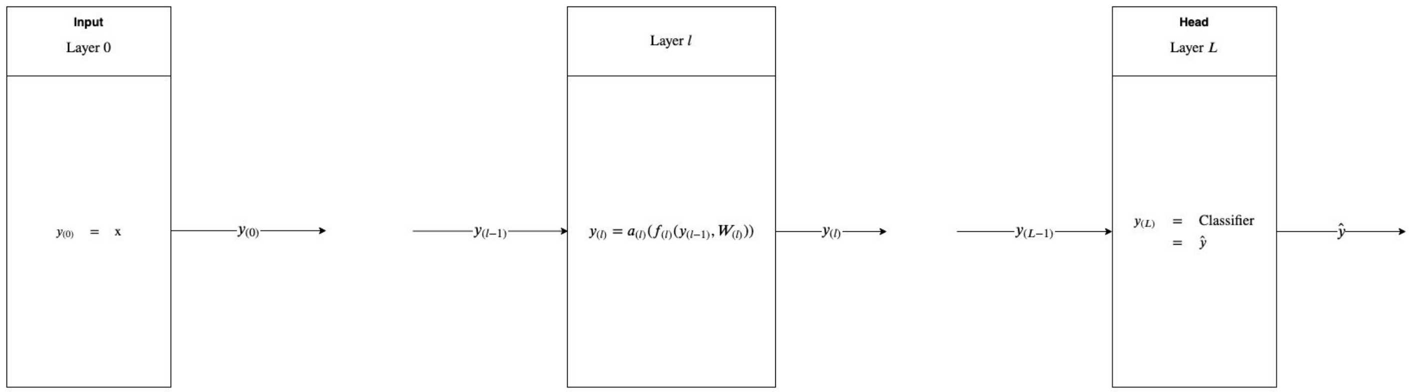
Interpretting Representations: Preview

We have described an L layer (Sequential) Neural Network as

- a sequence of tranformations of the input
    - each transformation a *layer* $1 \le l \le (L-1)$, producing a new *representation* $\mathbf{y}_{(l)}$
- that feed the final representation $\mathbf{y}_{(L-1)}$ to a *head* (classifier, regressor)

Layers


Input
Layer 0
$y_{(0)} = \mathrm{x}$
$y_{(0)}$
$y_{(l-1)}$
Layer l
$y_{(l)} = a_{(l)}(f_{(l)}(y_{(l-1)}, W_{(l)}))$
$y_{(l)}$
$y_{(L-1)}$
Head
Layer L
$y_{(L)} = \text{Classifier} = \hat{y}$
$\hat{y}$

Is it possible to *interpret* each representation $\mathbf{y}_{(l)}$?

- What do the new "synthetic features" mean ?
- Is there some structure among the new features ?
    - e.g., does each feature encode a "concept"

We will briefly introduce the topic of Interpretation.

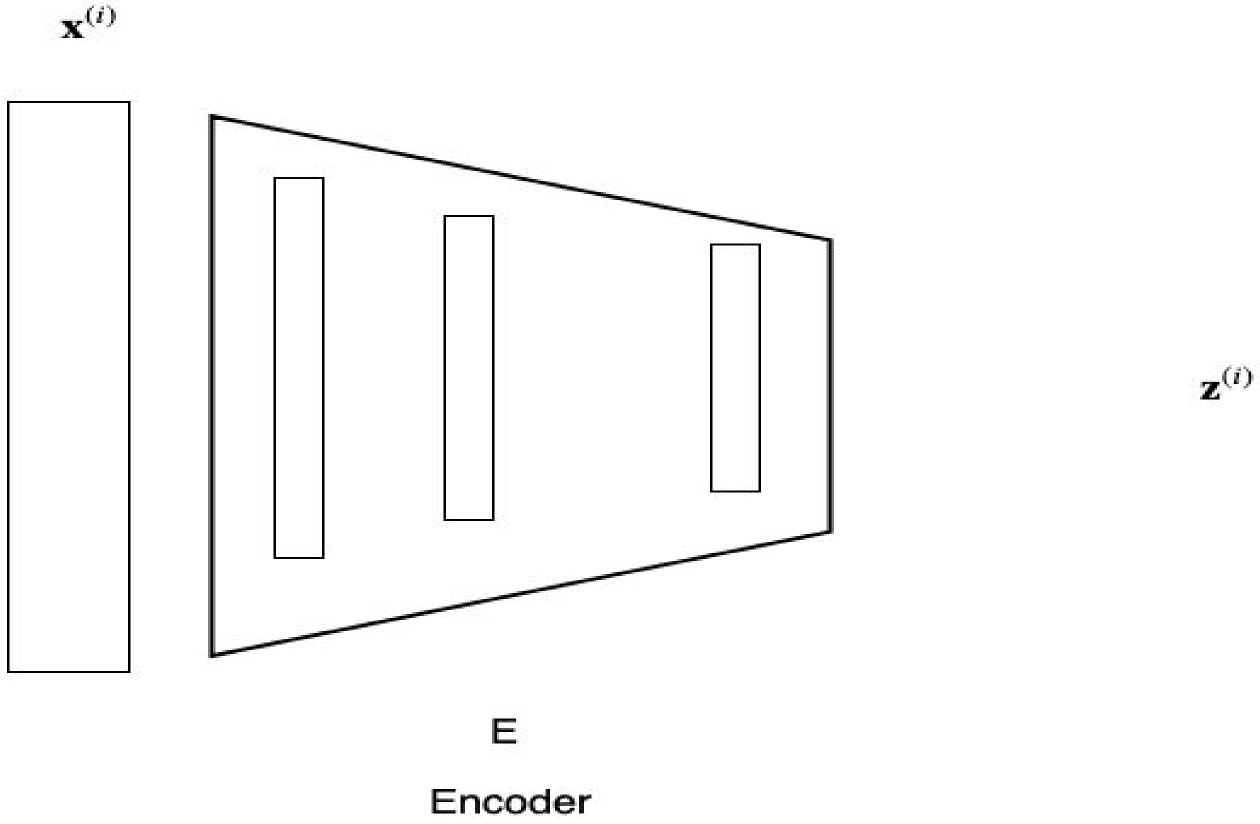
A deeper dive will be the subject of a later lecture.

Our goal, for the moment, is to motivate Autoencoders.

Interpretation 1: Clustering of examples

One way to try to interpet $\mathbf{y}_{(l)}$ is relative to a dataset $\langle \X, \y \rangle = [\x^\ip, y^\ip | 1 \le i \le m]$

- Compute $\mathbf{y}_{(l)}^{\mathbf{(i)}}$ by presenting $\mathbf{x}^{\mathbf{(i)}}$ to the NN
- Create a scatter plot (of dimension $n_{(l)} = |\mathbf{y}_{(l)}|$)
    - locate $\mathbf{y}_{(l)}^{\mathbf{(i)}}$ in the $n_{(l)}$-dimensional plot
    - label it with it's label $\mathbf{y}^{\mathbf{(i)}}$



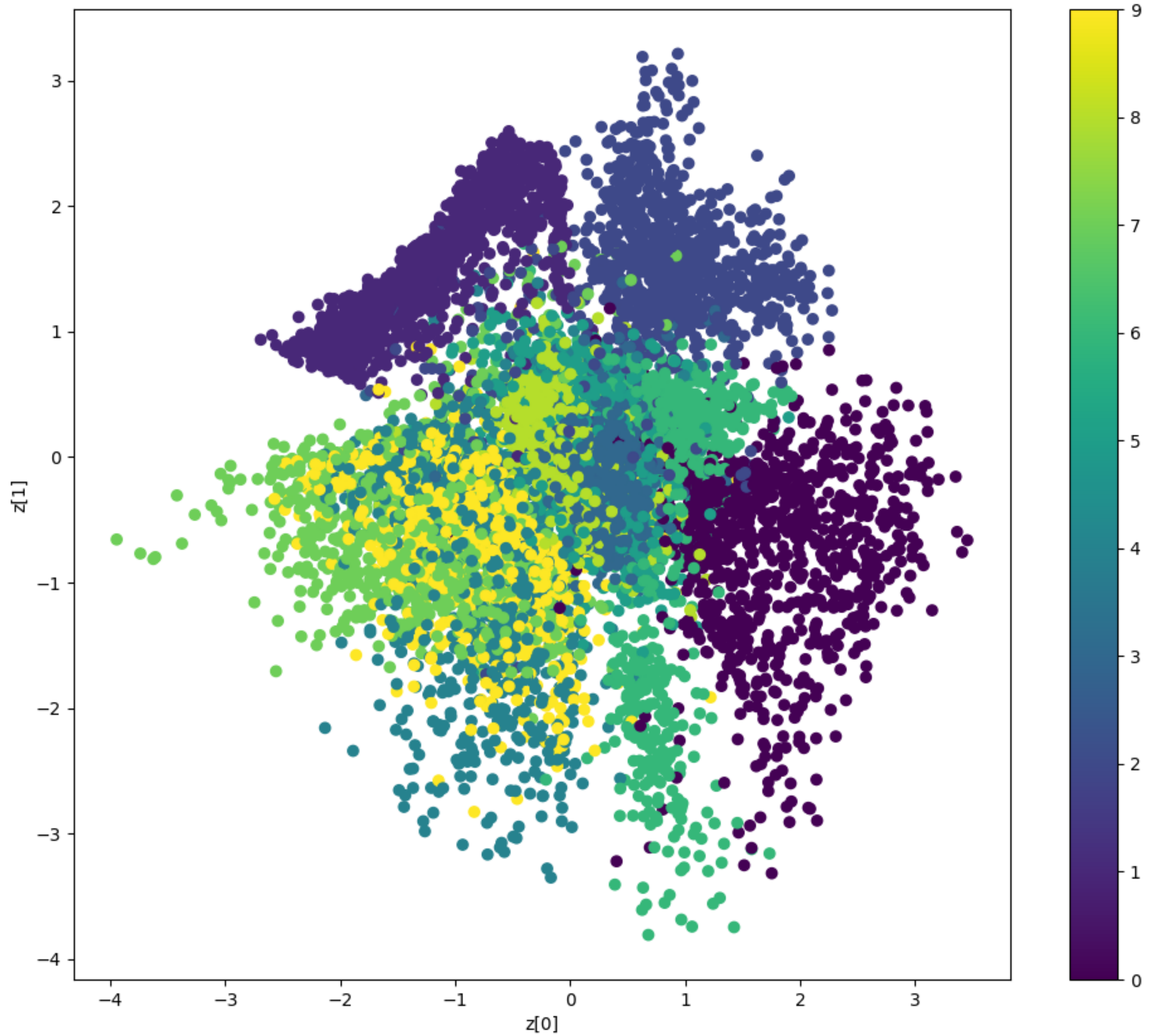
$\mathbf{x}^{(i)}$
$\mathbf{z}^{(i)}$
E
Encoder

Do examples with identical labels form recognizable clusters ?

If so, perhaps we can interpret synthetic feature $\mathbf{y}_{(l),j}$

- according to how variation in $\mathbf{y}_{(l),j}$ affects the set of examples $\mathbf{X}$

MNIST clustering produced by a VAE



z[0]
z[1]
−4
−3
−2
−1
0
1
2
3

- Each point is an example $\mathbf{x^{(i)}}$
- The color corresponds to the label $\mathbf{y^{(i)}}$
- Axes are the first two synthetic features

You can see that some digits form tight clusters.

By understanding

- the clusters
- how the digit label's vary as a synthetic feature vaires

we might be able to infer meaning to the synthetic features.

The first two synthetic features may correspond to properties of those digits

- digits with "tops"
- digits with "curves"

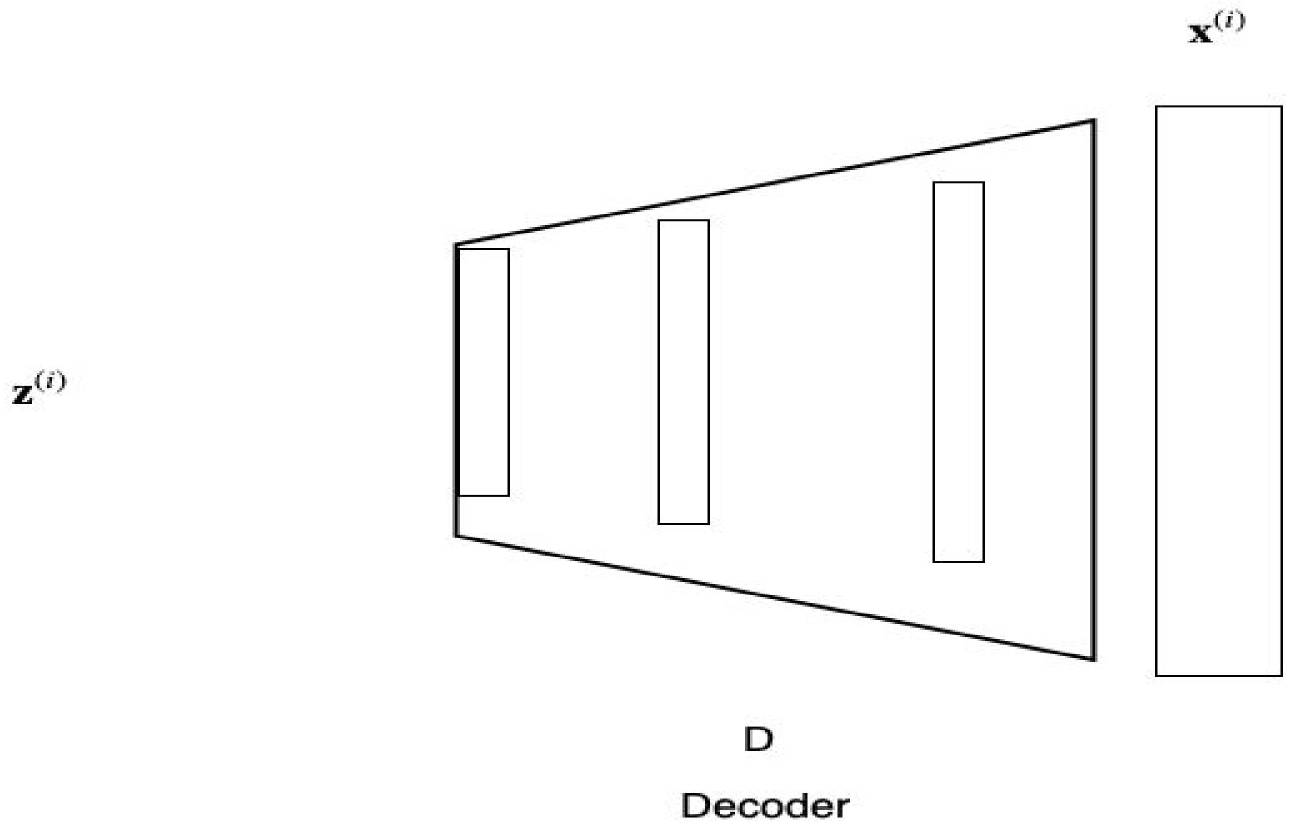
Note

This is not too different from trying to interpret Principal Components:

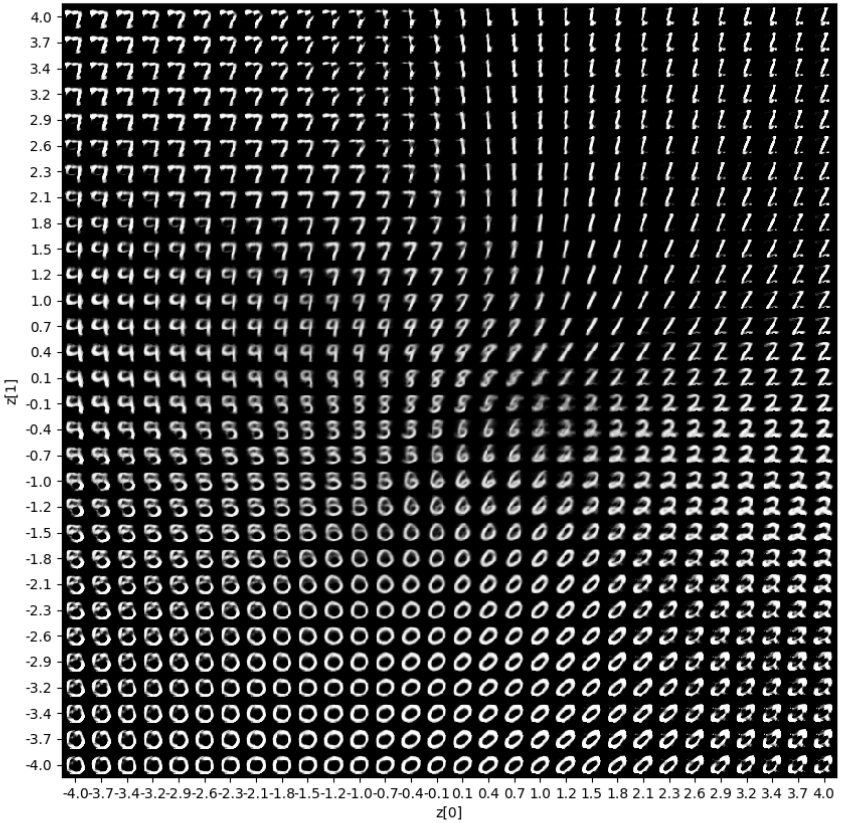
Interpretation 2: Examining the latent space

Another method

- Create an $n_{(l)}$ dimensional grid of evenly spaced values of $\mathbf{y}_{(l)}$
- Let $\mathbf{y}_{(l)}^{(i')}$ be such a value
  - **Note** this is **not necessarily** an example produced from an $\mathbf{x} \in \mathbf{X}$
- Map $\mathbf{y}_{(l)}^{(i')}$ to some value in the input representation $\mathbf{x}^{(i')}$
  - **Note** this is **not necessarily** an example from $\mathbf{X}$
  - But presenting $\mathbf{x}^{(i')}$ to the NN results in $\mathbf{y}_{(l)} = \mathbf{y}_{(l)}^{(i')}$


$\mathbf{x}^{(i)}$
$\mathbf{z}^{(i)}$
D
Decoder

MNIST clustering produced by a VAE



- Axes are the first two synthetic features
- For $\mathbf{y}_{(l)}$ at a given grid point:
    - find a value in the input representation that maps to this grid point

Note that there is *no reason* to expect that the inversion of an arbitrary representation *looks like* a digit

- it merely has the correct shape
- unless we impose some constraints

This is **not** just a different view of the first plot:

- we are able to infer a pseudo-input for a grid point $\mathbf{y}_{(l)}^{(i')}$ that corresponds to **no actual input** in $\mathbf{X}$
    - For example, we infer a digit from an uninhabited region of the grid of the first plot

Some observations (with possible intepretation)

- Does the first synthetic feature control slant ?
    - Examine 0's along bottom row
- Does the second synthetic feature control "curviness" ?
    - Examine the 2's column at the edge, from bottom to top

In order for this method to work, we must be able to *invert* $\mathbf{y}_{(l)}$.

We will show how to do this in a later lecture.

Deja vu: have we seen this before ?

These two methods of interpretation have been encountered in an earlier lecture

- mapping original features $\mathbf{x}^{(\mathbf{i})}$ to synthetic features $\tilde{\mathbf{x}}^{(\mathbf{i})}$
- inverting synthetic feature $\tilde{\mathbf{x}}^{(\mathbf{i})}$ to obtain original feature $\mathbf{x}^{(\mathbf{i})}$

Principal Component Analysis (PCA) !

PCA is an Unsupervised Learning task that can be used for

- dimensionality reduction
- clustering

The key to it's intepretability was the simplicity of transforming and inverting

$$
\begin{aligned}
\mathbf{X} &= U\Sigma V^T && \text{SVD decomposition of } \mathbf{X} \\
\tilde{\mathbf{X}} &= \mathbf{X}V && \text{tranformation to synthetic features} \\
\mathbf{X} &= \tilde{\mathbf{X}}V^T && \text{inverse tranformation to original features}
\end{aligned}
$$

The transformation V via matrix multiplication is *linear*.

We will explore *non-linear*, *invertable* transformations during our study of Autoencoders.

In [4]:

```
print("Done")
```

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Done
```