THERMONUCLEAR REACTION RATES, II[1]

✖2076

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There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact.

Mark Twain, *Life on the Mississippi*

INTRODUCTION

It has been eight years since the publication of *Thermonuclear Reaction Rates* by Fowler, Caughlan & Zimmerman (1967) (hereafter referred to as FCZ I) in volume 5 of this series. In the interim an enormous amount of experimental data has been obtained in nuclear laboratories throughout the world and new theoretical and computer-programmed techniques for analyzing these data and applying them to astrophysical circumstances have been developed. It has been 101 years since Mark Twain spoke so forthrightly, so it is indeed time for another "investment of fact." In this article we revise and update the charged particle reactions involving nuclei with $A \lesssim 30$ in FCZ I, except for the $H^2(d,\gamma)He^4$ reaction whose reaction rate is very uncertain. Reactions involving neutrons are treated as neutron-producing reactions but factors for calculating the reverse reaction rate are provided as for all reactions. We have not revised the rates of the (n,γ) reactions given in FCZ I. To avoid duplication the reader is referred to FCZ I for the basic ideas underlying the determination of nuclear reaction rates under astrophysical conditions.[4]

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[4] We repeat here the units we have adopted: 10^9 °K as the unit of temperature, $T_9 = T/(10^9\ °K)$ and 1 MeV $= 1.6022 \times 10^{-6}$ erg $= 1.6022 \times 10^{-13}$ J as the unit of energy, $E_6 = E/(10^6$ eV). In these units Boltzmann's constant is given by $k = 8.6171 \times 10^{-2}$ MeV/10^9 °K $= 1$ MeV/$(11.6049 \times 10^9$ °K). The atomic mass unit is given by $M_U = N_A^{-1} = 1.66053 \times 10^{-24}$ g $= 931.4812$ MeV/c^2, and Avogadro's number by $N_A = 6.0222 \times 10^{23}$ mole^{-1}. We did not think it necessary to adopt the small changes ($< 3 \times 10^{-5}$) in the fundamental constants recommended by Cohen & Taylor (1973) and Cohen (1974).

Reference to equations in that article will be preceded by I and definitions of many quantities used in equations in this article will be found only in FCZ I. The results of our analysis are given in Table 1. As in FCZ I we do not include factors for screening nor pycnonuclear effects (DeWitt, Graboske & Cooper 1973; Graboske et al 1973). Furthermore, we assume that the Maxwell-Boltzmann distribution holds for the relative velocities of the interacting particles, but see Clayton (1974).

We have in preparation a supplement to this article that will include a detailed description of the sources and analysis of the data used for each of the reactions given in Table 1. References to original material are not consistently included in the present article. A complete set of references for each reaction can be found for $A = 5$–10 in Ajzenberg-Selove & Lauritsen (1974); for $A = 11$–12 in Ajzenberg-Selove & Lauritsen (1968); for $A = 13$–15 in Ajzenberg-Selove (1970); for $A = 16$–17 in Ajzenberg-Selove (1971); for $A = 18$–20 in Ajzenberg-Selove (1972); and for $A = 21$–44 in Endt & Van der Leun (1973). The citations here refer in general to experimental, theoretical, or analytical techniques and not to data sources. Preprints of the supplement will be available in the near future and can be obtained by writing to us. In addition, we will accept inquiries for analysis of important and significant reactions ($A \leq 30$) that have not been included. Comparison with experimental data for reactions involving nuclei with $A > 30$ will be included in a series of theoretical papers using Hauser-Feshbach calculations now in preparation by various permutations and combinations of Woosley et al (1975).

We frequently receive inquiries from nuclear physicists asking where they should direct their efforts to make significant new measurements or to improve old ones. The many uncertain entries in Table 1 preceded by the factor (0 to 1), which is defined below, should serve to answer these questions in a succinct manner. The promised supplement will give details.

We will be most grateful if those who find errors in Table 1 will let us know.

INSTRUCTIONS FOR THE USE OF TABLE 1

The first column of Table 1 lists nuclear reactions in standard notation insofar as possible within the limitations of computer printout. The chemical symbol for the "target" nucleus followed by the integral mass number is placed before the first parenthesis. The incident "particles," one or two in number, are placed inside the parentheses *before* the comma. The emitted "particles," one or two in number, are placed inside the parentheses *after* the comma. The symbols used inside the parentheses are G for gamma ray, E− for electron, E+ for positron, NU for neutrino, N for neutron, P for proton, D for deuteron, T for triton, HE3 for the helium-3 nucleus, and A for the alpha particle. The "residual" nuclei, one or two in number, are represented by their chemical symbols and integral mass numbers placed after the parentheses.[5] The particles and nuclei to the left of the comma

[5] In the notation of FCZ I, the reactions tabulated are of the type $0+1 \rightarrow 2+3$, $0+1 \rightarrow 2+\gamma$, $0+1 \rightarrow 2+3+4$, $0+1 \rightarrow 2+3+4+5$, $0+1+2 \rightarrow 3+\gamma$.

are the primary interacting bodies and those to the right are the product bodies. These products become the primary bodies in the reverse reaction. The three reactions between pairs of C^{12} and O^{16} nuclei are followed by the compound nucleus in parentheses; the branching ratios for breakup into the *n*, *p*, and α-channels are given at the end of Table 1.

The second column of Table 1 lists the *Q*-value of the reaction in MeV. In exoergic ($Q > 0$) reactions, this is the energy released to the thermal bath of particles and radiation per reaction; in endoergic ($Q < 0$) reactions, this is the energy drawn from the thermal bath per reaction. Implicit in these statements is the assumption that the reaction products are thermalized before reacting again. For a discussion of the problem see Gryzinski (1958, 1959).

In exoergic reactions that produce neutrinos the energy released *exclusive of neutrino energy* is also listed. Except in the "big bang" and the formation of neutron stars, the *difference*

the neutrino carries away about half of the kinetic energy (*CM*) of the primary interacting particles when it is emitted with a positron and almost all of this energy when it is emitted without a positron. In endoergic reactions that produce neutrinos the absolute value of the negative *Q* listed is lost from the thermal bath and, in addition, the neutrino carries away the kinetic energy it receives. The energetics of neutrino processes are discussed in detail by Chiu (1968). In radioactive weak interactions such as beta decay ($\beta^{\pm}$) and electron capture (*EC*) the neutrino energy losses are

$$\langle E_\nu \rangle_\beta \approx \frac{W_0}{2}\left(1 - \frac{1}{w_0^2}\right)\left(1 - \frac{1}{4w_0} - \frac{1}{9w_0^2}\right) \qquad 1.$$

and

$$\langle E_\nu \rangle_{EC} \approx Q_{EC}, \qquad 2.$$

where $W_0 = w_0 m_e c^2 = Q_{\beta^-} + m_e c^2$ for electron decay and $W_0 = Q_{\beta^+} - m_e c^2$ for positron decay; the *Q*'s are the *atomic* mass difference between the parent and the daughter in the radioactive process. The approximation given in equation 1 fits the values given in Table 6.5 of Chiu (1968) to better than $\pm 1.5\%$.

Total *Q*-values, exclusive of neutrino losses, for the proton-proton (pp) chain and the CN cycle are listed under H1(P,E+NU)H2 and N14(P,G)O15, respectively. The value for H1(P,E+NU)H2 holds only for the Sun. A general expression of good accuracy is $Q = 13.116[1 + 1.412 \times 10^8 (1/X - 1)\exp(-4.998/T_9^{1/3})]$ MeV, where *X* is the mass fraction of hydrogen. The energy release for the CNO tri-cycle is not much different from that for the CN cycle. These energy values can be used when the other relevant reactions are in equilibrium with these reactions. Under these conditions the N^{14} abundance should be taken approximately equal to the original C+N abundance if the temperature and density are such that O^{16} does not interact in the time scale involved. If the O^{16} does interact and the NO reactions come to equilibrium, then the N^{14} abundance should be taken approximately equal to the original C+N+O abundance.

Table 1 Reaction rates per second per (mole/cm**3)**($N-1$)

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H1(E-,NU)N1	Q= -.782	1.55E-10*T932*EXP(-9.080/T9)*(1. + 0.084*T9)
H1(P,E+NU)H2	Q= 1.442	4.21E-15/T923*EXP(-3.380/T913)*(1.+0.123*T913+1.09*T923+0.938*T9)
	1.192	EXCLUSIVE OF NU-ENERGY
	Q= 13.811	PER PP-REACTION FOR ENTIRE PP-CHAIN IN SUN
H1(PE-,NU)H2	Q= 1.442	1.44E-20/T976*EXP(-3.380/T913)*(1.-0.729*T913+9.82*T923-2.58*T9)
	.001	EXCLUSIVE OF NU-ENERGY
H2(P,G)HE3	Q= 5.494	2.65E+03/T923*EXP(-3.720/T913)
		* (1.+0.112*T913+1.99*T923+1.56*T9+0.162*T943+0.324*T953)
	REV RATIO	1.63E+10*T932*EXP(-63.755/T9)
H2(P,N)2H1	Q= -2.225	3.35E+07*EXP(-3.720/T913-25.817/T9)*(1.+0.784*T913+0.346*T923+0.690*T9)
	REV RATIO	4.24E-10/T932*EXP(25.817/T9)
H2(D,N)HE3	Q= 3.269	3.97E+08/T923*EXP(-4.258/T913)
		* (1.+0.098*T913+0.876*T923+0.600*T9-0.041*T943-0.071*T953)
	REV RATIO	1.73E+00*EXP(-37.938/T9)
H2(D,P)H3	Q= 4.033	4.17E+08/T923*EXP(-4.258/T913)
		* (1.+0.098*T913+0.518*T923+0.355*T9-0.010*T943-0.018*T953)
	REV RATIO	1.73E+00*EXP(-46.802/T9)
H3(P,G)HE4	Q= 19.815	2.20E+04/T923*EXP(-3.869/T913)
		* (1.+0.108*T913+1.68*T923+1.26*T9+0.551*T943+1.06*T953)
	REV RATIO	2.61E+10*T932*EXP(-229.947/T9)

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H3(P,N)HE3	Q= -.764	7.07E+08*(1.-0.150*T912+0.098*T9)*EXP(-8.864/T9)
	REV RATIO	9.98E-01*EXP(8.864/T9)
H3(D,N)HE4	Q= 17.590	8.09E+10/T923*EXP(-4.524/T913-(T9/0.120)**2)
		* (1.+0.092*T913+1.80*T923+1.16*T9+10.52*T943+17.24*T953)
		+ 8.73E+08/T923*EXP(-0.523/T9)
	REV RATIO	5.54E+00*EXP(-204.130/T9)

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H3(T,2N)HE4       Q= 11.332   1.67E+09/T923*EXP(-4.872/T913)
                               * (1.+0.086*T913-0.455*T923-0.272*T9+0.148*T943+0.225*T953)
                  REV RATIO   3.38E-10/T932*EXP( -131.512/T9)

HE3(E-,NU)H3      Q=  -.019   7.71E-12*T932*(1. + 6.48*T9 + 7.48*T9**2 + 2.91*T9**3)*EXP(-0.216/T9)

HE3(P,E+NU)HE4    Q= 19.796   8.78E-13/T923*EXP(-6.141/T913)
                     10.156   EXCLUSIVE OF NU-ENERGY

HE3(D,P)HE4       Q= 18.354   6.67E+10/T923*EXP(-7.181/T913-(T9/0.315)**2)
                               * (1.+0.058*T913-1.14*T923-0.464*T9+3.08*T943+3.18*T953)
                               + 4.36E+08/T912*EXP(-1.720/T9)
                  REV RATIO   5.55E+00*EXP( -212.994/T9)

HE3(T,D)HE4       Q= 14.321   5.46E+09*T9A56/T932*EXP(-7.733/T9A13)
                              T9A = T9/(1.+0.128*T9)
                  REV RATIO   1.60E+00*EXP( -166.193/T9)

HE3(T,NP)HE4      Q= 12.096   7.71E+09*T9A56/T932*EXP(-7.733/T9A13)
                              T9A = T9/(1.+0.115*T9)
                  REV RATIO   3.39E-10/T932*EXP( -140.376/T9)

HE3(HE3,2P)HE4    Q= 12.860   5.96E+10/T923*EXP(-12.276/T913)
                               * (1.+ 0.034*T913-0.199*T923-0.047*T9+0.032*T943+0.019*T953)
                  REV RATIO   3.39E-10/T932*EXP( -149.240/T9)

HE4(NN,G)HE6      Q=   .971   (0 TO 1)*4.04E-11/T9**2 * (1.+0.138*T9)*EXP(-9.585/T9)
                  REV RATIO   1.08E+20*T9**3*EXP( -11.269/T9)

HE4(NP,G)LI6      Q=  3.698   4.62E-06/T9**2 * (1.+0.075*T9)*EXP(-19.353/T9)
                  REV RATIO   7.22E+19*T9**3*EXP( -42.919/T9)

HE4(T,G)LI7       Q=  2.467   5.27E+05/T923*EXP(-8.080/T913)
                  REV RATIO   1.11E+10*T932*EXP(  -28.626/T9)

HE4(T,N)LI6       Q= -4.784   1.80E+08*EXP(-55.516/T9)*(1.-0.261*T9A32/T932)
                               + 2.72E+09/T932*EXP(-50.050/T9)
                              T9A = T9/(1.+49.18*T9)
                  REV RATIO   9.35E-01*EXP(   55.516/T9)
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Table 1 *Continued*

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HE4(HE3,G)BE7    Q=  1.586   6.33E+06/T923*EXP(-12.826/T913)
                              * (1.+0.033*T913-0.350*T923-0.080*T9+0.056* 943+0.033*T953)
                 REV RATIO   1.11E+10*T932*EXP(  -18.410/T9)

HE4(AN,G)BE9     Q=  1.573   2.59E-06/(T9**2*(1 + 0.344*T9))*EXP(-1.062/T9)
                 REV RATIO   5.84E+19*T9**3*EXP( -18.256/T9)

HE4(2A,G)C12     Q=  7.275   2.49E-08/T9**3*EXP(-4.4109/T9) + (0 TO 1)*1.35E-07/T932*EXP(-24.811/T9)
                 Q= 14.436   IF C12(A,G)O16 ALWAYS FOLLOWS
                 REV RATIO   2.00E+20*T9**3*EXP( -84.424/T9)

LI6(P,G)BE7      Q=  5.606   5.87E+05/T923*EXP(-8.413/T913)
                 REV RATIO   1.19E+10*T932*EXP(  -65.062/T9)

LI6(P,HE3)HE4    Q=  4.020   3.80E+10*T9A56/T932*EXP(-8.413/T9A13) + 5.97E+09/T932*EXP(-18.301/T9)
                             T9A = T9/(1.+0.095*T9)
                 REV RATIO   1.07E+00*EXP(  -46.652/T9)

LI6(A,G)B10      Q=  4.460   1.27E+07/T923*EXP(-18.790/T913-(T9/1.327)**2)
                              * (1.+0.022*T913+1.00*T923+0.156*T9+1.39*T943+0.549*T953)
                              + 3.18E+03/T932*EXP(-3.484/T9) + 1.54E+04/T9*EXP(-7.212/T9)
                 REV RATIO   1.58E+10*T932*EXP(  -51.759/T9)

LI7(P,N)BE7      Q= -1.644   6.77E+09*(1.-0.903*T912 + 0.218*T9)*EXP(-19.080/T9)
                 REV RATIO   9.98E-01*EXP(   19.080/T9)

LI7(P,A)HE4      Q= 17.348   8.04E+08/T923*EXP(-8.471/T913-(T9/30.068)**2)
                              * (1.+0.049*T913+0.230*T923+0.079*T9-0.027*T943-0.023*T953)
                              + 1.54E+06/T932*EXP(-4.479/T9) + 1.07E+10/T932*EXP(-30.443/T9)
                 REV RATIO   4.69E+00*EXP( -201.321/T9)
                             REV RATIO APPLIES TO BOTH LI7(P,A)HE4 AND LI7(P,G)2HE4

LI7(D,N)2HE4     Q= 15.123   2.92E+11/T923*EXP(-10.259/T913)
                 REV RATIO   9.95E-10/T932*EXP( -175.504/T9)

LI7(T,2N)2HE4    Q=  8.866   8.81E+11/T923*EXP(-11.333/T913)
                 REV RATIO   1.22E-19/T9**3*EXP(-102.886/T9)
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LI7(HE3,NP)2HE4 Q=  9.630   1.11E+13/T923*EXP(-17.989/T913)
                REV RATIO   6.09E-20/T9**3*EXP(-111.750/T9)

LI7(A,G)B11     Q=  8.666   2.26E+08/T923*EXP(-19.161/T913-(T9/0.991)**2)
                             * (1.+0.022*T913+0.382*T923+0.058*T9+0.571*T943+0.221*T953)
                             + 1.51E+03/T932*EXP(-3.030/T9) + 3.45E+04/T912*EXP(-5.345/T9)
                REV RATIO   4.02E+10*T932*EXP( -100.563/T9)

LI7(A,N)B10     Q= -2.790   3.84E+08*EXP(-32.383/T9)
                REV RATIO   1.32E+00*EXP(   32.383/T9)

BE7(E-,NU+G)LI7 Q=   .862   1.34E-10/T912*(1.-0.537*T913+3.86*T923+1.20*T9+0.0027/T9
                             * EXP(2.515E-03/T9))
                     .049   EXCLUSIVE OF NU ENERGY
                            RATE MUST NOT EXCEED 1.51E-07/(RHO*(1.+X)/2.)
                            FOR T9 LESS THAN 0.001.

BE7(P,G)B 8     Q=   .137   4.35E+05/T923*EXP(-10.262/T913) + 3.30E+03/T932*EXP(-7.306/T9)
                REV RATIO   1.30E+10*T932*EXP(   -1.592/T9)

BE7(D,P)2HE4    Q= 16.768   1.07E+12/T923*EXP(-12.428/T913)
                REV RATIO   9.97E-10/T932*EXP( -194.585/T9)

BE7(T,NP)2HE4   Q= 10.510   2.91E+12/T923*EXP(-13.729/T913)
                REV RATIO   6.09E-20/T9**3*EXP(-121.966/T9)

BE7(HE3,2P)2HE4 Q= 11.274   6.11E+13/T923*EXP(-21.793/T913)
                REV RATIO   1.22E-19/T9**3*EXP(-130.830/T9)

BE7(A,G)C11     Q=  7.545   2.12E+08/T923*EXP(-23.212/T913-(T9/0.658)**2)
                             * (1.+0.018*T913+0.228*T923+0.029*T9+0.548*T943+0.175*T953)
                             + 7.27E+04/T932*EXP(-6.525/T9) + 1.23E+05/T9*EXP(-9.742/T9)
                REV RATIO   4.02E+10*T932*EXP(  -87.559/T9)

BE9(P,G)B10     Q=  6.585   1.33E+07/T923*EXP(-10.359/T913-(T9/0.846)**2)
                             * (1.+0.040*T913+1.52*T923+0.428*T9+2.15*T943+1.54*T953)
                             + 9.64E+04/T932*EXP(-3.445/T9) + 2.72E+06/T932*EXP(-10.620/T9)
                REV RATIO   9.73E+09*T932*EXP(  -76.422/T9)
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Table 1 *Continued*

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BE9(P,N)B 9      Q= -1.850   5.58E+07*(1.+0.042*T912+0.985*T9)*EXP(-21.466/T9)
                              + 1.02E+09/T932*EXP(-26.753/T9)
                 REV RATIO   9.98E-01*EXP(   21.466/T9)

BE9(P,D)2HE4     Q=   .651   2.11E+11/T923*EXP(-10.359/T913-(T9/0.520)**2)
                              * (1.+0.040*T913+1.09*T923+0.307*T9+3.21*T943+2.30*T953)
                              + 5.79E+08/T9*EXP(-3.046/T9) + 8.50E+08/T934*EXP(-5.800/T9)
                 REV RATIO   8.07E-11/T932*EXP(   -7.560/T9)

BE9(P,A)LI6      Q=  2.125   2.11E+11/T923*EXP(-10.359/T913-(T9/0.520)**2)
                              * (1.+0.040*T913+1.09*T923+0.307*T9+3.21*T943+2.30*T953)
                              + 4.51E+08/T9*EXP(-3.046/T9) + 6.70E+08/T934*EXP(-5.160/T9)
                 REV RATIO   6.18E-01*EXP(  -24.663/T9)

BE9(A,N)C12      Q=  5.702   4.62E+13/T923*EXP(-23.870/T913-(T9/0.049)**2)
                              * (1.+0.017*T913+8.57*T923+1.05*T9+74.51*T943+23.15*T953)
                              + 7.34E-05/T932*EXP(-1.184/T9) + 2.27E-01/T932*EXP(-1.834/T9)
                              + 1.26E+05/T932*EXP(-4.179/T9) + 2.40E+08*EXP(-12.732/T9)
                 REV RATIO   1.03E+01*EXP(  -66.167/T9)

B10(P,G)C11      Q=  8.691   4.61E+05/T923*EXP(-12.062/T913-(T9/4.402)**2)
                              * (1.+0.035*T913+0.426*T923+0.103*T9+0.281*T943+0.173*T953)
                              + 1.93E+05/T932*EXP(-12.041/T9) + 1.14E+04/T932*EXP(-16.164/T9)
                 REV RATIO   3.03E+10*T932*EXP( -100.862/T9)

B10(P,A)BE7      Q=  1.146   1.26E+11/T923*EXP(-12.062/T913-(T9/4.402)**2)
                              * (1.+0.035*T913-0.498*T923-0.121*T9+0.300*T943+0.184*T953)
                              + 2.59E+09/T9*EXP(-12.260/T9)
                 REV RATIO   7.54E-01*EXP(  -13.303/T9)

B11(P,G)C12      Q= 15.957   4.62E+07/T923*EXP(-12.095/T913-(T9/0.239)**2)
                              * (1.+0.035*T913+3.00*T923+0.723*T9+9.91*T943+6.07*T953)
                              + 7.89E+03/T932*EXP(-1.733/T9) + 9.68E+04/T915*EXP(-5.617/T9)
                 REV RATIO   7.01E+10*T932*EXP( -185.181/T9)

B11(P,N)C11      Q= -2.765   1.69E+08*(1.-0.048*T912 + 0.010*T9)*EXP(-32.084/T9)
                 REV RATIO   9.98E-01*EXP(   32.084/T9)
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B11(P,A)2HE4    Q=  8.682  2.59E+11/T923*EXP(-12.095/T913-(T9/2.02)**2)
                           * (1.+0.035*T913+1.22*T923+0.295*T9+2.15*T943+1.32*T953)
                           + 7.40E+06/T932*EXP(-1.733/T9) + 8.14E+09/T932*EXP(-7.177/T9)
                           + 1.71E+09/T923*EXP(-12.696/T9)
                REV RATIO  3.50E-10/T932*EXP( -100.758/T9)

C11(P,G)N12     Q=   .595  4.24E+04/T923*EXP(-13.658/T913-(T9/1.627)**2)
                           * (1.+0.031*T913+3.11*T923+0.665*T9+4.61*T943+2.50*T953)
                           + 8.84E+03/T932*EXP(-7.021/T9)
                REV RATIO  2.33E+10*T932*EXP(   -6.910/T9)

C12(P,G)N13     Q=  1.944  2.04E+07/T923*EXP(-13.690/T913-(T9/1.500)**2)
                           * (1.+0.030*T913+1.19*T923+0.254*T9+2.06*T943+1.12*T953)
                           + 1.08E+05/T932*EXP(-4.925/T9) + 2.15E+05/T932*EXP(-18.179/T9)
                REV RATIO  8.84E+09*T932*EXP(  -22.554/T9)

C12(A,G)O16     Q=  7.162  9.03E+07/T9**2*(1.+0.621*T923)**2/(1.+0.047/T923)**2
                          *EXP(-32.120/T913-(T9/5.863)**2) + 2.74E+07/T923*EXP(-32.120/T913)
                           + 1.25E+03/T932*EXP(-27.499/T9) + 1.43E-02*T9**5 * EXP(-15.541/T9)
                REV RATIO  5.13E+10*T932*EXP(  -83.110/T9)

C12(A,N)O15     Q= -8.502  2.48E+07*(1.+0.188*T912 + 0.015*T9)*EXP(-98.663/T9)
                REV RATIO  1.41E+00*EXP(   98.663/T9)

C13(P,G)N14     Q=  7.551  8.01E+07/T923*EXP(-13.717/T913-(T9/2.000)**2)
                           * (1.+0.030*T913+0.958*T923+0.204*T9+1.39*T943+0.753*T953)
                           + 1.35E+06/T932*EXP(-5.978/T9) + 2.66E+05/T932*EXP(-11.987/T9)
                           + 2.26E+06/T932*EXP(-13.463/T9)
                REV RATIO  1.19E+10*T932*EXP(  -87.625/T9)

C13(P,N)N13     Q= -3.003  1.88E+08*(1.-0.167*T912 + 0.037*T9)*EXP(-34.848/T9)
                REV RATIO  9.98E-01*EXP(   34.848/T9)

C13(A,N)O16     Q=  2.215  6.77E+15/T923*EXP(-32.329/T913-(T9/1.284)**2)
                           * (1.+0.013*T913+2.04*T923+0.184*T9)
                           + 3.82E+05/T932*EXP(-9.373/T9) + 1.41E+06/T932*EXP(-11.873/T9)
                           + 2.00E+09/T932*EXP(-20.409/T9) + 2.92E+09/T932*EXP(-29.283/T9)
                REV RATIO  5.79E+00*EXP(  -25.707/T9)
```

Table 1 *Continued*

```
C14(P,G)N15     Q= 10.207  6.80E+06/T923*EXP(-13.741/T913-(T9/5.721)**2)
                            * (1.+0.030*T913+0.503*T923+0.107*T9+0.213*T943+0.115*T953)
                            + 5.36E+03/T932*EXP(-3.811/T9) + 9.82E+04/T913*EXP(-4.739/T9)
                REV RATIO   9.00E+09*T932*EXP( -118.456/T9)

C14(P,N)N14     Q=  -.626  7.21E+05*(1.+0.361*T912+0.502*T9)*EXP(-7.269/T9)
                            + 1.18E+08*EXP(-11.348/T9)
                REV RATIO   3.33E-01*EXP(    7.269/T9)

N13(P,G)O14     Q=  4.626  5.99E+07/T923*EXP(-15.202/T913-(T9/1.181)**2)
                            * (1.+0.027*T913+1.71*T923+0.329*T9+2.10*T943+1.03*T953)
                            + 8.36E+05/T932*EXP(-6.313/T9)
                REV RATIO   3.57E+10*T932*EXP(  -53.688/T9)

N14(P,G)O15     Q=  7.298  5.08E+07/T923*EXP(-15.228/T913-(T9/ 3.090)**2)
                            * (1.+0.027*T913-0.778*T923-0.149*T9+0.261*T943+0.127*T953)
                            + 2.28E+03/T932*EXP(-3.011/T9) + 1.65E+04*T913*EXP(-12.007/T9)
                Q= 24.970 PER N14(P,G)O15 REACTION FOR FULL CN-CYCLE.
                      USE N14 = ORIGINAL C+N+(0 TO 1)*O ABUNDANCE (SEE TEXT)
                REV RATIO   2.70E+10*T932*EXP(  -84.692/T9)

N14(P,N)O14     Q= -5.927  6.74E+07*(1.+0.658*T912 + 0.379*T9)*EXP(-68.785/T9)
                REV RATIO   2.99E+00*EXP(   68.785/T9)

N14(P,A)C11     Q= -2.922  2.63E+16*T9A56/T932*EXP(-31.883/T9A13 - 33.911/T9)
                           T9A = T9/(1.+4.78E-02*T9+7.56E-03*T953/(1.+4.78E-02*T9)**(2./3.))
                REV RATIO   2.72E-01*EXP(   33.911/T9)

N14(A,G)F18     Q=  4.416  7.78E+09/T923*EXP(-36.031/T913-(T9/0.881)**2)
                            * (1.+0.012*T913+1.45*T923+0.117*T9+1.97*T943+0.406*T953)
                            + 2.36E-10/T932*EXP(-2.798/T9) + 2.03E+00/T932*EXP(-5.054/T9)
                            + 1.15E+04/T923*EXP(-12.310/T9)
                REV RATIO   5.42E+10*T932*EXP(  -51.247/T9)
```

```
N15(P,G)O16     Q= 12.128   9.78E+08/T923*EXP(-15.251/T913-(T9/0.450)**2)
                             * (1.+0.027*T913+0.219*T923+0.042*T9+6.83*T943+3.32*T953)
                             + 1.11E+04/T932*EXP(-3.328/T9) + 1.49E+04/T932*EXP(-4.665/T9)
                             + 3.80E+06/T932*EXP(-11.048/T9)
                REV RATIO   3.62E+10*T932*EXP( -140.741/T9)

N15(P,N)O15     Q= -3.536   3.51E+08*(1.+0.452*T912-0.191*T9)*EXP(-41.032/T9)
                REV RATIO   9.98E-01*EXP(    41.032/T9)

N15(P,A)C12     Q=  4.966   8.16E+11/T923*EXP(-15.251/T913-(T9/0.522)**2)
                             * (1.+0.027*T913+6.74*T923+1.29*T9)
                             + 1.29E+08/T932*EXP(-3.676/T9) + 3.14E+08*EXP(-7.974/T9)
                REV RATIO   7.06E-01*EXP(  -57.631/T9)

N15(A,G)F19     Q=  4.013   2.54E+10/T923*EXP(-36.211/T913-(T9/0.616)**2)
                             * (1.+0.012*T913+1.69*T923+0.136*T9+1.91*T943+0.391*T953)
                             + 9.83E-03/T932*EXP(-4.232/T9) + 7.59E+01/T932*EXP(-6.319/T9)
                             + 1.52E+03*T9*EXP(-9.747/T9)
                REV RATIO   5.54E+10*T932*EXP(  -46.569/T9)

O16(P,G)F17     Q=   .601   1.50E+08/(T923*(1.+2.13*(1.-EXP(-0.728*T923)[redacted] * EXP(-16.692/T913)
                REV RATIO   3.03E+09*T932*EXP(    -6.970/T9)

O16(P,A)N13     Q= -5.218   1.70E+19*T9A56/T932*EXP(-35.829/T9A13-60.556/T9)/(1.+5.*EXP(-2.396*T923)[redacted]
                            T9A = T9/(1.+0.224*T9)
                REV RATIO   1.72E-01*EXP(    60.556/T9)

O16(A,G)NE20    Q=  4.730   5.49E+09/T923*EXP(-39.756/T913) + 4.09E+01/T932*EXP(-10.359/T9)
                             + 3.92E+02/T932*EXP(-12.243/T9) + 8.05*T9**2*EXP(-20.093/T9)
                REV RATIO   5.65E+10*T932*EXP(  -54.891/T9)

O17(P,G)F18     Q=  5.609   7.97E+07*T9A56/T932*EXP(-16.712/T9A13) + 1.51E+08/T923*EXP(-16.712/T913)
                             * (1.+0.025*T913-0.051*T923-8.82E-03*T9) + 1.56E+05/T9*EXP(-6.272/T9)
                             + (0 TO 1, SAME AS O17(P,A)N14 ) * 1.31E+01/T932*EXP(-1.961/T9)
                            T9A = T9/(1.+2.69*T9)
                REV RATIO   3.66E+10*T932*EXP(  -65.092/T9)
```

Table 1 *Continued*

```
O17(P,A)N14      Q=  1.193  1.53E+07/T923*EXP(-16.712/T913-(T9/0.565)**2)
                             * (1.+0.025*T913+5.39*T923+0.940*T9+13.5*T943+5.98*T953)
                             + 2.92E+06*T9*EXP(-4.247/T9)
                             +                    (0 TO 1) *
                            (4.81E+10*T9*EXP(-16.712/T913-(T9/0.040)**2) + 5.05E-05/T932*EXP(-0.723/T9)
                             + (0 TO 1, SAME AS O17(P,G)F18 ) * 1.31E+01/T932*EXP(-1.961/T9)
                 REV RATIO   6.76E-01*EXP(  -13.845/T9)

O17(A,G)NE21     Q=  7.349  1.73E+17*FPT9A/GT9*T9A56/T932*EXP(-39.914/T9A13)
                             + 3.50E+15*FT9A/GT9*T9A56/T932*EXP(-39.914/T9A13)
                            T9A = T9/(1.+0.1646*T9)            GT9 = 1. + EXP(-10.106/T9)/3.
                            FT9A = EXP(-(0.786/T9A)**3.51)     FPT9A = EXP(-(T9A/1.084)**1.69)
                 REV RATIO   8.63E+10*T932*EXP(  -85.281/T9)

O17(A,N)NE20     Q=   .588  1.03E+18/GT9*T9A56/T932*EXP(-39.914/T9A13)
                            T9A = T9/(1.+.0268*T9+.0232*T953/(1.+.0268*T9)**(2./3.))
                            GT9 = 1. + EXP(-10.106/T9)/3.
                 REV RATIO   1.86E+01*EXP(   -6.819/T9)

O18(P,G)F19      Q=  7.993  2.39E+08/T923*EXP(-16.729/T913) + 1.80E+05/T923*EXP(-6.342/T9)
                             +                    (0 TO 1) *
                            3.96E+09/T923*EXP(-16.729/T913-(T9/0.138)**2)
                             * (1.+0.025*T913+5.88*T923+1.03*T9+29.80*T943+13.21*T953)
                             + 3.09E+02/T932*EXP(-1.662/T9)
                            DIVIDE ALL TERMS BY GT9 = 1. + 5.*EXP(-23.002/T9)
                 REV RATIO   9.20E+09*T932*EXP(  -92.756/T9)

O18(P,A)N15      Q=  3.980  2.13E+11/T923*EXP(-16.729/T913-(T9/1.318)**2)
                             * (1. +0.025*T913+1.68*T923+0.292*T9+1.86*T943+0.824*T953)
                             + 9.64E+08/T9*EXP(-7.361/T9)
                             +                    (0 TO 1) *
                            7.88E+14/(T923*(0.439*(1.+5.18*T923)**2 + 0.561)
                             * EXP(-16.729/T913-0.534*T923)
                             + 1.99E+13/T923*EXP(-16.729/T913-(T9/0.138)**2)
                             * (1. + 0.025*T913+5.98*T923+1.04*T9+30.70*T943+13.61*T953)
                             + 1.40E+06/T932*EXP(-1.662/T9) + 2.34E+06/T932*EXP(-2.915/T9)
                            DIVIDE ALL TERMS BY GT9 = 1.+5.*EXP(-23.002/T9)
                 REV RATIO   1.66E-01*EXP(  -46.187/T9)
```

```
O18(A,G)NE22     Q=  9.668  7.22E+17*FPT9A/GT9*T9A56/T932*EXP(-40.056/T9A13)
                             + 3.61E+14*FT9A/GT9*T9A56/T932*EXP(-40.056/T9A13)
                           T9A = T9/(1.+0.0483*T9+0.00569*T953 /(1.+0.0483*T9)**(2./3.))
                           GT9=1.+5.*EXP(-23.002/T9)          FT9A = EXP(-( 0.431/T9A)**3.89)
                           FPT9A = EXP(-(T9A/0.576)**1.87)
                 REV RATIO 5.85E+10*T932*EXP( -112.191/T9)

O18(A,N)NE21     Q=  -.698  7.22E+17*FT9A/GT9*T9A56/T932*EXP(-40.056/T9A13)
                             + 150.31/GT9*EXP(-8.101/T9)
                           T9A = T9/(1.+0.0483*T9+0.00569*T953 /(1.+0.0483*T9)**(2./3.))
                           GT9 = 1. + 5.*EXP(-23.002/T9)      FT9A = EXP(-(0.431/T9A)**3.89)
                 REV RATIO 7.84E-01*EXP(    8.101/T9)

F19(P,G)NE20     Q= 12.845  6.04E+07/T923*EXP(-18.113/T913-(T9/0.416)**2)
                             * (1. + 0.023*T913 + 2.06*T923 + 0.332*T9 + 3.16*T943 + 1.30*T953)
                             + 6.32E+02/T932*EXP(-3.752/T9) + 7.56E+04/T927*EXP(-5.722/T9)
                           DIVIDE ALL TERMS BY GT9=1.+4.*EXP(-2.090/T9)+7.*EXP(-16.440/T9)
                 REV RATIO 3.70E+10*T932*EXP( -149.063/T9)

F19(P,N)NE19     Q= -4.021  1.27E+08*(1.-0.147*T912 + 0.069*T9)*EXP(-46.659/T9)
                 REV RATIO 9.98E-01*EXP(   46.659/T9)

F19(P,A)O16      Q=  8.115  3.20E+11/T923*EXP(-18.113/T913-(T9/0.416)**2)
                             * (1.+0.023*T913 + 2.16*T923 + 0.348*T9 + 3.47*T943 + 1.42*T953)
                             + 5.23E+06/T932*EXP(-3.752/T9) + 3.88E+08/T918*EXP(-6.232/T9)
                           DIVIDE ALL TERMS BY GT9=1.+4.*EXP(-2.090/T9)+7.*EXP(-16.440/T9)
                 REV RATIO 6.54E-01*EXP(  -94.172/T9)

F19(A,P)NE22     Q=  1.675  4.50E+18/T923*EXP(-43.467/T913-(T9/0.637)**2)+7.98E+04*T932*EXP(-12.760/T9)
                 REV RATIO 6.36E+00*EXP(  -19.435/T9)

NE20(P,G)NA21    Q=  2.432  9.55E+06*EXP(-19.447/T913)/(T9**2*(1.+0.0127/T923)**2)
                             +2.05E+08/T923*EXP(-19.447/T913)*(1.+2.67*EXP(-SQRT(T9/0.210)))
                             + 1.80E+01/T932*EXP(-4.242/T9) + 1.02E+01/T932*EXP(-4.607/T9)
                             + 3.60E+04/T914*EXP(-11.249/T9)
                 REV RATIO 4.63E+09*T932*EXP(  -28.227/T9)
```

Table 1 *Continued*

```
NE20(A,G)MG24    Q=  9.315   4.11E+11/T923*EXP(-46.766/T913-(T9/2.219)**2)
                              * (1. + 0.009*T913 + 0.882*T923 + 0.055*T9 + 0.749*T943 + 0.119*T953)
                              + 5.27E+03/T932*EXP(-15.869/T9) + 6.51E+03*T912*EXP(-16.223/T9)
                              +                 (0 TO 1) *
                             4.21E+01/T932*EXP(-9.115/T9) + 3.20E+01/T923*EXP(-9.383/T9)
                             DIVIDE ALL TERMS BY GT9 = 1. + 5.*EXP(-18.960/T9)
                 REV RATIO   6.01E+10*T932*EXP( -108.094/T9)

NE21(P,G)NA22    Q=  6.740   3.42E+08/T923*EXP(-19.462/T913) + 1.56E+05/T932*EXP(-5.584/T9)
                              + 8.92E+05/T932*EXP(-5.810/T9) + 5.74E+06/T943*EXP(-8.131/T9)
                 REV RATIO   1.06E+10*T932*EXP(  -78.220/T9)

NE21(A,G)MG25    Q=  9.885   4.94E+15/GT9*T9A56/T932*EXP(-46.890/T9A13)       T9A = T9/(1.+0.0537*T9)
                             GT9 = 1. + 1.5*EXP(-4.068/T9) + 2.0*EXP(-20.258/T9)
                 REV RATIO   4.06E+10*T932*EXP( -114.717/T9)

NE21(A,N)MG24    Q=  2.553   4.94E+19*T9A56/T932*EXP(-46.890/T9A13) + 2.66E+07/T932*EXP(-22.049/T9)
                             T9A = T9/(1.+0.0537*T9)
                             DIVIDE ALL TERMS BY GT9 = 1.+1.5*EXP(-4.068/T9) + 2.0*EXP(-20.258/T9)
                 REV RATIO   1.29E+01*EXP(  -29.631/T9)

NE22(P,G)NA23    Q=  8.793   1.14E+09/T923*EXP(-19.475/T913) + 8.96E+03/T932*EXP(-4.840/T9)
                              + 6.52E+04/T932*EXP(-5.319/T9) + 8.33E+05/T912*EXP(-7.462/T9)
                 REV RATIO   4.67E+09*T932*EXP( -102.043/T9)

NE22(A,G)MG26    Q= 10.613   4.16E+19*FPT9A/GT9*T9A56/T932*EXP(-47.004/T9A13)
                              + 2.08E+16*FT9A/GT9*T9A56/T932*EXP(-47.004/T9A13)
                             T9A = T9/(1.+0.0548*T9)            GT9 = 1.+5.0*EXP(-14.791/T9)
                             FT9A = EXP(-(0.197/T9A)**4.82)     FPT9A = EXP(-(T9A/0.249)**2.31)
                 REV RATIO   6.15E+10*T932*EXP( -123.166/T9)

NE22(A,N)MG25    Q=  -.480   4.16E+19*FT9A/GT9*T9A56/T932*EXP(-47.004/T9A13)
                              + 1.44E-04/GT9*EXP(-5.574/T9)
                             T9A = T9/(1.+0.0548*T9)            GT9 = 1.+5.0*EXP(-14.791/T9)
                             FT9A = EXP(-(0.197/T9A)**4.82)
                 REV RATIO   5.44E-01*EXP(     5.574/T9)
```

```
NA23(P,G)MG24   Q= 11.692   2.93E+08/T923*EXP(-20.766/T913-(T9/0.297)**2)
                             * (1. + .020*T913 + 1.61*T923 + 0.226*T9 + 4.94*T943 + 1.76*T953)
                             + 9.34E+01/T932*EXP(-2.789/T9) + 1.89E+04/T932*EXP(-3.434/T9)
                             + 5.10E+04*T915*EXP(-5.510/T9)
                            DIVIDE ALL TERMS BY GT9 = 1. + 1.5*EXP(-5.105/T9)
                REV RATIO   7.49E+10*T932*EXP( -135.679/T9)

NA23(P,N)MG23   Q= -4.839   9.29E+08*(1. - 0.881*T9A32/T932)*EXP(-56.157/T9)
                            T9A = T9/(1.+0.141*T9)
                REV RATIO   9.98E-01*EXP(   56.157/T9)

NA23(P,A)NE20   Q=  2.377   6.79E+09/T923*EXP(-20.766/T913-(T9/0.131)**2)
                             * (1. + 0.020*T913 + 8.21*T923 + 1.15*T9 + 44.36*T943 + 15.84*T953)
                             + 2.75E+00/T932*EXP(-1.990/T9) + 1.10E+04/T943*EXP(-3.207/T9)
                             + 1.84E+06*T913*EXP(-5.170/T9)
                             + (0 TO 1)*3.06E-12/T932*EXP(-0.447/T9)
                            DIVIDE ALL TERMS BY GT9 = 1.+1.5*EXP(-5.105/T9)
                REV RATIO   1.25E+00*EXP(  -27.585/T9)

MG24(P,G)AL25   Q=  2.270   9.79E+10/T923*EXP(-22.019/T913-(T9/0.168)**2 )
                             * (1. + 0.019*T913+6.50*T923+0.861*T9+29.67*T943+9.99*T953)
                             + 8.26E+02/T932*EXP(-2.492/T9) + 1.89E+03*T914*EXP(-4.261/T9)
                            DIVIDE ALL TERMS BY GT9 = 1. + 5.*EXP(-15.882/T9)
                REV RATIO   3.13E+09*T932*EXP(  -26.346/T9)

MG24(A,G)SI28   Q=  9.985   4.78E+01/T932*EXP(-13.506/T9) + 1.97E+03/T932*EXP(-15.246/T9)
                             + 9.32E+02*T9*EXP(-17.425/T9)
                             +                  (0 TO 1)*
                            1.72E-09/T932*EXP(-5.028/T9) + 1.25E-03/T932*EXP(-7.929/T9)
                             + 2.43E+01/T9*EXP(-11.523/T9)
                            DIVIDE ALL TERMS BY GT9 = 1. + 5.*EXP(-15.882/T9)
                REV RATIO   6.27E+10*T932*EXP( -115.872/T9)
```

Table 1 *Continued*

```
MG25(P,G)AL26    Q=  6.307   2.70E+08/T923*EXP(-22.031/T913-(T9/0.678)**2)
                              * (1.+ 0.019*T913+3.05*T923+0.404*T9+7.12*T943+2.40*T953)
                              + 4.66E+03/T932*EXP(-3.529/T9) + 9.07E+04/T9*EXP(-5.006/T9)
                              +                  (0 TO 1) *
                             (7.40E+09/T923*EXP(-22.031/T913-(T9/0.129)**2)
                              * (1. + 0.019*T913+7.08*T923+0.937*T9+37.34*T943+12.57*T953)
                              + 3.54E+03/T932*EXP(-2.095/T9)
                             DIVIDE ALL TERMS BY GT9 = 1. + 10.*EXP(-13.180/T9)/3.
                 REV RATIO   1.03E+10*T932*EXP(  -73.187/T9)

MG25(A,G)SI29    Q= 11.127   8.97E+17/GT9*T9A56/T932*EXP(-53.410/T9A13)
                             T9A = T9/(1.+0.0963*T9)          GT9 = 1.+10.*EXP(-13.180/T9)/3.
                 REV RATIO   1.90E+11*T932*EXP( -129.125/T9)

MG25(A,N)SI28    Q=  2.653   3.59E+20/GT9*T9A56/T932*EXP(-53.410/T9A13)
                             T9A = T9/(1.+0.063*T9)           GT9 = 1.+10.*EXP(-13.180/T9)/3.
                 REV RATIO   2.00E+01*EXP(  -30.786/T9)

MG26(P,G)AL27    Q=  8.271   3.23E+08/T923*EXP(-22.042/T913-(T9/0.313)**2)
                              * (1. + 0.019*T913+3.55*T923+0.469*T9+9.45*T943+3.18*T953)
                              + 7.76E+02/T932*EXP(-3.265/T9) + 3.88E+04/T932*EXP(-3.781/T9)
                              + 1.80E+04*T934*EXP(-3.505/T9)
                              +                  (0 TO 1) *
                             (4.33E+10/T923*EXP(-22.042/T913-(T9/0.122)**2)
                              * (1. + 0.019*T913+7.31*T923+0.967*T9+40.04*T943+13.47*T953)
                              + 3.57E+03/T932*EXP(-2.010/T9)
                             DIVIDE ALL TERMS BY GT9 = 1. + 5.*EXP(-20.990/T9)
                 REV RATIO   3.14E+09*T932*EXP(  -95.982/T9)

MG26(A,G)SI30    Q= 10.643   4.18E+19*FPT9A/GT9*T9A56/T932*EXP(-53.505/T9A13)
                              + 4.71E+20*FT9A/GT9*T9A56/T932*EXP(-53.505/T9A13)
                             T9A = T9/(1.+0.248*T9)           GT9 = 1.+5.*EXP(-20.990/T9)
                             FT9A = EXP(-(0.664/T9A)**4.19)   FPT9A = EXP(-(T9A/0.870)**2.01)
                 REV RATIO   6.38E+10*T932*EXP( -123.509/T9)
```

```
MG26(A,N)SI29    Q=    .033  2.93E+20/GT9*T9A56/T932*EXP(-53.505/T9A13)
                             T9A = T9/(1.+0.0628*T9)          GT9 = 1.+5.*EXP(-20.990/T9)
                 REV RATIO   1.68E+00*EXP(    -.385/T9)

AL27(P,G)SI28    Q= 11.585   1.57E+08/T923*EXP(-23.261/T913-(T9/0.157)**2)
                              * (1. + 0.018*T913+5.68*T923+0.712*T9+26.45*T943+8.43*T953)
                              + 2.20E+00/T932*EXP(-2.269/T9) + 1.22E+01/T932*EXP(-2.517/T9)
                              + 4.14E+03*T985*EXP(-3.270/T9)
                              +                (0 TO 1) *
                             3.34E-07/T932*EXP(-0.858/T9) + 5.60E-07/T932*EXP(-0.968/T9)
                             DIVIDE ALL TERMS BY GT9=1.+EXP(-9.792/T9)/3.+2.*EXP(-11.773/T9)/3.
                 REV RATIO   1.13E+11*T932*EXP( -134.446/T9)

AL27(P,A)MG24    Q=  1.601   1.10E+08/T923*EXP(-23.261/T913-(T9/0.157)**2)
                              * (1. + 0.018*T913 + 12.85*T923 + 1.61*T9 + 89.87*T943 + 28.66*T953)
                              + 1.29E+02/T932*EXP(-2.517/T9) + 5.66E+03*T972*EXP(-3.421/T9)
                              +                (0 TO 1) *
                             2.00E-05/T932*EXP(-0.858/T9) + 2.81E-05/T932*EXP(-0.968/T9)
                             DIVIDE ALL TERMS BY GT9 = 1. + EXP(-9.792/T9)/3. + 2.*EXP(-11.773/T9)/3.
                 REV RATIO   1.81E+00*EXP(  -18.575/T9)

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C12+C12 (MG24)   Q= 13.931   1.26E+27*T9A56/T932*EXP(-84.165/T9A13)/(EXP(-0.010*T9A**4)
                              + 5.56E-03*EXP(1.685*T9[redacted]                T9A = T9/(1.+0.067*T9)

C12+O16 (SI28)   Q= 16.754   1.72E+31*T9A56/T932*EXP([redacted]594/T9A13)/(EXP(-0.180*T9A**2)
                              + 1.06E-03*EXP(2.562*T9[redacted]                T9A = T9/(1.+0.055*T9)

O16+O16 (S32)    Q= 16.541   3.61E+37*T9A56/T932*EXP(-135.930/T9A13)/(EXP(-0.032*T9A**4)
                              + 3.89E-04*EXP(2.659*T9A23))          T9A = T9/(1.+0.067*T9)
```

BRANCHING RATIO (Q-VALUE)

	N-CHANNELS	P-CHANNELS	A-CHANNELS
C12+C12 T9 BELOW 3	0.00(-2.599)	0.50(2.240)	0.50(4.617)
T9 ABOVE 3	0.05	0.30	0.65
C12+O16	0.10(-0.423)	0.50(5.169)	0.40(6.770)
O16+O16	0.14(1.453)	0.80(7.677)	0.29(9.593)

Two-body Interactions

In the case of two-body primary interactions the third column of Table 1 gives $N_A\langle 01\rangle$ in reactions per sec per (mole/cm^3), or more simply in cm^3 sec^{-1} mole^{-1}.[6] As in FCZ I, $\langle 01\rangle$ is identical to $\langle \sigma v\rangle_{01}$, the Maxwellian averaged product of cross section times velocity for the two interacting nuclei, 0 and 1. This average is proportional to the Laplace transform of σE with $s = 1/kT$ and is given by

$$\langle \sigma v\rangle = \frac{(8/\pi)^{1/2}}{M^{1/2}(kT)^{3/2}} \int \sigma E \exp(-E/kT)\, dE, \qquad 3.$$

where M is the reduced mass of the interacting particles. In reference to the notation of FCZ I we also have $N_A\langle 01\rangle = [01]/\rho$. We tabulate $N_A\langle 01\rangle$ because it is only a function of temperature[7] and not of density and is most convenient for use in reaction chain differential equations that use X, the mass fraction, or $Y = X/A$, the number of moles g^{-1}. The number of moles cm^{-3} is given by ρY. Thus from equations I-3, I-4, and I-5

$$\frac{dY_0/dt}{\rho Y_0 Y_1} = \frac{dY_1/dt}{\rho Y_0 Y_1} = -N_A\langle 01\rangle \text{ cm}^3 \text{ sec}^{-1} \text{ mole}^{-1}. \qquad 4.$$

Thus, multiply $N_A\langle 01\rangle$ by $\rho Y_1 = \rho X_1/A_1$ in mole cm^{-3} to obtain the interaction rate per sec, or reciprocal of the mean lifetime for 0 interacting with 1, and permute for the reciprocal of the mean lifetime for 1 interacting with 0. Multiply $N_A\langle 01\rangle$ by $\rho N_A Y_0 Y_1/(1+\delta_{01}) = \rho N_A Y_0 Y_1/N_0!\,N_1!$ to obtain reactions g^{-1} sec^{-1}. N_I is the number of particles of type $I = 0$, 1 involved. Multiply this result by 1.6022×10^{-6} Q_6 to obtain energy generation in erg g^{-1} sec^{-1}. To obtain reactions cm^{-3} sec^{-1} or erg cm^{-3} sec^{-1} multiply by an additional ρ.

For the rate of reverse reactions between *ground states* of the reacting nuclei multiply $N_A\langle 01\rangle$ by REV RATIO to obtain $N_A\langle 23\rangle$ or $N_A^2\langle 234\rangle$. For photodisintegrations induced by gamma radiation multiply $N_A\langle 01\rangle$ by REV RATIO to obtain $\lambda_\gamma(2) = 1/\tau_\gamma(2)$, the interaction rate per sec or the reciprocal of the mean lifetime to photodisintegration, for 2 from $\gamma + 2 \rightarrow 0 + 1$. The REV RATIOS listed are calculated using statistical weight factors for ground states only and are defined by equations I-10 and I-16 as given and equation I-20 modified to include a factor N_A on each side of the equation. When calculating reverse reaction rates at high temperature, $T_9 > 1$, see the section on reverse reaction rates and partition functions below. REV RATIOS are not listed for reactions that produce neutrinos.

[6] In general, as indicated at the top of Table 1, the reaction rates, direct or reverse, are given per second per (mole/cm^3)$^{N-1}$, where N is the number of interacting particles, including the target, *exclusive of photons* (gamma radiation). Recall that in the reverse reaction the residual nucleus of the direct reaction becomes the target.

[7] Fractional powers of temperature such as $T_9^{M/N}$, where M and N are integers, are given in Table 1 as *T9MN*, for example, *T923* $\equiv T_9^{2/3}$. Integral powers of temperature are given in standard computer form, for example, *T9**3* $\equiv T_9^3$.

Three-Body Interactions

In the case of three-body primary interactions the third column of Table 1 gives $N_A^2\langle 012\rangle$ in reactions per sec per (mole/cm^3)^2 or, more simply, in cm^6 sec^{-1} mole^{-2}. Multiply $N_A^2\langle 012\rangle$ by $\rho^2 Y_1 Y_2(1+\delta_{01}+\delta_{20})/(1+\delta_{01}+\delta_{12}+\delta_{20}+2\delta_{012}) = \rho^2 Y_1 Y_2 N_0/N_0!\,N_1!\,N_2!$ to obtain the interaction rate per sec for 0 and permute for 1 and 2. N_I equals the number of particles of type $I = 0$, 1, 2 involved. For example, N_α equals 3 in $He^4(2\alpha,\gamma)C^{12}$ and $N_0!\,N_1!N_2!$ equals 6. Recall that O! equals 1. Multiply $N_A^2\langle 012\rangle$ by $\rho^2 N_A Y_0 Y_1 Y_2/N_0!\,N_1!\,N_2!$ for reactions gm^{-1} sec^{-1}. Multiply this result by $1.6022\times 10^{-6}\ Q_6$ to obtain the energy generation in erg gm^{-1} sec^{-1}. To obtain reactions cm^{-3} sec^{-1} or erg cm^{-3} sec^{-1} multiply by an additional ρ.

All of the reactions involving three interacting particles (012) included in Table 1 result in the production of a residual nucleus (3) plus a gamma ray. Thus multiply $N_A^2\langle 012\rangle$ by REV RATIO to obtain $\lambda_\gamma(3) = 1/\tau_\gamma(3)$, the interaction rate per sec or the reciprocal of the mean lifetime to photodisintegration, for 3 from $\gamma+3\rightarrow 0+1+2$.

Reactions Producing Four Particles

Several two-body reactions in Table 1 result in the production of four particles. In these cases multiplication of the direct reaction rate by REV RATIO results in $N_A^3\langle 2345\rangle$ in reactions per sec per (mole/cm^3)^3 or, more simply, cm^9 sec^{-1} mole^{-3}. Multiply $N_A^3\langle 2345\rangle$ by $\rho^3 Y_3 Y_4 Y_5 N_2/N_2!\,N_3!\,N_4!\,N_5!$ to obtain the reaction rate per sec for 2 and permute for 3, 4, and 5. N_I is the number of particles of type $I = 2$, 3, 4, 5 involved. Multiply $N_A^3\langle 2345\rangle$ by $\rho^3 N_A Y_2 Y_3 Y_4 Y_5/N_2!\,N_3!\,N_4!\,N_5!$ to obtain reactions gm^{-1} sec^{-1} and so on as given previously.

Atomic Masses

The following atomic masses should be used in $Y = X/A$: $A_n = 1.008665$; $A_H = 1.007825$; $A_D = 2.014103$; $A_T = 3.016050$; $A_{He^3} = 3.016030$; and $A_{He^4} = 4.002603$ rounded off to the user's taste. All others are equal to the appropriate integral mass number within an error of no more than 3 parts in 1000. In our Q-value calculations we have used mass excesses given by Ajzenberg-Selove & Busch (1971) except for these revisions: Be^8, 4.94177 MeV; O^{15}, 2.8551 MeV; and Na^{21}, -2.18478 MeV.

Electron and Positron Capture

The expressions given in Table 1 for electron capture by H^1, He^3, and Be^7 are to be multiplied by n_{e^-}/N_A to obtain the reaction rate per sec. The electron density is designated by n_{e^-} and *at low temperature*

$$n_{e^-}/N_A = \rho Y_{e^-} = \rho/\mu_{e^-} = \rho[0.99935+0.98512X_H - \textstyle\sum X_{A,Z}\langle 0.99935-2Z/A\rangle]/2$$
$$\approx \rho(1+X_H)/2, \qquad 5.$$

where X_H is the mass fraction of hydrogen, $\sum X_{A,Z}$ is that of all atoms (A, Z) with

$A > 4$, and ρ is total density in g cm^{-3}. The last approximation must obviously not be used in neutron stars. The entry for the reaction $H^1(pe^-, \nu)H^2$ is to be multiplied by $\rho^2 N_A Y_{e^-} Y_H^2/2 \approx \rho^2 N_A(1+X_H)X_H^2/4$ to obtain the number of reactions gm^{-1} sec^{-1}.

At higher temperatures ($T_9 > 1$) electron-positron pair formation sets in, the electron density is increased, and positron capture by nuclei becomes possible. Non-degenerate expressions for the electron and positron density are given in equations 10, 11, 12, and 17 of Fowler & Hoyle (1964). In their notation $n_0 \approx N_A \rho(1+X_H)/2$ and $n_\pm = n_{e\pm}$. Higher order terms in the temperature must also be included in addition to those given in Table 1 for electron capture reactions and these can be found in equations A20 and A20′ of Fowler & Hoyle (1964). Degenerate situations are discussed by Bahcall (1964) and Chiu (1968) and semidegenerate situations by Wagoner (1969).

Beta Decay

The mean decay rates for a number of electron and positron emitters are given in Table 15B of Wagoner (1969). Rates for other cases can be determined from half-lives given in *Nuclear "Wallet Cards"* (Ajzenberg-Selove & Busch 1971) by using $\lambda_\beta = \ln 2/t_{1/2}$. The modification of these rates under degenerate conditions is discussed by Chiu (1968) on p. 303 et seq, and under semidegenerate conditions by Wagoner (1969) in Table 14B.

Reverse Reaction Rates and Partition Functions

In Table 1 the listed REV RATIO gives the ratio of the rate of the reverse reaction to that quoted for the direct rate assuming that all the interacting nuclei are in their ground states. The appropriate equations for obtaining these ratios are equations I-10, I-16, and I-20. The generalization for the reactions in Table 1 in which four (or more) particles are produced is quite straightforward if it is recalled that a factor $N_I!$ must be introduced in the numerator of REV RATIO for each type of particle I, where N_I is the number of particles of type I produced in the direct reaction and a factor $N_K!$ must be introduced in the denominator for each type of particle K, where N_K is the number of particles of type K consumed in the direct reaction.

Under stellar circumstances the *bound* excited states of nuclei will be populated in general according to the appropriate statistical factor; the population in the ith state is given by

$$P^i = \frac{g^i}{G}\exp(-E^i/kT), \qquad 6.$$

where the partition function is given by

$$G = \sum_i g^i \exp(-E^i/kT) \qquad 7.$$

and $g^i = 2J^i + 1$ is the statistical weight of the state with spin J^i (in units $\hbar$) and E^i is the excitation energy of the state above the ground state. A nucleus is designated by the subscript I; its excited states by the superscript i. A quantity of

considerable interest is the normalized partition function

$$\mathcal{G} = G/g^0 = 1/P^0, \qquad 8.$$

where g^0 is the statistical weight of the ground state because the reciprocal of $\mathcal{G}$ is just the fraction, P^0, of the nuclei populating the ground state ($E^0 = 0$) at equilibrium.

The population can be taken to be that for statistical equilibrium at temperature T because the *initial* rate for populating an excited state from the ground state by photoexcitation alone[8] can be shown to be

$$\lambda_i = \frac{1}{\tau^i} \equiv \frac{1}{n^i_{eq}} \frac{dn^i}{dt} = \frac{G}{g^0} \frac{\lambda^i_{\text{spon}}}{1-\exp(-E^i/kT)} \geqq \lambda^i_{\text{spon}}, \qquad 9.$$

which is always larger than the spontaneous decay rate, λ^i_{spon}, of the excited state and at high temperature ($kT > E^i$) is larger by the factor GkT/g^0E^i. The exponential term in the denominator incorporates the effect of stimulated emission and assumes a black-body or Planck radiation spectrum. There are situations such as those associated with shock wave phenomena (Hoyle & Fowler 1973, Colgate 1974) where the time to equilibrate nuclei, ions, atoms, electrons, and radiation may be long and of considerable importance. Except for long-lived isomeric states of nuclei, $\tau^i \leqq 1/\lambda^i_{\text{spon}}$ is $< 10^{-9}$ sec for the excited states of the nuclei discussed in this paper. This is to be compared with the hydrodynamic time scale of explosive nucleosynthesis, $\tau_{\text{hyd}} \sim 10^3\, \rho^{-1/2}$ sec, which is 10^{-3} sec even for $\rho = 10^{12}$ g cm^{-3}. Many nuclear processes can be slow compared to hydrodynamic time scales but photoexcitation is *not*, in the sense that the equilibrium population to be reached is low when the rate for photoexcitation is at a minimum ($E^i \gg kT$). Long-lived isomeric states, just like radioactive nuclei, must be treated as individual nuclear species in the handling of reaction chains under stellar circumstances.

Excited states of a nucleus that are not bound and do not decay primarily to the ground state can be taken as in equilibrium at a given time with those of their particle decay products, which are in greatest abundance at that time. The time to reach equilibrium after a sharp change in ρ and/or T, say, is of the order of $\hbar/\Gamma_c$, where Γ_c is the partial width for decay into the components of the appropriate channel (c). These times are nuclear times ($\sim 10^{-21}$ sec) divided by appropriate penetration factors, which can indeed be small, so that the equilibration time can be $\gg 10^{-21}$ sec; nonetheless, these times are usually very short compared to the "burning" times for transformation from the initial components to the final products. Toward the end of burning these excited states come into equilibrium with the final product or products. The case of one final product occurs in radiative capture and refers to the ground state or a *bound* excited state of the compound (final) nucleus in which case the unbound excited state does eventually reach equilibrium with the ground state and, in fact, with all the bound excited states. We emphasize that, during the initial stages of a radiative capture reaction, excited states that are not bound can be populated in much greater numbers than calculated from equilibrium

[8] Excitation by inelastic particle scattering (n, n'; p, p'; α, α'; etc) increases the rate.

with the ground state and the bound excited states. The opposite is of course true in the initial stages of photodisintegration, the reverse of radiative capture.

In calculating the abundance of a nucleus produced in nuclear burning one is interested in all those excited states of a nucleus that will decay to the ground state after the final cooling of any stellar event. Thus the appropriate upper limit on the summation over states for G or $\mathscr{G}$ is the excitation just above the point where the states become unbound and decay primarily by particle emission. For example, the nucleus N^{13} has only one bound state, its ground state with $J = 1/2$, and thus $G = g^0 = 2\mathscr{G} = 2$.

The above argument shows that the REV RATIOS needed in stellar reaction calculations should be based on the partition functions of the nuclei involved and not just on the statistical weights of the ground states. Thus a problem arises. In general, laboratory measurements yield σ and thus $N_A\langle\sigma v\rangle$ for the reaction from the *ground* state of the *target* nucleus (0) in the incident channel to *all bound* states of the *residual* nucleus (3) in the outgoing channel. Particles (1) and (2) are usually n, p, or α, which have excited states of such high excitation that we can neglect them for $T_9 \leqq 10$. Thus the REV RATIO in Table 1 should be corrected by dividing by $\mathscr{G}_3 = G_3/g_3^0$ for the residual nucleus. The reverse reaction-rate ratio as given in the table has the factor g_3^0 in the denominator, not the correct astrophysical factor G_3.

Certain of the heavier nuclei treated in this review such as F^{19}, Ne^{21}, Na^{23}, Mg^{25}, and Al^{27} have low-lying states between 0.1 and 1 MeV and these states will be significantly populated in the temperature range $1 \leqq T_9 \leqq 10$. Thus the target nuclei as well as the residual nuclei populate excited states under high temperature circumstances. In principle the interactions of these states can be determined by studying transitions to them in the reverse reaction. This often cannot be done, however, because the residual nucleus in the direct reaction (target nucleus in the reverse reaction) is radioactive with a short lifetime. We have attempted, where possible, to include the effects of these excited target states by using either inelastic scattering data as discussed by Bahcall & Fowler (1969a) or theoretical Hauser-Feshbach calculations following Michaud & Fowler (1970). For example, when the inelastic scattering-strength function, $S_{\text{inel}} = (2J_r+1)(\Gamma_1\Gamma_1'/\Gamma)_r$, has been measured at a resonance, r, as well as the reaction-strength function, $S_{\text{reac}} = (2J_r+1)(\Gamma_1\Gamma_2/\Gamma)_r$, then we increase the latter quantity by the factor

$$f = 1 + \left(\frac{\Gamma_1'}{\Gamma_1}\right)_r = 1 + \frac{S_{\text{inel}}}{(2J_r+1)\Gamma_r}\left(\frac{\Gamma}{\Gamma_1}\right)_r^2. \qquad 10.$$

Γ_r is the full width of the resonance, Γ_1 is the partial width for the incoming channel, Γ_1' is the partial width for the inelastic scattering channel, and Γ_2 is the partial width for the outgoing reaction channel. Γ_r is usually known but frequently $(\Gamma_1/\Gamma)_r$ is not. Fortunately in many of these unknown cases it is strongly indicated on theoretical grounds that $(\Gamma_1/\Gamma)_r \approx 1$ and f can be evaluated approximately.

In cases such as $Na^{23}(p, \alpha)Ne^{20}$, where we have been able to correct for the effects of excited states in the target nucleus, we express the results for $N_A\langle\sigma v\rangle$ in such a form that all terms are to be divided by $\mathscr{G}_0 = G_0/g_0^0$, the normalized partition function for the target nucleus. When this factor is properly included in the numerator of the reverse reaction-rate ratio and $\mathscr{G}_3 = G_3/g_3^0$ is properly included in

the denominator, then, in the reverse reaction rate $\mathcal{G}_0$ cancels out and all terms are divided by $\mathcal{G}_3$ as expected from the reciprocity principle. To be explicit and completely general, in these cases multiply the REV RATIO in Table 1 by $\mathcal{G}_0\mathcal{G}_1/\mathcal{G}_2\mathcal{G}_3$. The reaction rates then involve the number densities of the interacting particles summed over all of their bound states. After "cooling" this will be the number density of the ground state.

When information concerning the interactions of the excited states of target nuclei is unavailable, one can in principle follow only the reactions of the ground state of such nuclei; the reaction rates involving their bound excited states, which will eventually decay to the ground state, are unknown. Under these circumstances we recommend the equal strength approximation of Bahcall & Fowler (1969a), which assumes that all states have the same $N_A\langle\sigma v\rangle$ as the ground state. Thus the rate for the correct statistical population is just that for the ground state. Once again, then, the REV RATIO in Table 1 should be corrected by $\mathcal{G}_0 = G_0/g_0^0$ and, in complete generality, by $\mathcal{G}_0\mathcal{G}_1/\mathcal{G}_2\mathcal{G}_3$. This means that in expressions like $n_0n_1\langle\sigma v\rangle_{01}$ or $n_2n_3\langle\sigma v\rangle_{23}$ the number densities, n_I, are the sums over all bound states. The extension to three or four interacting particles or three or four product particles should be obvious.

The partition functions involving the nuclei treated in this paper are defined and tabulated in Bahcall & Fowler (1970) (see their Table 2 and especially their note added in proof). The G listed in their Table 2 should be divided by the g^0 for the ground state listed in their Table 1 to obtain $\mathcal{G}$, the normalized partition function. Inspection of the table will reveal the temperatures below which the $\mathcal{G} \approx 1$ and our values for REV RATIO apply without substantial correction. We have used new measurements to yield the corrected values $\mathcal{G}(F^{19}) = 1+4\exp(-2.090/T_9)+7\exp(-16.440/T_9)$ and $\mathcal{G}(Ne^{21}) = 1+1.5\exp(-4.068/T_9)+2.0\exp(-20.258/T_9)$. Note that $\mathcal{G}(T_9)$ is designated by $GT9$ in Table 1.

Stimulated and Induced Reaction Rates

We have implicitly included stimulated gamma-ray emission in equilibrium and reverse reaction-rate considerations in equation 9. However, we have not included in Table 1 the increase in reaction rates, direct and reverse, due to stimulated gamma-ray emission and a number of induced processes. Here we can only catalog in a schematic way these various enhancement mechanisms that are significant at high temperature.

Given a black-body radiation environment, every radiation process is enhanced by $[1-\exp(-\hbar\omega_p/kT)]^{-1}$ where $\hbar\omega_p$ is the energy of the primary gamma ray emitted. Thus one needs to know in detail the various primary gamma-ray transitions that can result from the decay of resonant states in the compound nucleus. In nuclear processes it must be recalled that the excited states resulting from the primary gamma-ray emission may be unbound to particle emission and thus stimulated processes also occur here. For example, an initial two-particle channel may transform to a final two-particle channel via the primary emission of a gamma ray. Such a transformation will be enhanced under stellar circumstances by the factor given above.

At high temperature various "induced" processes occur. The compound nucleus

produced as an intermediate state in a reaction can interact via inelastic and superelastic scattering with $\gamma, n, p, \alpha \ldots$ to produce the ground state of the compound nucleus, bound excited states that decay to the ground state, or excited states that break up into particle channels. All such processes can be treated as three reacting bodies in the incident channel and by considering the reactions in Table 1 as the first stage in these processes. For discussion of specific examples the reader is referred to Shaw & Clayton (1967) and to Truran & Kozlovsky (1969).

Remarks to Users

It should be appreciated at this point by the reader and user that our primary purpose is *not* to provide reaction rate expressions that can be employed most economically and efficiently in computer programs. Rather it is our primary purpose to provide expressions that accurately represent the available experimental data supplemented where necessary by accepted theoretical considerations, particularly in regard to extrapolation to low energies and thus low temperatures. In general we present expressions that represent the available data to $\lesssim 25\%$ accuracy over the temperature range 10^6 °K $\leqq T \leqq 10^{10}$ °K or $10^{-3} \leqq T_9 \leqq 10$. Space does not permit us to enter into the complex problem of the evaluation of the accuracy of primary data in this article.[9] In general the uncertainties range from $\sim 10\%$ for the pp reaction up to as much as a factor of 3 in some cases. The typical uncertainties are of the order of 50%. In cases where the available data only set upper limits on cross sections or resonance strengths, we have introduced the factor (0 to 1) in the expressions presented. Once the factor (0 to 1) appears it is to be included in *all* terms that follow it. Terms for which the factor must be the same are grouped together in brackets. We have also introduced this factor where resonance strengths have been evaluated by using dimensionless reduced widths $\theta_p^2 \leqq 1$ for protons and $\theta_\alpha^2 \leqq 0.1$ for $T = 0$ resonances, $\theta_\alpha^2 \leqq 10^{-4}$ for $T = 1$ resonances for alpha particles. T is the isotopic spin as defined by Chiu (1968), p. 318. When expressions that include the factor (0 to 1) are critical in determining ultimate results the user must refer to the original references cited and make his own judgment. Better still, make the calculations twice, once without the term in question and once with its full value. If a compromise is desired set (0 to 1) = 0.1. We will be happy to respond to inquiries in this regard. We will also be happy to provide numerical evaluations of all our expressions *but only* for our standard T_9 grid from 0.001(0.001) 0.016(0.002) 0.020(0.005) 0.030(0.01) 0.16(0.02) 0.20(0.05) 0.50(0.10) 1.00(0.25) 2.00(0.50) 4.00(1.00) to 10.00. A final word of advice: the user of Table 1 must be diligent in reading the parentheses correctly.

ANALYSIS OF THE EXPERIMENTAL DATA

Continuum Reaction Rates

The study of explosive nucleosynthesis on a short, *hydrodynamic* time scale (Fowler & Hoyle 1964) has created considerable interest in high-temperature reaction rates

[9] In any case we wish to remain on good terms with our hard-working colleagues in nuclear laboratories.

above $T_9 \sim 1$, which are in general related to reaction cross sections at energies above $E_6 \sim 1$. In this energy range nuclear reactions proceed through many resonances that are separated by intervals not much greater than their widths or that actually overlap to form a continuum. Frequently it is found that sharp resonances are superimposed on a continuum background. In any case the contribution to the reaction rate of the background and all but one or two of the low-lying resonances, which must be treated individually, can be summed and fitted to an expression of the form

$$\begin{aligned} N_A\langle\sigma v\rangle_{\rm cont} &= \text{const}\, T^m \exp(-C/kT) \\ &= BT_9^m \exp(-11.605C_6/T_9)\ \text{cm}^3\ \text{sec}^{-1}\ \text{mole}^{-1}, \end{aligned} \tag{11.}$$

where B and C are free least square parameters; we have arbitrarily limited $|m|$ to integers, half integers, or rational fractions involving integers < 10. In FCZ I we restricted ourselves to $m = 0$ but we have found many reactions require $m \neq 0$ for fits to $\lesssim 25\%$. The effective continuum threshold energy, C, can represent a true threshold in an endoergic reaction with $Q < 0$ in which case $C = |Q|$. In an exoergic reaction the cross section usually increases rapidly with energy as the penetration factor in the incident channel increases and flattens out either when the incident-channel partial width becomes equal to the sum of other partial widths or when the Coulomb barrier is reached in the incident channel. C is then roughly the energy at which this occurs.

Equation 11 is the rate that corresponds to a cross-section dependence on energy given by

$$\sigma(E) = 2\sigma(2C)\left(\frac{C}{E}\right)\left(\frac{E}{C} - 1\right)^{m+1/2} \qquad (E \geqq C), \tag{12.}$$

with $\sigma(2C)$ equal to the cross section at $E = 2C$ and $\sigma = 0$ for $E \leqq C$. From equation 3 one finds

$$\begin{aligned} N_A\langle\sigma v\rangle_{\rm cont} &= N_A \Gamma(m+\tfrac{4}{2})\sigma(2C)\left(\frac{32C}{\pi M}\right)^{1/2}\left(\frac{kT}{C}\right)^m \exp(-C/kT) \\ &= 1.888 \times 10^9\, \Gamma(m+\tfrac{3}{2})\sigma_b(2C_6)\left(\frac{C_6}{A}\right)^{1/2}\left(\frac{T_9}{11.605C_6}\right)^m \\ &\quad \times \exp(-11.605C_6/T_9)\ \text{cm}^3\ \text{sec}^{-1}\ \text{mole}^{-1}, \end{aligned} \tag{13.}$$

where Γ is the gamma function, $\sigma_b(2C_6)$ is the cross section at $2C_6$ in barns, and $A = MN_A$ is the reduced mass in atomic mass units. Note that for the limiting case of a single sharp resonance, $m = -\frac{3}{2}$, σ diverges at $E = C$ in such a way that σ must be treated as a δ-function in the customary manner (see equations I-62 and I-66).

Equation 12 for the cross section exhibits a maximum, characteristic of many reactions, for $-\frac{1}{2} \leqq m \leqq \frac{1}{2}$, at

$$E_{\max} = \frac{2C}{1-2m}. \tag{14.}$$

For $m < -\frac{1}{2}$, the cross section exhibits a singularity at $E = C$. For $m > \frac{1}{2}$, the cross section rises monotonically from the threshold at $E = C$. A case of special interest is that for $m = 0$, which applies to an endoergic reaction ($Q < 0$, $C = |Q|$) for s-wave neutrons (orbital angular momentum, $l = 0$). In this case the cross section rises as $(E-C)^{1/2}$ above threshold and drops off as $E^{-1/2}$ at high energy ($\sigma v \to$ const). For $m = -\frac{1}{2}$, σE exhibits a step function at $E = C$. The computer programmer will need definitions of $T9^m$ where $|m| = \frac{1}{2}, \frac{3}{2}, \frac{7}{2}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{5}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{8}{5}, \frac{7}{6}, \frac{2}{7}$, and $\frac{1}{8}$ for complete coverage of the reactions listed in Table 1. In addition, powers of $T9A$ (see below) that should be defined are $\frac{3}{2}, \frac{1}{3}, \frac{2}{3}$, and $\frac{5}{6}$.

Sharp Resonance Reaction Rates

In the case of a sharp resonance at energy, E_r, for which the full width at half maximum, Γ_r, is much less than the resonance energy, that is, $\Gamma_r \ll E_r$, the reaction rate is given by equation 11 with $m = -\frac{3}{2}$ and $C = E_r$. Equation I-62 shows that the relevant experimental parameter is $(\omega\gamma)_r$, which is determined from a thin target experiment by equation I-66. In many experimental determinations thick targets are used, in which case the resonance results in a step in the excitation curve. The increase in yield per incident particle, Y_r, in crossing the resonance step determines $(\omega\gamma)_r$ through

$$Y_r = \int n\sigma\, dx = \int_r (\sigma/\varepsilon) dE \approx (1/\varepsilon_r) \int_r \sigma\, dE, \qquad 15.$$

so that

$$(\omega\gamma)_r = (ME_r/\pi^2\hbar^2)\varepsilon_r Y_r, \qquad 16.$$

where n is the number of *reacting* target nuclei per cm^3, dx is an element of target thickness, and $\varepsilon = (1/n)\, dE/dx$ is the stopping cross section in the target per reacting target nucleus. We assume that $\varepsilon \approx \varepsilon_r \equiv \varepsilon(E_r)$ over the interval of several Γ_r near E_r.

From equation I-62 one obtains

$$N_A\langle\sigma v\rangle = N_A\left(\frac{8}{\pi MkT}\right)^{1/2} \varepsilon_r Y_r(E_r/kT) \exp(-E_r/kT). \qquad 17.$$

This leads to an important consideration concerning the use of thick targets in determining upper limits on reaction rates due to low energy "near-threshold" resonances. The experimentalist uses a thick target and gradually decreases the voltage on his accelerator and thereby the energy of his incident beam of particles. If he detects a yield above background, he may be able to lower his beam energy until the step at E_r is reached and all is fine and dandy—equation 17 can be used to yield $N_A\langle\sigma v\rangle$ even though the standard deviation of the measurement may be substantial. In many cases, however, a limit is reached below which the accelerator cannot deliver a stable, well-focused beam of incident particles of sufficient intensity, and background effects make it possible to set only an upper limit on Y_r. The resulting upper limit on $N_A\langle\sigma v\rangle$ then depends critically, as equation 17 shows, on the still unknown E_r relative to kT where T is the temperature of interest. For example, there will be a factor of $\sim 10^3$ involved in the upper limit on $N_A\langle\sigma v\rangle$ for a

given one on Y_r depending on whether $E_r = kT$ or 10 kT. (We neglect the slow variation of ε with E or ε_r with E_r at low energy.) Dwarakanath (1974) has discussed this matter extensively in connection with the bearing of his low-energy measurements on $He^3(He^3, 2p)He^4$ on the solar neutrino problem. The ultimate solution to the problem involves the determination of the resonance energy, if there is a resonance, by measuring the excitation of the resonant state in the compound nucleus in some other reaction or in showing that no resonance exists between the threshold and the lowest energy used in the direct measurements (Parker et al 1973). Indirect measurements of the properties of any state that does occur in this range may yield a value rather than an upper limit for $N_A\langle\sigma v\rangle$ (Rolfs et al 1975). The high-energy tails of subthreshold states can also be treated in this manner (Dyer & Barnes 1974).

Wide Resonance Reaction Rates

Wide resonances, $\Gamma_r > 0.1\ E_r$, cannot be treated accurately in the manner used for sharp resonances. Examples occur in the reactions $H^3(d, n)He^4$ where the standard tabulated values in the CM system are $\Gamma_r = 0.081$ MeV at $E_r = 0.064$ MeV and $Be^9(p, d)2He^4$ or $Be^9(p, \alpha)Li^6$, where $\Gamma_r = 0.120$ MeV at $E_r = 0.296$ MeV. For resonance energies below the Coulomb and/or centrifugal barrier, complications arise from the fact that the product, σE, is distorted from the symmetrical bell shape of the pure Lorentz factor $(\Gamma_r/2)^2/[(E-E_r)^2+(\Gamma_r/2)^2]$ by the rapid variation of the incoming and outgoing channel transition probabilities over the width of the resonance. The resonance shape is sharpened below the resonance energy and broadened above, and indeed the position of the maximum in σE may be shifted above E_r.

After a number of attempts we have found that the following procedure yields the most accurate results when compared with numerical integration over the resonance using equation 3. The low-energy tail of the resonance is treated as discussed in the next section on nonresonant reaction rates. For the resonant contribution we find that equation 11 is quite satisfactory if m is chosen somewhat greater than $-\frac{3}{2}$ and C is chosen as the centroid of the resonance area rather than the energy at the maximum in the product σE. Actually we determine B, m, and C as free parameters in a least squares fit of equation 11 to the cross section obtained by numerical integration using equation 3. In some cases such as $C^{12}(p, \gamma)N^{13}$ and $C^{13}(p, \gamma)N^{14}$ it is accurate enough to retain $m = -\frac{3}{2}$ and treat only B and C as free parameters.

Nonresonant Reaction Rates

As in FCZ I we have usually expressed the nonresonant *cross-section factor*, $S(E)$, as the first three terms of a truncated Maclaurin series in the center-of-momentum energy E according to the equation

$$S(E) \equiv \sigma E \exp 2\pi\eta = \sigma E \exp (E_G^{1/2}/E^{1/2})$$
$$= S(0)+S'(0)E+\tfrac{1}{2}S''(0)E^2, \qquad 18.$$

where $\eta = Z_0 Z_1 e^2/\hbar v$ and $E_G = (2\pi\alpha Z_0 Z_1)^2(Mc^2/2)$ is the "Gamow energy." Then $N_A\langle\sigma v\rangle$ is determined using equations I-51 (without the factor ρ)–I-59. A thorough discussion of the validity of this treatment of low-energy, charged-particle reactions has been given by Critchfield (1972). When low-energy experimental data are available the free parameters in equation 18, namely, $S(0)$, $S'(0)$, and $S''(0)$, can be determined by a least squares analysis.[10] In many cases treated in this paper such data are not available and it is necessary to determine the contribution of the tails of measured resonances to the low-energy resonant or, strictly speaking, off-resonance cross-section factor. In doing this we have found that it is necessary to include not only the resonance behavior, as was done in equations I-76 and I-77, but also the energy dependence of the width for the incoming and outgoing channels, namely, Γ_1 and Γ_2 of equation I-61. When the outgoing channel involves two particles the results are

$$S(0) = \frac{\pi\sigma_r E_r}{4K^2_{2l+1}(x)P_l(E_r)} \frac{P_L(Q)}{P_L(Q+E_r)} \frac{\Gamma_r^2/4}{E_r^2+\Gamma_0^2/4}, \qquad 19.$$

where P_l is the penetration factor as defined by Vogt (1968, equation 61) in the incoming channel for the appropriate orbital angular momentum, $l\hbar$, in that channel, and P_L is the penetration factor in the outgoing channel for the appropriate orbital angular momentum, $L\hbar$, in that channel. When several l's (or L's) were possible under the conservation of angular momentum and parity, we employed the minimum possible value for l (or L) except where available experimental evidence indicated that higher values make the major contribution to Γ_1 (or Γ_2). In the above expression for $S(0)$ we have used the asymptotic expression for P_l near zero energy given by

$$P_l(E) = \frac{\pi \exp\left[-(E_G/E)^{1/2} - \alpha_l E\right]}{4K^2_{2l+1}(x)}, \qquad E \to 0, \qquad 20.$$

where $K_{2l+1}(x)$ is the modified Bessel function of order $2l+1$ and $x = (8MR\ Z_0 Z_1 e^2/\hbar^2)^{1/2} = 0.525\ (AZ_0 Z_1 R_f)^{1/2}$ in the incoming channel. All other symbols in equations 19 and 20 are defined in FCZ I; α_l is defined in Burbidge et al (1957), p. 560. The radii, R_f in fermis, are taken from Michaud & Fowler (1970).

In equation 19 we have correctly used $\Gamma_0^2/4$ in the denominator rather than $\Gamma_r^2/4$ as in equation I-76. $\Gamma_0 \equiv \Gamma(E=0)$ is the sum of all partial widths at zero energy in the incoming channel. If this sum at resonance, $\Gamma_r = \Gamma(E_r)$, is mainly due to contributions from outgoing channels which do not vary rapidly with energy, then $\Gamma_0 \approx \Gamma_r$. However, if Γ_r is mainly due to the width of the incident channel that varies rapidly with energy in general and goes to zero at zero energy, then $\Gamma_0 \approx 0$ and the last part of the right-hand side of equation 19 becomes $\Gamma_r^2/4E_r^2$.

[10] We have not included the term $5/12\tau \propto T^{1/3}$ from equation I-52 in the entries in Table 1 unless the terms in $S'(0)$ and/or $S''(0)$ are known. The factor $1+5/12\tau$ can be calculated easily for any reaction using equations I-54 and I-55. Such factors are typically of the order $1+0.05T_9^{1/3}$.

For the derivatives of the cross-section factor at zero energy the results are

$$S'(0) = S(0)\left[\frac{2}{E_r} - \alpha_l - \alpha_L + \frac{1}{2}\left(\frac{E'_G}{Q^3}\right)^{1/2}\right], \quad 21.$$

where E'_G is the Gamow energy as defined in equation I-46 but for the outgoing channel and

$$S''(0) = S(0)\left[\frac{2}{E_r^2} + \left(\frac{S'(0)}{S(0)}\right)^2 - \frac{3}{4}\left(\frac{E'_G}{Q^5}\right)^{1/2}\right]. \quad 22.$$

The use of α_L in equation 21 is a poor approximation for Q equal to a substantial fraction of the Coulomb barrier in the outgoing channel, but its use is sufficiently accurate for our purposes.

When the outgoing channel involves gamma radiation the following replacements should be made: in equation 19 replace the ratio of the P_L's by $[Q/(Q+E_r)]^{2L+1}$, where L is now the multipole order of the radiation. Because electric or magnetic dipole radiation usually dominates, $L=1$ and $2L+1=3$; in equation 21 set $\alpha_L = 0$ and replace the last term in the square brackets with $(2L+1)/Q$; in equation 22 replace the last term in the square brackets by $-(2L+1)/Q^2$.

In the customary random-phase approximations the contributions of each resonance to $S(0)$, $S'(0)$, and $S''(0)$ are added to obtain overall values. However, states of the same spin and parity can add coherently in either a constructive or destructive fashion at low energy. Such cases must be treated individually using the proper sum rules as given, for example, by Rolfs & Rodney (1974a).

In cases such as $Mg^{24}(\alpha,\gamma)$ no nonresonant term is included. This is because the nonresonant term made a substantial contribution to $N_A\langle\sigma v\rangle$ only at such low temperatures that the lifetime of Mg^{24}, even for $\rho X_{He} = 10^5$, was greater than the lifetime of the "universe." We dropped all terms that made a substantial contribution only at temperatures such that $N_A\langle\sigma v\rangle < 10^{-21}$ cm^3 sec^{-1} mole^{-1} for proton interactions and $< 10^{-23}$ for alpha-particle reactions.

Cutoff for Nonresonant Reaction Rates Used with Resonant or Continuum Rates

The truncated Maclaurin series we have used for $S(E)$ diverges positively or negatively (infrequent) at high energy and so $S_{eff}(T)$ defined by equation I-52 diverges at high temperature. For example, the expansion for $S(E)$ due to a resonance using the parameters given in equations 19, 21, and 22 falls below the true $S(E)$ at E somewhat less than E_r and becomes equal to $\sim 3\Gamma_r^2/2E_r^2$ times the true value at E_r. It then becomes equal to the true value at $\sim 4E_r/3$ and continues to rise while the true $S(E)$ decreases. In FCZ I we stipulated an upper limit on T for the use of the nonresonant rate. In the interim we have found that this limitation was not always applied by users, and therefore in this article we include cutoff factors in nonresonant reaction rates so that they can be used in combination with resonant or continuum rates at all temperatures. The resonant or continuum rates are automatically cut off at low temperatures by the factor $\exp(-E_r/kT)$ or $\exp(-C/kT)$.

It will be clear that the cutoff temperature for the nonresonant rate must be somehow related to the resonance energy, E_r, or the continuum threshold, C, because just below and above these energies the resonance or continuum terms adequately represent the cross section. This can be done quite straightforwardly through equation I-56 or, more explicitly, through equation I-60, which relates the temperature to the effective interaction energy below the Coulomb barrier for charged particles. Because the next term in the expansion for $S(E)$ would be the term in $E^3 \approx E_0^3 \propto T^2$, we have chosen the cutoff factor as

$$f_{\text{cutoff}} = \exp -(T_9/T_{9co})^2. \tag{23.}$$

In certain cases the cutoff temperature, T_{9co}, can be determined by a detailed comparison of our analytical expressions with numerically integrated values for $N_A\langle\sigma v\rangle$. As a rough rule we have found that the following modification of equation I-60 is quite satisfactory:

$$\begin{aligned} T_{9co} &= \frac{23.46}{(W \ln 4)^{1/2}} E_{6r}^{3/2} \\ &= 19.92 W^{-1/2} E_{6r}^{3/2}, \end{aligned} \tag{24.}$$

with $W = Z_0^2 Z_1^2 A$. In the continuum case replace E_{6r} by C_6. When the nonresonant cross section is due to the tails of many resonances, including those lumped in the continuum, the cutoff energy is taken as the energy of the resonance that makes the largest contribution at low energy, or the continuum threshold energy, if the contribution of the continuum resonances is the largest. Usually the first strong and not-too-narrow resonance determines T_{9co}.

In Figure 1, the deviation of the analytic expressions given in Table 1 from the numerically integrated values are shown in four cases where such detailed comparison is possible. In these cases T_{9co} has been chosen by trial and error to give a "balanced" plus-and-minus deviation near the temperature where the nonresonant term is cut off and the other terms take over. It is interesting that the maximum positive error occurs at T_{9co}. In the cases shown it has been possible to adjust the nonresonant term to be "exact" at low temperature and to adjust the resonant and/or continuum terms to be "exact" at high temperature. Figure 1 also shows for the sum of the reactions $Be^9(p, d)2He^4 + Be^9(p, \alpha)Li^6$ the deviation when we use our standard cutoff prescription given in equation 24. In this case the resonance is a wide one and our standard prescription does not yield a balanced deviation. However, the error is still only $\sim 40\%$ at a temperature rather high for the occurrence of Be^9-burning. Figure 1 includes the deviation for $C^{12}(p, \gamma)N^{13}$ when the cutoff factor is omitted, indicating the divergence at high temperature. The reader must recall that Figure 1 shows the deviation of our analytical expressions from the available cross-section data smoothed by integration into $N_A\langle\sigma v\rangle$ and does not illustrate the possible experimental error in that data. We claim only to fit the available data to $\pm 25\%$ in most cases. This is about as well as can be done for rates that may vary by 20 or more orders of magnitude over the range of temperature under consideration.

Alternative Nonresonant Reaction Rates

Calculation of the rates of nuclear processes at elevated temperatures ($T_9 > 1$) requires knowledge of cross sections at interaction energies in the MeV range. In this range the cross-section factor varies considerably and a truncated expansion intended for use just above zero energy, as given in equation 18, is not at all satisfactory. Because the major variation is usually an exponential decrease with increasing energy we have found it convenient to adjust the experimental data for $\sigma(E)$ and thus for $S(E)$ to expressions of the form

$$S(E) = S(0)q(E)\exp(-\alpha E), \qquad 25.$$

where the function $q(E)$ is normalized to $q(E=0)=1$. Frequently there is not enough low-energy data to determine α. In such cases we use an average over α_l

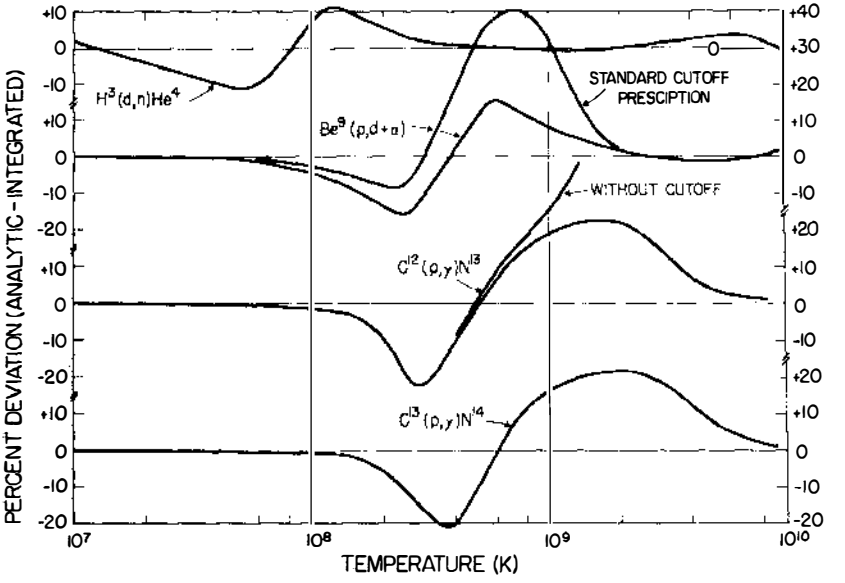

Figure 1 The percentage of deviation of the analytic expression for $N_A\langle\sigma v\rangle$ in Table 1 from the values given by numerical integration for the reactions indicated. In these cases it has been possible to choose the cutoff temperature for the nonresonant rate in such a way that a balanced plus-and-minus deviation is obtained. The standard cutoff prescription given by equation 24 does not always achieve this, as indicated for $Be(p,d)2He^4$ plus $Be^9(p,\alpha)Li^6$. If the cutoff is not applied, the result is disastrous, as indicated in the case of $C^{12}(p,\gamma)N^{13}$. The deviations shown are smaller than the uncertainty in the experimental data in most cases and must be considered in the light of the many orders of magnitude variation in the quantities being compared. For example, $N_A\langle\sigma v\rangle$ for $C^{12}(p,\gamma)N^{13}$ varies by 23 orders of magnitude over the range $10^7 \leqq T \leqq 10^{10}$ °K. This figure is to be compared with Figure 4 in FCZ I.

(see equation 20 et seq) for the first few partial waves believed to contribute to the reaction under consideration. Because $l = 0, 1, 2$, partial waves usually make the major contributions; because α_l decreases uniformly with increasing l we find that the use of $\alpha = \alpha_1$ is a good rule of thumb.

Consider the simplest case covered by equation 25, constant $q(E) = 1$. When $S(0)\exp(-E_G^1/E^{1/2} - \alpha E)$ replaces σE in the integrand in equation 3 it will be clear that an effective temperature

$$T_A = \frac{T}{1+\alpha kT} \qquad 26.$$

replaces T in the temperature dependence of the integral of equation 3 but not of course in the factor $(kT)^{-3/2}$. The final result is

$$N_A\langle 01\rangle = 7.8327 \times 10^9 (Z_0 Z_1/A)^{1/3} S(0)(T_{9A}^{5/6}/T_9^{3/2}) \times \exp(-4.2487\, Z_0^{2/3} Z_1^{2/3} A^{1/3}/T_{9A}^{1/3}), \qquad 27.$$

and the cutoff term, if any, involves T_{9A}. Equation 27 replaces $[01]/\rho$ given by equation I-51. An example of its use is found in the entry for $Mg^{26}(\alpha, n)Si^{29}$ in Table 1 where $\alpha = 0.729\ MeV^{-1}$ and $\alpha kT = 0.0628 T_9$.

A fairly frequent case, $q(E) = \exp(-\beta E^2)$, has been discussed in detail by Fowler (1974). Here a very good approximation results when one of the E-factors in βE^2 is replaced by E_o, the effective interaction energy, given by equation I-56. Then equation 26 becomes

$$T_A = \frac{T}{1+\alpha kT + \beta E_o kT/(1+\alpha kT)^{2/3}}. \qquad 28.$$

In full numerical detail

$$T_{9A} = \frac{T_9}{1+0.0862\alpha T_9 + 0.0105(Z_0^2 Z_1^2 A)^{1/3}\beta T_9^{5/3}/(1+0.0862\alpha T_9)^{2/3}} \qquad 29.$$

for α in MeV^{-1} and β in MeV^{-2}. An example of the use of equation 27 with T_{9A} given by equation 29 is found in the entry for $O^{17}(\alpha, n)Ne^{20}$ in Table 1 where $\alpha = 0.311\ MeV^{-1}$ and $\beta = 0.235\ MeV^{-2}$.

In the general case a fair approximation is to set $q(E) = q(E_{oA})$ where E_{oA} is given by equation I-56 with T_9 replaced by T_{9A}. This occurs, for example, in $O^{16}(p, \alpha)N^{13}$. In cases such as $O^{16}(p, \gamma)F^{17}$, E is replaced by E_o since $\alpha = 0$.

In rare cases the nonresonant cross-section factor is best represented by a power-law expression, $S(E) \propto E^m$, as in $O^{17}(p, \alpha)N^{14}$ where $m = 5/2$. Again it is recommended that E be replaced by $E_o \propto T^{2/3}$ so that the final temperature dependence for the reaction rate is given by $N_A\langle 01\rangle \propto T^{2(m-1)/3} \times \exp(-\text{const}/T^{1/3}) \Rightarrow T\exp(-\text{const}/T^{1/3})$ for $m = 5/2$.

Direct Radiative Capture

In many of the reactions producing gamma rays we have used theoretical estimates for direct capture made for us by T. A. Tombrello or C. Rolfs. Fortunately, in

many (p, γ) and (α, γ) reactions the direct capture is described by an approximately constant cross-section factor, although this is not always true. The estimates are made using the theory described in Christy & Duck (1961), Tombrello (1965), and Rolfs (1973). In general there is no cutoff for nonresonant direct-capture rates.

Reactions Involving C^{12} and O^{16} Pairs

Experimental measurements in the past few years have yielded a plethora of total cross-section data on reactions induced by $C^{12}+C^{12}$, $C^{12}+O^{16}$, and $O^{16}+O^{16}$. Information on the n, p, and α yield in these reactions has also become available.

The experimental data yield the cross sections that are directly relevant to the rates of these processes under explosive time scales at high temperature, $T_9 > 2$ for $C^{12}+C^{12}$, $T_9 > 4$ for $C^{12}+O^{16}$, and $T_9 > 6$ for $O^{16}+O^{16}$. However, to ascertain the rates under conditions of quasistatic equilibrium at lower temperatures it is necessary to parameterize the available data in terms of a reasonable theory of the reaction mechanisms. The problems raised are discussed in detail by Fowler (1974).

We have chosen a theoretical model that has the merit of great simplicity—a generalized "black-body" model (Michaud & Fowler 1970) incorporating a complex "square-well" potential with a repulsive real component. The standard black-body model assumes that the nuclear interaction vanishes beyond the nuclear radius, R_1, taken as an empirical parameter. R_1 is the radius of the square-well potential that gives results equivalent to those for a more "realistic" Woods-Saxon potential. Within this radius the wave function is taken as a traveling, ingoing wave described by $\phi \propto \exp(-iKr)$, $r \leqq R_1$, where K is a wave number appropriate to the nuclear interior, that is, $K^2 = 2M(V+E)/\hbar^2$ with M the reduced mass, E the interaction energy, and V the nuclear potential energy taken positive for an attractive potential and negative for a repulsive potential. We have taken $V = V_1 + iW_1$ and made a least squares analysis of the experimental data to determine R_1, V_1, and W_1 using transmission functions defined by Vogt (1968) in the Hauser-Feshbach theory described by Michaud & Fowler (1970). The "best fits" yield the following empirical parameters:

	R_1 (fermi)	V_1 (MeV)	W_1 (MeV)
$C^{12}+C^{12}$	7.50	−5.8	0.75
$C^{12}+O^{16}$	7.25	−5.0	1.0
$O^{16}+O^{16}$	7.96	−7.5	0.5

The smoothed-out, average cross-section factor for the sum of all reaction channels determined in this way is compared with the experimental data for $C^{12}+C^{12}$ in Figure 2. Similar results were obtained for $C^{12}+O^{16}$ and $O^{16}+O^{16}$. More detailed models of the reactions that assume $V_1 = V_1(r)$ usually incorporate a strong repulsive core at small r. Our constant V_1 is an average given by $V_1 \approx \int V_1(r) r dr / \int r dr$ and indeed our values for V_1 do represent the average value of the potentials used by Michaud (1973), for example.

The problem then reduces to fitting this average cross-section factor to a

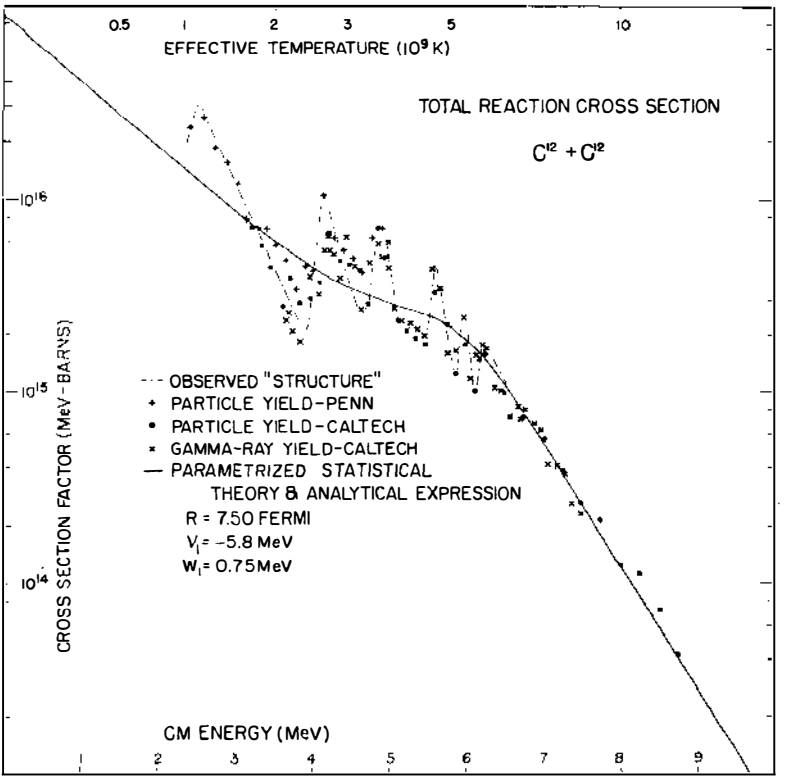

Figure 2 The cross-section factor for all reactions from $C^{12}+C^{12}$. The solid curve is a least squares adjustment to the available data of a parameterized statistical theory based on a generalized "black-body" model described in the text. This curve is also fitted to a few percent by the analytical expression in equation 30 with parameters given in the text. Even so, it goes without saying that the extrapolation to low energy given by a particular model is uncertain, probably by a factor of 3 either way.

reasonable analytical function of interaction energy. Almost perfect fits with deviations less than $\pm 2\%$ were obtained with $m > 0$, $\alpha > 0$, $\beta > 0$, $0 < b \ll 1$ in the expression

$$S(E) = \frac{S(0)\exp(-\alpha E)}{\exp(-\gamma E^m)+b\exp(+\beta E)}, \qquad 30.$$

which is asymptotic to $S(0)\exp(-\alpha E)$ at low energy and to $(1/b)S(0)\exp[-(\alpha+\beta)E]$ at high energy. The increase in the negative slope of ln $S(E)$ vs E from α at low energy to $(\alpha+\beta)$ at high energy is characteristic of the observed cross-section factor as seen in Figure 2. The transition behavior around 5 MeV is determined by the empirical parameters γ and m in the term $\exp(-\gamma E^m)$. Our results are:

	$S(0)$ (MeV barns)	α (MeV^{-1})	b	β (MeV^{-1})	γ (MeV^{-m})	m
$C^{12}+C^{12}$	8.83×10^{16}	0.772	5.56×10^{-3}	0.697	5.01×10^{-5}	6
$C^{12}+O^{16}$	1.15×10^{21}	0.643	1.06×10^{-3}	0.837	6.26×10^{-3}	3
$O^{16}+O^{16}$	2.31×10^{27}	0.780	3.89×10^{-4}	0.681	9.05×10^{-6}	6

The total reaction rates given for these interactions in Table 1 are based on the substitution of the above parameters into equation 30 with E in the denominator terms replaced by E_{oA}. It must be emphasized that our extrapolation for $S(E)$ to low energy, $E < 2$ MeV for $C^{12}+C^{12}$ for example, is rather uncertain. This means that the quoted reaction rates are uncertain below the temperatures given in the second paragraph of this section. The problem has been discussed in some detail by Michaud (1973) and Fowler (1974).

The main products of the C^{12} and O^{16} pair reactions are n, p, and α, with the appropriate residual nuclei in both ground and excited states. After surveying the experimental literature we recommend the following fractional yields:

		n-channels	p-channels	α-channels
$C^{12}+C^{12}$	$T_9 < 3$	0.00(−2.599)	0.50(2.240)	0.50(4.617)
	$T_9 \geqq 3$	0.05	0.30	0.65
$C^{12}+O^{16}$		0.10(−0.423)	0.50(5.169)	0.40(6.770)
$O^{16}+O^{16}$		0.14(1.453)	0.80(7.677)	0.29(9.593)

The quantities in brackets are the ground state Q-values in MeV. The entries for $O^{16}+O^{16}$ add up to greater than unity because three-body channels such as $Si^{30}+2p$, $Al^{27}+p+\alpha$, and $Mg^{24}+2\alpha$ are open and deuteron production is included with the neutron production.

Neutron-producing Reactions

Table 1 includes a number of exoergic ($Q > 0$) neutron-producing reactions that have been analyzed in a straightforward manner using the techniques previously discussed: $H^2(d,n)He^3$; $H^3(d,n)He^4$; $H^3(t,2n)He^4$; $He^3(t,np)He^4$; $Li^7(d,n)2He^4$; $Li^7(t,2n)2He^4$; $Li^7(He^3,np)2He^4$; $Be^7(t,np)2He^4$; $Be^9(\alpha,n)C^{12}$; $C^{13}(\alpha,n)O^{16}$; $O^{17}(\alpha,n)Ne^{20}$; $Ne^{21}(\alpha,n)Mg^{24}$; $Mg^{25}(\alpha,n)Si^{28}$; and $Mg^{26}(\alpha,n)Si^{29}$.

A number of endoergic ($Q < 0$) neutron-producing reactions with thresholds at $E_{th} = |Q|$ have been analyzed by Bahcall & Fowler (1969b) in terms of a truncated Maclaurin series (three terms) in $E^{1/2}$ for the neutron cross-section factor $\mathcal{S}(E^{1/2})$. Their results are included in Table 1 as well as new results for $Li^7(\alpha,n)B^{10}$ for which $\mathcal{S}$ = const. Cases that cannot be treated in the manner of Bahcall & Fowler (1969b) and require more rapidly varying analytical expressions are discussed by Howard et al (1974) and Fowler (1974). In addition several reactions, $H^2(p,n)2H^1$, $He^4(t,n)Li^6$, and $Na^{23}(p,n)Mg^{23}$, have required treatment rather different from the other endoergic reactions but using analytical expressions that have already been described.

Two endoergic neutron-producing reactions, $O^{18}(\alpha,n)Ne^{21}-0.698$ MeV and

$Ne^{22}(\alpha,n)Mg^{25} - 0.480$ MeV, have required special treatment. In these reactions the energy dependence above the neutron-production threshold is determined by the barrier penetration factor in the alpha-particle channel. The neutron partial width exceeds the alpha-particle partial width within ~ 25 keV above threshold and so we can take the threshold as the energy above which the standard treatment for a charged particle reaction holds. The problem is discussed in Appendix C of Fowler & Hoyle (1964), who show that the usual reaction rate must be multiplied by the integral

$$F(T) = \frac{1}{\pi^{1/2}} \int_{x_t}^{\infty} \exp(-x^2)\,dx \Rightarrow 1/2 \quad \text{for} \quad x_t \Rightarrow 0. \tag{31.}$$

In equation 31 the threshold value for the variable in the integrand is

$$x_t = 2(E_t - E_o)/\Delta E_o \tag{32.}$$

where $E_t = |Q|$ is the threshold energy, $E_o \propto T^{2/3}$ is the effective interaction energy defined by equation I-56, and $\Delta E_o \propto T^{5/6}$ is the effective energy interval defined by equation I-58. These effective values are determined by the maximum and full width at $1/e$ times maximum in the product of the Gamow and Maxwell-Boltzmann exponentials in equation 3 with $S(E)\exp(-E_G^{1/2}/E^{1/2})$ from equation 18 replacing σE. In the standard treatment for exoergic charged-particle reactions, $E_t = 0$ and $2E_o > \Delta E_o$ over the temperature range of interest, so that the lower limit on the integral in equation 31 is large and negative and $F(T) \approx 1$.

It will be clear at low temperatures where $E_t \gg E_o$ and ΔE_o, that $F(T) \Rightarrow 0$ and at high temperatures where energies well above threshold are involved that $F(T) \Rightarrow 1$ even for endoergic reactions. These asymptotic values put constraints on the type of analytic function of T to be used to approximate $F(T)$. We are primarily interested in the rise near threshold from low temperature to high temperature as E_o increases through the value E_t and x_t decreases through zero. Evaluation of the integral about this point, where it is equal to 1/2, yields

$$F(T_9) = \exp -(\theta_9/T_9)^n, \tag{33.}$$

where

$$n = \frac{8}{3\pi^{1/2}\ln 2}\frac{\varepsilon_o}{\delta_o}\left(\frac{\varepsilon_o}{E_t}\right)^{1/4} = 2.171\,\frac{\varepsilon_o}{\delta_o}\left(\frac{\varepsilon_o}{E_t}\right)^{1/4} \tag{34.}$$

and

$$\theta_9 = (\ln 2)^{1/n}\left(\frac{E_t}{\varepsilon_o}\right)^{3/2}, \tag{35.}$$

with

$$\varepsilon_o = E_o/T_9^{2/3} = 0.12204\,(Z_0^2 Z_1^2 A)^{1/3}\ \text{MeV}/(10^9\ ^\circ\text{K})^{2/3} \tag{36.}$$

and

$$\delta_o = \Delta E_o/T_9^{5/6} = 0.23682\,(Z_0^2 Z_1^2 A)^{1/6}\ \text{MeV}/(10^9\ ^\circ\text{K})^{5/6}. \tag{37.}$$

When necessary T_{9A} replaces T_9 in the above equations.

The threshold factor given by equation 33 holds only for T_9 near θ_9 and is much too small at low temperatures. If one thinks of the reactions in reverse, $Ne^{21}(n,\alpha)O^{18}$ and $Mg^{25}(n,\alpha)Ne^{22}$, which are exoergic, it will be clear, because there are no selection rules against s-wave neutrons, that $N_A\langle\sigma v\rangle_{\text{reverse}} = \text{const}$ and $N_A\langle\sigma v\rangle_{\text{direct}} = \text{const}\exp(-E_t/kT)$. Such a term has been evaluated theoretically and added to the rate of the reactions under discussion.

The onset of the neutron channel in the two interactions $O^{18}+\alpha$ and $Ne^{22}+\alpha$ has a profound effect on the two competing reactions $O^{18}(a,\gamma)Ne^{22}$ and $Ne^{22}(\alpha,\gamma)Mg^{26}$. Once the neutron channel is well above threshold the probability for neutron emission is large compared to gamma-ray emission or to alpha-particle reemission and the neutron channel cross section is approximately equal to the total reaction cross section for $E > E_t$. The gamma-ray cross section is a small fraction of this. However, below the threshold, $E < E_t$, the probability for neutron emission is zero, and because of the Coulomb barrier, the probability of alpha-particle reemission is small compared to gamma-ray emission. Thus the gamma-ray channel cross section is equal to the total reaction cross section, which can be obtained by extrapolating the observed neutron channel cross section to low energy, disregarding the threshold.

It will be clear that a cutoff factor corresponding to the threshold factor in equation 31 must be applied to the high-temperature reaction rate for the (α, γ) reactions when (α, n) becomes energetically possible. This cutoff factor is

$$F'(T) = \frac{1}{\pi^{1/2}}\int_{-\infty}^{x_t} \exp(-x^2)dx \Rightarrow 1/2 \quad \text{for} \quad x_t \Rightarrow 0. \tag{38.}$$

Now $F'(T) \Rightarrow 1$ at low temperature where $x_t \Rightarrow +\infty$ and $F'(T) \Rightarrow 0$ at high temperature where x_t is large and negative. Proceeding as in the case of $F(T)$ one finds

$$F'(T_9) = \exp -(T_9/\theta'_9)^{n'}, \tag{39.}$$

where

$$n' = \frac{8\ln 2}{3\pi^{1/2}}\frac{\varepsilon_o}{\delta_o}\left(\frac{\varepsilon_o}{E_t}\right)^{1/4} = 1.043\frac{\varepsilon_o}{\delta_o}\left(\frac{\varepsilon_o}{E_t}\right)^{1/4} = 0.4805n \tag{40.}$$

and

$$\theta'_9 = (\ln 2)^{-1/n'}\left(\frac{E_t}{\varepsilon_o}\right)^{3/2} = (\ln 2)^m\theta_9, \tag{41.}$$

with

$$m = -1/n - 1/n'. \tag{42.}$$

At high temperature the rate for gamma emission will be equal to a small fraction of the neutron emission rate including the $F(T)$ factor. In the case of the $O^{18}(\alpha,\gamma)Ne^{22}$ reaction the available experimental evidence indicates a rate $\sim 5\times 10^{-4}$ that of the $O^{18}(\alpha,n)Ne^{21}$ reaction. We have used this same ratio in a number of cases, namely, $O^{17}+\alpha$, $Ne^{22}+\alpha$, $Mg^{25}+\alpha$, and $Mg^{26}+\alpha$. Because of the high Q of the $Ne^{21}(\alpha,n)Mg^{24}$ reaction we have used the ratio 10^{-4} from estimates made by S. E. Woosley (personal communication, 1974) for the ratio in this case.

In the cases of $O^{17}(\alpha,\gamma)Ne^{21}$, $Mg^{25}(\alpha,\gamma)Si^{29}$, and $Mg^{26}(\alpha,\gamma)Si^{30}$ we have noted in Hauser-Feshbach calculations that the cross sections are a substantial fraction of the corresponding (α,n) cross sections at low energy but drop to a fraction $\sim 5\times 10^{-4}$ at the onset of $O^{17}(\alpha,n_1)Ne^{20*}-1.046$ MeV, $Mg^{25}(\alpha,n_2)Si^{28**}-1.965$ MeV, and $Mg^{26}(\alpha,n_1)Si^{29*}-1.240$ MeV, respectively, where Ne^{20*} is the first excited state of Ne^{20} at 1.634 MeV, Si^{28**} is the second excited state of Si^{28} at 4.618 MeV, and Si^{29*} is the first excited state of Si^{29} at 1.273 MeV. The appropriate factors, $F'(T_9)$, have been applied in these cases.

Contribution of Bound States

In three reactions of considerable astrophysical importance, $C^{12}(\alpha,\gamma)O^{16}$, $O^{18}(p,\alpha)N^{15}$, and $Ne^{20}(p,\gamma)Na^{21}$, the tail of a bound state in the compound nucleus makes a major contribution to the low-energy, nonresonant cross section and thus to the cross-section factor. This contribution can be represented approximately as a function of energy by

$$S(E) = \text{const}\left|\frac{1}{E+E_b+i\Gamma/2}+\frac{1}{E_{\text{int}}}\right|^2 \Rightarrow \frac{\text{const}}{(E+E_b)^2+\Gamma^2/4} \quad \text{for} \quad E_{\text{int}}^{-1}=0, \tag{43.}$$

where $E_b > 0$ is the binding energy (taken positive) of the state relative to the mass-energy equivalent of the components of the incident channel, Γ is the width of the state contributed by the outgoing and all competing channels, and E_{int}^{-1} is proportional to the amplitude, taken constant, of other states of the same spin and parity as the bound state in question. The contribution of these states can interfere coherently with that of the bound state. Again we replace E by E_o.

The simplest case is $Ne^{20}(p,\gamma)Na^{21}$, where the width of the bound state is $\Gamma = \Gamma_\gamma = 0.31$ eV, whereas the binding energy is $E_b = 7.1$ keV, and there seems to be no interference from other states (Rolfs et al 1975). Thus in this case $S \propto (0.5586T_9^{2/3}+0.0071)^{-2}$ because $E_o = 0.5586T_9^{2/3}$ for $Ne^{20}+p$. Then, because $N_A\langle 01\rangle \propto S/T_9^{2/3}$, the factor that occurs in the denominator of the first entry for $Ne^{20}(p,\gamma)$ in Table 1 is $T_9^2(1+0.0127/T_9^{2/3})^2$. Because E_b is so small compared to the effective reaction energy for significant temperatures, we have $S(E)$ approximately proportional to E^{-2} for $E \gg E_b$ in this case. The question quite naturally arose concerning how accurate is the substitution of E_o for E, which was made to obtain the analytic formula we have employed. Accordingly we carried out a numerical integration to determine $N_A\langle 01\rangle$ accurately for $S(E)\propto (E+0.0071)^{-2}$ and found that our analytic expression was good to $\sim 1\%$ at low temperature ("noise" in the numerical integration) and was low by only 5% near $T_9 = 1$ and by 10% near $T_9 = 10$.

In the case of $O^{18}(p,\alpha)N^{15}$ the 7.90-MeV state in F^{19} is bound by 0.09282 MeV relative to $O^{18}+p$ for which $E_o = 0.4805T_9^{2/3}$ and has a measured width given by $\Gamma = \Gamma_\alpha = 0.210$ MeV. We neglect interference terms. The upshot is $S \propto [(0.4805T_9^{2/3}+0.09282)^2+0.0110]^{-1}$ and the factor that occurs in the denominator

of the third term for $O^{18}(p,\alpha)N^{15}$ in Table 1 becomes $[0.439(1+5.18T_9^{2/3})^2+0.561]$ when normalized to unity for $T_9 = 0$. The reduced proton width for this state is not known so its contribution must perforce include the factor (0 to 1).

The importance of the bound state in O^{16} at 7.11867-MeV excitation ($J^\pi = 1^-$) just below the threshold for $C^{12}(\alpha,\gamma)O^{16}$ was discussed in detail in FCZ I. It is now known to be bound by 0.04295 MeV. In FCZ I it was necessary to estimate the nonresonant contribution by using a theoretical calculation for the reduced alpha-width of this state, $\theta_\alpha^2 = 0.085 \pm 0.040$. Measurement by Dyer & Barnes (1974) of the cross section of the $C^{12}(\alpha,\gamma)O^{16}$ reaction from just above the 3.160-MeV (LAB) resonance ($J^\pi = 1^-$) down to 1.88 MeV (LAB) has shown constructive interference of the two 1^- states and enabled them to extrapolate the cross section to low energy with some confidence using several theoretical methods that give substantially the same results (Koonin, Tombrello & Fox 1974; Humblet, Dyer & Zimmerman 1975). We have fitted their results below 1 MeV (LAB) to equation 43, neglecting $\Gamma \approx \Gamma_\gamma = 0.056$ eV. A good fit over this limited range of energy is obtained for $E_{\rm int} = 1.4420$ MeV and the upshot, with $E_o = 0.9226T_9^{2/3}$ MeV, is the occurrence of the factors $(1+0.621T_9^{2/3})^2/[T_9^2(1+0.047/T_9^{2/3})^2]$, the first entry for $C^{12}(\alpha,\gamma)O^{16}$ in Table 1. The accuracy of this analytical approximation has been tested against numerical integration and found to deviate by no more than 1% over the range $0.01 \leqq T_9 \leqq 10$. Further discussion of the $C^{12}(\alpha,\gamma)O^{16}$ reaction rate will follow in due course. The Dyer & Barnes (1974) results yield $N_A\langle\sigma v\rangle$ equal to 0.62 that given in FCZ I for $E_{o6} \sim 0.3$ and $T_9 \sim 0.2$.

Three-Body Reactions

Table 1 includes four reactions induced by the interaction of three bodies, namely, $He^4(nn,\gamma)He^6$, $He^4(np,\gamma)Li^6$, $He^4(\alpha n,\gamma)Be^9$, and $He^4(2\alpha,\gamma)C^{12}$, all of which are of interest in connection with attempts (unsuccessful!) to bridge the mass gaps at $A = 5$ and 8 in "big bang" nucleosynthesis. The last reaction is of course important as the first stage in helium burning in red giant stars.

Direct or indirect experimental evidence is available on the reverse of all these reactions except $He^4(nn,\gamma)He^6$. When the photodisintegration cross section, σ_γ, is known as a function of gamma-ray energy, E_γ, the photodisintegration rate per nucleus per second is given by

$$\lambda_\gamma = \frac{c}{\pi^2(\hbar c)^3}\int_{E_t}^{\infty} \sigma_\gamma E_\gamma^2 \exp(-E_\gamma/kT)\, dE_\gamma \ \text{sec}^{-1}, \qquad 44.$$

where E_t is the threshold energy. Both spontaneous and stimulated emission (in reverse) have been taken into account. This expression is to be compared with equation 3. Reciprocity through the use of the tabulated REV RATIOS makes it possible to determine the rate of the direct reactions involving three bodies that are under discussion. Equation 44 holds of course for photodisintegration into any number of bodies—2, 3, 4, ...

These reactions proceed through two stages, for example, $He^4 + He^4 \rightleftarrows Be^8$ and $Be^8(\alpha,\gamma)C^{12}$, and one or more excited states in the compound nucleus serve as

resonances. For a single sharp resonance equation 44 yields

$$\lambda_\gamma \approx \frac{2J_r+1}{2J_0+1}\,\frac{\Gamma_{\rm rad}(\Gamma-\Gamma_{\rm rad})}{\hbar\Gamma}\exp(-E_r/kT)\ \sec^{-1}, \qquad 45.$$

where E_r is the energy of the resonant state relative to the ground state in the compound nucleus, J_r is the spin of the resonant state, J_0 is the spin of the ground state, $\Gamma_{\rm rad}$ is the overall radiation width of the state, including gamma-ray transitions to *all* lower states and electron-positron pair emission, if it occurs, and Γ is the total width including radiation and particle emission. $\Gamma \gg \Gamma_{\rm rad}$ is usually the case so $\lambda_\gamma \propto \Gamma_{\rm rad}/\hbar$.

As discussed in FCZ I, equation 45 is used for the rate of $C^{12}(\gamma,2\alpha)He^4$, which yields the rate of $He^4(2\alpha,\gamma)C^{12}$ and the latest discussion of this important case is given by Barnes & Nichols (1973). Experimental data are available on $\sigma_\gamma(E_\gamma)$ for $Li^6(\gamma,np)He^4$ and $Be^9(\gamma,n\alpha)He^4$. The excited states at 5.366 and 1.665 MeV, respectively, seem to establish the threshold energy in these reactions above which the logarithm of the cross section varies slowly and linearly with $(E_\gamma - E_t)$. The integral in equation 44 is thus easy to calculate. We have made a rough calculation for $He^6(\gamma,nn)He^4$ in analogy with $Li^6(\gamma,np)He^4$ and have been able to set a reasonable upper limit on the rate of this reaction and its reverse.

RINGING THE CHANGES SINCE FCZ I

We conclude with a few remarks concerning important processes of nucleosynthesis and energy generation, stressing the changes in reaction rates that have occurred since the publication of FCZ I.

"Big Bang" Nucleosynthesis

As noted previously, a number of three-body reactions involving neutrons, protons, and alpha particles have been analyzed because of their possible importance in bridging the mass gap at mass 5 and 8. It is known that Be^7 is produced in significant amounts during "big bang" nucleosynthesis and some C^{11} results from $Be^7(\alpha,\gamma)C^{11}$. Thus we have included a theoretical estimate for $C^{11}(p,\gamma)N^{12}(e^+\nu)C^{12}$ based on analogy with $C^{12}(p,\gamma)N^{13}$. In addition, $C^{11}(\alpha,p)N^{14}$ can be calculated from the rate for $N^{14}(p,\alpha)C^{11}$ in Table 1 which is based on experimental data. These reactions have been considered as more efficacious in reaching $A \geqq 12$ than $C^{11}(n,\gamma)C^{12}$ because the latter must compete with the much faster break-up reaction $C^{11}(n,\alpha)2He^4$.

It is well known that "big bang" nucleosynthesis results in the production of Li^7 in amounts comparable to the solar system abundance of this rare nucleus both directly as Li^7 and indirectly from the decay of Be^7. In studying this mechanism of synthesis it is necessary to include not only all reactions that produce Li^7 and Be^7 but also those that destroy them. Thus, in addition to H and He^4 reactions with these nuclei, Table 1 includes rates of their interactions with D, T, and He^3, which exist in the early stages of the "big bang" in significant amounts. For neutron interactions with these nuclei see FCZ I.

Many of our preliminary estimates for the reaction rates given in Table 1 were employed by Wagoner (1969) in his study of "big and little bangs." There is no reason to believe that our final results will change in any significant way his conclusion that no heavy elements beyond helium, with the exception perhaps of Li^7, are produced in the "big bang" in amounts of interest. "Little bang" synthesis does depend on the rates of these reactions but not in a critical manner.

Controlled Thermonuclear Fusion

Many of the reactions involving the use of D, T, He^3, and the lithium and boron isotopes in controlled thermonuclear fusion are also of interest under astrophysical circumstances. Thus we have taken some pains to fit the accurate experimental results for the D, T, and He^3 reactions with considerable care as illustrated for $H^3(d,n)He^4$ in Figure 1. Our study of the literature has revealed that much more precise measurements of reactions involving lithium and boron isotopes are warranted because these may well prove to be of interest in connection with laser-induced fusion.

The Proton-Proton Chain

The coefficient of the basic pp reaction, $H^1(p,e^+\nu)H^2$, given in Table 1 is 27% greater than that given in FCZ I. This results from use of the most recent value for the half lifetime of the neutron (12.5%), new radiative corrections (1.6%), and meson exchange effects with soft-core potentials (11.4%). This increase results in a significant decrease in the high-energy neutrino flux expected from the Sun but not enough to bring observation and theoretical expectations into agreement (Ulrich 1974). New measurements have been made on many of the pp chain reactions since the publication of FCZ I and these measurements yield the appropriate entries in Table 1. The result has been only small changes in the cross sections and reaction rates but considerable reduction in the experimental uncertainties. In a most important experiment Dwarakanath (1974) has shown that resonance (Fowler 1972) almost certainly does not occur in $He^3(He^3,2p)He^4$, so that the production of B^8 neutrinos via $He^4(He^3,\gamma)Be^7(p,\gamma)B^8(e^+\nu)2He^4$ should occur as calculated using the non-resonant rate for $He^3(He^3,2p)He^4$. The solar neutrino problem is still with us!

The CNO Tri-Cycle

Major changes have occurred in some of the reactions involving protons and the isotopes of carbon, nitrogen, and oxygen but fortunately not in the primary reactions of the CN cycle. However, Rolfs & Rodney (1974a) found that the cross-section factor for $N^{15}(p,\gamma)O^{16}$ is 2.5 times larger than that given in FCZ I. $N^{15}(p,\alpha)C^{12}$ has not been remeasured and thus remains unchanged. This means that leakage from the CN cycle to oxygen isotopes occurs in somewhat less than 1000 cycles under conditions in upper main-sequence stars.

In addition, Rolfs & Rodney (1974b) have found that the rate for $O^{17}(p,\alpha)N^{14}$ is probably not much faster than $O^{17}(p,\gamma)F^{18}(e^+\nu)O^{18}$, so that a *tri-cycle* occurs among the CNO isotopes involving the recycling reaction $O^{18}(p,\alpha)N^{15}$, as well as $O^{17}(p,\alpha)N^{14}$ and $N^{15}(p,\alpha)C^{12}$. Among other things this means that O^{17} survives

hydrogen burning in greater abundance than heretofore thought and thus can serve as a source of neutrons in helium burning via $O^{17}(\alpha, n)Ne^{20}$.

Because of its importance in the fast CN cycle we have included an estimate for $N^{13}(p, \gamma)O^{14}$ based on analogy with $C^{12}(p, \gamma)N^{13}$—the extra proton in N^{13} is just a spectator in the reaction.

The NeNa and MgAl Cycles

Table 1 presents for the first time the rates of all the reactions involved in these cycles and the leakage from them. We assume in the MgAl cycle a time scale long compared to the mean lifetime of the ground state of Al^{26}, 1.04×10^5 yr. The most important new result is the experimental confirmation by Rolfs et al (1975) of a theoretical conjecture by Marion & Fowler (1957) that Ne^{21} is produced rapidly by $Ne^{20}(p, \gamma)Na^{21}(e^+ \nu)Ne^{21}$ through the off-resonant contribution of a bound state in Na^{21}. Ne^{21} may burn through $Ne^{21}(p, \gamma)Na^{22}$ more slowly than it is created so that it may survive hydrogen burning sufficiently to serve as a neutron source during helium burning via $Ne^{21}(\alpha, n)Mg^{24}$.

Helium Burning

Major experimental efforts have been made since the publication of FCZ I to elucidate the nature and rate of the basic helium-burning reactions, $3He^4 \rightarrow C^{12}$, $C^{12}(\alpha, \gamma)O^{16}$, $O^{16}(\alpha, \gamma)Ne^{20}$, ... The result of the work of many investigators [see references in Barnes & Nichols (1973)] has led to a change relative to FCZ I in the rate for $3He^4 \rightarrow C^{12}$ via the 7.655-MeV state in C^{12} by a factor $1.41 \exp(-0.117/T_9)$, which includes the change given in the notes added in proof. This is equal to 0.78 at the important red giant temperature, $T_9 = 0.2$—not a great change but the uncertainty in the new value is considerably less than before. The corresponding factor for the $C^{12}(\alpha, \gamma)O^{16}$ reaction rate, as noted previously, is 0.62. Significant changes have been made in $O^{16}(\alpha, \gamma)$, and new (α, γ) and (α, n) rates involving oxygen, neon, and magnesium isotopes have been included. A major decrease by several orders of magnitude is indicated for the important $N^{14}(\alpha, \gamma)F^{18}$ reaction so that it does not ignite before the onset of the $3\alpha \rightarrow C^{12}$ reaction in helium burning. The rate for $Ne^{20}(\alpha. \gamma)$ is still very uncertain at helium-burning temperatures.

Carbon, Oxygen, and Silicon Burning

Table 1 gives reliable rates for carbon burning, oxygen burning, and the first stages of silicon burning under explosive conditions. The importance of the photodisintegration rate of Mg^{24} is confirmed as determining the time scale for the freeing of neutrons, protons, and alpha particles in silicon burning. We emphasize once again that there is still considerable uncertainty (a factor of ~ 3 either way) in the reaction rates for quasistatic burning of carbon and oxygen and with this final hint to the experimental nuclear physicist we conclude this "Handbuch der Kernastrophysik."

NOTES ADDED IN PROOF The coefficient in the first term for $Hc^4(2A, G)C^{12}$ in Table 1 should be changed from $2.49E-08$ to $3.00E-08$. Questions have arisen

concerning the rate of the $N^{15}(P, A)C^{12}$ reaction, and new measurements are under way in several laboratories.

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