How to get logistic sigmoid function from logit function?

first of all, p is the probability that an event occurs.

$\frac{p}{1-p}$ represents the odds.

↳ Use for comparison ⇒ if $p = 0.75$ (75%), then $1-p = 0.25$.

so $\frac{p}{1-p} = \frac{0.75}{0.25} = 3$ ⇒ The event is 3 times more likely to occur than not occur.

it means 선형관계

So set $\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) = z$, ····> 실수값

$$\frac{p}{1-p} = e^z, \quad p = e^z(1-p)$$
$$= e^z - p \cdot e^z, \quad p + p e^z = e^z, \quad p(1+e^z) = e^z,$$

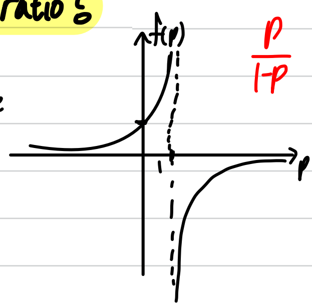
$$p = \frac{e^z}{1+e^z},$$ 우변의 분모와 분자를 e^z로 나누면

$$= \frac{1}{\frac{1}{e^z} + 1} = \boxed{\frac{1}{1+e^{-z}}}$$ ····> (0,1) 사이의 값

Standardization

cf) Why do we use logit instead of odds ratio?

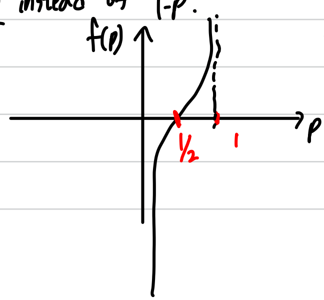
As you know, Odds ratio $= \frac{p}{1-p}$, and it is shaped like



$\frac{p}{1-p}$

We can't approach linearly because near $p=1$, It isn't differentiable.

So we use $\text{logit} = \ln\left(\frac{p}{1-p}\right)$ instead of $\frac{p}{1-p}$.



⇒ 중심점이 0(zero)라 해석상의 용이함도 있다함

⇒ differentiable. ⇒ Gradient 이용 가능

cf 2) Why do we transfer logit to sigmoid?

As you see above, logic function is focused on Real number $(-\infty, \infty)$
But what we interest in is P, probability of specific situation, which is $[0, 1)$
So we use inverse funcion of logit, which is sigmoid function,