S&P Managed Risk 2.0 Indices

介紹與實作

國立中山大學財管所菁英業師課程作業報告

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官網文件

- [S&P Managed Risk 2.0 Index Series Methodology]
- [Understanding the S&P Managed Risk 2.0 Indices]

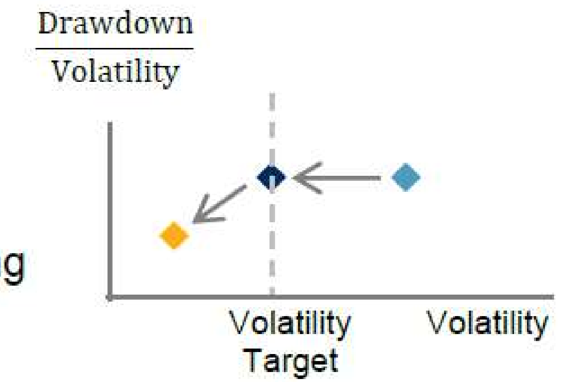
資產類別

Index	Underlying Indices Equity	Fixed Income	Cash Equivalent
S&P 500 Managed Risk 2.0 Index	S&P 500	S&P U.S. Treasury Bond Current 5-Year Index	S&P U.S Treasury Bill 0-3 Month Index
S&P 400 Managed Risk 2.0 Index	S&P MidCap 400		
S&P 600 Managed Risk 2.0 Index	S&P SmallCap 600		
S&P EM 100 Managed Risk 2.0 Index	S&P EM 100		
S&P Emerging Plus LargeMidCap Managed Risk 2.0 Index	S&P Emerging Plus LargeMidCap		
S&P EPAC Ex. Korea LargeMidCap Managed Risk 2.0 Index	S&P EPAC Ex. Korea LargeMidCap		

MR2.0指數模型架構

APPLY MANAGED RISK OVERLAY

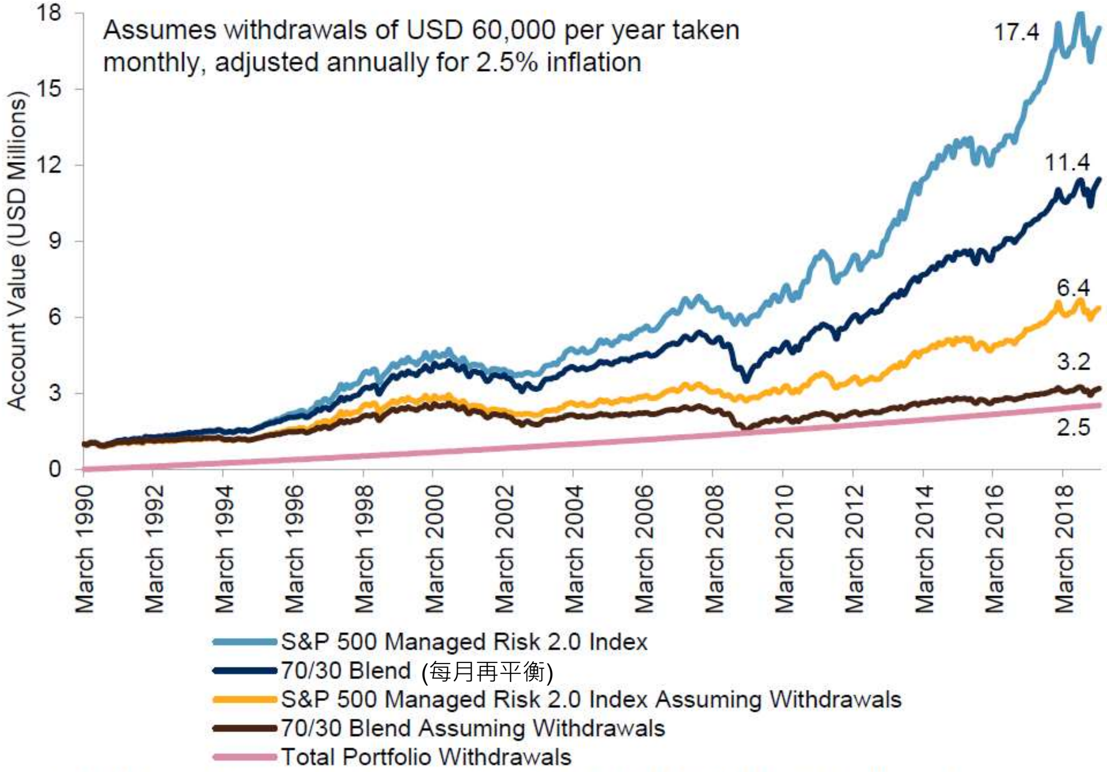
- ✓ **Volatility management:** seeks to stabilize portfolio volatility around a target level
- ✓ **Capital protection strategy:** seeks to defend against losses during sustained market declines.
- ✓ Risk management calculations are performed daily. (每日調整)



股+債 Target Volatility 資產配置模型	+	投資組合保險 Self-financing put option and hedge allocation	+	殖利率反轉機制 (債部位轉現金)	=	**S&P Managed Risk 2.0 Indices**

MR2.0指數績效表現

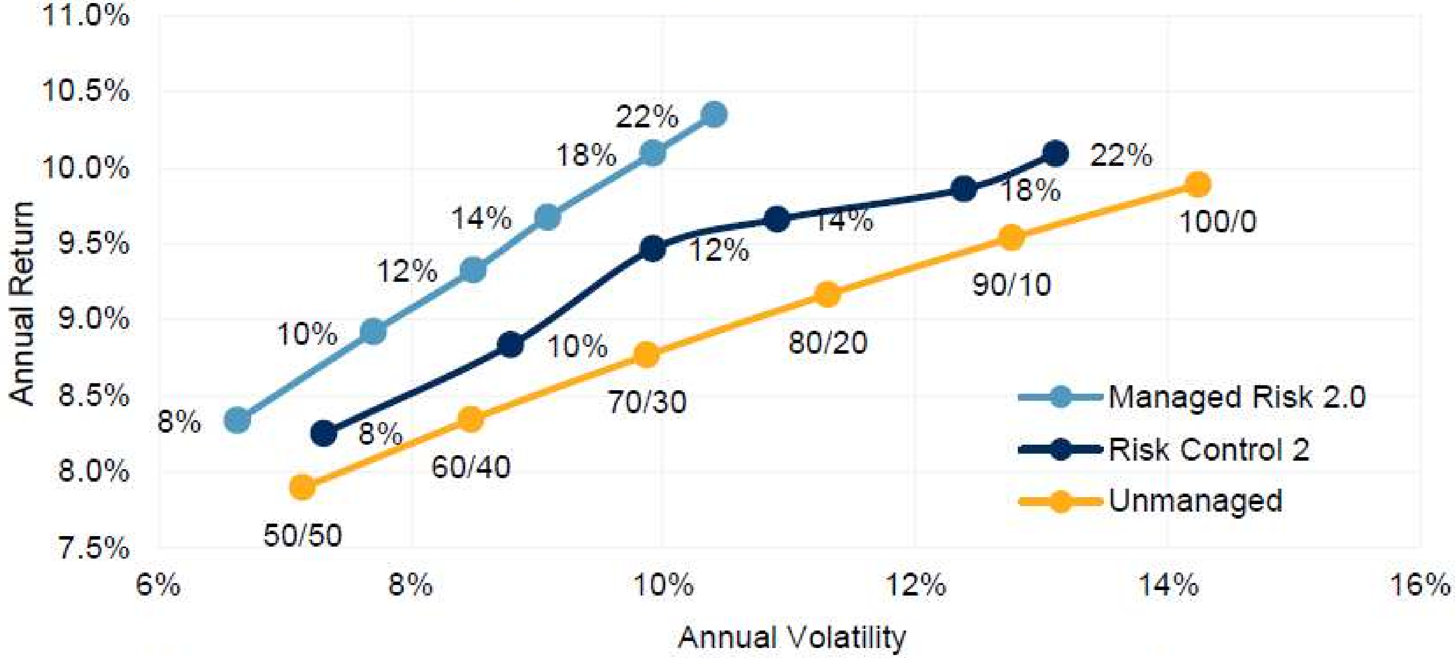
Exhibit 1: Accumulation and Decumulation Portfolio Value



Source: S&P Dow Jones Indices LLC. Data from March 31, 1990, to March 29, 2019. Index performance based on daily total return in USD. Past performance is no guarantee of future results. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

MR2.0指數績效表現

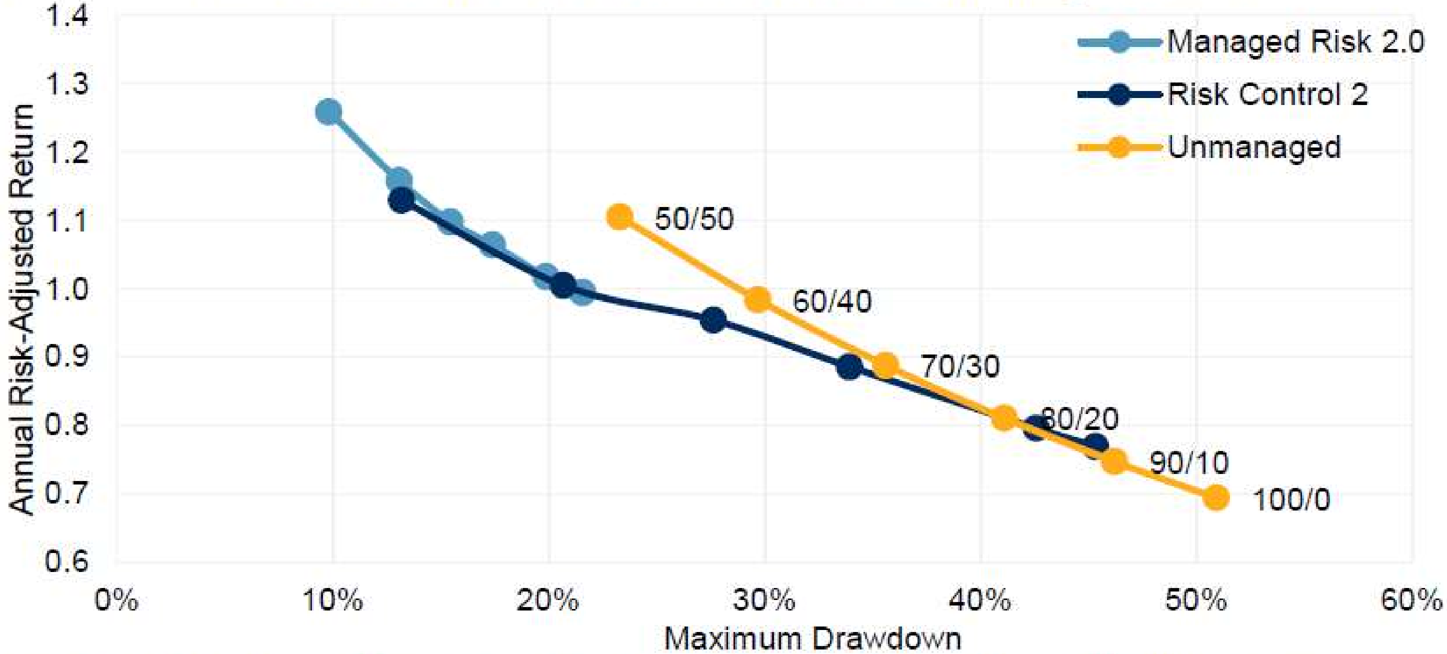
Exhibit 3: Historical Efficient Frontier – Managed Risk 2.0 Indices and Risk Control 2.0 Indices versus Unmanaged Portfolios



Source: S&P Dow Jones Indices LLC. Data from March 31, 1990, to March 29, 2019. Index performance based on daily total return in USD. Past performance is no guarantee of future results. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

MR2.0指數績效表現

Exhibit 4: Historical Sharpe Ratio and Maximum Drawdown – Managed Risk 2.0 Indices and Risk Control 2.0 Indices versus Unmanaged Portfolios



Source: S&P Dow Jones Indices LLC. Data from March 31, 1990, to March 29, 2019. Index performance based on daily total return in USD. Past performance is no guarantee of future results. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

MR2.0指數績效表現

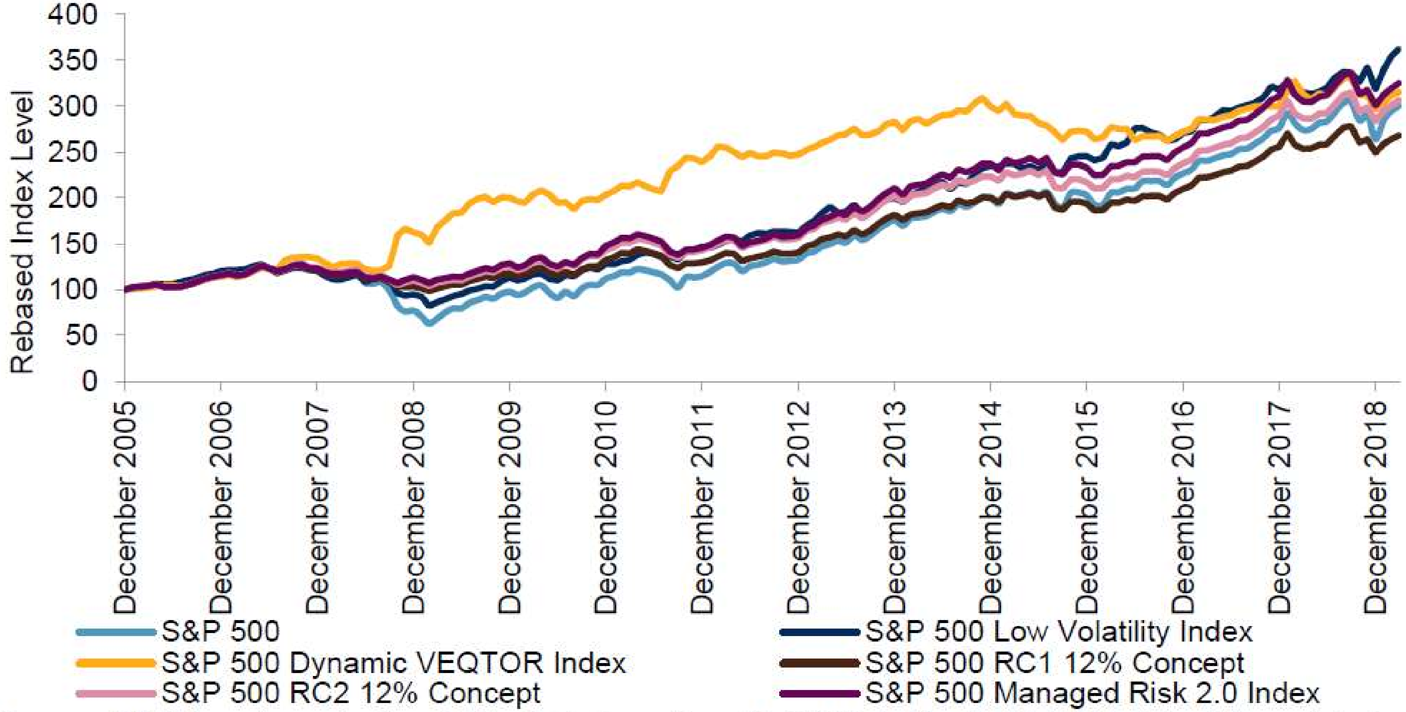
Exhibit 15: Statistical Summary

METRIC	25 YEARS		20 YEARS		15 YEARS	
	S&P 500 MANAGED RISK 2.0 INDEX	S&P 500	S&P 500 MANAGED RISK 2.0 INDEX	S&P 500	S&P 500 MANAGED RISK 2.0 INDEX	S&P 500
MOMENTS (ANNUALIZED)						
Return (%)	10.43	9.80	7.40	6.04	9.13	8.57
Volatility (%)	10.15	14.47	9.38	14.52	9.35	13.61
Skewness	-0.15	-0.20	-0.11	-0.16	-0.16	-0.23
Excess Kurtosis	-0.19	-0.15	-0.26	-0.16	-0.24	-0.07
RATIOS						
Sharpe Ratio	0.79	0.51	0.60	0.29	0.84	0.54
Sortino Ratio	1.69	1.01	1.26	0.60	1.56	0.92
MAR Ratio	0.48	0.19	0.34	0.12	0.57	0.17
PERFORMANCE RELATIVE TO S&P 500						
Monthly Alpha (%)	0.33	-	0.31	-	0.31	-
T-Stats of Alpha	4.11	-	3.46	-	3.04	-
Beta to the S&P 500	0.62	-	0.56	-	0.60	-

Source: S&P Dow Jones Indices LLC. Data from Dec. 31, 1992, to March 29, 2019. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

MR2.0指數績效表現

Exhibit 16: Historical Cumulative Return



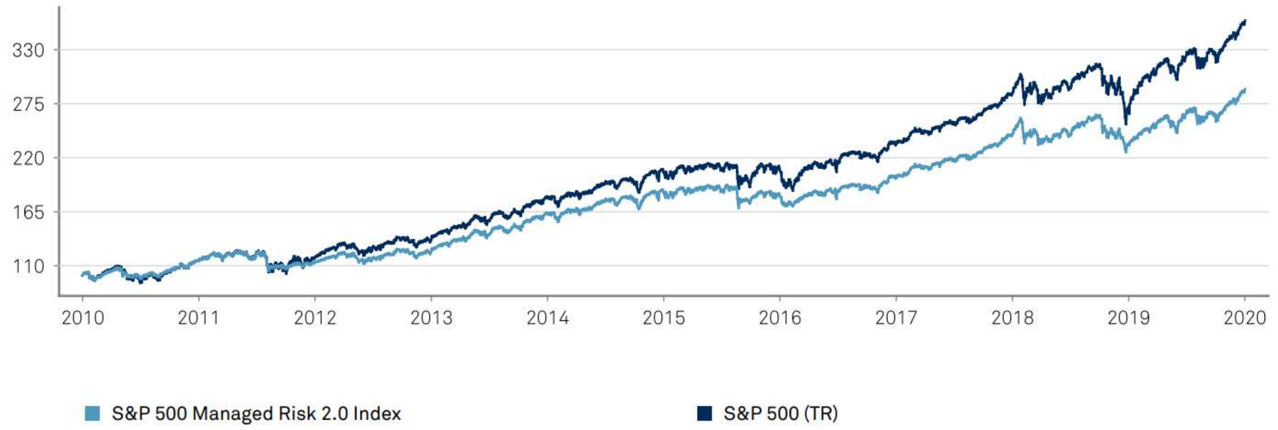
Source: S&P Dow Jones Indices LLC. Data from Dec. 30, 2005, to March 29, 2019. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

MR2.0指數 FACT SHEET(2019/12/31)

資料來源：https://us.spindices.com/indices/strategy/sp-500-managed-risk-20-index

Historical Performance

* Data has been re-based at 100

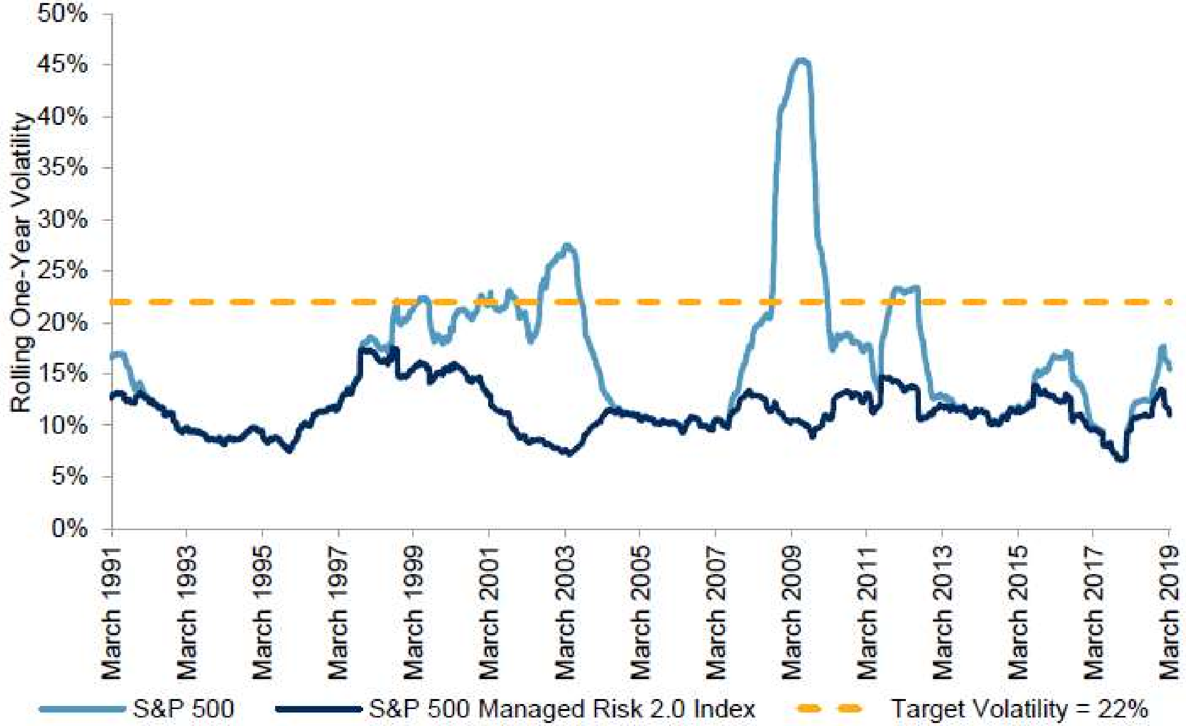


Quick Facts

WEIGHTING METHOD	Risk weighted
REBALANCING FREQUENCY	Daily
CALCULATION FREQUENCY	Real time
CALCULATION CURRENCIES	USD
LAUNCH DATE	January 23, 2017
FIRST VALUE DATE	March 28, 1990

MR2.0指數波動度走勢

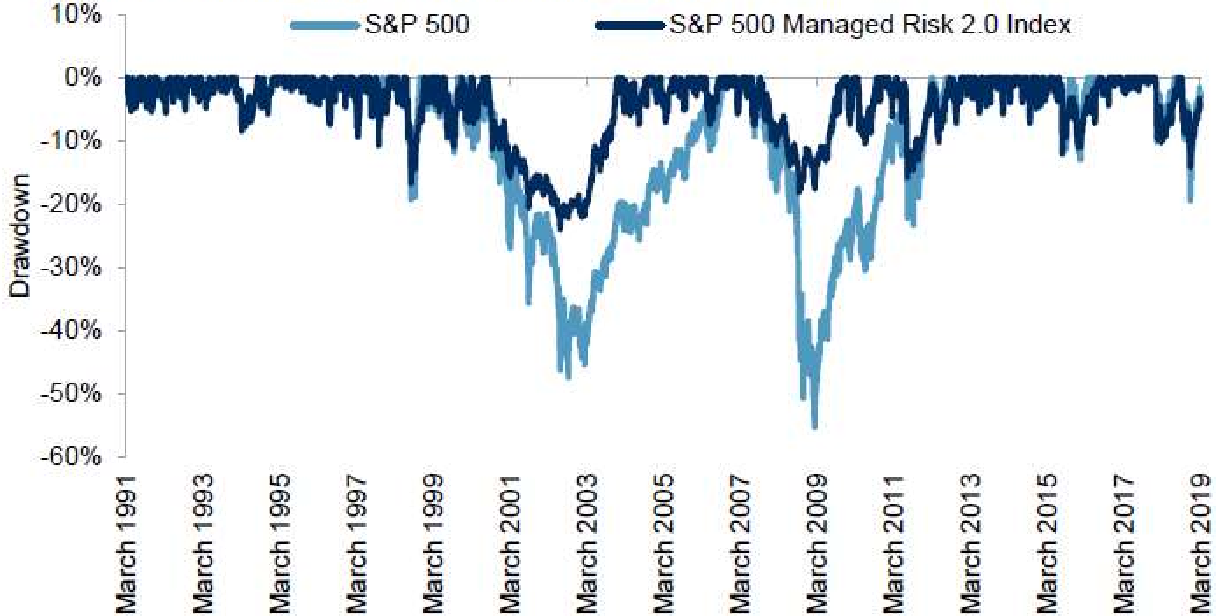
Exhibit 11: Low and Stable Volatility



Source: S&P Dow Jones Indices LLC. Data as of March 29, 2019. Volatility is calculated as standard deviation of daily total return over the past 250 trading days, which is then annualized. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

MR2.0指數最大回撤走勢

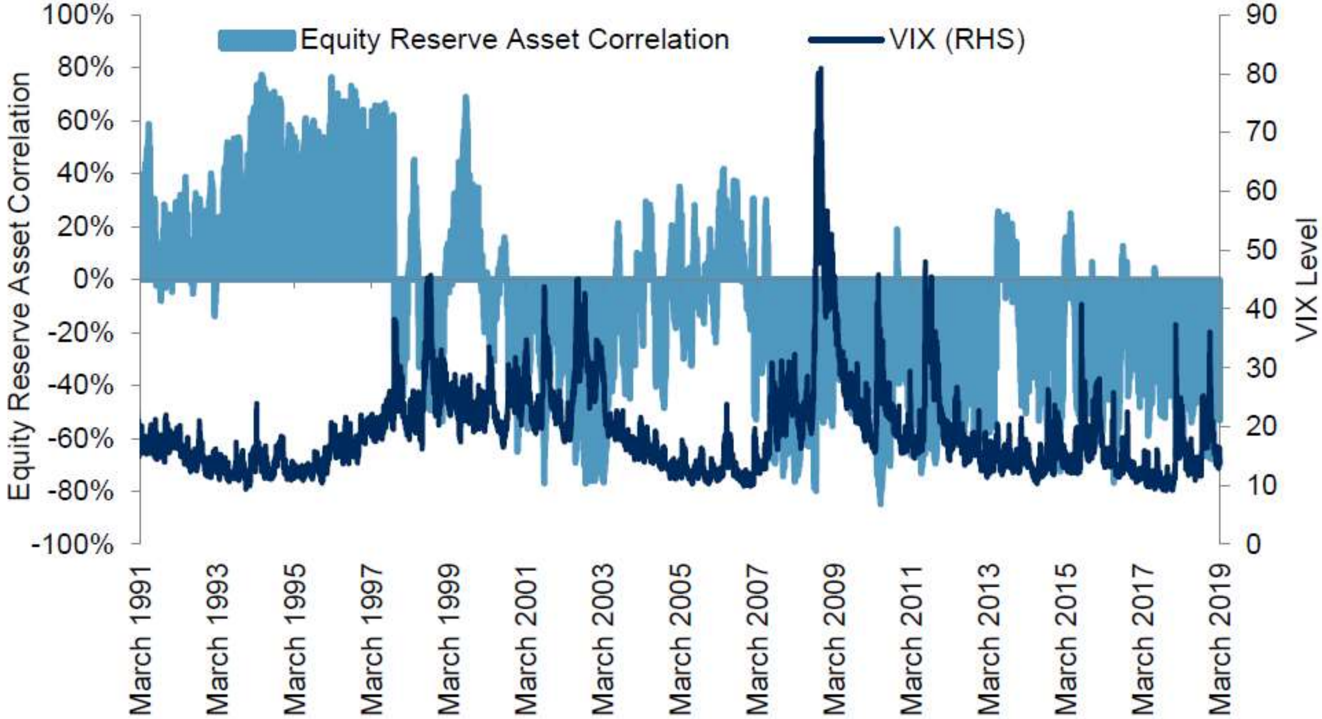
Exhibit 12: Shorter and Shallower Drawdowns



Source: S&P Dow Jones Indices LLC. Data as of March 29, 2019. Drawdown is calculated as cumulative return since the most recent high water mark. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

股票與債券相關係數走勢

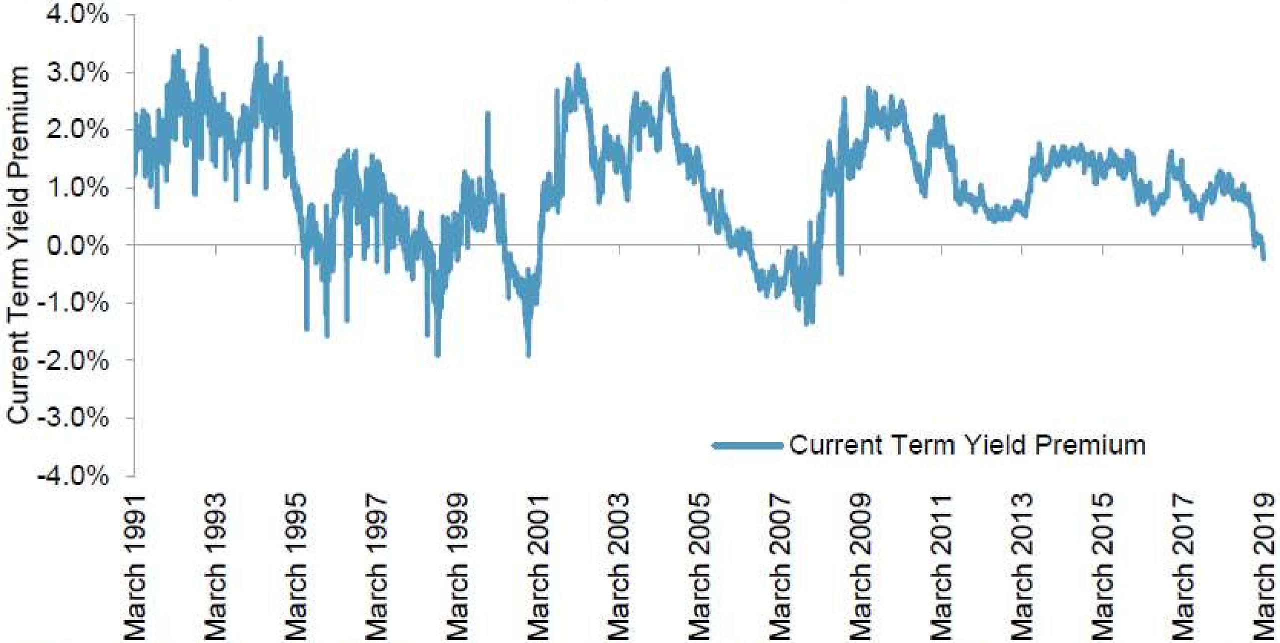
Exhibit 6: Correlation between Reserve Asset and Equities Tends to Be Negative during Periods of Market Stress



Source: S&P Dow Jones Indices LLC. Data as of March 29, 2019. Equity reserve asset correlation is defined as the average of the short-term and long-term exponentially weighted correlations with decay factors equal to 0.94 and 0.97. Past performance is no guarantee of future results. Chart is provided for illustrative purposes.

殖利率利差走勢

Exhibit 7: Term Yield Premium Tends to Be Positive Over Time

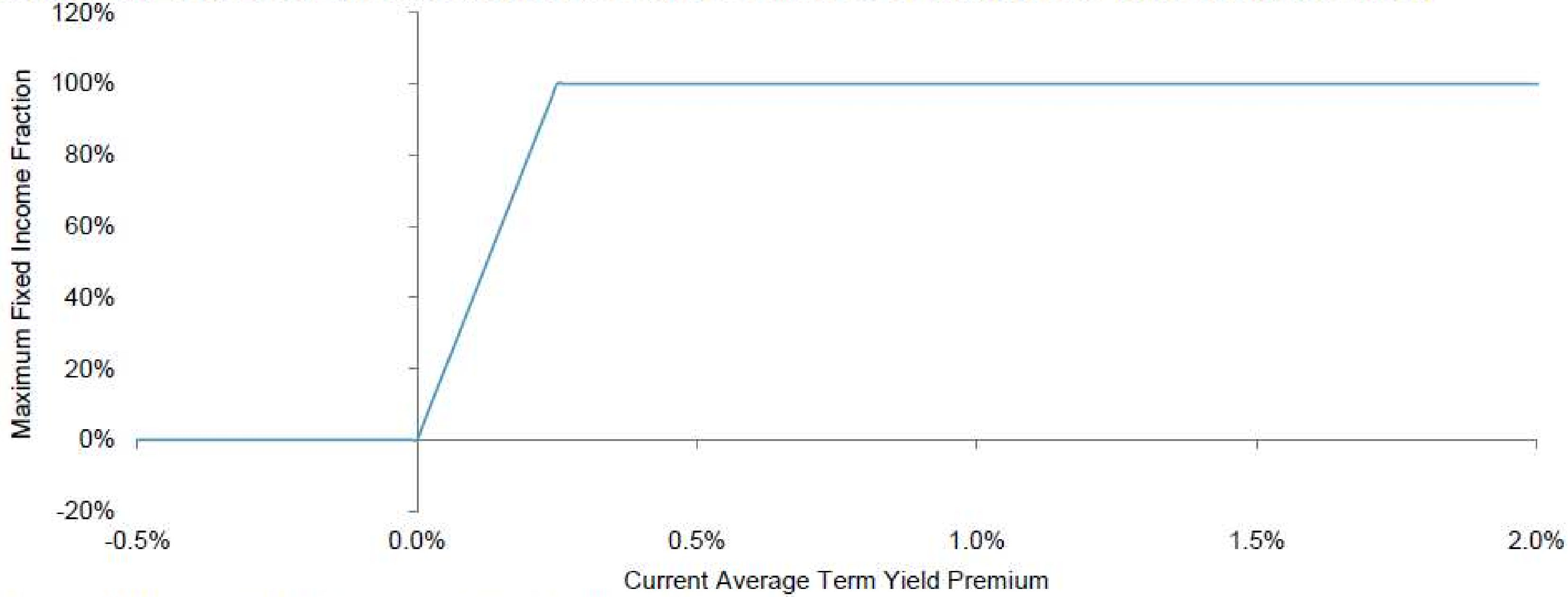


Source: S&P Dow Jones Indices LLC. Data as of March 29, 2019. The current term yield premium is calculated as the spread between the yield-to-maturity of the reserve asset and the cash money market rate. Past performance is no guarantee of future results. Chart is provided for illustrative purposes.

- Term Yield Premium = Treasury Bond 5 Years - Treasury Bill 0-3 Months
- 當殖利率利差反轉時，會將配置在債券的權重轉往現金部位

當殖利率反轉時，會配置權重於現金部位

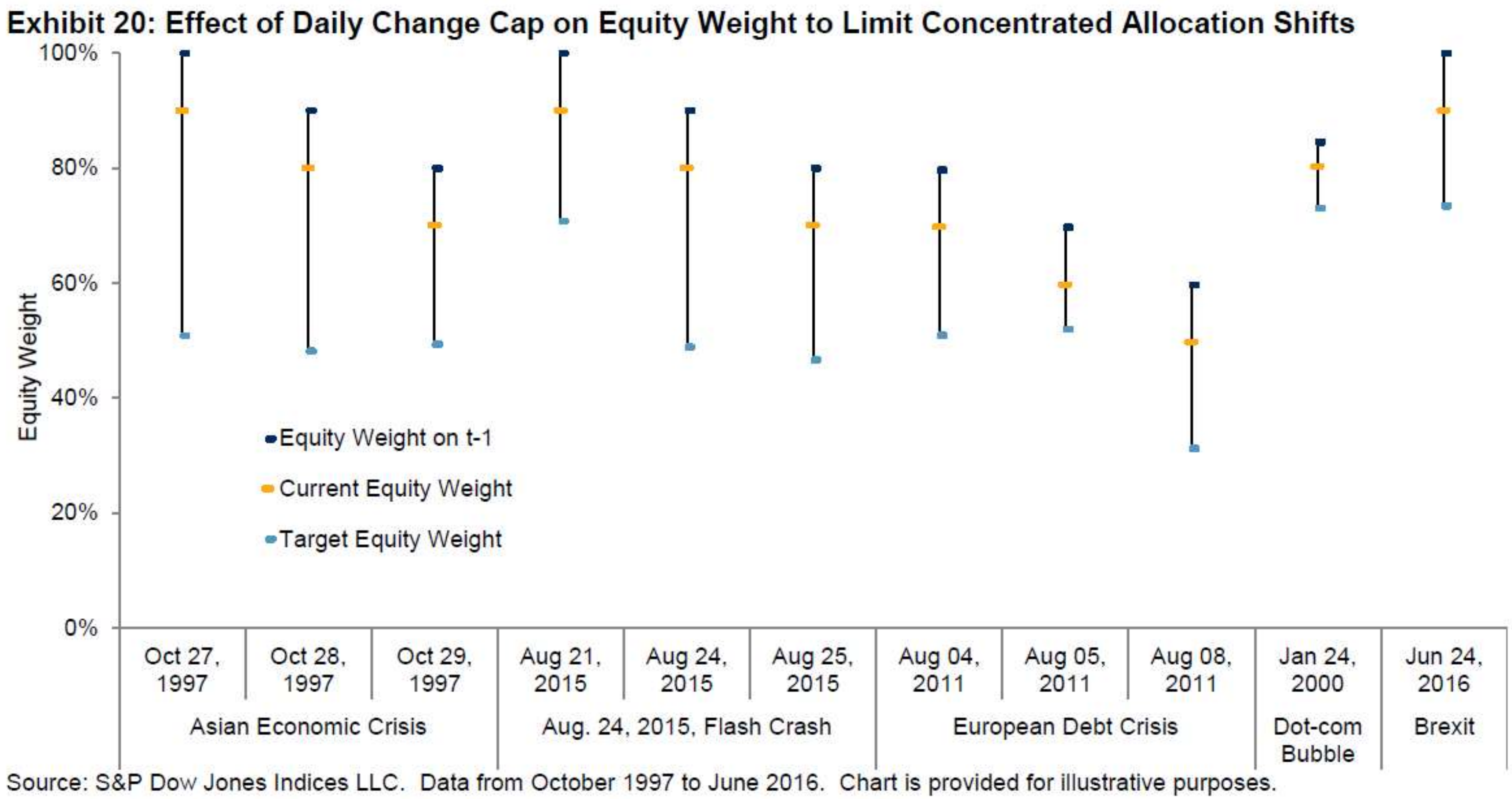
Exhibit 19: Substitute Allocation from Reserve Asset to Cash When Term Yield Premium Is Low



Source: S&P Dow Jones Indices LLC. Chart is provided for illustrative purposes.

權重限制

- 權重每日最多只能變動10%



(極端事件發生日期)

MR2.0指數計算方式

Total Return Index Calculations

On any business day *t*, the total return index value is calculated as:

$$IndexTR_t = IndexTR_{t-1} * \left(\begin{array}{l} w_{E,t-3} * \left(\frac{E_t}{E_{t-1}} - 1 \right) + w_{B,t-3} * \left(\frac{B_t}{B_{t-1}} - 1 \right) \\ + (1 - w_{E,t-3} - w_{B,t-3}) * \left(\frac{C_t}{C_{t-1}} - 1 \right) \end{array} \right) \quad (1)$$

where:

$IndexTR_{t-1}$ = The total return index level on *t-1*.

- 問題1: 為何要用2天前的權重?
- 問題2: 此處公式應該有錯，累積報酬率應為(1+R)的概念

PUT的履約價(K)計算方式

$$K_t = \begin{cases} K_{t-1} + \dfrac{\Delta t}{\tau^+}(k * A_t - K_{t-1}) \ldots if \ldots K_{t-1} \le k * A_t \\ K_{t-1} + \dfrac{\Delta t}{\tau^-}(k * A_t - K_{t-1}) \ldots if \ldots k * A_t < K_{t-1} \le A_t \\ A_t \ldots otherwise \end{cases} \quad (2)$$

直覺：
以$0.8 * A_t$值作為履約價均值
低於履約價均值時反應較快
高於履約價均值時反應較慢

(大空頭，指數直接跌破前一個交易日履約價時之情況)

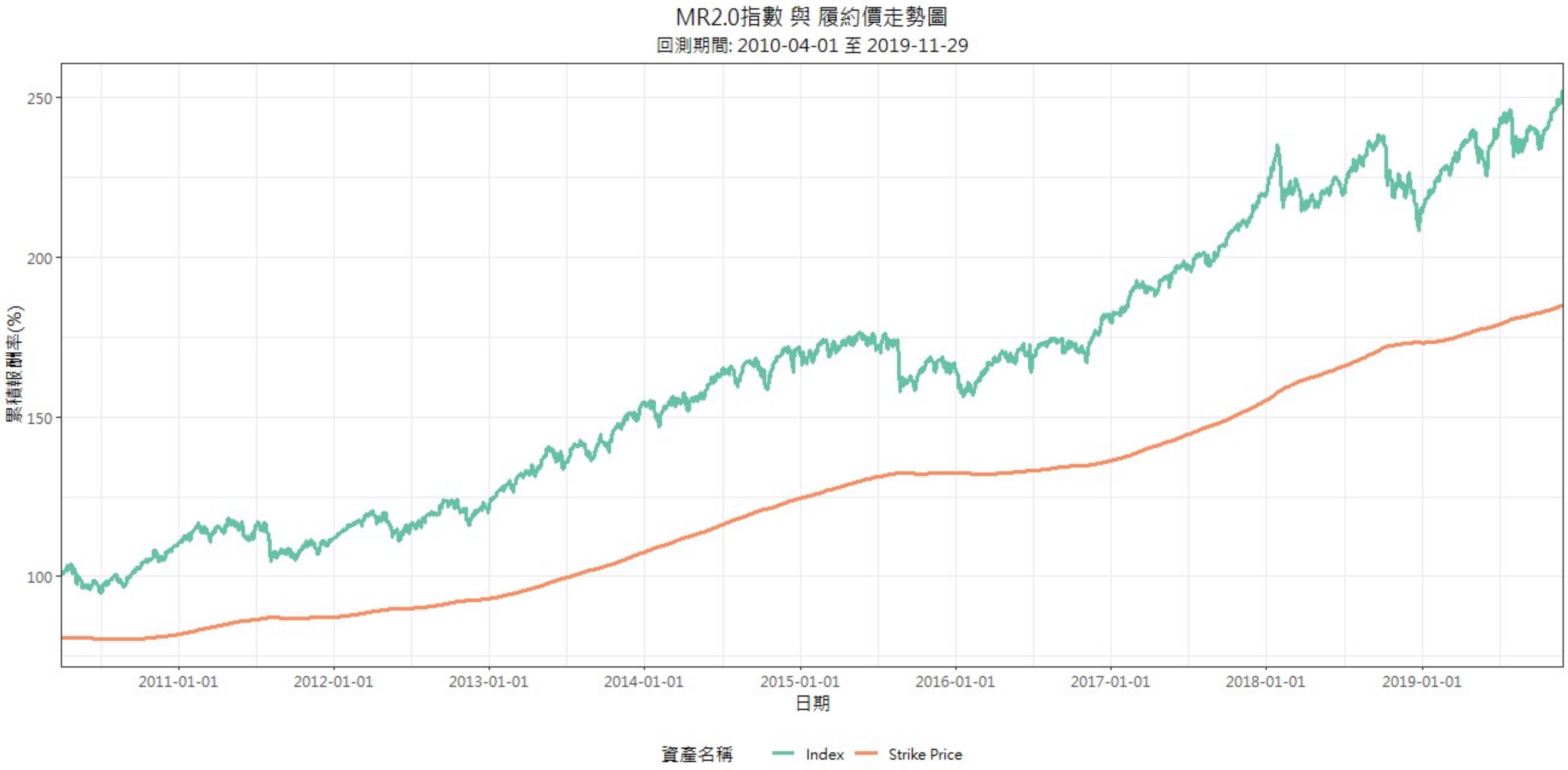
參數值

- $\mathrm{k} = 0.8$
- $\Delta\mathrm{t} = 1/252$
- $\tau^+ = 0.75$
- $\tau^- = 2$

- 問題: A是股票指數還是MR2.0指數?

- 目前採用MR2.0指數

A與K關係(實際回測結果)



計算避險部位比率

$$P_0(K) = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

To avoid borrowing funds, a portion of the index is sold to finance the put option position. This requires solving the equation below:

$$P_t = V(\underbrace{A_t - P_t}_{S_0}, K_t) \qquad (3)$$

where:

P_t = Put option premium

V = Black Scholes put option price calculated as:

$$P_t = V(\underline{A_t - P_t}, K_t) = K_t * N\left(\frac{1}{\sigma\sqrt{M}}\ln\frac{K_t}{\underline{A_t - P_t}} + \frac{1}{2}\sigma\sqrt{M}\right) - (\underline{A_t - P_t}) * N\left(\frac{1}{\sigma\sqrt{M}}\ln\frac{K_t}{A_t - P_t} - \frac{1}{2}\sigma\sqrt{M}\right) \qquad (4)$$

where N is the standard cumulative normal density function.

The net hedge allocation is calculated as:

$$H_t = -\boxed{\frac{K_t}{A_t} * N\left(\frac{1}{\sigma\sqrt{M}}\ln\frac{K_t}{A_t - P_t} + \frac{1}{2}\sigma\sqrt{M}\right)} \quad \text{表示避險部位要佔總資產多少比率} \qquad (5)$$

- 假設無風險利率為0，距到期日(M)為5年，波動度(σ)為0.22

求解Target Volatility 配置權重

$$tw_{E,t} = (1+H_t)^* \boxed{vmw_{E,t}} \tag{6a}$$

$$tw_{B,t} = (1+H_t)^* \boxed{vmw_{B,t}} - H_t * \frac{M}{\boxed{D}} \tag{6b}$$

債券的duration 約為4.75

Portfolio Constraints

Volatility Constraints. Both the short- and long-term variance of the portfolio should not exceed the current target variance.

$$tw_{E,t}{}^2 * EquityVariance_{S,t} + tw_{B,t}{}^2 * FIVariance_{S,t} + 2 * tw_{E,t} * tw_{B,t} * Covariance_{S,t} \le \sigma_t^2 \tag{7a}$$

$$tw_{E,t}{}^2 * EquityVariance_{L,t} + tw_{B,t}{}^2 * FIVariance_{L,t} + 2 * tw_{E,t} * tw_{B,t} * Covariance_{L,t} \le \sigma_t^2 \tag{7b}$$

Position Constraints. Leveraged and short positions are avoided.

$$tw_{E,t} + tw_{B,t} \le 1 \tag{7c}$$

$$tw_{E,t} \ge 0 \tag{7d}$$

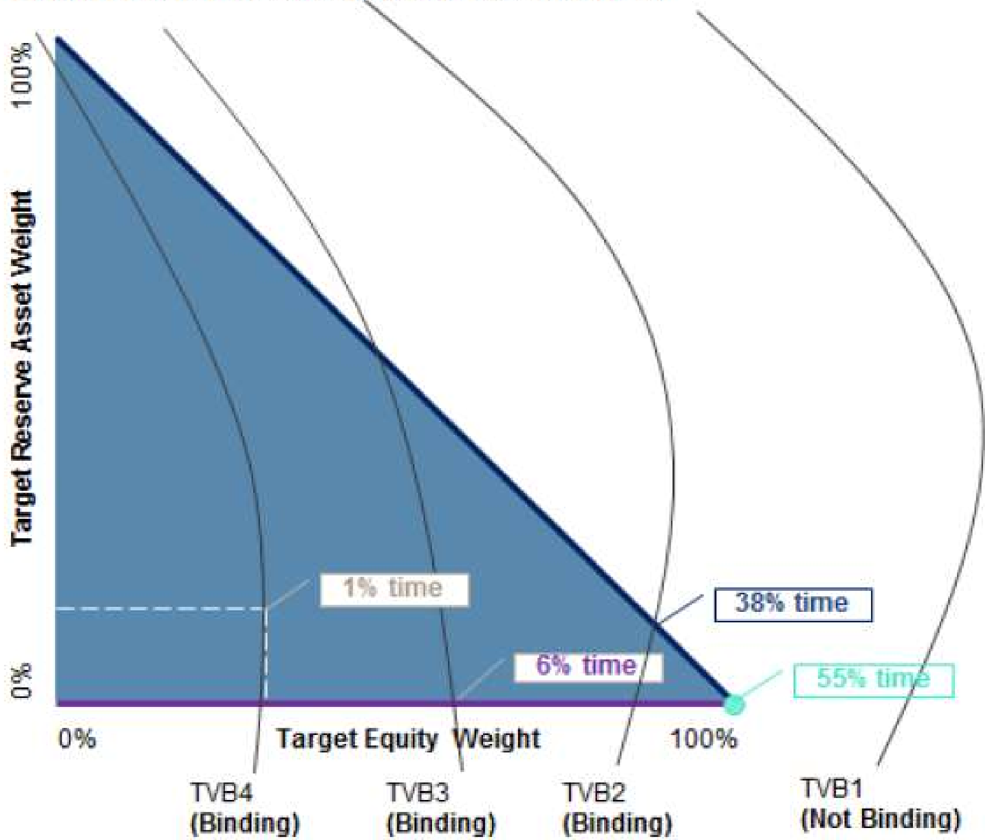
$$tw_{B,t} \ge 0 \tag{7e}$$

求解目標：

- 在符合限制式條件下，極大化$vmw_{E,t}$
- 若有相同的$vmw_{E,t}$之組合，則優先挑選最大的$vmw_{B,t}$

求解Target Volatility 配置權重

Exhibit 9: When Volatility Constraints Are Binding (45% of the Time), a Negative Correlation between Equities and the Reserve Asset Generally Led to Higher Equity Allocation Levels (39% of the Time)



Source: S&P Dow Jones Indices LLC. Data from March 28, 1990, to March 29, 2019. Chart is provided for illustrative purposes.

在符合限制式條件下，極大化$vmw_{E,t}$
若有相同的$vmw_{E,t}$之組合，則優先挑選最大的$vmw_{B,t}$

長短期變異數與共變異數計算方式

- 短期衰退因子(λ_S= 0.94)
- 長期衰退因子(λ_L= 0.97)

The short-term equity variance measure at time *t*:

問題：報酬率假設為0？

$$EquityVariance_{S,t} = \begin{cases} \lambda_S * EquityVariance_{S,t-1} + (1-\lambda_S) * \left[\ln\left(\frac{E_t}{E_{t-\boxed{n}}}\right)\right]^2 \ldots if \ldots t > T_0 \\ \sum_{i=m+1}^{T_0} \frac{\alpha_{S,i,m}}{WF_S} * \left[\ln\left(\frac{E_i}{E_{i-n}}\right)\right]^2 \ldots if \ldots t = T_0 \end{cases} \quad \text{(A.1a)}$$

n=1

(初始值)

The long-term equity variance measure at time *t*:

$$EquityVariance_{L,t} = \begin{cases} \lambda_L * EquityVariance_{L,t-1} + (1-\lambda_L) * \left[\ln\left(\frac{E_t}{E_{t-n}}\right)\right]^2 \ldots if \ldots t > T_0 \\ \sum_{i=m+1}^{T_0} \frac{\alpha_{L,i,m}}{WF_L} * \left[\ln\left(\frac{E_i}{E_{i-n}}\right)\right]^2 \ldots if \ldots t = T_0 \end{cases} \quad \text{(A.1b)}$$

債券變異數計算方式與股票相同

長短期變異數與共變異數計算方式

- 初始值計算方式

$\alpha_s =$	$(1-\lambda_S)\lambda_S{}^{59}$		$(1-\lambda_S)\lambda_S{}^{3}$	$(1-\lambda_S)\lambda_S{}^{2}$	$(1-\lambda_S)\lambda_S$	$(1-\lambda_S)$
	T-60	…	T-4	T-3	T-2	T-1

初始值計算期間為60個交易日

$$WF_s = (1-\lambda_S)\lambda_S{}^{59} + \cdots + (1-\lambda_S)\lambda_S{}^{3} + (1-\lambda_S)\lambda_S{}^{2} + (1-\lambda_S)\lambda_S + (1-\lambda_S)$$

$$EquityVariance_{S,t} = \begin{cases} \lambda_S * EquityVariance_{S,t-1} + (1-\lambda_S) * \left[\ln\left(\dfrac{E_t}{E_{t-n}}\right)\right]^2 \ldots if \ldots t > T_0 \\ \displaystyle\sum_{i=m+1}^{T_0} \boxed{\dfrac{\alpha_{S,i,m}}{WF_S}} * \left[\ln\left(\dfrac{E_i}{E_{i-n}}\right)\right]^2 \ldots if \ldots t = T_0 \end{cases}$$

標準化使權重和為1

長短期變異數與共變異數計算方式

The short-term covariance measure at time *t*:

$$Covariance_{S,t}=\begin{cases}\lambda_S * Covariance_{S,t-1}+(1-\lambda_S)*\left[\ln\left(\frac{E_t}{E_{t-n}}\right)\right]\left[\ln\left(\frac{B_t}{B_{t-n}}\right)\right] \ldots if \ldots t>T_0 \\ \sum_{i=m+1}^{T_0}\frac{\alpha_{S,i,m}}{WF_S}*\left[\ln\left(\frac{E_t}{E_{t-n}}\right)\right]\left[\ln\left(\frac{B_i}{B_{i-n}}\right)\right] \ldots if \ldots t=T_0\end{cases} \qquad \text{(A.3a)}$$

初始值計算期間為60個交易日

The long-term covariance measure at time *t*:

$$Covariance_{L,t}=\begin{cases}\lambda_L * Covariance_{L,t-1}+(1-\lambda_L)*\left[\ln\left(\frac{E_t}{E_{t-n}}\right)\right]\left[\ln\left(\frac{B_t}{B_{t-n}}\right)\right] \ldots if \ldots t>T_0 \\ \sum_{i=m+1}^{T_0}\frac{\alpha_{L,i,m}}{WF_L}*\left[\ln\left(\frac{E_t}{E_{t-n}}\right)\right]\left[\ln\left(\frac{B_i}{B_{i-n}}\right)\right] \ldots if \ldots t=T_0\end{cases} \qquad \text{(A.3b)}$$

初始值計算期間為60個交易日

放寬槓桿限制

原本的限制式：

$$tw_{E,t} + tw_{B,t} \le 1$$

Zero volatility managed weights imply a long fixed income weight of:

$$tw_{B,t} = -H_t * \frac{M}{D}$$

This may exceed one if M > D and H ≈ -1. Leveraged and short positions are avoided, but in this case a small amount of leverage is permitted in the volatility managed fixed income weight in order to include zero volatility in the solution domain. This requires a small relaxation of the constraint (7c) by replacing it with:

放寬後：

$$\theta * tw_{E,t} + tw_{B,t} \le \theta \qquad \text{(C.1)}$$

where:

$$\theta = \max\left(-H\frac{M}{D}, 1\right) \qquad \text{(C.2)}$$

Note that this is merely a technical consideration since the final fixed income weight $w_{B,t}$ is capped and thus will not result in a leveraged portfolio holding.

計算vmw_E的Boundary

$$tw_{E,t} = (1 + H_t) * vmw_{E,t} \tag{6a}$$

$$tw_{B,t} = (1 + H_t) * vmw_{B,t} - H_t * \frac{M}{D} \tag{6b}$$

將上述(6a)與(6b)式子代入：

Maximum Leverage Boundary. The first boundary considered is the maximum leverage boundary corresponding to constraint (B.1):

$$\theta * tw_{E,t} + tw_{B,t} = \theta$$

得vmw_B與vmw_E關係式(D.3)

$$vmw_{B,t} = \frac{\theta + H\frac{M}{D}}{1 + H} - \theta * vmw_{E,t} \tag{D.3}$$

計算vmw_E的Boundary

$$\boxed{vmw_{B,t}} = \frac{\theta + H\frac{M}{D}}{1+H} - \theta * vmw_{E,t} \qquad \text{(D.3)}$$

將(D.3)代入(6b)

$$\boxed{tw_{E,t}} = (1+H_t) * vmw_{E,t} \qquad \text{(6a)}$$

$$\boxed{tw_{B,t}} = (1+H_t) * \boxed{vmw_{B,t}} - H_t * \frac{M}{D} \qquad \text{(6b)}$$

將(6a)及(6b)代入(7a)及(7b)

二元一次方程式，採用公式解
此處的 x 即為 vmw_E

Volatility Constraints. Both the short- and long-term variance of the portfolio should not exceed the current target variance.

$$\boxed{tw_{E,t}{}^2} * EquityVariance_{S,t} + \boxed{tw_{B,t}{}^2} * FIVariance_{S,t}$$
$$+ 2 * \boxed{tw_{E,t}} * \boxed{tw_{B,t}} * Covariance_{S,t} \le \boxed{\sigma_t^2} \qquad \text{(7a)}$$

target volatility

$$tw_{E,t}{}^2 * EquityVariance_{L,t} + tw_{B,t}{}^2 * FIVariance_{L,t}$$
$$+ 2 * tw_{E,t} * tw_{B,t} * Covariance_{L,t} \le \sigma_t^2 \qquad \text{(7b)}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

計算vmw_E的Boundary

邊界條件1

$$vmw_{E,t}=\frac{(\theta * FIVariance_{S,t}-Covariance_{S,t})*\frac{\theta+H\frac{M}{D}}{1+H}}{EquityVariance_{S,t}-2\theta * Covariance_{S,t}+\theta^2 * FIVariance_{S,t}}$$

依短期變異數及短期共變異數計算出的vmw_E邊界

$$\pm\frac{\sqrt{(EquityVariance_{S,t}-2\theta * Covariance_{S,t}+\theta^2 * FIVariance_{S,t})*\sigma_t^2-(EquityVariance_{S,t} * FIVariance_{S,t}-Covariance_{S,t}{}^2)\left(\frac{\theta+H\frac{M}{D}}{1+H}\right)^2}}{EquityVariance_{S,t}-2\theta * Covariance_{S,t}+\theta^2 * FIVariance_{S,t}} \quad (D.1)$$

邊界條件2

$$vmw_{E,t}=\frac{(\theta * FIVariance_{L,t}-Covariance_{L,t})*\frac{\theta+H\frac{M}{D}}{1+H}}{EquityVariance_{L,t}-2\theta * Covariance_{L,t}+\theta^2 * FIVariance_{L,t}}$$

依長期變異數及長期共變異數計算出的vmw_E邊界

$$\pm\frac{\sqrt{(EquityVariance_{L,t}-2\theta * Covariance_{L,t}+\theta^2 * FIVariance_{L,t})*\sigma_t^2-(EquityVariance_{L,t} * FIVariance_{L,t}-Covariance_{L,t}{}^2)\left(\frac{\theta+H\frac{M}{D}}{1+H}\right)^2}}{EquityVariance_{L,t}-2\theta * Covariance_{L,t}+\theta^2 * FIVariance_{L,t}} \quad (D.2)$$

問題：此處自己推導後，發現與說明手冊公式不相同

計算vmw_E的Boundary

計算角解，令$tw_{E,t} = 1$

$$\theta \, tw_{E,t} = (1 + H_t) * vmw_{E,t} \qquad (6a)$$

Maximum Equity Corner. The final boundary point is the corner at the intersection of constraints (7c) and (7e):

邊界條件3 $$vmw_{E,t} \; \cancel{vmw_{B,t}} = \frac{\cancel{1} \; \theta}{1 + H} \qquad (D.18)$$

計算vmw_E的Boundary

計算角解，令$tw_{E,t} \geq 0$

$$tw_{E,t} \geq 0 \tag{7d}$$

邊界條件4

$$\cancel{tw_{E,t}} = (1 + H_t) * vmw_{E,t} \tag{6a}$$

0

$\Rightarrow \quad vmw_E \geq 0$

計算vmw_E的Boundary

- vmw_E須符合下列邊界條件，從可行域中挑選最大的vmw_E

邊界條件1

$$vmw_{E,t} = \frac{(\theta * FIVariance_{S,t} - Covariance_{S,t}) * \frac{\theta + H\frac{M}{D}}{1+H}}{EquityVariance_{S,t} - 2\theta * Covariance_{S,t} + \theta^2 * FIVariance_{S,t}}$$

$$\pm \frac{\sqrt{(EquityVariance_{S,t} - 2\theta * Covariance_{S,t} + \theta^2 * FIVariance_{S,t}) * \sigma_t^2 - (EquityVariance_{S,t} * FIVariance_{S,t} - Covariance_{S,t}^2)\left(\frac{\theta + H\frac{M}{D}}{1+H}\right)^2}}{EquityVariance_{S,t} - 2\theta * Covariance_{S,t} + \theta^2 * FIVariance_{S,t}} \quad (D.1)$$

(此處手冊提供公式有誤，需自行推導)

邊界條件2

$$vmw_{E,t} = \frac{(\theta * FIVariance_{L,t} - Covariance_{L,t}) * \frac{\theta + H\frac{M}{D}}{1+H}}{EquityVariance_{L,t} - 2\theta * Covariance_{L,t} + \theta^2 * FIVariance_{L,t}}$$

$$\pm \frac{\sqrt{(EquityVariance_{L,t} - 2\theta * Covariance_{L,t} + \theta^2 * FIVariance_{L,t}) * \sigma_t^2 - (EquityVariance_{L,t} * FIVariance_{L,t} - Covariance_{L,t}^2)\left(\frac{\theta + H\frac{M}{D}}{1+H}\right)^2}}{EquityVariance_{L,t} - 2\theta * Covariance_{L,t} + \theta^2 * FIVariance_{L,t}} \quad (D.2)$$

(此處手冊提供公式有誤，需自行推導)

邊界條件3 $vmw_{E,t}$ $\cancel{vmw_{B,t}} = \frac{\cancel{1}\ \theta}{1+H}$

邊界條件4 $vmw_E \geq 0$

計算vmw_B

- vmw_E須符合下列邊界條件，從可行域中挑選最大的vmw_E
- 依vmw_E與vmw_B關係式，計算出vmw_B

$$vmw_{B,t} = \frac{\theta + H\frac{M}{D}}{1+H} - \theta * vmw_{E,t} \qquad \text{(D.3)}$$

計算$tw_{E,t}$及$tw_{B,t}$

- 將最佳的vmw_E與vmw_B代入(6a)及(6b)，即可得$tw_{E,t}$及$tw_{B,t}$

$$tw_{E,t} = (1+H_t)^* \, vmw_{E,t} \tag{6a}$$

$$tw_{B,t} = (1+H_t)^* \, vmw_{B,t} - H_t{}^* \frac{M}{D} \tag{6b}$$

最後權重調整(Final Asset Weights)

- 處理殖利率利差為負之狀況

The average term yield premium is calculated as an exponentially weighted moving average of the spread between the yield to maturity and the cash money market rate.

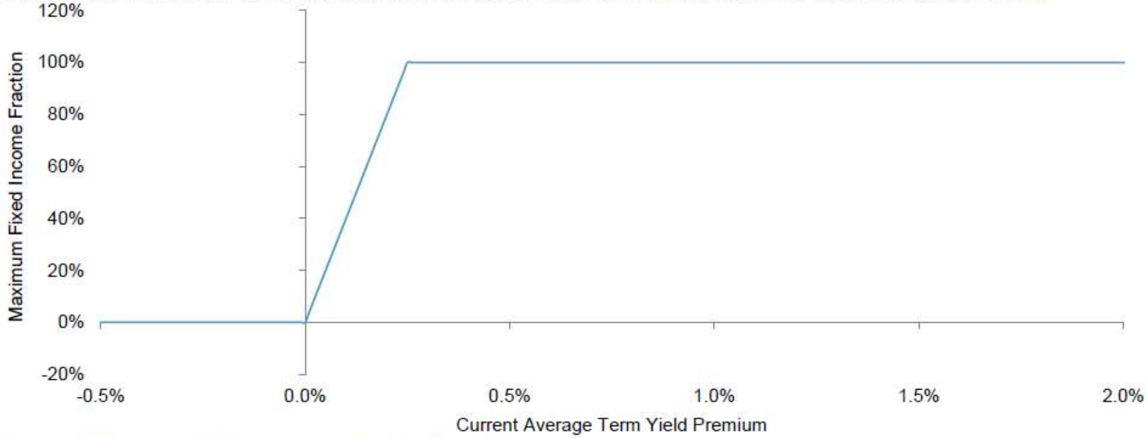
$$s_t = \left(1 - \frac{\Delta t}{\tau}\right) s_{t-1} + \frac{\Delta t}{\tau} \underbrace{(r_B - r)}_{\text{殖利率利差}} \tag{8}$$

The fraction of the non-equity component of the index invested in the fixed income index is capped by the maximum fixed income fraction, determined from the prior day average term yield premium as:

$$\omega_t = \min\left(1, \max\left(0, \frac{s_{t-1} - \underline{s}}{\bar{s} - \underline{s}}\right)\right)$$

$\bar{s} = 0.25\%$
$\underline{s} = 0\%$

Exhibit 19: Substitute Allocation from Reserve Asset to Cash When Term Yield Premium Is Low



Source: S&P Dow Jones Indices LLC. Chart is provided for illustrative purposes.

最後權重調整(Final Asset Weights)

- 權重調整限制

Daily Change Cap. The size of the daily adjustment in the final asset weights towards the target asset weights is limited by the asset allocation change fraction, calculated as:

$$\varphi_t = \min\left(1, \frac{\delta}{\left|tw_{E,t} - w_{E,t-1}\right|}, \frac{\delta}{\left|\min(tw_{B,t}, \omega_t * (1 - tw_{E,t})) - w_{B,t-1}\right|}\right) \tag{10}$$

- 最後權重調整

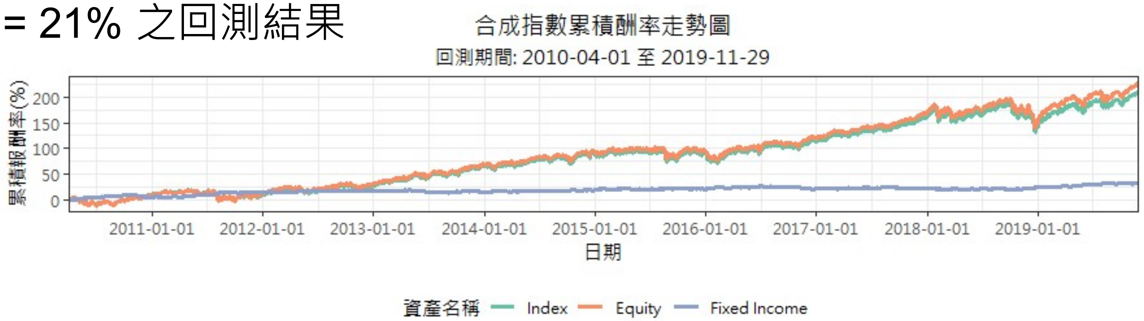
Final Asset Weights. Apply the maximum fixed income fraction and the asset allocation change fraction, as defined in (9) and (10), to the target asset weights to determine the daily final asset weights as:

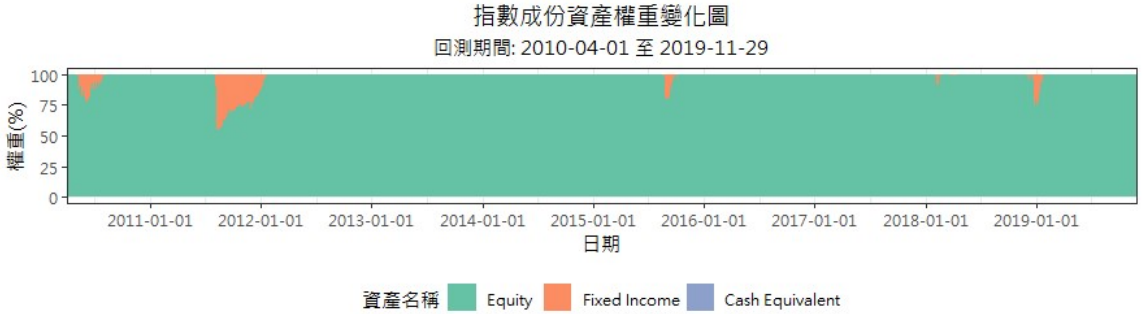
$$w_{E,t} = \varphi_t * tw_{E,t} + (1 - \varphi_t) * w_{E,t-1} \tag{11a}$$

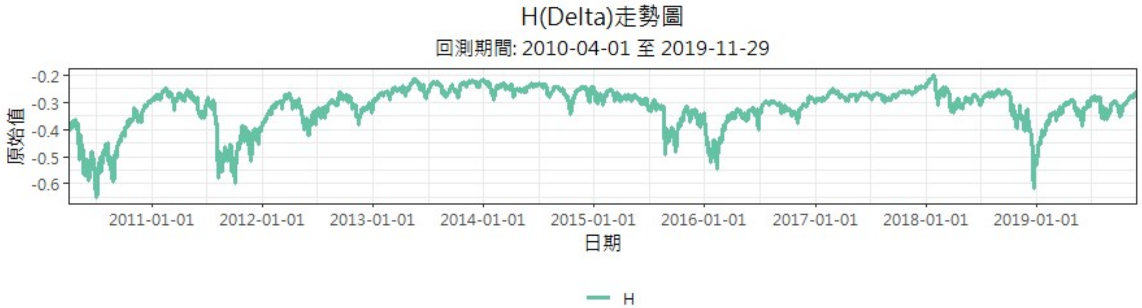
$$w_{B,t} = \varphi_t * \min(tw_{B,t}, \omega_t * (1 - tw_{E,t})) + (1 - \varphi_t) * w_{B,t-1} \tag{11b}$$

實作回測結果(未考慮最後權重調整)

- Target Volatility = 21% 之回測結果



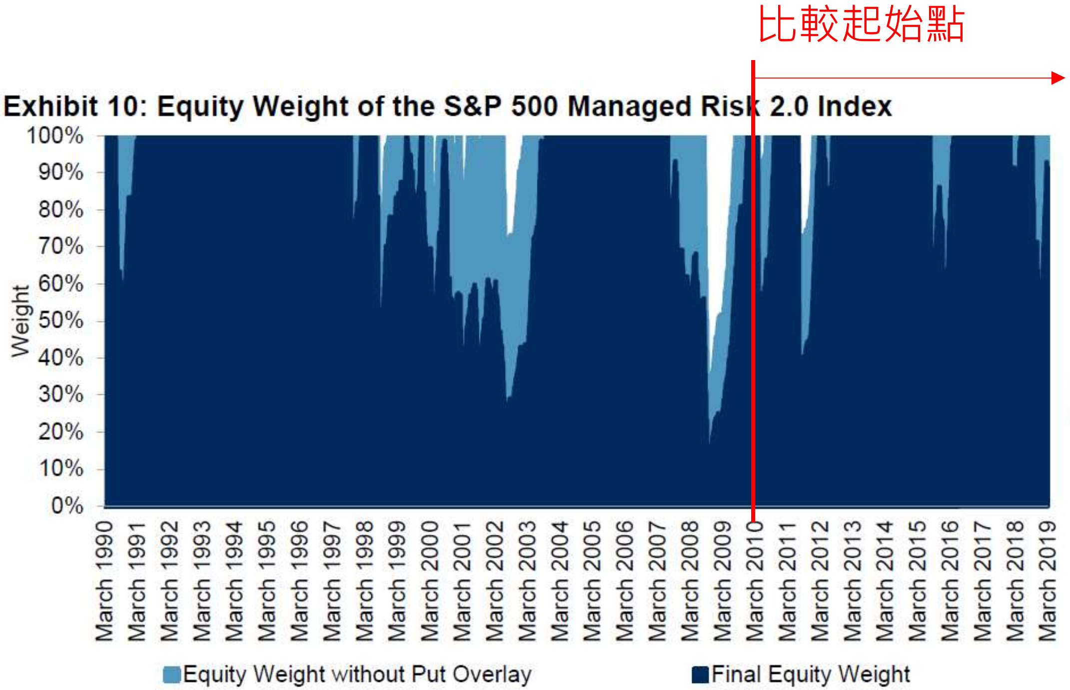




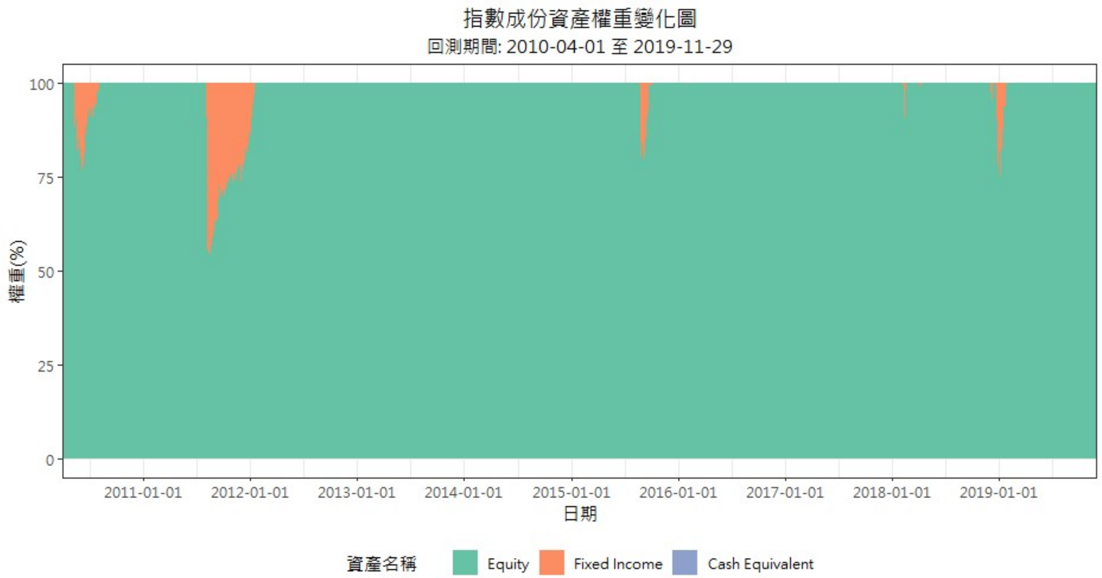
實作回測績效(未考慮最後權重調整)

年份	指數名稱	累積報酬率	年化報酬率	報酬率標準差	夏普比率	最大回撤率
2010	Equity	8.37%	11.13%	19.04%	0.65	-15.63%
2010	Fixed Income	5.07% 勝	6.70%	4.92%	1.34 勝	-4.90% 勝
2010	Index	8.94%	11.90%	17.92%	0.72	-14.65%
2011	Equity	0.97%	0.97%	23.26%	0.16	-18.64%
2011	Fixed Income	9.31%	9.31%	4.39%	2.05	-2.13% 勝
2011	Index	-1.71%	-1.71%	18.27%	0.00	-17.95%
2012	Equity	14.23%	14.23%	12.61%	1.12	-9.58%
2012	Fixed Income	2.31%	2.31%	2.55%	0.91	-2.17%
2012	Index	14.10%	14.10%	12.60%	1.11	-9.58%
2013	Equity	29.08%	29.08%	10.80%	2.42	-5.58%
2013	Fixed Income	-2.15%	-2.15%	3.36%	-0.63	-4.86%
2013	Index	29.08%	29.08%	10.80%	2.42	-5.58%
2014	Equity	14.69%	14.69%	11.34%	1.26	-7.28%
2014	Fixed Income	3.21%	3.21%	3.15%	1.02	-1.41%
2014	Index	14.69%	14.69%	11.34%	1.26	-7.28%
2015	Equity	1.40%	1.40%	15.46%	0.17	-12.04%
2015	Fixed Income	1.22%	1.22%	3.80%	0.34	-2.33% 勝
2015	Index	0.99%	0.98%	15.03%	0.14	-11.99%
2016	Equity	13.67%	13.50%	12.93%	1.04	-9.08%
2016	Fixed Income	0.42%	0.42%	3.24%	0.14	-4.86%
2016	Index	13.67%	13.50%	12.93%	1.04	-9.08%
2017	Equity	21.83%	21.64%	6.64%	2.98	-2.58%
2017	Fixed Income	0.82%	0.81%	2.59%	0.32	-2.24%
2017	Index	21.83%	21.64%	6.64%	2.98	-2.58%
2018	Equity	-5.18%	-5.20%	17.04%	-0.23	-19.36%
2018	Fixed Income	1.60%	1.60%	2.46%	0.66	-2.23% 勝
2018	Index	-6.22%	-6.25%	16.83%	-0.30	-19.29%
2019	Equity	27.47%	30.32%	12.82%	2.13	-6.62%
2019	Fixed Income	5.78%	6.32%	3.42%	1.81	-2.07%
2019	Index	26.17%	28.87%	12.44%	2.10	-6.62%
完整期間	Equity	226.08%	12.96%	14.80%	0.90	-19.36%
完整期間	Fixed Income	31.13%	2.83%	3.44%	0.83 勝	-5.52% 勝
完整期間	Index	210.01%	12.37%	13.84%	0.91	-19.29%

權重變化比較

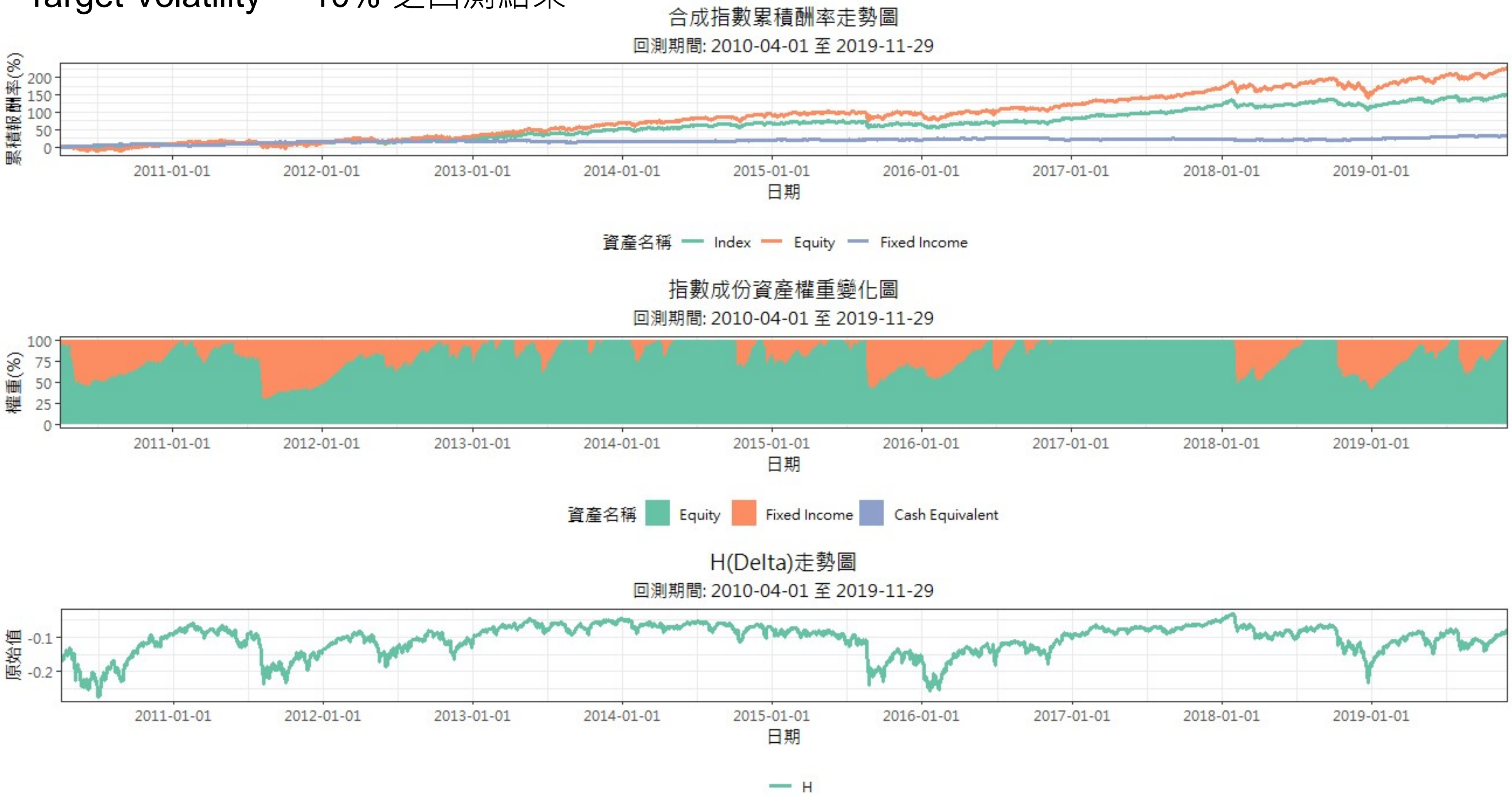


Source: S&P Dow Jones Indices LLC. Data as of March 29, 2019. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.



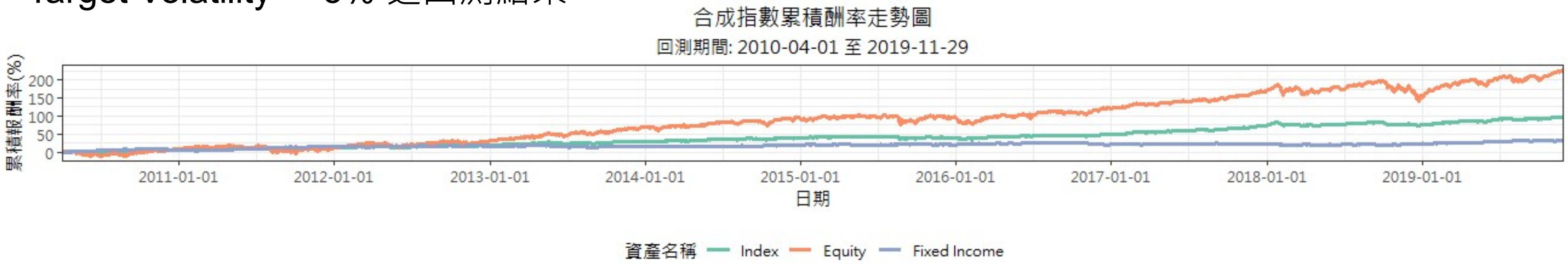
Target Volatility 為最重要的參數

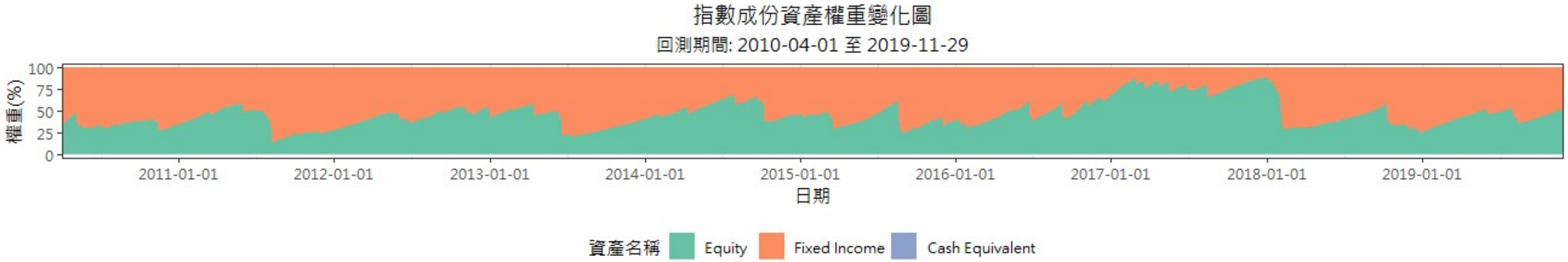
- Target Volatility = 10% 之回測結果

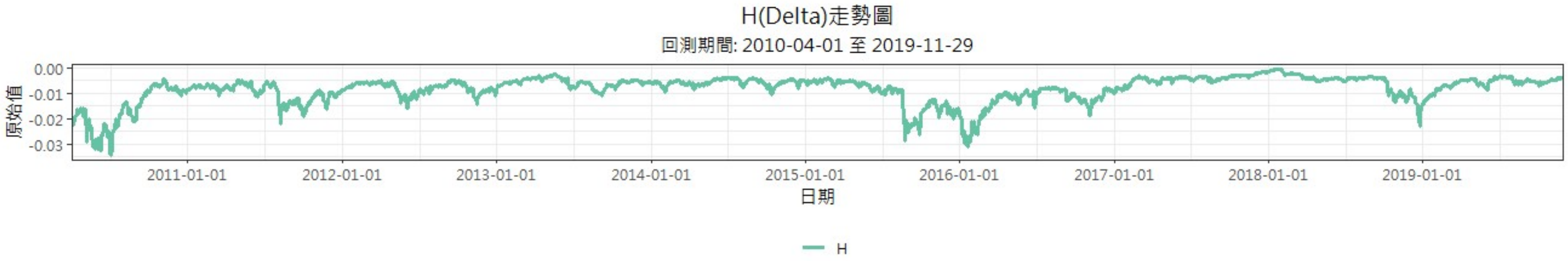


Target Volatility 為最重要的參數

- Target Volatility = 5% 之回測結果







Target Volatility 為最重要的參數

回測期間：2010/04/01-2019/11/29

- Target Volatility = 21% 之回測績效結果

指數名稱	累積報酬率	年化報酬率	報酬率標準差	夏普比率	最大回撤率
Equity	226.08%	12.96%	14.80%	0.90	-19.36%
Fixed Income	31.13%	2.83%	3.44%	0.83	-5.52%
Index	210.01%	12.37%	13.84%	0.91	-19.29%

- Target Volatility = 10% 之回測績效結果

指數名稱	累積報酬率	年化報酬率	報酬率標準差	夏普比率	最大回撤率
Equity	226.08%	12.96%	14.80%	0.90	-19.36%
Fixed Income	31.13%	2.83%	3.44%	0.83	-5.52%
Index	150.25%	9.92%	10.23%	0.98	-12.38%

- Target Volatility = 5% 之回測績效結果

指數名稱	累積報酬率	年化報酬率	報酬率標準差	夏普比率	最大回撤率
Equity	226.08%	12.96%	14.80%	0.90	-19.36%
Fixed Income	31.13%	2.83%	3.44%	0.83	-5.52%
Index	97.24%	7.25%	5.49%	1.30	-6.40%

結論

- Target Volatility為S&P Managed Risk 2.0 Indices的基本核心架構
- Self-financing put option and hedge allocation做為強化擇時的機制
- 模型考慮殖利率反轉狀況，避免由股票部位轉債券部位避險時依然遭受損失