1 Regret Analysis

1.1 Martingale Sequence

A sequence of random variables is called Martingale if the conditional expectation of next vatiable w.r.t all the previous variables is equal to present variable

$$E(X_{t+1}|X_t, ..., X_2, X_1) = X_t$$

General expectation must also be bounded

$$E(X_i) \leq C$$

1.2 Azuma Hoefding Inequality

If Y_t is Martingale and diffrence between consecutive terms is bounded i.e. $|Y_t - Y_{t-1}| \leq c$ then we can say that

$$P\left[Y_t > c\sqrt{2t\log\frac{1}{\gamma}}\right] \leq \gamma \tag{1}$$

This is simpler version of the original inequality and can also be viewed as - $Y_t \leq c\sqrt{2t\log\frac{1}{\gamma}}$ holds with probability $1-\gamma$

1.3 Continuation

We can write the final equation of Regret Analysis as -

$$reg_t \leq \frac{a \cdot \beta_t}{b + c \cdot \beta_t}$$

Consider $Y_t = \sum_{s=1}^{t} reg_s = \sum_{s=1}^{t}(R \cdot (x_* - x_t))$ as Martingale. This makes sense because in any bandit problem involving building we construct confidence sets using all the previous variables and pick next action i.e. x_t is a function of $x_1, \mu_1, ..., x_{t-1}, \mu_{t-1}$. To strengthen this statment few bandit papers just solve for regret assuming martingale and in Improved Algorithms paper he says a similar term to be martingale. We also know that -

$$|Y_t - Y_{t-1}| = reg_t \leq \frac{a \cdot \beta_t}{b + c \cdot \beta_t} \leq \frac{a \cdot \beta_T}{b + c \cdot \beta_T}$$

By using above described inequality with confidence δ

$$P\left[Y_T > \frac{a \cdot \beta_T}{b + c \cdot \beta_T} \cdot \sqrt{2T\log\frac{1}{\delta}}\right] \leq \delta$$

So we can say with confidence $1 - 2\delta$ - As we have used δ confidence already to get bound on reg_t

$$R(T) = Y_T <= \frac{a \cdot \beta_T}{b + c \cdot \beta_T} \cdot \sqrt{2T\log\frac{1}{\delta}}$$

2 Refered Papers

1. Aldo Pacchiano, Stochastic Bandits with Linear Constraints
   https://arxiv.org/pdf/2006.10185.pdf

2. Yasin Abbasi-Yadkor, Improved Algorithms for Linear Stochastic Banditsm
   https://papers.nips.cc/paper/2011/file/e1d5be1c7f2f456670de3d53c7b54f4a-Paper.pdf