18 In the 17th century, Kepler proposed his 'law of harmonies' for planetary motion. This law suggested that the ratio of the square of the orbital period T to the cube of the mean radius R has the same value for all the planets that orbit the Sun.

Mathematically his 'law of harmonies' can be written

$$T^2 = KR^3$$

where K is a constant.

(a) The table shows data for Earth and Mars.

Planet	***T*/s**	***R*/m**
Earth	3.16×10^7	1.50×10^{11}
Mars	5.93×10^7	2.28×10^{11}

Show that, for Earth and for Mars, K has a value of about $3.0 \times\times 10^{-19}\,s^2\,m^{-3}$.

(3)

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(b) Kepler's law of harmonies was derived later by Newton. Newton applied his law of gravitation to a planet moving in an approximately circular orbit around the Sun.

Determine a value for K by applying Newton's law of gravitation to a planet of mass m moving in a circular orbit about the Sun.

mass of Sun = 1.99×10^{30} kg

(3)

K = ... $s^2\,m^{-3}$

(c) Jupiter is the most massive planet in the solar system and has many moons. Kepler's law of harmonies applies to the orbiting moons. However, the value of K for the moons is not the same as the value that applies to planets orbiting the Sun.

Ganymede is Jupiter's largest moon. Ganymede has an orbital radius of 1.07×10^9 m and an orbital period of 172 hours.

Another moon, Io, has an orbital radius of 4.22×10^8 m.

Calculate the orbital period T_I of Io about Jupiter.

(2)

T_I = ...

(Total for Question 18 = 8 marks)