

EEE 187: Robotics

Summary 3: Legged Locomotion

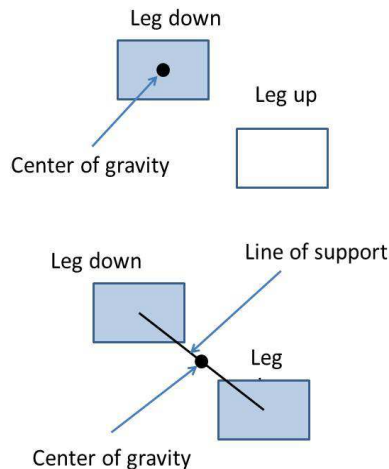


Fig. 1. Illustration of static walking

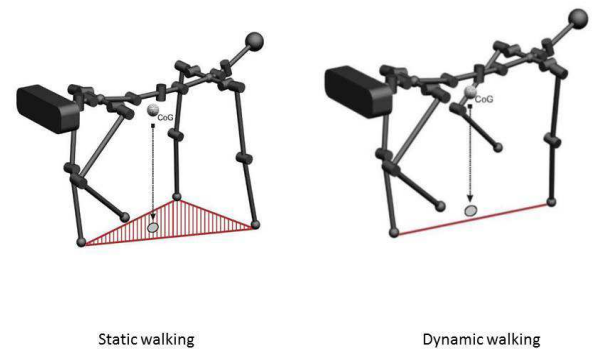


Fig. 2. Comparison between static and dynamic walking

LEGGED LOCOMOTION

Legged robots are inspired by biological systems (animals, insects). Compared with wheeled locomotion, legged locomotion is more complex and more difficult to control. Legged locomotion needs to be stable. Two types of stability are discussed in the literature:

- Static stability: a statically stable robot can stand without falling over. The key is to have enough legs to provide support. Humans are not statically stable, standing up appears to be effortless, but we are actively using active control to achieve balance. For us, balancing is largely unconscious and must be learned.
- Dynamic stability: dynamic stability allows a robot (or animal) to be stable while moving. A bicycle is a good example for dynamic stability. Another example is one-legged hopping robots, they can hop in place or move to other destinations, and not fall over, but they cannot stop and stay standing without falling over.

Condition for stability

The condition for static stability is very simple: the robot's center of gravity must fall under the robot polygon of support. For a two legged robot, the polygon of support is a straight line as shown in figure 1. Figure 2 shows a comparison between static and dynamic walking.

How many degrees of freedom are needed to move one leg?

At least two: one to lift the leg and the other one to move it forward. The following terminology is used

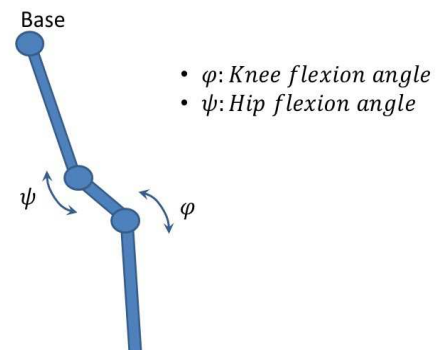


Fig. 3. Knee and hip flexion angles

- φ : knee flexion angle.
- ψ : hip flexion angle.

Figure 3 illustrates knee and hip flexion angles.

Definitions

Gait: manner of walking or running

Gait: distinct sequence of lift and release events of the individual legs. The gait depends on the number of legs.

What is the number of distinct events N for a walking robot machine with k legs? A distinct event sequence is a change

from one state to another and back

$$N = (2k - 1)! \quad (1)$$

For a biped ($k = 2$), the number of distinct event sequences (changes of state) is

$$N = (4 - 1)! = 6 \quad (2)$$

For a hexapod (six legged robot):

$$N = 11! = 39916800 \quad (3)$$

It is possible to determine the sequence for biped robots

- leg down: LD
- leg up: LU

With two legs (biped), we have four different states:

- {1}: both legs down (LD, LD)
- {2}: left leg up, right leg down (LU, LD)
- {3}: left leg down, right leg up (LD, LU)
- {4}: both legs up (LU, LU)

The six distinct event sequences for a biped robot are as follows

- **turning on right leg:**

$$\{1\} \Rightarrow \{2\} \Rightarrow \{1\} \\ (LD, LD) \Rightarrow (LU, LD) \Rightarrow (LD, LD)$$

- **turning on left leg:**

$$\{1\} \Rightarrow \{3\} \Rightarrow \{1\} \\ (LD, LD) \Rightarrow (LD, LU) \Rightarrow (LD, LD)$$

- **hopping with two legs:**

$$\{1\} \Rightarrow \{4\} \Rightarrow \{1\} \\ (LD, LD) \Rightarrow (LU, LU) \Rightarrow (LD, LD)$$

- **walking/running:**

$$\{2\} \Rightarrow \{3\} \Rightarrow \{2\} \\ (LU, LD) \Rightarrow (LD, LU) \Rightarrow (LU, LD)$$

- **hopping on right leg:**

$$\{2\} \Rightarrow \{4\} \Rightarrow \{2\} \\ (LU, LD) \Rightarrow (LU, LU) \Rightarrow (LU, LD)$$

- **hopping on left leg:**

$$\{3\} \Rightarrow \{4\} \Rightarrow \{3\} \\ (LD, LU) \Rightarrow (LU, LU) \Rightarrow (LD, LU)$$

WALKING PHASES

Walking is moving by putting forward each foot in turn, not having both feet off the ground at once. In running, there is a flight phase where the robot is not touching the ground. The swing leg is the leg performing the step. The stance foot is the foot that supports the weight of the robot. Human walking is highly energy efficient. Walking is cyclic, it is a periodic process. It walking gait consists of four different phases 4:

- Double support phase (DS): both feet are on the ground

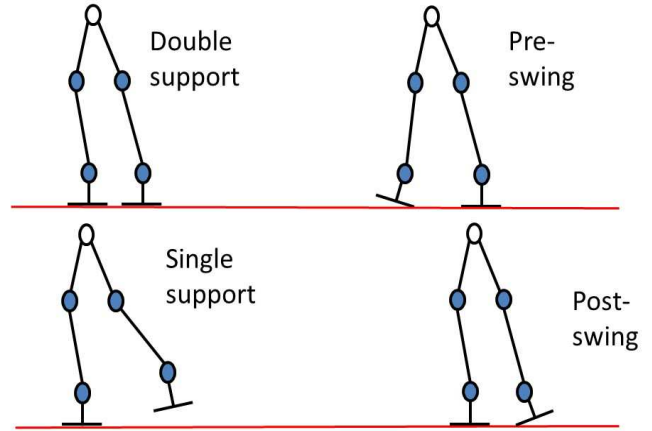


Fig. 4. Walking phases

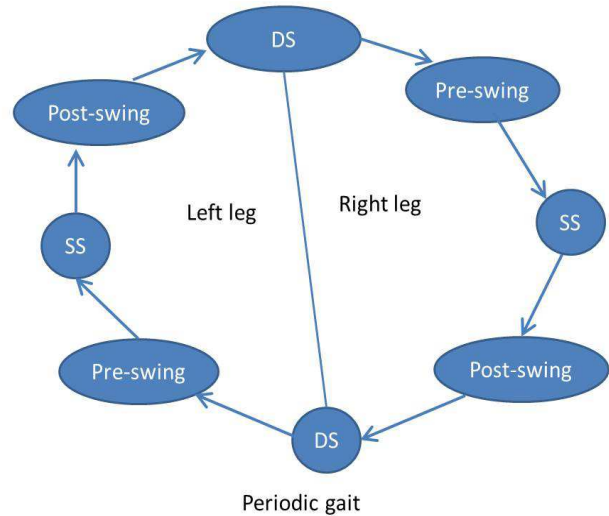


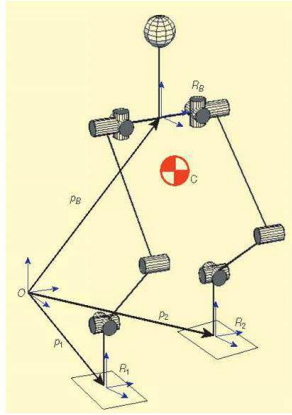
Fig. 5. Gait phases for a biped

- Pre-swing phase: The heel of the rear foot is lifting. The biped is still in double support.
- Single support phase (SS): One foot only is on the ground
- Post-swing phase: The toe of the front foot is declining towards the floor; the biped is in double support because the heel of the front foot is touching the ground.

The single support phase is the key phase in the walking process. Most walking models focus on this phase. Biped robots usually have 12 DOF and three complex joints per leg as shown in figure 6. Static walking is very slow with small steps, therefore we can ignore the dynamic forces. In static walking, the robots center of gravity is inside polygon of support.

WALKING BASED ON TRAJECTORY

The single support phase is the key phase in the walking process. Most walking models focus on this phase. The walking pattern of the biped walking robot is generated using



The most common number of degrees of freedom for walking robot is 12 DOF:

- Hip: 3 DOF
- Knee: 1 DOF
- Ankle: 2 DOF

Asimo from Honda has 26 DOF

Fig. 6. A biped robot with 12 DOF

trajectories that need to satisfy certain constraints to achieve stability. There exist several methods that can be used to achieve dynamic stability, including:

- Trial and error
- Observing human walking pattern
- Inverted pendulum model
- Zero moment point

The Zero moment point (ZMP) (Vukobratovic 1990) is among the most successful methods.

Zero Moment Point and the cart-table model

Zero Moment Point (ZMP) is the point on the ground about which the net moment of the total forces applied to the biped is zero. The zero moment point can be used to prove the dynamic stability of the robot. The gait is stable if the ZMP remains within the foot-print polygons. The cart table model shown in figure 7 is used to model the single support phase where a) shows the walking robot and b) shows its simplified model. This model is used here to explain the ZMP. The model can be summarized as follows:

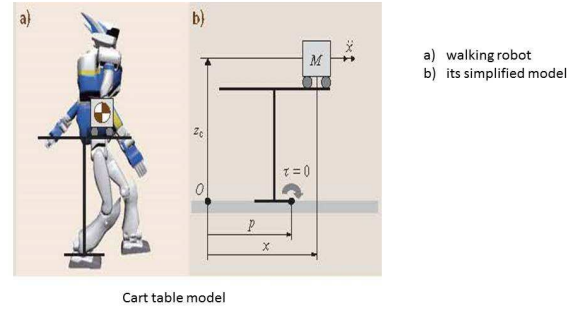
- The simplified model consists of a running cart on a massless table.
- The cart has mass M and its position is (x, z_c) corresponds to the center of mass of the robot.
- The table is assumed to have the same support polygon as the robot.

During the single support phase, the distributed floor reaction can be replaced by a single force acting at the zero moment point, which is equivalent to the Center of Pressure. Using the zero moment condition, the ZMP of the cart table model is

$$P_x = x - \frac{z_c}{g} \ddot{x} \quad (4)$$

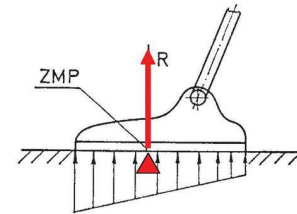
$$P_y = y - \frac{z_c}{g} \ddot{y} \quad (5)$$

where P_x, P_y represent the coordinates of the ZMP.



- Simplified model consists of a running cart on a massless table.
- The cart has mass M and its position is (x, z_c) corresponds to the center of mass of the robot.
- The table is assumed to have the same support polygon as the robot.

Fig. 7. The cart-table model



The distributed floor reaction can be replaced by a single force R acting at the zero moment point.

ZMP=center of pressure

Fig. 8. Distributed floor reaction

Algorithm

Generating the biped walking pattern using ZMP consists of three phases:

- Calculate the ZMP
- Generate trajectories for the feet such that locomotion is stable according to ZMP
- The motion of the joints is determined using the inverse kinematics from the body and the feet trajectories. Example: use of hip actuators.

Figure 9 illustrates the motion generation process.

MULTI-LEGGED ROBOTS

Compared with bipeds, multi-legged robots have a wider choice of foot placing to maintain static balance. From this reason, many research works have concentrated on gait planning for statically stable walking rather than treating dynamic stability. For the six legged robot shown in figure 10, the support polygon is a triangle, which corresponds to stable static walking (static stability requires that at any time, three legs are on the ground). For quadruped robots to walk keeping static stability (figure 11), the robot must lift and place only one leg at each step. This is called creeping gait. In this case,

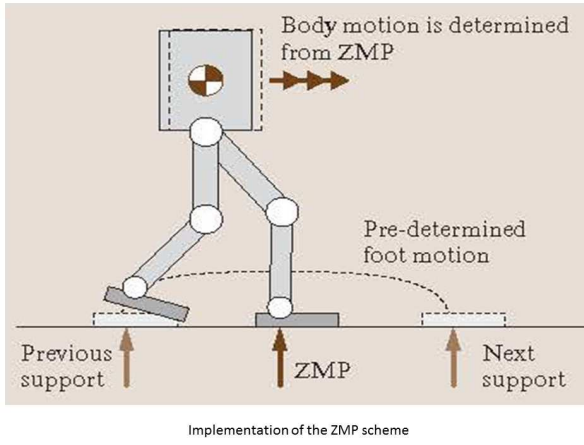


Fig. 9. Implementation of the ZMP

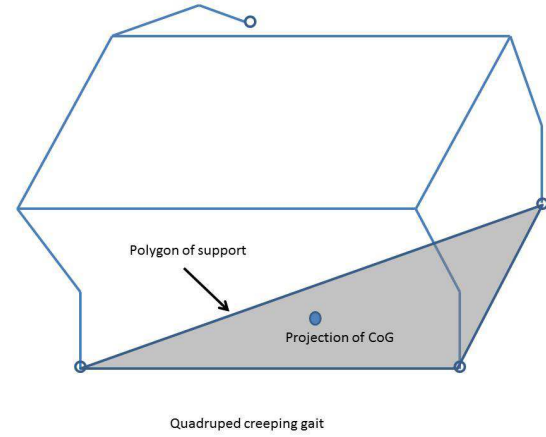


Fig. 11. The creeping gait

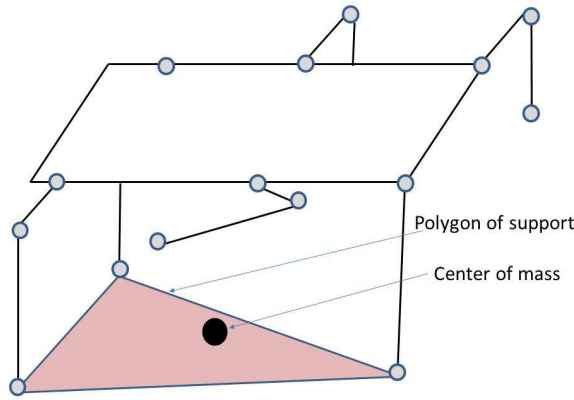


Fig. 10. A six legged robot

there are 6 gaits as illustrated in figure 12. We begin with leg 1 as the first swing leg.

Gait Diagram

The gait diagram is a way to describe the gait sequence of a multi-legged robot. The x-axis corresponds to the time and the y-axis corresponds to the leg number. Three important variables are defined:

- Walk cycle time is T .
- Duty factor of leg i : β_i
- Phase of leg i : ϕ_i

The duty factor and the phase are defined below:

$$\beta_i = \frac{\text{support period of leg } i}{T} \quad (6)$$

and

$$\phi_i = \frac{\text{touchdown time of leg } i}{T} \quad (7)$$

Wave gate

The wave gate is a particular gait that is statically stable. The wave gait for a six legged robot is shown in figure 13. According to McGhee and Frank, 1968, the wave gait presents

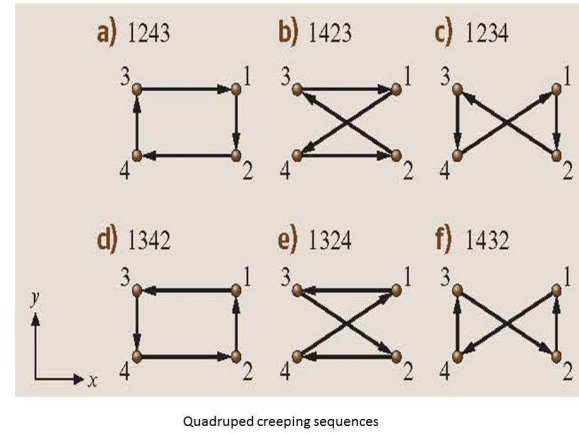


Fig. 12. The six sequences of the creeping gait

maximum longitudinal stability. It is also slow compared to other gaits.

Four legged robots

- Static walking: The wave gait shown in figure 11 represents static walking for a four legged robot. The robot lifts and places one leg at each step. This is called creeping gait. There 6 sequences in this case as shown in figure 12 where leg 1 is the swing leg.
- Dynamic walking: Figure 15 shows some basic running gaits (trot, pace and bound gaits) with their gait diagrams. Clearly, these represent dynamic walking gaits for a four legged robot.

Stability margin

For bipeds, it is difficult to calculate stability margins; it is easier for static stability. Figure 16 shows two definitions of static stability for a triangular polygon of support

- Stability margin: let d_1, d_2, d_3 be the distance from the center of gravity to the triangle of support edges. The

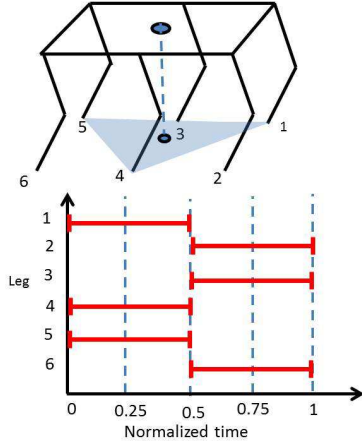
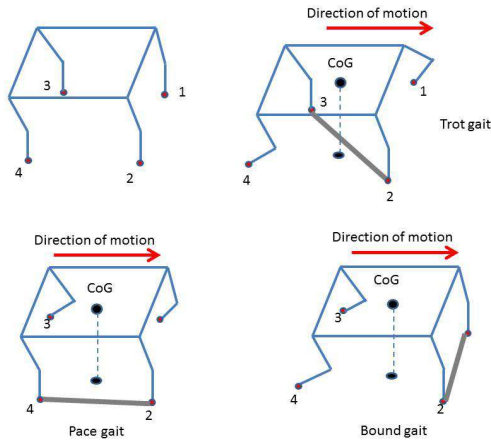
Fig. 13. Wave gait for a six legged robot with $\beta = 0.5$ 

Fig. 14. Trot, pace and bound gaits. Note that for trot gait, diagonal legs move together

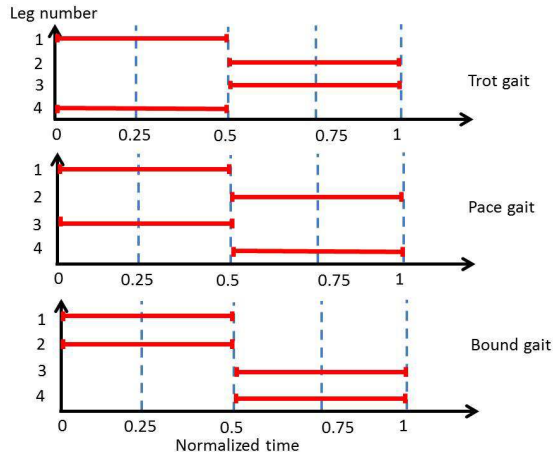


Fig. 15. Gait diagram for the trot, pace and bound gaits

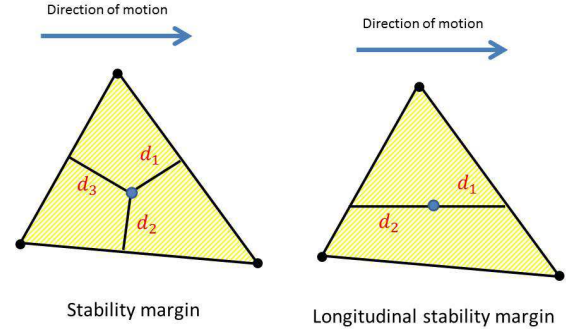


Fig. 16. Two definitions for stability margins

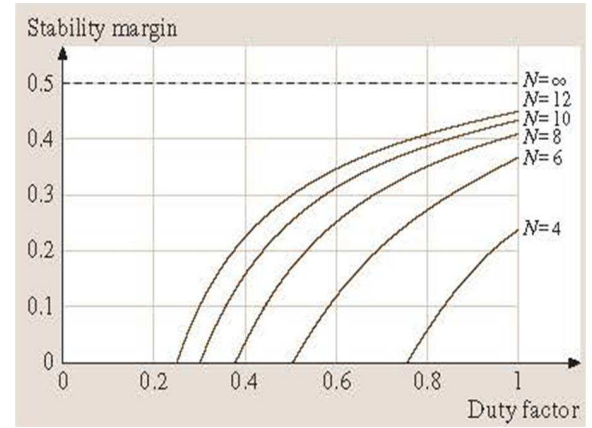


Fig. 17. Duty factor and stability margin

stability margin is defined as the minimum distance from the center of gravity to the edges:

$$S_m = \min \{d_1, d_2, d_3\} \quad (8)$$

- Longitude stability margin: the stability margin is defined as the minimum distance along the direction of motion:

$$S_l = \min \{d_1, d_2\} \quad (9)$$

Performance criteria

- Duty factors: The duty factor is related to stability. Figure 17 shows the duty factor as function of the stability margin with the number of legs as the parameter.

- Stability improves when the duty factor increases
- Stability improves with the number of legs, but the most important “jump is between $N = 4$ and $N = 6$.

Duty factors can be used to make the distinction between walks and runs. For example, for a biped, $\beta < 0.5$ for running.

- Froude number:

$$Fr = \frac{v^2}{gh} \quad (10)$$

- v is the walking or running speed,
- g is the acceleration due to gravity,
- h is the height of hip joint from the ground. In particular, most animals change their gait from walking to running at a speed equivalent to a Froude number of $F_r = 1$.
- Specific resistance: a dimensionless number that is used to evaluate the energy efficiency of a mobile robot

$$\varepsilon = \frac{E}{Mgd} \quad (11)$$

- E is the total energy consumption for a travel of distance d ,
- M is the total mass of the vehicle,
- g is the acceleration due to gravity

The specific resistance is an indicator of the smoothness of the locomotion.