# Image Restoration

# IMAGE RESTORATION

- **The main objective of restoration is to improve the quality of a digital image which has been degraded due to Various phenomena like:**
- **Motion**
- **Improper focusing of Camera during image acquisition.**
- **Noise**

# What is Image Restoration

- The purpose of image restoration is to restore a degraded/distorted image to its original content and quality.

- Restoration involves following process:-
- Modeling of Degradation
- Applying the inverse process to recover the original image

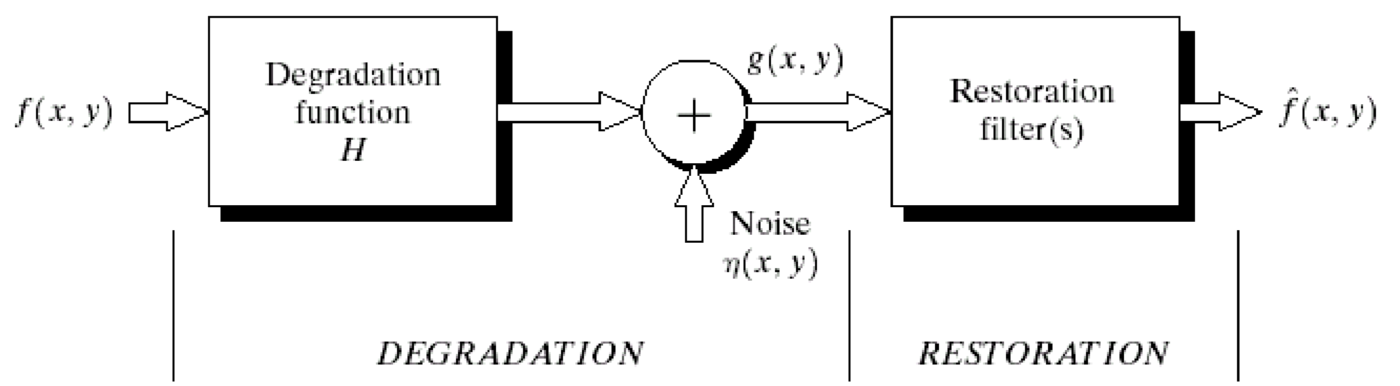


**FIGURE 5.1** A model of the image degradation/ restoration process.

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# Explanation of diagram

- We will assume that a degradation function exists, which, together with additive noise, operates on the input image $f(x,y)$ to produce a degraded image $g(x,y)$.
- The objective of restoration is to obtain an estimate for the original image from its degraded version $g(x,y)$ while having some knowledge about the degradation function $H$ and the noise

$\eta(x,y)$.

# Model for image degradation/restoration process

- the degraded image in the **spatial domain** is

$$g(x,y) = \underbrace{h(x,y) \otimes}_{\text{convolution}} f(x,y) + \eta(x,y)$$

  - Therefore, in the **frequency domain** it is

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

# Noise models

- The Principal source of noise in digital images arise during **image Acquisition and transmission.**
- In Acquiring images with Camera and Light levels are major factor affecting the amount of noise in resulting image.
- Images are corrupted during transmission due to interferences in the channel used for transmission.for example:image is transmitted using a wireless network might be corrupted due to atmospheric disturbances.

# *Degradation models : noise only*

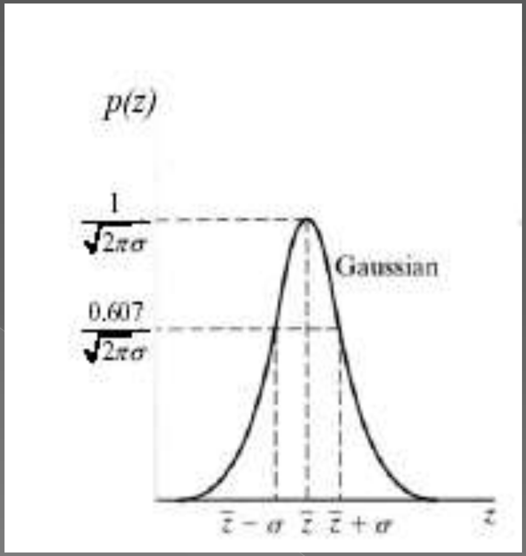
$$g(x,y) = f(x,y) + \eta(x,y)$$
$$G(u,v) = F(u,v) + N(u,v)$$

- Some important noise probability density functions
- – Gaussian noise
- – Rayleigh noise
- -Erlang gamma noise
- – Exponential noise
- – Impulse

# Gaussian Noise

- The PDF of a Gaussian random variable z given by:

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}}$$




*z* represents intensity,

*ž* is the mean (average) value of *z* ,

$\sigma$ is its standard deviation.

$\sigma^2$ is the variance of *z*.

# Source of Gaussian Noise

- The Gaussian noise arises in an image due to factors such as **electronic circuit noise** and **sensor noise** due to poor illumination. The images acquired by image **scanners** exhibit this phenomenon.

# Rayleigh noise

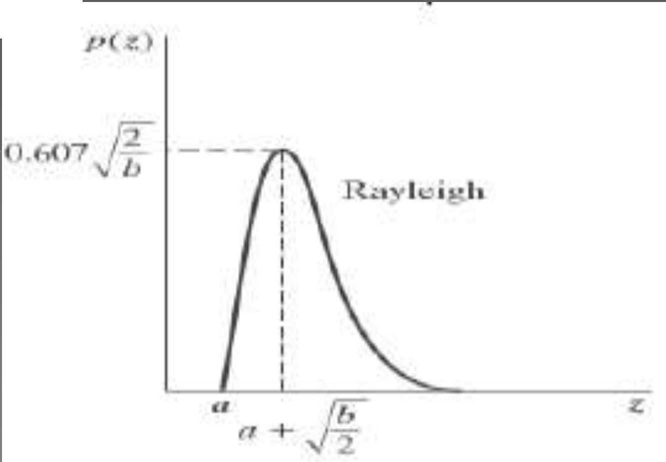
- **Rayleigh** noise is specified as

$$p(z)=\begin{cases}\frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & z\ge a\\ 0 & z<a\end{cases}$$

The mean and variance are given by

$$\bar{z}=a+\sqrt{\pi b/4}$$

$$\sigma^2=\frac{b(4-\pi)}{4}$$

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# Rayleigh noise

- Rayleigh noise PDF is helpful in characterizing noise phenomena in range imaging. (e.g-X-ray,ultraviolet imaging which depend upon the frequency of light )

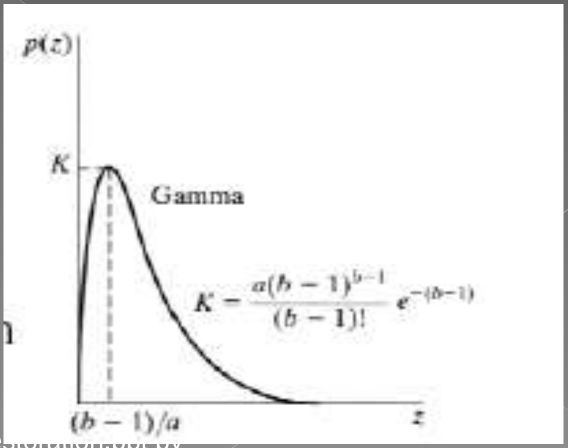
# Erlang noise(Gamma noise)

- **Erlang** noise is specified as

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

**Here *a* > 0 and *b* is a positive integer. The mean and variance are given by**

$$\bar{z} = b/a$$
$$\sigma^2 = b/a^2$$



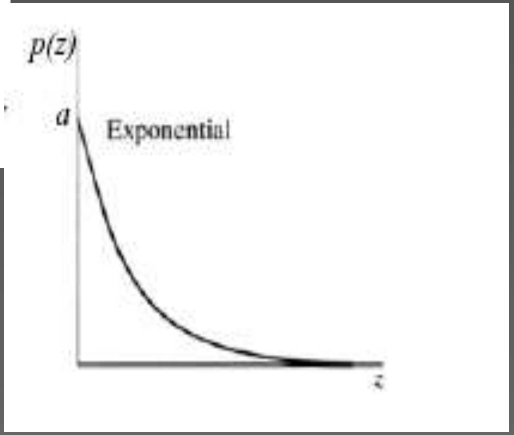
# Erlang noise(Gamma noise)

- Gamma noise finds in laser imaging.

# Exponential noise

**Exponential** noise is specified as

$$p(z) = \begin{cases} ae^{-az} & z \ge 0 \\ 0 & z < 0 \end{cases}$$




Here *a* > 0. The mean and variance are given by

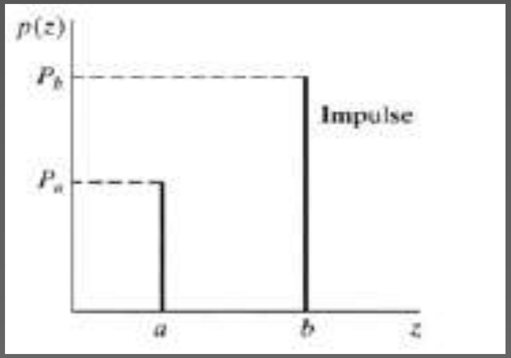
$$\bar{z} = 1/a$$
$$\sigma^2 = 1/a^2$$

Exponential pdf is a special case of Erlang pdf with *b* =1.Used in laser imaging.

# Impulse (salt-and-pepper)

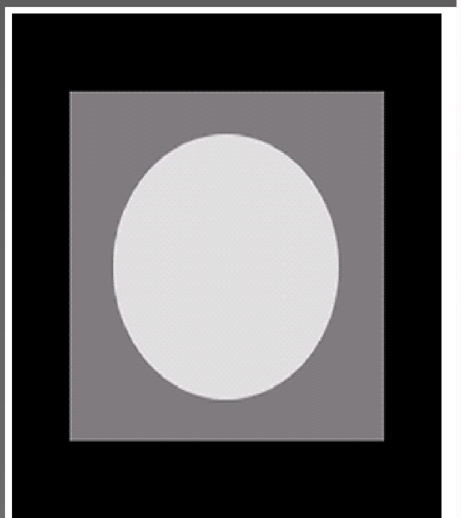
- **Impulse (salt-and-pepper)** noise (bipolar) is specified as

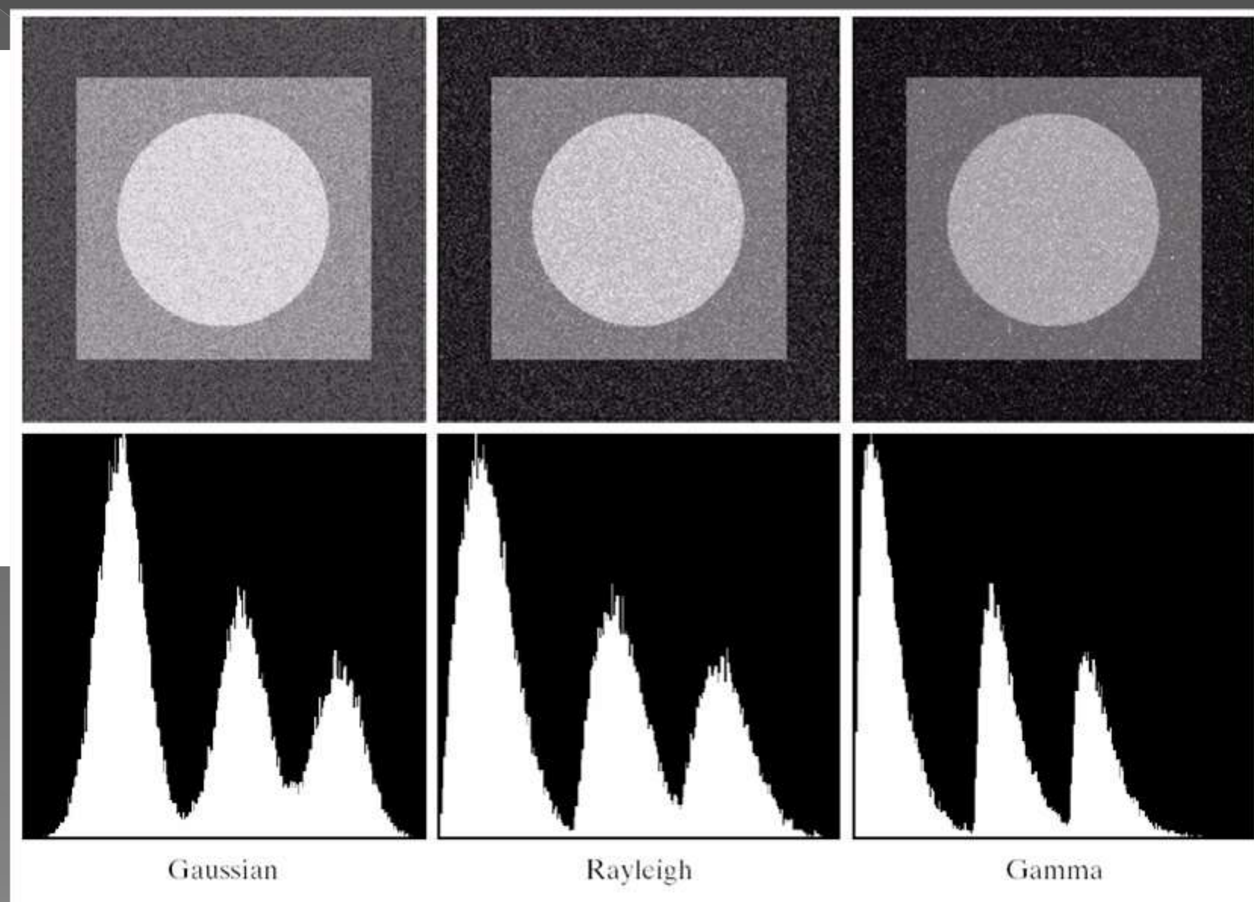
$$p(z) = \begin{cases} P_a & z = a \\ P_b & z = b \\ 0 & otherwise \end{cases}$$




If *b*>*a*, intensity *b* will appear as a light dot on the image and *a* appears as a dark dot If either *Pa* or *Pb* is zero the noise is called *unipolar*.If neither probability is zero, and especially if they are approximatly equal, impulse noise value will resemble salt and peeper granules randomely distributed over the image.for this reason , bipolar impulse noise is called salt and peeper noise.

# Noise models – examples



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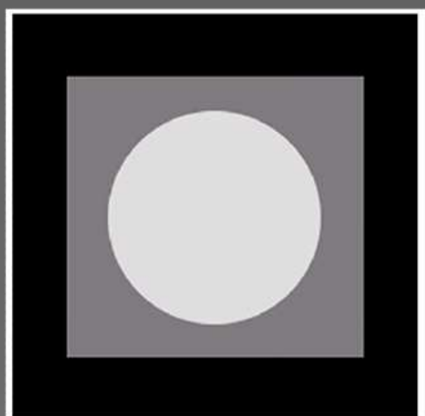
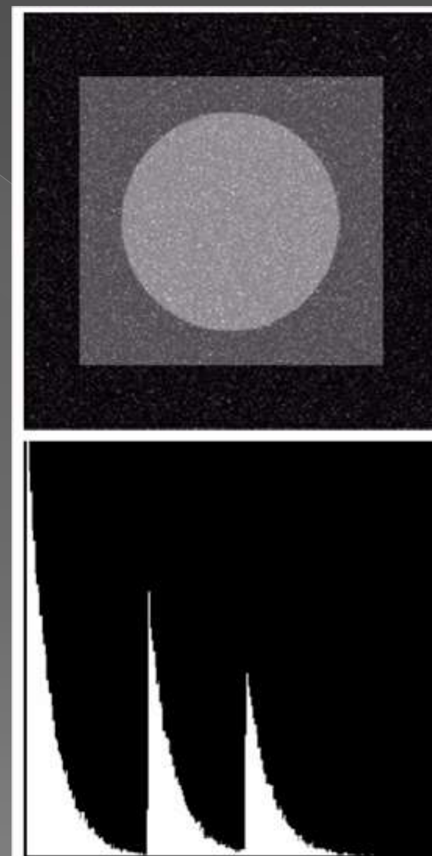
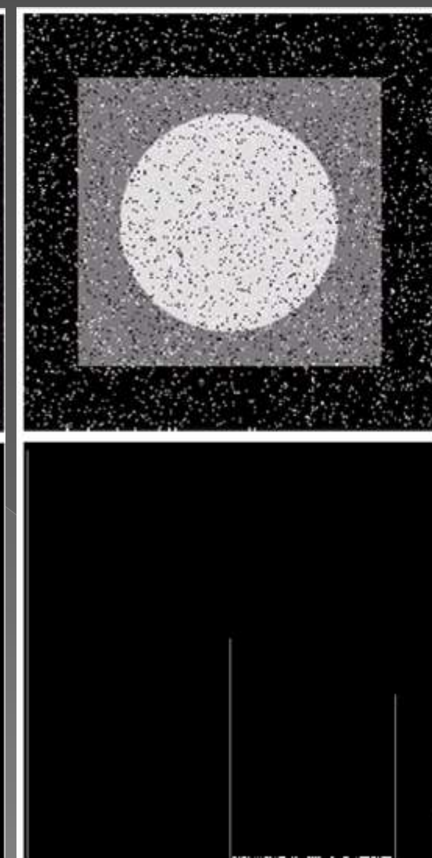




Exponential



Salt & Pepper

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# Noise Removal Restoration Method in spatial domain

- Mean filters
  - Arithmetic mean filter
  - Geometric mean filter
  - Harmonic mean filter
  - Contra-harmonic mean filter
- Order statistics filters
  - Median filter
  - Max and min filters
  - Mid-point filter
  - alpha-trimmed filters
- Adaptive filters
  - Adaptive local noise reduction filter
  - Adaptive median filter

# Mean filters

- **Arithmetic mean filter**:

$$\hat{f}(x,y)=\frac{1}{mn}\sum_{(s,t)\in S_{xy}} g(s,t)$$

The arithmetic mean filter computed the average value of the corrupted image *g*(*x*,*y*) in the area defined by *Sxy*. Let *Sxy* represent the set of coordinates in a rectangular neighborhood of size *m* x *n*, centered at the point (*x*,*y*).

**Effect:The Mean filter simply smoothes the variations in an image.noise is reduced as a result of blurring**

# Geometric mean filter:

$$\hat{f}(x,y) = \left[ \prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

**Effect:**

Geometric mean filter achieves smoothing comparable to the arithmetic mean filter but it preserves more details (It means loss less image detail in the process)

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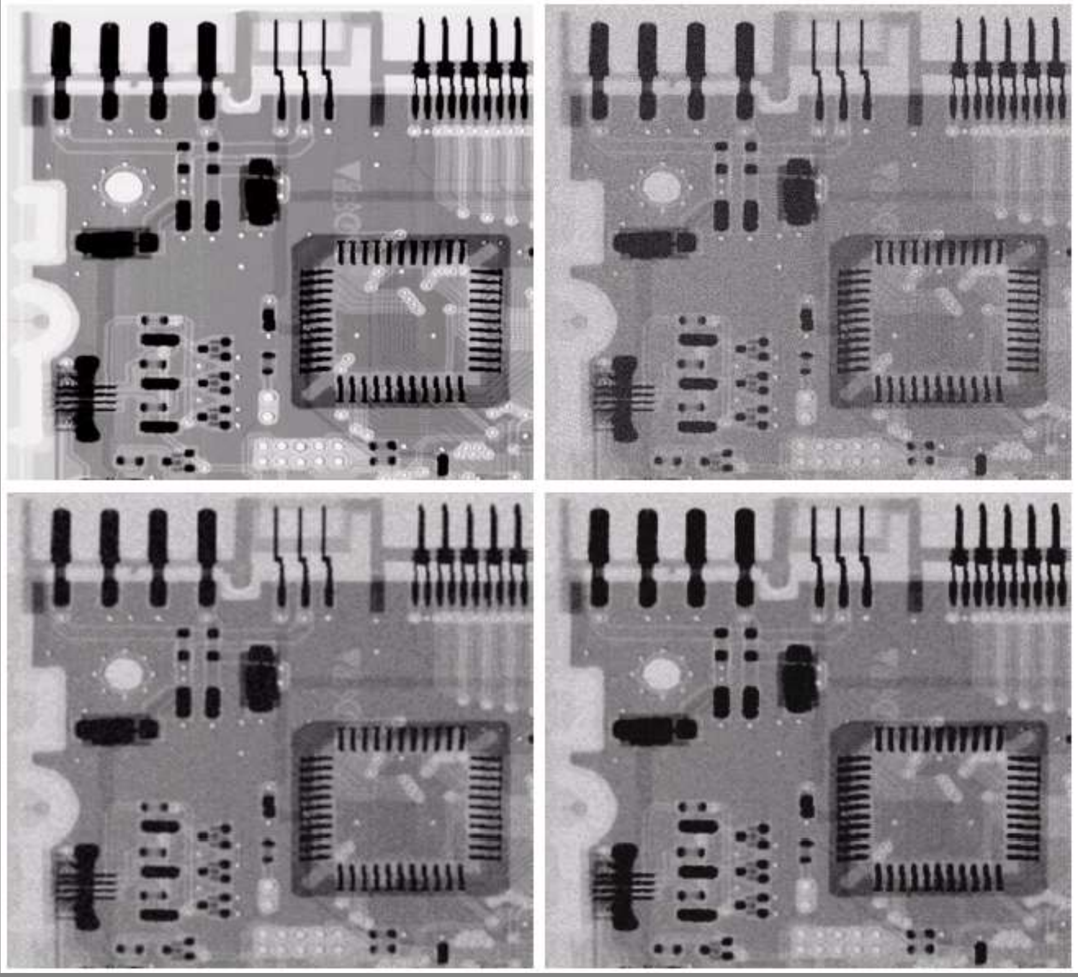


a b
c d

**FIGURE 5.7** (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

# Harmonic mean filter:

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

**Effect:**

Harmonic mean filter works well for salt noise and other types of noise (such as Gaussian) but fails for pepper noise.

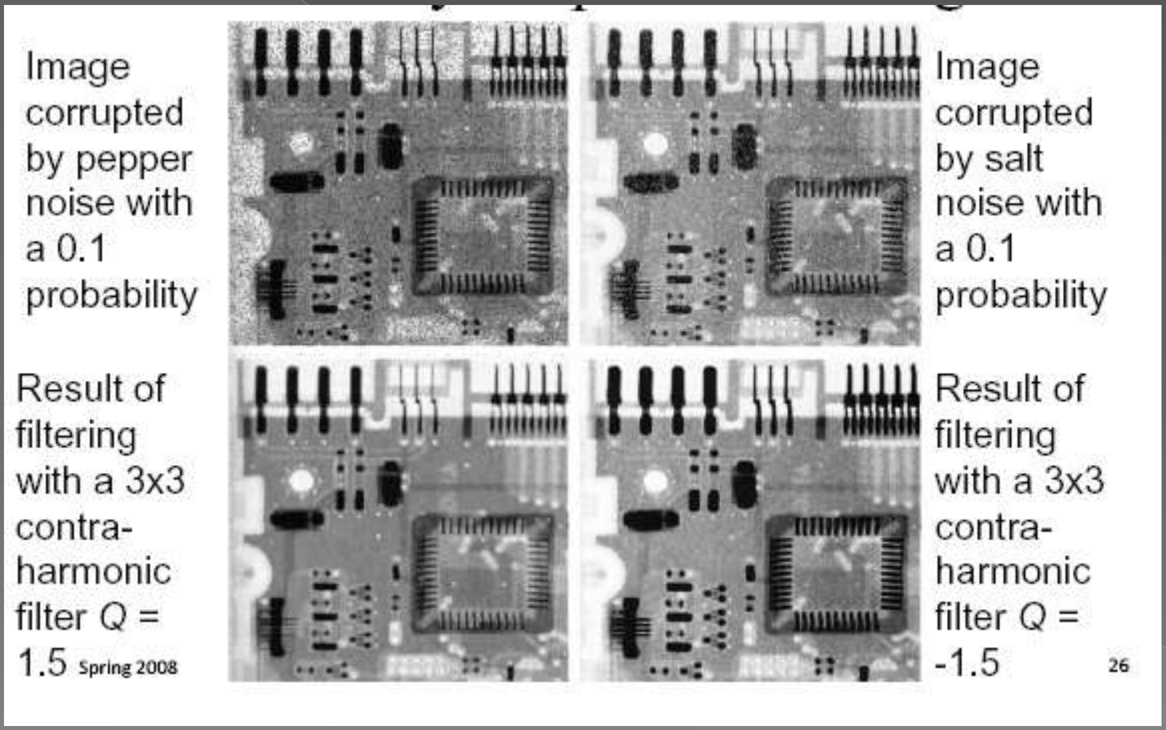
# Contraharmonic mean filter:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Here $Q$ is the **order** of the filter. This filter is well suited for reducing the effects of salt-pepper noise. For positive values of $Q$, eliminates pepper noise; for negative values of $Q$, it eliminates salt noise. This filter cannot reduce both simultaneously.

Notice that contraharmonic filter reduces to the arithmetic mean filter when $Q = 0$ and to the harmonic mean filter if $Q = -1$.

# Contraharmonic mean filter:

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# Order-statistic filters

- **Median filter**:It replaces the pixel value by the median of the intensity levels in the neighborhood of that pixel:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

- **Effect:**
- Median filters provide excellent results for certain types of noise with considerably less blurring than linear smoothing filters of the same size. These filters are very effective against both bipolar and unipolar noise. The same filter can be
- applied more than once to yield better results.

# Median Filter

Effective for removing salt-and-paper (impulsive) noise.

$$\hat{f}(x,y) = \underset{(s,t)\in S_{x,y}}{median}\left\{g(s,t)\right\}$$

a b
c d

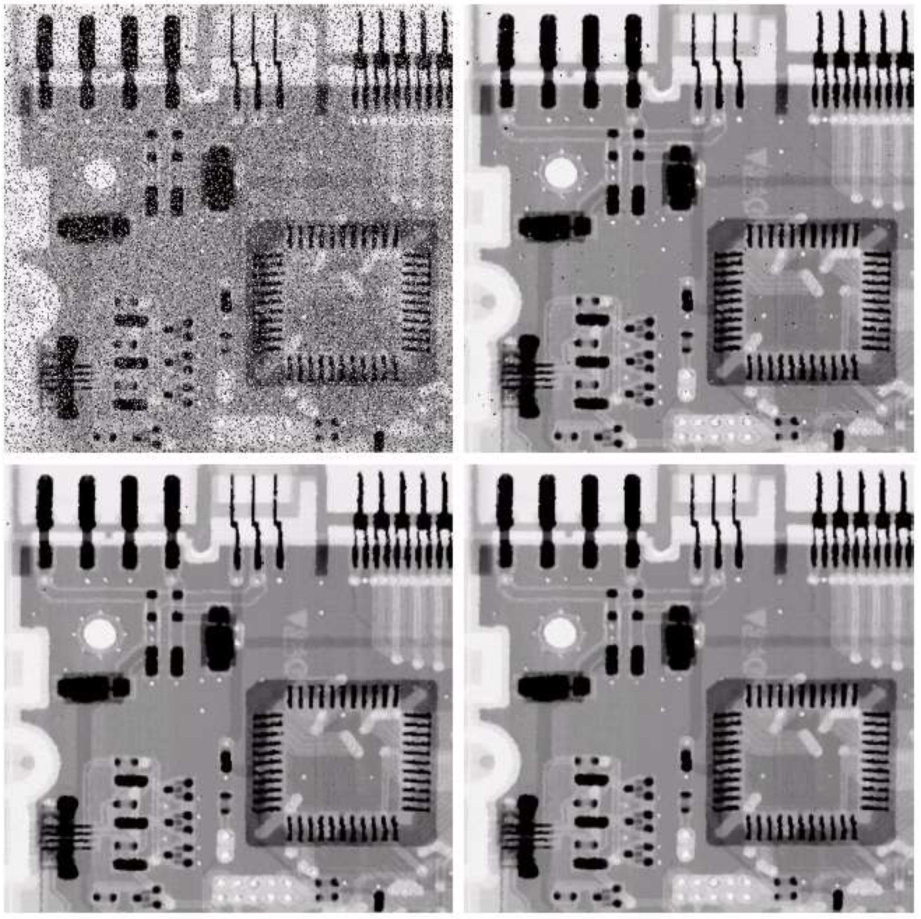

**FIGURE 5.10** (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size $3 \times 3$. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

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# Max and Min Filters

- **Max filters**:

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

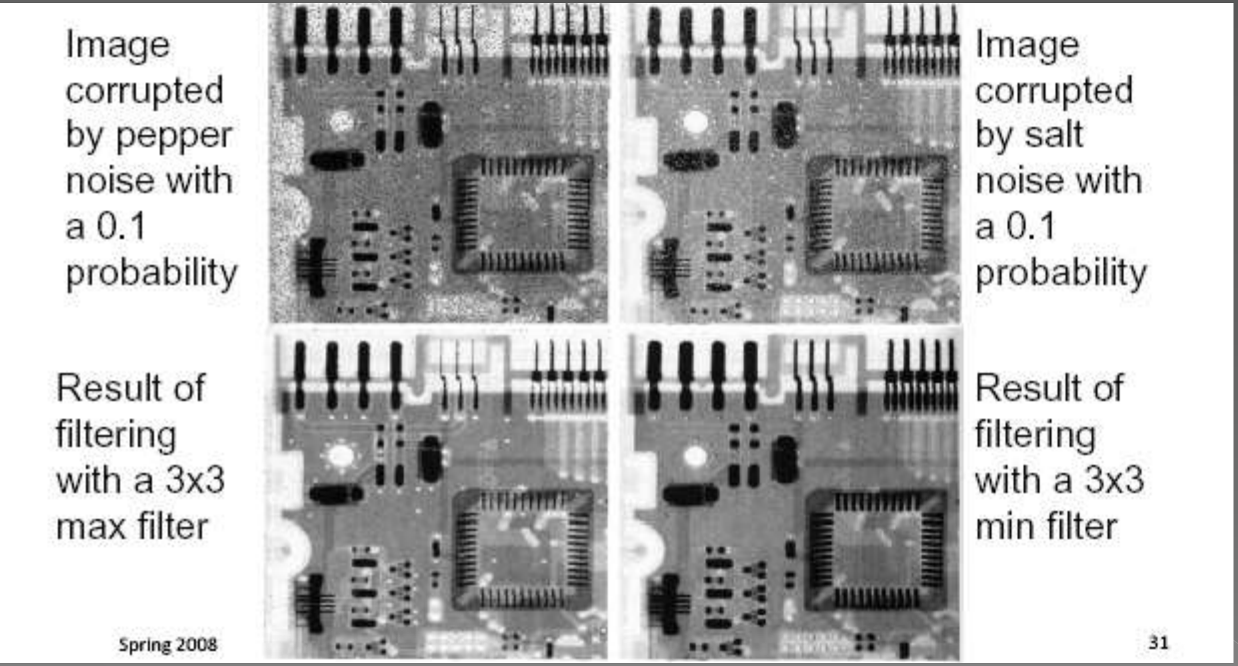
This filter is useful for finding the brightest points in an image; therefore, its effective against pepper noise.

**Min filters:**

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

This filter is useful for finding the darkest points in an image;therefore, its effective against salt noise.( it reduces the salt noise because it will eliminate the higher gray values in the subimage.

# Max and Min Filters

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# Midpoint filter:

- It computes the midpoint between the maximum and minimum values of intensities:

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

This filter is a combination of order statistics and averaging and works best for Gaussian and uniform noise contaminations.

# Alpha-trimmed mean filter:

- if we delete *d*/2 highest intensity values and *d*/2 lowest intensity values of g(s,t) in the neighbourhood $s_{xy}$., denote the rest as $g_r$(s.t), a filter that averages what is left is alpha-trimmed mean filter:

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

*d* can range from 0 to *mn*-1. When *d* = 0, this filter reduces to the arithmetic mean filter, when *d* = *mn*-1, this filter reduces to a median filter. For other values of *d*, the filter is useful in situation  with noise of multiple types, such as a combination of salt-and-pepper and Gaussian noise.

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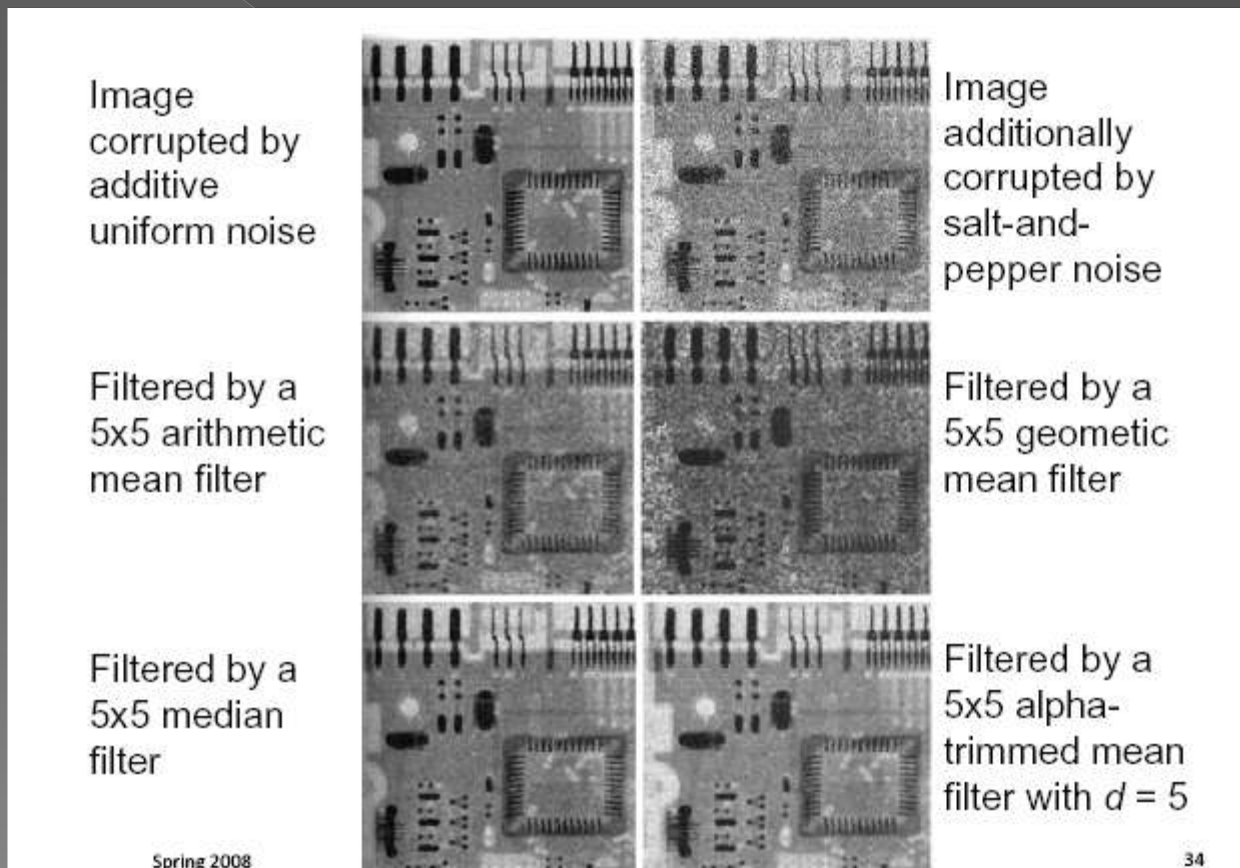


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# Adaptive filters

- Adaptive filters are those filters whose behavior changes based on the statistical characteristics of the image inside the filter region defined by a rectangular window size $S_{xy}$.
- It is better than the mean filter and order statistics filter.
- **Two types of filter**
  - Adaptive local noise reduction filter
  - Adaptive median filter

# Adaptive local noise reduction filter

- It uses two statistical parameters, mean and variance for the elimination of noise.
- **Mean Parameter:** It gives the average gray value.
- **Variance**: It provides the estimate of the contrast in the image.
- **the adaptive filter is:**

$$\hat{f}(x,y) = g(s,t) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(s,t) - m_L\right]$$

- The response of filter is based on four quantities:-
- G(x,y) the value of noisy image at (x,y).
- $\sigma_\eta^2$ the variance of noise corrupting f(x,y) to form g(x,y).
- $m_L$ local mean of pixels in the $S_{xy}$
- $\sigma_L^2$ The local variance of the pixels in $S_{xy}$

$$\hat{f}(x,y) = g(s,t) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(s,t) - m_L\right]$$

- The behavior of the **Adaptive filter is obtained as:**
- 1. If $\sigma_\eta^2$ is zero, the filter returns the value of $g(x,y)$;
  2. If $\sigma_L^2 > \sigma_\eta^2$ (typical for edges that needs to be preserved) the filter returns the value close to $g(x,y)$;
  3. If two variances are equal, the filter returns the arithmetic mean value for pixels in the region $S_{xy}$.

$$\hat{f}(x,y) = g(s,t) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(s,t) - m_L\right]$$

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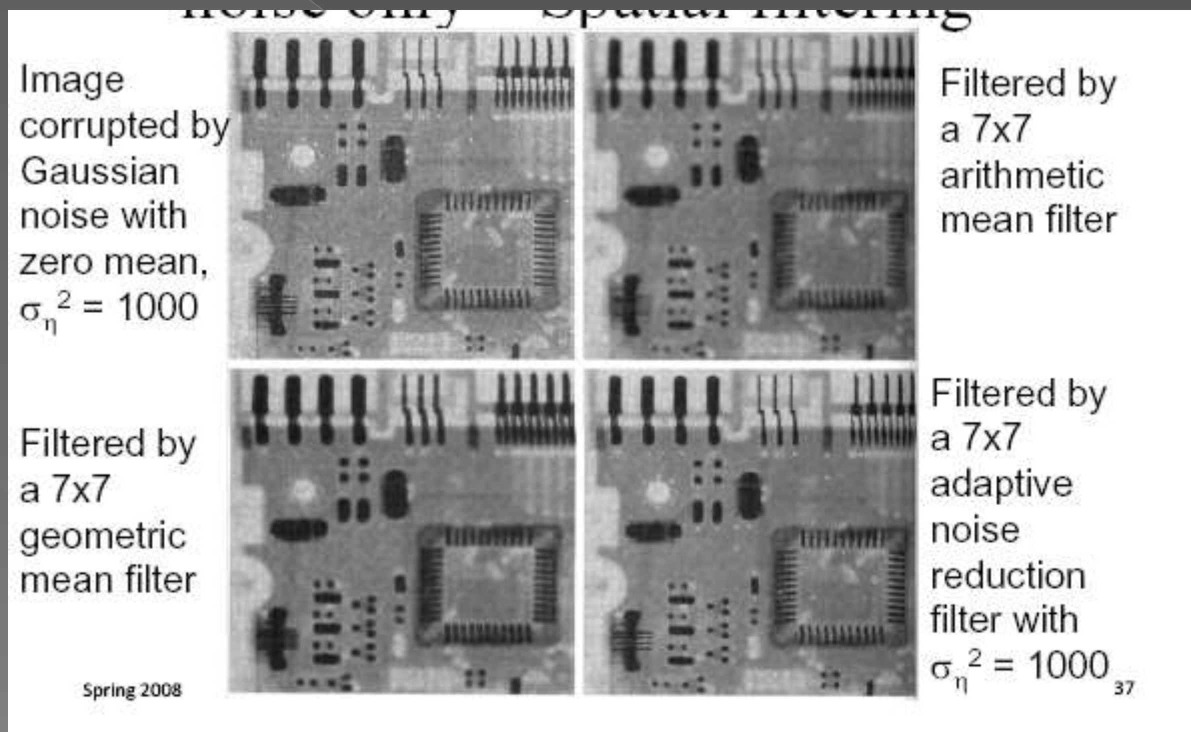


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# Effect

- Adaptive filter achieves approximately the same performance in noise cancellation but adds much less blurring than the mean filters.
- Adaptive filtering typically yields considerably better results in overall performance at the price of filter complexity.

# Adaptive median filter

- It can handle impulse noise with larger probabilities than traditional median filter. It operates on a rectangular region $S_{xy}$, whose ***size is changing***. Window size is variable to improve efficiency
- Adaptive median filter has 3 goals:
- to remove impulse noise,
- To provide smoothing
- to reduce distortion

# Implementation

- Three cases were implemented:
- With Salt and Pepper noise alone
- With non impulsive noise alone
- With both included
- Variations in the window size were introduced

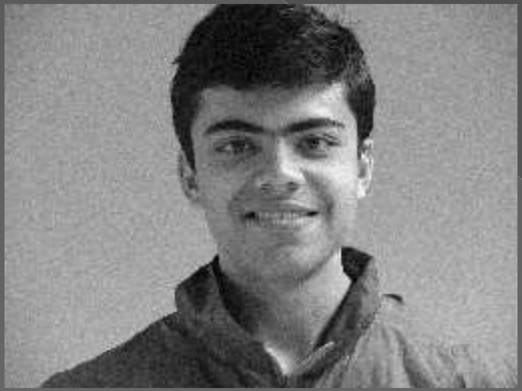
# Results

**Salt and Pepper**

**Standard Median output**

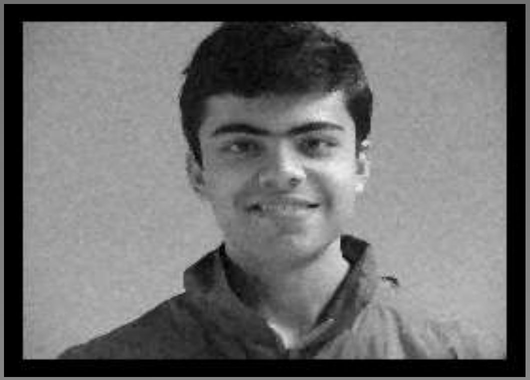
Adaptive median Output

# With Non-impulsive noise



Gaussian Noise

Standard Median output



Adaptive Median Output

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# With both types of noise

Gaussian and impulsive Noise

Standard Median output

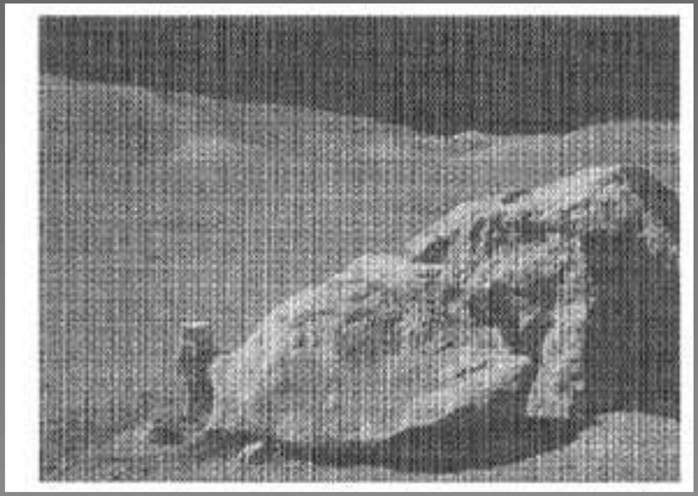
Adaptive Median output

# Effect:

- **The adaptive median filter successfully removes impulsive noise from images. It does a reasonably good job of smoothening images that contain non-impulsive noise.**
- **When both types of noise are present, the algorithm is not as successful in removing impulsive noise and its performance deteriorates.**

# Periodic noise

- This noise typically comes from electrical or electromechanical interference during image acquisition.
- It can be reduced via frequency domain filtering.
- The image is corrupted by sinusoidal noise of various frequencies.
- The parameters of periodic noise are estimated by inspection of Fourier spectrum of the image.



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# Periodic noise reduction filters.

- Bandreject filters
- Bandpass filters
- Notch aFilters

# Bandreject filters

- Bandreject filters remove or attenuate frequencies about the origin of the Fourier transform.
- Ideal Bandreject Filter:

$$H(u,v)=\begin{cases}1 & \text{if } D(u,v)<D_0-W/2\\ 0 & \text{if } D_0-W/2\le D(u,v)\le D_0+W/2\\ 1 & \text{if } D(u,v)>D_0+W/2\end{cases}$$

Where

$D(u,v)$ :distance from the origin of the centered freq.

$D$o :Radial centre

$W$- width of the band

# Butterworth bandreject filter

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