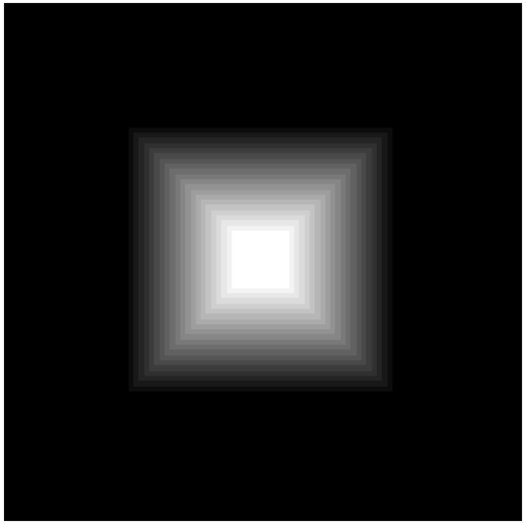
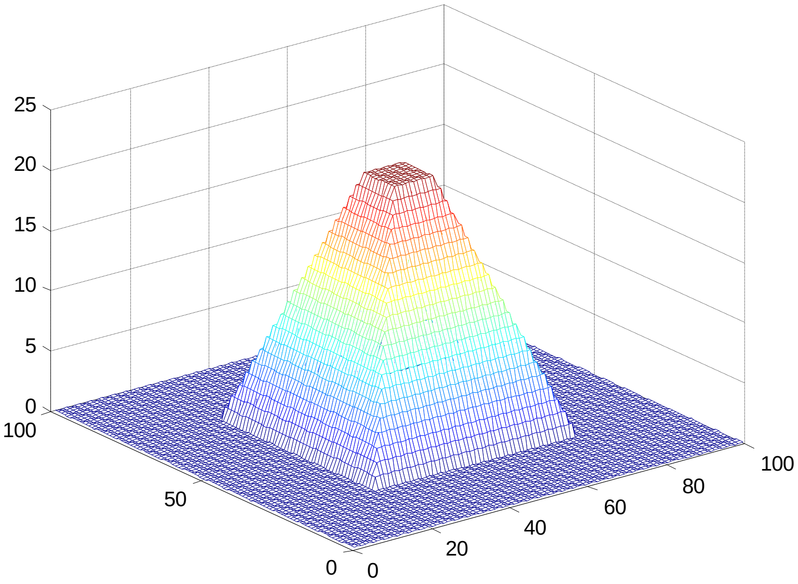
Segmentation by Morphological Watersheds

Introduction

- Based on visualizing an image in 3D

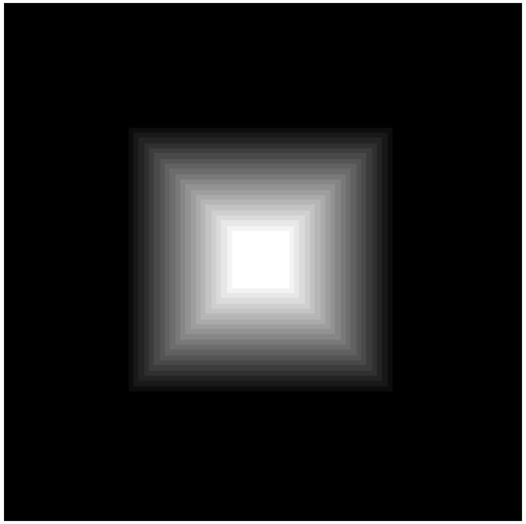


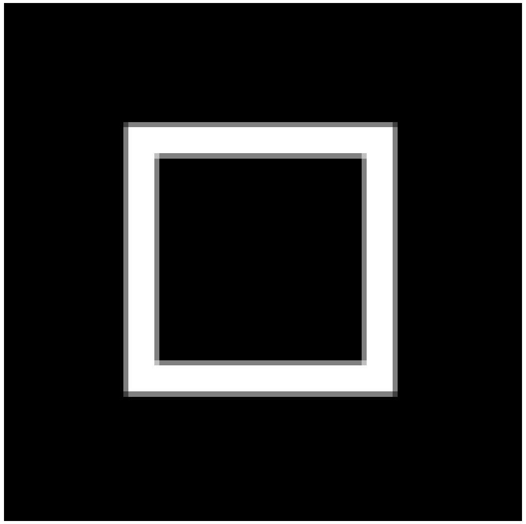
imshow(I,[ ])



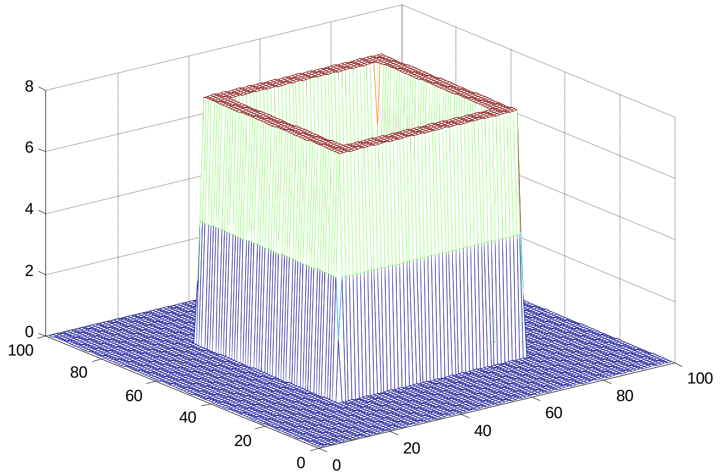
mesh(I)

Introduction





- Instead of working on an image itself, this technique is often applied on its gradient image.
  - In this case, each object is distinguished from the background by its up-lifted edges

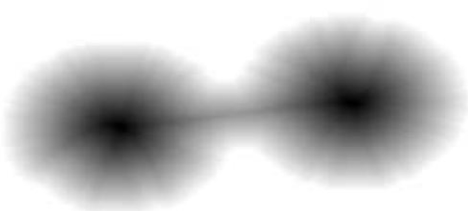


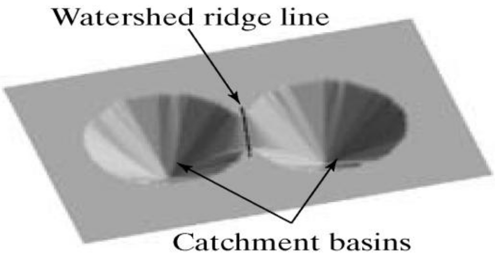
Basic Definitions

- I: 2D gray level image
- D_I: Domain of I
- Path P of length l between p and q in I
  - A (l+1)-tuple of pixels ($p_0=p,p_1,\ldots,p_l=q$) such that p_i,p_{i+1} are adjacent (4 adjacent, 8 adjacent, or m adjacent,
- l(P): The length of a given path P
- Minimum
  - A minimum M of I is a connected plateau of pixels from which it is impossible to reach a point of lower altitude without having to climb

Basic Definitions

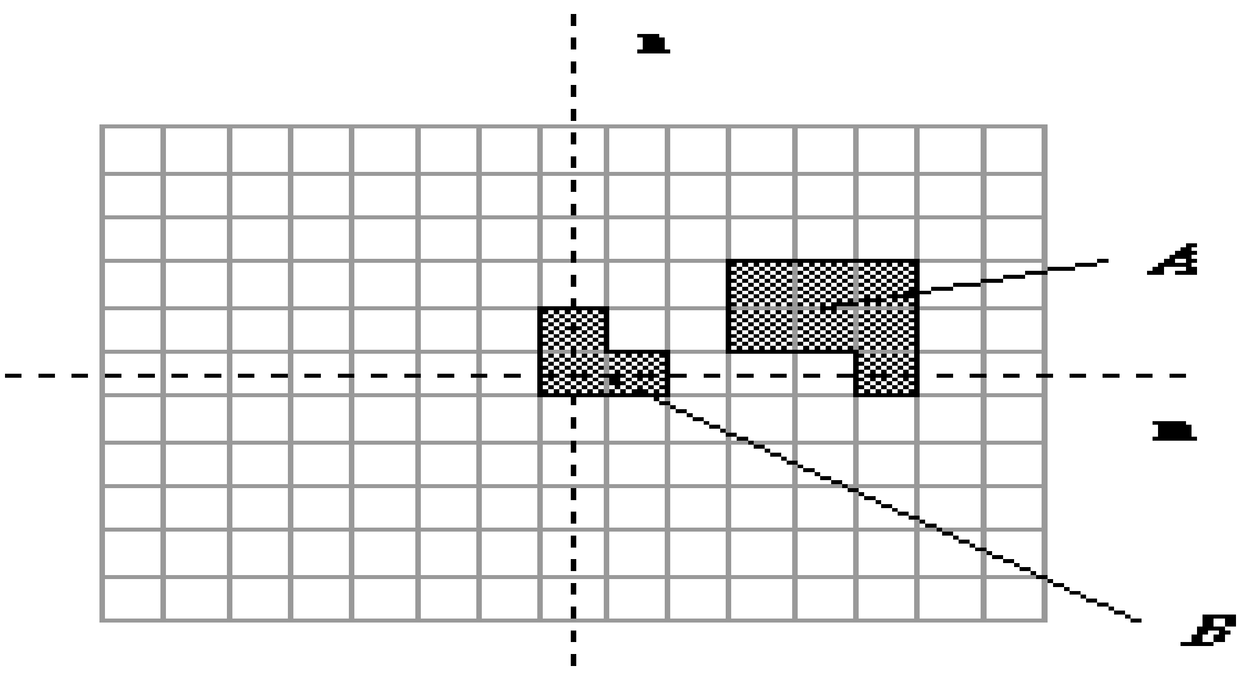
- Instead of working on an image itself, this technique is often applied on its gradient image.
- Three types of points
  - Points belonging to a regional minimum
  - Catchment basin / watershed of a regional minimum
    - Points at which a drop of water will certainly fall to a single minimum
  - Divide lines / Watershed lines
    - Points at which a drop of water will be equally likely to fall to more than one minimum
    - Crest lines on the topographic surface
- This technique is to identify all the third type of points for segmentation





Basic Definitions

We defined an image as an (amplitude) function of two, real (coordinate) variables $a(x,y)$ or two, discrete variables $a[m,n]$. An alternative definition of an image can be based on the notion that an image consists of a set (or collection) of either continuous or discrete coordinates.



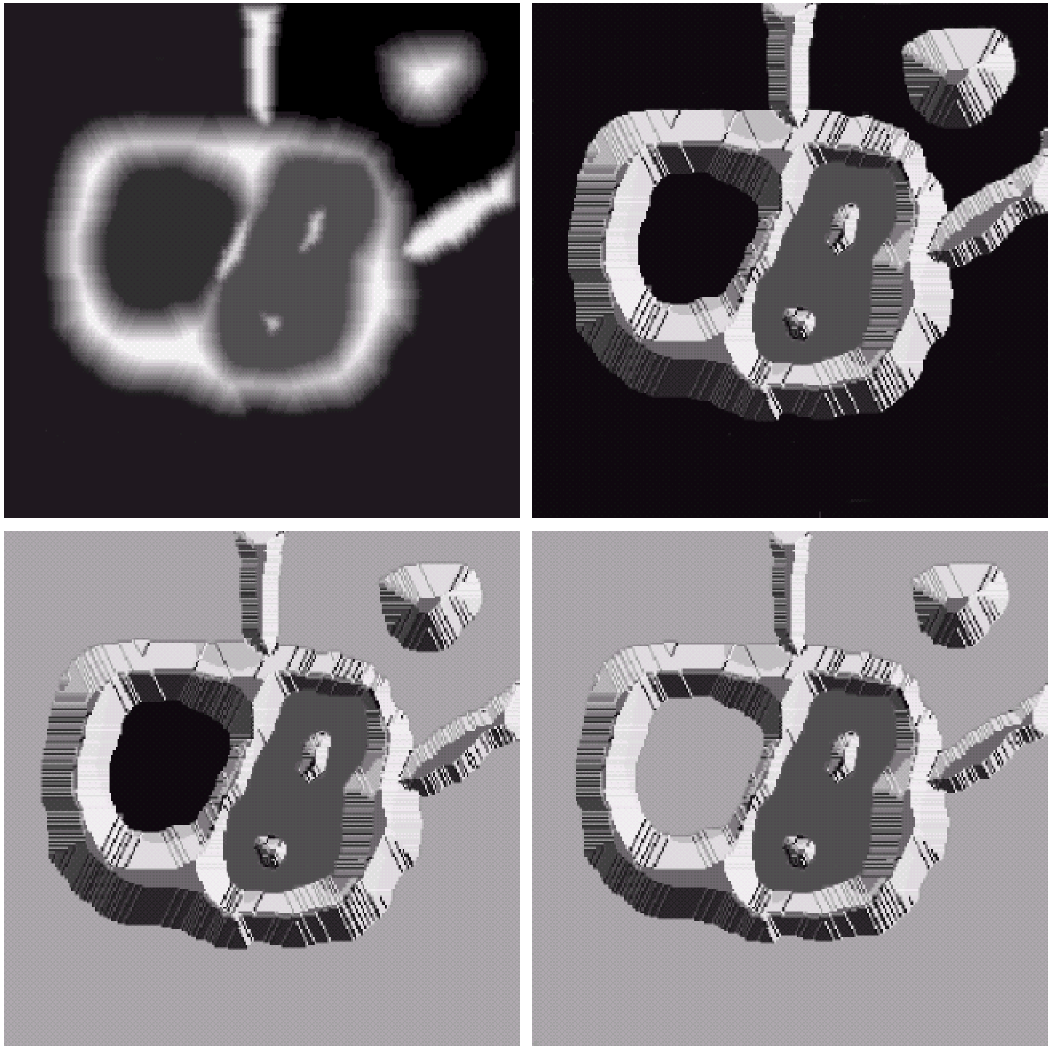
Fundamental Definitions – Erosion and Dilation

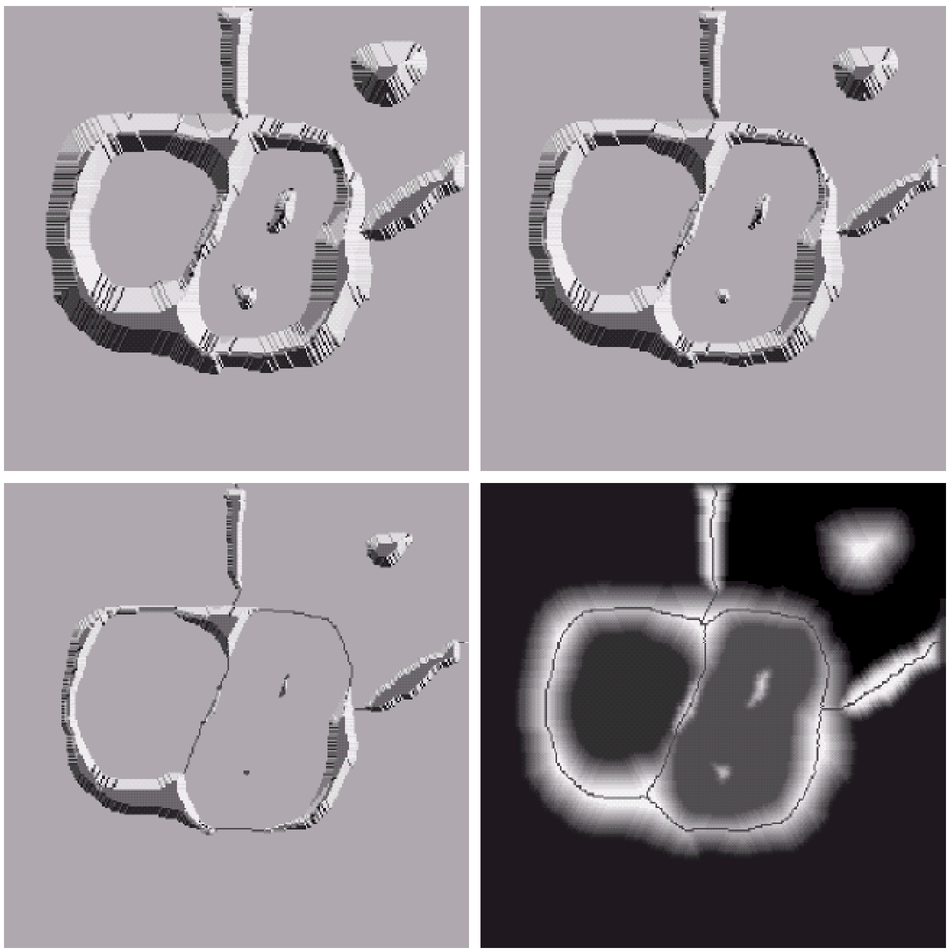
While either set A or B can be thought of as an "image", A is usually considered as the image and B is called a structuring element. The structuring element is to mathematical morphology what the convolution kernel is to linear filter theory.

Dilation, in general, causes objects to dilate or grow in size; *erosion* causes objects to shrink. The amount and the way that they grow or shrink depend upon the choice of the structuring element. Dilating or eroding without specifying the structural element makes no more sense than trying to lowpass filter an image without specifying the filter.

Basic Steps

1. Piercing holes in each regional minimum of I
2. The 3D topography is flooded from below gradually
3. When the rising water in distinct catchment basins is about to merge, a dam is built to prevent the merging





e f
g h

FIGURE 10.44 *(Continued)* (e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

3. The dam boundaries correspond to the watershed lines to be extracted by a watershed segmentation algorithm
   - Eventually only constructed dams can be seen from above

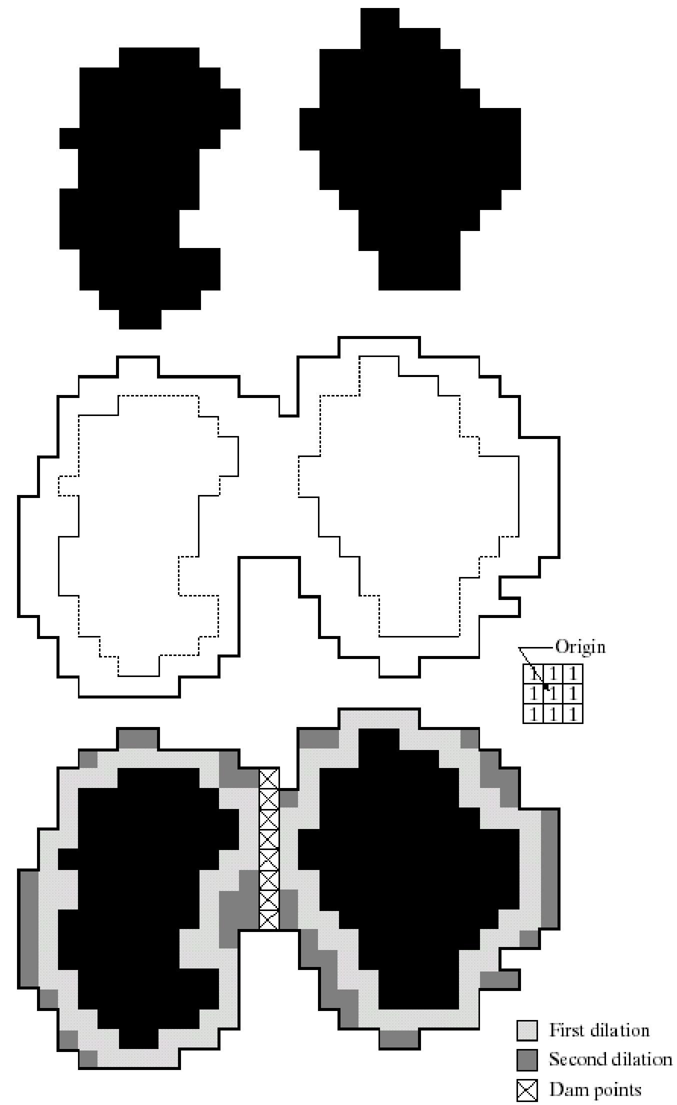
Dam Construction

- Based on binary morphological dilation
- At each step of the algorithm, the binary image in obtained in the following manner

1. Initially, the set of pixels with minimum gray level are 1, others 0.
2. In each subsequent step, we flood the 3D topography from below and the pixels covered by the rising water are 1s and others 0s.

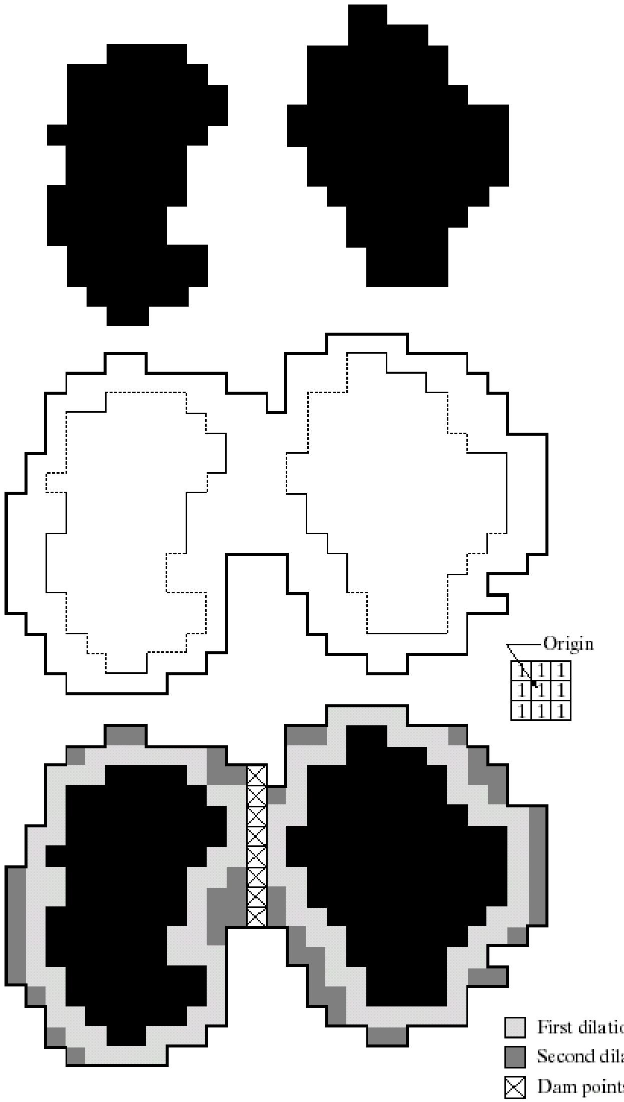
Notations

- M1, M2:
  - Sets of coordinates of points in the two regional minima
- $C_{n-1}(M_1)$, $C_{n-1}(M_2)$
  - Sets of coordinates of points in the catchment basins associated with M1 M2 at stage n-1 of flooding (catchment basins up to the flooding level)
- C[n-1]
  - Union of $C_{n-1}(M_1)$, $C_{n-1}(M_2)$



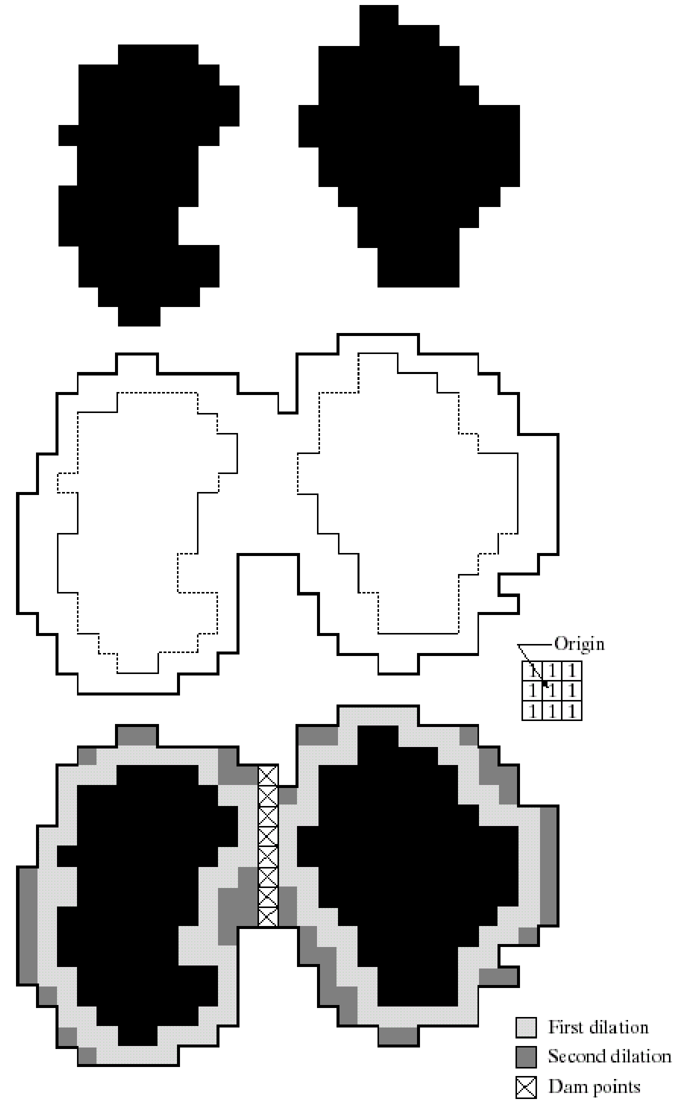
Dam Construction

- At flooding step n-1, there are two connected components. At flooding step n, there is only one connected component
  - This indicates that the water between the two catchment basins has merged at flooding step n
  - Use “q” to denote the single connected component
- Steps
  - Repeatedly dilate $C_{n-1}(M_1)$, $C_{n-1}(M_2)$ by the 3×3 structuring element shown, subject to the following condition
    - Constrained to q (center of the structuring element can not go beyond q during dilation)



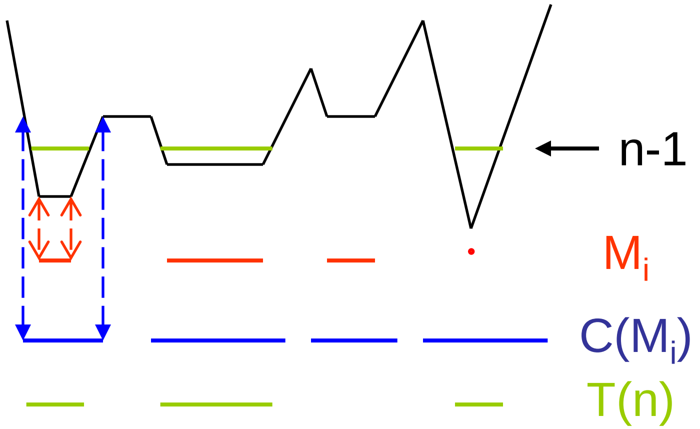
Dam Construction

- The dam is constructed by the points on which the dilation would cause the sets being dilated to merge.
  - Resulting one-pixel thick connected path
- Setting the gray level at each point in the resultant path to a value greater than the maximum gray value of the image. Usually max+1



Watershed Transform

- Denote $M_1, M_2, \ldots, M_R$ as the sets of the coordinates of the points in the regional minima of an (gradient) image g(x,y)
- Denote $C(M_i)$ as the coordinates of the points in the catchment basin associated with regional minimum M_i.
- Denote the minimum and maximum gray levels of g(x,y) as *min* and *max*
- Denote T[n] as the set of coordinates (s,t) for which g(s,t) < n
- Flood the topography in integer flood increments from n=*min*+1 to n=*max*+1
- At each flooding, the topography is viewed as a binary image

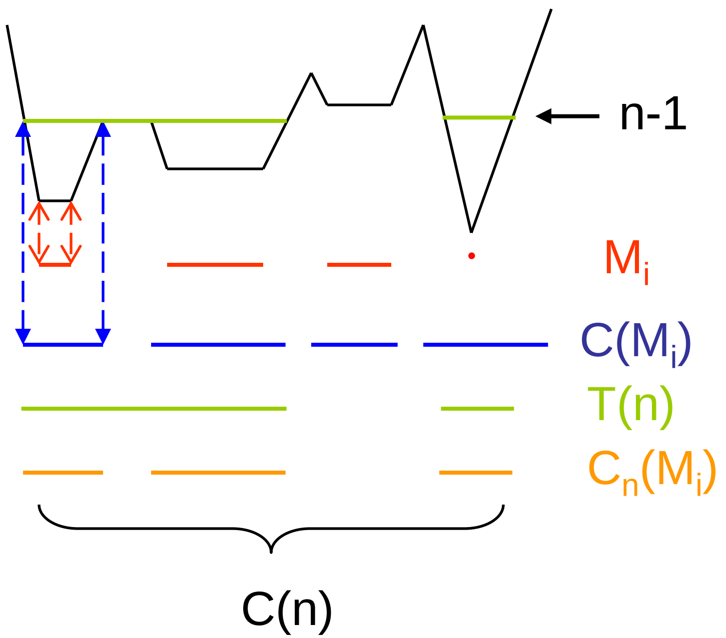


Watershed Transform

- Denote $C_n(M_i)$ as the set of coordinates of points in the catchment basin associated with minimum M_i at flooding stage n.
  - $C_n(M_i) = C(M_i) \cap T[n]$
  - $C_n(M_i) \subseteq T[n]$
- Denote C[n] as the union of the flooded catchment basin portions at stage n:

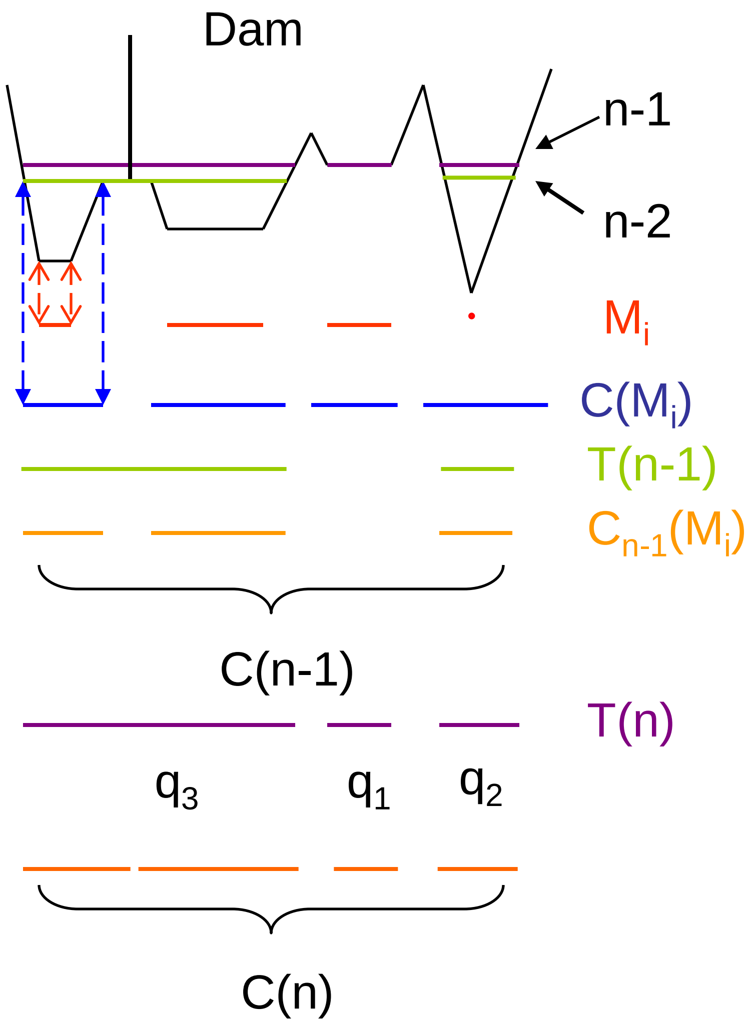
$$C[n] = \bigcup_{i=1}^{R} C_n(M_i) \text{ and } C[\max+1] = \bigcup_{i=1}^{R} C(M_i)$$

- Initialization
  - Let $C[min+1]=T[min+1]$
- At each step n, assume C[n-1] has been constructed. The goal is to obtain C[n] from C[n-1]



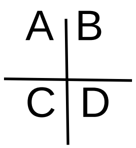
Watershed Transform

- Denote Q[n] as the set of connected components in T[n].
- For each $q \in Q[n]$, there are three possibilities
  1. $q \cap C[n-1]$ is empty (q_1)
     - A new minimum is encountered
     - q is incorporated into C[n-1] to form C[n]
  2. $q \cap C[n-1]$ contains one connected component of C[n-1] (q_2)
     - q is incorporated into C[n-1] to form C[n]
  3. $q \cap C[n-1]$ contains more than one connected components of C[n-1] (q_3)
     - A ridge separating two or more catchment basins has been encountered
     - A dam has to be built within q to prevent overflow between the catchment basins
  4. Repeat the procedure until n=max+1



Examples 1
Watershed Transform of Binary Image

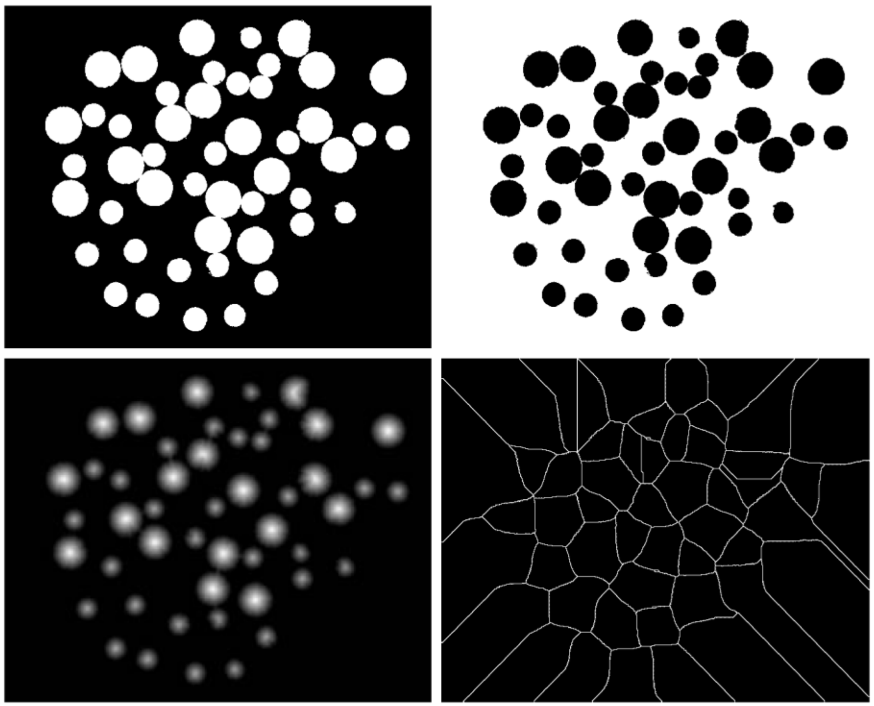
A: Original image
B: Negative of image A
C: Distance transform of B
D: Watershed transform of C



Distance transform of a binary image is defined by the distance from every pixel to the nearest non-zero valued pixel

1	1	0	0	0
1	1	0	0	0
0	0	0	0	0
0	0	0	0	0
0	1	1	1	0

0.00	0.00	1.00	2.00	3.00
0.00	0.00	1.00	2.00	3.00
1.00	1.00	1.41	2.00	2.24
1.41	1.00	1.00	1.00	1.41
1.00	0.00	0.00	0.00	1.00



Examples 2

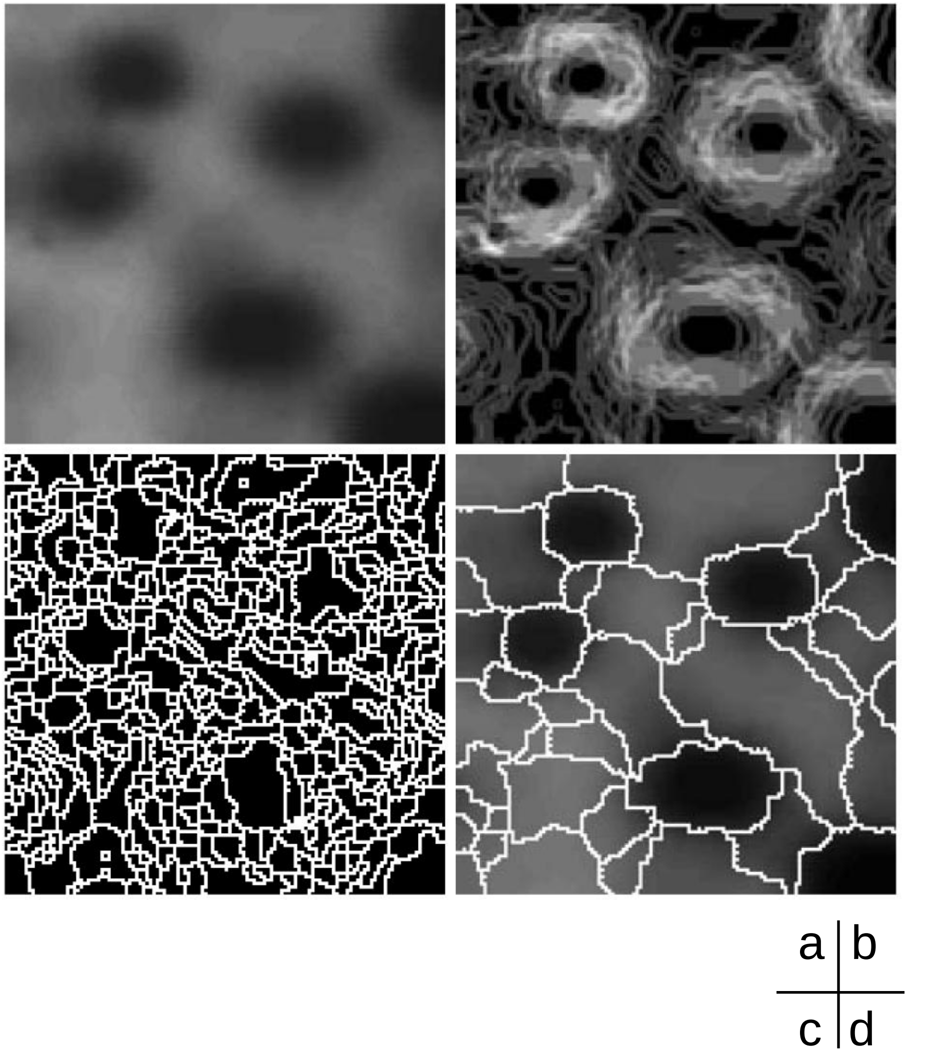
a: Original image

b: Gradient image of image a

c: Watershed lines obtained from image b (oversegmentation)

➔ Each connected region contains one local minimum in the corresponding gradient image

d: Watershed lines obtained from smoothed image b

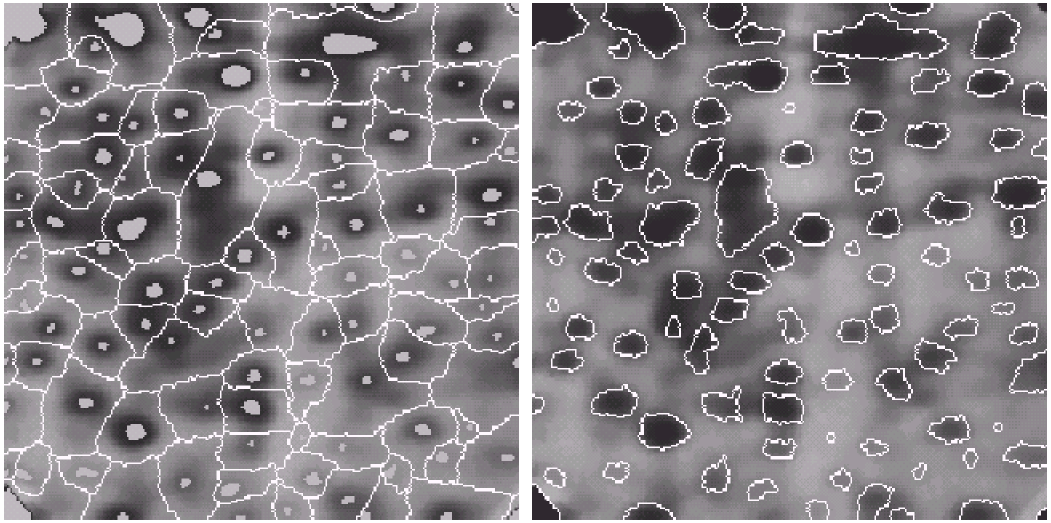


The Use of Markers

- Internal markers are used to limit the number of regions by specifying the objects of interest
  - Like seeds in region growing method
  - Can be assigned manually or automatically
  - Regions without markers are allowed to be merged (no dam is to be built)
- External markers those pixels we are confident to belong to the background
  - Watershed lines are typical external markers and they belong the same (background) region

Watershed Based Image Segmentation

1. Use internal markers to obtain watershed lines of the gradient of the image to be segmented.
2. Use the obtained watershed lines as external markers
3. Each region defined by the external markers contains a single internal marker and part of the background
4. The problem is reduced to partitioning each region into two parts: object (containing internal markers) and a single background (containing external markers)
   – Global thresholding, region growing, region splitting and merging, or watershed transform



a b

FIGURE 10.48 (a) Image showing internal markers (light gray regions) and external markers (watershed lines). (b) Result of segmentation. Note the improvement over Fig. 10.47(b). (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

MATLAB Example

- A: Original image f
- B: Direct watershed transform result using the following commands
  - L=watershed(g)
  - wr = L ==0
  - g is the gradient image of A
- C: shows all of the regional minima of g using "rm=imregionalmin(g)"
- D: internal markers obtained by
  - im = imextendedmin(g,2)
  - fim = f;
  - fim(im) = 175;
- E: External markers using
  - Lim = watershed(bwdist(im))
  - em = Lim ==0
- F: Modified gradient image obtained from internal and external markers
  - g2 = imimposemin(g, im | em)
- G: Final segmentation result
  - L2 = watershed(g2)
  - f2 = f;
  - f2(L2 == 0) = 255