Quantitative Macroeconomics - Home Work 4

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1 A simple wealth model

First, I proof when the CRRA function is equal to log-utility. If we fit $\sigma = 1$, we obtain the log-utility, so to arrive this result we need use L'Hopital's rule:
Taking limit when $\sigma \to 1$:

$$\lim_{\sigma \to 1} \left(\frac{c^{1-\sigma} - 1}{1 - \sigma} \right) = \frac{0}{0} \tag{1}$$

So, since our result is an indetermination, we apply L'Hopital with respect to σ:

$$\frac{\partial (c^{1-\sigma} - 1)}{\partial (1 - \sigma)} \Longrightarrow \frac{\ln(c) c^{1-\sigma}(-1)}{-1} \tag{2}$$

Applying limit again:

$$\lim_{\sigma \to 1} [ln(c)c^{1-\sigma}] \Longrightarrow \ln(c)c^0 \Longrightarrow \ln(c) \tag{3}$$

σ measure the degree of relative risk aversion. By assumption of the problem $r = 4\% < \rho = 6\%$, this implies that individuals want to consume more today instead tomorrow, so under this specification the consumption should be decreasing with the time. So, taking the first derivative with respect to c, we can arrive easily to the usual Euler equation:

$$u_c(c_t) = \beta(1 + r)u_c(c_{t+1}) \Longrightarrow \frac{u_c(c_t)}{u_c(c_{t+1})} \beta(1 + r) \tag{4}$$

Since the assumption of preferences hold, assuming CRRA and taking the definition of β, we can rewrite the equation as:

$$\left(\frac{c_{t+1}}{c_t} \right)^{\sigma} = \left(\frac{1 + r}{1 + \rho} \right) \Longrightarrow \left(\frac{c_{t+1}}{c_t} \right) = \left(\frac{1 + r}{1 + \rho} \right)^{\frac{1}{\sigma}} \tag{5}$$

Replacing for the values of parameters given by the problem:

$$\left(\frac{c_{t+1}}{c_t} \right) = \left(\frac{1 + 0.04}{1 + 0.06} \right)^{\frac{1}{\sigma}} < 1 \tag{6}$$

Therefore, consumption is decreasing over time.

2 Solving the ABHI Model

The recursive formulation of the problem of the agent is, where we have two state variables: assets today and income (a,y).

$$\begin{aligned} \max_{a'} \quad & V(a,y) = u(wy + (1+r)a - a') + \beta \sum_{y} \pi_{y'|y} V(a'.y') \\ \text{subject to} \quad & a' \geq \bar{A}_i \\ & \text{for i} = 1,2 \text{ and } c \geq 0 \end{aligned} \tag{7}$$

Where the borrowing limit, $\bar{A}_i$ can be:
1)$\bar{A}_1 \geq -\dfrac{y_i}{r}$
2)$\bar{A}_2 \geq 0$
Taking the derivative to respect to a':

$$\frac{V(a,y)}{\partial a'} = -qu_c(wy + (1+r)a - a') - \lambda_a = \beta \sum_{y} \pi_{y'|y} \frac{V(a',y')}{\partial a'} \tag{8}$$

We need two extra conditions:

1. Dual feasibility: $\lambda_a(a' + \bar{A}) = 0$
2. Slackness condition: $\lambda_a(a' + \bar{A}) = 0$

We apply the envelope theorem, that is, the derivative of the value function with respect to state variable is the derivative of the utility with respect to that state variable. So, we arrive to:

$$V'(a,y) = u_c(c)(1+r) \Longrightarrow V'(a',y') = u_c(c')(1 + r_{t+1}) \tag{9}$$

Replacing into 8, we arrive to:

$$u_c(c) = -\lambda_a + \beta(1+r) \sum_{y'} \pi_{y'|y} u_c(c') \tag{10}$$

So we obtain for each case of utility specifications:

- Quadratic utility:
$$u_c(c) = \frac{1+r}{1+\rho} \sim_{y'} \pi_{y'|y} u_c(c') \tag{11}$$

- CRRA utility
$$1 = \frac{1+r}{1+\rho} \sum_{'} \pi_{y'|y} \left(\frac{u_c(c')}{u_c(c)} \right) \tag{12}$$

2.1 The infinitely-lived household economy

This question is solved in the python code *Wealth model.ipynb* uploaded in the same box that the present pdf.

2.2 the life-cycle model

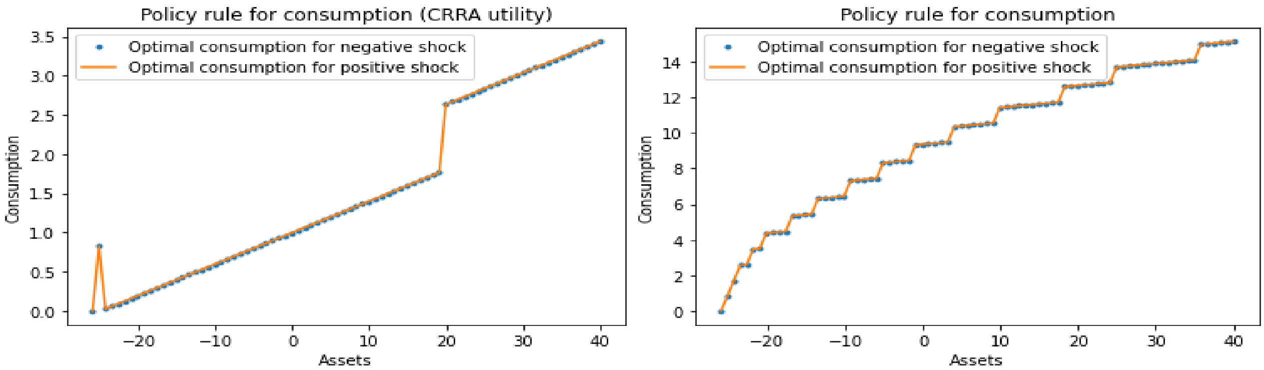
This question is solved in the python code *Wealth model.ipynb* uploaded in the same box that the present pdf

2.3 Partial equilibrium

2.3.1 With certainty

This question is solved in the python code *Partial Equilibrium.ipynb* uploaded in the same box that the present pdf.
Let we fit $\sigma = 2$ and $\bar{c} = 100$

(a) CRRA Utility function (b) Quadratic Utility function



(c) CRRA Utility function time profile (d) Quadratic Utility function time profile

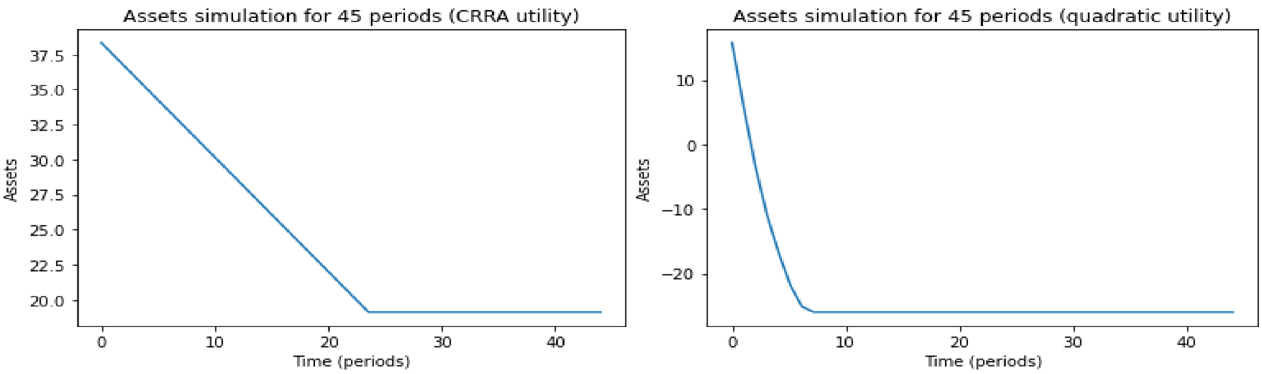


Figure 1: Consumption for different levels of assets

The figure 1 correspond to the part 1 of the exercise of certainty. For the utility function CRRA, we see in (c) that the consumption is decreasing with the assets in both cases the consumption go to zero. For the quadratic utility (b) there are no more differences in the policy rule for consumption, also we can observe the concavity in the quadratic because the Inada condition does not hold. In both case the agents starts with a large amount of assets, that consume in the first periods and then. they need to borrow and consume zero to return the debt.

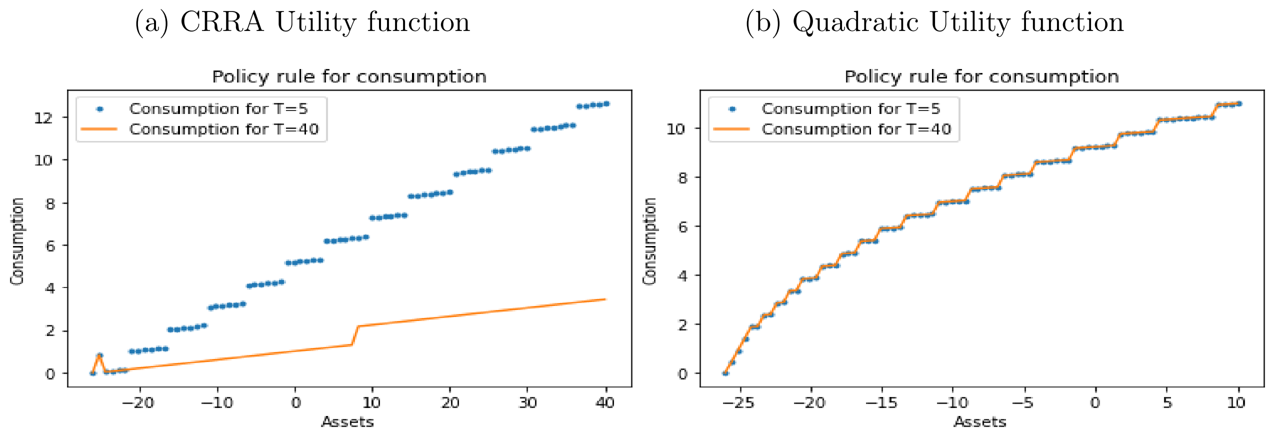


Figure 2: Consumption for different levels of assets at age 5 and 40

In the figure 2, we observe that young people in CRRA form consume more, this have sense since in the previous exercise we showed that people wanted to consume more today. In the Quadratic for, due to the form of the function both individuals consume similar. This result is always for given initial level of assets, we can put restrictions over the borrow.

2.3.2 With uncertainty

1. Now, we change the value of $\sigma_y = 0.1$ and γ keeping constant.

   So as we can observe in figure 3 and 4, we can no observe a clear differences between both, I think i due to the parameters. Con respect to the time paths, we can observe in both cases decrease with the time, so the individual is paying all of debts.

2. Present and compare representative simulated time paths of consumption for the certainty equivalence and precautionary saving economy T=45

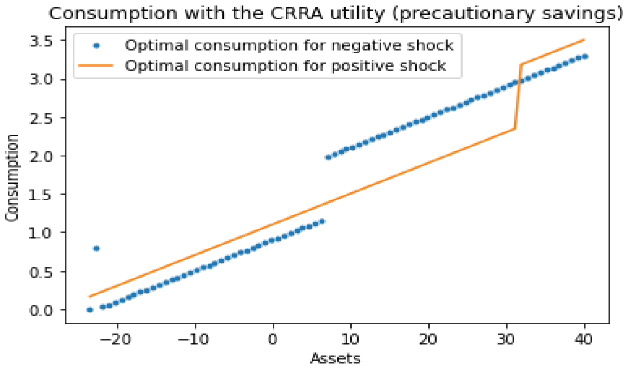
3. Now, we change and plot for different values of $\sigma = \{2, 5, 20\}$

   When we increases sigma the consumption level decreases with respect to infinity life. In figure 5 we can also compare between 5 and 20, there is no big differences between both utility functions.

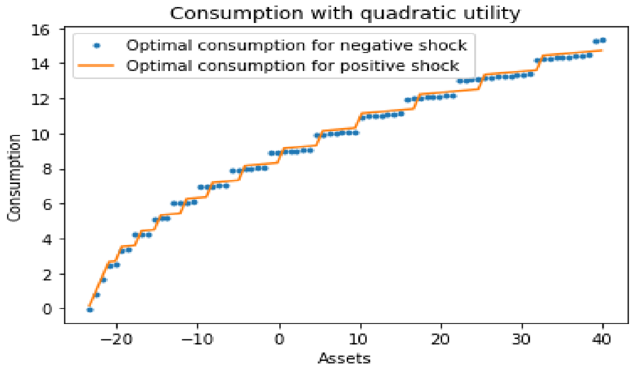
4. Increase the variance shock from $\sigma_y = 0.1$ to $\sigma_y = 0.5$

   In figure 6 only the slope change for the CRRA comparing with infinitive life whereas in the case on quadratic the functions are practically the same.

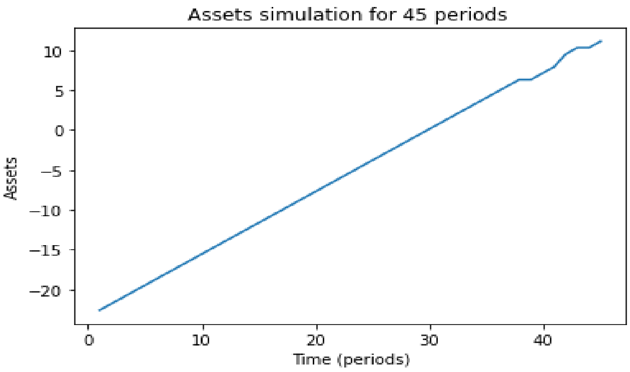
(a) CRRA Utility function



(b) Quadratic Utility function



(c) CRRA Utility function time profile



(d) Quadratic Utility function time profile

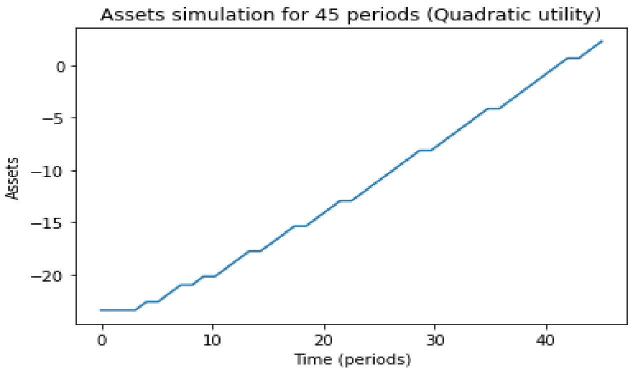
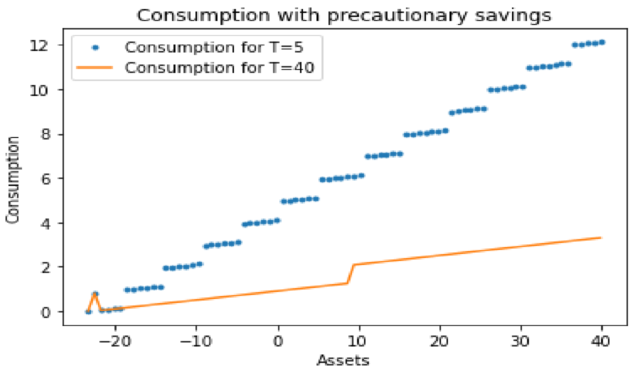


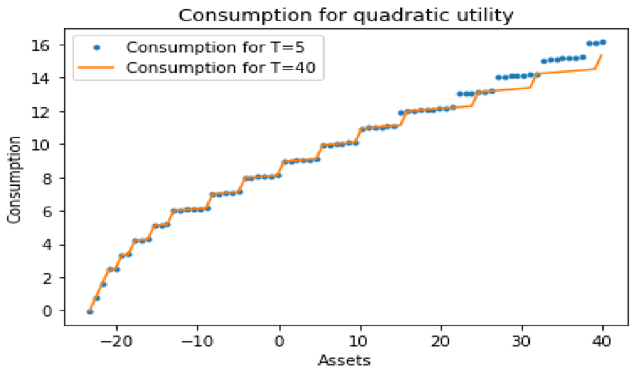
Figure 3: Consumption for different levels of assets and and shock y for an infinitive life economy

5. Increase the persistence of the income shocks from $\gamma = 0$ to $\gamma = 0.95$

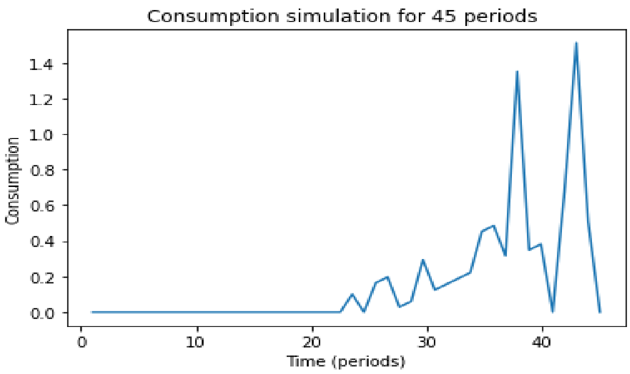
(a) CRRA Utility function



(b) Quadratic Utility function



(c) CRRA Utility function time profile



(d) Quadratic Utility function time profile

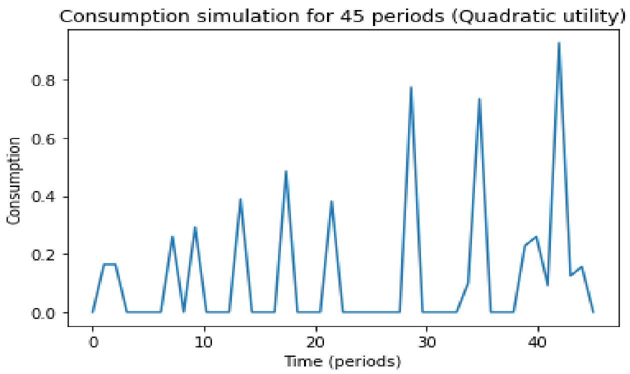
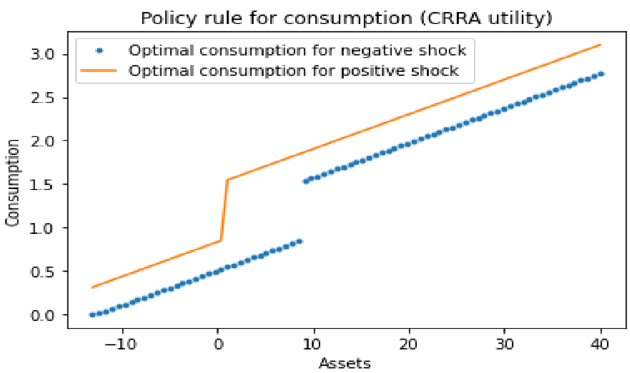
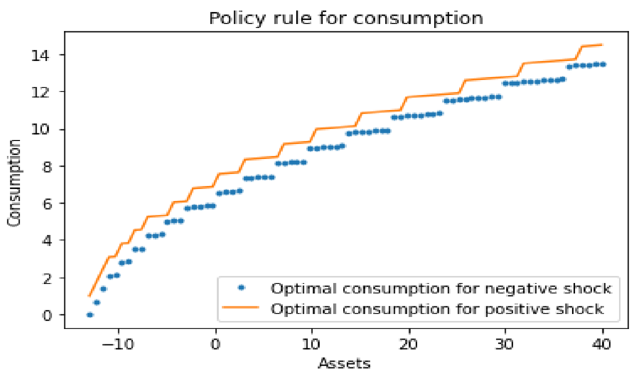


Figure 4: Consumption for different levels of assets and shock y and age 5 and 40

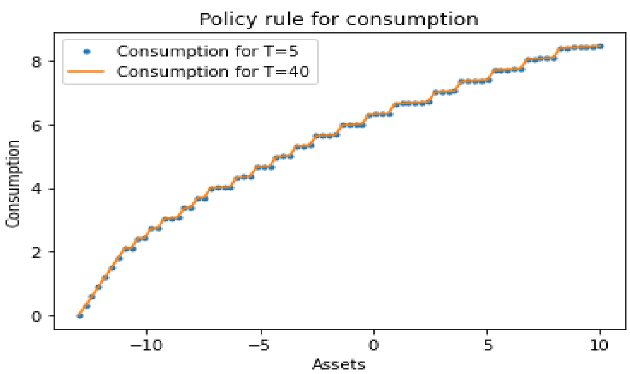
(a) CRRA Utility function



(b) Quadratic Utility function time profile



(c) CRRA Utility function



(d) Quadratic Utility function

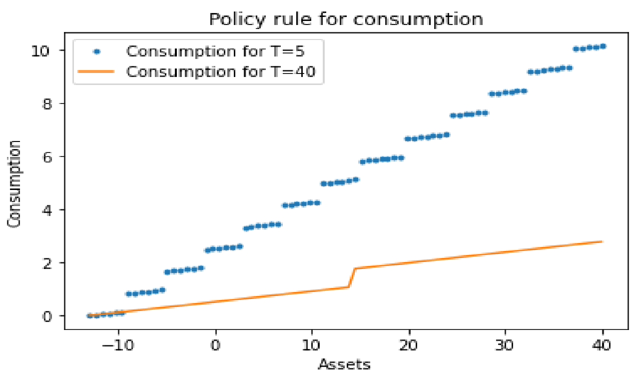
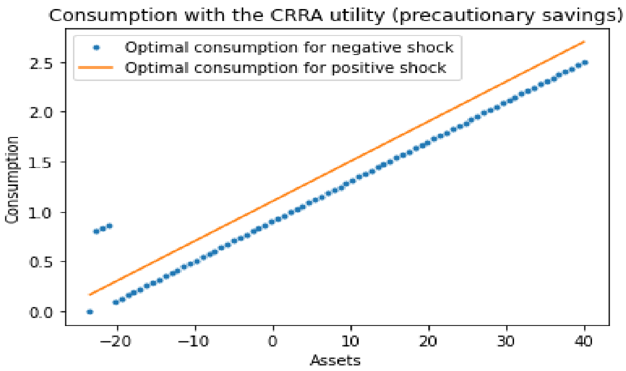
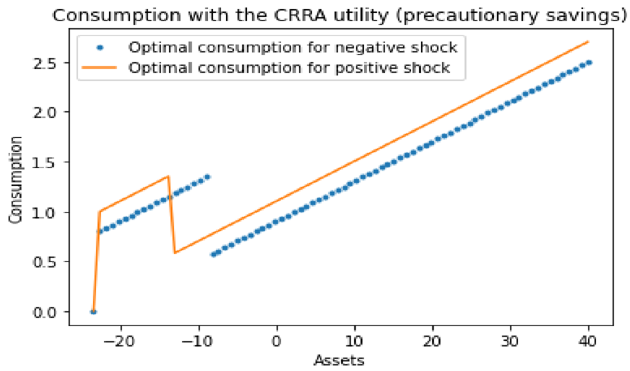


Figure 7: Consumption for different levels of assets and shock y and age 5 and 40 and $\sigma_y = 0.5$

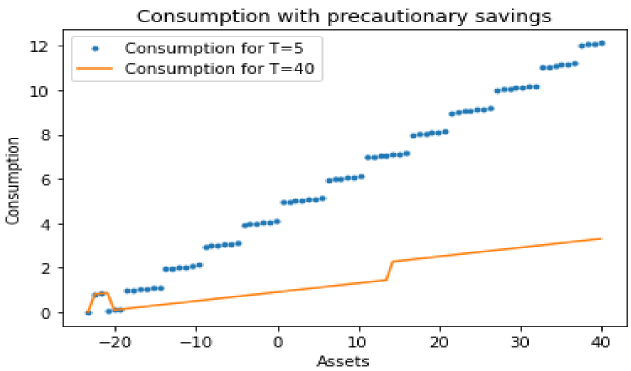
(a) CRRA Utility function $\sigma = 5$



(b) CRRA Utility function $\sigma = 20$



(c) CRRA Utility function $\sigma = 5$



(d) CRRA Utility function $\sigma = 20$

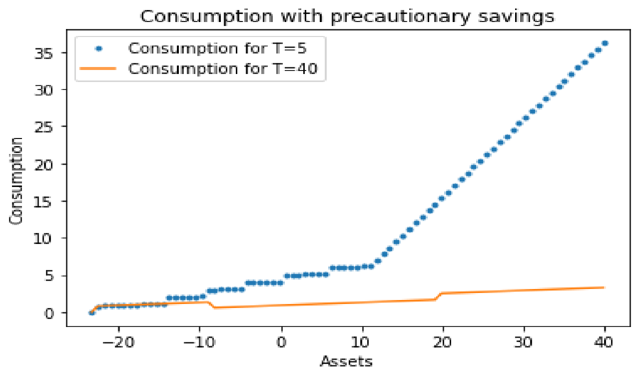


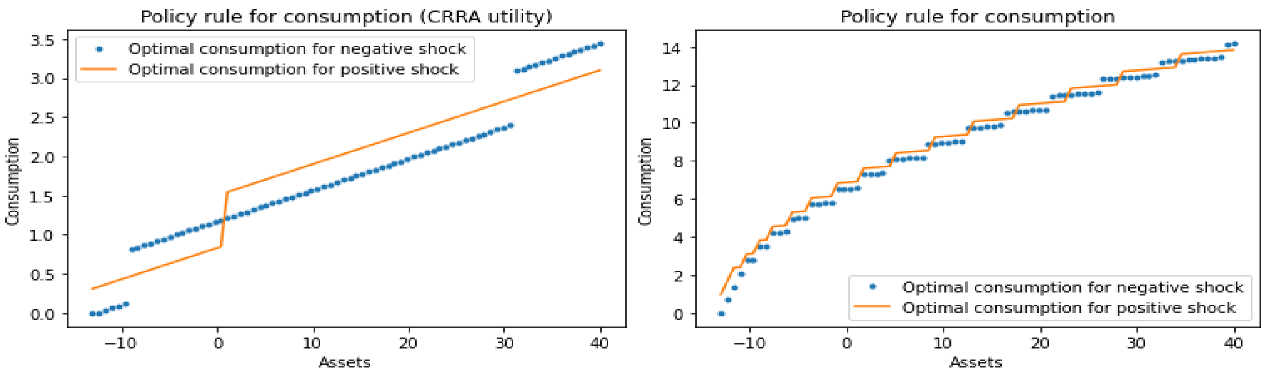
Figure 5: Consumption for different levels of assets and and shock y with differents levels of σ and with T=45

In figure 7, the reaction to a positive and negative shock are very different since the variance now is more higher, the shock is more persistent for both case.

3 General Equilibrium: The simple ABHI model

The General Equilibrium is solved directly in the python code *General Equilibrium.ipynb* uploaded in the same box that the present pdf

(a) CRRA Utility function (b) Quadratic Utility function time profile



(c) CRRA Utility function (d) Quadratic Utility function

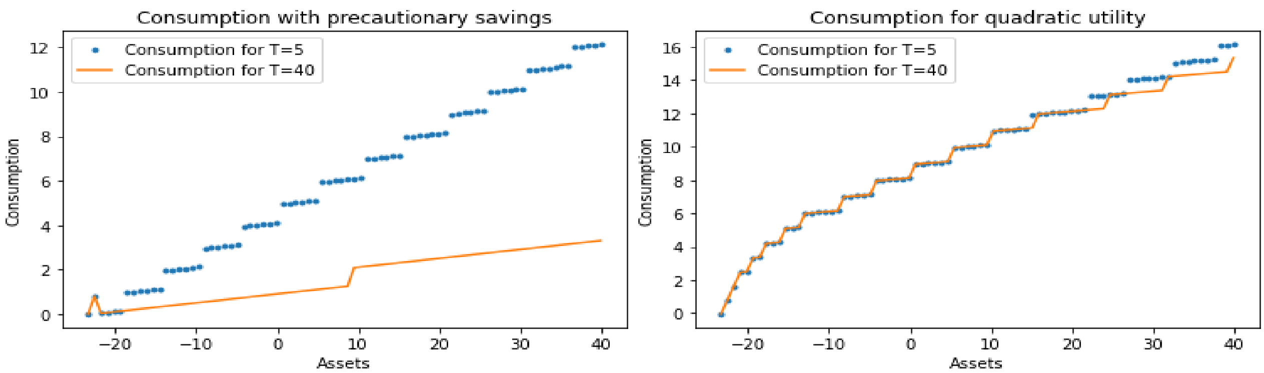


Figure 6: Consumption for different levels of assets and shock y and age 5 and 40