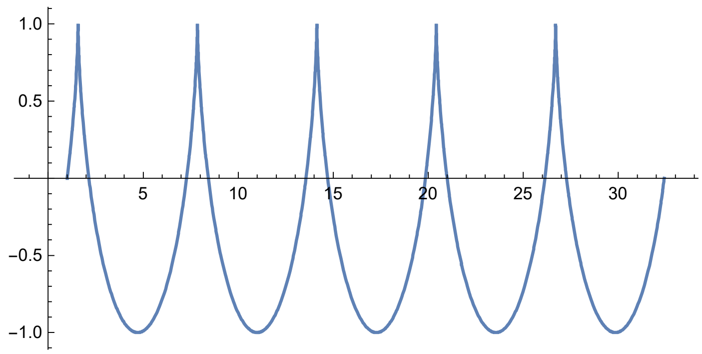
In[ ]:=
```
s[t_] := {Cos[t] + t, Sin[t]};
s[π / 4]
```

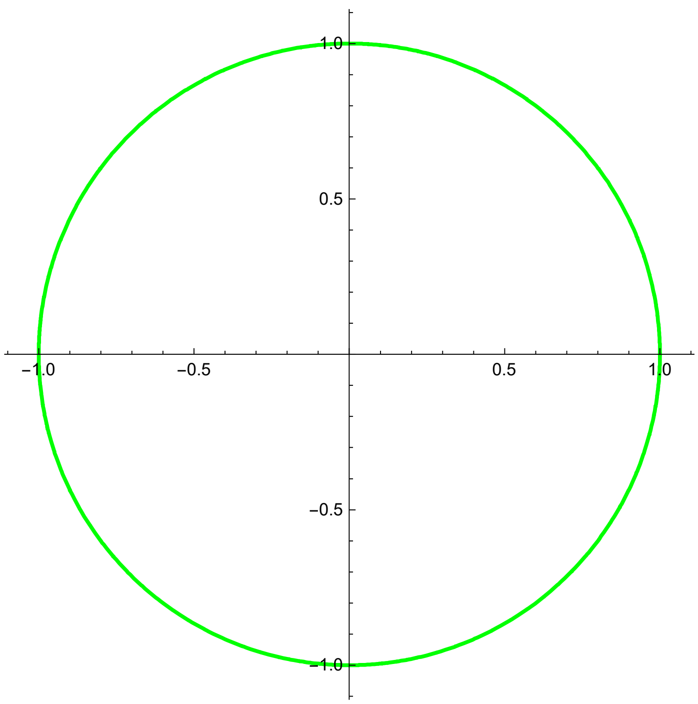
Out[ ]= $\left\{\frac{1}{\sqrt{2}} + \frac{\pi}{4}, \frac{1}{\sqrt{2}}\right\}$

In[ ]:=
```
ParametricPlot[s[t], {t, 0, 10 π}, AspectRatio → 1 / 2]
```

Out[ ]= 

In[ ]:=
```
Clear[s, t]
```

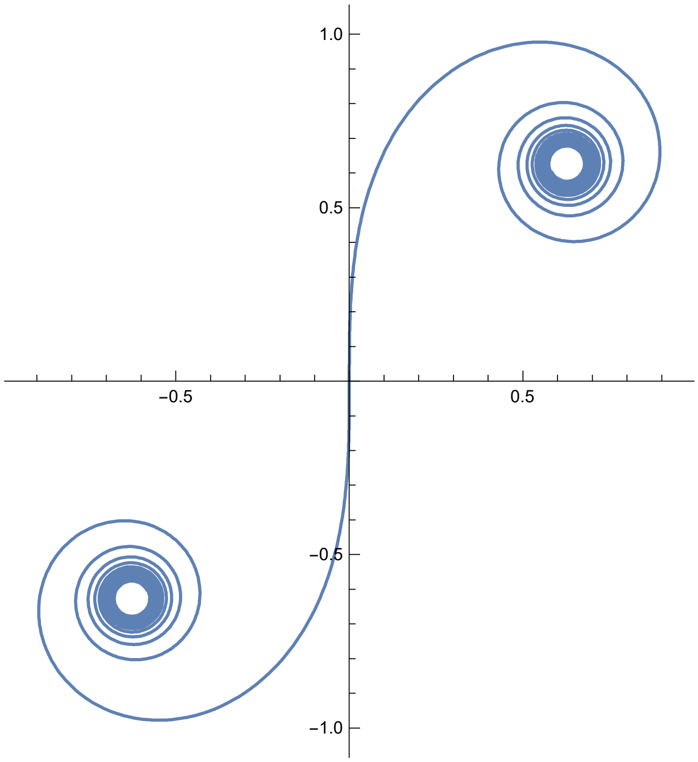
In[ ]:=
```
s[t_] := {Cos[t], Sin[t]};
ParametricPlot[s[t], {t, 0, 2 π}, PlotStyle → Directive[Thick, Green]]
```

Out[ ]= 

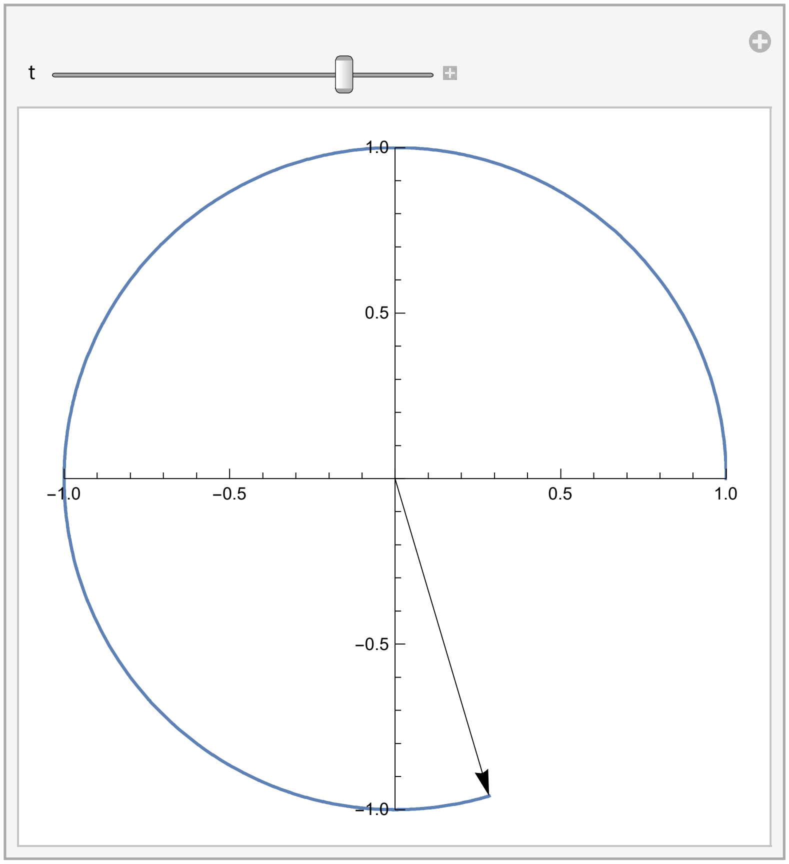
In[ ]:=
```
Clear[s, t]
```

In[ ]:=
```
c[t_] = {∫_0^t Sin[u^2] ⅆu, ∫_0^t Cos[u^2] ⅆu};
ParametricPlot[c[t], {t, -10, 10}]
```

Out[ ]=

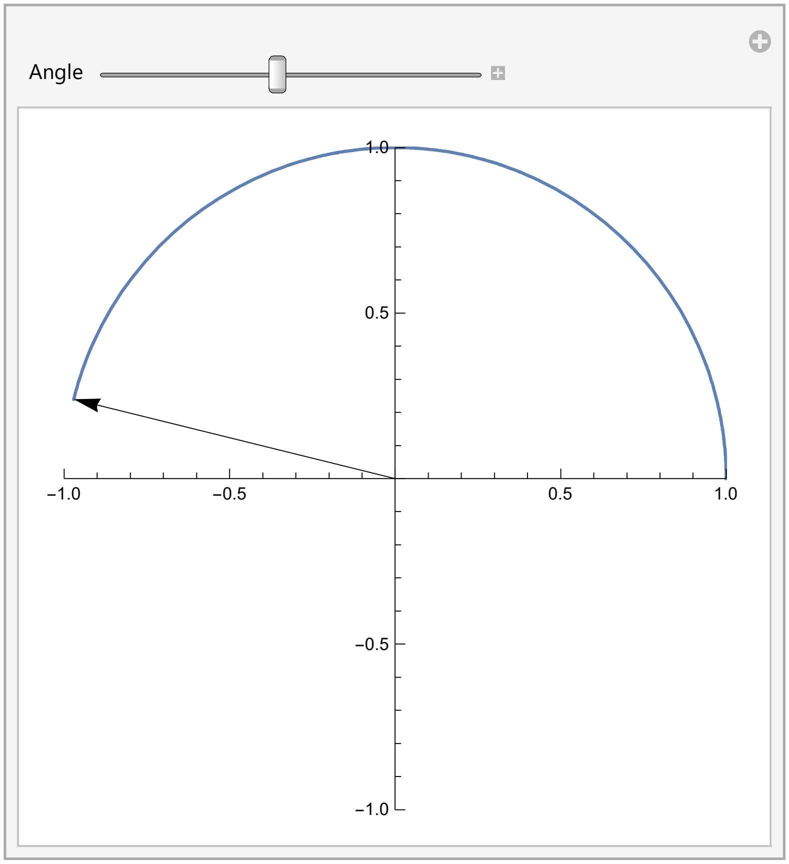


In[ ]:=
```
Manipulate[Show[ParametricPlot[{Cos[θ], Sin[θ]}, {θ, 0, t}, PlotRange → 1],
  Graphics[Arrow[{{0, 0}, {Cos[t], Sin[t]}}]]], {{t, 5}, 0, 2 π}]
```

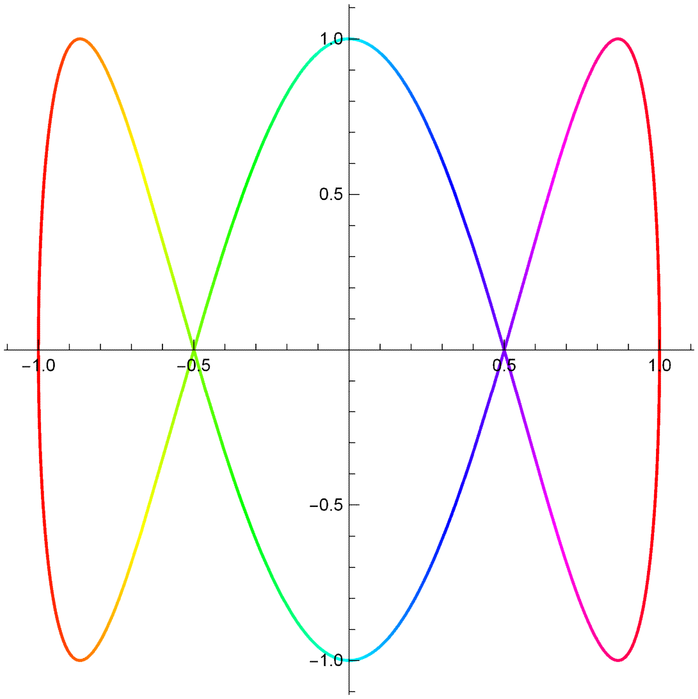
Out[ ]=


In[ ]:=
```
Manipulate[Show[ParametricPlot[{Cos[θ], Sin[θ]}, {θ, 0, t}, PlotRange → 1],
  Graphics[Arrow[{{0, 0}, {Cos[t], Sin[t]}}]]], {{t, 5, "Angle"}, 0, 2 π}]
```

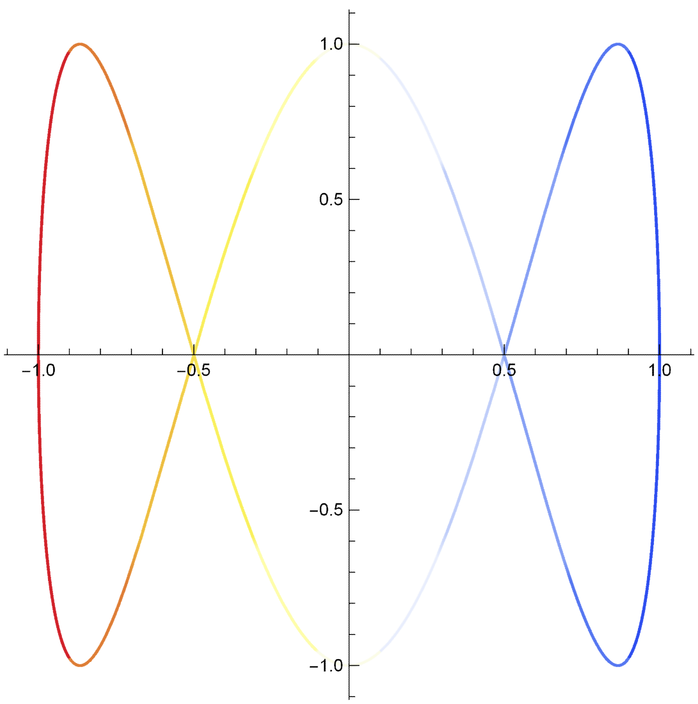
Out[ ]=



In[ ]:= ParametricPlot[{Cos[t], Sin[3 t]}, {t, 0, 2 π}, ColorFunction → Hue]

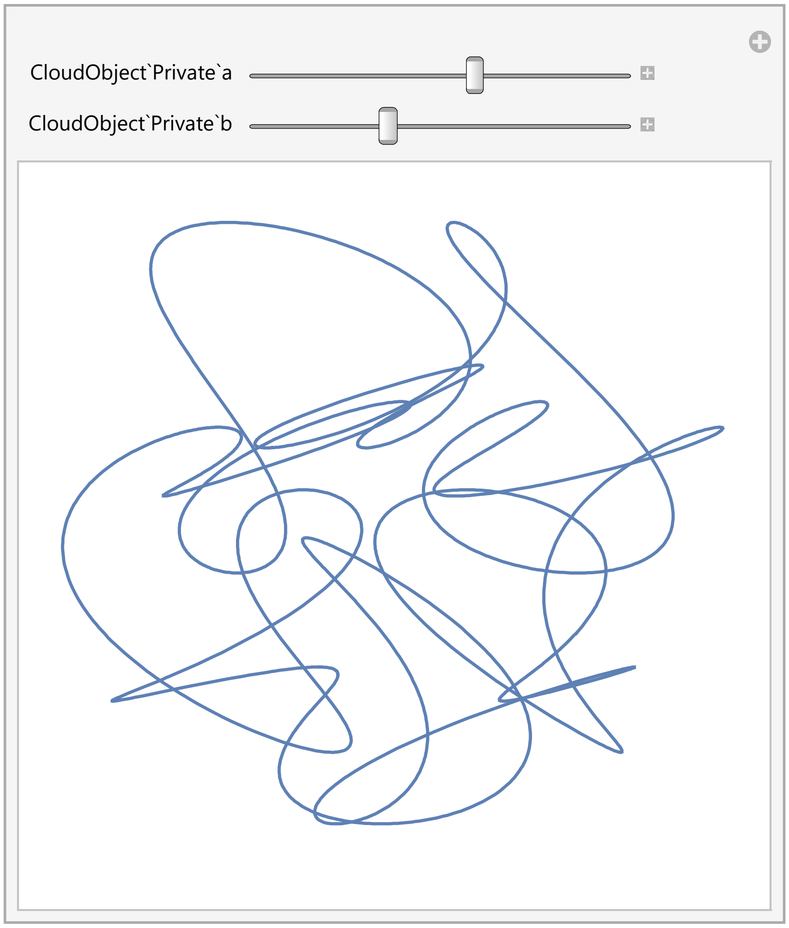
Out[ ]=


In[ ]:= ParametricPlot[{Cos[t], Sin[3 t]}, {t, 0, 2 π},
ColorFunction → Table[ColorData["TemperatureMap"][1 − k], {k, 0, 1, .1}]]

Out[ ]=


In[ ]:=
```
Manipulate[ParametricPlot[
  {Cos[t] + (1 / 2) Cos[7 t] + (1 / 2) Sin[a * t], Sin[t] + (1 / 2) Sin[7 t] + (1 / 2) Cos[b * t]},
  {t, 0, 2 π}, Axes → False, PlotRange → 2], {{a, 17}, 5, 25}, {{b, 12}, 5, 25}]
```

Out[ ]=



In[ ]:=
```
s[t_] := {Cos[t] + t, Sin[t]};
D[s[t], t]
```

Out[ ]= {1 - Sin[t], Cos[t]}

In[ ]:= ∂_t s[t]

Out[ ]= {1 - Sin[t], Cos[t]}

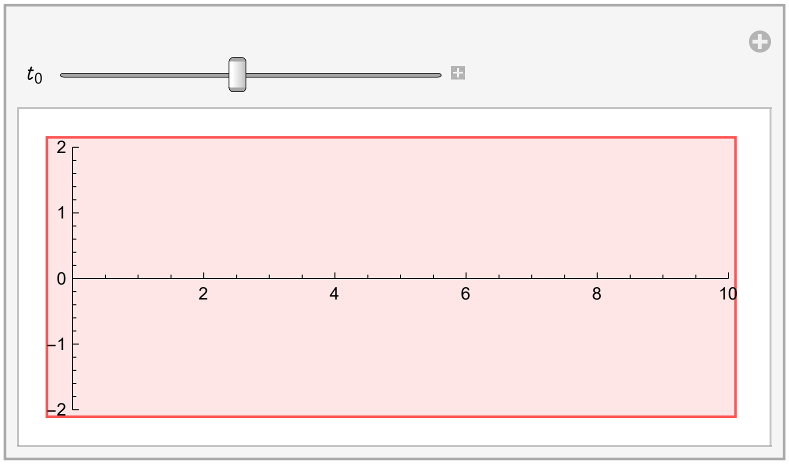
In[ ]:=
```
s'[t]
```

Out[ ]= {1 - Sin[t], Cos[t]}

In[ ]:=
```
Clear[s, t];
```

In[ ]:=
```
s[t_] := {t + Cos[t], Sin[t]};
Manipulate[Show[ParametricPlot[s[t], {t, 0, 10}, PlotRange → {{0, 10}, {-2, 2}}],
  Graphics[Arrow[{s[t0], s[t0] + s'[t0]}]]], {{t0, 4, "t0"}, 0, 10}]
```

Out[ ]=



In[ ]:=
```
Clear[s, t];
s[t_] := {Cos[t] + t, Sin[t]};

Norm[s'[3]]
```

Out[ ]= $\sqrt{\text{Cos}[3]^2 + (1 - \text{Sin}[3])^2}$

In[ ]:=
```
N[Norm[s'[3]]]
```

Out[ ]= 1.31063

In[ ]:=
```
Norm[s'[3]] // N
```

Out[ ]= 1.31063

In[ ]:=
```
Simplify[Norm[s'[t]], t ∈ Reals]
```

Out[ ]= $\sqrt{2 - 2\,\text{Sin}[t]}$

In[ ]:=
```
Clear[s, t]
```

In[ ]:=
```
unitTangent[s_, t_] := Simplify[Normalize[D[s, t]], t ∈ Reals]
```

In[ ]:=
```
s[t_] := {Cos[t] + t, Sin[t]};
unitTangent[s[t], t]
```

Out[ ]= $\left\{\frac{\sqrt{1 - \text{Sin}[t]}}{\sqrt{2}}, \frac{\text{Cos}[t]}{\sqrt{2 - 2\,\text{Sin}[t]}}\right\}$

In[ ]:=
```
unitTangent[s[t], t] /. t → 1.2
```
Out[ ]= {0.184338, 0.982863}

In[ ]:=
```
? unitTangent
```
Out[ ]= Missing[UnknownSymbol, unitTangent]

In[ ]:=
```
unitTangent[s_, t_] := Simplify[Normalize[D[s, t]], t ∈ Reals]
s[t_] := {t + Cos[t], Sin[t]};
```

In[ ]:=
```
unitTangent[s[t], t]
```
Out[ ]= $\left\{\frac{\sqrt{1-\mathrm{Sin}[t]}}{\sqrt{2}}, \frac{\mathrm{Cos}[t]}{\sqrt{2-2\,\mathrm{Sin}[t]}}\right\}$

```
unitNoramal[s_, t_] := FullSimplify[Normalize[D[unitTangent[s, t], t]], t ∈ Reals];
```

In[ ]:=
```
unitNormal[s[t], t]
```
Out[ ]= unitNormal[{t + Cos[t], Sin[t]}, t]

In[ ]:=
```
? unitTangent
```
Out[ ]=

Symbol

CloudObject`Private`unitTangent

Definitions unitTangent[s_, t_] := Simplify[Normalize[∂_t s], t ∈ ℝ]

Full Name CloudObject`Private`unitTangent

In[ ]:=
```
? unitNormal
```
Out[ ]=

Symbol

CloudObject`Private`unitNormal

Full Name CloudObject`Private`unitNormal

In[ ]:=
```
Clear[unitTangent, unitNormal, s, t]
```
In[ ]:=
```
s[t_] := {Cos[t] + t, Sin[t]};
```
In[ ]:=
```
unitTangent[s_, t_] := Simplify[Normalize[D[s, t]], t ∈ Reals]
```

In[ ]:=
```
unitTangent[s[t], t]
```

Out[ ]= $\left\{\frac{\sqrt{1-\text{Sin}[t]}}{\sqrt{2}}, \frac{\text{Cos}[t]}{\sqrt{2-2\,\text{Sin}[t]}}\right\}$

In[ ]:=
```
unitNormal[s_, t_] := Simplify[Normalize[D[unitTangent[s, t], t]], t ∈ Reals]
```

In[ ]:=
```
unitNormal[s[t], t]
```

Out[ ]= $\left\{-\frac{\text{Cos}[t]}{\sqrt{2-2\,\text{Sin}[t]}}, \frac{\sqrt{1-\text{Sin}[t]}}{\sqrt{2}}\right\}$

In[ ]:=
```
? unitTangent
```

Out[ ]= Missing[UnknownSymbol, unitTangent]

In[ ]:=
```
? unitNormal
```

Out[ ]= Missing[UnknownSymbol, unitNormal]

In[ ]:=
```
Clear[unitTangent, unitNormal, s, t]
```

In[ ]:=
```
unitTangent[s_, t_] := Simplify[Normalize[D[s, t]], t ∈ Reals]
```

In[ ]:=
```
s[t_] := {Cos[t] + t, Sin[t]};
unitTangent[s[t], t]
```

Out[ ]= $\left\{\frac{\sqrt{1-\text{Sin}[t]}}{\sqrt{2}}, \frac{\text{Cos}[t]}{\sqrt{2-2\,\text{Sin}[t]}}\right\}$

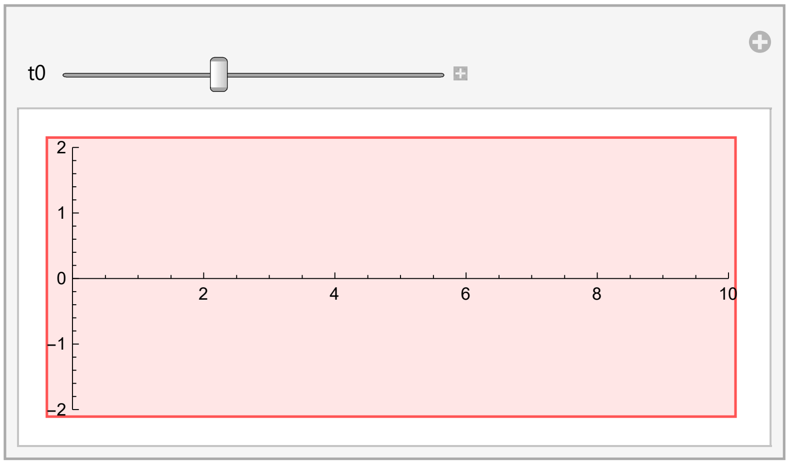
In[ ]:=
```
unitNormal[s_, t_] := Simplify[Normalize[D[unitTangent[s, t], t]], t ∈ Reals]
```

In[ ]:=
```
unitNormal[s[t], t]
```

Out[ ]= $\left\{-\frac{\text{Cos}[t]}{\sqrt{2-2\,\text{Sin}[t]}}, \frac{\sqrt{1-\text{Sin}[t]}}{\sqrt{2}}\right\}$

In[ ]:=
```
ut[t_] = unitTangent[s[t], t];
un[t_] = unitNormal[s[t], t];
Manipulate[Show[ParametricPlot[s[t], {t, 0, 10}, PlotRange → {{0, 10}, {-2, 2}}],
  Graphics[{Blue, Arrow[{s[t0], s[t0] + ut[t0]}]}],
  Graphics[{Red, Arrow[{s[t0], s[t0] + un[t0]}]}]], {{t0, 4}, 0, 10}]
```

Out[ ]=



In[ ]:=
```
κ[s_, t_] := Simplify[Norm[D[unitTangent[s, t], t]] / Norm[D[s, t]], t ∈ Reals]
```

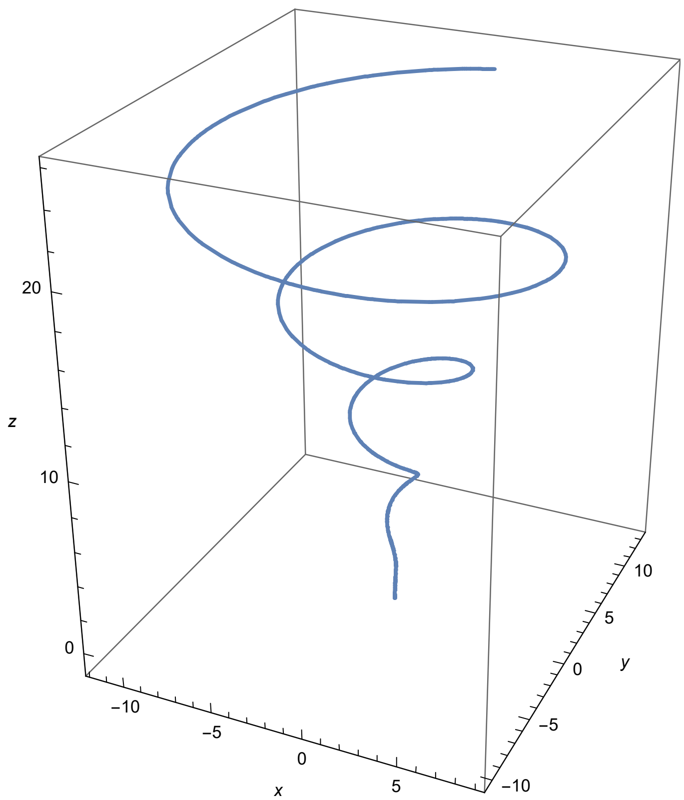
In[ ]:=
```
κ[s[t], t]
```

Out[ ]=
$$\frac{1}{2\sqrt{2-2\,\mathrm{Sin}[t]}}$$

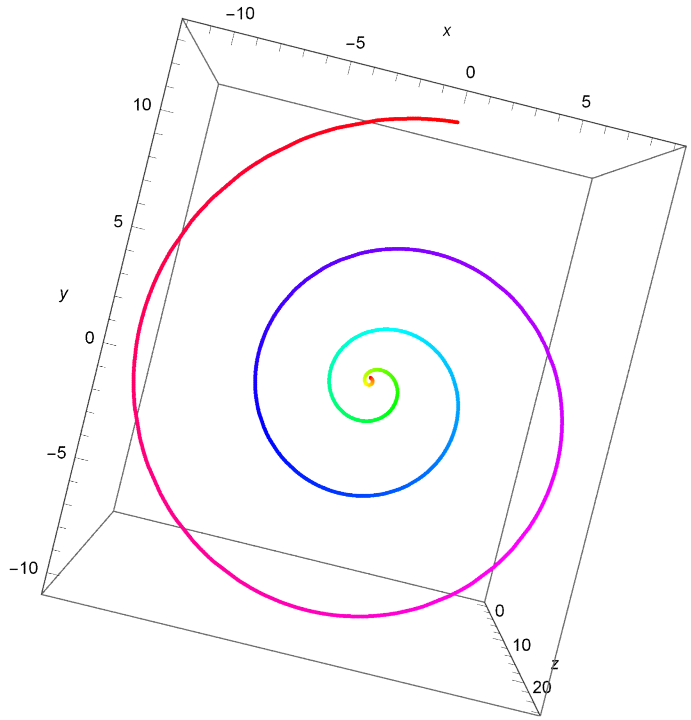
```
Clear[s, t];
```

In[ ]:=
```
s[t_] := {(t^2 / 50) * Sin[t], (t^2 / 50) * Cos[t], t};
ParametricPlot3D[s[t], {t, 0, 8 π}, AxesLabel → {x, y, z}]
```

Out[ ]=

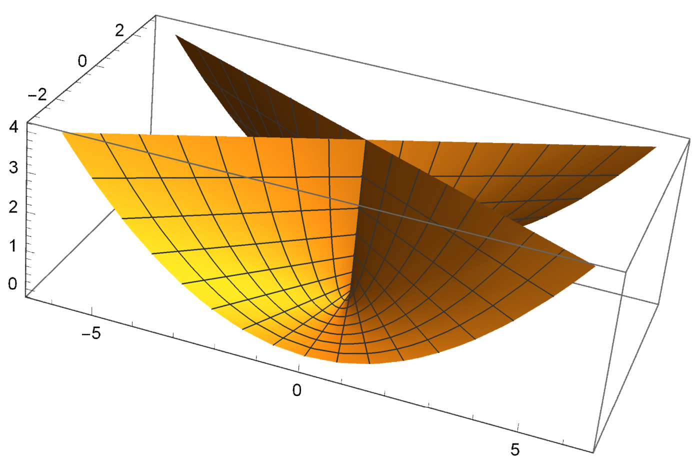


In[ ]:=
```
s[t_] := {(t^2 / 50) * Sin[t], (t^2 / 50) * Cos[t], t};
ParametricPlot3D[s[t], {t, 0, 8 π}, AxesLabel → {x, y, z}, ColorFunction → Hue]
```

Out[ ]=


In[ ]:=
```
Clear[𝒜, u, v]
```

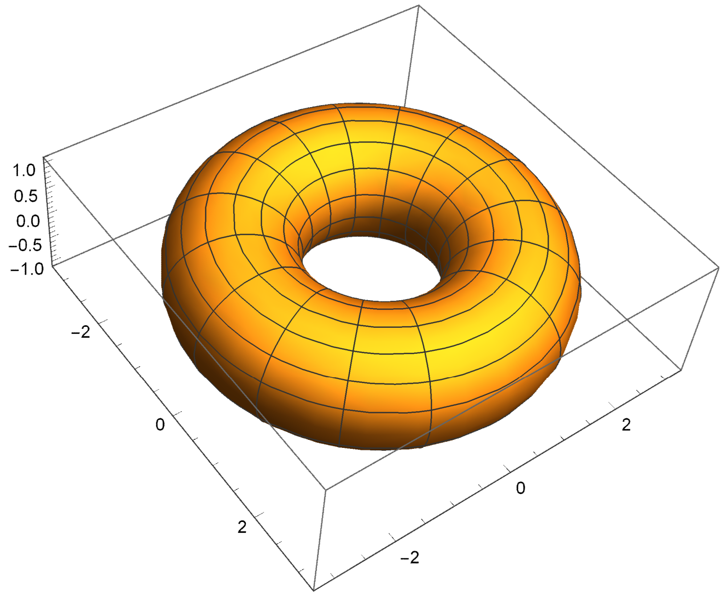
In[ ]:=
```
𝒜 = {u * v, u, v^2};
ParametricPlot3D[𝒜, {u, -3, 3}, {v, -2, 2}]
```

Out[ ]=


In[ ]:=
```
Clear[𝒜, u, v]
```

In[•]:=
```
𝒜 = {Cos[u] * (2 + Cos[v]), Sin[u] * (2 + Cos[v]), Sin[v]};
ParametricPlot3D[𝒜, {u, 0, 2 π}, {v, 0, 2 π}]
```

Out[•]=



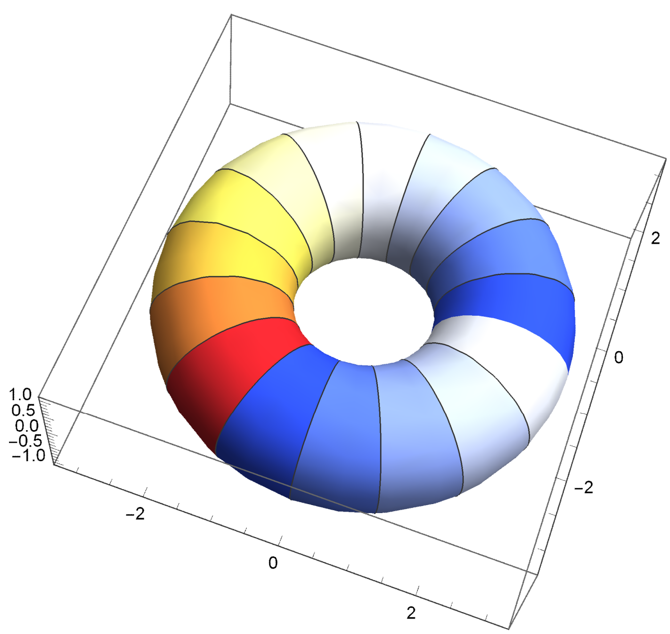
In[•]:=
```
? 𝒜
```

Out[•]=
```
Missing[UnknownSymbol, 𝒜]
```

In[•]:=
```
Clear[𝒜]
```

In[•]:=
```
𝒜 = {Cos[u] * (2 + Cos[v]), Sin[u] * (2 + Cos[v]), Sin[v]};
ParametricPlot3D[𝒜, {u, 0, 2 π}, {v, 0, 2 π}, MeshFunctions → {#4 &},
 MeshShading → Table[ColorData["TemperatureMap"][k], {k, 0, 1, .1}], Lighting → "Neutral"]
```

Out[•]=



In[ ]:=
```
? 𝒜
```

Out[ ]=

Symbol

Global`𝒜

Assignment 𝒜 = {Cos[u] (2 + Cos[v]), (2 + Cos[v]) Sin[u], Sin[v]}

Full Name Global`𝒜

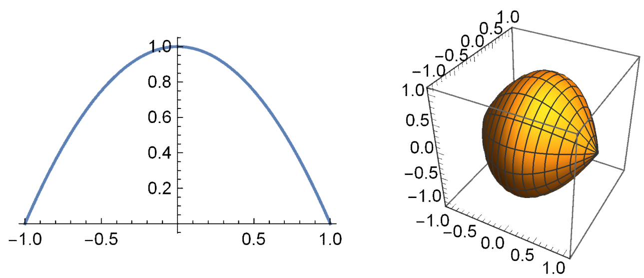
In[ ]:=
```
∂v 𝒜
```

Out[ ]=
```
{-Cos[u] Sin[v], -Sin[u] Sin[v], Cos[v]}
```

In[ ]:=
```
Clear[f]
```

In[ ]:=
```
f[x_] := 1 - x^2;
GraphicsRow[
 {Plot[f[x], {x, -1, 1}], RevolutionPlot3D[f[x], {x, -1, 1}, RevolutionAxis → "X"]}]
```

Out[ ]=

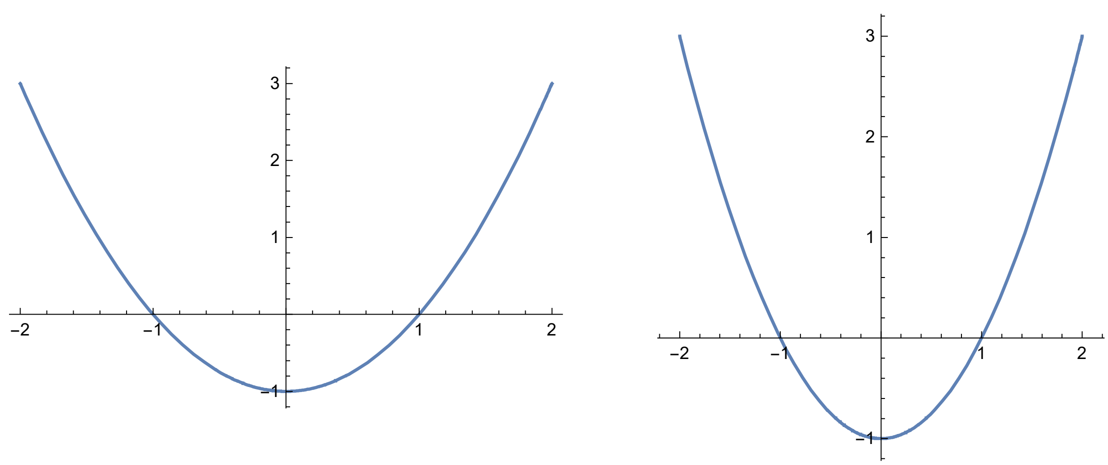


Exercise

(1)

In[ ]:=
```
GraphicsRow[{Plot[x^2 - 1, {x, -2, 2}], ParametricPlot[{x, x^2 - 1}, {x, -2, 2}]}]
```

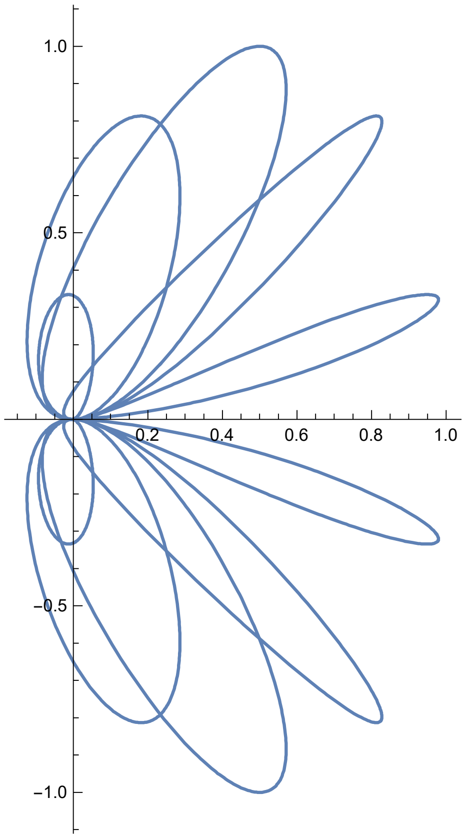
Out[ ]=



(2)

In[ ]:=
```
ParametricPlot[
 {(Sin[4 t] + Sin[5 t]) / 2 Sin[5 t], (Cos[4 t] - Cos[6 t]) / 2 Sin[5 t]}, {t, 0, 2 π}]
```

Out[ ]=



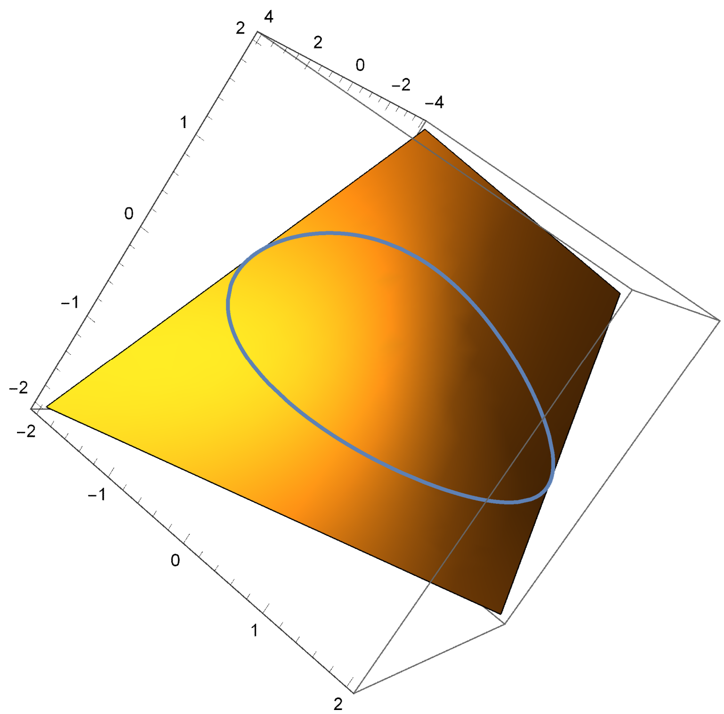
In[ ]:=
```
{(Sin[4 t] + Sin[5 t]) / 2 Sin[5 t], (Cos[4 t] - Cos[6 t]) / 2 Sin[5 t]} // TrigFactor
```

Out[ ]= $\left\{2\,\mathrm{Cos}\left[\frac{t}{2}\right]^2 (1 + 2\,\mathrm{Cos}[t])\,(1 - 2\,\mathrm{Cos}[t] + 2\,\mathrm{Cos}[2\,t])\,(1 + 2\,\mathrm{Cos}[t] + 2\,\mathrm{Cos}[2\,t])\,(1 + 2\,\mathrm{Cos}[3\,t])\,\mathrm{Sin}\left[\frac{t}{2}\right]^2,\ (1 + 2\,\mathrm{Cos}[2\,t] + 2\,\mathrm{Cos}[4\,t])^2\,\mathrm{Sin}[t]^3\right\}$

3 (a)

In[ ]:=
```
Show[ContourPlot3D[z == x * y, {x, -2, 2}, {y, -2, 2}, {z, -4, 4}, Mesh → None],
 ParametricPlot3D[{2 Sin[t], Cos[t], Sin[2 t]}, {t, 0, 2 π}]]
```

Out[ ]=



(b)

In[ ]:=
```
TrigExpand[Sin[2 t]] // TraditionalForm
```

Out[ ]//TraditionalForm=

$2 \sin(t) \cos(t)$

(5)

In[ ]:=
```
ParametricPlot3D[{Cos[u] * (Cos[v] + 2), Sin[u] * (Cos[v] + 2), Sin[v]},
  {u, 0, 2 π}, {v, 0, 2 π}, MeshFunctions → {#5 &},
  MeshShading → Table[ColorData["TemperatureMap"][k], {k, 0, 1, .1}]]
```

Out[ ]=