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1. Problem description and Overall hypothesis

A bond graph model is proposed for the Prusa i3 3D type printer. The 3 axes of this type of 3D printer and the extrusion mechanism are driven by 4 stepper motors, respectively. The main target is to make the bond graph model of the movements on the X, Y, and Z axes.

As shown in Fig. 1, the X-axis direction motion load is mainly composed of the mass of the heated bed and the mass of the workpiece. Since the mass of the workpiece is limited to the size of printer and material, it is reasonable to ignore it. So the load in X-axis direction is mainly the mass of the heated bed.

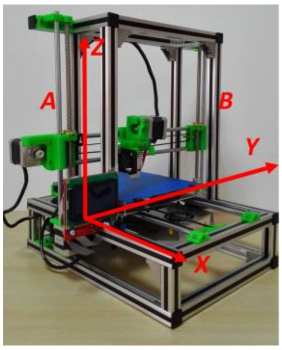

Figure 1

The X-axis power flow is as follows:

X-axis motor power → stepper motor → timing belt → X-axis load

The movement of the Y-axis and the Z-axis together constitute the movement of the extrusion mechanism in the plane of the gantry.

Due to the design of the mechanism, the Z-axis motion usually causes resistance due to the unsynchronization of the driving side (A side) and the supporting side (B side), which affects the normal operation of the printer. The modeling process assumes that the organization works smoothly and ignores this imbalance. At the same time, the change of the Z-axis load caused by the extrusion mechanism moving in the Y-axis direction is ignored. After simplification, it is considered that the Z-axis motion is decoupled from the Y-axis, and the load in the Z-axis direction is mainly the mass of the extrusion mechanism, the Y-direction motor and its bracket, and the gravity is not neglected.

The Z-axis power flow is as follows:

Z-axis motor power → stepper motor → lead screw → Z-axis load

The movement of the Y-axis is similar to the movement of the X-axis. The load is mainly the mass of the extrusion mechanism (including a stepper motor), ignoring the gravity received by the mechanism.

The Y-axis power flow is as follows:

Y-axis motor power → stepper motor → timing belt → Y-axis load

After simplification, the motions on X, Y, and Z axes are decoupled and can be modeled separately.

2. Modeling of X、Y、Z Direction Motion

The simplified X-axis model is shown in Figure 2 below. The ideal model of the timing belt can be seen in the following diagram 3. The stiffness can be divided into three sections, ignoring the damping.

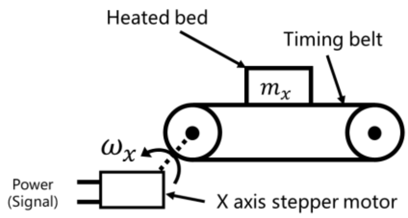

Figure 2

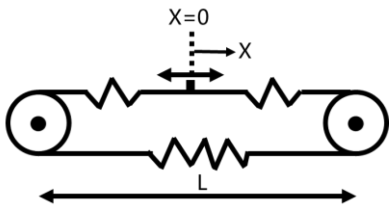

Figure 3

During the transmission, the stiffness of the timing belt causes a loss of part of the speed of the load, and the stiffness of the timing belt changes as the load moves. The stiffness of the timing belt after fully pre-tensioning is given by the following equation.

$$k = \frac{EA}{0.5l + x} + \frac{1}{\frac{1}{\frac{EA}{(0.5l - x)}} + \frac{1}{\frac{EA}{l}}} = EA\left(\frac{1}{0.5l + x} + \frac{1}{1.5l - x}\right) \quad \left(-\frac{1}{2}l < x < \frac{1}{2}l\right) \tag{1}$$

Taking the stiffness at X = 0 as a typical value, the model is further simplified to the form of Figure 4 below, where V(t) is the linear velocity of the tight edge of the timing belt.

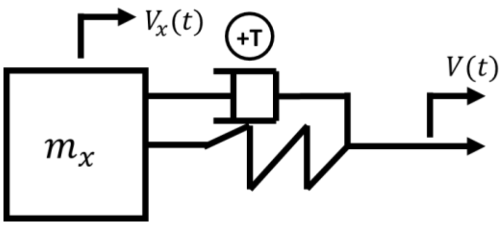

Figure 4

For a stepper motor, first use a relatively simple model (actually a DC motor model) as shown in Figure 5 below, and the core is a gyrator type transducer. The loss element R and the energy storage element I are included, and the moment of inertia J of the motor shaft is taken into account.

The X-axis bond diagram can be established by combining the above models. After simplification, as shown in Figure 6 where r is the radius of the timing pulley.

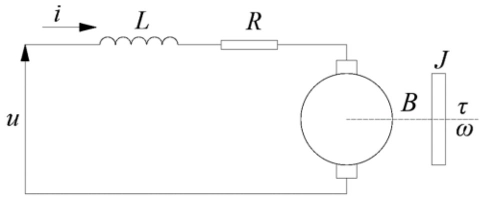

Figure 5

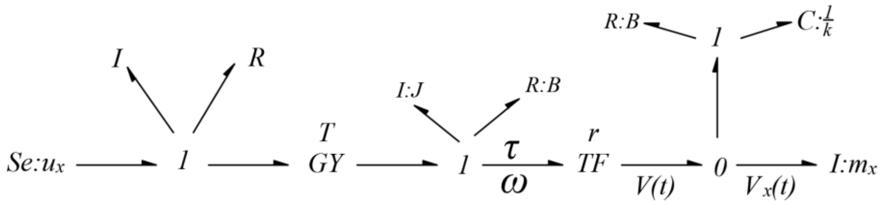

Figure 6 X axis bond graph

The simplified X-axis model is built. In fact, the movements of the Y-axis and the X-axis are identical after simplification, with only the difference in load. That is, its bonding graph is as shown in Figure 7.

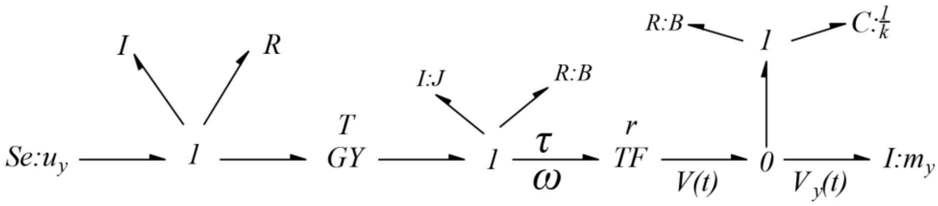

Figure 7 Y axis bond graph

The Z-axis is similar to the X-axis Y-axis but the Z-axis load continues to be affected by gravity. The schematic is as follows.

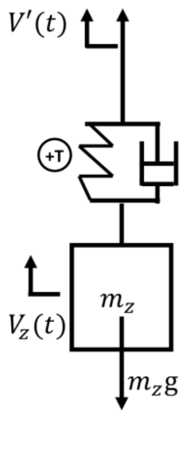

Figure 8

Add the simplified stepper motor model and use the transformer model as the ideal lead screw, where t is the lead screw ratio. The simplified bond diagram is shown in Figure 9.

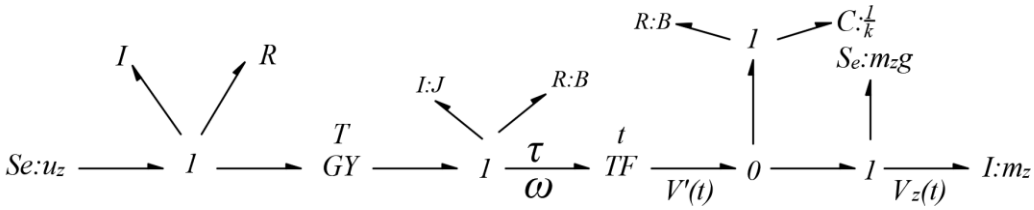

Figure 9

3. Stepper motor bond graph

The stepper motor is neither a DC motor nor an AC motor, it moves according to a given signal. The ideal effect can also be obtained by using the open-loop control method when using a stepping motor, so proper modeling of its characteristics helps to better control. It is difficult to model the stepping motor directly from the perspective of electromagnetic field. Therefore, the mapping of the bonding diagram is based on the differential equation of the stepping motor proposed by Hoang Le-Huy et al.

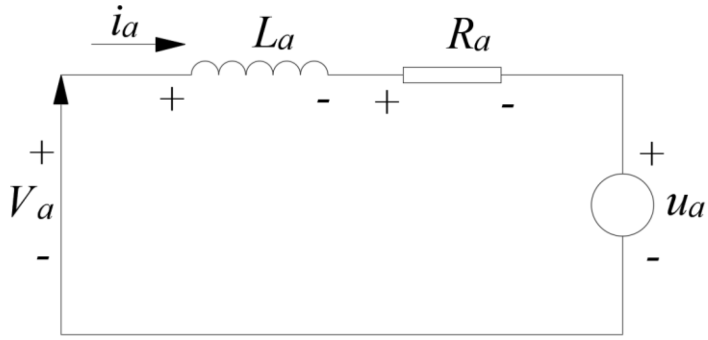

Figure 10

$$u_a(\theta) = -p\psi_m \sin(p\theta)\frac{d\theta}{dt} \tag{2}$$

$$u_b(\theta) = p\psi_m \sin(p\theta - \frac{\pi}{2})\frac{d\theta}{dt} \tag{3}$$

$$T_e = -p\psi_m[i_a \sin(p\theta) - i_b \sin(p\theta - \frac{\pi}{2})] - T_{dm}\sin(2p\theta) \tag{4}$$

$$T_e = J\frac{d\omega}{dt} + B\omega + T_L \tag{5}$$

The bonding graph model of the stepping motor can be established by combining the circuit of Fig. 10 with the equations (2) - (5).

Where V_a is the A-phase voltage; L_a is the A-phase self-inductance; θ is the mechanical relative rotation angle (the A-phase is positive to the A-axis and the initial position is the A-phase winding inductance is the largest); ω is the rotor mechanical angular velocity; m is the phase number ; J is the motor rotor inertia; T_L is the load torque; B is the total friction coefficient including the motor and the load; p is the magnetic pole number, and the calculation formula is as follows, $step$ is the step angle.

$$p = \frac{360}{2m * step} \tag{6}$$

T_{dm} is the maximum value of the Detent Torque, and its is usually not clearly specified. Generally, 1% to 10% of the holding torque is a typical value.

The maximum flux linkage (ψ_m) is also not always specified. This parameter can be obtained experimentally by driving the motor to a constant speed N and by measuring the maximum open-circuit winding voltage E_m. And the parameter ψ_m is then computed by the following relation:

$$\psi_m = \frac{30}{\pi}\left(\frac{E_m}{N}\right) \tag{7}$$

The electromagnetic part of the stepper motor bond graph is shown below.

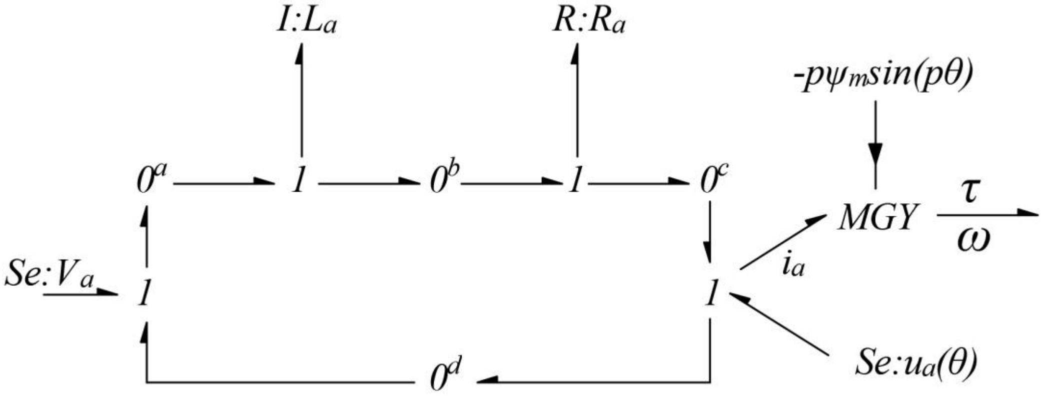

Figure 11

After simplifying and combining the mechanical parts, the stepping motor bonding graph shown in the figure below is obtained.

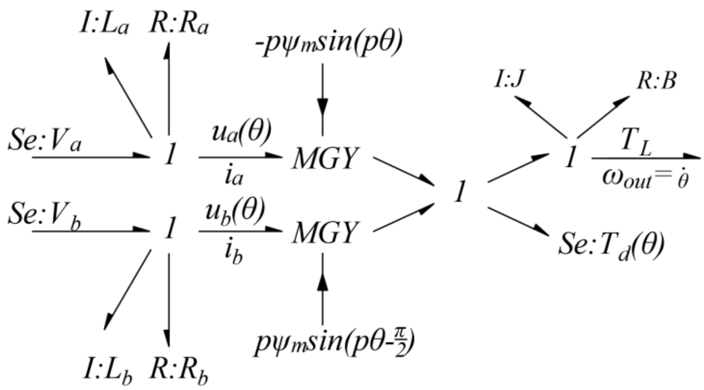

Figure 12

Combined with the bond graph given above, a more complete bond diagram of the X, Y, and Z axes can be obtained. Since the models of the three axes are similar, only the Z axis is drawn as an illustration.

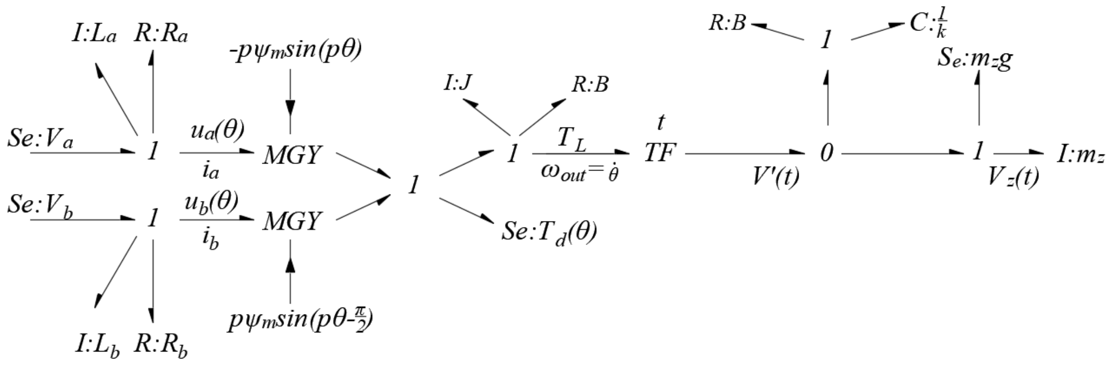

Figure 13

4. Causal Strokes and Diagram

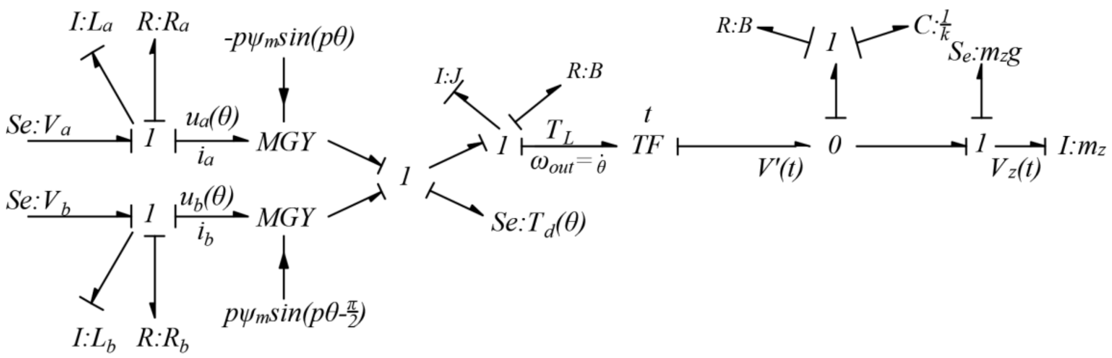

Figure 14

The process of adding causal strokes is smooth. The complete bond graph of Z axis motion is shown in Fig 14.

The circuit part and the mechanical part of the motor can be drawn separately and combined as shown below. Follow the standard pattern on the textbook, the bond graph model can be converted to block diagram like this. For X and Y motions, we can just replace the mass, stiffness, damping and transformer modulus with needed parameters, and let the mg be zero. The block diagram of Z axis is shown in Fig 15.

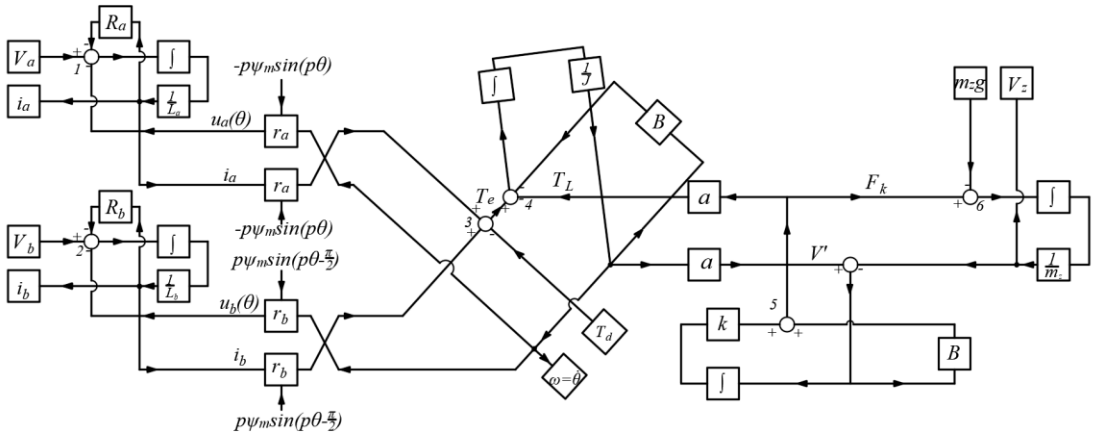

Figure 15 Z axis block diagram

Take the Z axis as an example to verify from node to node.

Node 1(same as Node 2):

$$\frac{1}{L_a}\int[V_a - i_a R_a - u_a(\theta)]dt = i_a \tag{8}$$

Node 3:

$$T_e = -p\psi_m i_a \sin(p\theta) + p\psi_m i_b \sin(p\theta - \frac{\pi}{2}) - T_d \tag{9}$$

Node 4:

$$\frac{1}{J}\int(T_e - B\frac{d\theta}{dt} - T_L)dt = \frac{d\theta}{dt} \tag{10}$$

Node 5:

$$F_k = k\int(V' - V_z)dt + B(V' - V) \tag{11}$$

Node 6:

$$V_z = \frac{1}{m_z}\int(F_k - m_z g)dt \tag{12}$$

5. Model Parameters

First, the dimensions of the printer frame are shown below.

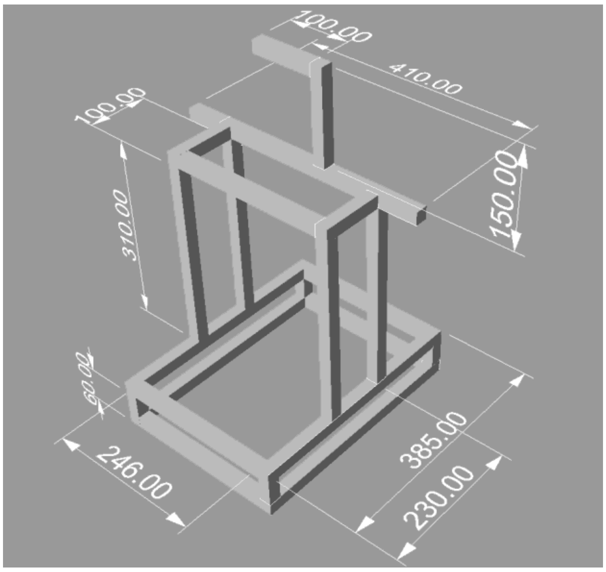

Figure 16 Frame size of 3D printer

Table 1

Parameters	X	Y	Z
R_a/R_b	2.8(Ω)	2.8(Ω)	2.8(Ω)
L_a/L_b	6.5×10^{-3} (H)	6.5×10^{-3} (H)	6.5×10^{-3} (H)
r_a/r_b	eq(2)、(3)	eq(2)、(3)	eq(2)、(3)
B_m	0.001	0.001	0.001
B_L	0.1	0.1	0.4
J	6.1×10^{-6}(kg • m^2)	6.1×10^{-6}(kg • m^2)	6.83×10^{-6}(kg • m^2)

a	1×10^-2^(m)	1×10^-2^(m)	1.27×10^-3^(m)
k	2.1×10^{6}(N/m)	3×10^{6}(N/m)	3×10^{8}(N/m)
m	0324(kg)	0.6(kg)	1.06(kg)
p	50	50	50
φ_m	0.0015(V • s)	0.0015(V • s)	0.0015(V • s)
T_{dm}	0.045(N • m)	0.045(N • m)	0.045(N • m)

R_a/R_b are resistors of phase A and phase B. L_a/L_b are inductances of phase A and phase B. r_a/r_b are the transfer factor of motor， the expressions are (2) and (3).

Most of motor parameters can be obtained from the manual or calculated form previous mentioned formulas. The damping of stepper motor(B_m)and transmission mechanism (B_L)can be tested out or just choose one to satisfy the experience vibration times.

The moment of inertia of the motor shaft is taken as 5.8×10^{-6}kg•m^2 with reference to its manual. The moment of inertia of the coupling can be taken as 7.0 x 10^{-7} kg•m^2 according to the product manual. The inertia of the lead screw is estimated to be 3.3 × 10^{-7} kg • m^2, and the inertia of the two timing pulleys is estimated to be 3 × 10^{-7} kg • m^2.

$$J_{x,y}=5.8\times10^{-6}+0.3\times10^{-6}=6.1\times10^{-6}\,kg{\bullet}m^2$$

$$J_z=5.8\times10^{-6}+0.7\times10^{-6}+0.33\times10^{-6}=6.83\times10^{-6}\,kg{\bullet}m^2$$

a is the radius of the synchronous pulley in the X and Y axis motion, which is taken as 10 mm. a is the lead screw transmission ratio in the Z-axis motion, and the calculation formula is as follows, where P is the lead of the lead screw.

$$a=\frac{P}{2\pi}=\frac{8\times10^{-3}}{2\pi}=1.27\times10^{-3}(m/rad)$$

T_{dm} is the maximum value of the Detent Torque, the size is usually not clearly stated. Generally, 1% to 10% of the holding torque is a typical value, and 10% is taken here. The holding torque is 4.5 kg • cm, and the maximum positioning torque is 0.45 kg • cm. This gives an expression of the detent torque T_d .

$$T_d=T_{dm}\sin(2p\theta)$$

For the stiffness in the case of the X-axis and the Y-axis, the formula of the stiffness k of the synchronous belt is estimated according to the conclusion of PART1. A 6mm wide 2GT timing belt can be selected for calculation. Where l is the center distance of the timing pulley, E is the elastic modulus of the timing belt, and A is the sectional area of the timing pulley, and the estimated value is 4×10^{-6} m^2. The elastic modulus of the core glass fiber 70 GPa was used as the calculated value of the elastic modulus. The center distance in the X direction is 0.35 m, and the center distance in the Y direction is 0.25 m.

$$k_x=\frac{8EA}{3l_x}=\frac{8\times7\times10^{10}\times4\times10^{-6}}{3\times0.35}=2.1\times10^{6}\,N/m$$

$$k_y = \frac{8EA}{3l_y} = \frac{8\times7\times10^{10}\times4\times10^{-6}}{3\times0.25} = 3\times10^6 N/m$$

For the Z axis, the value is the stiffness of the lead screw in the axial direction. According to the mechanical design manual, this value is 300 (N•μm^{-1}).

$$k_z = 3\times10^8 N/m$$

The X-axis load is mainly the quality of the hot bed. Usually, the hot bed is made of an aluminum substrate and is similarly regarded as an aluminum plate. The hot bed size of prusa3 is 200mm × 200mm × 3mm, and its quality can be estimated.

$$m_x = 0.2\times0.2\times0.003\times2.7\times10^3 = 0.324kg$$

The Y-axis load is mainly the mass of the extrusion mechanism (including one motor), which can be estimated according to the reference information of the motor and the extrusion mechanism.

$$m_y = 0.6kg$$

The load in the Z-axis direction is mainly the mass of the extrusion mechanism (including one motor) and the Y-direction motor and its two polished rod holders.

$$m_z = m_e + m_y + 2\times\frac{1}{4}\pi D^2 L\rho = 0.35 + 0.6 + 2\times\frac{1}{4}\pi(6\times10^{-3})^2\times0.24\times7.8\times10^3 = 1.06kg$$

So far, all the parameters in the parameter table are obtained, and Simulink modeling is started.

6. Simulink Modeling

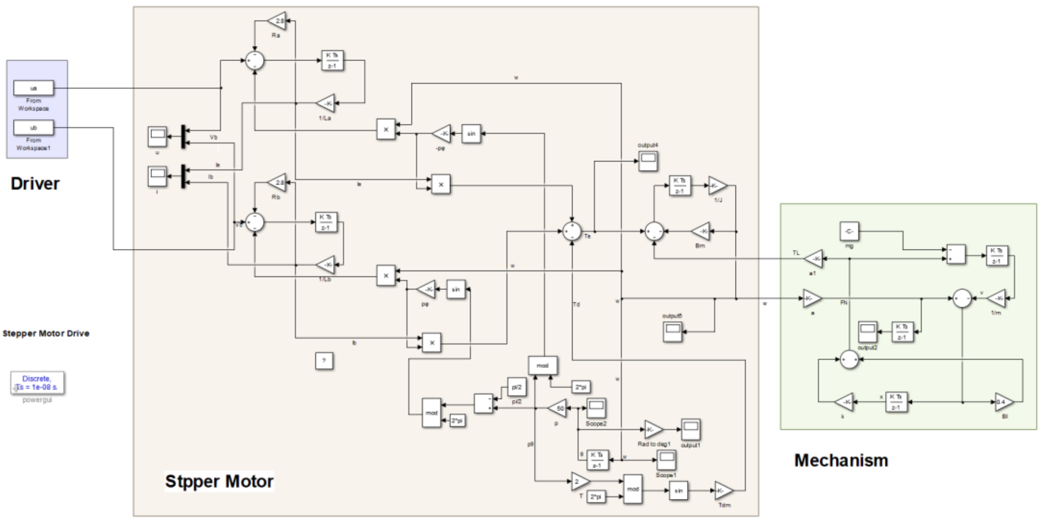

Figure 17

Fig17 shows the simulink model of Z axis motion.

The leftmost part is the drive signal. Its source is the stepper motor drive model in simulink. In order to improve the simulation speed, I saved the output signal so I can get it directly when necessary. The middle part is the stepper motor. And the rightmost part is the mechanical transmission part with load.

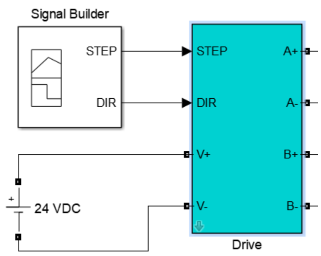

Figure 18

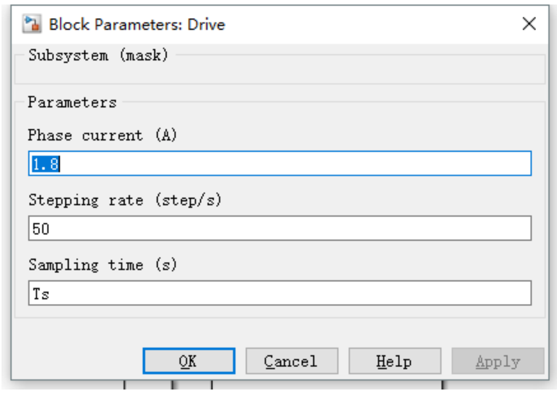

Figure 19

Fig18 and Fig19 show the settings of drive signal. Phase current is 1.8A, frequency if 50Hz. And the pulse continues 0.45s during the simulation.

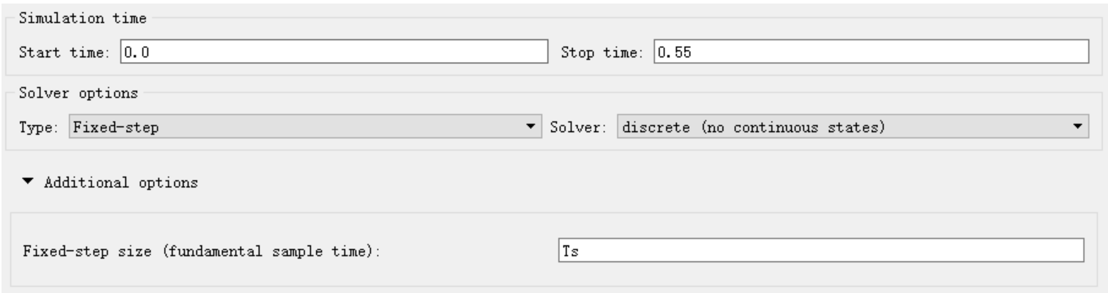

Figure 20

Fig20 shows the settings of the simulation. I chose the discrete solver with fixed step. The step, Ts, of it is 10^{-8}. If the step is larger, the simulation may diverge.

7. Simulation result and analysis

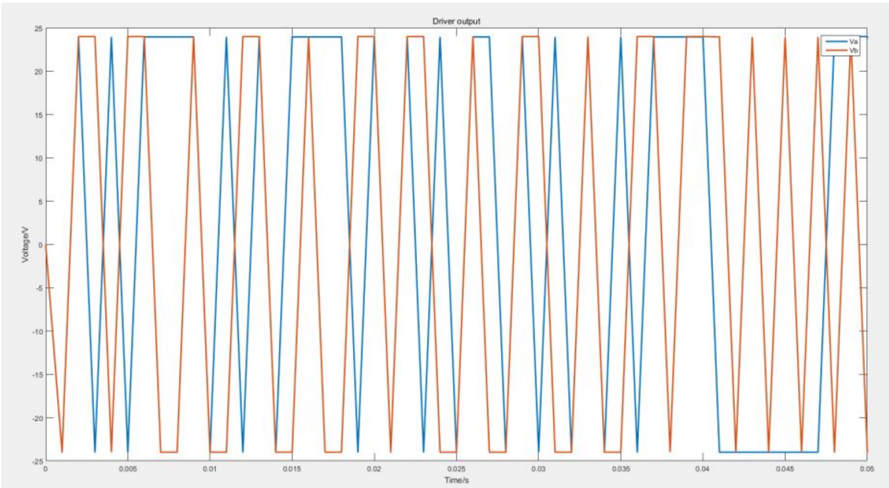

Figure 21

Fig 21 shows the output signal of the stepper motor driver, it is very complicated, so as to realize the subdivision drive to ensure no lost step.

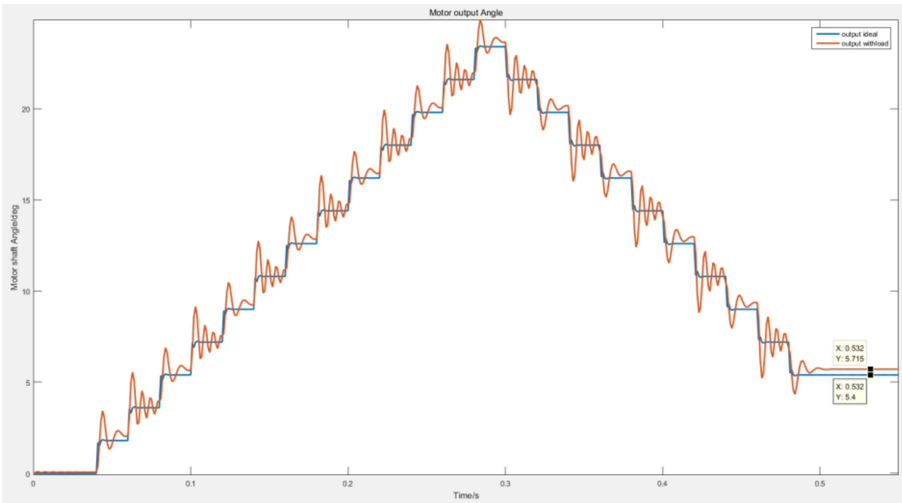

Figure 22

Fig 22 is angular displacement figure of the motor. In the beginning the drive does not start the step directly because it takes about 0.05s to establish the magnetic field and torque. With 50 Hz frequency , motor steps 23 times in 0.45s.

The blue line represents the ideal rotate situation without load. There is no large oscillation. Stepper motor can be stabilized very quickly and the final stable position is also accurate. It Stables at 5.4°.

The orange line represents the case with a Z-axis load, which is oscillating but can be stabilized at 5.715° for a long enough time. However, there will be an error about 0.3°, compared with the ideal case.

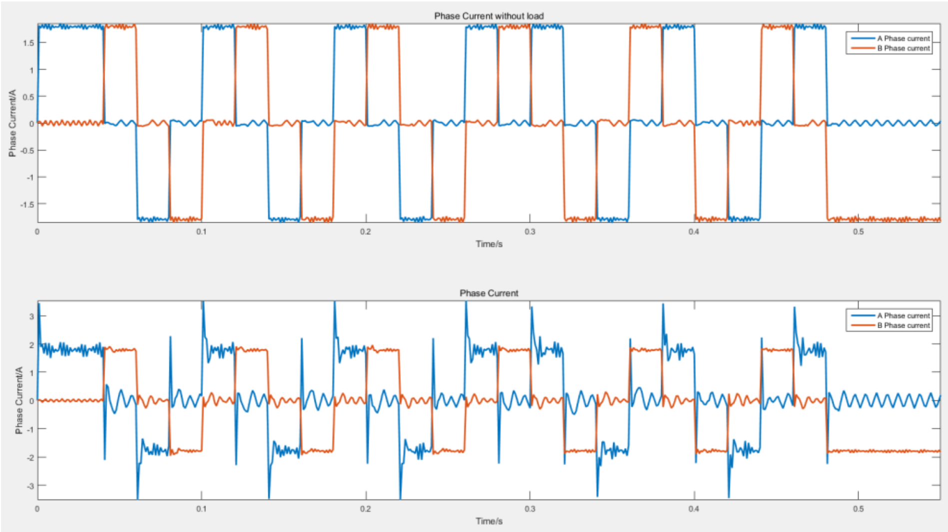

Figure 23

Fig 23 is about phase current. We can see that the current of each phase changes smoothly when there is no load. When the load is considered, the phase A current changes drastically.

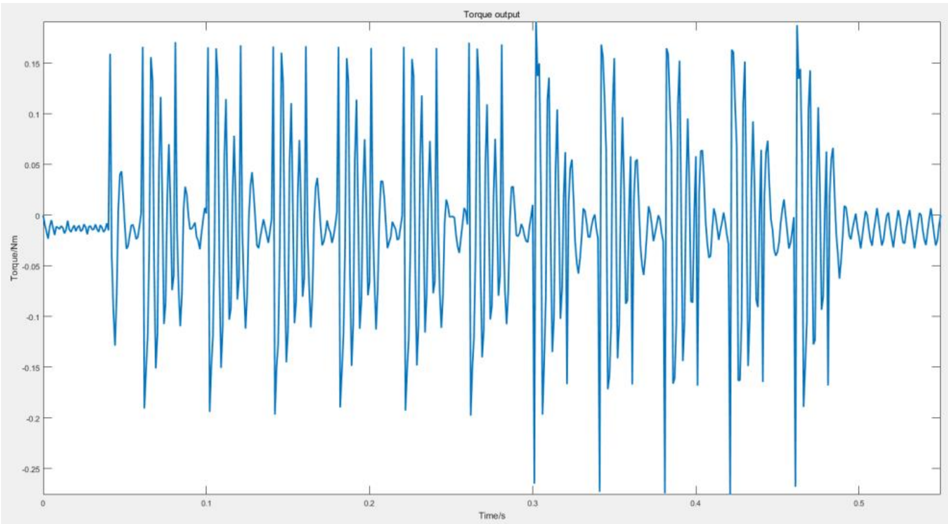

Figure 24

Fig 24 is about output torque of the stepper motor. Its peak value is about 0.25Nm, which also meets the load carrying capacity of the selected motor.

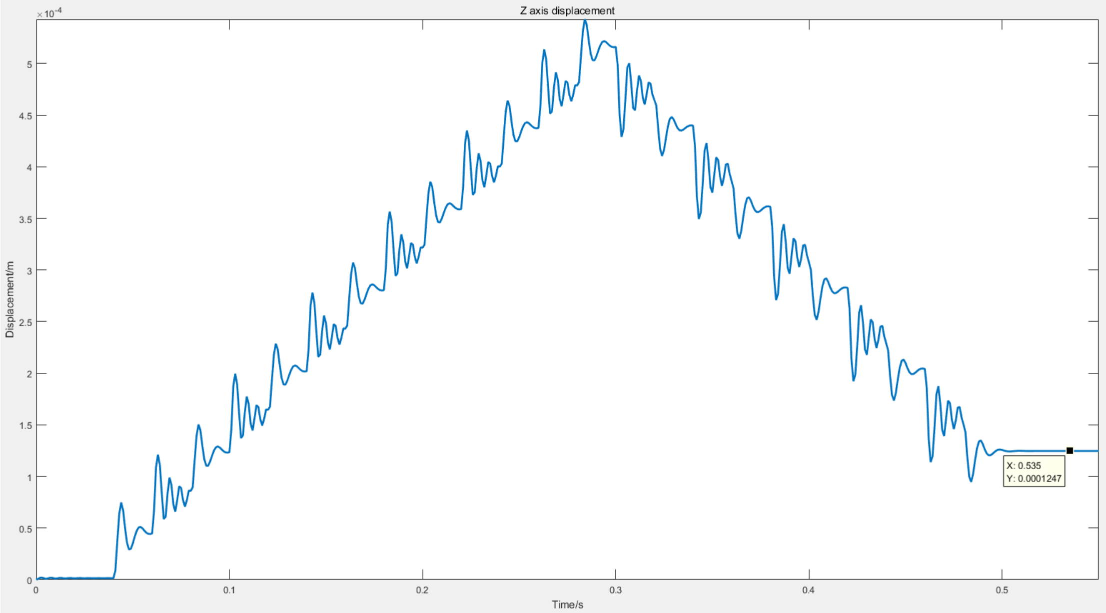

Figure 25

$$\Delta Z = 5.715^{\circ} \times \frac{8mm}{360^{\circ}} = 0.127$$

This is the displacement of the Z-axis load. We can see the unit of Vertical axis can be 10^{-4}m. It is very small because the lead of the selected lead screw is 8mm. The stepper motor only rotated 5.4° during the simulation, so the displacement of the load is very small. And because of the rigidity of the screw, the position will also have some deviation.

8. Conclusion and outlook

Conclusion:

1. Stepper motor drive start will take time to establish magnetic field and torque.
2. Load will increase the oscillation of the motor phase current, output torque and angular displacement.
3. Open loop control of the stepper motor can also get good results.
4. Step size setting may completely affect the results of the simulation.

Outlook:

1. Study stepper motor lost steps conditions
2. Optimize the parameters of the motor to make it more stable.
3. Choose the optimum drive and velocity curve based on this model.

9. References

[1] Le-Huy H , Brunelle P , Sybille G . Design and implementation of a versatile stepper motor model for simulink's SimPowerSystems[C]// 2008 IEEE International Symposium on Industrial Electronics. IEEE, 2008.

[2] Karnopp D C , Margolis D L , Rosenberg R C . System Dynamics: Modeling, Simulation, and Control of Mechatronic Systems, 5th Edition[J]. 2012.