Regression

Given examples $\langle \mathbf{X}, \mathbf{y} \rangle$ a *regression task* is to predict

- a continous $\mathbf{y}$
- from a vector of features $\mathbf{x}$

This differs from a *Classification* task (e.g., predicting the digit represented by an image)

- where the $\mathbf{y}$ are *discrete* values

To be concrete: imagine we need to predict the Price $\hat{\mathbf{y}}$ of a house given only its Size $\mathbf{x}$.

We could imagine an approach similar to the KNN algorithm used for classification

- compare $\mathbf{x}$ to each $\mathbf{x}^{(\mathbf{i})}$ in the training set $\mathbf{X}$
  - measure the "distance" from $\mathbf{x}$ to $\mathbf{x}^{(\mathbf{i})}$ to come up with a weight
- predict $\hat{\mathbf{y}}$ as the weighted average of the $\mathbf{y}^{i}p$

A strong criticism of KNN is that Θ, the parameters, comprised all m training examples

- large
- memorization versus generalization

The fact that $\mathbf{y}$ is *continous* rather than discrete opens the possibiity of a *numerical* relationship between features $\mathbf{x}$ and labels $\mathbf{y}$.

We will take advantage of this in our first Regression model.

Linear Regression

Our first predictor/estimator/model is called Linear Regression.

Linear Regression restricts the form of relationship between $\mathbf{y}$ and $\mathbf{x}$ to

$$\hat{\mathbf{y}} = \Theta^T \cdot \mathbf{x}$$

That is: the predicted $\hat{\mathbf{y}}$ is a linearly-weighted (with weights from vector Θ) sum of features $\mathbf{x}$.

Anyone who has fit a straight line to a cloud of points has performed Linear Regression.

A straight line has intercept Θ_0 and slope Θ_1

$$\hat{\mathbf{y}} = \Theta_0 + \Theta_1 * \mathbf{x}_1$$

In our example

- we expect the Price to increase with Size $\mathbf{x}_1$
    - Θ_1 tells us how much each extra unit of Size increases the Price

Rather than writing the intercept Θ_0 as a separate term we can modify $\mathbf{x}$ and $\boldsymbol{\Theta}$

$$\begin{aligned} \boldsymbol{\Theta}^T &= (\Theta_0, \Theta_1) \\ \mathbf{x}'^T &= (1, \mathbf{x}_1) \end{aligned}$$

so that the straight line may be written as

$$\hat{\mathbf{y}} = \boldsymbol{\Theta}^T \cdot \mathbf{x}'$$

Because the size of Θ^T and $\mathbf{x}$ must match

- we augmented $\mathbf{x}$ with a "constant" feature 1
    - that corresponds to the intercept

The real power of Linear Regression can be seen when there is more than 1 non-constant feature.

- Predict Price given features Size, Number of bedrooms, Number of bathrooms, Proximity to transportation
- Θ_j tells us how much each unit increase in feature $\mathbf{x}_j$ affects Price.

The prediction $\mathbf{y}$ is linear in each feature $\mathbf{x}_j$, hence the name *linear* regression

Anyone recognize this expression: $\Theta^T \cdot \mathbf{x}$?

It's our friend the dot product, as promised in the introductory lecture.

Watch out, this will be a regularly recurring character in our series.

Linear Regression in matrix form

We will typically augment $\mathbf{x}$ with the leading "constant feature 1" to capture the intercept.

$$\begin{aligned} \mathbf{\Theta}^T &= (\mathbf{\Theta}_0, \mathbf{\Theta}_1, \ldots, \mathbf{\Theta}_n) \\ \mathbf{x}'^T &= (1, \mathbf{x}_1, \ldots, \mathbf{x}_n) \end{aligned}$$

We do this for each example in $\mathbf{X}$ so that $\mathbf{X}$ becomes

$$\mathbf{X}' = \begin{pmatrix} 1 & \mathbf{x}_1^{(1)} & \dots & \mathbf{x}_n^{(1)} \\ 1 & \mathbf{x}_1^{(2)} & \dots & \mathbf{x}_n^{(2)} \\ \vdots & \vdots & \dots & \vdots \\ 1 & \mathbf{x}_1^{(m)} & \dots & \mathbf{x}_n^{(m)} \end{pmatrix}$$

We sometimes refer to $\mathbf{X}$ as the *design matrix*.

So we could simultaneously obtain our prediction for *all* training examples by the matrix product

$$\hat{\mathbf{y}} = \mathbf{X}' \Theta$$

Using matrix notation

- mimics an implementation using a language(such as `numPy`) with matrix arithmetic
- allows us to evaluate examples in parallel

Examples

Some examples

- Predict the Price of a stock given Earnings ($||\mathbf{x}|| = 1$)
- Predict the Price of a stock given Earnings, Dividend, and Sales ($||\mathbf{x}|| = 3$)