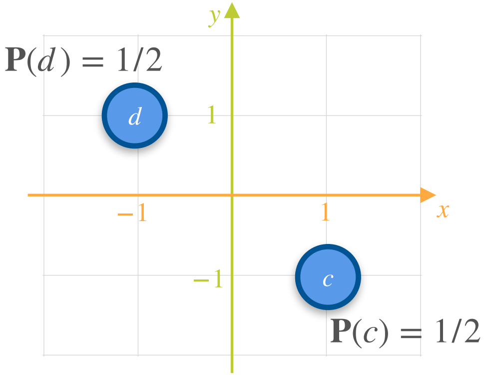

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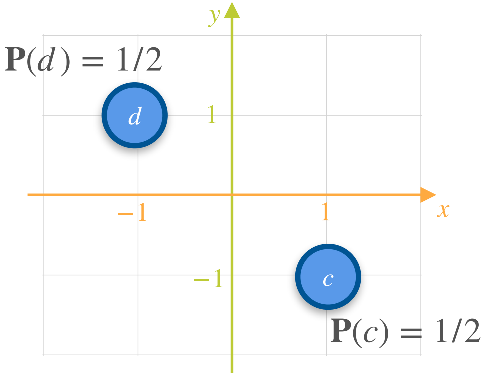

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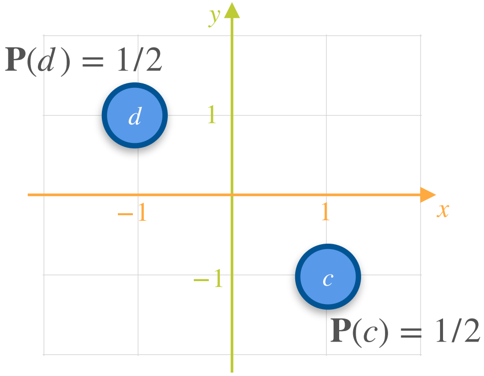

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1				
-1				

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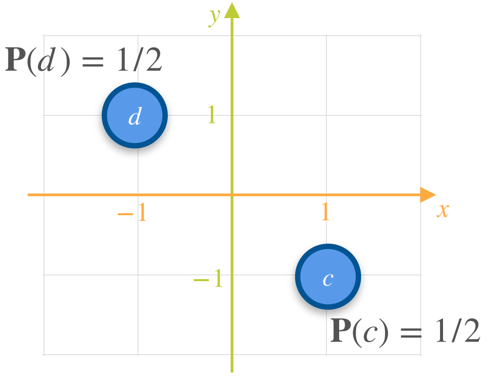

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1	-1			
-1	1			

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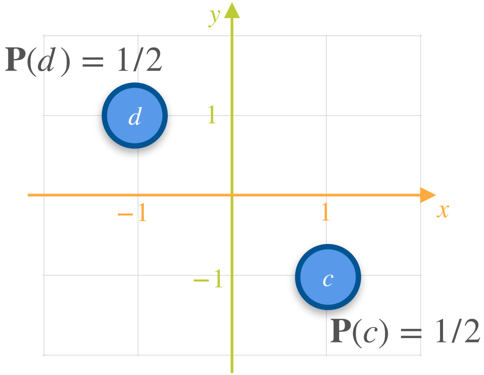

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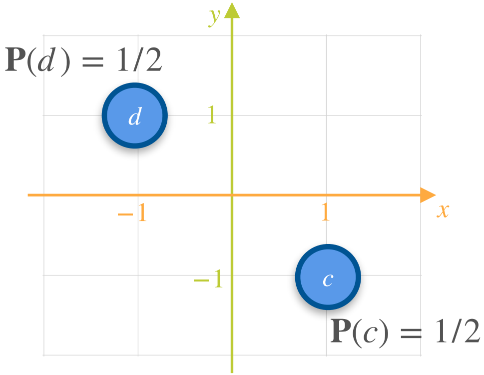

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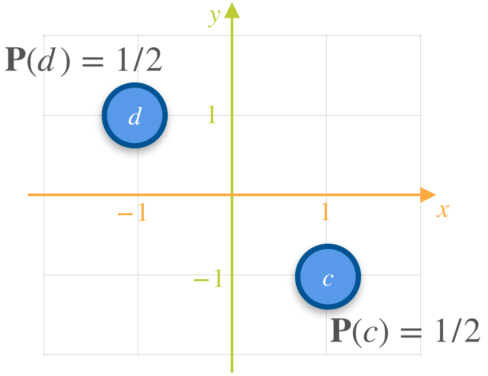
$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

x	y	$(x_i - \mu_x)$	$(y_i - \mu_y)$	$(x_i - \mu_x)(y_i - \mu_y)$
1	-1	1	-1	
-1	1	-1	1	

Covariance of a Probability Distribution: Motivation

GAME 2



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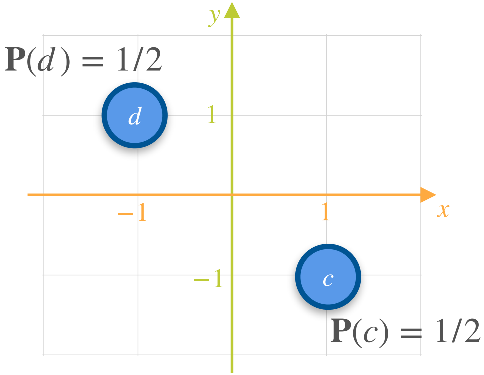
Covariance of a Probability Distribution: Motivation

GAME 2



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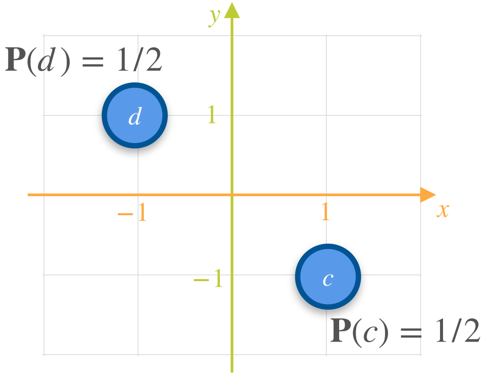
$$\sum (x_i - \mu_x)(y_i - \mu_y) \quad =$$

Covariance of a Probability Distribution: Motivation

GAME 2



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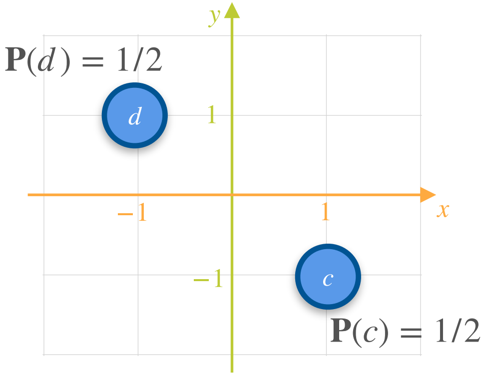

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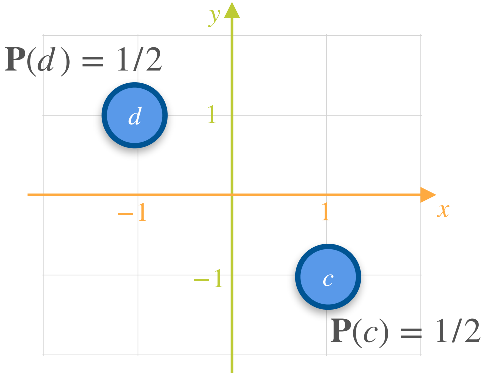
$$\sum (x_i - \mu_x)(y_i - \mu_y) \quad = -2$$

Covariance of a Probability Distribution: Motivation

GAME 2



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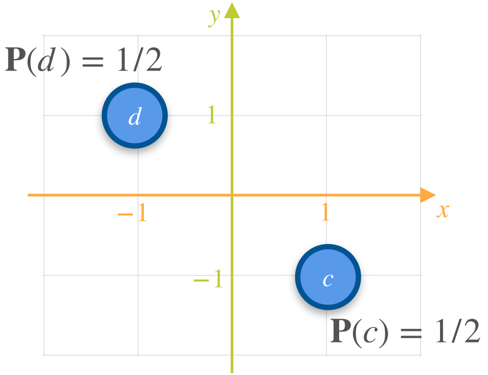
Covariance of a Probability Distribution: Motivation

GAME 2



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Covariance of a Probability Distribution: Motivation

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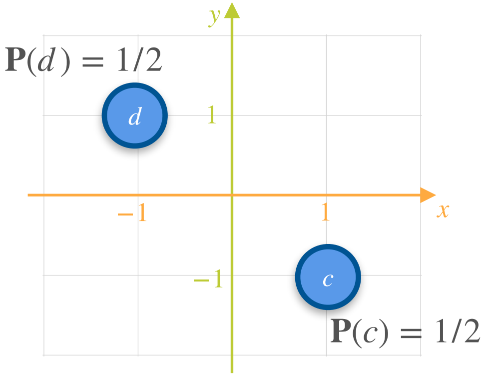

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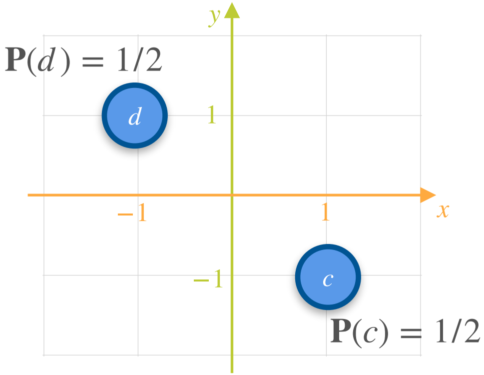
$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n} = \frac{-2}{2}$$

Covariance of a Probability Distribution: Motivation

GAME 2



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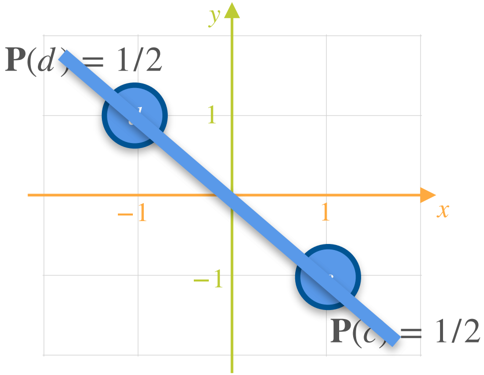
Covariance of a Probability Distribution: Motivation

GAME 2



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Covariance of a Probability Distribution: Motivation

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Covariance of a Probability Distribution: Motivation

GAME 3

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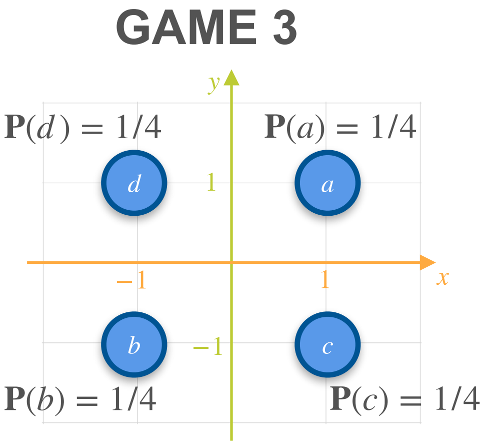
Covariance of a Probability Distribution: Motivation



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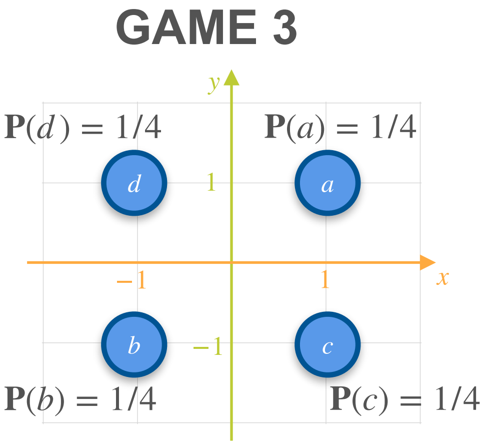
Covariance of a Probability Distribution: Motivation



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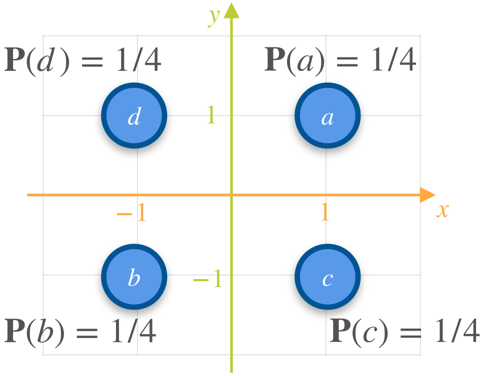
Covariance of a Probability Distribution: Motivation

GAME 3



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Covariance of a Probability Distribution: Motivation

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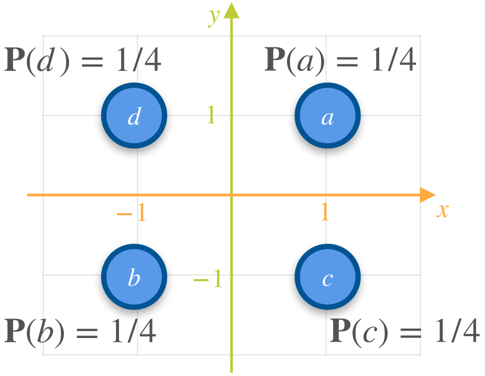

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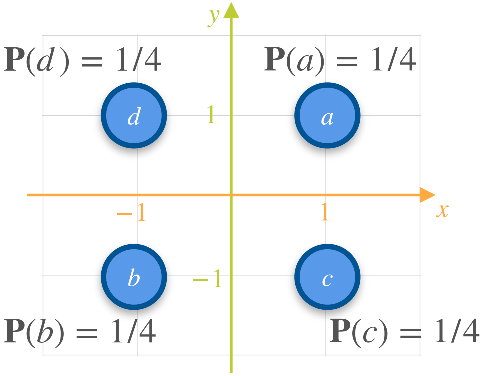
Covariance of a Probability Distribution: Motivation

GAME 3



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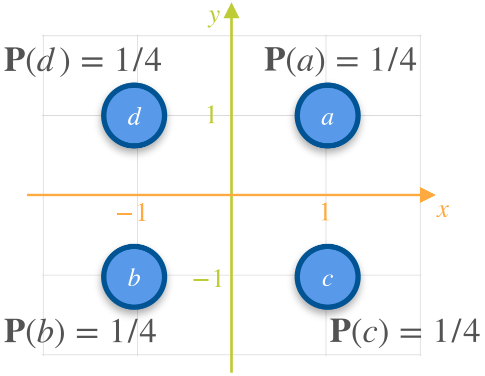
1				
1				
-1				
-1				

Covariance of a Probability Distribution: Motivation

GAME 3



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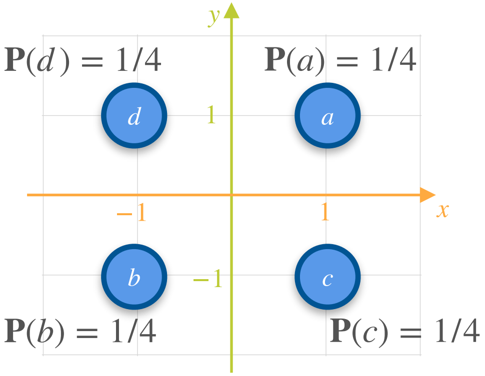
Covariance of a Probability Distribution: Motivation

GAME 3



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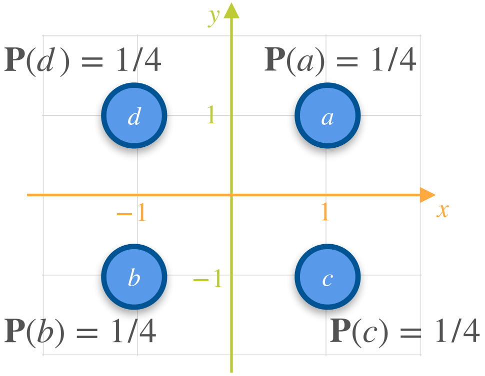
Covariance of a Probability Distribution: Motivation

GAME 3



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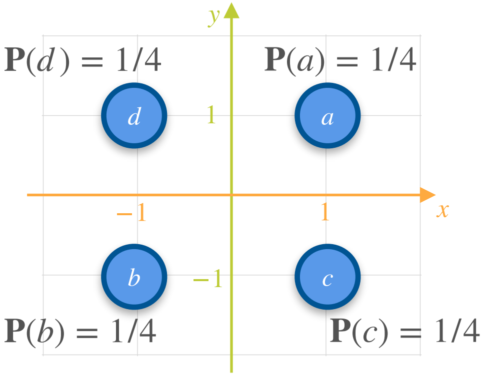
Covariance of a Probability Distribution: Motivation

GAME 3



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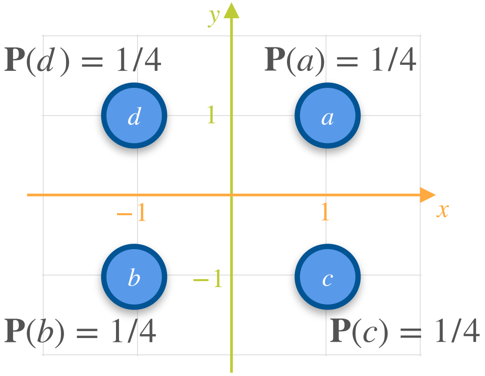
Covariance of a Probability Distribution: Motivation

GAME 3



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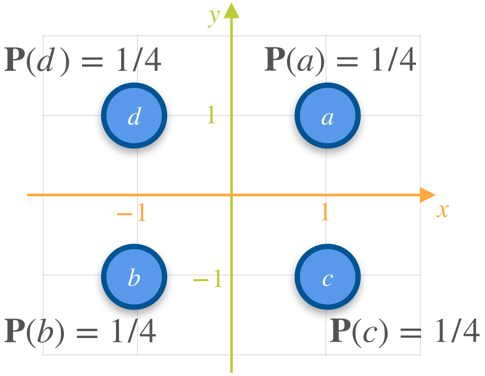
Covariance of a Probability Distribution: Motivation

GAME 3



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Covariance of a Probability Distribution: Motivation

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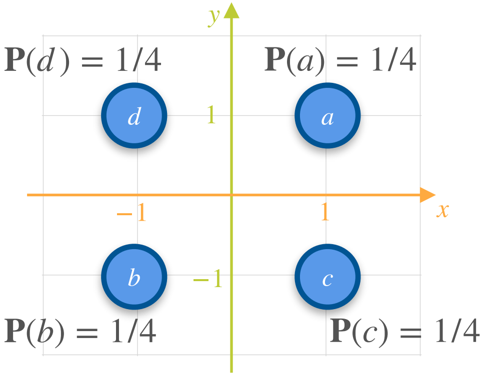

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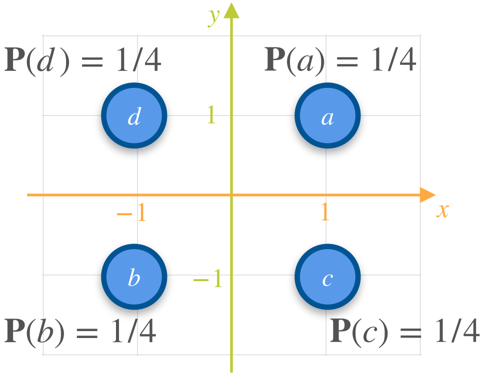
Covariance of a Probability Distribution: Motivation

GAME 3



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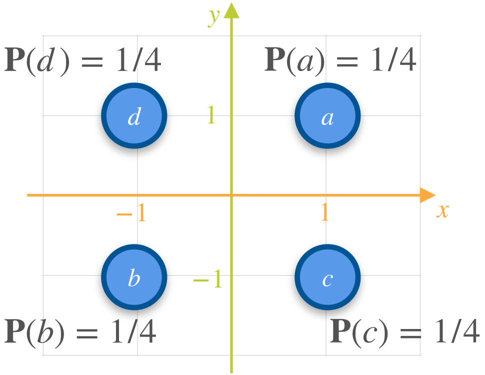
$$Cov(X, Y) = \sum (x_i - \mu_x)(y_i - \mu_y) = 0$$

Covariance of a Probability Distribution: Motivation

GAME 3



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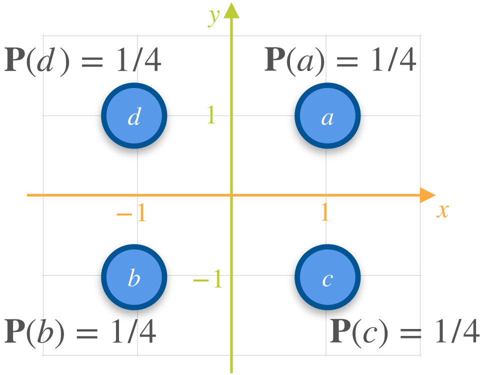
$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n} = 0$$

Covariance of a Probability Distribution: Motivation

GAME 3



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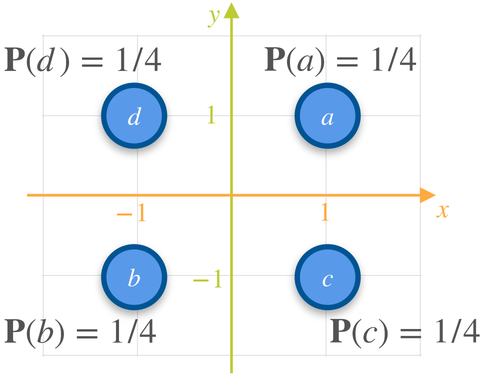
$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n} = \frac{0}{4}$$

Covariance of a Probability Distribution: Motivation

GAME 3



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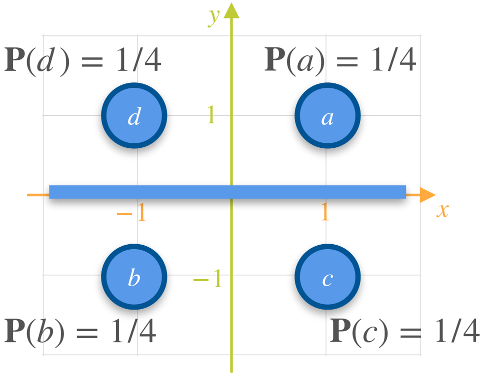
$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n} = \frac{0}{4} = 0$$

Covariance of a Probability Distribution: Motivation

GAME 3



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Covariance of a Probability Distribution: Motivation

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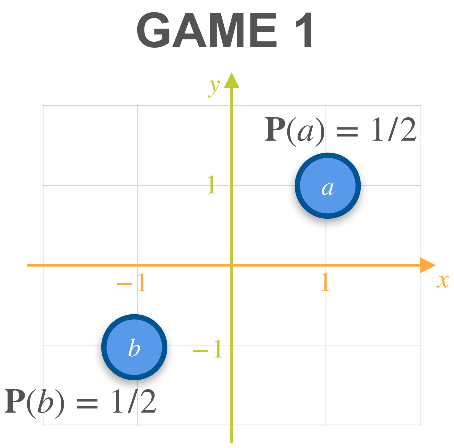
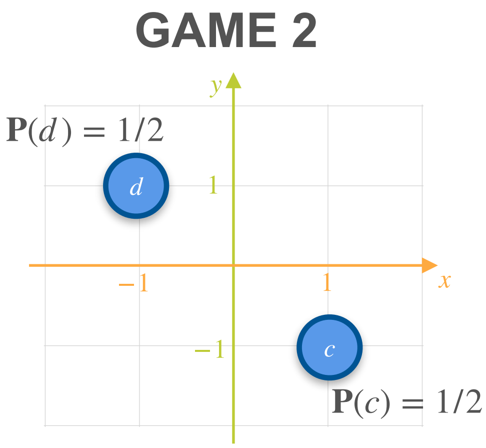
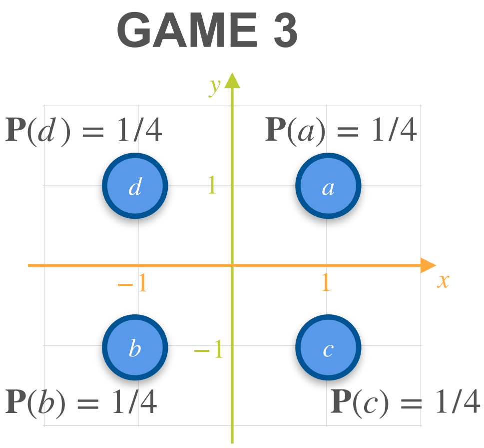
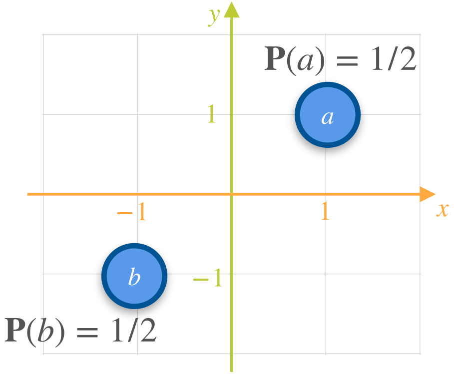






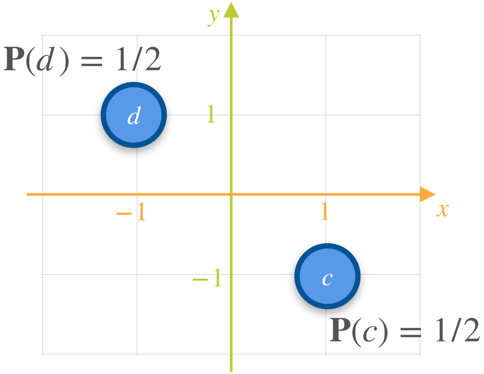
Covariance of a Probability Distribution: Motivation

GAME 1



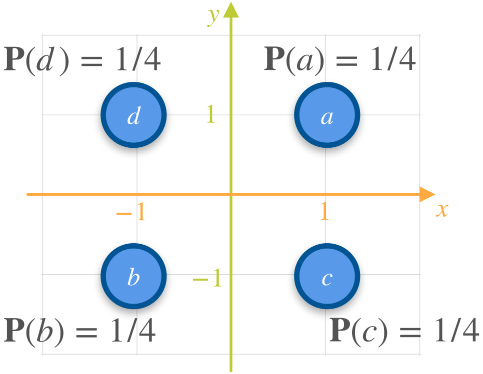
$$Cov(X, Y) = 1$$

GAME 2



$$Cov(X, Y) = -1$$

GAME 3



$$Cov(X, Y) = 0$$

Covariance of a Probability Distribution: Motivation

Covariance of a Probability Distribution: Motivation

GAME 4

Covariance of a Probability Distribution: Motivation

GAME 4

a: Both players win $1 each

Covariance of a Probability Distribution: Motivation

GAME 4

a: Both players win \$1 each

b: Both players lose \$1 each

Covariance of a Probability Distribution: Motivation

GAME 4

a: Both players win \$1 each

b: Both players lose \$1 each

c: Neither players wins nor lose anything

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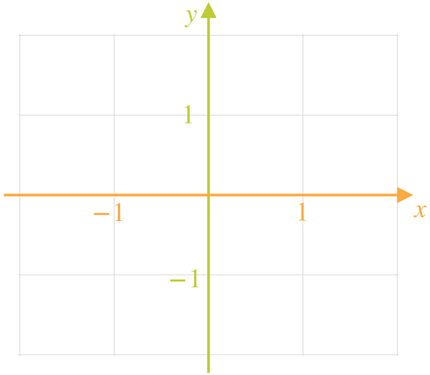

Covariance of a Probability Distribution: Motivation

GAME 4

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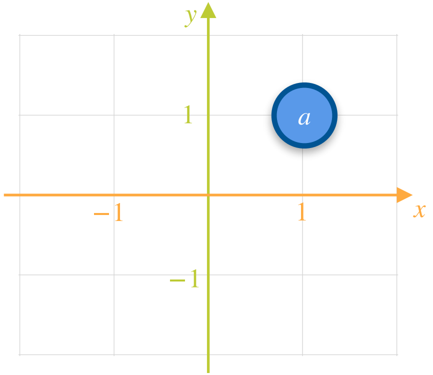

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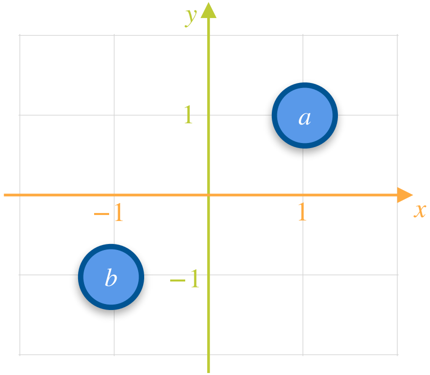

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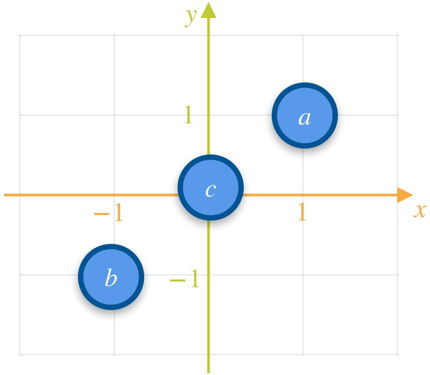

Covariance of a Probability Distribution: Motivation

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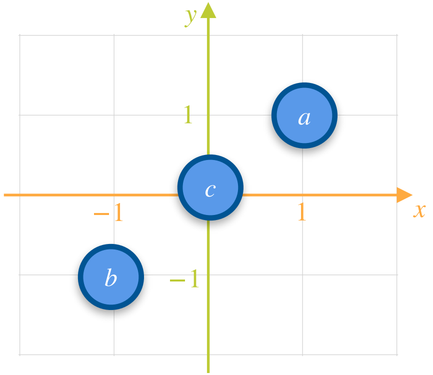

Unequal Probabiliites

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Covariance of a Probability Distribution: Motivation

GAME 4

a: Both players win \$1 each $\quad \mathbf{P}(a) = 1/2$

b: Both players lose \$1 each

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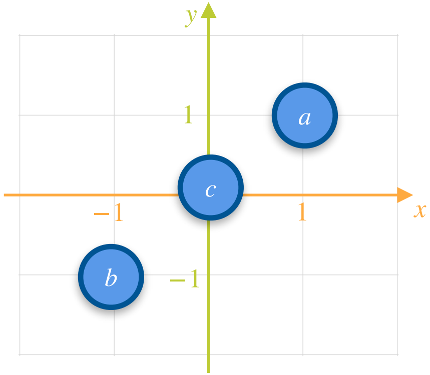

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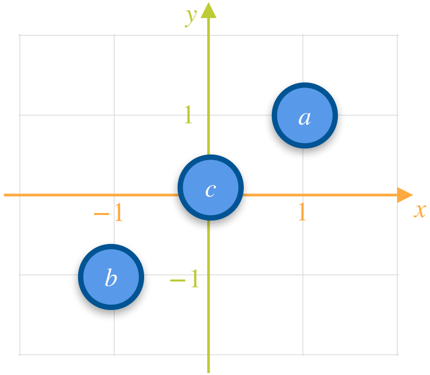

Unequal Probabiliites

Covariance of a Probability Distribution: Motivation

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a: Both players win \$1 each $\mathbf{P}(a) = 1/2$

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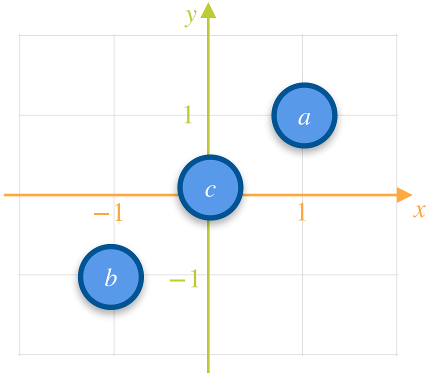

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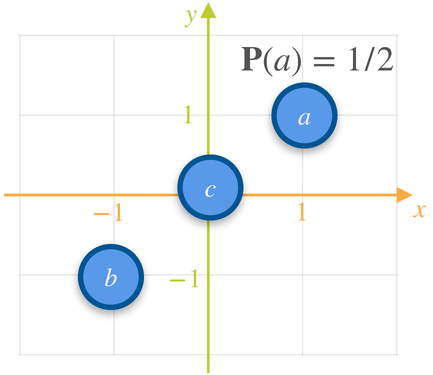

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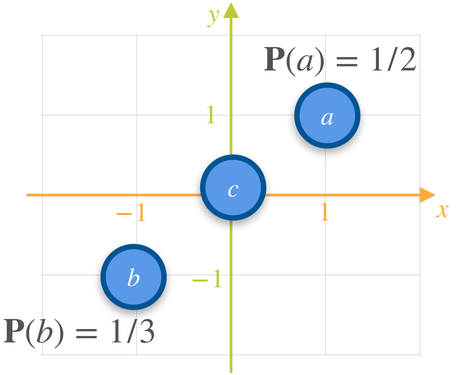

Unequal Probabiliites

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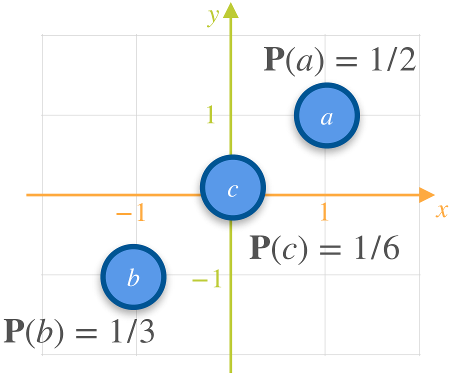

Unequal Probabiliites

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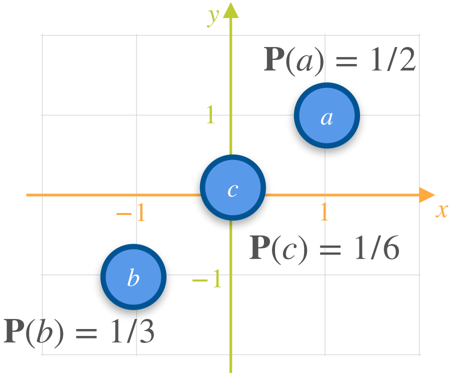

Unequal Probabiliites

$\mathbb{E}[X_4] =$

Covariance of a Probability Distribution: Motivation

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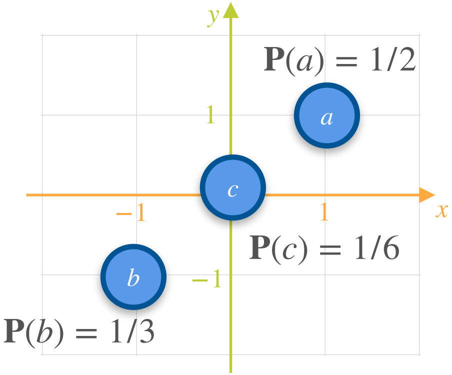

Unequal Probabiliites

$$\mathbb{E}[X_4] = \frac{1}{2}(1) + \frac{1}{6}(0) + \frac{1}{3}(-1) =$$

Covariance of a Probability Distribution: Motivation

GAME 4

a: Both players win \$1 each $\mathbf{P}(a) = 1/2$

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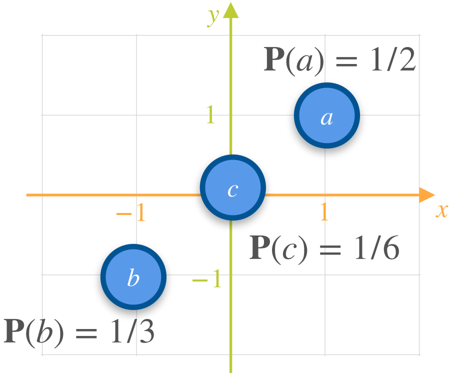

Unequal Probabiliites

$$\mathbb{E}[X_4] = \frac{1}{2}(1) + \frac{1}{6}(0) + \frac{1}{3}(-1) = \frac{1}{6}$$

Covariance of a Probability Distribution: Motivation

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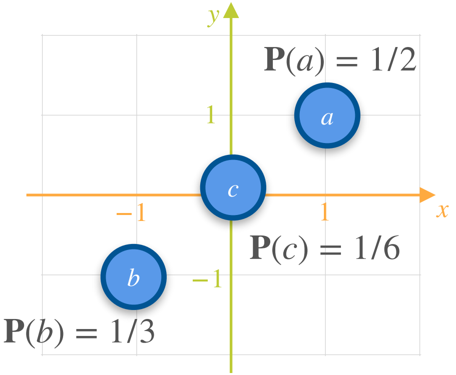

Unequal Probabiliites

$$\mathbb{E}[X_4] = \frac{1}{2}(1) + \frac{1}{6}(0) + \frac{1}{3}(-1) = \frac{1}{6}$$

$$\mathbb{E}[Y_4] =$$

Covariance of a Probability Distribution: Motivation

GAME 4

a: Both players win \$1 each $\mathbf{P}(a) = 1/2$

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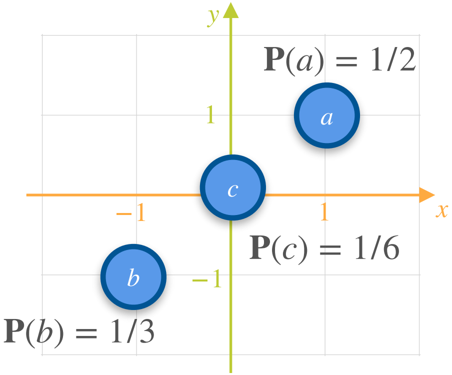

Unequal Probabiliites

$$\mathbb{E}[X_4] = \frac{1}{2}(1) + \frac{1}{6}(0) + \frac{1}{3}(-1) = \frac{1}{6}$$

$$\mathbb{E}[Y_4] = \frac{1}{2}(1) + \frac{1}{6}(0) + \frac{1}{3}(-1) = \frac{1}{6}$$

Covariance of a Probability Distribution: Motivation

GAME 4

Unequal Probabiliites

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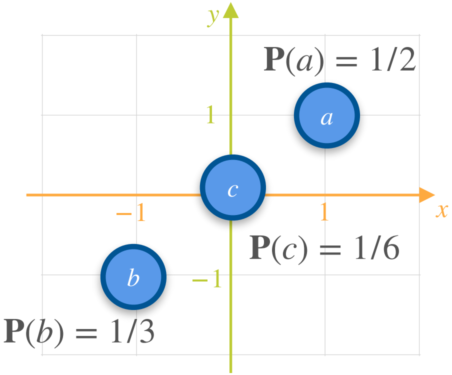

$$\mathbb{E}[X_4] = \frac{1}{6}$$

$$\mathbb{E}[Y_4] = \frac{1}{6}$$

Covariance of a Probability Distribution: Motivation

GAME 4

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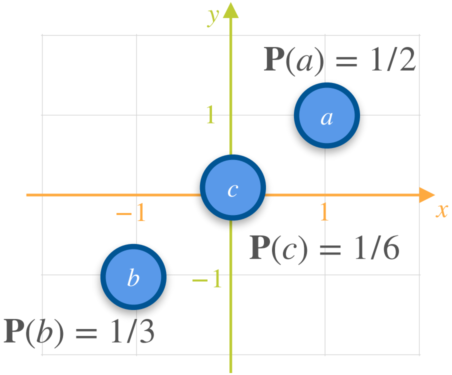

$$\mathbb{E}[X_4] = \frac{1}{6}$$

$$\mathbb{E}[Y_4] = \frac{1}{6}$$

Unequal Probabiliites

$$Var(X_4) =$$

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Covariance of a Probability Distribution: Motivation

GAME 4

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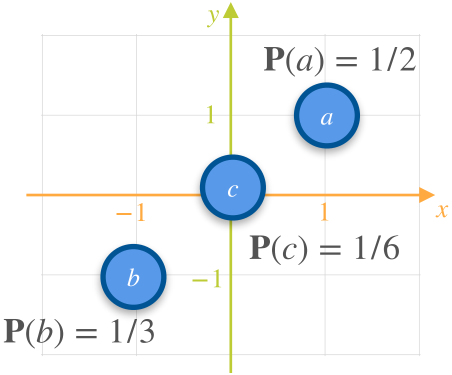

$$\mathbb{E}[X_4] = \frac{1}{6}$$

$$\mathbb{E}[Y_4] = \frac{1}{6}$$

Unequal Probabiliites

$$Var(X_4) = \sum_{n=1}^{N} (\mathbb{E}[X_4] - \mu_x)^2 \cdot \mathbf{P}(x_i)$$

Covariance of a Probability Distribution: Motivation

GAME 4

a: Both players win \$1 each $\mathbf{P}(a) = 1/2$

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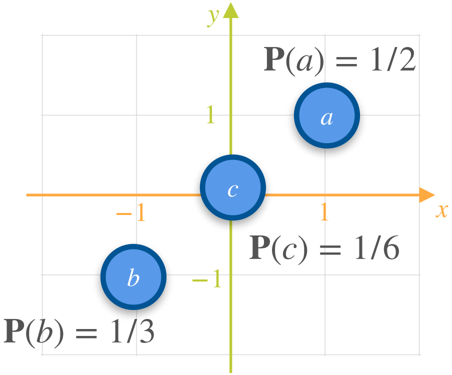

$$\mathbb{E}[X_4] = \frac{1}{6}$$

$$Var(X_4) =$$

$$\mathbb{E}[Y_4] = \frac{1}{6}$$

Covariance of a Probability Distribution: Motivation

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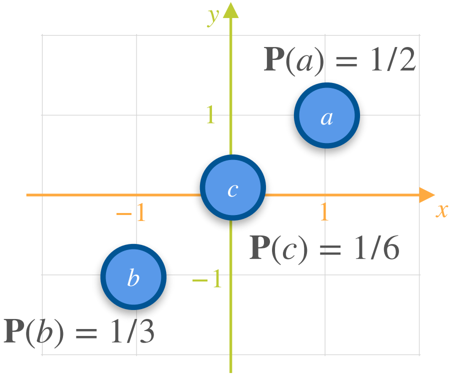

$$\mathbb{E}[X_4] = \frac{1}{6}$$

$$\mathbb{E}[Y_4] = \frac{1}{6}$$

Unequal Probabiliites

$$Var(X_4) = \sum_{n=1}^{N} (\mathbb{E}[X_4] - \mu_x)^2 \cdot \mathbf{P}(x_i)$$

$$Var(X_4) = \frac{1}{2}\left(1 - \frac{1}{6}\right)^2 + \frac{1}{6}\left(0 - \frac{1}{6}\right)^2 + \frac{1}{3}\left(-1) - \frac{1}{6}\right)^2$$

Covariance of a Probability Distribution: Motivation

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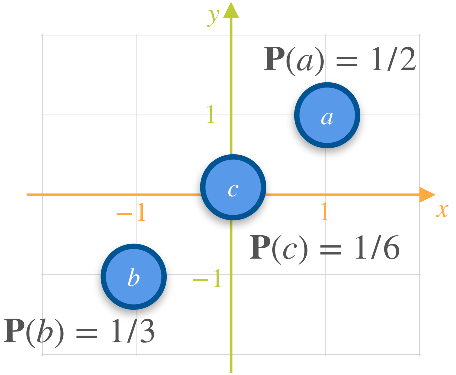

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Unequal Probabiliites

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$$= \ 0.806$$

Covariance of a Probability Distribution: Motivation

GAME 4

Unequal Probabiliites

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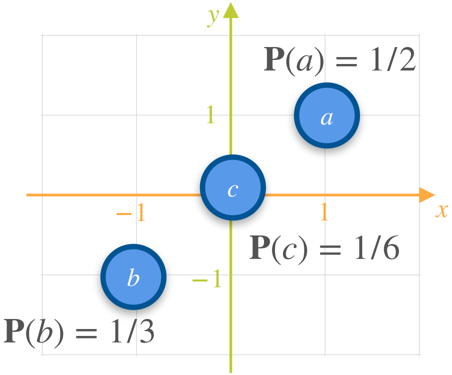

$$\mathbb{E}[X_4] = \frac{1}{6} \qquad Var(X_4) = 0.806$$

$$\mathbb{E}[Y_4] = \frac{1}{6}$$

Covariance of a Probability Distribution: Motivation

GAME 4

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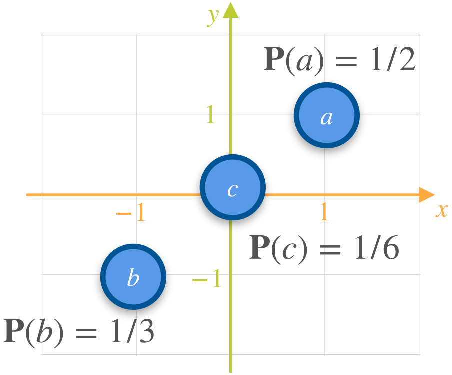

$$\mathbb{E}[X_4] = \frac{1}{6} \qquad Var(X_4) = 0.806$$

$$\mathbb{E}[Y_4] = \frac{1}{6} \qquad Var(Y_4) = 0.806$$

Covariance of a Probability Distribution: Motivation

GAME 4

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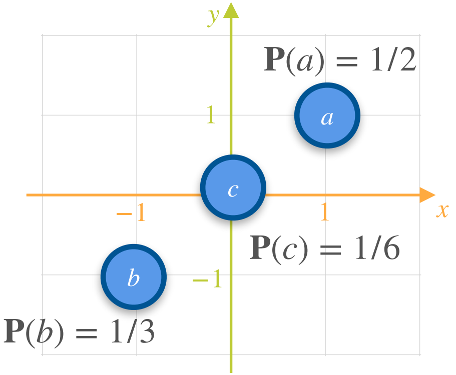

$$\mathbb{E}[X_4] = \frac{1}{6} \qquad Var(X_4) = 0.806$$

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$$\boxed{Cov(X, Y) = ?}$$

Covariance of Probability Distributions

Covariance of Probability Distributions

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n}$$

Covariance of Probability Distributions

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n} = \frac{1}{n} \sum (x_i - \mu_x)(y_i - \mu_y)$$

Covariance of Probability Distributions

$$Cov(X, Y) = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{n} = \boxed{\frac{1}{n}} \sum (x_i - \mu_x)(y_i - \mu_y)$$

equal probabilities

Covariance of Probability Distributions

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equal probabilities

$$Cov(X, Y) = \sum p_{XY}(x_i, y_i)(x_i - \mu_x)(y_i - \mu_y)$$

Covariance of Probability Distributions

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Covariance of Probability Distributions

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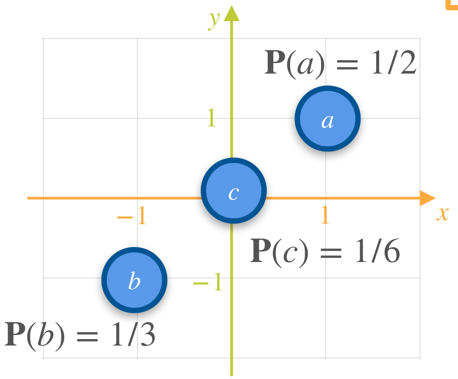
unequal probabilities

$$Cov(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

Covariance of a Probability Distribution: Motivation

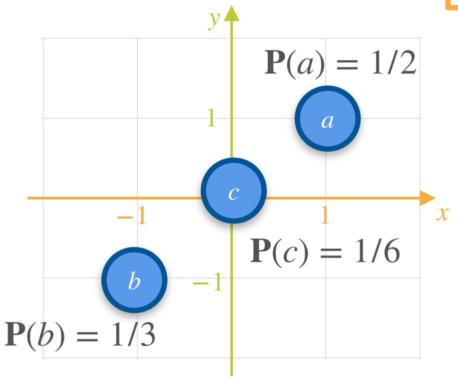
GAME 4

Unequal Probabiliites



Covariance of a Probability Distribution: Motivation

GAME 4



Unequal Probabiliites

$$Cov(X, Y) = \sum p_{XY}(x_i, y_i)(x_i - \mu_x)(y_i - \mu_y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

Covariance of a Probability Distribution: Motivation

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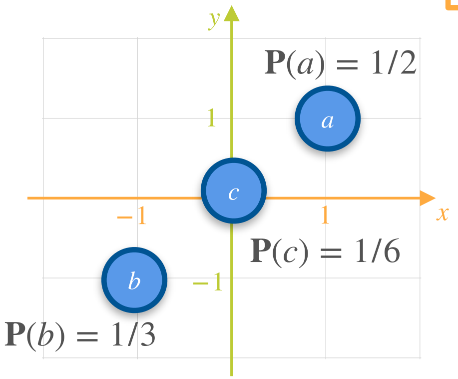


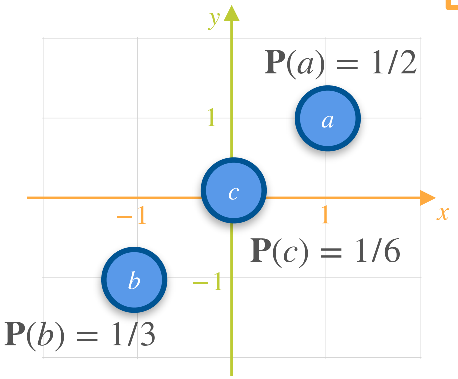
Unequal Probabiliites

$$Cov(X, Y) = \sum p_{XY}(x_i, y_i)(x_i - \mu_x)(y_i - \mu_y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

$$\mu_x = \mu_y = \mathbb{E}[X_4] = \mathbb{E}[Y_4] = 1/6$$

Covariance of a Probability Distribution: Motivation

GAME 4



Unequal Probabiliites

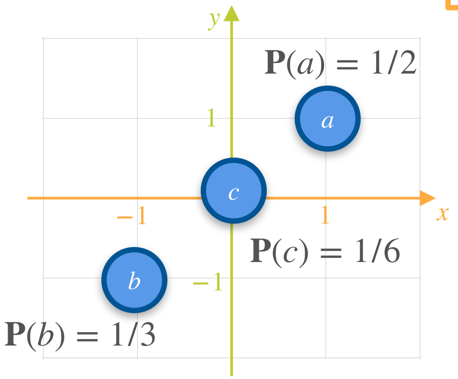
$$Cov(X, Y) = \sum p_{XY}(x_i, y_i)(x_i - \mu_x)(y_i - \mu_y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

$$\mu_x = \mu_y = \mathbb{E}[X_4] = \mathbb{E}[Y_4] = 1/6$$

$$Var(X_4) = Var(Y_4) = 0.806$$

Covariance of a Probability Distribution: Motivation

GAME 4



Unequal Probabiliites

$$Cov(X, Y) = \sum p_{XY}(x_i, y_i)(x_i - \mu_x)(y_i - \mu_y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

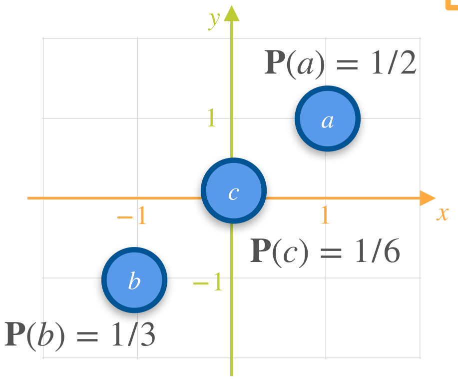
$$= \frac{1}{2}\left(1 - \frac{1}{6}\right)^2 + \frac{1}{6}\left(0 - \frac{1}{6}\right)^2 + \frac{1}{3}\left(-1) - \frac{1}{6}\right)^2$$

$$\mu_x = \mu_y = \mathbb{E}[X_4] = \mathbb{E}[Y_4] = 1/6$$

$$Var(X_4) = Var(Y_4) = 0.806$$

Covariance of a Probability Distribution: Motivation

GAME 4



Unequal Probabiliites

$$Cov(X, Y) = \sum p_{XY}(x_i, y_i)\boxed{(x_i - \mu_x)(y_i - \mu_y)} = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

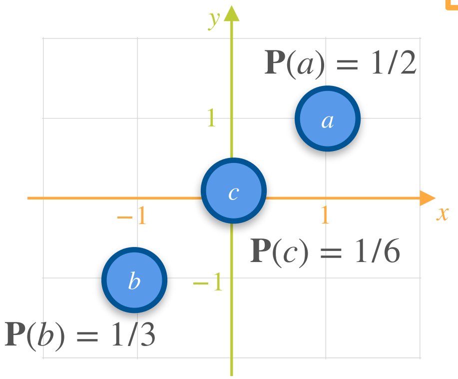
$$= \frac{1}{2}\boxed{\left(1 - \frac{1}{6}\right)^2} + \frac{1}{6}\left(0 - \frac{1}{6}\right)^2 + \frac{1}{3}\left((-1) - \frac{1}{6}\right)^2$$

$$\mu_x = \mu_y = \mathbb{E}[X_4] = \mathbb{E}[Y_4] = 1/6$$

$$Var(X_4) = Var(Y_4) = 0.806$$

Covariance of a Probability Distribution: Motivation

GAME 4



$$\mu_x = \mu_y = \mathbb{E}[X_4] = \mathbb{E}[Y_4] = 1/6$$

$$Var(X_4) = Var(Y_4) = 0.806$$

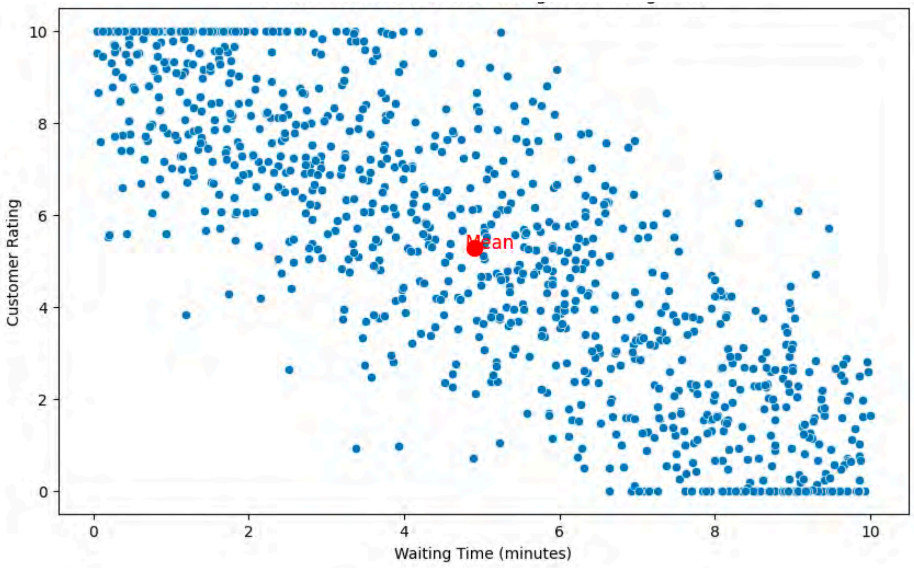
Unequal Probabiliites

$$Cov(X, Y) = \sum p_{XY}(x_i, y_i)\boxed{(x_i - \mu_x)(y_i - \mu_y)} = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

$$= \frac{1}{2}\boxed{\left(1 - \frac{1}{6}\right)^2} + \frac{1}{6}\left(0 - \frac{1}{6}\right)^2 + \frac{1}{3}\left(- 1) - \frac{1}{6}\right)^2$$

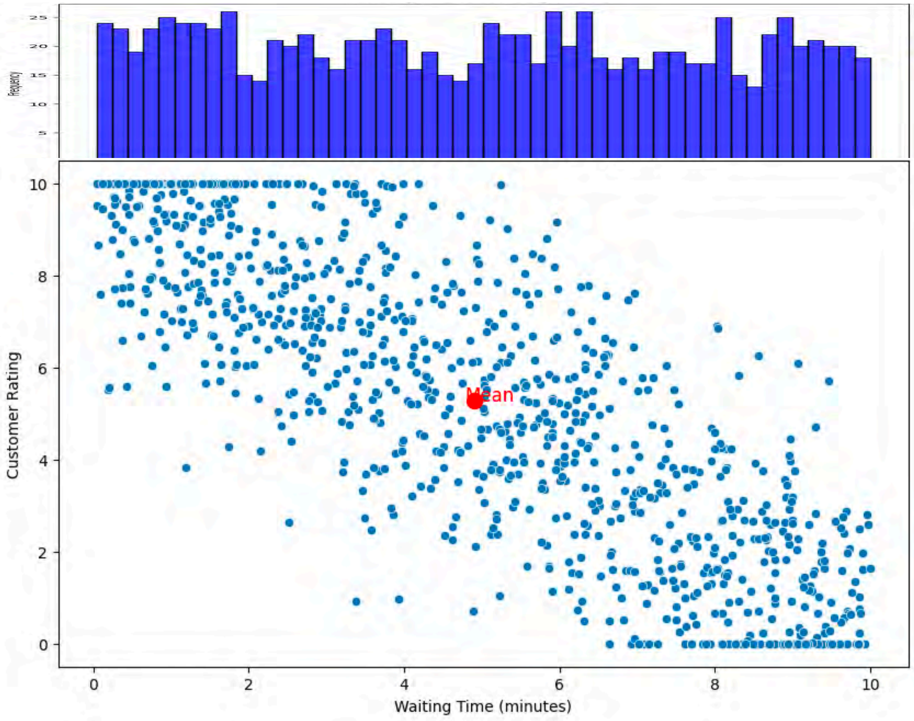
$$Cov(X, Y) = 0.806$$

Covariance?

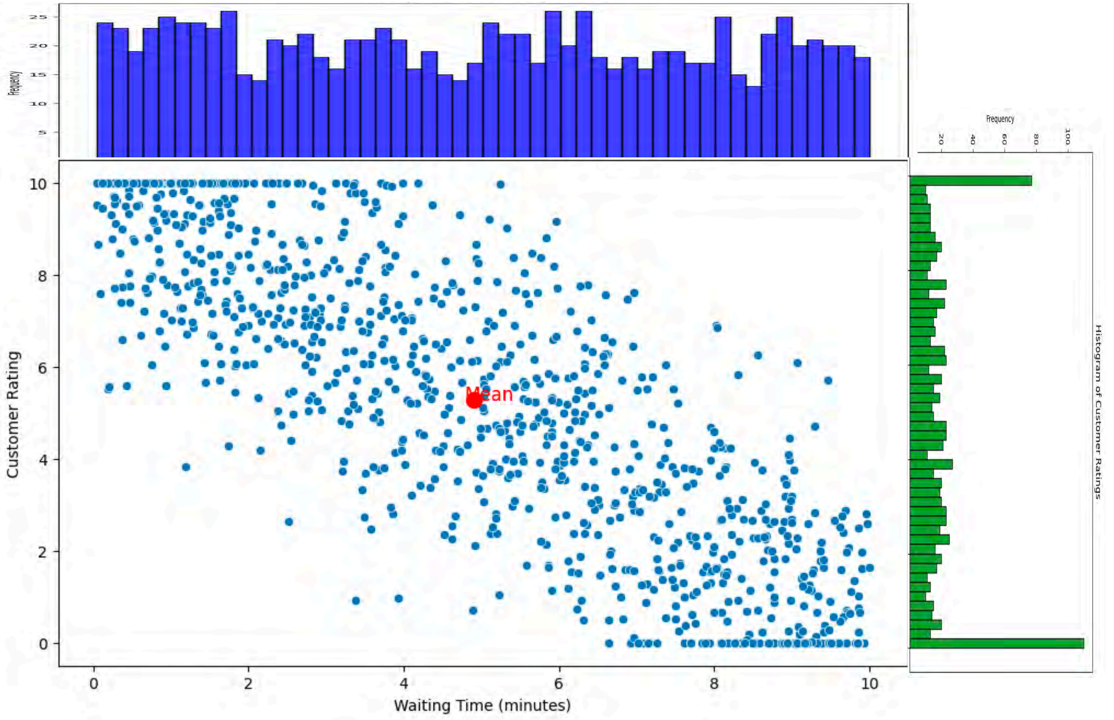


Mean
Customer Rating
Waiting Time (minutes)
0
2
4
6
8
10
0
2
4
6
8
10

Covariance?


Frequency
25
20
15
10
5
Customer Rating
10
8
6
4
2
0
Mean
0
2
4
6
8
10
Waiting Time (minutes)

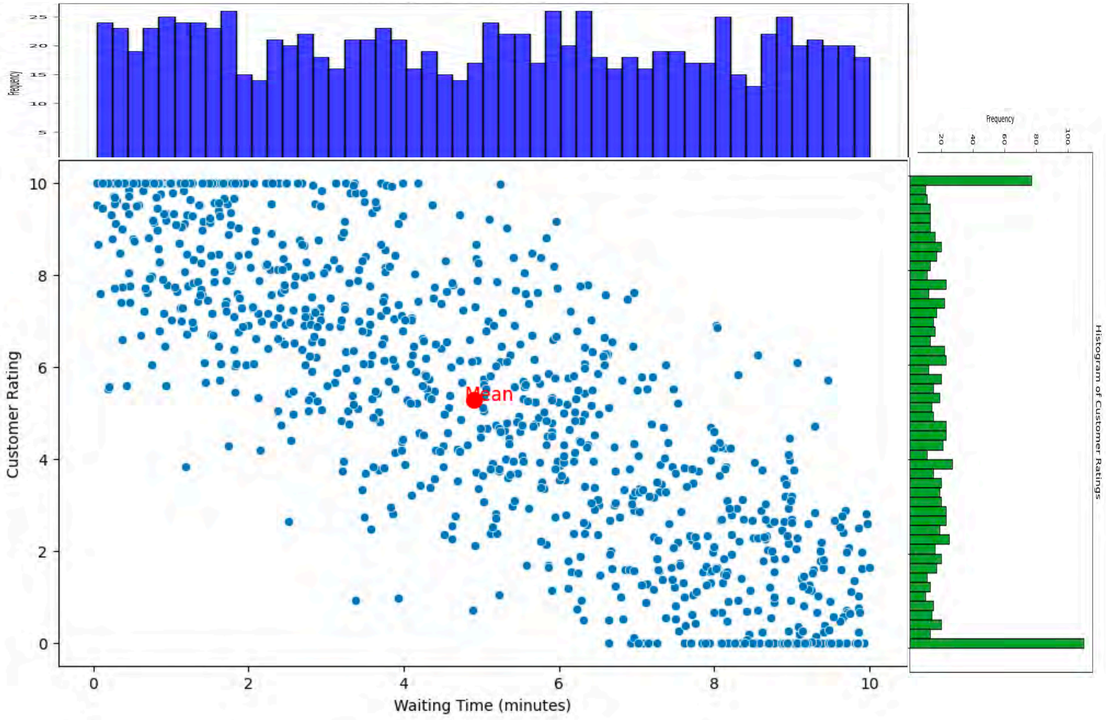
Covariance?



Frequency
Customer Rating
Waiting Time (minutes)
Mean
Histogram of Customer Ratings

Covariance?

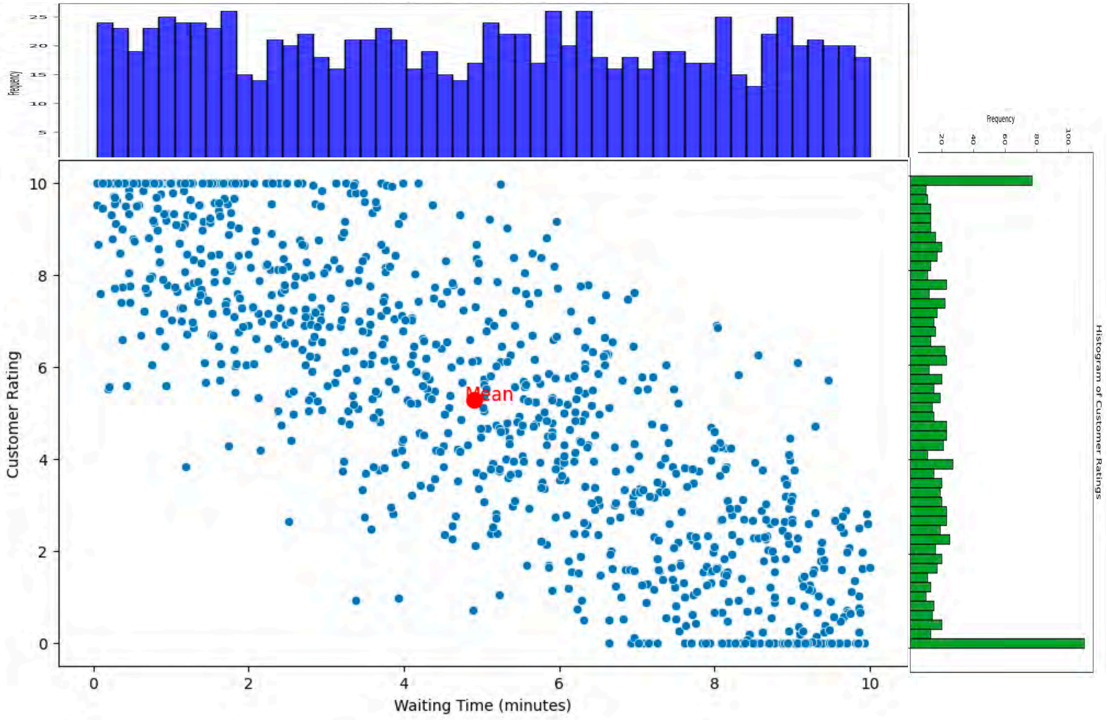
$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$



Covariance?

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$



Covariance?

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

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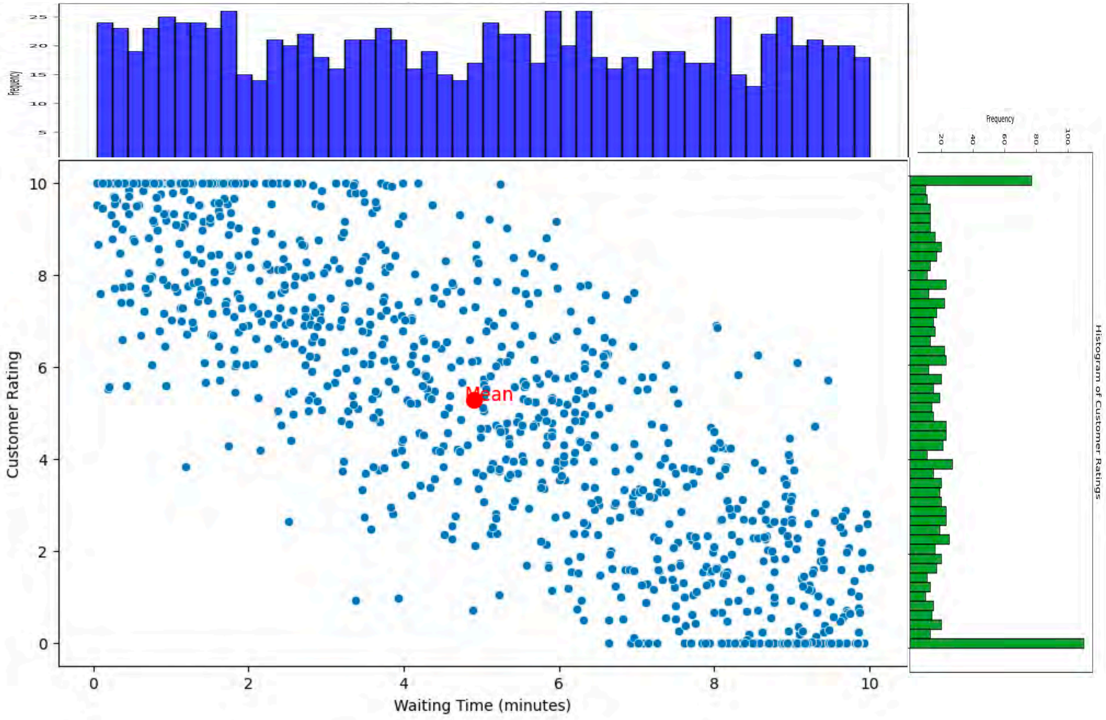


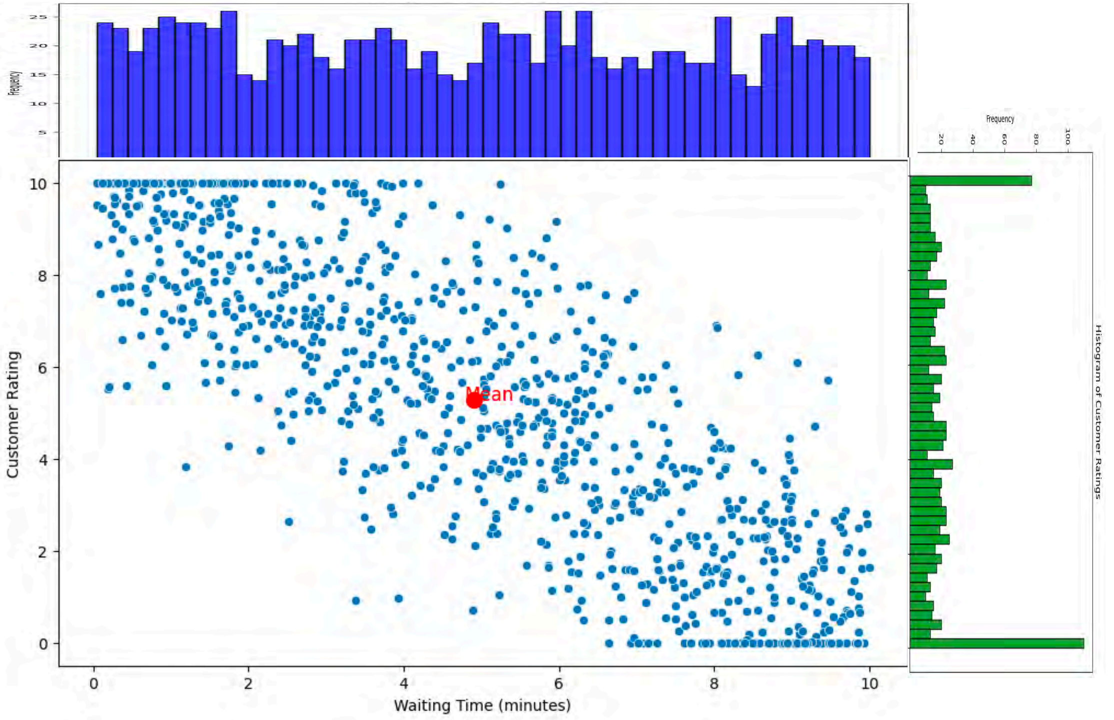
Covariance?

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

$$Cov(X, Y) = -7.878$$



Covariance?

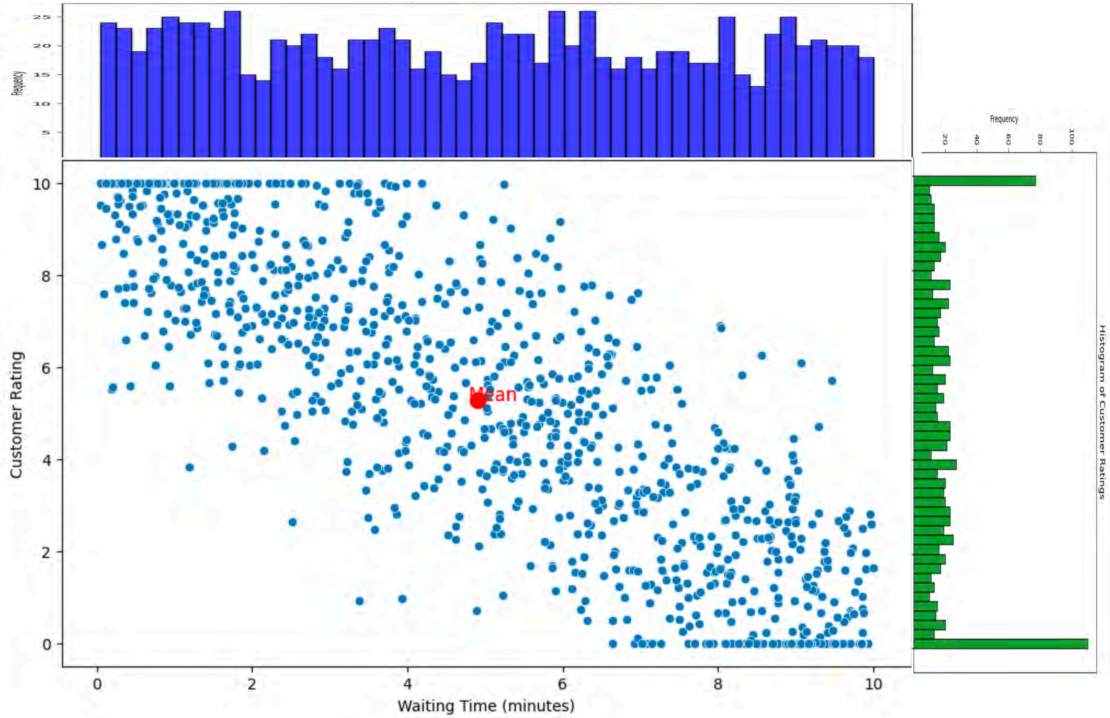
$\mathbb{E}(X) = 4.903$

$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$

$Cov(X, Y) = 18.014 - (4.903)(5.280)$

$Cov(X, Y) = 18.014 - (4.903)(5.280)$

$Cov(X, Y) = -7.878$



Covariance?

$$\mathbb{E}(X) = 4.903$$

$$\mathbb{E}(Y) = 5.280$$

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

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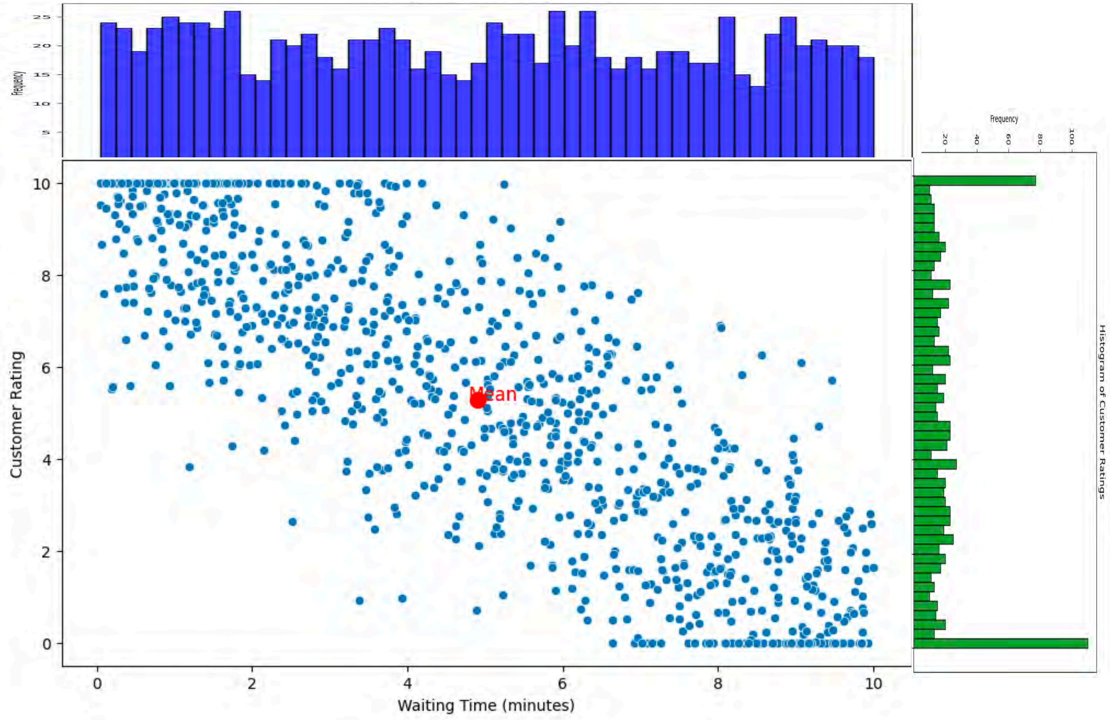

Covariance?

$$\mathbb{E}(X) = 4.903$$

$$\mathbb{E}(Y) = 5.280$$

$$\mathbb{E}(XY) = 18.014$$

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

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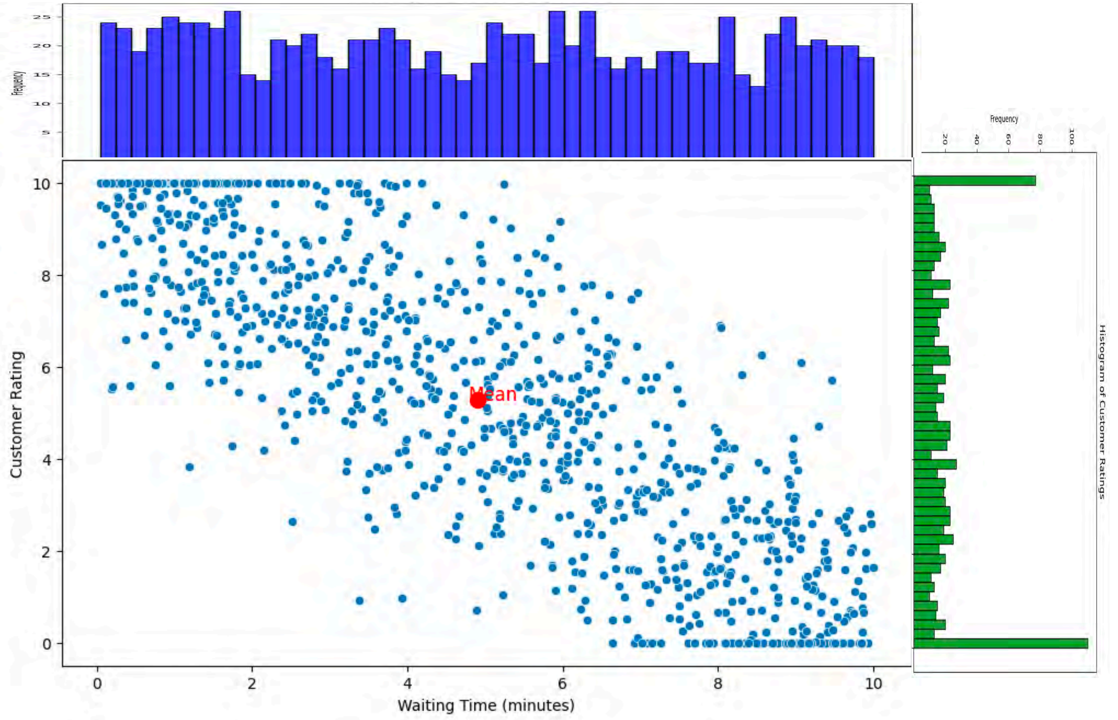


Covariance of a Joint Continuous Distribution

Covariance of a Joint Continuous Distribution

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

Covariance of a Joint Continuous Distribution

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$\mathbb{E}(X) = 4.903$$

Covariance of a Joint Continuous Distribution

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$\mathbb{E}(X) = 4.903$$

$$\mathbb{E}(Y) = 5.280$$

Covariance of a Joint Continuous Distribution

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$\mathbb{E}(X) = 4.903$$

$$\mathbb{E}(Y) = 5.280$$

$$\mathbb{E}(XY) = 18.014$$

Covariance of a Joint Continuous Distribution

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$\mathbb{E}(X) = 4.903$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

$$\mathbb{E}(Y) = 5.280$$

$$\mathbb{E}(XY) = 18.014$$

Covariance of a Joint Continuous Distribution

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$\mathbb{E}(X) = 4.903$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

$$\mathbb{E}(Y) = 5.280$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

$$\mathbb{E}(XY) = 18.014$$

Covariance of a Joint Continuous Distribution

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$\mathbb{E}(X) = 4.903$$

$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

$$\mathbb{E}(Y) = 5.280$$

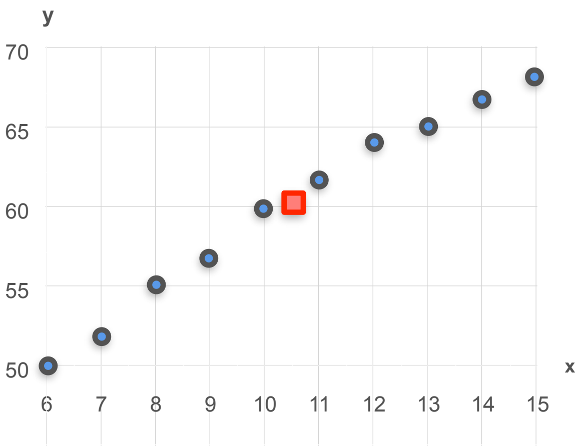
$$Cov(X, Y) = 18.014 - (4.903)(5.280)$$

$$\mathbb{E}(XY) = 18.014$$

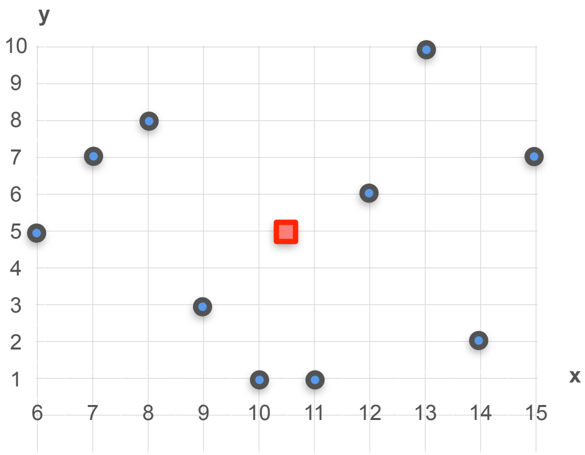
$$Cov(X, Y) = -7.878$$

Covariance Matrix

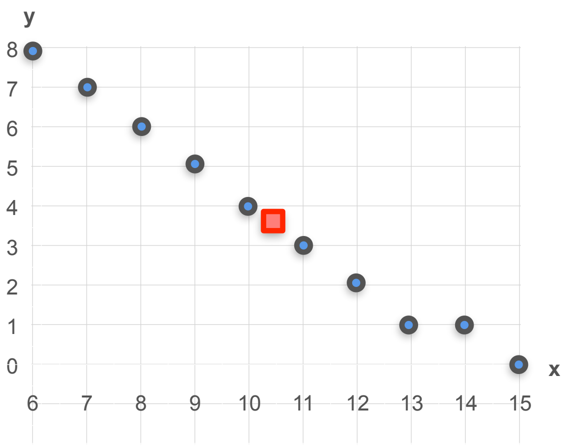
Covariance Matrix



Age vs Height

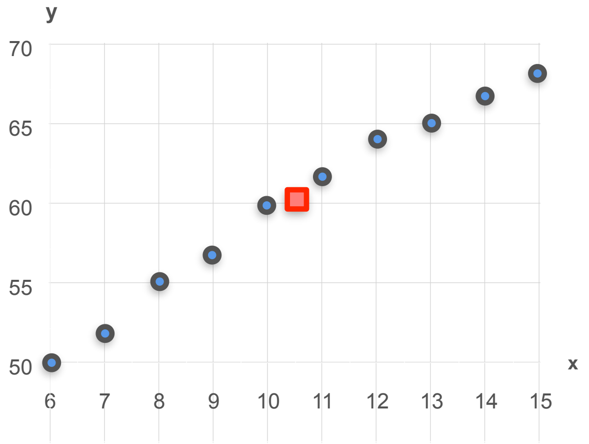


Age vs Grades



Age vs Naps per Day

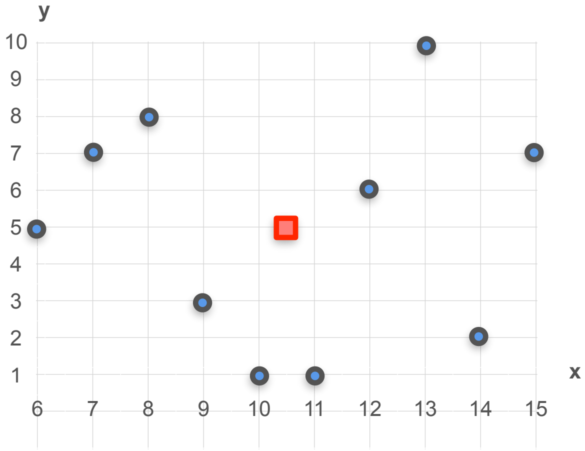
Covariance Matrix



Age vs Height

$Var(X) = 9.17$

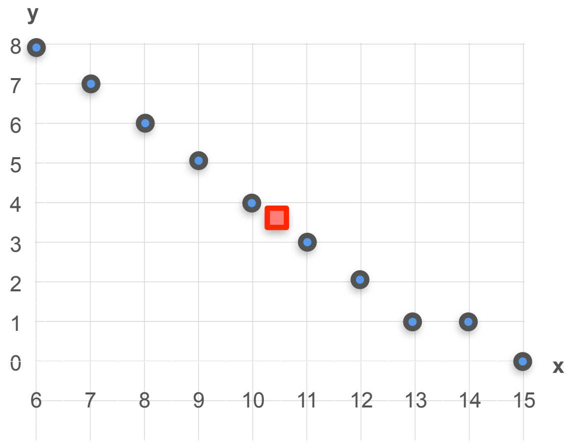
$Var(Y) = 39.56$



Age vs Grades

$Var(X) = 9.17$

$Var(Y) = 9.78$

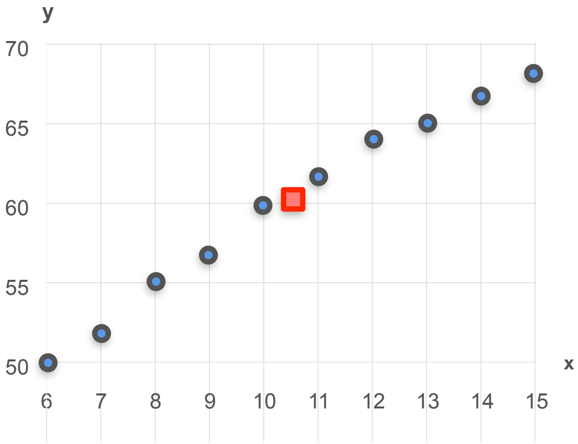


Age vs Naps per Day

$Var(X) = 9.17$

$Var(Y) = 7.57$

Covariance Matrix

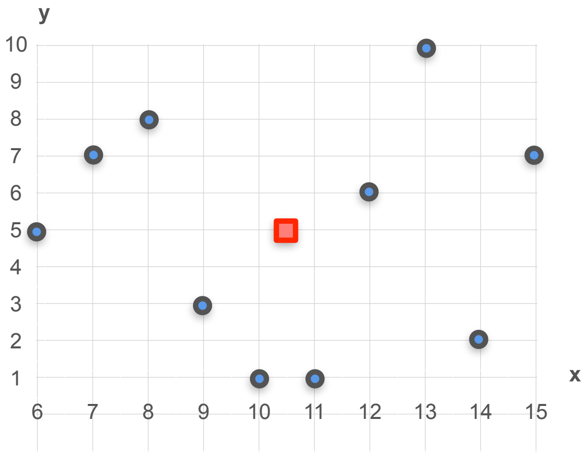


Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$

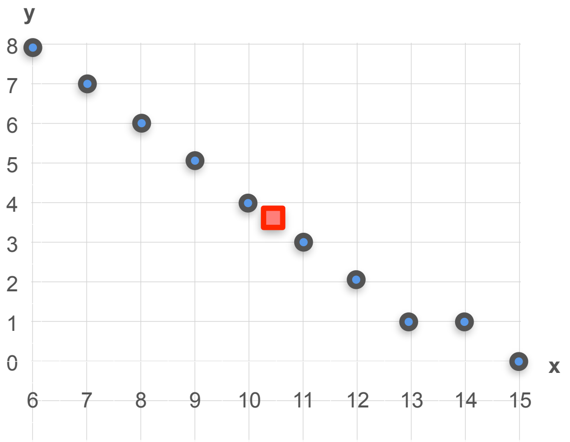


Age vs Grades

$Var(X) = 9.17$

$Var(Y) = 9.78$

$Cov(X, Y) = 0.1$



Age vs Naps per Day

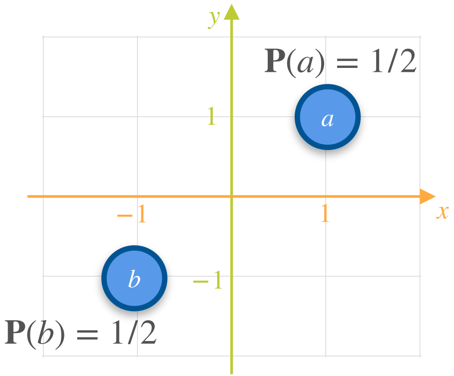
$Var(X) = 9.17$

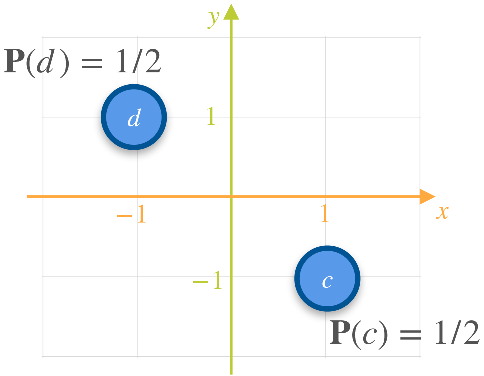
$Var(Y) = 7.57$

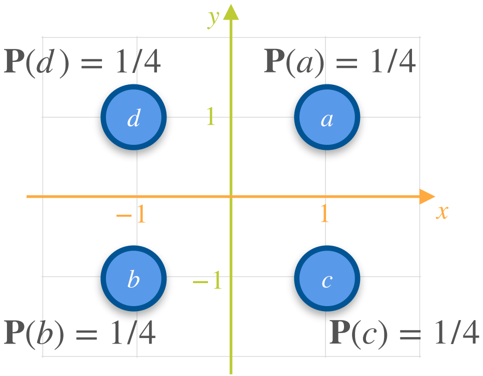
$Cov(X, Y) = -7.45$

Covariance Matrix

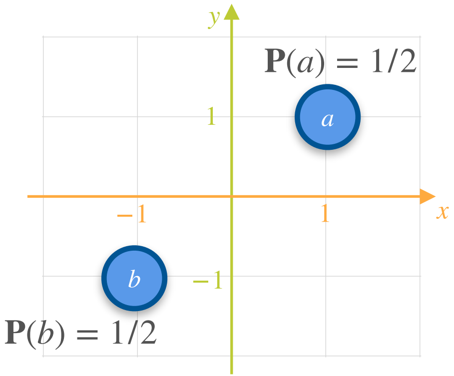
Covariance Matrix



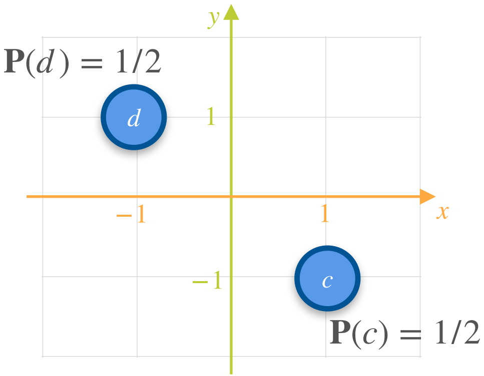




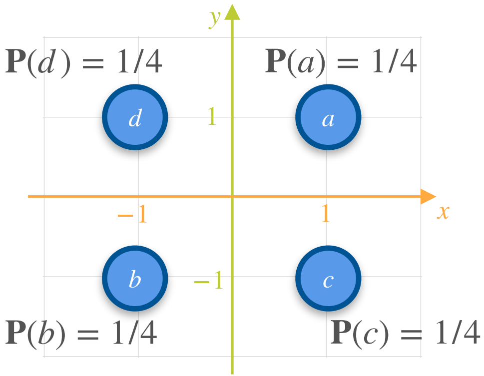
Covariance Matrix



$$Var(X) = Var(Y) = 1$$

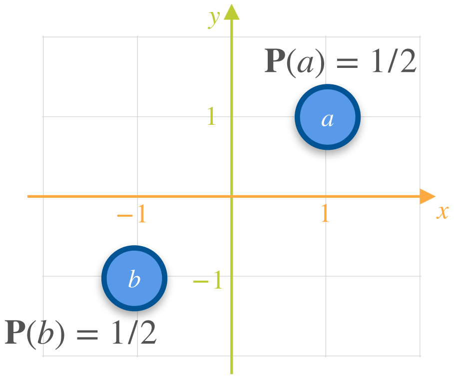


$$Var(X) = Var(Y) = 1$$

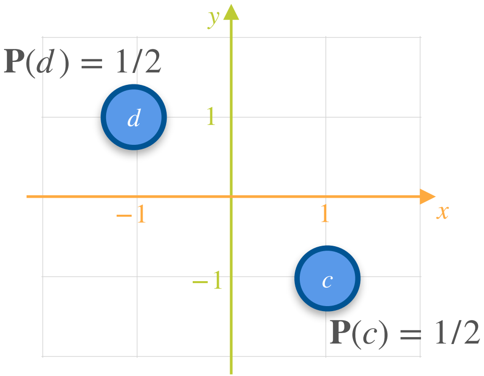


$$Var(X) = Var(Y) = 1$$

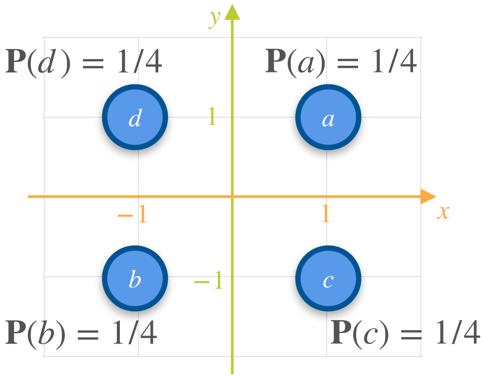
Covariance Matrix



$$Var(X) = Var(Y) = 1$$
$$Cov(X, Y) = 1$$



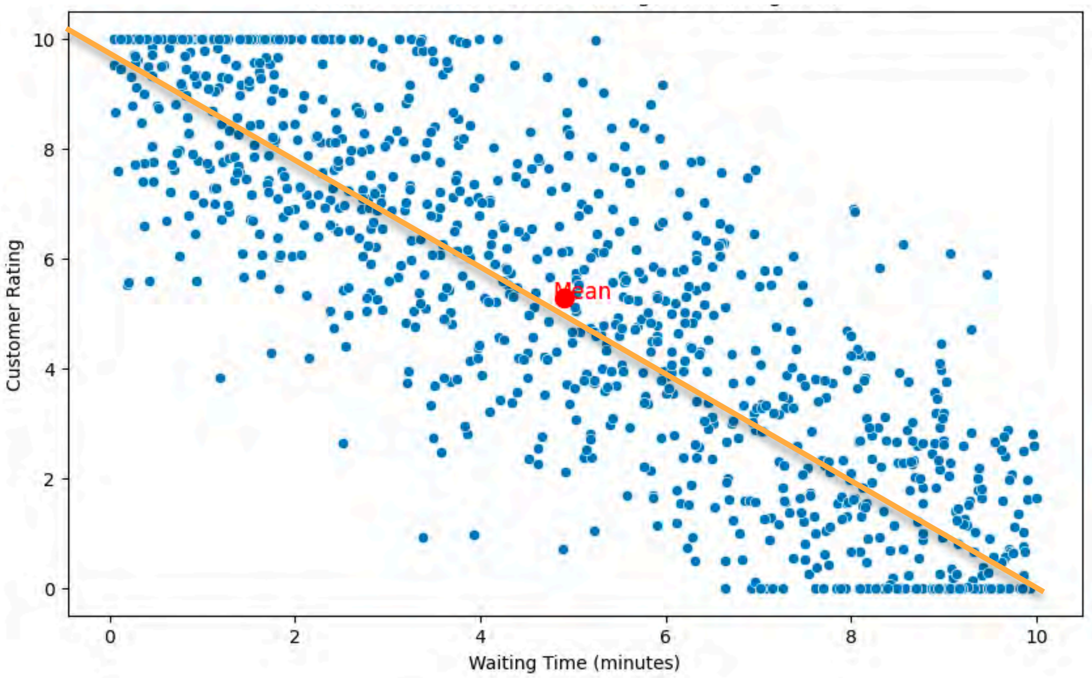
$$Var(X) = Var(Y) = 1$$
$$Cov(X, Y) = -1$$



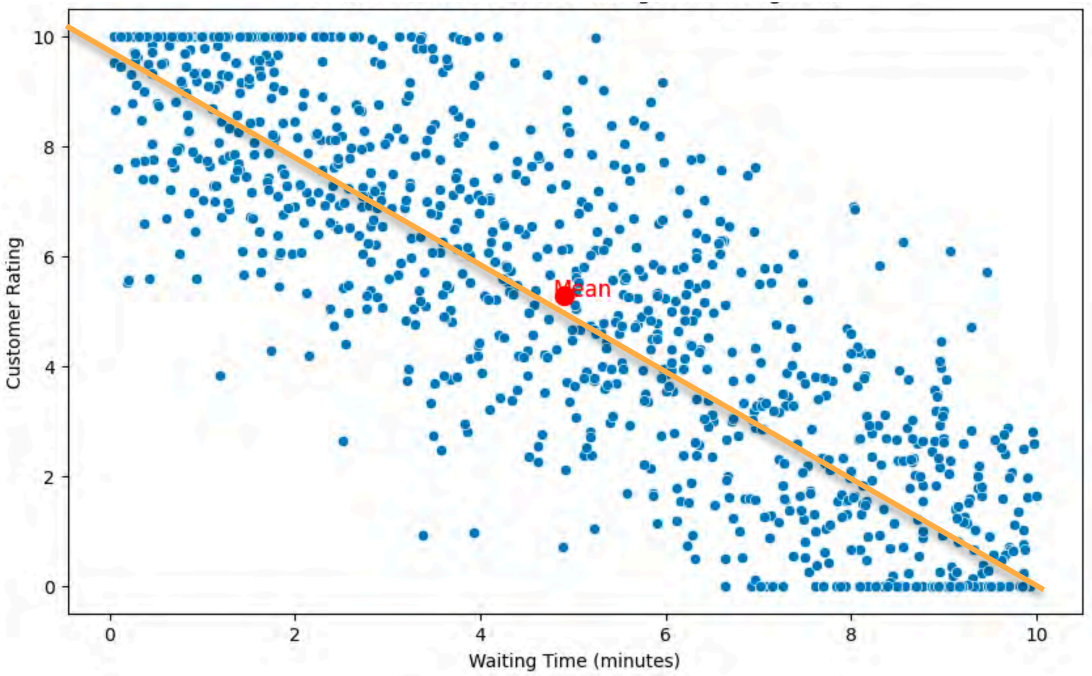
$$Var(X) = Var(Y) = 1$$
$$Cov(X, Y) = 0$$

Covariance Matrix

Covariance Matrix



Covariance Matrix

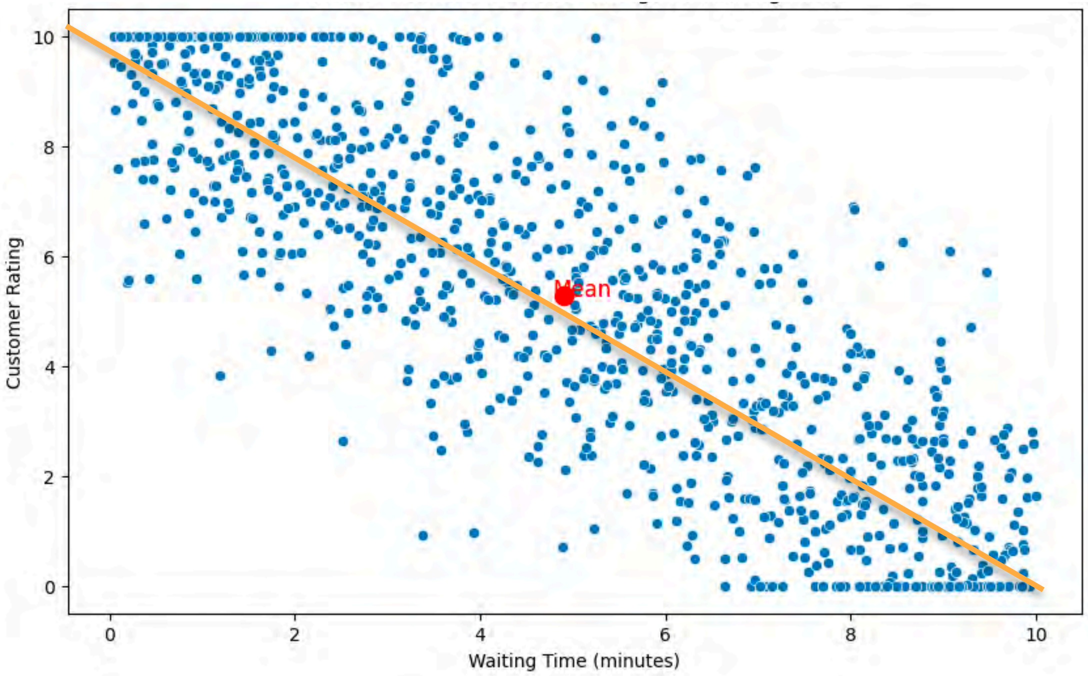


$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

$$Cov(X, Y) = -7.878$$

Covariance Matrix



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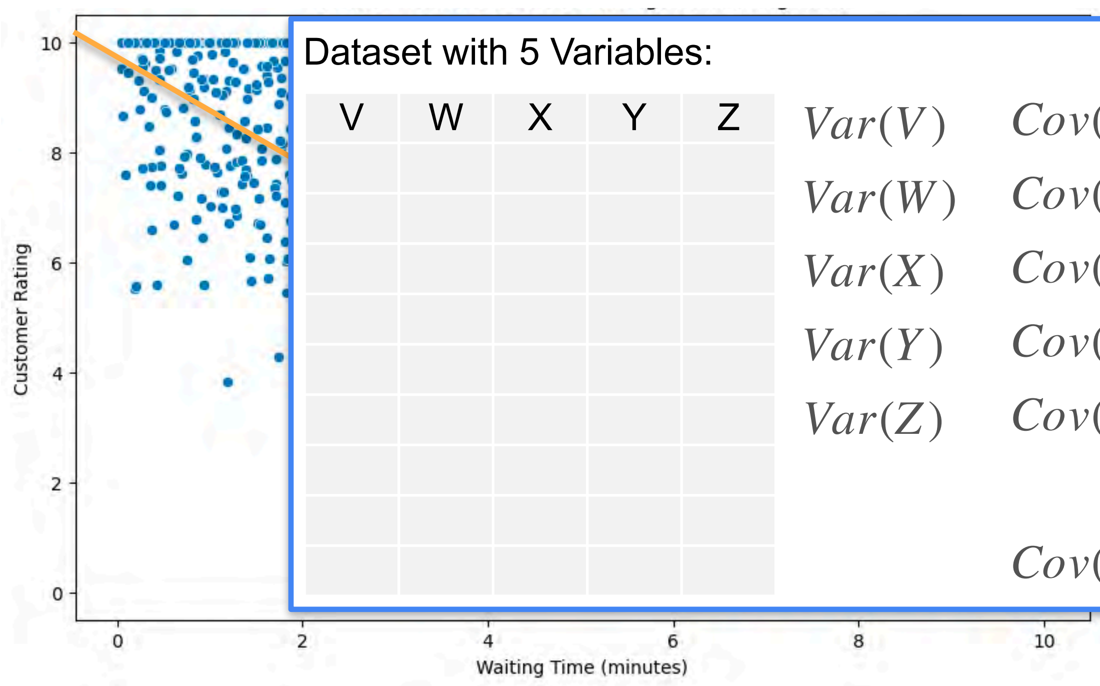
Covariance Matrix



Dataset with 5 Variables:

V	W	X	Y	Z

$Var(V)$ $Cov(V, W)$

$Var(W)$ $Cov(V, X)$

$Var(X)$ $Cov(V, Y)$

$Var(Y)$ $Cov(V, Z)$

$Var(Z)$ $Cov(W, X)$

$\vdots$

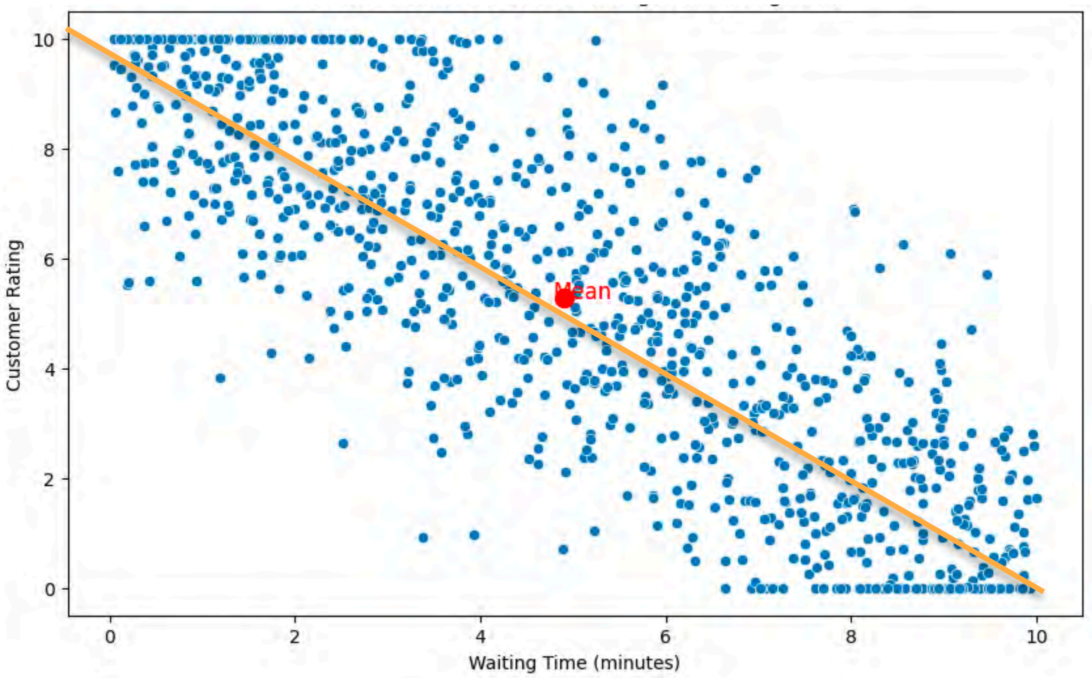
$Cov(Y, Z)$

$ar(X) = 8.526$

$ar(Y) = 10.163$

$ov(X, Y) = -7.878$

Covariance Matrix



$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

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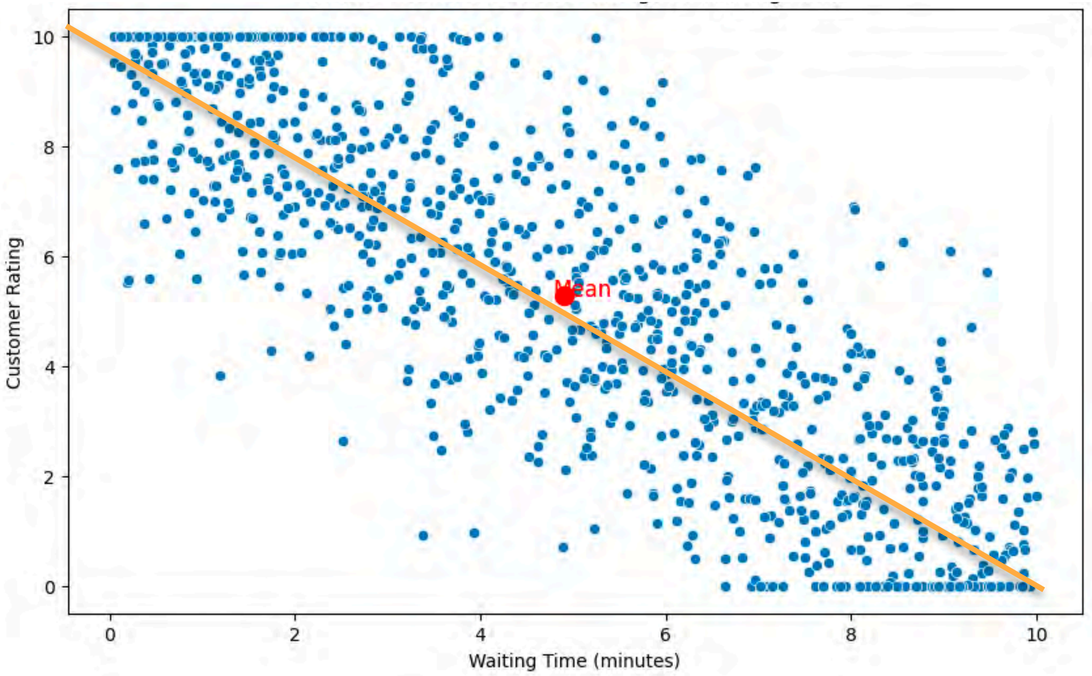
Covariance Matrix



$$Var(X) = 8.526$$

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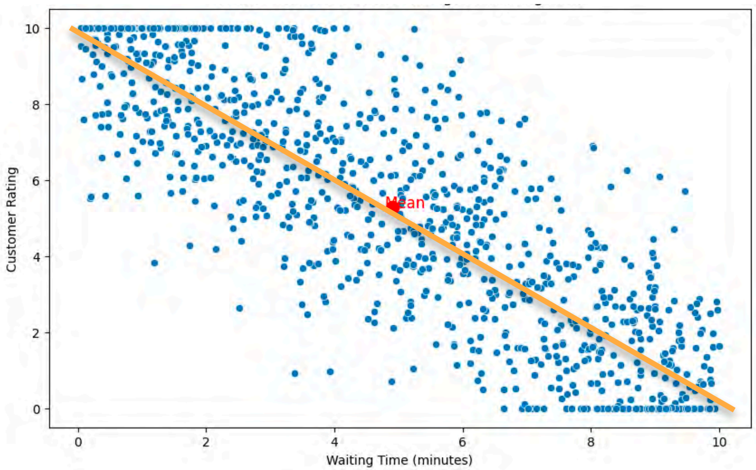
Covariance Matrix

Covariance Matrix



$$Var(X) = 8.526$$

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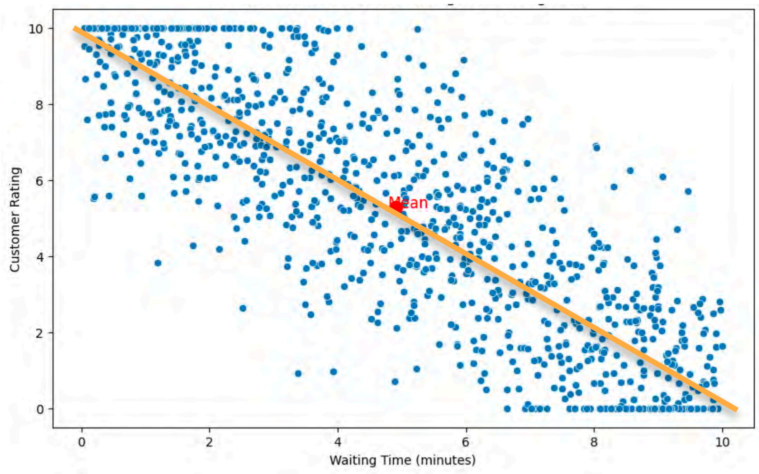
Covariance Matrix



$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

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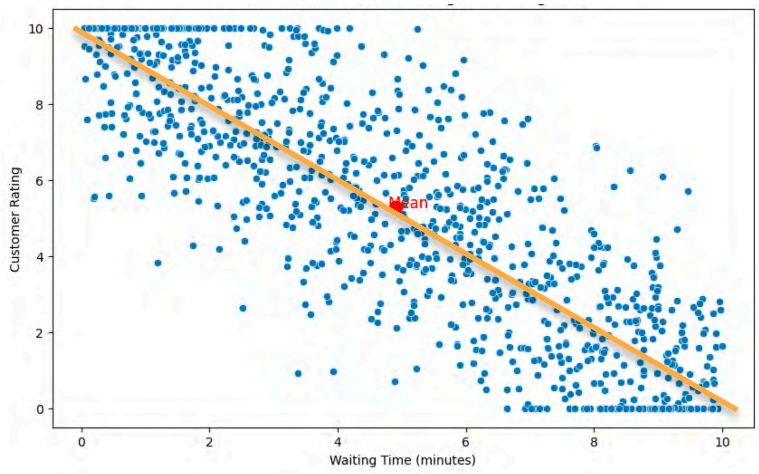
Covariance Matrix



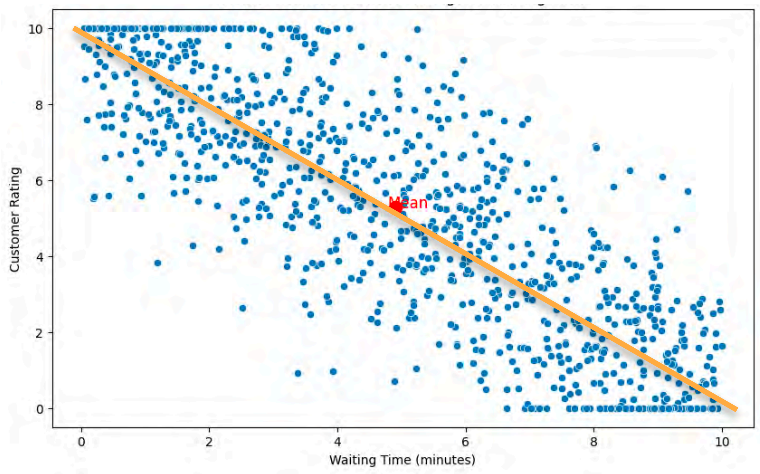
$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

$$Cov(X, Y) = -7.878$$

	X	Y
X		
Y		

Covariance Matrix



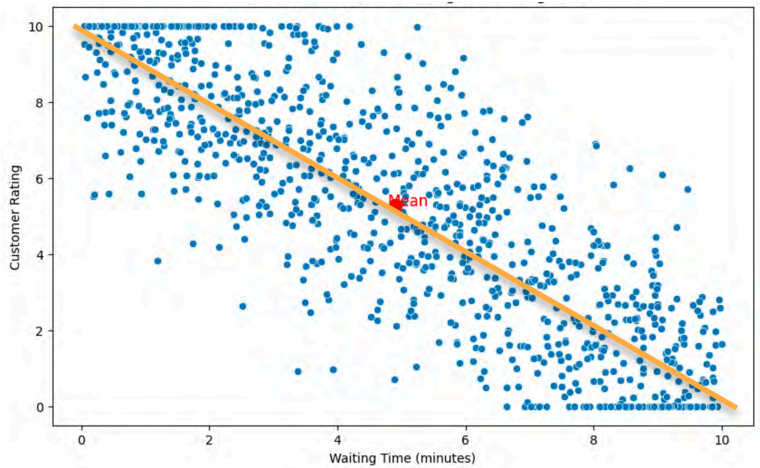
$Var(X) = 8.526$

$Var(Y) = 10.163$

$Cov(X, Y) = -7.878$

	X	Y
X	$Var(X)$	
Y		$Var(Y)$

Covariance Matrix



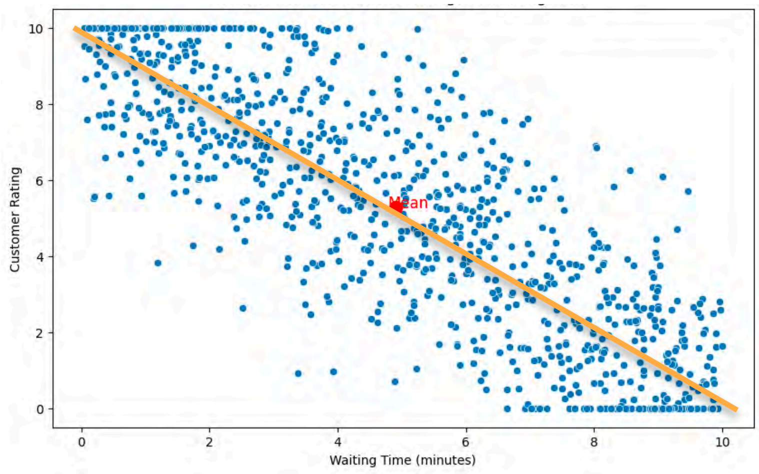
$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

$$Cov(X, Y) = -7.878$$

	X	Y
X	$Var(X)$	$Cov(X, Y)$
Y	$Cov(X, Y)$	$Var(Y)$

Covariance Matrix



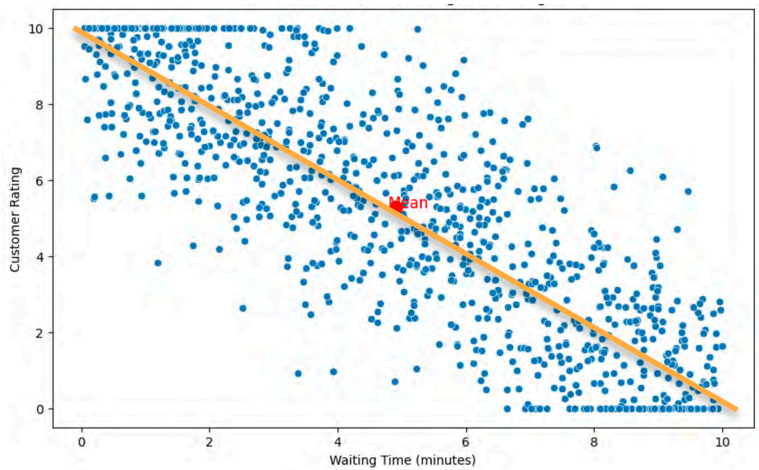
$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

$$Cov(X, Y) = -7.878$$

	X	Y
X	$Var(X)$	$Cov(X, Y)$
Y	$Cov(X, Y)$	$Var(Y)$

Covariance Matrix



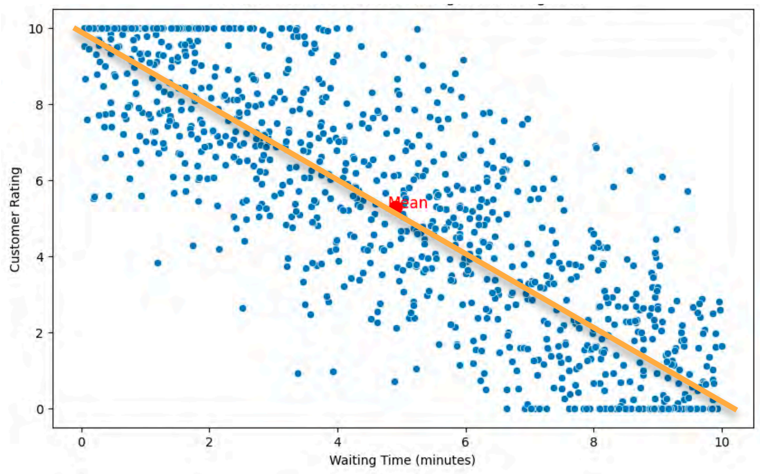
	X	Y
X	$Var(X)$	$Cov(X, Y)$
Y	$Cov(X, Y)$	$Var(Y)$

$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

$$Cov(X, Y) = -7.878$$

Covariance Matrix



$$
\begin{array}{c|cc} & X & Y \\ \hline X & Var(X) & Cov(X,Y) \\ Y & Cov(X,Y) & Var(Y) \end{array}
$$

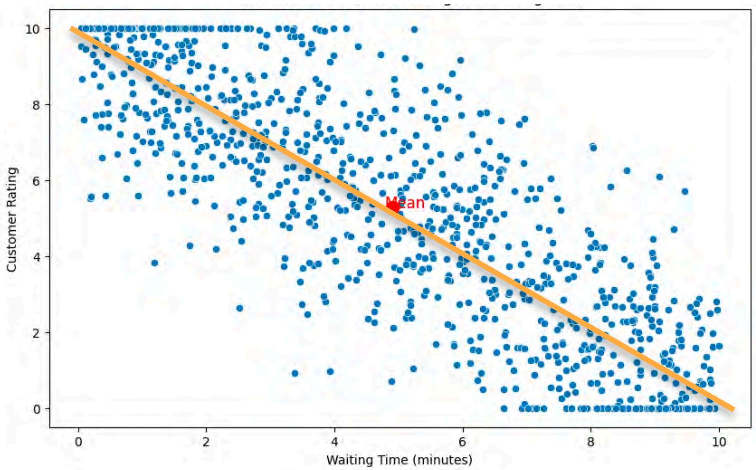
$$
\begin{bmatrix} Var(X) & Cov(X,Y) \\ Cov(X,Y) & Var(Y) \end{bmatrix}
$$

$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

$$Cov(X,Y) = -7.878$$

Covariance Matrix



$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

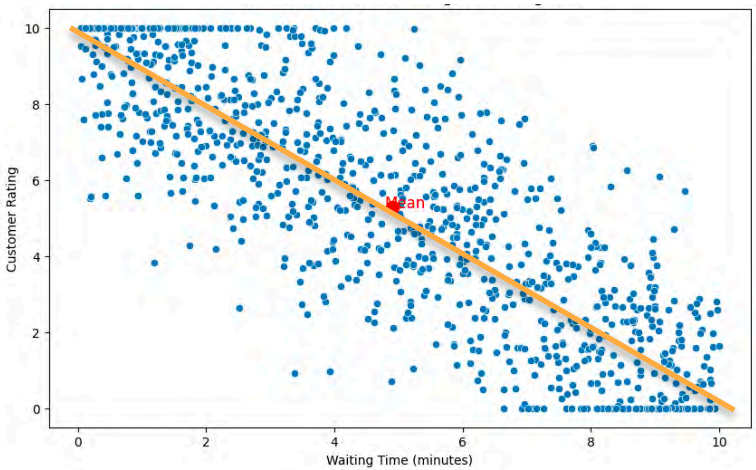
$$Cov(X, Y) = -7.878$$

	X	Y
X	$Var(X)$	$Cov(X, Y)$
Y	$Cov(X, Y)$	$Var(Y)$

8.526	−7.878
−7.878	10.163

$$\begin{bmatrix} Var(X) & Cov(X, Y) \\ Cov(X, Y) & Var(Y) \end{bmatrix}$$

Covariance Matrix



$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

$$Cov(X, Y) = -7.878$$

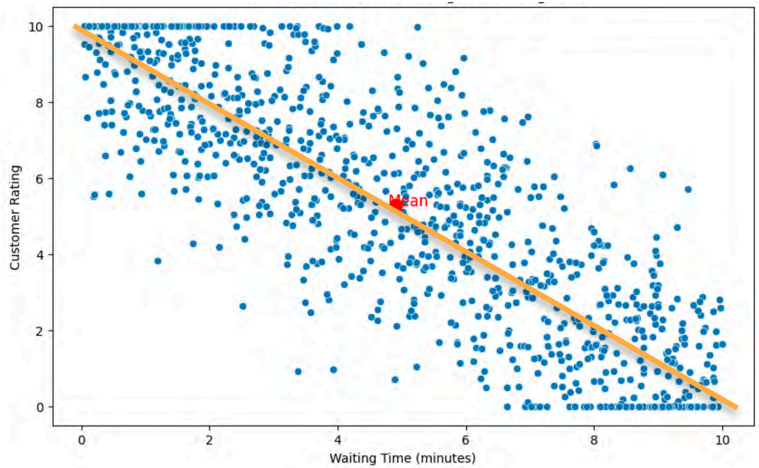
	X	Y
X	$Var(X)$	$Cov(X, Y)$
Y	$Cov(X, Y)$	$Var(Y)$

8.526	−7.878
−7.878	10.163

$$\begin{bmatrix} Var(X) & Cov(X, Y) \\ Cov(X, Y) & Var(Y) \end{bmatrix}$$

$$\begin{bmatrix} 8.534 & -7.878 \\ -7.878 & 10.173 \end{bmatrix}$$

Covariance Matrix



$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

$$Cov(X, Y) = -7.878$$

	X	Y
X	$Var(X)$	$Cov(X, Y)$
Y	$Cov(X, Y)$	$Var(Y)$

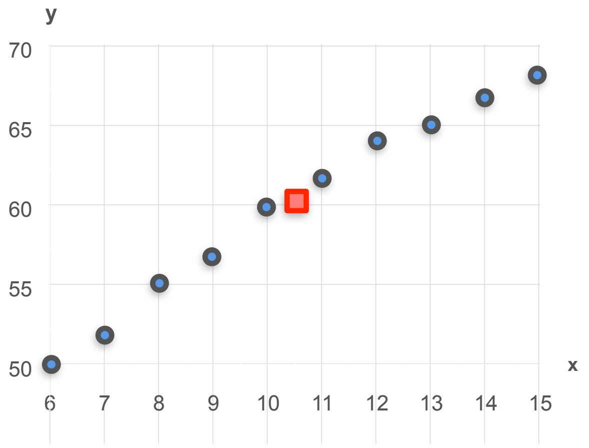
8.526	−7.878
−7.878	10.163

$$\begin{bmatrix} Var(X) & Cov(X, Y) \\ Cov(X, Y) & Var(Y) \end{bmatrix}$$

$$\begin{bmatrix} 8.534 & -7.878 \\ -7.878 & 10.173 \end{bmatrix}$$

Covariance Matrix

Covariance Matrix



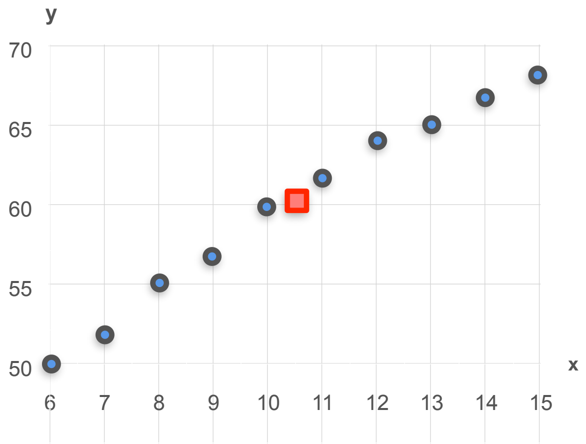
Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

Covariance Matrix



Age vs Height

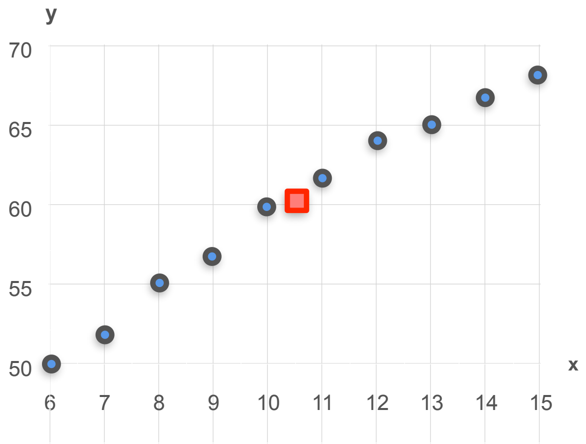
$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

$$\begin{bmatrix} Var(X) & Cov(X, Y) \\ Cov(X, Y) & Var(Y) \end{bmatrix}$$

Covariance Matrix



Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

$$\begin{bmatrix} Var(X) & Cov(X, Y) \\ Cov(X, Y) & Var(Y) \end{bmatrix}$$

$$\begin{bmatrix} 9.17 & 17 \\ 17 & 39.56 \end{bmatrix}$$

Covariance Matrix

$$Var(X) = 1$$
$$Var(Y) = 1$$
$$Cov(X, Y) = 1$$

$$Var(X) = 1$$
$$Var(Y) = 1$$
$$Cov(X, Y) = -1$$

Covariance Matrix

$$\begin{bmatrix} Var(X) & Cov(X,Y) \\ Cov(X,Y) & Var(Y) \end{bmatrix}$$

$$Var(X) = 1$$
$$Var(Y) = 1$$
$$Cov(X,Y) = 1$$

$$Var(X) = 1$$
$$Var(Y) = 1$$
$$Cov(X,Y) = -1$$

Covariance Matrix

$$\begin{bmatrix} Var(X) & Cov(X, Y) \\ Cov(X, Y) & Var(Y) \end{bmatrix}$$

$Var(X) = 1$

$Var(Y) = 1$

$Cov(X, Y) = 1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$Var(X) = 1$

$Var(Y) = 1$

$Cov(X, Y) = -1$

Covariance Matrix

$$\begin{bmatrix} Var(X) & Cov(X,Y) \\ Cov(X,Y) & Var(Y) \end{bmatrix}$$

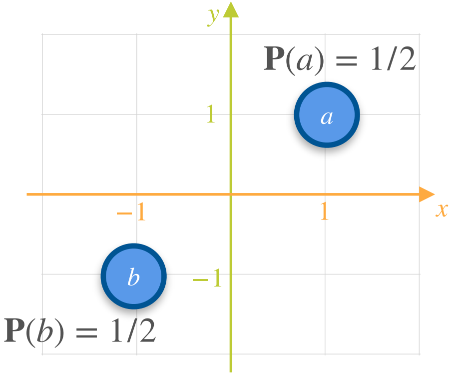
$$Var(X) = 1$$
$$Var(Y) = 1$$
$$Cov(X,Y) = 1$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

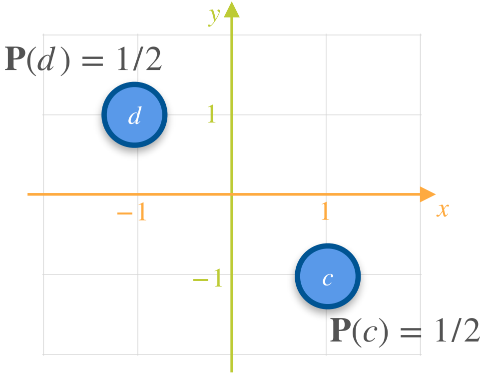
$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$Var(X) = 1$$
$$Var(Y) = 1$$
$$Cov(X,Y) = -1$$

Covariance Matrix



$$\begin{bmatrix} Var(X) & Cov(X,Y) \\ Cov(X,Y) & Var(Y) \end{bmatrix}$$



$$Var(X) = 1$$
$$Var(Y) = 1$$
$$Cov(X,Y) = 1$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$Var(X) = 1$$
$$Var(Y) = 1$$
$$Cov(X,Y) = -1$$

Covariance of a Joint Continuous Distribution

Covariance of a Joint Continuous Distribution

Dataset with 3 Variables:

X	Y	Z

$Var(X)$ $\quad$ $Cov(X, Y)$

$Var(Y)$ $\quad$ $Cov(X, Z)$

$Var(Z)$ $\quad$ $Cov(Y, Z)$

Covariance of a Joint Continuous Distribution

Dataset with 3 Variables:

X	Y	Z

$Var(X)$ $\quad Cov(X, Y)$

$Var(Y)$ $\quad Cov(X, Z)$

$Var(Z)$ $\quad Cov(Y, Z)$

	X	Y	Z
X			
Y			
Z			

Covariance of a Joint Continuous Distribution

Dataset with 3 Variables:

X	Y	Z

$Var(X)$ $Cov(X, Y)$

$Var(Y)$ $Cov(X, Z)$

$Var(Z)$ $Cov(Y, Z)$

	X	Y	Z
X			
Y			
Z			

Covariance of a Joint Continuous Distribution

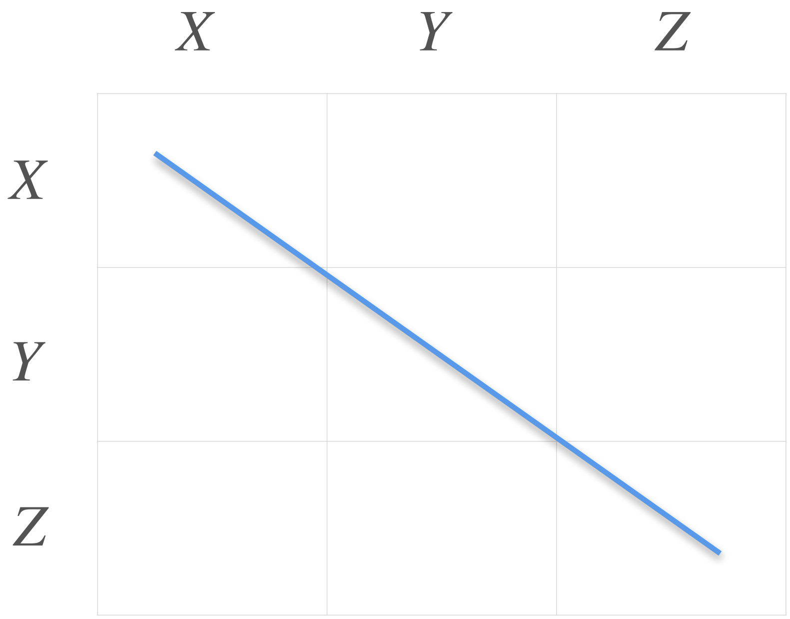
Dataset with 3 Variables:

X	Y	Z

$Var(X)$ $\quad Cov(X, Y)$

$Var(Y)$ $\quad Cov(X, Z)$

$Var(Z)$ $\quad Cov(Y, Z)$



Covariance of a Joint Continuous Distribution

Dataset with 3 Variables:

X	Y	Z

$Var(X)$ $\quad$ $Cov(X, Y)$

$Var(Y)$ $\quad$ $Cov(X, Z)$

$Var(Z)$ $\quad$ $Cov(Y, Z)$

	X	Y	Z
X	$Var(X)$		
Y		$Var(Y)$	
Z			$Var(Z)$

Covariance of a Joint Continuous Distribution

Dataset with 3 Variables:

X	Y	Z

$Var(X)$ $\quad$ $Cov(X,Y)$

$Var(Y)$ $\quad$ $Cov(X,Z)$

$Var(Z)$ $\quad$ $Cov(Y,Z)$

$$\begin{array}{c|ccc} & X & Y & Z \\ \hline X & Var(X) & Cov(X,Y) & Cov(X,Z) \\ Y & Cov(X,Y) & Var(Y) & Cov(Y,Z) \\ Z & Cov(X,Z) & Cov(Y,Z) & Var(Z) \end{array}$$

Covariance of a Joint Continuous Distribution

Covariance of a Joint Continuous Distribution

Dataset with 5 Variables:

V	W	X	Y	Z

$Var(V)$ $Cov(V,W)$

$Var(W)$ $Cov(V,X)$

$Var(X)$ $Cov(V,Y)$

$Var(Y)$ $Cov(V,Z)$

$Var(Z)$ $Cov(W,X)$

$\vdots$

$Cov(Y,Z)$

Covariance of a Joint Continuous Distribution

Dataset with 5 Variables:

V	W	X	Y	Z

$Var(V)$ $Var(W)$ $Var(X)$ $Var(Y)$ $Var(Z)$

$Cov(V, W)$ $Cov(V, X)$ $Cov(V, Y)$ $Cov(V, Z)$ $Cov(W, X)$ $\vdots$ $Cov(Y, Z)$

	V	W	X	Y	Z
V					
W					
X					
Y					
Z					

Covariance of a Joint Continuous Distribution

Dataset with 5 Variables:

V	W	X	Y	Z

$Var(V)$ $Cov(V, W)$

$Var(W)$ $Cov(V, X)$

$Var(X)$ $Cov(V, Y)$

$Var(Y)$ $Cov(V, Z)$

$Var(Z)$ $Cov(W, X)$

$\vdots$

$Cov(Y, Z)$

	V	W	X	Y	Z
V					
W					
X					
Y					
Z					

Covariance of a Joint Continuous Distribution

Dataset with 5 Variables:

V	W	X	Y	Z

$Var(V)$ $Cov(V, W)$

$Var(W)$ $Cov(V, X)$

$Var(X)$ $Cov(V, Y)$

$Var(Y)$ $Cov(V, Z)$

$Var(Z)$ $Cov(W, X)$

$\vdots$

$Cov(Y, Z)$

	V	W	X	Y	Z
V	$Var(V)$				
W		$Var(W)$			
X			$Var(X)$		
Y				$Var(Y)$	
Z					$Var(Z)$

Covariance of a Joint Continuous Distribution

Dataset with 5 Variables:

V	W	X	Y	Z

$Var(V)$

$Var(W)$

$Var(X)$

$Var(Y)$

$Var(Z)$

$Cov(V, W)$

$Cov(V, X)$

$Cov(V, Y)$

$Cov(V, Z)$

$Cov(W, X)$

$\vdots$

$Cov(Y, Z)$

	V	W	X	Y	Z
V	$Var(V)$	$Cov(V, W)$	$Cov(V, X)$	$Cov(V, Y)$	$Cov(V, Z)$
W	$Cov(V, W)$	$Var(W)$	$Cov(W, X)$	$Cov(W, Y)$	$Cov(W, Z)$
X	$Cov(V, X)$	$Cov(W, X)$	$Var(X)$	$Cov(X, Y)$	$Cov(X, Z)$
Y	$Cov(V, Y)$	$Cov(W, Y)$	$Cov(X, Y)$	$Var(Y)$	$Cov(Y, Z)$
Z	$Cov(V, Z)$	$Cov(W, Z)$	$Cov(X, Z)$	$Cov(Y, Z)$	$Var(Z)$

Covariance of a Joint Continuous Distribution

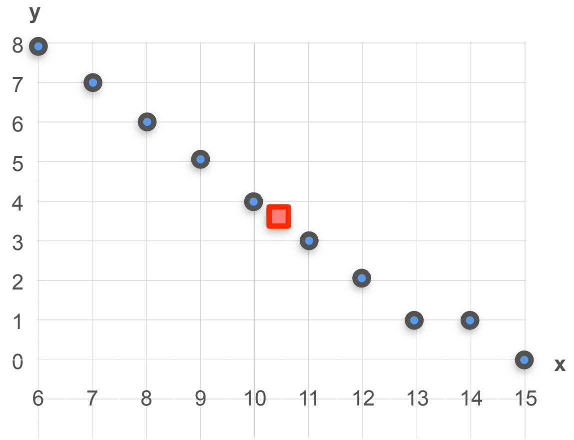
$$\Sigma =$$

$$\begin{array}{c|ccccc} & V & W & X & Y & Z \\ \hline V & Var(V) & Cov(V,W) & Cov(V,X) & Cov(V,Y) & Cov(V,Z) \\ W & Cov(V,W) & Var(W) & Cov(W,X) & Cov(W,Y) & Cov(W,Z) \\ X & Cov(V,X) & Cov(W,X) & Var(X) & Cov(X,Y) & Cov(X,Z) \\ Y & Cov(V,Y) & Cov(W,Y) & Cov(X,Y) & Var(Y) & Cov(Y,Z) \\ Z & Cov(V,Z) & Cov(W,Z) & Cov(X,Z) & Cov(Y,Z) & Var(Z) \end{array}$$

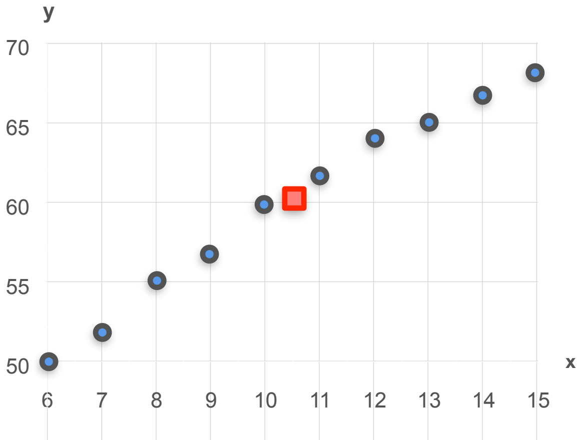
Covariance Matrix

Correlation Coefficient

Correlation Coefficient

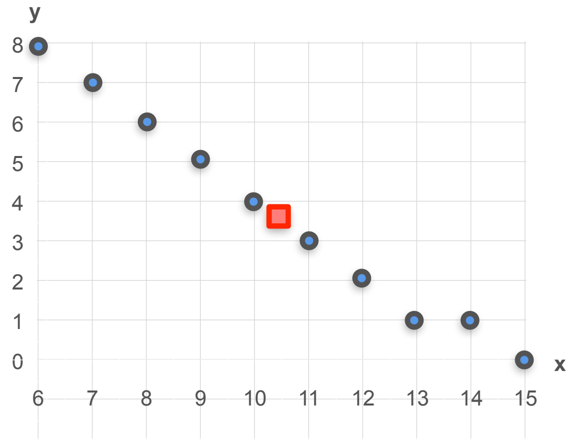


Age vs Naps per Day



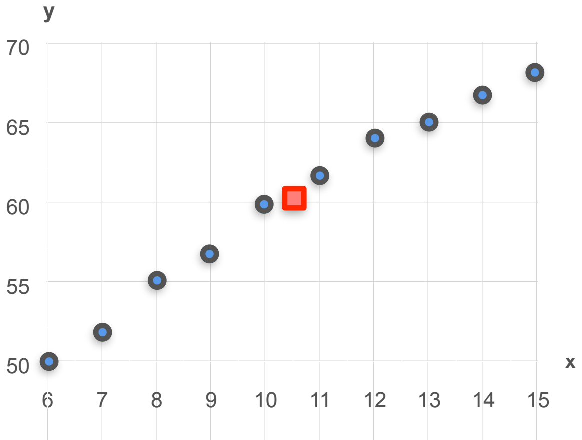
Age vs Height

Correlation Coefficient



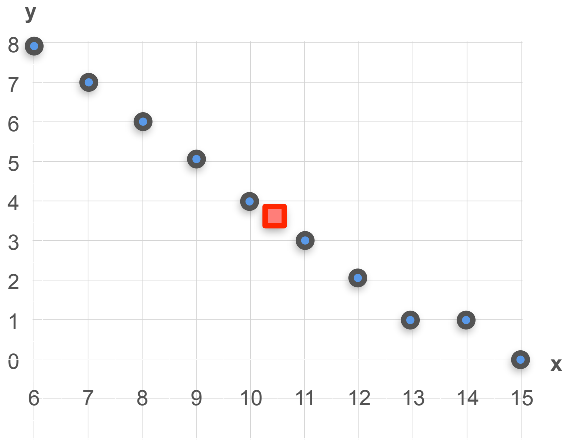
Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$



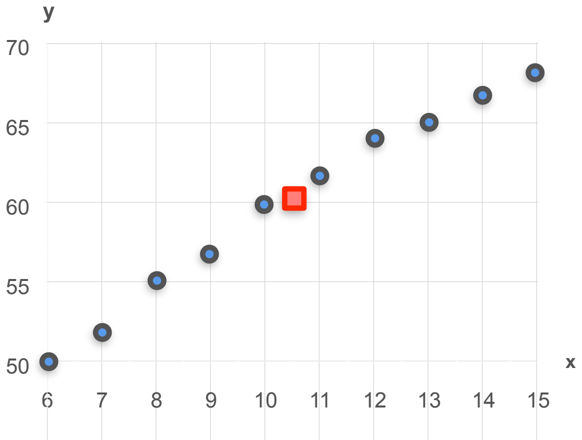
Age vs Height

Correlation Coefficient



Age vs Naps per Day

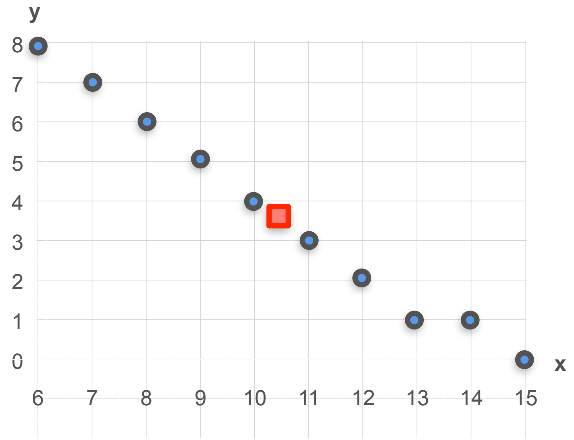
$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$



Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$

Correlation Coefficient

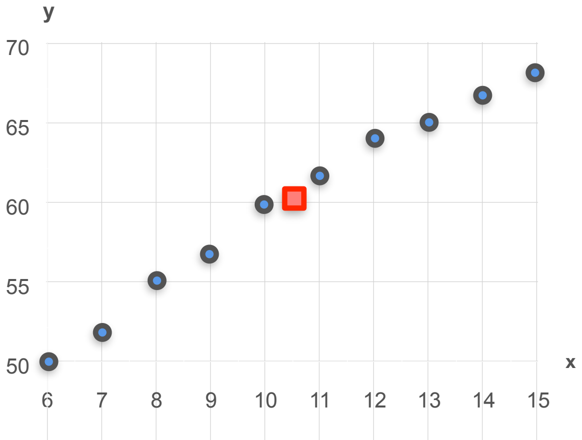


Age vs Naps per Day

$$Var(X) = 9.17$$

$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$



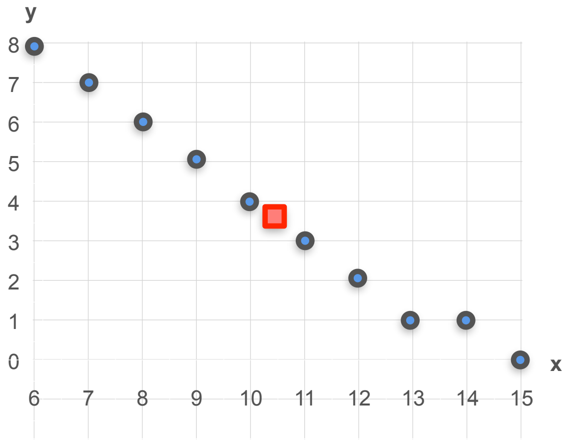
Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

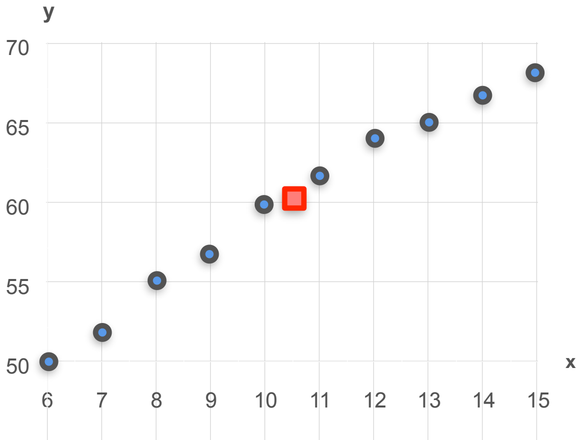
$$Cov(X, Y) = 17$$

Correlation Coefficient



Age vs Naps per Day

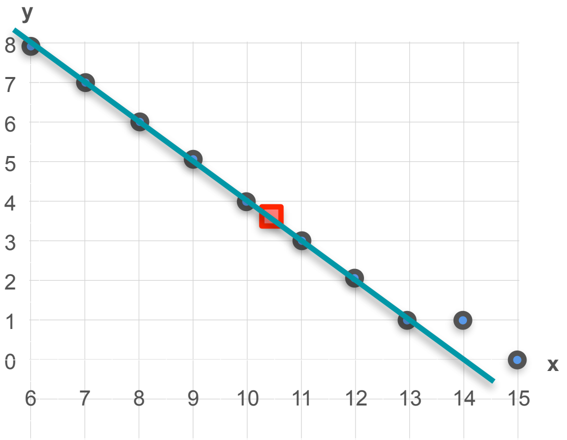
$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$
$$\boxed{Cov(X, Y) = -7.45}$$



Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$
$$\boxed{Cov(X, Y) = 17}$$

Correlation Coefficient

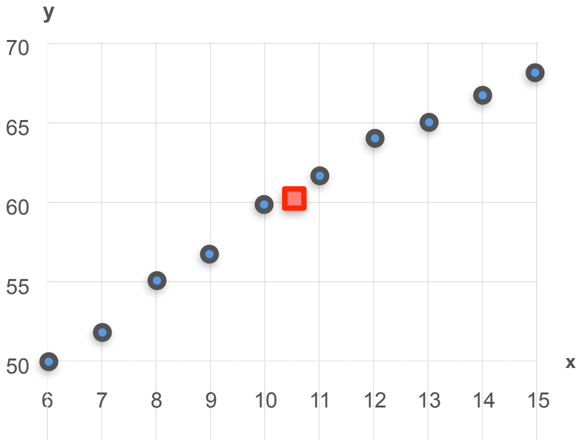


Age vs Naps per Day

$$Var(X) = 9.17$$

$$Var(Y) = 7.57$$

$$\boxed{Cov(X, Y) = -7.45}$$



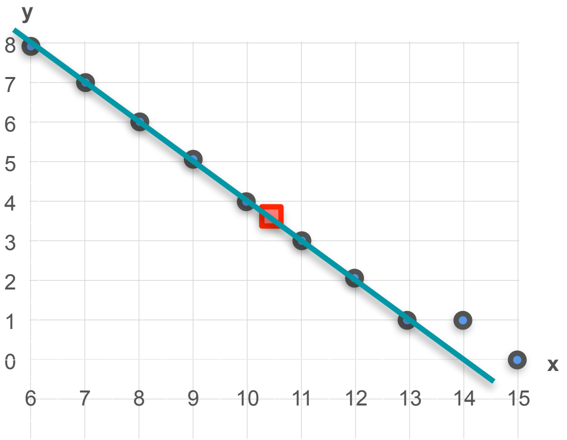
Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

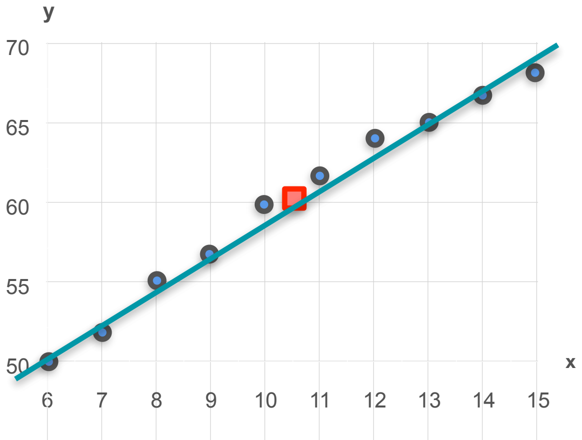
$$\boxed{Cov(X, Y) = 17}$$

Correlation Coefficient



Age vs Naps per Day

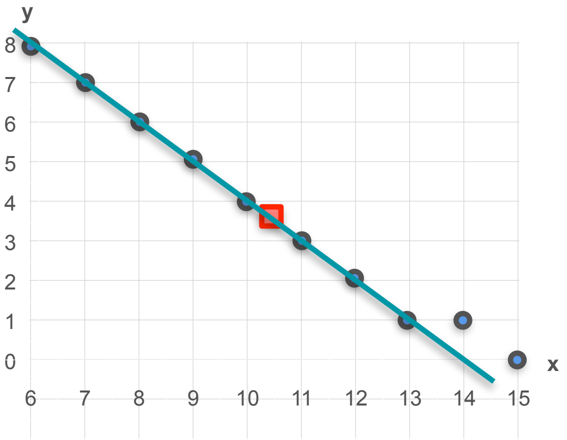
$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$
$$\boxed{Cov(X, Y) = -7.45}$$



Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$
$$\boxed{Cov(X, Y) = 17}$$

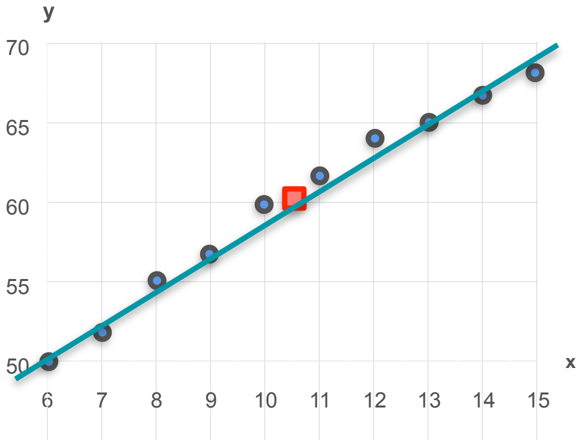
Correlation Coefficient



Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$

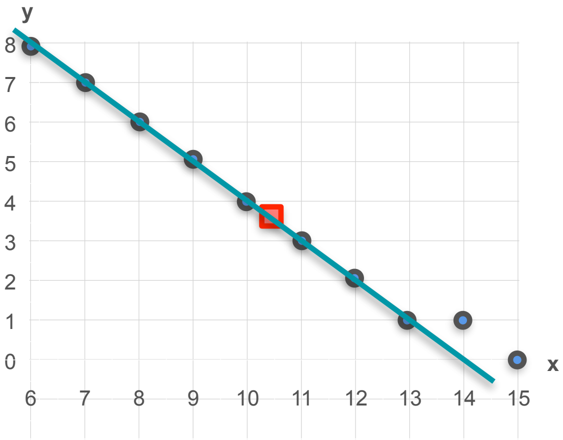


Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

Correlation Coefficient



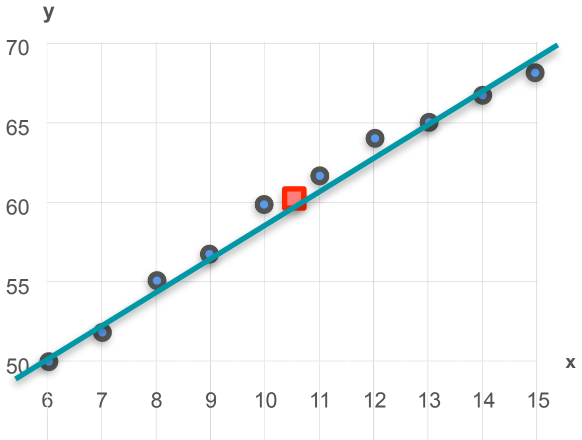
Age vs Naps per Day

$$Var(X) = 9.17$$

$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$

Is the correlation here stronger?



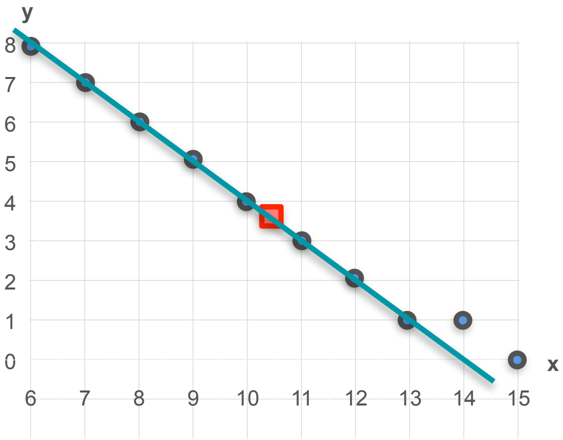
Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

Correlation Coefficient



Age vs Naps per Day

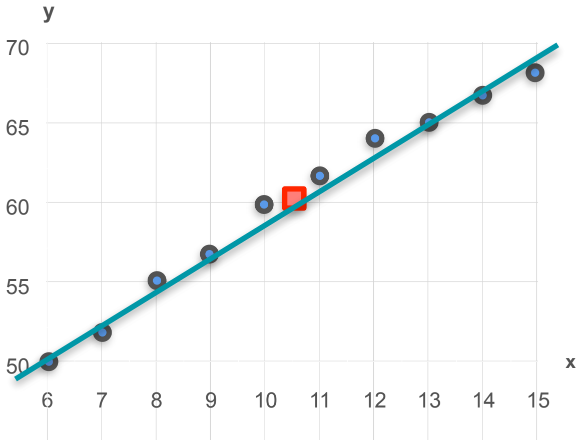
$$Var(X) = 9.17$$

$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$

Is the correl n

here stro ?



Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$

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Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

Correlation Coefficient

Age vs Naps per Day

$Var(X) = 9.17$

$Var(Y) = 7.57$

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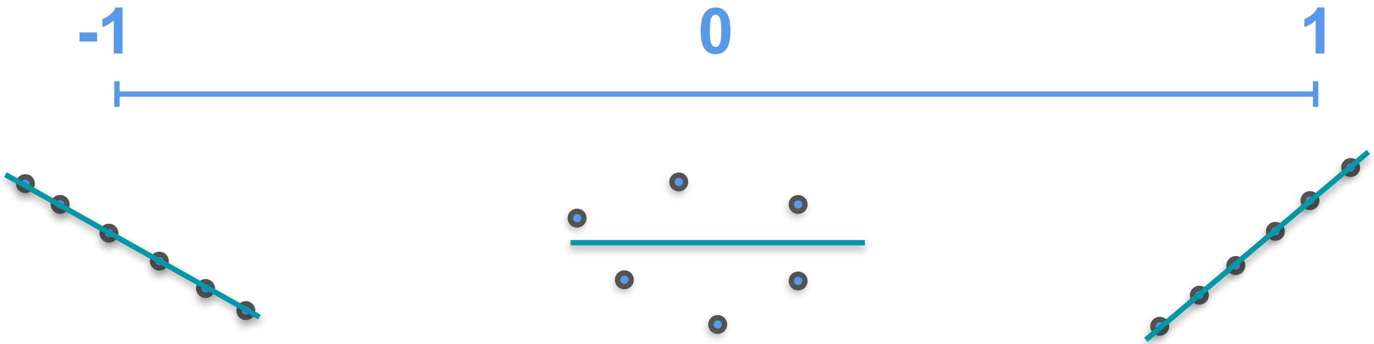
Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$



Correlation Coefficient

Age vs Naps per Day

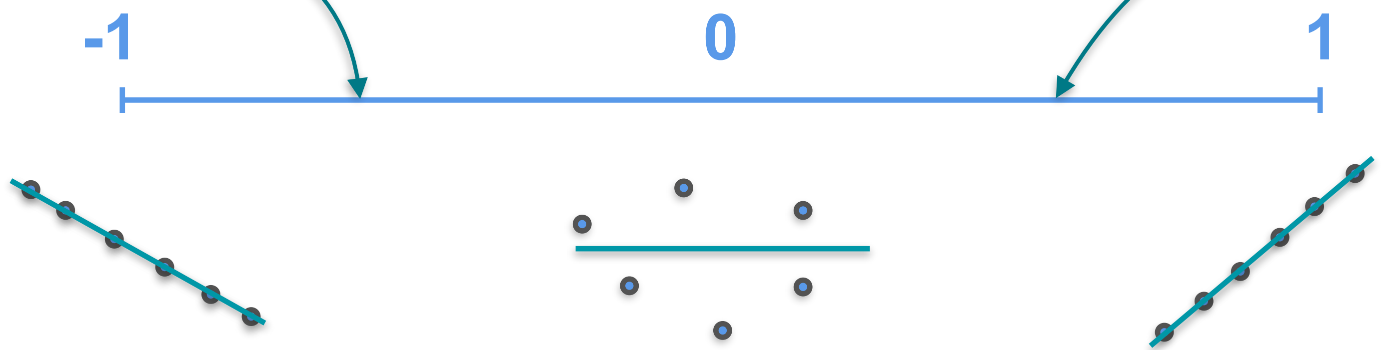
$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$

Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$



Correlation Coefficient

Age vs Naps per Day

$Var(X) = 9.17$

$Var(Y) = 7.57$

$Cov(X, Y) = -7.45$

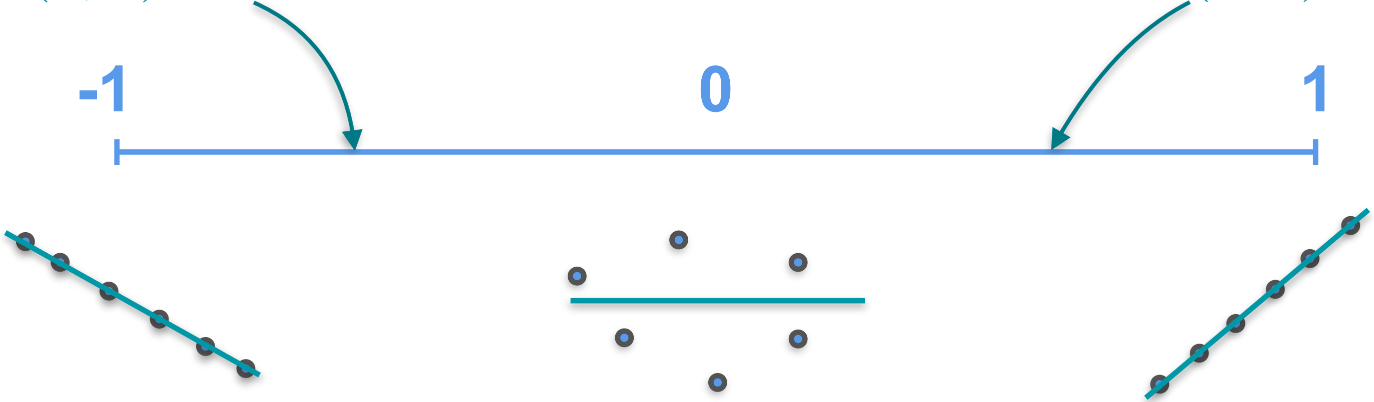
Correlation Coefficient

Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$



Correlation Coefficient

Age vs Naps per Day

$Var(X) = 9.17$

$Var(Y) = 7.57$

$Cov(X, Y) = -7.45$

Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$

Correlation Coefficient

Correlation Coefficient

Age vs Naps per Day

$Var(X) = 9.17$

$Var(Y) = 7.57$

$Cov(X, Y) = -7.45$

Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$

Correlation Coefficient =

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$
$$Cov(X, Y) = -7.45$$

Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$
$$Cov(X, Y) = 17$$

Correlation Coefficient $=$ $Cov(X, Y)$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$

$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$

Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\quad}$$

Correlation Coefficient

Age vs Naps per Day

$Var(X) = 9.17$

$Var(Y) = 7.57$

$Cov(X, Y) = -7.45$

Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$

-1 0 1

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sigma_x}$$

Correlation Coefficient

Age vs Naps per Day

$Var(X) = 9.17$

$Var(Y) = 7.57$

$Cov(X, Y) = -7.45$

Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$

-1 0 1

Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sigma_x \cdot}$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$
$$Cov(X, Y) = -7.45$$

Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$
$$Cov(X, Y) = 17$$

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sigma_x \cdot \sigma_y}$$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$

$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$

Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sigma_x \cdot \sigma_y} =$

Correlation Coefficient

Age vs Naps per Day

$Var(X) = 9.17$

$Var(Y) = 7.57$

$Cov(X, Y) = -7.45$

Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sigma_x \cdot \sigma_y} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$

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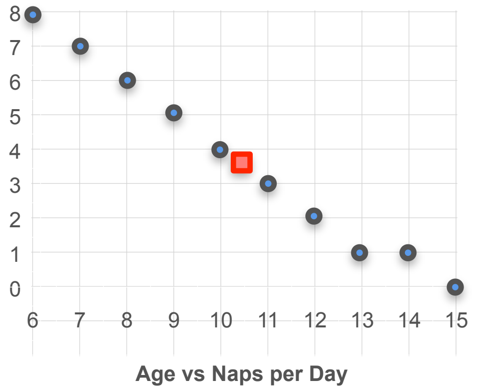

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

Correlation Coefficient

Age vs Naps per Day

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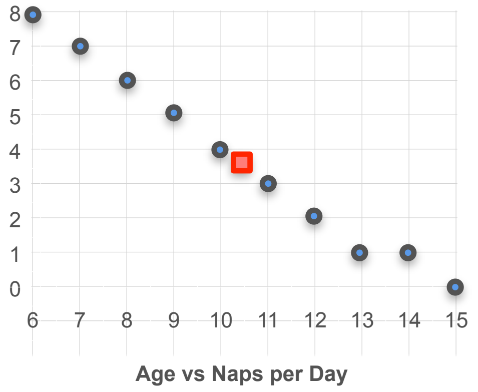

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

$$=$$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$
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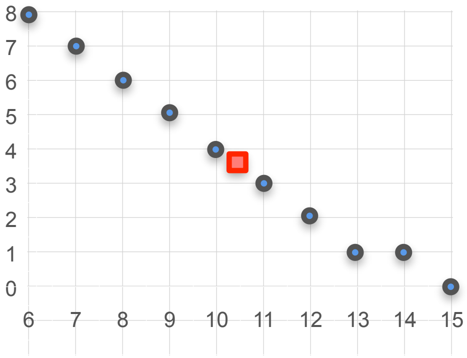

Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

$$= \frac{-7.45}{\sqrt{9.17} \cdot \sqrt{7.57}}$$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$

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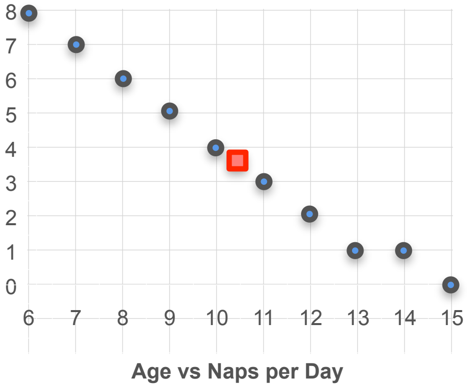


Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

$$= \frac{-7.45}{\sqrt{9.17} \cdot \sqrt{7.57}}$$

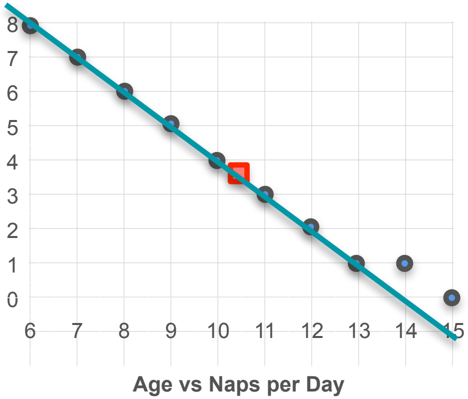
$$\approx -0.894$$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$



$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

$$= \frac{-7.45}{\sqrt{9.17} \cdot \sqrt{7.57}}$$

$$\approx -0.894$$

Correlation Coefficient

Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$

Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

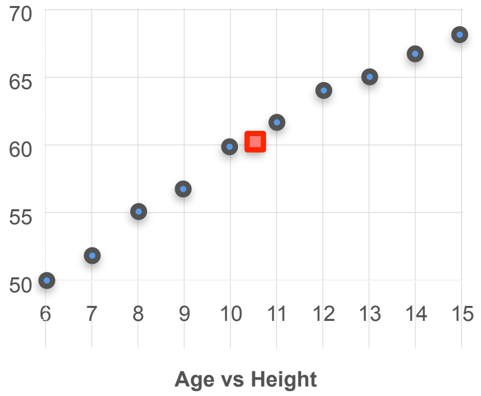
Correlation Coefficient

Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$



Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

Correlation Coefficient

Age vs Height

$$Var(X) = 9.17$$

$$Var(Y) = 39.56$$

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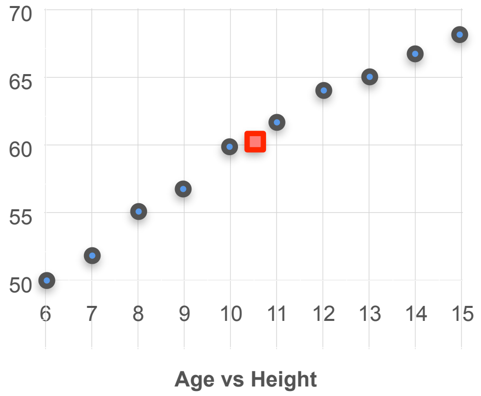


$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

$$=$$

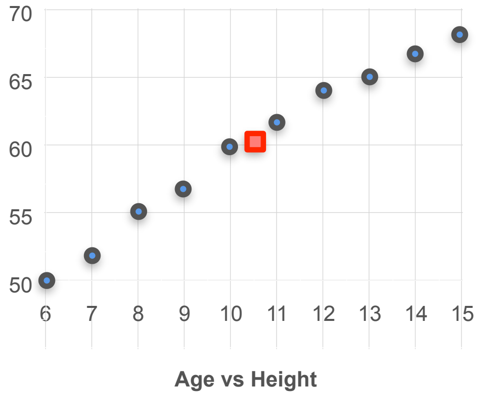
Correlation Coefficient

Age vs Height

$Var(X) = 9.17$

$Var(Y) = 39.56$

$Cov(X, Y) = 17$



$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

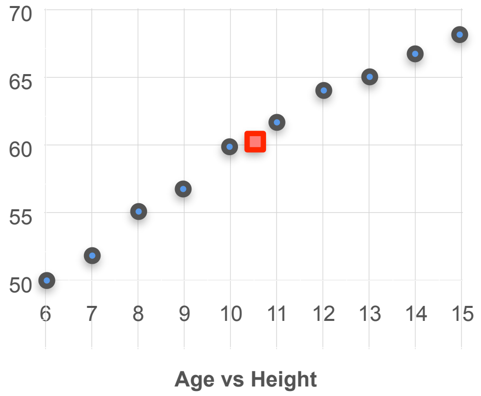
$$= \frac{17}{\sqrt{9.17} \cdot \sqrt{39.56}}$$

Correlation Coefficient

Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$



Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

$$= \frac{17}{\sqrt{9.17} \cdot \sqrt{39.56}}$$

$$\approx 0.893$$

Correlation Coefficient

Age vs Height

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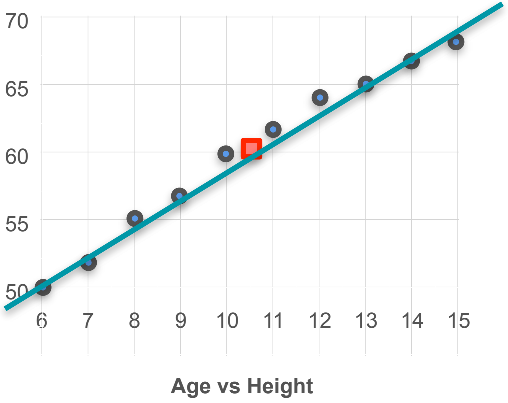

Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

$$= \frac{17}{\sqrt{9.17} \cdot \sqrt{39.56}}$$

$$\approx 0.893$$

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$

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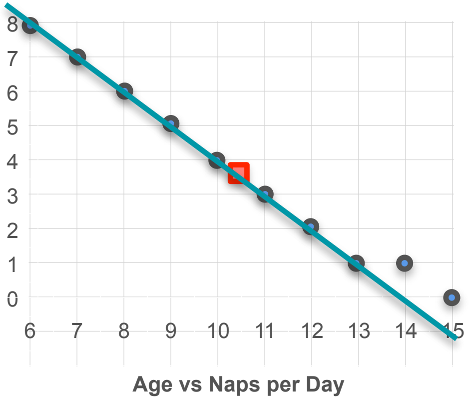

Age vs Height

$$Var(X) = 9.17$$
$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$

$$\approx 0.893$$

Correlation Coefficient

Age vs Naps per Day

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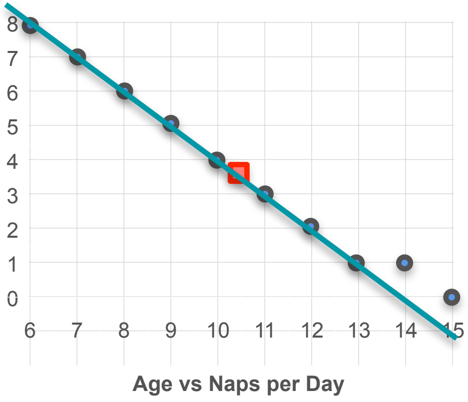

Age vs Height

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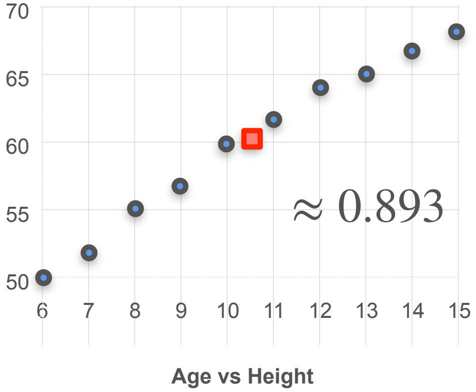
$$Var(Y) = 39.56$$

$$Cov(X, Y) = 17$$



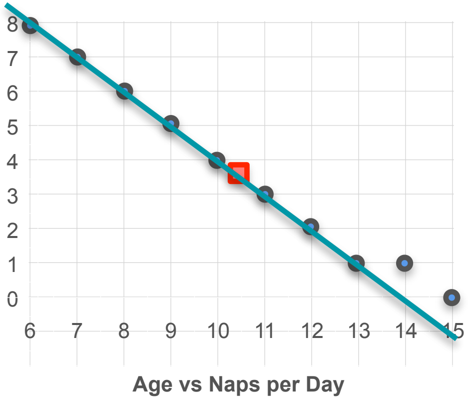
Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$

$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$



Age vs Height

$$Var(X) = 9.17$$

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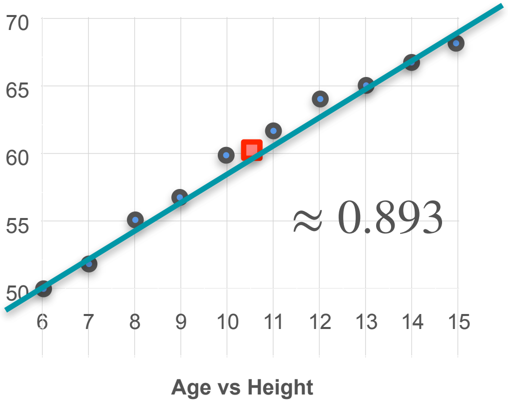


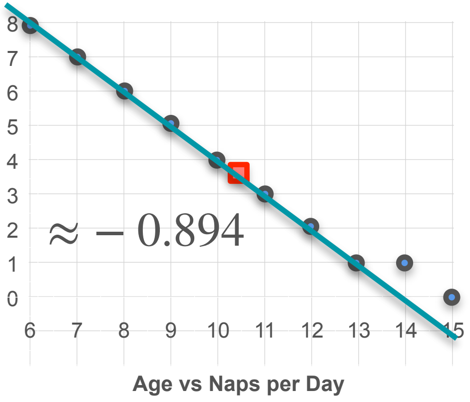
≈ 0.893

Correlation Coefficient

Age vs Naps per Day

$$Var(X) = 9.17$$
$$Var(Y) = 7.57$$

$$Cov(X, Y) = -7.45$$



Age vs Height

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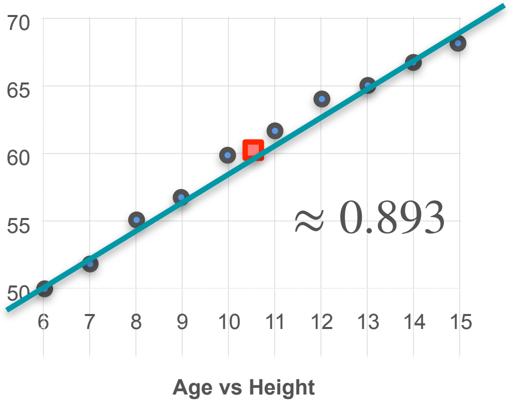
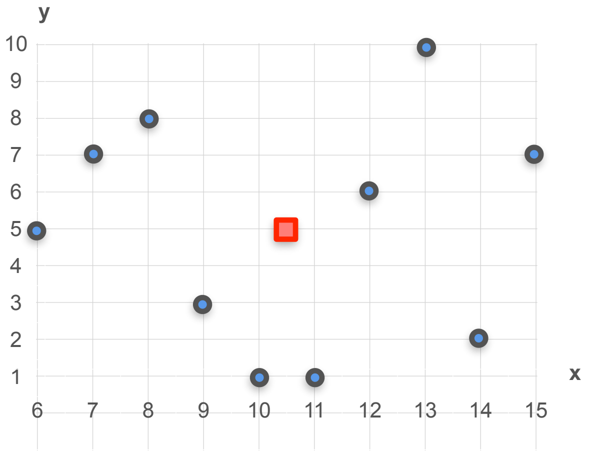


Correlation Coefficient

Correlation Coefficient



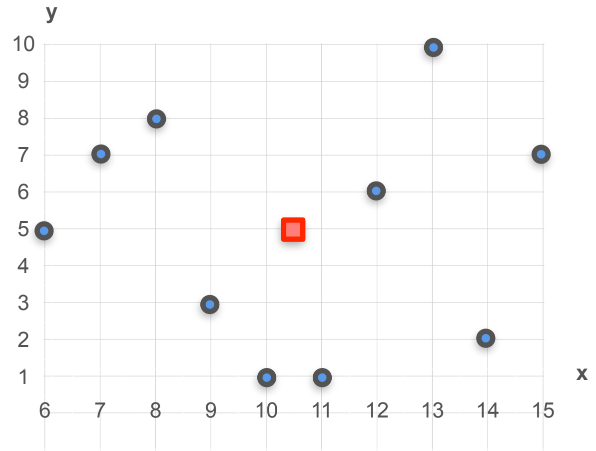
Age vs Grades

$$Var(X) = 9.17$$

$$Var(Y) = 9.78$$

$$Cov(X, Y) = 0.1$$

Correlation Coefficient



Age vs Grades

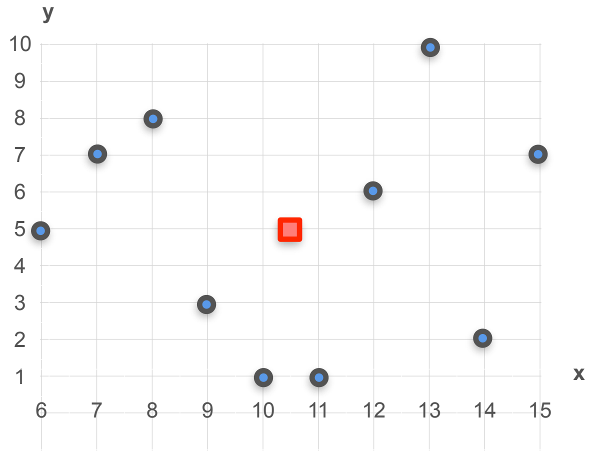
$$Var(X) = 9.17$$

$$Var(Y) = 9.78$$

$$Cov(X, Y) = 0.1$$

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

Correlation Coefficient



Age vs Grades

$$Var(X) = 9.17$$

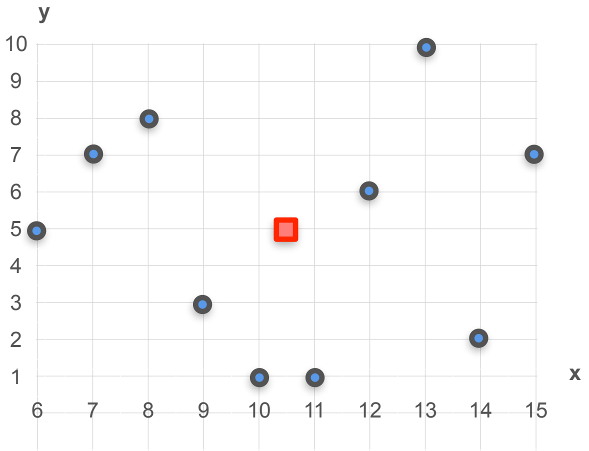
$$Var(Y) = 9.78$$

$$Cov(X, Y) = 0.1$$

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

$$=$$

Correlation Coefficient



Age vs Grades

$$Var(X) = 9.17$$

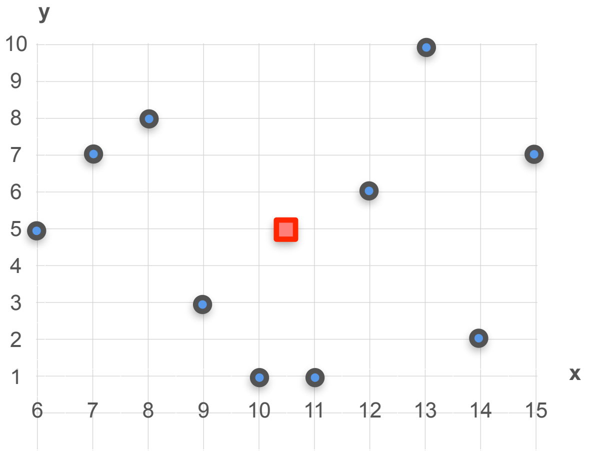
$$Var(Y) = 9.78$$

$$Cov(X, Y) = 0.1$$

Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

$$= \frac{0.1}{\sqrt{9.17} \cdot \sqrt{9.78}}$$

Correlation Coefficient



Age vs Grades

$$Var(X) = 9.17$$

$$Var(Y) = 9.78$$

$$Cov(X, Y) = 0.1$$

Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

$$= \frac{0.1}{\sqrt{9.17} \cdot \sqrt{9.78}}$$

$$\approx 0.01$$

Correlation Coefficient

Correlation Coefficient

$$\text{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

Correlation Coefficient

Correlation Coefficient $= \dfrac{Cov(X,Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

$=$

Correlation Coefficient

$$\textbf{Correlation Coefficient} = \frac{Cov(X,Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

$$= \frac{-7.878}{\sqrt{8.562} \cdot \sqrt{10.163}}$$

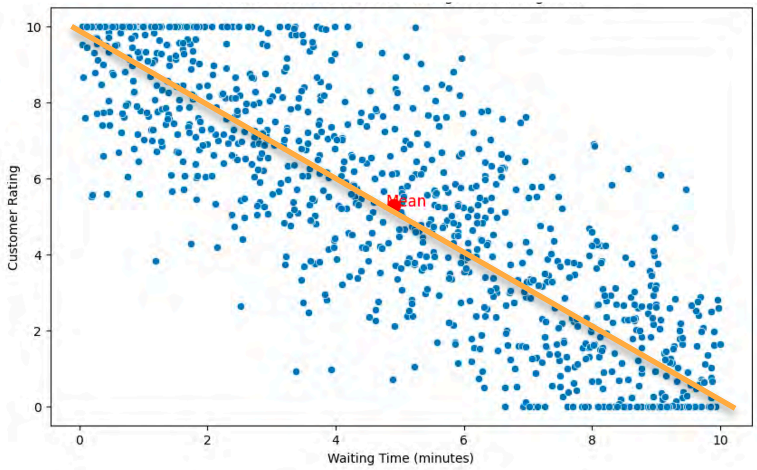
Correlation Coefficient

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

$$= \frac{-7.878}{\sqrt{8.562} \cdot \sqrt{10.163}}$$

$$\approx -0.845$$

Correlation Coefficient



$$Var(X) = 8.526$$

$$Var(Y) = 10.163$$

$$Cov(X, Y) = -7.878$$

Correlation Coefficient $= \dfrac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$

$$= \frac{-7.878}{\sqrt{8.562} \cdot \sqrt{10.163}}$$

$$\approx -0.845$$

Correlation Coefficient

Correlation Coefficient

Correlation Coefficient $=$

Correlation Coefficient

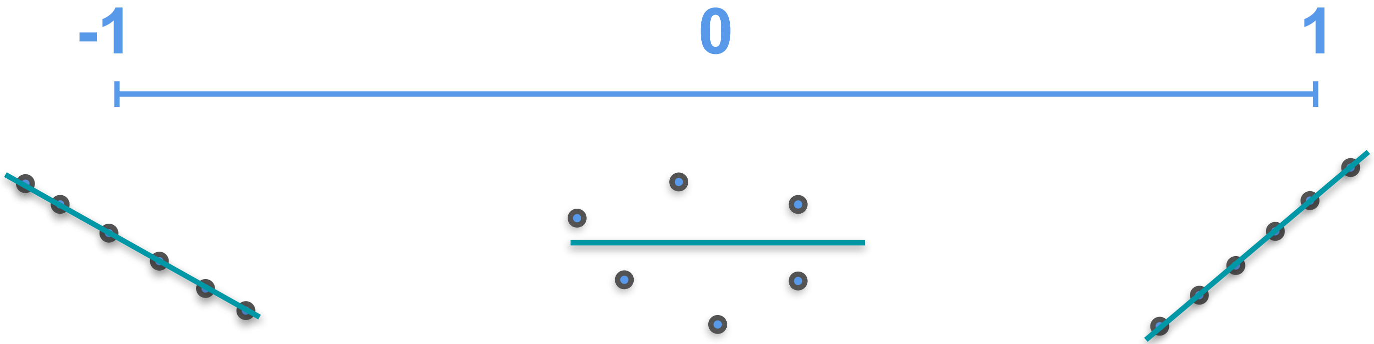
$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sigma_x \cdot \sigma_y}$$

Correlation Coefficient

$$\textbf{Correlation Coefficient} = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y} = \frac{Cov(X,Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$

Correlation Coefficient

$$\textbf{Correlation Coefficient} = \frac{Cov(X, Y)}{\sigma_x \cdot \sigma_y} = \frac{Cov(X, Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}}$$



Probability Distributions with Multiple Variables

DeepLearning.AI

Multivariate Gaussian Distribution

Multivariate Gaussian Distribution

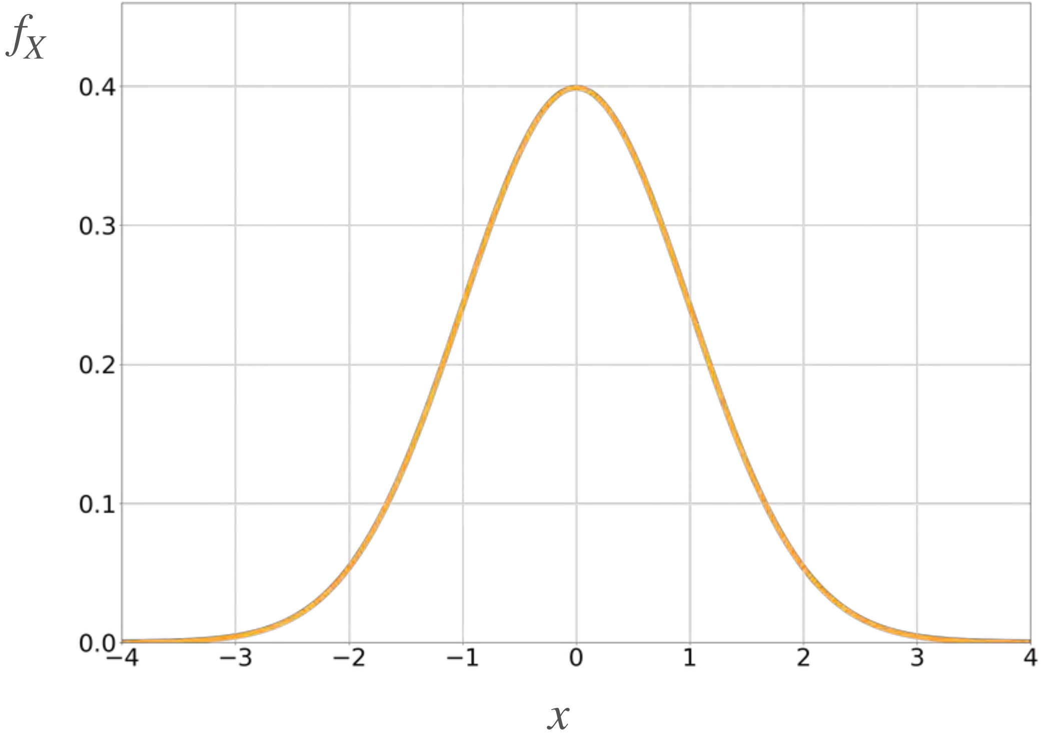
Multivariate Gaussian Distribution

For a single variable, X

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Parameters:

- μ: center of the bell
- σ: spread of the bell



Multivariate Gaussian Distribution

For a single variable, X

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Parameters:

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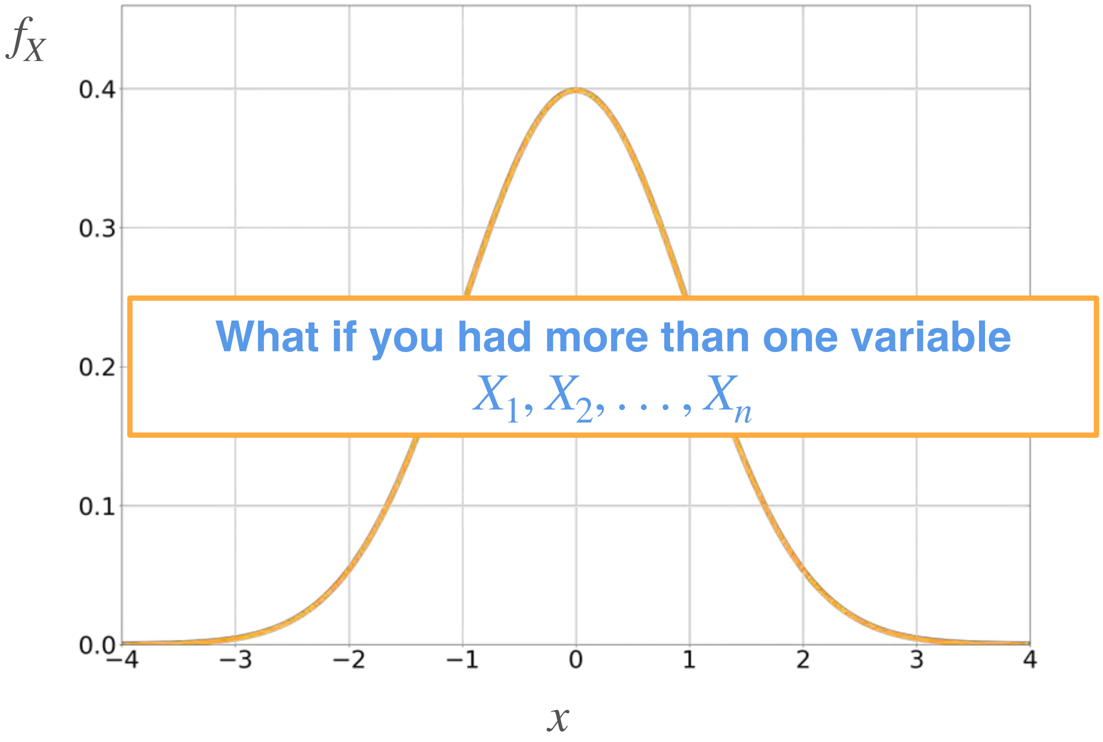


Multivariate Gaussian Distribution: An Example

Multivariate Gaussian Distribution: An Example

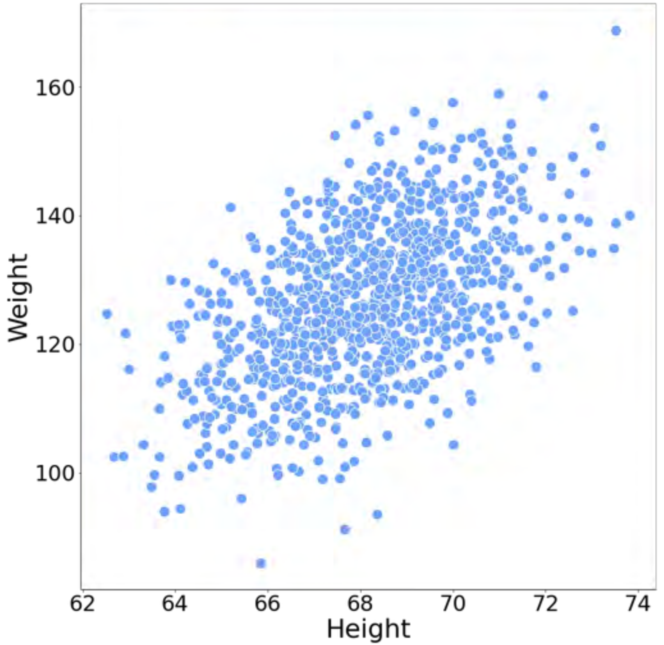
Two variables

Multivariate Gaussian Distribution: An Example

Two variables

H : Height of an adult in inches

W : Weight of an adult in pounds

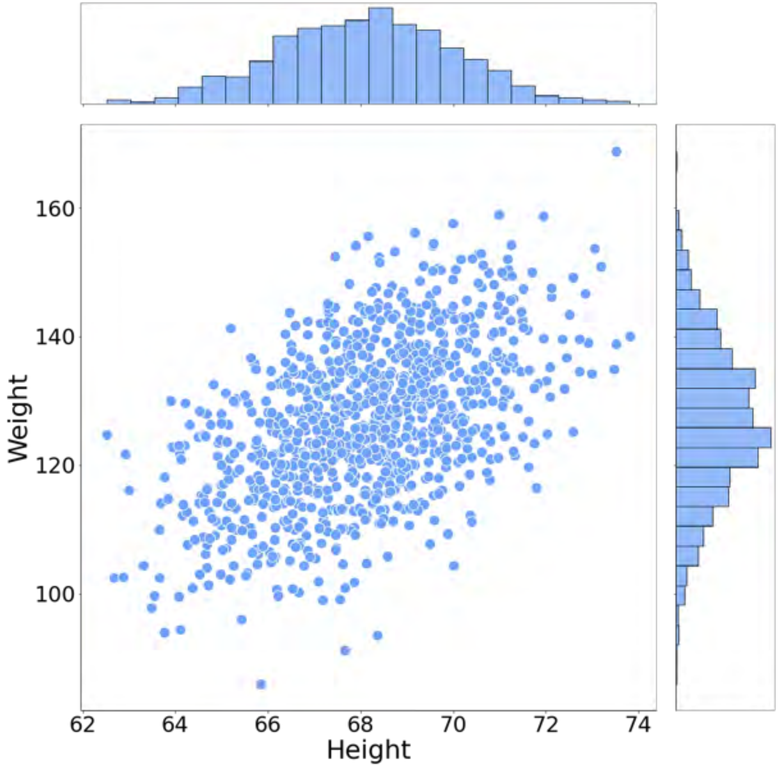


Multivariate Gaussian Distribution: An Example

Two variables

H : Height of an adult in inches

W : Weight of an adult in pounds



Multivariate Gaussian Distribution: An Example

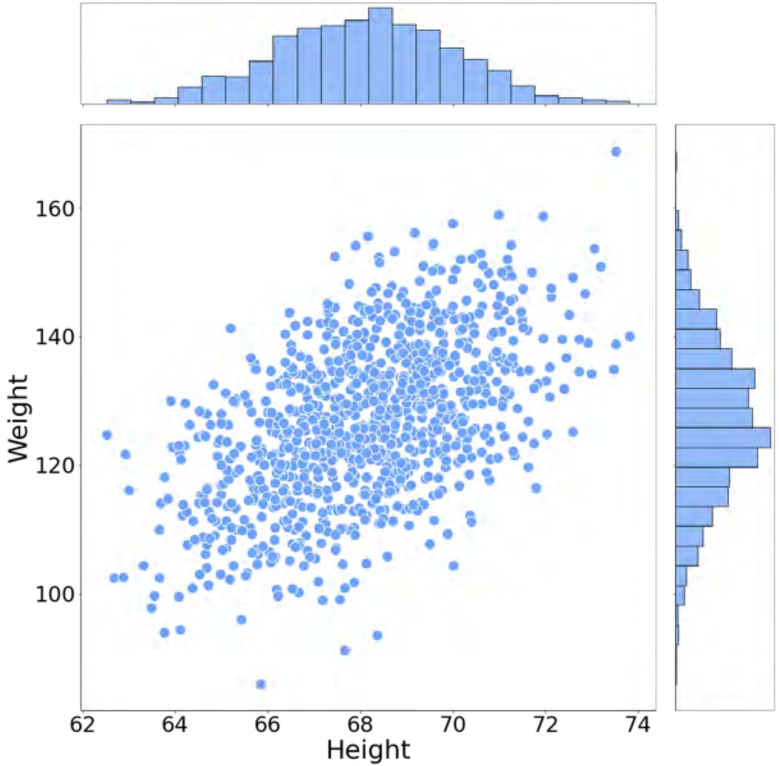
Two variables

H : Height of an adult in inches

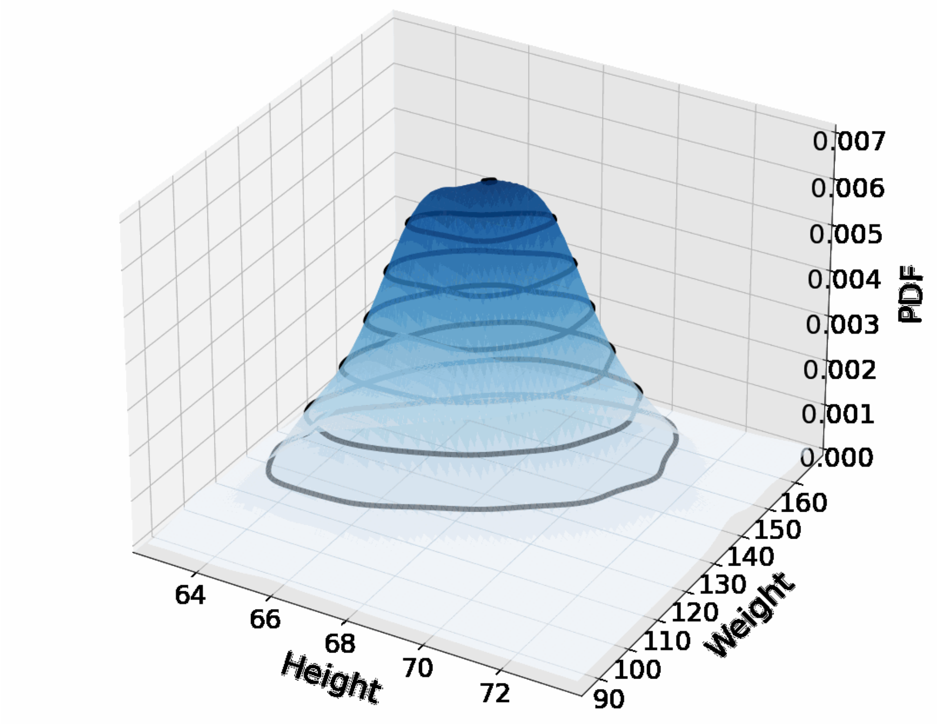
$$H \sim \mathcal{N}(\mu_H, \sigma_H)$$

W : Weight of an adult in pounds

$$W \sim \mathcal{N}(\mu_W, \sigma_W)$$



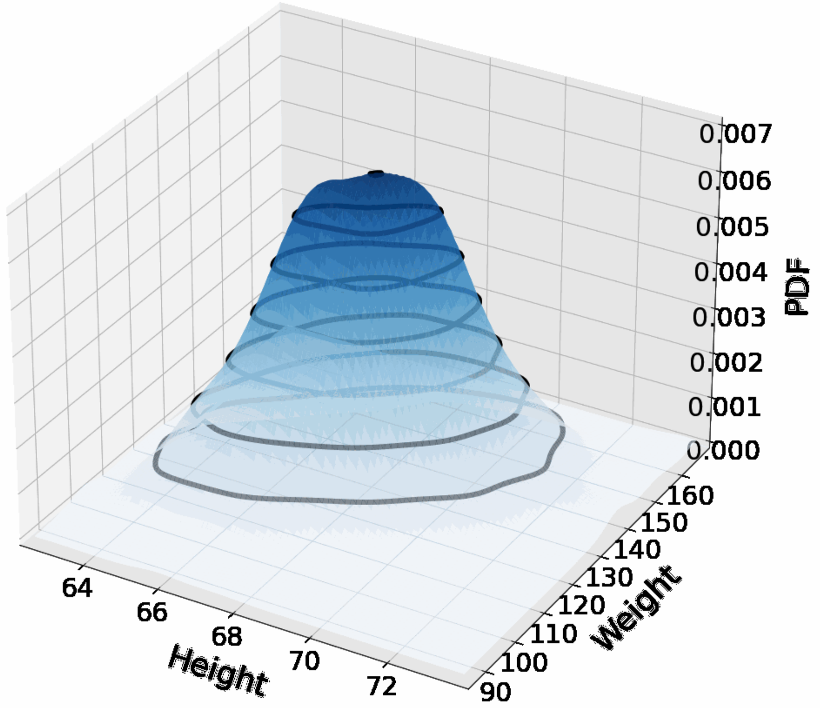
Multivariate Gaussian Distribution: An Example



If W, H were **independent**

$$f_{HW}(h, w) = f_H(h) f_W(w)$$

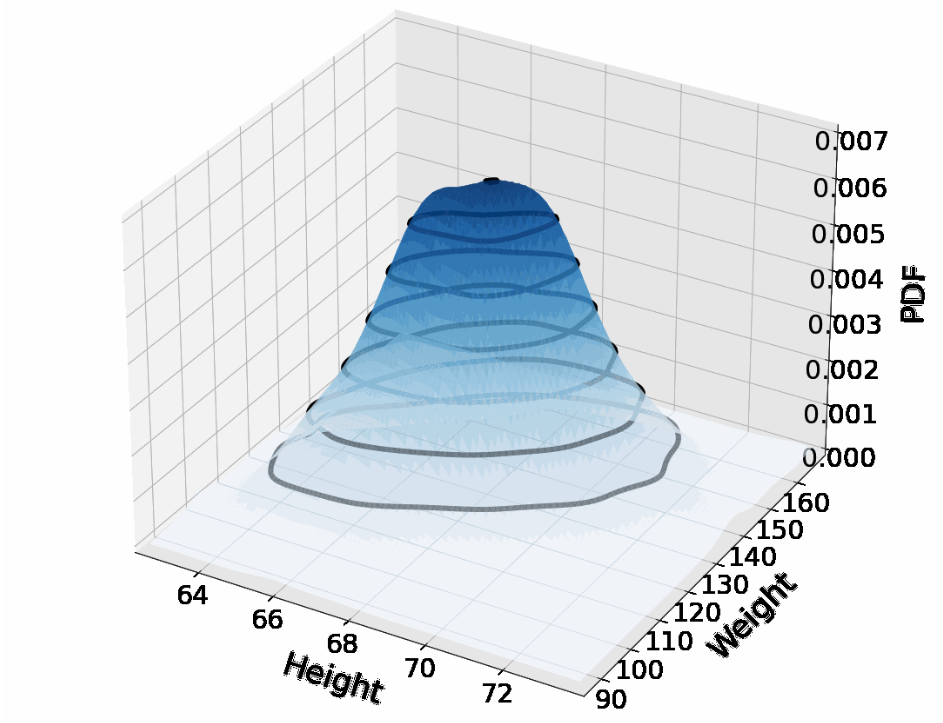
Multivariate Gaussian Distribution: An Example



If W, H were independent

$$f_{HW}(h, w) = f_H(h) f_W(w)$$

Multivariate Gaussian Distribution: An Example

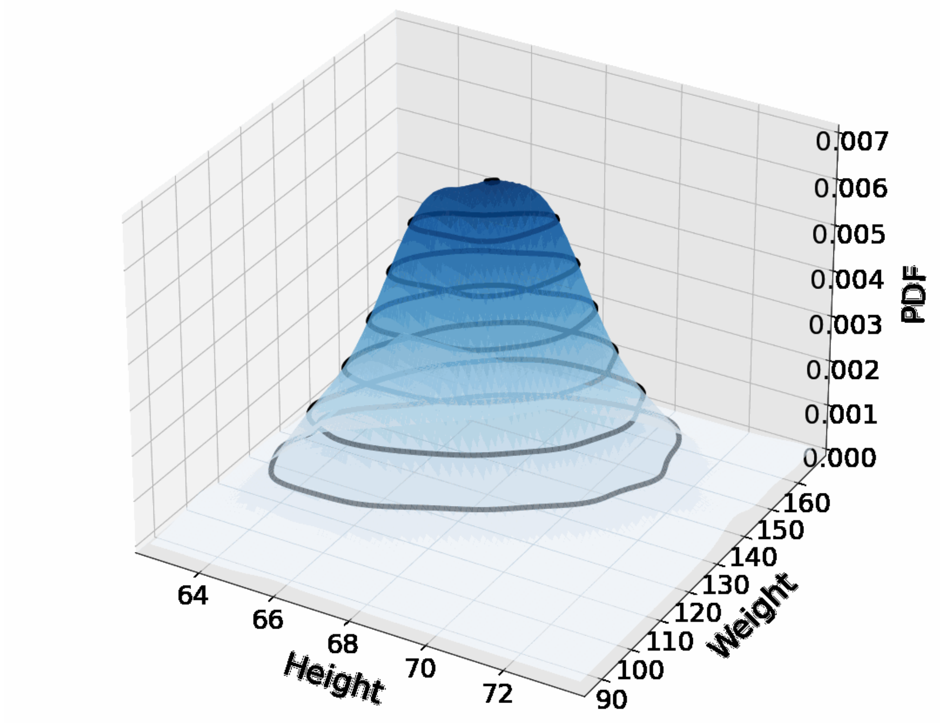


If W, H were **independent**

$$f_{HW}(h, w) = f_H(h) f_W(w)$$

$$= \frac{1}{\sqrt{2\pi}\sigma_H} e^{-\frac{1}{2}\frac{(h-\mu_H)^2}{\sigma_H^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_W} e^{-\frac{1}{2}\frac{(w-\mu_W)^2}{\sigma_W^2}}$$

Multivariate Gaussian Distribution: An Example



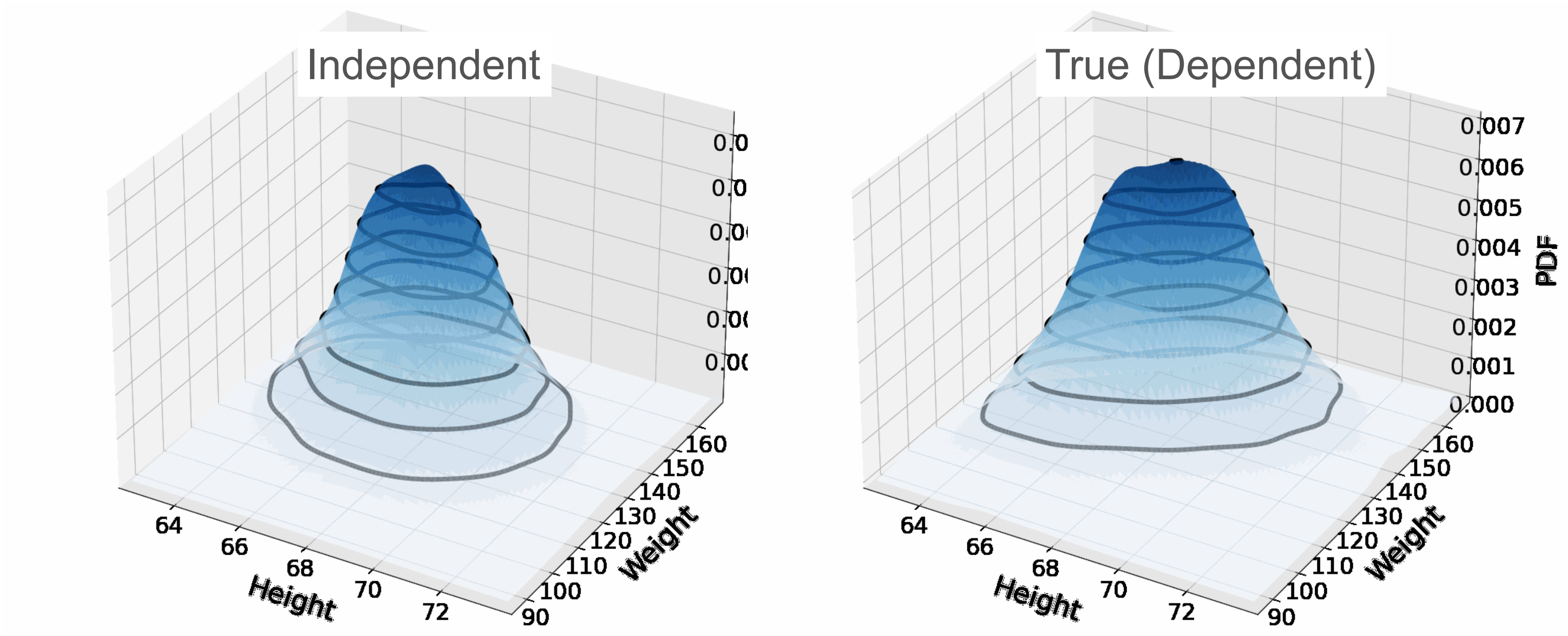
If W, H were **independent**

$$f_{HW}(h, w) = f_H(h) f_W(w)$$

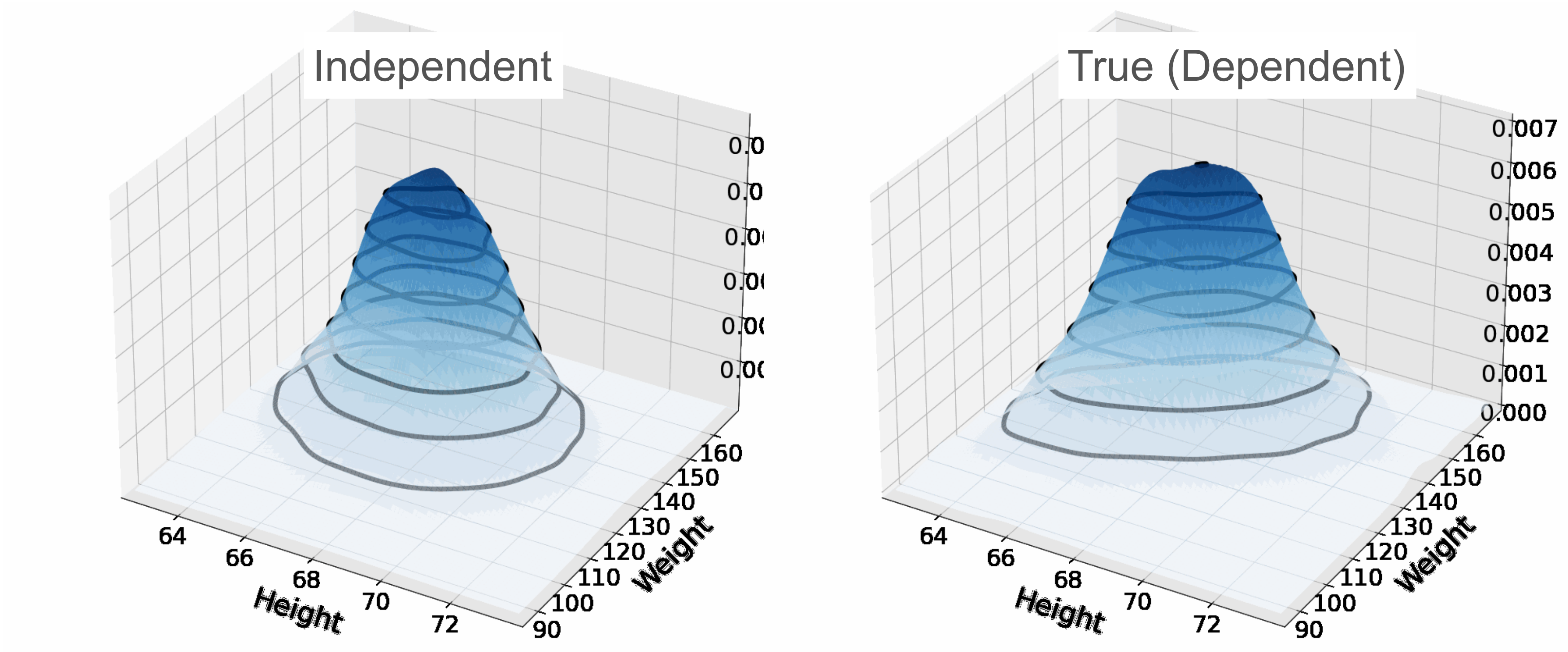
$$= \frac{1}{\sqrt{2\pi}\sigma_H} e^{-\frac{1}{2}\frac{(h-\mu_H)^2}{\sigma_H^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_W} e^{-\frac{1}{2}\frac{(w-\mu_W)^2}{\sigma_W^2}}$$

$$= \frac{1}{2\pi\sigma_H\sigma_W} e^{-\frac{1}{2}\left(\frac{(h-\mu_H)^2}{\sigma_H^2} + \frac{(w-\mu_W)^2}{\sigma_W^2}\right)}$$

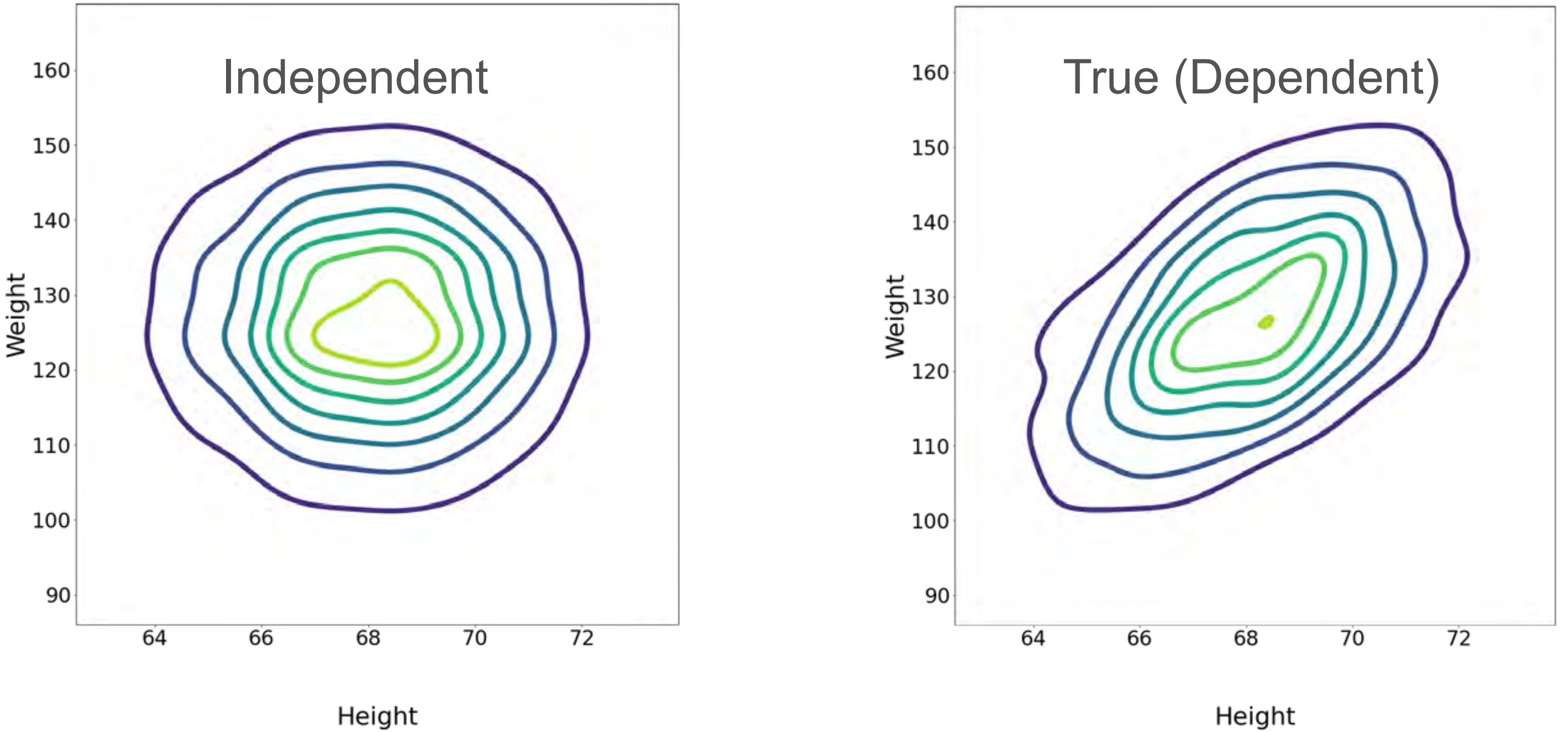
Multivariate Gaussian Distribution: An Example



Multivariate Gaussian Distribution: An Example



Multivariate Gaussian Distribution: An Example



Multivariate Gaussian Distribution: An Example

If W, H were **independent**

$$f_{HW}(h, w) = f_H(h) f_W(w)$$

Multivariate Gaussian Distribution: An Example

If W, H were **independent**

$$f_{HW}(h,w) = f_H(h) f_W(w)$$

$$= \frac{1}{2\pi\sigma_H\sigma_W} e^{-\frac{1}{2}\left(\frac{(h-\mu_H)^2}{\sigma_H^2} + \frac{(w-\mu_W)^2}{\sigma_W^2}\right)}$$

Multivariate Gaussian Distribution: An Example

If W, H were **independent**

$$f_{HW}(h, w) = f_H(h) f_W(w)$$

$$= \frac{1}{2\pi\sigma_H\sigma_W} e^{-\frac{1}{2}\left(\boxed{\frac{(h-\mu_H)^2}{\sigma_H^2} + \frac{(w-\mu_W)^2}{\sigma_W^2}}\right)}$$

$$\left\| \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} \\ \frac{w-\mu_W}{\sigma_W} \end{bmatrix} \right\|_2^2$$

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Multivariate Gaussian Distribution: An Example

If W, H were **independent**

$$f_{HW}(h,w) = f_H(h) f_W(w)$$

$$= \frac{1}{2\pi\sigma_H\sigma_W} e^{-\frac{1}{2}\left(\boxed{\frac{(h-\mu_H)^2}{\sigma_H^2} + \frac{(w-\mu_W)^2}{\sigma_W^2}}\right)}$$

$$\left\| \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} \\ \frac{w-\mu_W}{\sigma_W} \end{bmatrix} \right\|_2^2 = \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} & \frac{w-\mu_W}{\sigma_W} \end{bmatrix} \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} \\ \frac{w-\mu_W}{\sigma_W} \end{bmatrix}$$

Multivariate Gaussian Distribution: An Example

If W, H were **independent**

$$f_{HW}(h,w) = f_H(h) f_W(w)$$

$$= \frac{1}{2\pi\sigma_H\sigma_W} e^{-\frac{1}{2}\left(\frac{(h-\mu_H)^2}{\sigma_H^2} + \frac{(w-\mu_W)^2}{\sigma_W^2}\right)}$$

$$\left\| \begin{bmatrix} \dfrac{h-\mu_H}{\sigma_H} \\ \dfrac{w-\mu_W}{\sigma_W} \end{bmatrix} \right\|_2^2 = \begin{bmatrix} \dfrac{h-\mu_H}{\sigma_H} & \dfrac{w-\mu_W}{\sigma_W} \end{bmatrix} \begin{bmatrix} \dfrac{h-\mu_H}{\sigma_H} \\ \dfrac{w-\mu_W}{\sigma_W} \end{bmatrix}$$

$$\begin{bmatrix} h-\mu_h \\ w-\mu_w \end{bmatrix} = \begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_h \\ \mu_w \end{bmatrix}$$

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$$\left\| \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} \\ \frac{w-\mu_W}{\sigma_W} \end{bmatrix} \right\|^2 = \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} & \frac{w-\mu_W}{\sigma_W} \end{bmatrix} \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} \\ \frac{w-\mu_W}{\sigma_W} \end{bmatrix}$$

$$\begin{bmatrix} h-\mu_h \\ w-\mu_w \end{bmatrix} = \begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_h \\ \mu_w \end{bmatrix}$$

Multiply by diagonal matrix

Multivariate Gaussian Distribution: An Example

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$$\left\| \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} \\ \frac{w-\mu_W}{\sigma_W} \end{bmatrix} \right\|_2^2 = \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} & \frac{w-\mu_W}{\sigma_W} \end{bmatrix} \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} \\ \frac{w-\mu_W}{\sigma_W} \end{bmatrix}$$

$$= ([h \quad w] - [\mu_H \quad \mu_W]) \begin{bmatrix} \frac{1}{\sigma_H^2} & 0 \\ 0 & \frac{1}{\sigma_W^2} \end{bmatrix} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix} \right)$$

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$$= \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix} \right)^T \begin{bmatrix} \sigma_H^2 & 0 \\ 0 & \sigma_W^2 \end{bmatrix}^{-1} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix} \right)$$

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$$= ([h \quad w] - [\mu_H \quad \mu_W]) \begin{bmatrix} \frac{1}{\sigma_H^2} & 0 \\ 0 & \frac{1}{\sigma_W^2} \end{bmatrix} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix} \right)$$

Covariance matrix!
(Σ)

$$= \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix} \right)^T \boxed{\begin{bmatrix} \sigma_H^2 & 0 \\ 0 & \sigma_W^2 \end{bmatrix}}^{-1} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix} \right)$$

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Covariance matrix!
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$$= \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix} \right)^T \boxed{\begin{bmatrix} \sigma_H^2 & 0 \\ 0 & \sigma_W^2 \end{bmatrix}}^{-1} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \boxed{\begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix}} \right)$$

$\boldsymbol{\mu}$

Multivariate Gaussian Distribution: An Example

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$\det(\Sigma)^{1/2}$

$$\left\| \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} \\ \frac{w-\mu_W}{\sigma_W} \end{bmatrix} \right\|_2^2 = \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} & \frac{w-\mu_W}{\sigma_W} \end{bmatrix} \begin{bmatrix} \frac{h-\mu_H}{\sigma_H} \\ \frac{w-\mu_W}{\sigma_W} \end{bmatrix}$$

$$= ([h \quad w] - [\mu_H \quad \mu_W]) \begin{bmatrix} \frac{1}{\sigma_H^2} & 0 \\ 0 & \frac{1}{\sigma_W^2} \end{bmatrix} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix} \right)$$

Covariance matrix!

(Σ)

$$= \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix} \right)^T \boxed{\begin{bmatrix} \sigma_H^2 & 0 \\ 0 & \sigma_W^2 \end{bmatrix}}^{-1} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \boxed{\begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix}} \right)$$

μ

Multivariate Gaussian Distribution: An Example

If W, H were **independent**

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$$= \frac{1}{2\pi\sigma_H\sigma_W} \exp\left(-\frac{1}{2}\left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix}\right)^T \begin{bmatrix} \sigma_H^2 & 0 \\ 0 & \sigma_W^2 \end{bmatrix}^{-1} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \begin{bmatrix} \mu_H \\ \mu_W \end{bmatrix}\right)\right)$$

Multivariate Gaussian Distribution: An Example

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$$= \frac{1}{2\pi \det \Sigma^{1/2}} \exp\left(-\frac{1}{2}\left(\begin{bmatrix} h \\ w \end{bmatrix} - \boldsymbol{\mu}\right)^T \Sigma^{-1} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \boldsymbol{\mu}\right)\right)$$

Multivariate Gaussian Distribution: An Example

Dependent case:

$$f_{HW}(h,w) = f_H(h)f_W(w)$$

$$= \frac{1}{2\pi\sigma_H\sigma_W} e^{-\frac{1}{2}\left(\frac{(h-\mu_H)^2}{\sigma_H^2} + \frac{(w-\mu_W)^2}{\sigma_W^2}\right)}$$

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Multivariate Gaussian Distribution: An Example

Dependent case:

$$f_{HW}(h,w) = \cancel{f_H(h) f_W(w)}$$

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$$\Sigma = \begin{bmatrix} \sigma_H^2 & Cov(H,W) \\ Cov(H,W) & \sigma_W^2 \end{bmatrix}$$

$$= \frac{1}{2\pi \det \Sigma^{1/2}} \exp\left(-\frac{1}{2}\left(\begin{bmatrix} h \\ w \end{bmatrix} - \mu\right)^T \Sigma^{-1} \left(\begin{bmatrix} h \\ w \end{bmatrix} - \mu\right)\right)$$

Multivariate Gaussian Distribution: General Definition

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$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

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covariance matrix
spread of the bell

$|\Sigma|$ determinant of the covariance matrix

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$$\boldsymbol{x} = [x_1 \quad x_2 \quad \cdots \quad x_n]^T$$

Mean vector

$$\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_n]^T$$

$$f_X(x_1, x_2, \ldots, x_n) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T}$$

covariance matrix
spread of the bell

$|\Sigma|$ determinant of the covariance matrix

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$$f_X(x_1, x_2, \dots, x_n) = \frac{1}{(2\pi)^{n/2} \left|\Sigma\right|^{\frac{1}{2}}} e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T \Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu})}$$

covariance matrix
spread of the bell

$|\Sigma|$ determinant of the covariance matrix

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$$\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_n]^T$$

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Rescaling / standardization

covariance matrix
spread of the bell

$|\Sigma|$ determinant of the covariance matrix

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$$\boldsymbol{x} = [x_1 \quad x_2 \quad \cdots \quad x_n]^T$$

Mean vector

$$\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_n]^T$$

$$f_X(x_1, x_2 \ldots \quad {}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})$$

- **For univariate, we work with scalar values and variances**
- **For multivariate, we work with vectors and the covariance matrix**

random

$$X = [X_1 \quad X_2 \quad \cdots \quad X_n]$$

/

ion

covariance matrix
spread of the bell

$|\Sigma|$ determinant of the covariance matrix

Multivariate Gaussian Distribution: Conditionals

Probability Distributions with Multiple Variables

DeepLearning.AI

Conclusion