CS6100 - Midsem - Q3

KNN:

The discription in the qsn. is bit ambigous on what to be considered as value of KNN(S, R) so I am solving it for 2 cases

Case-1 : KNN(S, R) = Sum of distances of all edges

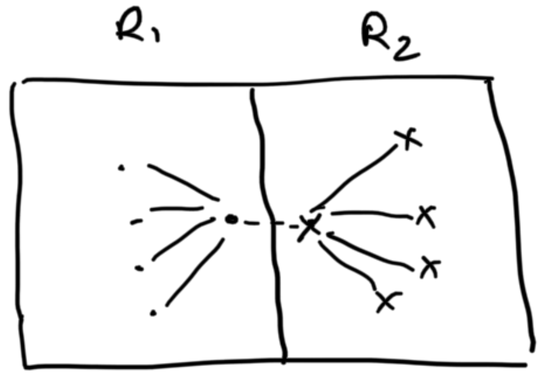
If the partitions S_1 & S_2 are far away then the nearest neighbours of vertices wont change even after union so $KNN(S, R) = KNN(S_1, R) + KNN(S_2, R)$. If not, (i.e the partitions are interfearing with each other) then the only chance is that for some vertex (or vertices) got a better neighbour than before and distance is reduced. So $KNN(S, R) \leq KNN(S_1, R) + KNN(S_2, R)$

In any case $KNN(S, R) \leq KNN(S_1, R) + KNN(S_2, R)$ hence it is sub additive. It is also Geometric subaditive as we can always write S_1 as $S \cap R_1$ and S_2 as $S \cap R_2$.

Case-2 : KNN(S, R) = The graph (i.e. the set of vertices and edges).

we also restate subadditivity for sets as
$$KNN(S, R) \subseteq KNN(S_1, R) + KNN(S_2, R).$$
Consider edges sets for analysis as we are

partitioning vertices in the split, these sets will always add up. It can be easily seen that $KNN(S,R)$ can have new edges if one vertex in S_1 have a vertex from S_2 which is nearer than its k^{th} distant vertex. So $KNN(S,R)$ will not be a subset of $KNN(S_1,R) + KNN(S_2,R)$. Similar example will also prove that KNN is not geometric subadditive consider -



If dot and cross are close to boundary we get a new edge which isn't present in both subsets.

Cycle Cover: $CC(S,R)$ = min. weight cycle cover

Consider 2 partitions S_1 & S_2. let C_1 & C_2 be min weight cycle covers in them. let C be min weight cycle cover in S. we can see that $C_1 + C_2$ is also a cycle cover in S by the definition

so $wt(C)$ must be $\leq wt(C_1) + wt(C_2)$

$\therefore CC(S,R) \leq CC(S_1,R) + CC(S_2,R)$

Hence CC is subadditive and also geometric subadditive.