IITM-CS6100 : Topics in Design and analysis of Algorithms — Given on: Mar 23

Problem Set #2 — Due on : May 15, 23:55 — Evaluation Due on : May 20

- Turn in your solutions electronically at the moodle page. The submission should be a pdf file typeset either using LaTeX or any other software that generates pdf. No handwritten solutions are accepted.
- Collaboration is encouraged, but all write-ups must be done individually and independently. For each question, you are required to mention the set of collaborators, if any.
- Submissions will be checked for **plagiarism**. Each case of plagiarism will be reported to the institute disciplinary committee (DISCO).

1. (8 points) Let $P = \{p_1, \ldots, p_n\}$ be a set of n points from the unit square $[0,1]^2$. A triangulation τ of P is a maximal planar graph with P as the vertex set (i.e., the locations of points in P should be an embedding of the graph). A minimum weight triangulation (denoted by mwt) is a triangulation with minimum total edge weight. Let MWT denote the corresponding Euclidean functional. The convex hull (denoted by CH) of P is the smallest convex set (i.e. polygon) containing P. Let $|CH(P)|$ denote the number of vertices in the convex hull of P. Suppose Q_1, Q_2, Q_3 and Q_4 be a partition of $[0,1]^2$ of equal sized squares. Show that

$$MWT(P, [0,1]^2) \leq \sum_{i=1}^{4} MWT(P \cap Q_i, Q_i) + \sum_{i=1}^{4} O(|CH(P \cap Q_i)|).$$

2. (16 points) In the vehicle routing problem, we are given a set of depots $D = \{d_1, \ldots, d_k\}$ from $[0,1]^d$ and a set of n points $P = \{p_1, \ldots, p_n\}$ (called customers) which are again points from $[0,1]^d$. The job is to compute minimum cost k vertex disjoint cycles such that every cycle has exactly one depot and every depot is in exactly one cycle. Let MDP denote the corresponding Euclidean functional.
   (a) (7 points) Formally define the functional MDP. Prove that MDP is subadditive but not superadditive.
   (b) (9 points) Define a canonical boundary functional MDP_B for MDP and show that it is superadditive. Show that the boundary functional is also a smooth Euclidean functional.

3. (6 points) Refer to phase 3 of the algorithm for computing Hamiltonian cycle given in Page 6 of Frieze's survey. Show that for any tractable graph, the phase 3 of the algorithm always succeeds in computing a Hamiltonian cycle, if exists.