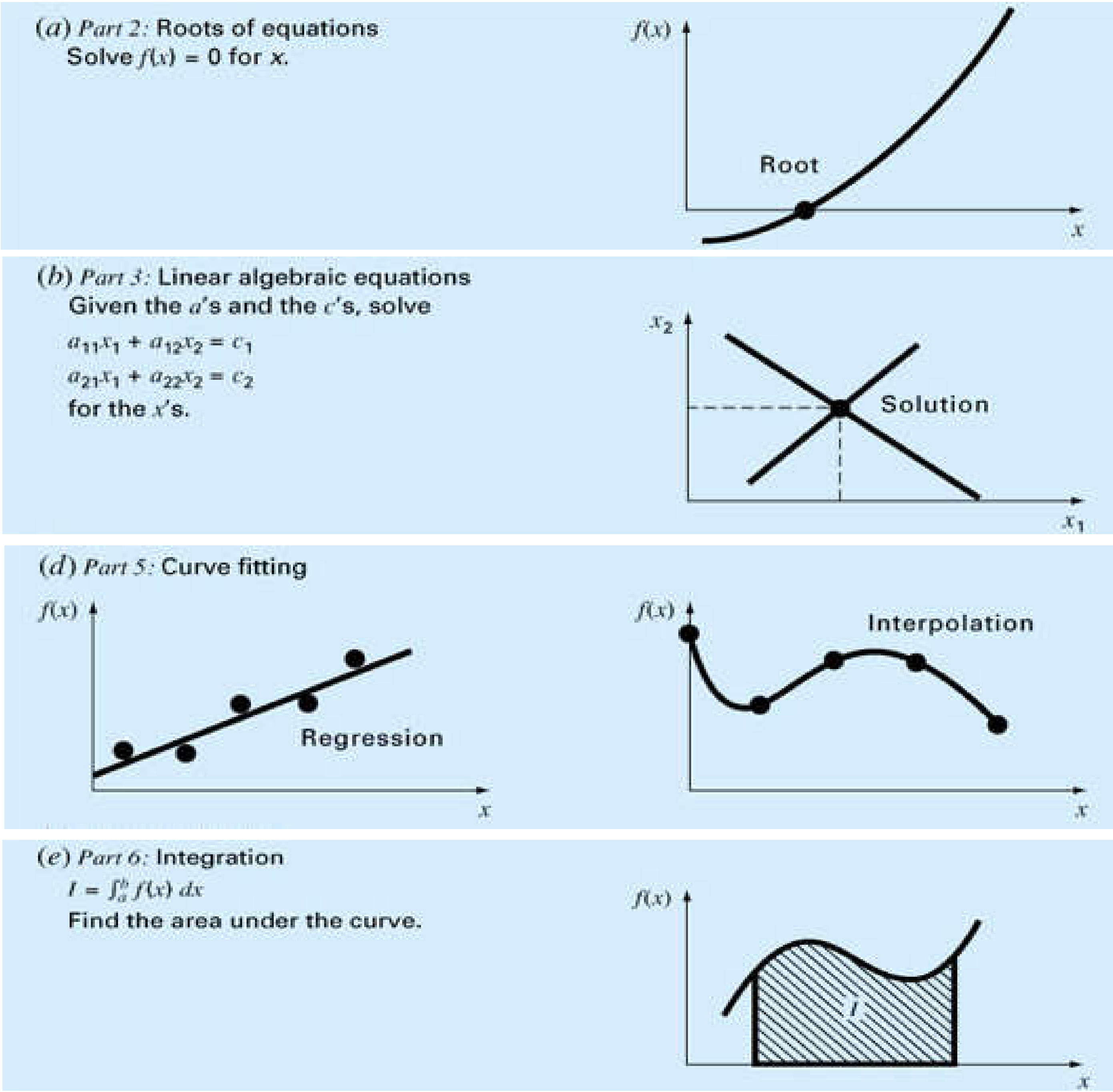
Mathematical Modeling and Engineering Problem solving

Chapter 1

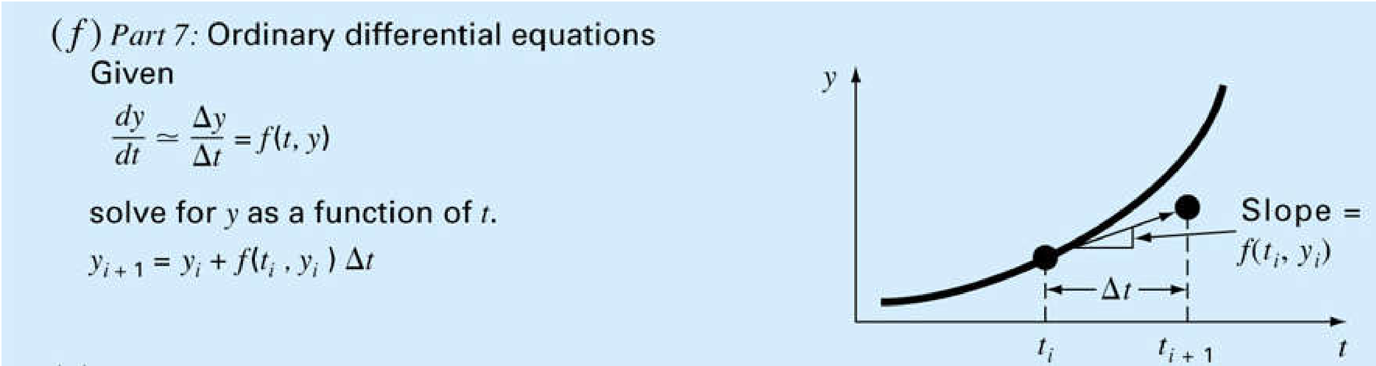
Numerical Methods

- Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations.

- Pre-computer Era:
  - ❖Analytical Solution
  - ❖Graphical
  - ❖Calculator



Every part in this book requires some mathematical background



Computers are great tools,

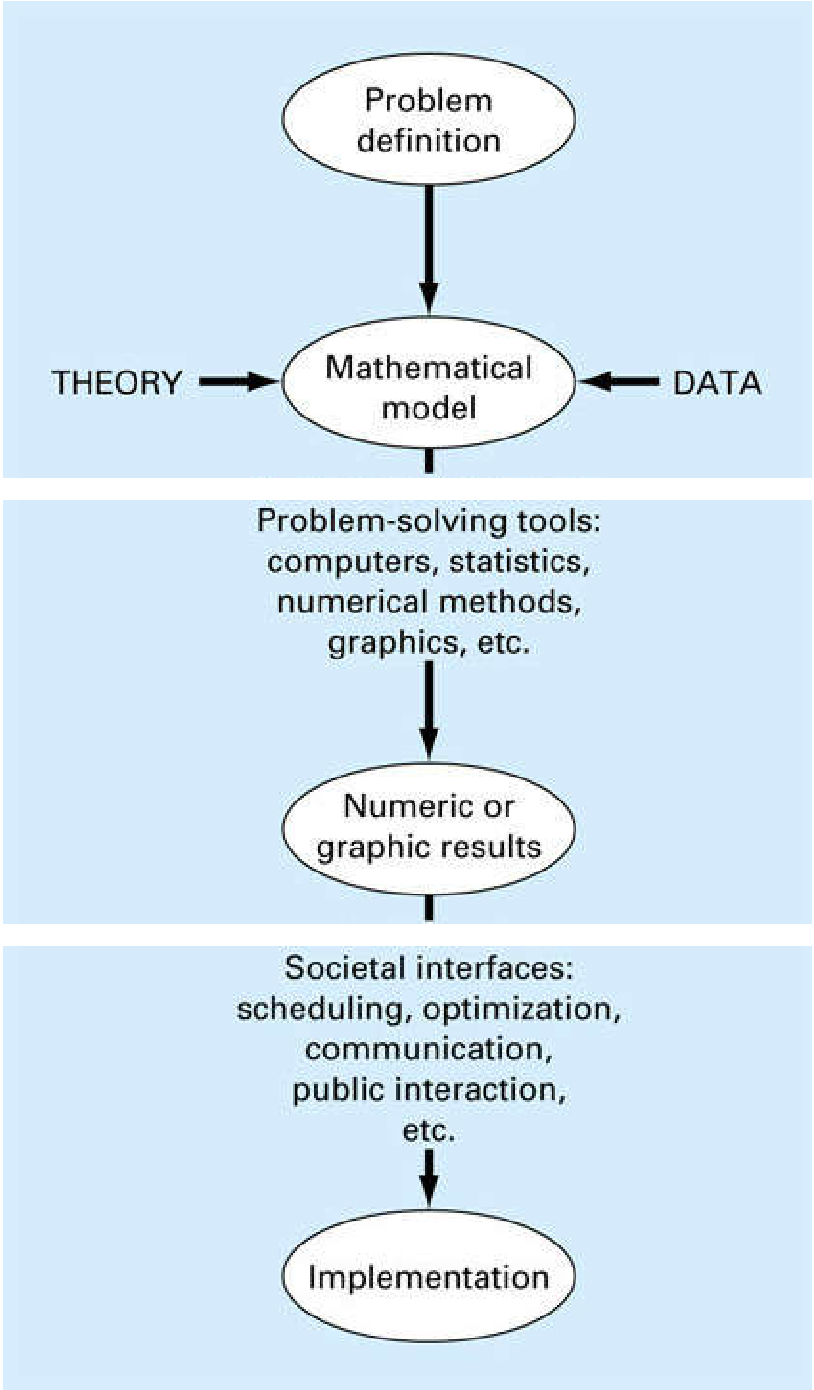
however, without fundamental understanding of engineering problems, they will be useless.

Benefits of Simulations

Cost savings by minimizing material usage.
Increased speed to market through reduced product development time.

Optimized structural performance with thorough analysis
Eliminate expensive trial-and-error.

The Engineering Problem Solving Process



Newton's 2nd law of Motion

- *"The time rate change of momentum of a body is equal to the resulting force acting on it."*

- Formulated as **F = m.a**

  F = net force acting on the body

  m = mass of the object (kg)

  a = its acceleration (m/s^2)

- Some complex models may require more sophisticated mathematical techniques than simple algebra
  - Example, modeling of a falling parachutist:

$$F = F_D + F_U$$

F_U = Force due to air resistance = $-cv$ **(*c = drag coefficient*)**

F_D = Force due to gravity = mg

$$\frac{dv}{dt} = \frac{F}{m}$$

$$F = F_D + F_U$$

$$F_D = mg$$

$$F_U = -cv$$

$$\frac{dv}{dt} = \frac{mg - cv}{m}$$

- This is a first order ordinary **differential equation.** We would like to solve for v (velocity).

- It can **not** be solved using algebraic manipulation

- <u>Analytical Solution:</u>

  If the parachutist is initially at rest (v=0 at t=0), using calculus dv/dt can be solved to give the result:

$$\frac{dv}{dt} = g - \frac{c}{m}v$$

Independent variable

Dependent variable

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

Forcing function

Parameters

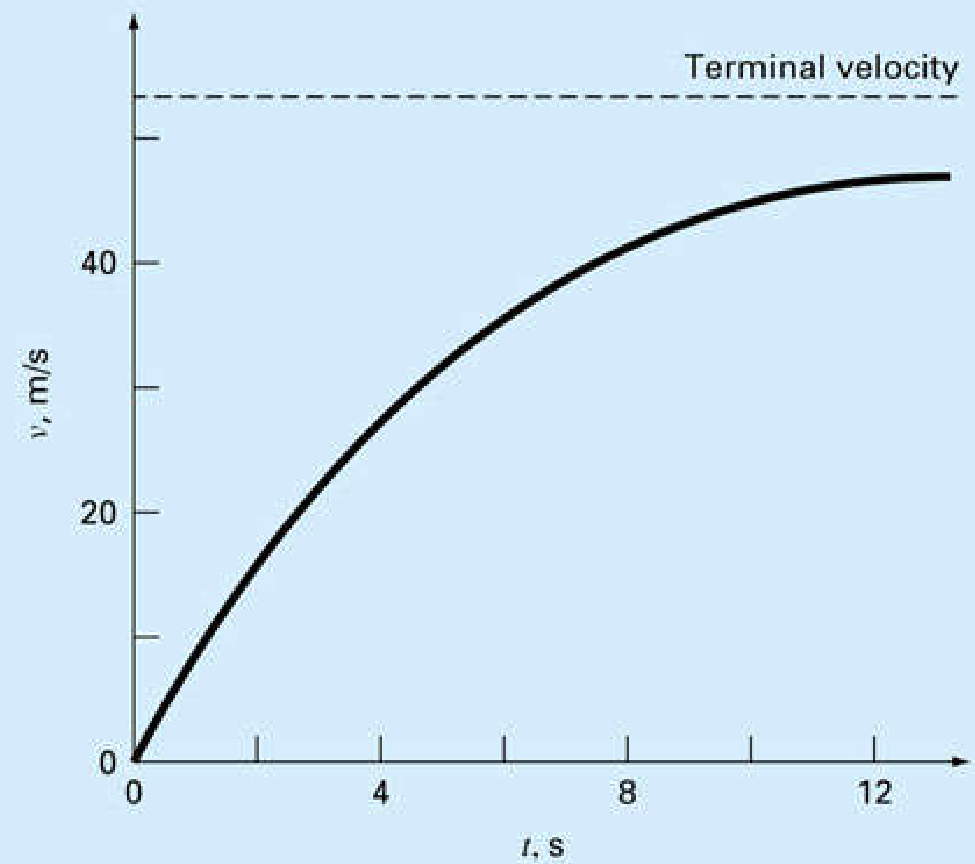
Analytical Solution

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

If ***v(t)*** could not be solved **analytically**, then we need to use a numerical method to solve it

g = 9.8 m/s^2 c =12.5 kg/s
m = 68.1 kg

t (sec.)	V (m/s)
0	0
2	16.40
4	27.77
8	41.10
10	44.87
12	47.49
∞	53.39



Numerical Solution

$$\frac{dv}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} \ldots\ldots\ldots \frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

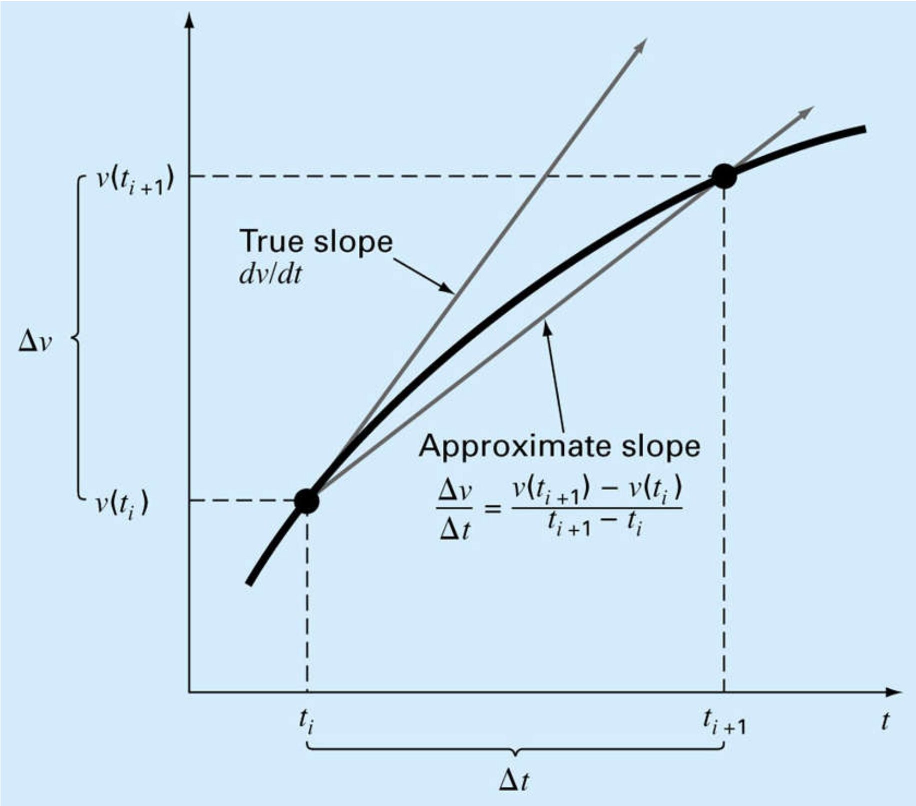
$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m} v(t_i)$$

This equation can be rearranged to yield

$$v(t_{i+1}) = v(t_i) + [g - \frac{c}{m} v(t_i)](t_{i+1} - t_i)$$

t (sec.)	V (m/s)
0	0
2	19.60
4	32.00
8	44.82
10	47.97
12	49.96
∞	53.39

Δt = 2 sec



To minimize the error, use a smaller step size, Δt
No problem, if you use a computer!

Analytical vs. Numerical solution

m=68.1 kg c=12.5 kg/s
g=9.8 m/s

t (sec.)	V (m/s)
0	0
2	16.40
4	27.77
8	41.10
10	44.87
12	47.49
∞	53.39

$\Delta t = 2$ sec

t (sec.)	V (m/s)
0	0
2	19.60
4	32.00
8	44.82
10	47.97
12	49.96
∞	53.39

$\Delta t = 0.5$ sec

t (sec.)	V (m/s)
0	0
2	17.06
4	28.67
8	41.95
10	45.60
12	48.09
∞	53.39

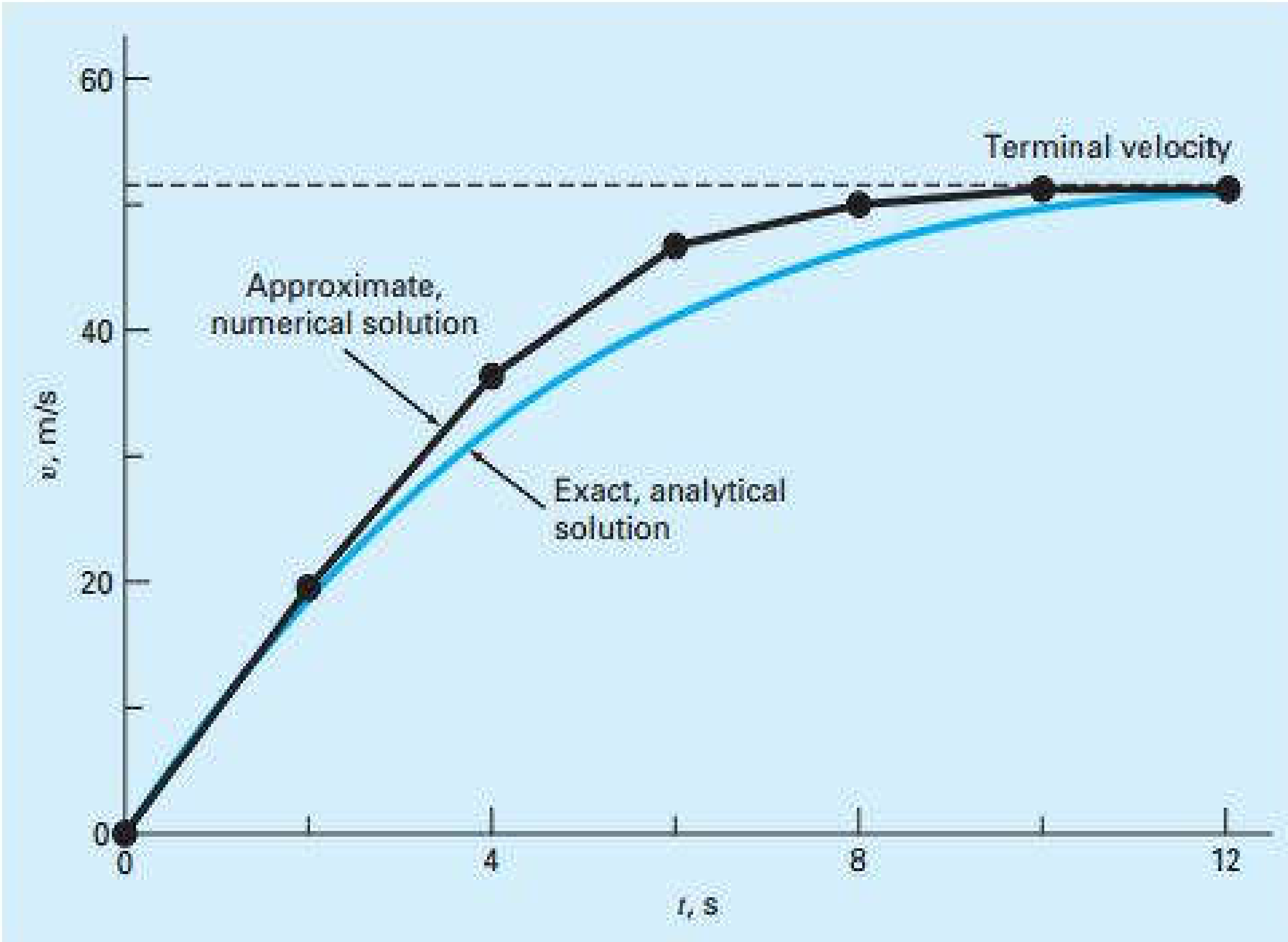
$\Delta t = 0.01$ sec

t (sec.)	V (m/s)
0	0
2	16.41
4	27.83
8	41.13
10	44.90
12	47.51
∞	53.39

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

$$v(t_{i+1}) = v(t_i) + [g - \frac{c}{m} v(t_i)]\Delta t$$

CONCLUSION: If you want to minimize the error, use a smaller step size, Δt



60
40
20
0
v, m/s
0
4
8
12
t, s
Terminal velocity
Approximate,
numerical solution
Exact, analytical
solution