

How does the NN "learn" the transformations ?

The matrix \mathbf{W} contains the "patterns" that serve to recognize the synthetic features created by each layer

- $\mathbf{W}_{(l),j}$ are the weights /pattern for feature $\mathbf{y}_{(l),j}$

How are these patterns discovered ?

The answer is: exactly as we did in Classical Machine Learning

- Define a loss function that is parameterized by \mathbf{W} :

$$\mathcal{L} = L(\hat{\mathbf{y}}, \mathbf{y}; \mathbf{W})$$

- Per example loss $\mathcal{L}^{(i)}$

- Average loss $\mathcal{L} = \frac{1}{m} \sum_{i=1}^m \mathcal{L}^{(i)}$

- Our goal is to find \mathbf{W}^* the "best" set of weights

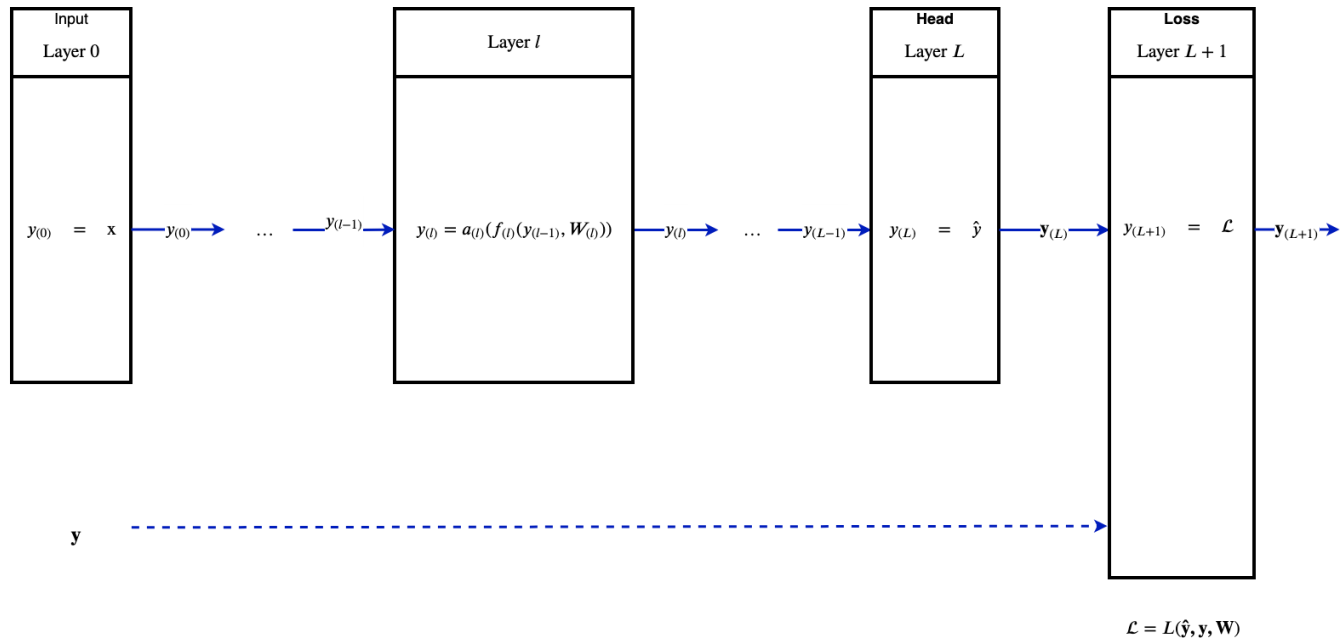
$$\mathbf{W}^* = \underset{\mathbf{W}}{\operatorname{argmin}} L(\hat{\mathbf{y}}, \mathbf{y}; \mathbf{W})$$

- Find \mathbf{W}^* using Gradient Descent !

Very much in spirit of the multi-layer architecture

- We add a new layer (L+1) to compute the loss \mathcal{L} !

Additional Loss Layer (L+1)



Gradient Descent review

Gradient Descent is an iterative method for finding the minimum of a function.

- See the [Gradient Descent lecture \(Gradient_Descent.ipynb\)](#) in the Classical ML part of the course for more details

Let's review Gradient Descent using our current notation

- We start with an initial guess for \mathbf{W} and iteratively improve it.
- Compute the loss \mathcal{L} given the current \mathbf{W}
 - Average loss of the m examples in the training examples
- Compute the gradient

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}}$$

- Update \mathbf{W} in the direction of the *negative* of the gradient
- Scaled by a learning rate α

$$\mathbf{W} = \mathbf{W} - \alpha * \frac{\partial \mathcal{L}}{\partial \mathbf{W}}$$

A unit change in \mathbf{W} increases \mathcal{L} by $\frac{\partial \mathcal{L}}{\partial \mathbf{W}}$

- That's why there is a negative sign: we proceed in the direction *opposite* the one that increases \mathcal{L}
- We move only a fraction $\alpha \leq 1$ of the (negative) of the gradient
- To avoid the possibility of over-shooting the minimum

W is a multi-dimensional vector, not a scalar

- So the gradient is multi-dimensional
- See a Deeper Dive for background on matrix gradients

The loss \mathcal{L} is averaged over all m training examples.

This can be expensive to compute.

We can approximate \mathcal{L} by *sampling* from the m training examples

- Choose a *random subset* (of size $m' \leq m$) of examples: $I = \{i_1, \dots, i_{m'}\}$
- Approximate \mathcal{L} on I

$$\mathcal{L} \approx \frac{1}{|I|} \sum_{i \in I} \mathcal{L}^{(i)}$$

Minibatch gradient descent

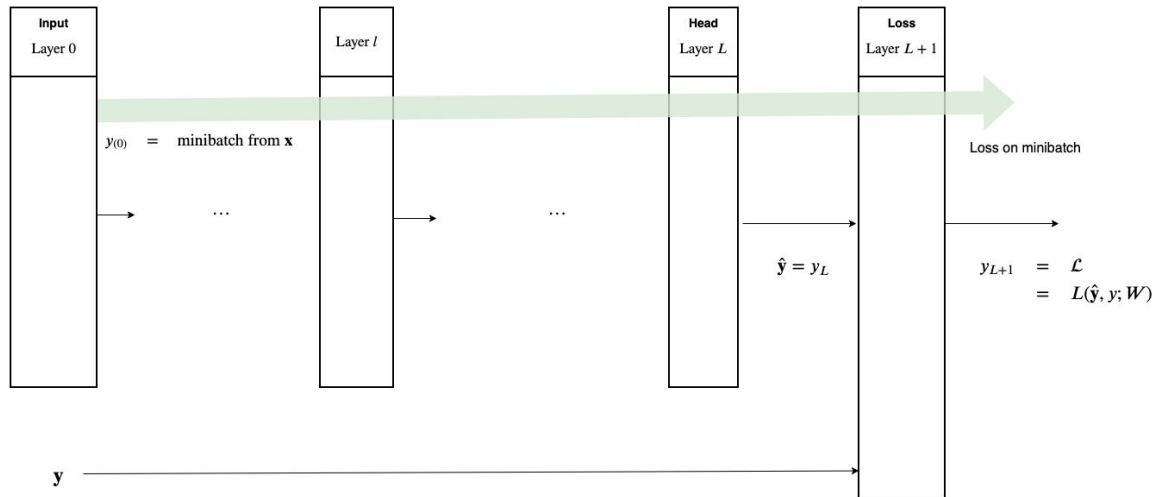
- Divides the m training examples
- Into $b = m/m'$ disjoint batches of $m' \leq m$ examples each

- Iterates over the batches
 - Approximate the loss on the current batch
 - Update \mathbf{W} according to

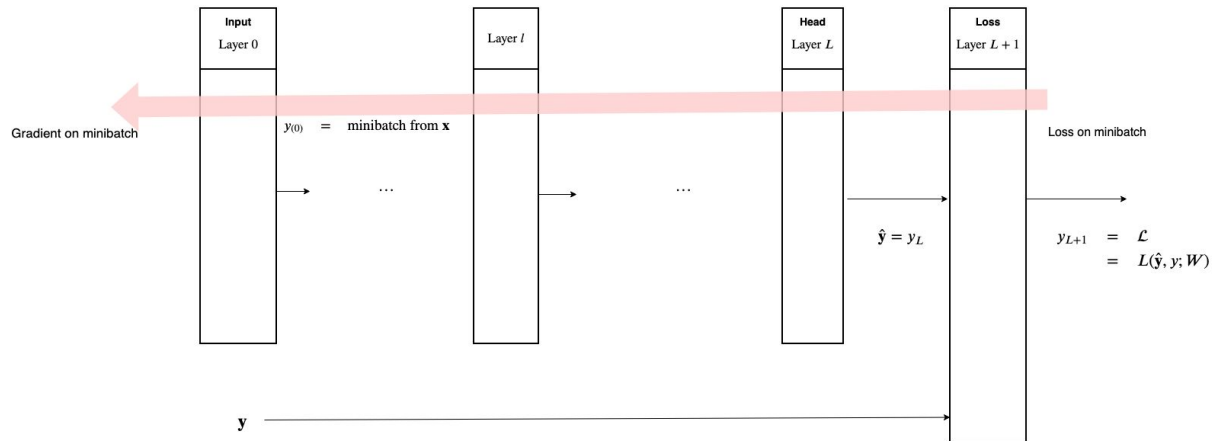
$$\mathbf{W} = \mathbf{W} - \alpha * \frac{\partial \mathcal{L}}{\partial \mathbf{W}}$$

- Repeat until all the batches have been processed

Minibatch: Forward Pass From minibatch to Loss



Minibatch: Backwards Pass From minibatch Loss to Gradient



Thus, Minibatch Gradient Descent

- Examines *all* m training examples
- In batches of size $m' \leq m$
- Resulting in $b = m/m'$ updates to \mathbf{W} for each complete pass through the m training examples

Minibatch Gradient Descent is faster than a single batch of size m

- Update \mathbf{W} b times, rather than once
- A complete pass through the b mini-batches is called an *epoch*

The Training loop

A single epoch of Gradient Descent encounters all m examples and makes b updates

We may need additional epochs to continue to drive down the Loss.

This iterative process is called the *training loop*.

Here is some pseudo-code:


```
initialize(W) # Training loop to implement mini-batch SGD for epoch in range(n_epochs):` for X_batch,  
y_batch in next_batch(X_train, y_train, batch_size, shuffle=True): # Forward pass y = NN(X_batch) # Loss  
calculation loss = loss_fn(y, y_batch) # Backward pass grads = gradient(loss, W) # Update W = W - grads *  
learning_rate
```

It used to be the case that this fairly standard training loop was coded for each problem.

Just as `sklearn` wrapped common code into a high-level API

- We will use a toolkit that hides the training loop behind a high level API

Scaling the inputs

Many times in this course we have pointed out that some models are *scale sensitive*.

Neural Networks are not *mathematically* sensitive but tend to be so *in practice*.

It is *highly recommended* to scale your data so their absolute values are around 1.0 or at least somewhat small.

Gradient Descent is the root of the problem:

- Two features on different scales can cause the optimizer to favor one over the other
- Activations can *saturate*
 - Output of dot product (Dense layer) is in the "flat*" area of the activation
 - Zero derivative: no learning
- The Cost/Loss may be large in initial epochs when the target values are too different from the dot products
 - *Large* gradients: unstable learning
 - Weights are typically initialized to values less than 1.0, leading to small dot products

Remember: if you re-scale the inputs, you will need to invert the transformation when communicating the results

In [5]: `print("Done")`

Done