DESIGN OF ROBOTS PROJECT

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1 Introduction

This report presents the analysis of the 3-RRR planar parallel manipulator. The screw theory analysis, the complexity analysis and the inverse kinematics of this robot has been caluclated. Modelling has been done in CATIA. An animation has also been done in MATLAB. The file named `RRRAssembly_sans_analysis` is the final file the reader must look into. The simulation can be played using the replay, we have not created any movie. There is another file named `RRRAssembly`, we used this to do the structural analysis whose results are presented in this report. The reader can also find a matlab animation in the folder named `matlab animation` .

2 Task Defintion

We have to design a 3-degrees of freedom (DOF) planar parallel manipulator which can translate along x and y and rotate about z axis only.In the following sections we perform the screw theory and complexity analysis of few of the models that satisfy the task definiton.

3 Screw theory based analysis

3.1 Screw theory analysis of 3-RRR

The schematic diagram of a limb of a 3-RRR planar robot is presented here. There are 3 revolute joints with joint centres at A_i, B_i and C_i and their axis of rotation is along z direction. The joint at A_i is attached to fixed base at F_i. Here, $i = 1, 2, 3$. The kinematic study is done here. The joint axes are in blue.

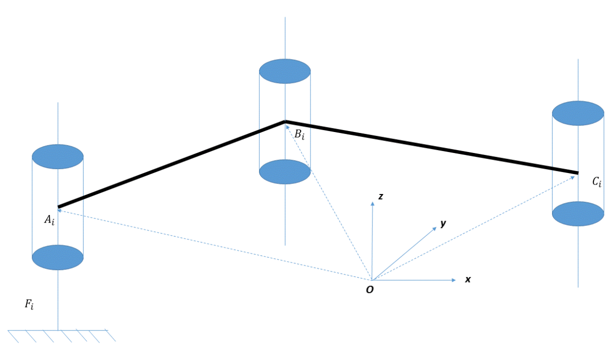

Figure 3.1: Simple diagram of a i^{th} limb of 3-RRR parallel manipulator

Steps in Kinematic study:

1. **Limb Twist System:** In the diagram we see that each limb has 3 revolute joints. In screw theory, each revolute joint can be represented as a zero pitch twist (ϵ_0). For the given system, the twist of each limb can be written as:

$$T_i = span\{\epsilon_{0A_i}, \epsilon_{0B_i}, \epsilon_{0C_i}\}$$

$$\epsilon_{0A_i} = \begin{bmatrix} \hat{z} \\ OA_i \times \hat{z} \end{bmatrix}, \epsilon_{0B_i} = \begin{bmatrix} \hat{z} \\ OB_i \times \hat{z} \end{bmatrix}, \epsilon_{0C_i} = \begin{bmatrix} \hat{z} \\ OC_i \times \hat{z} \end{bmatrix}$$

i=1,2,3.

2. **Limb Constraint Wrench System:** The elements of the Wrench system are reciprocal to the Limb Twist system. The twist system has 3 zero pitch wrenches along z axis. The reciprocity conditions are:

   - Two zero pitch screws are reciprocal when their axes are coplanar.
   - Two infinite pitch screws are always reciprocal.
   - One zero pitch and an infinite pitch screw are reciprocal if their axes are orthogonal.

   Using these conditions, a zero pitch wrench(a pure force) along z axis (τ_{0z}) is reciprocal to the entire system and infinite pitch wrenches (pure moment) along x ($\tau_{\infty x}$) and y ($\tau_{\infty y}$) are also reciprocal to the limb twist system.

$$W_{ci} = T_i^{\perp} = span\{\tau_{0z}, \tau_{\infty x}, \tau_{\infty y}\}$$

i=1,2,3.

3. **Constraint Wrench System:** The Constraint Wrench System is the sum of the limb constraint wrench systems. As the limb constraint wrench systems are same linear combinations for each limb, the net result of the addition spans the same set as the limb constraint wrench system.

$$W_c = \Sigma W_{ci} = span\{\tau_{0z}, \tau_{\infty x}, \tau_{\infty y}\}$$

4. **Moving Platform Twist:** The Moving Platform Twist is reciprocal to the Constraint Wrench System. Recalling the reciprocity conditions, we can conclude the following:

   - Among infinite pitch twists, the ones reciprocal to the Constraint Wrench System will be $\epsilon_{\infty x}$ and $\epsilon_{\infty y}$.
   - Among zero pitch twists, the one reciprocal to the Constraint Wrench System will be ϵ_{0z}.

$$T = W_c^{\perp} = span\{\epsilon_{\infty x}, \epsilon_{\infty y}, \epsilon_{0z}\}$$

5. **Actuation Wrench System:** We assume that the joint with centre A_i is actuated. The Actuation Wrench System is spanned by wrenches which are reciprocal to all the joints except the actuated joint (A_i) in each limb. In this case, the zero pitch wrenches (pure force) passing through B_iC_i $i = \{1, 2, 3\}$ span the actuation wrench system.

$$W_a = span\{\tau_{0B_1C_1}, \tau_{0B_2C_2}, \tau_{0B_3C_3}\}$$

3.2 Screw theory analysis of 3-RRP

The schematic diagram of a limb of a 3-RRP planar robot is presented here. There are 2 revolute joints and 1 prismatic joint centres at A_i, B_i and C_i, respectively. The axis of rotation is along z direction for revolute joints. The joint at A_i is attached to fixed base at F_i. Here, $i = 1, 2, 3$. The kinematic study is done here. The joint axes are in blue.

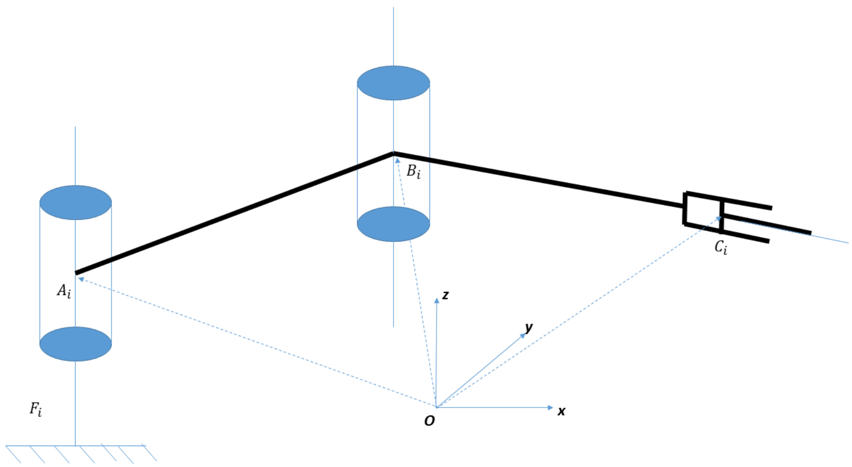

Figure 3.2: Simple diagram of a i^{th} limb of 3-RRP parallel manipulator

Steps in Kinematic study:

1. **Limb Twist System:** In the diagram we see that each limb has 2 revolute joints and 1 prismatic joint. In screw theory, each revolute joint can be represented as a zero pitch twist (ϵ_0) and a prismatic joint is represented as an infinite pitch twist (ϵ_∞). For the given system, the twist of each limb can be written as:

$$T_i = span\{\epsilon_{0A_i}, \epsilon_{0B_i}, \epsilon_{\infty C_i}\}$$

$$\epsilon_{0A_i} = \begin{bmatrix} \hat{z} \\ OA_i \times \hat{z} \end{bmatrix}, \epsilon_{0B_i} = \begin{bmatrix} \hat{z} \\ OB_i \times \hat{z} \end{bmatrix}, \epsilon_{0C_i} = \begin{bmatrix} 0 \\ \hat{bc_i} \end{bmatrix}$$

i=1,2,3.
$\hat{bc_i}$ is unit vector along B_iC_i

2. **Limb Constraint Wrench System:** The elements of the Wrench system are reciprocal to the Limb Twist system. The reciprocity conditions are:

   - Two zero pitch screws are reciprocal when their axes are coplanar.
   - Two infinite pitch screws are always reciprocal.
   - One zero pitch and an infinite pitch screw are reciprocal if their axes are orthogonal.

   Using these conditions, a zero pitch wrench(a pure force) along z axis (τ_{0z}) is reciprocal to the entire system and infinite pitch wrenches (pure moment) along x ($\tau_{\infty x}$) and y ($\tau_{\infty y}$) are also reciprocal to the limb twist system.

$$W_{ci} = T_i^{\perp} = span\{\tau_{0z}, \tau_{\infty x}, \tau_{\infty y}\}$$

i=1,2,3.

3. **Constraint Wrench System:** The Constraint Wrench System is the sum of the limb constraint wrench systems. As the limb constraint wrench systems are same linear combinations for each limb, the net result of the addition spans the same set as the limb constraint wrench system.

$$W_c = \Sigma W_{ci} = span\{\tau_{0z}, \tau_{\infty x}, \tau_{\infty y}\}$$

4. **Moving Platform Twist:** The Moving Platform Twist is reciprocal to the Constraint Wrench System. Recalling the reciprocity conditions, we can conclude the following:
   - Among infinite pitch twists, the ones reciprocal to the Constraint Wrench System will be $\epsilon_{\infty x}$ and $\epsilon_{\infty y}$.
   - Among zero pitch twists, the one reciprocal to the Constraint Wrench System will be ϵ_{0z}.

$$T = W_c^{\perp} = span\{\epsilon_{\infty x}, \epsilon_{\infty y}, \epsilon_{0z}\}$$

5. **Actuation Wrench System:** We assume that the prismatic joint with centre C_i is actuated. The Actuation Wrench System is spanned by wrenches which are reciprocal to all the joints except the actuated joint (C_i) in each limb. In this case, the zero pitch wrenches (pure force) along A_iB_i and passing through A_i and B_i ($i = \{1, 2, 3\}$) span the actuation wrench system.

$$W_a = span\{\tau_{0A_1B_1}, \tau_{0A_2B_2}, \tau_{0A_3B_3}\}$$

3.3 Screw theory analysis of 3-RPP

The schematic diagram of a limb of a 3-RPP planar robot is presented here. There are 1 revolute joint and 2 prismatic joints centres at A_i, B_i and C_i, respectively. The axis of rotation is along z direction for revolute joints. The joint at A_i is attached to fixed base at F_i. Here, $i = 1, 2, 3$. The kinematic study is done here. The joint axes are in blue.
Note: $A_iB_i \perp B_iC_i$

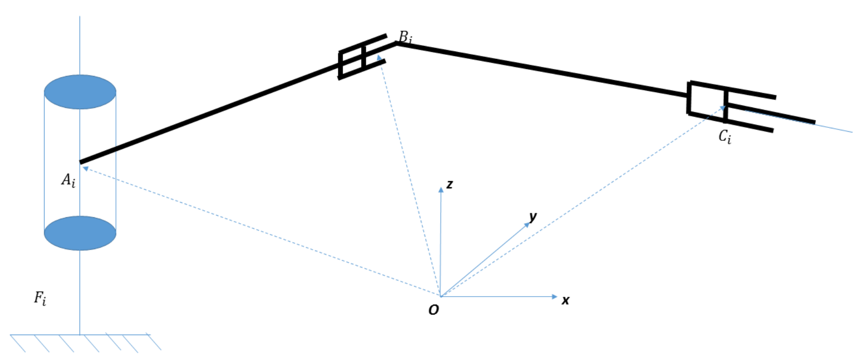

Figure 3.3: Simple diagram of a i^{th} limb of 3-RPP parallel manipulator

Steps in Kinematic study:

1. **Limb Twist System:** In the diagram we see that each limb has 1 revolute joint and 2 prismatic joints. In screw theory, each revolute joint can be represented as a zero pitch twist (ϵ_0) and a prismatic joint is represented as an infinite pitch twist (ϵ_∞). For the given system, the twist of each limb can be written as:

$$T_i = span\{\epsilon_{0A_i}, \epsilon_{\infty B_i}, \epsilon_{\infty C_i}\}$$

$$\epsilon_{0A_i} = \begin{bmatrix} \hat{z} \\ OB_i \times \hat{z} \end{bmatrix}, \epsilon_{0B_i} = \begin{bmatrix} 0 \\ \hat{ab_i} \end{bmatrix}, \epsilon_{0C_i} = \begin{bmatrix} 0 \\ \hat{bc_i} \end{bmatrix}$$

   i=1,2,3.
   $\hat{ab_i}$ is unit vector along A_iB_i
   $\hat{bc_i}$ is unit vector along B_iC_i

2. **Limb Constraint Wrench System:** The elements of the Wrench system are reciprocal to the Limb Twist system. The reciprocity conditions are:

   - Two zero pitch screws are reciprocal when their axes are coplanar.
   - Two infinite pitch screws are always reciprocal.
   - One zero pitch and an infinite pitch screw are reciprocal if their axes are orthogonal.

   Using these conditions, a zero pitch wrench(a pure force) along z axis (τ_{0z}) is reciprocal to the entire system and infinite pitch wrenches (pure moment) along x ($\tau_{\infty x}$) and y ($\tau_{\infty y}$) are also reciprocal to the limb twist system.

$$W_{ci} = T_i^{\perp} = span\{\tau_{0z}, \tau_{\infty x}, \tau_{\infty y}\}$$

   i=1,2,3.

3. **Constraint Wrench System:** The Constraint Wrench System is the sum of the limb constraint wrench systems. As the limb constraint wrench systems are same linear combinations for each limb, the net result of the addition spans the same set as the limb constraint wrench system.

$$W_c = \Sigma W_{ci} = span\{\tau_{0z}, \tau_{\infty x}, \tau_{\infty y}\}$$

4. **Moving Platform Twist:** The Moving Platform Twist is reciprocal to the Constraint Wrench System. Recalling the reciprocity conditions, we can conclude the following:

   - Among infinite pitch twists, the ones reciprocal to the Constraint Wrench System will be $\epsilon_{\infty x}$ and $\epsilon_{\infty y}$.
   - Among zero pitch twists, the one reciprocal to the Constraint Wrench System will be ϵ_{0z}.

$$T = W_c^{\perp} = span\{\epsilon_{\infty x}, \epsilon_{\infty y}, \epsilon_{0z}\}$$

5. **Actuation Wrench System:** We assume that the first prismatic joint of each limb is actuated. The Actuation Wrench System is spanned by wrenches which are reciprocal to all the joints except the actuated joint in each limb. In this case, a zero pitch wrench (pure force) along A_iB_i span the actuation wrench system. And this is same for each of the limbs. So, the system

$$W_a = span\{\tau_{0A_1B_1}, \tau_{0A_2B_2}, \tau_{0A_3B_3}\}$$

4 Complexity Analysis of few design Alternatives

We perform the complexity analysis of a few design alternatives like 3-RRR,3-PPR,3-PRR,3-RRP,3-RPR. The complexity indices used are the joint number complexity, the joint type complexity, the loop complexity and the link diversity. Before presenting the results we give a brief idea about these indices.

4.1 Joint number complexity

$$K_N = 1 - \exp^{-q_n N}$$

N=number of joints used in the topology.
q_n=resolution parameter$\rightarrow$ $q_n = -\frac{\ln 0.1}{N_{max}}$
N_{max}=The maximum number of joints in among all the topologies under consideration

4.2 Joint type complexity

$$K_J = \frac{1}{n}(\sum n_x K_x)$$

n=total number of joint pairs
n_x= number of pairs of type x (revolute, prismatic, spherical, etc)
K_x= geometric complexity related to the x pair

4.3 Loop complexity

$$K_N = 1 - \exp^{-q_l L}$$

$L = l - l_m$, here l is the number of kinematic loops and l_m is the minimum number of loops required
q_l=resolution parameter$\rightarrow$ $q_l = -\frac{\ln 0.1}{L_{max}}$
L_{max}=The maximum number of loops in among all the topologies under consideration

4.4 Link Diversity

$K_b = \frac{B}{B_{max}}$, $B = -\sum b_i \log_2 b_i$, $b_i = \frac{M_i}{\sum M_i}$
B_{max}=$\log_2$=2.32 bits, the maximum possible entropy
b_i=frequency of occurence of a link topology
M_i=number of instances of each type of joint constraint
c=number of distinct joint-constraint types used in a concept
The following table summarises the complexity evaluation table for the design alternatives like 3-RRR, 3-RPP, 3-PRR,3-RRP, 3-RPR.

topology	n_p	n_r	K_N	K_L	K_J	K_B	K
3RRR	0	9	0.9	0.9	0.52	0	**0.58**
3RPP	6	3	0.9	0.9	0.84	0	0.66
3PRR	3	6	0.9	0.9	0.68	0.43	0.7275
3RRP	6	3	0.9	0.9	0.68	0.43	0.7275
3RPR	3	6	0.9	0.9	0.68	0.43	0.7275

As can be seen, the 3RRR topology has the least complexity, so we proceed to model the same in CATIA.

5 Inverse Kinematics of the 3 RRR Robot and the Kinematic Jacobians

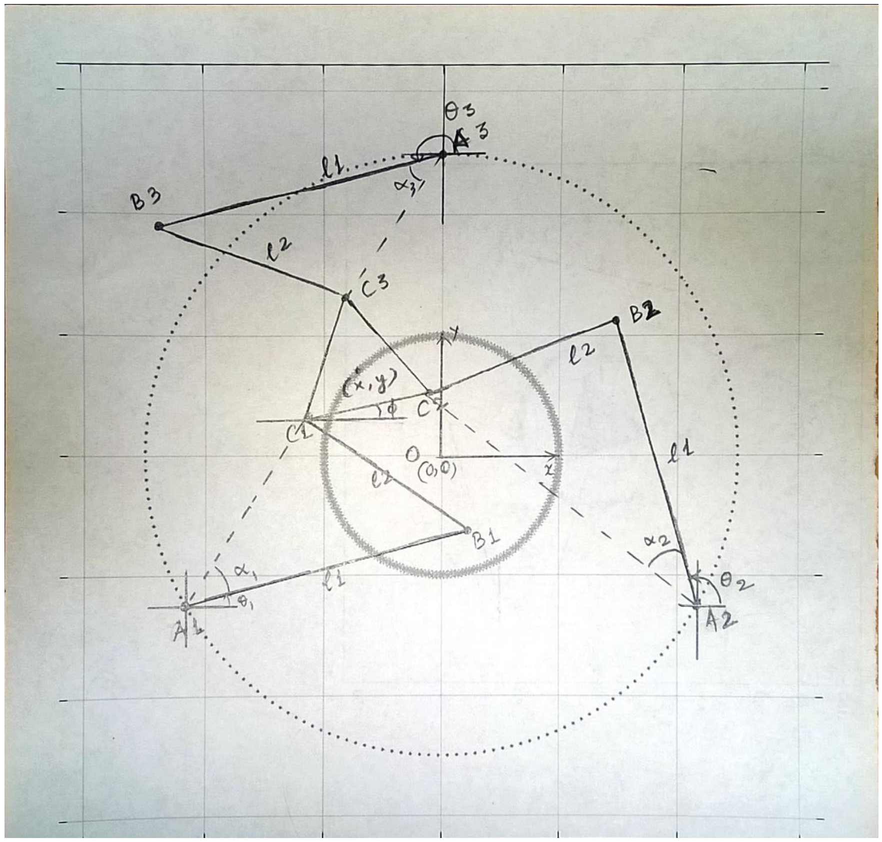

Figure 5.1: Simple diagram of a i^{th} limb of 3-RRR parallel manipulator

The Figure 4 shows the diagram used to find the inverse kinematic model of the 3RRR manipulator. The problem is to find the actuated joint angles θ_1,θ_2 and θ_3 as a function of the the end effector postion (x, y) and orientation ϕ. In this case, the centre of mass of the moving platform is assumed to be the position of the end effector.

note: The length of each side in the moving platform is a and the Radius of the big circle in the the Figure 4 is R, the points A_1, A_2 and A_3 lie on this circle and form an equilateral triangle. Using coordinate geometry we get the following results:

Limb 1. $A_1 = \big(-\frac{\sqrt{3}R}{2}, -\frac{R}{2}\big)$

$C_1 = \big(x - \frac{a}{\sqrt{3}}\cos(\phi + \frac{\pi}{6}), y - \frac{a}{\sqrt{3}}\sin(\phi + \frac{\pi}{6})\big)$

$\alpha_1 = \arccos\frac{A_1C_1^2 + l_1^2 - l_2^2}{2A_1C_1.l_1}$

$\theta_1 = a\tan 2\big(C_1(2) - A_1(2), C_1(1) - A_1(1)\big) - \alpha_1$

Limb 2. $A_2 = \big(+\frac{\sqrt{3}R}{2}, -\frac{R}{2}\big)$

$C_2 = \big(x - \frac{a}{\sqrt{3}}\cos(\phi + \frac{\pi}{6}) + a\cos\phi, y - \frac{a}{\sqrt{3}}\sin(\phi + \frac{\pi}{6}) + a\sin\phi\big)$

$\alpha_2 = \arccos\frac{A_2C_2^2 + l_1^2 - l_2^2}{2A_2C_2.l_1}$

$\theta_2 = a\tan 2\big(C_2(2) - A_2(2), C_2(1) - A_2(1)\big) - \alpha_2$

Limb 3. $A_3 = \big(0, R\big)$

$C_3 = \big(x - \frac{a}{\sqrt{3}}\cos(\phi + \frac{\pi}{6}) + a\cos(\phi + \frac{\pi}{3}), y - \frac{a}{\sqrt{3}}\sin(\phi + \frac{\pi}{6}) + a\sin(\phi + \frac{\pi}{3})\big)$

$\alpha_3 = \arccos \frac{A_3C_3^2 + l_1^2 - l_2^2}{2A_3C_3.l_1}$
$\theta_3 = a\tan 2\big(C_3(2) - A_3(2), C_3(1) - A_3(1)\big) - \alpha_3$

Using the screw theory analysis of the 3 RRR robot presented in Section 2.1, we find out the kinematic jacobians associated with this robot.
Note that the moving platform twist can be represented as:

$$t = \begin{bmatrix} \omega \\ \dot{p} \end{bmatrix}$$

and the vector of joint velocities is:

$$\dot{\theta} = \begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_2} \\ \dot{\theta_3} \end{bmatrix}$$

Taking the reciprocal products between the actuation wrench system and the moving platform twists, we obtain:

$$\begin{bmatrix} (r_{C_1} \times f_{B_1C_1})^T & f_{B_1C_1}^T \\ (r_{C_2} \times f_{B_2C_2})^T & f_{B_2C_2}^T \\ (r_{C_3} \times f_{B_3C_3})^T & f_{B_3C_3}^T \end{bmatrix} t = \begin{bmatrix} (A_1C_1 \times f_{B_1C_1})^T.k & 0 & 0 \\ 0 & (A_2C_2 \times f_{B_2C_2})^T.k & 0 \\ 0 & 0 & (A_3C_3 \times f_{B_3C_3})^T.k \end{bmatrix} .\dot{\theta}$$

Taking the reciprocal products between the constraint wrench system and the moving platform twists, we obtain:

$$\begin{bmatrix} (r_{P_{xy}} \times \hat{z})^T & \hat{z}^T \\ \hat{x}^T & 0_3^T \\ \hat{y}^T & 0_3^T \end{bmatrix} t = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$r_{P_{xy}}$=distance of a point P on the xy Plane. Without loss of generality it can be set to zero because we want to show that the force is perpendicular to the xy plane and it can pass through any point in the plane which also includes the origin.
Rewriting the reciprocal products between the constraint wrench system and the moving platform twists, we obtain:

$$\begin{bmatrix} 0_3^T & \hat{z}^T \\ \hat{x}^T & 0_3^T \\ \hat{y}^T & 0_3^T \end{bmatrix} t = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Combining the equations one obtains:

$$\begin{bmatrix} (r_{C_1} \times f_{B_1C_1})^T & f_{B_1C_1}^T \\ (r_{C_2} \times f_{B_2C_2})^T & f_{B_2C_2}^T \\ (r_{C_3} \times f_{B_3C_3})^T & f_{B_3C_3}^T \\ 0_3^T & \hat{z}^T \\ \hat{x}^T & 0_3^T \\ \hat{y}^T & 0_3^T \end{bmatrix} t = \begin{bmatrix} (A_1C_1 \times f_{B_1C_1})^T.k & 0 & 0 \\ 0 & (A_2C_2 \times f_{B_2C_2})^T.k & 0 \\ 0 & 0 & (A_3C_3 \times f_{B_3C_3})^T.k \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} .\dot{\theta}$$

$\implies \mathbf{A}.t = \mathbf{B}.\dot{\theta}$
Here,

$$\mathbf{A} = \begin{bmatrix} (r_{C_1} \times f_{B_1C_1})^T & f_{B_1C_1}^T \\ (r_{C_2} \times f_{B_2C_2})^T & f_{B_2C_2}^T \\ (r_{C_3} \times f_{B_3C_3})^T & f_{B_3C_3}^T \\ 0_3^T & \hat{z}^T \\ \hat{x}^T & 0_3^T \\ \hat{y}^T & 0_3^T \end{bmatrix}$$

and

$$\mathbf{B} = \begin{bmatrix} (A_1C_1 \times f_{B_1C_1})^T.k & 0 & 0 \\ 0 & (A_2C_2 \times f_{B_2C_2})^T.k & 0 \\ 0 & 0 & (A_3C_3 \times f_{B_3C_3})^T.k \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

6 Optimisation

The matlab function `fmincon()` can be used to find the optimal lengths of the links that always give a inverse condition number of the forward($\mathbf{A}$) and reverse jacobians($\mathbf{B}$) larger than 0.1. This concept is used as a constraint in the constraint functions. For the matrix A, the 3^{rd} column to 5^{th} column and the 1^{st} to 3^{rd} rows must be extracted to find the condition number using the matlab function `cond()`. For the matrix B, we need not include it in the constraints because the first 3 rows of matrix B form an identity matrix.

7 Catia Model

In our catia model we parameterized each and every geometrical parameter for ease of use. This enabled us to change the lenghts, radius, etc very easily without much change required in the DMU Kinematics. We, include below a few pictures of the parts of the Robot.

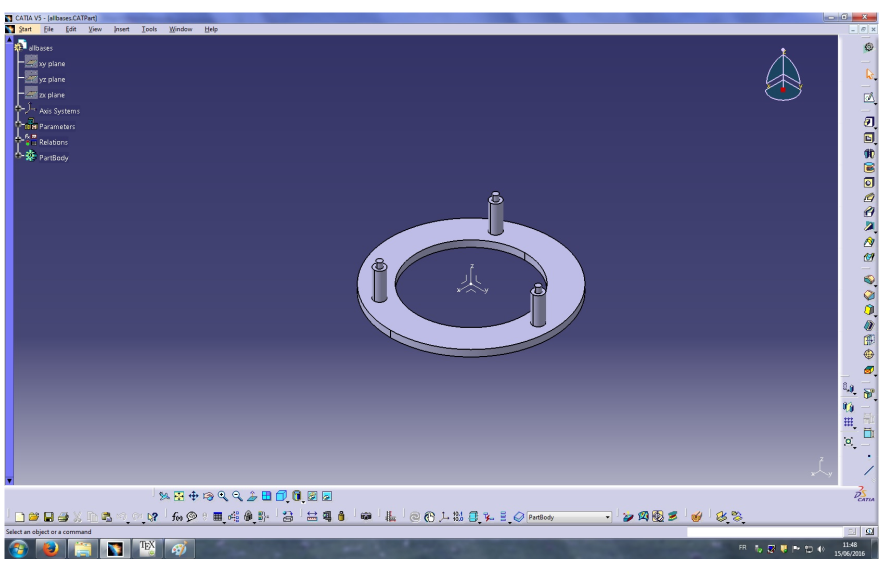

Figure 7.1: The base of the system

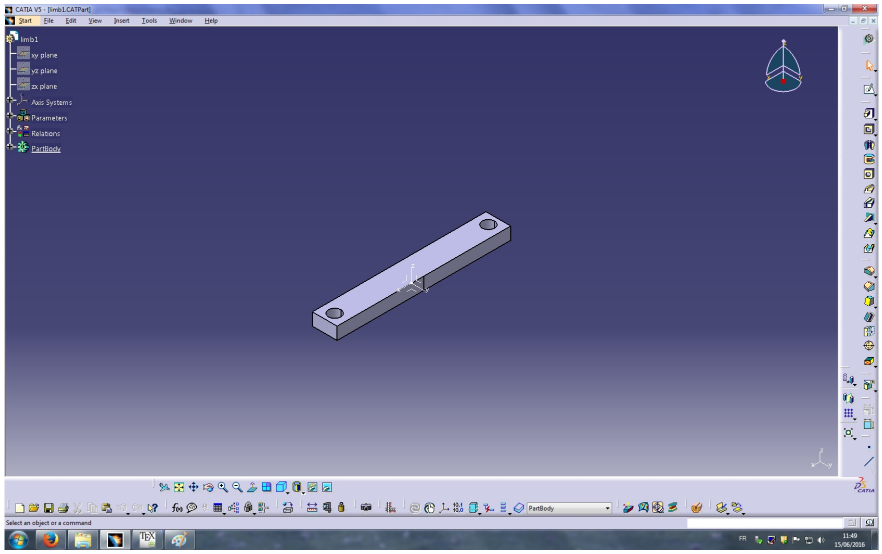

Figure 7.2: The first link of each limb

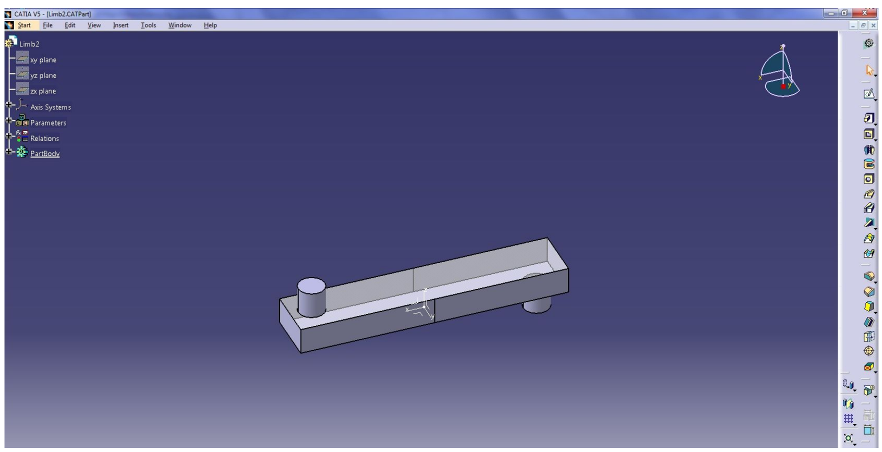

Figure 7.3: The second link of each limb

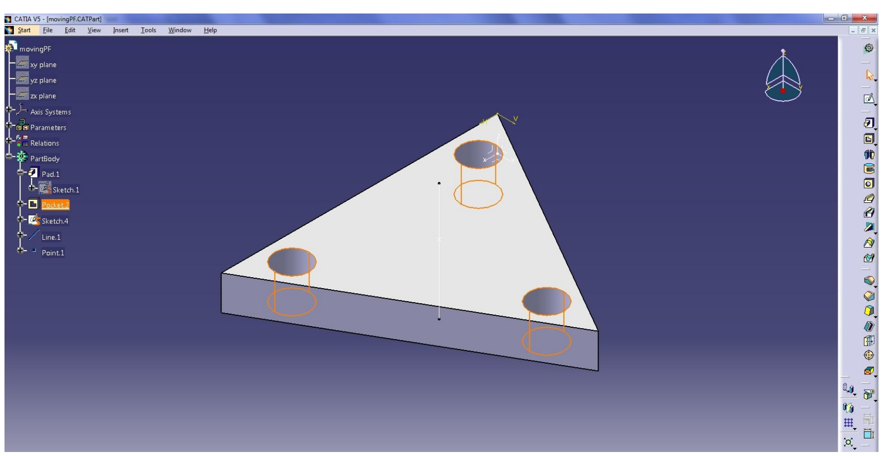

Figure 7.4: The moving platform

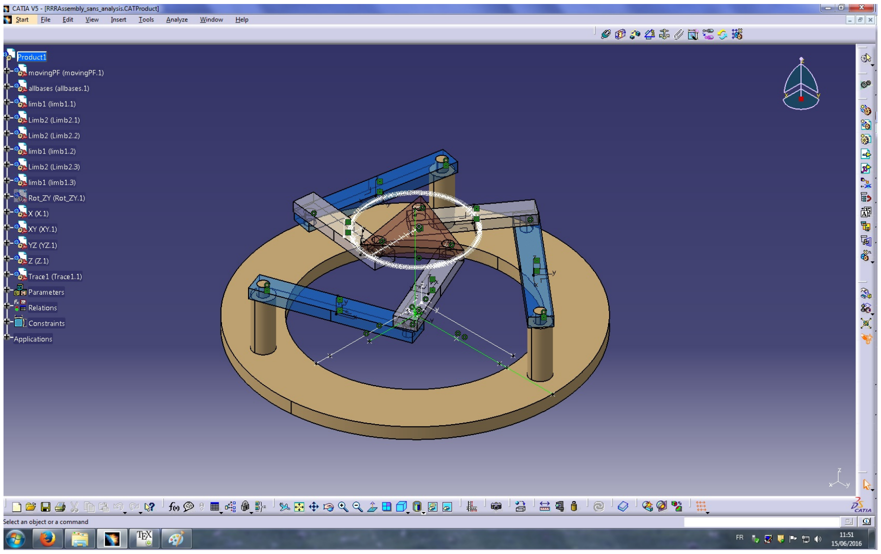

Figure 7.5: The robot: An assembly of all the parts

8 Structural Analysis

Presented below are the images obtained from the Structural analysis of the Robot, keeping all other parts fixed, a 10 newton force was applied on the moving platform, in a direction perpendicular to the plane and into the plane. The results are in the images below.

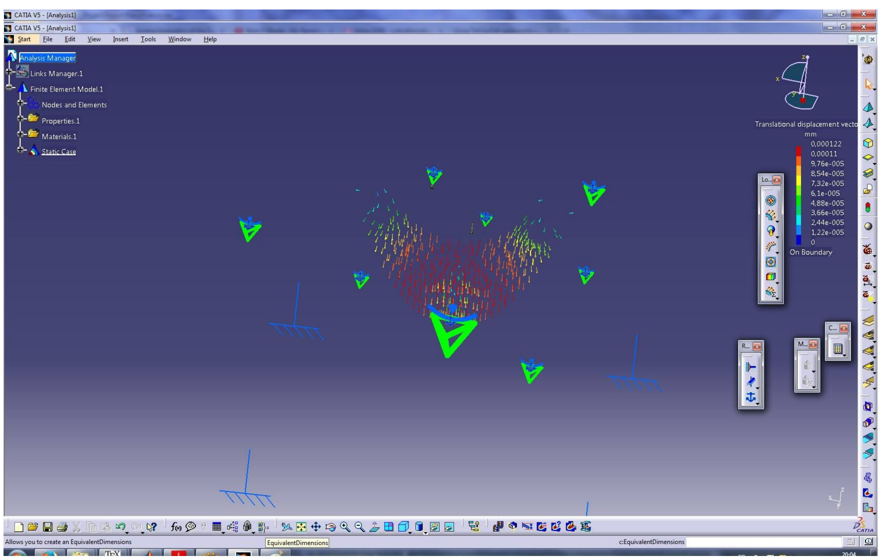

Figure 8.1: Structural Analysis: As can be seen in the data, the translational disp vctor is less that 0.1 mm

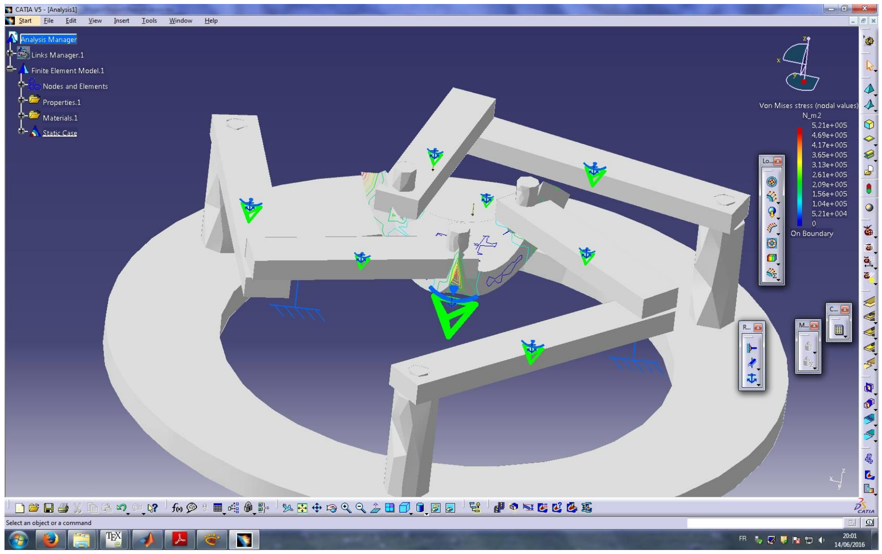

Figure 8.2: Structural Analysis