Quantitative Macroeconomics
What is the labour response to a technology shock and what is the European Central Bank's response in the New Keynesian Model?

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December 21, 2020

1 Introduction

The main objective of this paper is to see how agents react to a shock in worker productivity, as well as to give me some considerable insight into the research topic I want to carry out for my thesis.
In recent years we assist in the increase of publications related to monetary policy, which suggests the topic has aroused interest among researchers in this field. One of them was carried out by John Taylor on the interest rate Taylor (1993). The New Keynesian Model (NKM) was developed as an alternative option of the Real Business Cycle Model (RBC), which was a dynamic model assuming perfect competition; instead, the Keynesian models proposed were static but based on imperfect competition. So, the New neoclassical model was basically a combination of the dynamic aspects of Real Business Cycles with the imperfect competition and nominal rigidity of new Keynesian models. Real Business Cycles plays a role in explaining macroeconomic fluctuations from technology shocks, since monetary policy is nonexistent. From the literature, the sign of response of output and employment to a positive technology shock is, in general, ambiguous Galí (2015), so in the present study we try to see what happens with the hours worked when the economy is affected by technology shocks. Also, we try to clear what happens when the Central Bank take care of either inflation or the output gap. The sign depends mostly on the calibration of parameter values, including the interest rule coefficients, that we will describe in section 4. Because technological shocks affect the precise ones through the Phillips Curve, what we will try to define in this paper is the magnitude of that effect, and how it affects employment; we will observe how it affects the Central Bank's decisions when making monetary policy decisions, either to try to smooth the effect of the output gap or to correct the inflation derived from the shock. From now on and until the end, we will develop this work by following these steps: in section 2 we make a brief review of previous literature dealing with technological shocks; in section 3 we develop the theoretical framework of the model, and we will do it analytically, drawing out the equations that we will later use to solve it in Matlab; in section 4 we collect the equations, describe the calibration, the methodology Lombardo & Sutherland (2007) that we use to solve the problem and also we define the steady states to which the economy is going to converge after the shock. Finally in section 5 we show the two results with the different Central Bank performances and the variable responses.

2 Review of literature

There is a strong macroeconomics literature that try to answer the question of what is the labour response to a technology shock and how the European Central Bank responds. The New Keynesian Model (NKM) has microeconomics foundations, for example Clarida et al. (1999) it develops its monetary policy research in this model. Ireland (2004), in his paper, defined the more basic New Keynesian Model which consists of only three equations: the first one corresponds to a log-linearization of the Household problem Euler equation or, as McCallum & Nelson (1997) named it, "the expectational IS curve". The Euler equation connects consumption and production with the inflation and the return of nominal bonds (real interest rate). The second equation is the new version of Phillips Curve that will be replace by our auxiliary variables, so these describe the behaviour of monopolistic competitive firms problem, the firms set their prices randomly, as suggested Calvo (1983). Lastly, the final equation is the monetary policy rule that Taylor (1993) developed is his paper. These three equations interact in a dynamic context, where we observe the behaviour of output, inflation and the type of interest. Blanchard & Galí (2007) found that the standard NKM doesn't imply a trade-off between stabilizing inflation and output gap, so stabilizing inflation is equivalent to stabilizing the welfare relevant output, and this is called *the divine coincidence*, that means: if we have a technology shock and we want to stabilizing the output gap, we only need to stabilize the inflation, which is corrected by the introduction of a real imperfection *real wage rigidities* to the model. We know from Gali (1999) that when a technology shock has occurred, the hours worked decrease. Another interesting paper is Casares et al. (2014) that the more important shocks to determinant the unemployment fluctuations are *push shocks, demand shifts and monetary policy shocks*.

3 New Keynesian Model

The model detailed here consists of three equations; the policy monetary in this kind of model is made by a target of inflation and employment rate through the play with the interest rate. As we will see our model does not have capital, and this is a common practice in the NKM. It is difficult to understand, because until now in all the courses of macroeconomics that we have attended, the capital has a fundamental role in investment and economic fluctuations. One of the main causes is the complexity when modelling the model, and another is that capital introduces one more variable to be solved and it is oriented to the past, while the family equations and the NKPC of the log-linearized model are oriented to the future.

3.1 Household problem

The household choose sequences for the control variables: c_t, N_t and B_t to maximize the expected utility function:

$$\begin{aligned} \max_{c_t, B_{t+1}, N_t} \quad & U(c_t, N_t) = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \} \\ \text{subject to} \quad & \int_0^1 p_t^i c_t^i d_i + R_t^{-1} B_{t+1} = B_t + W_t N_t + D_t \end{aligned} \tag{1}$$

where p_t^i is the price of good i and c_t is now a consumption index given by:

$$c_t = \left[\int_0^1 (c_t^{i\nu} d_i)\right]^{1/\nu} \tag{2}$$

with c_t^i representing the quantity of good i consumed by the household in period t, for all $t = 0, 1, \ldots$ We are assuming the existence of a continuum of good represented in the interval $[0, 1]$. The discount factor from household's utility satisfies $\beta \in (0, 1)$ while that we assumed that the single-period utility function of c_t and N_t denotes hours of work are strictly increasing and concave, W_t is the nominal wage, B_t represents purchases of noe-period bonds and D_t is a lump-sum component income. So in order to solve the problem, we need to take the FOC:

$$\frac{\partial U(c_t^i, N_t)}{\partial c_t} = \beta^t c_t^{-\sigma} \left(\frac{c_t}{c_t^i}\right)^{1-\nu} - \lambda_t p_t^i = 0 \tag{3}$$

$$\frac{\partial U(c_t^i, N_t)}{\partial N_t} = -\beta_t N_t^{\phi} + \lambda_t W_t = 0 \tag{4}$$

$$\frac{\partial U(c_t^i, N_t)}{\partial B_{t+1}} = -\lambda_t R_t^{-1} + \mathbb{E}_t \lambda_{t+1} = 0 \tag{5}$$

So, we transform the equation 3 as follow:

$$\beta^t c_t^{-\sigma} \left(\frac{c_t}{c_t^i}\right)^{1-\nu} = \lambda_t p_t^i$$

multiplying in both side by c_t^i

$$\beta^t c_t^{-\sigma} \left(\frac{c_t}{c_t^i}\right)^{1-\nu} c_t^i = \lambda_t p_t^i c_t^i \Longrightarrow \beta^t c_t^{-\sigma} c_t^{1-\nu} (c_t^i)^{1-(1-\nu)} = \lambda_t c_t^i p_t^i \Longrightarrow$$

$\beta^t c_t^{-\sigma} c_t^{1-\nu} \int_0^1 (c_t^i)^{\nu} d_i = \lambda_t \int_0^1 c_t^i p_t^i d_i \Longrightarrow \beta^t c_t^{-\sigma} c_t^{1-\nu} c_t^{\nu} = \lambda_t p_t c_t \Longrightarrow \beta^t c_t^{-\sigma} = \lambda_t p_t$

Finally, we get:

$$\left(\frac{c_t}{c_t^i}\right)^{1-\nu} = \frac{p_t^i}{p_t} \tag{6}$$

Solving, equation 4, we get:

$$\frac{N_t^{\phi}}{c_t^{-\sigma}} = \frac{W_t}{p_t} \Longrightarrow N_t^{\phi} = w_t c_t^{-\sigma} \tag{7}$$

where $w_t = \dfrac{W_t}{p_t}$, that means the real wage.

Replacing equation 3 with respect to c_t and c_{t+1} in 5, we obtain the Euler equation:

$$c_t^{-\sigma} R_t^{-1} = \mathbb{E}_t \beta_t c_{t+1}^{-\sigma} \tag{8}$$

3.2 Firms' problem

There is a bunch of firms $\jmath$ that produce various final consumer goods. They face a demand curve where the price they set is markup above marginal cost, which is dependent on the elasticity of demand ϵ. This means that if the demand becomes more inelastic, firms can charge a markup over marginal cost and extract some consumer surplus. We use ϵ since is a more realistic assumption and it allows firms to set prices above marginal cost today so that if they cannot change their prices in the next few periods, they are not operating at a huge loss. If the shock hits them, they can cover their asset in the future because they are setting their prices based on what they expect prices a couple of months for now. If we have *nominal rigidity*, that is a nominal variable that can't move as much as normally be able to in a flexible price equilibrium. So, in our case nominal rigidity is called sticky prices.The equation of the firm problem results in:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[p_s^i c_s^i - W_s \frac{c_s^i}{A_s} \right] \tag{9}$$

Where $c_t^i = \left(\frac{p_s}{p_s^i}\right)^{\epsilon} c_s$ is the demand curve.
Replacing c_s^i into 9, we obtain, for simplicity we define ϵ.*

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[p_s^i \left(\frac{p_s}{p_s^i}\right)^{\epsilon} c_s - \frac{W_s}{A_s} \left(\frac{p_s}{p_s^i}\right)^{\epsilon} c_s \right]$$

Now, consider that there is a fraction of firms $\jmath_i$ that can't change their prices each period, then there is a fraction $\jmath_{-i}$ of firms that can change prices each period. Although firms know φ, they do not if they will be able to change prices next period. This particular framework is when we are assuming Calvo prices; we set that some firms can change their prices at period t with probability φ, but the rest keep sticky price more time Calvo (1983), then the firm function change as follow:

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta(1-\varphi))^{s-t} \left[p_s^i \left(\frac{p_s}{p_s^i}\right)^{\epsilon} c_s - \frac{W_s}{A_s} \left(\frac{p_s}{p_s^i}\right)^{\epsilon} c_s \right] + \mathbb{E}_t \sum_{s=t}^{\infty} (\beta\varphi)^{s-t} \left[p_s^i \left(\frac{p_s}{p_s^i}\right)^{\epsilon} c_s - \frac{W_s}{A_s} \left(\frac{p_s}{p_s^i}\right)^{\epsilon} c_s \right]$$

Taking the FOC, with respect to p_t^i, we obtain:

$$\frac{\partial y_t}{\partial p_t^i} = \mathbb{E}_t \sum_{s=t}^{\infty} (\beta(1-\varphi))^{s-t} \left[(1-\epsilon) p_t^i p_s^{\epsilon} c_s + \frac{W_s}{A_s} \epsilon p_s^{-\epsilon-1} p_s^{\epsilon} c_s \right] = 0 \Longrightarrow \tag{10}$$

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta(1-\varphi))^{s-t} \left[(1-\epsilon) p_s^i p_s^{\epsilon} c_s + \frac{W_s}{A_s} \epsilon p_s^{\epsilon} c_s \right] = 0 \Longrightarrow$$

We put the above equation in real terms, so that:

*See subsection 1 to see the process.

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta(1-\varphi))^{s-t} \left[(1-\epsilon)\frac{p_t^i}{p_t}\frac{p_s^\epsilon}{p_t^\epsilon} c_s + \frac{w_s}{A_s}\epsilon \frac{p_s^\epsilon}{p_t^\epsilon} c_s \right] = 0$$

As we can observe in the equation above, firms have to be forward-looking when setting their prices, because they know each period in the future, so if we develop the sum above, φ is raised to higher power $\varphi = \{\varphi\}_{s=0}^{t}$, by assumption $\varphi \in [0,1]$, so if you rise it to higher and higher numbers, it is getting smaller and smaller, meaning that the probability that the firm can change the price as $t \to \infty$, the firms will be able to change prices again. This is according to the theory in RBC: in long run the prices are flexible Aguirre (n.d.). Another assumption is that firms have rationals expectations, so firms know if the central bank will want to inflate the money supply to try to stimulate the output.

We define $x_t = \dfrac{p_t^i}{p_t}$ is the ratio of the price that firm j charges for their goods relative to all others prices of the rest of the firms p_t. Since $\dfrac{p_s}{p_t}$ is the ratio of prices between two different periods, we can rewrite the equation in terms of inflation.

Therefore, we obtain:

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta(1-\varphi))^{s-t} \left[(1-\epsilon)x_t \left(\prod_{j=t+1}^{s} (1+\pi_j)^\epsilon \right) c_s + \frac{w_s}{A_s}\epsilon \left(\prod_{j=t+1}^{s} (1+\pi_j)^\epsilon \right) c_s \right] = 0 \tag{11}$$

We define two auxiliary variables to help us to solve the problem:

$$\Phi_t^1 \equiv \mathbb{E}_t \sum_{s=t}^{\infty} (\beta(1-\varphi))^{s-t} \left[\left(\prod_{j=t+1}^{s} (1+\pi_j)^\epsilon \right) c_s \right]$$

Using the recursive method, we arrive:

$$\Phi_t^1 = c_t + \beta(1-\varphi)\mathbb{E}_t(1+\pi_{t+1})^\epsilon \Phi_{t+1}^1 \tag{12}$$

The second one is:

$$\Phi_t^2 \equiv \mathbb{E}_t \sum_{s=t}^{\infty} (\beta(1-\varphi))^{s-t} \left[\left(\prod_{j=t+1}^{s} (1+\pi_j)^\epsilon \right) c_s \right]$$

Again, using recursive formulation:

$$\Phi_t^2 = c_t \frac{w_t}{A_t} + \beta(1-\varphi)\mathbb{E}_t(1+\pi_{t+1})^\epsilon \Phi_{t+1}^2 \tag{13}$$

Thus, we replace the auxiliary equations into the equation 11, so we can rewrite it as:

$$(\epsilon - 1)x_t \Phi_t^1 = \epsilon \Phi_t^2 \tag{14}$$

Using the ideal price index, define in the appendix A.1

$$p_t^{1-\epsilon} = \varphi(p_t^i)^{1-\epsilon} + (1-\varphi)p_{t-1}^{1-\epsilon}$$

We repeat the process, dividing by $p_t^{1-\epsilon}$, and we get:

$$1 = \varphi(x_t)^{1-\epsilon} + (1-\varphi)(1+\pi_t)^{\epsilon-1} \tag{15}$$

In order to determine the economy's response to a technology shock we must first specify a process for the technology parameter a_{t+1}, and derive the implied process for the natural rate. We assume the following AR(1) process for a_{t+1}.

$$A_{t+1} = A\exp(a_{t+1}) \tag{16}$$

$$a_{t+1} = \rho a_t + \epsilon_{t+1}$$

Where $\rho \in [0,1)$ and ϵ_{t+1} is a zero it means with noise process. The demand should be equal to the supply, so we define the market clearing condition:

$$c_t = A_t N_t \tag{17}$$

Also, we need to define the Fisher equation. Letting R denote the real interest rate, i denote the nominal interest rate, and let π denote the inflation rate, a linear approximation, but the Fisher equation is often written as an equality:

$$1 + \imath_t = (1+\pi_t)R_t \tag{18}$$

The Central Bank can't control anything besides inflation, thus its objective function is to choose a policy that minimizes inflation while it is closing to the output gap, but they need a policy tool to do that. We define the policy function that is the Taylor rule; so, with this tool, the Central Bank sets the interest rate to manage real economy activity. Then, we define the following Taylor rule.

$$\imath_t = \bar{\imath} + h_\pi \pi_t + h_y \tilde{c}_t{}^* \tag{19}$$

Where $\bar{\imath}$ is the target of interest in the economy, π is the inflation, h_y is a measure of the percentage deviation of the NKM output and the flexible price output of RBC model, Galí (2015), $\imath$ is the nominal interest rate, being equal to the current inflation rate and the target real interest rate in the economy $\bar{\imath}$, h_π is a measure of the percentage deviation of inflation for their target rate of inflation (2% in Central Bank).

Both h_π and h_y tell us how concerned the Central Bank is in controlling the inflation or the output, and we assume that they are non-negative coefficients. In Europe, the Central Bank has the fundamental mission of worrying about inflation, but in times of crisis it can ignore this and look towards growth, and this is the main difference with the US Federal Reserve. Therefore we can face two possible types of policymaker:

*See appendix section 2

$h_\pi > h_y$ They care more about stabilising inflation keeping the current inflation rate close to it target rate then they do minimizing the output gap. Stabilising inflation deal stabilise also the output gap, Blanchard & Galí (2007).

$h_\pi < h_y$ They care about trying to boost the economy, at the cost that inflation can oscillate more than the target.

The h's terms also depend on the state of the economy; as we comment before, if we are in recession, and inflation is relatively low, then the Central Bank possibly will implement the second policy, because it is more important to boost the output. So, they set monetary aggregate to achieve a nominal interest rate $\imath$, consistent with the Taylor rule 19. In our case, as we have already said above, we design and execute both scenarios to see how the different response variables react. Notice that, if $h_\pi \pi_t + h_y \tilde{c_t} = 0$, the inflation is equal to the target. Also, if the output gap is zero, the Taylor rule is:

$$\imath_t = \bar{\imath} + \pi_t \Longrightarrow \pi_t = \imath_t - \bar{\imath}$$

So, there is no needed for the Central Bank to use a monetary policy to influence the business cycles. That is the perfect scenario for RBC Aguirre (n.d.).

4 Calibration, Equations of the model and Methodology

We already have all the pieces of our puzzle, consisting of the 10 equations with 10 unknown variables, and we present them in the next table:

Table 1: Unknown Variables

c	Consumption
N	Units of labour employees in the firms
R	Real interest rate
w	Real wage
A	Labour productivity
x	Ratio of prices with respect of firm i that can change the price in period t to rest of the firms.
π	The inflation rate
Φ^1	Auxiliary variable
Φ^2	Auxiliary variable
$\imath$	Nominal interest rate

Also, in Table 2 we summarize the equations that we will use to solve our problem.
We can observe in the LHS the equations developed above and in the RHS we have performed the log-linearization of the model. Also, we solve the equation to the steady state. Recall, that in the steady state $c_t = c_{t+1} = c_s$, so we applied this rule for each variable.

Table 2: Equations of the problem

$\mathrm{N}_t^{\phi} = w_t c_t^{-\sigma}$	$0 = \phi \ln N_t - \ln w_t + \ln c_t$
$c_t^{-\sigma} R_t^{-1} = \mathbb{E}_t \beta_t c_{t+1}^{-\sigma}$	$0 = \ln A_t + \ln N_t - lnc_t$
$\Phi_t^1 = c_t + \beta(1-\varphi)\mathbb{E}_t(1+\pi_{t+1})^{\epsilon}\Phi_{t+1}^1$	$0 = \ln(1+\pi_t) + \ln R_t - \ln(1+\imath_t)$
$\Phi_t^2 = c_t \frac{w_t}{A_t} + \beta(1-\varphi)\mathbb{E}_t(1+\pi_{t+1})^{\epsilon}\Phi_{t+1}^2$	$0 = \ln \frac{\epsilon}{1-\epsilon} + \ln \Phi_t^2 - \ln \Phi_t^1 - \ln x_t$
$(\epsilon-1)x_t\Phi_t^1 = \epsilon\Phi_t^2$	$0 = 1 \text{ - } \varphi \exp\{(1-\epsilon)\ln x_t\} - (1-\varphi)\exp\{(\epsilon-1)\ln(1+\pi_t)\}$
$1 = \varphi(x_t)^{1-\epsilon} + (1-\varphi)(1+\pi_t)^{\epsilon-1}$	$\ln A_{t+1} - \ln A* = \rho \ln A_t - \rho \ln A * + \epsilon_{t+1}$
$A_{t+1} = A\exp(a_{t+1})$	$\mathrm{E}_t\beta\exp\{-\sigma\ln c_{t+1}\} = \exp\{-\sigma \ln c_t\}\exp\{-\ln R_t\}$
$c_t = A_t N_t$	$\beta(1-\varphi\mathbb{E}_t)\exp\{\epsilon(1+\pi_{t+1}) + \ln\Phi_{t+1}^1\} = \exp\{\Phi_t^1\} - \exp\{\ln c_t\}$
$1+\imath_t = (1+\pi_t)R_t$	$\beta(1-\varphi\mathbb{E}_t)\exp\{\epsilon(1+\pi_{t+1}) + \ln\Phi_{t+1}^2\} = \exp\{\Phi_t^2\} - \exp\{\ln c_t - \ln w_t - \ln A_t\}$
$\imath_t = \bar{\imath} + h_\pi \pi_t + h_y \tilde{c}_t$	$0 = 1 + \;+ \mathrm{h}_\pi \exp\{\ln(1+\pi_t)\} - h_{pi} + h_y * \left(exp\left(\ln c_t - \frac{1}{\phi+\sigma}\left(\ln\left(\frac{\epsilon-1}{\epsilon}\right) + (1+\phi)\ln A_t\right)\right) - 1\right) - \exp\{\ln(1+\imath_t)\}$

Below, we describe the equations for the steady state:*

$$R_{ss} = \beta^{-1} \tag{20}$$

$$A_{ss} = A^* \tag{21}$$

$$\pi_{ss} = \frac{1+\bar{\imath}+h_y\tilde{c}-\beta^{-1}}{\beta^{-1}-h_\pi} \tag{22}$$

$$\imath_{ss} = \bar{\imath} + h_\pi \pi_{ss} + h_y \tilde{c} \tag{23}$$

$$x_{ss} = \left(\frac{1-(1-\varphi)(1+\pi_{ss})^{\epsilon-1}}{\varphi}\right)^{\frac{1}{1-\epsilon}} \tag{24}$$

$$w_{ss} = \frac{\epsilon-1}{\epsilon} x_{ss} A_{ss} \tag{25}$$

$$N_{ss} = w_{ss}^{\frac{1}{\phi+\sigma}} A_{ss}^{-\frac{\sigma}{\phi+\sigma}} \tag{26}$$

$$c_{ss} = A_{ss} N_{ss} \tag{27}$$

$$\Phi_{ss}^1 = \frac{c_{ss}}{(1-\beta(1-\varphi)\mathbb{E}_t(1+\pi_{ss})^{\epsilon})} \tag{28}$$

$$\Phi_{ss}^2 = \frac{c_{ss}\frac{\omega_{ss}}{A_{ss}}}{(1-\beta(1-\varphi)\mathbb{E}_t(1+\pi_{ss})^{\epsilon})} \tag{29}$$

$$\tilde{c} = \frac{c_{ss}}{\left(\frac{\epsilon-1}{\epsilon}A_{ss}^{1+\phi}\right)^{\frac{1}{\phi+\sigma}}} - 1 \tag{30}$$

The methodology that we are going to use to perform our model is the one described in Lombardo & Sutherland (2007), he shows how two compute a second-order accurate solution of a non-linear rational expectation model using algorithms developed for the solution of

*See the appendix A3, for some developments

linear rational expectation model. So, this state-space representation can easily be used to compute impulse responses as well as conditional and unconditional forecast. Hence, we solve the first order condition, we square that solution and we solve the second order condition solution using the square solution that we solved before as the quadratic part, so that you solve a first order problem again.
Our baseline calibration is given by the following table: $\beta = 0; \sigma = 2; \phi = 1.5; \epsilon = 1.1; \varphi = 0.75; A^* = 1.1; \rho = 0.9; \bar{\imath} = 0.06; h_y = 0.8$ and $0; h_\pi = 2$

5 Results

The code was executed in:
Elapsed time is 0.593792 seconds, for $h_y = 0$
Elapsed time is 1.272356 seconds, for $h_y = 0.8$

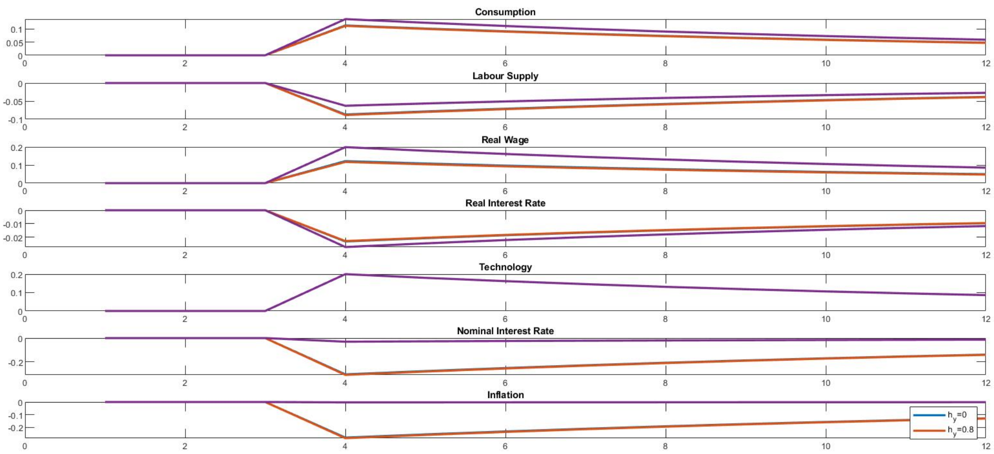

Figure 1: NKM Technology Shock

The effect of technology shock is shown in figure 1. In this case a technological positive shock drives to a persistent employment decline, and this response is consistent with the empirical evidence from Gali (1999) where a VAR model with long-run restrictions was used; he also provides empirical evidence that hours worked fall as a result of a positive technology shock in the United States. This figure 1 also shows the response of other variables described in 1. Notice that the blue line shows when $h_y = 0$, that is, the Central Bank is only concerned with inflation, while the red line shows that the Central Bank is interested in correcting the output gap at the expense of having a period of deflation. Through the monetary policy, the Central Bank partly accommodated the shock, with a lower nominal and real interest rate. However, as we can observe, this policy is not sufficient to correct the negative output gap, which is the cause of the drop of the inflation. So, under our calibration, the output increases, so it always is consistent with Galí (2004). Also notice that, when the Central Bank is concerned about inflation and not about the output gap, the consumption, which is equal to production in our case, because the markets are in balance *demand = consumption*, is higher, but the Central Bank maintains inflation almost at zero. We also observe that the real wage increases for both cases. Also, note that both real interest, nominal interest and inflation fall in line with the Fisher equation. In real life, one of the most important problems that Europe faces is not the shocks of technology affecting economic cycles, but it is that the parameter which we have described with the letter h. The Central Bank unlike the FED, only focuses on controlling inflation rather than looking at economic growth and financial stability. Therefore in the 2008 crisis while the FED adopted expansionary measures and helped the US economy, the Central Bank gave a series of stimuli but always looking at inflation, therefore these stimuli were insufficient and countries, such as Spain, paid the consequences by having to go into high interest rate debt. So much so that in 2011 it raised interest rates twice, even before the deflation that was generating the crisis.

5.1 Conclusion

This paper introduces a model with Calvo prices and technology shocks, the principal results are that the correlation of hours worked and productivity is negative for technology shocks, hours show a persistent decline in response to a positive technology shock. This result differs from the one of RBC models that interprets the bulk of aggregate fluctuations observed in the postwar U.S. economy as being consistent with the competitive equilibrium of a neoclassical growth model augmented with a labour-leisure choice and exogenous technology shocks, Kydland & Prescott (1982). However, our results are consistent with those models with imperfect competitions. Additionally, the *divine coincidence* described by Blanchard & Galí (2007) does not hold here since when the Central Bank takes care about the inflation, the output gap reflected in the consumption has a higher impact.

A Appendix

A.1 Price aggregation

$$p_t c_t = \int_0^1 p_t^i c_t^i d_i \Longrightarrow p_t = \int_0^1 p_t^i \left(\frac{c_t^i}{c_t}\right) d_i$$

From equation 6, we take this and replace into the above equation:

$$p_t = \int_0^1 p_t^i \left(\frac{p_t}{p_t^i}\right)^{\frac{1}{1-\nu}} d_i \Longrightarrow p_t^{1-\frac{1}{1-\nu}} = \int_0^1 (p_t^i)^{1-\frac{1}{1-\nu}} d_i$$

Finally, we obtain:

$$p_t^{-\frac{\nu}{1-\nu}} = \int_0^1 (p_t^i)^{-\frac{\nu}{1-\nu}} d_i$$

Where $1 - \epsilon = -\frac{\nu}{1-\nu} \Longrightarrow \epsilon = 1 + \frac{\nu}{1-\nu} = \frac{1}{1-\nu}$
So for simplicity we will use this equation to derive the optimal firm profit.

A.2 Development of $\tilde{c}$

We show what is $\tilde{c}_t$, so we recall the equation 10 and normalization $\varphi = 1$, when when we develop the the summatory, we obtain:

$$(1-\epsilon)(p_t^i)^{-\epsilon} p_t^{\epsilon} c_t + \frac{W_t}{A_t}\epsilon (p_t^i)^{-\epsilon-1} p_t^{\epsilon} c_t = 0 \Longrightarrow (\epsilon - 1) = \frac{W_t}{A_t p_t^i}\epsilon \Longrightarrow w_t = \frac{\epsilon - 1}{\epsilon} A_t$$

Where $w_t = \frac{W_t}{p_t^i}$. Now we replace the above equation into 7 and clear N from market clearing condition 17 an replace also into equation 7.

$$\left(\frac{c_t}{A_t}\right)^{\phi} = \frac{\epsilon - 1}{\epsilon} A_t c_t^{-\sigma} \Longrightarrow c_t^{\phi+\sigma} = \frac{\epsilon - 1}{\epsilon} A_t^{1+\phi}$$

Therefore, we describe:

$$\tilde{c}_t = \frac{c_t}{\left(\frac{\epsilon - 1}{\epsilon} A_t^{1+\phi}\right)^{\frac{1}{1+\phi}}} - 1$$

A.3 Steady State

Equation 22 come from the fact that:

$$1 + \bar{\imath} + h_\pi \pi_{ss} + h_y \tilde{c} = (1 + \pi_{ss})\beta^{-1} \Longrightarrow \pi_{ss} = \frac{1 + \bar{\imath} + h_y \tilde{c} - \beta^{-1}}{\beta^{-1} - h_\pi}$$

Equation 20 come from the fact that:

$$c_{ss}^{-\sigma} R_{ss}^{-1} = \mathbb{E}\beta_c c_{ss}^{-\sigma} \implies R_{ss} = \beta^{-1}$$

Equation 24 come from the fact that:

$$1 = \varphi(x_{ss})^{1-\epsilon} + (1-\varphi)(1+\pi_{ss})^{\epsilon-1} \implies \varphi(x_{ss})^{1-\epsilon} = 1-(1-\varphi)(1+\pi ss)^{\epsilon-1} \implies$$
$$x_{ss}^{1-\epsilon} = \frac{1-(1-\varphi)(1+\pi_{ss})^{\epsilon-1}}{\varphi} \implies x_{ss} = \left(\frac{1-(1-\varphi)(1+\pi_{ss})^{\epsilon-1}}{\varphi}\right)^{\dfrac{1}{1-\epsilon}}$$

Equation 28 come from the fact that:

$$\Phi_{ss}^1 = c_{ss} + \beta(1-\varphi)\mathbb{E}_t(1+\pi_{ss})^{\epsilon}\Phi_{ss}^1 \implies$$
$$\Phi_{ss}^1(1-\beta(1-\varphi)\mathbb{E}_t(1+\pi_{ss})^{\epsilon}) = c_{ss} \implies \Phi_{ss}^1 = \frac{c_{ss}}{(1-\beta(1-\varphi)\mathbb{E}_t(1+\pi_{ss})^{\epsilon})}$$

Equation 29 come from the fact that:

$$\Phi_{ss}^2 = c_{ss}\frac{\omega_{ss}}{A_{ss}} + \beta(1-\varphi)\mathbb{E}_t(1+\pi_{ss})^{\epsilon}\Phi_{ss}^1 \implies$$
$$\Phi_{ss}^2(1-\beta(1-\varphi)\mathbb{E}_t(1+\pi_{ss})^{\epsilon}) = c_{ss}\frac{\omega_{ss}}{A_{ss}} \implies \Phi_{ss}^2 = \frac{c_{ss}\dfrac{\omega_{ss}}{A_{ss}}}{(1-\beta(1-\varphi)\mathbb{E}_t(1+\pi_{ss})^{\epsilon})}$$

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