Chaos Control on a Boundary Layer Model

Luis Carlos González Sua
e-Robots Researcher
ITESM
Monterrey, Mexico
e-mail: A00798796@itesm.mx

Rogelio Soto Rodríguez
e-Robots Chief Advisor
ITESM
Monterrey, Mexico
e-mail: rsoto@itesm.mx

***Abstract*—Friction on airfoils causes several losses on cruise flight, however, this friction is necessary to avoid stall during take offs and landings, but on cruise flight is unnecessary and also counterproductive. It seems that the chaotic behavior on the boundary layer of an airfoil is responsible of the friction. In this article the beginnings to control the chaotic state of the boundary layer are proposed; in it describes the process of investigate a model, analyze it and check if its behavior is chaotic. Also includes the study of the model's response to different inputs in order to select the best variable for control and finally the design of a fuzzy controller to manipulate the model to a non-chaotic state.**

Keywords-component; boundary layer; chaos control; fuzzy logic

I. INTRODUCTION

"Where chaos begins, classical science stops" [1]; currently chaos is a relatively new science, it very beginnings date from 1960's when it was accidentally discovered by Edward Lorenz with his meteorological model.

The chaotic systems are characterized by their sensibility at the initial conditions, their nonlinearity and their stochastic appearance. Despite of all the characteristics that make them so hard to comprehend or analyze, there is still a possibility to control them but it is necessary the use of a very robust controller like the fuzzy ones.

Fuzzy have proven to be one of the most popular AI technologies [2]; it has been widely used in mass production, industrial and home products and it has spawned a very profitable industry [2]. Developed by Lofti A. Zadeh is basically an imprecise logical system, in which the truth-values are fuzzy subsets of the unit interval with linguistic labels [3].

Between the applications of chaotic systems, there is one of particular interest, the boundary layer; it based on the concept that a fluid in contact with a surface and with high Reynolds number can be divided in two unequal zones. At the big one the fluid is inviscid, and in the small one closer to the surface the effect of the friction is dominant [4]. In the small zone, there is a turbulent behavior that can be represented using a chaotic model.

Next the process of selection and analysis of a boundary layer chaotic model and its control is explained step by step.

II. SELECTION OF A BOUNDARY LAYER MODEL

In order to select a correct representative boundary layer model, it has to be an aerodynamical model, because there is the possibility to be implemented on an Unmanned Aerial Vehicle (UAV) for initial testing.

The American Institute of Aeronautics and Astronautics (AIAA) provides a plausible model, a forced boundary layer on a flat plate, that can be a representative model of an airfoil and seems to be chaotic, but it needs further analysis to verify its sensibility to the initial conditions.

The selected model was found on [13], and the model equation can be seen on (1), (2) and (3) were the variables X is related to the vertical velocity within the boundary layer, Y is related to the fullness of the streamwise velocity profile, and Z is related to the freestream conditions, as represented on Fig. 1. Because all the variables are velocity relative, the physical unit m/s is assumed. The constants α, Re and Δ are given, the constants σ and r are calculated as shown on (4) and (5)

$$dX/dt = \pi\alpha X^2 + \sigma XY + X(\sigma/4)\,(1-\Delta\cos Z^*) \tag{1}$$

$$dY/dt = -2\alpha(3\pi^2+\alpha^2)X^2 - rY \tag{2}$$

$$dZ^*/dt = \omega \tag{3}$$

Simulations of the model were made using SIMULINK® of MATLAB®, the model were simulated with different parameters and the initial conditions were slightly changed to prove the sensibility of the model. In Fig. 2 is shown the effect of the initial conditions sensibility, both models have the same parameters, $\alpha=0.4$, $Re=16000$ and $\Delta=0.7$. At the beginning of the simulation, both systems have similar behavior, but the differences became more evident as the simulation continues, having apparently two different systems. Also in order to verify presence of strange attractors in the system is necessary the plot of phase diagrams shown in Fig. 3. The tendency of the phase diagram shows that the system surrounds a point in the graphic, revealing the presence of a strange attractor at the center of the twister.

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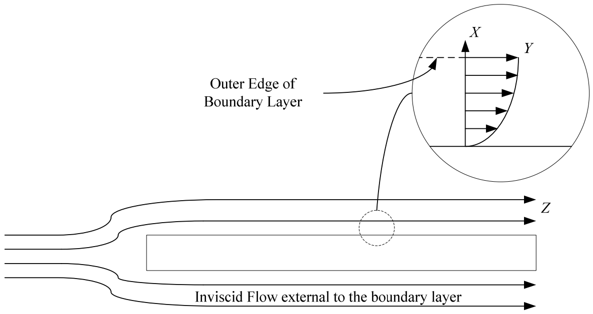

Figure 1 Boundary Layer Profile on a Flat Plate, on it shows which physical variables represent X, Y and Z.

Now that the simulations shows that the model is sensible to the initial conditions and have the presence of strange attractors, it seems to be chaotic and the next step is to analyze it in order to determine the system response to different inputs.

$$\sigma=(\pi^2\alpha)/(4\pi^2+\alpha^2) \tag{4}$$

$$r=\pi^2/Re \tag{5}$$

III. System's Response

In order to manipulate the system, an additional input has to be aggregated. The selected input to be introduced in the model is the Compensatory Delta (Δ_C); the additional input has been chosen to compensate the effect of Δ, because it is the responsible of the chaotic behavior. The resulting model equations are shown in (6), (7) and (8).

$$dX/dt=\pi\alpha X^2+\sigma XY+X(\sigma/4)\,(1-(\Delta+\Delta_c)\cos Z^*) \tag{6}$$

$$dY/dt=-2\alpha(3\pi^2+\alpha^2)X^2-rY \tag{7}$$

$$dZ^*/dt=\omega \tag{8}$$

Simulations were made with the resulting model and the most representative results are shown in Fig. 4, Fig. 5, and Fig. 6; they show a comparative of the model without the additional input ($\Delta=0.6$, $\Delta=0.7$ and $\Delta=0.7$ respectively) and with the additional input ($\Delta_C=0.55$, $\Delta_C=0.65$ and $\Delta_C=0.75$). For more results refer to [5].

Results indicates that the additional input can reduce the chaotic behavior of the system in some cases, however, for high values of Δ the additional input cannot compensate as shown in Fig. 5, so the right value of Δ_C has to be selected in order to control the system. Although Δ_C reduces the chaotic behavior, there is still some oscillation present on the response as shown on Fig. 4 and Fig. 6, so further analysis is needed to determine the best way to control the system.

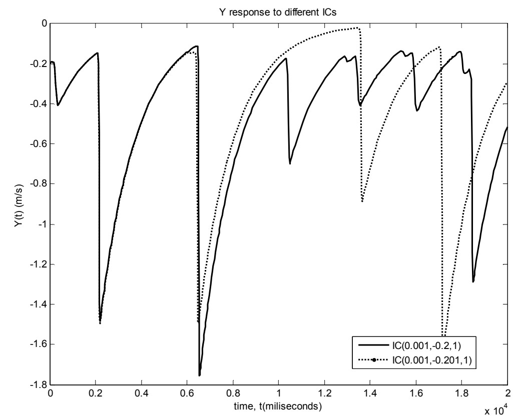

Figure 2 Y response to different ICs. The two systems have similar IC but the dotted one has *-0.201* instead of *-0.2*, look at the beginning the similar behavior but after some time they look like two totally different systems.

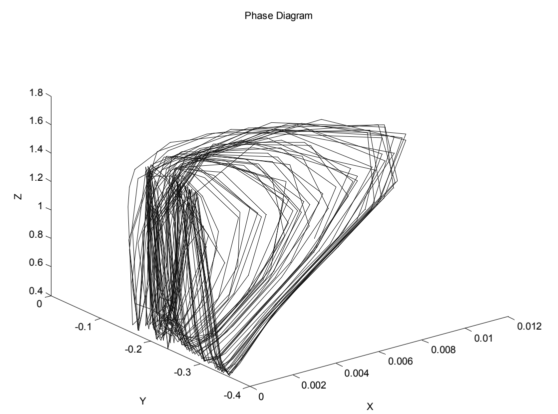

Figure 3. Phase Diagram. A Strange attractor can be visualized at the center of the twister.

IV. Designing the Controller

After the analysis of the system's response, the next step to do is to design a controller that can manipulate the system. The previous section shows that the additional input Δ_C, is capable of manipulate the system and compensate its behavior, so the logical step to do is to use this variable to control the system using a fuzzy logic controller. In Fig. 7 the block diagram of the controller is shown, the FLC (Fuzzy Logic Controller) has two inputs, the error (*e(t)*) and $\Delta(t)$, the output is $\Delta_C(t)$, the rest is similar to a convenient Closed Loop block diagram. The reason why is called FLC P+ is because is a Fuzzy Logic Controller with a proportional error entry and an additional input.

First, the controller need to identify the current value of Δ, Fig. 8 shows that the value of Δ is equal to the half of the amplitude of the output Z, that way we can use it to identify the current value of Δ, however, in order to simplify the simulations, this value is retrieved directly from the Model.

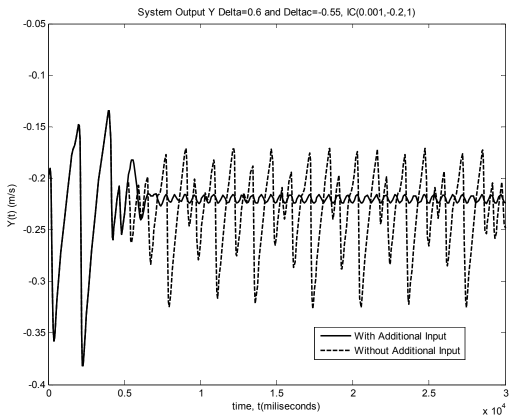

Figure 4. System Output Y *Δ=0.6* and Δ_C *=-0.55*, *IC(0.001, -0.2, 1)*. The solid line shows that the Additional Input Δ_C, can compensate the behavior of the system**.**

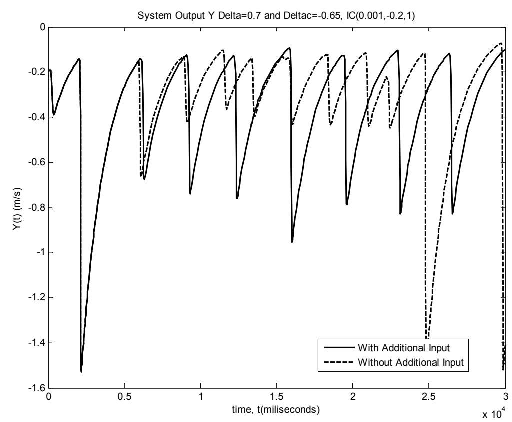

Figure 5. System Output Y *Δ=0.7* and Δ_C *=-0.65*, *IC(0.001, -0.2, 1)*. In this case, Δ_C cannot compensate the behavior.

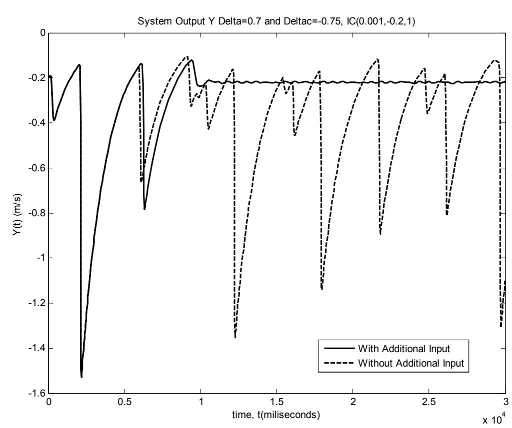

Figure 6. System Output Y *Δ=0.7* and Δ_C *=-0.75*, *IC(0.001, -0.2, 1)*. In this figure, it seems that Δ_C can compensate the behavior of the system. Also, the compensation is more drastic.

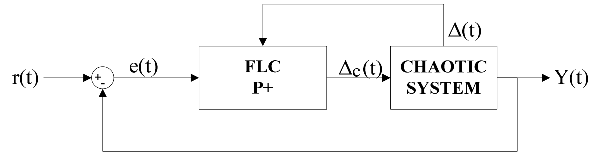

Figure 7. Closed Loop used for this application, notice that *Δ* also goes to the FLC(Fuzzy Logic Controller)

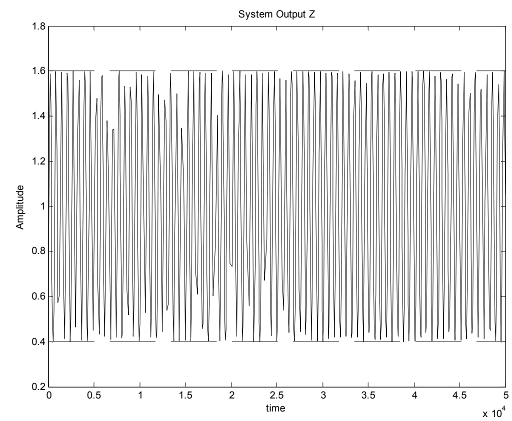

Figure 8. System Output Z. Half of the maximum amplitude is equal to the value of *Δ*.

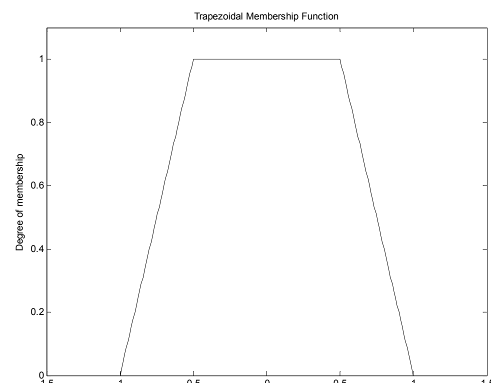

Figure 9. Trapezoidal Membership Function

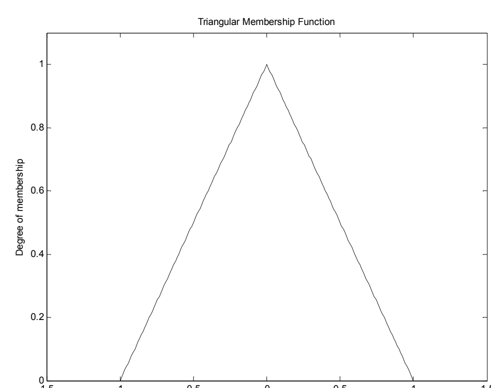

Figure 10. Triangular Membership Function

The type of membership functions selected for this controller are the pseudo-trapezoidal and the triangular shape membership functions, according to [6], the pseudo-trapezoidal membership functions are defined by (9), $a \leq b \leq c \leq d$, $a<d$, $0<H\leq 1$, $0\leq I(x) \leq 1$ is a nondecreasing function in $[a,b)$ and $0\leq D(x) \leq 1$ is a nonincreasing function in $(c,d]$.

If $b=c$ and $I(x)$ and $D(x)$ are as in (10) and (11) it becomes a triangular membership function defined by (12) and shown in Fig 10, thus the parameters of each membership function for the inputs and the output are defined in TABLE I, the range of input Δ is $(0,1)$, the range of *error* is $(-0.5,0.5)$ and the range of the output Δ_C is $(-1,1.5)$. Because of the centroid type defuzzification used in the FLC, the range of the output has to be slightly greater than the minimum-maximum range in order to reach each boundary.

$$\mu_{PTMF}(x)=\begin{cases} I(x),\ x \in [a,b) \\ H,\ x \in [b,c] \\ D(x),\ x \in (c,d] \\ 0,\ x \in R(a,d) \end{cases} \tag{9}$$

$$I(x)=(x-a)/(b-a) \tag{10}$$

$$D(x)=(x-d)/(c-d) \tag{11}$$

$$\mu_{TRIMF}(x)=\begin{cases} I(x),\ x \in [a,b) \\ H,\ x \in b \\ D(x),\ x \in (b,c] \\ 0,\ x \in R(a,c) \end{cases} \tag{12}$$

Once defined the input and outputs, the fuzzy rules of the controller has to be defined, TABLE II shows the rules proposed to the FLC P+, however the rules are subject to change in order to improve the performance of the FLC. The surface diagram shown on Fig. 11, indicates the relationship between the inputs and the output, shows a zone where the controller has to saturate the output and the irregularities in it presumes the nonlinearity of the system.

When the controller was established and tuned, the following step is to test if it can control the system, so the final controller connected as shown on Fig. 7, was simulated. Results are shown on the next section.

V. Simulation Results

In this section the results of most simulations will be shown; since the output Y is the most representative variable of the chaotic behavior of the system is one of the results to be showed, and also the Manipulated Variable is illustrated to observe the behavior of the controller in order to identify chattering or any other phenomena.

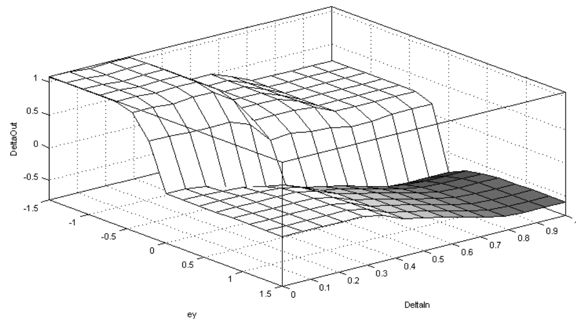

Figure 11. Surface Diagram of the controller. It shows the behavior of the controller for every input, in some cases the controller has to saturate the output.

The first pair of results exposed on Fig. 12 and Fig. 13 are the controlled Y Output of the system for $\Delta=0.55$ and the Controller Output respectively; the *2%* band [7] is represented by the dotted line. The set point was *-0.2199*, the *2%* boundary lies between *-0.2243* and *-0.2155*. The results show that the controller maintains the system in the 2% boundaries, suggesting that the system is controlled and the controller output does not oscillates too much so the actuator is not oscillating either.

The results shown on Fig. 14 and Fig. 15 shows the Controlled Output Y for $\Delta=0.75$ and the Controller Output for the same Δ, Notice that at this time the controller hardly maintains the system on the *2%* boundaries, but never reaches the *5%* boundaries (*5%* boundaries lies between *-0.2309* and *-0.2089*). Also notice that the controller output oscillates greater than the previous results indicating that in this case the actuators will be under high stress.

TABLE I Input/Output Membership Function Parameters

Input/Output	Membership Function	Type of MF and parameters
Δ	0.1	PT(0,0,0.1,0.2,1)
Δ	0.2	TRI(0.1,0.2,0.3,1)
Δ	0.3	TRI(0.2,0.3,0.4,1)
Δ	0.4	TRI(0.3,0.4,0.5,1)
Δ	0.5	TRI(0.4,0.5,0.6,1)
Δ	0.6	TRI(0.5,0.6,0.7,1)
Δ	0.7	TRI(0.6,0.7,0.8,1)
Δ	0.8	PT(0.7,0.8,1,1)
E	NB	PT(-0.5,-0.5,-0.1,-0.05,1)
E	NS	TRI(-0.1,-0.05,0,1)
E	Zero	TRI(-0.05,0,0.05,1)
E	PS	TRI(0,0.05,0.1,1)
E	PB	PT(0.05,0.1,0.5,0.5,1)
Δ_C	0.8	PT(-1,-1,-0.8,0.7,1)
Δ_C	0.7	TRI(-0.8,-0.7,-0.6,1)
Δ_C	0.6	TRI(-0.7,-0.6,-0.5,1)
Δ_C	0.5	TRI(-0.6,-0.5,-0.4,1)
Δ_C	0.4	TRI(-0.5,-0.4,-0.3,1)
Δ_C	0.3	TRI(-0.4,-0.3,-0.2,1)
Δ_C	0.2	TRI(-0.3,-0.2,-0.1,1)
Δ_C	0.1	TRI(-0.2,-0.05,0.1,1)
Δ_C	S	TRI(0,0.25,0.5,1)
Δ_C	M	TRI(0.25,0.5,0.75,1)
Δ_C	B	PT(0.5,0.8,1.5,1.5,1)

PT for Pseudo Trapezoidal
TRI for Triangular

TABLE II Rules of the FLC P+

Δ\e	NB	NS	Zero	PS	PB
0.1	G	M	0.1	0.1	0.1
0.2	G	M	0.2	0.1	0.1
0.3	G	M	0.3	0.2	0.1
0.4	M	P	0.4	0.3	0.2
0.5	M	P	0.5	0.4	0.3
0.6	M	P	0.6	0.5	0.4
0.7	P	P	0.7	0.6	0.5
0.8	P	P	0.8	0.7	0.6

Rules may be changed during tuning.

In order to show more situations, Fig. 16 shows an alternative simulation using a random generator for the system's *Δ*. Notice that even without a constant *Δ*, the controller can maintain the system between the boundaries of 2%, however, Fig. 17 shows the controller output and is visible the chattering.

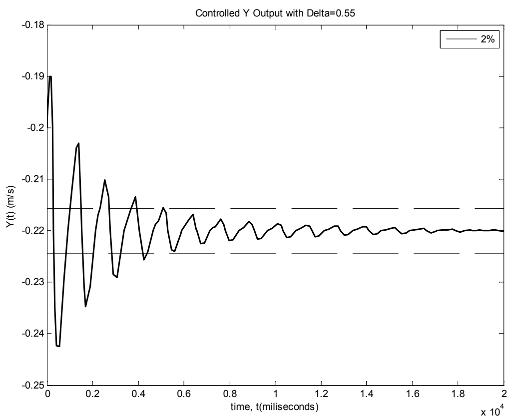

Figure 12. Controlled Y Output for *Δ=0.55*. The 2% band is represented by the dotted line.

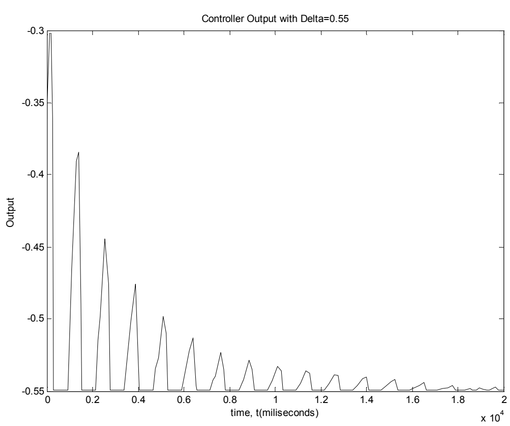

Figure 13. Controller Output for *Δ=0.55*. Notice that the output slightly oscillates.

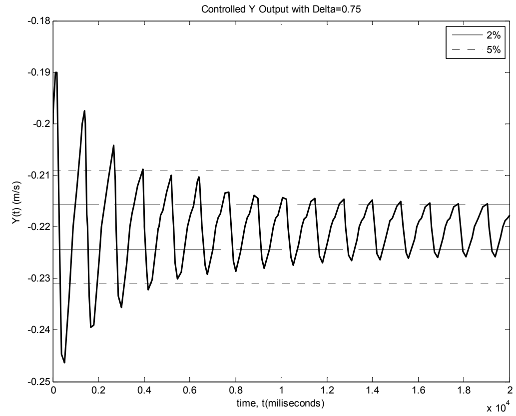

Figure 14. Controlled Output for *Δ=0.75*. The dotted line indicates the *2%* boundaries and the slashed dotted line represents the 5% boundaries.

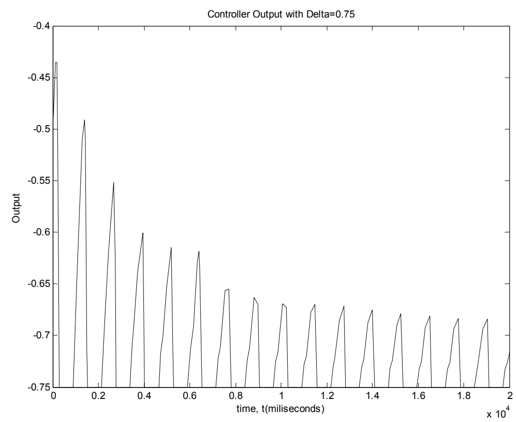

Figure 15. Controller Output for *Δ=0.75*. Notice that the oscillation increases.

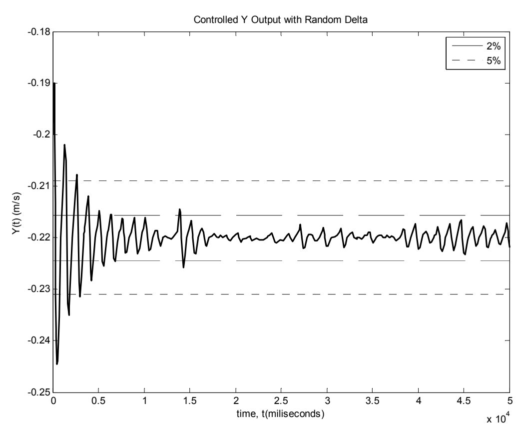

Figure 16. Controlled Y Output for Random *Δ* Between *0.2* and *0.8*. Notice that the variable remains in the *2%* boundaries.

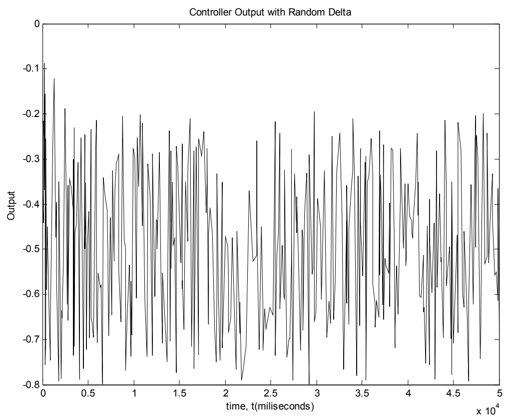

Figure 17. Controller Output for Random *Δ* between *0.2* and *0.8*. The manipulator is under stress because the system is constantly changing.

VI. Conclusions and Future Work

It's been demonstrated that a chaotic system can be controlled in some part, but it became harder when the chaotic behavior abruptly changes, also is even harder to make the systems goes to a reference, but maybe because of their chaotic behavior it's not recommended since it possibly causes instabilities or other aerodynamically issues.

The benefits of a research of this kind are numerous, the reduction of the friction on cruise flight can be translated as a reduction flying costs, as suggested in [8], and any reduction of the fuel consumption can be translated as less greenhouse gas emissions, millions of dollars in operation costs savings and a better use of aircrafts and aerial vehicles.

For future work can be proposed a better and more accurate model that can represent the behavior of an airfoil according to the National Advisory Committee for Aeronautics (NACA) standards, design a new controller for that model and then a possible implementation on an UAV. Also, other types of control strategies can be implemented in order to improve the controller response, robustness and performance.

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