EMARO, Erasmus Mundus Master
Nantes, France

Mobile Robot Control

Lab N^{o}1

Mobile Robot control

The aim of the lab is to use matlab simulink for the modeling, validation and control of particular mobile robots.

We will concentrate on kinematic modeling, on static and dynamic feedback using the [2,0] mobile robot, for the problem of target tracking.

We will study the robustness of the different control laws regarding the parameters variations.

The students will write a report including all commented simulation results and theoretical material (read carefully all questions). They will deliver also all the developed simulink blocks and matlab functions.

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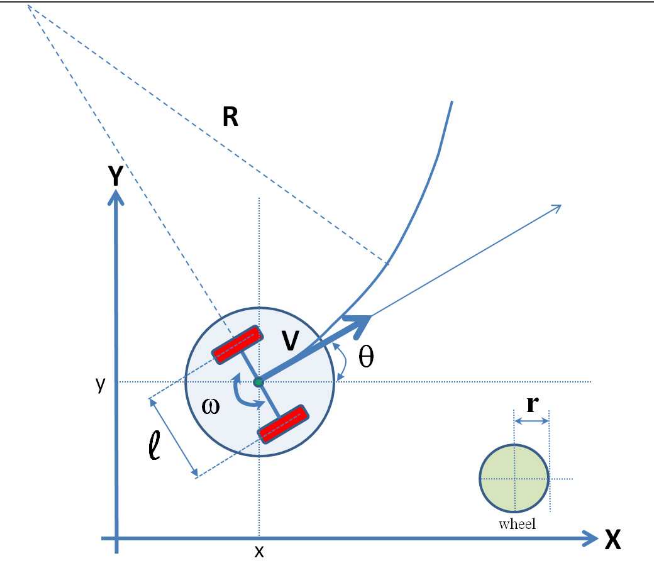
1 Direct Kinematic model

In this part, we will use the unicycle model :

Differential drive robot
Unicycle



$$\begin{cases} \dot{x} &= \mathbf{V} \cdot \cos\theta \\ \dot{y} &= \mathbf{V} \cdot \sin\theta \\ \dot{\theta} &= \omega \end{cases}$$

1.1 Theoretical model

We will consider the $[2, 0]$ mobile robot.

Considering the following definitions :
- $(\mathbf{V}, \omega)^T$: cartesian velocities of the mobile robot in m/s and rad/s
- $(x, y, \theta)^T$: posture coordinates of the mobile robot
- (Φ_1, Φ_2) the fixed wheel angles
- r : radius of the wheel in m
- L : distance of the fixed wheel in m from the mobile frame ($l = 2 \cdot L$)

Establish the following relations :

$$\begin{pmatrix} \mathbf{V} \\ \omega \end{pmatrix} = \begin{pmatrix} \cdots & \cdots \\ \cdots & \cdots \end{pmatrix} \cdot \begin{pmatrix} \dot{\Phi}_1 \\ \dot{\Phi}_2 \end{pmatrix} \tag{1}$$

1.2 Validation and Simulation

Develop a direct kinematic Simulink block representing the mobile robot equations, in terms of r and L (r and L must be parameters of the global block), such that :
- Inputs
  - Velocities of the fixed wheels $(\dot{\Phi}_1, \dot{\Phi}_2)$ in rad/s
- Ouputs
  - the posture coordinates of the mobile robot $(x, y, \theta)^T$ in m and rad
  - the cartesian velocities of the mobile robot $(\mathbf{V}, \omega)^T$ in m/s and rad/s

Build a simulink scheme using your block. Use a function generator to validate your block. check your results.

For the simulation purpose, you will consider :
- the wheel radius $r = 0,1m$
- the distance between the fixed wheels $2 \cdot L = 0,26m$

You can use the matlab function *plot_vehicle* (see the Mobrob webpage) to visualize the evolution of the robot, or just plot the position of the mobile robot.

2 Static decoupling control

This part concerns the simulation of the static decoupling control law of the robot $[2, 0]$.

This control law can be used for the tracking and regulation problems of the position (x, y) coordinates of a point P not lying on the axis of the rotation of the fixed wheels.

Suppose that this point is located on the x axis of the mobile such that : $d = 0.15m$.

The desired position motion is to execute a circular trajectory with radius $R = 2m$, and whose centre is at the origin of frame 0.

The desired angular velocity will be assumed constant and equal to $0.5rad/s$.

At $t = 0$, the posture of the robot is defined as :
- $x_0 = 2,30m$
- $y_0 = 0m$
- $\theta_0 = \pi\ rad$

2.1 Theoretical study

The desired trajectory that the point P of the robot should follow is defined by :

$$\mathbf{h}^d = \begin{pmatrix} R \cdot \cos(\omega^d \cdot t) \\ R \cdot \sin(\omega^d \cdot t) \end{pmatrix}$$

Consider the coordinates of the point P as :

$$\mathbf{h} = \begin{pmatrix} x + d \cdot \cos(\theta) \\ y + d \cdot \sin(\theta) \end{pmatrix}$$

For the general case, you will establish the global equation :

$$\dot{\mathbf{h}} = \mathbf{K}(\theta) \cdot \mathbf{u} \text{ with } \mathbf{u} = (\mathbf{V}, \omega)^T \text{ and } \mathbf{K}(\theta) = \begin{pmatrix} \cdots & \cdots \\ \cdots & \cdots \end{pmatrix}$$

Demonstrate that when using the control law $\mathbf{u} = \mathbf{K}^{-1}(\theta) \cdot \mathbf{W}$ with an auxiliary control command $\mathbf{W}$ such that :

$$\mathbf{W} = \dot{\mathbf{h}}^d + K_p \cdot (\mathbf{h}^d - \mathbf{h})$$

then the system is asymptotically stable. What is the remaining error ? What about the tracking error ?

What happen to the orientation of the mobile robot ?

2.2 Validation and Simulation

Build a simulink scheme using the block developed in the previous section.

For the simulation purpose, you will consider :
- the desired angular velocity $\omega^d = 0,5 rad/s$
- the gain matrix K_p to be tuned

Three cases will be studied :
- The values of the parameters of the robot (r and L) are well known,
- There is a $\pm 10\%$ error on the values of r and L (used in the control law) while the ones present in the direct kinematic model are not changing.

For each cases, you will study and comment the perfomances.

Make comments on the evolution of the orientation for each cases.

Consider now, the case of position regulation reduce to a fixed desired final position. Try to make different simulation. Conclude on this part.

3 Dynamic decoupling control

This part concerns the simulation of the dynamic decoupling control law of the robot $[2,0)]$. This control law can be used for the tracking and regulation problems of the position (x, y) coordinates of a point P .

The desired position motion is to execute two turns of a circular trajectory with radius $R = 2m$, and whose centre is at the origin of frame 0.

3.1 Theoretical study

The desired trajectory that the point P of the robot should follow is defined by :

$$\mathbf{h}^d = \begin{pmatrix} R \cdot \cos(\omega^d \cdot t) \\ R \cdot \sin(\omega^d \cdot t) \end{pmatrix}$$

Consider the coordinates of the point P as :

$$\mathbf{h} = \begin{pmatrix} x \\ y \end{pmatrix}$$

For the general case, you will establish the global equation :

$$\ddot{\mathbf{h}} = \mathbf{F}(\theta) \cdot \mathbf{u}'$$

with

$$\mathbf{u}' = (\dot{\mathbf{V}}, \omega)^T \text{ and } \mathbf{F}(\theta) = \begin{pmatrix} \cdots & \cdots \\ \cdots & \cdots \end{pmatrix}$$

Demonstrate that when using the control law $\mathbf{u} = \mathbf{F}^{-1}(\theta) \cdot \mathbf{W}$ with an auxiliary control command $\mathbf{W}$ such that :

$$\mathbf{W} = \ddot{\mathbf{h}}^d + K_d \cdot (\dot{\mathbf{h}}^d - \dot{\mathbf{h}}) + K_p \cdot (\mathbf{h}^d - \mathbf{h})$$

then the system is asymptotically stable. What is the remaining error ? What about the tracking error ?

What happen to the orientation of the mobile robot ?

3.2 Validation and Simulation

Build a simulink scheme using the block developed before.

For the simulation purpose, you will consider :
- the desired angular velocity $\omega^d = 0,5 rad/s$
- the gain matrices K_p and K_d to be tuned

Three cases will be studied :
- The values of the parameters of the robot (r and L) are well known,
- There is a $\pm 10\%$ error on the values of r and L (used in the control law) while the ones present in the direct kinematic model are not changing.

For each cases, you will study and comment the perfomances.

Make comments on the evolution of the orientation for each cases.

4 Lyapunov control law

This part concerns the simulation of the Lyapunov control law of the robot $[2,0)]$. This control law can be used for the posture tracking of the mobile robot.

4.1 Theoretical study

Consider the posture tracking error define by :

$$\xi^e = (\xi^d - \xi)$$

where
- ξ^d represents the desired posture of the mobile robot
- ξ represents the current posture of the mobile robot

So, we have :

$${}^m\xi^e = \begin{pmatrix} {}^m x^e \\ {}^m y^e \\ {}^m \theta^e \end{pmatrix} = {}^m\mathbf{R}_0 \cdot ({}^0\xi^d - {}^0\xi)$$

Consider the control error define by :

$$\mathbf{u}^e = (\mathbf{u}^d - \mathbf{u})$$

where
- $\mathbf{u}^d = (\mathbf{V}^d, \omega^d)^T$ represents the desired control velocities of the mobile robot
- $\mathbf{u} = (\mathbf{V}, \omega)^T$ represents the current control velocities of the mobile robot

Demonstrate that we can put the evolution of ${}^m\xi^e$ on the form :

$$ {}^m\dot{\xi}^e = \mathbf{A}(\xi^e, u^d) \cdot^m \xi^e + \mathbf{B}(\xi^e, u^d) \cdot \mathbf{u}^e $$

with

$$ \mathbf{A}(\xi^e, u^d) = \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \text{ and } \mathbf{B}(\xi^e, u^d) = \begin{pmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{pmatrix} $$

You will use the following control error :

$$ \mathbf{u}^e = \begin{pmatrix} \mathbf{V}^d \cdot (1 - \cos({}^m\theta^e)) - K_x \cdot^m x^e \\ -K_y \cdot \mathbf{V}^d \cdot \frac{\sin({}^m\theta^e)}{{}^m\theta^e} \cdot^m y^e - K_\theta \cdot^m \theta^e \end{pmatrix} $$

Consider the function $\mathbf{W} = \frac{{}^m x^{e2} + {}^m y^{e2} + \frac{{}^m\theta^{e2}}{K_y}}{2}$. Demonstrate that this is a Lyapunov function. Demonstrate that the system is asymptotically stable ? What is the domain of stability ?

4.2 Validation and Simulation

Build a simulink scheme using the block developed before.

For the simulation purpose, you will consider :
- the desired angular velocity $\omega^d = 0,5 rad/s$
- the gain K_x, K_y and K_θ to be tuned

Three cases will be studied :
- The values of the parameters of the robot (r and L) are well known,
- There is a $\pm 10\%$ error on the values of r and L (used in the control law) while the ones present in the direct kinematic model are not changing.

For each cases, you will study and comment the perfomances.

5 Extension

Consider now the mobile robot [1,1]. Restart all the lab.