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Influence of synchronized traffic light on the states of bus operating system

Shi-Gong Ge [a], Zhong-Jun Ding [a,*], Rui Jiang [b], Qin Shi [a], Reinhart Kühne [a], Jiancheng Long [a], Jian-Xun Ding [a], Bing-Hong Wang [c]

[a] *School of Transportation Engineering, Hefei University of Technology, Hefei, 230009, People's Republic of China*
[b] *School of Engineering Science, University of Science and Technology of China, Hefei, 230026, People's Republic of China*
[c] *Department of Modern Physics, University of Science and Technology of China, Hefei, 230026, People's Republic of China*

HIGHLIGHTS

- The paper investigates the bus operating system under the control of traffic light.
- The analytical equations for the critical passenger arrival rate and average velocity agree with the simulation results well.
- The average velocity in the free-flow and bunching state oscillates with the increase of T.
- The exact condition for the two states called lag and catch (LC) and variant LC state is presented.
- A monotonic decreasing curve instead of oscillation was observed under nonsynchronous traffic light.

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ABSTRACT

This paper investigates the bus operating system under the synchronized traffic light control strategies with cellular automation. Besides the insufficient capacity, the sufficient capacity is observed in the free-flow state of the phase diagram. An analytical equation for the critical passenger arrival rate is developed. The average velocity of free-flow and bunching state oscillates with the increase of signal period. With the increase of the ratio of green phase time, the oscillation amplitude of average velocity decreases while the oscillation frequency increases. At the same time, the critical passenger arrival rate and each region of the phase diagram also vary synchronously. An analytical equation for the average velocity is developed, which shows good agreement with the simulation results. Two states called lag and catch (LC) and variant LC state are observed. The exact condition for the LC state is presented. Finally, a monotonic decreasing curve instead of oscillation was observed under nonsynchronous traffic light. The results indicate that a proper signal period could improve the efficiency of bus system.

1. Introduction

The public bus plays an important role in urban system. The bus operating system is a typical many-body system which has attracted the interest of a community of physicists and others [1–8]. It was shown that the bus system also exhibits the dynamical phase transition similar to the traffic flow [2].

* Corresponding author.
E-mail address: dingzj@hfut.edu.cn (Z.-J. Ding).

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However, the bus operating system suffers from the problem of bus bunching, i.e., several buses traveling together. This makes the service inefficient since people need to wait more time. To understand this behavior, many models have been proposed and studied, including cellular automation (CA) models [1,3,4], car-following models [5,6], and time headway models [7,8]. Furthermore, some realistic elements were considered to discuss the bunching problem, such as open boundary [9] and the mixture of buses and cars in a two-lane traffic system [10].

Many strategies such as adjusting the waiting times and cruise speed were proposed to alleviate the bus bunching behavior. Attempting to achieve equal headways, the minimum, maximum and adaptive waiting time method were put forward by Gershenson et al. [11]. Ding et al. proposed the method that buses adjust their speeds adaptively based on the number of passengers waiting at the bus stops [12]. The system performance is improved significantly with this strategy.

The traffic signal is an essential element in urban traffic network. The impact of traffic light has been studied in detail on single road [13], single intersection [14] and urban traffic network [15]. The previous studies mainly concern on the flow of private car. In this paper, the synchronized traffic lights were introduced into the public bus operating system. How does the bus system's states change such as the bunching and phase separation under the control of traffic lights? How to reduce the bunching and improve the system efficiency by traffic light?

The bus system interfered by traffic light has been studied previously by Huang et al. [16]. However, Huang et al. mainly focus on the statistical distributions of time-headway through a mean-field analysis. The dynamical behavior of a single shuttle bus moving between the origin and the destination through one traffic signal has been studied by Nagatani [17]. However only one bus and one signal were considered in the paper. In this paper, a more realistic cellular automata model is developed. We mainly focus on the relationship between the system states or the average velocity and the signal period. The analytical results are also presented which agree with the simulation results well.

This paper is organized as follows. In Section 2, the model is described in detail. The simulation results and discussion are reported in Section 3. Finally, the conclusions are given in Section 4.

2. Model

Q4 In this section, a CA model for the bus operating system is presented. As shown in Fig. 1, the buses move on a periodic boundary lattice which is divided into L_{total} cells. Each cell represents a bus stop or a segment of road, which is occupied by at most one bus. It is assumed that there are N_s bus stops and the neighboring bus stops are separated uniformly by L cells. Thus the total number of cells is $L_{total} = (L+1)N_s$. There are N_{light} traffic lights and the neighboring lights are separated uniformly by s bus stops. Thus, $N_s = sN_{light}$. The gap between two neighboring traffic lights actually represents a road section between two intersections in urban road network. In order to simplify, each traffic light is located at the middle of two neighboring stops. Denote i as the number of bus, j the number of bus stop, M the bus capacity and N_b the total number of buses. From $t \to t+1$, the parallel update rules of the system are as follows.

1. Update of traffic lights:

   The traffic lights are assumed to switching between green and red synchronously. We denote the duration of green phase and red phase, respectively, as T_g and T_r, while $T = T_g + T_r$ denotes the period of traffic lights.
2. Passengers'arrival:

   $N_{ps}(j, t+1) = N_{ps}(j, t)+1$ for each bus stop site with probability λ. Here $N_{ps}(j, t)$ represents the number of passengers waiting at the bus stop j at time t, and λ is the passenger arrival rate.
3. Bus motion:

   (i) If the bus i is not at a bus stop site, there are two cases.

   If the traffic light in front of bus i immediately is red, then $v_i(t+1) = 0$, $x_i(t+1) = x_i(t)$. For other cases, $v_i(t+1) = \min[1, gap_i(t)]$, $x_i(t+1) = x_i(t) + v_i(t+1)$. Here $v_i(t)$ and $x_i(t)$ are the velocity and position of bus i at time t, $gap_i\,(t)$ is the gap to the preceding bus $i+1$.

   Let $N_{pb}(i, t)$ represent the number of passengers on bus i at time t. If $x_i(t+1)$ is a bus stop site (stop j), then this means that the bus i pulls in the bus stop j at time t+1. In this case, we suppose the number of passengers getting off the bus,

   $$O = \mu \cdot N_{pb}(i, t), \tag{1}$$

   therefore the number of passengers getting on the bus,

   $$I = \min(N_{ps}(j, t+1), M - (N_{pb}(i, t) - O)). \tag{2}$$

   Thus, the number of passengers on the bus,

   $$N_{pb}(i, t+1) = N_{pb}(i, t) - O + I, \tag{3}$$

   and the number of passengers waiting at the bus stop j,

   $$N_{ps}(j, t+1) = N_{ps}(j, t) - I. \tag{4}$$

   The total time that the bus i must stay at the bus stop,

   $$T_{in}(i) = \mathrm{int}[\max(\gamma I, \delta O)] + 1. \tag{5}$$

   Here μ is the proportion of passengers getting off buses. γ and δ are parameters indicating the average time it takes a passenger to get on and off the bus respectively. We take $\gamma > \delta$ to represent the fact that it will take more time getting on than off the bus. In the model, it is assumed that each bus has to stop at every bus stop even if passengers neither get off nor get on it.

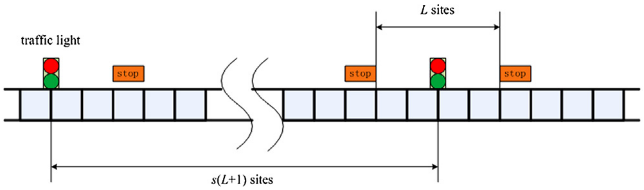

Fig. 1. The illustration of the bus operating system. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

(ii) If the bus i is at one bus stop site, then we check $T_{in}(i)$.
Only if $T_{in}(i) = 0$ and $gap_i(t) > 0$, then the bus can move. So $N_{pb}(i, t+1) = N_{pb}(i, t)$, $v_i(t+1) = 1$, $x_i(t+1) = x_i(t) + v_i(t+1)$. For other cases, the bus has to stay at the bus stop. Then $N_{pb}(i, t+1) = N_{pb}(i, t)$, $v_i(t+1) = 0$, $T_{in}(i) = \max(T_{in}(i) - 1, 0)$.

3. Simulation and discussion

In this section, numerical simulations are carried out. The parameters are set: $M = 100$, $N_s = 40$, $L = 20$, $\mu = 0.2$, $\gamma = 0.5$, $\delta = 0.3$. In accordance to the reality, the number of the buses is no more than twice of the number of the bus stops. In the initial state of the system, N_b buses are distributed randomly among the road with no passengers and there are no passengers at each bus stop.

3.1. Phase diagram

Fig. 2 shows the phase diagram in (N_b, λ) space where (a), (b) and (c) correspond to cases of $T = 2$, 18 and 20 respectively. The four states similar to that of Ref. [1] are also observed. For each figure, region I corresponds to free-flow state where all of buses move freely without bunching. The time-headway distribution of free-flow state is shown in Fig. 3. One can see that the time-headway distribution can be modified by the traffic light. The distribution is depleted at some regions while enhanced at other regions, i.e., the discontinuities of time-headway distribution.

When the system is in region II, some buses carry more passengers and others carry less passengers. The buses that carry less passengers will be slowed down by those carrying more. Because the buses carrying more passengers have to stay longer in the bus stops. Then the bus bunching occurs.

The space time plot and corresponding time-headway distribution of bunching state are shown in Fig. 4. One can see that there are a few large clusters and the distribution concentrates at one value in the system without traffic lights. However, much more smaller clusters are observed and the distribution for other time headways is enhanced with the increase of T. Because as the traffic light switches between green and red alternatively, the buses in one cluster may be released at different periods. Region IV corresponds to phase separation, i.e., some buses pile together with almost no gap between each other and other buses are well separated. Compared with that of region I and region II, the average velocity in region IV is lower.

In region III, free-flow state, bunching state and phase separation cannot exist stably. Different from the result of Ref. [1], from Fig. 5 one can see that the system evolves periodically as the following sequence: free-flow state → bunching state → phase separation → free-flow state.

It can be observed that the system is bistable in region V, either the bunching state or the phase separation state may be stable. If different random seeds are used, the system may be in different states even if the initial conditions are the same.

3.2. Average velocity and the critical passenger arrival rate

Fig. 6 shows the typical relationship between the average velocity v_{ave} and signal period T in region I. From Fig. 6, we can find an oscillating curve. The oscillation amplitude of v_{ave} decreases with the increase of T.

Next we present analytical equations for the average velocity v_{ave} against the signal period T. Q5

$$v_{ave} = \begin{cases} \dfrac{s(L+1)}{t_L} & \text{if } y = 0 \quad \text{(a)} \\ \dfrac{su(L+1)}{(\lfloor t_L/T \rfloor \times u + n)T} & \text{if } y > 0 \quad \text{(b)}. \end{cases} \tag{6}$$

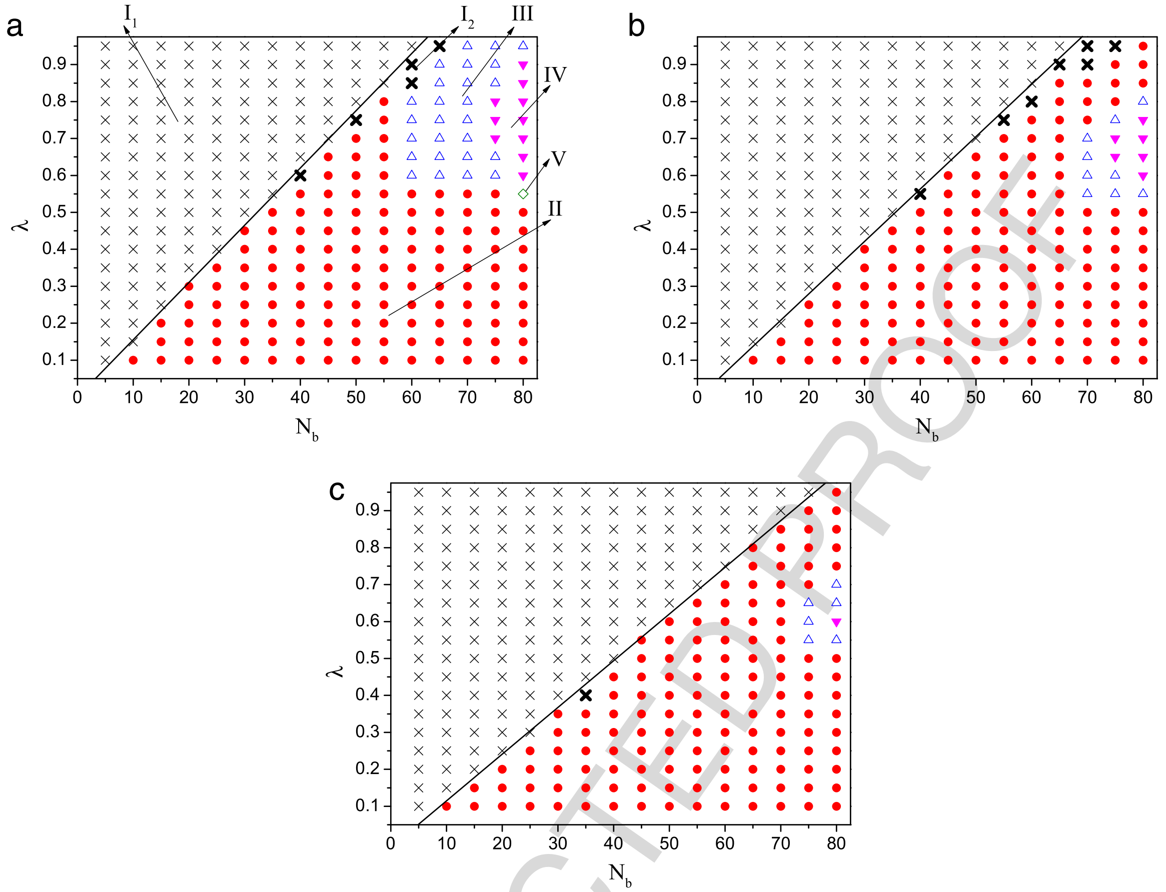

Fig. 2. (Color online) The phase diagram of the bus system for $s = 1$, (a) $T = 2, T_g = T_r = 1$, (b) $T = 18, T_g = T_r = 9$, (c) $T = 20, T_g = T_r = 10$.

Here,

$$t_L = s(L + \lfloor \gamma \mu M \rfloor + 2), \tag{7}$$

$$y = t_L \% T, \tag{8}$$

$$u = \begin{cases} \lfloor (nT)/y \rfloor & \text{if } T_r < y < T_g \\ \lceil T_g/y \rceil & \text{if } T_g \le y \text{ or } y \le T_r \end{cases} \tag{9}$$

$$n = \begin{cases} \min i,\ \{i \in N,\ (i \times T)\%y \le T_r\} & \text{if } T_r < y < T_g \\ 1 & \text{if } T_g \le y \text{ or } y \le T_r \end{cases} \tag{10}$$

where $\lfloor x \rfloor$ denotes the maximal integer which is less than or equal to x while $\lceil x \rceil$ denotes the minimal integer which is greater than or equal to x. "$t_L\%T$" represents the remainder after t_L is divided by T. "N" represents the set of natural numbers. t_L represents the time it takes one bus to move from one traffic light to the next one, excluding the stopping time at the intersection waiting the signal to turn green. It consists of two parts: the time that the bus is moving and the stopped time at the bus stop. The former time equals to $s(L+1)$, and the latter time equals to $s(\lfloor \gamma \mu M \rfloor + 1)$. Since all the buses are full, and the time the buses are forced to stop at each bus stop is a fixed value $t_{stop} = \lfloor \gamma \mu M \rfloor + 1$.

Now we explain how we derive Eqs. (6). Two cases are classified.

Case 1: $y = 0$

From Fig. 7(a) we can see that when the buses arrive at every traffic light, the traffic light is in green phase. The buses do not need to stop at any traffic light. In this case, the time that a bus spends in one road section is t_L. Thus we have Eq. (6)(a).

Case 2: $y > 0$

The buses are in the following recurrent state after the transient time: wait the signal to turn green in front of traffic light $\longrightarrow$ the green phase appears $\longrightarrow$ travel across u road sections and $u - 1$ green lights until meet another red light $\longrightarrow$ wait the signal to turn green in front of traffic light. Thus we only need to focus on one cycle.

As the example shown in Fig. 7(b), the bus spends $t_L = \lfloor t_L/T \rfloor \times T + y$ time steps for the first road section. Since $y < T_g$, the light in front of it is green and the bus continues moving. It spends $(2 \times \lfloor t_L/T \rfloor \times T + 2 \times y)$ time steps

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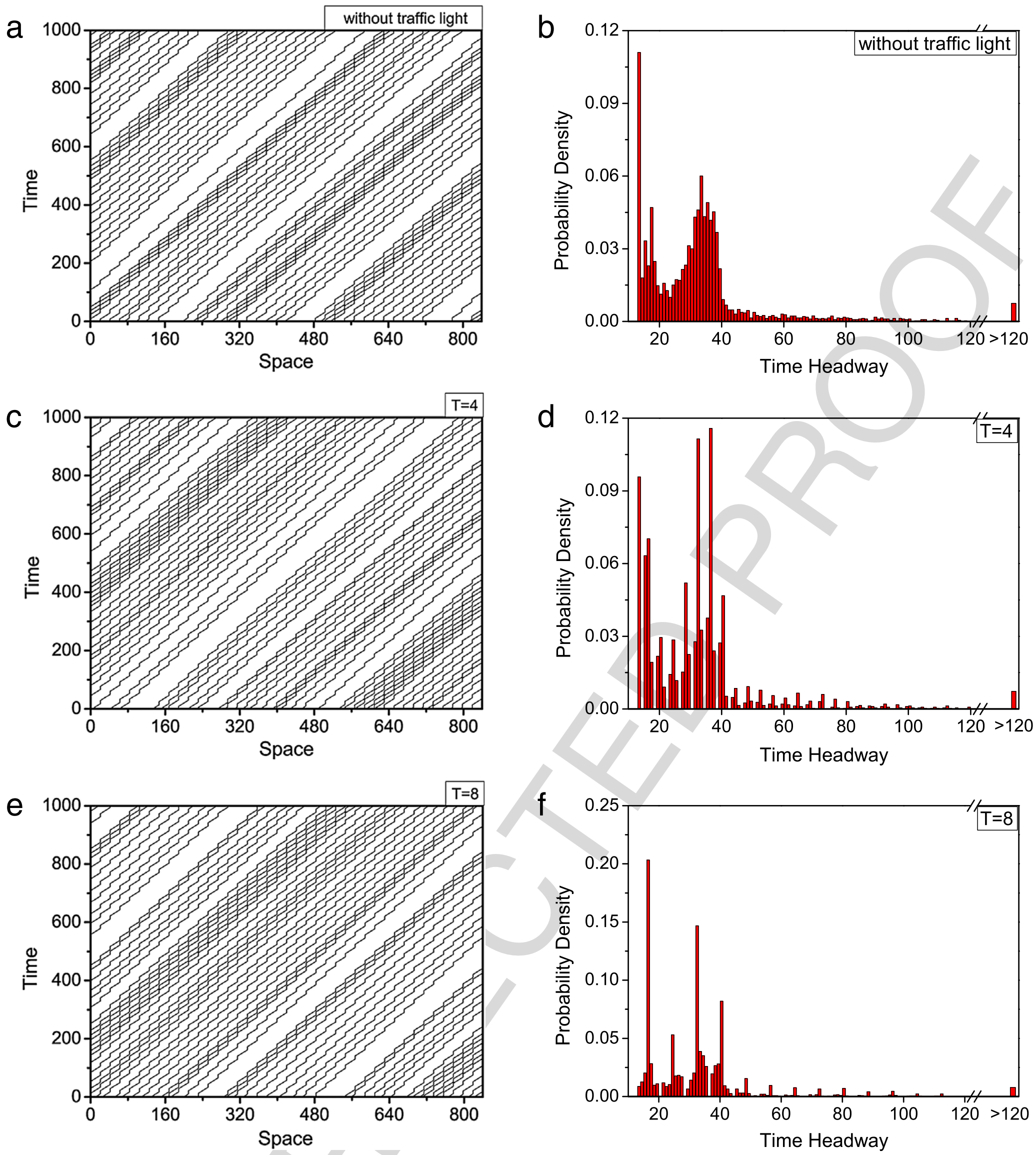

Fig. 3. (Color online) The space time plot of free flow state for $\lambda = 0.95$, $N_b = 40$, $s = 1$. (a) Without traffic lights, (c) $T = 4$, $T_g = T_r = 2$, (e) $T = 8$, $T_g = T_r = 4$. (b), (d) and (f) are the time-headway distributions corresponding to (a), (c) and (e) respectively.

when it arrives at the second road section. Since $T_g < 2 \times y$, it arrives in red phase and stops. One can see that where the bus meets the red light is determined by $u \times y$. The general condition that the bus will arrive in red phase is

$$(n-1)T + T_g < u \times y, \quad n = 1, 2 \ldots, \tag{11}$$

where both u and n are unknown. Fig. 7(c)–(e) show how to get n and u which are classified into two sub-cases. The method is that rolling y until its right end reaches the red phase.

Case 2.1: $T_g \le y$ or $y \le T_r$.

For $y \ge T_g$, from Fig. 7(c) we can see that the right end of y will touch the red phase in the first signal period, i.e., $n = 1$ and $T_g < (u \times y)$. Obviously, $u = 1 = \lceil T_g/y \rceil$.

For $y \le T_r$, when rolling y, it cannot step over the red phase of the first period as shown in Fig. 7(d). Thus n also equals 1, i.e., $n = 1$. From Fig. 7(d), we can obtain that $u = \lceil T_g/y \rceil$.

In these two cases, the distance the bus travels in one cycle is $us(L + 1)$. The total time the bus spends equals the travel time across u traffic lights plus the waiting time z in front of the last one, i.e.,

$$t = ut_L + z = u(\lfloor t_L/T \rfloor T + y) + z = u\lfloor t_L/T \rfloor T + uy + z = (u\lfloor t_L/T \rfloor + 1)T. \tag{12}$$

Here, $uy + z = T$ is used which is shown in Fig. 7(c) and (d). Thus we obtain Eq. (6)(b) for $T_g \le y$ or $y \le T_r$.

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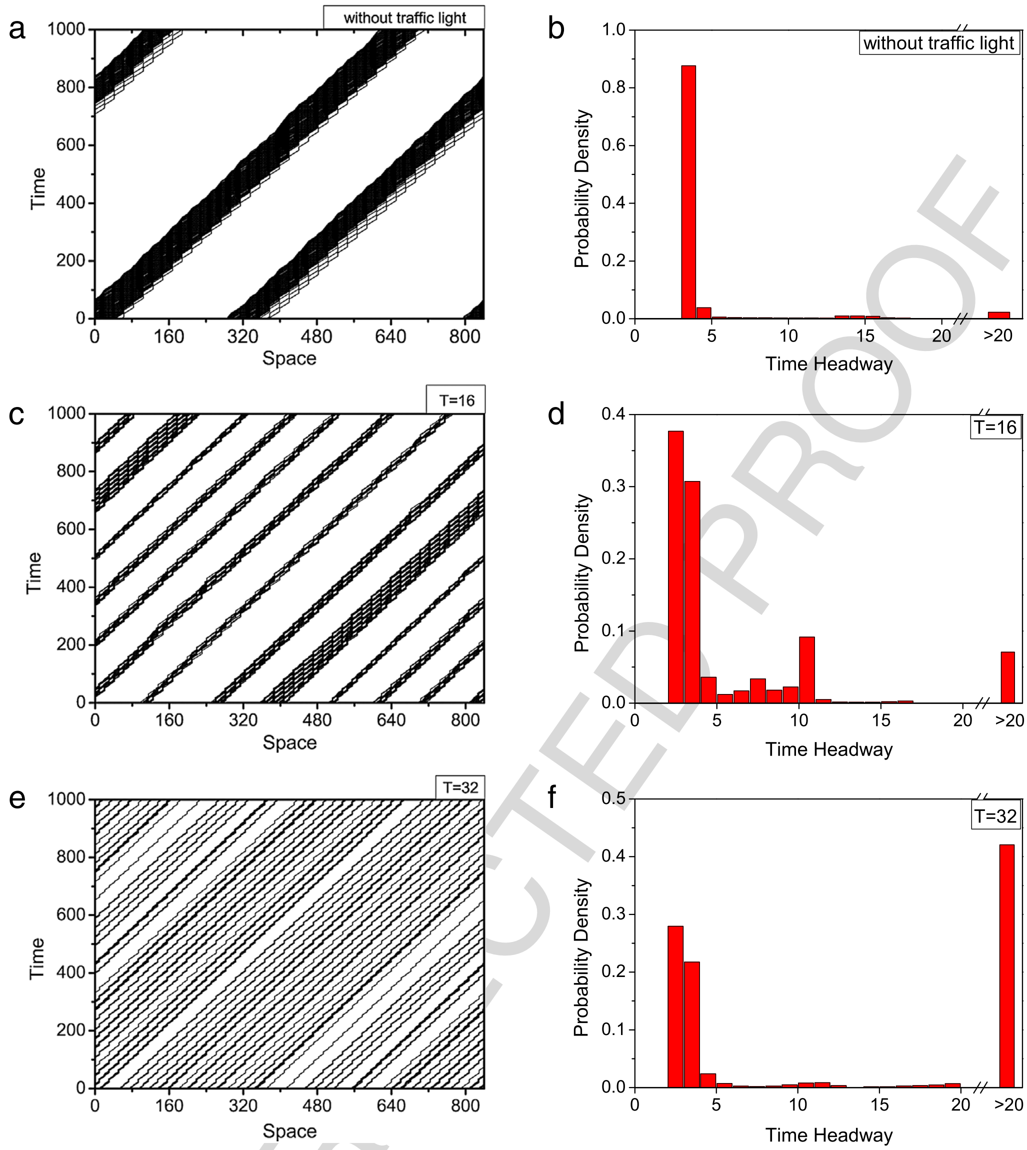

Fig. 4. The space time plot of bunching state for $\lambda = 0.1$, $N_b = 80$, $s = 1$. (a) Without traffic lights, (c) $T = 16$, $T_g = T_r = 8$, (e) $T = 32$, $T_g = T_r = 16$. (b), (d) and (f) are the time-headway distributions for (a), (c) and (e) respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Case 2.2: $T_r < y < T_g$.

Since $T_r < y$, when rolling y, it can step over the red phase as shown in Fig. 7(e). We can get n through the equation

$$n = \min i, \{i \in N, \ (i \times T)\%y \le T_r\}. \tag{13}$$

Here, $(i \times T)\%y$ is the shortest distance between the right end of y and $(i \times T)$. $(i \times T)\%y \le T_r$ means the right end of y is in the red phase. The number of y contained in n periods is $u = \lfloor (nT)/y \rfloor$.

In this sub-case, the distance the bus travels in one cycle also equals $us(L + 1)$. The total time the bus spends also equals the travel time across u traffic lights plus the waiting time in front of the last one, i.e.,

$$t = ut_L + z = u(\lfloor t_L/T \rfloor T + y) + z = u\lfloor t_L/T \rfloor T + uy + z = (u\lfloor t_L/T \rfloor + n)T. \tag{14}$$

However, here $uy + z = nT$ as shown in Fig. 7(e). Thus we obtain Eq. (6)(b) for $T_r < y < T_g$.

With the increase of total bus number, the system transforms from the free flow state into the bunching state. However, v_{ave} varies little. From Fig. 8, it can be observed that although the system is in the bunching state, there are still a few free flowing buses such as the leading buses in the bunching cluster. From the trajectories of buses in Fig. 8, it can be seen that

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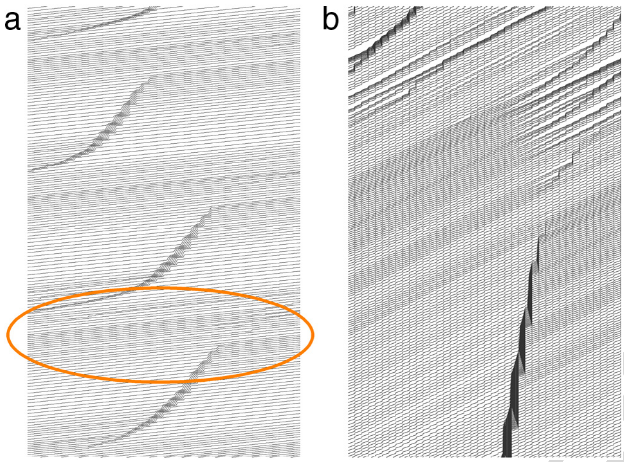

Fig. 5. (a) The space time plot of the bus system for $T = 2, T_g = T_r = 1, N_b = 70, \lambda = 0.85, s = 1$. The vertical direction corresponds to 20 000 time steps and only one trajectory of every five buses is shown for clarity reason. (b) Is the zoomed in view of (a).

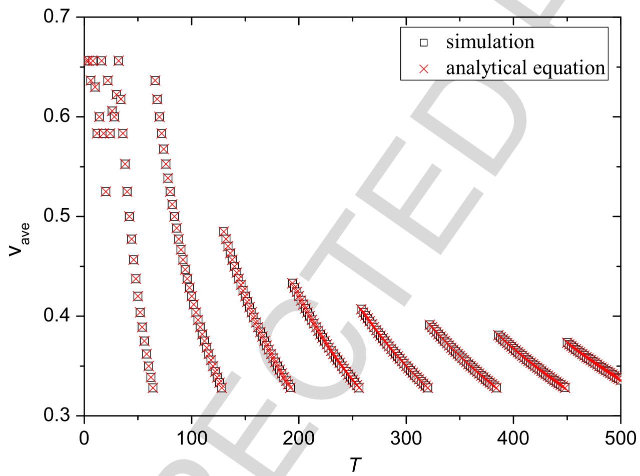

Fig. 6. (Color online) The average velocity v_{ave} against T for $N_b = 5$ and $\lambda = 0.95$. Here $T_g = T_r, s = 1$. The empty box and cross represent the simulation results and analytical equation, respectively.

almost all buses move at the same average speed. Thus, v_{ave} is determined by the velocities of these leading buses which are free-flowing and full. For the full buses, the time that they need to stop at the bus stops is also $t_{stop} = \lfloor \gamma \mu M \rfloor + 1$, which is the same as that in the free flow state. Hence Eq. (6) are also valid for the bunching state.

Fig. 6 shows that the analytical results are in good agreement with the simulation results. It can be observed that Eq. (6) are independent of the distances between the bus stop and the traffic light. The simulation results also indicate that the average velocity does not depend on the locations of traffic lights.

In Fig. 2, the solid line is the boundary between sufficient and insufficient transportation capability. On the left side of the boundary, the transportation capability is insufficient. The number of the passengers waiting at each bus stop will increase with time (see Fig. 9(a)). On the right side, the system capability is enough, the passengers waiting at the bus stops will fluctuate in certain range (see Fig. 9(b)).

Next, the equation for the boundary line is derived. In the system, the density of bus is $\rho = N_b/L_{total}$, and the flow of bus is $q_b = \rho v_{ave}$. When all of the buses are full, the transportation capability of passenger for each bus stop is

$$q_p = q_b \mu M = v_{ave} \mu M N_b / L_{total}. \tag{15}$$

If the passenger arrival rate $\lambda = q_p$, the transportation capability of the bus system satisfies passengers needs exactly. Thus, we get the critical passenger arrival rate λ_c against the bus number N_b, i.e., the equation for the boundary line in Fig. 2

$$\lambda_c = \frac{v_{ave} \mu M}{L_{total}} N_b. \tag{16}$$

The simulation results agree with Eq. (16) well which is not shown here.

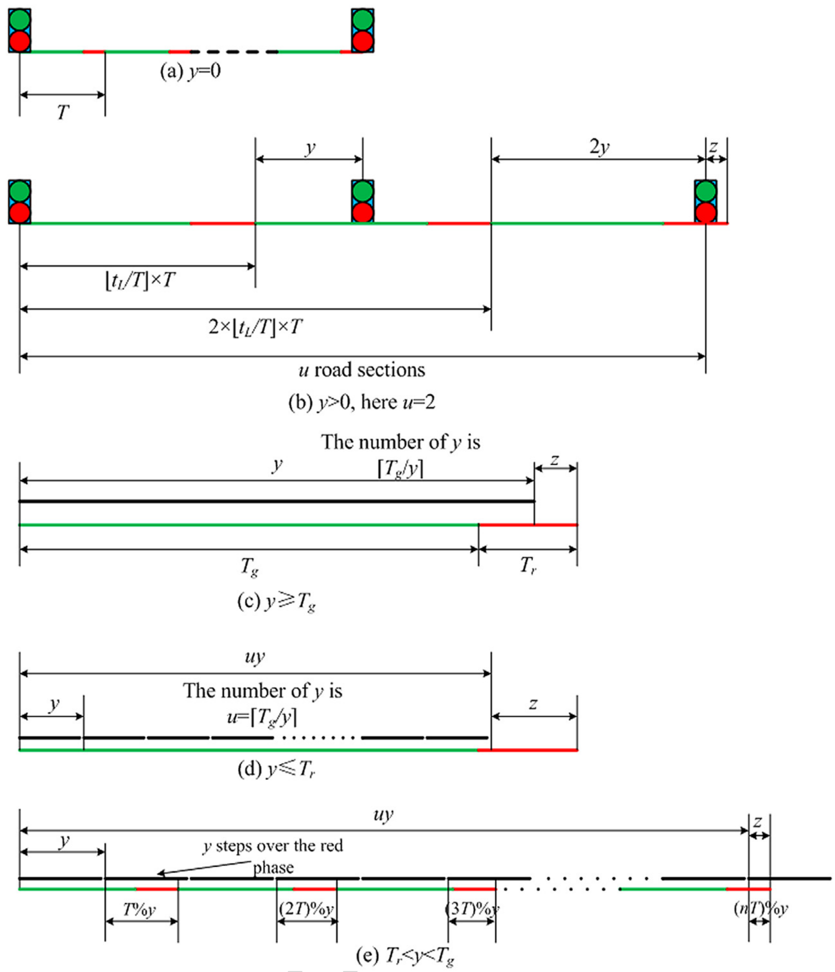

Fig. 7. The illustration of the analytical equations of the average velocity. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

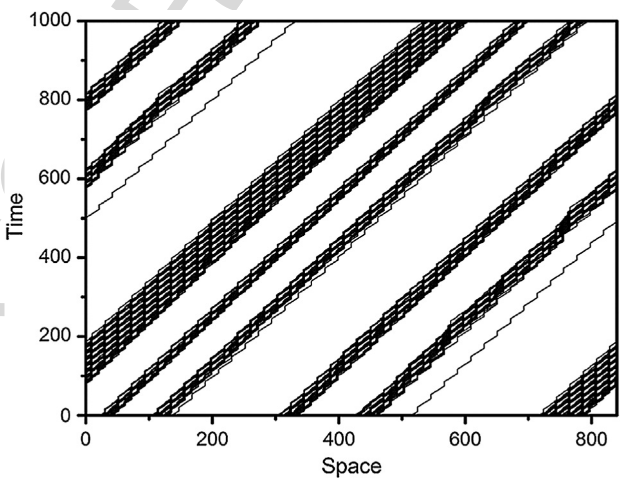

Fig. 8. The space time plot of the bus system for $T = 16$, $T_g = T_r = 8$, $N_b = 80$, $\lambda = 0.1$, $s = 1$.

The boundary of transportation capability coincides with the boundary between region I and region II in the system without traffic lights, i.e., the model of Jiang [1]. However, there is a little deviation in the system with traffic light. Region I is divided into two parts (I_1 and I_2) by the boundary line. In the left part of region I (region I_1), the system capability is

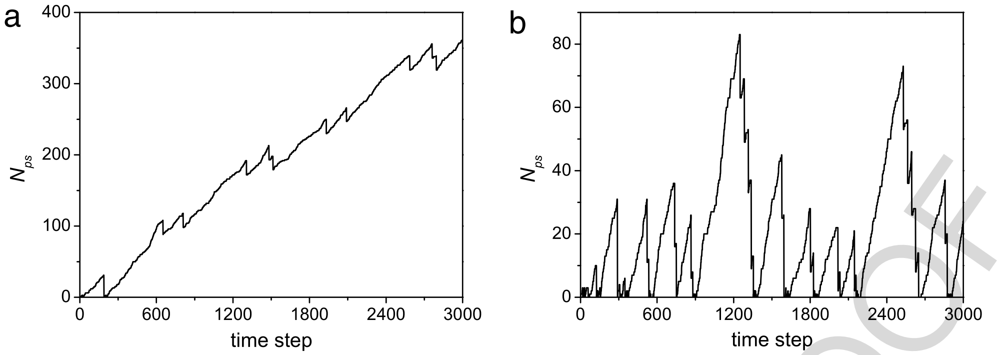

Fig. 9. The evolution of N_{ps} for (a) $T = 8, T_g = T_r = 4, s = 1, \lambda = 0.2, N_b = 5$ (b) $T = 8, T_g = T_r = 4, s = 1, \lambda = 0.2, N_b = 40$.

insufficient which is the same as Ref. [1]. However, in the right part of region I (region I_2), the system capability is sufficient, which does not appear in Ref. [1].

Because when the system is sufficient, the bus carrying less will catch the one carrying more and bunching will occur in Jiang's model. However, from Eq. (6) we can see that when t_L for the buses with less passengers decreases a little, their average velocity may not increase. Their average velocity is still the same as the bus with more passengers in the present model. Thus when the system capability is a little abundant, it is still in the free flow state.

Since slope of λ_c depends on the average velocity, the slope of λ_c and each region in the phase diagram also oscillates synchronously with the average velocity.

3.3. Effect of duration of green light T_g and s

Fig. 10 shows the relationship between the average velocity v_{ave} and the duration of green light T_g when the period of traffic lights $T = 20$, 40 and 80, respectively. From Fig. 10, we can find an increasing step function. The analytical equation (6) also agrees with the simulation results well.

Next, we analyze the monotonic property of analytical equation (6).

Case 1: $y = 0$

$v_{ave} = \frac{s(L+1)}{t_L}$, which is independent of T_g.

Case 2: $y > 0$

Case 2.1: $T_g \le y$ or $y \le T_r$

$$v_{ave} = \frac{su(L+1)}{(\lfloor t_L/T \rfloor \times u + 1)T} = \frac{s(L+1)}{T} \times \frac{1}{\lfloor t_L/T \rfloor + \frac{1}{u}} = \frac{s(L+1)}{T} \times \frac{1}{\lfloor t_L/T \rfloor + \frac{1}{\lceil T_g/y \rceil}}. \quad (17)$$

Obviously, v_{ave} increases or remains constant with the increase of T_g. $T_g = y, 2y, 3y \ldots$ correspond to the discontinuous point.

Case 2.2: $T_r < y < T_g$

$$v_{ave} = \frac{su(L+1)}{(\lfloor t_L/T \rfloor \times u + n)T} = \frac{s(L+1)}{T} \times \frac{1}{\lfloor t_L/T \rfloor + \frac{n}{u}} = \frac{s(L+1)}{T} \times \frac{1}{\lfloor t_L/T \rfloor + n/\lfloor (nT)/y \rfloor}. \quad (18)$$

We can find that n increases or remains constant with the increase of T_g by observing Eq. (13). When T_g increases, T_r decreases and we need a larger n to meet the equation. $\frac{n}{\lfloor (nT)/y \rfloor}$ decreases or remains constant with the increase of n. Thus v_{ave} increases or remains constant with the increase of T_g.

Fig. 11 shows the phase diagram for $T_g = 2, 4$ and 9, respectively. Similarly to previously, the slope of λ_c and the boundary between region I and region II also change synchronously with the average velocity.

Fig. 12 shows the relationship between v_{ave} and T under different ratio of green light. With the increase of the ratio, the oscillation amplitude of v_{ave} decreases while the oscillation frequency increases.

Fig. 13 shows the relationship between v_{ave} and T under different s for the simulation and analytical results, respectively. The analytical equation (6) also agrees with the simulation results well. When $s \neq 1$, the relationship is also an oscillation curve which includes higher and lower frequency part. The critical point moves rightwards with the increase of s. In the higher frequency part, the oscillation frequency decreases with the increase of s. However, There is no law for the oscillation frequency and amplitude of the lower frequency part.

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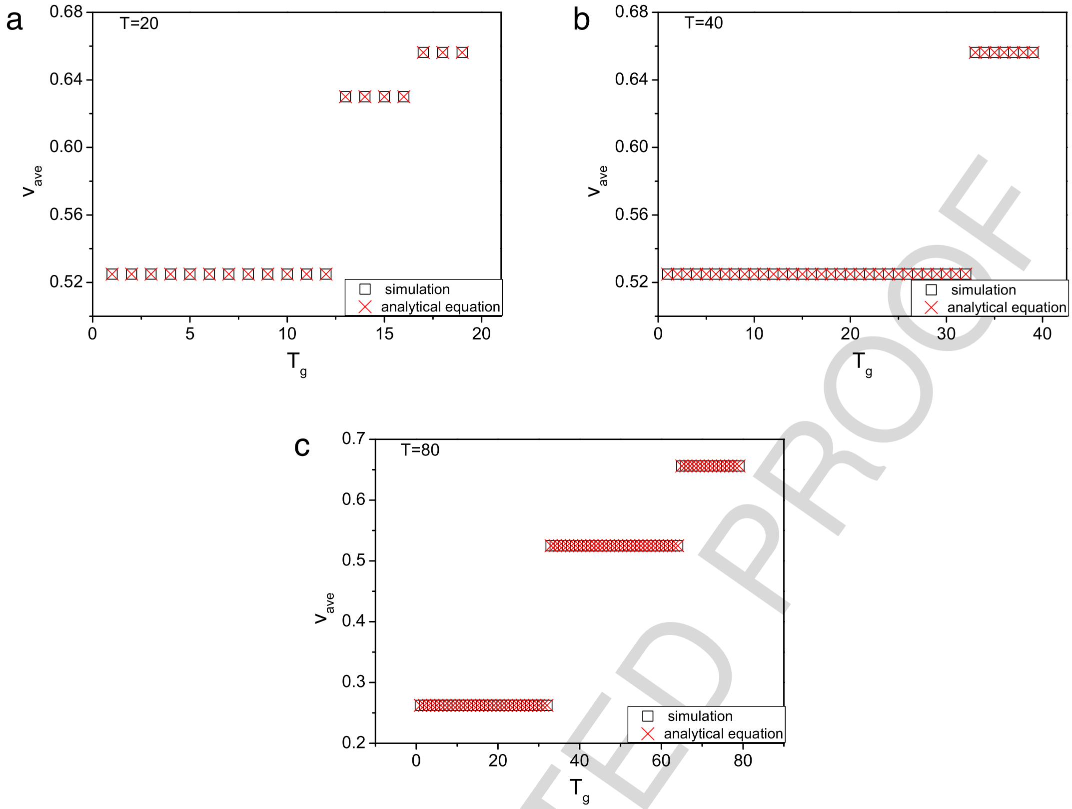

Fig. 10. The average velocity v_{ave} against T_g for $N_b = 5, \lambda = 0.95$ (a) $T = 20$, (b) $T = 40$, (c) $T = 80$. The empty box and cross represent the simulation results and analytical equation, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3.4. Lag and catch state and variant LC state

Two particular states are observed as shown in Fig. 14. Fig. 14(a) shows that there exist clusters of buses. At somewhere, the distance between buses is zero. Then, other buses lag behind the first bus. However, after some time they catch up with the first bus again. This process repeats over time, thus this state is named as lag and catch (LC for short) state.

Fig. 14(c) shows a variant LC state, in which the first bus in a LC cluster can be caught up by other buses in the cluster for several times. Then the first bus will leave the cluster behind farther and farther until it bumps into downstream cluster.

The LC state and variant LC state cannot coexist simultaneously. As shown in Fig. 15, the LC state can coexist stably with other states except variant LC state, while variant LC state can also coexist stably with other states except LC state.

Now we present the emergence condition of the LC cluster that involves only two buses.

Case 1: $y > 0$

Case 1.1: $t_u - 2 \geq (\lfloor t_L/T \rfloor \times u + n)T$
if $1 + \sum_{k=1}^{u} \theta_k t_w^r(k) < t_w^l$, the LC cluster might emerge. Otherwise the LC cluster will not occur.

Case 1.2: $t_u - 2 < (\lfloor t_L/T \rfloor \times u + n)T$

Case 1.2.1: $2\%T < T_g$, the LC state might emerge.

Case 1.2.2: $2\%T \geq T_g$, if $T - 2\%T + 1 \leq \lfloor \gamma \mu M \rfloor + 1$, the LC state might emerge. Otherwise, the LC state will not occur.

Case 2: $y = 0$

Since $t_w^l = 0$, $t_w^r + 1 > t_w^l$, the LC state will not occur.

Here, $t_w^l = T - (u \times t_L)\%T$ represents the total waiting time of the first bus in the cluster in front of red light in one cycle. $t_w^r(k) = T - t_k\%T$ represents the waiting time of the second bus at kth road section, and thus $\sum_{k=1}^{u} \theta_k t_w^r(k)$ is the total waiting time of the second bus. t_k represents the moment that the second bus arrives at kth road section. $t_{k+1} = t_k + t_L + \theta_k t_w^r(k)$, $t_1 = t_L + t_h$. If $t_k\%T \geq T_g$, then $\theta_k = 1$; otherwise $\theta_k = 0 (1 \leq k \leq u)$. $t_h = \lfloor \gamma \mu M \rfloor + 3$ is the time headway between the two buses after the first bus stop.

Next, we explain the condition in detail. Similar to that of v_{ave}, two cases are identified.

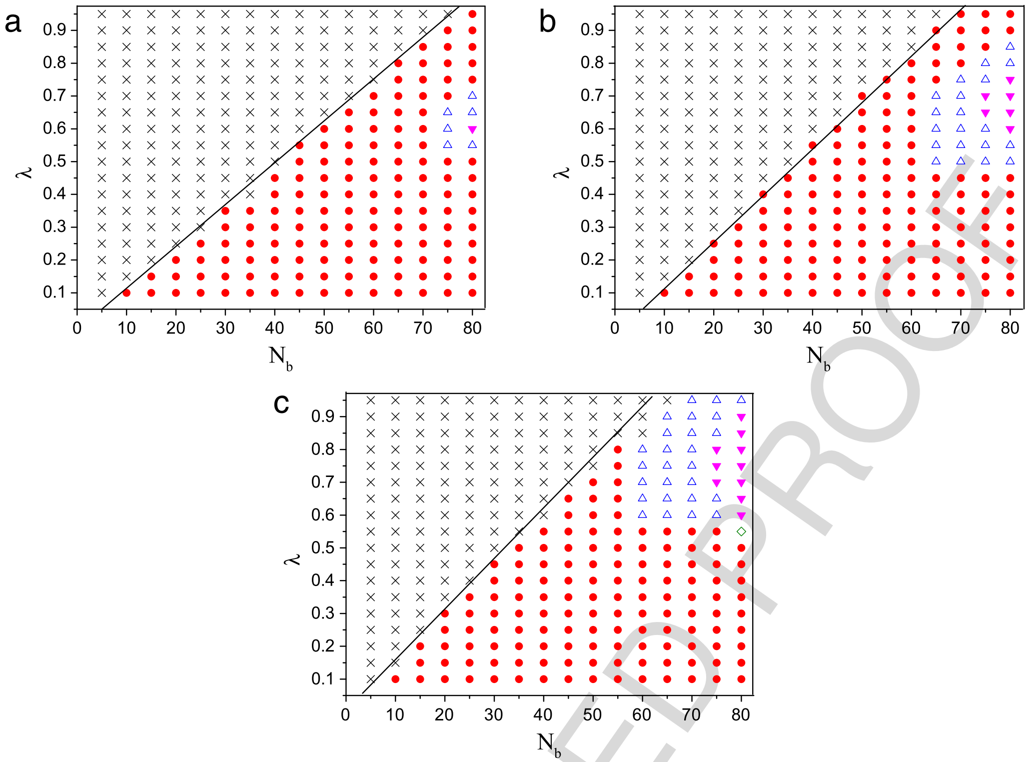

Fig. 11. (Color online) The phase diagram of the bus system for $s = 1$, $T = 10$, (a) $T_g = 2$, (b) $T_g = 4$, (c) $T_g = 9$.

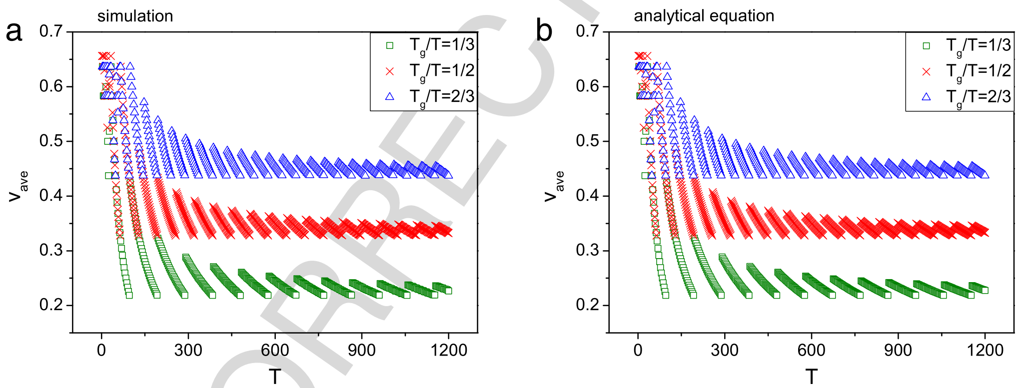

Fig. 12. The average velocity v_{ave} against T under different ratio of green light. (a) and (b) are results of simulation and analytical equation, respectively. Here, $N_b = 5$, $\lambda = 0.95$, $s = 1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Case 1: $y > 0$

As in the analysis of v_{ave}, the first bus is in the recurrent state and travels across u road sections in one cycle. As shown in Fig. 16, the first bus $i+1$ starts from the traffic light g when the green phase appears. Suppose that the second bus i catches up the first bus at the first station (i.e., stop j in Fig. 16). Then the gap between them is 0. After the first bus leaves the stop, the second bus enters. The gap between the two buses thus equals 1. During stoppage of the second bus, the gap between them increases gradually from 1 to $t_{stop} + 1$. When the first bus enters the next stop, the second bus is able to narrow the gap. The cycle ends when the first bus meets the red light (e.g., light $g+2$ in Fig. 16).

When the first bus stops at the red light, the second bus may catch up or not. Thus, two sub-cases are classified.

Case 1.1: $t_u - 2 \geq (\lfloor t_L/T \rfloor \times u + n)T$.

As shown in Fig. 16(a) and (b), the second bus does not catch up with the first bus at the red light (light $g+2$). Here $(\lfloor t_L/T \rfloor \times u + n)T$ is the total time the first bus spends in one cycle and $t_u - 2$ is the time the second bus arrives the place which is two sites in front of the last traffic light.

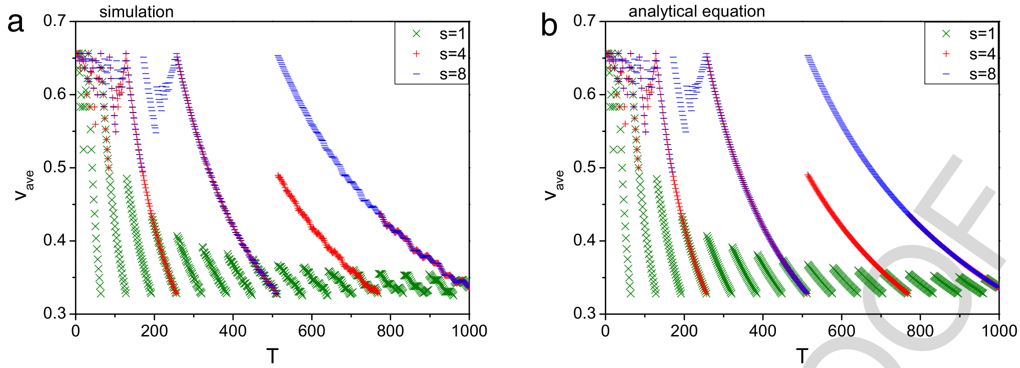

Fig. 13. The average velocity v_{ave} against T when $s = 1, 4$ and 8. (a) and (b) are simulation results and ones by analytical equation. Here, $T_g = T_r$, $N_b = 5$, $\lambda = 0.95$.

In this sub-case, if the second bus is able to catch up with the first bus at the first stop of next cycle (stop $j+2$ in Fig. 16), the LC cluster emerges (see Fig. 16(a)). This requires that the total waiting time of the second bus in front of red light must be smaller than that of the first bus by at least one time step, i.e., $1 + \sum_{k=1}^{u} \theta_k t_w^r(k) < t_w^l$. Otherwise the LC state will not occur (see Fig. 16(b)).

Case 1.2: $t_u - 2 < (\lfloor t_L/T \rfloor \times u + n)T$.

As shown in Fig. 16(c), the second bus catches up with the first bus at the red light (light $g+2$). There are two subsub-cases for this sub-case.

Case 1.2.1: $2\%T < T_g$.

The second bus will not be blocked by the subsequent red traffic light. The second bus will catch the first bus at the stop $j+2$ definitely and the LC state will emerge (see Fig. 16(c)).

Case 1.2.2: $2\%T \geq T_g$.

Since the duration of green phase T_g is so short that the second bus will be blocked by the subsequent red phase. In this case, if the gap between the two buses $(T - 2\%T + 1)$ is smaller than the stopping time of first bus at the bus stop $j+2$ $(t_{stop} = \lfloor \gamma \mu M \rfloor + 1)$, i.e., $T - 2\%T + 1 \leq \lfloor \gamma \mu M \rfloor + 1$, the second bus will catch the first bus at the stop $j+2$ and the LC state will emerge. Otherwise, the LC state will not occur (see Fig. 16(d)).

Case 2: $y = 0$

Since the first bus always meets the green light, the second bus cannot catch it. As shown in Fig. 16(e), the LC state will not occur.

If the second bus is blocked by the light $g+1$, two buses will be separated. Since the velocity of second bus is lower than the first one, free buses behind them might catch the second one. Then the variant LC state emerges.

When T increases, clusters with more than two buses will emerge in the LC state. However, the exact condition is very complex which will be studied in the future work. The average velocities of the buses in the LC state are the same as that of free flow state. Because the second bus can catch the first which are free. Hence, the Eqs. (6) and (16) are valid to the LC state. However, they are invalid to the variant LC state.

3.5. Nonsynchronous traffic lights

Next we discuss the case with nonsynchronous traffic lights. The periods of all the traffic lights T are still the same while the green phase starts randomly between $[0, T]$. Fig. 17 shows the relationship between the average velocity v_{ave} and the signal period T in the free flow state. With the increase of T, the average velocity v_{ave} decreases. We present an analytical equation for the average velocity v_{ave} against the signal period T.

$$v_{ave} = \frac{sL}{t_L + \frac{T_r^2}{2T}}. \tag{19}$$

Now, we explain how we derive Eq. (19). The distance between two neighboring traffic lights is $s \times L$. The average waiting time in front of each traffic light is $\frac{T_g}{T} \times 0 + \frac{T_r}{T} \times \frac{T_r}{2} = \frac{T_r^2}{2T}$. Thus the total time the bus spends between two neighboring traffic lights is $t_L + \frac{T_r^2}{2T}$ and we have Eq. (19). The analytical results agree with the simulation results well as shown in Fig. 17.

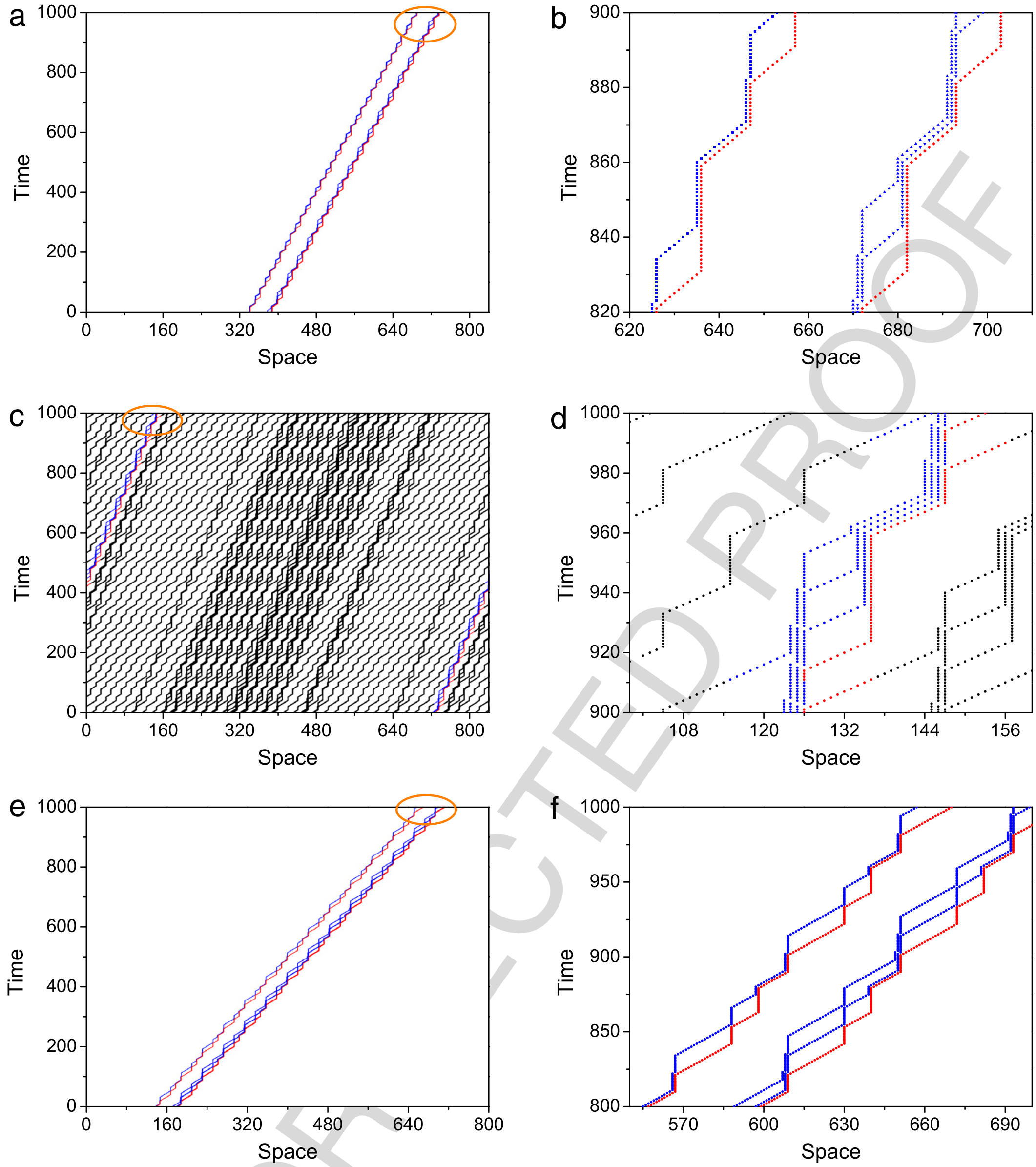

Fig. 14. (Color online) The space time plot for (a) $T = 60$, $T_g = 30$, $T_r = 30$, $N_b = 15$, $\lambda = 0.15$, $s = 1$, (c) $T = 80$, $T_g = 40$, $T_r = 40$, $N_b = 80$, $\lambda = 0.95$, $s = 1$, (e) $T = 80$, $T_g = 20$, $T_r = 60$, $N_b = 5$, $\lambda = 0.95$, $s = 2$. (b), (d) and (f) are the zoomed in view of (a), (c) and (e) respectively.

4. Conclusions

In this paper, the synchronized traffic lights were introduced into the bus operating model. The four states similar to that of Ref. [1] are also observed. Besides the insufficient capability, the sufficient capability is observed in the free-flow state region of the phase diagram.

An equation for the boundary between the sufficient and insufficient capability region was presented which agrees with the simulation results well. The boundary coincides with the boundary between region I and II in the model of Jiang. However, there is a little deviation in the present model.

The average velocity in the free-flow and bunching state oscillates and the oscillation amplitude decreases with the increase of traffic light period. With the increase of the ratio of green phase time, the oscillation amplitude of average velocity decreases while the oscillation frequency increases. Two parts, i.e., higher and lower frequency part were observed in the oscillation curve of average velocity. The critical point moves rightwards with the increase of s. An analytical equation for the average velocity is developed which shows good agreement with the simulation results. Since the slope of the capability

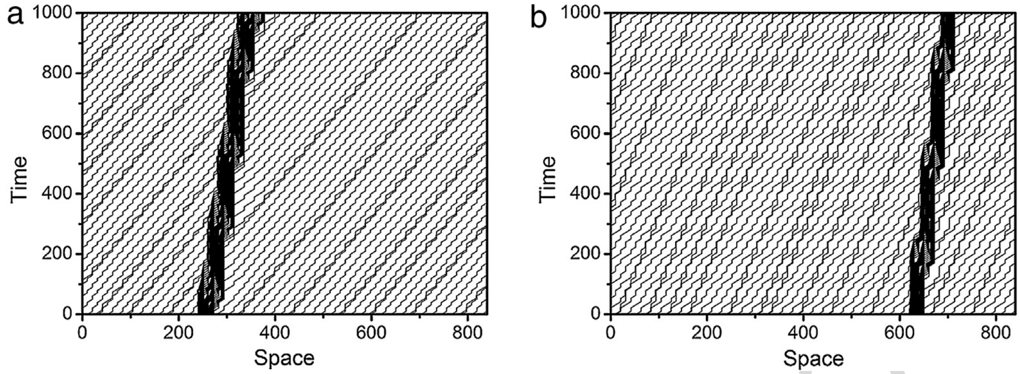

Fig. 15. The space time plot of bus system for (a) $T = 40, T_g = 20, T_r = 20, \lambda = 0.6, N_b = 80, s = 1$. The state is a coexistence of LC and phase separation state; (b) $T = 80, T_g = 40, T_r = 40, \lambda = 0.55, N_b = 80, s = 1$. The state is a coexistence of variant LC state and phase separation state.

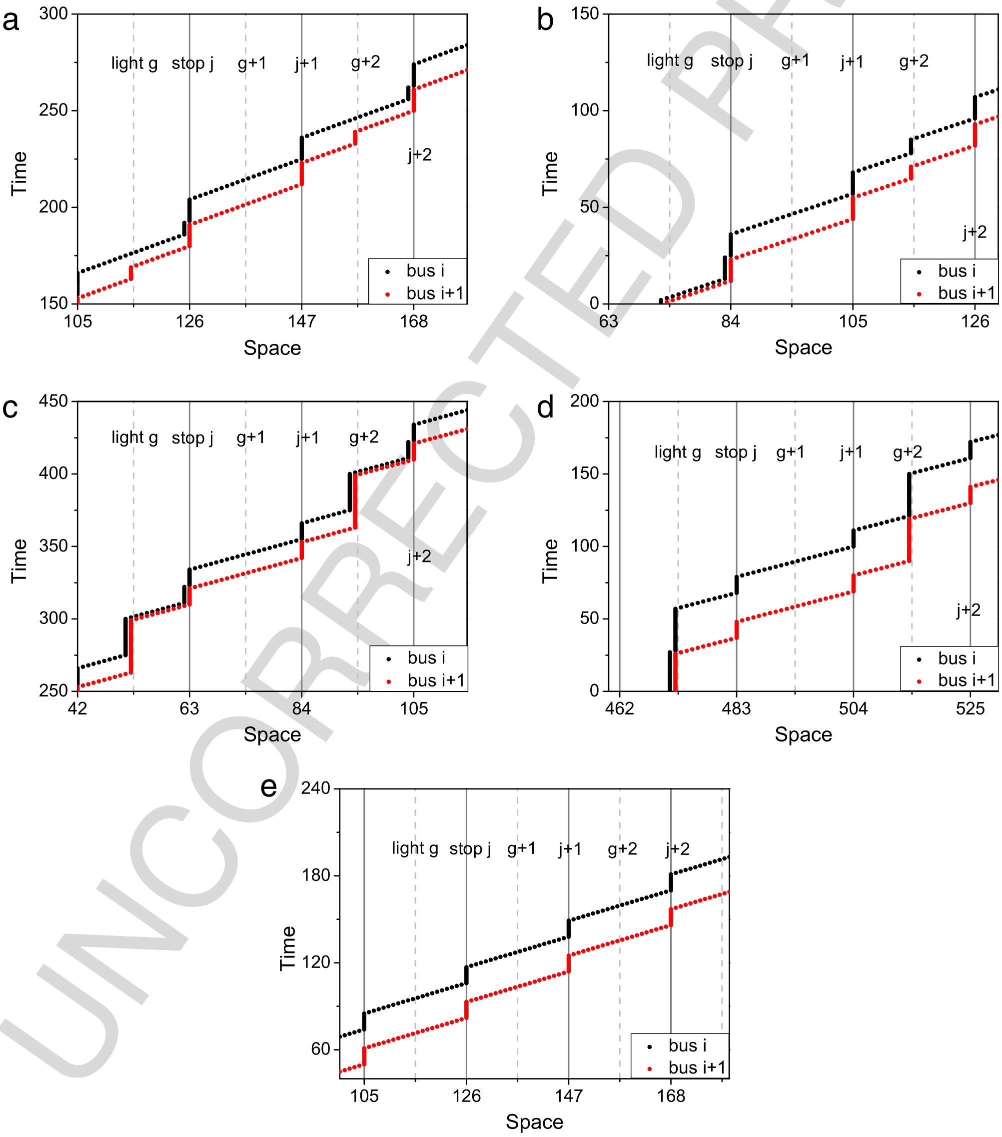

Fig. 16. The trajectories of the two buses in the LC state ((a) and (c)) and non LC state ((b), (d) and (e)). Here, (a) $T = 70, T_g = 50, T_r = 20, s = 1$ (b) $T = 14, T_g = 7, T_r = 7, s = 1$ (c) $T = 100, T_g = 50, T_r = 50, s = 1$ (d) $T = 31, T_g = 2, T_r = 29, s = 1$ (e) $T = 32, T_g = 16, T_r = 16, s = 1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

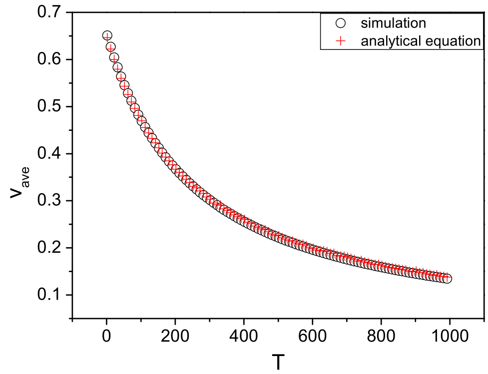

Fig. 17. The average velocity v_{ave} against T with nonsynchronous traffic lights. The parameters $s = 1$, $N_b = 5$, $\lambda = 0.95$ and $T_g = T_r$. Each data point are the average of 1000 independent runs. The empty box and cross represent the simulation results and analytical results, respectively.

line depends on the average velocity, the capability line and five regions also show the same oscillation behavior as the average velocity.

Two states called LC and variant LC state are observed. When the LC state could emerge is analyzed in detail. At last, the case with nonsynchronous traffic lights was discussed preliminarily. A monotonic decreasing curve instead of oscillation was observed for the average velocity.

Based on the studies, there are following recommendations. For traffic managers, too long signal periods should be avoided. Because it will cause LC and variant LC state with higher probability.

Further questions concern the development of the model. The synchronized traffic light can be extended to the green wave or adaptive one. The buses may be allowed to change lane in the two lane or multi-lane system.

Acknowledgments

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