Q2 - Endsem - CS6100 - TDAA

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Described algorithm is christofides and I refered different books while studying it for project (1.a)

a) $MST(G) \leq TSP(G)$ — ①

proof. If we remove one edge in TSP we get a spanning Tree which is always costlier thant MST

$\rightarrow MST(G) \leq ST(G) = TSP(G) - edge \leq TSP(G)$

$MM(G) \leq \frac{1}{2} TSP(G)$ — ②

proof. we can split TSP into 2 matchings by selecting alternative edges.

$\Rightarrow TSP(G) = M_1(G) + M_2(G)$

$\rightarrow$ W.R.T. $MM(G) \leq M_1(G), M_2(G)$

$\Rightarrow TSP(G) \geq 2\,MM(G)$

$\Rightarrow MM(G) \leq \frac{1}{2} TSP(G)$

$MM(O) \leq \frac{1}{2} TSP(G)$ — ③

proof. from ② we can say that $MM(O) \leq \frac{1}{2} TSP(O)$ and we can short cut part of $TSP(G)$ in O to get a hamiltonial cycle $HC(O)$.

$\rightarrow HC(O) \leq TSP(G)$ because of triangle inequality in shortcutting

$\Rightarrow MM(O) \leq \frac{1}{2} TSP(O) \leq \frac{1}{2} HC(O) \leq \frac{1}{2} TSP(G)$

2) $CHR(G) \leq MM(O) + MST(G)$ because shortcutting only decreases cost due to triangle inequality

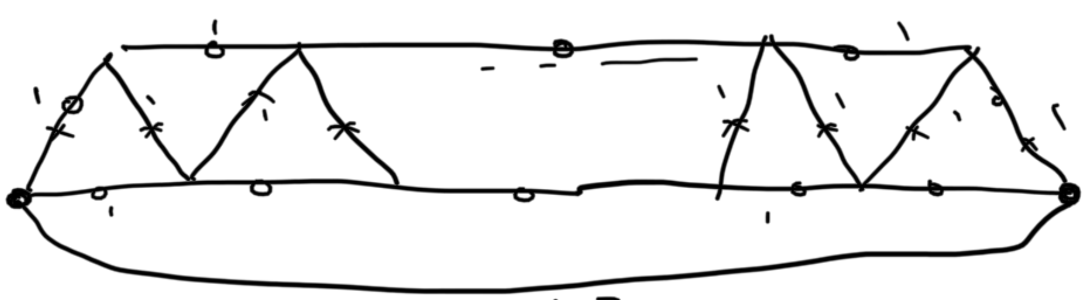
from ① & ③

$\Rightarrow CHR(G) \leq \frac{1}{2} TSP(G) + TSP(G)$

$CHR(G) \leq 1.5\, TSP(G)$

b) $CHR(G) \geq (1.5 - o(1))\, TSP(G)$

we can show there exists a G satisfying above condition by constructing a tight example of christofides for a 'n'



Assume for some large 'n' we construct graph like shown above

MST(G) : The edges crossed ('x')

and the odd degree vertices would be the left and right most point and joining them would be the MM(O).

here the cost is $(n-1) + \lceil n/2 \rceil$

which is 1.5 n. Here TSP is obtained by joining circled ('o') edges.

$$TSP = n$$

$$\Rightarrow CHR = 1.5\, TSP$$