1 Reorganize Figure 1

Hypothesis: The tiger is green. **Proved** Premises: 1. The mouse is green. 2. The squirrel likes the dog. 3. The tiger is blue. 4. If someone is blue, they eat the squirrel. 5. If someone is green then they like the squirrel. 6. If someone likes the dog and they are green then they are blue. 7. If the tiger is big then they are green. 8. If someone is blue and they eat the squirrel then the tiger is green. Ground Truth Proof: (1) Premise 3, 4 infers Tiger eats the squirrel (Inf 1); (2) Premise 3, Premise 8 and Inf 1 infers The tiger is green. (a) Selection Inference - Forward Chaining Iteration 1 Iteration 2 Hypothesis Premises Selection Inference Premises Selection Inference Premises Unknown Tiger is green. Tiger eats the squirrel. Mouse likes squirrel Premise 3, 4 Premise1. 5 Select wrong premise (b) LAMBADA - Backward Chaining Start from wrong premise Iteration 1 Selection Premise 7 Hypothesis Premise 7 Inference Selection Inference Premises Rerank Selection Unknown Need to prove tiger is big Tiger is green. Premise 6 Do not know Premise 8 Prioritize premise 7 with fewer conditions Premise 7: If tiger is big then they are green. Premise 8: If some one is blue and they eat the squirrel then they are green. (c) Bi-Chainer - Bidirectional Chaining Switch to Backward Chaining

Iteration 2 Iteration 1 Multiple deductions True Selection Inference Confusion Premise1, 5Mouse likes squirrel Premises Premises Premises Inference Hypothesis Selection Inference **Proved** Selection Premise 8 Premise 3, 4 Tiger eats the squirrel **Terminate**: all conditions in Premise 8 are proved Premise 3: The tiger is blue. Inf 1: Tiger eats the squirrel. Premise 8: If some one is blue and they eat the squirrel then they are *green*.

Figure 1: Bi-Chainer framework in bidirectional chaining (c) in comparison with the Selection-Inference framework in forward chaining (a) and the LAMBADA framework in backward chaining (b). Each iteration step involves rule selection and executing a logical deduction in forward chaining or a logical abduction in backward chaining.

2 Revised Section 3.1

Bidirectional chaining is a reasoning strategy that combines both forward and backward chaining to facilitate the inference process. It involves simultaneous exploration in both directions, starting from the available facts and working forward to derive new conclusions, while also starting from the goal and working backward to decompose the goal into sub-goals using applicable rules.

Figure 1 illustrates the application of bidirectional chaining in proving a hypothesis using a set of premises. Initially, forward chaining is employed to derive more definite facts and update the premises. In the forward chaining process, deductions are made based on selected premises, such as Premises 3 and 4 leading to the deduction "Tiger eats the squirrel", and Premises 1 and 5 establishing the deduction "Mouse likes squirrel". However, as multiple deductions are obtained, further forward chaining becomes confusing on which deduction to select to continue the chaining process. Therefore, the Confusion Check module triggers a switch to backward chaining. In the backward chaining phase, both Premise 7 and Premise 8 support the consequence of the hypothesis "Someone is green". However, Premise 8's conditions can all be proven using the intermediate deductions obtained from forward chaining. As a result, the hypothesis is successfully proved using bi-directional chaining.