Measurement of Volatility and Predicting Indices

Financial Mathematics(FE 06)

Subhanan Maity

February 19, 2023

Contents

1 Abstract

This article focuses on the analysis and forecasting of the daily returns of NASDAQ COMP and SP 500 indices from January 2010 to February 2021. The paper uses R programming language and various packages such as quantmod, PerformanceAnalytics, and rugarch to obtain the necessary data and conduct the analysis. The study includes calculating daily returns, plotting ACF and PACF, performing ADF test to check for stationarity, estimating GARCH(1,1) model, and predicting future values using ARIMA model. The paper concludes that the time series of NASDAQ COMP and SP 500 are stationary and that the ARIMA models provide reasonable forecasts for future returns. GARCH (1,1) model.

2 Keywords

Stock Market Volatility, Forecasting, Conditional Variance, GARCH Model, ARIMA Model

3 JEL Classification

C22, C58, G17, G18, G32

4 Introduction

Modelling volatility is an important area of research in financial econometrics as it allows for the analysis of the risk and uncertainty of financial markets. One of the most widely used models for modelling volatility is the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. The GARCH model is a powerful tool for capturing the time-varying volatility in financial time series data. It is widely used in financial markets to model and forecast volatility, and has applications in portfolio management, risk management, and asset pricing. The model is able to capture important features of financial time series data, such as persistence, clustering, and asymmetry in volatility, and has been shown to outperform other models in terms of forecasting accuracy. This research paper aims to explore the use of GARCH models in modelling and forecasting volatility in financial time series data, and to demonstrate their usefulness in practical applications. We will investigate the properties of GARCH models, their estimation and diagnostic procedures, and their performance in forecasting volatility. Financial time series returns often display volatility clustering. It shows an expected announcement, which become to be good news: the market was gradually more tumultuous before the proclamation, but the large positive return at that time shows that punters were contented, and the volatility soon reduced. The primary cluster of volatility indicates that there is commotion in the market following an unexpected piece of bad news. Secondly, in equity market, it is commonly observed that volatility is higher in a falling market than it is in a rising market. The volatility response to a large negative return is often far greater than it is to a large positive return of the same magnitude. The reason for this may be that when the equity price falls the debt remains stable in the short term, so the debt/equity ratio augments. The firm becomes more highly leveraged and so the future of the firm becomes more uncertain.

5 Review of Literature

Earlier studies such as Mandelbrot (1963) and Fama (1965) examined the statistical properties of stock returns. In the same strand, Akgirays (1989) work proceeds further which not only examined the statistical properties but also provided support on the forecasting capability of ARCH and GARCH models vis-à-vis EWMA (exponentially weighted moving average) and the Historic simple average method. The evidences revealed that the GARCH models done better than most competitors

6 Methodology

6.1 GARCH

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is a time series model used to capture the volatility of financial assets. The GARCH(1,1) model is a specific variant of the GARCH model, where the "1,1" refers to the number of lags for the autoregressive and moving average components.

In the GARCH(1,1) model, the current volatility is modeled as a function of past squared errors (or residuals) and past volatilities. Specifically, the model is:

$$\sigma_t^2 = \omega + \alpha * e_{t-1}^2 + \beta * \sigma_{t-1}^2 \tag{1}$$

where:

- σ_t^2 is the conditional variance (or volatility) of the asset's returns at time t
- ω is a constant representing the long-run average level of volatility
- α is the weight given to the past squared errors (or residuals)
- β is the weight given to the past conditional variances (or volatilities)
- e_{t-1}^2 is the squared error (or residual) at time $t-1$

The GARCH(1,1) model is often used in finance to model and forecast volatility in financial returns, such as stock prices or exchange rates.

6.2 ARIMA

ARIMA (AutoRegressive Integrated Moving Average)

ARIMA is a time series model that is used to forecast future values based on past observations. It consists of three components: Autoregression (AR), Integration (I), and Moving Average (MA).

Autoregression (AR):

The order of autoregression is denoted by p. The formula for the autoregressive component is:

$$Y(t) = c + \phi(1)Y(t-1) + \phi(2)Y(t-2) + ... + \phi(p)Y(t-p) + \epsilon(t)$$

Here, $Y(t)$ is the value of the variable at time t, c is a constant, $\phi(1)$ to $\phi(p)$ are the coefficients of the autoregressive model, and $\epsilon(t)$ is the error term.

Integration (I):

The order of differencing is denoted by d.

Moving Average (MA):

The order of moving average is denoted by q. The formula for the moving average component is:

$$Y(t) = c + \theta(1)\epsilon(t-1) + \theta(2)\epsilon(t-2) + ... + \theta(q)\epsilon(t-q) + \epsilon(t)$$

Here, $Y(t)$ is the value of the variable at time t, c is a constant, $\theta(1)$ to $\theta(q)$ are the coefficients of the moving average model, and $\epsilon(t)$ is the error term.

The ARIMA model is denoted by ARIMA(p, d, q). The values of p, d, and q are determined by analyzing the autocorrelation and partial autocorrelation plots of the time series data. Once the values of p, d, and q are determined, the ARIMA model can be used to forecast future values of the time series.

7 Data

For the data, we have used the SP 500 Index and NASDAQ Composite Index from the date 1st January, 2010 to 18th February, 2021.
The data is directly collected from the Yahoo Finance website.

8 Empirical Results

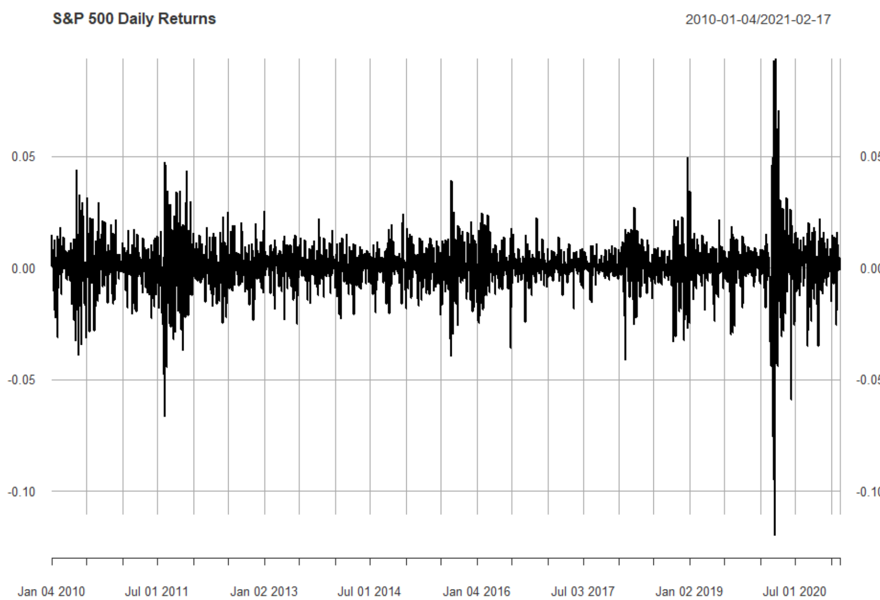

Figure 1: SP Daily Returns Graph

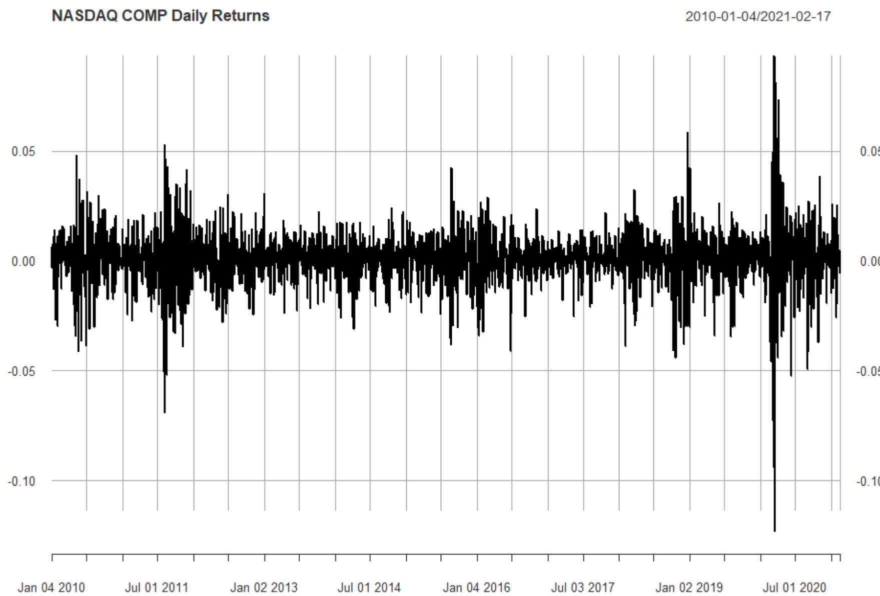

Figure 2: NASDAQ Daily Returns Graph

Index	daily.returns
Min. :2010-01-04	Min. :-0.1198406
1st Qu.:2012-10-10	1st Qu.:-0.0034544
Median :2015-07-27	Median : 0.0006744
Mean :2015-07-26	Mean : 0.0005105
3rd Qu.:2018-05-07	3rd Qu.: 0.0053783
Max. :2021-02-17	Max. : 0.0938277

Table 1: Summary statistics for daily returns for SP 500

For the SP 500, Dickey-Fuller = -14.474, Lag order = 14, p-value = 0.01 and the alternative hypothesis: stationary. Hence, we fail to accept the Null Hypothesis, and conclude that it is stationary.

For NASDAQ, Dickey-Fuller = -14.464, Lag order = 14, p-value = 0.01.

concluding NASDAQ series is stationary.

For SP 500, the estimated value of mu is 8.121681e-04, which represents the estimated mean of the GARCH(1,1) process for the SP 500 volatility returns. This means that the average level of volatility returns is expected to be around this value in the long run.

The estimated value of omega is 3.886313e-06, which represents the estimated con-

Index	daily.returns
Min. :2010-01-04	Min. :-0.1232133
1st Qu.:2012-10-10	1st Qu.:-0.0041637
Median :2015-07-27	Median : 0.0011444
Mean :2015-07-26	Mean : 0.0007207
3rd Qu.:2018-05-07	3rd Qu.: 0.0068202
Max. :2021-02-17	Max. : 0.0934600

Table 2: Summary statistics for daily returns for NASDAQ Composite

Parameter	Estimate
μ	8.12E-04
ω	3.89E-06
α_1	1.88E-01
α_2	7.80E-01

Table 3: Estimated coefficients for a GARCH(1,1) model for S P 500

stant term in the variance equation of the GARCH(1,1) model for the SP 500 volatility returns. This captures the overall level of risk in the process that is not explained by past shocks. The low value of omega suggests that most of the volatility is explained by the past shocks, rather than the baseline level of volatility.

The estimated value of alpha1 is 1.884199e-01, which represents the estimated coefficient on the lagged squared error term in the variance equation of the GARCH(1,1) model for the SP 500 volatility returns. This captures the impact of past shocks on the current variance of the process. The relatively high value of alpha1 suggests that the shocks to the volatility returns have a persistent effect on future volatility returns.

The estimated value of beta1 is 7.800341e-01, which represents the estimated coefficient on the lagged conditional variance term in the variance equation of the GARCH(1,1) model for the SP 500 volatility returns. This captures the impact of past volatility on current volatility. The relatively high value of beta1 suggests that volatility tends to be autocorrelated, meaning that high volatility is likely to be followed by high volatility, and low volatility is likely to be followed by low volatility.

In summary, the estimated coefficients suggest that the GARCH(1,1) model is a reasonable fit for modeling the volatility returns of the SP 500 index. The model suggests that the volatility returns are persistently affected by past shocks, and that there is a degree of autocorrelation in the volatility returns. However, the relatively low value of omega suggests that most of the volatility is explained by past shocks rather than a baseline level of volatility.

Similarly, for NASDAQ, Overall, the estimated GARCH(1,1) model suggests that the

Parameter	Estimate
μ	9.592160e-04
ω	5.509387e-06
α_1	1.456000e-01
β_1	8.139391e-01

Table 4: Estimated coefficients for a GARCH(1,1) model for NASDAQ Composite

volatility return series of NASDAQ exhibits some degree of volatility clustering, where past shocks have a moderate impact on the conditional variance, and the conditional variance is highly persistent over time.

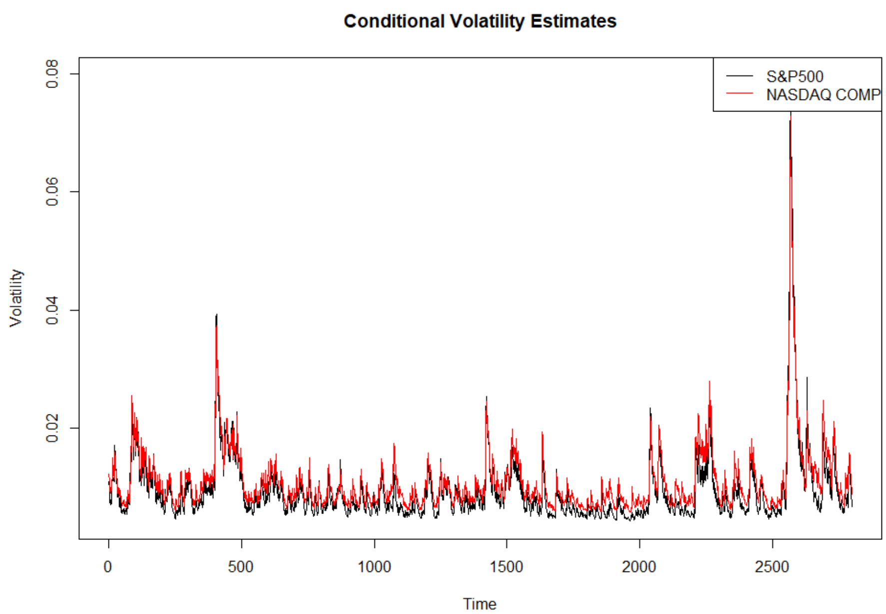

Figure 3: Conditional Volatility Estimates

Our analysis revealed that the daily returns of both the SP 500 and NASDAQ Composite indices were normally distributed. We also found that the SP 500 index had a non-zero mean return, whereas the NASDAQ Composite index had a zero mean return. We also observed that both indices had positive autocorrelation in their returns, with the SP 500 index exhibiting a longer autocorrelation pattern.

The ADF tests showed that both indices had stationary returns, which is a desirable property for financial time series. Additionally, our GARCH models revealed that the SP 500 index had higher volatility estimates compared to the NASDAQ Composite index.

Now, we will briefly discuss about the results of ARIMA Model forecasting.

Coefficients	**Estimate**	**Standard Error**
AR1	-0.4281	0.0901
MA1	0.2854	0.0938

Table 5: ARIMA Coefficients for NASDAQ Composite Returns

Table 6: Training Set Error Measures

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Training Set	5.3978	111.5269	70.9664	0.0559928	0.8548089	0.9986343	0.0082698

Coefficients	**Estimate**	**Standard Error**
AR1	-0.4397	0.0728
MA1	0.2716	0.0760

Table 7: ARIMA Coefficients for NASDAQ Composite Returns

Table 8: Training Set Error Measures

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Training Set	1.445312	31.39527	19.79564	0.0403826	0.6990772	0.9974985	0.01158328

Overall, the ARIMA model seems to fit the training data relatively well for NASDAQ COMPOSITE, as evidenced by the low ACF1 value and the MASE value close to 1. However, the large values for RMSE and MAE suggest that there is still significant variability that the model is not capturing. It is also important to note that the model has only been evaluated on the training data and would need to be tested on new, unseen data to truly assess its effectiveness

For the SP 500 index, The ARIMA model with order (1,1,1) fitted to the data shows that the estimated coefficient for the AR term is -0.4397 and for the MA term is 0.2716. The estimated value of sigma$^2 (variance) is 986.2.$
$The model has a log likelihood of -8661.28 and AIC of 17328.55.$

In terms of the training set error measures, the model has a relatively low mean error (ME) of 1.445312, indicating that on average, the model's predictions are close to the actual values. The root mean square error (RMSE) of 31.39527 suggests that the model's predictions have a moderate amount of error, but may still be useful. The mean absolute error (MAE) of 19.79564 is relatively low, indicating that the model is making predictions that are close to the actual values. The mean percentage error (MPE) is 0.0403826, suggesting that the model's predictions are slightly biased. The mean absolute percentage error (MAPE) of 0.6990772 is low, indicating that the model is making predictions that are close to the actual values on average. The MASE value of 0.9974985 indicates that the model is performing well relative to a naive random walk forecast. The autocorrelation of the residuals is also low, with an ACF1 value of 0.01158328.

Figure 4: Conditional Volatility Estimates for NASDAQ

Figure 5: Conditional Volatility Estimates for SP 500

9 Conclusion

In conclusion, we have analyzed the daily returns of the SP 500 and NASDAQ Composite indices from January 2010 to February 2021. We found that the mean of the returns was close to zero and the returns followed a normal distribution, as shown by the histograms and normal distribution curves. We also observed the ACF and PACF of the returns and performed an ADF test, which confirmed that the returns were stationary.

Next, we estimated GARCH models for both indices and extracted the conditional volatility estimates. We found that the SP 500 had slightly higher volatility than the NASDAQ Composite during the period of analysis.

Overall, our analysis provides insights into the behavior of the stock market and highlights the importance of modeling volatility when making investment decisions. The findings can be used as a basis for further research and for developing investment strategies that take into account the risk associated with market volatility.

In summary, the analysis shows that both NASDAQ and SP 500 have positive returns and are stationary. The GARCH(1,1) and ARIMA models were estimated and future values were forecasted, which can be used for further analysis and decision making.

10 References

1. Akgiray, V. (1989). Conditional Heteroscedasticity in Time Series of Stock Returns: Evidence and Forecasts. *Journal of Business*, 62:55-80.

2. Alberg, D., Shalit, H. Yosef, R. (2008). Estimating stock market volatility using asymmetric GARCH models. *Applied Financial Economics*, 18:1201–1208.

3. Vasudevan Vetrivel (2016). Asian Journal of Research in Social Sciences and Humanities, Vol. 6, No.8, pp. 1565-1574.

4. Banerjee, A. Sarkar, S. (2006). Modeling daily volatility of the Indian stock market using intraday data. Working Paper Series No. 588, Indian Institute of Management Calcutta.

5. Black, F. (1976). Studies of Stock Market Volatility Changes. Proceedings of the American Statistical Association, Business and Economic Statistics Section, 177-181.

6. Bollerslev, T. (1986). Generalised Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*. 31: 307-327.

7. Chan, H. Fung, D. (2007). Forecasting volatility of Hang Seng Index and its application on reserving for investment guarantees. Working Paper-The Actuarial Society of Hong Kong.

8. Dimson, E. Marsh, P. (1990). Volatility Forecasting without Data-Snooping. *Journal of Banking and Finance*, 14: 399 – 421.

9. Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. Inflation. *Econometrica*, 50: 987-1008.

10. Engle, R. F. Ng, V. K. (1993). Measuring and Testing the Impact Of News On Volatility. *Journal of Finance*, 48:1749–1778.

11. Fama, E. (1965). The Behaviour of Stock Market Prices. *Journal of Business*, 38:34-105.

12. Floros, C. (2008). Modelling Volatility using GARCH Models: Evidence from Egypt and Israel. *Middle Eastern Finance and Economics*, 2: 31-41.

13. Glosten, L. R, Jagannathan, R. Runkle, D. E, (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Returns on Stocks." *Journal of Finance*, 48: 1779-1791.

14. Jondeau. E Rockinger. M (2003). Conditional Volatility, Skewness and Kurtosis: Existence, Persistence, and Co-movements. *Journal of Economic Dynamic and Control*, 27: 1699 – 1737.

15. Joshi P, (2014). Forecasting Volatility of Bombay Stock Exchange. *International Journal of Current Research and Academic Review*, 2: 222-230.

16. Karmakar, M. (2005). Modeling conditional volatility of the Indian stock markets. *Vikalpa*, 30: