CS6100 - Midsem - Q2

Model-2 is not feasable. this is because a perturbed instance of an input should be a similar (not same) from the neighbourhood of the main one. when we do $q_i = \alpha \cdot P_i$ we are not in the neighbourhood as infitisimal shrinking is possible. Apart from that the problem it self is not changing (i.e. euclidean TSP doesn't change on multiplication with a constant factor)

$\rightarrow$ $Eu\,dis(\alpha A, \alpha B) = \alpha\; Eu\,dis(A, B)$

where as model-1 is suitable for smoothed analysis owing to the fact that each point is perturbed independantly by α_i which is chosen uniformly at random from $[0, \varepsilon]^2$

Two lemmata that are crucial anabyzing a Euclidean TSP algorithm is Interval lemma and tail bound.

this perturbation model satisfies both

Interval lemma:

$$Pr[X \in (t, t+\delta)] = \delta\varepsilon$$

(uniform distribution)

Tail bound:

$\Pr[x > d] \leq$ some low value

In our case perturbed instance x' is bounded (i.e. $\leq x_0 + \epsilon$) hence we can always say that the tour cost is bounded by a TSP bound in $[0, 1+\epsilon]^2$

$$\Pr[L > \sqrt{2}(1+\epsilon)n] = 0$$

As these 2 lemmata are bounded we can use these results to analyze the algorithm easily.