Database Design and Normal Forms

Database Design
- coming up with a "good" schema is very important

How do we characterize the "goodness" of a schema ?
If two or more alternative schemas are available
how do we compare them ?
What are the problems with "bad" schema designs ?

Normal Forms:
Each normal form specifies certain conditions
If the conditions are satisfied by the schema
certain kind of problems are avoided

Details follow….

An Example

student relation with attributes: studName, rollNo, sex, studDept
department relation with attributes: deptName, officePhone, hod

Several students belong to a department.
studDept gives the name of the student's department.

Correct schema:

Student

studName	<u>rollNo</u>	sex	studDept

Department

<u>deptName</u>	officePhone	HOD

Incorrect schema:

Student-Dept

studName	rollNo	sex	deptName	officePhone	HOD

What are the problems that arise ?

Problems with bad schema

Redundant storage of data:
Office Phone & HOD info - stored redundantly
- once with each student that belongs to the department
- wastage of disk space

A program that updates Office Phone of a department
- must change it at several places
  - more running time
  - error - prone

Transactions running on a database
- must take as short time as possible to increase transaction throughput

Update Anomalies

Another kind of problems with bad schema

Insertion anomaly:

No way of inserting info about a new department unless we also enter details of a (dummy) student in the department

Deletion anomaly:

If all students of a certain department leave and we delete their tuples, information about the department itself is lost

Update Anomaly:

Updating officePhone of a department

- value in several tuples needs to be changed
- if a tuple is missed - inconsistency in data

Normal Forms

First Normal Form (1NF) - included in the definition of a relation

Second Normal Form (2NF)

Third Normal Form (3NF)

Boyce-Codd Normal Form (BCNF)

defined in terms of functional dependencies

Fourth Normal Form (4NF) - defined using multivalued dependencies

Fifth Normal Form (5NF) or Project Join Normal Form (PJNF) defined using join dependencies

Functional Dependencies

A functional dependency (FD) $X \rightarrow Y$ [where ($X \subseteq R$, $Y \subseteq R$)]
(read as X *determines* Y)
is said to hold on a schema R if
in *any* instance r on R,
if two tuples t_1, t_2 ($t_1 \neq t_2$, $t_1 \in r$, $t_2 \in r$)
agree on X i.e. $t_1[X] = t_2[X]$
then they also agree on Y i.e. $t_1[Y] = t_2[Y]$

$t_1[X]$ – the sub-tuple of t_1 consisting of values of attributes in X

Note: If $K \subset R$ is a key for R then for any $A \in R$,
$K \rightarrow A$
holds because the above if …..then condition is vacuously true

Functional Dependencies – Examples

Consider the schema:
Student(studName, rollNo, sex, dept, hostelName, roomNo)

Since rollNo is a key, rollNo → {studName, sex, dept, hostelName, roomNo}

Suppose that each student is given a hostel room exclusively, then
hostelName, roomNo → rollNo

Suppose boys and girls are accommodated in separate hostels, then
hostelName → sex

Does Sex → hostelName?

FDs are additional constraints that can be specified by designers

Trivial / Non-Trivial FDs and Notation

An FD X → Y where $Y \subseteq X$
- called a *trivial* FD, as it always holds good

An FD X → Y where $Y \nsubseteq X$
- *non-trivial* FD

An FD X → Y where $X \cap Y = \Phi$
- *completely non-trivial* FD

Notational Convention:
(Low-end alphabets) A, B, C, D, ⋯ and their subscripted versions
-- denote individual attributes
(High-end alphabets) Z, Y, X, W,⋯ and their subscripted versions
--- denote sets of attributes

FDs – Examples

Consider the scheme preRequisite(preReqCourse, courseId)

Does preReqCourse → courseId ?

No, as a course might be pre-requisite for many courses

Does courseId → preReqCourse ?

No, a course may have many pre-requisite courses

So, it is possible that no FDs hold on some schema

FDs − Examples

Consider the scheme:

Student-dept(rollNo, name, sex, deptName, officePhone, Hod)

The key is rollNo, so
rollNo $\rightarrow$ name, sex, deptName, officePhone, Hod

Any more FDs hold?
deptName $\rightarrow$ officePhone, Hod
Hod $\rightarrow$ deptName, officePhone
(Assuming that each professor heads at most one department)

officePhone $\rightarrow$ deptName, Hod

No other FDs hold

Deriving new FDs

Given that a set of FDs F holds on R
we can infer that a certain new FD must also hold on R

For instance,
given that $X \rightarrow Y$, $Y \rightarrow Z$ hold on R
we can infer that $X \rightarrow Z$ must also hold

How to systematically obtain all such new FDs ?

Unless *all* FDs are known, a relation schema is not fully specified

Entailment Relation

We say that a set of FDs F $\models \{X \rightarrow Y\}$
(read as F *entails* $X \rightarrow Y$ or
F *logically implies* $X \rightarrow Y$
if in every instance r of R on which FDs F hold,
FD $X \rightarrow Y$ also holds.

Armstrong came up with several inference rules
for deriving new FDs from a given set of FDs

We define $F^+ = \{X \rightarrow Y \mid F \models X \rightarrow Y\}$
F^+: Closure of F

Armstrong's Inference Rules (1/2) (aka Armstrong's Axioms)

1. Reflexive rule

   $F \models \{X \rightarrow Y \mid Y \subseteq X\}$ for any X. Trivial FDs

2. Augmentation rule

   $\{X \rightarrow Y\} \models \{XZ \rightarrow YZ\}, Z \subseteq R$. Here, XZ denotes $X \cup Z$

3. Transitive rule

   $\{X \rightarrow Y, Y \rightarrow Z\} \models \{X \rightarrow Z\}$

4. Decomposition or Projective rule

   $\{X \rightarrow YZ\} \models \{X \rightarrow Y\}$

5. Union or Additive rule

   $\{X \rightarrow Y, X \rightarrow Z\} \models \{X \rightarrow YZ\}$

6. Pseudo transitive rule

   $\{X \rightarrow Y, WY \rightarrow Z\} \models \{WX \rightarrow Z\}$

Armstrong's Inference Rules (2/2)

Rules 4, 5, 6 are not really necessary.

For instance, Rule 5: $\{X \rightarrow Y, X \rightarrow Z\} \models \{X \rightarrow YZ\}$ can be *proved* using 1, 2, 3 alone

$$\left.\begin{array}{ll} 1) & X \rightarrow Y \\ 2) & X \rightarrow Z \end{array}\right\} \text{given}$$

3) $X \rightarrow XY$ Augmentation rule on 1
4) $XY \rightarrow ZY$ Augmentation rule on 2
5) $X \rightarrow ZY$ Transitive rule on 3, 4.

Similarly, 4, 6 can be shown to be unnecessary.
But it is useful to have 4, 5, 6 as <u>short-cut</u> rules

Sound and Complete Inference Rules

Armstrong showed that

Rules (1), (2) and (3) are sound and complete.
These are called Armstrong's Axioms (AA)

$F_{AA} = \{ X \rightarrow Y \mid X \rightarrow Y$ can be derived from F using AA $\}$

Soundness: ($F_{AA} \subseteq F^+$)

Every new FD $X \rightarrow Y$ derived from a given set of FDs F using Armstrong's Axioms is such that $F \models \{X \rightarrow Y\}$

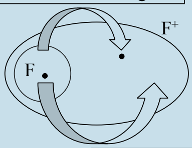
Completeness: ($F^+ \subseteq F_{AA}$)

Any FD $X \rightarrow Y$ logically implied by F (i.e. $F \models \{X \rightarrow Y\}$) can be derived from F using Armstrong's Axioms

Soundness and Completeness of AA

Completeness

Can be derived using AA $F^+ \subseteq F_{AA}$



derive using AA $F_{AA} \subseteq F^+$

Soundness

Proving Soundness

Suppose $X \rightarrow Y$ is derived from F using AA in some *n* steps.
If each step is correct then overall deduction would be correct.
Single step: Apply Rule (1) or (2) or (3)

Rule (1) – Reflexive Rule. Obviously results in correct FDs

Rule (2) – $\{X \rightarrow Y\} \vDash \{XZ \rightarrow YZ\}$, $Z \subseteq R$

Suppose $t_1, t_2 \in r$ agree on XZ
$\Rightarrow t_1, t_2$ agree on X
$\Rightarrow t_1, t_2$ agree on Y (since $X \rightarrow Y$ holds on r)
$\Rightarrow t_1, t_2$ agree as YZ

Hence Rule (2) gives rise to correct FDs

Rule (3) – $\{X \rightarrow Y, Y \rightarrow Z\} \vDash X \rightarrow Z$

Suppose $t_1, t_2 \in r$ agree on X
$\Rightarrow t_1, t_2$ agree on Y (since $X \rightarrow Y$ holds)
$\Rightarrow t_1, t_2$ agree on Z (since $Y \rightarrow Z$ holds)

Proving Completeness of Armstrong's Axioms (1/4)

Define X^+_F (closure of X wrt F)
$= \{A \mid X \rightarrow A$ can be derived from F using AA$\}$, $A \in R$
X^+_F is the set of all attributes that occur on
the rhs for an FD whose lhs is X, as per AA (wrt F)

Claim1:
$X \rightarrow Y$ can be derived from F using AA iff $Y \subseteq X^+$

(If) Let $Y = \{A_1, A_2, \ldots, A_n\}$. $Y \subseteq X^+$
$\Rightarrow X \rightarrow A_i$ can be derived from F using AA ($1 \leq i \leq n$)
By union rule, it follows that $X \rightarrow Y$ can be derived from F.

(Only If) $X \rightarrow Y$ can be derived from F using AA
By projective rule $X \rightarrow A_i$ ($1 \leq i \leq n$)
Thus by definition of X^+, $A_i \in X^+$
$\Rightarrow Y \subseteq X^+$

Completeness of Armstrong's Axioms (2/4)

Completeness:

$(F \vDash \{X \rightarrow Y\}) \Rightarrow X \rightarrow Y$ follows from F using AA

We will prove the contrapositive:

$X \rightarrow Y$ can't be derived from F using AA

$\Rightarrow F \nvDash \{X \rightarrow Y\}$

$\Rightarrow \exists$ a relation instance r on R st all the FDs of F hold on r but $X \rightarrow Y$ doesn't hold.

Consider the relation instance r with just two tuples:

$$r: \begin{array}{cc} \overbrace{1\ 1\ 1 \ldots 1}^{X^+ \text{ attributes}} & \overbrace{1\ 1\ 1 \ldots 1}^{\text{Other attributes}} \\ 1\ 1\ 1 \ldots 1 & 0\ 0\ 0 \ldots 0 \end{array}$$

Completeness Proof (3/4)

Claim 2: All FDs of F are satisfied by r

Suppose not. Let $W \rightarrow Z$ in F be an FD not satisfied by r

Then $W \subseteq X^+$ and $Z \nsubseteq X^+$

Let $A \in Z - X^+$

Now, $X \rightarrow W$ follows from F using AA as $W \subseteq X^+$ (claim 1)

$X \rightarrow Z$ follows from F using AA by transitive rule

$Z \rightarrow A$ follows from F using AA by reflexive rule as $A \in Z$

$X \rightarrow A$ follows from F using AA by transitive rule

By definition of closures, A must belong to X^+

- a contradiction.

Hence the claim.

$$r: \begin{array}{cc} 1\ 1\ 1 \ldots 1 & 1\ 1\ 1 \ldots 1 \\ \underbrace{1\ 1\ 1 \ldots 1}_{X^+} & \underbrace{0\ 0\ 0 \ldots 0}_{R - X^+} \end{array}$$

Completeness Proof (4/4)

Claim 3: $X \rightarrow Y$ is not satisfied by r

Suppose not

Because of the structure of r, $Y \subseteq X^+$

$\Rightarrow X \rightarrow Y$ can be derived from F using AA

contradicting the assumption about $X \rightarrow Y$

Hence the claim

Thus, whenever $X \rightarrow Y$ doesn't follow from F using AA,

F doesn't logically imply $X \rightarrow Y$

Armstrong's Axioms are complete.

$$r: \begin{array}{cc} 1\ 1\ 1 \ldots 1 & 1\ 1\ 1 \ldots 1 \\ \underbrace{1\ 1\ 1 \ldots 1}_{X^+} & \underbrace{0\ 0\ 0 \ldots 0}_{R - X^+} \end{array}$$

Consequence of Completeness of AA

$X^+ = \{A \mid X \to A$ follows from F using AA$\}$

$= \{A \mid F \models X \to A\}$

Similarly

$F^+ = \{X \to Y \mid F \models X \to Y\}$

$= \{X \to Y \mid X \to Y$ follows from F using AA$\}$

Computing closures

The size of F^+ can sometimes be exponential in the size of F.

For instance, $F = \{A \to B_1, A \to B_2, \ldots, A \to B_n\}$

$F^+ = \{A \to X\}$ where $X \subseteq \{B_1, B_2, \ldots, B_n\}$.

Thus $|F^+| = 2^n$

Computing F^+: computationally expensive

Fortunately, checking if $X \to Y \in F^+$

can be done by checking if $Y \subseteq X^+_F$

Computing attribute closure (X^+_F) is computationally easier

Computing X^+_F

We compute a sequence of sets $X_0, X_1, \ldots$ as follows:

$X_0 = X$; // X is the given set of attributes

$X_{i+1} = X_i \cup \{A \mid$ there is a FD $Y \to Z$ in F

such that $Y \subseteq X_i$ and $A \in Z\}$

To get new attributes into X_{i+1}, we use Transitive Rule and

we can only use that!

Since $X_0 \subseteq X_1 \subseteq X_2 \subseteq \ldots \subseteq X_i \subseteq X_{i+1} \subseteq \ldots \subseteq R$, and R is finite,

There is an integer i such that $X_i = X_{i+1} = X_{i+2} = \ldots$

X^+_F is equal to such X_i.

Computing X^+_F can be done in polynomial time

Attribute Closures − An Example

Consider a scheme R and the FDs: (Data redundancy exists in R)

R = (rollNo, name, advisorId, advisorName, courseId, grade)

FDs = { rollNo → name; rollNo → advisorId;
advisorId → advisorName;
rollNo, courseId → grade }

$\{\text{rollNo}\}^+ = \{\text{rollNo, name, advisorId, advisorName}\}$

$\{\text{rollNo, courseId}\}^+ = \{\text{rollNo, name, advisorId, advisorName, courseId, grade}\} = R$

So {rollNo, courseId} is the key for R.

Normal Forms – 2NF

Full functional dependency:
An FD X → A for which there is <u>no</u> proper subset Y of X such that Y → A
(A is said to be *fully functionally* dependent on X or)

2NF: A relation schema R is in 2NF if
every *non-prime* attribute is fully functionally dependent on any key of R

Prime attribute: A attribute that is part of some key
Non-prime attribute: An attribute that is not part of any key

Example 1: 2NF

student(rollNo, name, dept, sex, hostelName, roomNo, admitYear)

Assumptions:
Each student is allotted a single-occupancy room.
A room is identified by values of attributes hostelName, roomNo.
Boys and girls are accommodated in separate hostels.

Keys: rollNo, (hostelName, roomNo)
Not in 2NF as hostelName → sex

Decompose:
student(rollNo, name, dept, hostelName, roomNo, admitYear)
hostelDetail(hostelName, sex)
- These are both in 2NF

Example 2: 2NF

book(authorName, title, authorAffiliation, ISBN, publisher, pubYear)

Assumptions: A book has exactly one author.
Author can be uniquely identified by value of attribute authorName
AuthorAffiliation is the organization to which the author is *currently* associated with.
An author is associated with *exactly one* organization at any time.

Keys: (authorName, title), ISBN
Not in 2NF as authorName → authorAffiliation
(authorAffiliation is not fully functionally dependent on the first key)

Decompose:
book(authorName, title, ISBN, publisher, pubYear)
authorInfo(authorName, authorAffiliation) -- both in 2NF

Transitive Dependencies

Transitive dependency:
An FD X → Y in a relation schema R for which there is a set of attributes Z ⊆ R such that
X → Z and Z → Y and Z is not a subset of any key of R

studentDept(rollNo, name, dept, hostelName, roomNo, headDept)
Keys: rollNo, (hostelName, roomNo)
rollNo → dept; dept → headDept hold
So, rollNo → headDept is a transitive dependency

Head of the dept of dept D is stored redundantly in every tuple where D appears.

Relation is in 2NF but redundancy still exists.

Normal Forms – 3NF

Relation schema R is in 3NF if it is in 2NF and no non-prime attribute of R is transitively dependent on any key of R

studentDept(rollNo, name, dept, hostelname, roomNo, headDept) is not in 3NF

Decompose: student(<u>rollNo</u>, name, dept, <u>hostelName, roomNo</u>)
deptInfo(<u>dept</u>, headDept)

both in 3NF

Redundancy in data storage - removed

Another definition of 3NF

Relation schema R is in 3NF if for any nontrivial FD $X \rightarrow A$ either (i) X is a superkey or (ii) A is prime.

Suppose some R violates the above definition
$\Rightarrow$ There is an FD $X \rightarrow A$ for which both (i) and (ii) are false
$\Rightarrow$ X is not a superkey and A is non-prime attribute

Two cases arise:
1) X is contained in a key – A is not fully functionally dependent on this key
   - violation of 2NF condition
2) X is not contained in a key
   $K \rightarrow X$, $X \rightarrow A$ is a case of transitive dependency
   (K – any key of R)

Motivating example for BCNF

gradeInfo (rollNo, studName, course, grade)

Suppose the following FDs hold:
1) rollNo, course $\rightarrow$ grade
2) studName, course $\rightarrow$ grade
3) rollNo $\rightarrow$ studName
4) studName $\rightarrow$ rollNo

Keys:
(rollNo, course)
(studName, course)

(Assumption: No two students have the same name)

For 1, 2 lhs is a key. For 3, 4 rhs is prime; so gradeInfo is in 3NF

But studName is stored redundantly along with every course being done by the student.

Boyce - Codd Normal Form (BCNF)

Relation schema R is in BCNF if for every nontrivial FD $X \rightarrow A$, X is a *superkey* of R.

In gradeInfo, FDs 3, 4 are nontrivial but lhs is not a superkey
So, gradeInfo is not in BCNF

Decompose:
gradeInfo (rollNo, course, grade)
studInfo (rollNo, studName)

Redundancy allowed by 3NF is disallowed by BCNF

BCNF is stricter than 3NF
3NF is stricter than 2NF

Decomposition of a relation schema

If R doesn't satisfy a particular normal form, we decompose R into smaller schemas

What's a decomposition?

$R = (A_1, A_2, \ldots, A_n)$

$D = (R_1, R_2, \ldots, R_k)$ st $R_i \subseteq R$ and $R = R_1 \cup R_2 \cup \ldots \cup R_k$

(R_i's need not be disjoint)

Replacing R by $R_1, R_2, \ldots, R_k$ is the process of decomposing R

Ex: gradeInfo (rollNo, studName, course, grade)

R_1: gradeInfo (<u>rollNo, course</u>, grade)

R_2: studInfo (<u>rollNo</u>, studName)

Desirable Properties of Decompositions

Not all decomposition of a relational scheme R are useful

We require two properties to be satisfied

(i) Lossless join property
- the information in an instance r of R must be preserved in the instances $r_1, r_2, \ldots, r_k$ where $r_i = \Pi_{R_i}(r)$

(ii) Dependency preserving property
- if a set F of dependencies hold on R it should be possible to enforce F on an instance r by enforcing appropriate dependencies on each r_i

Lossless join property

F – set of FDs that hold on R

R – decomposed into $R_1, R_2, \ldots, R_k$

Decomposition is *<u>lossless</u>* wrt F if

for every relation instance r on R satisfying F,

$r = \Pi_{R_1}(r) * \Pi_{R_2}(r) * \ldots * \Pi_{R_k}(r)$

Lossless joins are also called non-additive joins

$R = (A, B, C)$; $R_1 = (A, B)$; $R_2 = (B, C)$

r:

A	B	C
a_1	b_1	c_1
a_2	b_2	c_2
a_3	b_1	c_3

Lossy join

r_1:

A	B
a_1	b_1
a_2	b_2
a_3	b_1

r_2:

B	C
b_1	c_1
b_2	c_2
b_1	c_3

Original info is distorted

$r_1 * r_2$:

A	B	C
a_1	b_1	c_1
a_1	b_1	c_3
a_2	b_2	c_2
a_3	b_1	c_1
a_3	b_1	c_3

Spurious tuples

Dependency Preserving Decompositions

Decomposition $D = (R_1, R_2, \ldots, R_k)$ of schema R *preserves* a set of dependencies F if

$$(\Pi_{R_1}(F) \cup \Pi_{R_2}(F) \cup \ldots \cup \Pi_{R_k}(F))^+ = F^+$$

Here, $\Pi_{R_i}(F) = \{ (X \rightarrow Y) \in F^+ \mid X \subseteq R_i, Y \subseteq R_i \}$
(called projection of F onto R_i)

Informally, any FD that logically follows from F must also logically follow from the union of projections of F onto R_i's
Then, D is called dependency preserving.

An example

Schema R = (A, B, C)
FDs $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow A\}$

Decomposition $D = (R_1 = \{A, B\}, R_2 = \{B, C\})$
$\Pi_{R_1}(F) = \{A \rightarrow B, B \rightarrow A\}$
$\Pi_{R_2}(F) = \{B \rightarrow C, C \rightarrow B\}$

$$(\Pi_{R_1}(F) \cup \Pi_{R_2}(F))^+ = \{A \rightarrow B, B \rightarrow A, B \rightarrow C, C \rightarrow B, A \rightarrow C, C \rightarrow A\} = F^+$$

Hence Dependency preserving

Testing for lossless decomposition property(1/6)

R – given schema with attributes $A_1, A_2, \ldots, A_n$
F – given set of FDs
D – $\{R_1, R_2, \ldots, R_m\}$ given decomposition of R

Is D a lossless decomposition?

Create an $m \times n$ matrix S with columns labeled as $A_1, A_2, \ldots, A_n$ and rows labeled as $R_1, R_2, \ldots, R_m$

Initialize the matrix as follows:
- set $S(i,j)$ as symbol b_{ij} for all i,j.
- if A_j is in the scheme R_i, then set $S(i,j)$ as symbol a_j, for all i,j

Testing for lossless decomposition property(2/6)

After S is initialized, we carry out the following process on it:

repeat
  for each functional dependency $U \rightarrow V$ in F **do**
    for all rows in S which agree on U-attributes **do**
      make the symbols in each V- attribute column
       the *same* in all the rows as follows:
        if any of the rows has an "a" symbol for the column
          set the other rows to the same "a" symbol in the column
        else // if no "a" symbol exists in any of the rows
          choose one of the "b" symbols that appears
          in one of the rows for the V-attribute and
          set the other rows to that "b" symbol in the column
until no changes to S

At the end, if there exists a row with all "a" symbols then D is lossless otherwise D is a lossy decomposition

Testing for lossless decomposition property(3/6)

R = (rollNo, name, advisor, advisorName, course, grade)
FD's = { rollNo → name; rollNo → advisor; advisor →advisorName
rollNo, course → grade}
D : { R_1 = (rollNo, name, advisor), R_2 = (advisor, advisorName),
R_3 = (rollNo, course, grade) }
Matrix S : (Initial values)

	rollNo	name	advisor	advisor Name	course	grade
R_1	a_1	a_2	a_3	b_{14}	b_{15}	b_{16}
R_2	b_{21}	b_{22}	a_3	a_4	b_{25}	b_{26}
R_3	a_1	b_{32}	b_{33}	b_{34}	a_5	a_6

Testing for lossless decomposition property(4/6)

R = (rollNo, name, advisor, advisorDept, course, grade)
FD's = { rollNo → name; rollNo → advisor; advisor → advisorName
rollNo, course → grade}
D : { R_1 = (rollNo, name, advisor), R_2 = (advisor, advisorName),
R_3 = (rollNo, course, grade) }

Matrix S : (After enforcing rollNo → name & rollNo → advisor)

	rollNo	name	advisor	advisor Name	course	grade
R_1	a_1	a_2	a_3	b_{14}	b_{15}	b_{16}
R_2	b_{21}	b_{22}	a_3	a_4	b_{25}	b_{26}
R_3	a_1	$\cancel{b_{32}}\,a_2$	$\cancel{b_{33}}\,a_3$	b_{34}	a_5	a_6

Testing for lossless decomposition property(5/6)

R = (rollNo, name, advisor, advisorDept, course, grade)

FD's = {rollNo → name; rollNo → advisor; advisor → advisorName
rollNo, course → grade}

D : { R_1 = (rollNo, name, advisor), R_2 = (advisor, advisorName), R_3 = (rollNo, course, grade) }

Matrix S : (After enforcing advisor → advisorName)

	rollNo	name	advisor	advisor Name	course	grade
R_1	a_1	a_2	a_3	$\cancel{b_{14}}a_4$	b_{15}	b_{16}
R_2	b_{21}	b_{22}	a_3	a_4	b_{25}	b_{26}
R_3	a_1	$\cancel{b_{32}}a_2$	$\cancel{b_{33}}a_3$	$\cancel{b_{34}}a_4$	a_5	a_6

No more changes. Third row with all *a* symbols. So a lossless join.

Testing for lossless decomposition property(6/6)

R – given schema. F – given set of FDs

The decomposition of R into R_1, R_2 is lossless wrt F if and only if
either $R_1 \cap R_2 \rightarrow (R_1 - R_2)$ belongs to F^+ or
$R_1 \cap R_2 \rightarrow (R_2 - R_1)$ belongs to F^+

Example:
gradeInfo (rollNo, studName, course, grade)
with FDs = {rollNo, course → grade; studName, course → grade;
rollNo → studName; studName → rollNo}
decomposed into
grades (rollNo, course, grade) and studInfo (rollNo, studName)
is lossless because
rollNo → studName

A property of lossless joins

D_1: $(R_1, R_2,\ldots, R_K)$ lossless decomposition of R wrt F

D_2: $(R_{i1}, R_{i2},\ldots, R_{ip})$ lossless decomposition of R_i wrt $F_i = \Pi_{R_i}(F)$

Then
$D = (R_1, R_2, \ldots , R_{i-1}, R_{i1}, R_{i2}, \ldots, R_{ip}, R_{i+1},\ldots, R_k)$ is a lossless decomposition of R wrt F

This property is useful in the algorithm for BCNF decomposition

Algorithm for BCNF decomposition

R – given schema. F – given set of FDs

D = {R} // initial decomposition
while there is a relation schema R_i in D that is not in BCNF do
{ let $X \rightarrow A$ be the FD in R_i violating BCNF;
Replace R_i by $R_{i1} = R_i - \{A\}$ and $R_{i2} = X \cup \{A\}$ in D;
}

Decomposition of R_i is lossless as

$$R_{i1} \cap R_{i2} = X,\ R_{i2} - R_{i1} = A \text{ and } X \rightarrow A$$

Result: a lossless decomposition of R into BCNF relations

Dependencies may not be preserved (1/2)

Consider the schema: townInfo ($\overset{S}{\text{stateName}}$, $\overset{T}{\text{townName}}$, $\overset{D}{\text{distName}}$)
with the FDs F: $ST \rightarrow D$ (town names are unique within a state)
$D \rightarrow S$ (district names are unique across states)

Keys: ST, DT – all attributes are prime
– relation is in 3NF
Relation is not in BCNF as $D \rightarrow S$ and D is not a key

Decomposition given by algorithm: R1: TD R2: DS
Not dependency preserving as $\Pi_{R1}(F)$ = trivial dependencies
$\Pi_{R2}(F) = \{D \rightarrow S\}$

Union of these doesn't imply $ST \rightarrow D$
$ST \rightarrow D$ can't be enforced unless we perform a join.

Dependencies may not be preserved (2/2)

Consider the schema: R (A, B, C)
with the FDs F: $AB \rightarrow C$ and $C \rightarrow B$
Keys: AB, AC – relation in 3NF (all attributes are prime)
– Relation is not in BCNF as $C \rightarrow B$ and C is not a key

Decomposition given by algorithm: R_1: CB R_2: AC
Not dependency preserving as $\Pi_{R_1}(F)$ = trivial dependencies
$\Pi_{R_2}(F) = \{C \rightarrow B\}$
Union of these does not entail $AB \rightarrow C$

All possible decompositions: {AB, BC}, {BA, AC}, {AC, CB}
Only the last one is lossless!

Lossless and dependency-preserving decomposition doesn't exist.

Equivalent Dependency Sets

F, G – two sets of FDs on schema R

F is said to <u>cover</u> G if $G \subseteq F^+$ (equivalently $G^+ \subseteq F^+$)

F is equivalent to G if $F^+ = G^+$ (or, F covers G and G covers F)

Note: To check if F covers G,

it's enough to show that for each FD $X \rightarrow Y$ in G, $Y \subseteq X^+_F$

Canonical covers or Minimal covers

It is of interest to reduce a set of FDs F into a 'standard' form F′ such that F′ is equivalent to F.

We define that a set of FDs F is in '*minimal form*' if

(i) the rhs of any FD of F is a single attribute

(ii) there are no redundant FDs in F

that is, there is no FD $X \rightarrow A$ in F

s.t $(F - \{X \rightarrow A\})$ is equivalent to F

(iii) there are no redundant attributes on the lhs of any FD in F

that is, there is no FD $X \rightarrow A$ in F s.t there is $Z \subset X$ for which

$F - \{X \rightarrow A\} \cup \{Z \rightarrow A\}$ is equivalent to F

Minimal Covers

useful in obtaining a lossless, dependency-preserving decomposition of a scheme R into 3NF relation schemas

Algorithm for computing a minimal cover

R – given Schema or set of attributes; F – given set of FDs on R

Step 1: $G := F$

Step 2: Replace every fd of the form $X \rightarrow A_1A_2A_3\ldots A_k$ in G by $X \rightarrow A_1$; $X \rightarrow A_2$; $X \rightarrow A_3$; … ; $X \rightarrow A_k$

Step 3: For each fd $X \rightarrow A$ in G do
    for each B in X do
        if $(G - \{X \rightarrow A\} + \{(X - B) \rightarrow A\})^+ = F^+$ then
            replace $X \rightarrow A$ by $(X - B) \rightarrow A$

Step 4: For each fd $X \rightarrow A$ in G do
    if $(G - \{ X \rightarrow A\})^+ = G^+$ then
        replace G by $G - \{ X \rightarrow A\}$

Computing Minimal Covers

Example from Elmasri and Navathe, Database Sytems (6^{th} edition)

Determine the minimal cover for $F = \{ B \to A, D \to A, AB \to D \}$

All rhs sets are single attributes. So, Step 2 changes nothing.

If $G = \{ B \to A, D \to A, B \to D \}$, we find that $G^+ = F^+$

In G, since $B \to D$, $AB \to AD$ and hence $AB \to D$

So $AB \to D$ belongs to G^+. Hence G covers F

In F, since $B \to A$, $B \to AB$.

Since $B \to AB$, $AB \to D$, we get $B \to D$. So $B \to D$ is in F^+.

Hence F covers G.

Finally, in G, we find that $B \to A$ can be obtained for the other two.

Hence, $\{ D \to A, B \to D \}$ is a minimal cover for F

3NF Decomposition Algorithm

R – given Schema; F – given set of fd's on R in *minimal form*

Use BCNF algorithm to get a lossless decomposition $D = (R_1, R_2, \ldots, R_k)$
Note: each R_i is already in 3NF (it is in BCNF in fact!)

Algorithm: Let G be the set of fd's not preserved in D
For each fd $Z \to A$ that is in G
Add relation scheme $S = (B_1, B_2, \ldots, B_s, A)$ to D. // $Z = \{B_1, B_2, \ldots, B_s\}$

As $Z \to A$ is in F which is a minimal cover,
there is no proper subset X of Z s.t $X \to A$. So Z is a key for S!
Any other fd $X \to C$ on S is such that C is in $\{B_1, B_2, \ldots, B_s\}$.
Such fd's do not violate 3NF because each B_j's is prime a attribute!
Thus any scheme S added to D as above is in 3NF.

D continues to be lossless even when we add new schemas to it!

Multi-valued Dependencies (MVDs) and 4NF

studCoursesAndFriends(<u>rollNo,courseNo,frndEmailAddr</u>)

A student enrolls for several courses and has several friends whose email addresses we want to record.

If rows (CS05B007, CS370, shyam@gmail.com) and
(CS05B007, CS376, radha@yahoo.com) appear then
rows (CS05B007, CS376, shyam@gmail.com)
(CS05B007, CS370, radha@yahoo.com) should also appear!

For, otherwise, it implies that having "shyam" as a friend has something to do with doing course CS370!

Causes a huge amount of data redundancy!
Since there are no non-trivial FD's, the scheme is in BCNF

We say that MVD rollNo $\to\to$ courseNo holds
(read as rollNo *multi-determines* courseNo)
By symmetry, rollNo $\to\to$ frndEmailAddr also holds

More about MVDs

Consider studCourseGrade(<u>rollNo,courseNo</u>,grade)

Note that rollNo →→ courseNo *does not* hold here even though courseNo is a multi-valued attribute of a student entity

If (CS05B007, CS370, A)
(CS05B007, CS376, B) appear in the data then
(CS05B007, CS376, A)
(CS05B007, CS370, B) will not appear !!

Attribute 'grade' depends on (rollNo,courseNo)

MVD's arise when two or more *unrelated* multi-valued attributes of an entity are sought to be represented together in a scheme.

More about MVDs

Consider

studCourseAdvisor(<u>rollNo,courseNo</u>,advisor)

Note that rollNo →→ courseNo *holds* here

If (CS05B007, CS370, Dr Ravi)
(CS05B007, CS376, Dr Ravi) appear in the data then
swapping courseNo values gives rise to existing rows only.

But, since rollNo → advisor and (rollNo, courseNo) is the key, this gets caught in checking for 2NF itself.

MVD Definition

Consider a scheme R(X, Y, Z),

An MVD X →→Y holds on R if, for in any instance of R,

the presence of two tuples

(xxx, y1y1y1, z1z1z1) and

(xxx, y2y2y2, z2z2z2)

guarantees the presence of tuples

(xxx, y1y1y1, z2z2z2) and

(xxx, y2y2y2, z1z1z1)

Note that every FD on R is also an MVD!

- the notion of MVD's generalizes the notion of FD's

Alternative definition of MVDs

Consider R(<u>X,Y,Z</u>)

Suppose that X $\rightarrow\rightarrow$ Y and by symmetry X $\rightarrow\rightarrow$ Z

Then, decomposition D = (XY, XZ) of R should be lossless

That is, for any instance *r* on R, $r = \Pi_{XY}(r) * \Pi_{XZ}(r)$

MVDs and 4NF

An MVD X $\rightarrow\rightarrow$ Y on scheme R is called *trivial* if either Y $\subseteq$ X or R = X $\cup$ Y. Otherwise, it is called *non-trivial.*

4NF: A relation R is in 4NF if it is in BCNF and for every nontrivial MVD X $\rightarrow\rightarrow$ A, X must be a superkey of R.

studCourseEmail(<u>rollNo,courseNo,frndEmailAddr</u>) is not in 4NF as

rollNo $\rightarrow\rightarrow$ courseNo and

rollNo $\rightarrow\rightarrow$ frndEmailAddr

are both nontrivial and rollNo is not a superkey for the relation

Join Dependencies and 5NF

A join dependency (JD) is generalization of an MVD

A JD $JD(R_1, R_2, \ldots, R_k)$ is said to hold on schema R if for every instance $r = *(\Pi_{R1}(r),\ \Pi_{R2}(r),\ \ldots,\Pi_{Rk}(r))$

Here, $R = R_1 \cup R_2 \cup \ldots \cup R_k$ and Natural join $*$ is a multi-way join.

A JD is difficult to detect in practice. It occurs in rare situations.

A relational scheme is said to be in 5NF wrt to a set of FDs, MVDs and JDs if it is in 4NF and for every non-trivial $JD(R_1,R_2,\ldots,R_k)$, each R_i is a superkey.

Join Dependencies – An Example

Consider the following relation:

studProjSkill(rollNo, skill, project) and the three relations

studSkill(rollNo, skill) // who has what skill

studProj(rollNo, project) // who is interested in what project

skillProj(project, skill) // which project requires what skills

Suppose there is a rule that:

If a student r1 has skill s1, and r1 is interested in project p1and project p1 requires skill s1 then (r1, s1, p1) *must be* in studProjSkill

In other words, studProjSkill = ∗ (studSkill, studProj, skillProj)

Then, we say JD(studSkill, studProj, skillProj) holds

Example - Observations

rollNo	skill
r1	s1
r1	s2

Size <= rs

rollNo	project
r1	p1
r1	p2

Size <= rp

project	skill
p1	s1
p2	s3

Size <= sp

rollNo	project	skill
r1	p1	s1

Size <= rps

There are no MVDs in 3-column table

#students = r, #projects = p, #skills =s
rps >> rp + sp + rs

Huge amount of data redundancy exists

Relational DB Design - Approaches

Two Approaches: Bottom-up and Top-down

Bottom-up Approach (aka Synthesis Approach)

- Keep all attributes in a universal relation
- Determine *all* the FDs, MVDs, applicable
- Use the algorithms discussed to decompose the universal relation
- Obtain a design using the algorithms discussed

Drawbacks of the approach

- Difficult to obtain *all* the FDs in a large DB with 100s of attributes
- Algorithms are non-deterministic
- Not popular in practice

Relational DB Design - Approaches

Top-down Approach (aka Analysis Approach)

- Represent Entities/Relationships as relations
  - Group attributes that belong naturally together
- Determine the FDs, MVDs, applicable among attributes
- Analyze the relations individually and also collectively
  - If necessary carry out decomposition to obtain desirable properties
- More popular approach
- Theoretical observations are applicable to both approaches