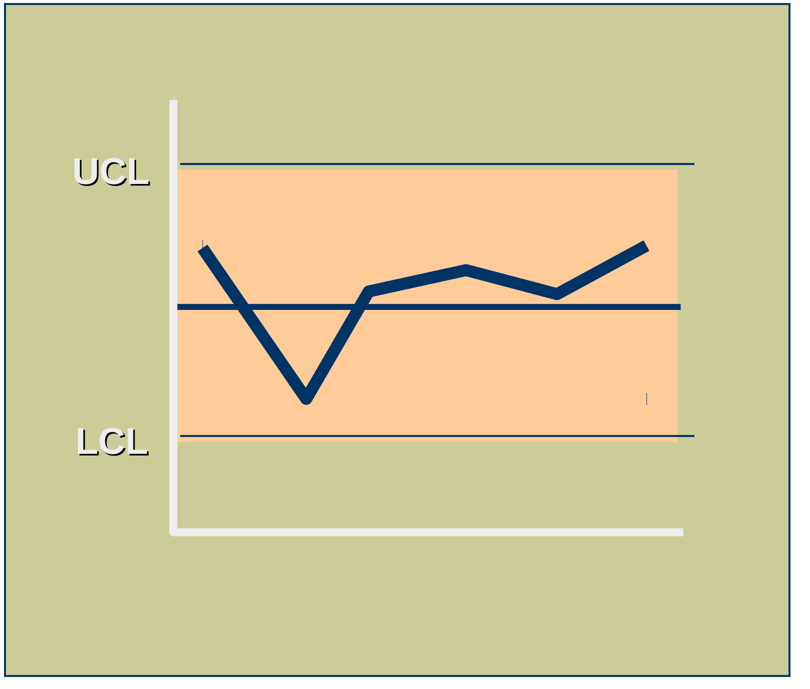
Statistical Process Control

Outline

- Basics of Statistical Process Control
- Control Charts
- Control Charts for Attributes
- Control Charts for Variables
- Control Chart Patterns
- SPC with Excel
- Process Capability

Basics of Statistical Process Control

- Statistical Process Control (SPC)
  - monitoring production process to detect and prevent poor quality
- Sample
  - subset of items produced to use for inspection
- Control Charts
  - process is within statistical control limits



Variability

- Random
  - common causes
  - inherent in a process
  - can be eliminated only through improvements in the system
- Non-Random
  - special causes
  - due to identifiable factors
  - can be modified through operator or management action

SPC in TQM

- SPC
  - tool for identifying problems and make improvements
  - contributes to the TQM goal of continuous improvements

Quality Measures

- **Attribute**
  - a product characteristic that can be evaluated with a discrete response
  - good – bad; yes - no
- **Variable**
  - a product characteristic that is continuous and can be measured
  - weight - length

Applying SPC to Service

- Nature of defect is different in services
- Service defect is a failure to meet customer requirements
- Monitor times, customer satisfaction

Applying SPC to Service (cont.)

- Hospitals
  - timeliness and quickness of care, staff responses to requests, accuracy of lab tests, cleanliness, courtesy, accuracy of paperwork, speed of admittance and checkouts
- Grocery Stores
  - waiting time to check out, frequency of out-of-stock items, quality of food items, cleanliness, customer complaints, checkout register errors
- Airlines
  - flight delays, lost luggage and luggage handling, waiting time at ticket counters and check-in, agent and flight attendant courtesy, accurate flight information, passenger cabin cleanliness and maintenance

Applying SPC to Service (cont.)

- Fast-Food Restaurants
  - waiting time for service, customer complaints, cleanliness, food quality, order accuracy, employee courtesy
- Catalogue-Order Companies
  - order accuracy, operator knowledge and courtesy, packaging, delivery time, phone order waiting time
- Insurance Companies
  - billing accuracy, timeliness of claims processing, agent availability and response time

Where to Use Control Charts

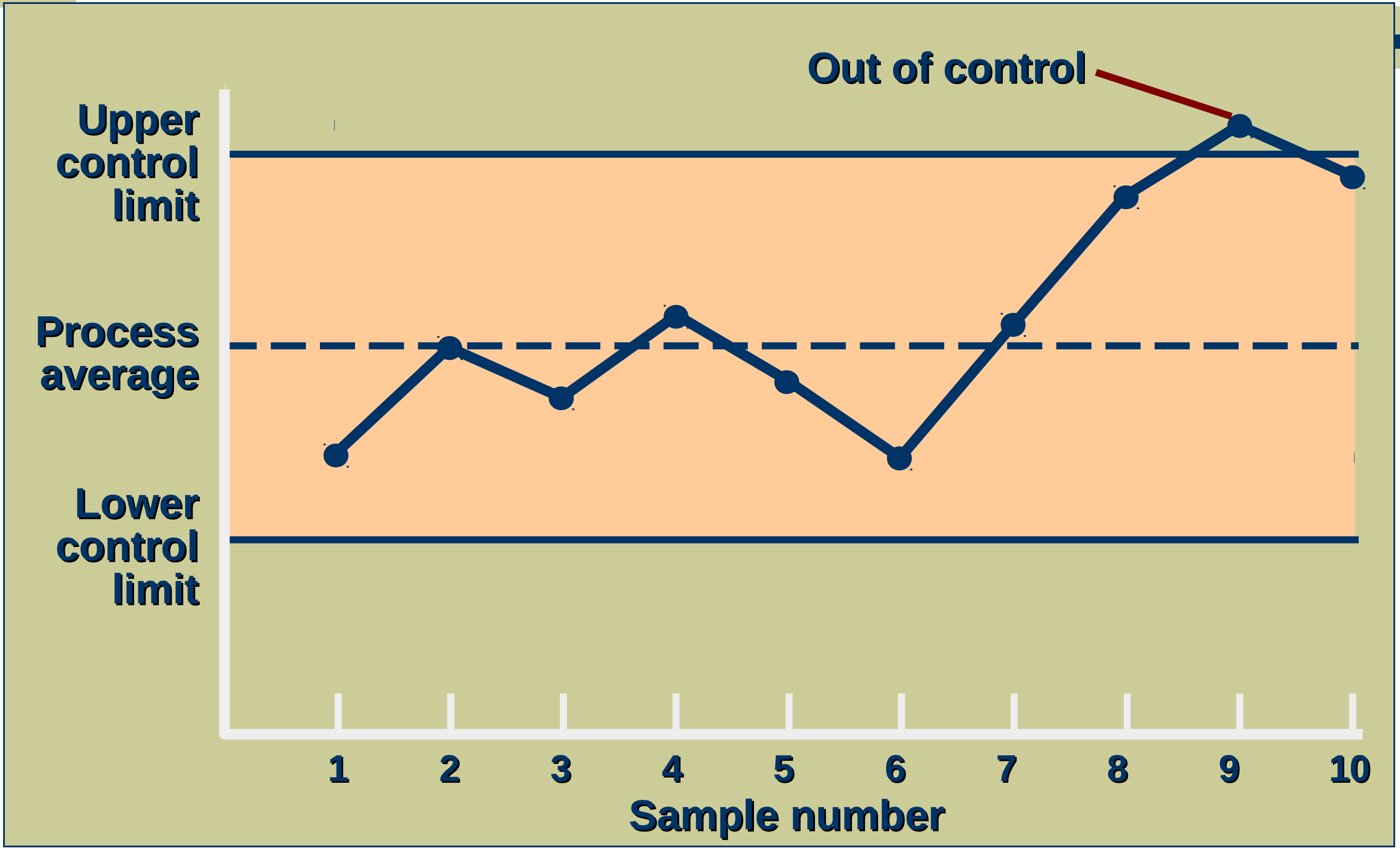
- Process has a tendency to go out of control
- Process is particularly harmful and costly if it goes out of control
- Examples
  - at the beginning of a process because it is a waste of time and money to begin production process with bad supplies
  - before a costly or irreversible point, after which product is difficult to rework or correct
  - before and after assembly or painting operations that might cover defects
  - before the outgoing final product or service is delivered

Control Charts

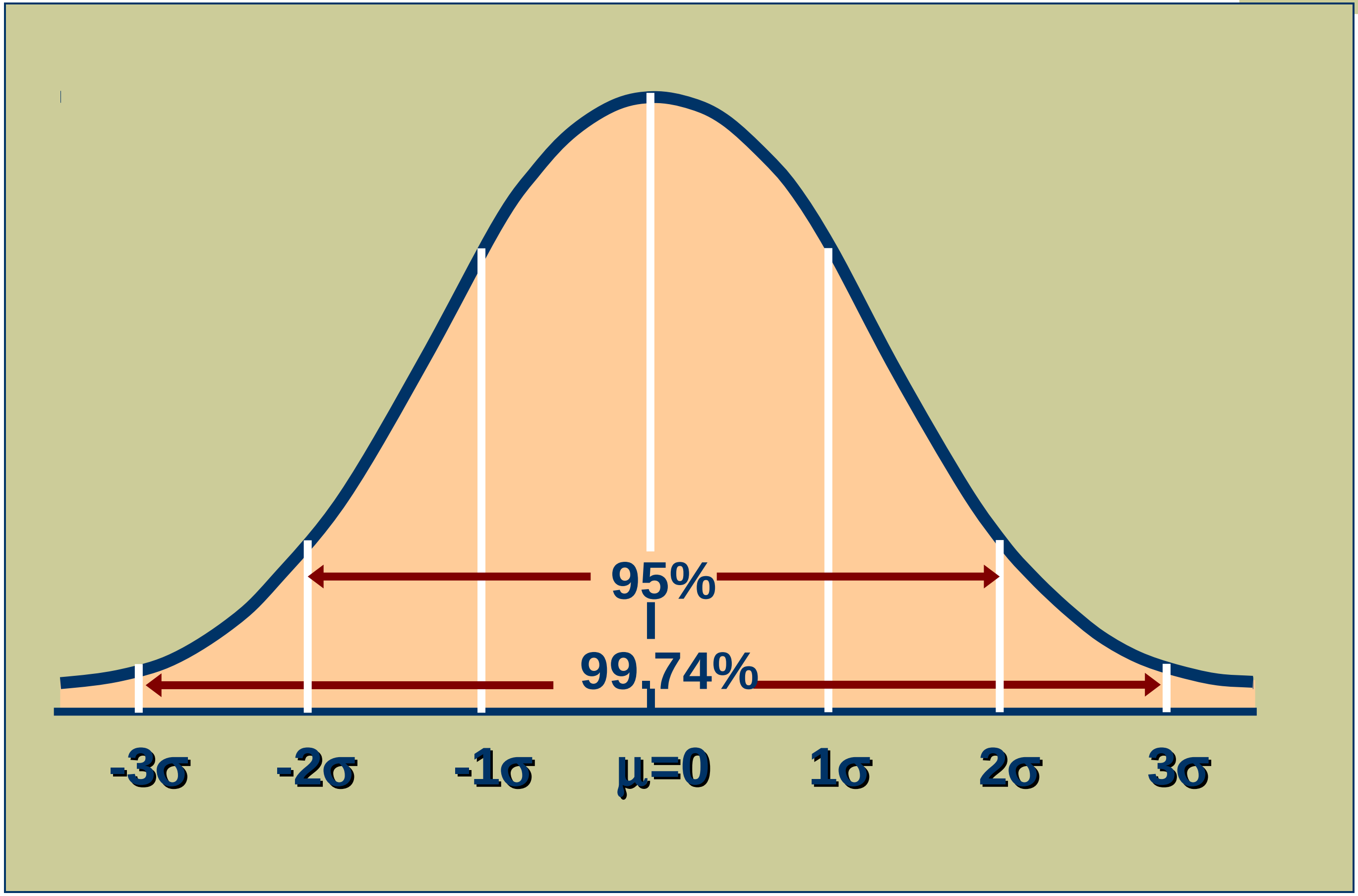
- A graph that establishes control limits of a process
- Control limits
  - upper and lower bands of a control chart

- Types of charts
  - Attributes
    - p-chart
    - c-chart
  - Variables
    - range (R-chart)
    - mean (x bar – chart)

Process Control Chart



Normal Distribution



A Process Is in Control If …

1. … no sample points outside limits
2. … most points near process average
3. … about equal number of points above and below centerline
4. … points appear randomly distributed

Control Charts for Attributes

- p-charts
  - uses portion defective in a sample
- c-charts
  - uses number of defects in an item

p-Chart

$$UCL = p + z\sigma_p$$

$$LCL = p - z\sigma_p$$

z = number of standard deviations from process average

p = sample proportion defective; an estimate of process average

σ_p = standard deviation of sample proportion

$$\sigma_p = \sqrt{\frac{p(1 - p)}{n}}$$

p-Chart Example

SAMPLE	NUMBER OF DEFECTIVES	PROPORTION DEFECTIVE
1	6	.06
2	0	.00
3	4	.04
:	:	:
:	:	:
20	18	.18
	200	

20 samples of 100 pairs of jeans

p-Chart Example (cont.)

$$\bar{p} = \frac{\text{total defectives}}{\text{total sample observations}} = 200 / 20(100) = 0.10$$

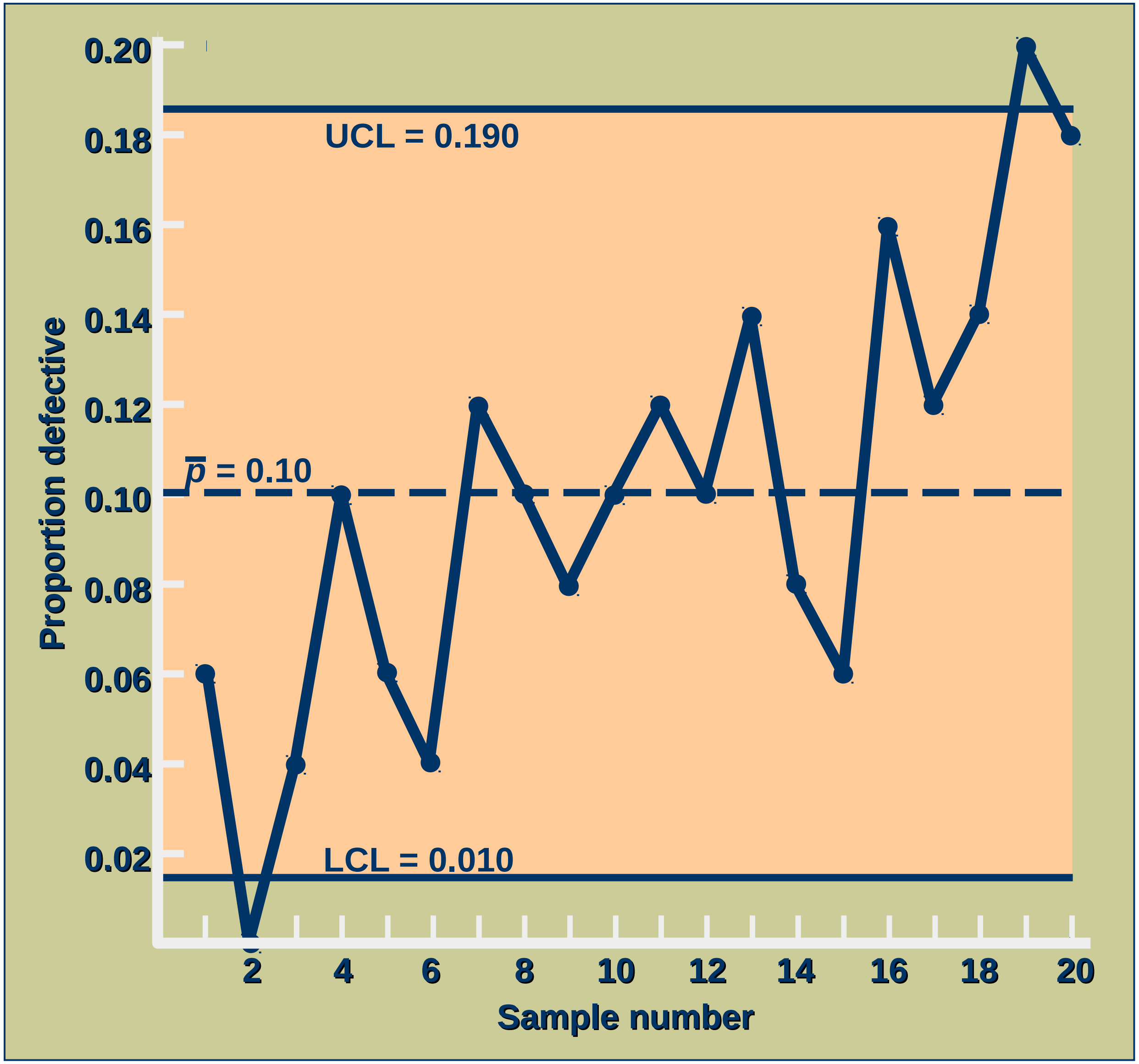
$$UCL = \bar{p} + z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 + 3\sqrt{\frac{0.10(1 - 0.10)}{100}}$$

$$UCL = 0.190$$

$$LCL = \bar{p} - z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 - 3\sqrt{\frac{0.10(1 - 0.10)}{100}}$$

$$LCL = 0.010$$

p-Chart Example (cont.)



c-Chart

$$UCL = \bar{c} + z\sigma_c$$

$$LCL = \bar{c} - z\sigma_c$$

$$\sigma_c = \sqrt{\bar{c}}$$

where

c = number of defects per sample

c-Chart (cont.)

Number of defects in 15 sample rooms

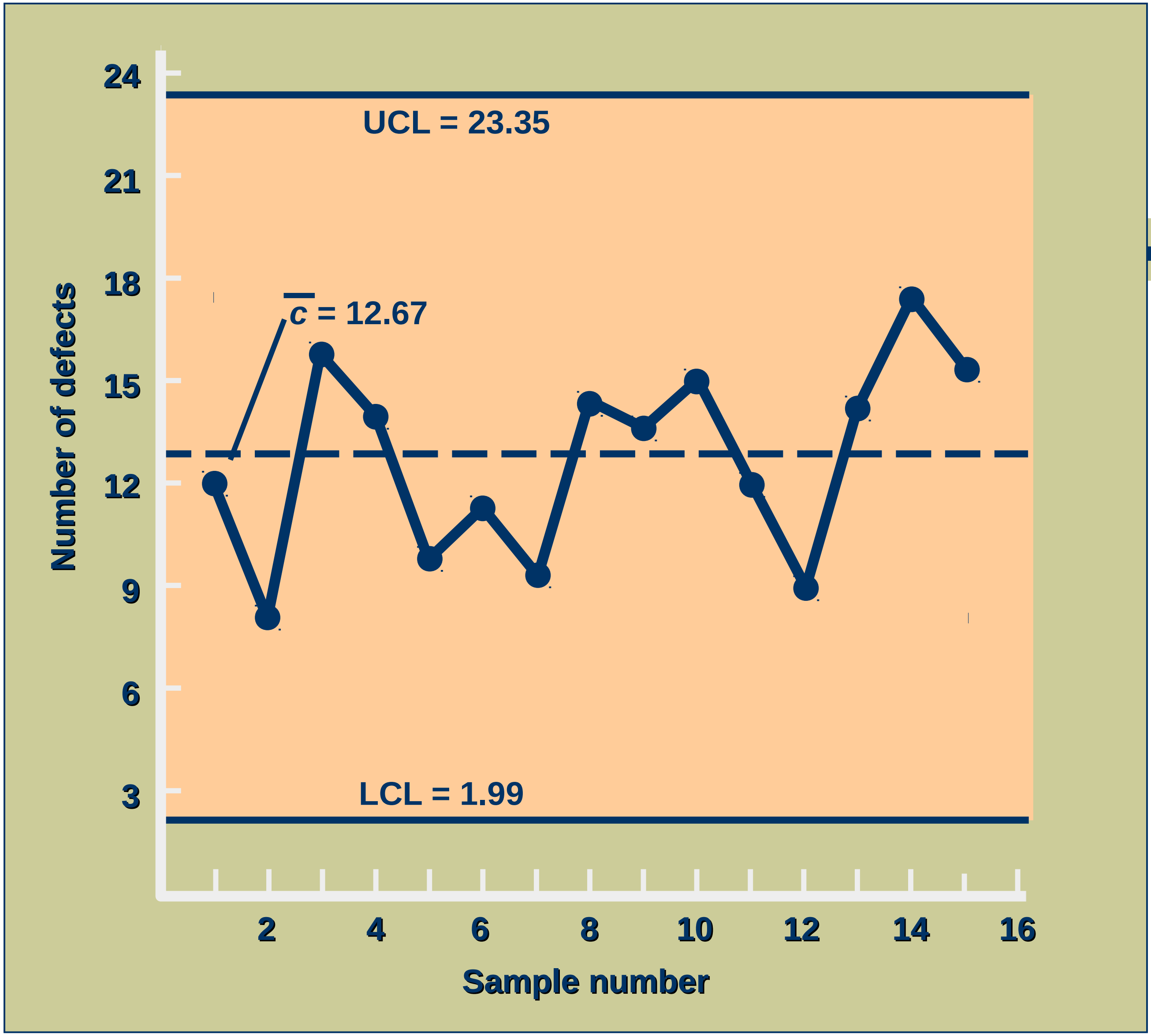
SAMPLE	NUMBER OF DEFECTS
1	12
2	8
3	16
:	:
:	:
15	15
	190

$$\bar{c} = \frac{190}{15} = 12.67$$

$$\begin{aligned} \text{UCL} &= \bar{c} + z\sigma_c \\ &= 12.67 + 3\sqrt{12.67} \\ &= 23.35 \end{aligned}$$

$$\begin{aligned} \text{LCL} &= \bar{c} + z\sigma_c \\ &= 12.67 - 3\sqrt{12.67} \\ &= 1.99 \end{aligned}$$

c-Chart (cont.)



24
21
18
15
12
9
6
3
Number of defects
UCL = 23.35
$\bar{c}$ = 12.67
LCL = 1.99
2
4
6
8
10
12
14
16
Sample number

Control Charts for Variables

- Mean chart ($\bar{x}$ -Chart)
  - uses average of a sample
- Range chart (R-Chart)
  - uses amount of dispersion in a sample

x-bar Chart

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots \bar{x}_k}{k}$$

$UCL = \bar{\bar{x}} + A_2\bar{R}$ $LCL = \bar{\bar{x}} - A_2\bar{R}$

where

$\bar{\bar{x}}$ = average of sample means

x-bar Chart Example

	OBSERVATIONS (SLIP- RING DIAMETER, CM)						
SAMPLE *k*	*1*	*2*	*3*	*4*	*5*	$\bar{x}$	*R*
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

Example 15.4

x- bar Chart Example (cont.)

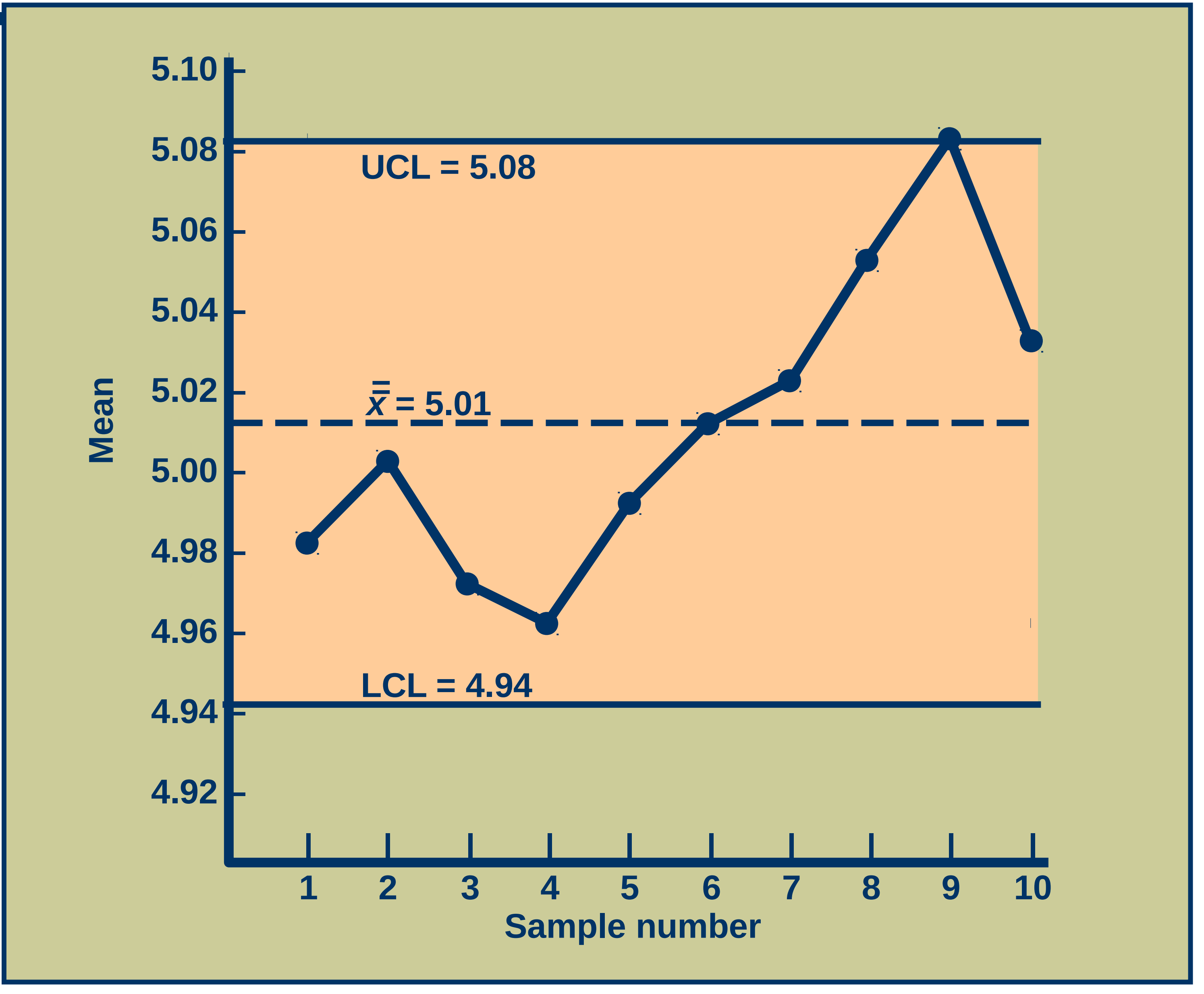
$$\bar{\bar{x}} = \frac{\sum \bar{x}}{k} = \frac{50.09}{10} = 5.01 \text{ cm}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 5.01 + (0.58)(0.115) = 5.08$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 5.01 - (0.58)(0.115) = 4.94$$

Retrieve Factor Value A_2

x- bar Chart Example (cont.)



R- Chart

$$UCL = D_4\bar{R} \qquad LCL = D_3\bar{R}$$

$$\bar{R} = \frac{\sum R}{k}$$

where

$\bar{R}$ = range of each sample

k = number of samples

R-Chart Example

	OBSERVATIONS (SLIP-RING DIAMETER, CM)						
SAMPLE *k*	*1*	*2*	*3*	*4*	*5*	$\bar{x}$	*R*
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

Example 15.3

R-Chart Example (cont.)

$$\bar{R} = \frac{\sum R}{k} = \frac{1.15}{10} = 0.115$$

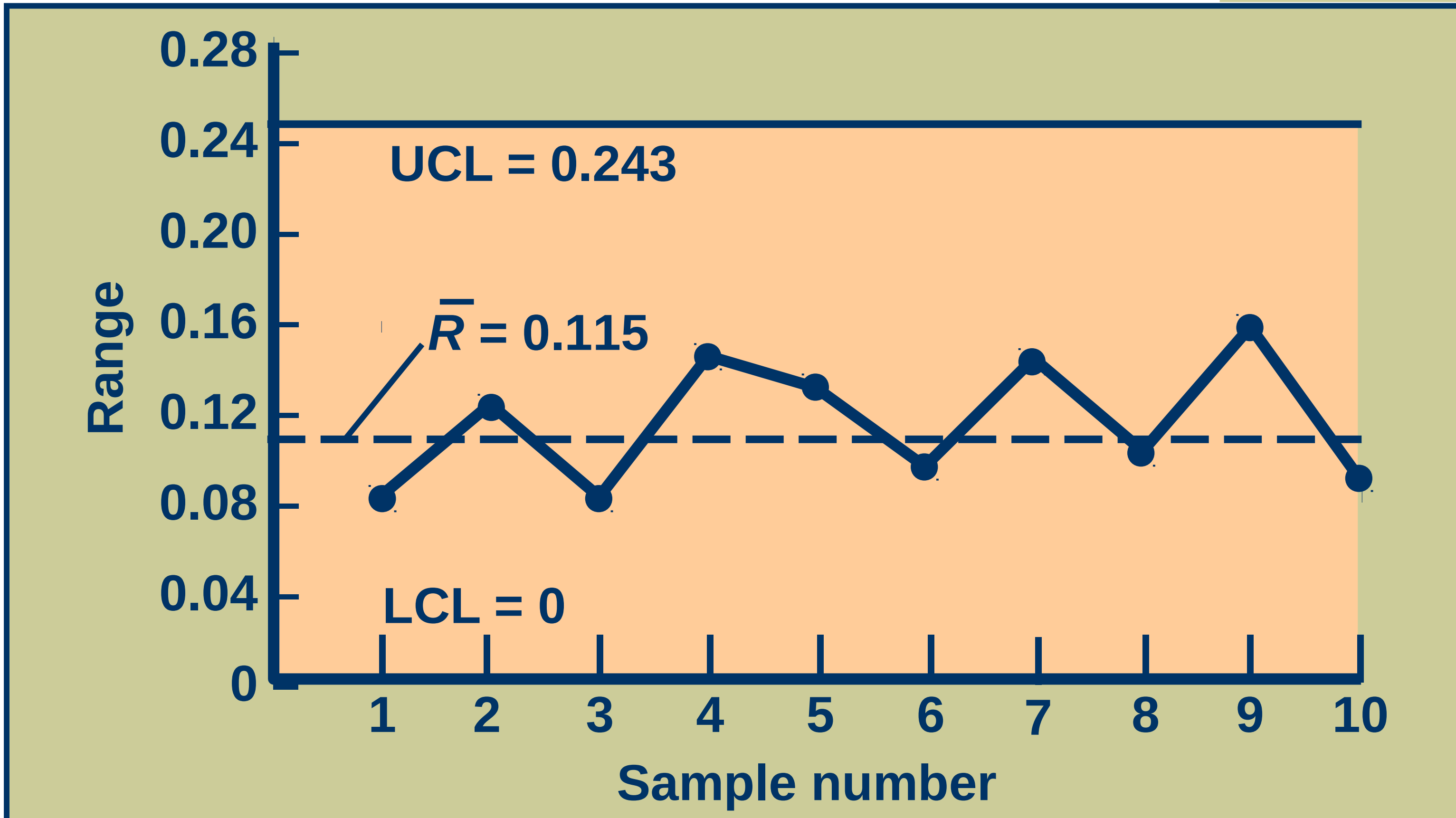
$$UCL = D_4\bar{R} = 2.11(0.115) = 0.243$$

$$LCL = D_3\bar{R} = 0(0.115) = 0$$

Retrieve Factor Values D_3 and D_4

Example 15.3

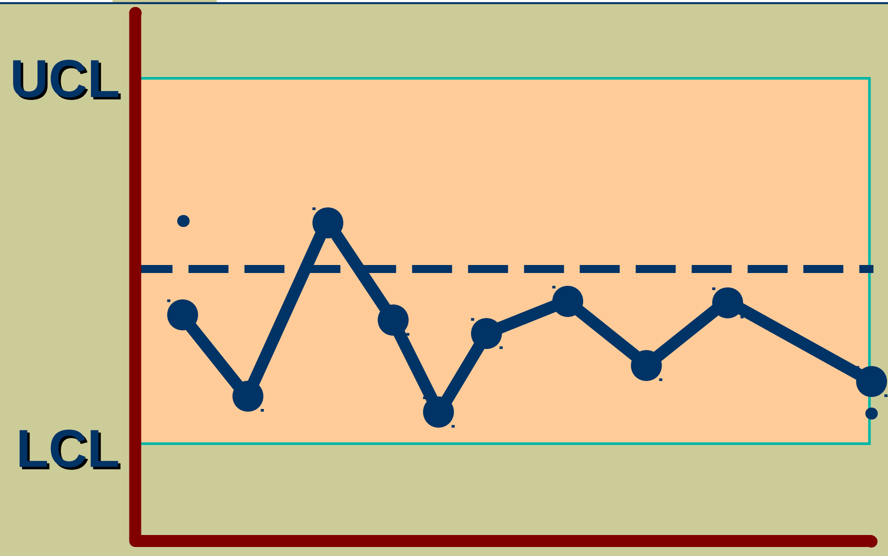
R-Chart Example (cont.)



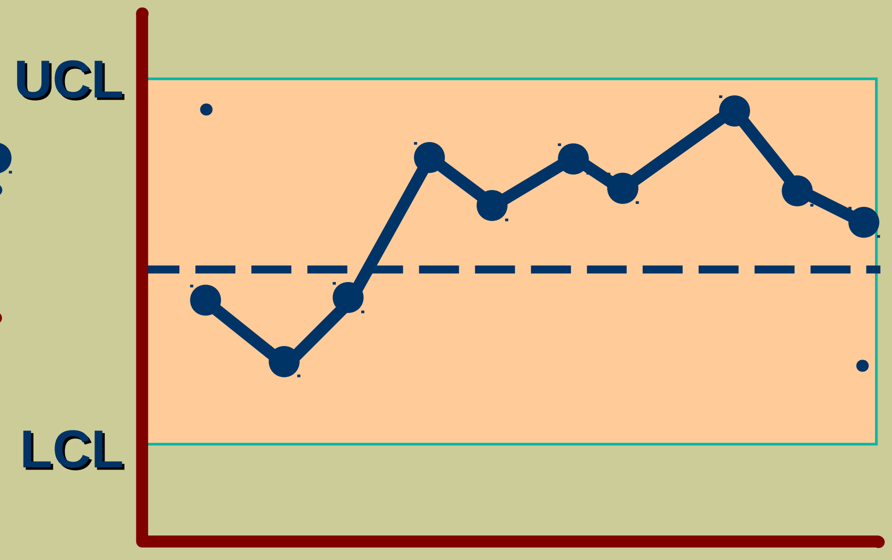
Using x- bar and R-Charts Together

- Process average and process variability must be in control.
- It is possible for samples to have very narrow ranges, but their averages is beyond control limits.
- It is possible for sample averages to be in control, but ranges might be very large.

Control Chart Patterns

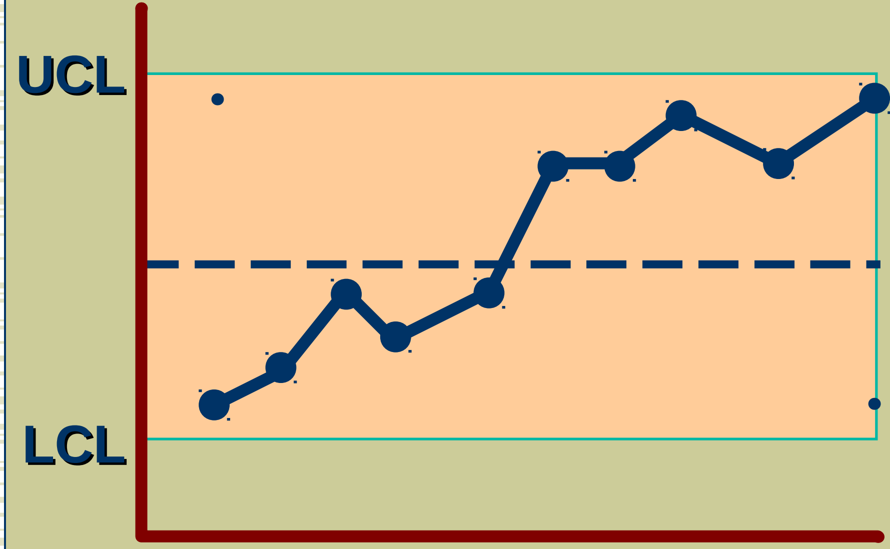


Sample observations consistently below the center line

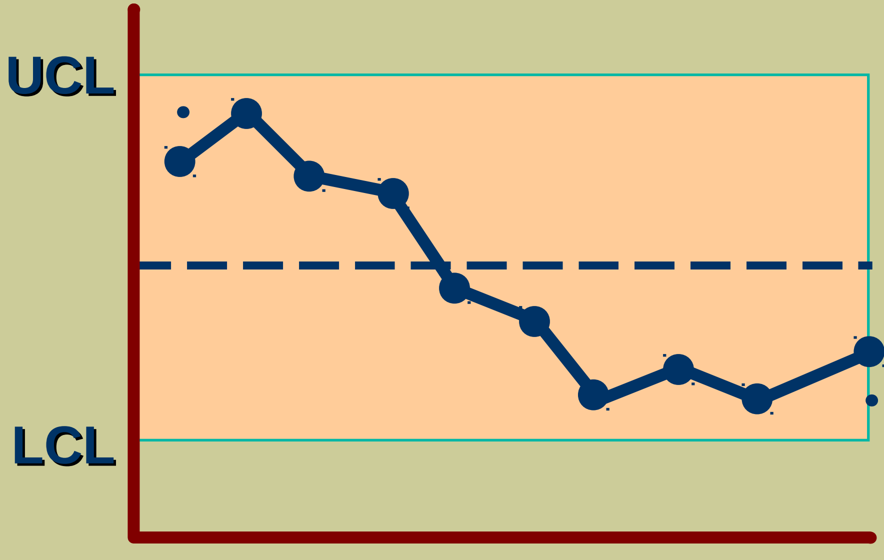


Sample observations consistently above the center line

Control Chart Patterns (cont.)

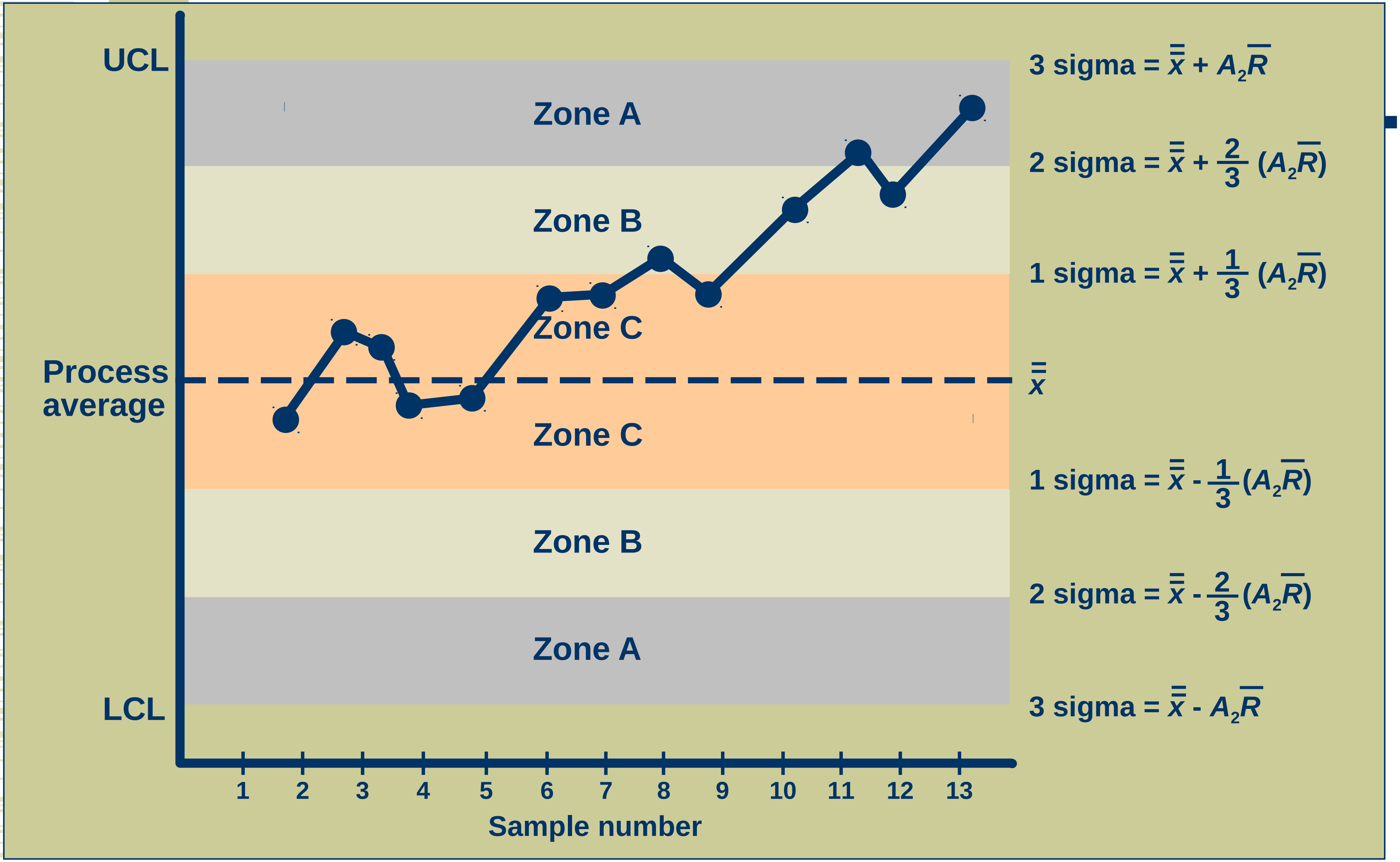


Sample observations consistently increasing



Sample observations consistently decreasing

Zones for Pattern Tests



Control Chart Patterns

- 8 consecutive points on one side of the center line
- 8 consecutive points up or down across zones
- 14 points alternating up or down
- 2 out of 3 consecutive points in zone A but still inside the control limits
- 4 out of 5 consecutive points in zone A or B

Performing a Pattern Test

SAMPLE	$\bar{x}$	ABOVE/BELOW	UP/DOWN	ZONE
1	4.98	B	—	B
2	5.00	B	U	C
3	4.95	B	D	A
4	4.96	B	D	A
5	4.99	B	U	C
6	5.01	—	U	C
7	5.02	A	U	C
8	5.05	A	U	B
9	5.08	A	U	A
10	5.03	A	D	B

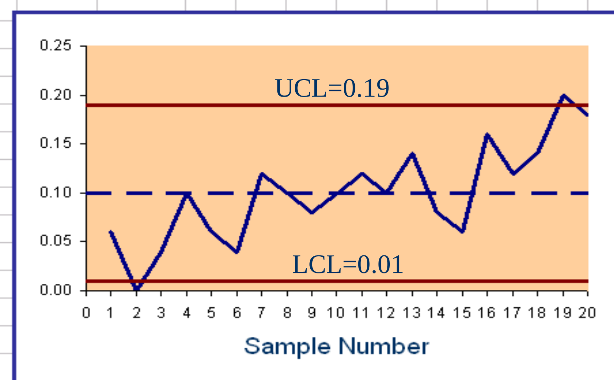
Sample Size

- Attribute charts require larger sample sizes
  - 50 to 100 parts in a sample
- Variable charts require smaller samples
  - 2 to 10 parts in a sample

SPC with Excel

Sample	Portion Defective	UCL	LCL	$\bar{p}$
0		0.190	0.010	0.100
1	0.06	0.190	0.010	0.100
2	0.00	0.190	0.010	0.100
3	0.04	0.190	0.010	0.100
4	0.10	0.190	0.010	0.100
5	0.06	0.190	0.010	0.100
6	0.04	0.190	0.010	0.100
7	0.12	0.190	0.010	0.100
8	0.10	0.190	0.010	0.100
9	0.08	0.190	0.010	0.100
10	0.10	0.190	0.010	0.100
11	0.12	0.190	0.010	0.100
12	0.10	0.190	0.010	0.100
13	0.14	0.190	0.010	0.100
14	0.08	0.190	0.010	0.100
15	0.06	0.190	0.010	0.100
16	0.16	0.190	0.010	0.100
17	0.12	0.190	0.010	0.100
18	0.14	0.190	0.010	0.100
19	0.20	0.190	0.010	0.100
20	0.18	0.190	0.010	0.100

$\bar{p}$ = 0.100
UCL= 0.190
LCL= 0.010

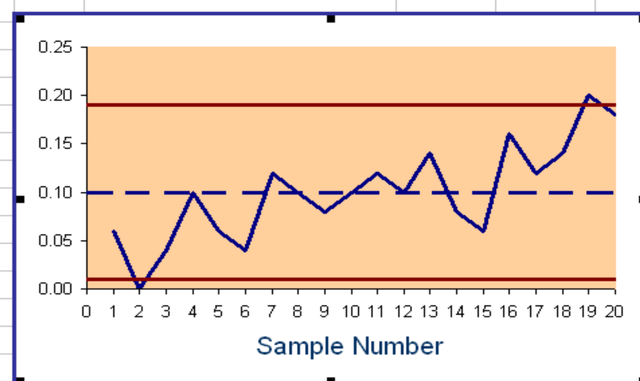


SPC with Excel: Formulas

Microsoft Excel - SPC_excel

Sample	Portion Defective	UCL	LCL	$\bar{p}$
0		=$H$3	=$H$4	=$H$2
1	0.06	=$H$3	=$H$4	=$H$2
2	0	=$H$3	=$H$4	=$H$2
3	0.04	=$H$3	=$H$4	=$H$2
4	0.1	=$H$3	=$H$4	=$H$2
5	0.06	=$H$3	=$H$4	=$H$2
6	0.04	=$H$3	=$H$4	=$H$2
7	0.12	=$H$3	=$H$4	=$H$2
8	0.1	=$H$3	=$H$4	=$H$2
9	0.08	=$H$3	=$H$4	=$H$2
10	0.1	=$H$3	=$H$4	=$H$2
11	0.12	=$H$3	=$H$4	=$H$2
12	0.1	=$H$3	=$H$4	=$H$2
13	0.14	=$H$3	=$H$4	=$H$2
14	0.08	=$H$3	=$H$4	=$H$2
15	0.06	=$H$3	=$H$4	=$H$2
16	0.16	=$H$3	=$H$4	=$H$2
17	0.12	=$H$3	=$H$4	=$H$2
18	0.14	=$H$3	=$H$4	=$H$2
19	0.2	=$H$3	=$H$4	=$H$2
20	0.18	=$H$3	=$H$4	=$H$2

$\bar{p}$ =	=AVERAGE(B3:B22)
UCL=	=H2+3*SQRT(H2*(1-H2)/100)
LCL=	=H2-3*SQRT(H2*(1-H2)/100)

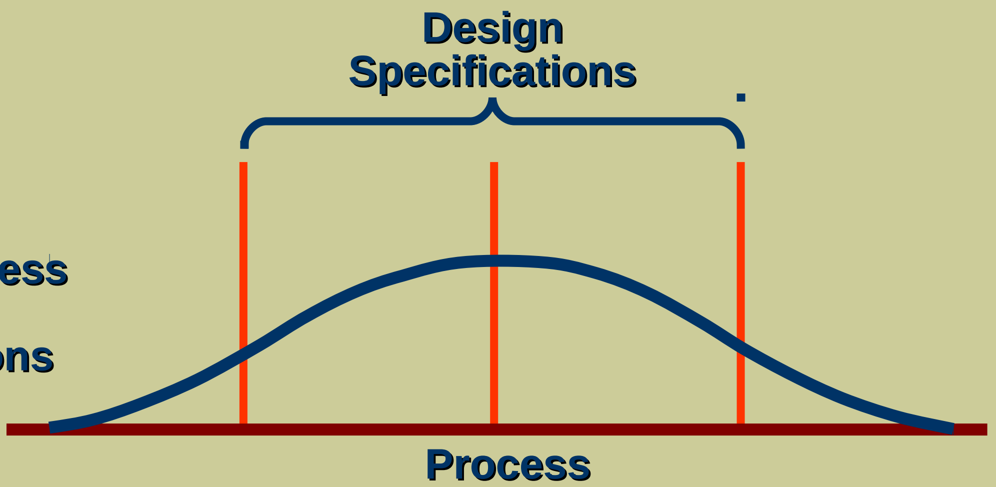


Process Capability

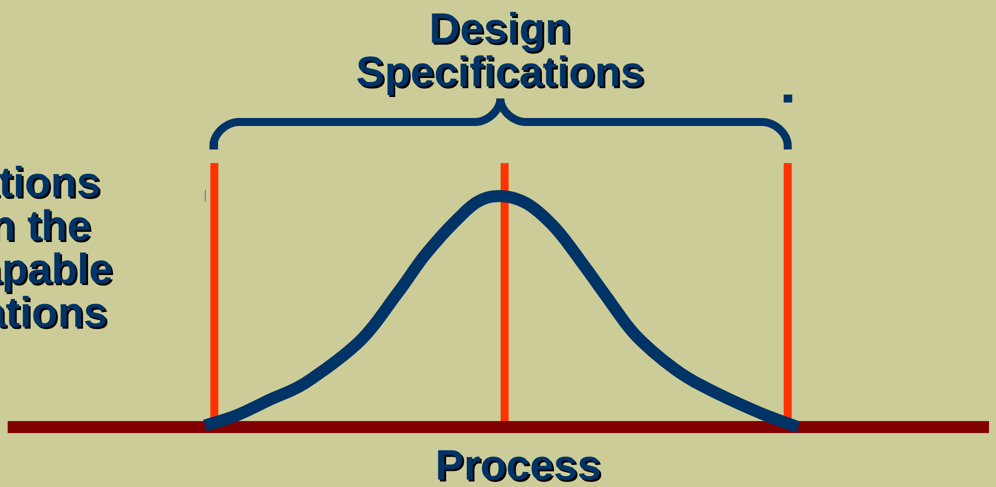
- Tolerances
  - design specifications reflecting product requirements
- Process capability
  - range of natural variability in a process what we measure with control charts

Process Capability

(a) Natural variation exceeds design specifications; process is not capable of meeting specifications all the time.

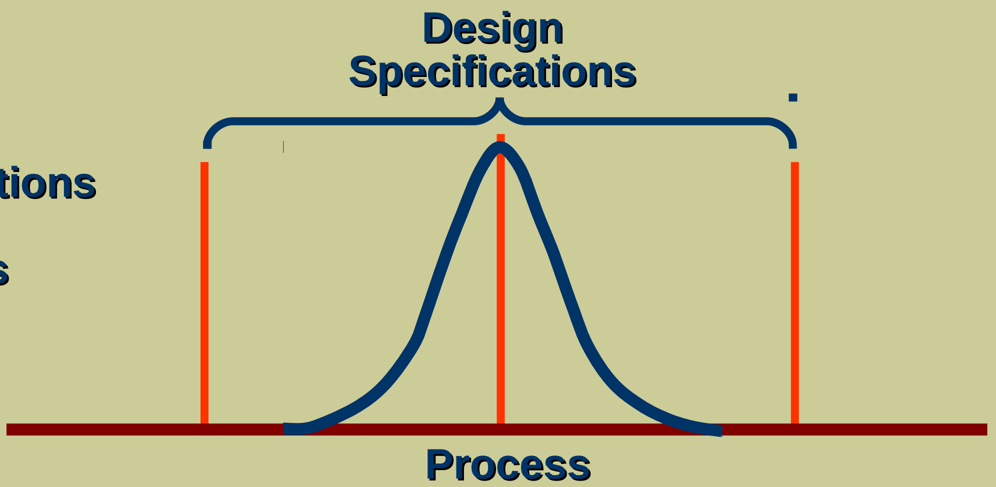


(b) Design specifications and natural variation the same; process is capable of meeting specifications most of the time.

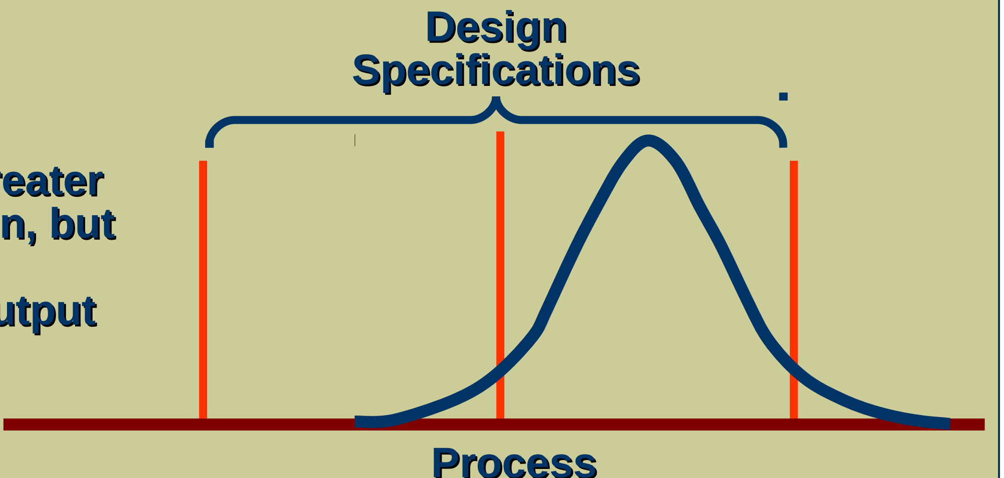


Process Capability (cont.)

(c) Design specifications greater than natural variation; process is capable of always conforming to specifications.



(d) Specifications greater than natural variation, but process off center; capable but some output will not meet upper specification.



Process Capability Measures

Process Capability Ratio

$$C_p = \frac{\text{tolerance range}}{\text{process range}}$$

$$= \frac{\text{upper specification limit - lower specification limit}}{6\sigma}$$

Computing C_p

Net weight specification = 9.0 oz ± 0.5 oz

Process mean = 8.80 oz

Process standard deviation = 0.12 oz

$$C_p = \frac{\text{upper specification limit} - \text{lower specification limit}}{6\sigma}$$

$$= \frac{9.5 - 8.5}{6(0.12)} = 1.39$$

Process Capability Measures

Process Capability Index

$$C_{pk} = \text{minimum}\left[\frac{\bar{\bar{x}} - \text{lower specification limit}}{3\sigma}, \frac{\text{upper specification limit} - \bar{\bar{x}}}{3\sigma}\right]$$

Computing C_{pk}

Net weight specification = 9.0 oz ± 0.5 oz

Process mean = 8.80 oz

Process standard deviation = 0.12 oz

$$C_{pk} = \text{minimum}\left[\frac{\bar{x} - \text{lower specification limit}}{3\sigma}, \frac{\text{upper specification limit} - \bar{x}}{3\sigma}\right]$$

$$= \text{minimum}\left[\frac{8.80 - 8.50}{3(0.12)}, \frac{9.50 - 8.80}{3(0.12)}\right] = 0.83$$

Appendix:
Determining Control Limits for x-bar and R-Charts

SAMPLE SIZE n	FACTOR FOR $\bar{x}$-CHART A_2	FACTORS FOR R-CHART D_3	D_4
2	1.88	0.00	3.27
3	1.02	0.00	2.57
4	0.73	0.00	2.28
5	0.58	0.00	2.11
6	0.48	0.00	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.44	0.18	1.82
10	0.11	0.22	1.78
11	0.99	0.26	1.74
12	0.77	0.28	1.72
13	0.55	0.31	1.69
14	0.44	0.33	1.67
15	0.22	0.35	1.65
16	0.11	0.36	1.64
17	0.00	0.38	1.62
18	0.99	0.39	1.61
19	0.99	0.40	1.61
20	0.88	0.41	1.59

Return