Ground-Based Spectroscopic Exposure Time Calculator Version 5 User Manual & Code Description

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1. Overview

This ETC is intended for use in optimizing the PFS observing strategy.

The ETC is written in standard C and does not link to any external libraries. This should make it easier for users to install and run, as well as avoiding legal issues with copyrighted code.

Suggestions for improvement, or additional effects that should be included, can be sent to the point of contact (Christopher Hirata).

This manual is deliberately verbose: it errs on the side of providing more information, and providing it repetitively. It also serves the dual role of reminding the author how everything works, hence §5 on data structures and functions. Most readers (who do not wish to write their own drivers) will need only the first several sections.

The user will note that this ETC does not live up to its title: one inputs an exposure time, and the code returns the observed flux limits and properties of galaxies (not the other way around). If you do not like the answer then you can increase the exposure time. The code was configured this way in part for simplicity, but also because each PFS exposure will have the same duration for all targets, regardless of field angle, expected [O II] flux, etc.

This code is the "sister project" to the WFIRST ETC and the functions share some common heritage. However for a ground-based survey a number of additional terms come into play, particularly involving atmospheric emission, absorption, and turbulence. Also the treatment of fiber spectroscopy requires some additional machinery. Therefore this code is built independently of the space-based ETC.

2. Compiling the Code

The code consists of a stand-alone C code (`gsetc.c`) and a `#included` header file (`modeldata.h`) containing data on atmospheric models (e.g. tables of OH lines). It can thus be compiled with the standard command:

```
gcc gsetc.c -lm -Wall -O3 -o gsetc.x
```

Additional compiler flags can be implemented as part of the compiler command, as described in §2.A.

A. Compiler options

The following compiler flags are supported:

- `-DDIFFRACTION_OFF`: Turns off the treatment of diffraction features in the telescope PSF; this is a minor effect in any practical application of the ETC. This is not normally recommended, but is useful if one wants to check for "double counting" of the subdominant diffraction contribution to the "observed" seeing.
- `-DHGCDTE_SUTR`: Switches the sampling options on the HgCdTe detectors to sampling up the ramp. We are likely to actually use this option in practice; it reduces overheads but at the expense of enhanced Poisson noise.
- `-DNO_OBJ_CONTINUUM`: Turns off the object continuum contribution to the noise. This is obviously not realistic and is only for testing purposes. This flag should **not** be used for any forecasts.
- `-DNATURAL_NLINES`: Forces N_{lines} to be the actual number of illuminated lines in the grating (overrides `NLINES` keyword).
- `−DMOONLIGHT_`: Enables Moonlight (if not set, assumes dark time).

The compiler options used are displayed at runtime before any prompts.

3. Running the Code

The code can be called from the command line. An example run is as follows:

```
./gsetc.x
Compiler flags: -DHGCDTE_SUTR
Enter spectrograph configuration file: data/PFS-v3.dat
Enter observational conditions [hexadecimal code; suggested=10003]:
   10003
Enter seeing [arcsec FWHM @ lambda=800nm]: 0.8
Enter zenith angle [degrees]: 26.0
Enter Galactic dust column [magnitudes E(B-V)]: 0.05
Enter field angle [degrees]: 0.675
Enter fiber astrometric offset [arcsec]: 0.12
Enter time per exposure [s]: 450
Enter number of exposures: 2
Enter systematic sky subtraction floor [rms per 1D pixel]: 0
Enter diffuse stray light [fraction of total]: 0
Enter output file for noise vector: out/ref.Noise.dat
Enter output file for ELG S/N curve: [- for no ELG S/N curve output]
   out/ref.SN.dat
Enter output file for continuum curve: [- for no continuum S/N curve
   output] out/ref.SNC.dat
Enter [OII] input catalogue file: [- for no OII computation]
   data/OIIgal.dat
Enter [OII] output catalogue file: out/ref.Gal.dat
Enter minimum SNR: 9.0
```

```
Fiber aperture factor [@800nm, r_eff=0.3"(exp)] = 0.45310478
Fiber aperture factor [@800nm,    point source] = 0.59488634
Computing noise vector ...
 Done.
Computing ELG SNR curve for fiducial parameters ...
 Done.
Computing continuum SNR curve for fiducial parameters ...
 Done.
Processing [OII] emitter catalog ...
Done.

 NUMBER OF AVAILABLE TARGETS:   24339
[i.e. objects where OII detected *if* targeted]

 zmin  zmax  ngal
 0.100 0.200     233
 0.200 0.300     561
 0.300 0.400     992
 0.400 0.500     688
 0.500 0.600     807
 0.600 0.700    1113
 0.700 0.800    1397
 0.800 0.900    1588
 0.900 1.000    1994
 1.000 1.100     999
 1.100 1.200    1995
 1.200 1.300    2302
 1.300 1.400    1309
 1.400 1.500    1540
 1.500 1.600     627
 1.600 1.700     503
 1.700 1.800     664
 1.800 1.900     943
 1.900 2.000    1051
 2.000 2.100     624
 2.100 2.200     705
 2.200 2.300     585
 2.300 2.400     813
 2.400 2.500     306
```

The code first lists any compiler flags that were turned on (in this case, `-DHGCDTE_SUTR`). It then proceeds to prompt the user for any inputs that are not specified as part of the configuration file.[1] These are as follows:

- The spectrograph configuration file, which contains all of the constant hardware parameters (see §3.A).
- The observational conditions assumption is a 5-digit hexadecimal number. In order from first to last digit, these are:
    - The sky emission line model.
    - The atmospheric line absorption model.
    - The atmospheric continuum absorption model.

[1] The tagged v1 of the code lacked a functioning front end. This was mostly fixed in v2, and v3 is fully functional.

    - The Moonlight model.
    - The sky continuum model.
- The other observing parameters – seeing, zenith angle, Galactic dust column, field angle, fiber astrometric offset, and observing time (including the number of exposures).
- The systematic sky subtraction error as a fraction (e.g. 0.01 implements a systematic error floor of 1% of the sky in each 1D pixel in the **stacked** exposure. This error is assumed uncorrelated; in cases where the true error is correlated, one should set a larger number by multiplying by the square root of the number of 1D pixels covering the correlation length).
- The diffuse stray light takes a fraction of the radiation incident on the detector, and smears it out uniformly across the entire 2D detector surface. This can be very significant even at low levels in the NIR, where the OH lines (if smeared out) are ~100 times brighter than the continuum.
- **If** the `−DMOONLIGHT_` flag is set, then the code will prompt the user for lunar parameters (otherwise dark time is assumed). These are:
    - The Moon-zenith angle [deg] – this is 0 when the Moon is at the top of the sky and 90 when the Moon is on the horizon. For values >90, the Moonlight is taken to be zero and the calculation is equivalent to dark time.
    - The Moon-target angle [deg] – this is 0 if the Moon is in the field of observation.[2]
    - The lunar phase – this is 0 for new Moon, ¼ for first quarter, ½ for full Moon, ¾ for last quarter, and 1 again for new Moon.[3]
- The files to which the code should write its various outputs. In most cases, a "−" indicates that an output should be skipped (e.g. continuum S/N may not be of interest for an ELG program).
    - The noise vector – noise variance in electrons squared per 1D pixel (e^2 pix^{-1}) in each arm.
    - The S/N curve for the [O II] doublet as a function of z for a reference case (flux 10^{-16} erg cm^{-2} s^{-1}, effective radius 0.3 arcsec, 1:1 line ratio, velocity width σ = 70 km s^{-1}).
    - The continuum S/N per 1D pixel for a reference case (point source, 22.5 mag AB or equivalently F_ν = 3.631 μJy).
    - The set of galaxies from a given input catalog on which [O II] is detected.
- The minimum matched filter[4] S/N at which one considers an [O II] detection successful.

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[2] Technically for Moon-target angles of <0.25° the target is occulted. The ETC does not treat this, since we don't anticipate ever using the spectrograph in this way!
[3] Some combinations are not actually realizable, e.g. we won't have a new Moon at zenith at night, but the code does not check for this.
[4] Explicitly: `snrType==2` in the language of §4.G.

Before providing the output files, the code computes the fiber aperture factors (i.e. fraction of light in the focal plane encircled by the fiber entrance) for a typical galaxy (r_{eff} = 0.3 arcsec, exponential) and a point source.

If the [O II] program is selected, then a table is returned including the properties of the "successful" galaxies from the input catalog. Of course, in practice success on a galaxy requires it to be targeted, which is not one of the functions of the ETC at the moment. However the output list of these successful galaxies can be cross-matched to the outputs of target selection to determine the efficiency and completeness of the target selection scheme.[5]

A. Configuration file

The hardware parameters are included in a configuration file. This currently contains parameters describing the telescope and fiber system, as well as each spectrograph arm. Finally there is a throughput table. Some inputs (indicated below) are not actually used in the present version of the ETC and are included for forward compatibility.

The configuration file consists mostly of keywords followed by data. Data follows on the same line as the keyword and is space-delimited, *except* for the `THRPUT` keyword, which denotes a throughput table (not practical to include in a single line). Lines beginning with the '#' character are considered comments and are ignored. Keywords need not be in any specific order except for the `ARMS` keyword (which must be specified before the throughput table or any keywords associated with specific arms) but most must be present (the exception is `VIGNET`, which describes vignetting in the telescope + corrector; if it is not present then vignetting is neglected). Spectrograph arm indices run from 0 to $N-1$, where N is the number of arms (specified by the `ARMS` keyword).

An example configuration file for a "generic" spectrograph on a 6-meter telescope is:

```
# "Generic" Spectrograph ... just as an example
#
OPTICS 6.0 0.5 1.0 18.0 18.0 18.0 18.0 18.0
SPOT 20 22 24 26 28 30
VIGNET 1.00 1.00 1.00 0.95 0.90
FIBER 60
ARMS 2
PARAM 0 500 800 4096 5
PARAM 1 750 1040 4096 5
CAMERA 0 2.0 100 15.0 173 13 60 3e-4 3 120
CAMERA 1 2.0 650 15.0 173 13 60 3e-4 3 120
NLINES 0 3000.0
NLINES 1 5000.0
THRPUT
 500 0.70 0.80 0.76 0.60 0.80
 600 0.70 0.80 0.76 0.80 0.90
```

[5] For the conceptual design phase of PFS, this is being done by an external script.

```
 700 0.70 0.80 0.76 0.80 0.90
 750 0.70 0.80 0.76 0.75 0.85
 775 0.70 0.80 0.38 0.75 0.83
 800 0.70 0.80 0.00 0.75 0.80
D
 750 0.70 0.80 0.00 0.75 0.92
 775 0.70 0.80 0.36 0.78 0.92
 800 0.70 0.80 0.72 0.80 0.86
 830 0.70 0.80 0.72 0.80 0.92
 900 0.70 0.80 0.72 0.80 0.90
 960 0.70 0.80 0.72 0.80 0.86
 980 0.70 0.80 0.72 0.80 0.84
1000 0.70 0.80 0.72 0.80 0.76
1020 0.70 0.80 0.72 0.80 0.58
1040 0.70 0.80 0.72 0.80 0.33
D
```

The keywords are:

- `OPTICS` (*required*): This is followed by 4 numbers:
    i. The outer diameter of the telescope (in meters).
    ii. The linear central obscuration fraction (dimensionless, <1).
    iii. The radius of the FOV (in degrees).
    iv. The EFL (in meters) at the center of the field.
    v. The EFL (in meters) at a field angle of ¼ of the radius.
    vi. The EFL (in meters) at a field angle of ½ of the radius.
    vii. The EFL (in meters) at a field angle of ¾ of the radius.
    viii. The EFL (in meters) at the edge of the field.
- `SPOT` (*required*): This is the geometric spot size delivered by the telescope + corrector, in μm RMS per axis. The spot size is specified at five positions linearly spaced from the center to the edge of the FOV.
- `VIGNET` (*optional*): The vignetting factor (1 for no vignetting, 0 for complete obscuration), specified at five positions linearly spaced from the center to the edge of the FOV.
- `FIBER` (*required*): The apparent fiber radius in μm. This is divided by the EFL in m to obtain the fiber radius in μrad projected onto the sky. Therefore in e.g. the case where the fiber input contains a lens, this should be the apparent fiber radius as seen by someone looking down on the focal plane, rather than the physical radius of the fiber core.
- `ARMS` (*required*): The number of arms in the spectrograph.
- `PARAM` (*one required per arm*): This is followed by 5 numbers:
    i. The index of the spectrograph arm to which the parameters should be applied.

ii. The minimum wavelength of the arm in nm (vacuum).[6]
iii. The maximum wavelength of the arm in nm (vacuum).
iv. The number of pixels in the dispersion direction between λ_{min} and λ_{max}.
v. The (integer) width of the trace in pixels that is co-added to produce a 1D spectrum.[7]

- `CAMERA` (*one required per arm*): This is followed by 10 numbers:
  i. The index of the spectrograph arm to which the parameters should be applied.
  ii. The *f*/ratio of the camera.
  iii. The thickness of the detector in μm.
  iv. The pixel scale of the detector in μm pixel^{-1}.
  v. The detector temperature in K.
  vi. The image quality of the spectrograph in μm RMS per axis.
  vii. The fiber diameter projected onto the focal plane in μm.
  viii. The dark current in e^- pixel^{-1} s^{-1}.
  ix. The read noise in e^- RMS per exposure.
  x. The separation of adjacent spectral traces in μm on the detector.
- `THRPUT` (*required*): A throughput table for each arm. The table rows are each given by λ (in nm vacuum) and then 5 numbers, which multiply together to form the throughput. Normally these are the telescope+corrector, fiber system, spectrograph optics, grating × dichroic, and detector, but since they are multiplied before any other operations a throughput term can be assigned to any one of the columns. The tables for consecutive arms are delimited by a single line containing the '`D`' character.
- `HGCDTE` (*optional*): This keyword is followed by the index of a spectrograph arm (example: "`HGCDTE 2`"). It indicates that this spectrograph arm contains a HgCdTe rather than silicon detector.
- `NLINES` (*optional*): This keyword is followed by the index of a spectrograph arm and the effective number of lines $N_{\rm lines}$, used in computing the wings of the line-spread function. If not present, $N_{\rm lines}$ defaults to 10^{12} (i.e. effectively infinite). A separate line is required for each arm.

B. Galaxy catalog file

The galaxy catalog file is simply a list of galaxies. The ETC runs through the input catalog and determines for each object whether the [O II] doublet would be detected under the specified observational conditions.

The catalog format is as follows:

---

[6] In the case of a multi-arm spectrograph, these ranges will generically overlap. The ETC will quadrature-sum the SNR expected in each arm for features that fall in the overlap region.

[7] The current version of the ETC does not support the "optimal" 2D→1D extraction as proposed by A. Bolton & D. Schlegel, "Spectro-perfectionism: An algorithmic framework for photon noise-limited extraction of optical fiber spectroscopy," *Proc. Ast. Soc. Pac.* **122**:248—257 (2010).

```
 256455 1.089 0.483 2.146 1.17307E-17 7.37706E-30   8.9
 256908 0.634 0.283 0.713 1.74279E-17 7.08814E-30   2.5
 256957 1.587 0.111 0.762 1.01542E-17 1.27201E-30  11.2
 257058 1.322 0.303 0.848 1.42750E-17 4.03373E-30   6.4
 257059 1.284 0.312 1.159 4.41801E-17 1.48668E-29  15.5
…
1993677 1.543 0.275 0.956 5.67933E-17 1.09924E-29  10.7
```

Note that the number of galaxies is not specified – it is simply the line count. The columns are:

I. An *identification number* for each galaxy.
II. The redshift z.
III. The effective radius $r_{\rm eff}$ in arcsec.
IV. The [O II] line ratio, F_{3727}/F_{3730}. (Atomic physics constrains this to be between the low-density limit of 0.667 and the high-density limit of 3.87. If the specified ratio is outside of this range, it is replaced with the nearest legal value.)
V. The observer-frame [O II] doublet flux, in erg cm^{-2} s^{-1}.
VI. The observer-frame underlying continuum, in erg cm^{-2} s^{-1} Hz^{-1} (for a 22.5 mag AB galaxy, this flux is 3.631×10^{-29} erg cm^{-2} s^{-1} Hz^{-1}).
VII. The 1σ line width in Å observed-frame, taken to be a Gaussian. [**Note:** This is not currently used; the ETC assumes σ = 70 km s^{-1}.]

C. Output files

The ETC is capable of providing several types of output files, as follows.

The noise vector is simply a table of the noise variance in each 1D pixel in a single exposure. Its format is

```
0    0 380.0354 6.12512e+01
0    1 380.1062 6.13847e+01
0    2 380.1770 6.17342e+01
0    3 380.2478 6.18007e+01
…
2 4095 1299.9597 4.31783e+02
```

The four columns are (i) the spectrograph arm number; (ii) the pixel number; (iii) the vacuum wavelength [nm] in the center of that pixel; and (iv) the noise variance in e^2 pix^{-1}. All pixels are zero-indexed.

The ELG S/N vector provides the S/N on a galaxy with reference profile (exponential, half-light radius = 0.30 arcsec) with a reference [O II] doublet (flux 10^{-16} erg cm^{-2} s^{-1}, 1:1 line ratio, velocity width σ = 70 km s^{-1}, no source continuum) as a function of redshift. Its format is

```
0.1000   409.98   410.28 0.397396   5.45075    6.9846    0.0000    0.0000    6.9846
0.1002   410.06   410.35 0.397409   5.45515    7.0432    0.0000    0.0000    7.0432
0.1004   410.13   410.43 0.397422   5.45954    7.0854    0.0000    0.0000    7.0854
0.1006   410.20   410.50 0.397435   5.46394    7.1381    0.0000    0.0000    7.1381
```

```
…
2.5000 1304.48 1305.43 0.473782  0.00000   0.0000   0.0000   0.0000   0.0000
```

The columns are: (i) the redshift; (ii) and (iii), the vacuum wavelengths [nm] of the two lines; (iv) the fiber aperture factor for the galaxy (extended source); (v) the effective collecting area [m^2], including throughput and vignetting but *not* the fiber aperture effect; (vi+) the S/N in each arm (one column for each, 3 in the example above); and lastly the total S/N. Note that the total number of columns is $7+N_{\rm arms}$.

The continuum S/N vector provides the S/N per pixel on a point source at 22.5 mag AB (F_ν = 3.631 μJy). Its format is

```
0    0   380.000   0.7801
0    1   380.071   0.7810
0    2   380.142   0.7805
0    3   380.212   0.7817
…
2 4095  1299.919   0.4059
```

The four columns are: (i) the spectrograph arm number; (ii) the pixel number; (iii) the vacuum wavelength [nm] in the center of that pixel; and (iv) the continuum S/N in that pixel (where continuum Poisson variance is counted as part of the "noise").

Finally, the source galaxy output file is simply a text catalog:

```
 257059 1.284 0.312 4.41801E-17   10.9744
 257420 0.934 0.668 2.22968E-16   28.0260
 257713 0.864 0.206 6.51522E-17   11.3564
 258804 1.091 0.435 7.90253E-17   10.0384
…
1993582 2.316 0.191 4.56476E-17   10.4812
```

The five columns are: (i) the galaxy identification number, for cross-matching purposes; (ii) the redshift z; (iii) the effective radius [arcsec]; (iv) the [O II] doublet flux [erg cm^{-2} s^{-1}]; and (v) the matched filter S/N for [O II] detection.

4. Detailed Methodology and Assumptions

A. Background cosmology

The background cosmology assumed for number density yields is the same as that of the FoMSWG[8] (chosen for consistency), itself derived from the WMAP 5-year

[8] A. Albrecht *et al.*, "Findings of the Joint Dark Energy Mission Figure of Merit Science Working Group" (2009). arXiv:0901.0721
http://wfirst.gsfc.nasa.gov/science/fomswg/fomswg_technical.pdf

parameter constraints.[9] The parameters are H_0 = 71.903 km/s/Mpc, Ω_m = 0.25648, and zero spatial curvature.

B. Sky brightness

The sky brightness consists of several pieces: the emission lines (mostly from vibrationally excited OH); the continuum emission; and, if present, moonlight.

1. *Emission lines*

The emission line component is specified by bits 16—19 of the `skytype` member in the `OBS_ATTRIB` structure. That is, we construct the hexadecimal digit:

```
(obs->skytype>>16) & 0xf
```

The following models are currently available (numbered by their hexadecimal digit):

Sky emission line models

Model	Description
0	No sky lines. Obviously unrealistic and for testing only.
1	Uses the UVES visible line atlas, and the theoretical OH line intensities in the NIR, normalized to a total brightness of 15.8 mag arcsec^{-2} in J band at X = 1.

The visible sky lines were obtained from the UVES atlas.[10] A total of 2816 lines were extracted from the atlas (after removing duplicates in wavelength overlap regions) and assigned an integrated intensity (in units of 10^{-16} erg cm^{-2} s^{-1} arcsec^{-2}). The linewidth was treated as a δ-function, which is appropriate for resolutions used in galaxy survey work. The line catalog in `modeldata.h` has the wavelength scale in nm in air; conversion to vacuum wavelengths is done at runtime in `gsGetNoise`. The UVES data were acquired at airmass X = 1.1, and are rescaled in proportion to X, as computed from

$$X = \frac{1}{\sqrt{1-0.96\sin^2\theta_{\rm ZO}}},$$

where $\theta_{\rm ZO}$ is the angle from the zenith to the direction of observation.

The UVES-based sky line model currently runs from 346.0—1043.0 nm air (346.1—1043.3 nm vacuum). For longer wavelength lines (1043.3—1500.0 nm vacuum) we use the theoretical OH intensities[11] normalized to produce a sky brightness

[9] J. Dunkley *et al.*, "Five-year Wilkinson Microwave Anisotropy Probe observations: Likelihoods and parameters from the WMAP data," *Astrophys. J.* **180**:306—329 (2009).
[10] R. Hanuschik, "A flux-calibrated, high-resolution atlas of optical sky emission from UVES," *Astron. Astrophys.* **407**:1157—1164 (2003).
[11] P. Rousselot *et al.*, "Night-sky spectral atlas of OH emission lines in the near-infrared," *Astron. Astrophys.* **354**:1134 (2000).

of 15.8 mag arcsec^{-2} Vega (16.6 AB) in the J band (1.18—1.32 μm).[12] The O_2 bands at ~1.27 μm are **not** currently included in the sky model.

2. *Continuum emission*

The sky continuum model is set by the last 4 bits in `skytype`. The following models are currently available (numbered by their hexadecimal digit):

Sky continuum models

Model	Description
0	No sky continuum. Obviously unrealistic and for testing only.
1	In the optical, uses the "typical" continuum from the UVES atlas documentation,[13] and rescaled by $X/1.1$. This ranges from $(0.7—1.7)\times10^{-17}$ erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1} and is smallest in the red. At $\lambda > 1.04$ μm, we switch to the NIR continuum model,[14] fit by $(2.0\lambda_{\mu m}-0.5)$ R Å^{-1} and multiplied by X. This is probably a **drastic** overestimate in the NIR, and is included **only** for reference purposes.
2	In the optical, uses the "typical" continuum from the UVES atlas documentation,[15] and rescaled by $X/1.1$. This ranges from $(0.7—1.7)\times10^{-17}$ erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1} and is smallest in the red. The NIR continuum model continues at the UVES level of 7×10^{-18} erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1}.
3	An analytic fit to the continuum in the sky spectrum provided by Jim Gunn.[16] The functional form in magnitudes AB per arcsec^2, corrected for atmospheric extinction, is $$m = 21.55 + c_1(\lambda - 600) - 0.55e^{-(\lambda-594)^2/200} - 0.175[1+\tanh(375-\lambda)] - \left(\tfrac{280}{\lambda}\right)^4,$$ where $c_1 = 5\times10^{-5}$ ($\lambda>600$) or $c_1 = -0.006$ ($\lambda<600$); here λ is in nm vacuum. This is rescaled by a factor of X. Gunn's model is itself based on several inputs, including the UVES lines and the Kitt Peak spectrum[17] with adjustments to provide the correct broadband fluxes in the bluer bands. The far-red continuum agrees with the measurements[18] in between the OH lines at $\lambda \sim 850$ nm.

[12] The UKIRT User Guide gives this as a "mean" level. http://www.jach.hawaii.edu/UKIRT/instruments/wfcam/user_guide/performance.html (Accessed October 4, 2011)

[13] R. Hanuschik, "A flux-calibrated, high-resolution atlas of optical sky emission from UVES," *Astron. Astrophys.* **407**:1157—1164 (2003), Table 3.

[14] R. Content, "Deep-sky infrared imaging by reduction of the background light: I. Sources of the background and potential suppression of the OH emission," *Astrophys. J.* **464**:412—425 (1996).

[15] R. Hanuschik, "A flux-calibrated, high-resolution atlas of optical sky emission from UVES," *Astron. Astrophys.* **407**:1157—1164 (2003), Table 3.

[16] J. Gunn, private communication, Nov. 4, 2011.

4	An analytic fit to the sky continuum provided in the public version of the BigBOSS proposal.[19] The continuum in units of 10^{-16} erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1} is: $$[0.035\sqrt{\tfrac{1000}{\lambda}} + 0.045e^{-(\lambda-594)^2/200}]10^{0.4k}$$ where the last term k = 0.05 + (280/λ)^4 corrects for atmospheric extinction. This is then rescaled by a factor of X.

Note that several types of units of continuum intensity are often used: they are related by the following conversions:

- 1 R Å^{-1} = 4.669×10^{-17} (1 μm/λ) erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1},
- 1 $S10_{AB}$ = 8.399×10^{-21} (1 μm/λ)^2 erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1},
- 1 μW m^{-2} sr^{-1} μm^{-1} = 2.350×10^{-18} erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1},
- 1 γ cm^{-2} s^{-1} arcsec^{-2} μm^{-1} = 1.986×10^{-16} (1 μm/λ) erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1},
- 1 MJy sr^{-1} = 7.046×10^{-18} (1 μm/λ)^2 erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1}, and
- 1 $nmgy_{AB}$ arcsec^{-2} = 1.089×10^{-18} (1 μm/λ)^2 erg cm^{-2} s^{-1} arcsec^{-2} Å^{-1}.

The sky emission continuum and the emission lines are multiplied by the transmission of the atmosphere, since the UVES data were "corrected" for atmospheric extinction. (The other models are adjusted to be consistent with this.) This has little effect in the far red/NIR.

3. *Moonlight*

Finally we require the moonlight model. This is specified by bits 4—7 of the `skytype` member in the `OBS_ATTRIB` structure. That is, we construct the hexadecimal digit:

```
(obs->skytype>>4) & 0xf
```

The following moonlight models are currently available (numbered by their hexadecimal digit):

Moonlight models

Model	Description
0	The two-component (Rayleigh + Mie) moonlight scattering model[20] at V band

[17] P. Massey & C. Foltz, "The spectrum of the night sky over Mount Hopkins and Kitt Peak: Changes after a decade," *Proc. Astron. Soc. Pac.* **112**:566—573 (2000).
[18] J. Noxon, "The near infrared nightglow continuum," *Plan. Sp. Sci.* **26**:191—192 (1978).
[19] http://bigboss.lbl.gov/docs/BigBOSS_NOAO_public.pdf

	(λ = 550 nm) is used. At other wavelengths, the components are scaled by $\sim\lambda^{-4}$ (Rayleigh) or $\sim\lambda^{-1.3}$ (Mie)[21] times a "solar-type" spectrum (here a 5800 K blackbody).

4. *Some tests*

We outline here several simple tests that have been performed with the sky model, since it is one of the largest uncertainties we face in forecasting the performance of PFS.

One simple "sanity check" is to compare to the Subaru narrow-band sky brightnesses, which are estimated at 6.3 $\mathrm{nmgy_{AB}/arcsec^2}$ (NB816) and 6.9 $\mathrm{nmgy_{AB}/arcsec^2}$ (NB921)[22] at lunar age 3 days. These filters lie in between the OH vibration bands and are thus much darker than broad *i* and *z* filters. The Gunn continuum + UVES line spectrum predicts a *zenith* brightness during dark time of 3.8 and 5.2 $\mathrm{nmgy_{AB}/arcsec^2}$, respectively, given the filter transmission curves, of which 2.4 $\mathrm{nmgy_{AB}/arcsec^2}$ is continuum (the moonlight model adds ~0.1 $\mathrm{nmgy_{AB}/arcsec^2}$ and hence is not expected to be significant).[23] The agreement is at the level of a factor of 1.3—1.6, or ~20% in predicted S/N for background limited exposures. Using the UVES-derived continuum model instead of Gunn's model would have predicted a brightness of 5.3 or 7.8 $\mathrm{nmgy_{AB}/arcsec^2}$. Given the known variability of atmospheric emission and the notorious difficulty of the far-red continuum measurements (and the many reasons why narrow band sky brightnesses may not agree with the model even if its continuum level is correct), we do not view these differences as being especially worrisome, and instead have designed the PFS cosmology program with large margins.

A comparison of moonlight model #0 (plus the UVES line spectrum and Gunn continuum) with SDSS imaging data in light-gray (lunar age −11 days) but photometric time is shown in Figure 1. The data were obtained from the SDSS[24,25,26] supernova

---

[20] K. Krisciunas & B. Schaefer, "A model of the brightness of moonlight," *Proc. Ast. Soc. Pac.* **103**:1033—1039 (1991).

[21] As recommended by the CFHT Redeye Manual, Chapter 7: http://www.cfht.hawaii.edu/Instruments/Detectors/IR/Redeye/Manual/chapter7.htm l (Accessed October 6, 2011).

[22] From the Suprime Cam website: http://www.naoj.org/Observing/Instruments/SCam/exptime.html (Accessed March 5, 2012).

[23] From the Suprime Cam website: http://www.naoj.org/Observing/Instruments/SCam/sensitivity.html (Accessed March 5, 2012).

[24] J. Gunn *et al.*, "The Sloan Digital Sky Survey photometric camera," *Astron. J.* **116**:3040—3081 (1998).

[25] D. York *et al.*, "The Sloan Digital Sky Survey: Technical summary," *Astron. J.* **120**:1579—1587 (2000).

[26] J. Gunn *et al.*, "The 2.5 m telescope of the Sloan Digital Sky Survey," *Astron. J.* **131**:2332—2359 (2006).

survey[27] (Run 5744, Camcol 1, acquired on the night of 2005 October 19—20) and downloaded from the Data Archive Server[28]. Generally good agreement is found in the *g*, *r*, *i*, and *z* bands. If anything, moonlight brightness is moderately overpredicted at small target-Earth-Moon angles (θ_{OM}<45°) although this may not be representative since the small-angle scattering has a large contribution from the Mie component (expected to be highly variable). Also, in the *z* band the moonlight even in the gibbous phase is a subdominant contribution to sky brightness, so it seems unlikely that the model can be tested in the *z* or redder bands using broadband imaging.

The *u* band is not shown but the agreement there is much worse – the model over-predicts the *u* band sky brightness by a factor of ~2. For both this reason and theoretical reasons (e.g. the neglect of multiple scatterings should be a poor description of *u* band sky illumination) use of the ETC for *u* band observations in gray or bright time is not recommended. (Of course, such observations are not recommended anyway as they are a highly inefficient use of telescope time!)

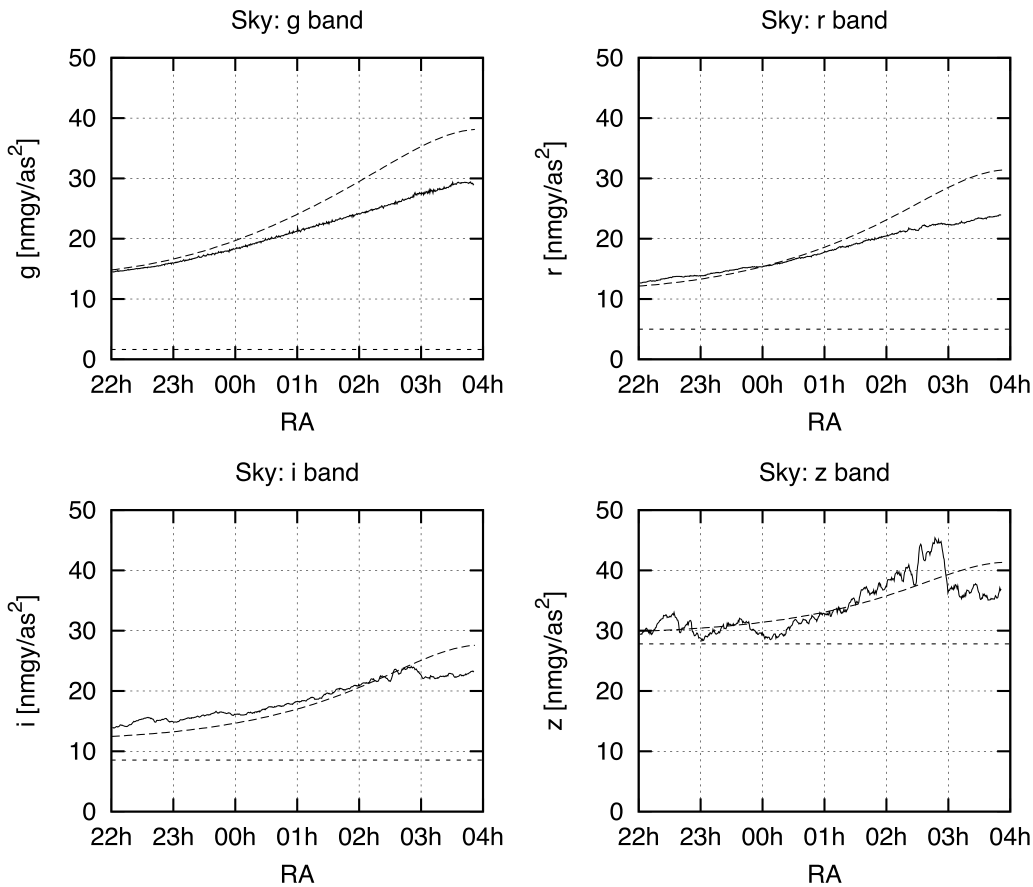

Figure 1: A comparison of the moonlight model (#0) to a sample observation from SDSS. The plotted data are from an SDSS run along the celestial equator. The solid curves show the sky brightness estimated by the photometric pipeline, the dashed curves are the

[27] J. Frieman *et al.*, "The Sloan Digital Sky Survey-II Supernova Survey: Technical Summary," *Astron. J.* **135**:338—347 (2008).
[28] http://das.sdss.org/

model with moonlight, and the short-dashed curves are the model without moonlight. The models are not fits (they were overplotted as-is). The Moon was −11 days from new and located at RA 04h, Dec 24°N. The telescope was "parked" at fixed altitude/azimuth and drift-scanned while the Earth rotated; the actual time series is 6 hours long. The sky brightness in the *g*, *r*, *i*, and *z* bands is shown on the vertical scale in $\mathrm{nmgy_{AB}/arcsec^2}$. Note that at this lunar phase the *g*, *r*, and sometimes *i* bands are dominated by moonlight. There is generally good agreement with the model, but deviations up to ~25% can be seen. In the *g* and *r* band, the moonlight is less than predicted at small angles, implying that the atmospheric Mie scattering (dust or aerosol) component on this night was less than in the model. In the *i* and especially *z* bands, rapid variability of the OH emission forest (not captured by our simple model) can be seen.

C. Atmospheric transmission

The ETC is designed to support a range of atmospheric transmission models. At present there is only one option but this is expandable in the future. Two types of contribution must be considered, and are treated separately: continuum absorption (or scattering) and line absorption (e.g. H_2O bands).

The atmospheric continuum opacity model is specified by bits 8—11 of the `skytype` member in the `OBS_ATTRIB` structure. That is, we construct the hexadecimal digit:

```
(obs->skytype>>8) & 0xf
```

The present description of the atmospheric continuum opacity models is in the table below (here k is the standard extinction term in magnitudes per airmass).

Atmospheric continuum opacity models

Model	Description
0	Mauna Kea optical extinction model from the Gemini website, linearly interpolated.[29] At λ > 900 nm, this model keeps k = 0.05. In the blue the extinction rises to k = 0.12 (550 nm) and k = 0.30 (380 nm).

Since (unlike the emission lines) atmospheric absorption occurs principally in the lower levels of the atmosphere, we have taken the airmass to be $X = \sec\theta_{Z0}$.

The line absorption model is specified by bits 12—15 of the `skytype` member in the `OBS_ATTRIB` structure. That is, we construct the hexadecimal digit:

```
(obs->skytype>>12) & 0xf
```

The line opacity model options are shown in the table below. The line opacity currently does not include an airmass dependence.

[29] http://www.gemini.edu/?q=node/10790 (Accessed October 3, 2011). Further referenced to S. Béland, O. Boulade, and T. Davidge, *CFHT Bulletin* **19**:16 (1988) for λ<400 nm, and the CFHT Observers Manual for λ>400 nm.

Atmospheric line opacity models

Model	Description
0	Kitt Peak model[30], tabulated over the range $500 < \lambda < 1500$ nm and smoothed to 25 km s^{-1} RMS resolution. For $\lambda < 500$ nm the line opacity is neglected.
1	The Kitt Peak model is used for $\lambda < 900$ nm. At longer wavelengths, we switch to a simulated ATRAN[31] Mauna Kea transmission spectrum with 3.0 mm precipitable H_2O at zenith.[32]

D. Galactic extinction

Galactic (Milky Way) extinction is assumed to follow the $R_V = 3.1$ law typical of high Galactic latitude lines of sight. The reddening $E(B-V)$ is an input to the ETC. The extinction curve is obtained from B. Draine.[33]

E. Noise model

The ETC constructs a noise model consisting of the expected variance (in e^- squared) in each 1D pixel (λ-pixel). The 1D spectrum is obtained by combining W pixels in the cross-dispersion direction. The fraction of radiation from a fiber that lands within the W-pixel window is $f_{\rm trace}$, which is obtained from the spectrograph MTF (§4.J). For the purposes of sky brightness modeling, we use not $f_{\rm trace}$ but $f_{\rm trace,1}$, which includes sky brightness that leaks over from the two adjacent spectra.

The model includes the following contributions to the variance:

- *Dark current*: For $I_{\rm dark}$ e^- pix^{-1} s^{-1}, the variance per λ-pixel is $I_{\rm dark}Wt$.
- *Read noise*: For $\sigma_{\rm r}$ e^- RMS per pixel, the variance per λ-pixel is $\sigma_{\rm r}^2W$.
- *Sky continuum*: The continuum is converted to photons per second per λ-pixel per m^2 per arcsec^2. This is then multiplied by the effective area including all telescope + spectrograph throughput terms, the fiber solid angle, the exposure time, and $f_{\rm trace,1}$.
- *Sky lines*: Each sky line is converted to a total number of counts in the detector (again by converting input brightness units from the 10^{-12} erg m^{-2} s^{-1} arcsec^{-2} to photons m^{-2} s^{-1} arcsec^{-2}, and then multiplying by $A_{\rm eff}f_{\rm trace,1}\Omega_{\rm fiber}t$). This light is then distributed over a box of length `SP_PSF_LEN` λ-pixels, with a probability distribution given by the Fourier transform of the spectrograph MTF.
- *Systematic sky subtraction errors*: In each λ-pixel, one adds to the variance in the $i^{\rm th}$ pixel the term

[30] K. Hinkle, L. Wallace & W. Livingston, "Atmospheric transmission above Kitt Peak, 0.5 to 5.5 microns," *Bull. AAS* **35**:1260 (2003).
[31] S. Lord, NASA Technical Memorandum 103957 (1992).
[32] http://www.gemini.edu/sciops/ObsProcess/obsConstraints/atm-models/mktrans_zm_30_10.dat (Accessed January 10, 2012).
[33] ftp://ftp.astro.princeton.edu/draine/dust/mix/kext_albedo_WD_MW_3.1_60_D03.all

$$\sigma^2_{i,\mathrm{sys}} = N_{\mathrm{exp}}\left(f_{\mathrm{sys}} \max_{j=i-1}^{i+1} Q_{\mathrm{sky},j} \right)^2,$$

where Q_{sky} is the collected number of sky photoelectrons, N_{exp} is the number of exposures (to prevent this signal from averaging down), and f_{sys} is the systematic floor. This is treated as uncorrelated between λ-pixels; for correlated noise one should multiply the user-provided f_{sys} by the square root of the correlation length (in λ-pixels). The value of Q_{sky} to which the systematic floor is scaled is taken to be the maximum of the 3 pixels centered on the one being measured. This prevents "perfect" subtraction in the wings of a bright atmospheric line.

- *Diffuse stray light*: The total number N_γ of sky photons received on the entire detector is computed. Then we add to each pixel a contribution

$$\sigma^2_{i,\mathrm{diff}} = f_{\mathrm{diff}} \frac{W}{N^2_{\mathrm{pix}}} N_\gamma,$$

where f_{diff} is the diffuse stray light fraction and N_{pix} is the (1D) number of pixels in the detector.

For HgCdTe detectors, if the `-DHGCDTE_SUTR` flag is set, "Poisson-like" contributions (sky or dark current shot noise) contain an additional factor of 1.2 associated with the linear SUTR fitting algorithm.[34] [If this flag is **not** set, it is assumed that we are using Fowler sampling instead of SUTR in the HgCdTe channel(s). If sufficient readout time is available, or forced on us by the CCD channels, this may be advantageous. This choice also couples to the sky subtraction problem since the sky brightness will vary during the exposure.]

F. 2D to 1D projection

Before any following operations, the ETC assumes that we first project the 2D spectrum into a 1D spectrum. This is done by an unweighted coaddition of W pixels in the slit direction.

Note that this is not an optimal method for two reasons: (i) unweighted (or tophat) integration over the slit direction is suboptimal in the presence of read noise; and (ii) if the sky and source have spectral structure (which they do), one can improve upon direct 2D→1D projection[35]: e.g. a source emission line partially blended with a bright sky line should really be extracted in the crescent-shaped region containing source line and dark sky.

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[34] The factor of 1.2 is appropriate for many samples. See B. Rauscher *et al*, "Detectors for the James Webb Space Telescope Near-Infrared Spectrograph: I. Readout mode, noise model, and calibration considerations," *Pub. Astron. Soc. Pac.* **119**:768—786 (2007).

[35] A. Bolton & D. Schlegel, "Spectro-perfectionism: An algorithmic framework for photon noise-limited extraction of optical fiber spectroscopy," *Proc. Ast. Soc. Pac.* **122**:248—257 (2010).

This of course introduces some conservatism into the ETC. However, the conservatism introduced by (i) is not large. In the sky-dominated case, there is no associated loss, i.e. the reduction in SNR is by a factor $f_{\rm loss} = 1$. In the read noise-limited case, the fractional loss of SNR due to tophat weighting versus matched filter weighting is given by

$$f_{\rm loss} = \frac{\sum_{i=1}^{W} P_i}{\sqrt{W \sum_{\text{all } i} P_i^2}},$$

where P_i is the projection of the PSF in the slit direction (in units of probability per spatial pixel). For the "best" choice of W, this loss is $f_{\rm loss} = 0.9435$ for a Gaussian and 0.9761 for a uniform disk (idealized image of a fiber). In the real case, the losses are somewhere in between, so clearly (i) is an issue far below the accuracy of any ETC for a system that has not yet been built!

We do not yet have a single estimate for (ii).

G. Line sensitivity

There are several possible definitions for the sensitivity to an emission line. Even given that we are extracting the line from a 1D spectrum, there are three choices:

- One may construct the SNR for a *single line* (one member of the [O II] doublet), or for the *overall feature* (the RSS of the SNRs in the case of well-separated lines).
- If we desire the SNR of a single line, we may ask for either line (or the higher SNR line; or, if our interest is in using the doublet structure as confirmation of the redshift, we may ask for the lower SNR line).
- A line may be either detected in an *optimal filter* (inverse noise variance weighted) or a *uniform filter* (so that the detection criteria do not vary rapidly with redshift as source lines sweeps over the OH emission forest).

Clearly the overall feature optimal filter will give the highest SNR. It is of the form

$${\rm SNR}_{\rm opt} = \sqrt{\sum_i \frac{S_i^2}{N_i}},$$

where S_i is the expected signal counts in the $i^{\rm th}$ λ-pixel, and N_i is the noise variance in the $i^{\rm th}$ λ-pixel. The uniform filter is constructed without the inverse noise variance weighting:

$${\rm SNR}_{\rm uni} = \frac{\sum_i S_i^2}{\sqrt{\sum_i S_i^2 N_i}}.$$

The type of SNR is described by the `snrType` parameter. The options are:

- 0: Optimal filter, higher SNR member of [O II] doublet.
- 1: Uniform filter, higher SNR member of [O II] doublet.
- 2: Optimal filter for overall [O II] feature.

H. Galaxy profiles

The galaxy profiles will be taken to be exponential, as this is characteristic of late-type galaxies that host emission lines. A normalized exponential profile with half-light radius $r_{\rm eff}$ [rad] is given by:

$$f(\mathbf{x}) = \frac{1}{2\pi r_s^2} \exp\frac{-|\mathbf{x}|}{r_s}.$$

The scale radius can be found numerically to be $r_s = r_{\rm eff}$ / 1.67834. The Fourier transform is

$$\tilde{f}(\mathbf{u}) = \left(1 + 4\pi^2 r_s^2 |\mathbf{u}|^2\right)^{-3/2}.$$

I. Point spread functions and geometric throughput

The geometric throughput $f_{\rm geo}$ is the integral of the galaxy image f convolved with the PSF G over the fiber aperture:

$$f_{\rm geo} = \int_{\rm fiber} [f * G](\mathbf{x}) d^2\mathbf{x} = \int_{\mathrm{R}^2} \tilde{f}(\mathbf{u})\tilde{G}(\mathbf{u}) \frac{RJ_1(2\pi Ru)}{u} d^2\mathbf{u},$$

where the ~ denotes a Fourier transform, * denotes convolution, **u** is the spatial frequency in cycles per arcsec, and R is the fiber radius in arcsec (obtained from the physical diameter and the EFL). This can be decomposed into an angular average of the MTF (i.e. averaged over directions of **u**) to give

$$f_{\rm geo} = 2\pi R \int_0^\infty \left\langle \tilde{f}(\mathbf{u})\tilde{G}(\mathbf{u}) \right\rangle J_1(2\pi Ru) du.$$

The MTF $\tilde{G}(\mathbf{u})$ is composed of the product of several pieces:

$$\tilde{G}(\mathbf{u}) = \tilde{G}_{\rm seeing}(\mathbf{u}) \times \tilde{G}_{\rm diffraction}(\mathbf{u}) \times \tilde{G}_{\rm aberrations}(\mathbf{u}) \times \tilde{G}_{\rm offset}(\mathbf{u}).$$

These are modeled as follows:

- The seeing is taken to be a Kolmogorov profile with FHWM $\theta_{\rm FWHM}(\lambda)$. The seeing is defined at a reference wavelength of $\lambda_{\rm vac}$ = 800 nm, and scaled in proportion to $\sim \lambda^{-1/5}$. Explicitly, the MTF from seeing is $\tilde{G}_{\rm seeing}(\mathbf{u}) = \exp[-(\theta_{\rm FWHM} u/0.465)^{5/3}]$.

- For large ground-based telescopes, diffraction contributes negligibly to the size of the image core. However diffraction features may still be significant in the far tails of the PSF, which are a source of lost radiation. The amount of radiation diffracted to large radii is proportional to the perimeter-to-area ratio ς of the aperture; the leading term is $\ln\tilde{G}_{\rm diffraction}(\mathbf{u}) = -\frac{4}{\pi}\theta_{\rm D}u$, where the diffraction scale length is $\theta_{\rm D} = \varsigma\lambda/4$ radians. The perimeter-to-area ratio is $\varsigma = 4(1-\upsilon)/D_{\rm outer}$, where we have neglected the contribution of the diffraction spikes. There are higher-order terms in $\ln\tilde{G}_{\rm diffraction}(\mathbf{u})$ – i.e. terms involving u^2, u^3, etc. – but we do not include them as they are expected to be insignificant, and in any case are subdominant to the aberration terms.
- The aberrations are taken to be Gaussian with some RMS spot size per axis σ (in physical units). So if the EFL is F, we have $\tilde{G}_{\rm aberrations}(\mathbf{u}) = \exp[-2\pi^2(\sigma/F)^2u^2]$. The value of σ is interpolated from a table as a function of field angle in the configuration file.
- The fiber center is offset from the galaxy by some decenter distance δ (arcsec). After isotropic averaging this is equivalent to convolving the PSF with a circular ring of radius δ. The corresponding Fourier transform is $\tilde{G}_{\rm offset}(\mathbf{u}) = J_0(2\pi\delta u)$.

Tracking errors, jitter, and changes in distortion over an exposure are not explicitly included. We recommend adding them in quadrature to σ in the configuration file.

J. Spectrograph image quality

The PSF of the spectrograph is needed in order to forecast the appearance of emission lines. We work with its Fourier transform, the MTF. We include the fiber diameter and detector effects in the MTF, even though they are technically not part of the spectrograph optics, since they are functionally equivalent. The MTF then has a number of contributions:

$$\mathrm{MTF}(u) = \mathrm{MTF}_{\rm fiber}(u) \times \mathrm{MTF}_{\rm ab}(u) \times \mathrm{MTF}_{\rm def}(u) \times \mathrm{MTF}_{\rm cd}(u) \times \mathrm{MTF}_{\rm pix}(u) \times \mathrm{MTF}_{\rm scat}(u),$$

where u is the spatial frequency in cycles per pixel. We denote the pixel scale by P. The contributions are the fiber diameter $\mathrm{MTF}_{\rm fiber}$; the aberrations $\mathrm{MTF}_{\rm ab}$; the defocus within the detector $\mathrm{MTF}_{\rm pix}$; the charge diffusion $\mathrm{MTF}_{\rm cd}$; the pixelization $\mathrm{MTF}_{\rm pix}$; and the scattering from a grating with a finite number of lines, $\mathrm{MTF}_{\rm scat}$.

The fiber effect is

$$\mathrm{MTF}_{\rm fiber}(u) = \frac{J_1(2\pi ru)}{\pi ru},$$

where r is the effective fiber radius projected onto the detector in pixel units. It is generally smaller than the physical fiber core radius by the ratio of the camera to the collimator focal length. The aberrations are treated as a Gaussian at σ μm RMS per axis:

$$\mathrm{MTF}_{\rm ab}(u) = \exp[-2\pi^2(\sigma/P)^2u^2].$$

Charge diffusion is incorporated by including it in σ via quadrature summation.

The pixel MTF is a standard tophat, $\mathrm{MTF}_{\mathrm{pix}}(u) = j_0(\pi u)$.

For silicon CCD detectors, we include the defocus within the CCD, which is significant at long wavelengths in a fast beam. The contribution to the MTF from such defocus is

$$\mathrm{MTF}_{\mathrm{def}}(u) = \frac{\int w(z)\varphi(uz/2nfP)\,dz}{\int w(z)\,dz},$$

where z is the depth in the detector, n is the real part of the index of refraction of the detector, f is the f/ratio, $w(z)$ is the un-normalized weight function for distribution of depths in the detector, and $\varphi(x)$ is the 2D Fourier transform of the unit circular tophat. We use

$$w(z) = \begin{cases} e^{-z/\ell} & 0 < z < t \\ 0.3e^{-z/\ell} & t < z < 2t \\ 0 & \text{otherwise} \end{cases},$$

where t is the detector thickness and the 0.3 accounts for reflectivity at the silicon-to-vacuum transition after passage through the detector. Subsequent reflections at the AR coat of the illuminated surface are not considered, nor are fringing effects (which we expect to be small in a sufficiently fast beam – they are primarily an issue for sky subtraction rather than raw sensitivity). The Fourier transform is ideally a Bessel function, but for speed we use the approximation

$$\varphi(x) = \frac{J_1(2\pi x)}{\pi x} \approx \frac{1}{6}\cos(\sqrt{3}\pi x) + \frac{1}{2}\cos(\pi x) + \frac{1}{3},$$

which is equivalent to treating the circular beam as composed of a weighted combination of rays at 0%, 50%, and 86.6% of the maximum angle of incidence. This does not take into account central obscuration of the beam, which would increase defocusing effects, but we again expect this to be small (it is a correction to a correction). For HgCdTe detectors the finite detector thickness is neglected.

The absorption length in silicon is an empirical model containing both direct and indirect band gap absorption with allowance for phonon absorption and (if energetically allowed) emission and stimulated emission, and temperature-dependent band gap.[36] The real part of the index of refraction is interpolated from tabulated data and does not currently contain temperature dependence (although we do not expect this to be important).[37,38]

---

[36] K. Rajkanan, R. Singh, J. Shewchun, "Absorption coefficient of silicon for solar cell calculations," *Solid State Electronics* **22**:793 (1979)

[37] http://snap.lbl.gov/ccdweb/ccd_data/complex_index.dat

We finally come to the scattering from the grating. A grating has a true number of lines across the pupil given by

$$N_{\rm lines,true} = \frac{P}{f\Delta\lambda},$$

where $\Delta\lambda$ is the dispersion in wavelength units per pixel. An imperfect grating will have an effective number of lines $N_{\rm lines} < N_{\rm lines,true}$. The contribution to the MTF from scattering off of the grating is then

$$\mathrm{MTF}_{\rm scat}(u) = \exp\left(-\frac{\lambda}{N_{\rm lines}\Delta\lambda}u\right).$$

This gives a Lorentzian profile with damping wings determined by $N_{\rm lines}$. For a perfect grating the core shape is not reproduced perfectly, but in practice the core of the profile is swamped by the finite fiber size and geometric optics contributions.

5. Data structures and functions

This section provides a complete listing of the internal data structures and functions in the code (including low-level utilities). It may be of use to users who wish to write their own driver routines.

A. Data structures

```
typedef struct {
  double D_outer, EFL[5];      /* Outer diameter & effective focal
   length (meters) */
  double fiber_ent_rad;        /* Fiber entrance radius (microns) */
  double centobs;              /* Central obscuration */
  double rfov;                 /* Field of view radius (degrees) */
  double rms_spot[5];          /* RMS spot size per axis: 5 values
   from center to edge (microns) */
  double vignette[5];          /* Vignetting factor: 5 values from
   center to edge */
  int N_arms;                  /* Number of arms */

  double lmin[MAXARM];         /* Min wavelength in nm */
  double lmax[MAXARM];         /* Max wavelength in nm */
  long npix[MAXARM];           /* Number of pix from min to max
   wavelength */
  double dl[MAXARM];           /* Wavelength spacing, nm/pix, derived
   */
```

[38] D. Groom *et al.*, “Quantum efficiency of a back-illuminated CCD imager: an optical approach,” *Proc. SPIE* **3649**:80 (1999)

```
  long width[MAXARM];          /* Width of trace used in actual
   analysis, pixels */
  double fratio[MAXARM];       /* Camera f/ratio in ith arm */
  double thick[MAXARM];        /* Camera detector thickness (microns)
   */
  double pix[MAXARM];          /* Camera pixel scale (microns) */
  double temperature[MAXARM]; /* Camera temperature (K) */
  double rms_cam[MAXARM];      /* Camera rms spot per axis (microns)
   */
  double diam[MAXARM];         /* Geometric fiber image diameter
   (microns) */
  double dark[MAXARM];         /* Dark current (e/pix/s) */
  double read[MAXARM];         /* Read noise (e rms) */
  double sep[MAXARM];          /* Trace spacing (microns) */
  double nline[MAXARM];        /* Effective number of lines */

  int N_thr;                   /* Number of points in throughput grid
   */
  int istart[MAXARM+1];        /* Position at which the i_arm grid
   begins; istart[N_arms] = N_thr */
  double l[MAXNTHR];           /* Wavelength at grid points (nm) */
  double T[MAXNTHR];           /* Throughput at grid points
   (excluding atmosphere & fiber geom) */

  int Dtype[MAXARM];           /* =0 for Si, 1 for HgCdTe */

  double sysfrac;              /* Sky subtraction systematic (per
pixel per exposure) */
  double diffuse_stray;        /* Diffuse stray light amplitude */

} SPECTRO_ATTRIB;
```

This structure describes the attributes of the hardware (telescope, fiber system, and spectrograph).

The `SPECTRO_ATTRIB` structure contains the following parameters describing the telescope:

- The outer diameter `D_outer` of the primary mirror, and the effective focal length `EFL` at the plane of the fiber inputs.
- The radius of the fiber inputs, `fiber_ent_rad`, in μm. For fibers with lenslets at the entrance (as planned for some versions of PFS) this is the apparent diameter of the entrance to the fiber as seen after the corrector, rather than the physical fiber core radius.
- The fractional central obscuration `centobs`, in linear dimensions (the area obscuration is `centobs*centobs`).
- The radius of the field of view in degrees, `rfov`.
- The spot size delivered by the telescope + corrector, in units of μm RMS per axis. Five values are specified, linearly spaced from the center of the field to the edge in steps of `rfov`÷4. These form an array, `rms_spot[5]`.

- The vignetting factor for the telescope + corrector (1 = no vignetting), *not* including the central obscuration. Five values are specified, linearly spaced from the center of the field to the edge in steps of `rfov`÷4. These form an array, `vignette[5]`.

Next comes the parameter `N_arms` describing the number of arms per spectrograph. The data for each arm is specified next:

- The wavelength range of the `i`th arm is `lmin[i]` < λ (nm vac) < `lmax[i]`.
- The number of pixels in the spectral direction in the `i`th arm is `npix[i]`.
- The wavelength spacing in nm vacuum per pixel is a derived parameter but is stored as `dl[i]` for the `i`th arm.
- The width of the trace as used when compressing the spectrum down to 1D is not a hardware parameter, but is included here as `width[i]` for the `i`th arm.
- The spectrograph optics parameters are given next: for the `i`th arm, these are the f/ratio seen by the detector, `fratio[i]`; the detector thickness `thick[i]` in μm; the pixel scale `pix[i]` in μm; the detector temperature `temperature[i]` in Kelvin; the spot size `rms_cam[i]` in μm RMS per axis; and the diameter of the geometric fiber image, `diam[i]` in μm. For silicon detectors, we treat detector characteristics as optics parameters since the beam can pass through the detector instead of interacting at the surface.
- The dark current for the `i`th arm, `dark[i]` (in e^- pixel^{-1} s^{-1}), and the RMS read noise `read[i]`.
- The separation of adjacent spectral traces, `sep[i]`, in μm.
- The effective number of lines $N_{\rm lines}$, used in determining the spectrograph MTF.

Then `SPECTRO_ATTRIB` contains a throughput table. This is a 1D vector with the throughput curves for all of the arms concatenated. The total number of throughput points is `N_thr`. The throughput points for the `i`th arm start at `istart[i]` and end at `istart[i+1]-1`. Note that `N_arm+1` values in `istart[]` must be specified. The throughput points are specified by a wavelength `l[j]` (in nm vacuum) and a throughput `T[j]`. Here "throughput" denotes all optical terms: mirror coatings, corrector absorption and AR coatings, losses in the fiber, FRD, the spectrograph (including vignetting in a reflective camera), grating, and detector QE. Specifically **excluded** from this definition are Galactic extinction, atmospheric extinction, central obscuration, and vignetting in the telescope+corrector.

Then `SPECTRO_ATTRIB` contains the `Dtype` vector, which contains 0 entries for silicon detectors and 1 entries for HgCdTe detectors.

Finally, parameters are specified that control two nasty effects: `sysfrac` describes the systematic sky subtraction floor (as a fraction of the sky brightness per pixel taken as the maximum of that λ-pixel and its neighbors), and `diffuse_stray` describes the amplitude of diffuse stray light (e.g. if this is 0.02, then 2% of the light on the spectrograph is spread over the focal plane as an additional source of background).

```
typedef struct {
```

```
  double seeing_fwhm_800; /* Seeing FWHM @ 800 nm */
  double elevation;       /* Elevation in meters above sea level */
  double zenithangle;     /* Angle from the zenith, in degrees */
  double lunarphase;      /* Lunar phase: 0.0 (new Moon) --> 0.5
   (full Moon) --> 1.0 (new Moon) */
  double lunarangle;      /* Angle from the Moon to the line of
   sight, in degrees */
  double lunarZA;         /* Angle from the Moon to the zenith, in
   degrees; set to >90 for no Moon */
  double EBV;             /* Dust reddening column, E(B-V) */
  unsigned long skytype;  /* Bitmask for assumptions on atmospheric
   conditions */
} OBS_ATTRIB;
```

This structure describes the attributes of the observing conditions. The parameters are:

- The seeing, `seeing_fwhm_800`. This is measured in arcsec FWHM, at a reference wavelength of 800 nm. (The Kolmogorov scaling FWHM $\sim \lambda^{-0.2}$ is assumed.)
- The `elevation` of the observing site in meters above sea level.
- The zenith angle ("`zenithangle`") in degrees (0 for observations at zenith; 90 at horizon).[39]
- The phase of the Moon, `lunarphase`. This is 0 at new Moon; ¼ at first quarter; ½ at full Moon; ¾ at last quarter; and 1 at new Moon again.
- The angle `lunarangle` between the Moon and the direction of observation, in degrees.
- The lunar zenith angle `lunarZA` in degrees. This is 0 if the Moon is at the zenith, and 90 at moonrise or moonset. Values ≥90 indicate that the Moon is below the horizon.
- The Galactic dust column `EBV`, measured by the color excess $E(B-V)$ in magnitudes.
- The atmospheric conditions or model are specified by the 32 bits in `skytype`.

B. Special functions

```
double getJ0(double x);
double getJ1(double x);
double geterf(double x);
```

These are the standard special functions $J_0(x)$, $J_1(x)$, and erf x. The Bessel functions are accurate to several parts in 10^8, and the error function to much better accuracy.[40]

---

[39] This is defined in vacuum – the "observed" zenith angle (corrected for atmospheric refraction) is slightly less, but the difference is not significant for the purposes of the ETC.

[40] The Bessel functions are implemented using a combination of the integral form at $|x|<3$ and the formulae of M. Abramowitz & I. Stegun, "Handbook of Mathematical

C. Conversion functions

```
double gs_n_air(double lambda_vac);
double gs_vac2air(double lambda_vac);
double gs_air2vac(double lambda_air);
```

These functions return the index of refraction of air and perform conversions between vacuum and air wavelengths. All inputs and outputs are in nanometers. Conversions use the Edlén equation:[41]

$$10^8(n-1) = 6432.8 + \frac{2949810}{146-\sigma^2} + \frac{25540}{41-\sigma^2},$$

where σ is the vacuum wavenumber in cycles μm^{-1}.

D. Material properties

```
double gsOP_Si_abslength(double lambda, double T);
double gsOP_Si_indexreal(double lambda, double T);
```

These functions return the absorption mean free path in silicon and the (real part of) the index of refraction. The wavelength is in nanometers, the temperature is in Kelvin, and the output absorption length is in microns.

E. Routines to compute the foreground absorption

```
double Galactic_Alambda__EBV(double lambda);
```

The Galactic reddening law, $A_\lambda/E(B-V)$, given λ in nm (in vacuum, although the difference is not significant).

```
double gsAtmContOp(OBS_ATTRIB *obs, double lambda, unsigned long
   flags);
```

This returns the continuum opacity of the atmosphere in magnitudes per airmass at the stated wavelength (in nm vacuum). Observing conditions are specified by `obs`.

The `flags` are not currently used.

```
double gsAtmTrans(OBS_ATTRIB *obs, double lambda, unsigned long
   flags);
```

This returns the atmospheric transmission at wavelength λ (in nm vacuum). This is a fraction, i.e. between 0 and 1. Both continuum and absorption lines (e.g. H_2O) are included. Observing conditions are specified by `obs`.

Functions" (Dover: New York, 1964), Eqs. 9.4.3 & 9.4.6. The error function uses either the Taylor expansion or an integral transformation described in the comments.

[41] B. Edlén, "The dispersion of standard air," *J. Opt. Soc. Am.* **43**:339 (1953)

The `flags` are not currently used.

F. Routines to calculate throughput & related quantities

```
double gsGeometricThroughput(SPECTRO_ATTRIB *spectro, OBS_ATTRIB
   *obs, double lambda, double r_eff, double decent, double
   fieldang, unsigned long flags);
```

This routine calculates the geometric throughput, i.e. the fraction of the radiation from an object of specified $r_{\rm eff}$ (arcsec) that is encircled within the geometrical radius of the fiber. The input wavelength λ is in nanometers (vacuum) and the field angle is `fieldang` degrees. Telescope and seeing parameters are in `spectro` and `obs` respectively. The fiber center is assumed to be decentered by `decent` arcsec from the astrometric center of the target.

The `flags` are not currently used.

```
double gsAeff(SPECTRO_ATTRIB *spectro, OBS_ATTRIB *obs, int i_arm,
   double lambda, double fieldang);
```

This returns the effective area $A_{\rm eff}$ of the system (telescope + corrector + fibers + spectrograph + detector) in square meters. The atmosphere and Galaxy are not included.

Telescope and seeing parameters are in `spectro` and `obs` respectively, and we consider a point at `fieldang` degrees from the boresight. The wavelength of observation is λ (nm vacuum) and only light that goes into the detector in `i_arm` is included. Usually only one arm has nonzero $A_{\rm eff}$ at a given λ, but at the transitions between arms the finite width cutoff of the dichroic results in nonzero $A_{\rm eff}$ for two adjacent arms.

```
double gsSpectroMTF(SPECTRO_ATTRIB *spectro, OBS_ATTRIB *obs, int
   i_arm, double lambda, double u);
```

This returns the MTF of the spectrograph in arm `i_arm`, at wavelength λ nm in vacuum, and at a spatial frequency of u cycles per pixel. The finite size of the fiber and the pixelization effects are included in this calculation even though from an optics perspective they are not technically part of the MTF. Telescope and seeing parameters are in `spectro` and `obs` respectively.

```
void gsSpectroDist(SPECTRO_ATTRIB *spectro, OBS_ATTRIB *obs, int
   i_arm, double lambda, double pos, double sigma, int N, double
   *fr);
```

This function computes the probability distribution for a photon in the spectrograph to land in a particular pixel, projected down into 1 dimension. The number of pixels considered is `N`, and the output goes to `fr[0..N-1]`. The center of the spot is at position `pos` (in pixel units). We consider arm `i_arm`, at wavelength λ nm in vacuum. Telescope and seeing parameters are in `spectro` and `obs` respectively.

```
double gsFracTrace(SPECTRO_ATTRIB *spectro, OBS_ATTRIB *obs, int
   i_arm, double lambda, int tr);
```

If `tr`=0, this function computes the probability for a photon to land in the 1D trace (of width `spectro->width[i_arm]` pixels). We consider arm `i_arm`, at wavelength λ nm in vacuum. Telescope and seeing parameters are in `spectro` and `obs` respectively. If `tr`=1, the routine returns the probability to land in either the intended trace *or* an adjacent trace.

G. Routines to compute the signal and the noise

```
void gsGetNoise(SPECTRO_ATTRIB *spectro, OBS_ATTRIB *obs, int i_arm,
   double fieldang, double *Noise, double t_exp, unsigned long
   flags);
```

Returns to `Noise`[0 … `spectro->npix[i_arm]`−1] the noise variance in e^- squared per λ-pixel in an exposure of length $t_{\rm exp}$ seconds, for the specified spectrograph configuration (`spectro`), observational conditions (`obs`), spectrograph arm (`i_arm`), and field position (`fieldang` in degrees). The `flags` are currently for plumbing/forward compatibility – they are not actually used, rather all relevant conditions are specified inside `spectro` or `obs`.

```
void gsGetSignal(SPECTRO_ATTRIB *spectro, OBS_ATTRIB *obs, int
   i_arm, double lambda, double F, double sigma_v, double r_eff,
   double decent, double fieldang, double *Signal, double t_exp,
   unsigned long flags);
```

Returns to `Signal`[0 … `spectro->npix[i_arm]`−1] the expected number of collected e^- per λ-pixel from an emission line. Most of the spectrum is empty but the entire 1D vector is generated anyway. The emission line is taken to have wavelength λ (nm vacuum); flux F (erg cm^{-2} s^{-1}, measured before entering the Milky Way Galaxy); velocity dispersion σ_v (1σ width, km s^{-1}); be from an object with exponential profile and half-light radius $r_{\rm eff}$ (arcsec); be decentered from the geometric center of the fiber by `decent` (arcsec); and be at field position `fieldang` (degrees from on-axis). The exposure time is $t_{\rm exp}$ seconds. The `flags` are currently for plumbing/forward compatibility – they are not actually used, rather all relevant conditions are specified inside `spectro` or `obs`.

```
double gsGetSNR(SPECTRO_ATTRIB *spectro, OBS_ATTRIB *obs, int i_arm,
   double lambda, double F, double sigma_v, double r_eff, double
   decent, double fieldang, double *Noise, double t_exp, unsigned
   long flags, int snrType);
```

This routine computes the SNR for an emission line object. The emission line is taken to have wavelength λ (nm vacuum); flux F (erg cm^{-2} s^{-1}, measured before entering the Milky Way Galaxy); velocity dispersion σ_v (1σ width, km s^{-1}); be from an object with exponential profile and half-light radius r_{eff} (arcsec); be decentered from the geometric center of the fiber by `decent` (arcsec); and be at field position `fieldang` (degrees from on-axis). The exposure time is t_{exp} seconds. The `flags` are currently for plumbing/forward compatibility – they are not actually used, rather all relevant conditions are specified inside `spectro` or `obs`. There are several possible definitions of SNR, as indicated by `snrType`. Right now the options are:

- `snrType == 0`: 1D optimal filter.
- `snrType == 1`: Uniform matched filter.

See §4.G for complete definitions.

```
double gsGetSNR_OII(SPECTRO_ATTRIB *spectro, OBS_ATTRIB *obs, int
   i_arm, double z, double F, double sigma_v, double r_eff, double
   src_cont, double ROII, double decent, double fieldang, double
   *Noise, double t_exp, unsigned long flags, int snrType);
```

This is the same as `gsGetSNR`, *except* that it is specifically configured for the [O II] λ = 3727, 3730 Å doublet. Therefore instead of a wavelength λ, the redshift z is provided. The source continuum in erg cm^{-2} s^{-1} Hz^{-1} ("`src_cont`") and the line flux ratio F_{3727}/F_{3730} ("`ROII`") are provided. Also the `snrType` codes are slightly different:

- `snrType == 0`: SNR for the higher-significance of the two doublet members, 1D optimal filter, assume 1:1 doublet ratio.
- `snrType == 1`: SNR for the higher-significance of the two doublet members, uniform matched filter, assume 1:1 doublet ratio.
- `snrType == 2`: SNR for the combined doublet, 1D optimal filter, assume 1:1 doublet ratio.

See §4.G for complete definitions.

```
void gsGetSNR_Continuum(SPECTRO_ATTRIB *spectro, OBS_ATTRIB *obs,
   int i_arm, double mag, double r_eff, double decent, double
   fieldang, double *Noise, double t_exp, unsigned long flags,
   double *out_SNR_curve);
```

This routine computes the S/N **per pixel** for an object with a white spectrum (constant magnitude AB, or constant F_ν in Jy). The inputs include the magnitude `mag` in the AB system. The `flags` are currently for plumbing/forward compatibility – they are not actually used, rather all relevant conditions are specified inside `spectro` or `obs`. The noise vector `Noise` must have been previously calculated, probably using `gsGetNoise`. The output is to the vector `out_SNR_curve` of length `spectro->npix[i_arm]`.

H. I/O functions

```
void gsReadSpectrographConfig(char FileName[], SPECTRO_ATTRIB
   *spectro);
```

This function reads a spectrograph configuration file from `FileName`, and stores it in the data structure `spectro`. Some basic error checking is done but it is not foolproof.

6. Version History

Version 1: November 15, 2011

An idealized version of the code that runs on the [O II] doublet was constructed and tagged.

Version 2: January 2, 2012

Included source continuum contribution in the noise, and support for variable [O II] doublet ratio.

Version 3: January 28, 2012

Added support for continuum S/N computations.
Added support for Lorentzian grating wings.
Implemented systematic sky subtraction errors.

Version 4: March 4, 2012

Added support for diffuse stray light.

Version 5: March 11, 2012

Fixed a memory allocation error that caused the program to not run successfully on some 32-bit platforms.

7. Acknowledgements

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8. Acronyms

AR	Anti-Reflection
CCD	Charge Coupled Device
EFL	Effective Focal Length
ELG	Emission Line Galaxy
ETC	Exposure Time Calculator

FoMSWG	Figure of Merit Science Working Group
FOV	Field of View
FRD	Focal Ratio Degradation
FWHM	Full Width at Half Maximum
MTF	Modulation Transfer Function
PFS	Prime Focus Spectrograph
PSF	Point Spread Function
QE	Quantum Efficiency
RMS	Root Mean Square
RSS	Root Sum Square
SDSS	Sloan Digital Sky Survey
SNR	Signal-to-Noise Ratio
UVES	Ultraviolet and Visual Echelle Spectrograph
WFIRST	Wide Field InfraRed Space Telescope
WMAP	Wilkinson Microwave Anisotropy Probe