Spherical Inward Propagating Carbon-Oxygen Two Step Flame of Type Ia Supernova

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ABSTRACT

The explosion of the supernova is one of the most important subjects in the field of the astrophysics. For the influence of Rayleigh-Taylor instability and turbulence, there exists some unburned bubbles in the supernova flame. According to the Zeldovich detonation mechanism, when the reaction gradient is very small, flame spontaneous propagating speed approaches or even exceeds the sound speed, that pressure wave will be coupled with the reaction wave and develops into detonation in final. We use the Zeldovich detonation mechanism to research the acceleration effect of the spherical inward propagating Carbon-Oxygen two step flame of type Ia supernova, and derive the relationship between Oxygen flame spontaneous propagating speed and flame radius. The simulation results show that, for the Carbon-Oxygen white dwarf that density is 3.5×10^7 g/cm^3, Reynolds number is 5.8×10^{13}, integral length scale is 100 km and turbulent velocity is 629 m/s, the Oxygen flame spontaneous propagating speed gets up to the sound speed when the flame radius is 1.5 times of the flame thickness, and then keeps supersonic. The supersonic combustion generates shock wave, which interacts with the main flame and promotes the detonation development of the main flame.

Subject headings: supernova, detonation, flame spontaneous propagating speed

1. Introduction

Supernova is the last stage of the nuclear burning of the large mass main sequence star that is end as an explosion (Hoyle & Fowler 1960). Type Ia supernova is a member of the family of supernova, whose predecessor is Carbon-Oxygen white dwarf. The white dwarf absorbs the materials around it for gravity, transforming gravitational potential energy into thermal energy. When the mass increases to the critical mass, thermal nuclear reaction is ignited, which develops to deflagration, and to detonation in final (Hillebrandt & Niemeyer 2000). The C-O white dwarfs almost have the same mass when they explode all their bodies, so the luminosities of the type Ia supernova have little difference, and they are looked as the standard candles of the universe to measure the large scale distance. In 2011, Saul PerlmutterBrian P. Schmidt and Adam G. Riess were award the Nobel prize for that they discover the accelerating expansion phenomenon of the universe through observing the distant supernova (Schmidt et al. 1998). However, we do not know how the supernova explode from the first principle, the detonation mechanism of the supernova has a great significance for the development of astronomy and cosmology (Chapman 1899; Höflich et al. 1995). At present, people mostly focus on DDT (deflagration to detonation) model to research the ignition of detonation, that the laminar flame transforms to the turbulent flame through self-acceleration, and develops into detonation furtherly (Jouguet 1905; Lee & Oppenheim 1969). The key point of the research is to reveal the mechanism of the flame acceleration. Zeldovich reveals the detonation mechanism through comparing the flame spontaneous propagating speed with the sound speed. The simulation result shows that, the self-ignition of the mixture in preheat zone can induce detonation. When the temperature gradient of the mixture in preheat zone is small, that the temperature is almost homogeneous, and the whole preheat zone starts to react almost simultaneously. In this situation, the flame speed is very large, when it gets up to the sound speed, the pressure wave couples with the reaction wave, strengthening each other, and developing into detonation finally. There are many factors influencing the nuclear reaction transforming into detonation (Zel'Dovich 1985; Zel'Dovich et al. 1970). Firstly, Rayleigh-Taylor and Kelvin Helmholtz instability affects the combustion of the C-O white dwarf (Taylor 1950). On the other hand, strong turbulent flow has an obvious effect on flame acceleration. The simulation result of Ropke (2007) shows that, for the C-O white dwarf that the density is 1×10^7 g/cm^3, the fluctuation of the flame speed can be up to 10^3 km/s, which is the same order as the sound speed of the nucleon, and provides favorable conditions for the development of the detonation (Röpke 2007). The combustion of the C-O white is a two-step reaction, that the Carbon reacts firstly and then ignites the reaction of the Oxygen. Before, people pay much attention on the combustion of the Carbon and have little concern on the combustion of the Oxygen (Woosley et al. 2009, 2011). This paper will research how the Oxygen combustion promote the development of the detonation. We use Zeldovich mechanism in the supernova detonation model, researching the unburned bubbles that are formed by Rayleigh-Taylor instability and turbulence respectively. Firstly, we use spherical inward propagating laminar flame model to research the flame acceleration of unburned bubble that is formed by Rayleigh-Taylor instability, and then use spherical inward propagating turbulent flame model to research the flame acceleration of unburned bubble that is formed by turbulence.

2. Spherical inward propagating laminar flame

2.1. Spherical inward propagating laminar flame formed by Rayleigh-Taylor instability

During the burning of the white dwarf, the hot product goes up at the effect of the gravity, which is Rayleigh-Taylor instability. The main flame surface wrinkles under the effect of the Rayleigh-Taylor instability, that forms a series of unburned bubbles insert in the main flame surface. According to the simulation results of the Bell (2004), the width of the bubble is about 163 cm , and the height is about 328 cm (Bell et al. 2004).

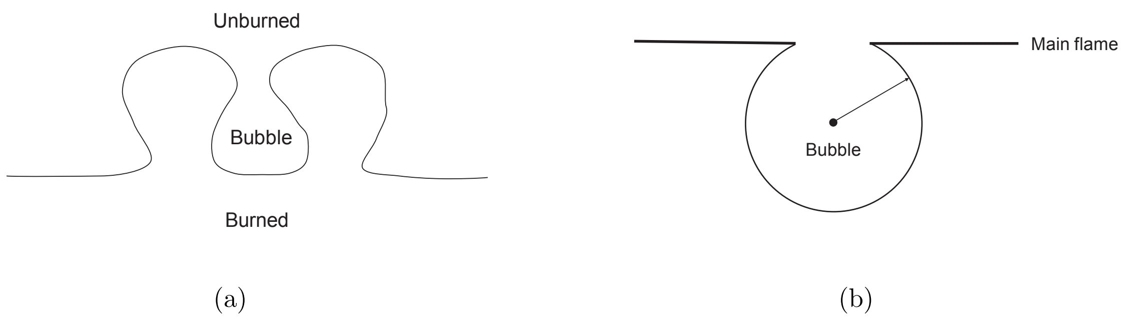

Fig. 1.— (a) The wrinkled flame forms unburned bubble; (b) Assuming the bubble as a sphere.

The thing in the bubble is unburned mixture of Carbon and Oxygen, and that outside of the bubble is the hot product. The ignition temperature of Carbon-Carbon nuclear reaction is lower than that of the Oxygen-Oxygen reaction, so Carbon-Carbon nuclear reaction starts from the margin of the unburned bubbles, and the heat released from the Carbon-Carbon reaction ignites the Oxygen-Oxygen nuclear reaction, forming the inward propagating spherical two step flame structure that Carbon flame leads the Oxygen flame. Then, we will discuss the spherical inward propagating Oxygen flame in detail.

2.2. Spontaneous propagating speed of the Oxygen flame

For combustible mixture, when the gradient of the preheat temperature is very small, the time interval of the reaction between the two adjacent is short. A small reaction gradient may induce a detonation at this situation. Zel'dovich (1980) proposed the concept of flame spontaneous propagating velocity from the point of the ignition delay time (Zeldovich 1980).

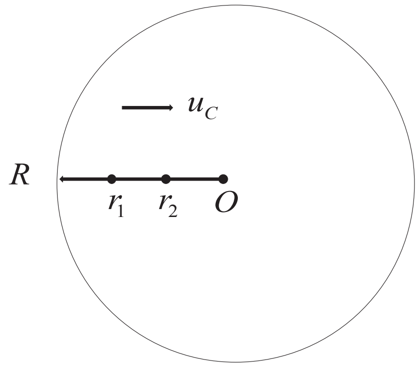

Fig. 2.— Spherical inward propagating flame

As shown in figure 2, for spherical inward propagating flame, assume that the flame travels from R, the flame passes the r_1 firstly, and the propagating distance is $R - r_1$; then, the flame passes the r_2, and the propagating distance is $R - r_2$. t_1 and t_2 respectively indicate the time at which Oxygen begins to burn at the point r_1 and r_2, and the spontaneous propagating speed of Oxygen can be defined as:

$$u_{sp(O)} = \frac{(R - r_2) - (R - r_1)}{t_2 - t_1} = \frac{r_1 - r_2}{t_2 - t_1}. \tag{1}$$

Since the oxygen flame follows the carbon flame, at any point, the time at which the oxygen begins to burn is the time that Carbon flame needs to propagate plus the ignition delay time of the oxygen at that point. Since the spherical flame is affected by the curvature effect, the flame temperature of Carbon varies with the flame radius, and that the flame speed of the Carbon and the ignition delay time of the Oxygen can be expressed as a function of the flame radius. The time that Carbon flame propagating from R to r_1 can be expressed as $t = \int_R^{r_1} \frac{-dr}{u_C(r)}$ (since radius decreases, dr is negative). The Oxygen ignition delay time at the point r_1 and r_2 are $\tau(r_1)$ and $\tau(r_2)$ respectively, so the moment of Oxygen ignition at the point r_1 and r_2 are:

$$t_1 = \int_R^{r_1} \frac{-dr}{u_C(r)} + \tau(r_1), \tag{2}$$

$$t_2 = \int_R^{r_2} \frac{-dr}{u_C(r)} + \tau(r_2). \tag{3}$$

Taking the above equations into the definition of the Oxygen spontaneous propagating speed, setting $r_2 = r$, $r_1 = r + dr$, and letting dr tend to zero, we will obtain the expression of the flame spontaneous propagating speed of the one dimensional spherical inward propagating flame:

$$u_{sp(O)} = \frac{1}{1/u_C(r) - d\tau/dr}. \tag{4}$$

It can be seen from the above formula, the spontaneous propagating speed of Oxygen flame is closely related to the Carbon flame speed and the ignition delay time of Oxygen. Next, we will deduce the relationship between Carbon flame speed $u_C(r)$ and flame radius, as well as that between Oxygen ignition delay time $\tau(r)$ and flame radius.

2.2.1. The Carbon flame speed at different radius

For one dimensional spherical flame, assuming that the thermodynamic parameters are constant, the unsteady energy equation and the component equation of the carbon-carbon nucleus reaction in the spherical coordinate system can be written as follows:

$$\rho C_p \frac{\partial \tilde{T}}{\partial \tilde{t}} = \frac{1}{\tilde{r}^2} \frac{\partial}{\partial \tilde{r}} (\tilde{r}^2 \lambda \frac{\partial \tilde{T}}{\partial \tilde{r}}) + \rho q_C \tilde{\omega}_C, \tag{5}$$

$$\rho \frac{\partial \tilde{Y}}{\partial \tilde{t}} = \frac{1}{\tilde{r}^2} \frac{\partial}{\partial \tilde{r}} (\tilde{r}^2 \rho D \frac{\partial \tilde{Y}}{\partial \tilde{r}}) - \rho \tilde{\omega}_C, \tag{6}$$

where ρ is density, c_p is specific heat, λ is heat transfer coefficient, q_C is Carbon-Carbon nuclear reaction energy release rate, $\tilde{\omega}_C$ is Carbon-Carbon nuclear rate, D is mass transfer coefficient. Based on the above equations, Chen (2010) has done a detailed theoretical derivation (Chen et al. 2010), and then we will give the dimensionless flame temperature and flame speed as a function of the flame radius directly:

$$T_f = \frac{1}{u_C(R) - \frac{2}{R}} \omega_C. \tag{7}$$

$$(u_C - \frac{2}{R}) \ln[(u_C - \frac{2}{R})^2] = (Ze_C - 2)\frac{2}{R}(\frac{1}{Le} - 1), \tag{8}$$

The dimensionless process:

$$u_C = \frac{\tilde{u}}{S_{\rm C}}\ ,\ r = \frac{\tilde{r}}{\delta_f}\ ,\ T = \frac{\tilde{T} - T_0}{T_{ad} - T_0},\ t = \frac{\tilde{t}}{\delta_f / S_C}, \tag{9}$$

where $T_{ad} = T_0 + Y_C q_C / C_p$, δ_f is the flame thickness of the Carbon, T_{ad} is the adiabatic flame temperature of Carbon.

Ze_C and Le both are constant, for the white dwarf whose density is 3.5×10^7 g/cm^3 , flame temperature of the Carbon is 3.2×10^9 K , $Ze_C = 19.0$, $Le = 10^5$ (Woosley et al. 2009). (7) shows the relationship between the flame temperature and the flame radius, and (8) shows that between the flame speed and the flame radius. We can obtain the numerical solution using the iterative method. As the figure 3(a),(b) shown, with the flame propagating inward, the Carbon flame temperature and speed increases gradually, when propagating to the radius that is nearly a flame thickness, the flame speed increases to 9.8 times of the original laminar flame speed.

2.2.2. The ignition delay time of Oxygen at different radius

In this part, we need to obtain the relationship between the Oxygen ignition delay time and the flame radius. The Oxygen ignition delay time is related to the initial preheating temperature and the concentration of Oxygen component. The initial preheating temperature can be approximated to the flame temperature of Carbon. The flame temperature increases with the decrease of the radius (8), and we can get the relationship between the Oxygen ignition delay time and the flame radius. The Oxygen-Oxygen reaction rate can be written as the product of the density and mass fraction rate of change.

$$\omega_O(Y_O, T) {=} \rho_O \frac{dY_O}{dt}. \tag{10}$$

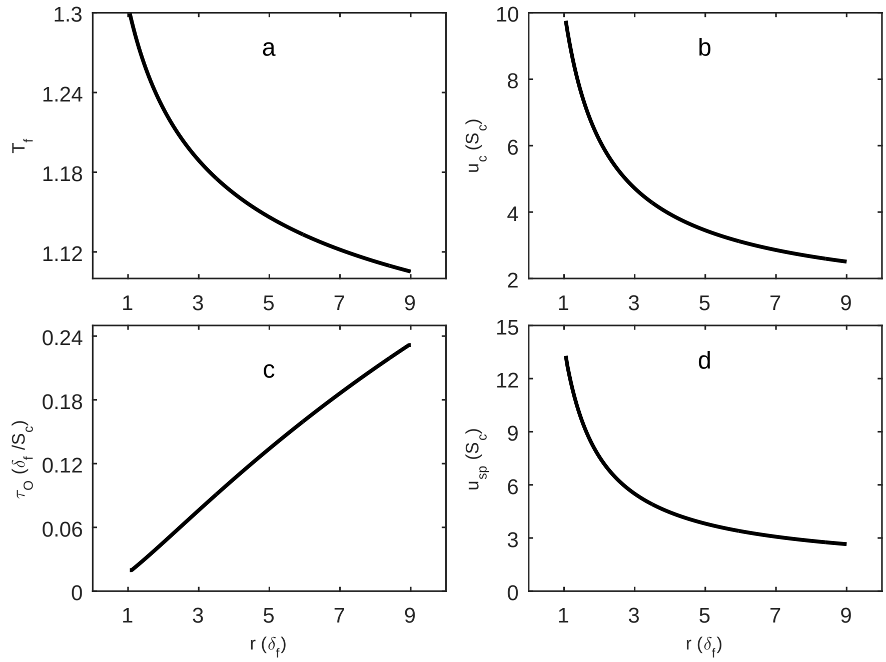

Fig. 3.— a, b, c, d, shows the Carbon flame temperature, Carbon flame speed, Oxygen ignition delay time and Oxygen spontaneous propagating speed respectively. The abscissa axis is the flame position, normalized by the Carbon flame thickness. (a) With the flame propagating inwardly, the Carbon flame temperature goes up because of the curvature effect when $Le > 1$; (b) Carbon flame accelerating due to the increasing flame temperature; (c) Oxygen ignition delay time decreases due to the increasing flame temperature; (d) Oxygen flame spontaneous propagating speed increases under the coupled effect of the Carbon flame speed and the Oxygen ignition delay time.

During the ignition, due to the ignition delay time is very short, it can be considered that the ignition temperature of Oxygen is the Carbon flame temperature, and the reaction rate can be written as the form $\omega_O = \omega_O(Y_O, T_f)$. Deforming (10), we can get:

$$dt = \rho_O \frac{dY_O}{\omega_O(Y_O, T_f)}, \tag{11}$$

integral both side from the initial moment to the end of the ignition:

$$\int_0^\tau dt = \int_{Y_{t=0}}^{Y_{t=\tau}} \rho_O \frac{dY_O}{\omega_O(Y_O, T_f)}. \tag{12}$$

The left side of the equation is the Oxygen ignition delay time. On the right side of the equation, we set the initial Oxygen mass fraction as the unit $Y_{t=0} = 1$, and set the Oxygen mass fraction $Y_{t=\tau} = 1 - \Delta Y$ at the end of the ignition. Thus, we can obtain:

$$\tau = \int_1^{1-\Delta Y} \rho_O \frac{dY_O}{\omega_O(Y_O, T_f)}. \tag{13}$$

In this paper, we use the ignition delay time definition of the Williams (Williams 1985). It is assumed that the time that the mass fraction of Oxygen decreases from the initial value to a certain value is the ignition delay time. Generally, the mass fraction change ΔY is five percent. When setting the mass fraction change, the ignition delay time is only negatively correlated to the flame temperature which affects the Oxygen-Oxygen reaction rate $\omega_O(T) = 4.27 \times 10^{26} \times \rho_O^2 Y_O^2 \exp(-135.93/T_9^{1/3}), (T_9 = T/10^9)$ (Fowler et al. 1975). It is easily to understand from the physics, it consumes a certain quality of combustible mixture, the higher the temperature is, the faster the reaction rate is and the shorter the time required.

We substitute the numerical solution of the Carbon flame speed into (7), then we can obtain the numerical solution of the Carbon flame temperature. Substituting the numerical solution of the Carbon flame temperature into (13), we will obtain the ignition delay time of Oxygen. As the figure 3a shown, with the Carbon flame propagating inward, the Carbon flame temperature goes up and the Oxygen ignition delay time decreases.

2.2.3. Numerical solution of the Oxygen flame spontaneous propagating speed

Now, we have obtained the numerical solution of the Carbon flame speed and the Oxygen ignition delay time at different radius, substituting them into (4), and we will obtain the numerical solution of the Oxygen flame spontaneous propagating speed. As the figure 3d shown, with the flame propagating inward, the Oxygen flame spontaneous propagating speed increases gradually. When propagating to the radius which is close to one Carbon flame thickness, the Oxygen flame spontaneous speed increases to 13.3 times of the Carbon laminar flame speed. For the C-O white dwarf that the density is 4.66×10^4 cm/s, the Carbon laminar flame speed is 4.66×10^4 cm/s (Woosley et al. 2009), the Oxygen flame spontaneous propagating speed is 6.2 km/s, which is much smaller than the sound speed of the nucleon based on ideal gas state equation 1500 km/s. Thus, it can not form the detonation, and then, we will consider the accelerating effect of the strengthened flame in turbulent combustion.

3. Spherical inward propagating strengthened flame

3.1. Spherical inward propagating strengthened flame formed by turbulence

For turbulent combustion, the vortex wrinkles the flame surface and there exists some unburned bubbles. The size of the bubble is related to the turbulent integral length scale and turbulent strength, which is between the C-O white dwarf turbulent integral scale 100 km and the minimum vortex scale 10^{-3} cm. Small scale vortexes coming into the preheating zone of the flame and having a strengthending effect on the flame propagation, it will enhance the flame speed and flame thickness. We will consider the acceleration effect based on the strengthened turbulent flame.

Turbulent combustion is divided into three categories according to the minimum scale of the vortex (Law 2010), flame thickness and the thickness of the reaction zone, that we use Karlovitz number to measure it. $Ka_f = (\delta_f/\eta)^2$, $Ka_{\rm r} = (\delta_r/\eta)^2$, where δ_f, $\delta_r$and η represent flame thickness, the thickness of reaction zone and the minimum scale of vortex. When $Ka_f < 1$, the minimum scale of vortex is larger than the flame thickness, and the acceleration of the flame speed is attributed to the folds of the flame surface. When $Ka_f > 1$ $Ka_r < 1$, the minimum scale of vortex is smaller than flame thickness but larger than the thickness of the reaction zone, and a small part of the vortex will enter into the preheating zone of the flame where the acceleration of the flame speed is attributed to the folds of the flame surface by large scale vortex and the flame strengthening of the thermal diffusion in preheating zone by small scale vortex, which is called the thin reaction zone model. When $Ka_r > 1$, the minimum scale of vortex is smaller than the thickness of the reaction zone, where the flame structure is destroyed and forms a distributed reaction model.

Woosley (2009) discusses the turbulent combustion models of the C-O white dwarf. If a detonation can occur, the turbulent combustion model should between the thin reaction zone model and the distributed reaction model (Woosley et al. 2009). For thin reaction model, the flame is not destroyed. Even if there were no folds of the flame surface by large vortex, the flame speed and the flame thickness will increase because of the strengthening of the thermal diffusion in preheating zone by small scale vortex. The speed and thickness of strengthened flame without folded can be measured by equivalent heat diffusion coefficient, that is $u_t/u_f = \delta_t/\delta_f = \sqrt{\alpha_t/\alpha_f}$. We assume that turbulence has the same strengthening effect on heat transfer and momentum transfer in the preheat zone, then $\alpha_t/\alpha_f = \nu_t/\nu_f$, where u, δ, α, ν represent flame speed, flame thickness, heat diffusion coefficient, and viscosity coefficient, and the subscript t and f correspond to the case of turbulence and laminar flame. The condition for the thin reaction model / distributed reaction model is the minimum scale of the vortex larger / smaller than the thickness of the reaction zone.

For the turbulent combustion model that is between these two model, we can assume that the minimum scale of the vortex is equal to the thickness of the reaction zone. According to the Zimont (1979), $\sqrt{\nu_t/\nu} = Ka$ (Zimont 1979). Thus, Ka=Ze^2 (for Ze=δ_f/δ_r). In this situation, the Reynolds is 5.8×10^{13}, integral length scale is 100 km, and the turbulent velocity is 629m/s. The intensity of the turbulence in the C-O white dwarf flame is easy to achieve. For the white dwarf that the density is 3.5×10^7 g/cm^3, laminar flame speed is 4.66×10^4 cm/s, flame thickness is 0.09 cm, the strengthened flame speed is $u_t = 168$ km/s, strengthened flame thickness is $\delta_t = 32.5$ cm. The strengthened flame speed calculated above is the same order as the carbon turbulence flame speed simulated by Aspden (2010) (Aspden et al. 2010). Then, we consider the flame acceleration effect based on the strengthened flame speed, the Oxygen flame spontaneous propagating speed can be supersonic.

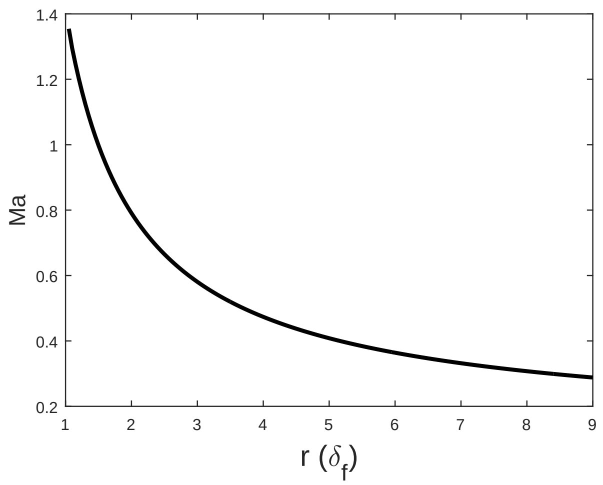

Fig. 4.— Ma number of the Oxygen flame spontaneous propagating speed

As the figure 4 shown, with the Oxygen flame propagating inward, the Mach number of the Oxygen flame spontaneous propagating speed increases. When the flame propagates to the radius that is 1.5 times of the Carbon flame thickness (48.8 cm), the flame speed is equal to the local sound speed 1627 km/s, and then propagates in supersonic. If the turbulence is stronger, the turbulent velocity is larger, and the minimum scale of the vortex will be smaller than the thickness of the reaction zone. The flame structure will be broken and the acceleration effect for spherical flame fails for that. If the turbulence is weaker, the effect of the small scale vortex on the flame preheating zone is weakened, and the oxygen flame needs to propagate to a smaller radius to reach the sound speed.

3.2. Main flame acceleration by the shock wave

For the C-O white dwarf that the density is 3.5×10^7 g/cm^3, laminar flame speed is 4.66×10^4 cm/s, Reynolds number is 5.8×10^{13}, integral length scale is 100 km and turbulent velocity is 629m/s, when the flame propagates to the radius that is equal to the Carbon flame thickness, the Mach number of the Oxygen flame spontaneous propagating speed approaches to 1.4, and the ratio of the temperature at two side of the flame is $T_2/T_1 = 1.22$ ($T_2/T_1 = [2\gamma Ma^2 - (\gamma-1)][(\gamma-1)Ma^2+2]/[(\gamma+1)^2 Ma^2]$). Assuming that the shock wave reflects after converged, the energy in the sphere with a radius of one flame thickness is uniformly spread to the sphere that is swept by the shock wave propagating outward. When the shock wave propagates to the main flame surface (boundary of the unburned bubble), the temperature ratio of the main flame is $\lambda_T = (T_2/T_1 - 1)/R_b^3 + 1$ (R_b is the radius of the unburned bubble). For the speed of the main flame is proportional to the 1/2 power of the reaction rate $u \propto \sqrt{\omega_C(T)}$ (for $\omega_C(T) \propto \exp(-84.165/T_9^{1/3})$), we can obtain the acceleration effect of a single shock wave to the main flame $u/u_t = \exp[42.083/T_9^{1/3} \cdot (1 - \lambda_T^{-1/3})]$ ($T_9 = 3.2$). Thus, the acceleration effect of multiple shock waves can be expressed as $u_n/u_t = \exp[42.083/T_9^{1/3} \cdot (1 - \lambda_T^{-n/3})]$, where $n = ft_{main}$ is the interaction times of the shock waves to the main flame surface, f is the frequency of the shock wave, t_{main} is the propagating time of the flame surface. Let's make an example, for the bubble that the radius is 9 times of the Carbon flame thickness ($R_b = 2.93$ m), the generating frequency can be measured by the ratio of the turbulent velocity at this scale to the radius, that is $f = u_{R_b}/R_b \approx 6.6$ Hz, ($u_{R_b} = S_t(R_b/l)^{1/3} \approx 19.4$ m). The speed of the main flame can accelerate 0.28% by a single shock wave interaction, and that can be supersonic by 858 shock waves interaction, where the whole time is 129 seconds. when we change the scale of the bubble, the results are shown in table 1. we can see that, when the bubble is larger, the generating frequency and acceleration effect is lower, and it need more interaction times for the main flame to accelerate to supersonic.

4. simulation

In the section 2.1.1, we just consider the composition equation and energy equation in constant density to simplify the analysis. In this section, we will make a simulation including the whole governing equations, that are continuity equation, momentum equation, energy equation, composition equation, reaction rate equation and state equation in the spherical coordinates, the equations are as follows:

Continuity equation:

$$\frac{\partial \tilde{\rho}}{\partial \tilde{t}} + \frac{\partial(\tilde{\rho}\tilde{u})}{\partial \tilde{r}} + \frac{2(\tilde{\rho}\tilde{u})}{\tilde{r}} = 0, \tag{14}$$

Momentum equation:

$$\frac{\partial(\tilde{\rho}\tilde{u})}{\partial \tilde{t}} + \frac{\partial(\tilde{\rho}\tilde{u}^2 + \tilde{P})}{\partial \tilde{r}} + \frac{2(\tilde{\rho}\tilde{u}^2)}{\tilde{r}} = 0, \tag{15}$$

Energy equation:

$$\frac{\partial(\tilde{\rho}\tilde{E}_s)}{\partial \tilde{t}} + \frac{\partial(\tilde{\rho}\tilde{u}\tilde{E}_s + \tilde{u}\tilde{P})}{\partial \tilde{r}} + \frac{2(\tilde{\rho}\tilde{u}\tilde{E}_s + \tilde{u}\tilde{P})}{\tilde{r}} = \lambda(\frac{\partial^2 \tilde{T}}{\partial \tilde{r}^2} + \frac{2}{r}\frac{\partial \tilde{T}}{\partial \tilde{r}}), \tag{16}$$

Composition equation:

$$\frac{\partial(\tilde{\rho}\tilde{Y})}{\partial \tilde{t}} + \frac{\partial(\tilde{\rho}\tilde{u}\tilde{Y})}{\partial \tilde{r}} + \frac{2(\tilde{\rho}\tilde{u}\tilde{Y})}{\tilde{r}} = \tilde{\rho}D(\frac{\partial^2 \tilde{Y}}{\partial \tilde{r}^2} + \frac{2}{r}\frac{\partial \tilde{Y}}{\partial \tilde{r}}) - \tilde{\omega}, \tag{17}$$

Table 1. Different scale bubbles speed up the main flame

$R_b(\delta_t)$ ($\delta_t = 32.5$ cm)	Frequency(Hz)	Heat effect	Acceleration effect	No. of interaction	Propagating time(s)
3	14	0.93%	9.2%	28	2.0
5	10	0.19%	1.9%	132	13.3
9	6.7	0.03%	0.31%	790	118

Note. — R_b is the radius of the bubble, normalized by the strengthened flame thickness; For the larger bubble, the generating frequency is lower, the heat effect to the main flame is weaker bucause of the degradation of energy, so is the acceleration effect to the main flame; Thus, it needs more interaction times and propagating time for the main flame speeds up to supersonic.

Reaction rate equation:

$$\tilde{\omega} = A\tilde{\rho}\tilde{Y}\exp(-Ea/\tilde{T}_9^{1/3}), \tag{18}$$

State equation:

$$\tilde{P} = \frac{\tilde{\rho}\tilde{E}}{3}, \tag{19}$$

where $\tilde{\rho}$ is the density, $\tilde{u}$ is the flow velocity, $\tilde{P}$ is pressure, $\tilde{E}_s$ is the total energy, $\tilde{T}$ is the temperature, λ is the heat diffuse coefficient, $\tilde{Y}$ is the composition, D is the mass diffuse coefficient, $\tilde{\omega}$ is the reaction rate of the Carbon. Then we make the equations non-dimensional: $r = \frac{\tilde{r}}{\delta_f}$, $t = \frac{\tilde{t}}{\delta_f/S_L}$, $u = \frac{\tilde{u}}{S_L}$, $\rho = \frac{\tilde{\rho}}{\tilde{\rho}_0}$, $P = \frac{\tilde{P}}{\tilde{P}_0}$, $T = \frac{\tilde{T}-\tilde{T}_0}{\tilde{T}_b-\tilde{T}_0}$, $E_s = \frac{\tilde{E}_s}{\tilde{E}_{s0}}$, where δ_f is the Carbon flame thickness, S_L is the Carbon flame speed, T_b is the adiabatic flame temperature, $\tilde{\rho}_0$, $\tilde{P}_0$, $\tilde{T}_0$, $\tilde{E}_{s0}$ is the initial density, pressure, temperature and total energy. Thus we obtain the non-dimensional equations: Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial r} + \frac{2(\rho u)}{r} = 0, \tag{20}$$

Momentum equation:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + A_1 P)}{\partial r} + \frac{2\rho u^2}{r} = 0, \tag{21}$$

Energy equation:

$$\frac{\partial(\rho E_s)}{\partial t} + \frac{\partial(\rho u E_s + uP)}{\partial r} + \frac{2(\rho u E_s + uP)}{\tilde{r}} = A_2(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}), \tag{22}$$

Composition equation:

$$\frac{\partial(\rho Y)}{\partial t} + \frac{\partial(\rho u Y)}{\partial r} + \frac{2(\rho u Y)}{r} = \frac{\rho}{Le}(\frac{\partial^2 Y}{\partial r^2} + \frac{2}{r}\frac{\partial Y}{\partial r}) - \omega, \tag{23}$$

Reaction rate equation:

$$\omega = \rho Y \exp(-Ea/\tilde{T}_9^{1/3} + Ea/\tilde{T}_{b,9}^{1/3}), \tag{24}$$

State equation:

$$P = \rho E, \tag{25}$$

5. Conclusion and discussion

The paper considers the spherical inward propagating Carbon-Oxygen two step flame, and derives the formula of the Oxygen flame spontaneous propagating speed $u_{sp(O)}(r) = \frac{1}{1/u_C(r) - d\tau/dr}$. As the formula shown, the Oxygen flame spontaneous propagating speed is affected by the Carbon flame speed and the Oxygen ignition delay time. Then we talk about the acceleration effect of the spherical inward propagating Carbon-Oxygen two step flame in the case of laminar flame formed by Rayleigh-Taylor instability and strengthened flame formed by turbulence. In the laminar case, the basic speed is the laminar flame speed which is so small that the final speed is also subsonic, and there are no shock wave and detonation; In the turbulent case, for the C-O white dwarf that the density is 3.5×10^7 g/cm^3, Reynolds number is 5.8×10^{13}, integral length scale is 100 km, and the turbulent velocity is 629m/s, the Oxygen flame accelerates to supersonic when it propagates to the radius that is 1.5 times of the flame thickness (48.8 cm), and generates the shock wave. The main flame accelerates under the effect of the shock wave, and speed up to supersonic finally.

A. Appendix:

REFERENCES

Aspden, A., Bell, J., & Woosley, S. 2010, The Astrophysical Journal, 710, 1654

Bell, J., Day, M., Rendleman, C., Woosley, S., & Zingale, M. 2004, The Astrophysical Journal, 608, 883

Chapman, D. L. 1899, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 47, 90

Chen, Z., Gou, X., & Ju, Y. 2010, Combustion Science and Technology, 182, 124

Fowler, W. A., Caughlan, G. R., & Zimmerman, B. A. 1975, Annual Review of Astronomy and Astrophysics, 13, 69

Hillebrandt, W., & Niemeyer, J. C. 2000, Annual Review of Astronomy and Astrophysics, 38, 191

Höflich, P., Khokhlov, A., & Wheeler, J. 1995, The Astrophysical Journal, 444, 831

Hoyle, F., & Fowler, W. A. 1960, The Astrophysical Journal, 132, 565

Jouguet, E. 1905, J. de mathematiques Pures et Appliquees, 1, 2

Law, C. K. 2010, Combustion physics (Cambridge university press)

Lee, J. H., & Oppenheim, A. 1969

Röpke, F. 2007, The Astrophysical Journal, 668, 1103

Schmidt, B. P., Suntzeff, N. B., Phillips, M. M., et al. 1998, The Astrophysical Journal, 507, 46

Taylor, G. 1950in , The Royal Society, 192–196

Williams, F. 1985, Combustion theory: the theory of chemically reacting flow systems Benjamin Cummings

Woosley, S., Kerstein, A., & Aspden, A. 2011, The Astrophysical Journal, 734, 37

Woosley, S., Kerstein, A., Sankaran, V., Aspden, A., & Röpke, F. 2009, The Astrophysical Journal, 704, 255

Zeldovich, Y. B. 1980, Combustion and Flame, 39, 211

Zel'Dovich, Y. B. 1985, The Mathematical Theory of Combustion and Explosions

Zel'Dovich, Y. B., Librovich, V., Makhviladze, G., & Sivashinskil, G. 1970, Journal of Applied Mechanics and Technical Physics, 11, 264

Zimont, V. 1979, Combustion, explosion, and shock waves, 15, 305

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