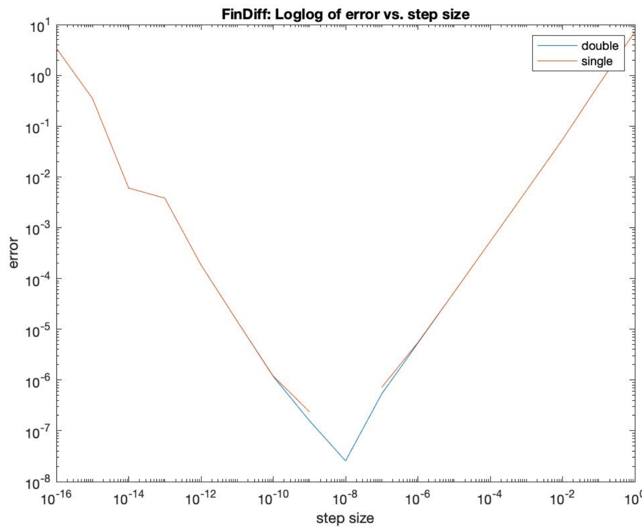
README and source code are in two other files.

1 Problem 1

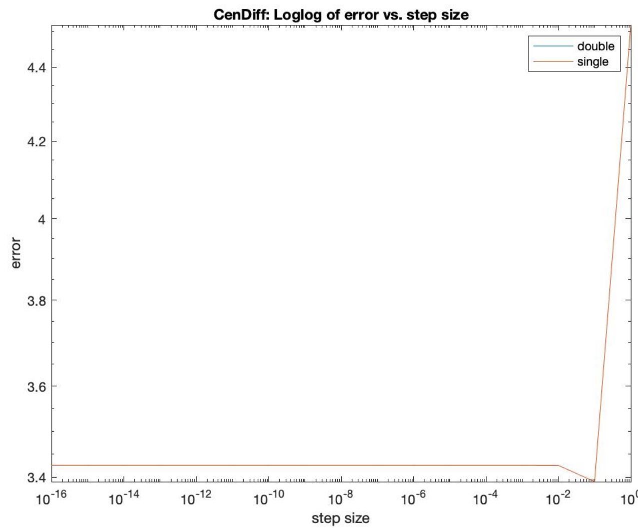
1.1 (a)-(b)



The minimum value is around 10^{-8}. It occurs when $h \approx \sqrt{10^{-16}} = 10^{-8}$.

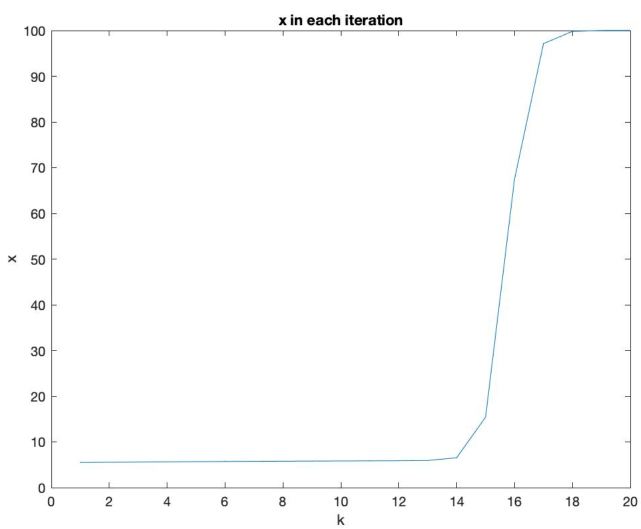
1.2 (c)

Double-precision curve coincides with the single-precision curve.



2 Problem 2

x converges to 6 for the first 14 iterations but after $x_k > 6$, the value goes to 100. This is because when $x_k > 6$, $(1130 - 3000/x_{k-1})/x_k$ becomes smaller. Then, the sequence no longer converges to 6.



3 Problem 3

3.1 (a)

Problem 3

(a) $x = \dfrac{2c}{-b \mp \sqrt{b^2-4ac}}$ vs. $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$ax^2 + bx + c = 0 \Rightarrow a + \frac{b}{x} + \frac{c}{x^2} = 0. \Rightarrow a + c\left(\frac{b}{cx} + \frac{1}{x^2}\right) = 0$$

$$c\left(\frac{b}{cx} + \frac{1}{x^2}\right) + c\left(\frac{b}{2c}\right)^2 = -a + c\left(\frac{b}{2c}\right)^2$$

$$c\left(\frac{b}{cx} + \frac{1}{x^2} + \left(\frac{b}{2c}\right)^2\right) = -a + \frac{b^2}{4c}$$

$$c\left(\frac{1}{x} + \frac{b}{2c}\right)^2 = \frac{b^2-4ac}{4c}$$

$$\frac{1}{x} + \frac{b}{2c} = \pm\sqrt{\frac{b^2-4ac}{4c^2}} = \pm\frac{\sqrt{b^2-4ac}}{2c}$$

$$x = \frac{2c}{-b \pm \sqrt{b^2-4ac}}$$

The alternative formula is more accurate when a is much smaller than b, or $a = 0$.

3.2 (b)

coefficients			reference solutions		approximate solutions	
a	b	c	root 1	root 2	root 1	root 2
6	5	-4	$1/2$	$-4/3$	**0.500000000000000**	**-1.333333333333333**
6e154	5e154	-4e154	$1/2$	$-4/3$	**Inf**	**-Inf**
0	1	1	-1	-	**NaN**	**-Inf**
1	-1e5	1	9.999999999e4	1.0000000001e-5	**9.999999999000000e+04**	**1.000000338535756e-05**
1	-4	3.999999	2.00100000000007	1.99899999999993	**2.001000000000070**	**1.998999999999930**
1e-155	-1e155	1e155	1	-	**Inf**	**-Inf**

3.3 (c)

The program checks if $a = 0$, and $abs(a) > maxint$. It handles linear equations and overflow (underflow) problems separately. The source code is in **.m** file.

coefficients			reference solutions		approximate solutions	
a	b	c	root 1	root 2	root 1	root 2
6	5	-4	1/2	$-4/3$	**0.500000000000000**	**-1.333333333333333**
6e154	5e154	-4e154	1/2	$-4/3$	**0.500000000000000**	**-1.333333333333333**
0	1	1	-1	-	**-1**	**-**
1	-1e5	1	9.999999999e4	1.0000000001e-5	**9.999999999000000e+04**	**1.000000338535756e-05**
1	-4	3.999999	2.00100000000007	1.99899999999993	**2.001000000000070**	**1.998999999999930**
1e-155	-1e155	1e155	1	-	**1**	**-**

4 Problem 4

Problem 4

(a) $x_k = \alpha = 1$

$g(x_k) = 1 + (1-1)^2 = 1 = x_{k+1} = x_{k+2} \cdots$

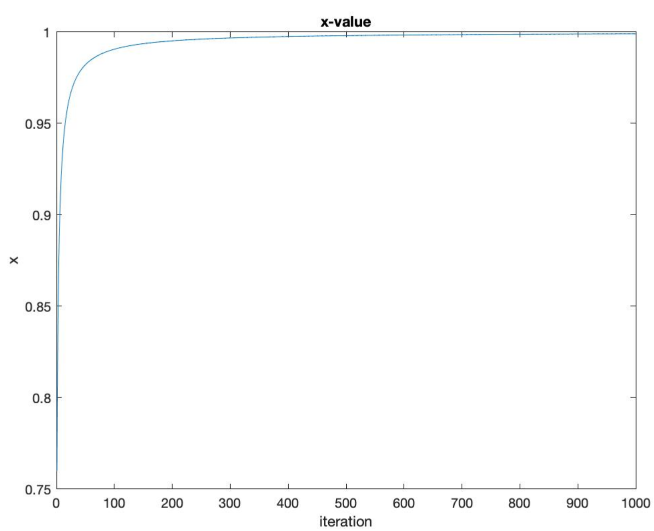
Hence, $\alpha = 1$ is a fixed point.

$|g'(1)| = |1 + 2(\cancel{1}-1)| = 1$.

(b) let $x_k = 1-\varepsilon$. $x_{k+1} = g(x_k) = (1-\varepsilon) + (1-\varepsilon-1)^2 = (1-\varepsilon)+\varepsilon^2$

$$\frac{|x_{k+1}-1|}{|x_k-1|} = \frac{|1-\varepsilon+\varepsilon^2-1|}{|1-\varepsilon-1|} = \frac{|\varepsilon^2-\varepsilon|}{|\varepsilon|} = 1-\varepsilon \quad \text{for } 0<\varepsilon<1$$

Hence, sublinear.



5 Problem 5

Problem 5

(a) Consider secant method

$$x_{k+1} = x_k - f(x_k) \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$$

$$= \frac{\cancel{x_k f(x_k)} - x_k f(x_{k-1}) - \cancel{x_k f(x_k)} + x_{k-1} f(x_k)}{f(x_k) - f(x_{k-1})}.$$

$$= \frac{x_{k-1} f(x_k) - x_k f(x_{k-1})}{f(x_k) - f(x_{k-1})}.$$

Given that $f(x) = 0 \quad \forall x$, the secant method is mathematically equivalent to

$$x_{k+1} = \frac{x_{k-1} f(x_k) - x_k f(x_{k-1})}{f(x_k) - f(x_{k-1})}.$$

if no numerical error is considered.

(b)

Consider

$$x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$$

given in 5.5.4.

When x_k converges to a stationary point, $x_k - x_{k-1}$ is nearly zero. In this case, the above formula results in numerical instability.

When $f(x_k)$ is nearly a constant, the formula in part(a) has a numerically unstable numerator.

6 Problem 6

Problem 6

$$J(x) = \begin{bmatrix} 1 & 0 \\ x_2 & x_1 \end{bmatrix}$$

$\det J = x_1$

Let $x = [0 \quad 1]^T$. Then, $\det J(x) = 0$.

Newton's method fails because J^{-1} does not exist.